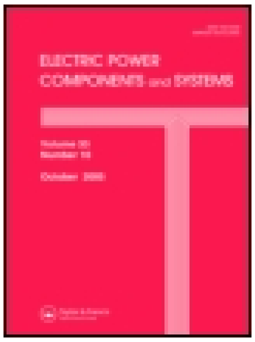

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A New Method to Determine Incremental Costs of Transmission Lightning Protection Systems

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CONTENTS

Abstract—This paper investigates the relationship between the cost associated to lightning protection systems (LPS) and the back-flashover rate (BFR) in high voltage transmission lines. The fundamental research question raised is how to determine the incremental cost of the mitigation measures associated to a prescribed back flashover rate. The interaction of tower supplemental grounding and tower insulation design is analyzed satisfying a given reliability level at minimum overall investment cost. A new optimization model relating back flashover phenomena to expenditure in LPS is presented expressly accounting the dynamic behavior of the tower footings being suitable to be applied using any existing BFR evaluation methodology. To illustrate the method, practical study case based on the step-by-step Anderson-EPRI method is presented. Despite the EPRI method has limitations, its simplicity is useful to illustrate the calculation of incremental cost curves of LPS without resorting to simulation software. Researchers are encouraged to apply the method using other BFR evaluation methodologies.

1. INTRODUCTION

Overvoltages of atmospheric origin are one of the main causes of faults and breakdowns in power systems. The insulation Back Flashover (BF) is the outcome of a complex electromagnetic phenomena. It occurs when a lightning stroke terminates on the shielding system of the transmission line and voltages surges that arise between phase conductors and tower's cross arms are much greater than the specified insulation level. Despite uncertainties of the BF phenomena are still challenging researchers, many improved techniques have been introduced to determine the performance of transmission lines due direct lightning and certain maturity has been reached in this topic. Several methods to determine transmission line reliability indexes, such as back-flashover rates (BFR), have been widely discussed in literature. Contributions range from early approaches [1, 2] to specialized software for the simulation and analysis of transients in power systems [3].

Diverse standardized procedures have been developed in the last years helping designers to identify the corresponding

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NOMENCLATURE

Greek Symbols

α	grounding cost function fitting parameter
β	grounding cost function fitting parameter
ΔW_{Tk}	additional weight of the tower at section k in kg
η_k	flash density at section k in flashes cloud-to-ground per km^2-yr
η_k^{max}	maximum flash density at section k in flashes cloud-to-ground per km^2-yr
γ	grounding cost function fitting parameter
ν,μ	Karush-Khun-Tucker factors
μ_k	annual direct strokes at section k per 100 km
L	Lagrangian
λ	Lagrange multiplier
ℓ_k	horizontal radial wire length in m
ρ_k	soil resistivity at section k in Ωm
σ_k^0	fixed indirect costs at section k in US$/m
σ_k	incremental cost of insulation in US$/m
Θ	Back Flashover Rate function fitting parameter

Symbols

BFR	Back Flashover Rate
B_k	horizontal distance between the groundwires at section k in m
C_{G0k}	fixed grounding cost in US$
C_{Gk}	grounding cost at section k
C_{Ik}	Cost of insulation of length w_k at section k in US$
CFO_k	critical flashover voltage at section k in kV
d	trench depth in m
$f(I)$	log-normal lightning current probability density distribution
G_0	earth ionization electric field gradient in kV/m
h_k	height of the shield/groundwires at section k in m
h_{gk}	height of each groundwire at section k in m,
h_{pk}	height of each phase conductor of the tower at section k in m
I_{ck}	critical lightning current amplitude at section k in kA
I_k	lightning current amplitude at section k in kA
I_{mk}	maximum shielding failure current at section k in kA
k	denotes section k
k_e	cost of mechanical excavation, backfilling and compaction in US$/m^3
k_c	ground conductor cost in US$/m
k_m	factor for double circuit lattice EHV systems
k_t	tower steel cost in US$/kg
n	number of sections
N_k	number of towers at section k
n_r	number of electrode radial wires
N_T	total number of towers of the line
m	fitting polynomial order
L_T	transmission line length in km
L_k	section k length in km
P_{ck}	cumulative probability at section k
$\mathcal{R}$	set of impulse impedances
R_k	impulse grounding impedance at section k in Ω
R_k^e	steady-state supplemental electrode impedance at section k in Ω
R_k^b	steady-state footing tower-impedance at section k in Ω
R_k^0	steady-state grounding impedance at section k
s_g	trench width in m
$S(I)$	maximum and minimum exposure distances for the shield wires in m
t	time flashover in µs
t_f	crest times in µs
t_g	individual force in kg
t_p	transverse force in kg
T_k	back-flashover rate at section k in outages/100km-yr
T	calculated global outage rate for the entire transmission line in outages/100km-yr
T^*	prescribed global outage rate for the entire transmission line in outages/100km-yr
$\sqsupseteq$	set of insulation dry lengths
W_{Tk}	tower weight at section k in kg
W_k	shadow length at section k in m
w_k	insulation dry arc length at section k in m
w_k^{max}	maximum insulation length at section k in m
w_k^b	insulation length specified at power-frequency at section k in m

Incremental factors

$\partial C_{Gk}/\partial R_k$	incremental grounding costs with respect to R_k at section k in US$/$\Omega$
$\partial C_{Gk}/\partial w_k$	incremental grounding costs with respect to w_k at section k in US$/m
$\partial T_k/\partial R_k$	incremental BFR with respect to R_k at section k in outages per 100km /Ω
$\partial T_k/\partial w_k$	incremental BFR with respect to w_k at section k in outages per 100km/m
$\partial C_{Ik}/\partial w_k$	incremental cost of insulation at section k in US$/m

mitigation measures, i.e. the lightning protection systems (LPS), with the aim of reduce the effects of the BF phenomenon [4, 5]. These procedures encompass the specification of insulation levels, supplemental grounding electrodes, the installation of additional shield wires and line arresters. Designers tend to specify supplemental grounding electrodes as the main mitigation measure to reduce the BFR. However, these costs could be sizable in transmission lines with adverse lightning conditions. Furthermore high costs of line arresters, the inclusion of additional shield wires and insulation lengths could have important impacts on overall transmission structural costs. Thereby, it is worth to investigate the relationship between outage rates and the required investment costs to reduce it.

Grossner & Hileman analyzed in the last sixties the effects of the back-flashover phenomena from economic standpoint [6]. As a fundamental contribution, they stated that the cost of grounding and insulation associated to a prescribed BFR can be optimized. Hence, for each reliability level, there is an incremental cost for the mitigation

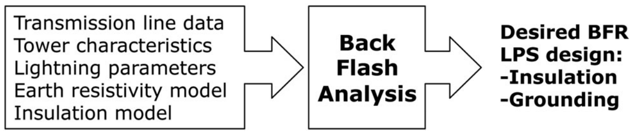

FIGURE 1. Basic backflashover analysis chain.

measures, i.e. the lightning protection systems required to guarantee a desirable back-flashover outage rate.

Nowadays, the overestimation of transmission infrastructure cost entails regulatory implications. Average costs above incremental costs required to assure a given reliability level could be considered inefficient under modern Performance Based-Regulation (PBR) schemes of transmission utilities [7]. This economic view is gaining relevance in recent times. According to [8] designing protection measures (LPS) against lightning effects is in dire need of scientific guidance since it is not only necessary to minimize line outages but also to reduce the mitigation costs of lightning effects to the minimum. Marginal costing could be relevant to detect unwarranted expenses in lightning protection. In many cases, transmission utilities tend to adopt standard solutions for the entire line with some exceptions in zones with high resistivity or lightning activity. On account of this, reliability of transmission lines due to BF phenomena can be certainly improved at minimum investment cost [9].

In this paper, an economic optimization model relating back flashover phenomena to expenditure in lightning protection systems is described. Thus, the balance between use of supplementary grounding and additional insulation for a given reliability level is achieved using the concept of the incremental cost associated with tower grounding and tower insulation [6]. The proposed technique accounts the dynamic behavior of the tower footings being suitable to be applied using any existing BFR evaluation methodology.

It is important to highlight that this paper does not propose any new method to evaluate back-flashover rates for design purposes. The method can use any existing BFR evaluation method to evaluate incremental costs of the lightning protection systems. For the sake of simplicity, a practical study case based in the well-known Electric Power Research Institute (EPRI) method is presented to illustrate the method. The effect of the quality of existing BFR methods in the results are out of scope of the paper and matter of further research.

The paper is organized in the following manner. Section 2 is devoted to review the state of art. Section 3 describes existing methods to compute outage rates due to back-flashover. Section 4 presents the proposed optimization model. Results

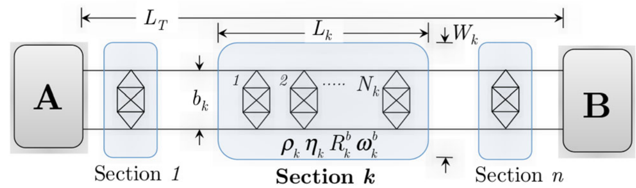

FIGURE 2. Transmission line scheme.

obtained from a case study are discussed in Section 5. Conclusions are drawn in Section 6.

2. BACKGROUND

2.1. Lightning Protection System Optimization Methods

Contributions on optimization methods for lightning protection design are certainly scarce. The seminal Grossner & Hileman optimization model [6] was based upon an incremental/marginal costing approach but with immature methods to evaluate the BFR and disregarding the effect of non-linear grounding impedances. Later, some few contributions were introduced in order to select the best grounding/insulation scheme for an appropriate outage rate by using mathematical programing. In [10–12], tower grounding optimization procedures were presented and solved using linear-integer programing. Latter contributions are based upon a discrete decision variables and they do not include insulation specification. An approach based on the Simplex Nelder-Mead optimization method is presented in [13]. More recently, mixed-integer single and multiple objective optimization approaches were also discussed to solve the insulation and grounding specification problem [14, 15].

In all previous methods, there is no specific model to link steady-state and dynamic grounding impedances. This is crucial since the costing model relies on a verifiable steady state grounding impedance and the assessment of the outage rates will depend on an adequate model of impulse impedances. Currently, exhaustive research is devoted to ground impulse modeling. The dynamic model used in [5, 16, 17] relating impulse and steady state is now widely accepted and used for standardized BFR calculations. Moreover, these detailed grounding models as well as BFR calculation procedures allow us to update the concept of incremental cost of a prescribed back flashover rate (BFR) [6].

In this paper, in order to fulfill the research gap, the relationship between the back flashover rate and the cost of the mitigation measures, i.e. the lightning protection systems (grounding and insulation) is determined expressly taking in account the nonlinear effect of grounding impedances. The proposed method can be applied by using any existing methodology to evaluate the back-flashover rates. In this case, the flashover rates are determined with the EPRI's method [18]. This method was selected due to its simplicity being useful to illustrate the proposal since no simulation software is needed. Accordingly, the interested reader can replicate the results in a straightforward manner. However, on account of the EPRI method limitations any other BFR method can be used instead. In the following, a review of existing BFR methods is provided.

2.2. Review of Existing BFR Methods

Literature is exhaustive about procedures to evaluate the BFR and certain maturity has been reached in this area. Early analytic approaches can be found in [1, 2, 19]. An overview of more recent analytic methods can be seen in [20, 21]. A number of electromagnetic-based models have been also discussed from simulations performed in the EMTP platform [3, 22, 23]. Some aspects of overhead transmission lines lightning performance estimation in engineering practice can be revisited in [24]. Many utilities are still using step-by-step procedures developed by EPRI [18], Institute of Electrical and Electronics Engineers (IEEE) [25–27] and the International Council on Large Electric Systems (CIGRE) [28]. There are some available software to calculate BFR as IEEE-Flash and LPTL [29, 30].

Despite the step-by-step Anderson/EPRI [18] two point method has limitations, it is still widely used in the industry [4]. An insightful comparison between CIGRE and IEEE methods is provided by [31]. A discussion about the quality —under and over estimation– of simplified methods with respect to advanced electromagnetic models can be found in [32]. All existing procedures for BFR calculation, either those based upon simplification formulas or advanced computational simulations, are based on the same input data structure shown in Figure 1.

In this paper, for clarity of presentation, only grounding and insulation specifications are considered as previously done in the original Hileman's method [6]. Further research is encouraged to include additional design considerations in the optimization approach. Furthermore, without loss of generality, economical aspects of the BFR calculation are illustrated using the EPRI simplified step-by-step procedure whose input data scheme is shown in Figure 1. Interested readers are able to replicate the results without resorting to advanced computer modeling programs.

It must be stressed that this paper does not discuss the quality of existing BFR evaluation methods. For the sake of simplicity, we use the EPRI method to illustrate the optimization procedure, however any other more detailed BFR method can be used for this purpose.

3. THE BFR CALCULATION MODEL

Long transmission lines traverse vast areas with non-homogeneous earth resistivity and diverse lightning activity. Therefore, lightning performance due back-flashover phenomena is strongly dependent on individual performance of each tower, rather than by the performance of a group of towers with averaged on-site resistivity. For this reason, the reliability associated with the back-flashover effect should be evaluated using a composite model. According to IEEE Std. 1243-1997 (Eq. (20)) [5], the overall back-flashover rate of the line T could be computed separately for each tower or section with similar resistivity and flash density characteristics. The results may then be combined in specific outage rates per section T_k in order to determine the composite performance by the following equation:

$$T = \sum_{k=1}^{n} T_k \frac{L_k}{L_T} \tag{1}$$

where T_k is the back-flashover rate at section k in outages/100 km-yr. Line outages due to BF are also denoted # or tripouts along the text, tables and figures.

To apply Eq. (1) let us define a transmission line of length L_T (km) with n sections of L_k (km) each as shown in Figure 2. Each line section k encompasses a number of towers N_k with similar geometry sketch, flash density η_k (flashes cloud-to-ground per km^2-yr), earth apparent resistivity ρ_k (Ωm), tower footing impedance R_k^b (Ω) and number of insulators, i.e. the base insulation length specified at power-frequency, contamination and switching regime, w_k^b (m). The resistivity profile of the line is in general a known parameter and lightning parameters associated with each section or tower are now available from detectors [33, 34]. The total number of towers of the line is $N_T = \sum_{k=1}^{n} N_k$. In the most specific case, if only one tower is selected at each section, then the number of sections equals the number of towers of the line ($N_T = n$). Thus, according to Eq. (1) procedures for back-flashover rate calculation should be applied separately for each section or tower.

The back-flashover rate T_k is the product of two components: the number of annual direct strokes to the line per 100 km μ_k and the cumulative probability P_{ck} of actual

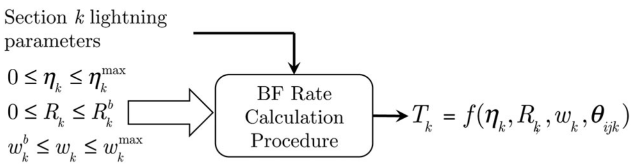

FIGURE 3. Backflashover rate function parametrization.

current stroke I_k exceeds a critical lightning current amplitude I_{ck} (kA) enough to provoke insulation flashover: [17, 18].

$$T_k = 0.6\mu_k P_{ck}[I_k > I_{ck}] \tag{2}$$

In this paper, it is assumed that the transmission line is perfect shielded and therefore the outage rate due to direct impact in phases is zero.

For each section k, Eq. (3) reflects the number of flashes that might strike the line (the tower, any phase conductor or any overhead ground wire) per each 100 km per year μ_k depends on the flash density η_k (flashes/km^2/yr) and the shadow length W_k (m) depicted in Figure 2.

$$\mu_k = 0.1\eta_k W_k \tag{3}$$

The value of W_k can take different values depending on the BFR procedure used. For instance, according to [18] the shadow is $W_k = B_k + 4h_k^{1.09}$. More appropriately [17] uses $W_k = 28h_k^{0.6} + B_k$. In both formulas h_k is the height of the groundwires (m) and B_k is the horizontal distance between the groundwires at section k in (m). The Electro Geometric Model (EGM) can be also used to determine the flash frequency μ_k at section k [17] by means of the following expression:

$$\mu_k = 0.1\eta_k \left[2\int_0^{I_{mk}} f(I)S(I)dI + 2\int_{I_{mk}}^{\infty} f(I)S'(I)dI + B_k \right] \tag{4}$$

where for each section k, I_{mk} is the maximum shielding failure current (kA), $f(I)$ is the log-normal probability density function of the lightning current amplitude distribution, $S(I)$ is the maximum and minimum exposure distances (m) for the shield wires. As indicated above, we consider here that the line is perfectly shielded.

The second component of Eq. (2) is the log-normal probability cumulative function of a lightning current amplitude distribution I_k that will exceed the critical current I_{ck} required to produce de BF phenomenon at section k:

$$P_{ck}[I_k > I_{ck}] = \int_{I_{ck}}^{\infty} f(I)dI \approx \frac{1}{(1 + (\frac{I_{ck}}{31})^{2.6}} \tag{5}$$

The value of the critical current I_{ck} relies on the adopted model for insulation breakdown, the surge impedances of the shield wires, the tower and the grounding system of each tower R_k. According to [5, 18], the critical flashover voltage (kV) can be expressed as $CFO_k = 400w_k + 710w_k/t^{0.75}$ where w_k is the insulation dry arc length (m) and t is the time flashover (µs). If the amplitude I and the crest times t_f of lightning strokes are considered, conditional probabilities should be taken into account and the BFR can be determined in its more general way as [17]:

$$T_k = 0.6\mu_k \int_0^{\infty}\int_{I_{ck}}^{\infty} f(I_k|t_f)f(t_f)dI_k dt_f \tag{6}$$

The BFR value T_k can be determined at each section k using any referenced procedure or program, either by Eq. (2) or by Eq. (6). The relationship between T_k and a number of variables of interest such as η_k, ρ_k, w_k, R_k can be parameterized from the set of solutions. In this instance, the control variables of the optimization problem are the impulse grounding impedances R_k and the insulation length w_k. Environmental variable η_k is known in each section and not subject of optimization. In this process, rest of variables used to compute T_k remains fixed. Then the parametric BFR function is given by the following equation:

$$\begin{aligned} &T_k = f(\eta_k, w_k, R_k, \Theta_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{m}\Theta_{ij}\eta_k w_k^i R_k^j \\ &0 \le \eta_k \le \eta_k^{max} \\ &w_k^b \le w_k \le w_k^{max} \\ &0 \le R_k \le R_k^0 \end{aligned} \tag{7}$$

where, m is the polynomial order and factors Θ_{ij}, $\forall i,j = 1, \ldots, m$, are the fitting parameters. The parametrization of T_k as a function of a number of variables, in this case η_k,R_k,w_k, can be carried out using any BFR calculation procedure as illustrated in Figure 3. R_k is the impulse grounding resistance at section k.

When the lightning performance function $T_k(\eta_k$, R_k, w_k, $\Theta_{ij})$ is defined from the parameterization process, the critical lightning current I_{ck} (kA) can be also derived from Eq. (5) for a given point (η_k,R_k,w_k) as:

$$I_{ck}(\eta_k, R_k, w_k) = 31\sqrt{\frac{T_k(\eta_k, R_k, w_k)}{0.6\mu_k}5/13} \tag{8}$$

where μ_k is the annual frequency of flashes cloud-to-ground per 100 km intercepted by section k and T_k is lightning performance function. The shield wires of a transmission line passing above the earth can be said to throw an electric shadow on the land beneath.

At this point, as the lightning performance function $T_k(\eta_k$,R_k,$w_k)$ is smooth and continuous, its first derivative

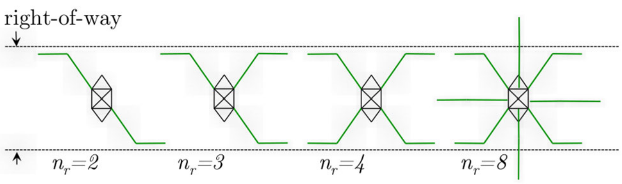

FIGURE 4. Supplemental grounding electrodes with n_r radial wires.

are also continuous over the range under consideration in the parameterization process. Thus, the incremental flashover rate of the transmission line $\partial T_k/\partial R_k$ with respect to the impulse grounding resistance R_k and the insulation level w_k can be written as, respectively:

$$\frac{\partial T_k}{\partial R_k} = \sum_{i=0}^{m}\sum_{j=1}^{m} j\Theta_{ij}\eta_k w_k^i R_k^{j-1} \tag{9}$$

$$\frac{\partial T_k}{\partial w_k} = \sum_{i=1}^{m}\sum_{j=0}^{m} i\Theta_{ij}\eta_k w_k^{i-1} R_k^{j} \tag{10}$$

Equations (9) and (10) will be necessary later to solve the non linear optimization problem through the Lagrangian function.

4. ECONOMICS OF MITIGATION OF THE BACK-FLASHOVER

The design of a transmission line should take into account some predefined basic factors such as the operating voltage level, number of circuits, the route of the transmission line and the desired current capacity of the line. In this context, the designer may choose structural details, the geometry of the structure, the structure height, the exact placement of the shield wires in order to avoid direct strokes upon the phase conductors. The amount and type of insulation generally is also determined by requirements set forth by power frequency voltages, contamination and switching overvoltages.

Lightning activity and back-flashover phenomena has a critical influence on the reliability of the system. A direct stroke terminates on a double-circuit transmission line may produce a N-2 contingency with important economic consequences on power system operation. In this instance, additional redundancy and investments are required to ensure system security. Therefore, despite the construction costs of the line are significant higher than the mitigation measures required to reduce the back-flash rate, the designer should balance the costs of higher insulation levels and improved grounding against the benefits of improved reliability. The tradeoff between adding insulation and improve grounding should be tackled from economic viewpoint. In the following, grounding and insulation costs are defined.

4.1. Grounding Costs

Economic models required to evaluate grounding expenditure in the context of the BF cost-effective design are twofold. On one hand, steady-state grounding impedances determined using standardized formulas [18] or specialized simulation programs [35]. On the other hand, the BF analysis also requires a detailed dynamic model for grounding electrode resistances [17, 27]. In the following, it is developed a general model to determine the incremental cost functions of supplemental grounding electrode expenditure in transmission lines.

4.1.1. Steady-State Grounding Cost Model. Each section k should be provided of a grounding means able to divert any lightning current and short-circuit currents into earth. Tower footings of the transmission tower are usually built from reinforced concrete, and these footings can be regarded as an effective grounding mechanism. As steel towers require large footings due to mechanical reasons, suitable value grounding resistances can be obtained in areas of low soil resistivity. In the instance of high soil resistivity, self-impedance of tower footing is significantly high and supplemental ground electrodes should be added to the tower footings. On account of this, let us define R_k^b and R_k^e as the power-frequency (steady-state) self-impedance and supplemental electrode resistance of section k, respectively. Thus, the power-frequency grounding resistance R_k^0 can be defined as a function of natural and artificial resistances, i.e. $R_k^0 = f(R_k^b, R_k^e)$. Thus, when $R_k^b \gg R_k^e$ the effective tower grounding R_k^0 equals the supplemental electrode resistance R_k^e.

There are different types of supplemental grounding electrodes with different geometries and structures suitable to be installed in transmission towers [8]. In some cases, if multilayer earth resistivity patterns are available numerical methods can be applied to assess impedance values of grounding footings and supplemental electrodes. In this paper, for sake of simplicity, a single, two, three, four and eight-star horizontal radial wires of length ℓ_k (m) each is considered to illustrate the optimization methodology.

In Figure 4 n_r wires horizontal arrangements electrodes are depicted. Legs of each electrode have the same length ℓ_k. Usually a four-wire electrode ($n_r = 4$) is used. The four-wire electrode is not formed as a star. Rather, the wires are

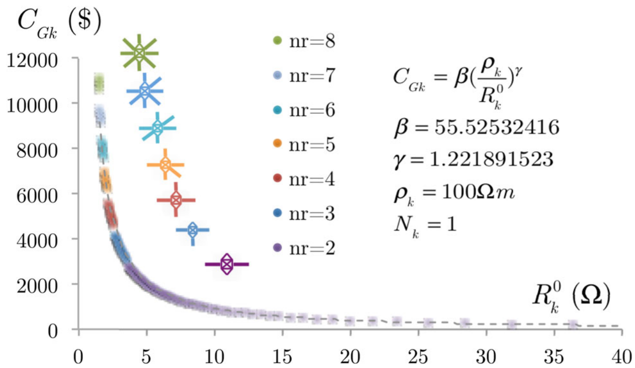

FIGURE 5. Electrode cost C_{Gk} vs. steady state resistance R_k^0 (n_r=1, … ,8 horizontal radial wires).

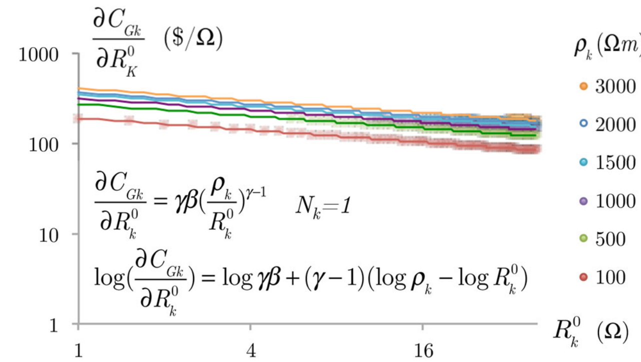

FIGURE 6. Steady state incremental grounding cost per resistivity level.

routed parallel to and close to edge of the right-of way, with short radial sections connecting these to the footings. The use of $n_r = 8$ radial counterpoise wires for power line towers is not usual. This is because the counterpoise must be within the right-of way. In some special cases, counterpoises can run outside the right-of way. So, the power-frequency resistance of the supplemental electrode with at least 8-arms is computed according to the Sunde's equations [19]:

$$R_k^e = \frac{1}{n_r}\frac{\rho_k}{\pi \ell_k}\left[\log_e \frac{2\ell_k}{r_g} - 1 + \sum_{k=1}^{n_r-1} \log_e \frac{1+\sin\left(\frac{\pi k}{n_r}\right)}{\sin\left(\frac{\pi k}{n_r}\right)}\right] \tag{11}$$

where ρ_k is the single-layer earth resistivity (Ωm) at section k, and n_r is the number of horizontal radial wires. The Sunde's equations are appropriate to model the power-frequency resistance of horizontal electrodes. However, the use of vertical electrodes passing through two or more earth layers will require a more precise cost model. This model is out of the scope of the paper. In addition in this paper we assume a natural tower footing resistance R_k^b higher than supplemental electrode resistance R_k^e, so the corresponding power-frequency grounding resistance at section k is given by:

$$R_k^0 = R_k^b \,||\, R_k^e \approx R_k^e \tag{12}$$

The expenditure required for installing supplemental electrodes is a linear function of the number of radial wires connected to the tower, the earth conductor length, exothermic connections, excavation and backfilling. A per-tower cost C_{G0k} just to get equipment to the site should be considered. Contractors might settle different price points for soft soil and hard soil due to the labor that it will take. Some soils are different to excavate than others. Excavation tasks can be manual or mechanical. Manual excavation technique is suitable for soft soils. Mechanical excavation technique is suitable for hard soils and subject to access of the required machinery at the working site. The total installation cost function of a supplemental electrode at section k is then expressed as

$$C_{Gk} = C_{G0k} + n_r N_k \ell_k (k_e s_g d + k_c) \tag{13}$$

where C_{G0k} is a fixed grounding cost in US\$, N_k is the number of towers in section k, n_r is the number of radial wires with length ℓ_k, the trench width (m) s_g and depth (m) d. Entry k_e is the cost associated with manual or mechanical excavation, backfilling and compaction (US\$per m^3). Trenches for laying the counterpoise wires are never dug by hand unless an excavator cannot be brought to site. The width of the trench is usually the width of the bucket of an excavator (about 0.45 m). In general, these costs already include contractor profit. Finally, k_c is the ground conductor cost (US\$per m). This cost should include procurement, transport, installation in trench and connection to tower structure.

Notice that in Eqs. (12) and (13) increasing resistivity and increasing length lead to lower power-frequency grounding resistances and therefore higher construction costs. Combining Eqs. (12) and (13), the total cost of concentrated grounding electrodes (when the counterpoise length ℓ_k ranges from 0 to 30 meters) can be parameterized via Ordinary Least Squares as a function of the resistance R_k^0 (steady-state), resistivity ρ_k as:

$$\begin{aligned} &C_{Gk} = f(R_k^0) = \beta\left(\frac{\rho_k}{R_k^0(R_k, w_k)}\right)^\gamma \\ &0 \le R_k^0 \le R_k^b \\ &\gamma > 1 \end{aligned} \tag{14}$$

where β, γ are fitting parameters, R_k^0 is the power-frequency tower grounding resistance (including supplemental electrodes) and R_k^b the tower grounding self-impedance (base resistance). Later, it will be shown that R_k^0 should be written as function of its dynamic resistance R_k and the insulation level of the line w_k.

Practical application of Eq. (14) is carried out here considering eight electrode types (n_r=2, … ,8.), a trench width of s_g=0.45m, a trench depth of $d = 1$m, installation cost of 1/0 soft drawn bare copper wire k_e=10 US$/m and excavation/backfill/compaction cost of k_c=50 US$/m^3, the parameters were adjusted with R^2=0.99.

As indicated above, fitting parameters β and γ should be determined using only for concentrated electrodes whose counterpoises do not surpass 30 m length. However, without loss of generality, fitting parameters can be also parameterized from more a complex electrode geometry.

Figure 5 shows the electrode cost as a function of the steady-state grounding resistance for different electrode types, from n_r=2 to n_r=8. The grounding cost function given in Eq. (14) is depicted only for a soil resistivity of ρ_k=100 Ωm to illustrate the costs of grounding when the soil resistivity ρ_k is low. So, in this case simple 2-arm electrodes yield low resistances and optimization is not required. However, if Eq. (14) is plot using high a resistivity value ρ_k (not included due space constraints) the curve moves to the upper right hand of the graph and more counterpoises are needed to get a low grounding resistance. In this case the optimization procedure indicates the adequate number of counterpoises required by each tower.

At this point, it is important to clarify whether grounding costs are relevant or not when compared with the cost of the entire line. General considerations about the capital cost of the lightning protection with respect to the total tower costs can be found in [4]. The foregoing reference is only focused in the cost of the shielding system disregarding the costs of supplemental electrodes to control the BF phenomenon. For example, a typical tension 345-kV double circuit line would be a weight of 8420 kg where 10% corresponds to two shield wires (860 kg). Assuming a cost of material of 12 US$/kg [36], the construction cost of each tower is estimated about US$100,000 and the cost of the shielding system are almost US$10000 (10%). However, consider now that each tower is provided with an eight wire radial grounding electrode (n_r=8x30m). According to Figure 5, the expenditure in grounding electrodes could reach US$11200, almost 12% of the entire tower cost. As a result, we must emphasize that cost of supplemental grounding must not be overlooked in the transmission line design process.

The incremental cost of the steady-state grounding resistance $\partial C_{Gk}/\partial R_k^0$ can be derived of 14 as:

$$\frac{\partial C_{Gk}}{\partial R_k^0} = \frac{-\gamma\beta\rho_k^\gamma}{(R_k^0)^{\gamma+1}} \tag{15}$$

Note that fixed costs C_{G0k} are not included in the incremental cost formula since its derivative is zero.

In Figure 6 the incremental cost of the steady state grounding scheme is depicted for different soil resistivity values. For an electrode of steady state resistance 0-40 ohm, installed in a 100 Ωm resistivity, the incremental cost ranges 46-160 \$/Ω. For a 3000 Ωm soil resistivity, the incremental cost ranges 145-500 \$/Ω.

The concept of incremental cost of grounding resistance was originally suggested by [6] in 1967. However, in that paper no details were given about the dependence of the cost with respect to the impulse resistance and insulator length. In this paper, an exact formulation is developed to fulfill the research gap. Next section elaborates some considerations about how link grounding impulse resistances with the proposed grounding cost model.

4.1.2. Considerations about Grounding Impulse Resistances. An appropriate dynamic model for the tower grounding footing is crucial to assess the overvoltages across the insulation strings when the lightning stroke terminates on the tower or the shield wires. Hence, the influence of the surge footing resistance on the tower top voltage is dependent on the lightning current magnitude and the crest time. In general, if large counterpoises of effective length $\ell_k > 30$ m are installed, the expected behavior of the footing resistance is frequency-dependent and a more detailed model is needed. Conversely, if the grounding system is concentrated, i.e. deployed within a radius of 30 m with respect to the center of the tower [16], this dependence is circumscribed only to current amplitude of the lightning stroke [17]. Literature is vast on how to evaluate and measure impulse resistances. A review of research efforts in this topic can be found in [8, 37]

According to [17, 27], the dynamic grounding impulse resistances associated with a concentrated supplemental electrode ($\ell_k < 30$m) is expressed as

$$R_k = \frac{R_k^0}{\sqrt{1 + \frac{I_{ck}(R_k, w_k)}{I_{gk}}}} \tag{16}$$

where

$$I_{gk} = \frac{\rho_k G_0}{2\pi (R_k^0)^2}$$

where $I_{ck}(R_k, w_k)$ (kA) is the current magnitude of the lightning flash.

For the sake of simplicity, in this paper, only concentrated electrodes are considered. However, the method allows the use any formula to determine impulse resistances for different electrode geometry and line parameters.

Notice from Eq. (16) that the critical current magnitude ($I_{ck}(R_k, w_k)$) also depends on both the impulse and steady-state resistances (R_k and R_k^0) and the insulation length (w_k) as indicated in Eq. (8). Parameter G_0 is earth ionization electric field gradient (kV/m) and ρ_k (Ωm) is the earth resistivity. Therefore, the grounding cost Eq. (14) can be rewritten as function of the impulse resistances R_k (Eq. (16)).

$$
\begin{aligned}
&C_{Gk}(R_k, w_k) = f(R_k) = N_k \beta \left(\frac{\rho_k}{R_k^0(R_k)}\right)^{\gamma} \\
&0 \leq R_k^0(R_k) \leq R_k^b \\
&\gamma > 1
\end{aligned}
\tag{17}
$$

resistance and insulation level. This cost function is continuous in both impulse grounding resistance R_k and insulator length w_k. This aspect was not considered in the formulation introduced by [6] where the incremental costs of grounding were determined only as function of steady state resistance R_k^0 (Eq. (18)). Note that Eq. (18) is derived from Eq. (16).

As a key contribution of this paper, it is introduced the concept of incremental costs of the grounding system [6] as a function of impulse grounding resistance R_k and insulator length w_k. So by calculating the corresponding derivatives of Eq. (17), Eqs. (19) and (20) express the incremental grounding costs $\partial C_{Gk}/\partial R_k$ and $\partial C_{Gk}/\partial w_k$, respectively as:

$$
\frac{\partial C_{Gk}}{\partial R_k} = -\frac{N_k G_0 \beta \gamma \rho_k R_k \left[\rho_k \Big/ \left(\frac{G_0 \rho_k R_k^2}{G_0 \rho_k - 62\pi R_k^2 (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{5}{13}}}\right)^{\frac{1}{2}}\right]^{\gamma} \left(13 G_0 \rho_k T_k^2 (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{8}{13}} - 93\pi\mu_k \frac{\partial T_k}{\partial R_k} R_k^3\right)}{13 T_k^2 \left[G_0 \rho_k - 62\pi R_k^2 (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{5}{13}}\right]^2 (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{8}{13}} \frac{G_0 \rho_k R_k^2}{G_0 \rho_k - 62\pi R_k^2 (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{5}{13}}}}
\tag{19}
$$

$$
\frac{\partial C_{Gk}}{\partial w_k} = \frac{93 G_0 \mu_k N_k R_k^4 \beta \gamma \rho_k \pi \frac{\partial T_k}{\partial R_k} \left[\rho_k \Big/ \left(\frac{G_0 R_k^2 \rho_k}{G_0 \rho_k - 62 R_k^2 \pi (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{5}{13}}}\right)^{\frac{1}{2}}\right]^{\gamma}}{13 T_k^2 \left[G_0 \rho_k - 62 R_k^2 \pi (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{5}{13}}\right]^2 (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{8}{13}} \left(\frac{G_0 R_k^2 \rho_k}{G_0 \rho_k - 62 R_k^2 \pi (\frac{3\mu_k - 5T_k}{5T_k})^{\frac{5}{13}}}\right)}
\tag{20}
$$

where

$$
R_k^0(R_k) = \sqrt{\frac{\rho_k G_0 R_k}{\rho_k G_0 - 2\pi I_{ck}(R_k, w_k)(R_k)^2}}
\tag{18}
$$

and

$$
I_{ck}(R_k, w_k) = 31\sqrt{\frac{T_k(R_k, w_k)}{0.6\mu_k} 5/13}
$$

where μ_k is the annual frequency of flashes cloud-to-ground per 100 km per year intercepted by section k.

The grounding cost function Eq. (17) give us a specific formulation of the relations among electrode cost, impulse

Note that Eqs. (19) and (20) depend on the specified black-flashover rate T_k given in Eq. (7) as well as its first derivatives $\partial T_k/\partial R_k$ and $\partial T_k/\partial w_k$ established in Eqs. (9) and (10), respectively.

4.2. Additional Insulation Costs

Adding more insulators in each phase of the transmission line to reduce the back-flashover rate implies to modify the geometry of the tower. Using a longer insulator string increases height of the tower and therefore more steel to withstand transversal external loadings, e.g. wind forces, and may also necessitate redesigning the whole tower and this may significantly increase its weight, and also require redesigning the footings. Also, this would increase the phase-to-phase spacing, and hence increase the electric and magnetic fields at ground level, as well as the induced voltages in parallel metallic objects (pipe lines, telecom lines, fences, etc.). Further, a wider right-of-way would be needed, thus requiring the acquisition of more land and the cutting of more trees. In view of the above, an utility usually have only two options: a basic length of the insulator string that can withstand the applicable pollution level, and one alternative design which is longer than the above by two or three disc insulators per string. The optimization issue is how many insulators can be added to each tower according to additional costs.

As a result, higher towers will require more steel according the re-design needs. Depending on the characteristics of all project, designer should develop any formulation to account the cost changes due to tower redesign. Due to lack of space this paper does not provide a general method to measure the costs accrued due tower resize.

For the sake of simplicity, we use here the method recommended by EPRI to estimate additional steel needs when the height of a tower changes. In [4], a general formula is given to determine the tower weight at each section k, W_{Tk} (kg) needed to withstand external forces:

$$W_{Tk} = 1.9k_m\left(\sum_g h_{gk}t_g + \sum_p h_{pk}t_p\right) \tag{21}$$

where k_m is 0.014 for double circuit lattice Extra High Voltage (EHV) systems, 0.021 for single circuit horizontal EHV systems, h_g is the height of each groundwire, h_{pk} is the height of each phase conductor of the tower (bundle phases), and t_g and t_p are the individual transverse force (kg) located at attachment height h_{gk} and h_{pk}, respectively. Thus, when the insulator length in all phases varies, the additional weight of the tower ΔW_{Tk} should be recalculated accounting the displacements in all conductors.

Reference [4] provides an additional example of lightning line protection costing, including the peak power loss from induced currents and deriving a per-avoided-momentary-operation value.

In practice, the additional tower weight could be determined by installing a standard insulator disk ANSI 52-3 (0.146 m) in all phases. Assuming a tower steel cost k_t in US\$/kg and a factor σ_k^0 to account indirect costs, the incremental cost of insulation σ_k (in US\$/m) at section k is:

$$\sigma_k = \frac{k_t \Delta W_{Tk}}{0.146} + \sigma_k^0 \approx \frac{\partial C_{Ik}}{\partial w_k} \tag{22}$$

In general, according to [36] the cost of steel averages by 2011 k_t=6-12 US\$/kg for 345 kV suspension and tension towers, respectively. Variations on ΔW_{Tk} may range between 100 and 1000 kg and indirect strikes due to foundation redesign can reach $\sigma_k^0 = 7000$ US\$/m. So, the incremental costs for additional insulation would range between 3000 and 15000 US\$/m. These values strongly depend on the tower type: suspension, angle suspension or tension tower. The exact economical impact of insulation length change should be assessed using a specialized transmission design program. In this paper, for the sake of simplicity, we will assume an incremental cost of 7000 US\$/m due to insulation length change. The total insulation cost in US\$ for a section k with N_k towers is given by:

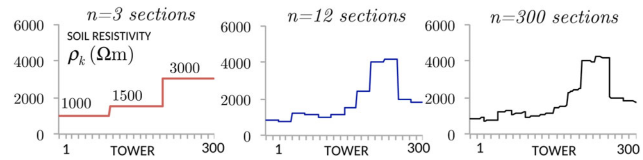

FIGURE 7. Resistivity profile of the line.

$$\begin{aligned} C_{Ik} &= \int_{w_k^b}^{w_k} N_k\sigma_k dw_k = N_k\sigma_k(w_k - w_k^b) \\ w_k^b &\le w_k \le w_k^{max} \end{aligned} \tag{23}$$

where, w_k is the additional insulator length in all phases at section k in m and w_k^b is the basic insulator length for switching surges/60 Hz at section k in m.

4.3. The Proposed Optimization Model

The optimization problem aims to find the set of impulse resistances $\mathcal{R} = \{R_1, ..., R_k, ..., R_n\}$ and the set of insulation dry lengths $\mathcal{W} = \{w_1, ..., w_k, ..., w_n\}$ such that minimize the total investment costs in supplementary grounding electrodes (C_G) and tower insulation (C_I) defined in Eqs. (17) and (23) subject to a prescribed global outage rate for the entire transmission line T^* equal to the composite model defined in Eq. (1):

$$\min_{\mathcal{W},\mathcal{R}} \quad \sum_{k=1}^{n} N_k\beta\left[\frac{\rho_k}{R_k^0}\right]^\gamma + N_k\sigma_k(w_k - w_k^b) \tag{24}$$

subject to:

$$T^* = \sum_{k=1}^{n} T_k\frac{L_k}{L_T} \tag{25}$$

$$w_k^b \le w_k \le w_k^{max} \quad \forall k = 1, ..., n \tag{26}$$

$$0 \le R_k \le R_k^b \quad \forall k = 1, ..., n \tag{27}$$

where:

$$R_k^0 = \sqrt{\frac{\rho_k G_0 R_k}{\rho_k G_0 - 62\pi\sqrt{\frac{T_k}{0.6\mu_k}}5/13R_k^2}} \tag{28}$$

i / j	1	2	3	4	5
1	0.765642556	0.008388985	0.010379005	−1.74E-05	−1.20E-06
2	−0.00801268	1.05E-05	−8.30E-05	−5.47E-09	1.24E-08
3	3.16E-05	−4.80E-07	2.70E-07	1.83E-10	−4.36E-11
4	−5.47E-08	1.59E-09	−4.12E-10	−3.11E-15	6.34E-14
5	3.48E-11	−1.45E-12	2.44E-13	−3.79E-16	−3.20E-17

TABLE 1. BFR fitting parameters Θ_{ij}.

Cost functions	Number of sections: n			
	1	3	12	300
C_G, (10^6 US$)	3.22	2.54	2.31	2.31
C_I, (10^6 US$)	0.00	0.25	0.37	0.37
C_T, (10^6 US$)	3.22	2.79	2.68	2.67
$\lambda = \frac{\partial C_T}{\partial T_k}, \left(\frac{\text{US\$}10^6}{\frac{\text{tripouts}}{100\text{km-yr}}}\right)$	−2.90	−2.53	−2.41	−2.41

TABLE 2. Summary of results for T^*=1.1.

$$T_k = \sum_{i=1}^{m}\sum_{j=1}^{m}\Theta_{ij}\eta_k w_k^i R_k^j \tag{29}$$

The objective function and its first and second derivatives are continuous over the range under consideration. As the lowest investment cost in grounding and insulation is desired for achieving the prescribed line performance, the problem can be interpreted as one of finding the minimum point on the overall cost function under the restriction that the sum of the individual section performances equal the desired line performance T^*.

In view of the mathematical properties which have been attributed to the grounding and insulation cost functions, this problem is suited to solution by means of Lagrangian multiplier theory [38] which makes possible a straightforward mathematical approach to solving for the extremums of a continuous function when restrictions are imposed. The Lagrangian function is given by:

$$\begin{aligned}\mathscr{L} = &\sum_{k=1}^{n} C_{Gk} + \sum_{k=1}^{n} C_{Ik} + \lambda\left(\sum_{k=1}^{n}\frac{T_k L_k}{L_T} - T^*\right)\\ &+\sum_{k=1}^{n}\mu_{1k}\left[w_k - w_k^{max}\right] + \sum_{k=1}^{n}\nu_{1k}\left[w_k^b - w_k\right]\\ &+\sum_{k=1}^{n}\mu_{2k}\left[R_k - R_k^0\right] + \sum_{k=1}^{n}\nu_{2k}\left[-R_k\right]\end{aligned} \tag{30}$$

For an extremum of the cost function to exist the partial derivatives of the following quantity must be zero. The first-order optimality conditions are given by Eqs. (31)–(38).

$$\frac{\partial\mathscr{L}}{\partial w_k} = \frac{\partial C_{Gk}}{\partial w_k} + \frac{\partial C_{Ik}}{\partial w_k} + \lambda\frac{L_k}{L_T}\frac{\partial T_k}{\partial w_k} + \mu_{1k} - \nu_{1k} = 0 \tag{31}$$

$$\frac{\partial\mathscr{L}}{\partial R_k} = \frac{\partial C_{Gk}}{\partial R_k} + \lambda\frac{L_k}{L_T}\frac{\partial T_k}{\partial R_k} + \mu_{2k} - \nu_{2k} = 0 \tag{32}$$

$$\frac{\partial\mathscr{L}}{\partial\lambda} = \sum_{k=1}^{n} T_k\frac{L_k}{L_T} - T^* = 0 \tag{33}$$

$$\frac{\partial\mathscr{L}}{\partial\mu_{1k}} = w_k - w_k^{max} \le 0, \mu_{1k}(w_k - w_k^{max}) = 0 \tag{34}$$

$$\frac{\partial\mathscr{L}}{\partial\nu_{1k}} = w_k^b - w_k \le 0, \nu_{1k}(w_k^b - w_k) = 0 \tag{35}$$

$$\frac{\partial\mathscr{L}}{\partial\mu_{2k}} = R_k - R_k^0 \le 0, \mu_{2k}(R_k - R_k^0) = 0 \tag{36}$$

$$\frac{\partial\mathscr{L}}{\partial\nu_{2k}} = -R_k \le 0, \nu_{2k}(-R_k) = 0 \tag{37}$$

$$\mu_{1k} \ge 0, \nu_{1k} \ge 0, \mu_{2k} \ge 0, \nu_{2k} \ge 0, \tag{38}$$

where λ is the Lagrangian multiplier or the incremental cost of the back-flashover rate in US$per outages/100 km-yr. Entries μ and ν are the Karush-Khun-Tucker (KKT) factors associated to binding constraints. Derivatives calculated in Eqs. (9), (10), (22), (19) and (20) should be included in Eqs. (31) and (32), respectively.

The system incremental cost can be also obtained from previously defined partial derivatives $\partial C_{Gk}/\partial R_k$ (Eq. (19)) and $\partial T_k/\partial R_k$ (Eq. (9)) as $\lambda = -\partial C_{Gk}/\partial T_k L_T/L_k$. The system stated in Eqs. (31)–(38) has $6n+1$ equations with unknowns $6n+1$, that are n impulse resistances ($R_1, \ldots, R_n$), n insulation lengths ($w_1, \ldots, w_n$), $4n$ KKT coefficients ($\mu_{11}, \ldots, \mu_{1n}$, $\mu_{21}, \ldots, \mu_{2n}$, $\nu_{11}, \ldots, \nu_{1n}$, $\nu_{21}, \ldots, \nu_{2n}$) plus the Lagrange multiplier λ. Any Newton-based method can be used to solve the system of non-linear equations.

5. CASE STUDY AND RESULTS DISCUSSION

The proposed optimization model was applied using as case-study the well-known EPRI's 345 kV transmission line ([18], Chapter 12). In this book, the Anderson's two-point method [18] was applied to determine the black-flashover rate over a vertical double-circuit tower subject to an average cloud-to-ground flash density $\eta = 3.6$ flashes/km^2-yr, a fixed insulation

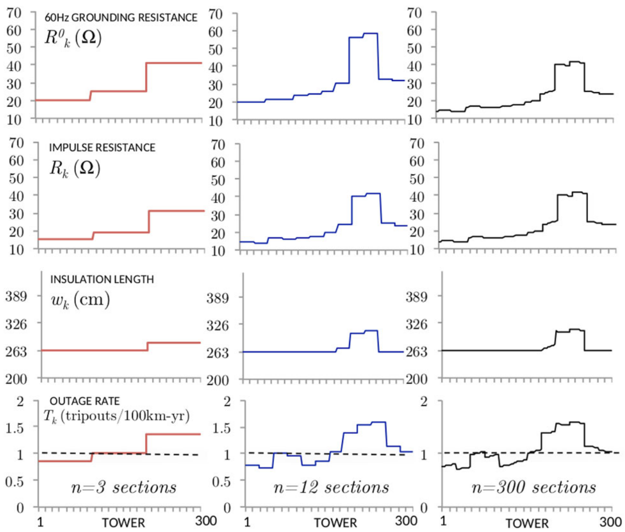

FIGURE 8. Optimal solution for $T^*=1.1$ outages/ 100 km-yr.

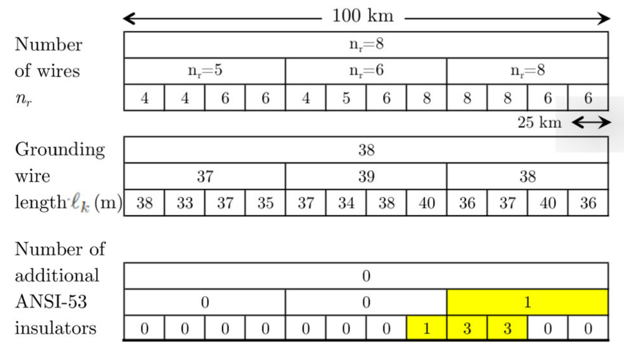

FIGURE 9. Electrode type and length selected for $T^*=1.1$ outages/100 km-yr.

length w^b=2.63m and the same impulse grounding resistance for all towers, $R = 20$ Ω. In this example the calculated BFR was 1.1 outages/100 km-yr.

Hereafter, the two-point procedure is parameterized according to Eq. (7) and the optimization model posed in Eqs. (31)–(38) is solved in order to determine the best grounding/insulation arrangement as well as the incremental costs of the BFR. The interested reader can replicate the results by running the parameterization and optimization programs in the following repository: https://github.com/pmdeoliveiradejesus/BackFlashoverOptimization (EPCS folder).

5.1. Data Setup and Modeling Premises

The proposed method is applied to a double circuit 345 kV line with length L_T=100km and N_T=300 towers with six phases and two groundwires. The prescribed outage rate T^* for the entire line is 1.1 outages/100 km-yr. For the sake of simplicity, it is assumed all towers have the same geometry and structure as indicated in [18]. The economic analysis was applied considering four different cases, when the line segmented in $n = 1$, 3, 12 and 300 sections. Therefore, the number of towers per section is $N_k = 300$, 100, 25 and 1. In all cases, the average values for resistivity and flash density are the same $\rho = \sum_{k=1}^{n} \rho_k = 1833$ Ωm and $\eta = \sum_{k=1}^{n} \eta_k = 3.6$ flashes/km^2-yr. The average values of η_k for 1-100, 101-200 and 201-300 are 1000, 1500 and 3000 Ωm, respectively. The resistivity profile for $n = 1$, 3, 12 and 300 sections is shown in Figure 7.

The basic insulation distance is the same in all towers $w^b_k = 2.63$m and w_{max}=4.45m. Steady state footing resistance in all towers is $R^b_k = 100$ Ω. Fitting factors Θ_{ij} of the BFR formula given in Eq. (7) were adjusted with $m = 5$ via least squares approach from results obtained of multiples runs of the two-point method [18] with R_k, w_k and η_k ranging 1-50 Ω, 2-6 meters and 0.1-8 flashes/km^2-yr. BFR fitting parameters for the test case are listed in Table 1. The Pearson coefficient of regression was 0.95.

The grounding cost model is given by Figure 6 corresponding to eight electrode types (n_r=2, … ,8.) of maximum length 30 m, a trench width of s_g=0.5m, a trench depth of $d = 1$m, installation cost of 1/0 soft drawn bare copper wire k_e=10 US\$/m and excavation/backfill/compaction cost of k_c=50 US\$/m^3, the adjusted cost parameters are $\beta = 55.52532416$ and $\gamma = 1.221891523$. Soil ionization electric field G_0 is set in 400 kV/m [28]. The insulation cost model is given by Eq. (22) where the incremental cost of adding insulation σ_k is set in 15000 US\$/m. Grounding and insulation costs were taken from the examples previously discussed in Section 4.2 and Section 4.1.

5.2. *Optimization Solution for a Fixed $T^*=1.1$*

The optimization problem stated in Eqs. (24)–(27) is solved by finding the roots of the non-linear system posed in Eqs. (31)–(38) with all fixed parameters defined in Section 5.1. Matlab's Fsolve script was used to get the solutions. The optimization problem has $6n + 1$ equations with $6n + 1$ unknowns, for the number of sections defined in each case $n = 1$, 3, 12, 300. The goal is to specify the set of n impulse grounding resistances $\mathcal{R} = \{R_1, ..., R_k, ..., R_n\}$ and the set of n insulation lengths $\mathcal{W} = \{w_1, ..., w_k, ..., w_n\}$, the $4n$ KKT coefficients and the incremental cost of the BFR (the Lagrange multiplier λ). Hence, four optimization problems with size 7, 22, 73 and 1801 state variables are defined and solved.

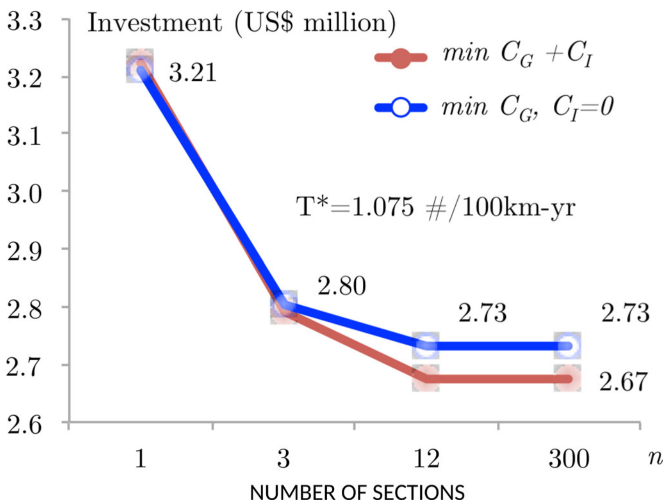

FIGURE 10. Optimal investment for T^{*}=1.1 outages/100 km-yr from $n = 1$ to $n = 300$ sections, with and without insulation optimization.

Results for $n = 1$,3, 12 and 300 are summarized in Table 2.

For $n = 1$ (uniform resistivity and uniform flash density in all sections of the line), the best solution corresponds to a supplemental electrode of eight wires n_r=8, with $\ell = 28.38$m each in all towers of the line. The optimal cost is \$3.22 million to assure a BFR of 1.1. No additional insulation is required. This means that any expenditure above this cost is ineffective to hold a the prescribed BFR.

Technical results show that uniform steady-state and dynamic resistances required are $R_1^0 = 24.72$ ohms, R_1=20 ohms (impulse), respectively. As indicated above, no additional insulation is needed, then insulator length remains unchanged $w_1 = w_1^b = 2.63$m.

Incremental grounding and BFR costs defined in Eqs. (9), (10), (19) and (20) are $\partial C_{G1}/\partial R_1 = -892.34$ US\$/Ω, $\partial C_{G1}/\partial w_1 = -1668.86$ US\$/m, $\partial T_1/\partial R_1 = 0.0922$ outages/100 km/Ω, $\partial T_1/\partial w_1 = -1.30$ outages/100 km/m, respectively.

The incremental cost of the BFR of the transmission line is $\lambda = \frac{\partial C_T}{\partial T_k} = -2.90$ million US\$/outages/100 km-yr. This means that under uniform design (no variations of earth resistivity and flash density along a 100 km line) a reduction of 0.01 outages per 100 km-yr in the actual BFR (1.1) will imply an additional investment of US\$29000 in mitigation measures (LPS) in all towers, supplemental grounding electrodes in this case.

Simulations considering uniform resistivity and uniform flash densities are unrealistic. More precise solutions can be obtained considering non uniform resistivity and flash density patterns.

To the extent that the number of sections increases, the overall cost decreases from US\$2.53 million with $n = 3$ to US\$2.40 million with $n = 3$. Unlike case $n = 1$, insulation is optimized for $n = 3$ (US\$0.25 million), $n = 12$ (US\$0.37 million) and $n = 300$ (US\$0.37 million). Differences observed between the general case $n = 12$ and the very detailed case $n = 300$ are quite small. Then no added value is given by the exhaustive tower by tower analysis. As the resistivity and flash density are not homogeneous through the line, best cost solutions are obtained with high resolution analysis $n = 12$ and 300. However, in this case a very high resolution $n = 300$ does not provide added value since sections 1 to 12 (when $n = 12$) have low variability. Then the optimal overall investment cost is about US\$2.67 million for a desired outage rate of 1.1 outages/100 km-yr.

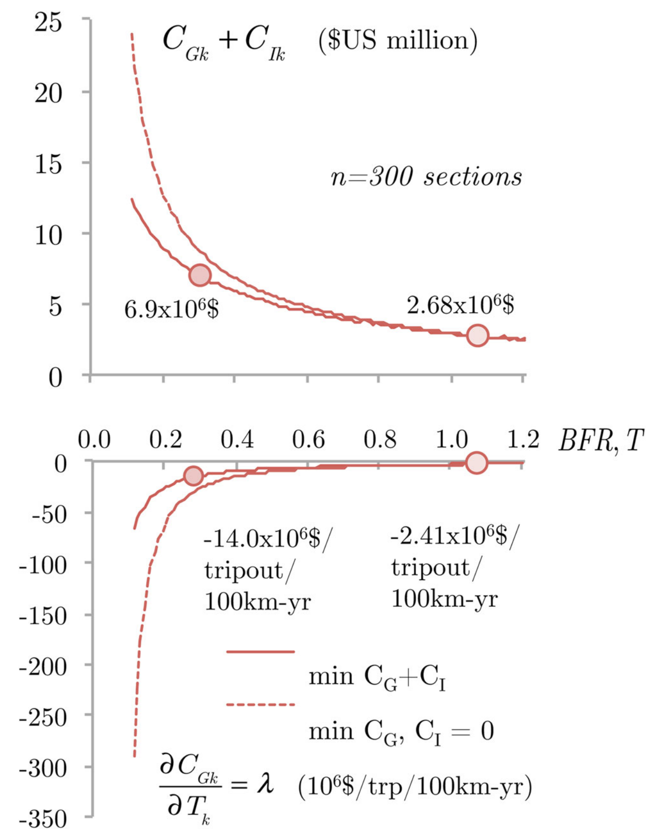

FIGURE 11. Total and incremental cost of T^{*} between 0.1 and 1.2 outages/100 km-yr ($n = 300$).

On account of insulation and grounding are optimized, the incremental cost of the BFR of the transmission line drops to −2.40 million US\$/outages/100 km-yr. This means that under no-uniform design (twelve sections along a 100 km line) a reduction of 0.01 outages per 100 km-yr in the actual BFR (1.1) will imply an additional investment of US\$24000 in mitigation measures (LPS) in all towers, supplemental grounding electrodes in this case.

Technical results R_k^0, R_k, w_k, and T_k for $n=3$, 12 and 300 are shown in Figure 8. Note f how high resistivity and high flash densities zones are associated with high outage rates. Conversely, lower outage rates are then seen in zones with lower resistivity and flash densities. This means that it is not necessary make additional expenses in grounding schemes in adverse zones in order to get lower outage rates.

The selected supplemental grounding electrodes types n_r and length ℓ_k are specified in Figure 9. Only sections 7, 8 and 9 (when $n=12$) are needing additional insulation specifications to reach the optimum US$2.67 million for a desired outage rate of 1.1.

If the optimization problem is solved with no insulation choice, i.e. $w_{max} = 2.63$, investment in supplemental grounding is about US$2.73 million as seen in Figure 10. As a result, the economical impact of adding insulation is low in this particular case: US$60000 (2.2%).

5.3. Total and Incremental Cost Curves

The core contribution of the paper is the determination of the relationship between the grounding and insulation costs and the desired reliability levels. To do so, it is introduced a new parameterization process for the BFR function as a function of grounding and insulation specifications. The incremental cost curves of the test case [18] are obtained from the solution of successive optimization formulations ranging the desired outage rate T^* from 0.1 to 1.2 outages/100 km-yr. Total cost results considering both insulation and grounding optimization as well as solutions with no insulation optimization are depicted in Figure 11.

For a desired reliability level of $T^*=1.1$, it is observed that the efficient total cost is about US$2.68 million (US$8900 per tower). Is this amount relevant in the context of the total tower cost? As indicated by [4], a typical suspension double-circuit tower 345 kV of 8400 kg would cost about $67000 (considering a steel cost of k_t=8 $/kg). This implies for this particular case that the efficient cost of the grounding/insulation system required to guarantee a prescribed BFR of 1.1 is almost 13% of the total tower cost. As seen in Figure 11, it seems that change of insulation does not provide a significant reduction upon the overall cost. In this case no changes in the basic insulation level (w_k=2.63m) are required.

However, if the reliability requirement is high, e.g. $T^*=0.3$ outages/100 km-yr, the choice of additional insulation becomes important in order to achieve a cost-effective solution. In this case, the best solution is about US$6.9 million (US$23000 per tower where 10% corresponds to insulation changes) representing a 34% of the total tower cost.

At this high reliability level ($T^*=0.3$), infrastructure costs are relevant since Lagrange multipliers (the incremental cost of the BFR) change from -2.40 million US$/outages/100 km-yr ($T^*=1.1$) to -14.0 million US$/outages/100 km-yr ($T^*=0.3$). This means that a small reduction of 1% in the BFR -from 0.3 to 0.27- will require an additional investment of US$140000 in mitigation measures (US$466 per tower).

5.4. Final Discussion and Further Research

The proposed procedure not only allows us to determine the efficient costs in lightning protection system devices to achieve a desired reliability level but also permits to identify the corresponding incremental costs at each investment level. The author would like to stand that expenditure on lightning protection system devices matters. Specification of standardized electrodes to mitigate back-flashover effects can lead to over-investments not allowable under an efficient regulation. The proposed approach demonstrates that there is an optimal LPS design for a given reliability level.

Results discussed in this paper are based upon the parametrization of the BFR function for the EPRI's two-point procedure. This procedure is still widely used in the power industry [18] and it can be easily replicated by the reader to validate the results of this work. However, the proposed procedure can use any other BFR evaluation method. Thus, further research must be carried out in order to integrate existing BFR estimating processes. Another drawback of this formulation is regarding the soil resistivity model. Two-layer resistivity models should be integrated in order to consider horizontal grounding electrodes. These aspects are out of scope of this proposal which is focused in the economic interpretation of the design process to control the effects of the back-flashover phenomenon.

6. CONCLUSIONS

This paper investigates the relationship between the cost associated to lightning protection systems and the back-flashover rate (BFR) in high voltage transmission lines. The core contribution of the paper is the determination of the relationship between the grounding and insulation costs and desired reliability levels of transmission lines. To do so, it is introduced a new parameterization process for the BFR function as a function of grounding and insulation specifications. The proposal allows to find the total and the incremental cost of the mitigation measures in grounding improvement and insulation adequacy in order to fulfill a prescribed back flashover rate (BFR).

The proposed approach is general since the cost structure of the mitigation measures can be adapted to specific economic conditions of each country. Furthermore, the optimization procedure can use any existing BFR evaluation method to evaluate the incremental costs of the lightning protection systems. The impact produced by considering other existing BFR methods in the calculation of optimal cost curves are out of scope of the paper and matter of current research.

From a practical study case based on the well-known Anderson-EPRI method we can obtain a cost-effective solution curves allowing utilities and regulators achieve better reliability indexes with strong incentives for infrastructure cost minimization.

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