Comparison of PID and LQR based Quad-copter Controller Designs

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***Abstract*— This project is intended to compare the Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR) based controller design techniques applied to the Vertical Motion of a Quad-Copter. The controller design for a quad-copter system is a very complex task as the system dynamics are highly non-linear in nature. The system is linearized for the vertical motion control about an operating point to which the LQR controller is designed in MATLAB and the PID controller is designed in SIMULINK and their responses are compared in terms of speed of response, overshoot, and so on.**

***Index Terms*—LQR, PID, Closed Loop Control, Position Control, Velocity Control, Non-linear System**

I. Introduction

The Quad-copter is a type of Unmanned Aerial Vehicle (UAV) which has numerous applications in fields of military, education, commercial, entertainment and others. Since, the aerial systems are highly non-linear, its control is an essential part of the design. Optimal Control design for aerial systems has thus trended recently and there is a lot of progress currently going on in this direction. Some commonly used control techniques are PID Control, Back-Stepping, H∞ Control, Kalman Filter, and so on.

There are several controllers designed using these techniques for various models of quad-copters. In [6], Pounds P. et. al, uses a disturbance model based on aerodynamic changes and rotor / propeller bending to design simple PID controller for the quad-copter system. Other strategies of controller design using PID control technique is proposed by Salih A. L. in [9]. In [5], Madani, T. et. al. divides a quad-copter system into multiple sub-systems and applies back-stepping algorithm and Lyapunov Stability theory to design a controller. Another design of back-stepping algorithm based controller is proposed by Soumelidis, A, et.al. in [8]. In [7], Raffo G. V., proposed the use of predictive state space control and non-linear H∞ control design technique for quad-copter regulation and tracking applications.

The objective of this project is to design a LQR controller considering an Infinite Horizon Problem for stabilizing the vertical motion of the quad-copter with an assumption that the quad-copter is always in hovering state. Further the LQR controller response is compared with the response of a PID controller designed using auto tuning.

The report organization is as follows: Section I gives an introduction about the current approaches for quad-copter system control. Section II discusses the highly non-linear system dynamics of a Quad-copter and derives the dynamic equations of the entire system followed by the linearization of the system in Section III. In section IV the system will be defined for vertical movement which will be used for controller design purposes. Further, in Section V an LQR based controller design using MATLAB will be discussed and in Section VI design of a conventional PID controller using Simulink modelling will be explained. Section VII will include the results and discussion of the experiments and comparison of the two controller designs. Finally, in Section VIII concluding remarks will be made for the results generated by this project.

II. Dynamic Modelling

The quad-copter is an aerial system operating in a 3 dimensional space and hence the quad-copter's attitude defined in its Body-axes frame needs to be referenced with respect to the Earth-axis frame. Here, we use the right hand rule to determine the co-ordinate frames of both the body and the Earth. The attitude of the quad-copter can be represented by 6 Degree of Freedom system, out of which 3 DOF determines the position and other 3 DOF determines the orientation of the quad-copter in 3D space. Thus, the overall system dynamics can be summarized using 12 different states. The position of the Quad-copter (x,y,z) is described with respect to Earth Axes Frame (North, East, Down). The linear speeds of the corresponding axes (x,y,z) is denoted by (u,v,w) in the quad-copter's body axes frame. The orientation of the quad-copter is defined by the roll, pitch and yaw angles (Φ,θ,Ψ) between the body-axes frame and the earth axes frame and the angular speeds (roll rate, pitch rate and yaw rate) of each axes is denoted by (p,q,r). Figure 1 shows the body axes of the Quad-copter system.

In order to determine the exact position and orientation of an aerial object, the co-ordinate axes of the body frame needs to be defined with respect to a known reference co-ordinate frame. Generally, for aerial objects, the reference frame is the Earth Co-ordinate Frame(O, x_E, y_E, z_E). Figure 2 shows the body axes represented by (O, x_b, y_b, z_b). In order to determine the transformation between the body axes frame and the earth axes frame, first, roll about 'Ox_b' through the angle '$\emptyset$' with angular velocity '$\dot{\emptyset}$'. Second, pitch about 'Oy_2' through the angle 'θ' with angular velocity '$\dot{\theta}$'. And finally, yaw about 'Oz_1' through the angle 'ψ' with angular velocity '$\dot{\psi}$'.

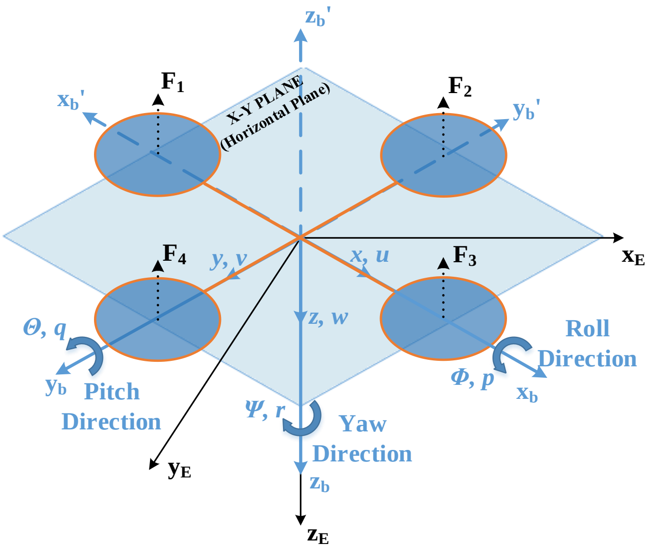

Figure 1: Quadcopter Body Co-ordinate Frame

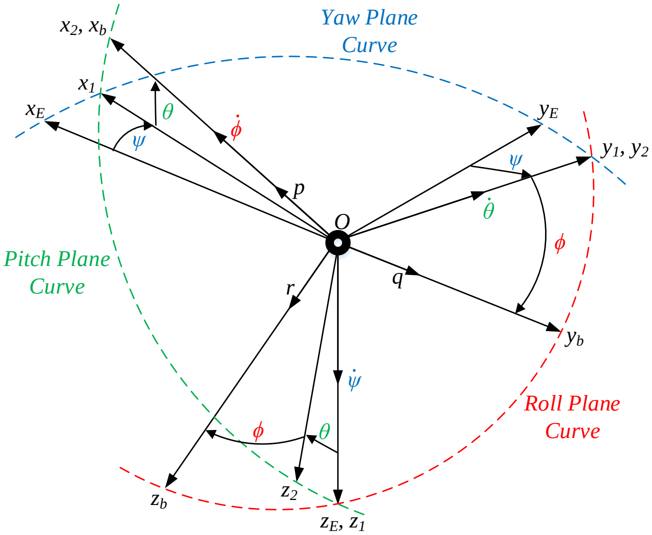

Figure 2: Transformation from Body Co-ordinate Frame to Reference Earth Co-ordinate Frame.

The transformation analysis of an object flying in 3-Dimensional space with respect to the reference axis is given in [2]. Using similar analysis, the Linear Quantity Transformations of each of the Quad-copter's Body Axes about respective Earth Axes is given by:

Transformation about x Axis:

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\emptyset & sin\emptyset \\ 0 & -sin\emptyset & cos\emptyset \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \tag{1}$$

Transformation about y Axis:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \tag{2}$$

Transformation about z Axis:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} \tag{3}$$

Thus the linear quantity transformation from Earth Axis frame (x_E, y_E, z_E) to the quad-copter body frame (x_b, y_b, z_b) can be given as:

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = D \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} \tag{4}$$

where, 'D' is the direction cosine matrix and is given by:

$$D = \begin{bmatrix} cos\theta cos\psi & cos\theta sin\psi & -sin\theta \\ sin\emptyset sin\theta cos\psi - cos\emptyset sin\psi & sin\emptyset sin\theta sin\psi + cos\emptyset cos\psi & sin\emptyset cos\theta \\ cos\emptyset sin\theta cos\psi + sin\emptyset sin\psi & cos\emptyset sin\theta sin\psi - sin\emptyset sin\psi & cos\emptyset cos\theta \end{bmatrix} \tag{5}$$

We measure the velocities of the quad-copter along each axis *(u, v, w)*. Thus, the change in position of the quad-copter with respect to the earth-axes frame $(\dot{x}, \dot{y}, \dot{z})$ can be represented as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = D^{-1} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{6}$$

The Equations which relate the rate of change of angles in each axis to the respective angular velocities on each of the axes are:

$$p = \dot{\emptyset} - \dot{\psi} sin\theta \tag{7}$$

$$q = \dot{\theta} cos\emptyset - \dot{\psi} sin\emptyset cos\theta \tag{8}$$

$$r = \dot{\psi} cos\emptyset cos\theta - \dot{\theta} sin\emptyset \tag{9}$$

Representing the above equations in matrix form to relate the rate of change angular velocities of each axis with respect to the change in angle, we get;

$$\begin{bmatrix} \dot{\emptyset} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & sin\emptyset tan\theta & cos\emptyset tan\theta \\ 0 & cos\emptyset & -sin\emptyset \\ 0 & sin\emptyset sec\theta & cos\emptyset sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{10}$$

Using the newton's second law we have the linear acceleration for a fixed frame as:

$$F = ma = m\dot{V} \tag{11}$$

where, 'F' is the Force acting on the object, 'a' is the acceleration, '$\dot{V}$' is the velocity vector denoting linear velocities of the quad-copter in each of the 3 axes. In order to determine the rate of change of the linear velocity vector (linear acceleration) with respect to the earth, we consider the rotation of the velocity vector as it changes its magnitude [3]. Thus, the resulting expression is represented as:

$$F|_E = F + \omega \times mV \tag{12}$$

where, '*w*' is the total angular velocity vector of the flying object and '×' indicates the cross product. Solving the above equation, we get:

$$F|_E = m\dot{V} + \omega \times mV \tag{13}$$

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + m \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} = m \begin{bmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{bmatrix} \tag{14}$$

The forces acting on each axis, F_x, F_y and F_z is the sum of thrust, weight force and aerodynamic forces acting on each axis. Here, ignoring the aerodynamic forces we have the weight forces which are dependent on the attitude of the quad-copter and the thrust always acts along the Z-axis to be considered. Thus,

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} W_x \\ W_y \\ W_z - T \end{bmatrix} \tag{15}$$

Considering the horizontal quad-copter position as the default, the weight force will then act on the z-axis and is given by $W_z = mg$, where 'm' is the mass of the quad-copter and 'g' is the gravitational force. Further, the conversion from the Earth fixed frame to the quad-copter body axes frame is done using the direction cosine matrix 'D'. Thus,

$$D\begin{bmatrix}0\\0\\mg\end{bmatrix} - \begin{bmatrix}0\\0\\T\end{bmatrix} = m\begin{bmatrix}\dot{u} + qw - rv\\ \dot{v} + ru - pw\\ \dot{w} + pv - qu\end{bmatrix} \tag{16}$$

Re-arranging the terms we get the three equations for the linear acceleration of the quad-copter as:

$$\dot{u} = rv - qw - gsin\theta \tag{17}$$
$$\dot{v} = pw - ru + gcos\theta sin\emptyset \tag{18}$$
$$\dot{w} = qu - pv + gcos\emptyset cos\theta - \frac{T}{m} \tag{19}$$

As explained in [4], if the motor dynamics are ignored, the total thrust of all rotors is proportional to the sum of the squares of the each propeller's angular speed. i.e.

$$T = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \tag{20}$$

where, $\Omega_1^2, \Omega_2^2, \Omega_3^2\ and\ \Omega_4^2$ are the speeds of each rotor and 'b' is the thrust co-efficient. Thus, the equations of linear acceleration changes and are given by:

$$\dot{u} = rv - qw - gsin\theta \tag{21}$$
$$\dot{v} = pw - ru + gcos\theta sin\emptyset \tag{22}$$
$$\dot{w} = qu - pv + gcos\emptyset cos\theta - \frac{b}{m}(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \tag{23}$$

Using similar analysis, we can determine the equations for the angular accelerations of the quad-copter considering the moments of motor dynamics, aerodynamic and gyroscopic forces. The equations for the angular accelerations on each axis can be represented as:

$$\dot{p} = \frac{lb}{I_x}(\Omega_2^2 - \Omega_4^2) - qr\frac{I_z - I_y}{I_x} - \frac{H_z}{I_x}q \tag{24}$$
$$\dot{q} = \frac{lb}{I_y}(\Omega_1^2 - \Omega_3^2) - pr\frac{I_x - I_z}{I_y} - \frac{H_z}{I_y}p \tag{25}$$
$$\dot{r} = \frac{d}{I_z}(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \tag{26}$$

where, I_x,I_y and I_z are the moments of inertia of the quad-coptor along each axis, 'l' distance of the propeller from the center of gravity of the quad-copter and 'd' is the drag factor of all rotors. Also, H_x, H_y *and* H_z are the total angular momentum of spinning masses with angular rates in *x, y and z* direction of the body frame.

Finally, the equation for the mechanical dynamics of the DC motors used in the rotors is given by:

$$J_r\dot{\omega_m} = k_i i - d_m\omega_m - f(\omega_m) \tag{27}$$

where, 'J_r' is the moment of inertia of the rotor, 'ω_m' is the motor angular rate, 'k_i' back emf constant, 'i' is the current flowing through the motors, 'd_m' is the bearing damping constant and '$f(\omega_m)$' is the non-linear drag torque function. Ignoring the non-linearities and solving the above equation further, the dynamic equation to express the motor angular rate is given by:

$$\dot{\omega_m} = -\frac{d_m}{(J_m + J_t)}\omega_m + \frac{k_i}{(J_m + J_t)R}e \tag{28}$$

where, 'J_m', Moment of inertia of rotors motor, 'J_t' moment of inertia of an arbitrary load, 'R' is the resistance of motor coils and windings and 'e' is the control input voltage applied to the motors.

III. Linearization

In order to linearize the set of non-linear equations of the quad-copter's system dynamics, we define an operating point / equilibrium point to be used as a reference for linearization. Consider, one such equilibrium point to be when the quad-copter is hovering still and the earth axes frame and the quad-copter's body axes frame are in full alignment. In such scenario, the equilibrium state values are:

$$x_0 = 0;\ \emptyset_o = 0;\ \ u_0 = 0;\ p_0 = 0$$
$$y_0 = 0;\ \theta_o = 0;\ \ v_0 = 0;\ q_0 = 0$$
$$z_0 = 0;\ \psi_o = 0;\ \ z_0 = 0;\ r_0 = 0$$
$$-\Omega_{10} = \Omega_{20} = -\Omega_{30} = \Omega_{40} = 463.1\ rad/sec$$

Differentiating the state equations and linearizing at the equilibrium states, we get the linearized system state equations as:

$$\dot{x} = u \tag{29}$$
$$\dot{y} = v \tag{30}$$
$$\dot{z} = w \tag{31}$$
$$\dot{\emptyset} = p \tag{32}$$
$$\dot{\theta} = q \tag{33}$$
$$\dot{\psi} = r \tag{34}$$
$$\dot{u} = -g\theta \tag{35}$$
$$\dot{v} = g\emptyset \tag{36}$$
$$\dot{w} = -2\Omega_0\frac{b}{m}(\Omega_1 + \Omega_3 - \Omega_2 - \Omega_4) \tag{37}$$
$$\dot{p} = 2\Omega_0\frac{lb}{I_x}(\Omega_2 - \Omega_4) \tag{38}$$
$$\dot{q} = 2\Omega_0\frac{lb}{I_y}(\Omega_1 - \Omega_3) \tag{39}$$
$$\dot{r} = 2\Omega_0\frac{d}{I_z}(\Omega_1 + \Omega_3 + \Omega_2 + \Omega_4) \tag{40}$$

Similarly, linearizing the motor dynamics equation about the equilibrium point gives:

$$\dot{\Omega_j} = -10\Omega_j + 7E \tag{41}$$

where, j = 1, 2, 3, 4 for a quad-copter system with 4 motors.

IV. Vertical Movement System

The vertical movement is defined when the quad-copter travels along the z-axis (yaw movement) without affecting its attitude in the other two axes (no change in pitch and roll movement). Thus, changes occur only in the quad-copter's vertical position and speed. This movement can be represented by using only a subset of system's linearized state equations:

$$\dot{z} = w \tag{42}$$
$$\dot{w} = -2\Omega_0\frac{b}{m}(\Omega_1 + \Omega_3 - \Omega_2 - \Omega_4) \tag{43}$$
$$\dot{\Omega_1} = -10\Omega_1 + 7E \tag{44}$$
$$\dot{\Omega_2} = -10\Omega_2 - 7E \tag{45}$$
$$\dot{\Omega_3} = -10\Omega_3 + 7E \tag{46}$$
$$\dot{\Omega_4} = -10\Omega_4 - 7E \tag{47}$$

Consider $b = 1.5108x10^{-5}kg.m$ and $m = 1.32\ kg.$, the state-space system to represent the vertical position and speed is given as:

$$\dot{X} = AX + BU \tag{48}$$

where the state vector X, A and B matrices used for system analysis are given by equations (49), (50) and (51) and the control input $U = E$.

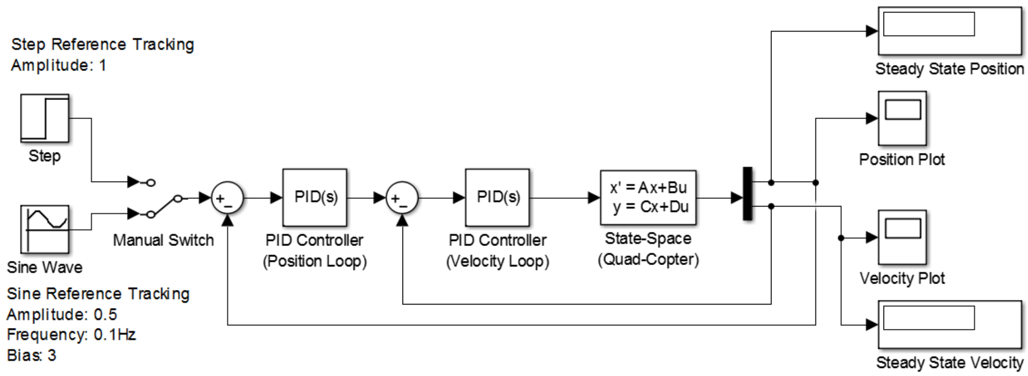

Figure 3: Simulink Model for PID controller.

$$X = \begin{bmatrix} z \\ w \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{bmatrix} \tag{49}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0106 & 0.0106 & -0.0106 & 0.0106 \\ 0 & 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix} \tag{50}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 7 \\ -7 \\ 7 \\ -7 \end{bmatrix} \tag{51}$$

In order to monitor the vertical attitude of the system we used two system states at the output: position (z) and linear speed (w). Thus the output matrix observes the states (z and w). The C matrix is modified for observing only one output at a time by making the other output 0.

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{52}$$

Since, the output does not have any direct effect due to changes in the input, D = 0 is a valid assumption.

V. LQR Controller

A linearized system in state space form can be represented by the generalized equation as:

$$\dot{x} = Ax + Bu \qquad \ldots\ t \geq t_0 \tag{53}$$

$$y = Cx \tag{54}$$

where, *A*, *B* and *C* represent the system state matrix, the input matrix and the output matrix respectively, '$\dot{x}$' and 'x' are the time derivative of the state and the current state and 'u' is the input that drives the system.

For the Quad-copter system linearized for vertical motion control, the *A*, *B* and *C* matrices are given in equations (50), (51) and (52) respectively.

As per the Continuous-time Linear Quadratic Regulator [10] of an infinite horizon problem, the cost function of a linear system given by:

$$J(t_0) = \int_0^{\infty} (x^T Q x + u^T R u) dt \tag{55}$$

can be minimized by following the control input policy of:

$$u = -K_\infty x \tag{56}$$

Here, 'K_∞' is the controller Kalman Gain given by the expression:

$$K_\infty = -R^{-1} B^T S_\infty \tag{57}$$

and S_∞ is the solution of the Algebraic Riccati Equation (ARE) of an Infinite horizon problem for a continuous time system. The expression for obtaining the solution of ARE is given by:

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{58}$$

The solution of S obtained using the above equation is considered to be S_∞ for an infinite horizon problem.

Further, in order for the state vector to follow a specific reference trajectory, the control input can be modified as

$$u = -K_\infty x + r \tag{59}$$

where, r denotes the reference trajectory.

VI. PID Controller

The Quad-copter system for vertical motion control given by equation (48) can be represented by a simplified transfer function in the Laplace domain. This simplified transfer function is used for the PID controller design. Since the control input 'u' has a direct relationship with the position and speed, a Dual Cascaded PID controller is designed for precision velocity and position control.

The simplified transfer function of the system with position output is given as:

$$G_z(s) = -\frac{0.0424}{s^2(s+10)} \tag{60}$$

and the transfer function of the system with velocity output is given as:

$$G_z(s) = -\frac{0.0424}{s(s+10)} \tag{61}$$

The PID controller model was designed using Simulink. The model of the system with cascaded PID controllers is as shown in figure 3. The transfer function for the PID controller is:

$$C(s) = \frac{K_D s^2 + K_P s + K_I}{s} \tag{62}$$

where, K_P, K_I *and* K_D are the controller (Proportional, Integral and Derivative) gains respectively. The values for these controller gains were obtained by using the PID auto-tune feature of Simulink PID block. The response time and control robustness were adjusted to obtain the controlled output with minimum overshoot, damping and settling time. First the velocity loop was tuned and then the position loop.

The PID controller gains for the Velocity loop are set as: $K_{p-w} = -12.69$, $K_{I-w} = -0.33$, and $K_{D-w} = -1.08$ with the filter co-efficient being set to 340.56 by auto-tuning. The PID controller gains for the position loop are set as: $K_{p-z} =$

1.44, $K_{I-z} = 0.017$, and $K_{D-z} = 3.45$ with the filter co-efficient being set to 2.87.

By altering the filter co-efficient, the derivative control on the system changes which either introduces more damping or increases the settling time to large value. It was observed that only a single controller for the position control was able to achieve the desired position. However, for highly unstable system like a Quad-copter, the velocity control in cascade with position control allows precise control of the system.

VII. Results and Discussion

LQR controller design: Q and R selection

One of the important considerations while designing an LQR controller, is the selection of the intermediate state and input weighting matrices Q and R as given in equation (55). Since there is no standard procedure to determine the values of Q and R, arbitrary values were taken and the system responses were observed. For analyzing the vertical motion control of the quad-copter system, first, the selection of the intermediate state weighting matrix Q is done keeping R constant such that the position state (z) is affected. Figure 4 shows the system state responses for different Q matrices. For Q - 100000 and Q – 1000000000, only the position state element was set to given Q. All remaining diagonal elements of the Q matrix were 1.

As visible in figure 4, the response of the system states for Q – 1000000000 shows that the position state converges very fast as compared to other Q values. However, there is large undershoot in velocity state. Hence, this Q value is not best suitable value. Also, by using a diagonal Q matrix with gain 10; weighting is added in all states. As a result, the system state response becomes very slow which confirms that the assumption made for only modifying the Q element of the position state and keeping the other diagonal elements to 1 is valid. Following similar procedure, the control input weighting was selected by experimenting different values and observing the state behavior as shown in figure 2.

Based on the above observations, the most efficient value of Q and R matrices were found to be:

$$Q = \begin{bmatrix} 10000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \; and \; R = 1 \qquad (63)$$

and the Kalman Gain for the system determined using the LQR function of MATLAB is then given by:

$$K = [-3162.30 \quad -1507.00 \quad 1.3062 \quad -1.3062 \quad 1.3062 \quad -1.3062] \qquad (64)$$

The control input was calculated using equation (56), (57) and (59) along with the controller gain 'K' which was then applied to the system for regulation as well as reference tracking of the quad-copter position state from a random initial condition defined as $x(t_0) = [5\ 10\ 0\ 0\ 0\ 0]^T$. The system regulation response can be seen in figure 5 marked by the blue curve.

Reference Tracking:

In order to verify if the controller is capable of following the reference trajectory for Quad-copter's vertical motion control, first a constant reference tracking position was set. The initial conditions were set as $x(t_0) = [5\ 0\ 0\ 0\ 0\ 0]^T$. The reference input was set to 1 unit. As shown in figure 6, both the LQR and PID controllers were successful in achieving the desired reference set-point. The LQR controller response is faster and does not generate any overshoot / undershoot as compared to the PID controller. Also, on close observation it was verified that the LQR controller was able to exactly match the reference input however, the PID controller does not reach the exact reference value and has small error. A small disturbance was then added to the position state with a magnitude of +3 units at time t = 10 seconds. Again, the LQR controller was able to stabilize the system states much faster as compared to PID controller.

The performance of the LQR and PID controller was then verified for tracking a sine reference input. Figure 7 shows the position and velocity state trajectories for tracking a sine reference input with a peak-to-peak amplitude of 1 unit, frequency as 0.1Hz and a bias of 3 units. The position state trajectory has a small phase lag for both the LQR and PID controller which may be introduced due to the closed loop system dynamics.

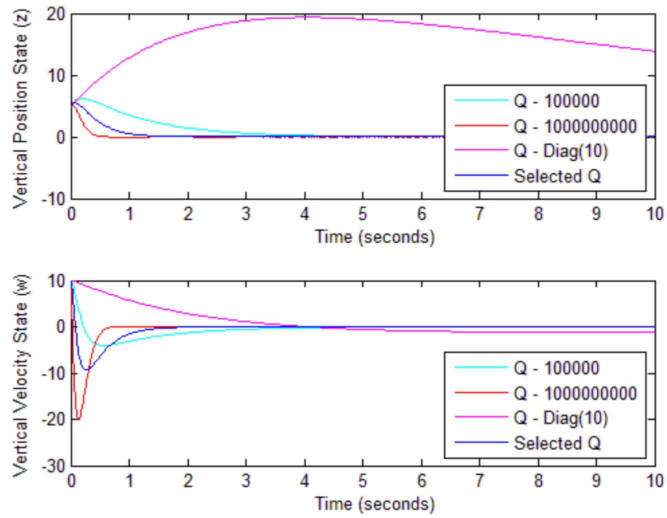

Figure 4: Effect of Variations in Intermediate states weighting matrix 'Q' on system states.

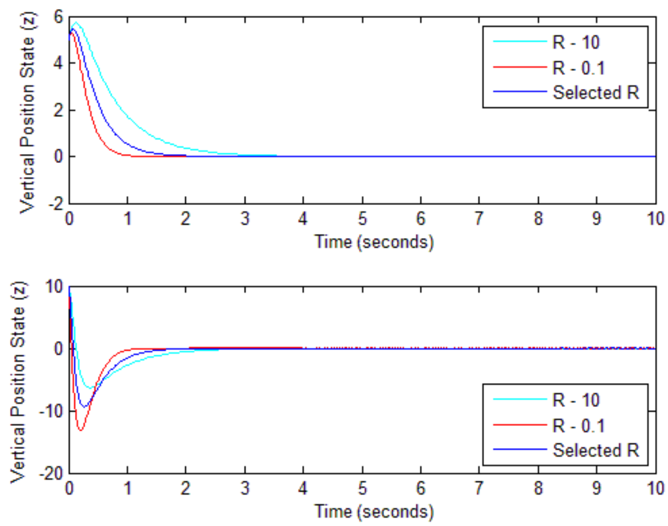

Figure 5: Effect of Variations in Input weighting matrix 'R' on system states.

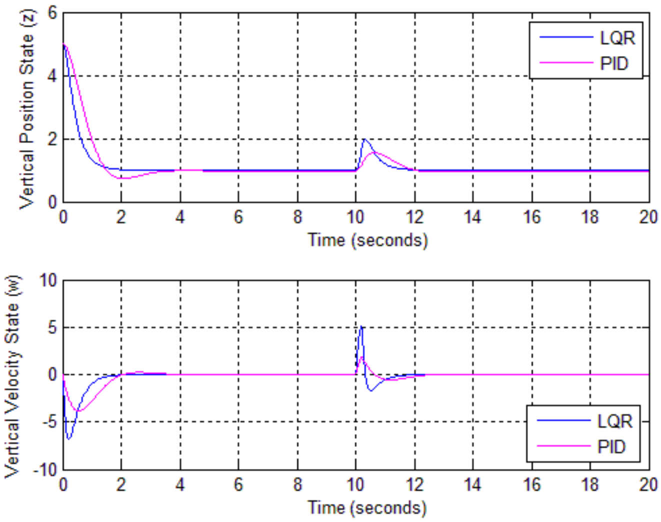

Figure 6: Quad-copter's Position and Velocity States: Tracking Constant Reference Input.

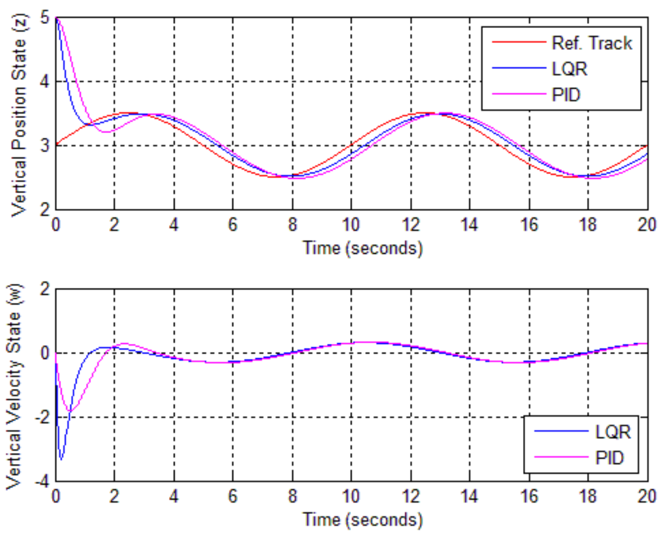

Figure 7: Quad-copter's Position and Velocity States: Tracking Sinusoidal Reference Input.

A range of frequencies for the sinusoidal reference input were applied which revealed that the bandwidth of the system is very low. Even with the frequency of 0.1Hz there was an attenuation of 0.1 units at the output of the closed loop system. This is justified by the system dynamics which indicate that the system would not be responsive to fast changes in the reference position changes.

VIII. Conclusion

In this project, the objective of verifying and comparing the performance LQR and PID controller was successfully achieved. The system dynamics revealed that the system is highly non-linear. When using the LQR controller, the computational costs increase as number of feedback gains are directly proportional to number of states which makes it unfit for applications with fast parameter updates. LQR is dependent on state information availability which might not be possible every-time. On the contrary, the PID controller does provide a satisfactorily fast control with less robustness as compared to LQR. Thus, selection of a controller largely depends on the available system information and level of robustness desired. The comparison of the LQR and PID controller performances for the vertical motion control of the quad-copter system revealed that the LQR controller provides faster convergence to stability and a more precise control as compared to the PID controller for both constant and sine reference position tracking.

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