ANALYSIS OF MEANS (ANOM)

In many situations, you need to examine differences among more than two groups. When you are interested in comparing multiple group means, you can use the analysis of means (ANOM) as an alternative to the one-way analysis of variance F test described in Section 11.1.

The ANOM provides a "confidence interval type of approach" that allows you to determine which, if any, of the c groups has a mean significantly different from the overall average of all the group means combined. Instead of looking at the lower and upper limits of a confidence interval, you will be studying which of the c group means are not contained in an interval formed between a *lower decision line* and an *upper decision line*. Any individual group mean not contained in this interval is deemed significantly higher than the overall average of all groups if it lies above the upper decision line. Similarly, any group mean that falls below the lower decision line is declared significantly lower than the overall group average.

As with confidence interval estimation, to compute the upper and lower decision lines (UDL and LDL) for ANOM, you must add and subtract a measure of sampling error around the statistic of interest. That is,

> ## FORMAT FOR ALL CONFIDENCE INTERVALS
>
> $$\text{statistic} \pm \text{sampling error}$$
>
> $$\text{statistic} \pm (\text{critical value})(\text{standard error of the statistic})$$

For ANOM, this is demonstrated as follows:

> ## OBTAINING THE UDL AND LDL
>
> $$\overline{\overline{X}} \pm h_{c,n_j}\sqrt{\frac{S_p^2(c-1)}{n}}$$
>
> where
>
> c = number of groups in the study
>
> j = representation for a particular group; $j = 1, 2, \ldots, c$
>
> n_j = sample size for group j
>
> n = total number of observations where all of the n_j sample sizes are equal; $n = n_1 + n_2 + \cdots + n_c$
>
> $\overline{\overline{X}}$ = overall or grand mean; $\overline{\overline{X}} = (\overline{X}_1 + \overline{X}_2 + \cdots + \overline{X}_c)/c$
>
> S_p^2 = pooled variance, an estimate of the inherent variability in the data computed by averaging the c group variances; $S_p^2 = (S_1^2 + S_2^2 + \cdots + S_c^2)/c$
>
> h_{c,n_j} = critical value of Nelson's h statistic with c groups and n_j equal observations per group obtained from the table of the h statistic

so that

> ## UPPER DECISION LINE FOR ANOM
>
> $$\text{UDL} = \overline{\overline{X}} + h_{c,n_j}\sqrt{\frac{S_p^2(c-1)}{n}}$$

and

> **LOWER DECISION LINE FOR ANOM**
>
> $$\text{LDL} = \bar{\bar{X}} - h_{c,n_j}\sqrt{\dfrac{S_p^2(c-1)}{n}}$$

To demonstrate how the ANOM procedure is used, refer to the Using Statistics scenario on page 381. The production manager of the Perfect Parachutes Company produces parachutes that are woven using a synthetic fiber purchased from one of four different suppliers. Because the strength of these fibers is an important characteristic in ensuring the quality of the parachutes, it is necessary for the production manager to conduct an experiment in order to decide whether the synthetic fibers provided by each of the four suppliers result in parachutes of equal strength.

A sample of five parachutes woven from the synthetic fibers provided by each of the four suppliers is selected. The strength of a parachute is measured by placing it in a testing device that pulls on both ends of the parachute until it tears apart. The amount of force required to tear the parachute is measured on a tensile-strength scale, where the larger the value, the stronger the parachute. The tensile strengths for these four groups of five parachutes are organized and stored in `Parachute` and are presented in Figure 11.3 on page 386, along with the sample mean and sample standard deviation for each group.

What can the production manager of the Perfect Parachutes Company conclude about the fibers provided by the four suppliers? Are they equal in strength? Are the differences you observe in Figure 11.3 in the sample means for tensile strength sufficiently small for you to conclude that the synthetic fibers provided by the four suppliers are not significantly different, or should you be more concerned about safety here?

To use ANOM to study the above experimental data, you must first verify that the assumptions of the procedure are not significantly violated.

One important assumption of ANOM is that the samples of parachutes woven from synthetic fibers provided by each of the four suppliers are taken from production processes where the tensile strengths are approximately normally distributed. This assumption implies that the tensile strengths of the four populations of parachutes woven from fibers provided by the suppliers are each approximately bell-shaped and symmetrical in appearance. With larger sample sizes, you could assess the validity of this assumption by using stem-and-leaf displays, box-plots, and normal probability plots. Figure 11.4 on page 386 shows a scatter plot of the tensile strengths for the four suppliers, and you can visually assess how the tensile strengths distribute and cluster within their respective groups as well as across groups. From this scatter plot, it appears that fibers provided by Suppliers 2 and 3 may be a bit stronger than those provided by Suppliers 1 and 4.

A second important assumption of ANOM is that the populations from which each of the c samples of size n were selected do not differ with respect to variability in tensile strength. The scatter plot of Figure 11.4 offers a visual impression of the viability of this important assumption. The spread in the tensile strengths of fibers provided by Suppliers 1 and 4 appears to be very slightly wider than those provided by Suppliers 2 and 3.

Given the small sample sizes and the slight visual differences depicted in the scatter plot, it would be difficult to conclude that there is an overwhelmingly significant violation in these two important assumptions for using the ANOM procedure. You could examine the three major characteristics of numerical data—central tendency, variation, and shape—using the summary measures shown in Figure 11.3 and the scatter plot shown in Figure 11.4, and you could then conclude that there is insufficient evidence of any major violation in the assumptions.

With that conclusion, you can proceed to study whether any of the groups appear to have significantly above-average or below-average tensile strengths. At this point, you can say that the parachutes woven from fibers taken from the four suppliers may have the same amount of variability in tensile strength as well as the same population shapes (i.e., approximate normal distributions). If there are any real, significant differences in the suppliers' synthetic fibers, these differences would have to be observed in their central tendencies. You can now answer the

question "Are any of the mean tensile strengths significantly above or below the overall or combined average tensile strength of synthetic fibers provided by all four suppliers?"

To apply the ANOM procedure, you first need to compute the key statistics from the experiment as well as the critical value of Nelson's h statistic. Using the worksheet results shown in Figure 11.3, you compute $\overline{\overline{X}}, n,$ nd S_p^2:

$$\overline{\overline{X}} = (\overline{X}_1 + \overline{X}_2 + \cdots + \overline{X}_c)/c = (19.52 + 24.26 + 22.84 + 21.16)/4 = 21.945$$

$$n = n_1 + n_2 + \cdots + n_c = (5 + 5 + 5 + 5) = 20$$

$$S_p^2 = (S_1^2 + S_2^2 + \cdots + S_c^2)/c = (7.237 + 3.683 + 4.553 + 8.903)/4 = 6.094$$

The critical values of Nelson's h statistic for obtaining the 95% UDL and LDL are provided in the following table.

Selected Critical Values of Nelson's h Statistic for Obtaining 95% Upper and Lower Decision Lines (UDL and LDL)

Sample Size per Group, n_j	Number of Groups, c				
	3	4	5	6	7
4	2.79	2.85	2.88	2.91	2.94
5	2.67	2.74	2.79	2.83	2.87
6	2.60	2.68	2.74	2.79	2.83
7	2.55	2.65	2.71	2.77	2.80
8	2.52	2.62	2.69	2.74	2.79
10	2.49	2.59	2.66	2.72	2.76
12	2.46	2.56	2.64	2.70	2.75
15	2.43	2.55	2.62	2.68	2.73
20	2.40	2.53	2.61	2.66	2.71
"infinity"	2.34	2.47	2.56	2.62	2.68

Note: *Values in italics were obtained through linear interpolation.*

Source: *Table 2 of L. S. Nelson, "Exact Critical Values for Use with the Analysis of Means,"* Journal of Quality Technology, *15(1), 1983. Reprinted with permission of the American Society for Quality.*

Given that you have four fiber suppliers ($c = 4$) and a sample of five woven parachutes per group ($n_j = 5$), the critical value of the statistic h_{c,n_j} for the parachute tensile strength experiment is $h_{4,5} = 2.74$.

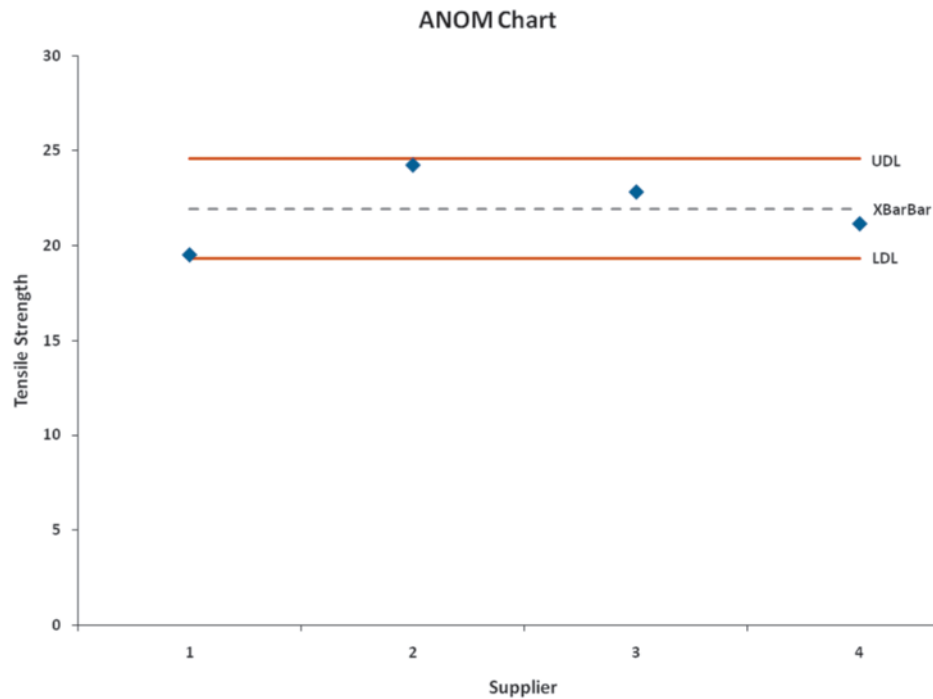You can now compute the UDL and LDL as follows:

$$\text{UDL} = \overline{\overline{X}} + h_{c,n_j}\sqrt{\frac{S_p^2(c-1)}{n}}$$

$$= 21.945 + (2.74)\sqrt{\frac{(6.094)(3)}{20}}$$

$$= 21.945 + 2.620$$

$$= 24.565$$

and

$$\text{LDL} = \overline{\overline{X}} - h_{c,n_j}\sqrt{\frac{S_p^2(c-1)}{n}}$$

$$= 21.945 - (2.74)\sqrt{\frac{(6.094)(3)}{20}}$$

$$= 21.945 - 2.620$$

$$= 19.325$$

The following line graph shows the graphical display for the ANOM.

The mean tensile strengths for fibers provided by each supplier are plotted on the vertical axis. From top to bottom, the three horizontal lines represent the UDL, $\bar{\bar{X}}$ (the overall, or grand, mean tensile strength for the fibers provided by all four suppliers combined), and the LDL.

From this figure, you observe that none of the four group means falls outside the UDL and the LDL. You conclude that there is no evidence of a significant difference in the mean tensile strength of parachutes woven from synthetic fibers provided by any of the four suppliers when compared to the overall mean tensile strength of all parachutes, regardless of fiber supplier. This particular finding may disappoint you because the graphical display above shows how close Supplier 2 came to being recognized as having had superior synthetic fibers when compared to the overall average of all suppliers, while Supplier 1 came even closer to being flagged as having had inferior synthetic fibers when compared to the overall average.

In this subsection, you have been introduced to ANOM, a "confidence interval type of approach" that allows you to determine which, if any, of the c groups has a mean significantly different from the overall average of all the group means combined. ANOM is actually a simpler procedure to use than the one-way analysis of variance F test described in Section 11.1. Both procedures are used for comparing multiple groups with numerical data, and both depend on the same set of assumptions. In particular, ANOM compares each of the c means against the overall mean of all c groups combined, while the one-way analysis of variance F test attempts to determine whether any of the c group means differ from each other. With the parachute tensile strength data, the one-way analysis of variance F test actually determined that significant differences in the mean tensile strengths existed. Subsequent analysis described in Section 11.1 led to the further important conclusion that the parachutes woven from the synthetic fibers of Supplier 2 had a significantly greater tensile strength than parachutes woven from the fibers provided by Supplier 1.