4

Series Impedance of Overhead and Underground Lines

The determination of the series impedance for overhead and underground lines is a critical step before the analysis of a distribution feeder can begin. The series impedance of a single-phase, two-phase (V-phase), or three-phase distribution line consists of the resistance of the conductors and the self and mutual inductive reactances resulting from the magnetic fields surrounding the conductors. The resistance component for the conductors will typically come from a table of conductor data such as that found in Appendix A.

4.1 Series Impedance of Overhead Lines

The inductive reactance (self and mutual) component of the impedance is a function of the total magnetic fields surrounding a conductor. Figure 4.1 shows conductors 1 to n with the magnetic flux lines created by currents flowing in each of the conductors.

The currents in all conductors are assumed to be flowing out of the page. It is further assumed that the sum of the currents will add to zero. That is:

$$I_1 + I_2 + \cdots + I_i + \cdots + I_n = 0 \tag{4.1}$$

The total flux linking conductor i is given by:

$$\lambda_i = 2 \cdot 10^{-7} \cdot \left(I_1 \cdot \ln \frac{1}{D_{i1}} + I_2 \cdot \ln \frac{1}{D_{i2}} + \cdots + I_i \cdot \ln \frac{1}{GMR_i} + \cdots + I_n \cdot \ln \frac{1}{D_{in}} \right) \text{ W-T/m} \tag{4.2}$$

where

D_{in} = Distance between conductor i and conductor n (ft)

GMR_i = geometric mean radius of conductor i (ft)

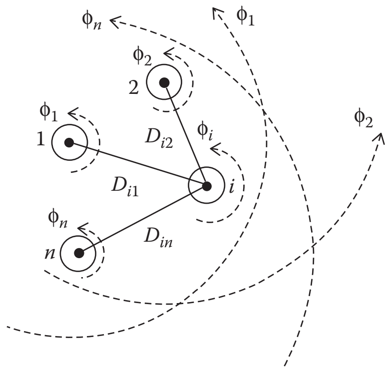

FIGURE 4.1
Magnetic fields.

The inductance of conductor i consists of the "self-inductance" of conductor i and the "mutual inductance" between conductor i and all of the other $n-1$ conductors. By definition:

$$\text{Self-inductance: } L_{ii} = \frac{\lambda_{ii}}{I_i} = 2 \cdot 10^{-7} \cdot \ln \frac{1}{GMR_i} \text{ H/m} \tag{4.3}$$

$$\text{Mutual inductance: } L_{in} = \frac{\lambda_{in}}{I_n} = 2 \cdot 10^{-7} \cdot \ln \frac{1}{D_{in}} \text{ H/m} \tag{4.4}$$

4.1.1 Transposed Three-Phase Lines

High-voltage transmission lines are usually assumed to be transposed (each phase occupies the same physical position on the structure for one-third of the length of the line). In addition to the assumption of transposition, it is assumed that the phases are equally loaded (balanced loading). With these two assumptions, it is possible to combine the "self" and "mutual" terms into one "phase" inductance [1].

$$\text{Phase inductance: } L_i = 2 \cdot 10^{-7} \cdot \ln \frac{D_{eq}}{GMR_i} \text{ H/m} \tag{4.5}$$

where

$$D_{eq} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}} \text{ ft} \tag{4.6}$$

D_{ab}, D_{bc}, and D_{ca} are the distances between phases.

Assuming a frequency of 60 Hz, the phase inductive reactance is given by:

$$\text{Phase reactance: } x_i = \omega \cdot L_i = 0.12134 \cdot \ln \frac{D_{eq}}{GMR_i} \ \Omega/\text{mile} \tag{4.7}$$

The series impedance per phase of a transposed three-phase line consisting of one conductor per phase is given by:

$$\text{Series impedance: } z_i = r_i + j \cdot 0.12134 \cdot \ln \frac{D_{eq}}{GMR_i} \Omega/\text{mile} \tag{4.8}$$

4.1.2 Untransposed Distribution Lines

Because distribution systems consist of single-phase, two-phase, and untransposed three-phase lines serving unbalanced loads, it is necessary to retain the identity of the self- and mutual impedance terms of the conductors in addition to taking into account the ground return path for the unbalanced currents. The resistance of the conductors is taken directly from a table of conductor data. Equations 4.3 and 4.4 are used to compute the self- and mutual inductive reactances of the conductors. The inductive reactance will be assumed to be at a frequency of 60 Hz, and the length of the conductor will be assumed to be 1 mile. With those assumptions, the self- and mutual impedances are given by:

$$\bar{z}_{ii} = r_i + j0.12134 \cdot \ln \frac{1}{GMR_i} \Omega/\text{mile} \tag{4.9}$$

$$\bar{z}_{ij} = j0.12134 \cdot \ln \frac{1}{D_{ij}} \Omega/\text{mile} \tag{4.10}$$

In 1926, John Carson published a paper in which he developed a set of equations for computing the self- and mutual impedances of lines, taking into account the return path of the current through the ground [2]. Carson's approach was to represent a line with the conductors connected to a source at one end and grounded at the remote end. Figure 4.2 illustrates a line

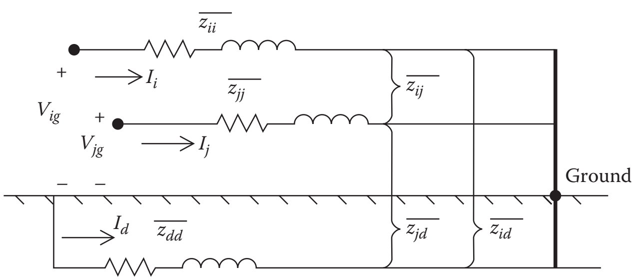

FIGURE 4.2
Two conductors with dirt return path.

consisting of two conductors (i and j) carrying currents (I_i and I_j) with the remote ends of the conductors tied to the ground. A fictitious "dirt" conductor carrying current I_d is used to represent the return path for the currents.

In Figure 4.2, Kirchhoff's voltage law (KVL) is used to write the equation for the voltage between conductor i and the ground.

$$V_{ig} = \bar{z}_{ii} \cdot I_i + \bar{z}_{ij} \cdot I_j + \bar{z}_{id} \cdot I_d - \left(\bar{z}_{dd} \cdot I_d + \bar{z}_{di} \cdot I_i + \bar{z}_{dj} \cdot I_j\right) \tag{4.11}$$

Collect terms in Equation 4.11:

$$V_{ig} = \left(\bar{z}_{ii} - \bar{z}_{di}\right) \cdot I_i + \left(\bar{z}_{ij} - \bar{z}_{dj}\right) \cdot I_j + \left(\bar{z}_{id} - \bar{z}_{dd}\right) \cdot \tag{4.12}$$

From Kirchhoff's Current Law:

$$\begin{aligned} I_i + I_j + I_d &= 0 \\ I_d &= -I_i - I_j \end{aligned} \tag{4.13}$$

Substitute Equation 4.13 into Equation 4.12 and collect terms:

$$V_{ig} = (\bar{z}_{ii} + \bar{z}_{dd} - \bar{z}_{di} - \bar{z}_{id}) \cdot I_i + (\bar{z}_{ij} + \bar{z}_{dd} - \bar{z}_{dj} - \bar{z}_{id}) \cdot I_j \tag{4.14}$$

Equation 4.14 is of the general form:

$$V_{ig} = \hat{z}_{ii} \cdot I_i + \hat{z}_{ij} \cdot I_j \tag{4.15}$$

where

$$\hat{z}_{ii} = \bar{z}_{ii} + \bar{z}_{dd} - \bar{z}_{di} - \bar{z}_{id} \tag{4.16}$$

$$\hat{z}_{ij} = \bar{z}_{ij} + \bar{z}_{dd} - \bar{z}_{dj} - \bar{z}_{id} \tag{4.17}$$

In Equations 4.16 and 4.17, the "hat" impedances are given by Equations 4.9 and 4.10. Note that in these two equations, the effect of the ground return path is being "folded" into what will now be referred to as the "primitive" self- and mutual impedances of the line. The "equivalent primitive circuit" is shown in Figure 4.3.

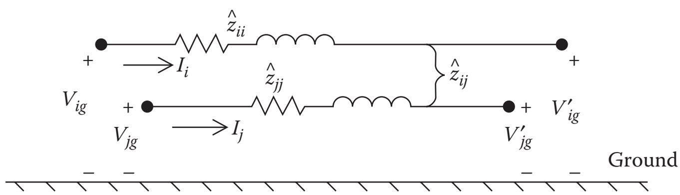

FIGURE 4.3
Equivalent primitive circuit.

Substituting Equations 4.9 and 4.10 of the "hat" impedances into Equations 4.16 and 4.17, the primitive self-impedance is given by:

$$\hat{z}_{ii} = r_i + jx_{ii} + r_d + jx_{dd} - jx_{dn} - jx_{nd}$$

$$\hat{z}_{ii} = r_d + r_i + j0.12134 \cdot \left(\ln\frac{1}{GMR_i} + \ln\frac{1}{GMR_d} - \ln\frac{1}{D_{id}} - \ln\frac{1}{D_{di}} \right) \quad (4.18)$$

$$\hat{z}_{ii} = r_d + r_i + j0.12134 \cdot \left(\ln\frac{1}{GMR_i} + \ln\frac{D_{id} \cdot D_{dj}}{GMR_d} \right)$$

In a similar manner, the primitive mutual impedance can be expanded:

$$\hat{z}_{ij} = jx_{ij} + r_d + jx_{dd} - jx_{dj} - jx_{id}$$

$$\hat{z}_{ij} = r_d + j0.12134 \cdot \left(\ln\frac{1}{D_{ij}} + \ln\frac{1}{\mathrm{GMR}_d} - \ln\frac{1}{D_{dj}} - \ln\frac{1}{D_{id}} \right) \quad (4.19)$$

$$\hat{z}_{ij} = r_d + j0.12134 \left(\ln\frac{1}{D_{ij}} + \ln\frac{D_{dj} \cdot D_{id}}{\mathrm{GMR}_d} \right)$$

The obvious problem in using Equations 4.18 and 4.19 is the fact that we do not know the values of the resistance of dirt (r_d), the geometric mean radius of dirt (GMR_d), and the distances from the conductors to dirt (D_{nd}, D_{dn}, D_{md}, D_{dm}). This is where John Carson's work bails us out.

4.1.3 Carson's Equations

Because a distribution feeder is inherently unbalanced, the most accurate analysis should not make any assumptions regarding the spacing between conductors, conductor sizes, and transposition. In Carson's 1926 paper, he developed a technique whereby the self- and mutual impedances for *ncond* overhead conductors can be determined. The equations can also be applied to underground cables. In 1926, this technique was not met with a lot of enthusiasm because of the tedious calculations that would have to be done on the slide rule and by hand. With the advent of the digital computer, Carson's equations have now become widely used.

In his paper, Carson assumes the earth as an infinite, uniform solid, with a flat uniform upper surface and a constant resistivity. Any "end effects" introduced at the neutral grounding points are not large at power frequencies, and therefore are neglected.

Carson made use of conductor images—that is, every conductor at a given distance above ground has an image conductor at the same distance below ground. This is illustrated in Figure 4.4.

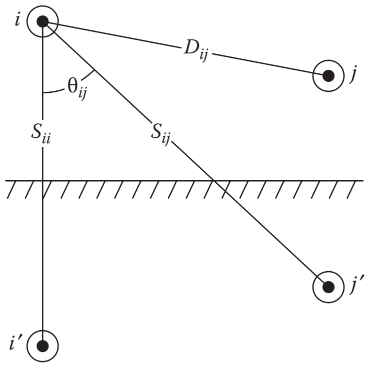

FIGURE 4.4
Conductors and images.

Referring to Figure 4.4, the original Carson equations are given in Equations 4.20 and 4.21.

Self-impedance:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j\left(X_i + 2\omega G \cdot \ln \frac{S_{ii}}{RD_i} + 4\omega Q_{ii} G \right) \Omega/\text{mile} \tag{4.20}$$

Mutual impedance:

$$\hat{z}_{ij} = 4\omega P_{ij} G + j\left(2\omega G \cdot \ln \frac{S_{ij}}{D_{ij}} + 4\omega Q_{ij} G \right) \Omega/\text{mile} \tag{4.21}$$

where
- $\hat{z}_{ii}$ = self-impedance of conductor i in Ω/mile
- $\hat{z}_{ij}$ = mutual impedance between conductors i and j in Ω/mile
- r_i = resistance of conductor i in Ω/mile
- $\omega = 2\pi f$ = system angular frequency in radians per second
- $G = 0.1609347 \times 10^{-3}$ Ω/mile
- RD_i = radius of conductor i in ft
- GMR_i = geometric mean radius of conductor i in ft
- f = system frequency in Hertz
- ρ = resistivity of earth in Ω-meters
- D_{ij} = distance between conductors i and j in ft (see Figure 4.4)
- S_{ij} = distance between conductor i and image j in ft (see Figure 4.4)
- θ_{ij} = angle between a pair of lines drawn from conductor i to its own image and to the image of conductor j (see Figure 4.4)

$$X_i = 2\omega G \cdot \ln \frac{RD_i}{GMR_i}\ \Omega/\text{mile} \tag{4.22}$$

$$P_{ij} = \frac{\pi}{8} - \frac{1}{3\sqrt{2}} k_{ij} \cos(\theta_{ij}) + \frac{k_{ij}^2}{16} \cos(2\theta_{ij}) \cdot \left(0.6728 + \ln \frac{2}{k_{ij}}\right) \tag{4.23}$$

$$Q_{ij} = -0.0386 + \frac{1}{2} \cdot \ln \frac{2}{k_{ij}} + \frac{1}{3\sqrt{2}} k_{ij} \cos(\theta_{ij}) \tag{4.24}$$

$$k_{ij} = 8.565 \cdot 10^{-4} \cdot S_{ij} \cdot \sqrt{\frac{f}{\rho}} \tag{4.25}$$

4.1.4 Modified Carson's Equations

Only two approximations are made in deriving the "Modified Carson Equations." These approximations involve the terms associated with P_{ij} and Q_{ij}. The approximations use only the first term of the variable P_{ij} and the first two terms of Q_{ij}.

$$P_{ij} = \frac{\pi}{8} \tag{4.26}$$

$$Q_{ij} = -0.03860 + \frac{1}{2} \ln \frac{2}{k_{ij}} \tag{4.27}$$

Substitute X_i (Equation 4.22) into Equation 4.20:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j\left(2\omega G \cdot \ln \frac{RD_i}{GMR_i} + 2\omega G \cdot \ln \frac{S_{ii}}{RD_i} + 4\omega Q_{ii} G\right) \tag{4.28}$$

Combine terms and simplify:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j2\omega G\left(\ln \frac{S_{ii}}{GMR_i} + \ln \frac{RD_i}{RD_i} + 2Q_{ii}\right) \tag{4.29}$$

Simplify Equation 4.21:

$$\hat{z}_{ij} = 4\omega P_{ij} G + j2\omega G\left(\ln \frac{S_{ij}}{D_{ij}} + 2Q_{ij}\right) \tag{4.30}$$

Substitute expressions for P (Equation 4.27) and ω $(2 \cdot \pi \cdot f)$:

$$\hat{z}_{ii} = r_i + \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ii}}{GMR_i} + 2Q_{ii}\right) \tag{4.31}$$

$$\hat{z}_{ij} = \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ij}}{D_{ij}} + 2Q_{ij}\right) \tag{4.32}$$

Substitute expression for k_{ij} (Equation 4.25) into the approximate expression for Q_{ij} (Equation 4.27):

$$Q_{ij} = -0.03860 + \frac{1}{2}\ln\left(\frac{2}{8.565 \cdot 10^{-4} \cdot S_{ij} \cdot \sqrt{\frac{f}{\rho}}}\right) \tag{4.33}$$

Expand:

$$Q_{ij} = -0.03860 + \frac{1}{2}\ln\left(\frac{2}{8.565 \cdot 10^{-4}}\right) + \frac{1}{2}\ln\frac{1}{S_{ij}} + \frac{1}{2}\ln\sqrt{\frac{\rho}{f}} \tag{4.34}$$

Equation 4.34 can be reduced to:

$$Q_{ij} = 3.8393 - \frac{1}{2}\ln S_{ij} + \frac{1}{4}\ln\frac{\rho}{f} \tag{4.35}$$

or:

$$2Q_{ij} = 2Q_{ij} = 7.6786 - \ln S_{ij} + \frac{1}{2}\ln\frac{\rho}{f} \tag{4.36}$$

Substitute Equation 4.36 into Equation 4.31 and simplify:

$$\begin{aligned} \hat{z}_{ii} &= r_i + \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ii}}{GMR_i} + 7.6786 - \ln S_{ii} + \frac{1}{2}\ln\frac{\rho}{f}\right) \\ \hat{z}_{ii} &= r_i + \pi^2 fG + 4\pi fG\left(\ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \end{aligned} \tag{4.37}$$

Substitute Equation 4.36 into Equation 4.32 and simplify:

$$\hat{z}_{ij} = \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ij}}{D_{ij}} + 7.6786 - \ln S_{ij} + \frac{1}{2}\ln\frac{\rho}{f}\right)$$
$$\hat{z}_{ij} = \pi^2 fG + j4\pi fG\left(\ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.38}$$

Substitute in the values of π and G:

$$\hat{z}_{ii} = r_i + 0.00158836\cdot f + j0.00202237\cdot f\left(\ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.39}$$

$$\hat{z}_{ij} = 0.00158836\cdot f + j0.00202237\cdot f\left(\ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.40}$$

It is now assumed:

f = frequency = 60 Hertz
ρ = earth resistivity = 100 Ω-m

Using these approximations and assumptions, the "Modified Carson's Equations" are:

$$\hat{z}_{ii} = r_i + 0.09530 + j0.12134\left(\ln\frac{1}{GMR_i} + 7.93402\right)\ \Omega/\text{mile} \tag{4.41}$$

$$\hat{z}_{ij} = 0.09530 + j0.12134\left(\ln\frac{1}{D_{ij}} + 7.93402\right)\ \Omega/\text{mile} \tag{4.42}$$

It will be recalled that Equations 4.18 and 4.19 could not be used because the resistance of dirt, the GMR_d, and the various distances from conductors to dirt were not known. A comparison of Equations 4.18 and 4.19 to Equations 4.41 and 4.42 demonstrates that the Modified Carson's Equations have defined the missing parameters. A comparison of the two sets of equations shows that:

$$r_d = 0.09530\ \Omega/\text{mile} \tag{4.43}$$

$$\ln\frac{D_{id}\cdot D_{di}}{GMR_d} = \ln\frac{D_{dj}\cdot D_{id}}{GMR_d} = 7.93402 \tag{4.44}$$

The "Modified Carson's Equations" will be used to compute the primitive self- and mutual impedances of overhead and underground lines.

4.1.5 Primitive Impedance Matrix for Overhead Lines

Equations 4.41 and 4.42 are used to compute the elements of an *ncond* × *ncond* "primitive impedance matrix." An overhead four-wire grounded wye distribution line segment will result in a 4 × 4 matrix. For an underground-grounded wye line segment consisting of three concentric neutral cables, the resulting matrix will be 6 × 6. The primitive impedance matrix for a three-phase line consisting of *m* neutrals will be of the form:

$$\left[\hat{z}_{primitive}\right]=\left[\begin{array}{ccc|ccc}\hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an1} & \hat{z}_{an2} & \hat{z}_{anm}\\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn1} & \hat{z}_{bn2} & \hat{z}_{bnm}\\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn1} & \hat{z}_{cn2} & \hat{z}_{cnm}\\ \hline \hat{z}_{n1a} & \hat{z}_{n1b} & \hat{z}_{n1c} & \hat{z}_{n1n1} & \hat{z}_{n1n2} & \hat{z}_{n1nm}\\ \hat{z}_{n2a} & \hat{z}_{n2b} & \hat{z}_{n2c} & \hat{z}_{n2n1} & \hat{z}_{n2n2} & \hat{z}_{n2nm}\\ \hat{z}_{nma} & \hat{z}_{nmb} & \hat{z}_{nmc} & \hat{z}_{nmn1} & \hat{z}_{nmn2} & \hat{z}_{nmnm}\end{array}\right] \tag{4.45}$$

In partitioned form, Equation 4.45 becomes:

$$\left[\hat{z}_{primitive}\right]=\begin{bmatrix}\left[\hat{z}_{ij}\right] & \left[\hat{z}_{in}\right]\\ \left[\hat{z}_{nj}\right] & \left[\hat{z}_{nn}\right]\end{bmatrix} \tag{4.46}$$

4.1.6 Phase Impedance Matrix for Overhead Lines

For most applications, the primitive impedance matrix needs to be reduced to a 3 × 3 "phase frame" matrix consisting of the self- and mutual equivalent impedances for the three phases. A four-wire grounded neutral line segment is shown in Figure 4.5.

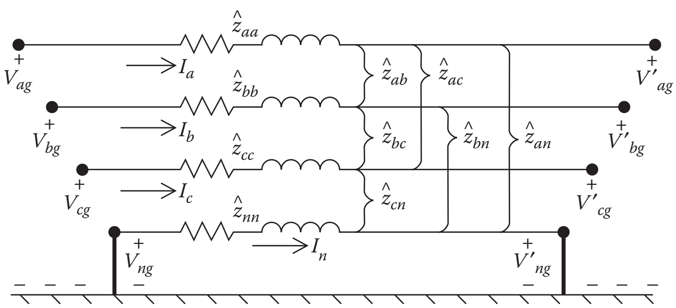

FIGURE 4.5
Four-wire grounded wye line segment.

One standard method of reduction is the "Kron" reduction [3]. It is assumed that the line has a multigrounded neutral (Figure 4.5). The Kron reduction method applies KVL to the circuit.

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \\ V_{ng} \end{bmatrix} = \begin{bmatrix} V'_{ag} \\ V'_{bg} \\ V'_{cg} \\ V'_{ng} \end{bmatrix} + \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn} \\ \hat{z}_{na} & \hat{z}_{nb} & \hat{z}_{nc} & \hat{z}_{nn} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \tag{4.47}$$

In partitioned form, Equation 4.47 becomes:

$$\begin{bmatrix} [V_{abc}] \\ [V_{ng}] \end{bmatrix} = \begin{bmatrix} [V'_{abc}] \\ [V'_{ng}] \end{bmatrix} + \begin{bmatrix} [\hat{z}_{ij}] & [\hat{z}_{in}] \\ [\hat{z}_{ng}] & [\hat{z}_{nn}] \end{bmatrix} \cdot \begin{bmatrix} [I_{abc}] \\ [I_n] \end{bmatrix} \tag{4.48}$$

Because the neutral is grounded, the voltages V_{ng} and V'_{ng} are equal to zero. Substituting those values into Equation 4.48 and expanding results in:

$$[V_{abc}] = [V'_{abc}] + [\hat{z}_{ij}] \cdot [I_{abc}] + [\hat{z}_{in}] \cdot [I_n] \tag{4.49}$$

$$[0] = [0] + [\hat{z}_{nj}] \cdot [I_{abc}] + [\hat{z}_{nn}] \cdot [I_n] \tag{4.50}$$

Solve Equation 4.50 for $[I_n]$:

$$[I_n] = -[\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \cdot [I_{abc}] \tag{4.51}$$

Note in Equation 4.51 that once the line currents have been computed, it is possible to determine the current flowing in the neutral conductor. Because this will be a useful concept later on, the "neutral transformation matrix" is defined as:

$$[t_n] = -[\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \tag{4.52}$$

Such that:

$$[I_n] = [t_n] \cdot [I_{abc}] \tag{4.53}$$

Substitute Equation 4.51 into Equation 4.49:

$$\begin{aligned} [V_{abc}] &= [V'_{abc}] + \left([\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]\right) \cdot [I_{abc}] \\ [V_{abc}] &= [V'_{abc}] + [z_{abc}] \cdot [I_{abc}] \end{aligned} \tag{4.54}$$

where

$$[z_{abc}] = [\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \tag{4.55}$$

Equation 4.55 is the final form of the "Kron" reduction technique. The final phase impedance matrix becomes:

$$[z_{abc}] = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \Omega/\text{mile} \tag{4.56}$$

For a distribution line that is not transposed, the diagonal terms of Equation 4.56 will not be equal to each other and the off-diagonal terms will not be equal to each other. However, the matrix will be symmetrical.

For two-phase (V-phase) and single-phase lines in grounded wye systems, the Modified Carson's Equations can be applied, which will lead to initial 3×3 and 2×2 primitive impedance matrices. Kron reduction will reduce the matrices to 2×2 and a single element. These matrices can be expanded to 3×3 "phase frame" matrices by the addition of rows and columns consisting of zero elements for the missing phases. For example, for a V-phase line consisting of phases *a* and *c*, the phase impedance matrix would be:

$$[z_{abc}] = \begin{bmatrix} z_{aa} & 0 & z_{ac} \\ 0 & 0 & 0 \\ z_{ca} & 0 & z_{cc} \end{bmatrix} \Omega/\text{mile} \tag{4.57}$$

The phase impedance matrix for a phase *b* single-phase line would be:

$$[z_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & z_{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} \Omega/\text{mile} \tag{4.58}$$

The phase impedance matrix for a three-wire delta line is determined by the application of Carson's equations without the Kron reduction step.

The phase impedance matrix can be used to accurately determine the voltage drops on the feeder line segments once the currents have been determined. Because no approximations (transposition, for example) have been made regarding the spacing between conductors, the effect of the mutual coupling between phases is accurately taken into account. The application of the Modified Carson's Equations and the phase frame matrix leads to the most accurate model of a line segment. Figure 4.6 shows the general

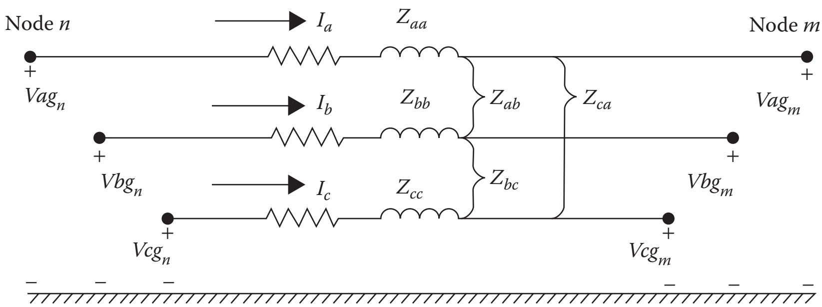

FIGURE 4.6
Three-phase line segment model.

three-phase model of a line segment. Keep in mind that for V-phase and single-phase lines, some of the impedance values will be zero.

The voltage equation in matrix form for the line segment is:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.59}$$

where $Z_{ij} = z_{ij} \cdot \text{length}$.

Equation 4.59 can be written in "condensed" form as:

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}] \cdot [I_{abc}] \tag{4.60}$$

4.1.7 Sequence Impedances

Mostly, the analysis of a feeder will use only the positive and zero sequence impedances for the line segments. There are two methods for obtaining these impedances. The first method incorporates the application of the Modified Carson's Equations and the Kron reduction to obtain the phase impedance matrix.

The definition for line-to-ground phase voltages as a function of the line-to-ground sequence voltages is given by Carson [2]:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \cdot \begin{bmatrix} V_{0g} \\ V_{1g} \\ V_{2g} \end{bmatrix} \tag{4.61}$$

where $a_s = 1.0\underline{/120}$.

In condensed form, Equation 4.61 becomes:

$$[VLG_{abc}] = [A_s] \cdot [VLG_{012}] \tag{4.62}$$

where

$$[A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \tag{4.63}$$

The phase line currents are defined in the same manner:

$$[I_{abc}] = [A_s] \cdot [I_{012}] \tag{4.64}$$

Equation 4.62 can be used to solve for the sequence line-to-ground voltages as a function of the phase line-to-ground voltages.

$$[VLG_{012}] = [A_s]^{-1} \cdot [VLG_{abc}] \tag{4.65}$$

where

$$[A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s & a_s^2 \\ 1 & a_s^2 & a_s \end{bmatrix} \tag{4.66}$$

Equation 4.60 can be transformed to the sequence domain by multiplying both sides by $[A_s]^{-1}$ and also substituting in the definition of the phase currents as given by Equation 4.62.

$$\begin{aligned} [VLG_{012}]_n &= [A_s]^{-1} \cdot [VLG_{abc}]_n \\ [VLG_{012}]_n &= [A_s]^{-1} \cdot [VLG_{abn}]_m + [A_s]^{-1} \cdot [Z_{abc}] \cdot [A_s] \cdot [I_{012}] \\ [VLG_{012}]_n &= [VLG_{012}]_m + [Z_{012}] \cdot [I_{012}] \end{aligned} \tag{4.67}$$

where

$$[Z_{012}] = [A_s]^{-1} \cdot [Z_{abc}] \cdot [A_s] = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \tag{4.68}$$

Equation 4.67 in expanded form is given by:

$$\begin{bmatrix} V_{0g} \\ V_{1g} \\ V_{2g} \end{bmatrix}_n = \begin{bmatrix} V_{0g} \\ V_{1g} \\ V_{2g} \end{bmatrix}_m + \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{02} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{4.69}$$

Equation 4.68 is the defining equation for converting phase impedances to sequence impedances. In Equation 4.68, the diagonal terms of the matrix are the "sequence impedances" of the line such that:

Z_{00} = zero sequence impedance
Z_{11} = positive sequence impedance
Z_{22} = negative sequence impedance

The off-diagonal terms of Equation 4.68 represent the mutual coupling between sequences. In the idealized state, these off-diagonal terms would be zero. In order for this to happen, it must be assumed that the line has been transposed. For high-voltage transmission lines, this will generally be the case. When the lines are transposed, the mutual coupling between phases (off-diagonal terms) are equal, and consequently the off-diagonal terms of the sequence impedance matrix become zero. Because distribution lines are rarely if ever transposed, the mutual coupling between phases is not equal, and as a result, the off-diagonal terms of the sequence impedance matrix will not be zero. This is the primary reason that distribution system analysis uses the phase domain rather than symmetrical components.

If a line is assumed to be transposed, the phase impedance matrix is modified so that the three diagonal terms are equal and all of the off-diagonal terms are equal. A different method to compute the sequence impedances is to set the three diagonal terms of the phase impedance matrix equal to the average of the diagonal terms of Equation 4.56 and the off-diagonal terms equal to the average of the off-diagonal terms of Equation 4.56. When this is done, the self- and mutual impedances are defined as:

$$z_s = \frac{1}{3} \cdot \left(z_{aa} + z_{bb} + z_{cc} \right) \ \Omega/\text{mile} \tag{4.70}$$

$$z_m = \frac{1}{3} \left(z_{ab} + z_{bc} + z_{ca} \right) \ \Omega/\text{mile} \tag{4.71}$$

The phase impedance matrix is now defined as:

$$[z_{abc}] = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} \Omega/\text{mile} \tag{4.72}$$

When Equation 4.68 is used with this phase impedance matrix, the resulting sequence matrix is diagonal (off-diagonal terms are zero). The sequence impedances can be determined directly as:

$$z_{00} = z_s + 2 \cdot z_m \ \ \Omega/\text{mile} \tag{4.73}$$

$$z_{11} = z_{22} = z_s - z_m \ \Omega/\text{mile} \tag{4.74}$$

A second method that is commonly used to determine the sequence impedances directly is to employ the concept of geometric mean distances (GMDs). The GMD between phases is defined as:

$$D_{ij} = GMD_{ij} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}} \ \text{ft} \tag{4.75}$$

The GMD between phases and neutral is defined as:

$$D_{in} = GMD_{in} = \sqrt[3]{D_{an} \cdot D_{bn} \cdot D_{cn}} \ \text{ft} \tag{4.76}$$

The GMDs as defined previously are used in Equations 4.41 and 4.42 to determine the various self- and mutual impedances of the line resulting in:

$$\hat{z}_{ii} = r_i + 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{GMR_i}\right) + 7.93402 \right] \Omega/\text{mile} \tag{4.77}$$

$$\hat{z}_{nn} = r_n + 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{GMR_n}\right) + 7.93402 \right] \Omega/\text{mile} \tag{4.78}$$

$$\hat{z}_{ij} = 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{D_{ij}}\right) + 7.93402 \right] \Omega/\text{mile} \tag{4.79}$$

$$\hat{z}_{in} = 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{D_{in}}\right) + 7.93402 \right] \Omega/\text{mile} \tag{4.80}$$

Equations 4.77 through 4.80 will define a matrix of order $ncond \times ncond$ where *ncond* is the number of conductors (phases plus neutrals) in the line segment. Application of the Kron reduction (Equation 4.55) and the sequence impedance transformation (Equation 4.68) leads to the following expressions for the zero, positive, and negative sequence impedances:

$$z_{00} = \hat{z}_{ii} + 2 \cdot \hat{z}_{ij} - 3 \cdot \left(\frac{\hat{z}_{in}^2}{\hat{z}_{nn}} \right) \Omega/\text{mile} \tag{4.81}$$

$$z_{11} = z_{22} = \hat{z}_{ii} - \hat{z}_{ij}$$

$$z_{11} = z_{22} = r_i + j0.12134 \cdot \ln\left(\frac{D_{ij}}{GMR_i}\right)\ \Omega/\text{mile} \tag{4.82}$$

Equations 4.81 and 4.82 are recognized as the standard equations for the calculation of the line impedances when a balanced three-phase system and transposition are assumed.

Example 4.1

An overhead three-phase distribution line is constructed as shown in Figure 4.7. Determine the phase impedance matrix and the positive and zero sequence impedance matrices of the line. The phase conductors are 336,400 26/7 ACSR (Linnet), and the neutral conductor is 4/0 6/1 ACSR.

Solution: From the table of standard conductor data (Appendix A), it is found that:

336,400 26/7 ACSR: $GMR = 0.0244$ ft

Resistance = 0.306 Ω/mile

4/0 6/1 ACSR: $GMR = 0.00814$ ft

Resistance = 0.5920 Ω/mile

An effective way of computing the distance between all conductors is to specify each position on the pole in Cartesian coordinates using complex number notation. The ordinate will be selected as a point on the ground directly below the leftmost position. For the line in Figure 4.7, the positions are:

$$d_1 = 0 + j29 \quad d_2 = 2.5 + j29 \quad d_3 = 7.0 + j29 \quad d_4 = 4.0 + j25$$

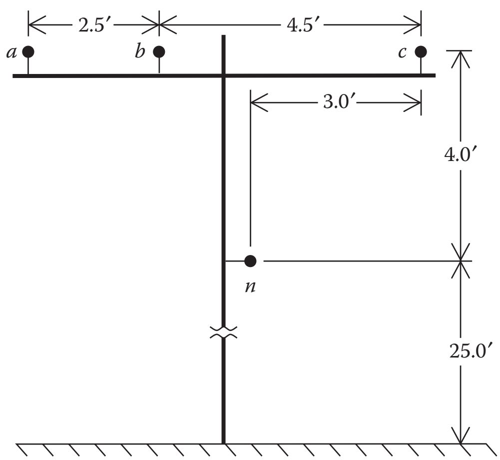

FIGURE 4.7
Three-phase distribution line spacings.

The distances between the positions can be computed as:

$$D_{12} = |d_1 - d_2| \quad D_{23} = |d_2 - d_3| \quad D_{31} = |d_3 - d_1|$$

$$D_{14} = |d_1 - d_4| \quad D_{24} = |d_2 - d_4| \quad D_{34} = |d_3 - d_4|$$

For this example, phase *a* is in position 1, phase *b* is in position 2, phase *c* is in position 3, and the neutral is in position 4.

$$D_{ab} = 2.5' \quad D_{bc} = 4.5' \quad D_{ca} = 7.0'$$

$$D_{an} = 5.6569' \quad D_{bn} = 4.272' \quad D_{cn} = 5.0'$$

The diagonal terms of the distance matrix are the GMRs of the phase and neutral conductors.

$$D_{aa} = D_{bb} = D_{cc} = 0.0244, \; D_{nn} = 0.00814$$

Applying the Modified Carson's Equation for self-impedance (Equation 4.41), the self-impedance for phase *a* is:

$$\begin{aligned}\hat{z}_{aa} &= 0.0953 + 0.306 + j0.12134 \cdot \left(\ln \frac{1}{0.0244} + 7.93402 \right) \\ &= 0.4013 + j1.4133 \; \Omega/\text{mile}\end{aligned}$$

Applying Equation 4.42 for the mutual impedance between phases *a* and *b*:

$$\hat{z}_{ab} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{2.5} + 7.93402 \right) = 0.0953 + j0.8515 \; \Omega/\text{mile}$$

Applying the equations for the other self- and mutual impedance terms results in the primitive impedance matrix.

$$[\hat{z}] = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\ 0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.6873 + j1.5465 \end{bmatrix} \Omega/\text{mile}$$

The primitive impedance matrix in partitioned form is:

$$[\hat{z}_{ij}] = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0943 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 \end{bmatrix} \Omega/\text{mile}$$

$$[\hat{z}_{in}] = \begin{bmatrix} 0.0953 + j0.7524 \\ 0.0953 + j0.7865 \\ 0.0953 + j0.7674 \end{bmatrix} \Omega/\text{mile}$$

$$[\hat{z}_{nn}] = [0.6873 + j1.5465] \ \Omega/\text{mile}$$

$$[\hat{z}_{nj}] = [0.0953 + j0.7524 \quad 0.0953 + j0.7865 \quad 0.0953 + j0.7674] \ \Omega/\text{mile}$$

The "Kron" reduction of Equation 4.55 results in the "phase impedance matrix."

$$[z_{abc}] = [\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]$$

$$[z_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

The neutral transformation matrix given by Equation 4.52 is:

$$[t_n] = -\left([\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]\right)$$

$$[t_n] = [-0.4292 - j0.1291 \quad -0.4476 - j0.1373 \quad -0.4373 - j0.1327]$$

The phase impedance matrix can be transformed into the "sequence impedance matrix" with the application of Equation 4.66.

$$[z_{012}] = [A_s]^{-1} \cdot [z_{abc}] \cdot [A_s]$$

$$[z_{012}] = \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & -0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & -0.0723 - j0.0060 \\ 0.0256 + j0.0115 & 0.0723 - j0.0059 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

In the sequence impedance matrix, the 1,1 term is the zero sequence impedance, the 2,2 term is the positive sequence impedance, and the 3,3 term is the negative sequence impedance. The 2,2 and 3,3 terms are equal, which demonstrates that for line segments, the positive and negative sequence impedances are equal. Note that the off-diagonal terms are not zero. This implies that there is mutual coupling between sequences. This is a result of the nonsymmetrical spacing between phases. With the off-diagonal terms being nonzero, the three sequence networks representing the line will not be independent. However, it is noted that the off-diagonal terms are small relative to the diagonal terms.

In high-voltage transmission lines, it is usually assumed that the lines are transposed and that the phase currents represent a balanced three-phase set. The transposition can be simulated in Example 4.1 by replacing the diagonal terms of the phase impedance matrix with the average value of the diagonal terms (0.4619+j1.0638) and replacing each off-diagonal term with the average of the off-diagonal terms (0.1558+j0.4368). This modified phase impedance matrix becomes:

$$[z1_{abc}] = \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix} \Omega/\text{mile}$$

Using this modified phase impedance matrix in the symmetrical component transformation equation results in the modified sequence impedance matrix.

$$[z1_{012}] = \begin{bmatrix} 0.7735 + j1.9373 & 0 & 0 \\ 0 & 0.3061 + j0.6270 & 0 \\ 0 & 0 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

Note now that the off-diagonal terms are all equal to zero, which means that there is no mutual coupling between sequence networks. It should also be noted that the modified zero, positive, and negative sequence impedances are exactly equal to the exact sequence impedances that were first computed.

The results of this example should not be interpreted to mean that a three-phase distribution line could be assumed to have been transposed. The original phase impedance matrix should be used if the correct effect of the mutual coupling between phases is to be modeled.

4.1.8 Parallel Overhead Distribution Lines

It is fairly common in a distribution system to find instances where two distribution lines are "physically" parallel. The parallel combination may have both distribution lines constructed on the same pole, or the two lines may run in parallel on separate poles but on the same right-of-way. For example, two different feeders leaving a substation may share a common pole or right-of-way before they branch out to their own service area. It is also possible that two feeders may converge and run in parallel until again they branch out into their own service areas. The lines could also be underground circuits sharing a common trench. In all of the cases, the question arises as to how the parallel lines should be modeled and analyzed.

Two parallel overhead lines on one pole are shown in Figure 4.8.

Note in Figure 4.8 the phasing of the two lines.

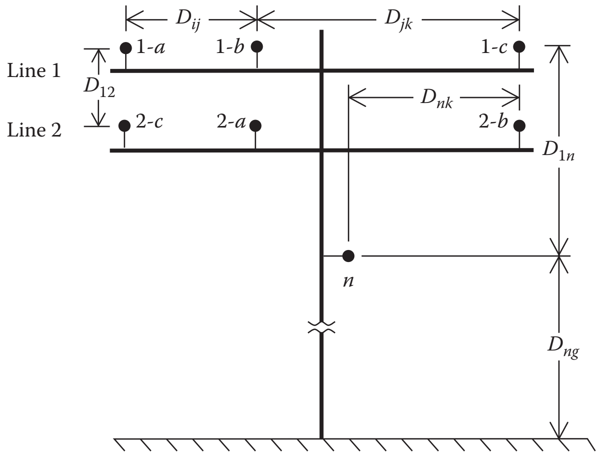

FIGURE 4.8
Parallel overhead lines.

The phase impedance matrix for the parallel distribution lines is computed by the application of Carson's equations and the Kron reduction method. The first step is to number the phase positions as follows:

Position	1	2	3	4	5	6	7
Line-Phase	1-a	1-b	1-c	2-a	2-b	2-c	Neutral

With the phases numbered, the 7×7 primitive impedance matrix for 1 mile can be computed using the Modified Carson's Equations. It should be pointed out that if the two parallel lines are on different poles, most likely each pole will have a grounded neutral conductor. In this case, there will be 8 positions, and position 8 will correspond to the neutral on line 2. An 8×8 primitive impedance matrix will be developed for this case. The Kron reduction will reduce the matrix to a 6×6 phase impedance matrix. With reference to Figure 4.8, the voltage drops in the two lines are given by:

$$\begin{bmatrix} v1_a \\ v1_b \\ v1_c \\ v2_a \\ v2_b \\ v2_c \end{bmatrix} = \begin{bmatrix} z11_{aa} & z11_{ab} & z11_{ac} & z12_{aa} & z12_{ab} & z12_{ac} \\ z11_{ba} & z11_{bb} & z11_{bc} & z12_{ba} & z12_{bb} & z12_{bc} \\ z11_{ca} & z11_{bb} & z11_{cc} & z12_{ca} & z12_{cb} & z12_{cc} \\ z21_{aa} & z21_{ab} & z21_{ac} & z22_{aa} & z22_{ab} & z22_{ac} \\ z21_{ba} & z21_{bb} & z21_{bc} & z22_{ba} & z22_{bb} & z22_{bc} \\ z21_{ca} & z21_{cb} & z21_{cc} & z22_{ca} & z22_{cb} & z22_{cc} \end{bmatrix} \cdot \begin{bmatrix} I1_a \\ I1_b \\ I1_c \\ I2_a \\ I2_b \\ I2_c \end{bmatrix} \tag{4.83}$$

Partition Equation 4.83 between the third and fourth rows and columns, so that series voltage drops for 1 mile of line are given by:

$$[v]=[z]\cdot[I]=\begin{bmatrix}[v1]\\ [v2]\end{bmatrix}=\begin{bmatrix}[z11] & [z12]\\ [z21] & [z22]\end{bmatrix}\cdot\begin{bmatrix}[I1]\\ [I2]\end{bmatrix}\text{ V} \tag{4.84}$$

Example 4.2

Two parallel distribution lines are on a single pole (Figure 4.9).

The phase conductors are:

Line 1: 336,400 26/7 ACSR: $GMR_1 = 0.0244'$ $r_1 = 0.306\ \Omega/\text{mile}$ $d_1 = 0.721''$
Line 2: 250,000 AA: $GMR_2 = 0.0171'$ $r_2 = 0.41\ \Omega/\text{mile}$ $d_2 = 0.567''$
Neutral: 4/06/1 ACSR: $GMR_n = 0.00814'$ $r_n = 0.592\ \Omega/\text{mile}$ $d_n = 0.563''$

Determine the 6×6 phase impedance matrix.
Define the conductor positions according to the phasing:

$$d_1 = 0 + j35 \quad d_2 = 2.5 + j35 \quad d_3 = 7 + j35$$
$$d_4 = 2.5 + j33 \quad d_5 = 7 + j33 \quad d_6 = 0 + j33$$
$$d_7 = 4 + j29$$

Using $D_{ij} = |d_i - d_j|$, the distances between all conductors can be computed. Using this equation, the diagonal terms of the resulting spacing matrix will be zero. It is convenient to define the diagonal terms of the spacing matrix as the GMR of the conductors occupying the position. Using this approach, the final spacing matrix is:

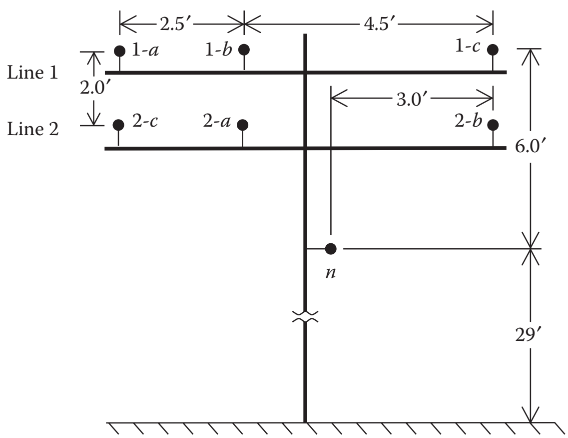

FIGURE 4.9
Example parallel OH lines.

$$[D] = \begin{bmatrix} 0.0244 & 2.5 & 7 & 3.2016 & 7.2801 & 2 & 7.2111 \\ 2.5 & 0.0244 & 4.5 & 2 & 4.9244 & 3.2016 & 6.1847 \\ 7 & 4.5 & 0.0244 & 4.9244 & 2 & 7.2801 & 6.7082 \\ 3.2016 & 2 & 4.9244 & 0.0171 & 4.5 & 2.5 & 4.2720 \\ 7.2801 & 4.9244 & 2 & 4.5 & 0.0171 & 7 & 5 \\ 2 & 3.2016 & 7.2801 & 2.5 & 7 & 0.0171 & 5.6869 \\ 7.2111 & 6.1847 & 6.7082 & 4.2720 & 5 & 5.6569 & 0.0081 \end{bmatrix}$$

The terms for the primitive impedance matrix can be computed using the Modified Carson's Equations. For this example, the subscripts i and j will run from 1 to 7. The 7×7 primitive impedance matrix is partitioned between rows and columns 6 and 7. The Kron reduction will now give the final phase impedance matrix. In partitioned form, the phase impedance matrices are:

$$[z_{11}]_{abc} = \begin{bmatrix} 0.4502 + j1.1028 & 0.1464 + j0.5334 & 0.1452 + j0.4126 \\ 0.1464 + j0.5334 & 0.4548 + j1.0873 & 0.1475 + j0.4584 \\ 0.1452 + j0.4126 & 0.1475 + j0.4584 & 0.4523 + j1.0956 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{12}]_{abc} = \begin{bmatrix} 0.1519 + j0.4848 & 0.1496 + j0.3931 & 0.1477 + j0.5560 \\ 0.1545 + j0.5336 & 0.1520 + j0.4323 & 0.1502 + j0.4909 \\ 0.1531 + j0.4287 & 0.1507 + j0.5460 & 0.1489 + j0.3955 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{21}]_{abc} = \begin{bmatrix} 0.1519 + j0.4848 & 0.1545 + j0.5336 & 0.1531 + j0.4287 \\ 0.1496 + j0.3931 & 0.1520 + j0.4323 & 0.1507 + j0.5460 \\ 0.1477 + j0.5560 & 0.1502 + j0.4909 & 0.1489 + j0.3955 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{22}]_{abc} = \begin{bmatrix} 0.5706 + j1.0913 & 0.1580 + j0.4236 & 0.1559 + j0.5017 \\ 0.1580 + j0.4236 & 0.5655 + j1.1082 & 0.1535 + j0.3849 \\ 0.1559 + j0.5017 & 0.1535 + j0.3849 & 0.5616 + j1.1212 \end{bmatrix} \Omega/\text{mile}$$

4.2 Series Impedance of Underground Lines

Figure 4.10 shows the general configuration of three underground cables (concentric neutral or tape-shielded) with an additional neutral conductor.

The Modified Carson's Equations can be applied to underground cables in much the same manner as for overhead lines. The circuit in Figure 4.10 will result in a 7×7 primitive impedance matrix. For underground circuits

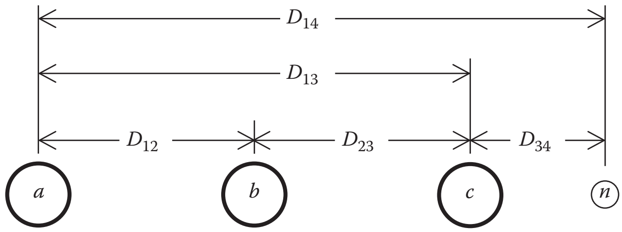

FIGURE 4.10
Three-phase underground with additional neutral.

that do not have the additional neutral conductor, the primitive impedance matrix will be 6×6.

Two popular types of underground cables are the "concentric neutral cable" and the "tape shield cable." To apply the Modified Carson's Equations, the resistance and GMR of the phase conductor and the equivalent neutral must be known.

4.2.1 Concentric Neutral Cable

Figure 4.11 shows a simple detail of a concentric neutral cable. The cable consists of a central "phase conductor" covered by a thin layer of nonmetallic semiconducting screen to which is bonded the insulating material. The insulation is then covered by a semiconducting insulation screen. The solid strands of concentric neutral are spiraled around the semiconducting screen with a uniform spacing between strands. Some cables will also have an insulating "jacket" encircling the neutral strands.

In order to apply Carson's equations to this cable, the following data needs to be extracted from a table of underground cables (Appendices A and B).

d_c = phase conductor diameter (in.)
d_{od} = nominal diameter over the concentric neutrals of the cable (in.)
d_s = diameter of a concentric neutral strand (in.)

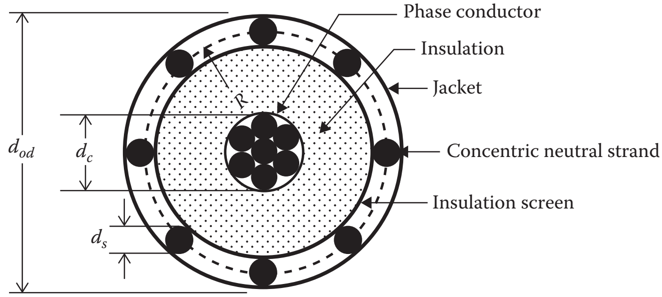

FIGURE 4.11
Concentric neutral cable.

GMR_c = geometric mean radius of the phase conductor (ft)
GMR_s = geometric mean radius of a neutral strand (ft)
r_c = resistance of the phase conductor (Ω/mile)
r_s = resistance of a solid neutral strand (Ω/mile)
k = number of concentric neutral strands

The GMRs of the phase conductor and a neutral strand are obtained from a standard table of conductor data (Appendix A). The equivalent GMR of the concentric neutral is computed using the equation for the GMR of bundled conductors used in high-voltage transmission lines [2].

$$GMR_{cn} = \sqrt[k]{GMR_s \cdot k \cdot R^{k-1}} \text{ ft} \tag{4.85}$$

where

R = radius of a circle passing through the center of the concentric neutral strands

$$R = \frac{d_{od} - d_s}{24} \text{ ft} \tag{4.86}$$

The equivalent resistance of the concentric neutral is:

$$r_{cn} = \frac{r_s}{k} \ \Omega/\text{mile} \tag{4.87}$$

The various spacings between a concentric neutral and the phase conductors and other concentric neutrals are as follows:

Concentric Neutral to Its Own Phase Conductor

$D_{ij} = R$ (Equation 4.86)

Concentric Neutral to an Adjacent Concentric Neutral

D_{ij} = center-to-center distance of the phase conductors

Concentric Neutral to an Adjacent Phase Conductor

Figure 4.12 shows the relationship between the distance between centers of concentric neutral cables and the radius of a circle passing through the centers of the neutral strands.

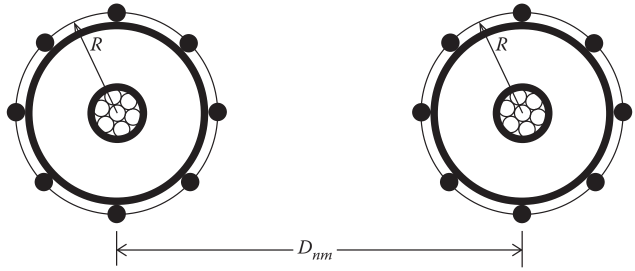

FIGURE 4.12
Distances between concentric neutral cables.

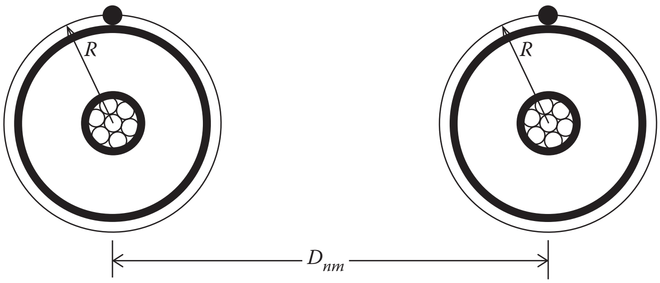

FIGURE 4.13
Equivalent neutral cables.

The GMD between a concentric neutral and an adjacent phase conductor is given by:

$$D_{ij} = \sqrt[k]{D_{nm}^k - R^k} \text{ ft} \tag{4.88}$$

where D_{nm} = center-to-center distance between phase conductors.

The distance between cables will be much greater than the radius R; so a good approximation of modeling the concentric neutral cables is shown in Figure 4.13. In this figure, the concentric neutrals are modeled as one equivalent conductor (shown in black) directly above the phase conductor.

In applying the Modified Carson's Equations, the numbering of conductors and neutrals is important. For example, a three-phase underground circuit with an additional neutral conductor must be numbered as:

1 = phase *a* Conductor #1
2 = phase *b* Conductor #2
3 = phase *c* Conductor #3
4 = neutral of Conductor #1
5 = neutral of Conductor #2
6 = neutral of Conductor #3
7 = additional neutral conductor (if present)

Example 4.3

Three concentric neutral cables are buried in a trench with spacings as shown in Figure 4.14.

The concentric neutral cables of Figure 4.14 can be modeled as shown in Figure 4.15. Notice the numbering of the phase conductors and the equivalent neutrals.

The cables are 15 kV, 250,000 CM stranded all aluminum with k = 13 strands of #14 annealed coated copper wires (1/3 neutral). The outside

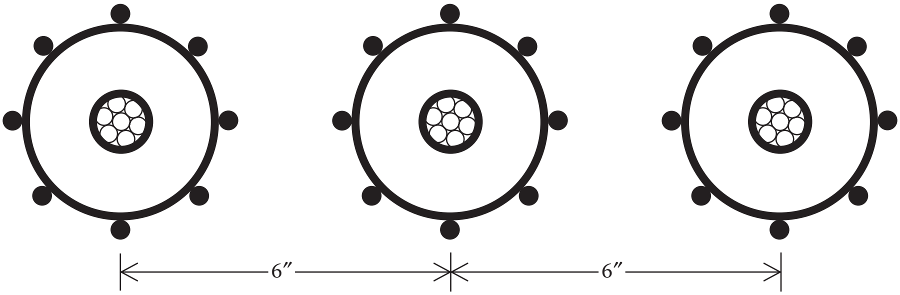

FIGURE 4.14
Three-phase concentric neutral cable spacing.

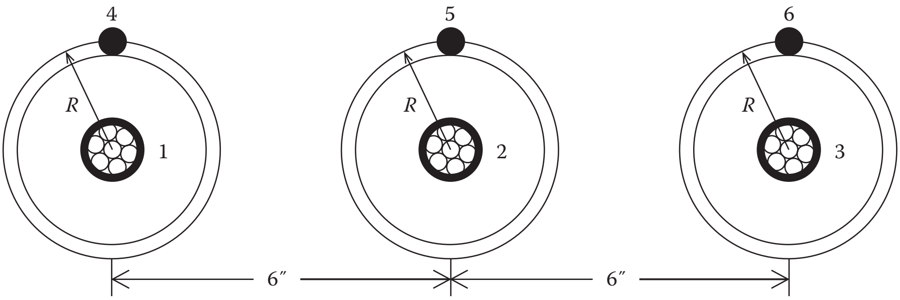

FIGURE 4.15
Three-phase equivalent concentric neutral cable spacing.

diameter of the cable over the neutral strands is 1.29 in. (Appendix B). Determine the phase impedance matrix and the sequence impedance matrix.

Solution: The data for the phase conductor and neutral strands from a conductor data table (Appendix A) are:

250,000 AA phase conductor:

$GMR_p = 0.0171$ ft
Diameter = 0.567 in.
Resistance = 0.4100 Ω/mile

14 copper neutral strands:

$GMR_s = 0.00208$ ft
Resistance = 14.87 Ω/mile
Diameter (d_s) = 0.0641 in.

The radius of the circle passing through the center of the strands (Equation 4.82) is:

$$R = \frac{d_{od} - d_s}{24} = 0.0511\,\text{ft}$$

The equivalent GMR of the concentric neutral is computed by:

$$GMR_{cn} = \sqrt[k]{GMR_s \cdot k \cdot R^{k-1}} = \sqrt[13]{0.00208 \cdot 13 \cdot 0.0511^{13-1}} = 0.0486\,\text{ft}$$

The equivalent resistance of the concentric neutral is:

$$r_{cn} = \frac{r_s}{k} = \frac{14.8722}{13} = 1.1438\ \Omega/\text{mile}$$

The phase conductors are numbered 1, 2, and 3. The concentric neutrals are numbered 4, 5, and 6.

A convenient method of computing the various spacings is to define each conductor using Cartesian coordinates. Using this approach, the conductor coordinates are:

$$d_1 = 0 + j0 \quad d_2 = 0.5 + j0 \quad d_3 = 1 + 1j0$$

$$d_4 = 0 + jR \quad d_5 = 0.5 + jR \quad d_6 = 1 + jR$$

The spacings of off-diagonal terms of the spacing matrix are computed by:

For: $n = 1$ to 6 and $m = 1$ to 6

$$D_{n,m} = |\,d_n - d_m\,|$$

The diagonal terms of the spacing matrix are the GMRs of the phase conductors and the equivalent neutral conductors:

For $i = 1$ to 3 and $j = 4$ to 6

$$D_{i,i} = GMR_p$$

$$D_{j,j} = GMR_s$$

The resulting spacing matrix is:

$$[D] = \begin{bmatrix} 0.0171 & 0.5 & 1 & 0.0511 & 0.5026 & 1.0013 \\ 0.5 & 0.0171 & 0.5 & 0.5026 & 0.0511 & 0.5026 \\ 1 & 0.5 & 0.0171 & 1.0013 & 0.5026 & 0.0511 \\ 0.0511 & 0.5026 & 1.0013 & 0.0486 & 0.5 & 1 \\ 0.5026 & 0.0511 & 0.5026 & 0.5 & 0.0486 & 0.5 \\ 1.0013 & 0.5026 & 0.0511 & 1 & 0.5 & 0.0486 \end{bmatrix} \text{ft.}$$

The self-impedance for the cable in position 1 is:

$$\hat{z}_{11} = 0.0953 + 0.41 + j0.12134 \cdot \left(\ln \frac{1}{0.0171} + 7.93402 \right) = 0.5053 + j1.4564\ \Omega/\text{mile}$$

The self-impedance for the concentric neutral for Cable #1 is:

$$\hat{z}_{44} = 0.0953 + 1.144 + j0.12134 \cdot \left(\ln \frac{1}{0.0486} + 7.93402 \right) = 1.2391 + j1.3296 \ \Omega/\text{mile}$$

The mutual impedance between Cable #1 and Cable #2 is:

$$\hat{z}_{12} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.5} + 7.93402 \right) = 0.0953 + j1.0468 \ \Omega/\text{mile}$$

The mutual impedance between Cable #1 and its concentric neutral is:

$$\hat{z}_{14} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.0511} + 7.93402 \right) = 0.0953 + j1.3236 \ \Omega/\text{mile}$$

The mutual impedance between the concentric neutral of Cable #1 and the concentric neutral of Cable #2 is:

$$\hat{z}_{45} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.5} + 7.93402 \right) = 0.0953 + j1.0468 \ \Omega/\text{mile}$$

Continuing the application of the Modified Carson's Equations results in a 6 × 6 primitive impedance matrix. This matrix in partitioned (Equation 4.33) form is:

$$\left[\hat{z}_{ij}\right] = \begin{bmatrix} 0.5053 + j1.4564 & 0.0953 + j1.0468 & 0.0953 + j.9627 \\ 0.0953 + j1.0468 & 0.5053 + j1.4564 & 0.0953 + j1.0468 \\ 0.0953 + j.9627 & 0.0953 + j1.0468 & 0.5053 + j1.4564 \end{bmatrix} \Omega/\text{mile}$$

$$\left[\hat{z}_{in}\right] = \begin{bmatrix} 0.0953 + j1.3236 & 0.0953 + j1.0468 & 0.0953 + j.9627 \\ 0.0953 + j1.0462 & 0.0953 + j1.3236 & 0.0953 + j1.0462 \\ 0.0953 + j.9626 & 0.0953 + j1.0462 & 0.0953 + j1.3236 \end{bmatrix} \Omega/\text{mile}$$

$$\left[\hat{z}_{nj}\right] = \left[\hat{z}_{in}\right]$$

$$\left[\hat{z}_{nn}\right] = \begin{bmatrix} 1.2393 + j1.3296 & 0.0953 + j1.0468 & 0.0953 + j.9627 \\ 0.0953 + j1.0468 & 1.2393 + j1.3296 & 0.0953 + j1.0468 \\ 0.0953 + j.9627 & 0.0953 + j1.0468 & 1.2393 + j1.3296 \end{bmatrix} \Omega/\text{mile}$$

Using the Kron reduction results in the phase impedance matrix:

$$\left[z_{abc}\right] = \left[\hat{z}_{ij}\right] - \left[\hat{z}_{in}\right] \cdot \left[\hat{z}_{nn}\right]^{-1} \cdot \left[\hat{z}_{nj}\right]$$

$$\left[z_{abc}\right] = \begin{bmatrix} 0.7981 + j0.4467 & 0.3188 + j0.0334 & 0.2848 - j0.0138 \\ 0.3188 + j0.0334 & 0.7890 + j0.4048 & 0.3188 + j0.0334 \\ 0.2848 - j0.0138 & 0.3188 + j0.0334 & 0.7981 + j0.4467 \end{bmatrix} \Omega/\text{mile}$$

The sequence impedance matrix for the concentric neutral three-phase line is determined using Equation 4.68.

$$[z_{012}] = [A_s]^{-1} \cdot [z_{abc}] \cdot [A_s]$$

$$[z_{012}] = \begin{bmatrix} 1.4140 + j0.4681 & -0.0026 - j0.0081 & -0.0057 + j0.0063 \\ -0.0057 + j0.0063 & 0.4876 + j0.4151 & -0.0265 + j0.0450 \\ -0.0026 - j0.0081 & 0.0523 + j0.0004 & 0.4876 + j0.4151 \end{bmatrix} \Omega/\text{mile}$$

4.2.2 Tape-Shielded Cables

Figure 4.16 shows a simple detail of a tape-shielded cable. The cable consists of a central "phase conductor" covered by a thin layer of nonmetallic semiconducting screen to which is bonded the insulating material. The insulation is covered by a semiconducting insulation screen. The shield is bare copper tape helically applied around the insulation screen. An insulating "jacket" encircles the tape shield.

Parameters of the tape-shielded cable are:

d_c = diameter of phase conductor (in.): Appendix A

d_s = outside diameter of the tape shield (in.): Appendix B

d_{od} = outside diameter over jacket (in.): Appendix B

T = thickness of copper tape shield in mils: Appendix B

Once again, the Modified Carson's Equations will be applied to calculate the self-impedances of the phase conductor and the tape shield as well as the mutual impedance between the phase conductor and the tape shield. The resistance and GMR of the phase conductor are found in a standard table of conductor data (Appendix A).

The resistance of the tape shield is given by:

$$r_{shield} = 1.0636 \cdot 10^9 \cdot \frac{\rho m_{20}}{d_s \cdot T} \ \Omega/\text{mile} \tag{4.89}$$

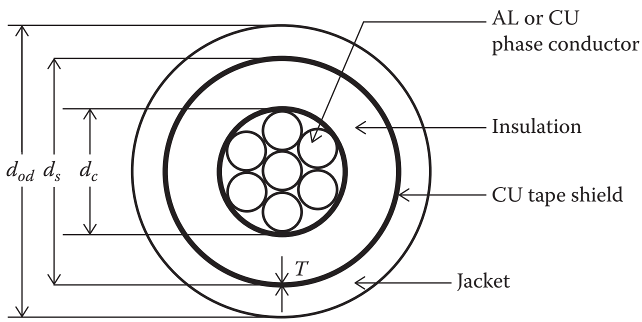

FIGURE 4.16
Tape-shielded cable.

The resistance of the tape shield given in Equation 4.89 assumes a resistivity (ρm_{20}) of $1.7721 \cdot 10^{-8}$ Ω-m and a temperature of 20°C. The outside diameter of the tape shield d_s is given in inches, and the thickness of the tape shield T is in mils.

The GMR of the tape shield is the radius of a circle passing through the middle of the shield and is given by:

$$GMR_{shield} = \frac{\frac{d_s}{2} - \frac{T}{2000}}{12} \text{ ft} \tag{4.90}$$

The various spacings between a tape shield and the conductors and other tape shields are as follows:

Tape Shield to its Own Phase Conductor

$$D_{ij} = GMR_{shield} = \text{radius to midpoint of the shield (ft)} \tag{4.91}$$

Tape Shield to an Adjacent Tape Shield

$$D_{ij} = \text{Center-to-center distance of the phase conductors (ft)} \tag{4.92}$$

Tape Shield to an Adjacent Phase or Neutral Conductor

$$D_{ij} = D_{nm} \text{ ft} \tag{4.93}$$

where D_{nm} = center-to-center distance between phase conductors.

Example 4.4

A single-phase circuit consists of a 1/0 AA, 220 mil insulation tape-shielded cable and a 1/0 CU neutral conductor (Figure 4.17). The single-phase line is connected to phase *b*. Determine the phase impedance matrix.

Cable data: 1/0 AA
Outside diameter of the tape shield = d_s = 0.88 in.
Resistance = 0.97 Ω/mile
GMR_p = 0.0111 ft
Tape shield thickness = T = 5 mils
Resistivity = $\rho m_{20} = 1.7721 \cdot 10^{-8}$ Ω-m

Neutral data: 1/0 Copper, 7 strand
Resistance = 0.607 Ω/mile
GMR_n = 0.01113 ft
Distance between cable and neutral = D_{nm} = 3 in.

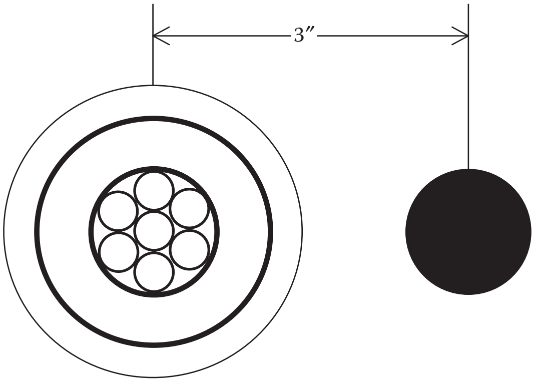

FIGURE 4.17
Single-phase tape shield with neutral conductor.

The resistance of the tape shield is computed according to Equation 4.89:

$$r_{shield} = \frac{1.0636 \cdot 10^9 \cdot \rho m_{20}}{d_s \cdot T} = \frac{18.8481}{0.88 \cdot 5} = 4.2836\ \Omega/\text{mile}$$

The GMR of the tape shield is computed according to Equation 4.90:

$$GMR_{shield} = \frac{\frac{d_s}{2} - \frac{T}{2000}}{12} = \frac{\frac{0.88}{2} - \frac{5}{2000}}{12} = 0.0365\ \text{ft}$$

The conductors are numbered such that:
#1 = 1/0 AA conductor
#2 = tape shield
#3 = 1/0 copper ground

The spacings used in the Modified Carson's Equations are:

$$D_{12} = GMR_{shield} = 0.0365$$

$$D_{13} = \frac{3}{12} = 0.25$$

The self-impedance of Conductor #1 is:

$$\hat{z}_{11} = 0.0953 + 0.97 + j0.12134 \cdot \left(\ln \frac{1}{0.0111} + 7.93402 \right) = 1.0653 + j1.5088\ \Omega/\text{mile}$$

The mutual impedance between Conductor #1 and the tape shield (Conductor #2) is:

$$\hat{z}_{12} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.0365} + 7.93402 \right) = 0.0953 + j1.3645\ \Omega/\text{mile}$$

The self-impedance of the tape shield (Conductor #2) is:

$$\hat{z}_{22} = 0.0953 + 4.2786 + j0.12134 \cdot \left(\ln \frac{1}{0.0365} + 7.93402 \right) = 4.3739 + j1.3645\ \Omega/\text{mile}$$

Continuing on the final primitive impedance matrix is:

$$\left[\hat{z}\right] = \begin{bmatrix} 1.0653 + j1.5088 & 0.0953 + j1.3645 & 0.0953 + 1.1309 \\ 0.0953 + j1.3645 & 4.3739 + j1.3645 & 0.0953 + j1.1309 \\ 0.0953 + j1.1309 & 0.0953 + j1.1309 & 0.7023 + j1.5085 \end{bmatrix} \Omega/\text{mile}$$

In partitioned form, the primitive impedance matrix is:

$$\left[\hat{z}_{ij}\right] = 1.0653 + j1.5088$$

$$\left[\hat{z}_{in}\right] = \left[0.0953 + j1.3645 \quad 0.0953 + j1.1309\right]$$

$$\left[\hat{z}_{nj}\right] = \begin{bmatrix} 0.0953 + j1.3645 \\ 0.0953 + j1.1309 \end{bmatrix}$$

$$\left[\hat{z}_{nn}\right] = \begin{bmatrix} 4.3739 + j1.3645 & 0.0953 + j1.1309 \\ 0.0953 + j1.1309 & 0.7023 + j1.5085 \end{bmatrix} \Omega/\text{mile}$$

Applying Kron's reduction method will result in a single impedance, which represents the equivalent single-phase impedance of the tape shield cable and the neutral conductor.

$$z_{1p} = \left[\hat{z}_{ij}\right] - \left[\hat{z}_{in}\right] \cdot \left[\hat{z}_{nn}\right]^{-1} \cdot \left[\hat{z}_{nj}\right]$$

$$z_{1p} = 1.3218 + j0.6744\ \Omega/\text{mile}$$

Because the single-phase line is on phase b, the phase impedance matrix for the line is:

$$\left[z_{abc}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.3218 + j0.6744 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Omega/\text{mile}$$

4.2.3 Parallel Underground Distribution Lines

The procedure for computing the phase impedance matrix for two overhead parallel lines has been presented in Section 4.1.8. Figure 4.18 shows two concentric neutral parallel lines each with a separate grounded neutral conductor.

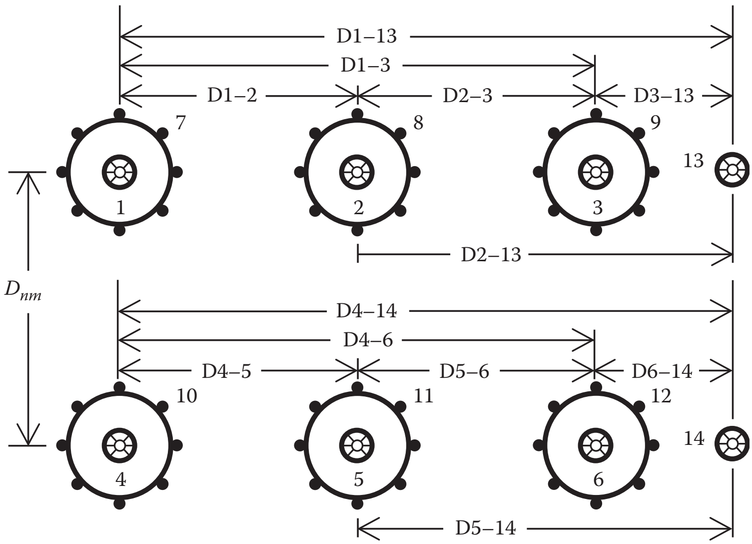

FIGURE 4.18
Parallel concentric neutral underground lines.

The process for computing the 6 × 6 phase impedance matrix follows exactly the same procedure as for the overhead lines. In this case, there are a total of 14 conductors (6 phase conductors, 6 equivalent concentric neutral conductors, and 2 grounded neutral conductors). Applying Carson's equations will result in a 14 × 14 primitive impedance matrix. This matrix is partitioned between the sixth and seventh rows and columns. The Kron reduction is applied to form the final 6 × 6 phase impedance matrix.

Example 4.5

Two concentric neutral three-phase underground parallel lines are shown in Figure 4.19.

Cables (both lines): 250 kcmil, 1/3 neutral
Extra neutral: 4/0 Copper
Determine the 6 × 6 phase impedance matrix.

Solution: From Appendix B for the cables:

Outside diameter: $d_{od} = 1.29'$
Neutral strands: $k = 13$ #14 copper strands
From Appendix A for the conductors:

$$250 \text{ kcmil Al}: \; GMR_c = 0.0171', \; r_c = 0.41 \; \Omega/\text{mile}, \; d_c = 0.567''$$

$$\#14 \text{ Copper}: \quad GMR_s = 0.00208', \; r_s = 14.8722 \; \Omega/\text{mile}, \; d_s = 0.0641''$$

$$4/0 \text{ Copper}: \; GMR_n = 0.1579', \; r_n = 0.303 \; \Omega/\text{mile}, \; d_n = 0.522''$$

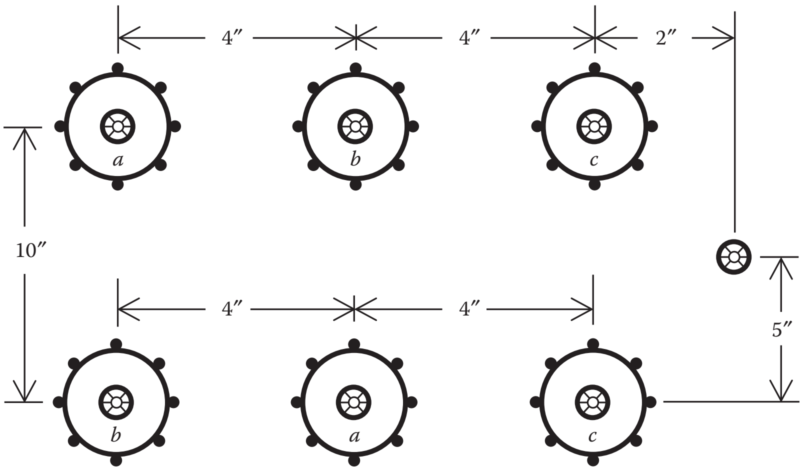

FIGURE 4.19
Parallel concentric neutral three-phase lines.

The radius of the circle to the center of the strands is:

$$R_b = \frac{d_{od} - d_s}{24} = \frac{1.29 - 0.0641}{24} = 0.0511'$$

The equivalent GMR of the concentric neutral strands is computed as:

$$GMR_{eq} = \sqrt[k]{GMR_s \cdot k \cdot R_b^{k-1}} = \sqrt[13]{0.00208 \cdot 13 \cdot 0.05111^{12}} = 0.0486'$$

The positions of the six cables and extra neutral using Cartesian coordinates with the phase *a* cable in line 1 (top line) as the ordinate are shown below. Note the phasing in both lines.

$$\text{Phase } a, \text{ line } 1: \; d_1 = 0 + j0 \quad \text{Phase } b, \text{ line } 1: \; d_2 = \frac{4}{12} + j0$$

$$\text{Phase } c, \text{ line } 1: \; d_3 = \frac{8}{12} + j0$$

$$\text{Phase } a, \text{ line } 2: \; d_4 = \frac{4}{12} - j\frac{10}{12} \quad \text{Phase } b, \text{ line } 2: \; d_5 = 0 - j\frac{10}{12}$$

$$\text{Phase } c, \text{ line } 2: \; d_6 = \frac{8}{12} - j\frac{10}{12}$$

Equivalent neutrals:

$$\text{Phase } a, \text{ line } 1: \ d_7 = d_1 + jR_b \quad \text{Phase } b, \text{ line } 1: \ d_8 = d_2 + jR_b$$

$$\text{Phase } c, \text{ line } 1: d_9 = d_3 + jR_b$$

$$\text{Phase } a, \text{ line } 2: \ d_{10} = d_4 + jR_b \quad \text{Phase } b, \text{ line } 2: \ d_{11} = d_5 + jR_b$$

$$\text{Phase } c, \text{ line } 2: d_{12} = d_6 + jR_b$$

Extra neutral:

$$d_{13} = \frac{10}{12} - j\frac{5}{12}$$

The spacing matrix defining the distances between conductors can be computed by:

$$i = 1 \text{ to } 13 \qquad j = 1 \text{ to } 13$$

$$\mathrm{D}_{i,j} = \left| d_i - d_j \right|$$

The diagonal terms of the spacing matrix are defined as the appropriate GMR:

$$D_{1,1} = D_{2,2} = D_{3,3} = D_{4,4} = D_{5,5} = D_{6,6} = GMR_c = 0.0171'$$

$$D_{7,7} = D_{8,8} = D_{9,9} = D_{10,10} = D_{11,11} = D_{12,12} = GMR_{\text{eq}} = 0.0486'$$

$$D_{13,13} = GMR_n = 0.01579'$$

The resistance matrix is defined as:

$$r_1 = r_2 = r_3 = r_4 = r_5 = r_6 = 0.41 \ \Omega/\text{mile}$$

$$r_7 = r_8 = r_9 = r_{10} = r_{11} = r_{12} = \frac{r_s}{k} = \frac{14.8722}{13} = 1.144 \ \Omega/\text{mile}$$

$$r_{13} = r_n = 0.303 \ \Omega/\text{mile}$$

The primitive impedance matrix (13 × 13) is computed using Carson's equations:

$$i = 1 \text{ to } 13 \qquad j = 1 \text{ to } 13$$

$$zp_{i,j} = 0.0953 + j0.12134 \cdot \left(\ln\left(\frac{1}{D_{i,i}} \right) + 7.93402 \right)$$

$$zp_{i,i} = r_i + 0.0953 + j0.12134 \cdot \left(\ln\left(\frac{1}{D_{i,j}} \right) + 7.93402 \right)$$

Once the primitive impedance matrix is developed, it is partitioned between the sixth and seventh rows and columns, and the Kron reduction method is applied to develop the 6×6 phase impedance matrix. The phase impedance matrix in partitioned form is:

$$[z_{11}]_{abc} = \begin{bmatrix} 0.6450+j0.4327 & 0.1805+j0.0658 & 0.1384+j0.0034 \\ 0.1805+j0.0658 & 0.6275+j0.3974 & 0.1636+j0.0552 \\ 0.1384+j0.0034 & 0.1636+j0.0552 & 0.6131+j0.4081 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{12}]_{abc} = \begin{bmatrix} 0.1261-j0.0086 & 0.1389+j0.071 & 0.0782-j0.0274 \\ 0.1185-j0.0165 & 0.1237-j0.0145 & 0.0720-j0.0325 \\ 0.1083-j0.0194 & 0.1074-j0.0246 & 0.0725-j0.0257 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{21}]_{abc} = \begin{bmatrix} 0.1261-j0.0086 & 0.1185-j0.0165 & 0.1083-j0.0195 \\ 0.1389+j0.0071 & 0.1237-j0.0145 & 0.1074-j0.0246 \\ 0.0782-j0.0274 & 0.072-j0.0325 & 0.0725-j0.0257 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{22}]_{abc} = \begin{bmatrix} 0.6324+j0.4329 & 0.1873+j0.0915 & 0.0776-j0.0233 \\ 0.1873+j0.0915 & 0.6509+j0.4508 & 0.0818-j0.0221 \\ 0.0776-j0.0233 & 0.0818-j0.0221 & 0.8331+j0.6476 \end{bmatrix} \Omega/\text{mile}$$

4.3 Summary

This chapter has been devoted to presenting methods for computing the phase impedances and sequence impedances of overhead lines and underground cables. Carson's equations have been modified to simplify the computation of the phase impedances. When using the Modified Carson's Equations, there is no need to make any assumptions, such as transposition of the lines. By assuming an untransposed line and including the actual phasing of the line, the most accurate values of the phase impedances, self and mutual, are determined. It is highly recommended that no assumptions be made in the computation of the impedances. Because voltage drop is a primary concern on a distribution line, the impedances used for the line must be as accurate as possible. This chapter also included the process of applying Carson's equations to two distribution lines that are physically parallel. This same approach would be taken when there are more than two lines physically parallel.

Problems

4.1 The configuration and conductors of a three-phase overhead line is shown in Figure 4.20.

Phase conductors: 556,500 26/7 ACSR
Neutral conductor: 4/0 ACSR

1. Determine the phase impedance matrix $[z_{abc}]$ in Ω/mile.
2. Determine the sequence impedance matrix $[z_{012}]$ in Ω/mile.
3. Determine the neutral transformation matrix $[t_n]$.

4.2 Determine the phase impedance $[z_{abc}]$ matrix in Ω/mile for the two-phase configuration in Figure 4.21.

Phase conductors: 336,400 26/7 ACSR
Neutral conductor: 4/0 6/1 ACSR

4.3 Determine the phase impedance $[z_{abc}]$ matrix in Ω/mile for the single-phase configuration shown in Figure 4.22.

Phase and Neutral Conductors: 1/0 6/1 ACSR

4.4 Create the spacings and configurations of Problems 4.1, 4.2, and 4.3 in the distribution analysis program WindMil. Compare the phase impedance matrices to those computed in the previous problems.

4.5 Determine the phase impedance matrix $[z_{abc}]$ and sequence impedance matrix $[z_{012}]$ in Ω/mile for the three-phase pole configuration in Figure 4.23. The phase and neutral conductors are 250,000 all aluminum.

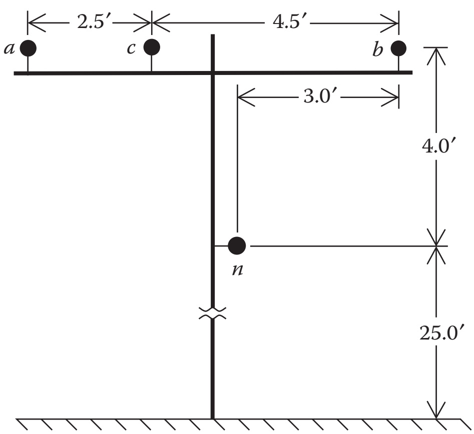

FIGURE 4.20
Three-phase configuration for Problem 4.1.

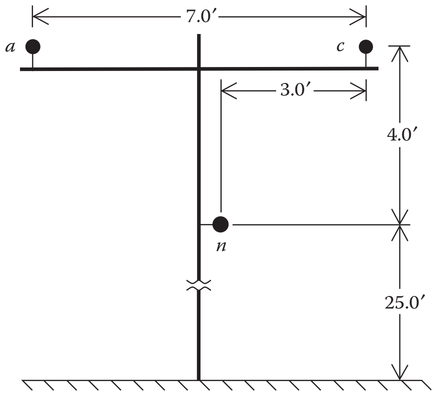

FIGURE 4.21
Two-phase configuration for Problem 4.2.

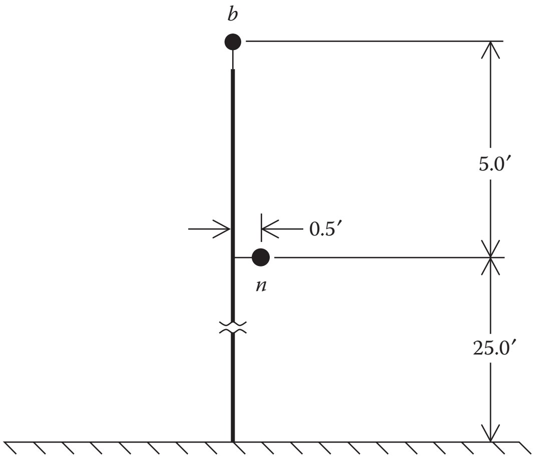

FIGURE 4.22
Single-phase pole configuration for Problem 4.3.

4.6 Compute the positive, negative, and zero sequence impedances in Ω/1000 ft using the GMD method for the pole configuration shown in Figure 4.23.

4.7 Determine the $[z_{abc}]$ and $[z_{012}]$ matrices in Ω/mile for the three-phase configuration shown in Figure 4.24. The phase conductors are 350,000 all aluminum, and the neutral conductor is 250,000 all aluminum.

4.8 Compute the positive, negative, and zero sequence impedances in Ω/1000 ft for the line of Figure 4.24 using the average self- and mutual impedances defined in Equations 4.70 and 4.71.

4.9 A 4/0 aluminum concentric neutral cable is to be used for a single-phase lateral. The cable has a full neutral (see Appendix B). Determine the

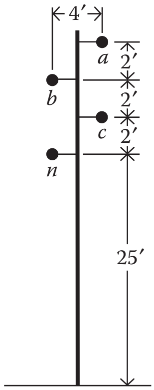

FIGURE 4.23
Three-phase pole configuration for Problem 4.5.

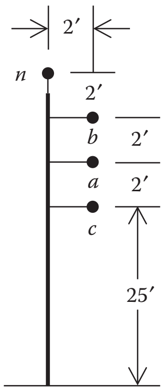

FIGURE 4.24
Three-phase pole configuration for Problem 4.7.

impedance of the cable and the resulting phase impedance matrix in Ω/mile, assuming the cable is connected to phase *b*.

4.10 Three 250,000 CM aluminum concentric cables with one-third neutrals are buried in a trench in a horizontal configuration (see Figure 4.14). Determine the $[z_{abc}]$ and $[z_{012}]$ matrices in Ω/1000 ft assuming phasing of *c*–*a*–*b*.

4.11 Create the spacings and configurations of Problems 4.9 and 4.10 in Windmil. Compare the values of the phase impedance matrices to those computed in the previous problems. In order to check the phase impedance matrix, it will be necessary for you to connect the line to a balanced three-phase source. A source of 12.47 kV works fine.

4.12 A single-phase underground line is composed of a 350,000 CM aluminum tape-shielded cable. A 4/0 copper conductor is used as the neutral. The cable and neutral are separated by 4 in. Determine the phase impedance matrix in Ω/mile for this single-phase cable line assuming phase *c*.

4.13 Three one-third neutral 2/0 aluminum jacketed concentric neutral cables are installed in a 6-in. conduit. Assume the cable jacket has a thickness of 0.2 in. and the cables lie in a triangular configuration inside the conduit. Compute the phase impedance matrix in Ω/mile for this cabled line.

4.14 Create the spacing and configuration of Problem 4.13 in WindMil. Connect a 12.47 kV source to the line, and compare results to those of 4.13.

4.15 Two three-phase distribution lines are physically parallel as shown in Figure 4.25.

Line # 1 (left side): Phase conductors = 266,800 26/7 ACSR
Neutral conductor = 3/0 6/1 ACSR
Line # 2 (right side): Phase conductors = 300,000 CON LAY Aluminum
Neutral conductor = 4/0 CLASS A Aluminum

a. Determine the 6 × 6 phase impedance matrix.
b. Determine the neutral transform matrix.

4.16 Two concentric neutral underground three-phase lines are physically parallel as shown in Figure 4.26.

Line # 1 (top): Cable = 250 kcmil, 1/3 neutral
Additional neutral: 4/0 6/1 ACSR
Line #2 (bottom): Cable = 2/0 kcmil, 1/3 neutral
Additional neutral: 2/0 ACSR

a. Determine the 6 × 6 phase impedance matrix.
b. Determine the neutral transform matrix.

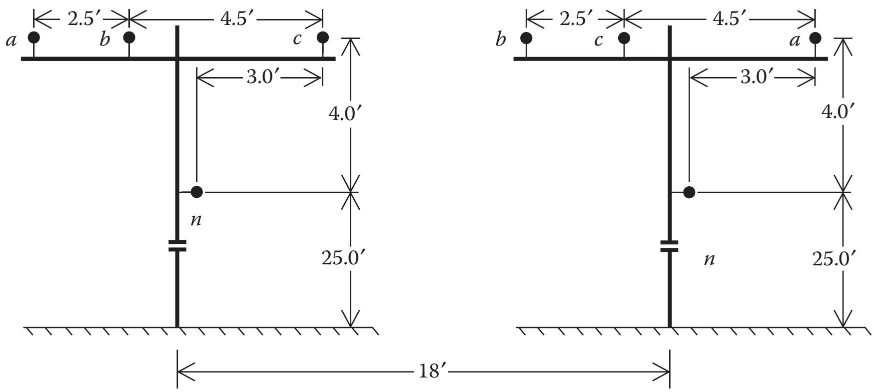

FIGURE 4.25
Parallel OH lines.

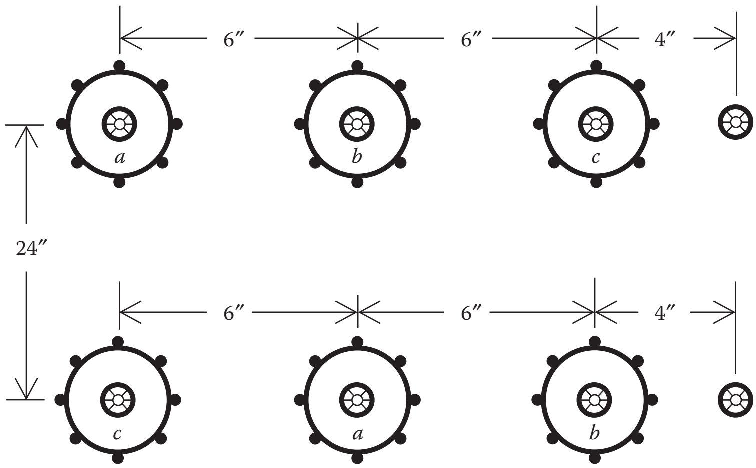

FIGURE 4.26
Parallel concentric neutral three-phase lines for Problem 4.16.

WindMil Assignment

Follow the method outlined in the User's Manual to build a system called "System 1" in WindMil that will have the following components:

- 12.47 kV line-to-line source. The "Bus Voltage" should be set to 120 V
- Connect to the node and call it Node 1
- A 10,000 ft long overhead three-distribution line as defined in Problem 4.1. Call this line OH-1.
- Connect a node to the end of the line and call it Node 2.
- A wye-connected unbalanced three-phase load is connected to Node 2 and is modeled as constant PQ load with values of:
    - Phase *a*–*g*: 1000 kVA, Power factor = 90% lagging
    - Phase *b*–*g*: 800 kVA, Power factor = 85% lagging
    - Phase *c*–*g*: 1200 kVA, Power factor = 95% lagging

Determine the voltages on a 120 V base at Node 2 and the current flowing on the OH-1 line.

References

1. Glover, J. D. and Sarma, M., *Power System Analysis and Design*, 2nd Edition, PWS-Kent Publishing, Boston, MA, 1994.
2. Carson, J. R., Wave propagation in overhead wires with ground return, *Bell System Technical Journal*, Vol. 5, pp. 539–554, 1926.
3. Kron, G., Tensorial analysis of integrated transmission systems, part I, the six basic reference frames, *Transactions of the American Institute of Electrical Engineers*, Vol. 71, pp. 814–882, 1952.

5

Shunt Admittance of Overhead and Underground Lines

The shunt admittance of a line consists of the conductance and the capacitive susceptance. The conductance is usually ignored because it is very small compared to the capacitive susceptance. The capacitance of a line is the result of the potential difference between conductors. A charged conductor creates an electric field that emanates outward from the center of the conductor. Lines of equipotential are created that are concentric to the charged conductor. This is illustrated in Figure 5.1.

In Figure 5.1, a difference in potential between two points (P1 and P2) is a result of the electric field of the charged conductor. When the potential difference between the two points is known, the capacitance between the two points can be computed. If there are other charged conductors nearby, the potential difference between the two points will be a function of the distance to the other conductors and the charge on each conductor. The principle of superposition is used to compute the total voltage drop between two points and then the resulting capacitance between the points. Understand that the points can be points in space or the surface of two conductors or the surface of a conductor and the ground.

5.1 General Voltage Drop Equation

Figure 5.2 shows an array of N positively charged solid round conductors. Each conductor has a unique uniform charge density of q cb/m.

The voltage drop between conductor i and conductor j as a result of all of the charged conductors is given by:

$$V_{ij} = \frac{1}{2\pi\varepsilon}\left(q_1 \ln\frac{D_{1j}}{D_{1i}} + \cdots + q_i \ln\frac{D_{ij}}{RD_i} + \cdots + q_j \ln\frac{RD_j}{Dij} + \cdots + q_N \ln\frac{D_{Nj}}{D_{Ni}}\right) \quad (5.1)$$

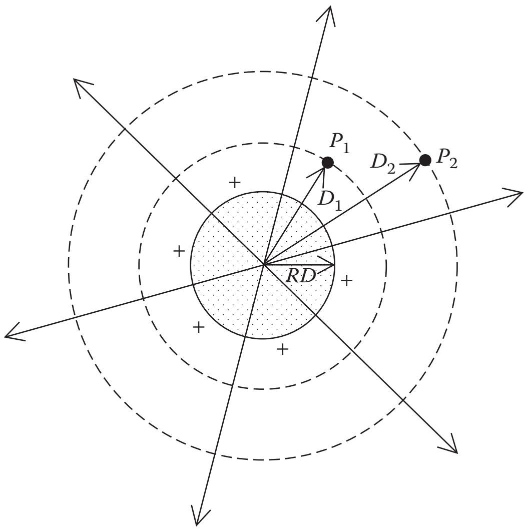

FIGURE 5.1
Electric field of a charged round conductor.

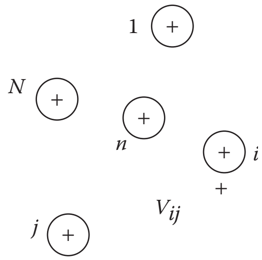

FIGURE 5.2
Array of round conductors.

Equation 5.1 can be written in general form as:

$$V_{ij} = \frac{1}{2\pi\varepsilon}\sum_{n=1}^{N} q_n \ln\frac{D_{nj}}{D_{ni}} \tag{5.2}$$

where $\varepsilon = \varepsilon_0\varepsilon_r$ = permittivity of the medium
ε_0 = permittivity of free space = $8.85 \cdot 10^{-12}$ μF/m
ε_r = relative permittivity of the medium
q_n = charge density on conductor n cb/m
D_{ni} = distance between conductor n and conductor i (ft)
D_{nj} = distance between conductor n and conductor j (ft)
D_{nn} = radius (RD_n) of conductor n (ft)

5.2 Overhead Lines

The method of conductors and their images is employed in the calculation of the shunt capacitance of overhead lines. This is the same concept that was used in Chapter 4 in the general application of Carson's equations. Figure 5.3 illustrates the conductors and their images, and it will be used to develop a general voltage drop equation for overhead lines.

In Figure 5.3, it is assumed that:

$$\begin{aligned} q_i' &= -q_i \\ q_j' &= -q_j \end{aligned} \tag{5.3}$$

Appling Equation 5.2 to Figure 5.3:

$$V_{ii} = \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} + q_i' \ln\frac{RD_i}{S_{ii}} + q_j \ln\frac{S_{ij}}{D_{ij}} + q_j' \ln\frac{D_{ij}}{S_{ij}} \right) \tag{5.4}$$

Because of the assumptions of Equation 5.3, Equation 5.4 can be simplified to:

$$V_{ii} = \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} - q_i \ln\frac{RD_i}{S_{ii}} + q_j \ln\frac{S_{ij}}{D_{ij}} - q_j \ln\frac{D_{ij}}{S_{ij}} \right)$$

$$V_{ii} = \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} + q_i \ln\frac{S_{ii}}{RD_i} + q_j \ln\frac{S_{ij}}{D_{ij}} + q_j \ln\frac{S_{ij}}{D_{ij}} \right)$$

$$V_{ii} = \frac{1}{2\pi\varepsilon}\left(2 \cdot q_i \ln\frac{S_{ii}}{RD_i} + 2 \cdot q_j \ln\frac{S_{ij}}{D_{ij}} \right) \tag{5.5}$$

where S_{ii} = distance from conductor i to its image i' (ft)

S_{ij} = distance from conductor i to the image of conductor j (ft)

D_{ij} = distance from conductor i to conductor j (ft)

RD_i = radius of conductor i in ft.

Equation 5.5 gives the total voltage drop between conductor i and its image. The voltage drop between conductor i and the ground will be one-half of that given in Equation 5.5.

$$V_{ig} = \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} + q_j \ln\frac{S_{ij}}{D_{ij}} \right) \tag{5.6}$$

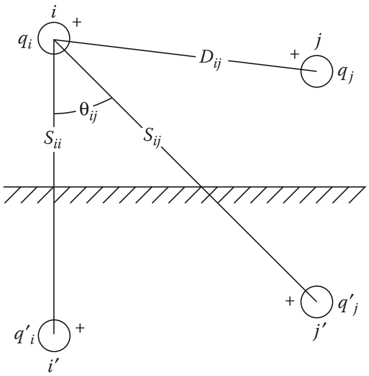

FIGURE 5.3
Conductors and images.

Equation 5.6 can be written in a general form as:

$$V_{ig} = P_{ii} \cdot q_i + P_{ij} \cdot q_j \tag{5.7}$$

where P_{ii} and P_{ij} are the self- and mutual "potential coefficients."

For overhead lines, the relative permittivity of air is assumed to be 1.0 so that:

$$\begin{aligned} \varepsilon_{\text{air}} &= 1.0 \times 8.85 \times 10^{-12}\ \text{F/m} \\ \varepsilon_{\text{air}} &= 1.4240 \times 10^{-2}\ \text{mF/mile} \end{aligned} \tag{5.8}$$

Using the value of permittivity in μF/mile, the self- and mutual potential coefficients are defined as:

$$\hat{P}_{ii} = 11.17689 \cdot \ln \frac{S_{ii}}{RD_i}\ \text{mile/μF} \tag{5.9}$$

$$\hat{P}_{ij} = 11.17689 \cdot \ln \frac{S_{ij}}{D_{ij}}\ \text{mile/μF} \tag{5.10}$$

Note In applying Equations 5.9 and 5.10, the values of RD_i, S_{ii}, S_{ij}, and D_{ij} must be in the same units. For overhead lines, the distances between conductors are typically specified in feet, whereas the value of the conductor diameter from a table will typically be in inches. Care must be taken to ensure that the radius in feet is used in applying the two equations.

For an overhead line of *n-cond* conductors, the "primitive potential coefficient matrix" $\left[\hat{P}_{primitive}\right]$ can be constructed. The primitive potential coefficient matrix will be an *n-cond* × *n-cond* matrix. For a four-wire grounded wye line, the primitive coefficient matrix will be of the form:

$$\left[\hat{P}_{primitive}\right]=\begin{bmatrix} \hat{P}_{aa} & \hat{P}_{ab} & \hat{P}_{ac} & \bullet & \hat{P}_{an} \\ \hat{P}_{ba} & \hat{P}_{bb} & \hat{P}_{bc} & \bullet & \hat{P}_{bn} \\ \hat{P}_{ca} & \hat{P}_{cb} & \hat{P}_{cc} & \bullet & \hat{P}_{cn} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \hat{P}_{na} & \hat{P}_{nb} & \hat{P}_{nc} & \bullet & \hat{P}_{nn} \end{bmatrix} \tag{5.11}$$

The dots (•) in Equation 5.11 are partitioning the matrix between the third and fourth rows and columns. In partitioned form, Equation 5.11 becomes:

$$\left[\hat{P}_{primitive}\right]=\begin{bmatrix} \left[\hat{P}_{ij}\right] & \left[\hat{P}_{in}\right] \\ \left[\hat{P}_{nj}\right] & \left[\hat{P}_{nn}\right] \end{bmatrix} \tag{5.12}$$

Because the neutral conductor is grounded, the matrix can be reduced using the "Kron reduction" method to an n-phase × n-phase phase potential coefficient matrix $[P_{abc}]$.

$$[P_{abc}]=\left[\hat{P}_{ij}\right]-\left[\hat{P}_{in}\right]\cdot\left[\hat{P}_{nn}\right]^{-1}\cdot\left[\hat{P}_{jn}\right] \tag{5.13}$$

The inverse of the potential coefficient matrix will give the n-phase × n-phase capacitance matrix $[C_{abc}]$.

$$[C_{abc}]=[P_{abc}]^{-1}\,\mu F \tag{5.14}$$

For a two-phase line, the capacitance matrix of Equation 5.14 will be 2 × 2. A row and column of zeros must be inserted for the missing phase. For a single-phase line, Equation 5.14 will result in a single element. Again, rows and columns of zero must be inserted for the missing phase. In the case of the single-phase line, the only nonzero term will be that of the phase in use.

Neglecting the shunt conductance, the phase shunt admittance matrix is given by:

$$[y_{abc}]=0+j\cdot\omega\cdot[C_{abc}]\,\mu S/\text{mile} \tag{5.15}$$

where $\omega=2\cdot\pi\cdot f=376.9911$.

Example 5.1

Determine the shunt admittance matrix for the overhead line in Example 4.1. Assume that the neutral conductor is 25 ft above the ground.

The diameters of the phase and neutral conductors from the conductor table (Appendix A) are:

$$\text{Conductor}: \ 336,400 \ 26/7 \ ACSR \ \ d_c = 0.721 \text{ in.}, \quad RD_c = 0.03004 \text{ ft}$$

$$4/0 \ \ 6/1 \ ACSR: d_s = 0.563 \text{ in.}, \quad RD_s = 0.02346 \text{ ft}$$

Using the Cartesian coordinates in Example 4.1, the image distance matrix is given by:

$$S_{ij} = | d_i - d_j^* |$$

where d_j^* = the conjugate of d_j.

For the configuration, the distances between conductors and images in matrix form are:

$$[S] = \begin{bmatrix} 58 & 58.0539 & 58.4209 & 54.1479 \\ 58.0539 & 58 & 58.1743 & 54.0208 \\ 58.4209 & 58.1743 & 58 & 54.0833 \\ 54.1479 & 54.0208 & 54.0833 & 50 \end{bmatrix} \text{ft}$$

The self-primitive potential coefficient for phase *a* and the mutual primitive potential coefficient between phases *a* and *b* are:

$$\hat{P}_{aa} = 11.17689 \ln \frac{58}{0.03004} = 84.5600 \text{ mile/}\mu\text{F}$$

$$\hat{P}_{ab} = 11.17689 \ln \frac{58.0539}{2.5} = 35.1522 \text{ mile/}\mu\text{F}$$

Using Equations 5.9 and 5.10, the total primitive potential coefficient matrix is computed to be:

$$\left[\hat{P}_{primitive}\right] = \begin{bmatrix} 84.5600 & 35.1522 & 23.7147 & 25.2469 \\ 35.1522 & 84.5600 & 28.6058 & 28.3590 \\ 23.7147 & 28.6058 & 84.5600 & 26.6131 \\ 25.2469 & 28.3590 & 26.6131 & 85.6659 \end{bmatrix} \text{mile/}\mu\text{F}$$

Because the fourth conductor (neutral) is grounded, the Kron reduction method is used to compute the "phase potential coefficient matrix." Because only one row and column need to be eliminated, the $\left[\hat{P}_{nn}\right]$ term is a single element, so that the Kron reduction equation for this case can be modified to:

$$P_{ij} = \hat{P}_{ij} - \frac{\hat{P}_{in} \cdot \hat{P}_{jn}}{\hat{P}_{44}}$$

where $i = 1, 2, 3$ and $j = 1, 2, 3$.

For example, the value of P_{cb} is computed to be:

$$P_{cb} = \hat{P}_{32} - \frac{\hat{P}_{34} \cdot \hat{P}_{42}}{\hat{P}_{44}} = 28.6058 - \frac{26.6131 \cdot 28.359}{85.6659} = 19.7957$$

Following the Kron reduction, the phase potential coefficient matrix is:

$$[P_{abc}] = \begin{bmatrix} 77.1194 & 26.7944 & 15.8714 \\ 26.7944 & 75.1720 & 19.7957 \\ 15.8714 & 19.7957 & 76.2923 \end{bmatrix} \text{mile/}\mu\text{F}$$

Invert $[P_{abc}]$ to determine the shunt capacitance matrix:

$$[C_{abc}] = [P]^{-1} = \begin{bmatrix} 0.015 & -0.0049 & -0.0019 \\ -0.0049 & 0.0159 & -0.0031 \\ -0.0019 & -0.0031 & 0.0143 \end{bmatrix} \mu\text{F/mile}$$

Multiply $[C_{abc}]$ by the radian frequency to determine the final three-phase shunt admittance matrix.

$$[y_{abc}] = j \cdot 376.9911 \cdot [C_{abc}] = \begin{bmatrix} j5.6711 & -j1.8362 & -j0.7033 \\ -j1.8362 & j5.9774 & -j1.169 \\ -j0.7033 & -j1.169 & j5.3911 \end{bmatrix} \mu\text{S/mile}$$

5.2.1 The Shunt Admittance of Overhead Parallel Lines

The development of the shunt admittance matrix for parallel overhead lines is similar to the steps taken to create the phase impedance matrix. The numbering of the conductors must be the same as that which was used in developing the phase impedance matrix. To develop the shunt admittance matrix for overhead lines, it is necessary to know the distance from each conductor to the ground, and it will be necessary to know the radius in feet for each conductor.

The first step is to create the primitive potential coefficient matrix. This will be an *n-cond* × *n-cond* matrix, where *n-cond* is the total number of phase and ground conductors. For the lines in Figure 4.8, *n-cond* will be 7; for two lines each with its own grounded neutral, *n-cond* will be 8.

The elements of the primitive potential coefficient matrix are given by:

$$\begin{aligned}\hat{P}_{ii} &= 11.17689 \cdot \ln \frac{S_{ii}}{RD_i} \\ \hat{P}_{ij} &= 11.17689 \cdot \ln \frac{S_{ij}}{D_{ij}}\end{aligned} \quad \text{mile/μF} \tag{5.16}$$

where S_{ii} = distance in ft from a conductor to its image below ground
S_{ij} = distance in ft from a conductor to the image of an adjacent conductor
D_{ij} = distance in ft between two overhead conductors
RD_i = radius in ft of conductor i

The last one or two rows and columns of the primitive potential coefficient matrix are eliminated by using Kron reduction. The resulting voltage equation is:

$$\begin{bmatrix} V1_{ag} \\ V1_{bg} \\ V1_{cg} \\ V2_{ag} \\ V2_{bg} \\ V2_{cg} \end{bmatrix} = \begin{bmatrix} P11_{aa} & P11_{ab} & P11_{ac} & P12_{aa} & P12_{ab} & P12_{ac} \\ P11_{ba} & P11_{bb} & P11_{bc} & P12_{ba} & P12_{bb} & P12_{bc} \\ P11_{ca} & P11_{cb} & P11_{cc} & P12_{ca} & P12_{cb} & P12_{cc} \\ P21_{aa} & P21_{ab} & P21_{ac} & P22_{aa} & P22_{ab} & P22_{ac} \\ P21_{ba} & P21_{bb} & P21_{bc} & P22_{ba} & P22_{bb} & P22_{bc} \\ P21_{ca} & P21_{cb} & P21_{cc} & P22_{ca} & P22_{cb} & P22_{cc} \end{bmatrix} \cdot \begin{bmatrix} q1_a \\ q1_b \\ q1_c \\ q2_a \\ q2_b \\ q2_c \end{bmatrix} \text{V} \tag{5.17}$$

In short hand form Equation 5.17 is:

$$[V_{LG}] = [P] \cdot [q] \tag{5.18}$$

The shunt capacitance matrix is determined by:

$$[q] = [P]^{-1} \cdot [V_{LG}] = [C] \cdot [V_{LG}] \tag{5.19}$$

The resulting capacitance matrix is partitioned between the third and fourth rows and columns.

$$[C] = [P]^{-1} = \begin{bmatrix} [C11] & [C12] \\ [C21] & [C22] \end{bmatrix} \tag{5.20}$$

The shunt admittance matrix is given by:

$$[y] = j\omega \cdot [C] \cdot 10^{-6} = \begin{bmatrix} [y11] & [y12] \\ [y21] & [y22] \end{bmatrix} \text{S} \tag{5.21}$$

where $\omega = 2 \cdot \pi \cdot \text{frequency}$.

Example 5.2

Determine the shunt admittance matrix for the parallel overhead lines in Example 4.2.

The position coordinates for the seven conductors and the distance matrix are defined in Example 4.2. The diagonal terms of the distance matrix (Example 4.2) must be the radius in feet of the individual conductors. For this example:

$$D_{1,1} = D_{2,2} = D_{3,3} = \frac{d_1}{24} = \frac{0.721}{24} = 0.0300'$$

$$D_{4,4} = D_{5,5} = D_{6,6} = \frac{d_2}{24} = \frac{0.567}{24} = 0.0236'$$

$$D_{7,7} = \frac{d_n}{24} = 0.0235'$$

The resulting distance matrix is:

$$[D] = \begin{bmatrix} 0.0300 & 2.5000 & 7.0000 & 3.2016 & 7.2801 & 2.0000 & 7.2111 \\ 2.5000 & 0.0300 & 4.5000 & 2.0000 & 4.9244 & 3.2016 & 6.1847 \\ 7.0000 & 4.5000 & 0.0300 & 4.9244 & 2.0000 & 7.2801 & 6.7082 \\ 3.2016 & 2.0000 & 4.9244 & 0.0236 & 4.5000 & 2.5000 & 4.2720 \\ 7.2801 & 4.9244 & 2.0000 & 4.5000 & 0.0236 & 7.0000 & 5.0000 \\ 2.0000 & 3.2016 & 7.2801 & 2.5000 & 7.0000 & 0.0236 & 5.6569 \\ 7.2111 & 6.1847 & 6.7082 & 4.272 & 5.0000 & 5.6569 & 0.0235 \end{bmatrix} \text{ft.}$$

The distances between conductors and conductor images (image matrix) can be determined by:

$$S_{i,j} = \left| d_i - d_j^* \right|$$

For this example, the image matrix is:

$$[S] = \begin{bmatrix} 70.000 & 70.045 & 70.349 & 68.046 & 68.359 & 68.000 & 64.125 \\ 70.045 & 70.000 & 70.145 & 68.000 & 68.149 & 68.046 & 64.018 \\ 70.349 & 70.145 & 70.000 & 68.149 & 68.000 & 68.359 & 64.070 \\ 68.046 & 68.000 & 68.149 & 66.000 & 66.153 & 66.047 & 62.018 \\ 68.359 & 68.149 & 68.000 & 66.153 & 66.000 & 66.370 & 62.073 \\ 68.000 & 68.046 & 68.359 & 66.047 & 66.370 & 66.000 & 62.129 \\ 64.125 & 64.018 & 64.070 & 62.018 & 62.073 & 62.129 & 60.000 \end{bmatrix} \text{ft.}$$

The distance and image matrices are used to compute the 7 × 7 potential coefficient matrix by:

$$Pp_{i,j} = 11.17689 \cdot \ln\left(\frac{S_{i,j}}{D_{i,j}}\right)$$

The primitive potential coefficient matrix is partitioned between the sixth and seventh rows and columns, and the Kron reduction method produces the 6 × 6 potential matrix. This matrix is then inverted and multiplied by $\omega = 376.9911$ to give the shunt admittance matrix. The final shunt admittance matrix in partitioned form is:

$$[y11] = \begin{bmatrix} j6.2992 & -j1.3413 & -j0.4135 \\ -j1.3413 & j6.5009 & -j0.8038 \\ -j0.4135 & -j0.8038 & j6.0257 \end{bmatrix} \mu\text{S/mile}$$

$$[y12] = \begin{bmatrix} -j0.7889 & -j0.2992 & -j1.6438 \\ -j1.4440 & -j0.5698 & -j0.7988 \\ -j0.5553 & -j1.8629 & -j0.2985 \end{bmatrix} \mu\text{S/mile}$$

$$[y21] = \begin{bmatrix} -j0.7889 & -j1.4440 & -j0.5553 \\ -j0.2992 & -j0.5698 & -j1.8629 \\ -j1.6438 & -j0.7988 & -j0.2985 \end{bmatrix} \mu\text{S/mile}$$

$$[y22] = \begin{bmatrix} j6.3278 & -j0.6197 & -j1.1276 \\ -j0.6197 & j5.9016 & -j0.2950 \\ -j1.1276 & -j0.2950 & j6.1051 \end{bmatrix} \mu\text{S/mile}$$

5.3 Concentric Neutral Cable Underground Lines

Most underground distribution lines consist of one or more concentric neutral cables. Figure 5.4 illustrates a basic concentric neutral cable with the center conductor being the phase conductor, and the concentric neutral strands displaced equally around a circle of radius R_b.

Referring to Figure 5.4, the following definitions apply:

R_b = Radius of a circle passing through the centers of the neutral strands
d_c = Diameter of the phase conductor
d_s = Diameter of a neutral strand
k = Total number of neutral strands

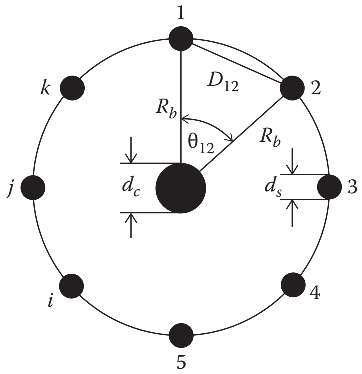

FIGURE 5.4
Basic concentric neutral cable.

The concentric neutral strands are grounded so that they are all at the same potential. Because of the stranding, it is assumed that the electric field created by the charge on the phase conductor will be confined to the boundary of the concentric neutral strands. In order to compute the capacitance between the phase conductor and ground, the general voltage drop of Equation 5.2 will be applied. Because all of the neutral strands are at the same potential, it is only necessary to determine the potential difference between the phase conductor p and strand 1.

$$V_{p1} = \frac{1}{2\pi\varepsilon}\left(q_p \ln\frac{R_b}{RD_c} + q_1 \ln\frac{RD_s}{R_b} + q_2 \ln\frac{D_{12}}{R_b} + \cdots + q_i \ln\frac{D_{1i}}{R_b} + \cdots + q_k \ln\frac{D_{k1}}{R_b}\right) \tag{5.22}$$

where $RD_c = \dfrac{d_c}{2}$

$RD_s = \dfrac{d_s}{2}$

It is assumed that each of the neutral strands carries the same charge such that:

$$q_1 = q_2 = q_i = q_k = -\frac{q_p}{k} \tag{5.23}$$

Equation 5.22 can be simplified:

$$V_{p1} = \frac{1}{2\pi\varepsilon}\left[q_p \ln\frac{R_b}{RD_c} - \frac{q_p}{k}\left(\ln\frac{RD_s}{R_b} + \ln\frac{D_{12}}{R_b} + \cdots + \ln\frac{D_{1i}}{R_b} + \cdots + \ln\frac{D_{1k}}{R_b}\right)\right]$$

$$V_{p1} = \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{RD_s \cdot D_{12} \cdot D_{1i},\ldots,D_{1k}}{R_b^k}\right)\right] \tag{5.24}$$

The numerator of the second ln term in Equation 5.24 needs to be expanded. The numerator represents the product of the radius and the distances between strand *i* and all of the other strands. Referring to Figure 5.4, the following relations apply:

$$\theta_{12} = \frac{2 \cdot \pi}{k}$$

$$\theta_{13} = 2 \cdot \theta_{12} = \frac{4 \cdot \pi}{k}$$

In general, the angle between strand #1 and any other strand #*i* is given by:

$$\theta_{1i} = (i-1) \cdot \theta_{12} = \frac{(i-1) \cdot 2\pi}{k} \tag{5.25}$$

The distances between the various strands are given by:

$$\begin{aligned} D_{12} &= 2 \cdot R_b \cdot \sin\left(\frac{\theta_{12}}{2}\right) = 2 \cdot R_b \cdot \sin\left(\frac{\pi}{k}\right) \\ D_{13} &= 2 \cdot R_b \cdot \sin\left(\frac{\theta_{13}}{2}\right) = 2 \cdot R_b \cdot \sin\left(\frac{2\pi}{k}\right) \end{aligned} \tag{5.26}$$

The distance between strand 1 and any other strand *i* is given by:

$$D_{1i} = 2 \cdot R_b \cdot \sin\left(\frac{\theta_{1i}}{2}\right) = 2 \cdot R_b \cdot \sin\left[\frac{(i-1) \cdot \pi}{k}\right] \tag{5.27}$$

Equation 5.27 can be used to expand the numerator of the second log term of Equation 5.24.

$$\begin{aligned} RD_s \cdot D_{12},\ldots,D_{1i},\ldots,D_{1k} = RD_s \cdot R_b^{k-1}\Bigg[2\sin\left(\frac{\pi}{k}\right) \cdot 2\sin\left(\frac{2\pi}{k}\right),\ldots,2\sin \\ \times\left\{\left(\frac{(i-1)\pi}{k}\right)\right\},\ldots,2\sin\left\{\left(\frac{(k-1)}{k}\right)\right\}\Bigg] \end{aligned} \tag{5.28}$$

The term inside the bracket in Equation 5.28 is a trigonometric identity that is merely equal to the number of strands *k* [1]. Using that identity, Equation 5.18 becomes:

$$V_{p1} = \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{k \cdot RD_s \cdot R_b^{k-1}}{R_b^k}\right)\right]$$

$$V_{p1} = \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{k \cdot RD_s}{R_b}\right)\right] \tag{5.29}$$

Equation 5.29 gives the voltage drop from the phase conductor to neutral strand #1. Care must be taken such that the units for the various radii are the same. Typically, underground spacings are given in inches; so the radii of the phase conductor (RD_c) and the strand conductor (RD_s) should be specified in inches.

Because the neutral strands are all grounded, Equation 5.29 gives the voltage drop between the phase conductor and ground. Therefore, the capacitance from the phase to ground for a concentric neutral cable is given by:

$$C_{pg} = \frac{q_p}{V_{p1}} = \frac{2\pi\varepsilon}{\ln\frac{R_b}{RD_c} - \frac{1}{k}\ln\frac{k \cdot RD_s}{R_b}}\ \mu\text{F/mile} \tag{5.30}$$

where $\varepsilon = \varepsilon_0\varepsilon_r$ = permittivity of the medium

ε_0 = permittivity of free space = 0.01420 μF/mile

ε_r = relative permittivity of the medium

The electric field of a cable is confined to the insulation material. Various types of insulation material are used, and each will have a range of values for the relative permittivity. Table 5.1 gives the range of values of relative permittivity for four common insulation materials [2].

TABLE 5.1

Typical Values of Relative Permittivity (ε_r)

Material	**Range of Values of Relative Permittivity**
Polyvinyl chloride (PVC)	3.4–8.0
Ethylene-propylene rubber (EPR)	2.5–3.5
Polyethylene (PE)	2.5–2.6
Cross-linked polyethlyene (XLPE)	2.3–6.0

Cross-linked polyethlyene is a very popular insulation material. If the minimum value of relative permittivity is assumed (2.3), the equation for the shunt admittance of the concentric neutral cable is given by:

$$y_{ag} = 0 + j\frac{77.3619}{\ln\frac{R_b}{RD_c} - \frac{1}{k}\ln\frac{k \cdot RD_s}{R_b}}\ \mu\text{S/mile} \tag{5.31}$$

Example 5.3

Determine the three-phase shunt admittance matrix for the concentric neutral line in Example 4.3 in Chapter 4.

From Example 4.3:

$$R_b = R = 0.0511\,\text{ft} = 0.631\,\text{in.}$$

Diameter of the 250,000 AA phase conductor = 0.567 in.

$$RD_c = \frac{0.567}{2} = 0.2835\ \text{in.}$$

Diameter of the #14 CU concentric neutral strand = 0.0641 in.

$$RD_s = \frac{0.0641}{2} = 0.03205\ \text{in.}$$

Substitute into Equation 5.24:

$$y_{ag} = j\frac{77.3619}{\ln\left(\frac{R_b}{RD_c}\right) - \frac{1}{k}\cdot\ln\left(\frac{k \cdot RD_s}{R_b}\right)}$$

$$y_{ab} = j\frac{77.3619}{\ln\left(\frac{0.6132}{0.2835}\right) - \frac{1}{13}\cdot\ln\left(\frac{13 \cdot 0.03205}{0.6132}\right)} = j96.6098\ \mu\text{S/mile}$$

The phase admittance for this three-phase underground line is:

$$[y_{abc}] = \begin{bmatrix} j96.6098 & 0 & 0 \\ 0 & j96.6098 & 0 \\ 0 & 0 & j96.6098 \end{bmatrix} \mu\text{S/mile}$$

5.4 Tape-Shielded Cable Underground Lines

A tape-shielded cable is shown in Figure 5.5.

Referring to Figure 5.5, R_b is the radius of a circle passing through the center of the tape shield. As with the concentric neutral cable, the electric field is confined to the insulation so that the relative permittivity of Table 5.1 will apply.

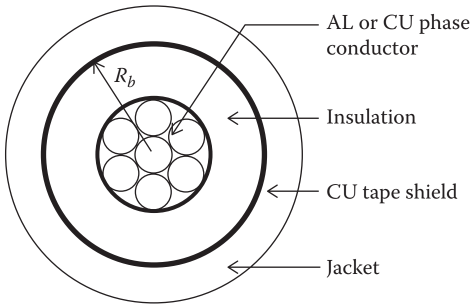

FIGURE 5.5
Tape-shielded conductor.

The tape-shielded conductor can be visualized as a concentric neutral cable where the number of strands k has become infinite. When k in Equation 5.24 approaches infinity, the second term in the denominator approaches zero. Therefore, the equation for the shunt admittance of a tape-shielded conductor becomes:

$$y_{ag} = 0 + j\frac{77.3619}{\ln\frac{R_b}{RD_c}}\,\mu\text{S/mile} \tag{5.32}$$

Example 5.4

Determine the shunt admittance of the single-phase tape-shielded cable in Example 4.4 in Chapter 4. From Example 4.4, the outside diameter of the tape shield is 0.88 in. The thickness of the tape shield (T) is 5 mils. The radius of a circle passing through the center of the tape shield is:

$$T = \frac{5}{1000} = 0.005$$

$$R_b = \frac{d_s - T}{2} = \frac{0.88 - 0.005}{2} = 0.4375\,\text{in.}$$

The diameter of the 1/0 AA phase conductor = 0.368 in.

$$RD_c = \frac{d_p}{2} = \frac{0.368}{2} = 0.1840\,\text{in.}$$

Substitute into Equation 5.25:

$$y_{bg} = j\frac{77.3619}{\ln\left(\frac{R_b}{RD_c}\right)} = j\frac{77.3619}{\ln\left(\frac{0.4375}{0.184}\right)} = j89.3179$$

The line is on phase *b* so that the phase admittance matrix becomes:

$$[y_{abc}]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & j89.3179 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mu\text{S/mile}$$

5.5 Sequence Admittance

The sequence admittances of a three-phase line can be determined in much the same manner as the sequence impedances were determined in Chapter 4. Assume that the 3 × 3 admittance matrix is given in S/mile. Then the three-phase capacitance currents as a function of the line-to-ground voltages are given by:

$$\begin{bmatrix} Icap_a \\ Icap_b \\ Icap_c \end{bmatrix}=\begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ba} & y_{bb} & y_{bc} \\ y_{ca} & y_{cb} & y_{cc} \end{bmatrix}\cdot\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} \tag{5.33}$$

$$[Icap_{abc}]=[y_{abc}]\cdot[VLG_{abc}] \tag{5.34}$$

Applying the symmetrical component transformations:

$$[Icap_{012}]=[A_s]^{-1}\cdot[Icap_{abc}]=[A_s]^{-1}\cdot[y_{abc}]\cdot[A_s]\cdot[VLG_{012}] \tag{5.35}$$

From Equation 5.35, the sequence admittance matrix is given by:

$$[y_{012}]=[A_s]^{-1}\cdot[y_{abc}]\cdot[A_s]=\begin{bmatrix} y_{00} & y_{01} & y_{02} \\ y_{10} & y_{11} & y_{12} \\ y_{20} & y_{21} & y_{22} \end{bmatrix} \tag{5.36}$$

For a three-phase overhead line with unsymmetrical spacing, the sequence admittance matrix will be full. That is, the off-diagonal terms will be nonzero. However, a three-phase underground line with three identical cables will only have the diagonal terms since there is no "mutual capacitance" between phases. In fact, the sequence admittances will be exactly the same as the phase admittances.

5.6 The Shunt Admittance of Parallel Underground Lines

For underground cable lines using either concentric neutral cables or tape-shielded cables, the computation of the shunt admittance matrix is quite simple. The electric field created by the charged phase conductor does not link to adjacent conductors because of the presence of the concentric neutrals or the tape shield. As a result, the shunt admittance matrix for parallel underground lines will consist of diagonal terms only.

The diagonal terms for concentric neutral cables are given by:

$$y_{ii} = 0 + j\frac{77.3619}{\ln\frac{R_b}{RD_i} - \frac{1}{k}\cdot\ln\frac{k\cdot RD_s}{R_b}}\cdot 10^{-6}\ \text{S/mile} \tag{5.37}$$

where R_b = radius in ft of circle going through the center of the neutral strands
RD_i = radius in ft of the center phase conductor
RD_s = radius in ft of the neutral strands
k = number of neutral strands

The diagonal terms for tape-shielded cables are given by:

$$y_{ii} = 0 + j\frac{77.3619}{\ln\frac{R_b}{RD_i}}\cdot 10^{-6}\ \text{S/mile} \tag{5.38}$$

where R_b = radius in ft of circle passing through the center of the tape shield
RD_i = radius in ft of the center phase conductor

Example 5.5

Compute the shunt admittance matrix (6 × 6) for the concentric neutral underground configuration in Example 4.5.

From Example 4.5:

Diameter of the central conductor: $d_c = 0.567''$

Diameter of the strands: $d_s = 0.641''$

Outside diameters of concentric neutral strands: $d_{od} = 1.29''$

Radius of circle passing through the strands: $R_b = \frac{d_{od} - d_s}{24} = 0.0511'$

Radius of central conductor: $RD_c = \frac{d_c}{24} = \frac{0.567}{24} = 0.236'$

Radius of the strands: $RD_s = \frac{d_s}{24} = \frac{0.0641}{24} = 0.0027'$

Because all cables are identical, the shunt admittance of a cable is:

$$y_c = 0 + j \cdot \frac{77.3619}{\ln\left(\frac{R_b}{RD_c}\right) - \frac{1}{k} \cdot \ln\left(\frac{k \cdot RD_s}{R_b}\right)} = 0 + j \cdot \frac{77.3619}{\ln\left(\frac{0.0511}{0.0236}\right) - \frac{1}{13} \cdot \ln\left(\frac{13 \cdot 0.0027}{0.0511}\right)}$$

$$y_c = 0 + j \cdot 96.6098\ \mu\text{S/mile}$$

The phase admittance matrix is:

$$[y]_{abc} = \begin{bmatrix} j96.6098 & 0 & 0 & 0 & 0 & 0 \\ 0 & j96.6098 & 0 & 0 & 0 & 0 \\ 0 & 0 & j96.6098 & 0 & 0 & 0 \\ 0 & 0 & 0 & j96.6098 & 0 & 0 \\ 0 & 0 & 0 & 0 & j96.6098 & 0 \\ 0 & 0 & 0 & 0 & 0 & j96.6098 \end{bmatrix} \mu\text{S/mile}$$

5.7 Summary

Methods for computing the shunt capacitive admittance for overhead and underground lines have been presented in this chapter. The development of computing the shunt admittance matrix for parallel overhead and underground lines is included.

Distribution lines are typically so short that the shunt admittance can be ignored. However, there are cases of long, lightly loaded overhead lines where the shunt admittance should be included. Underground cables have a much higher shunt admittance per mile than overhead lines. Again, there will be cases where the shunt admittance of an underground cable should be included in the analysis process. When the analysis is being done using a computer, the approach to take is to model the shunt admittance for both overhead and underground lines, rather than making a simplifying assumption when it is not necessary.

Problems

5.1 Determine the phase admittance matrix $[y_{abc}]$ and sequence admittance matrix $[y_{012}]$ in μS/mile for the three-phase overhead line of Problem 4.1.

5.2 Determine the phase admittance matrix in μS/mile for the two-phase line of Problem 4.2.

5.3 Determine the phase admittance matrix in μS/mile for the single-phase line of Problem 4.3.

5.4 Verify the results of Problems 5.1, 5.2, and 5.3 using WindMil.

5.5 Determine the phase admittance matrix and sequence admittance matrix in μS/mile for the three-phase line of Problem 4.5.

5.6 Determine the phase admittance matrix in μS/mile for the single-phase concentric neutral cable of Problem 4.9.

5.7 Determine the phase admittance matrix and sequence admittance matrix for the three-phase concentric neutral line of Problem 4.10.

5.8 Verify the results of Problems 5.6 and 5.7 using WindMil.

5.9 Determine the phase admittance matrix in μS/mile for the single-phase tape-shielded cable line of Problem 4.12.

5.10 Determine the phase admittance for the three-phase tape-shielded cable line of Problem 4.13.

5.11 Verify the results of Problem 5.9 and 5.10 using WindMil.

5.12 Determine the shunt admittance matrix for the parallel overhead lines of Problem 4.15.

5.13 Determine the shunt admittance matrix for the underground concentric neutral parallel lines of Problem 4.16.

WindMil Assignment

Add to the WindMil System 1 a single-phase line connected to Node 2. Call this "System 2." The single-phase line is on phase *b* and is defined in Problem 4.3. Call this line OH-2. At the end of the line, connect a node and call it Node 3. The load at Node 3 is 200 kVA at a 90% lagging power factor. The load is modeled as a constant impedance load.

Determine the voltages at the nodes on a 120-V base and the currents flowing on the two lines.

References

1. Glover, J. D. and Sarma, M., *Power System Analysis and Design*, 2nd Edition, PWS-Kent Publishing, Boston, MA, 1995.
2. T. P. Arnold and Mercier, C. D. (eds), *Power Cable Manual*, 2nd Edition, Southwire Company, Carrollton, GA, 1997.