

Problem of Time

From standard quantum mechanics to quantum gravity

Philip Caesar Flores

Theoretical Physics Group, National Institute of Physics, University of the Philippines Diliman, Quezon City

15 December 2020

Overview

- 1 Introduction
 - Properties of time
 - Problem of time
- 2 Quantum time problem
 - Time-energy uncertainty
 - Time observable
 - Perspectives on the QTP
- 3 Relativistic QM
 - Localization problem
 - One time + multiple position

- 4 QFT and second quantization
 - Number-phase uncertainty
 - Super many time theory
- 5 Quantum gravity
 - Frozen formalism problem
 - Problem of time is multifaceted
- 6 Summary

Introduction

Properties of time Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Localization probler

One time + multiple position

quantization Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

Time is what prevents everything happening at once.
(John Wheeler)

Some relevant aspects of what time means include the following properties

Introduction Properties of time

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QM

Localization problem One time + multiple position

quantization

Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

Time is what prevents everything happening at once.
(John Wheeler)

- Some relevant aspects of what time means include the following properties
 - 1 Time as an ordering: notion of present that separates future and past notions

Properties of time

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QM

Localization problem One time + multiple

quantization Number-phase uncertainty

Super many time theory

Guantum gravity Frozen formalism problem

Problem of time is multifaceted

Summary

Time is what prevents everything happening at once.
(John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Time as an ordering: notion of present that separates future and past notions
 - Causation: one phenomenon at an earlier time brings about another phenomenon at a later time

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012), pp. 757–786

Philip Caesar Flores Problem of Time 15 December 2020 3/38

Properties of time

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QM

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Guantum gravity Frozen formalism problem

Problem of time is multifaceted

Summary

Time is what prevents everything happening at once.
(John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Time as an ordering: notion of present that separates future and past notions
 - Causation: one phenomenon at an earlier time brings about another phenomenon at a later time
 - 3 Temporal logic: basic logic with "and then"and "at a time t"constructs

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012), pp. 757–786

Philip Caesar Flores Problem of Time 15 December 2020 3/38

Properties of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is

Time is what prevents everything happening at once. (John Wheeler)

Some relevant aspects of what time means include the following properties

Properties of time

Perspectives on the QTP

Number-phase uncertainty

Super many time theory

Frozen formalism problem

Time is what prevents everything happening at once. (John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Change in time: a parameter with respect to which change is manifest

Properties of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Time is what prevents everything happening at once. (John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Change in time: a parameter with respect to which change is manifest
 - Modeled by the real line

Properties of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Time is what prevents everything happening at once. (John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Change in time: a parameter with respect to which change is manifest
 - Modeled by the real line
 - Habitually taken to be monotonic

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012),

Philip Caesar Flores Problem of Time 15 December 2020 4/38

Properties of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is

Time is what prevents everything happening at once. (John Wheeler)

Some relevant aspects of what time means include the following properties

Properties of time

Perspectives on the QTP

Number-phase uncertainty

Super many time theory

Frozen formalism problem

Time is what prevents everything happening at once. (John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Freedom in prescribing a timefunction: e.g choice of year zero and tick duration

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012),

Philip Caesar Flores Problem of Time 15 December 2020 5/38

Properties of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Time is what prevents everything happening at once. (John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Freedom in prescribing a timefunction: e.g choice of year zero and tick duration
 - A good timefunction is globally valid

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012),

5/38

Properties of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Time is what prevents everything happening at once. (John Wheeler)

- Some relevant aspects of what time means include the following properties
 - Freedom in prescribing a timefunction: e.g choice of year zero and tick duration
 - A good timefunction is globally valid
 - Must be operationally meaningful

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012),

5/38

Problem of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is

Physical theory of time carries three fundamental problems

Problem of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is

- Physical theory of time carries three fundamental problems
 - Asymmetry of the "direction of time"

Describer

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

quantization

Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

- Physical theory of time carries three fundamental problems
 - 1 Asymmetry of the "direction of time"
 - Travelling backwards through time does not appear to be precluded by current physical theories

Proportion of time

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Perspectives on the QTP

Relativistic QI

One time + multiple

quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

- Physical theory of time carries three fundamental problems
 - Asymmetry of the "direction of time"
 - 2 Travelling backwards through time does not appear to be precluded by current physical theories
 - 3 Lack of a consistent quantum description

Lack of a consistent quantum description

Problem of time

Time observable Perspectives on the QTP

Number-phase uncertainty

Super many time theory

Frozen formalism problem

poses a fundamental problem in quantum mechanics

leads to a number of problems when trying to replace general relativity and quantum theory with a single framework

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012), pp. 757-786.

Bryce S DeWitt. "Quantum theory of gravity. I. The canonical theory". In: *Physical Review* 160.5 (1967), p. 1113.

John Archibald Wheeler and Cécile DeWitt-Morette, Battelle rencontres: 1967 Lectures in mathematics and physics. WA Benjamin, 1968.

Chris J Isham et al. Quantum gravity 2: a second Oxford symposium. Oxford University Press, USA, 1981.

KV Kuchař and CG Torre. Conceptual Problems of Quantum Gravity ed A Ashtekar and J Stachel. 1991. Carlo Rovelli. Quantum gravity. Cambridge university press, 2004.

L Smolin. "Problem of Time Course, available in video form at http://pirsa. org". In: C08003 (2008).

15 December 2020 7/38

History of the quantum time problem (QTP)

Introductio

Properties of time

Quantum time

Time-energy uncertainty

Perspectives on the QTP

Relativistic Q

Localization problem One time + multiple position

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

Summary

Heisenberg's uncertainty relations

There are limitations on the possible accuracy of measurements

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- Consequence of replacing classical numbers by operators
- Time-energy uncertainty principle is accepted as valid event though it is not deduced from the commutation relation of operators

Yakir Aharonov and David Bohm. "Time in the quantum theory and the uncertainty relation for time and energy". In: *Physical Review* 122.5 (1961), p. 1649.

History of the quantum time problem (QTP)

Problem of time

Time-energy uncertainty

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Heisenberg's uncertainty relations

There are limitations on the possible accuracy of measurements

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- Different types of time-energy uncertainty relation can be deduced in specific contexts (can be summarized into 8 different interpretations)
- No unique universal relation that could stand on equal footing with the position-momentum uncertainty relation

Paul Busch. "The time-energy uncertainty relation". In: Time in quantum mechanics. Springer, 2008,

History of the quantum time problem (QTP)

Introduction

Properties of time

Quantum time

Time-energy uncertainty

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

Summary

Heisenberg's uncertainty relations

There are limitations on the possible accuracy of measurements

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Pauli's theorem:

- It is generally impossible to construct a self-adjoint time operator \hat{T} , that is canonically conjugate with the Hamiltonian
- The spectrum of the Hamiltonian must always be continuous and unbounded from below

Introduction

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

One time + multiple

quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

Decay time: the time at which a radioactive particle decays is inherently random

15 December 2020 11/38

introduction

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Polotiviotio Oli

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

- Decay time: the time at which a radioactive particle decays is inherently random
- Arrival time: time it takes to arrive at a given location in the configuration space

inti oddectioi

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic Ol

Localization problem
One time + multiple

quantization

Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

- Decay time: the time at which a radioactive particle decays is inherently random
- Arrival time: time it takes to arrive at a given location in the configuration space
- Tunneling time: time it takes to emerge on the other side of a potential barrier

introductioi

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QN

Localization problem One time + multiple

quantization Number-phase uncertainty

Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

- Decay time: the time at which a radioactive particle decays is inherently random
- Arrival time: time it takes to arrive at a given location in the configuration space
- Tunneling time: time it takes to emerge on the other side of a potential barrier
- **Escape time:** time it takes to escape the potential well

Interference in time

introduction

Properties of time Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem
Problem of time is
multifaceted

Summary

Lindner experiment

The double slit is realized in the time-energy domain where the role of the slits is played by windows in time of attosecond duration.

- Nonrelativistic Schrodinger theory cannot be used to predict interference phenomena in time
- Implies the existence of a time observable

F. Lindner et al. "Attosecond Double-Slit Experiment". In: *Phys. Rev. Lett.* 95 (4 July 2005), p. 040401. Lawrence P Horwitz. "Quantum interference in time". In: *Foundations of Physics* 37.4-5 (2007), pp. 734–746.

Philip Caesar Flores Problem of Time 15 December 2020 12/38

Time as a quantum observable

Introduction

Problem of time

Quantum time problem

Time-energy uncertainty
Time observable

Perspectives on the QTP

Relativistic QM

ocalization problem
One time + multiple
osition

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem
Problem of time is
multifaceted

Time as a quantum observable

Introduction

Properties of time Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

Weak form of the quantum time problem

Is the result of any measurement of time involving a quantum object, such as time of arrival (TOA) and tunneling time of a quantum particle, describable in terms of a resolution of the identity?

Time as a quantum observable

Introduction

Properties of time Problem of time

Quantum time problem

Time-energy uncerta

Perspectives on the QTP

Relativistic QI

Localization problem
One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem
Problem of time is
multifaceted

Summary

Weak form of the quantum time problem

Is the result of any measurement of time involving a quantum object, such as time of arrival (TOA) and tunneling time of a quantum particle, describable in terms of a resolution of the identity?

Strong form of the quantum time problem

Is every time measurement distribution derivable from a time operator? Or does time measurement distribution has an underlying ideal distribution generated from a time operator?

Perspectives on the QTP

Introduction

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

quantization Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

The weak and strong forms of the quantum time problem are not equivalent.

⇒ How is time treated in quantum mechanics?.

Eric A Galapon, Roland F Caballar, and Ricardo T Bahague Jr. "Confined quantum time of arrivals". In: *Physical review letters* 93.18 (2004), p. 180406.

V Delgado and JG Muga. "Arrival time in quantum mechanics". In: *Physical Review A* 56.5 (1997), p. 3425.

Norbert Grot, Carlo Rovelli, and Ranjeet S Tate. "Time of arrival in quantum mechanics". In: *Physical Review A* 54.6 (1996), p. 4676.

Paul Busch; Marian Grabowski; Pekka Lahti. *Operational quantum physics*. Lecture notes in physics., New series m, Monographs; m31. Springer, 2001.

Marco Toller. "Localization of events in space-time". In: Physical Review A 59.2 (1999), p. 960.

Riccardo Giannitrapani. "Positive-operator-valued time observable in quantum mechanics". In: International Journal of Theoretical Physics 36.7 (1997), pp. 1575–1584.

Harald Atmanspacher and Anton Amann. "Positive-operator-valued measures and projection-valued measures of noncommutative time operators". In: *International Journal of Theoretical Physics* 37.2 (1998), pp. 629–650.

Philip Caesar Flores Problem of Time 15 December 2020 14/38

Problem of time

Time observable

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

1. Accept Pauli's theorem

It is not a property of a system in the same way as spin and mass are properties of a system. As such time is not a dynamical observable of a given system and not subject to the laws of quantum measurement theory. In particular, no time operator exists in the same way that a position or a momentum operator exists.

E. A. Galapon and P. C. M. Flores. "Time and particles". In: In preparation ().

Asher Peres. "Measurement of time by quantum clocks". In: American Journal of Physics 48.7 (1980), pp. 552-557.

Yakir Aharonov, Peter G Bergmann, and Joel L Lebowitz. "Time symmetry in the quantum process of measurement". In: Physical Review 134.6B (1964), B1410.

Yakir Aharonov and Lev Vaidman, "The two-state vector formalism; an updated review", In: Time in quantum mechanics. Springer, 2008, pp. 399-447.

An extensive review is subject for future work

Philip Caesar Flores

Problem of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

2. Disprove Pauli's theorem

It must be represented by an operator in the system Hilbert space and the distribution of measurement is dictated by the resolution of the identity provided by the operator. It is not only that time is an operator but that it has the additional property of being canonically conjugate with the system Hamiltonian such that it evolves in step with the parametric time, i.e. time observables are time operators.

E. A. Galapon and P. C. M. Flores. "Time and particles". In: In preparation ().

ntroduction

Properties of time

Quantum time

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

Summary

3. Bypass Pauli's theorem

In this view, it is possible that the aspect of time under consideration is represented by an operator but not necessarily conjugate to the Hamiltonian. Or that time measurement must always be taken in context with other systems such as a measuring instrument.

E. A. Galapon and P. C. M. Flores. "Time and particles". In: In preparation ().

Gordon R Allcock. "The time of arrival in quantum mechanics I. Formal considerations". In: *Annals of physics* 53.2 (1969), pp. 253–285.

Gordon R Allcock. "The time of arrival in quantum mechanics II. The individual measurement". In: *Annals of Physics* 53.2 (1969), pp. 286–310.

GR Allcock. "The time of arrival in quantum mechanics III. The measurement ensemble". In: Annals of Physics 53.2 (1969), pp. 311–348.

An extensive review is subject for future work

Philip Caesar Flores Problem of Time 15 December 2020 17/38

Problem of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

4. Time is multifaceted.

It has several aspects so that it is not expected that the same treatment applies to them all. This view accommodates the possibility that time can be a parameter labeling the system, a dynamical observable, and a measurable quantity but not necessarily represented by a time operator.

E. A. Galapon and P. C. M. Flores. "Time and particles". In: In preparation ().

Problem of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

4. Time is multifaceted.

It has several aspects so that it is not expected that the same treatment applies to them all. This view accommodates the possibility that time can be a parameter labeling the system, a dynamical observable, and a measurable quantity but not necessarily represented by a time operator.

- quantum time
- coordinate time
- time of arrival

- Wheeler-DeWitt time
- Leibniz time
- time and memory

E. A. Galapon and P. C. M. Flores. "Time and particles". In: In preparation ().

Space and time on equal footing

ntroduction

Properties of time Problem of time

Quantum time problem

Time-energy uncertainty
Time observable
Perspectives on the QTP

Relativistic QN

Localization problem One time + multiple

quantization Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

The wavefunctions $\psi(x, t)$ is a square-integrable function over spatial variables parametrized by an external time t.

The probability that an event will take place at (x, t) in a frame S should be equal to the probability that the same event will happen at (x', t') in a frame S'

Space and time on equal footing

Introduction

Properties of time Problem of time

Quantum time problem

Time-energy uncertainty
Time observable
Perspectives on the QTP

Relativistic QN

Localization problem One time + multiple

quantization

Number-phase uncertainty

Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Summary

The wavefunctions $\psi(x, t)$ is a square-integrable function over spatial variables parametrized by an external time t.

- The probability that an event will take place at (x, t) in a frame S should be equal to the probability that the same event will happen at (x', t') in a frame S'
- t' is no longer a parameter because it is dependent on both x and t

Space and time on equal footing

ntroduction

Properties of time Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QN

Localization problem One time + multiple

quantization

Number-phase uncertainty

Super many time theory

Guantum gravity Frozen formalism problem

Problem of time is multifaceted

Summary

The wavefunctions $\psi(x, t)$ is a square-integrable function over spatial variables parametrized by an external time t.

- The probability that an event will take place at (x, t) in a frame S should be equal to the probability that the same event will happen at (x', t') in a frame S'
- t' is no longer a parameter because it is dependent on both x and t
- The transformed function loses its interpretation as the description of a state because Hilbert spaces associated with different times are distinct

Relativistic canonical commutation relations

Introduction

Properties of time

Quantum time

Time observable

Perspectives on the QTP

Relativistic QM

Localization problem

One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

Summary

Covariant commutation relation

We should expect that the Heisenberg uncertainty relations can be written in a compact form

$$[\hat{\mathbf{x}}^{\mu},\hat{\mathbf{p}}^{\nu}]=-i\hbar\mathbf{g}^{\mu\nu}$$

where, $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

- The time operator is supposed to be the x^0 component of the 4-position operator \hat{x}^{μ} .
- The Hamiltonian is supposed to be the p^0 component of the 4-momentum operator \hat{p}^{ν} .

Hrvoje Nikolić. "Time in relativistic and nonrelativistic quantum mechanics". In: *International Journal of Quantum Information* 7.03 (2009), pp. 595–602.

Localization in relativistic quantum mechanics

Introduction

Properties of time

Quantum tim

Time observable
Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Summary

Localization problem

Consists of finding the operator representative of the 4-position and/or its eigenstates.

- Such an operator should be able to simultaneously answer when and where a particle is
- Solutions of the Klein-Gordon and Dirac equations cannot provide a well-defined local probability distribution

Andrés J Kálnay. "The localization problem". In: *Problems in the Foundations of Physics*. Springer, 1971, pp. 93–110.

Juan León. "Time-of-arrival formalism for the relativistic particle". In: Journal of Physics A: Mathematical and General 30.13 (1997), p. 4791.

Lawrence P Horwitz. Relativistic quantum mechanics. Vol. 180. Springer, 2015, pp. 1-7.

Theodore Duddell Newton and Eugene P Wigner. "Localized states for elementary systems". In: *Reviews of Modern Physics* 21.3 (1949), p. 400.

Philip Caesar Flores Problem of Time 15 December 2020 21/38

Covariant canonical commutation relations

Introduction

Properties of time

Quantum tim

Time-energy uncertaint

Perspectives on the QTP

Relativistic QN

Localization problem

One time + multiple position

quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem
Problem of time is

Summary

Covariant commutation relation

We should expect that the Heisenberg uncertainty relations can be written in a compact form

$$[\hat{x}^{\mu},\hat{p}^{\nu}]=-i\hbar g^{\mu\nu}$$

where, $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

- Consider a transformation to a primed frame moving with velocity v in the x-direction
- Low-energy: (ignore QED corrections)
 - What happens to the transformation connecting the operators in the two frames?
 - Are the commutators invariant?

Frank R Tangherlini. "Canonical commutation relations and special relativity". In: *Physica Scripta* 77.6 (2008), p. 065008.

What if time is still an external parameter?

Introduction

Properties of time

Quantum time

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

Quantization

Number-phase uncertainty

Super many time theory

Quantum gravity

Frozen formalism problem
Problem of time is
multifaceted

Summary

N-particle system

Consider a system composed of N particles, and let the coordinates of each particle be r_1, r_2, \ldots, r_N . The probability amplitude of the system is $|\psi(t, r_1, \ldots, r_n)|^2$.

- Contradiction with the relativity of simultaneity
- In another reference system, all particles in the system would no longer share a common time coordinate
- The original Hamiltonian would no longer correspond to the total energy

Sin-Itiro Tomonaga – Nobel Lecture. NobelPrize.org. Nobel Media AB 2020. Tue. 20 Oct 2020. https://www.nobelprize.org/prizes/physics/1965/tomonaga/lecture/

Philip Caesar Flores Problem of Time 15 December 2020 23/38

ZY Wang, B Chen, and CD Xiong. "Time in quantum mechanics and quantum field theory". In: *Journal of Physics A: Mathematical and General* 36.18 (2003), p. 5135.

What if time is still an external parameter?

Introduction

Properties of time

Quantum tim

Time observable
Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

quantization Number-phase uncertainty

Number-phase uncertaint Super many time theory

Quantum gravity

Frozen formalism problem Problem of time is multifaceted

Summary

N-particle system

Consider a system composed of N particles, and let the coordinates of each particle be r_1, r_2, \ldots, r_N . The probability amplitude of the system is $|\psi(t, r_1, \ldots, r_n)|^2$.

Many-time theory:

- Separate time coordinate for every particle
- Can compute probability that particle 1 is at r_1 at time t_1 , etc.
- Space coordinates can be taken as dynamic variables while the time coordinate cannot

Paul Adrien Maurice Dirac. "Relativistic quantum mechanics". In: *Proceedings of the Royal Society of London* 136.829 (1932), pp. 453–464.

Felix Bloch and Arnold Nordsieck. "Note on the radiation field of the electron". In: *Physical Review* 52.2 (1937), p. 54.

Philip Caesar Flores Problem of Time 15 December 2020 24/38

Recap of QFT:

Perspectives on the QTP

QFT and second quantization

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is

- In standard quantum mechanics, position is an operator while time is a parameter
- We are now presented with two options:
 - promote time to an operator
 - demote position to a parameter

Recap of QFT:

Introduction

Properties of time

Quantum time

Time observable

Perspectives on the QTP

Relativistic QI

One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem
Problem of time is
multifaceted

Summary

- \Rightarrow operators: $\hat{x}, \hat{p} \longrightarrow \hat{\Psi}(x, t), \hat{\Pi}(x, t)$
- ⇒ canonical commutation relations:

$$[\hat{x},\hat{p}]=i\hbar\longrightarrow\left[\hat{\Psi}(x,t),\hat{\Pi}(x',t)\right]=i\hbar\delta(x-x')$$

⇒ are there no problems regarding time in QFT? the time-energy uncertainty manifests itself as the phase-number uncertainty

Number-phase uncertainty

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Phase operator

The existence of a Hermitian phase operator $\hat{\phi}$ was first postulated by Dirac in his description of the quantized electromagnetic field.

The creation and annihilation operators were factored in the following form

$$\hat{a} = \exp\left(i\hat{\phi}\right)\sqrt{\hat{N}}$$
 $\hat{a}^{\dagger} = \sqrt{\hat{N}}\exp\left(-i\hat{\phi}\right)$

It follows that $|\hat{N}, \hat{\phi}| = i\hbar$

DT Pegg and SM Barnett. "Unitary phase operator in quantum mechanics". In: EPL (Europhysics Letters) 6.6 (1988), p. 483.

Paul Adrien Maurice Dirac, "The quantum theory of the emission and absorption of radiation", In: Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character 114.767 (1927), pp. 243-265.

Number-phase uncertainty

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Phase operator

The existence of a Hermitian phase operator $\hat{\phi}$ was first postulated by Dirac in his description of the quantized electromagnetic field.

- The approach has two problems:
 - The operator $\exp(i\hat{\phi})$ is not unitary
 - The number operator has a spectrum bounded below
 - ⇒ these are the same problems encountered by the time operator which led Pauli to the conclusion of the non-existence of a time operator

DT Pegg and SM Barnett, "Unitary phase operator in quantum mechanics". In: EPL (Europhysics Letters)

Christopher Gerry, Peter Knight, and Peter L Knight. Introductory quantum optics. Cambridge university

Super many time theory

Introductio

Properties of time

Quantum tin problem

Time observable
Perspectives on the QTP

Perspectives on the Q1

Relativistic QI

Localization problem One time + multiple position

quantization Number-phase uncertainty

Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

Summary

An extension of Dirac's many time theory to quantum field theory was done by Tomonaga

⇒ was able to show that infinities in the field theory of the electron and electromagnetic fields, and quantum electrodynamics can be regarded as a correction to the mass and energy of electrons

Sin-itiro Kanesawa Suteo; Tomonaga. "On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields. IV *". In: *Progress of Theoretical Physics* 3 (1 Jan. 1950).

S.-i. Kanesawa S.; Tomonaga. "On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields. V: Case of Interacting Electromagnetic and Meson Fields". In: *Progress of Theoretical Physics* 3 (1 Mar. 1948).

Sin-Itiro Tomonaga – Nobel Lecture. NobelPrize.org. Nobel Media AB 2020. Tue. 20 Oct 2020. https://www.nobelprize.org/prizes/physics/1965/tomonaga/lecture/

See: University of Tsubaka, Bibliography: Dr. Sin-Itiro Tomonaga https://bit.ly/2JyRzwg
Philip Caesar Flores Problem of Time 15 December 2020 29/38

S. Tomonaga. "On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields". In: *Progress of Theoretical Physics* 1 (2 Feb. 1949).

Frozen time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Wheeler-DeWitt equation

Physical states are annihilated by the Hamiltonian of the theory

$$\hat{H}\Psi[g_{ik},\phi]=0$$

where, \hat{H} is a constraint quadratic in momenta conjugate to g_{ik} .

- ⇒ The wave function of the Universe is in an eigenstate of its Hamiltonian.
- ⇒ Hamiltonian does not generate time translations of the physical states with respect to an external time.

Bryce S DeWitt, "Quantum theory of gravity, I. The canonical theory", In: Physical Review 160.5 (1967). p. 1113.

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012),

Philip Caesar Flores

Problem of Time

Frozen time

Problem of time

Time observable

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem

Page and Wootters mechanicsm

Consider an ancillary system in addition to the system S of interest. Could we then have a setup in which the global system is time independent, yet system S is time dependent and obeys the Schrödinger equation?

Use a clock as the ancillary system, and correlate its initial state with the initial state of S

Don N Page and William K Wootters. "Evolution without evolution: Dynamics described by stationary observables", In: Physical Review D 27,12 (1983), p. 2885.

Alexander RH Smith and Mehdi Ahmadi. "Quantizing time: Interacting clocks and systems". In: Quantum

Perspectives on the QTP

Number-phase uncertainty

Super many time theory

Frozen formalism problem

Problem of time is multifaceted

it has become more common to suggest that the problem of time is the Frozen Formalism Problem.

Edward Anderson. "Problem of time in quantum gravity". In: Annalen der Physik 524.12 (2012), pp. 757-786.

KV Kuchař and CG Torre. Conceptual Problems of Quantum Gravity ed A Ashtekar and J Stachel. 1991. Chris J Isham. "Canonical quantum gravity and the problem of time". In: Integrable systems, quantum

groups, and quantum field theories. Springer, 1993, pp. 157–287.

Philip Caesar Flores Problem of Time 15 December 2020 32/38

Introduction

Problem of time

Quantum time problem

Time observable
Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

quantization Number-phase uncertainty

Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Summary

it has become more common to suggest that the problem of time is the Frozen Formalism Problem.

A more long-standing point of view is that the POT contains a number of further facets.

Edward Anderson. "Problem of time in quantum gravity". In: *Annalen der Physik* 524.12 (2012), pp. 757–786.

KV Kuchař and CG Torre. Conceptual Problems of Quantum Gravity ed A Ashtekar and J Stachel. 1991. Chris J Isham. "Canonical quantum gravity and the problem of time". In: Integrable systems, quantum groups, and quantum field theories. Springer, 1993, pp. 157–287.

Philip Caesar Flores Problem of Time 15 December 2020 32/38

Introductio

Problem of time

Quantum tin problem

Time observable
Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

Number-phase uncertainty Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Summary

- it has become more common to suggest that the problem of time is the Frozen Formalism Problem.
- A more long-standing point of view is that the POT contains a number of further facets.
- These facets interfere with each other

Edward Anderson. "Problem of time in quantum gravity". In: *Annalen der Physik* 524.12 (2012), pp. 757–786.

KV Kuchař and CG Torre. Conceptual Problems of Quantum Gravity ed A Ashtekar and J Stachel. 1991. Chris J Isham. "Canonical quantum gravity and the problem of time". In: Integrable systems, quantum groups, and quantum field theories. Springer, 1993, pp. 157–287.

Philip Caesar Flores Problem of Time 15 December 2020 32/38

Introduction

roblem of time

Quantum tir problem

Time observable
Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

quantization

Number-phase uncertainty Super many time theory

Quantum gravity Frozen formalism problem

Problem of time is multifaceted

Summary

- it has become more common to suggest that the problem of time is the Frozen Formalism Problem.
- A more long-standing point of view is that the POT contains a number of further facets.
- These facets interfere with each other
- All arise from a common cause: the mismatch of the notions of time in GR and Quantum Theory

Edward Anderson. "Problem of time in quantum gravity". In: *Annalen der Physik* 524.12 (2012), pp. 757–786.

KV Kuchař and CG Torre. Conceptual Problems of Quantum Gravity ed A Ashtekar and J Stachel. 1991. Chris J Isham. "Canonical quantum gravity and the problem of time". In: Integrable systems, quantum groups, and quantum field theories. Springer, 1993, pp. 157–287.

Philip Caesar Flores Problem of Time 15 December 2020 32/38

Problem of time

Time observable

Perspectives on the QTP

position

Number-phase uncertainty Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Introduction

Properties of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem

Problem of time is

multifaceted

ummary

Foliation dependence problem

How observers operationally determine the foliation for the physics that they experience

Introduction

Properties of time Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem
Problem of time is
multifaceted

Summary

Foliation dependence problem

How observers operationally determine the foliation for the physics that they experience

Constraint closure problem

Closure of a classical Poisson bracket algebroid does not entail closure of the corresponding quantum commutator algebroid

Problem of time

Time observable

Perspectives on the QTP

position

Number-phase uncertainty Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Problem of time

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is

multifaceted

Global problem of time

Time functions may not be globally defined in space and/or time.

Introduction

Properties of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple position

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity Frozen formalism problem

Problem of time is multifaceted

Summary

Global problem of time

Time functions may not be globally defined in space and/or time.

Multiple Choice Problem

Different choices of time variable may give inequivalent quantum theories, e.g. making different choices of sets of variables to quantize may give inequivalent quantum theories.

Problem of time

Time observable

Perspectives on the QTP

position

Number-phase uncertainty Super many time theory

Frozen formalism problem

Problem of time is multifaceted

Introduction

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem

Problem of time is

Problem of time multifaceted

Summary

Problem of beables/observables

Involves construction of a sufficient set of beables for the physics of one's model, which are then involved in the model's notion of evolution.

15 December 2020 35/38

Perspectives on the QTP

Number-phase uncertainty Super many time theory

Frozen formalism problem Problem of time is multifaceted

Problem of beables/observables

Involves construction of a sufficient set of beables for the physics of one's model, which are then involved in the model's notion of evolution.

Spacetime reconstruction problem

Quantum fluctuations are too numerous to be embedded within a single spacetime

Introduction

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

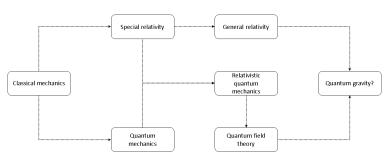
Localization problem
One time + multiple

QFT and second quantization

Number-phase uncertainty Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted



- Quantum gravity aims to unify the empirically successful theories of general relativity and quantum field theory
- Incompatible notions of time

Introduction

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem One time + multiple

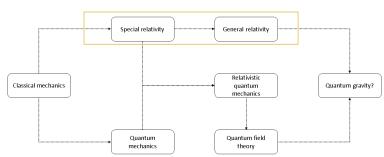
quantization Number-phase uncertainty

Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted

Summary



Time in relativity is dynamical:

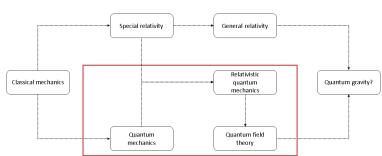
- SR treats time and space as components of a 4-vector
- Time in GR is influenced by the geometry of spacetime

Perspectives on the QTP

Number-phase uncertainty

Super many time theory

Frozen formalism problem



- Quantum mechanics treats time as an external parameter which governs the evolution of the system
- Quantization schemes lack any fundamental notion of time

introduction

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QI

Localization problem

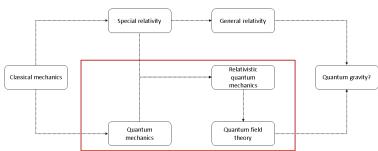
One time + multiple position

quantization Number-phase uncertainty

Super many time theory

Quantum gravit

Frozen formalism problem Problem of time is multifaceted



- How should the notion of time be reintroduced?
- Is time a fundamental concept or is it purely phenomenological?

Properties of time

Problem of time

Quantum time problem

Time observable

Perspectives on the QTP

Relativistic QN

Localization problem One time + multiple position

quantization

Number-phase uncertainty Super many time theory

Quantum gravity

Frozen formalism problem
Problem of time is
multifaceted

- lack of a consistent quantum description of time has far reaching consequences
- the problem is a very broad subject and a rich history

Thank You

A derivation of the time-energy uncertainty relation by Bohr

Let Δt , Δx be the extension of a wavepacket in time and space, respectively. Furthermore, let $\Delta \nu$ and $\Delta \sigma$ be the frequency and number widths.

"In the most favorable case"

$$\Delta t \Delta \nu = \Delta x \Delta \sigma = 1$$

$$\Delta t \Delta \left(\frac{E}{h}\right) = \Delta x \Delta \left(\frac{p}{h}\right) = 1$$

$$\Delta t \Delta E = \Delta x \Delta p = h$$

Philip Caesar Flores Probl

N Bohr. The quantum postulate and the recent development of atomic energy, talk at the Como Conference 1927, published in Nature (Supplement), 121, 580-590 (1928); reprinted in Atomic Theory and the Description of Nature.