Mercados Eléctricos
Despacho
Subastas
Contratos

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Ciclo de 1 semana – 6 sesiones

Sesión 4- 4 de diciembre de 2025

COMPETENCIA PERFECTA EJEMPLO - 1 GENERADOR

Perfect Competition

Competencia Perfecta: Un Generador - Demanda Elástica

$$P^{\max} = 100 \frac{\$}{\mathrm{MWh}}, m_D = 20 \frac{\$}{\mathrm{MW^2h}} \quad P^{\min} = 5 \frac{\$}{\mathrm{MWh}}, m_O = 15 \frac{\$}{\mathrm{MW^2h}}$$

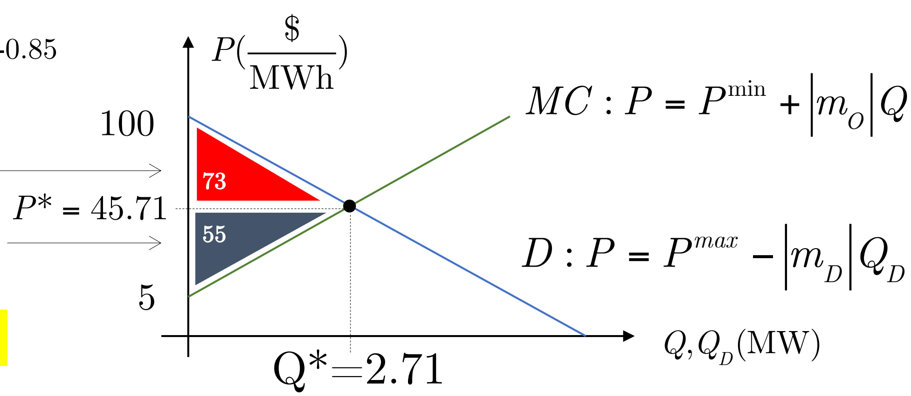
$$\begin{bmatrix} Q^* \\ Q_D^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 15 & 0 & -1 \\ 0 & -20 & -1 \\ -1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -5 \\ -100 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.7143 \\ 2.7143 \\ 45.7143 \end{bmatrix}$$

$$E_p(Q = 2.71) = 1 - \frac{100}{20 \cdot 2.7} = -0.85$$

$$CSP^* = P^{max} Q^* - \frac{1}{2}|m_D| Q^{*2} - P^* Q^*$$

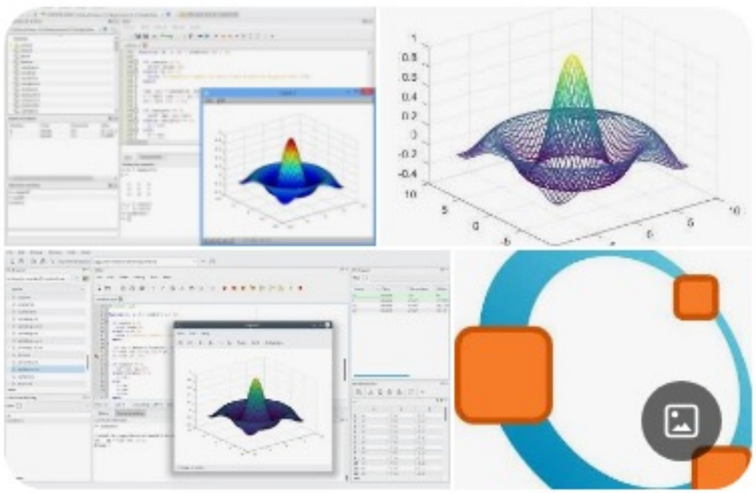
$$PSP^* = P^* Q^* - P^{\min} Q^* - \frac{1}{2}|m_O| Q^{*2}$$

SW*=128 \$/h



GNU Octave

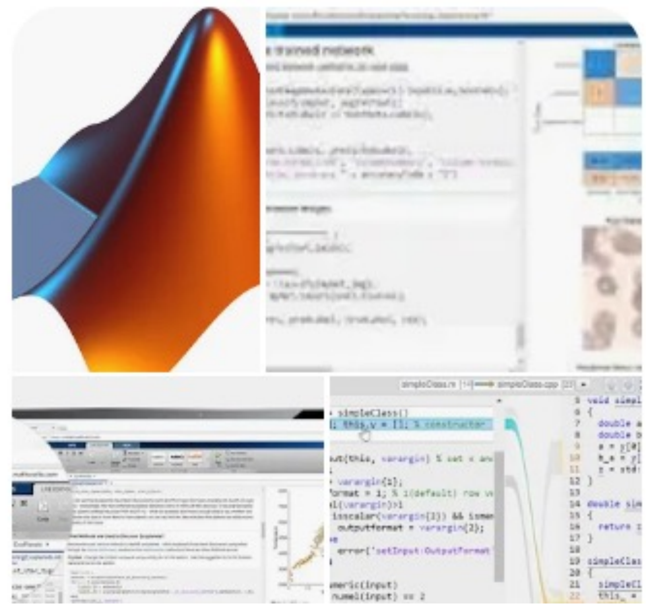
Programming language



https://octave.org/

MATLAB

Programming language



```
%Clearing the market (1 producer - 1 demand)
% cp: perfect competition
mO1=15;pmax=100;mD=20;p1min=5
B=[-p1min; -pmax; 0];
Acp=[mO1 0 -1;
0 -mD -1;
-1 1 0];
Solcp=inv(Acp)*B
Past=Solcp(3);
Qast=Solcp(1);
CSP = 0.5*Qast*(pmax-Past) % Area bajo la curva superior
CSPx= pmax*Qast-0.5*mD*Qast*Qast-Past*Qast
PSPx= Past*Qast-p1min*Qast-0.5*mO1*Qast*Qast
PSP = 0.5*Qast*(Past-p1min) % Area bajo la curva inferior
SW=PSP+CSP
```

ED_cp1.m

Octave Online

Variables

```
[3x3] Acp
[3x1] B
# CSP
# CSPx
# PSP
# PSPx
# Past
# Qast
# SW
[3x1] Solcp
# ans
# mD
# mO1
# p1min
# pmax
```

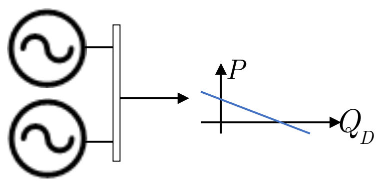
```
octave:1> %Clearing the market (1 producer - 1 demand)
% cp: perfect competition
mO1=15;pmax=100;mD=20;p1min=5
B=[-p1min; -pmax; 0];
Acp=[mO1 0 -1;
     0  -mD -1;
     -1  1 0];
Solcp=inv(Acp)*B
Past=Solcp(3);
Qast=Solcp(1);
CSP = 0.5*Qast*(pmax-Past) % Area bajo la curva superior
CSPx=  pmax*Qast-0.5*mD*Qast*Qast-Past*Qast
PSPx= Past*Qast-p1min*Qast-0.5*mO1*Qast*Qast
PSP = 0.5*Qast*(Past-p1min) % Area bajo la curva inferior
SW=PSP+CSP
p1min = 5
Solcp =

    2.7143
    2.7143
   45.7143

CSP = 73.673
CSPx = 73.673
PSPx = 55.255
PSP = 55.255
SW = 128.93
```

COMPETENCIA PERFECTA EJEMPLO - 2 GENERADORES

Competencia Perfecta: Dos Generadores - Demanda Elástica



$$P^{\max} = 100\,\frac{\$}{\mathrm{MWh}},\ m_D = 20\,\frac{\$}{\mathrm{MW^2h}} \qquad P_1^{\min} = 5\,\frac{\$}{\mathrm{MWh}},\ m_{O1} = 15\,\frac{\$}{\mathrm{MW^2h}}$$

$$P_2^{\min} = 10\,\frac{\$}{\mathrm{MWh}},\ m_{O2} = 10\,\frac{\$}{\mathrm{MW^2h}}$$

$$\begin{bmatrix} Q_1^* \\ Q_n^* \\ Q_D^* \\ \lambda \end{bmatrix} = \begin{bmatrix} |m_{O1}| & 0 & 0 & -1 \\ 0 & |m_{On}| & 0 & -1 \\ 0 & 0 & -|m_D| & -1 \\ -1 & -1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -P_1^{\min} \\ -P_n^{\min} \\ -P^{\max} \\ 0 \end{bmatrix} = \begin{bmatrix} 1.61 \\ 1.92 \\ 3.53 \\ 29.23 \end{bmatrix}$$

$$PSP_1^* = P^* \cdot Q_1^* - \int_0^{Q_1^*} P_1^{\min} + |m_{O1}| Q_1 \, dQ_1 = 19.57$$

$$PSP_2^* = P^* \cdot Q_2^* - \int_0^{Q_2} P_2^{\min} + |m_{O2}| Q_2 \, dQ_2 = 18.49$$

$$PSP^* = PSP_1^* + PSP_2^* = 38.06$$

$$CSP^* = \int_0^{Q_D^*} P^{\max} - |m_D| Q_D dQ_D - P^* \cdot Q_D^* = 125.2$$

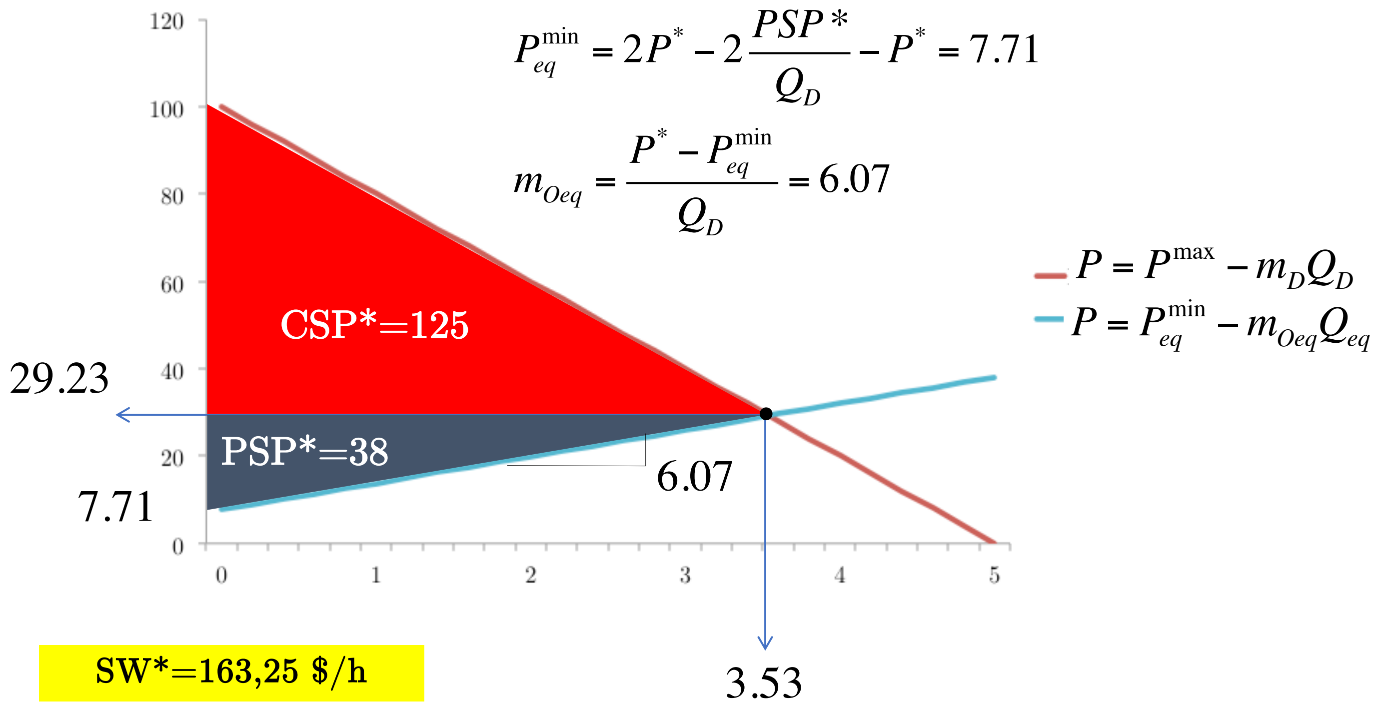
$$SW = CSP^* + PSP^* = 163.25$$

Perfect Competition

Competencia Perfecta: Dos Generadores - Demanda Elástica

$$P_{eq}^{\min} = 2P^* - 2\frac{PSP^*}{Q_D} - P^* = 7.71$$

$$m_{Oeq} = \frac{P^* - P_{eq}^{\min}}{Q_D} = 6.07$$



SW*=163,25 $/h

```
%Clearing the market (2 producers - 1 demand)
% cp: perfect competition
mO1=15;mO2=10;pmax=100;mD=20;p1min=5;p2min=10;
B=[-p1min; -p2min; -pmax; 0];
Acp=[mO1 0 0 -1;
     0 mO2 0 -1;
     0 0 -mD -1;
     -1 -1 1 0];
Solcp=inv(Acp)*B
Past=Solcp(4);
Qast1=Solcp(1);
Qast2=Solcp(2);
Qast=Solcp(3);
PSP1=Past*Qast1-p1min*Qast1-0.5*mO1*Qast1*Qast1
PSP2=Past*Qast2-p2min*Qast2-0.5*mO2*Qast2*Qast2
PSP=PSP1+PSP2
CSP = pmax*Qast-0.5*mD*Qast*Qast-Past*Qast
SW=PSP+CSP
%Elasticity at equilibrium point
DP=(pmax-Past);
DQ=0-Qast;
e=(DQ/Qast)/(DP/Past)
```

ED_cp2.m

Octave Online

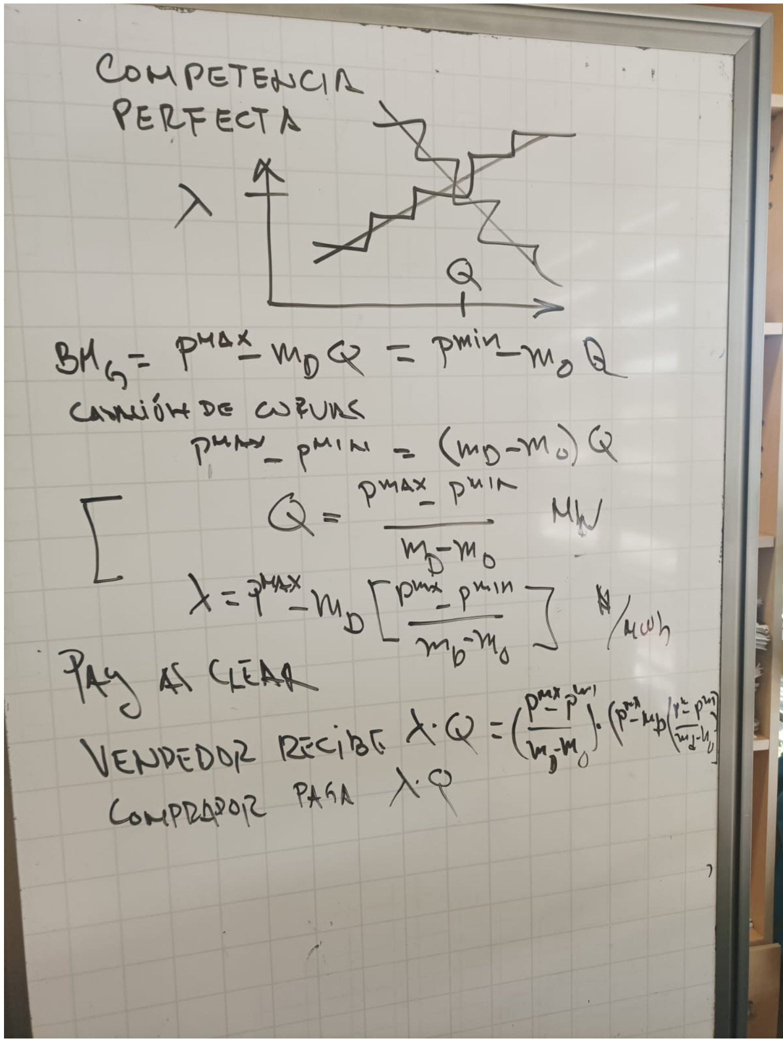
Variables

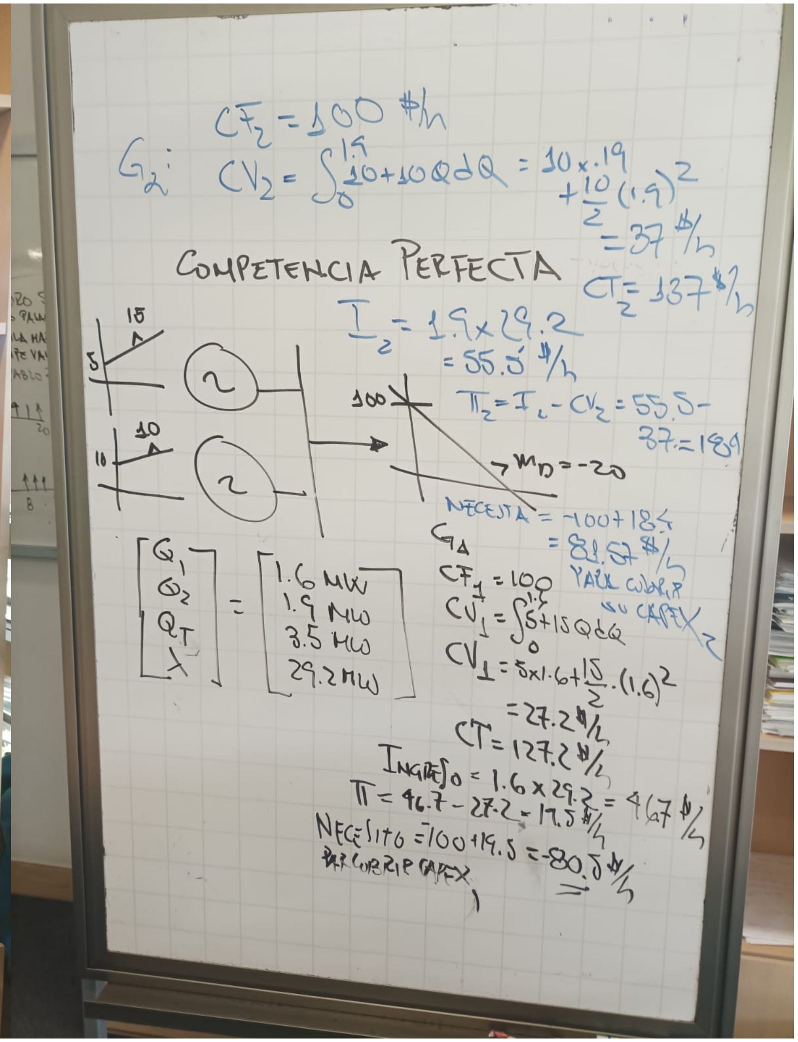
[4x4] Acp
[4x1] B
CSP
CSPx
DP
DQ
PSP
PSP1
PSP2
PSPx
Past
Qast
Qast1
Qast2
SW
[4x1] Solcp
ans
e
mD
m01
m02
p1min
p2min
pmax

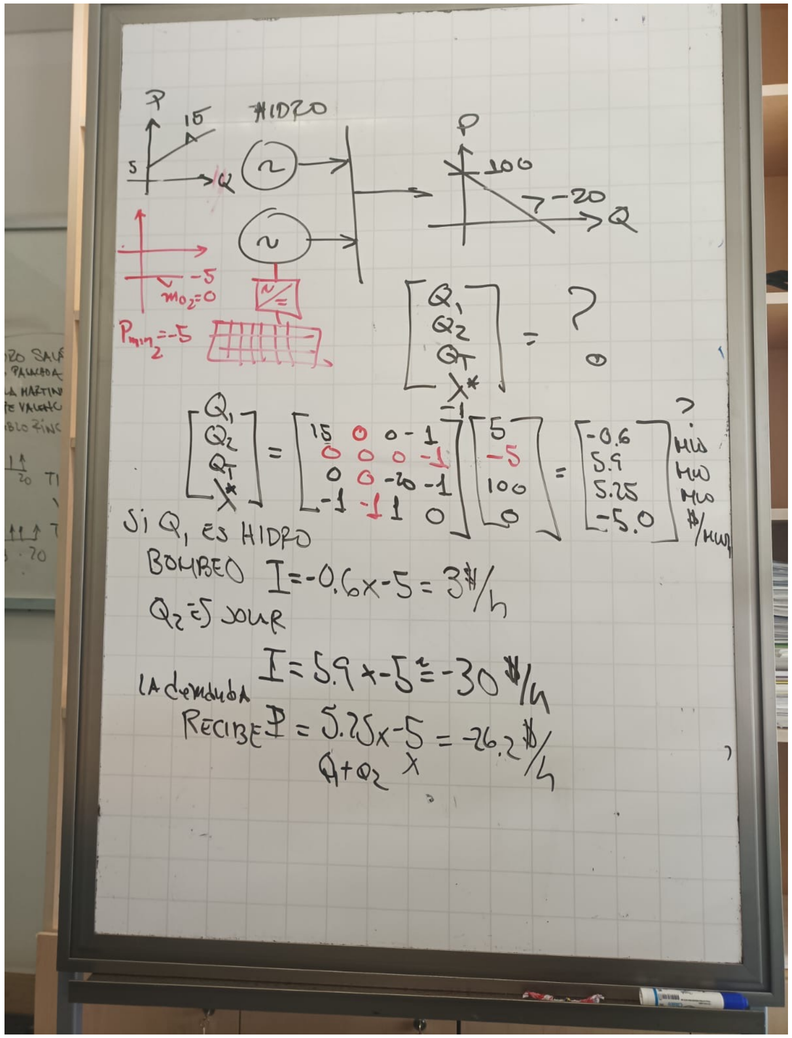
```
octave:12> %Clearing the market (2 producers - 1 demand)
% cp: perfect competition
m01=15;m02=10;pmax=100;mD=20;p1min=5;p2min=10;
B=[-p1min; -p2min; -pmax; 0];
Acp=[m01 0 0 -1;
     0 m02 0 -1;
     0 0 -mD -1;
     -1 -1 1 0];
Solcp=inv(Acp)*B
Past=Solcp(4);
Qast1=Solcp(1);
Qast2=Solcp(2);
Qast=Solcp(3);
PSP1=Past*Qast1-p1min*Qast1-0.5*m01*Qast1*Qast1
PSP2=Past*Qast2-p2min*Qast2-0.5*m02*Qast2*Qast2
PSP=PSP1+PSP2
CSP = pmax*Qast-0.5*mD*Qast*Qast-Past*Qast
SW=PSP+CSP
%Elasticity at equilibrium point
DP=(pmax-Past);
DQ=0-Qast;
e=(DQ/Qast)/(DP/Past)
Solcp =

    1.6154
    1.9231
    3.5385
   29.2308

PSP1 = 19.571
PSP2 = 18.491
PSP = 38.062
CSP = 125.21
SW = 163.27
e = -0.4130
```


COMPETENCIA PERFECTA
λ
Q
BM_G = P^MAX − m_D Q = P^min − m_O Q
CANALIÓN DE CURVAS
P^MAX − P^MIN = (m_D − m_O) Q
Q = (P^MAX − P^MIN) / (m_D − m_O) MW
λ = P^MAX − m_D [(P^MAX − P^MIN) / (m_D − m_O)] $/MWh
PAY AS CLEAR
VENDEDOR RECIBE λ·Q
COMPRADOR PAGA λ·Q


CF_2 = 100 $/h
G_2: CV_2 = ∫_0^1.9 10+10Q dQ = 10 x 1.9 + 10/2 (1.9)^2 = 37 $/h
COMPETENCIA PERFECTA
CT_2 = 137 $/h
I_2 = 1.9 x 29.2 = 55.5 $/h
Π_2 = I_2 − CV_2 = 55.5 − 37 = 18.5
m_D = −20
NECESITA = −100 + 18.5 = 81.5 $/h PARA CUBRIR SU CAPEX_2
15
5
10
10
100
G_1
CF_1 = 100
CV_1 = ∫_0^1.6 5+15Q dQ
CV_1 = 5x1.6 + 15/2 . (1.6)^2 = 27.2 $/h
CT = 127.2 $/h
Q_1 = 1.6 MW
Q_2 = 1.9 MW
Q_T = 3.5 MW
λ = 29.2 MW
INGRESO = 1.6 x 29.2 = 46.7 $/h
Π = 46.7 − 27.2 = 19.5 $/h
NECESITO = 100 + 19.5 = −80.5 $/h
PARA CUBRIR CAPEX



G2 es una planta PV
con m02 =
y pmin2 = -5 $/MWh

Perfect Competition

MAX SOCIAL WELFARE: CSP+PSP
WITH PRODUCTION CONSTRAINTS (MAX-MIN)

Lagrangiano:

$$L=\sum_{i=1}^{n}\left[FC+\rho_i^{\min}P_{Gi}+\frac{1}{2}\left|m_{Oi}\right|\left(P_{Gi}\right)^2\right]-\left[\rho^{\max}P_D-\frac{1}{2}\left|m_D\right|\left(P_D\right)^2\right]+\lambda\left[P_D-\sum_{i=1}^{n}P_{Gi}\right]$$

$$+\sum_{i=1}^{n}\mu_i\left[P_{Gi}-P_{Gi}^{\max}\right]+\sum_{i=1}^{n}\nu_i\left[P_{Gi}^{\min}-P_{Gi}\right]$$

$$\frac{\partial L}{\partial P_{Gi}}=\rho_i^{\min}+\left|m_{Oi}\right|P_{Gi}-\lambda+\mu_i-\nu_i=0\ \ i=1,\ldots,n$$

$$\frac{\partial L}{\partial P_D}=-\rho^{\max}+\left|m_D\right|P_D+\lambda=0$$

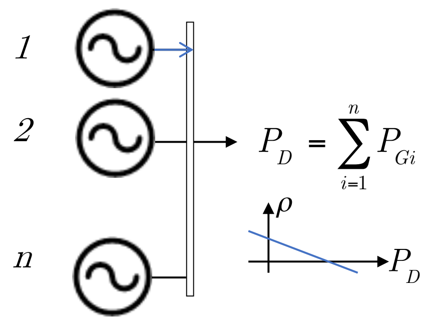
See Matlab program: EDLimits.m

$$\frac{\partial L}{\partial \lambda}=P_D-\sum_{i=1}^{n}P_{Gi}=0,$$

$$\frac{\partial L}{\partial \mu_i}=P_{Gi}-P_{Gi}^{\max}+\beta_i=0,\ \beta_i\geq 0, i=1,\ldots,n \qquad \frac{\partial L}{\partial \nu_i}=P_{Gi}^{\min}-P_{Gi}+\vartheta_i=0,\ \vartheta_i\geq 0, i=1,\ldots,n$$

$$\mu_i(P_{Gi}-P_{Gi}^{\max})=0, i=1,\ldots,n \qquad \nu_i(P_{Gi}^{\min}-P_{Gi})=0, i=1,\ldots,n$$

Perfect Competition



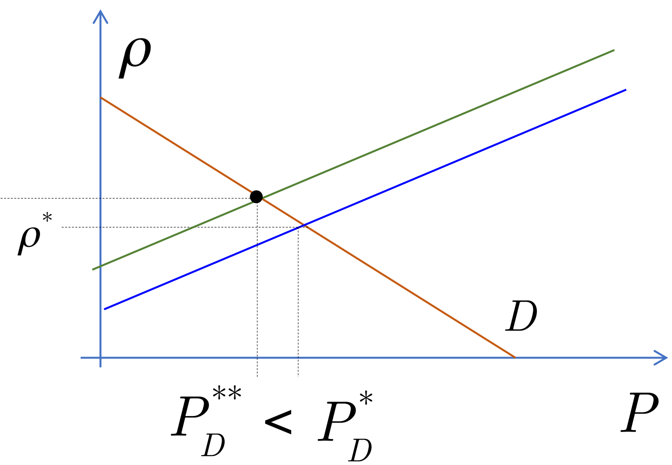
$$P_D = \sum_{i=1}^{n} P_{Gi}$$

$$P_{Gi} < P_{Gi}^{\max} \Rightarrow \mu_i = 0 \quad i = 1,\ldots.,n$$

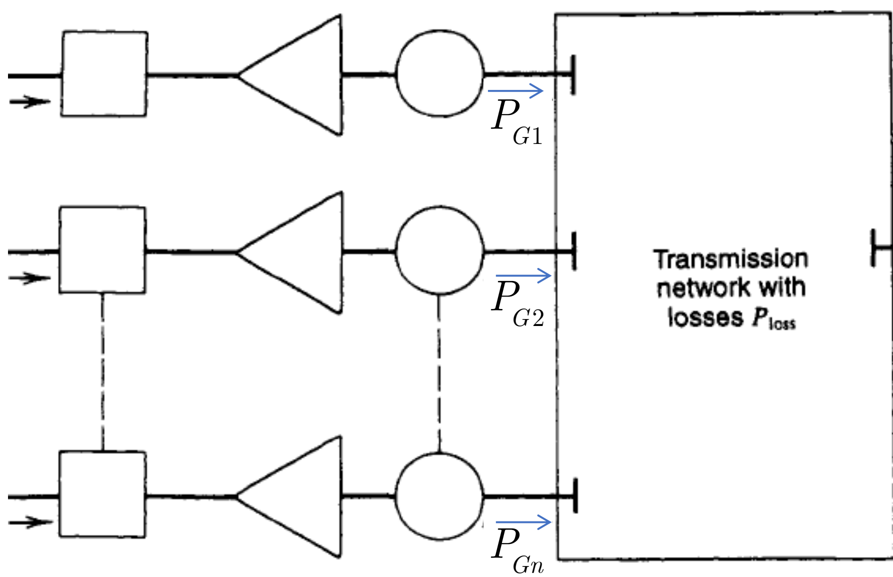
$$\rho^{**} = \lambda = \rho_1^{\min} + |m_{O1}| P_{G1}^{**} + \mu_1 = \frac{\partial SC}{\partial P_{G1}} > \rho^*$$

$$\rho^{**} = \lambda = \rho_2^{\min} + |m_{O2}| P_{G2}^{**} + \mu_2 = \frac{\partial SC}{\partial P_{G2}} > \rho^*$$

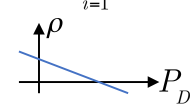
	Benefit	*Cost*
G1	$P_{G1}^{**}(\lambda - \mu_1)$	$\rho_1^{\min} P_{G1}^{**} + \frac{1}{2}\lvert m_{O1}\rvert \left(P_{G1}^{**}\right)^2$
G2	$P_{G2}^{**}(\lambda - \mu_1)$	$\rho_2^{\min} P_{G2}^{**} + \frac{1}{2}\lvert m_{O2}\rvert \left(P_{G2}^{**}\right)^2$
D	$\rho^{\max} P_D^{**} - \frac{1}{2}\lvert m_D\rvert \left(P_D^{**}\right)^2$	$P_D^{**}(\lambda - \mu_1)$



MAX SOCIAL WELFARE DISPATCH
WITH PRODUCTION CONSTRAINTS (MAX-MIN)
AND NETWORK LOSES

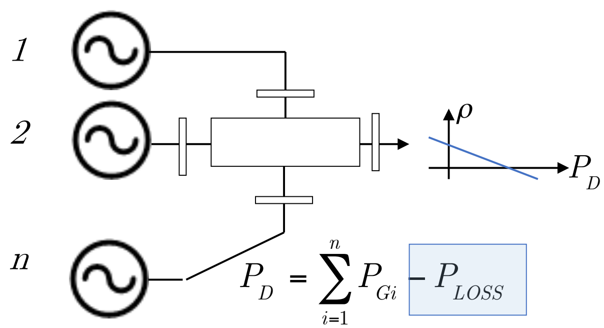


$$P_D = \sum_{i=1}^{n} P_{Gi} \boxed{- P_{LOSS}}$$



Perfect Competition

MAX SOCIAL WELFARE DISPATCH
WITH PRODUCTION CONSTRAINTS (MAX-MIN)
AND NETWORK LOSES



$$P_D = \sum_{i=1}^{n} P_{Gi} \boxed{- P_{LOSS}}$$

$$\sum_{i=1}^{n} P_{Gi} \cdot \lambda_{Gi} \neq P_D \lambda_D$$

Network Revenue:

$$NR = P_D \lambda_D - \sum_{i=1}^{n} P_{Gi} \cdot \lambda_{Gi}$$

$$\min SC = -SW = CSP + PSP - NR$$

$$\min TC - U = \sum_{i=1}^{n} \left[\int_0^{P_{Gi}} MC(P_{Gi})\, dP_{Gi} \right] - \int_0^{P_D} D(P_D)\, dP_D$$

$$\text{s.t.} \quad P_D = \sum_{i=1}^{n} P_{Gi} \boxed{- P_{LOSS}}$$

Perfect Competition

MAX SOCIAL WELFARE: CSP+PSP
WITH PRODUCTION CONSTRAINTS (MAX-MIN)
AND NETWORK LOSES

Lagrangiano:

$$L = \sum_{i=1}^{n}\left[FC + \rho_i^{\min} P_{Gi} + \frac{1}{2}\left|m_{Oi}\right|\left(P_{Gi}\right)^2\right] - \left[\rho^{\max} P_D - \frac{1}{2}\left|m_D\right|\left(P_D\right)^2\right] + \lambda\left[P_D - \sum_{i=1}^{n} P_{Gi} \boxed{- P_{LOSS}}\right]$$

$$+\sum_{i=1}^{n} \mu_i\left[P_{Gi} - P_{Gi}^{\max}\right] + \sum_{i=1}^{n} \nu_i\left[P_{Gi}^{\min} - P_{Gi}\right]$$

$$\frac{\partial L}{\partial P_{Gi}} = \rho_i^{\min} + \left|m_{Oi}\right| P_{Gi} - \lambda\left(1 - \boxed{\frac{dP_{LOSS}}{dP_{Gi}}}\right) + \mu_i - \nu_i = 0 \quad i = 1,\ldots,n$$

$$\frac{\partial L}{\partial P_D} = -\rho^{\max} + \left|m_D\right| P_D + \lambda = 0$$

Coeficiente
Incremental de Pérdidas del nodo de generación i

$$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{n} P_{Gi} \boxed{- P_{LOSS}} = 0,$$

$$\frac{\partial L}{\partial \mu_i} = P_{Gi} - P_{Gi}^{\max} + \beta_i = 0,\ \beta_i \geq 0, i = 1,\ldots,n \qquad \frac{\partial L}{\partial \nu_i} = P_{Gi}^{\min} - P_{Gi} + \vartheta_i = 0,\ \vartheta_i \geq 0, i = 1,\ldots,n$$

$$\mu_i(P_{Gi} - P_{Gi}^{\max}) = 0, i = 1,\ldots,n \qquad \nu_i(P_{Gi}^{\min} - P_{Gi}) = 0, i = 1,\ldots,n$$

Perfect Competition

MAX SOCIAL WELFARE: CSP+PSP
WITH PRODUCTION CONSTRAINTS (MAX-MIN)
AND NETWORK LOSES

$$P_{LOSS} = \mathbf{P_G B_{LOSS} P_G} + \mathbf{B}_0 \mathbf{P_G} + B_{00}$$

$$P_{LOSS} \approx \sum_{i=1}^{n} B_{ii} P_{Gi}^2 = \begin{bmatrix} P_{G1} & \cdots & P_{Gn} \end{bmatrix} \begin{bmatrix} B_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_{nn} \end{bmatrix} \begin{bmatrix} P_{G1} \\ \vdots \\ P_{Gn} \end{bmatrix}$$

See Bloss Formula: See Kirchmayer 1958, Wood & Wollemberg 1984

Perfect Competition

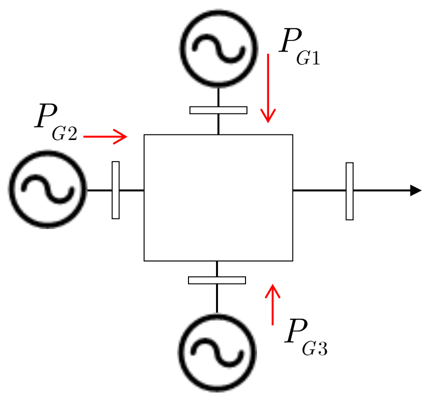
Wood & Wollemberg Example 3C [BUT WITH ELASTIC DEMAND]

$$F_1(P_1) = H_1(P_1) \times 1.1 = 561 + 7.92P_1 + 0.001562P_1^2 \text{ Ɍ/h}$$

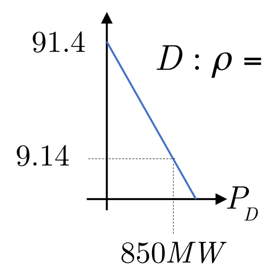
$$F_2(P_2) = H_2(P_2) \times 1.0 = 310 + 7.85P_2 + 0.00194P_2^2 \text{ Ɍ/h}$$

$$F_3(P_3) = H_3(P_3) \times 1.0 = 78 + 7.97P_3 + 0.00482P_3^2 \text{ Ɍ/h}$$

$$P_{\text{loss}} = 0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2$$



$$P_D = \sum_{i=1}^{n} P_{Gi} + P_{LOSS}$$



$$D : \rho = \rho^{max} - |m_D| P_D$$

MAX SOCIAL WELFARE: CSP+PSP
WITH PRODUCTION CONSTRAINTS (MAX-MIN)
AND NETWORK LOSES

Lagrangiano:

$$L = \sum_{i=1}^{3}\left[FC + \rho_i^{\min} P_{Gi} + \frac{1}{2}\left|m_{Oi}\right|\left(P_{Gi}\right)^2\right] - \left[\rho^{\max} P_D - \frac{1}{2}\left|m_D\right|\left(P_D\right)^2\right] + \lambda\left[P_D - \sum_{i=1}^{3} P_{Gi} + P_{LOSS}\right]$$

$$\frac{\partial L}{\partial P_{Gi}} = \rho_i^{\min} + \left|m_{Oi}\right| P_{Gi} - \lambda(1 - 2B_{ii}P_{Gi}) = 0,\ i = 1,\ldots,3$$

$$\frac{\partial L}{\partial P_D} = -\rho^{\max} + \left|m_D\right| P_D + \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{3} P_{Gi} + \sum_{i=1}^{3} B_{ii} P_{Gi}^2 = 0,$$

5 ecuaciones 5 incógnitas: $P_{G1}, P_{G2}, P_{G3}, P_D, \lambda$

$$\lambda_{Gi} = \lambda(1 - \frac{dP_{LOSS}}{dP_{Gi}}) = \lambda(1 - 2B_{ii}P_{Gi}), i = 1,\ldots,3$$

Perfect Competition

Network Revenue is not null

$$NR = P_D \lambda - \sum_{i=1}^{n} P_{Gi} \cdot \lambda_{Gi}$$

In general, taking into account generation and tranmission limits:

$$\lambda_{Gi} = \lambda\left(1 - \frac{dP_{LOSS}}{dP_{Gi}}\right) - \mu_i^G + \nu_i^G - \mu_i^T$$

$$pf_{Gi} = \frac{1}{\left(1 - \frac{dP_{LOSS}}{dP_{Gi}}\right)}$$

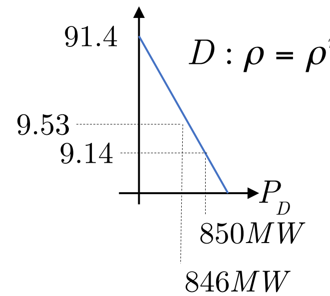
$$pf_{Gi} > 0 \Rightarrow \frac{dP_{LOSS}}{dP_{Gi}} > 0 \Rightarrow \Delta P_{Gi} \propto \Delta P_{LOSS}$$

Perfect Competition

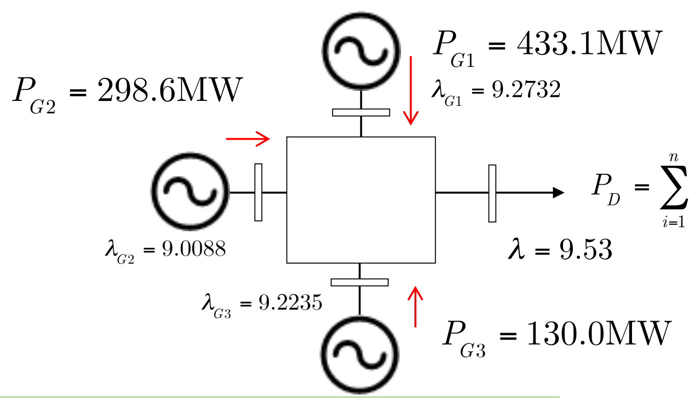
Wood & Wollemberg Example 3C [BUT WITH ELASTIC DEMAND]

$$\lambda_{G1} = \lambda - \lambda\frac{dLoss}{dP_{G1}} = \lambda(1+2B_{11}P_{G1}) = 9.53\underbrace{(1-0.0264)}_{\text{Penalty Factor}} = 9.2732$$

Penalty Factor
$PF_1 = 1/0.97 > 1$



$$D : \rho = \rho^{max} - |m_D| P_D$$



$$P_D = \sum_{i=1}^{n} P_{Gi} + P_{LOSS} = \overbrace{861.84}^{\text{Bulk Generation}} + \underbrace{15.68}_{\text{Losses}} = \underbrace{846.1\text{MW}}_{\text{Demand}}$$

See Matlab program: <u>EDLosses.m</u>

Social Cost = -3.5373e+04

Methods Dispatch Solution

When no Kuhn-Tucker coeficients are active, the ED problem is
Determined. Minimum is obteined from n non-linear equations
With n unknowns.

Linear programming (e.g. Simplex) for linearized cost and loss functions

In general, NonLinear Optimization Solvers should be applied.
Gradient
Interior Point
Newton

Tools: Minos/Gams, Frontline Solver
Matlab, SciPy, etc.

Other: Genetic Algorithm, Neural Networks

(See Zhu, 4.8, 4.9)

COMPETENCIA PERFECTA

SUBASTA PAY AS CLEAR

In a Pay-as-Clear market, **all** winning power plants receive the same price per megawatt-hour (MWh), regardless of what they originally bid. That price is determined by the **most expensive unit** needed to meet the demand (the "Marginal Unit").

The Demand (The Buyers)

- **Load 1:** 50 MW
- **Load 2:** 50 MW
- **Total Market Demand:**

$$50 + 50 = 100 \text{ MW}$$

The Supply (The Sellers) We have two power plants with different technologies and costs. To illustrate how the auction works effectively, we will assume **Plant A** (perhaps Solar or Wind) has a limitation (availability) or a strategy that restricts its bid volume, forcing the market to use the more expensive plant.

- **Plant A (Renewable/Cheap):**
  - **Capacity:** 100 MW
  - **Bid Offer:** 50 MW at **$20/MWh**
  - *(Note: Even though it has 100 MW capacity, let's assume availability is low or it only offers 50 MW into the auction).*
- **Plant B (Gas/Expensive):**
  - **Capacity:** 100 MW
  - **Bid Offer:** 100 MW at **$80/MWh**

2. The Auction (The Merit Order)

The System Operator (SO) ranks the bids from cheapest to most expensive to create the "Merit Order."

Rank	Generator	Quantity Offered	Bid Price	Cumulative Supply
1	Plant A	50 MW	**$20**	50 MW
2	Plant B	100 MW	**$80**	150 MW

3. Clearing the Market

The System Operator fills the 100 MW demand starting with the cheapest power.

1. **First 50 MW:** The operator takes all 50 MW from **Plant A** because it is the cheapest ($\$20$).
   - *Demand met:* 50 MW.
   - *Demand remaining:* 50 MW.
2. **Next 50 MW:** The operator must now turn to **Plant B**. They take 50 MW from Plant B to finish filling the demand.
   - *Demand met:* 100 MW.
   - *Demand remaining:* 0 MW.

The Market Clearing Price: The auction stops at **Plant B**. Because Plant B is the last unit needed to satisfy the demand (the Marginal Unit), its bid sets the price for *everyone*.

$$\text{Clearing Price} = \$80/\text{MWh}$$

4. The Result: Pay-as-Clear

This is the crucial part. Even though Plant A offered to sell its power for $20, it gets paid the Clearing Price of $80.

Why is this good for Plant A?

Plant A receives a **surplus** (profit). This creates an incentive to invest in cheap, efficient technologies (like wind, solar, or hydro).

- **Plant A Revenue:** $50 \text{ MW} \times \$80 = \$4,000$
- *If it were "Pay-as-Bid", they would have only made* $50 \times \$20 = \$1,000$.

The Final Payouts

Generator	Power Generated	Bid Price	**Paid Price**	Total Revenue
Plant A	50 MW	$20	**$80**	**$4,000**
Plant B	50 MW	$80	**$80**	**$4,000**
Plant B (Remaining)	50 MW	$80	N/A	$0 (Not dispatched)

1. Mathematical Formulation

Sets & Indices:

- $g \in \{1, 2\}$: The set of generators (Plant A, Plant B).
- $L \in \{1, 2\}$: The set of loads.

Parameters:

- C_g: Bid price of generator g ($/MWh).
- P_g^{max}: Maximum capacity of generator g (MW).
- D_l: Demand of load l (MW). Total Demand $D_{total} = \sum D_l$.

Variables:

- $x_g \geq 0$: Power generated by plant g (MW).

Objective Function: Minimize Total Cost:

$$\min Z = \sum_g C_g \cdot x_g$$

Constraints:

1. **Power Balance (Demand Constraint):** The total generation must equal the total load. The **dual variable** (shadow price) of this constraint is the **Clearing Price**.

$$\sum_g x_g = \sum_l D_l \qquad [\lambda]$$

2. **Capacity Constraints:**

$$x_g \leq P_g^{max} \quad \forall g$$

Gams

```
    s        Scenarios        / Scen1, Scen2 /;

Parameters
    Bid(g)        Bid Price ($ per MWh)
    Cap(g)        Capacity (MW)
    Demand        Total Demand (MW) / 100 /;

* Define Prices
Bid('PlantA') = 20;
Bid('PlantB') = 80;

* Define Variables
Positive Variable x(g);
Variable z;

* Define Equations
CostObj      Objective Function
BalEq        Supply Demand Balance
CapLim(g)    Capacity Limit;

CostObj..    z =e= sum(g, Bid(g) * x(g));
BalEq..      sum(g, x(g)) =e= Demand;
CapLim(g)..  x(g) =l= Cap(g);

Model Auction /all/;

* Reporting Parameter
Parameter Report(s, *);

* --- SCENARIO 1: Plant A restricted to 50MW ---
Cap('PlantA') = 50;
Cap('PlantB') = 100;

Solve Auction using lp minimizing z;

Report('Scen1', 'Gen_A') = x.l('PlantA');
Report('Scen1', 'Gen_B') = x.l('PlantB');
Report('Scen1', 'ClearingPrice') = BalEq.m;

* --- SCENARIO 2: Plant A full 100MW ---
* Note: To strictly ensure the price drops to 20 in LP,
* we assume A has margin or is the marginal unit.
Cap('PlantA') = 100;
Cap('PlantB') = 100;

Solve Auction using lp minimizing z;

Report('Scen2', 'Gen_A') = x.l('PlantA');
Report('Scen2', 'Gen_B') = x.l('PlantB');
Report('Scen2', 'ClearingPrice') = BalEq.m;

Display Report;
```

1. Mathematical Formulation

Sets & Indices:

- $g \in \{1, 2\}$: The set of generators (Plant A, Plant B).
- $L \in \{1, 2\}$: The set of loads.

Parameters:

- C_g: Bid price of generator g ($/MWh).
- P_g^{max}: Maximum capacity of generator g (MW).
- D_l: Demand of load l (MW). Total Demand $D_{total} = \sum D_l$.

Variables:

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Objective Function: Minimize Total Cost:

$$\min Z = \sum_g C_g \cdot x_g$$

Constraints:

1. **Power Balance (Demand Constraint):** The total generation must equal the total load. The **dual variable** (shadow price) of this constraint is the **Clearing Price**.

$$\sum_g x_g = \sum_l D_l \qquad [\lambda]$$

2. **Capacity Constraints:**

$$x_g \leq P_g^{max} \quad \forall g$$

```python
import numpy as np
from scipy.optimize import linprog

def solve_auction(scenario_name, cap_A, cap_B):
    # 1. Setup Data
    # Cost vector c = [Price_A, Price_B]
    c = [20, 80]

    # Equality Constraint: 1*A + 1*B = 100 (Demand)
    A_eq = [[1, 1]]
    b_eq = [100]

    # Bounds: 0 <= A <= cap_A, 0 <= B <= cap_B
    bounds = [(0, cap_A), (0, cap_B)]

    # 2. Solve LP
    # method='highs' is recommended for modern scipy versions
    res = linprog(c, A_eq=A_eq, b_eq=b_eq, bounds=bounds, method='

    # 3. Extract Results
    gen_A = res.x[0]
    gen_B = res.x[1]

    # The Clearing Price is the dual value (shadow price) of the e
    # Note: Scipy's 'highs' method stores equality duals in res.in
    # but strictly it's often easier to inspect the logic or use t
    if hasattr(res, 'eqlin'):
         # Negative sign convention sometimes used in optimization
        clearing_price = abs(res.eqlin.marginals[0])
    else:
        # Fallback for older scipy versions
        clearing_price = abs(res.con[0] if res.con else 0)

    # Output
    print(f"--- {scenario_name} ---")
    print(f"Capacity A: {cap_A} MW | Capacity B: {cap_B} MW")
    print(f"Dispatch:   Plant A = {gen_A:.1f} MW, Plant B = {gen_B
    print(f"Total Cost: ${res.fun:.2f}")
    print(f"Clearing Price (Shadow Price): ${clearing_price:.2f}/M
    print("-" * 30)

# Run Scenario 1: Plant A limited to 50 MW
solve_auction("Scenario 1 (Scarcity)", 50, 100)

# Run Scenario 2: Plant A has full 100 MW (plus epsilon to resolve
solve_auction("Scenario 2 (Abundance)", 100.001, 100)
```

Would you like to see how this scenario changes if Plant A bids its full 100 MW capacity?

1. The New Setup

- **Total Market Demand:** 100 MW (Load 1 + Load 2)
- **Plant A (Cheap):** Offers 100 MW at **$20/MWh**
- **Plant B (Expensive):** Offers 100 MW at **$80/MWh**

2. The New Merit Order

The System Operator ranks the bids again. This time, the "cheap" stack is much larger.

Rank	Generator	Quantity Offered	Bid Price	Cumulative Supply
1	Plant A	100 MW	**$20**	100 MW
2	Plant B	100 MW	**$80**	200 MW

3. Clearing the Market

The operator fills the 100 MW demand starting with the cheapest power.

1. **First 100 MW:** The operator takes the full 100 MW from **Plant A** because it is the cheapest ($20).
   - *Demand met:* 100 MW.
   - *Demand remaining:* 0 MW.

The Market Clearing Price: The auction stops at **Plant A**. Since Plant A is the last unit needed to satisfy the demand (the Marginal Unit), and no expensive power from Plant B was needed, the price is set by Plant A's bid.

$$\text{New Clearing Price} = \$20/\text{MWh}$$

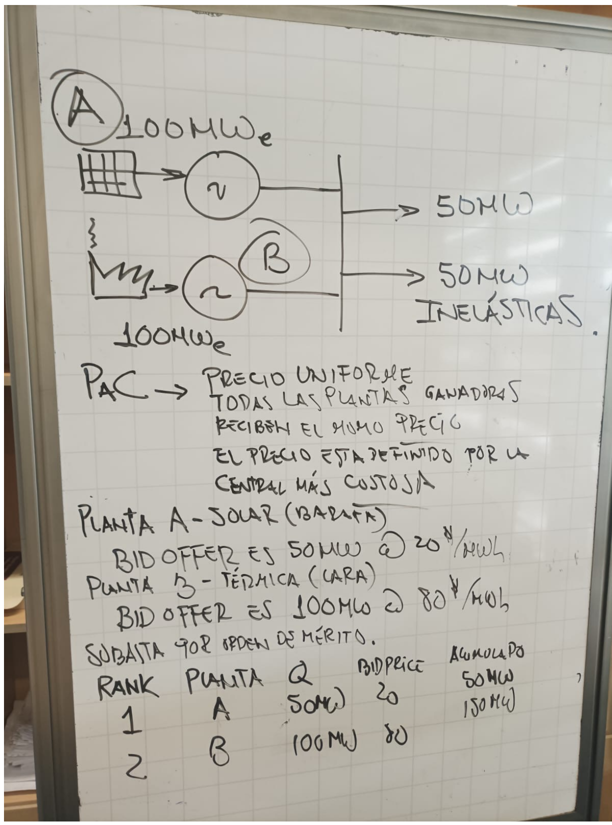
(Note: The price collapsed from $80 down to $20 simply because the expensive plant was not needed.)

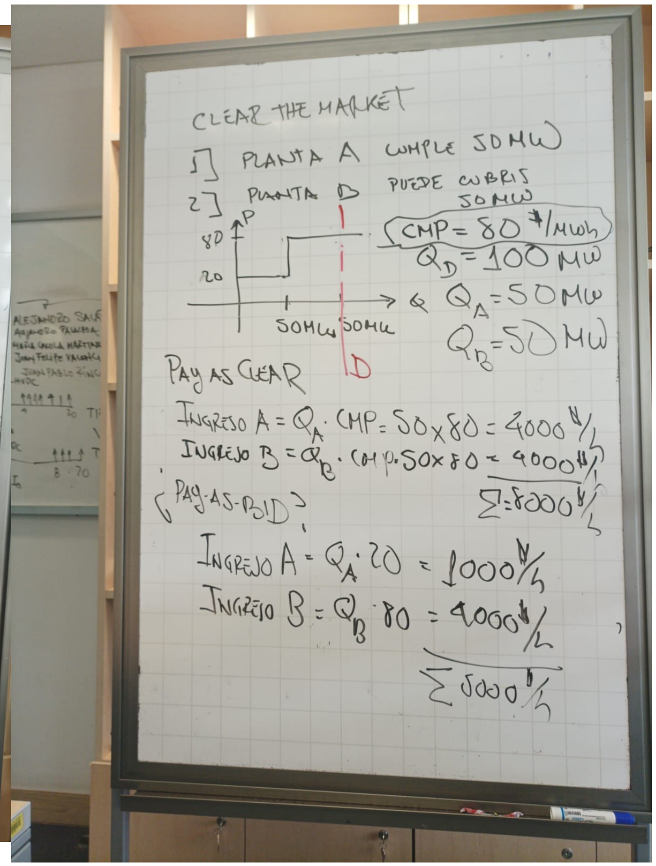
4. The Result: A Win for Consumers, A Loss for Plant A

This creates a fascinating paradox often seen in energy markets. By supplying *more* power, Plant A actually made *less* money than in the first example.

The Financial Comparison

Metric	Scenario 1 (Plant A restricted)	Scenario 2 (Plant A full capacity)
Clearing Price	**$80** / MWh	**$20** / MWh
Plant A Generation	50 MW	100 MW
Plant A Revenue	$4,000	**$2,000**
Plant B Revenue	$4,000	**$0**
Total Cost to Consumers	$8,000	**$2,000**


A
100MWe
50MW
B
50MW
INELÁSTICAS
100MWe
PAC → PRECIO UNIFORME
TODAS LAS PLANTAS GANADORAS
RECIBEN EL MISMO PRECIO
EL PRECIO ESTÁ DEFINIDO POR LA
CENTRAL MÁS COSTOSA
PLANTA A - SOLAR (BARATA)
BID OFFER ES 50MW @ 20$/MWh
PLANTA B - TÉRMICA (CARA)
BID OFFER ES 100MW @ 80$/MWh
SUBASTA POR ORDEN DE MÉRITO.
RANK PLANTA Q BIDPRICE ACUMULADO
1 A 50MW 20 50MW
2 B 100MW 80 150MW


CLEAR THE MARKET
1] PLANTA A CUMPLE 50MW
2] PLANTA B PUEDE CUBRIR 50MW
CMP = 80 $/MWh
QD = 100 MW
P
80
20
Q
50MW 50MW
D
QA = 50MW
QB = 50MW
Pay As CLEAR
INGRESO A = QA · CMP = 50x80 = 4000 $/h
INGRESO B = QB · CMP = 50x80 = 4000 $/h
Σ=8000 $/h
PAY-AS-BID
INGRESO A = QA · 20 = 1000 $/h
INGRESO B = QB · 80 = 4000 $/h
Σ 5000 $/h