Homework 2

Pratham Mehta

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Question 1: Matrix Expressions on Undirected Networks

Let A be the $n \times n$ adjacency matrix of a simple, undirected, loopless graph (so A is symmetric and $A_{ii} = 0$), and let $\mathbf{1}$ be the n-vector of all ones.

(a) Vector of degrees

The degree of node i is the number of neighbors it has. In matrix form, multiplying A by an all-ones vector simply *sums each row* of A, which is exactly "count how many 1s are in that row".

$$\boldsymbol{k} \;=\; A\mathbf{1}.$$

So, entry k_i equals $\sum_j A_{ij}$, i.e., the number of nodes connected to i.

(b) Number of edges

Summing all degrees counts each edge twice (once from each endpoint). So we divide by 2 to get the actual number of edges m.

$$m \;=\; \frac{1}{2}\sum_i k_i \;=\; \frac{1}{2}\mathbf{1}^\top \boldsymbol{k} \;=\; \frac{1}{2}\mathbf{1}^\top A\mathbf{1}.$$

(c) Common-neighbor matrix

A^2 counts common neighbors The (i, j) entry of A^2 equals $\sum_\ell A_{i\ell}A_{\ell j}$. This is 1 for each node ℓ that connects to both i and j, i.e., a common neighbor. so, for $i \neq j$:

$$N := A^2 \quad \Rightarrow \quad N_{ij} = \text{number of common neighbors of } i \text{ and } j.$$

(We often ignore the diagonal N_{ii} because it counts length-2 closed walks at i rather than "common neighbors with itself".)

(d) Number of triangles

The diagonal of A^3 counts length-3 closed walks starting and ending at the same node. Each triangle can be traversed in $3!/1! = 6$ distinct closed walks (3 starting points $\times$ 2 directions). So dividing by 6 converts closed-walk counts into triangles:

$$T \;=\; \frac{1}{6}\,\mathrm{trace}(A^3).$$

(e) Dolphin network post-processing and computations

Code in Jupter Notebook

Question 2: US Flight Network (BTS2025.csv)

All of Question 2 is in the Jupyter Notebook.