Comments for MEDB 5502, Week 11, Hierarchical models

Topics to be covered

- What you will learn
  - Hierarchical models
  - Hypothetical litter weights
  - Two sample t-test on first subset
  - Paired t-test on second subset
  - Variance components on third subset
  - Analysis of full dataset
  - Assumptions and complications

Hierarchical data

- Moving beyond the independence assumption
- Correlation within clusters

Speaker notes

Throughout this class, I have discussed the assumptions that you need for the t-test, the chi-square test, the ANOVA test, and so forth. Every single time, I mention the assumption of independence. It's often one that you can only check qualitatively. I mention special cases where you can't assume independence. In this lecture, I want to talk about one of those special cases: hierarchical data.

Hierarchical data has some additional grouping factor, often called a cluster. Measurements made within a cluster are correlated with one another, violating the assumption of independence.

Examples of hierarchical data, 1 of 2

- Body parts
  - Left eye/right eye
  - Teeth
  - Skin patches
- Human families
- Animal litters

Speaker notes

A simple example of hierarchical data is when you select a group of patients and then make measurements on two or more parts of their body. You might, for example, put an eye drop medication in the left eye and a placebo drop in the right eye. You might apply different types of sealents on different teeth in a mouth. You might put different food allergens on different parts of a patient’s back.

You might select families from a population and make measurements on two or more members of the same family. Since family members share the same environment and have very similar genetics, any comparison made within a family is likely to be more precise.

Likewise, measurements on the animals from the same litter will be precise because of a shared inter-uterine environment prior to birth and shared feeding from the same mother before weaning.

Examples of hierarchical data, 2 of 2

- Clinics/hospitals
- Communities
- Repeated measurements

Speaker notes

Patients treated at the same clinic or the same hospital will often have similar outcomes. This might be caused by the location of the clinic, which determines the types of patients that come in. it might also be caused by subtle treatment practices that are agreed upon within a clinic but which might vary from one clinic to another.

You might select entire communities and then sample people within each community. You will see some level of similarity within each community because of demographic similarities or because of common dietary or cultural practices.

Often, you take measurements repeatedly on an individual under different experimental conditions.

Longitudinal data (topic for next module)

- Measurements taken at different times
  - Emphasis in changes over time

Speaker notes

A special case that I want to handle separately is longitudinal data. This is similar to repeated measures data. With longitudinal data, often the emphasis is in changes that occur over time. Repeated measurements, in contrast, emphasize different treatments with the hope that the time gaps between the measurements are small enough that you don't see changes over time.

Between and within cluster comparisons

- Positive correlation
  - Improves precision of within cluster comparisons
  - Hurts precision of between cluster comparisons
- Example with litters
  - Medication administered during pregnancy
  - Medication administered after birth

Basic notation, 1 of 2

- Y_{ij}
  - i defines cluster
    - i=1,…,a
  - j defines individual within cluster
    - j=1,…,n

Basic notation, 2 of 2

- $Y_{ij} = \mu+\alpha_i+\epsilon_{ij}$
  - μ unknown constant
  - α_i is normally distributed
    - $SD(\alpha_i)=\sigma_{between}$
  - ϵ_{ij} is normally distributed
    - $SD(\epsilon_i)=\sigma_{within}$

Some basic results

- $SD(Y_{ij})=\sigma_{total} = \sqrt{\sigma^2_{between}+\sigma^2_{within}}$
- $SD(\bar{Y}_{..})=\sqrt{\frac{\sigma^2_{between}}{a}+\frac{\sigma^2_{within}}{an}}$
- $Corr(Y_{ij}, Y_{ik})=\frac{\sigma^2_{between}}{\sigma^2_{between}+\sigma^2_{within}}$
  - Intraclass correlation (ICC)

Expected mean squares, 1 of 2

- $MS(between) = \frac{1}{a-1}\Sigma_i n(\bar{Y}_{i.}-\bar{Y}_{..})^2$
  - $E[MS(between)] = a\sigma^2_{between}+\sigma^2_{within}$

Expected mean squares, 2 of 2

- $MS(within) = \frac{1}{a(n-1)}\Sigma_i\Sigma_j(\bar{Y}_{ij}-\bar{Y}_{i.})^2$
    - $E[MS(within)] = \sigma^2_{within}$

Variance components estimates

- $\hat\sigma_{between}=\frac{MS(between)-MS(within)}{a}$
- $\hat\sigma_{within}=MS(within)$

Break #1

- What you have learned
  - Hierarchical models
- What's coming next
  - Hypothetical litter weights

Description of litter weights data, 1 of 3

```
data_dictionary: litter-weights.sav

description: |
  Hypothetical data simulated to  illustrate analysis issues  associated with
random litter effects.

source: |
  Sobin, Christina; Golub, Mari (2020),  “Data for: Statistical Modeling with
Litter as a Random Effect in Mixed  Models to Manage Intralitter  Likeness”,
Mendeley Data, V1,  doi: 10.17632/bwptvj2cmz.1
```

Description of litter weights data, 2 of 3

```
ID:
  label: ID number for each animal
LITTER:
  label: ID number for each litter
SEX:
  label: Unspecified sex
  values:
  - 1
  - 2
```

Description of litter weights data, 3 of 3


```
GRP:
  label: Unspecified group
  values:
  - 1
  - 2
  - 3
WGTP21:
  label: Weight
  units: Unspecified
```

Break #2

- What you have learned
  - Hypothetical litter weights
- What's coming next
  - Two sample t-test on first subset

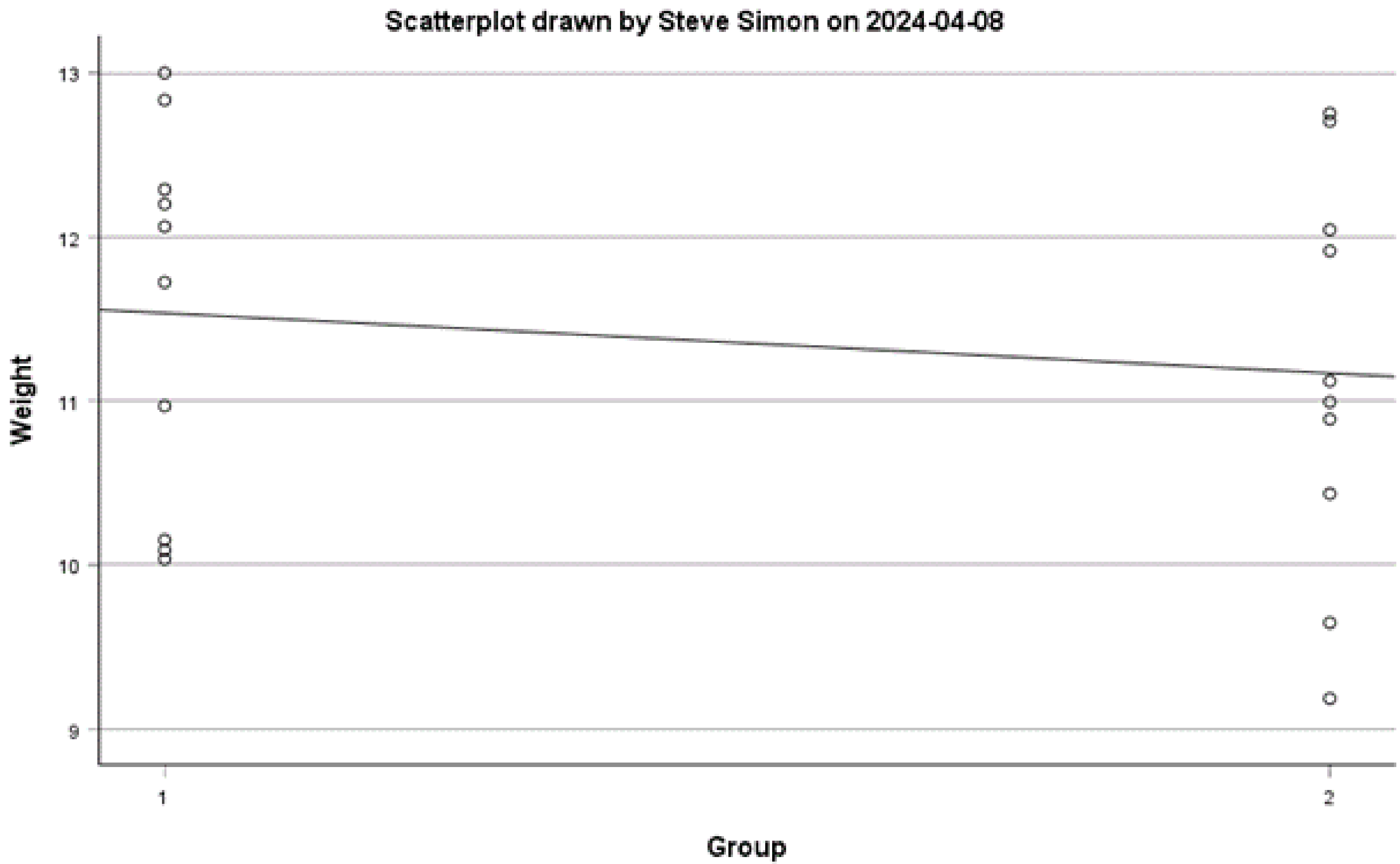
Subset1 of litter weights

- Consider only groups 1 and 2.
- Pick a male (sex=1) randomly from each litter
- Example of a between litter comparison

Listing of subset

	LITTER	GRP	SEX	WGTP21	var
1	1	2	1	12.71	
2	2	1	1	12.29	
3	3	1	1	12.83	
4	4	2	1	12.04	
5	5	1	1	11.72	
6	6	1	1	10.03	
7	7	2	1	12.75	
8	9	2	1	10.99	
9	11	2	1	9.65	
10	12	2	1	11.91	
11	13	2	1	10.43	
12	15	2	1	10.89	
13	18	2	1	11.12	
14	20	2	1	9.18	
15	22	1	1	10.15	
16	23	1	1	12.06	
17	24	1	1	12.20	
18	26	1	1	13.00	
19	27	1	1	10.97	
20	29	1	1	10.09	
21					

Graph of subset



Two-sample t-test

Group Statistics

	GRP	N	Mean	Std. Deviation	Std. Error Mean
WGTP21	1	10	11.5353	1.14225	.36121
	2	10	11.1683	1.20665	.38158

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means							
						Significance				95% Confidence Interval of the Difference	
		F	Sig.	t	df	One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference	Lower	Upper
WGTP21	Equal variances assumed	.014	.908	.698	18	.247	.494	.36695	.52543	-.73693	1.47083
	Equal variances not assumed			.698	17.946	.247	.494	.36695	.52543	-.73717	1.47107

Live demo, two sample t-test

Break #3

- What you have learned
  - Two sample t-test on first subset
- What's coming next
  - Paired t-test on second subset

Subset2 of litter weights

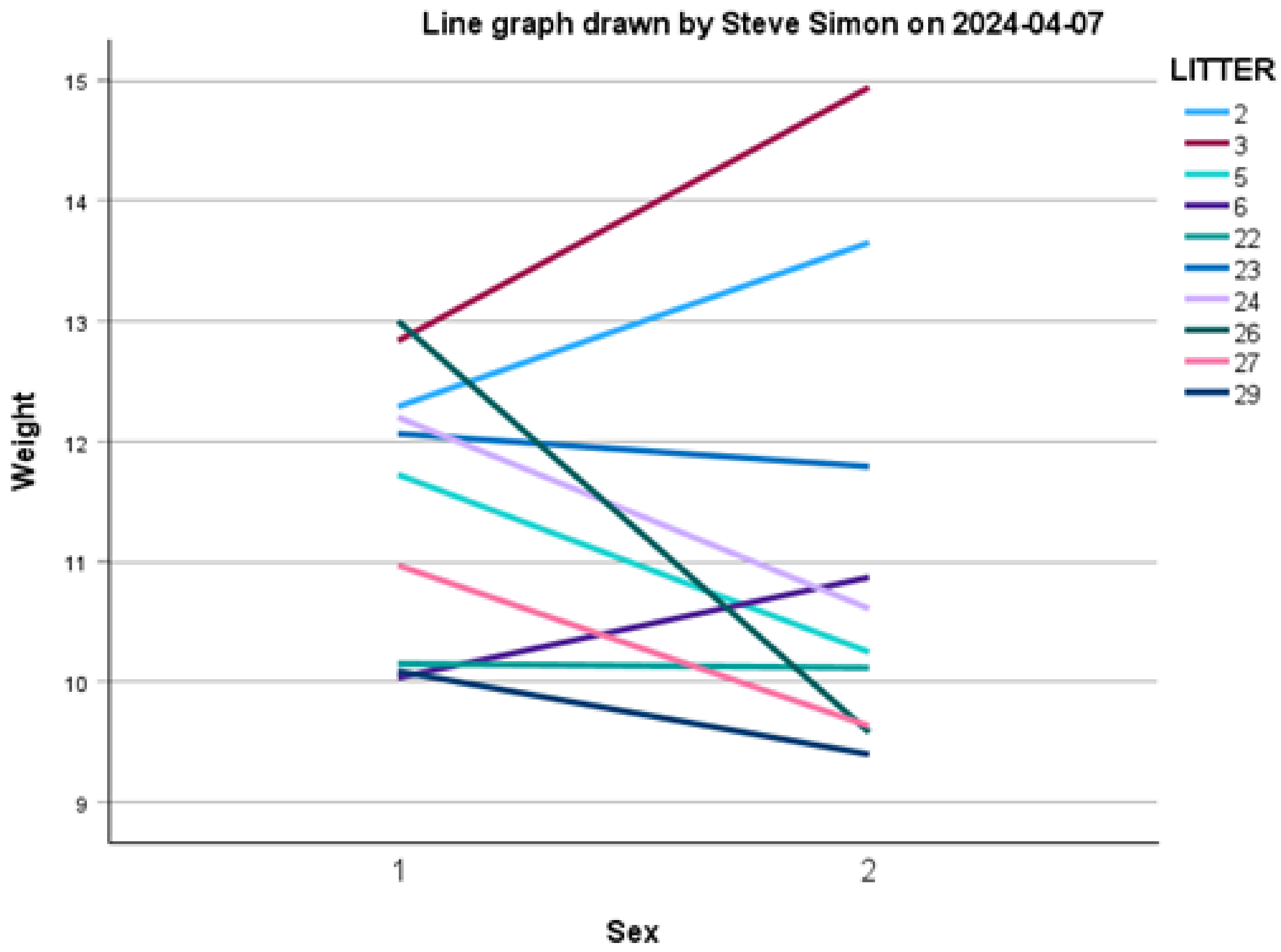
- Only group 1
- Randomly select one male and one female
- Example of a within litter comparison

Speaker notes

Listing of subset

	LITTER	SEX	GRP	WGTP21	var
1	2	1	1	12.29	
2	2	2	1	13.65	
3	3	1	1	12.83	
4	3	2	1	14.94	
5	5	1	1	11.72	
6	5	2	1	10.25	
7	6	1	1	10.03	
8	6	2	1	10.87	
9	22	1	1	10.15	
10	22	2	1	10.12	
11	23	1	1	12.06	
12	23	2	1	11.79	
13	24	1	1	12.20	
14	24	2	1	10.61	
15	26	1	1	13.00	
16	26	[illegible]	1	9.58	
17	27	1	1	10.97	
18	27	2	1	9.63	
19	29	1	1	10.09	
20	29	2	1	9.40	
21					

Graph of data



Restructuring information

Generated Variables

Name	WGTP21	1	WGTP21.1
		2	WGTP21.2

Processing Statistics

Cases In	20
Cases Out	10
Cases In/Cases Out	2.0
Variables In	4
Variables Out	4
Index Values	2

View of data after restructuring

	LITTER	GRP	weight_male	weight_female	var
1	2	1	12.29	13.65	
2	3	1	12.83	14.94	
3	5	1	11.72	10.25	
4	6	1	10.03	10.87	
5	22	1	10.15	10.12	
6	23	1	12.06	11.79	
7	24	1	12.20	10.61	
8	26	1	13.00	9.58	
9	27	1	10.97	9.63	
10	29	1	10.09	9.40	
11					

Paired t-test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	WGTP21.1	11.5353	10	1.14225	.36121
	WGTP21.2	11.0842	10	1.85849	.58771

Paired Samples Correlations

				Significance	
		N	Correlation	One-Sided p	Two-Sided p
Pair 1	WGTP21.1 & WGTP21.2	10	.501	.070	.140

Live demo, paired t-test and restructuring

Break #4

- What you have learned
  - Paired t-test on second subset
- What's coming next
  - Variance components on third subset

Third subset

- Only group 1
- Only males (three per litter)
- Litter is a random factor
  - Represents a sample from a larger population
  - Intent to generalize to that population

Partial listing of subset

	ID	LITTER	SEX	GRP	WGTP21	var
1	174	2	1	1	12.29	
2	130	2	1	1	14.46	
3	133	2	1	1	15.44	
4	27	3	1	1	12.83	
5	53	3	1	1	12.10	
6	3	3	1	1	12.26	
7	111	5	1	1	11.72	
8	170	5	1	1	11.80	
9	147	5	1	1	12.92	
10	164	6	1	1	10.03	
11	109	6	1	1	11.90	
12	139	6	1	1	12.55	
13	1	22	1	1	10.15	
14	72	22	1	1	10.23	
15	118	22	1	1	9.24	
16	96	23	1	1	12.06	
17	138	23	1	1	12.80	
18	146	23	1	1	13.46	
19	119	24	1	1	12.20	
20	71	24	1	1	10.90	
21	9	24	1	1	11.79	
22	48	26	1	1	13.00	

Variance components, 1 of 2

ANOVA

Source	Type I Sum of Squares	df	Mean Square
Corrected Model	35.506	9	3.945
Intercept	4226.405	1	4226.405
LITTER	35.506	9	3.945
Error	15.393	20	.770
Total	4277.305	30	
Corrected Total	50.899	29	

Dependent Variable: WGTP21

Variance components, 2 of 2

Expected Mean Squares

Source	Variance Component		
	Var(LITTER)	Var(Error)	Quadratic Term
Intercept	3.000	1.000	Intercept
LITTER	3.000	1.000	
Error	.000	1.000	

Dependent Variable: WGTP21
Expected Mean Squares are based on Type I Sums of Squares.
For each source, the expected mean square equals the sum of the coefficients in the cells times the variance components, plus a quadratic term involving effects in the Quadratic Term cell.

Variance Estimates

Component	Estimate
Var(LITTER)	1.058
Var(Error)	.770

Dependent Variable: WGTP21
Method: ANOVA (Type I Sum of Squares)

Speaker notes

The F-ratio is 3.945/0.770 = 5.1

The intraclass correlation is 1.058/(1.058+0.770) = 0.58

Live demo, Variance components

Break #5

- What you have learned
  - Variance components on third subset
- What's coming next
  - Analysis of full dataset

Analysis of full dataset

- Random factor
  - Litter Fixed factors
  - Sex
    - Within litter comparison
  - Group
    - Between litter comparison

Variance components, 1 of 2

ANOVA

Source	Type I Sum of Squares	df	Mean Square
Corrected Model	415.044	30	13.835
Intercept	25855.880	1	25855.880
SEX	23.426	1	23.426
GRP	66.368	2	33.184
LITTER	325.250	27	12.046
Error	160.520	149	1.077
Total	26431.444	180	
Corrected Total	575.564	179	

Dependent Variable: WGTP21

Variance components, 2 of 2

Expected Mean Squares

Source	Var(LITTER)	Var(Error)	Quadratic Term
	Variance Component		
Intercept	6.000	1.000	Intercept, SEX, GRP
SEX	.000	1.000	SEX
GRP	6.000	1.000	GRP
LITTER	6.000	1.000	
Error	.000	1.000	

Dependent Variable: WGTP21
Expected Mean Squares are based on Type I Sums of Squares.
For each source, the expected mean square equals the sum of the coefficients in the cells times the variance components, plus a quadratic term involving effects in the Quadratic Term cell.

Variance Estimates

Component	Estimate
Var(LITTER)	1.828
Var(Error)	1.077

Dependent Variable: WGTP21
Method: ANOVA (Type I Sum of Squares)

Speaker notes

The F-ratios are 23.426/1.077 = `r round(23.426/1.077, 1)` and 33.184/12.046 = 2.8.

The intraclass correlation is 1.828/(1.828+1.077) = 0.63.

Live demo, More variance components

Break #6

- What you have learned
  - Analysis of full dataset
- What’s coming next
  - Assumptions and complications

Assumptions

- Normality
  - Within clusters
  - Between clusters
- Independence
  - Only between clusters

Complications

- Unbalanced data
- Interactions
- Multiple hierarchies

Speaker notes

Summary

- What you have learned
  - Hierarchical models
  - Hypothetical litter weights
  - Two sample t-test on first subset
  - Paired t-test on second subset
  - Variance components on third subset
  - Analysis of full dataset
  - Assumptions and complications