MEDB 5501, Module08

2025-10-14

Topics to be covered

- What you will learn
    - The two-sample t-test
    - The t-distribution
    - Critical values and p-values
    - R code for the t-test
    - Confidence intervals
    - Sample size justification
    - R code for sample size justification
    - Your homework

Population model

- Population 1
  - $X_{11}, X_{12}, \ldots, X_{1N_1}$
  - X_{1i} are independent $N(\mu_1, \sigma_1)$
- Population 2
  - $X_{21}, X_{22}, \ldots, X_{2N_2}$
  - X_{2i} are independent $N(\mu_2, \sigma_2)$

Speaker notes

The two sample t-test is based on a population model where there are N_1 observations in the first population and N_2 observations in the second population. In general, the size of the two populations, N_1 and N_2 are assumed to be very large.

Sample values

- Sample 1
  - $X_{11}, X_{12}, \ldots, X_{1n_1}$
  - Calculate $\bar{X}_1$ and S_1
- Sample 2
  - $X_{21}, X_{22}, \ldots, X_{2n_2}$
  - Calculate $\bar{X}_2$ and S_2

Speaker notes

Because the populations are so large, you need to take a sample (hopefully a representative sample) from each population. With the sample, you can calculate sample statistics.

Hypothesis and test statistic

- $H_0 : \mu_1 - \mu_2 = 0$
- $H_1 : \mu_1 - \mu_2 \neq 0$
  - Accept H_0 if $\bar{X}_1 - \bar{X}_2$ is close to zero

Speaker notes

The null hypothesis for the two-sample t-test is that the population means, μ_1 and μ_2 are equal, which is the same as saying that the difference between the two population means is equal to zero.

The population means are unknown, but you can use the sample means,

How close is close?

- $T = \frac{\bar{X}_1 - \bar{X}_2}{se}$
- se = standard error
  - $se = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
- S_p = Pooled standard deviation - $S_p = \sqrt{\frac{n_1 \ S_1^2 + n_2 \ S_2^2}{n_1 + n_2}}$
- Not valid with heterogeneity
  - That is, $\sigma_1 \neq \sigma_2$

Speaker notes

You measure how close $\bar{X}_1 - \bar{X}_2$ is to zero by using the standard error. The standard error is a measure of how much sampling error you have when using $\bar{X}_1 - \bar{X}_2$ to estimate $\mu_1 - \mu_2$.

This standard error relies on equal variation in both groups. You'll hear more discussion of this issue later in the presentation.

Break #1

- What you have learned
  - The two-sample t-test
- What's coming next
  - The t-distribution

The t distribution

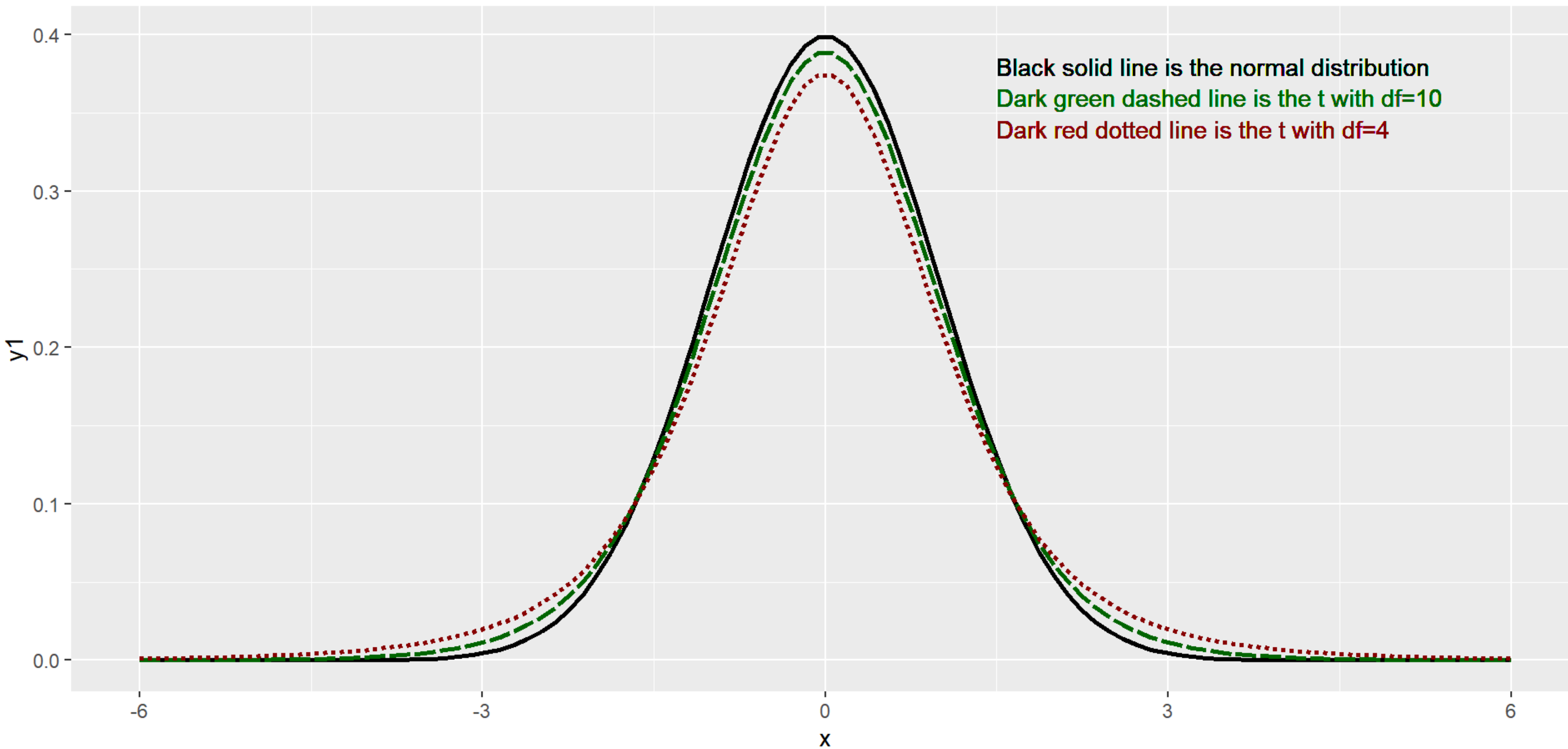
- $T = \frac{\bar{X}_1 - \bar{X}_2}{se}$
  - Variation in the numerator AND the denominator
  - Use a t-distribution, not a normal distribution
    - $df = n_1 + n_2 - 2$

Speaker notes

The test statistic, T, is the ratio of the difference in sample means to the standard error. This statistic has variation both in the numerator and the denominator. This produces a statistic that is not normally distributed, but close to normal. It is the t-distribution.

The t-distribution has degrees of freedom. A large degrees of freedom means very little sampling error in the denominator. It is the total sample size $(n_1 + n_2)$ minus two degrees of freedom associated with the two estimated means used in the standard deviation calculation.

Comparing the t and normal distributions



Steve Simon, 2024-10-06, CC0

Speaker notes

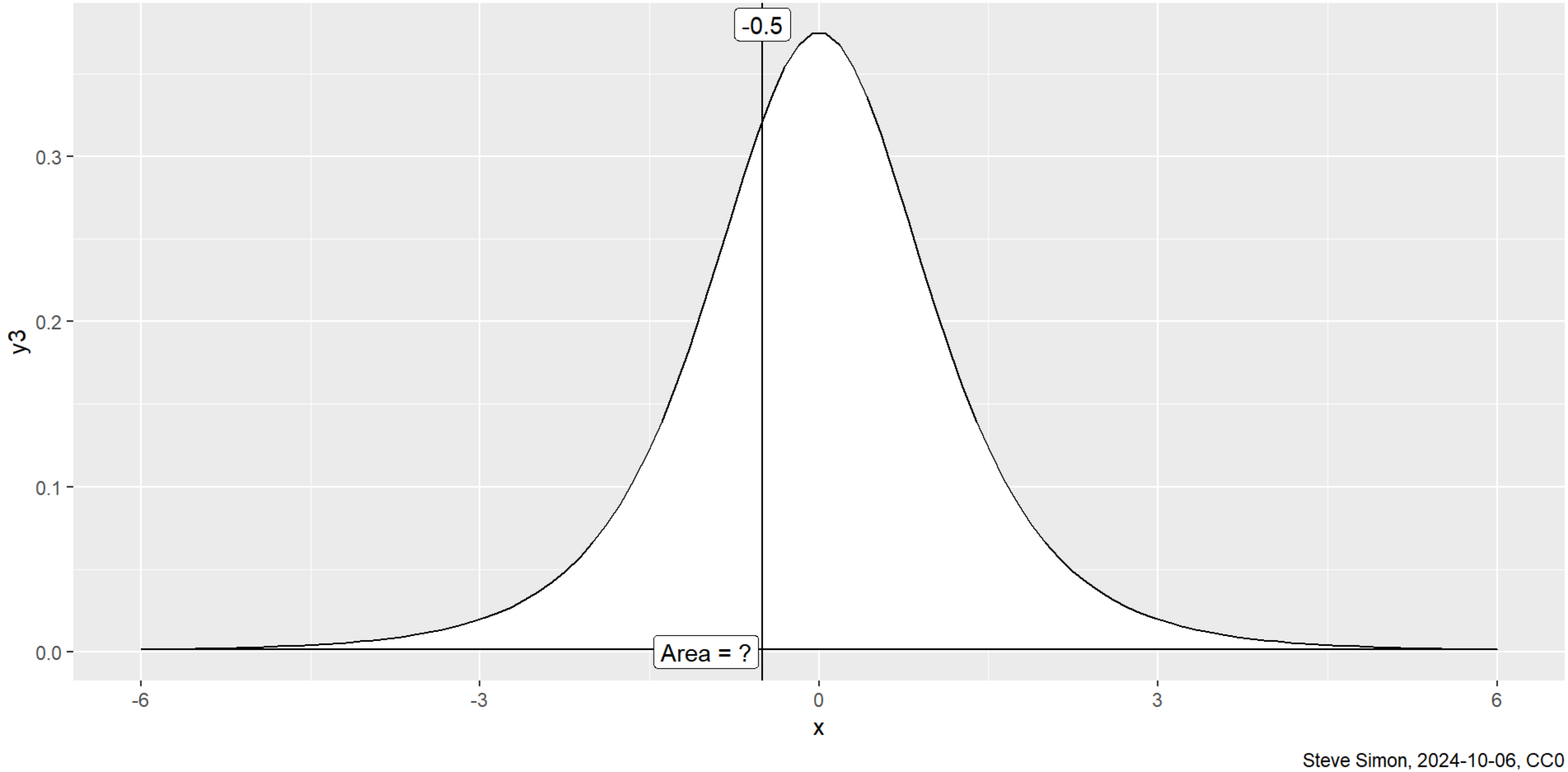
This graph compares the normal distribution to a t distribution with 10 degrees of freedom and a t distribution with 4 degrees of freedom. Both the normal and the t-distributions are symmetric. The t-distributions have a little bit less probability near zero and a bit more probability at the extremes.

P[t(4) < 1.5]


```
pt(1.5, 4)
```

```
[1] 0.896
```

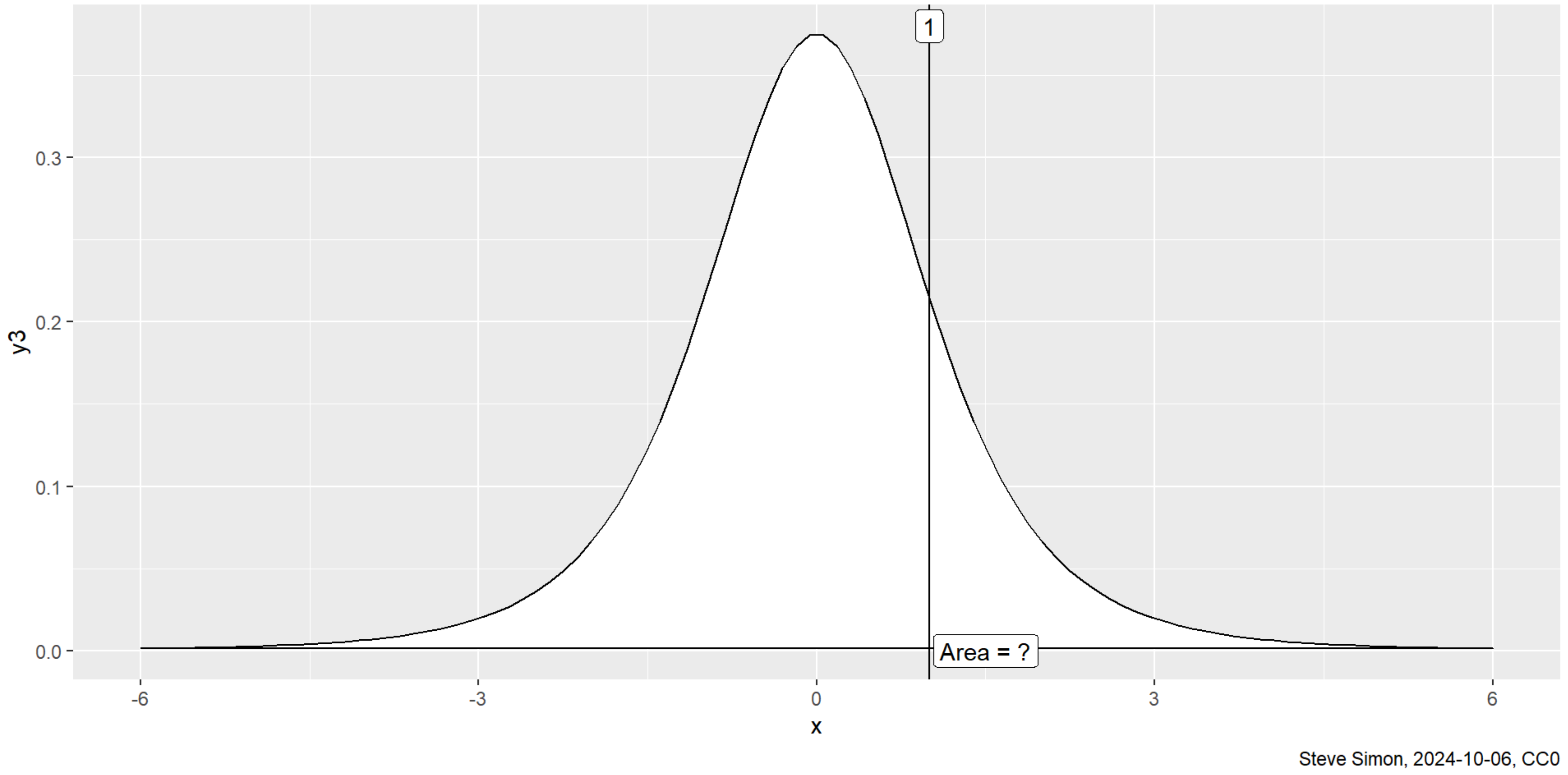
P[t(4) < -0.5]




```r
pt(-0.5, 4)
```

```
[1] 0.321665
```

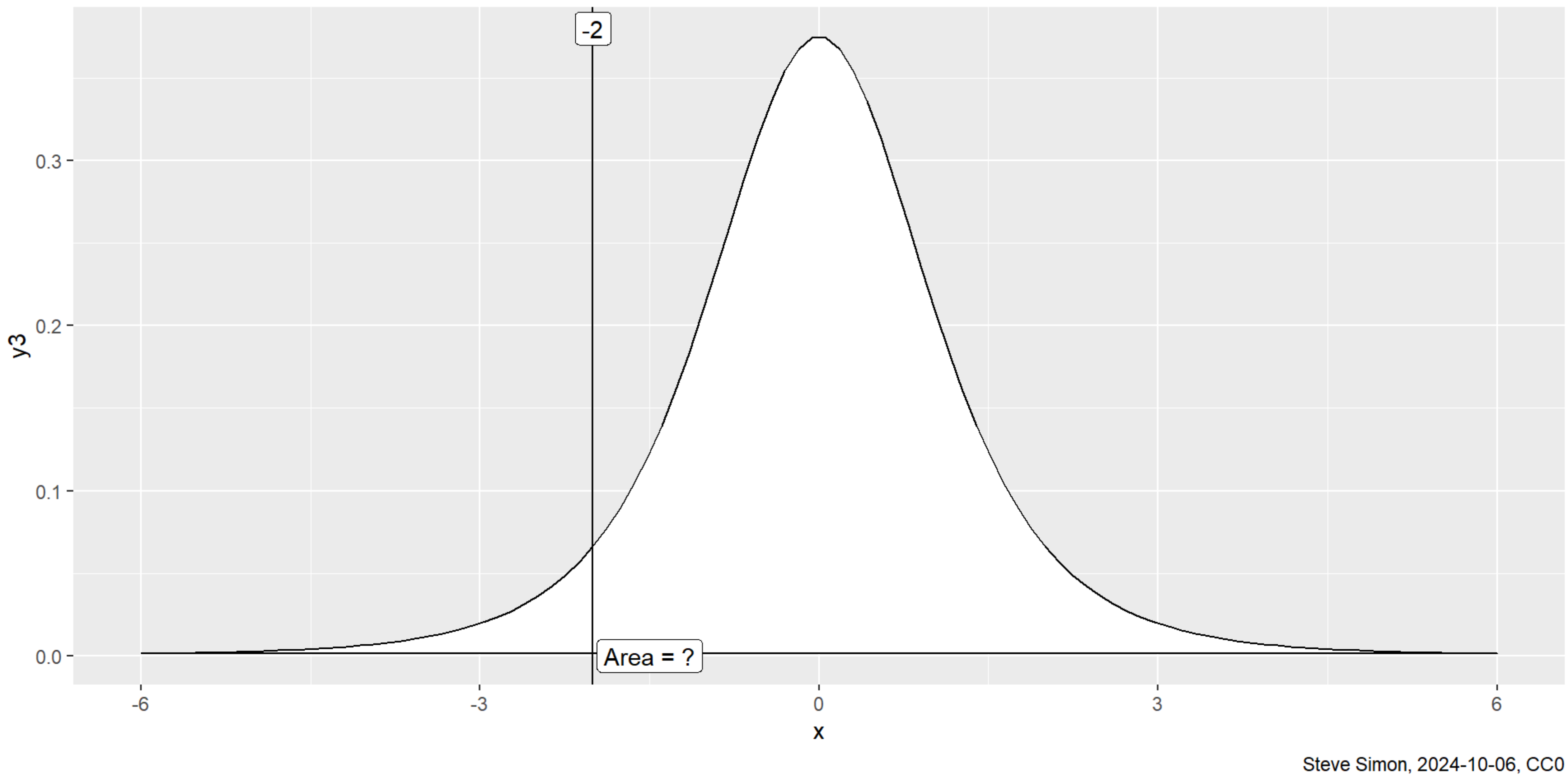
P[t(4) > 1]




```
1 - pt(1, 4)
```

[1] 0.1869505

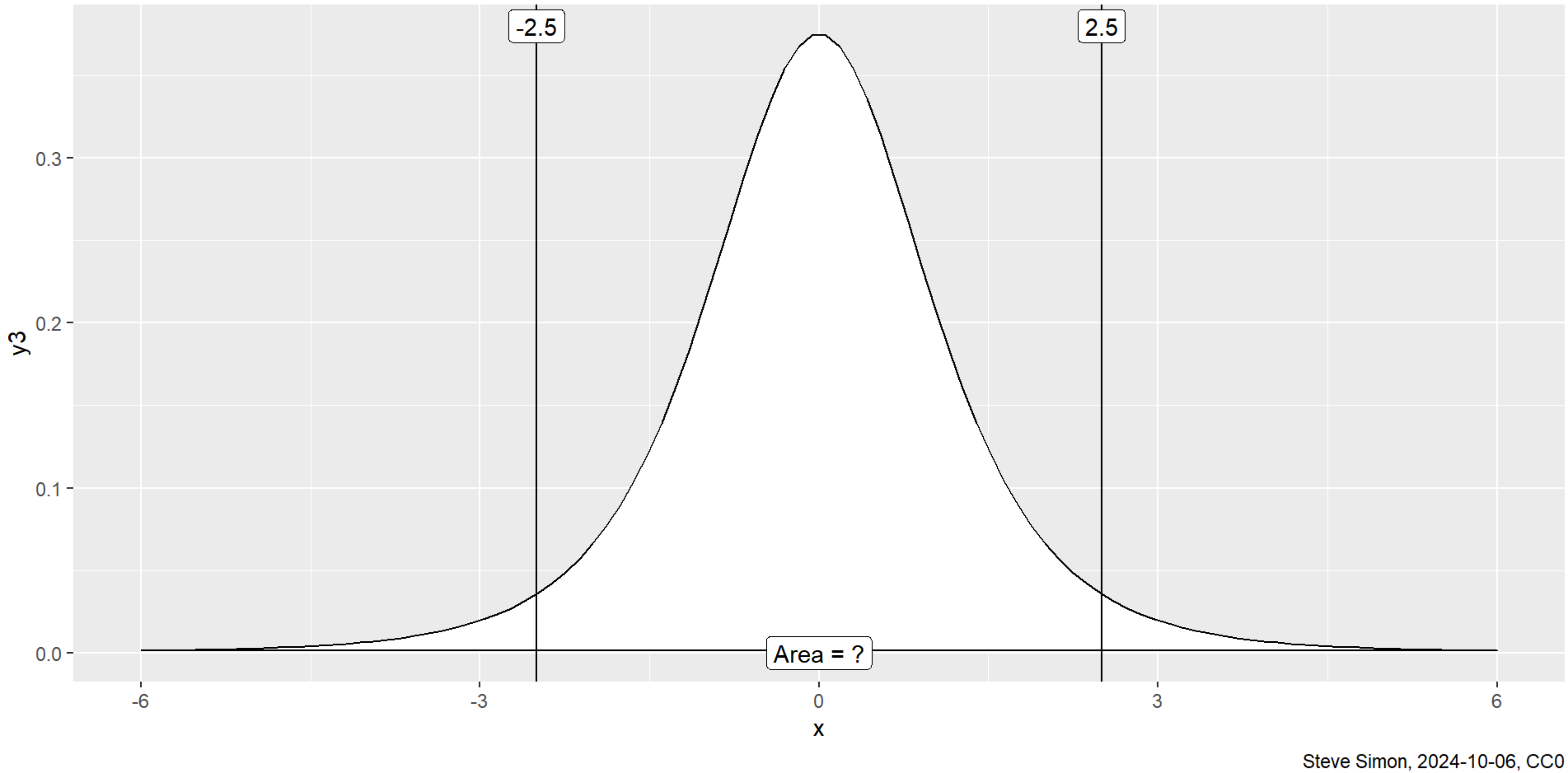
P[t(4) > -2]




```
1 - pt(-2, 4)
```

[1] 0.9419417

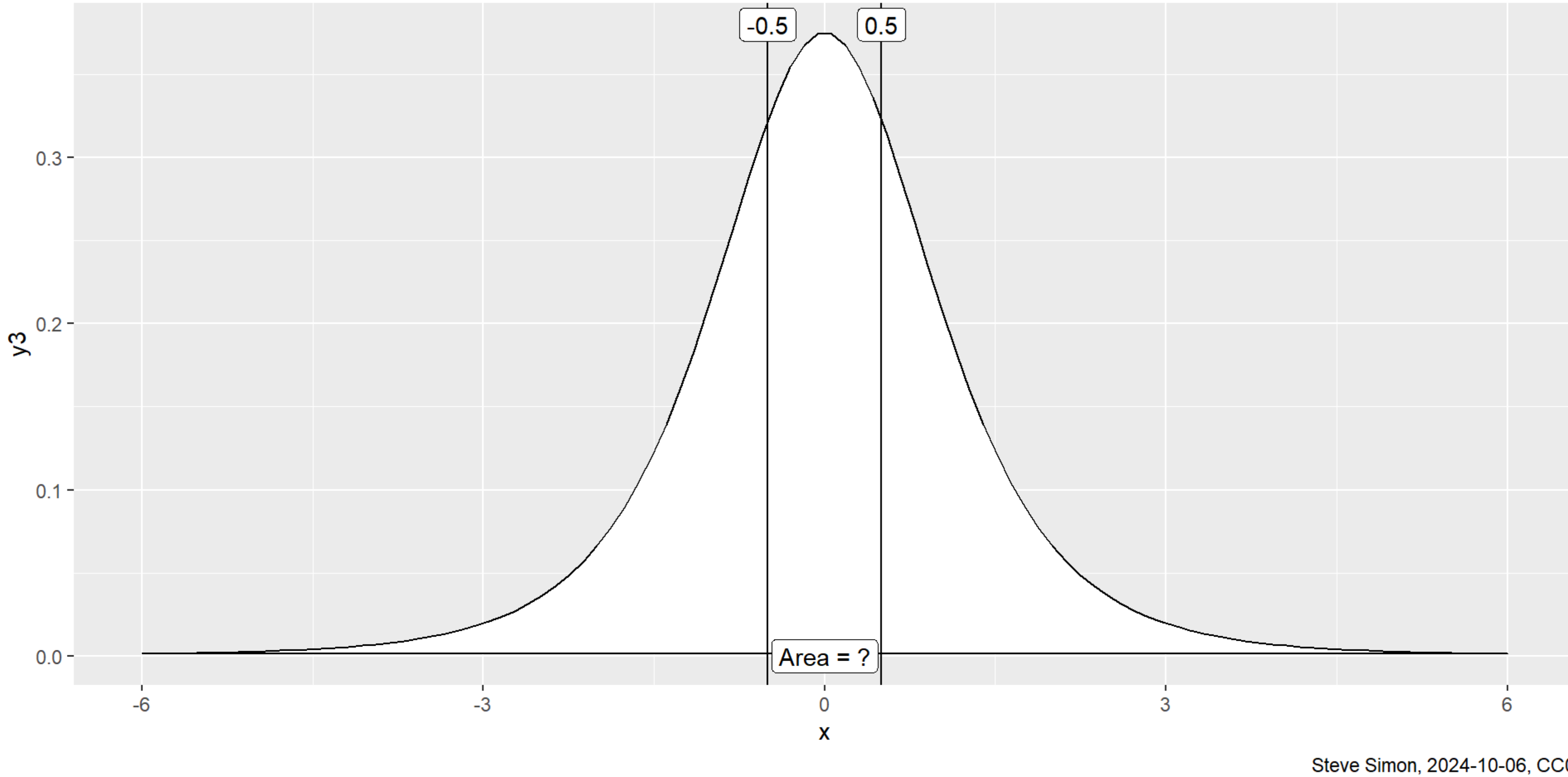
P[-2.5 < t(4) < 2.5]




```r
pt(2.5, 4) - pt(-2.5, 4)
```

```
[1] 0.9332335
```

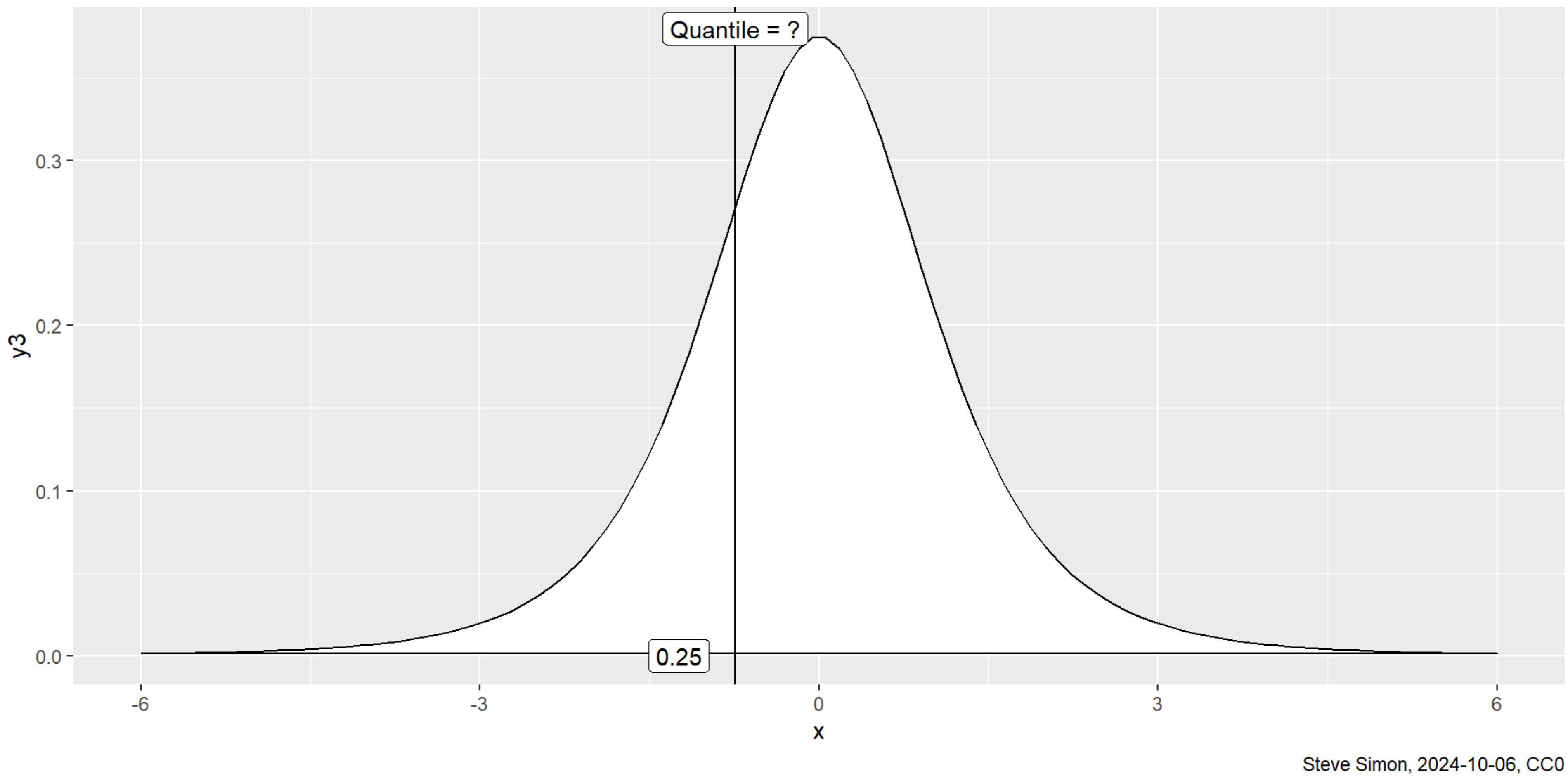
P[-0.5 < t(4) < 0.5]




```
pt(0.5, 4) - pt(-0.5, 4)
```

```
[1] 0.35667
```

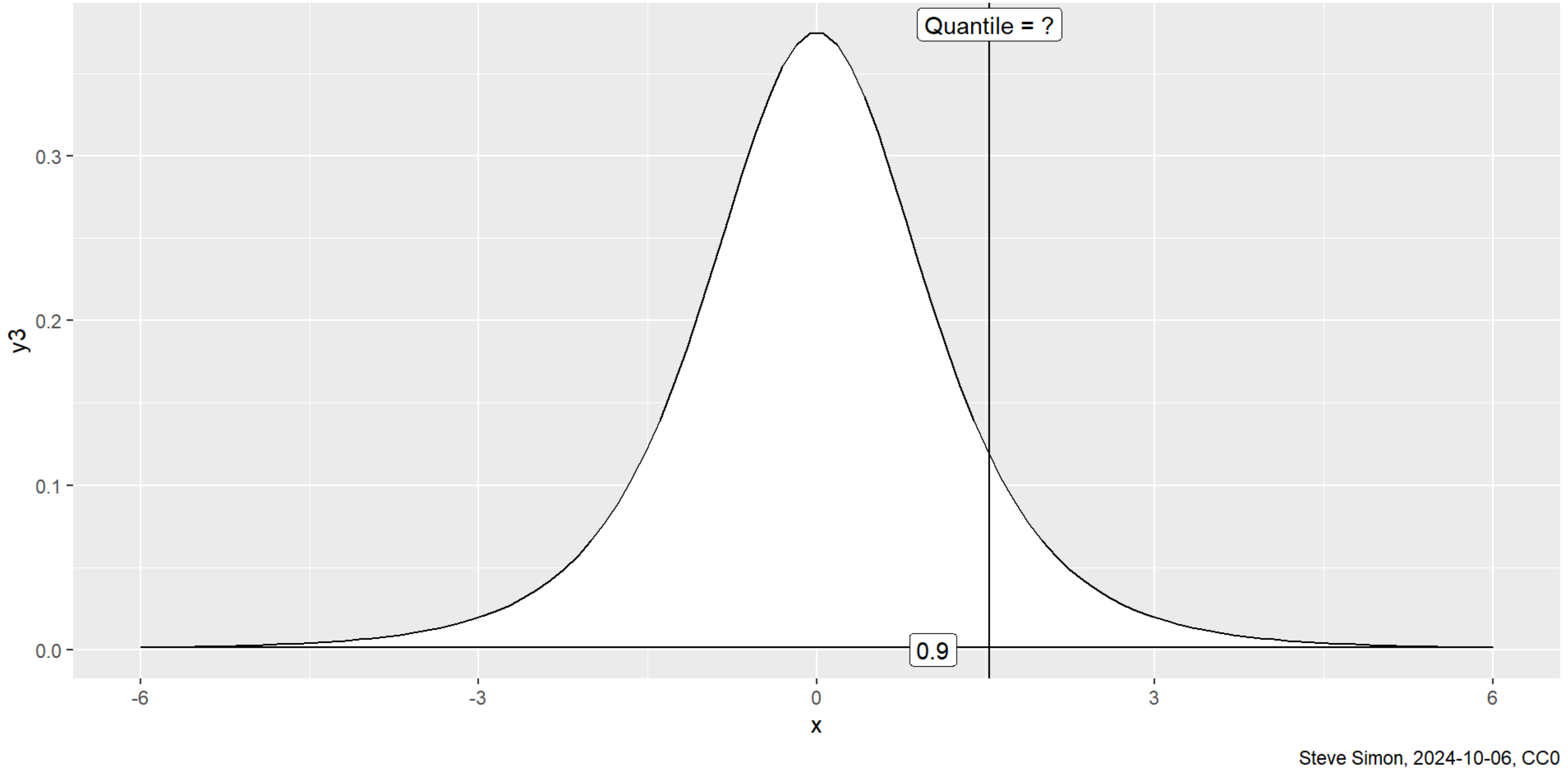
25th percentile of t(4)




```
qt(0.25, 4)
```

```
[1] -0.7406971
```

90th percentile of t(4)




```
qt(0.9, 4)
```

```
[1] 1.533206
```

Break #2

- What you have learned
  - The t-distribution
- What's coming next
  - Critical values and p-values

Type I and Type II errors

- Type I error, Rejecting the null hypothesis when the null hypothesis is true
  - α is the probability of a Type I error
- Type II error, Accepting the null hypothesis when the null hypothesis is false
  - β is the probability of a Type II error
  - Power = 1 - β

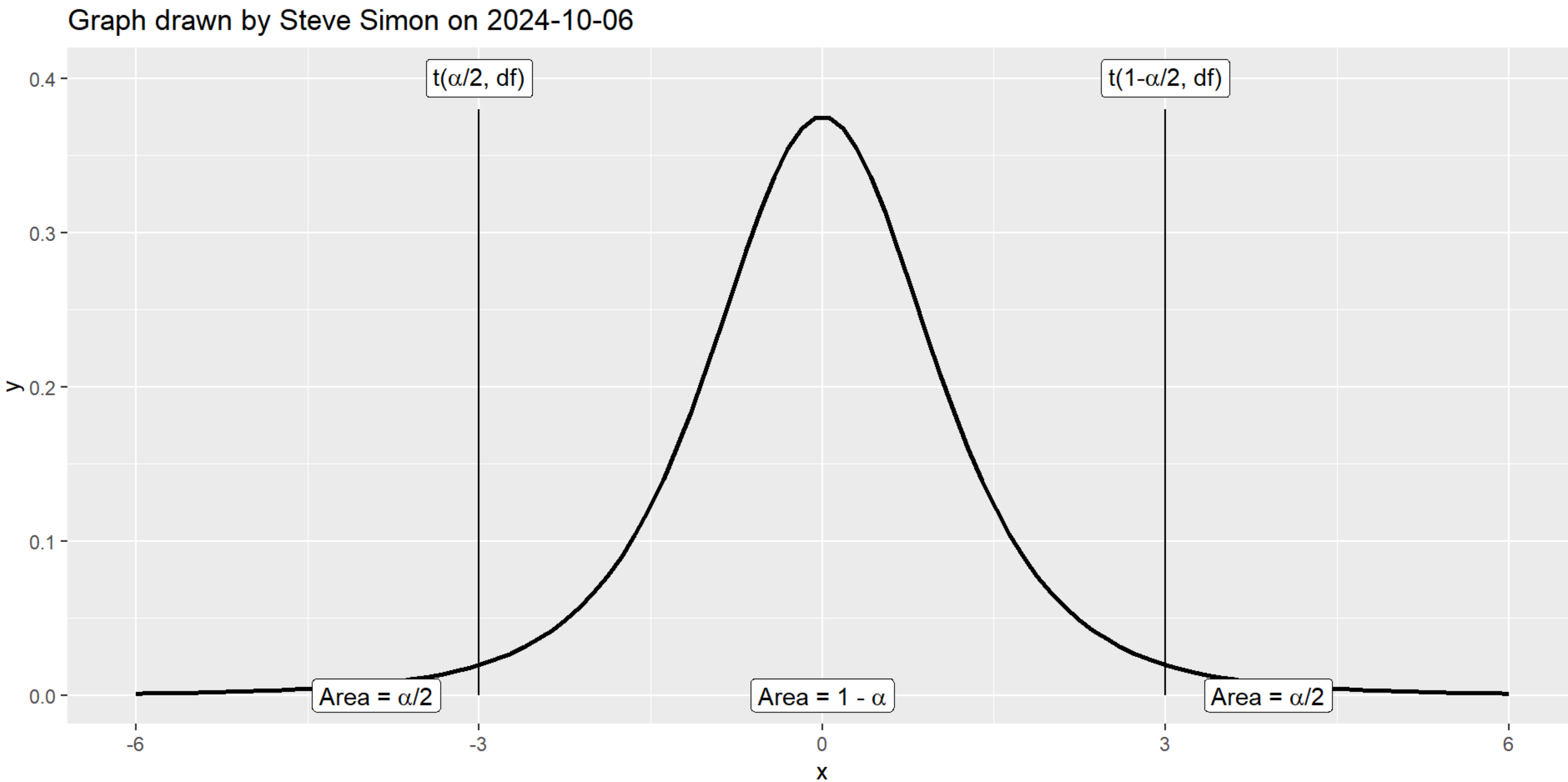
Critical values, 1

- $H_0 : \mu_1 - \mu_2 = 0$
- $H_1 : \mu_1 - \mu_2 \neq 0$
  - $T = \frac{\bar{X}_1 - \bar{X}_2}{se}$
  - Accept H_0 if $t(\alpha/2, df) < T < t(1 - \alpha/2, df)$

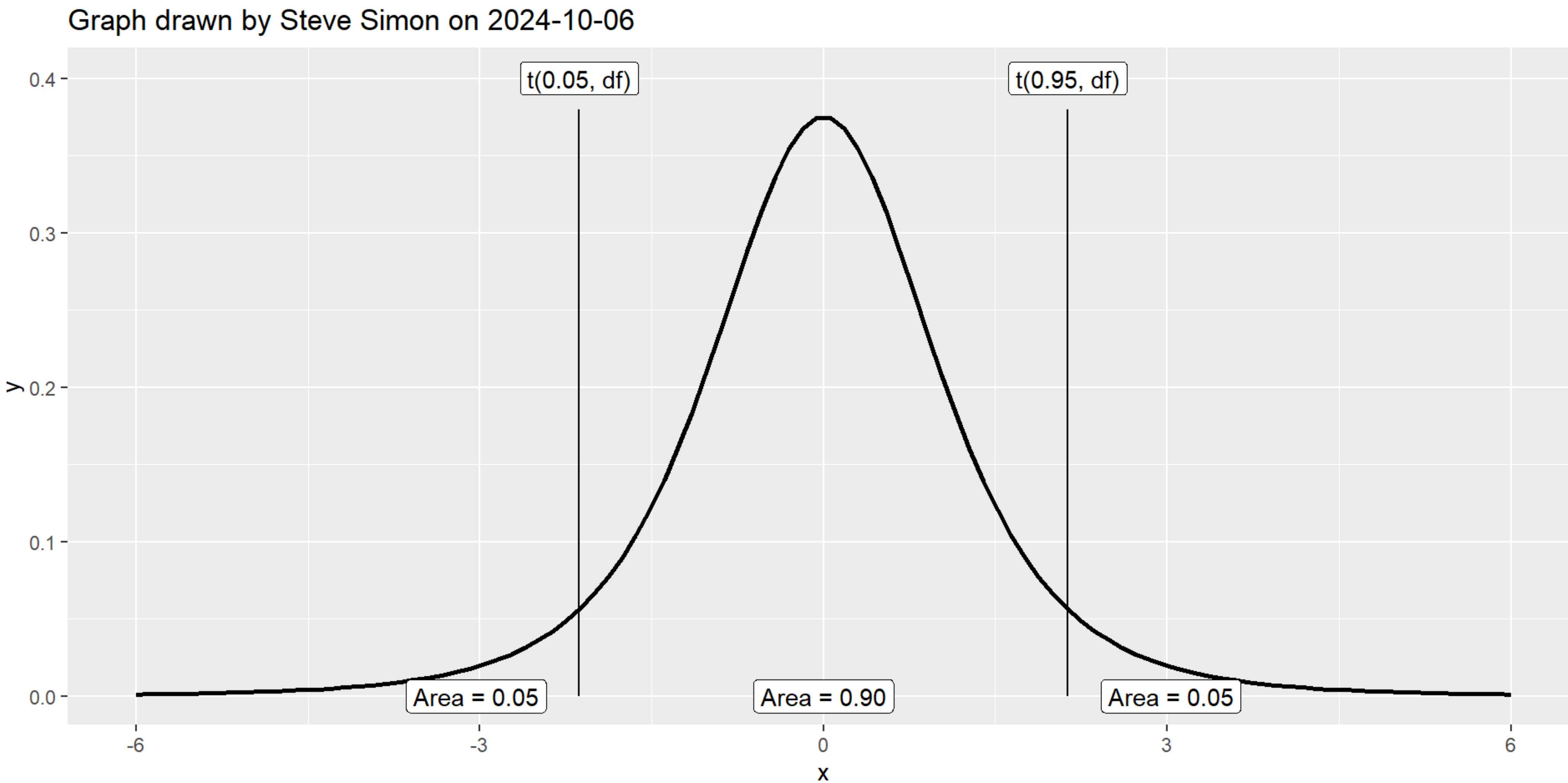
Speaker notes

The formal test of hypothesis looks at whether the test statistic T is close to zero. Close means that it falls between the $\alpha/2$ and $1-\alpha/2$ percentiles of a t distribution with $n_1 + n_2 - 2$ degrees of freedom.

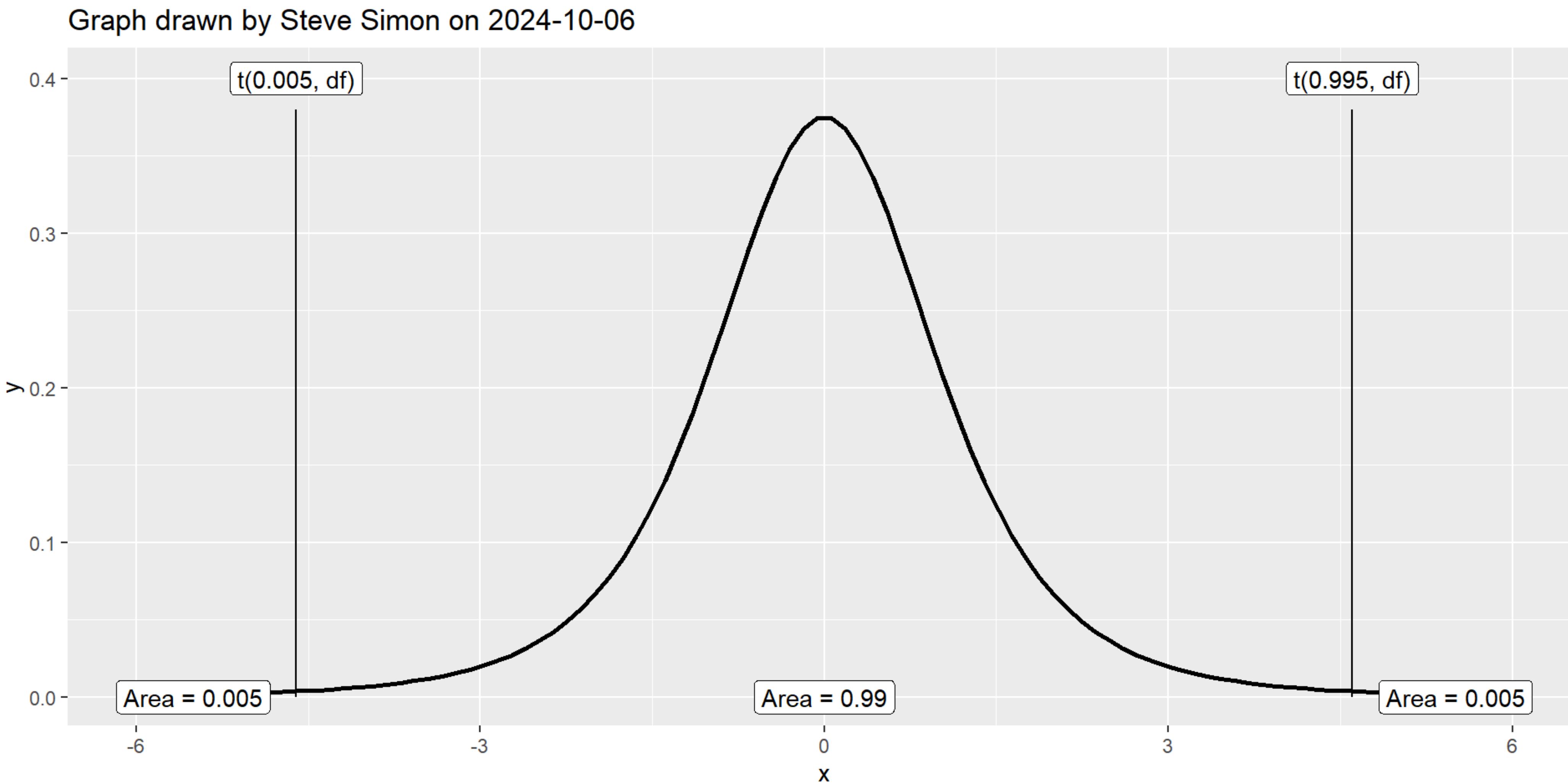
Critical values, 2



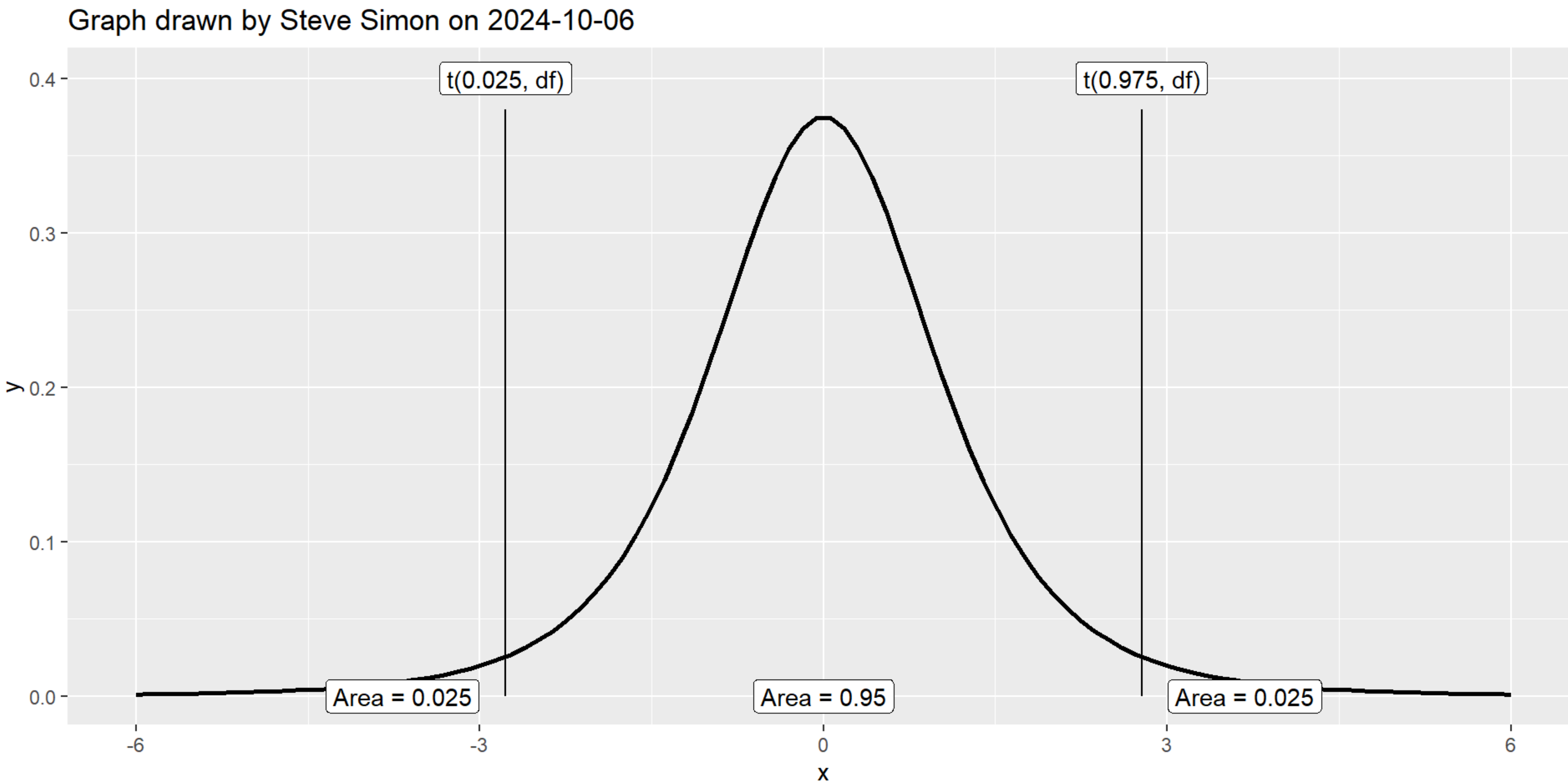
Critical values, 3



Critical values, 4



Critical values, 5



P-value

- p-value = $2P[t(n_1+n_2-2) > |T|]$
  - Probability of sample results or results more extreme
  - Accept H_0 if p-value > α
- Why the 2?
  - Measuring extremity in either direction

Postural sway data, 1

```
# A tibble: 2 × 4
  age     fb_mn fb_sd     n
  <chr>   <dbl> <dbl> <int>
1 Elderly  26.3  9.77     9
2 Young    18.1  4.09     8
```

Postural sway data, 2

- Calculate pooled standard deviation
  - $S_p = \sqrt{\frac{n_1\ S_1^2 + n_2\ S_2^2}{n_1 + n_2}}$
  - $S_p = \sqrt{\frac{9\ (9.77)^2 + 8\ (4.09)^2}{9 + 8}}$
  - S_p = 7.64

Postural sway data, 3

- Calculate standard error
  - se = $S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
  - se = $7.64\sqrt{\frac{1}{9} + \frac{1}{8}}$
  - se = 3.71

Postural sway data, 4

- Calculate T
  - T = $\frac{\bar{X}_1 - \bar{X}_2}{se}$
  - T = $\frac{26.3 - 18.1}{3.72}$
  - T = 2.2

Postural sway data, 5

- Calculate critical values
  - t(0.025, 15) = qt(0.025, 15) = -2.13
  - t(0.975, 15) = qt(0.975, 15) = 2.13
  - Since T is outside the two critical values, reject H_0

Postural sway data, 6

- Calculate p-value
  - p.value = $P[t(n_1 + n_2 - 2) > |T|]$
  - p.value = $2\ P[t(15) > |2.2|]$ = 2*(1-pt(2.2, 15)) = 0.044

Postural sway data, 7

```
    Two Sample t-test

data:  fbsway by age
t = 2.2044, df = 15, p-value = 0.04353
alternative hypothesis: true difference in means between group Elderly and group
Young is not equal to 0
95 percent confidence interval:
  0.2715453 16.1451214
sample estimates:
mean in group Elderly   mean in group Young
             26.33333              18.12500
```

Break #3

- What you have learned
  - Critical values and p-values
- What's coming next
  - R code for the t-test

postural-sway data dictionary

Refer to the data dictionary on my github site.

simon-5501-08-sway.qmd

Refer to the R program on my github site.

Break #4

- What you have learned
  - R code for the t-test
- What's coming next
  - Confidence intervals

Confidence interval for difference in means

- $\bar{X}_1 - \bar{X}_2 \pm t(1-\alpha/2, n_1+n_2-2)se$
  - Range of plausible values for $\mu_1 - \mu_2$

```
	Two Sample t-test

data:  fbsway by age
t = 2.2044, df = 15, p-value = 0.04353
alternative hypothesis: true difference in means between group Elderly and group
Young is not equal to 0
95 percent confidence interval:
  0.2715453 16.1451214
sample estimates:
mean in group Elderly   mean in group Young
             26.33333              18.12500
```

Speaker notes

The formula for a confidence interval for the difference between two means is shown here. The calculations are tedious, but not difficult.

Interpretation

- Statement about population mean difference ($\mu_1 - \mu_2$)
- Range of plausible values
- Not a probability statement
  - 95% confidence does not mean 95% probability
- If you collected 100 independent samples,
  - Roughly 95 would contain $\mu_1 - \mu_2$

Break #5

- What you have learned
  - Confidence intervals
- What's coming next
  - Sample size justification

Three things you need to justify your sample size

1. Research hypothesis
2. Measure of variability
3. Minimum clinically important difference (MCID)

Scenario

- Replicate postural sway study
  - Different populations
  - Same outcome measure
- Research hypothesis, $H_0 \ \mu_1 - \mu_2 = 0$
- Standard deviations: 9.77, 4.09
- MCID = 4

Scenario, R code

```r
sample_size_estimate <- power.t.test(
  n=NULL,
  delta=4,
  sd=9.8,
  sig.level=0.05,
  power=0.9,
  type="two.sample",
  alternative="two.sided")
```

Scenario, Output

```
     Two-sample t test power calculation

              n = 127.1097
          delta = 4
             sd = 9.8
      sig.level = 0.05
          power = 0.9
    alternative = two.sided

NOTE: n is number in *each* group
```

Break #6

- What you have learned
  - Sample size justification
- What's coming next
  - R code for sample size justification

simon-5501-08-sway.qmd

Refer to the R program on my github site.

Break #7

- What you have learned
  - R code for sample size justification
- What's coming next
  - Your homework

simon-5501-08-directions.md

Refer to the programming assignment on my github site.

Summary

- What you have learned
  - The two-sample t-test
  - The t-distribution
  - Critical values and p-values
  - R code for the t-test
  - Confidence intervals
  - Sample size justification
  - R code for sample size justification
  - Your homework