Implicit Neural Representations with Periodic Activation Functions

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Implicit (neural) representations

- Neural networks that parameterize such implicitly defined fun ctions.
- Model discrete signals using a continuous and differentiable r epresentation.

A class of functions Φ

- We are interested in a class of functions Φ that satisfy equation of the form
- $F(x, \Phi, \nabla_x \Phi, \nabla_x^2 \Phi, \ldots) = 0, \Phi: x \rightarrow \Phi(\mathrm{x})$
- F could be any constraint.

Example of $\Phi: x \rightarrow \Phi(\mathrm{x})$

- Image: Coordinate to RGB value
- One function for one image.
- The other way to save image.

Contribution

- **A continuous implicit neural representation using periodic activation functions that fits complicated signals, such as natural images and 3D shapes, and their derivatives robustly.**
- **An initialization scheme** for training these representations and validation that distributions of these representations can be learned using **hypernetworks.**
- **Demonstration of applications in:** image, video, and audio representation; 3D shape reconstruction; solving first-order differential equations that aim at estimating a signal by supervising only with its gradients; and solving second-order differential equations.

Formulation

- Find $\Phi(x)$ subject to $C_m(a(x), \Phi(x), \nabla\Phi(x), \dots) = 0, \forall x \in \Omega_m, m = 1, \dots M$
- This problem can be cast in a loss function that penalizes dev iations from each of the constraints on their domain Ω_m:

$$\mathcal{L} = \int_{\Omega} \sum_{m=1}^{M} \mathbf{1}_{\Omega_m}(x) \| C_m(a(x), \Phi(x), \nabla\Phi(x), \dots) \| \, dx$$

Formulation

- Periodic activations for implicit neural representations.
- $\Phi(x) = W_n(\phi_{n-1} \circ \phi_{n-2} \ldots \circ \phi_0)(x) + b_n$
- $x \rightarrow \phi_i(x_i) = \sin(W_i x_i + b_i)$
- I.e. MLP with sinusoidal activation function.
- **Non-monotonic: Initialize carefully.**

Fitting ground truth image

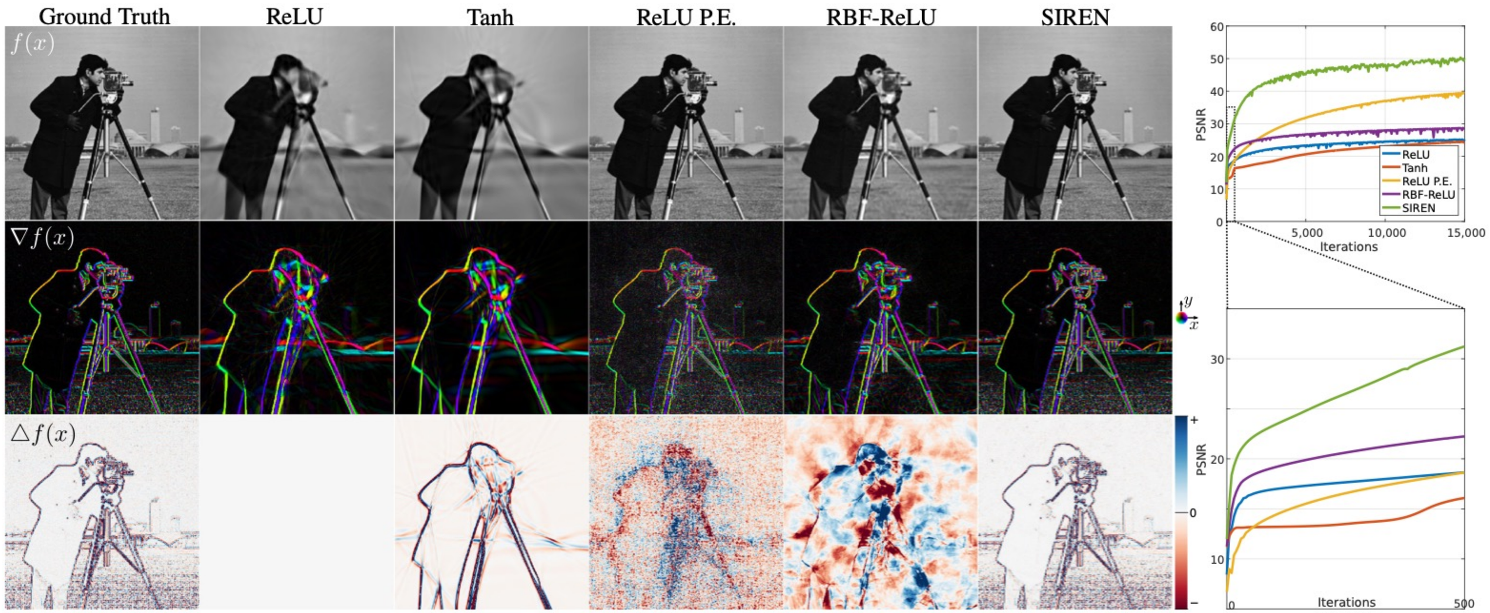

Figure 1: Comparison of different implicit network architectures fitting a ground truth image (top left). The representation is only supervised on the target image but we also show first- and second-order derivatives of the function fit in rows 2 and 3, respectively.

A principled initialization scheme

- $w_i \sim \mathcal{U}\left(-\frac{c}{\sqrt{n}}, \frac{c}{\sqrt{n}}\right), \rightarrow w_i x \sim \mathcal{N}(0, \frac{c^2}{6})$
- Hence, we propose $c = \sqrt{6}$, so that $w_i \sim \mathcal{U}\left(-\sqrt{\frac{6}{n}}, \sqrt{\frac{6}{n}}\right)$ and ensure standard deviation of weighted sum = 1

Shape representation

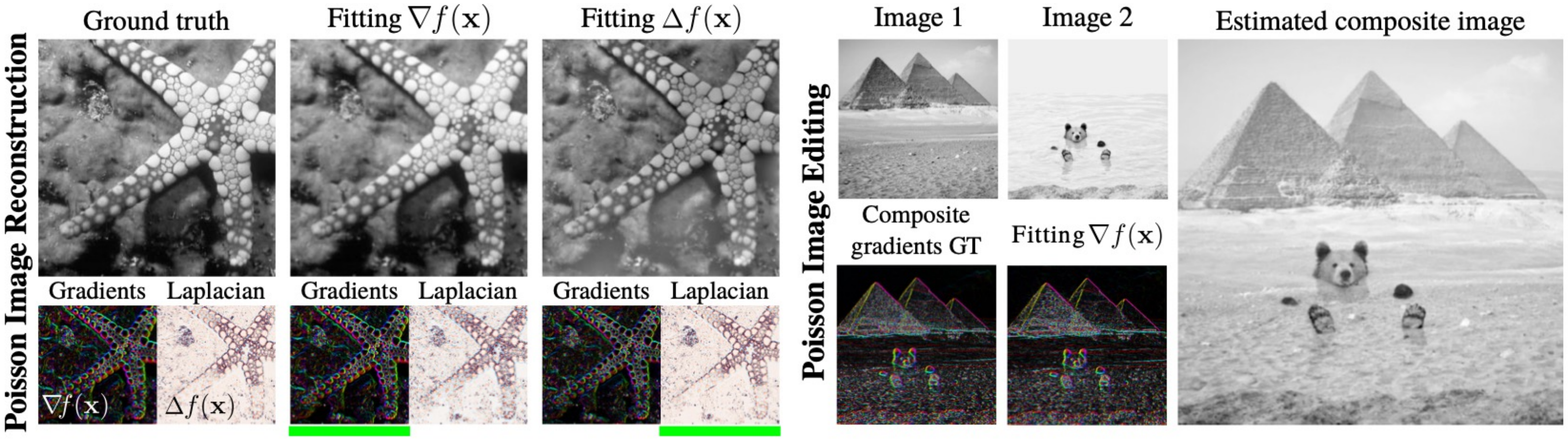

Figure 3: **Poisson image reconstruction:** An image (left) is reconstructed by fitting a SIREN, supervised either by its gradients or Laplacians (underlined in green). The results, shown in the center and right, respectively, match both the image and its derivatives well. **Poisson image editing:** The gradients of two images (top) are fused (bottom left). SIREN allows for the composite (right) to be reconstructed using supervision on the gradients (bottom right).

$$\mathcal{L}_{\text{grad.}} = \int_{\Omega} \|\boldsymbol{\nabla}_{\mathbf{x}}\Phi(\mathbf{x}) - \boldsymbol{\nabla}_{\mathbf{x}} f(\mathbf{x})\| \, d\mathbf{x}, \quad \text{or} \quad \mathcal{L}_{\text{lapl.}} = \int_{\Omega} \|\Delta\Phi(\mathbf{x}) - \Delta f(\mathbf{x})\| \, d\mathbf{x}. \tag{5}$$

Signed Distance Functions

- Definition:
- the signed distance function of a set Ω in a metric space dete rmines the distance of a given point x from the boundary of Ω, with the sign determined by whether x is in Ω.

A disk (top, in grey) and its signed distance function

Wikipedia, Signed distance function: https://en.wikipedia.org/wiki/Signed_distance_function

Representing Shapes with Signed Distance Functions

- Three constraints:
- 1. $\Phi(x) = 0$
- 2. $\|\nabla\Phi(x)\| = 1$
- 3. $\langle\nabla\Phi(x), n(x)\rangle = 1$

$$\mathcal{L}_{\text{sdf}} = \int_{\Omega} \big\| \, |\nabla_{\mathbf{x}} \Phi(\mathbf{x})| - 1 \big\| d\mathbf{x} + \int_{\Omega_0} \|\Phi(\mathbf{x})\| + \big(1 - \langle \nabla_{\mathbf{x}} \Phi(\mathbf{x}), \mathbf{n}(\mathbf{x}) \rangle\big) d\mathbf{x} + \int_{\Omega \setminus \Omega_0} \psi\big(\Phi(\mathbf{x})\big) d\mathbf{x}, \quad (6)$$

$$\psi(\mathbf{x}) = \exp(-\alpha \cdot |\Phi(\mathbf{x})|), \alpha \gg 1$$

Representing Shapes with Signed Distance Functions

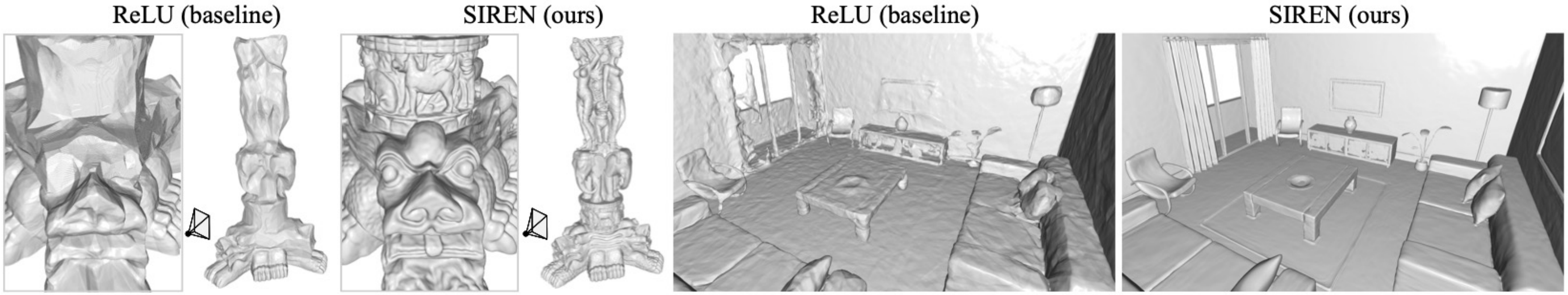

Figure 4: Shape representation. We fit signed distance functions parameterized by implicit neural representations directly on point clouds. Compared to ReLU implicit representations, our periodic activations significantly improve detail of objects (left) and complexity of entire scenes (right).