COORDINATION OF OVERCURRENT DIRECTIONAL RELAYS IN MESHED NETWORKS USING THE SIMPLEX METHOD

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Abstract - In this paper one formulates the coordination problem of a set of directional overcurrent relays installed in meshed networks. After analysing the topology of the network to identify the primary/back up pairs, the coordination problem is organized in terms of a linear programming problem to be solved using the Simplex method. As results, this application gives the instantaneous relay settings, the relay pick up taps and the time dial settings. The methodology is illustrated using a small network mainly conceived for didactic purposes.

I. INTRODUCTION

Modern societies are more and more dependent on electricity to fulfill the needs of industrial development and quality of life so that large interconnected power systems provide a very important service. However, several kinds of faults may occur and when those situations happen the faulted components must be readily identified and isolated in other to guarantee the energy supply to the largeest number of consumers as possible and to keep system stability. Relays detect such faults and are able to give a tripping order to circuit breakers. The operation of these relays must be coordinated in the sense that if a fault occurs one wishes that only the faulted component is isolated. This requires that, given a fault, only its primary relays operate. The corresponding back up relays should only operate if one of those primary ones fail to trip. The process of relay coordination gets more difficult as the network's structure is more complex and more reduced operating times are required in order to increase the system availability and stability.

Traditionally, the protection engineer used to spend most of his time on performing calculations and manipulating graphics in order to coordinate a set of relays according to some technical constraints. The problem was, however, difficult as:

- one would have to cope with large quantities of information;
- lots of calculations had to be done;
- during the process lots of settings had to be changed and their values had to be recalculated;
- changes in the settings of one relay could originate changes in several others;
- when coordination was not obtained one should consider the possibility of substituting some protection devices by others having different technical characteristics;

All these aspects suggested the use of computational procedures to obtain the relay settings so that coordination was guaranteed. The first efforts on this direction were reported by Albrecht et al [1] and by Begian [2]. In [3], Dwarakanath and Nowitz describe the application of Graph Theory to obtain a relative sequence to set directional overcurrent relays. However, it was only in the 80^{th} that a more systematic approach was developed by Damborg, Ramaswami, Venkata and Postforoosh [4,5,6,7]. In this procedure these authors also use graph concepts to identify a sequential order to set directional relays, as well as, the set of primary/back up relays. Using this information, an incremental iterative approach is adopted to identify the time dial settings of the relays. In [8] a minimization procedure to identify relay settings is described. In this algorithm, Urdaneta et al assume that both the pick up and time dial settings are continuous in a given range and formulate an optimization problem to evaluate the optimal values of these variables. The obtained pick up values are rounded to the nearest available tap.

In the proposed paper a Simplex based approach to the relay coordination problem is presented. This approach includes three modules as follows:

- in the first one, Graph Theory concepts are used to study the network's structure to obtain information useful to the coordination process. This study includes a procedure to build the graph's loop matrix and a module to identify the set of primary/ back up relays which contains, for each relay to be set, all its primary ones;
- the second one allows the user to obtain the values of fault currents after specifying the type of fault and its location;
- and, finally, a coordination procedure that identifies the values of the relay settings by solving a Linear Programming Problem using the Simplex Method. In order to reduce the dimension of the problem to be solved the Upper Bounding technique is used.

This algorithm is exemplified using a case study based on an 8 bus, 9 branches network.

II. NETWORK TOPOLOGY ANALYSIS

The structure of a network is analysed using Graph Theory concepts to obtain, in a systematic way, the set of primary relay/back up relay (pr/br) pairs.

A. Building the Simple Loop Matrix - SLM

This matrix is obtained after building the graph associated to the network being studied and assigning each branch an orientation in an arbitrary way. In such a graph one can identified two classes of loops:

- simple loops corresponding to closed paths where all nodes (except the first) are touched only once;
- multiple loops are closed paths built as superpositions of simple loops;

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In the set of simple loops one can identify several subsets. Any of these subsets is a set of fundamental loops if its cardinality is minimum and any loop of the graph can be built by composition of a number of fundamental loops.

The SLM matrix contains information about the branches integrating the simple loops of the graph. This matrix is obtained after selecting a tree - connected graph having no loops - of the graph and building an incidence matrix A, whose element a_{ij} is:

- 1 if branch j is incident to vertex i and directed outwards this vertex;
- -1 if branch j is incident to vertex i and directed towards this vertex;
- 0 if branch j is not incident to vertex i.

After eliminating from A the line corresponding to a reference node, this matrix can be partitioned according to (1). In this expression, A_{11} is a sub matrix of A that includes information about the branches not included in the tree of the graph previously referred.

$$A = [A_{11} | A_{12}] \qquad (1)$$

Matrix A can be used to build the fundamental loop matrix B_f according to (2). In this expression I is the identity matrix and B_{fb} is obtained using (3).

$$B_f = [I | B_{fb}] \qquad (2)$$

$$B_{fb}{}^t = -A_{12}{}^{-1} . A_{11} \qquad (3)$$

The lines of B_f are related to each of the fundamental loops and its columns to the branches of the graph. The SLM matrix is formed by all the linearly independent combinations of lines of B_f. Each line of SLM corresponds to a simple loop while each column is related to a branch of the graph.

B. The set of Primary / Back up relay (pr/br) pairs

The set of primary/back up relay pairs includes, for each relay to be set, all its primary relays. This set can be built using the augmented incidence matrix A_a given by (4). In this expression A is the incidence matrix built according to the rules indicated in II.A. Now, each line of A_a is related to a branch of the network while each column corresponds to a relay.

$$A_a = [A | -A] \qquad (4)$$

For each relay to be set it must be identified the bus i for which it is directed. Afterwards, one must identify the relays located in branches adjacent to bus i and directed outwards that bus. This algorithm can be readily implemented using the A_a matrix considering the following steps:

- for a relay k to be set, select line j of A_a related to the network branch where that relay is installed;
- in the line j of A_a identify all +1 elements and the respective columns;
- the relays associated to these columns are primary relays of k provided that they are not installed in branch j, that is, in the same branch of the relay k to be set;

III. FAULT CALCULATION MODULE

The coordination methodology assumes that a set of fault currents are available. These can correspond to any kind of faults namely, three phase faults or single to ground faults. Considering a three phase fault and once the fault location is specified, one can simulate, in an approximate way, the new operating conditions of the system building its impedance matrix. Using this matrix it is well known that its diagonal elements correspond to the Thevenin impedances seen from each system bus.

IV. COORDINATION PROCEDURE

A. Modelization of Overcurrent Directional Relay Operation

Overcurrent relays generally include an instantaneous unit and an inverse time equipment. The inverse time operation characteristics can be provided in terms of a family of curves depending on a parameter usually referred as the time dial, t_d. The mathematical modelization of this family of curves can be performed using multiple regression techniques in order to obtain an expression giving the operation time in function of the time dial and the current flowing through the relay. Using, for instance the operation curves of the CO9 overcurrent relay of Westinghouse, expression (5) can be used to evaluate its operation time, t_{op}. The relative errors of t_{op} values given by (4.1) regarding the actual operation times as specified by the relay curves are less than 4%.

$$t_{op} = \sum_{i=1}^{4} b_i . \frac{1}{(I-1)^i} + b_5 . t_d + \sum_{i=6}^{8} b_i . \frac{t_d}{(I-1)^{i-5}} \qquad (5)$$

According to (4.1), the operation time, t_{op}, depends on:

- I - current going through the relay referred to the pick up value;
- t_d - time dial;
- b_i - coefficients obtained using a regression process;

This model is linear with t_d. This means that once a fault current is available the relay t_{op} is given by (6).

$$t_{op} = \alpha + \gamma . t_d \qquad (6)$$

B. Instantaneous tap setting

The setting of the instantaneous unit of the overcurrent relay can be selected multiplying the Instantaneous Factor - usually 1.3 - by the largest fault current flowing through the relay if one simulates a fault in the bus for which the relay is directed. In setting this unit one can consider the network topology leading to the largest fault current flowing through the relay.

C. Pick up tap setting

The pick up current corresponds to the minimum current that, if exceeded, leads to the operation of the relay. According to [4] the user can select this value in a range determined by a minimum and a maximum value.

The minimum value is the largest between the current corresponding to the minimum available tap and the product of the larger load current by a security factor (usually 1.3 for phase protection and 0.05 for ground protection).

The maximum value is the minimum of the following two values:

- product of the fault current on the remote bus of the relay by a coefficient usually assuming the value 0.15. This current should be evaluated considering the most usual network configuration;
- product of the smaller fault current flowing through the relay by a factor assuming the value 0.55.

The upper bound guarantees that the relay is sensitive to all fault currents while lower one avoids relay operation for the maximum load conditions of the network. The pick up current should be selected by the user among the available pick-up tap values considering this range. The chosen value should be closer to the lower limit in order to increase the sensitivity of the relay.

D. Time dial setting

After simulating a set of faults, one can formulate a set of linear constraints as (7) relating, for each fault, the operation time of the back-up relay - $t_{op}(br)$ - and the operation time of the primary relay - $t_{op}(pr)$. Each of these operating times will be given by expressions as (6) assuming that the current exceeds the pick up value. On the other hand, the operation time is 0.0 if the current exceeds the instantaneous value. In this constraint CI represents the coordination interval corresponding to the minimum difference required for the operation time of any pair of primary/back up relays for a given fault.

$$t_{op}(br) - t_{op}(pr) \geq CI \qquad (7)$$

Regarding the objective function one can think of minimizing the sum of the time dials of all relays installed in the network. This function can be altered considering that, in some cases, the minimization of the operation times of some relays are far more important than others. In these cases, one can assign weights, w_i, to the time dial variables of those relays in the objective function leading to (8).

$$Z = \sum_{i=1}^{n} w_i \cdot t_{di} \qquad (8)$$

The set of linear constraints (7) together with this linear objective function originates the following Linear Programming Problem where n is the number of directional overcurrent relays to be set.

$$\min Z = \sum_{i=1}^{n} w_i \cdot t_{di} \qquad (9)$$

$$\text{sub} \quad t_{op}(bc) - t_{op}(p) \geq CI \quad \text{(all faults)} \qquad (10)$$

$$t_{di}{}_{min} \leq t_{di} \leq t_{di}{}_{max} \quad (i=1..,n) \qquad (11)$$

The solution of this linear problem can be obtained using the Upper Bounding technique in order to reduce the size of the inverse basic matrix inherent to the Simplex Method.

V. CASE STUDY

The proposed methodology will be illustrated using the 8 bus, 9 branch network sketched in figure 1. In tables I to IV one presents the system data (line, transformer and generator data, and load values). In buses 1, 4 and 6 there are also links to other networks modeled by short circuit powers of 400 MVAr each. In this figure one also indicates the location of the overcurrent directional relays.

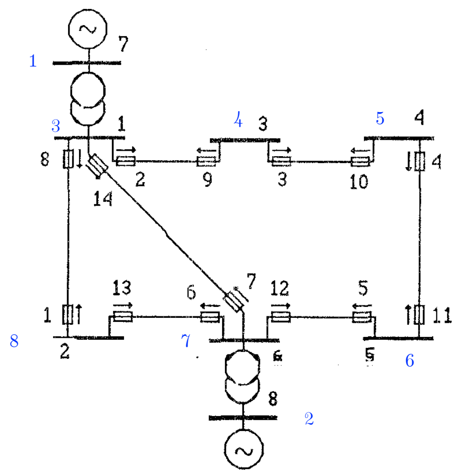

Fig. 1. Network scheme.

Table I - Line characeteristics.

extreme nodes		R(Ω/km)	X(Ω/km)	Y(S/km)	l(km)
1	2	0.004	0.05	0.0	100.0
1	3	0.0057	0.0714	0.0	70.0
3	4	0.005	0.0563	0.0	80.0
4	5	0.005	0.045	0.0	100.0
5	6	0.0045	0.0409	0.0	110.0
2	6	0.0044	0.05	0.0	90.0
1	6	0.005	0.05	0.0	100.0

Table II - Transformer data.

extreme nodes		Sn(MVA)	Vp(kV)	Vs(kV)	x(%)
7	1	150.0	10.0	150.0	4.0
8	6	150.0	10.0	150.0	4.0

Table III - Generator data.

node	Sn(MVA)	Vn(kV)	xsub(%)
7	150.0	10.0	15.0
8	150.0	10.0	15.0

Table IV - Load data.

node	P(MW)	Q(MVAr)
2	40.0	20.0
3	60.0	40.0
4	70.0	40.0
5	70.0	50.0

For the fourteen relays installed in the network the following data are also considered:

- 6 available instantaneous tap settings (10.0, 15.0, 20.0, 24.0, 30.0 and 40.0);
- 7 available pick up tap settings (0.5, 0.6, 0.8, 1.0, 1.5, 2.0 and 2.5);
- the ratio of the associated Current Transformer - CT - as indicated in table 6;
- the time dial varies in the range from 0.5 to 11.0;

The network topology module was used to identify the primary/back up relay pairs listed in table V.

Table V - List of primary/back up relay pairs.

pair no.	pr	br	pair no.	pr	br
1	1	6	11	14	9
2	7	13	12	8	9
3	12	13	13	5	4
4	2	7	14	9	10
5	8	7	15	4	3
6	6	14	16	3	2
7	12	14	17	10	11
8	13	8	18	2	1
9	6	5	19	14	1
10	7	5	20	11	12

Using these data one simulated a set of faults gathered in three groups:

- type I faults - located in the remote bus of each relay to be set;
- type II faults - located near each primary relay of a realy to be set;
- type III faults - located immediately after the zone protected in an instantaneous way by each of the primary realys to be set.

It should also be mentioned that these faults were simulated for the network presented in figure 1 and, also, considering that line 1-6 was disconnected. The 78 resulting faults were analysed to evaluate the currents flowing throught its primary relays and each of the corresponding back up relays. Assuming a 0.3 s CI, one formulates a linear programming problem integrating:

- 78 constraints, one for each simulated fault;
- 28 constraints representing the admissible range of the time dial of each relay;
- an objective function assuming that weights w_i are all set to 1.0;

Once the instantaneous and the pick up taps are fixed according to the procedure described in sections IV.B and IV.C, the algorithm described in section IV.D gives the t_d values presented in table VI.

Table VI - CT ratio, instantaneous and pick up taps, and time dial setting for each relay.

relay no.	CT ratio	instant. tap	pick up tap	time dial
1	240.0	10.0	0.5	5.5303
2	240.0	24.0	2.0	3.6875
3	160.0	20.0	1.5	3.6034
4	240.0	15.0	1.5	2.1313
5	240.0	8.0	1.0	1.7704
6	240.0	20.0	2.0	2.1447
7	160.0	20.0	1.5	3.0741
8	240.0	20.0	1.5	2.9114
9	160.0	12.0	1.5	1.7143
10	240.0	20.0	1.5	2.1795
11	240.0	15.0	1.0	3.9344
12	240.0	24.0	2.0	4.0490
13	240.0	10.0	0.5	6.1096
14	160.0	20.0	1.5	3.3468

VI. CONCLUSIONS

In this paper one summarizes some Graph Theory concepts leading to a network structure module used to obtain a set of primary/back up relay pairs. The paper also details the modelization of relay operation curves and presents the complete formulation of the coordination problem of a set of directional overcurrent relays. Traditionally, this was a complex task due to the required amount of information and calculations. In fact, the skills and experience of the engineer were key factors to a successful solution. Therefore, we would like to stress the importance of applications like this to help the protection's engineer solving complex tasks in a systematic and efficient way.

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