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RESEARCH ARTICLE

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Short-circuit current distribution analysis using a network model based on a 5×5 primitive matrix

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Summary

This paper analyzes the well-known short-circuit current distribution problem in electric distribution facilities from a new perspective. Fault current distribution, temporary overvoltages, and ground potential rise profile are determined according to a detailed network model based on 5×5 primitive matrices. Despite the use of a 5×5 network model entails additional data with respect to the traditional approach based on Carson's assumptions, the main advantage of the method lies on the enhanced representation of the ground loop. Thus, short-circuit current distribution can be determined either by using any existing closed-form expression for the ground loop representation or by incorporating impedance parameters adjusted from dynamic state estimation based on synchronized measurements. The proposed method was illustrated in a simple radial two bus and also applied on a large-scale 115/12 kV system with 360 buses. Results were compared and validated with the nodal admittance approach provided by the OpenDSS platform.

KEYWORDS

fault current distribution, ground potential rise, neutral to earth voltages, phasor measurement units, primitive matrix, short-circuit analysis, temporary overvoltages

1 | INTRODUCTION

Advanced short-circuit analysis is required not only to determine the fault current magnitude to be handled by the equipment in substations but also to assess how that current is distributed through the returning paths. Fault currents flowing through earth in substations and towers will provoke ground potential rises (GPRs) and temporary overvoltages (TOVs) that could endanger the personal safety and security of the system. Thus, advanced short-circuit analyses are required to characterize the effect of fault currents. These analyses can be performed resorting to approximated network models based upon symmetrical components[1,2] or exact calculations over a phase-domain framework.[2] Table 1 lists a number of advanced power flow and short circuit with different network models reported in the literature.

Most of advanced power system analyses—power flow, short-circuit, and state estimation—are founded on a 4×4 primitive matrix network model.[3,4] In this representation, hereinafter denoted "4×4" models, primitive impedance equations for multiphase, and neutrals/ground are usually corrected using a ground return impedance model and later reduced to an equivalent 4×4 matrix using the Kron's formula. In this instance, self and mutual impedances are finally described as a three-phase (*abc*) and neutral (*n*) representation. Several short-circuit solution approaches for fault current distribution have been introduced under the 4×4 paradigm, ie, double-sided elimination method,[5] driving-point impedance technique,[6] ladder circuit solution,[7-9] single division factor,[10] two-port theory,[11] and nodal admittance methods.[12-17] Special purpose application software such as OpenDSS,[16] WinIGS,[18] and CDEGS[19] include

TABLE 1 Summary of network models suitable to be used in advanced short-circuit analysis

Reference	Network Model		
	Seq: 0+-	4×4	5×5
IEEE80[1]	∘		
Meliopoulos[2]	∘	∘	
Dawalibi[5]		∘	
Gooi[6]		∘	
Popovic[7]		∘	
Levey[8,9]		∘	
Kiessling[10]		∘	
Buccheri[11]		∘	
Zou[12]		∘	
Penido[13-15]		∘	
OpenDSS[16]		∘	
Klucznik[17]		∘	
WinIGS[18]		∘	
CDEGS[19]		∘	
Acharya[20,22]		∘	
Oka[21]		∘	
Chen[23]		∘	
Kersting[3]		∘	∘
Anderson[24]		∘	∘
Ciric[25,26,28]			∘

specific modules to determine current fault distribution in substations and lines. Other general-purpose application tools such as electromagnetic transients program (EMTP)[20-22] and personal simulation program with integrated circuit emphasis (PSPICE)[23]: have been also used to tackle the problem.

Another possible mathematical description of the network is the complete primitive matrix model,[24] hereafter referred as "5×5." In this model, three-phase and neutral impedances are calculated without any correction due to earth effect and the fifth wire (the resistance and reactance of the ground loop) is intended to represent a fictitious ground conductor. As a result, the 5×5 model comprises an equivalent of three phases (*abc*), neutral (*n*), and grounding (*g*) impedance representation. This model has been applied by previous studies[25-27] to perform system studies such as such as power flow, optimal power flow, and state estimation. In Ciric, Ochoa et al,[28] the model developed in Ciric, Feltrin et al[25] is used to calculate short-circuit current distribution. However, in this valuable contribution, results about the fault current division, produced GPR, and TOV were not compared and validated with any existing procedure.

The impedance of the ground return path exhibits a nonlinear frequency dependence. The problem of how current carrying wires are affected by the ground effect was solved in 1926 by Carson.[29] The exact calculation of ground path impedances generally requires the evaluation of an infinite complex integral. Today, the integral evaluation using numerical integration could require considerable computational times.[30] Several approximate closed-form expressions for the integral have been proposed such as the standard Carson's series formula (generally taken as reference),[29] Deri formula,[31] Gary-Dubanton formulas,[32] Alvarado-Betancourt formulas,[33] Noda's double symmetry-plane formulas,[34] and the widespread used Carson's modified series formula.[2] Some recent research has shown that the standard Carson-series method can produce large errors as high as 7.7% with respect to the evaluation of an infinite complex integral.[30] Other methods could produce errors between 8% and 15%.

It is worth to recall that Carson considered the return current through the earth with uniform resistivity and to be of infinite extent. This assumption leads to a fixed resistance of 0.0953 Ω/mile at 60 Hz.[35] Even though closed-form equations have been sufficiently precise to design and operate existing power systems,[36] those assumptions should be reappraised in the context of next generation distribution systems with higher short-circuit levels due to increased penetration of distributed generators. These systems are characterized by having short and untransposed line sections and high resistivity variability.[27]

Nowadays, the deployment of phasor measurement units (PMUs) in power grids are enabling synchronized voltages and branch currents measurements.[37] PMU can also provide synchronized measurements of currents in neutral and shield wires.[38,39] The increased observability given by the PMU infrastructure could improve the estimation line impedance parameters, including self and mutual impedances of the ground loop.[27] This means that traditional earth approximations can be adjusted to real-world conditions considering a full observable distribution system. Under this perspective, the use of 5×5 primitive matrices with PMU-estimated parameters in advanced short-circuit studies is worth. The use of more detailed models such as discussed in this paper can improve relaying applications such as fault location in high impedance conditions.[40]

In this paper, a 5×5 primitive matrix model is applied to determine the GPR profile, temporary overvoltages, and fault current distribution through all returning paths in electric distribution systems. The main advantage of the method lies on an enhanced representation of the ground loop. Thus, short-circuit current distribution is determined in accordance with existing closed-form expressions or by incorporating impedance parameters adjusted via dynamic state estimation. The proposal was illustrated in a radial two bus. The current distribution method was also applied in the large-scale example previously reported in Sakis Meliopoulos.[2] Results obtained were compared and validated against the approximate sequence domain analysis and the exact model based upon "4×4" nodal admittance approach provided by the OpenDSS platform.[16]

The paper is organized in the following manner. Section 2 presents the methodology. Results obtained from two case studies are discussed and compared with standard sequence network analysis in Section 3. Conclusions are drawn in Section 4.

2 | METHODOLOGY

2.1 | The 5×5 primitive matrix network model

The detailed network model developed in De Oliveira-De Jesus and Antunes[27] is structured according to a three-phase line segment with equivalent neutral wire grounded at both sending i and receiving buses k is shown in Figure 1. The 5×5 primitive impedance matrix $\mathbf{z_{ik}}$ of length ℓ_{ik} in miles between buses 0 and 1 is given by the following:

$$\mathbf{z_{ik}} = \begin{bmatrix} z_{ik}^{aa} & z_{ik}^{ab} & z_{ik}^{ac} & z_{ik}^{an} & z_{ik}^{ag} \\ z_{ik}^{ba} & z_{ik}^{bb} & z_{ik}^{bc} & z_{ik}^{bn} & z_{ik}^{bg} \\ z_{ik}^{ca} & z_{ik}^{cb} & z_{ik}^{cc} & z_{ik}^{cn} & z_{ik}^{cg} \\ z_{ik}^{na} & z_{ik}^{nb} & z_{ik}^{nc} & z_{ik}^{nn} & z_{ik}^{ng} \\ z_{ik}^{ga} & z_{ik}^{gb} & z_{ik}^{gc} & z_{ik}^{gn} & z_{ik}^{gg} \end{bmatrix} \ell_{ik}. \tag{1}$$

The model depicted in Figure 1 has explicit representation for the ground loop, then self and mutual phase and neutral parameters of $\mathbf{z_{ik}}$ are determined without ground effect. According to Kersting,[3] for overhead lines, self and mutual impedances of phases (Ω/mile) without Carson's corrections are as follows:

$$z_{ik}^{qq} = r_{ik}^{qq} + k\log_e\left(\frac{1}{D_{ik}^{qq}}\right)i, \tag{2}$$

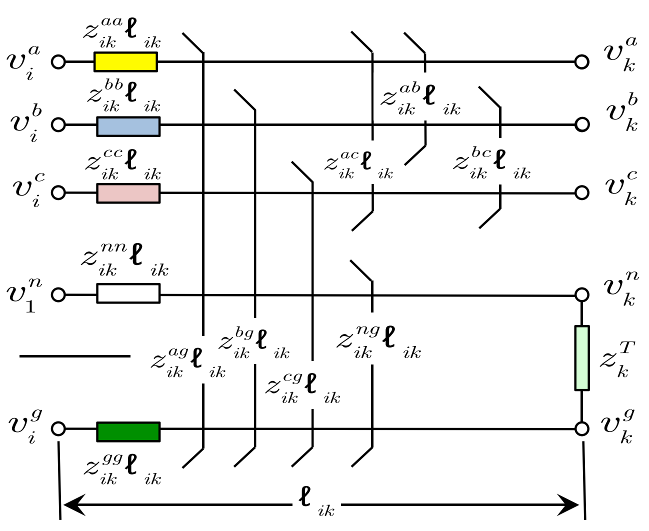

FIGURE 1 Two-terminal distribution line segment line

$$z_{ik}^{pq} = k\log_e\left(\frac{1}{D_{ik}^{pq}}\right)i, \tag{3}$$

$$\forall q, p = a, b, c, n, \tag{4}$$

where $i = \sqrt{-1}$ and

r_{ik}^{qq} is the resistance of phase or neutral conductor in Ω/mile,
D_{ik}^{qq} is the geometric mean radius of phase or neutral conductors in feet, and
D_{ik}^{pq} is the distance between two different conductors in feet.

Factor k is equal to $4\pi f\xi \times 10^{-4} = f\mu_0$ in Ω/miles where μ_0 is the permeability in free space in H/mile, with $\xi = 1.609$. Frequency f is given in Hz. For different kinds of underground cables, self and mutual impedances of phase/neutral conductors can be determined according to Rivas.[41]

The most appropriate theoretical value for the impedance of the ground path z_{ik}^{gg} is given by the Carson integral.[30] However, this procedure is expensive in computer times.[30] Instead, several approximate closed-form expressions such as the Deri formula,[31] Gary-Dubanton formulas,[32] Alvarado-Betancourt formulas,[33] and Noda's double symmetry-plane formulas[34] can be used. Network parameters of Equation 1 can be adjusted to real-world conditions by means of a dynamic system state estimator based upon synchronized phasor measurements.[27,38] Phase resistances are dependent on temperature at given operation point. Ground loop parameters depend on resistivity changes produced by moisture and temperature variations. Phase and ground loop self and mutual impedances can be included in the state vector as long as the system remains observable. For instance, the estimation process can include in the measurement vector and the state of the system vector all tower or pole footing resistances z_k^T.

In this paper, self and mutual impedances of the ground path in Ω/mile are defined according to traditional modified Carson's representation (equivalent depth of return method)[2] as follows:

$$z_{ik}^{gg} = \frac{1}{4}\pi k + k\log_e\left(\frac{D_e}{h^{qg}}\right)i \quad \forall q = a, b, c, n, \tag{5}$$

where the parameter $D_e = 2160\sqrt{\frac{\rho}{f}}$ is the equivalent distance earth to phase in feet, ρ is the soil resistivity in Ωm, and the h^{qg} is the height between phase/neutral conductors and the equivalent ground conductor in feet. This basic representation allows us to validate the proposal with standard sequence-based methods[2] and nodal admittance methods.[16]

The coupling between phase/neutral wires and the ground path is given by the following equation:

$$z_{ik}^{qg} = \frac{1}{2}\left(z_{ik}^{gg} + z_{ik}^{qq} - \bar{z}_{ik}^{qq}\right) \quad \forall q = a, b, c, n, \tag{6}$$

where the parameter $\bar{z}_{ik}^{qq}$ is the phase self-impedance corrected by Carson equations[3] as follows:

$$\bar{z}_{ik}^{qq} = r_{ik}^{qq} + \frac{1}{2}\pi k + k\left(\log_e\frac{D_e}{D_{ik}^{qq}}\right)i \tag{7}$$
$$\forall q = a, b, c, n.$$

Consider a general system with m buses as shown in Figure 2. The voltage source is reserved for the line segment 0-1. It comprises an ideal voltage source $\mathbf{V_0}$ and its equivalent Thévenin impedance matrix $\mathbf{z_{S01}}$. Nodes from 2 to m are regarded as load or generation nodes. So, the smallest number of buses is m=2 with a voltage source at sending bus 1 and a load/generation unit at the receiving bus 2.

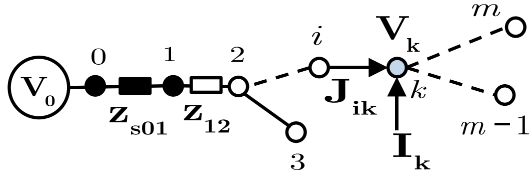

FIGURE 2 *m* segment network topology

The relationship between nodal voltages and branch currents can be established according to Kirchhoff laws by Equation 8 as follows:

$$\mathbf{V} = \mathbf{V_0} - \mathbf{T}^T \cdot \text{blkdiag}(\mathbf{z}) \cdot \mathbf{J}, \tag{8}$$

where $\mathbf{V}$=[$\mathbf{v_1}, \ldots, \mathbf{v_k}, \ldots \mathbf{v_m}]^T$ is the $(5m-5) \times 1$ state of the system vector on which each element of $\mathbf{V}$ is denoted a $\mathbf{v_k} = [v_k^a, v_k^b, v_k^c, v_k^n, v_k^g]^T$. The source voltage is defined as a $(5m-5)\times 1$ vector: $\mathbf{V_0}$=[$\mathbf{v_0}, \ldots, \mathbf{v_0}]^T$ where $\mathbf{v_0}$=[$v_0^a, v_0^b, v_0^c, v_0^n, v_0^g]^T$. The set of all primitive matrices are $\mathbf{z}$=[$\mathbf{z_{01}}, \mathbf{z_{12}} \ldots, \mathbf{z_{ik}} \ldots, \mathbf{z_{m\text{-}1,m}}]^T$. The operator `blkdiag` returns a block diagonal matrix based on specified square blocks.

The branch current vector is defined as follows:

$$\mathbf{J} = -\mathbf{T} \cdot \mathbf{M} \cdot \mathbf{I}, \tag{9}$$

where Equation 9 is defined as $\mathbf{J}$=[$\mathbf{j_{01}}, \ldots, \mathbf{j_{ik}}, \ldots, j_{m-1,m}]^T$. $\mathbf{I}$= [$i_1, \ldots, \mathbf{i_k}, \ldots \mathbf{i_m}]^T$ is the $(5m-5)\times 1$ injected current vector, where $\mathbf{i_k} = \left[i_k^a, i_k^b, i_k^c, i_k^u, i_k^u\right]^T$. Injected currents i_k^a, i_k^b, i_k^c will depend on fault current component i_{sck}^a, the line capacitance charging i_{ck}^a, and the loading/distributed generation model adopted at each phase i_{lk}^a. For instance, at phase a, the injected current is i_k^a=$i_{sck}^a+i_{ck}^a+i_{lk}^a$. Entry i_k^u of $\mathbf{i_k}$ is the unbalanced or fault current that is flowing through neutrals and earth paths $\left(-i_k^a - i_k^b - i_k^c\right)$. Matrix $\mathbf{M}$ groups the neutral to earth current divisors and $\mathbf{T}$ is the topology matrix built according the numbering scheme shown in Figure 2. Both matrices $\mathbf{M}$ and $\mathbf{T}$ have the same dimension $(5m-5)\times(5m-5)$ and should be structured according to the procedure established in De Oliveira-De Jesus and Antunes.[27] When substituting Equation 9 in Equation 8, we can observe that $\mathbf{T}^T\cdot$`blkdiag`$(\mathbf{z}) \cdot \mathbf{T} \cdot \mathbf{M}$ is a constant $(5m-5) \times (5m-5)$ impedance matrix, unique for each circuit as follows:

$$\mathbf{Z} = \quad \mathbf{T}^T \cdot \text{blkdiag}(\mathbf{z}) \cdot \mathbf{T} \cdot \mathbf{M}. \tag{10}$$

All vectors and matrices defined above have complex entries. Matrix $\mathbf{Z}$ can account the effect of neighboring circuits or parallel lines using appropriate Kron's reductions. Thus, the complete 5×5 primitive matrix model is expressed as the general Kirchhoff voltage law expression as follows:

$$\mathbf{V} = \mathbf{V_0} + \mathbf{Z} \cdot \mathbf{I}. \tag{11}$$

Equation 11 is constrained to radial systems feed by one three-phase voltage source $\mathbf{V_0}$ regarded as reference or slack bus. Injected currents $\mathbf{I}$ can be associated with load consumption (constant power, constant impedance, or constant current) or distributed generation injections.

2.2 | Short-circuit analysis - *m* buses

Under steady-state conditions, given the load/generation profile at each bus k=1, … , m, the solution of the set of $5m-5$ equations with $5m-5$ unknowns provide us the so-called power flow solution at unbalanced operation. If a fault occurs at given bus k and phase p, the system can be easily solvable by introducing additional equations and state variables. For instance, a single phase to neutral fault at given phase p and bus k through a resistance R_f must satisfy an additional equation $v_k^p - v_k^n = R_f \cdot i_{sck}^p$ where the short-circuit current i_{sck}^p should be regarded as an additional unknown variable.

The system posed in Equation 11 is nonlinear if injected currents $\left(i_k^a, i_k^b, i_k^c\right)$ are represented as voltage-dependent loads such as constant power, polynomial, or exponential. In this case, power flow solution under steady-state or fault conditions is straightforward by using any solver such as Newton. Otherwise, if injected load currents are linear, the set equations given in Equation 11 has direct solution and no iterative-based method is necessary.

Once Equation 11 is solved, the short-circuit current distribution is given by Equation 9. The grid to earth current at each bus k is given by the following:

$$i_k^{ng} = \frac{v_k^n - v_k^g}{z_k^T} \quad \forall k = 2, \ldots, m. \tag{12}$$

The GPR is a temporary overvoltage that appears between local neutrals and remote earth until the fault is effectively cleared. It can be defined as the maximum electrical potential that a ground grid may attain relative to a distant grounding point assumed to be at the potential of remote earth. The GPR can be determined at any node k by multiplying the portion of short-circuit currents flowing from neutral to ground (i_k^{ng}) through a ground resistance (z_k^{T}).[1]

$$GPR_k = i_k^{ng} z_k^T \quad \forall k = 2, \ldots, m. \tag{13}$$

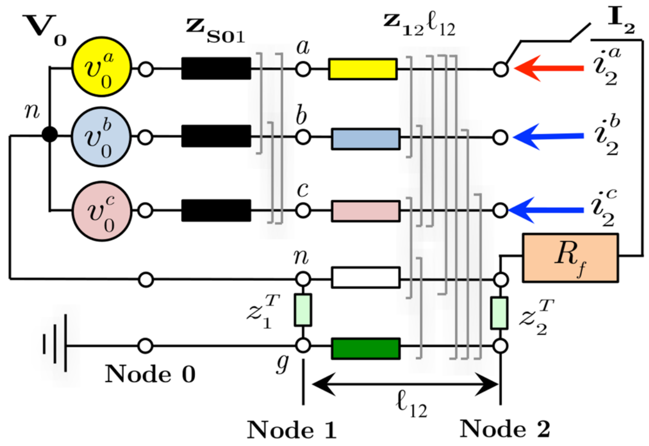

FIGURE 3 Two-bus system with a single phase to neutral fault

The coefficient of grounding (COG)[42] indicates the ratio of a phase-to-ground voltage with respect to post-fault line-to-line voltage (expressed as a percentage), at a selected location, during a fault to ground affecting one or two phases. The limit established in a previous study[42] for a system be considered effectively grounded is 80%. Thus, the COG is expressed by node and by phase as follows:

$$COG_k^p = \frac{v_k^p}{v_{LL}} \times 100 \quad \forall p = a, b, c, \quad k = 2, \ldots, m, \tag{14}$$

where v_k^p is the line to neutral voltages at phase p and bus k until the fault is cleared and v_{LL} is the post-fault line-to-line voltage magnitude. In this paper, for the sake of simplicity, all post-fault voltages are regarded as the nominal ones.

To illustrate the proposal, in the following detailed formulation and solution approaches for short-circuit distribution and GPR calculations are provided for two test systems. In this paper, only single-phase to neutral faults are considered.

2.3 | Short-circuit analysis—Two bus system

In this case, the number of nodes is m=2. Figure 3 shows a radial system with two line sections. The first segment 0-1 is reserved for the equivalent power source associated with the upstream substation. The second segment 1-2 corresponds to a three-phase line of length ℓ_{12}.

2.3.1 | Two-bus model description

According to the general model specified in Equation 8, the detailed matrix formulation for short-circuit current calculation is given by Equation 16 as follows:

$$\mathbf{v_2} = \mathbf{v_0} + \mathbf{Z} \cdot \mathbf{i_2}, \tag{15}$$

in matrix notation,

$$\begin{bmatrix} v_2^a \\ v_2^b \\ v_2^c \\ v_2^n \\ v_2^g \end{bmatrix} = \begin{bmatrix} v_0^a \\ v_0^b \\ v_0^c \\ v_0^n \\ v_0^g \end{bmatrix} + \mathbf{Z} \begin{bmatrix} i_2^a \\ i_2^b \\ i_2^c \\ i_2^u \\ i_2^u \end{bmatrix}, \tag{16}$$

where the system matrix $\mathbf{Z}$ has a dimension of 5×5. The main source is represented by the segment 0-1 through a Thévenin equivalent impedance matrix z_{S01} and an ideal three-phase voltage source $\mathbf{V_0}=\left[v_0^a, v_0^b, v_0^c, v_0^n, v_0^g\right]^T = \left[v_0 e^{0i} + v_0^n, v_0 e^{\frac{-2\pi}{3}i} + v_0^n, v_0 e^{\frac{2\pi}{3}i} + v_0^n, v_0^n, 0\right]^T$. Entry v_0 is the line-to-neutral nominal voltage. Entry v_0^n is the neutral voltage at origin with respect to local reference v_0^g, which is set equal to 0. Voltage neutral at sending bus v_1^n should be determined from the local ground impedance z_1^T and the sum of all returning ground currents as follows:

$$v_1^n = v_0^n = z_1^T J_{12}^{gg} = z_1^T \frac{v_2^g - v_2^n}{z_2^T}. \tag{17}$$

Note that neutral at bus 1 depends on grounding resistance value at bus 2. This model does not support null grounding impedances.

The Thévenin equivalent impedance $\mathbf{z_{S01}}$ is given by

$$\mathbf{z_{S01}} = \begin{bmatrix} z_{S01}^{aa} & z_{S01}^{ab} & z_{S01}^{ac} \\ z_{S01}^{ba} & z_{S01}^{bb} & z_{S01}^{bc} \\ z_{S01}^{ca} & z_{S01}^{cb} & z_{S01}^{cc} \end{bmatrix}. \tag{18}$$

Parameters of $\mathbf{z_{S01}}$ can be determined from single- and three-phase short-circuit level as well as by the relations R_1/X_1 and R_0/X_0 at bus 1.

The system impedance matrix is given by

$$\mathbf{Z} = (\mathbf{z_{S01}} + \mathbf{z_{12}}) \cdot \mathbf{M} = \mathbf{z_{02}} \cdot \mathbf{M} \tag{19}$$

where the matrix of split factors at the line segment 1-2 is

$$\mathbf{M} = \mathtt{diag}([1, \quad 1, \quad 1, \quad 1-\zeta_2, \quad \zeta_2]) \tag{20}$$

The split factor at given bus k is defined as the ratio between the neutral-to-ground and the short circuit current $\zeta_k = i_k^{ng}/i_k^u$.[1] Thus, the split factor at bus 2 (ζ_2) is

$$\zeta_2 = \frac{i_2^{ng}}{i_2^u} = \frac{z_{12}^{nn}\ell_{12} + z_{12}^{ag}\ell_{12} - z_{12}^{an}\ell_{12} - z_{12}^{ng}\ell_{12}}{z_1^T + z_2^T + z_{12}^{gg}\ell_{12} + z_{12}^{nn}\ell_{12} - 2z_{12}^{ng}\ell_{12}}. \tag{21}$$

The injected currents at bus 2 (I_2) comprise three components as follows: (a1) the load/generation-injected current at each phase of bus 1 $\left(i_{l2}^a, i_{l2}^b, i_{l2}^c\right)$, (b) the line charging due to shunt capacitances in the segment 1-2 $\left(i_{c2}^a, i_{c2}^b, i_{c2}^c\right)$, and (c) the short-circuit currents when the phase $a$is unintentionally connected to a grounded neutral (i_{sc2}^a) as seen in Figure 3. Thus, the injected currents per phase at bus 2 are as follows:

$$\begin{aligned} i_2^a &= i_{l2}^a + i_{c2}^a + i_{sc2}^a \\ i_2^b &= i_{l2}^b + i_{c2}^b \\ i_2^c &= i_{l2}^c + i_{c2}^c. \end{aligned} \tag{22}$$

The unbalanced load is written as $i_2^u = -i_2^a - i_2^b - i_2^c$. As the fault is at phase a, only injected current at this phase includes the short-circuit component i_{sc2}^a.

The injected load current at a given phase $p=a,b,c$ is $i_{l2}^p = (S_{G2}^p - S_{D2}^p)^*/(v_2^p - v_2^n)^*$, the apparent demanded power is given by $S_{D2}^p = P_{D2}^p + \mathrm{j}Q_{D2}^p$, and the apparent generated power is given by $S_{G2}^p = P_{G2}^p + \mathrm{j}Q_{G2}^p$. For the sake of simplicity, single-phase loads are always connected between phase and neutral. A general representation for nonlinear loads is given by the following expression:

$$P_{D2}^p = P_{Bk}^p \left(v_2^p - v_2^n\right)^\alpha, \tag{23}$$

where $=P_{Bk}^p$ is a base power parameter. Note that, if $\alpha = 0$, the model reflects a constant power load; if $\alpha = 1$, the model reflects a constant current load; and, if $\alpha = 2$, the model reflects a constant impedance load. The line capacitance charging model $\mathbf{i_{c2}}$ is obtained from the primitive admittance matrix of Sections 1 and 2, $\mathbf{y}_{12}$.[3]

Because of space restrictions, only single-phase to neutral fault analysis is considered in this paper.

2.3.2 | Single phase to neutral fault

When a single-phase a is unintentionally connected to a grounded neutral through a fault impedance R_f at the faulted bus 2, the following conditions should be also added to Equation 16 to find out the short-circuit currents:

$$v_2^a = R_f i_2^u + v_2^n. \tag{24}$$

If the fault impedance R_f=0, then neutral and phase a voltages at bus 2 are the same (v_2^a=v_2^n) and the resultant system (Equation 16 and Equation 24) has five complex equations with five complex state variables: the single-phase short circuit current i_{sc2}^a and the voltages v_2^a=v_2^n, v_2^b, v_2^c, and v_2^g. Conversely, if the fault impedance $R_f \neq 0$, the resultant system has six complex equations with six complex state variables: the single-phase short circuit current i_{sc2}^a and the voltages v_2^a, v_2^b, v_2^c, v_2^n, and v_2^g.

2.3.3 | Solution approach

The system of equations stated in Equation 16 is nonlinear. If system loads are regarded as constant power, then the solution can be assessed through an iterative method. As a special case, if loads are disconnected, the system of equations becomes linear and the solution is direct and no iterative procedure is required. Considering that Z_{ik} is the i, k element of the system impedance $\mathbf{Z}$, the fault impedance R_f=0 and $\psi_2 = z_1^T/z_2^T$, the direct solution of Equation 16 is given by Equation 25.

$$\begin{bmatrix} v_2^a = v_2^n \\ v_2^g \\ v_2^b \\ v_2^c \\ i_{sc2}^a \end{bmatrix} = \begin{bmatrix} \psi_2+1 & -\psi_2 & 0 & 0 & Z_{14}+Z_{15}-Z_{11} \\ \psi_2 & -\psi_2 & 1 & 0 & Z_{24}+Z_{25}-Z_{21} \\ \psi_2 & -\psi_2 & 0 & 1 & Z_{34}+Z_{35}-Z_{31} \\ \psi_2+1 & -\psi_2 & 0 & 0 & Z_{44}+Z_{45}-Z_{41} \\ 0 & 1 & 0 & 0 & Z_{54}+Z_{55}-Z_{51} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ e^{\frac{-2\pi}{3}i} \\ e^{\frac{2\pi}{3}i} \\ 0 \\ 0 \end{bmatrix}. \tag{25}$$

3 | ILLUSTRATIVE EXAMPLES

The proposed short circuit analysis approach was applied to two well-known examples. The first one is the Neutral-to-Earth test system.[43] The second one is the 115/12 kV large-scale system included in Sakis Meliopoulos,[2] Chapter 7. Both methods were also solved using the approximate sequence analysis[2] and by the OpenDSS[16] program based upon a detailed 4×4 primitive matrix model.

3.1 | NEV system

The neutral to earth voltage (NEV) test system[43] comprises three-phase with neutral main feeder connected to an ideal 12.47 kV (line-to-line) source. Root and ending nodes have the neutral solidly grounded through a 100 Ω resistance. The test feeder has 6000 ft length with 21 poles. For illustration purposes only, one section with two nodes (the source bus 1 and the ending bus 2) are considered. Single-phase to neutral fault is applied at bus 2 phase a.

3.1.1 | Data setup

System basic characteristics: the phase conductors used are 336 400 26/7 aluminum conductor steel reinforced (ACSR), geometrical mean radius (GMR) = 0.0244 ft, resistance = 0.306 Ω/mile, diameter = 0.721 inches. Neutral Conductor: 4/0 6/1 ACSR: GMR = 0.00814 ft, resistance = 0.592 Ω/mile, diameter = 0.563 inches. The feeder length ℓ_{12} is 1.13 miles long. Impedance at root 1 and remote bus 2 are fixed as z_1^T=z_2^T=100 Ω. The test case has an ideal voltage source with $\mathbf{z_{s01}}$=$\mathbf{0}$ (infinite bus). Short circuit analysis at ending bus 1 is performed assuming no loads and no shunt admittances $\mathbf{y_{01}}$. Constant power load load at phases b and c are 3500 kVA at 0.95 lagging power factor and 2500 kVA at 0.85 lagging power factor. Line capacitances were neglected. Programs used in the illustrative NEV example were coded in Octave/Matlab. The interested reader can get access to the programs in the following website: https://github.com/pmdeoliveiradejesus/55short-dist.git.

The elements of the primitive matrix $\mathbf{z_{12}}$ in Ω/mile are determined according Equations 2 to 7 as follows:

$z_{12}^{aa} = z_{12}^{bb} = z_{12}^{cc}$= 0.3060 + 0.4506i,

$z_{12}^{ag} = z_{12}^{bg} = z_{12}^{cg} = z_{12}^{ng}$ = - 0.2043i,

z_{12}^{ab} = -0.1112i, z_{12}^{bb} =, z_{12}^{ac} = - 0.2361i, z_{12}^{bc} = - 0.1825i,

z_{12}^{an} = -0.2103i, z_{12}^{bn} = - 0.1762i, z_{12}^{cn} = - 0.1953i,

z_{12}^{nn} = 0.5920 + 0.5838i, z_{12}^{gg} = 0.0953 + 0.5540i.

The system matrix **Z** is calculated according to Equation 19 assuming an ideal source at bus 1. According to Equation 21 calculated split factor at bus 2 (ζ_2) is 0.0034 + 0.0045i.

3.1.2 | Case A: Single-phase a to neutral fault—No loads

In this case, load effect is disregarded. According to Equation 24, it must be accomplished that v_2^a=v_2^n with R_f=0. Therefore, equations stated in Equation 16 are only expressed as a function of system voltages at bus 2 and the short-circuit current at phase a through the following system of linear equations:

$$\begin{bmatrix} v_2^a = v_2^n \\ v_2^b \\ v_2^c \\ v_2^n = v_2^a \\ v_2^g \end{bmatrix} = \begin{bmatrix} 1 + v_0^n \\ e^{\frac{-2\pi}{3}i} + v_0^n \\ e^{\frac{2\pi}{3}i} + v_0^n \\ v_0^n \\ v_0^g = 0 \end{bmatrix} + \mathbf{Z} \cdot \begin{bmatrix} i_{sc2}^a \\ 0 \\ 0 \\ -i_{sc2}^a \\ -i_{sc2}^a \end{bmatrix}. \tag{26}$$

As the system is linear, the solution of Equation 26 is straightforward. By applying Equation 25 with ψ=1, voltages at bus 2 and the phase a current is given by the following:

$$\begin{bmatrix} v_2^a = v_2^n \\ v_2^g \\ v_2^b \\ v_2^c \\ i_{sc2}^a \end{bmatrix} = \begin{bmatrix} 2084.0\angle-4.86^o\text{V} \\ 184.0\angle 68.69^o\text{V} \\ 8551.9\angle-133.68^o\text{V} \\ 8547.6\angle 130.28^o\text{V} \\ 3713.7\angle 121.78^o\text{A} \end{bmatrix}. \tag{27}$$

Voltage at neutral's source is determined according to Equation 28

$$v_1^n = v_2^g - v_2^n = 2081.7\angle 174.61^o\text{V}. \tag{28}$$

The short-circuit current has only one contribution, the power source. Thus, j_{12}^a=- i_{sc2}^a=3713.7 A∠ -58.22^o. Current division at bus 2 is shown in the second column of Table 2 and Figure 4A. Note that due to the high local grounding resistance, almost 100% of the fault current is returning through the neutral path. Despite the grid to ground current is quite small (20.8 A), the GPR reaches 2081.7 V at both source and load buses. The NEV test system has been defined with a grounding resistance of 100 ohms. This value is very common in real world. As a result, any small unbalanced current flowing to ground, e.g. 1 ampere, is able to produce a substantial potential rise of 100 V. The rise of potential affects nonfaulted phases such as v_2^b=8551.9 V and v_2^c=8547.6 V. In both cases, this result implies a deviation of 68.5% with

TABLE 2 Summary of results -NEV system

	Case A: No Loads			Case B: Loads	
	5×5	4×4 [16]	Seq. [2]	5×5	4×4 [16]
j_{12}^a (A)	3713.7	3713.7	3776.8	3911.8	3911.6
j_{12}^b (A)	0	0	-	364.4	364.4
j_{12}^c (A)	0	0	-	252.0	252.0
j_{12}^n (A)	3701.2	3701.1	3741.1	3568.3	3568.3
j_{12}^g (A)	20.817	20.816	36.17	20.061	20.061
COG_2^b	68.5%	68.5%	78.5%	66.0%	66.0 %
COG_2^c	68.5%	68.5%	82.2 %	66.0%	66.0 %
GPR_2 (V)	2081.7	2081.7	3617	2006	2006

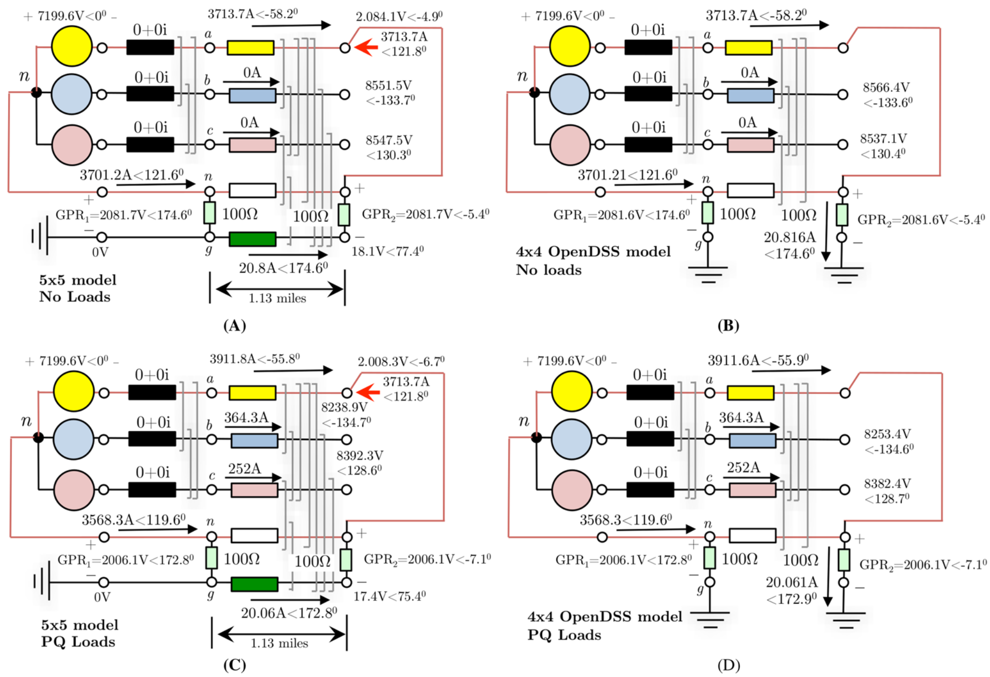

FIGURE 4 NEV test system—general solution sketch

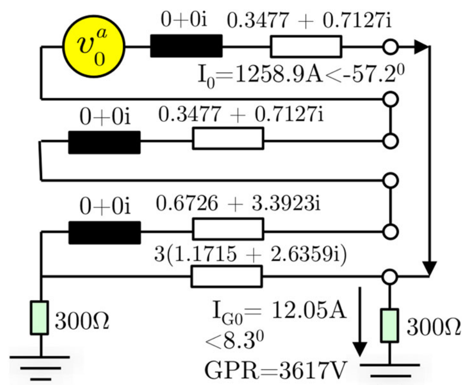

FIGURE 5 NEV test system—Sequence networks circuit

respect to the nominal line-to-neutral voltage. So, the system is effectively grounded since the COG is below 80%. If the NEV system is provided with footing resistances lower than 10 Ω, we can observe higher split factors and fault currents returning through natural earth.

Solutions obtained using the proposed method are compared with the sequence impedance model provided in Meliopoulos's book[2] and validated using the OpenDSS program.[16] Solutions are listed in the third and fourth column of Table 2. The OpenDSS solution is also depicted in Figure 4B. The interested reader can verify the solution using the source script included in Appendix A.

We must highlight that both approaches (4×4 and 5×5 models) yield the same solution. Consequently, both approaches are equivalent. The added value of the proposed method stems from the possible inclusion of estimated parameters into the 5×5 primitive matrix.

Table 2 also shows how the sequence model yield inaccurate results with respect to the exact solutions provided by the 5×5 and the OpenDSS model. Thus, from Figure 5, the connections of sequence networks for a single fault phase to neutral in bus 2 is illustrated. The zero-sequence current is 1258.9A $\angle$ -57.20^o, and the fault current is 3776.7A $\angle$ -57.20^o. This latter result is close to the correct one (3713.7$\angle$ -58.22^o). However, the calculated split factor fails. The sequence model dictates that a current of 36.17 A is flowing through a 100 Ω resistance at the origin bus 1 and at the ending bus 2. As a result, the expected GPR (3617 V) is much higher than the correct one (2081.7 V). This result implies an overestimation of the COG of the system.

3.1.3 | Case B: Single-phase a to neutral fault with constant power loads

This second example assumes the NEV system with two constant power loads at bus 2, phase b, and phase c. As the faulted phase a and neutral n have the same potential, it is further assumed that load at bus a is not capable to absorb/inject current during the fault. In this case, the system posed in Equation 29 is nonlinear since i_2^b and i_2^c depend on voltages v_2^b and v_2^c, respectively.

$$\begin{bmatrix} v_2^a = v_2^n \\ v_2^b \\ v_2^c \\ v_2^n = v_2^a \\ v_2^g \end{bmatrix} = \begin{bmatrix} 1 + v_0^n \\ e^{\frac{-2\pi}{3}i} + v_0^n \\ e^{\frac{2\pi}{3}i} + v_0^n \\ v_0^n \\ v_0^g = 0 \end{bmatrix} + \mathbf{Z} \cdot \begin{bmatrix} i_{sc2}^a \\ i_2^b \\ i_2^c \\ -i_{sc2}^a - i_2^b - i_2^c \\ -i_{sc2}^a - i_2^b - i_2^c \end{bmatrix}. \tag{29}$$

The solution of the nonlinear problem stated above is obtained using the general purpose Matlab's "fmincon" tool.

$$\begin{bmatrix} v_2^a = v_2^n \\ v_2^g \\ v_2^b \\ v_2^c \\ i_{sc2}^a \end{bmatrix} = \begin{bmatrix} 2008.3\angle -6.67^o\mathrm{V} \\ 174\angle 75.44^o\mathrm{V} \\ 8238.9\angle -134.64^o\mathrm{V} \\ 8392.3\angle 128.63^o\mathrm{V} \\ 3911.7\angle 124.11^o\mathrm{A} \end{bmatrix}. \tag{30}$$

As previously done in the first case A, the fifth and sixth column of Table 2 show solutions obtained using the proposed method and the OpenDSS program,[16] respectively. If the interested reader wants to reproduce the results, loads in the OpenDSS script included in A can be enabled in order to get the solution of Case B. For the sake of simplicity sequence analysis was omitted. Graphical solution is also presented in Figure 4C and Figure 4D, for the proposed method and the OpenDSS solution.

Summarizing the results shown in Table 2, we can observe that in case A, both methods (standard 4×4 model[16] and the proposed 5×5 model) yield the same solution. As expected, sequence domain analysis[2] applied in case A produces inaccurate results with respect to the phase-domain models. Likewise, in case B, results are also coincidental. It is worth to note that short-circuit level obtained in case B is slightly higher to the one observed in case A. The reason lies on the contribution of the unbalanced current at load-side considered in Case B.

3.2 | Large-scale test case

The proposed method was applied in the 115/12 kV power system used by Sakis Meliopoulos[2] to compute fault current distribution and ground potential using the sequence model. The system encompasses two lines and a substation. The substation is fed by a 25.5-mi long single circuit 115 kV subtransmission line. Loads are fed by a 10-mi long distribution 12 kV circuit. As shown in Figure 6, the substation has a 115/12 kV 20 MVA transformer with delta-wye connection. The subtransmission line comprises 235 towers whose shield wire is solidly grounded through a 30-Ω resistance electrode. The distribution line has 120 poles with neutral wire grounded through a 50-Ω resistance at each pole. In this case, the number of buses is $m = 359$. The equivalent impedances of the interconnected system are $Z_{S01}^{+}=Z_{S01}^{-}= 9.8\ \Omega$ and $Z_{S01}^{0}= 6.6\ \Omega$. The source station grounding resistance is 2 Ω. At 115/12 kV substation, power transformer low voltage neutral as well as incoming shield wires and outgoing neutrals are grounded to a 2 Ω substation ground mat. Soil resistivity is equal to 265 Ωm along the both lines and the substation under study.

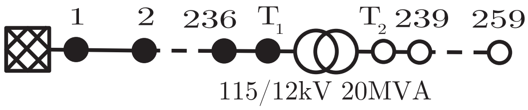

FIGURE 6 115/12 kV test system

TABLE 3 Summary of results—115/12 kV test system

Parameters	5×5	4×4 [16]	Seq. [2]
$J^a_{236-237}$ (A)	1493.1	1494.2	1470
$J^n_{237-237}$ (A)	370.9	371.5	544
$J^n_{237-238}$ (A)	279.9	278.2	544
i^{ng}_{237} (A)	852.3	854.4	926
$\lvert\zeta_{237}\rvert$	57.0 %	57.2 %	63.2 %
GPR_{237} (V)	1704.6	1708.9	1858

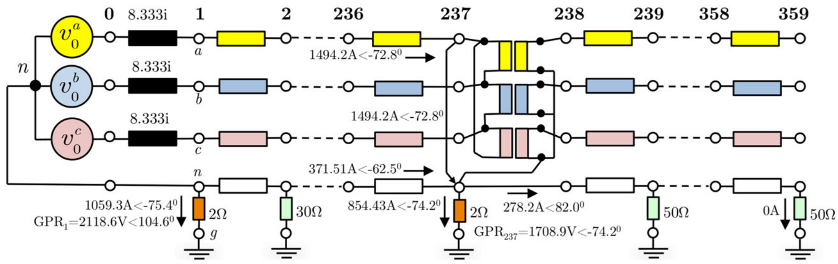

FIGURE 7 115/12 kV test system—fault current distribution (OpenDSS)

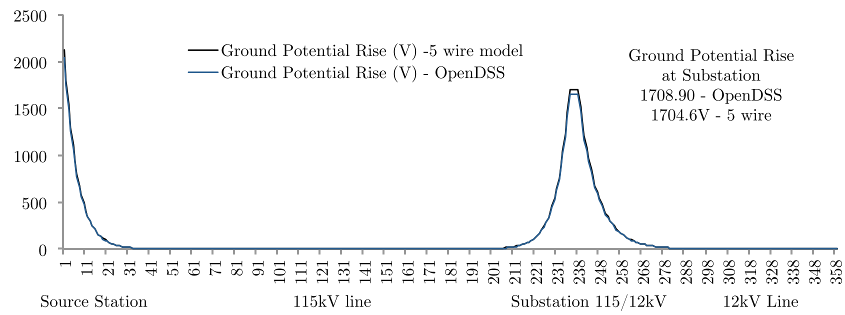

FIGURE 8 115/12 kV test system—fault current distribution (5 × 5 model)

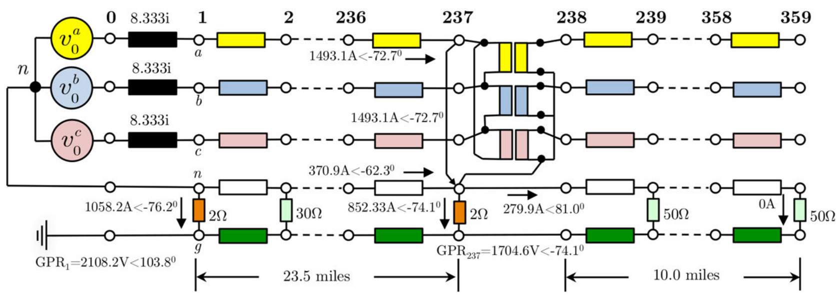

FIGURE 9 115/12 kV test system—ground potential rise (GPR) profile

The GPR and the fault current distribution along the entire system is determined considering a single-phase to neutral fault at high-voltage side of the substation, bus 237 (T1), phase a. Table 3 presents the results for a single fault at bus 237.

The solution provided by the 5 × 5 primitive matrix model matches with the solution provided by the OpenDSS platform. The split factor ζ_{237} is around 57%. Meliopoulos reported a division factor of 63% using the approximate sequence model approach.[2] Likewise, we can observe slight differences in the fault current and GPR magnitudes with the approximated sequence model. The fault current distribution patterns for the 5 × 5 model and the OpenDSS engine are also shown in Figures 7 and 8, respectively.

The GPR magnitude profile for the entire system is shown in Figure 9. Again, we can observe small differences between both solutions.

The examples discussed in this section aim to validate system solutions with two different network models but with the same earth model assumptions, ie, modified Carson's equations. The use of detailed 5 × 5 network models allows to introduce modifications in the primitive matrix by including parameters estimated via enhanced dynamic system state estimation. Further developments of the proposed methodology should be focused on distribution systems with widespread deployment of PMUs.

4 | CONCLUSIONS

In this paper, fault current distribution through groundwires, neutrals, and natural earth are determined according to a new method based upon a 5 × 5 primitive matrix network model. The method allows to evaluate temporary overvoltages at phases and GPR patterns at substations and line towers. The proposed methodology is suitable to be applied either by using closed-form expressions for the self and mutual impedance of the ground path or by incorporating impedance parameters adjusted by dynamic state estimation tools and synchronized measurements. The proposed method was successfully tested and validated with the exact nodal admittance approach provided by the OpenDSS platform under the earth modeling conditions (Carson's modified formula).

The advantage of using 5 × 5 primitive network models in systems with widespread deployment of PMUs can improve the estimation of system parameters, in particular the parameters of the ground path when distribution lines traverse vast zones with non homogeneous soil resistivity. The proposed method allows to assess how voltages and currents are distributed under unbalanced conditions with 5 × 5 primitive matrix network models.

5 | LIST OF SYMBOLS

Indexes

i	denotes bus i
k	denotes bus k
a	denotes phase a
b	denotes phase b
c	denotes phase c
n	denotes neutral n
g	denotes ground g
T	denotes a terminal ground
p, q	denotes any conductor phase (a, b, c) or neutral n

Symbols

α	load model constant
μ_0	permeability in free space in H/mile
ψ	ratio between z_1^T/z_2^T
ρ	soil resistivity in Ωm
ξ	constant to switch parameters from meters to miles (1.609)
ζ_k	split factor at bus k
COG_k	coefficient of grounding at bus k
D_e	equivalent distance earth to phase in feet
D_{ik}^{qq}	geometric mean radius of phase or neutral conductors in feet
D_{ik}^{pq}	distance between two different conductors in feet
f	system frequency in Hz
GPR_k	ground potential rise (neutral to earth voltage) at bus k in volts
h^{qg}	height between phase/neutral conductors and the equivalent ground conductor in feet
$\mathbf{I}$	system injected currents array in amperes
$\mathbf{i_k}$	system injected currents array between at bus k in amperes
i_{ck}^p	injected capacitive current at bus k and phase p
i_{lk}^p	injected load current at bus k and phase p
i_k^q	injected current at bus k and phase/neural/ground q
i_k^u	unbalanced current at bus k
i_{sck}^p	short-circuit current at phase p and bus k in amperes
i_k^{ng}	neutral to ground current at bus k in amperes
$\mathbf{J}$	system branch currents array in amperes
$\mathbf{j_{ik}}$	system branch currents array between bus i and k in amperes
j_{ik}^q	branch current between bus i and k and phase/neural/ground q in amperes
ℓ_{ik}	distance between bus i and bus k in miles
m	number of system buses
$\mathbf{M}$	current divisors matrix
P_B^p	base active power demanded at phase p bus k in MW
P_{Gk}^p	active power generated at phase p bus k in MW
P_{Dk}^p	active power demanded at phase p bus k in MW
Q_{Gk}^p	reactive power generated at phase p bus k in mvar
Q_{Dk}^p	reactive power demanded at phase p bus k in mvar
r_{ik}^{qq}	resistance of phase or neutral conductor in Ω/mile

R_1/X_1 positive sequence ratio
R_0/X_0 zero sequence ratio
R_f fault resistance in Ω
S^p_{Gk} apparent power generated at phase p bus k in MVA
S^p_{Dk} apparent power demanded at phase p bus k in MVA
$\mathbf{T}$ topology matrix
$\mathbf{V}$ system voltage array in volts
$\mathbf{V_0}$ source voltage array in volts
$\mathbf{v_k}$ system voltage array at bus k in volts
v^q_k complex voltage at phase/neutral/earth p and bus k
y^{aa}_{ik} self primitive shunt admittance between i and k, phase a (without earth effect correction) in siemens/mile
y^{ab}_{ik} mutual primitive shunt admittance between i and k, phases $a-b$ (without earth effect correction) in siemens/mile
$\mathbf{y_{ik}}$ 5×5 primitive shunt admittance matrix in siemens
$\mathbf{z}$ array of all primitive impedance matrices $i-k$ in Ω
$\mathbf{Z}$ system impedance matrix in Ω
$\mathbf{z_{ik}}$ 5×5 primitive series impedance matrix in Ω
z^{aa}_{ik} self primitive series impedance between i and k, phase a (without earth effect correction) in Ω/mile
z^{ab}_{ik} mutual primitive series impedance between i and k, phases $a-b$ (without earth effect correction) in Ω/mile
$\mathbf{z_{S01}}$ equivalent Thévenin impedance matrix in Ω
z^T_k grounding electrode impedance at bus k in Ω

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APPENDIX A: OPENDSS SCRIPT

```
! n=1 SHORT-CIRCUIT ANALYSIS November 15, 2018

! Full NEV approach (considering tower footings)
clear
New circuit.SOURCE_1 bus1=n1.1 bus2=n1.4
basekV=7.1996 pu=1 angle= 000
Z1=[.0000001, .0000001] phase=1
New Vsource.SOURCE_2 bus1=n1.2 bus2=n1.4
```

```
basekV=7.1996 pu=1 angle=-120
Z1=[.0000001, .0000001] phase=1
New Vsource.SOURCE_3 bus1=n1.3 bus2=n1.4
basekV=7.1996 pu=1 angle= 120
Z1=[.0000001, .0000001] phase=1
set earthmodel=carson
new wiredata.conductor Runits=mi Rac=0.306 GMRunits=ft
GMRac=0.0244 Radunits=in
new wiredata.neutral Runits=mi Rac=0.592 GMRunits=ft
GMRac=0.00814 Radunits=in
new linegeometry.4wire nconds=4 nphases=3 reduce=no
~ cond=1 wire=conductor units=ft x=-4 h=28
~ cond=2 wire=conductor units=ft x=-1.5 h=28
~ cond=3 wire=conductor units=ft x=3 h=28
~ cond=4 wire=neutral units=ft x=0 h=24
new line.line1 geometry=4wire length=6000 units=ft
bus1=n1.1.2.3.4 bus2=n2.1.2.3.4 Rho=100
!New Load.load1b.2 Phases=1 Bus1=n2.2.4 kVA=3500
pf=0.95 kV=12.47 conn=wye vminpu=0.1 vmaxpu=1.9
!New Load.load1c.3 Phases=1 Bus1=n2.3.4 kVA=2500
pf=0.85 kV=12.47 conn=wye vminpu=0.1 vmaxpu=1.9
New Reactor.SourceGround Phases=1 Bus1=n1.4 Bus2=n1.0 R=100.0
New Reactor.Load1Ground Phases=1 Bus1=n2.4 Bus2=n2.0 R=100.0
set voltagebases=[12.47]
calcvoltagebases ! **** let DSS compute voltage bases
New Fault.faseA Phases=1 Bus1=n2.1 Bus2=n2.4
solve !.
show voltages LN Nodes
show currents resid=yes elements
```