A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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CHAPTER 4.

Matching function

In slackish markets, all trades are mediated by a matching function. These markets therefore differ from Walrasian markets, where all trades are mediated via an auction, and will have sharply different properties.

In this chapter we introduce the matching function. This will be a key tool we use to model slackish markets and study slack and unemployment. The matching function summarizes the complex process through which buyers find sellers: for instance, how workers searching for jobs meet firms searching for employees, and how firms searching for customers meet consumers searching for goods and services.[1] Pissarides (2000, pp. 3–4) explains the role of the matching function wonderfully in the case of the labor market, but this book's argument is that his description in fact applies to the vast majority of markets. Slightly generalizing his words:

> Trade in [any] market is a nontrivial economic activity because of the existence of heterogeneities . . . and information imperfections. If all [buyers and sellers] were identical to each other, and if there was perfect information about their [attributes], trade would be trivial. But without homogeneity on either side of the market and with costly acquisition of information, [buyers and sellers] find it necessary to spend resources to find [desirable] trades. . . . The matching function gives the outcome of the investment of resources by [buyers and sellers] in the trading process as a function of inputs. It is a modeling device that captures the implications of the costly trading process without the need

[1] See Petrongolo and Pissarides (2001, 425–427) for a detailed history of the matching function.

to make ... the features that give rise to it explicit.

After having introduced a generic matching function and discussed its properties, we will cover several specific matching functions.

4.1. Why do we need a matching function?

At any point in time, in almost any market, we have buyers and sellers who want to trade, but are not able to trade immediately. On the labor market, there are workers who want to sell their labor and firms who want to buy labor and yet, even though these two groups coexist, it takes time for workers to find a job and for firms to fill a vacant job. On the housing market, families trying to sell their houses coexist with families looking for a house to buy—but it takes time for families to sell their homes and for other families to find a new home. On the car market, there are cars that sit in inventory and people who are looking to buy cars. It takes time for people to find a car, and it also takes time for the cars in inventory to be sold. On the service market, there may be babysitters who are looking for work, and at the same time, there are families looking to hire—it still takes time for them to find the right babysitter. There are restaurant tables that are empty and hungry people looking for a good restaurant to eat at.

The bottom line is that in almost all markets, buyers who want to buy and sellers who want to sell always coexist—it takes time for the transaction to occur. To model such markets, we cannot use a Walrasian market. In the Walrasian market, one can always buy and sell any quantity of a good at the market price, immediately and without constraints. So a Walrasian market ignores the fact that it takes time and effort to buy and sell in most real-world markets.

To model this complex trading process, we introduce a tool called the matching function. The matching function is an aggregate function that captures and summarizes, at the macro level, all the complexities of trading that happen at the micro level. It is very similar to the production function, which summarizes at the aggregate level all the production that happens at the micro level. It is a simple, well-behaved function that depends on only a few aggregate variables.

Most trading is quite complicated—it is hard for buyers and sellers to get together to execute a trade that both sides desire. For instance, on the labor market, a worker with specific skills would look for a job with specific requirements, whether it be a specific industry, specific location, or specific working conditions or responsibilities. Similarly, a firm advertising a vacant job would have specific needs and would be looking for a worker with the right skill set, the right experience, the right qualifications and character.

This complexity is not limited to the labor market. It would also occur when firms trade with other firms. When firms look for suppliers for a specific part in a product that they are building, they have a long list of technical requirements because that part is unique to their product. They also have additional constraints on the quality of the part produced by the supplier, its reliability, the supplier's location and how it operates. The production capacity of the supplier and its existing commitments also matter, as they determine whether the supplier can produce the part rapidly and in sufficient quantity.

And the complexity of trading is not confined to the labor market or to sophisticated goods and services traded by firms. Even for seemingly simple goods and services purchased in everyday life, trading is often complex because buyers have heterogeneous needs and sellers offer differentiated products. Consider something as simple as a haircut. Hairdressers specialize: for women, men, children; for budget or high-end styles. Hairdressers have more or less experience, are more or less skilled, and are more or less talkative. Hair salons have different numbers of chairs and existing customers so they are more or less busy. And customers' preferences differ. Hours of operation and locations may or may not fit a customer's schedule and daily routine. As a result, it takes time and effort for customers to obtain a suitable haircut, and conversely, for hairdressers to attract and retain customers.

This complexity exists in most markets, so the matching function can be applied to almost all markets in which trading is complex. The only exceptions are markets in which the goods that are traded are available in large quantity and entirely homogeneous. Then, as all goods are the same, finding the right seller is not relevant. An example is the market for common stocks, where all the shares are exactly the same. This is well-modeled by a Walrasian model. The market for commodities such as gold or oil is similar too. But very few markets operate that way; in practice, most goods or services sold are different and most buyers have specific needs. Even financial markets, for the most part, operate within an over-the-counter structure, which is better modeled using a matching function than a Walrasian apparatus (Hugonnier, Lester, and Weill 2025).

Furthermore, the complexity of the trade process is quite visible by the effort that both buyers and sellers put into trying to trade. If we look at the labor market, firms that are trying to fill a vacant job have to spend a lot of time and effort recruiting. Similarly, workers spend a lot of time and effort trying to find a job, for example by browsing LinkedIn and sending out their applications. On the product market, a buyer spends time doing market research before purchasing a good or service, either by reading customer reviews or going on websites like Yelp or TripAdvisor. Similarly, sellers also spend time and money on marketing and advertising to try and find consumers.

4.2. Theoretical properties of the matching function

The matching function is an aggregate function that summarizes all the micro-level complexities of the trading process. The idea of summarizing the complex trading process by a generic function—without modeling the trading process explicitly—goes back to Pissarides (1979, 1984, 1985, 1986). Before that, people usually modeled specific trading processes that resulted in trading probabilities between 0 and 1—often from the urn-ball model that is commonly used in probability. Even today, researchers usually assume specific functional forms for their matching functions—often a Cobb-Douglas function. But most matching functions share similar properties, reflecting natural properties of markets. We review these properties here, before discussing their implications for traders and market outcomes.

Consider a market for a specific good with $s > 0$ sellers and $b > 0$ buyers, open for one period. Each seller sells one good, and each buyer aims to purchase one good. The matching function m gives the number of trades in the period:

$$M = m(s, b).$$

The shape of the matching function is often characterized by the matching elasticity: the elasticity of the matching function with respect to the number of sellers, denoted η. Formally, the matching elasticity is defined by

$$\eta = \frac{s}{m(s, b)} \cdot \frac{\partial m(s, b)}{\partial s}.$$

The matching elasticity may be constant or not, depending on matching function.

A few restrictions are typically imposed on the matching function, to be realistic and simplify the analysis (Blanchard and Diamond 1989; Petrongolo and Pissarides 2001).

First, obviously, the matching function is assumed to be positive.

Second, the matching function is assumed to be increasing in both arguments, s and b. This means that if there are more sellers in the market, or more buyers in the market, there will be more trades. This is a natural assumption. If more goods are available, chances are that a greater number of buyers will find an appropriate good to buy. Conversely if there are more buyers on the market, chances are that a greater number of sellers will find a buyer for their good.

A third assumption is that the matching function has constant returns to scale:

$$m(zs, zb) = zm(s, b) \tag{4.1}$$

for any $z \geq 0$. This assumption critically simplifies the analysis, because it ensures that the trading probabilities can be expressed as functions of market tightness, as the next section shows. This assumption has also been tested extensively with labor market data and could not be rejected (Petrongolo and Pissarides 2001).

With constant returns to scale, the elasticity of the matching function with respect to the number of buyers is directly connected to the matching elasticity η:

$$\frac{b}{m(s,b)} \cdot \frac{\partial m(s,b)}{\partial b} = 1 - \eta. \tag{4.2}$$

This result is a direct application of Euler's homogeneous function theorem. For a function that has constant returns to scale (homogenous of degree 1), the theorem says that the sum of the function's elasticities with respect to all its variables is equal to 1.[2]

A fourth assumption is that the matching function is concave in both arguments, s and b. This means that the matching function exhibits diminishing marginal returns to the numbers of sellers and buyers, which seems natural.

A fifth typical assumption is that there are no trades if there are no buyers or no sellers. This is also natural, since we need people on both sides of the market for trades to occur.

A final, sixth assumption is that the number of trades that occur within the period considered is less than the numbers of sellers and buyers: $m(s,b) \leq \min(s,b)$. Indeed, there cannot be more goods sold than sellers, and more goods bought than buyers. This assumption is specific to discrete-time models, however. In a continuous-time model, the matching function gives the flow of trades occurring at any point in time. Hence there is no restriction on the level of $m(s,b)$.

4.3. Market tightness and trading probabilities

The matching function tells us that, during a certain period, not all sellers are able to sell their good, and not all buyers are able to buy a good. Therefore, we need to figure out the probabilities that a buyer is able to buy and that a seller is able to sell. The two trading possibilities are the probability to sell f and the probability to buy q, given by

$$f = \frac{M}{s}, \quad q = \frac{M}{b}.$$

The assumptions we made on the matching function have clear implications for how these trading probabilities behave.

4.3.1. Market tightness

Before we start, we introduce a new fundamental variable, the market tightness:

$$\theta = \frac{b}{s}.$$

[2] We can also obtain (4.2) by differentiating the constant-returns-to-scale condition (4.1) in elasticity with respect to z, using the results from appendix B. The differentiation should be around $z = 1$, for any (s,b). That process tells us that the sum of the matching function's elasticities with respect to the numbers of sellers and buyers equals 1.

The tightness can be defined on any market with sellers and buyers.

Since both trading probabilities will only depend on market tightness, tightness will play a crucial role in the book—it is going to summarize the state of any market.

4.3.2. Selling probability

We then look at the selling probability:

$$f = \frac{m(s,b)}{s} = m\left(\frac{s}{s}, \frac{b}{s}\right) = m\left(1, \frac{b}{s}\right),$$

since the matching function has constant returns to scale. We can re-express our selling probability as a function of market tightness:

$$f(\theta) = m(1, \theta). \tag{4.3}$$

Since the matching function is increasing in its two arguments, we infer that the selling probability is increasing in tightness. The tighter the market, the more likely you are to sell. This makes sense because the tighter a market is, the more buyers there are for each seller. And since the matching function is concave in its two arguments, we infer that the selling probability is concave in tightness. This means that a higher tightness leads to a higher selling probability, but with diminishing returns.

We can also see that when tightness is 0, the selling probability must also be 0. This is because, in the matching function, when any of the two arguments are zero, there is no trade. This is intuitive because if there are no buyers, there is no chance of selling anything, so the probability of selling has to be 0.

Lastly, we know that the selling probability is always between 0 and 1 because we have assumed that the matching function is always less than the minimum of its two arguments. Since $m(s,b) \leq \min(s,b)$, then $m(s,b) \leq s$ and $f \leq 1$.

4.3.3. Buying probability

Now, we shift our attention to the buying probability:

$$q = \frac{m(s,b)}{b} = m\left(\frac{s}{b}, \frac{b}{b}\right) = m\left(\frac{s}{b}, 1\right),$$

since the matching function has constant returns to scale. Using market tightness, we can rewrite the probability as:

$$q(\theta) = m\left(\frac{1}{\theta}, 1\right), \tag{4.4}$$

so that the buying probability only depends on tightness.

We can go over the properties of the buying probability in the same way. When tightness increases, $1/\theta$ decreases. Because the matching function is increasing in both arguments, the buying probability is decreasing in tightness. This means that a buyer in a tight market is less likely to be able to buy the good they want, since there are very few sellers and a lot of buyers, increasing competition. This is true in any tight market: it is a good time to be a seller but a bad time to be a buyer.

Another thing that we see is when we take the limit, the buying probability is 0 when tightness is infinite. This makes sense because when infinitely many buyers are competing to buy the goods they want, the probability to be able to buy that good is bound to go to 0.

Finally, we know that the buying probability is always between 0 and 1 because we have assumed that the matching function is always less than the minimum of its two arguments. Since $m(s, b) \leq \min(s, b)$, then $m(s, b) \leq b$ and $q \leq 1$.

4.3.4. Relation between the trading probabilities

There is a simple relationship between the two trading probabilities. The number of trades is $f(\theta)s = q(\theta)b$, so $f(\theta)/q(\theta) = b/s = \theta$. That is, for any matching function:

$$f(\theta) = \theta q(\theta). \tag{4.5}$$

From this result we also see that the elasticities of the trading probabilities with respect to tightness are necessarily related:

$$\epsilon^f_\theta = 1 + \epsilon^q_\theta.$$

From (4.4), and the results in appendix B on the elasticity of composite functions, we see that the elasticity of the buying probability is directly related to the matching elasticity η:

$$\epsilon^q_\theta = -\eta. \tag{4.6}$$

Thus, the elasticity of the selling probability is also related to the matching elasticity:

$$\epsilon^f_\theta = 1 - \eta. \tag{4.7}$$

Once again, these elasticities might or might not be constant, depending on the matching function.

4.4. Urn-ball matching function

We now study the urn-ball matching function. This matching function is useful because it has a simple microfoundation, which draws on the urn-ball model in probability theory.

4.4.1. Foundation

Several researchers proposed an urn-ball foundation to explain why trading probabilities might be less than 1 in a given period.

Butters (1977) used the urn-ball framework in the product market. In this model, firms advertise their products by dropping ads into customers' mailboxes. The probability to sell is less than 1 because several firms might drop their ad in the same mailbox, in which case the customer only buys from the cheapest firm. The probability to buy is also less than 1, because a customer might not receive any ad in their mailbox.

Hall (1979) used a similar setup in the labor market. In this model, firms simultaneously and randomly make job offers to job seekers. The probability to hire a job seeker is less than 1 because several firms might make a job offer to the same job seeker, who cannot accept more than one offer. The probability to find a job is also less than 1 because a job seeker might be unlucky and not receive any of the job offers.

Both of these situations correspond to an urn-ball setup. In Butters's example, the ads are the balls and the mailboxes are the urns. In Hall's example, the job offers are the balls and the job seekers are the urns.

In this section, we consider as an example the haircut market. There are b customers who are looking for a haircut, and s hairdressers. Each hairdresser can only accommodate one customer at a time, so if two customers pick the same hairdresser, only one will get a haircut. Because customers do not know where the others go, they don't know which hairdressers already serve a customer and which hairdressers are idle. Such coordination failure means that some customers cannot get a haircut and some hairdressers do not get customers.

4.4.2. Expression

We now derive the number of haircuts sold in a simple situation: b customers in need of a haircut each go to one of s open hair salons. The customers pick their hair salons simultaneously and randomly. That day, customers can only make one trip to the hair salon.

The probability that a specific customer goes to a specific hairdresser is $1/s$, and the probability that the customer does not go to the hairdresser is $1 - 1/s$. Accordingly, the probability that a specific hairdresser does not get a visit from any of the b customers is $(1 - 1/s)^b$. Conversely, the probability that a specific hairdresser gets at least one visit is $1 - (1 - 1/s)^b$. Since a hairdresser sells a haircut if at least one customer visits, this probability gives the probability to sell a haircut for a specific hairdresser, as well as the expected number of haircuts sold by the hairdresser (since the hairdresser sells either 0 or 1 haircut). Then, the expected number of haircuts sold by all hairdressers is just

$$M = s\left[1 - \left(1 - \frac{1}{s}\right)^b\right].$$

This is the expected number of trades that are going to occur on the haircut market. This matching function is a little cumbersome, but it can be greatly simplified when the number of hairdressers is large enough.

To simplify the above matching function, we use a linear approximation of the logarithm: $\ln(1 - x) \approx -x$ when x is small. This approximation allows us to rewrite the probability that a hairdresser gets no visit at all:

$$\left(1 - \frac{1}{s}\right)^b = \exp\left(b \cdot \ln\left(1 - \frac{1}{s}\right)\right) \approx \exp\left(b \cdot \frac{-1}{s}\right) = \exp\left(-\frac{b}{s}\right).$$

We can then rewrite our matching function, which is the expected number of haircuts sold:

$$m(s, b) = s\left[1 - \exp\left(-\frac{b}{s}\right)\right]. \tag{4.8}$$

This is the urn-ball matching function, which gives the number of trades for a given number of sellers, s, and buyers, b.

4.4.3. Properties

By looking at the matching function (4.8), we can verify that it satisfies the general properties mentioned in section 4.2.

First, we confirm that the matching function is positive, and if the number of sellers or buyers is zero, the matching function is zero.

Second, we see that the matching function has constant returns to scale:

$$m(zs, zb) = zs\left[1 - \exp\left(-\frac{zb}{zs}\right)\right] = zm(s, b),$$

since the z in the numerator and the z in the denominator of the fraction cancel out.

Third, we check that the matching function is increasing in both its arguments. We immediately see from (4.8) that if the number of buyers goes up, the number of haircuts also increases. Formally, the partial derivative of the matching function with respect to the number of buyers is simply:

$$\frac{\partial m}{\partial b} = \exp\left(-\frac{b}{s}\right) > 0.$$

Showing that the number of haircuts increases with the number of hairdressers is a little bit more tricky since s has two opposite influences on the matching function. We must calculate the derivative formally to check that it is positive. After simplifying, we get:

$$\frac{\partial m}{\partial s} = 1 - \left(1 + \frac{b}{s}\right)\exp\left(-\frac{b}{s}\right). \tag{4.9}$$

By using the fact that $\exp(x) \geq 1 + x$ for all x, we infer that $1 \geq (1 + x)\exp(-x)$ for all x, so we verify that $\partial m/\partial s \geq 0$. From this we conclude that the matching function is increasing in s. Thus, we confirm that the matching function is increasing in its two arguments.

Fourth, we verify that the matching function is concave in both arguments. Clearly $\partial m/\partial b$ is decreasing in b, so $\partial^2 m/\partial b^2 < 0$, which tells us the matching function is concave in the number of buyers. The second derivative of the matching function with respect to the number of sellers is

$$\frac{\partial^2 m}{\partial s^2} = -\frac{b^2}{s^3}\exp\left(-\frac{b}{s}\right);$$

since $\partial^2 m/\partial s^2 < 0$, the function is also concave with respect to the number of sellers.

Finally, the matching function is less than its two arguments. Since $\exp(-x) > 0$, it is clear that $m(s, b) \leq s$. Using again $\exp(x) \geq 1 + x$, we obtain

$$m(s, b) \leq s\left[1 - \left(1 - \frac{b}{s}\right)\right] = b.$$

So overall $m(s, b) \leq \min(s, b)$.

The trading probabilities given by the urn-ball matching function are illustrated in figure 4.1.

4.4.4. Matching elasticity

A key statistic to describe the shape of the matching function is the matching elasticity: the elasticity of the matching function with respect to the number of sellers. We compute the matching elasticity to better understand how the urn-ball matching function behaves.

We start from the definition of the matching elasticity:

$$\eta = \frac{s}{m(s, b)} \cdot \frac{\partial m}{\partial s}.$$

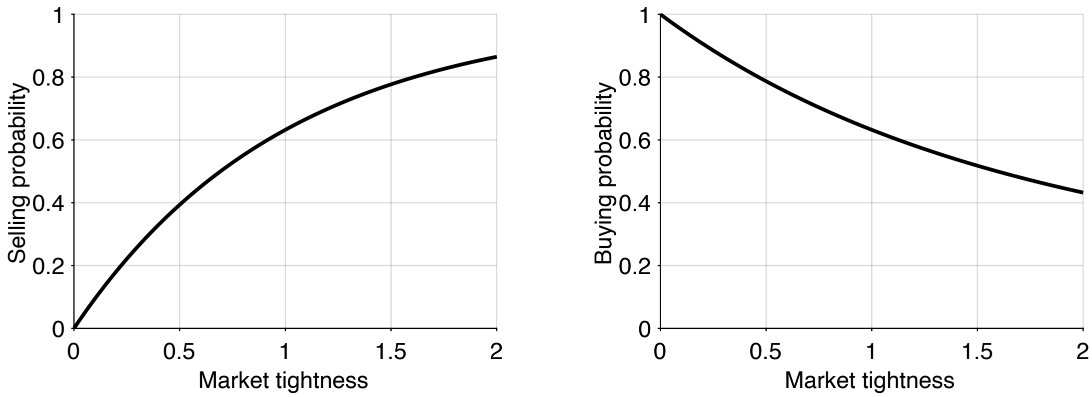


FIGURE 4.1. Selling and buying probabilities from urn-ball matching function

The urn-ball matching function is given by (4.8). The selling and buying probabilities are computed from the matching function with (4.3) and (4.4).

Using the partial derivative (4.9), we can write the matching elasticity as

$$\eta = \frac{s}{m(s,b)} \cdot \left[1 - \exp\left(\frac{-b}{s}\right)\right] - \frac{b}{m(s,b)} \cdot \exp\left(\frac{-b}{s}\right).$$

Given that the matching function $m(s,b)$ is given by (4.8), we can easily express the matching elasticity as a function of the tightness of the market:

$$\eta(\theta) = 1 - \frac{\theta}{\exp(\theta) - 1}.$$

The matching elasticity is an increasing function of tightness, growing from 0 when tightness is 0, to $1 - 1/e$ when tightness is 1, to 1 when tightness is infinite.

The property of the matching elasticity can be showed by using the definition of the exponential function via Taylor series:

$$\exp(\theta) = \sum_{n=0}^{\infty} \frac{\theta^n}{n!},$$

so

$$\frac{\exp(\theta) - 1}{\theta} = 1 + \sum_{n=1}^{\infty} \frac{\theta^n}{(n+1)!}.$$

All the terms in the sum are increasing in $\theta \geq 0$, so the function $[\exp(\theta) - 1]/\theta$ is increasing in θ, which tells us that $\eta(\theta)$ is increasing in θ. All the terms in the sum are 0 when $\theta = 0$, which tells us that $\eta(0) = 0$. And all the terms in the sum grow to ∞ when $\theta \to \infty$, which tells us that $\lim_{\theta \to \infty} \eta(\theta) = 1$.

The behavior of the elasticity means that when there are no buyers on the market ($\theta = 0$), adding more sellers does not lead to any more trades. The intuition is that the market is already extremely congested with sellers, so more sellers do not bring more trade: only more buyers would generate more trades. On the other hand, when there are almost no sellers ($\theta \to \infty$), adding 1% more sellers generates 1% more trades, even with a fixed number of buyers. In that case the number of trades is almost solely determined by the number of sellers, so numbers of sellers and trades grow proportionally.

4.5. Constant-elasticity-of-substitution matching function

Despite its simple microfoundation, the urn-ball matching function is not used often in theoretical work, because it is a little cumbersome. Instead, it is preferable to use the constant-elasticity-of-substitution (CES) matching function, as it is much more tractable. While it does not have a microfoundation as simple as the urn-ball process, it is possible to obtain a CES matching function from a Poisson queuing process in which sellers call buyers to advertise their goods (Stevens 2007), and from an Erdos-Renyi network in which sellers and buyers are connected (Angelis and Bramoulle 2023).

4.5.1. Expression

The CES matching function takes the following form:

$$m(s,b) = \left(s^{-\gamma} + b^{-\gamma}\right)^{-1/\gamma}. \tag{4.10}$$

The parameter $\gamma > 0$ governs the elasticity of substitution between s and b.

4.5.2. Properties

We can again check that all the general properties of a matching function are satisfied here. First, the CES matching function is positive and increasing in both its arguments. Second, when there are no buyers or no sellers, the number of matches is zero. Third, the CES function has constant returns to scale:

$$m(zs,zb) = \left[(zs)^{-\gamma} + (zb)^{-\gamma}\right]^{-1/\gamma} = zm(s,b).$$

To verify that the function is concave in both arguments, we compute its derivatives. For instance, the partial derivative with respect to b is

$$\frac{\partial m}{\partial b} = \left[1 + \left(\frac{b}{s}\right)^{\gamma}\right]^{-\frac{1+\gamma}{\gamma}}.$$

The partial derivative is not only positive but also decreasing in b, so $\partial^2 m/\partial b^2 < 0$. From this we infer that the CES matching function is concave in the number of buyers. The

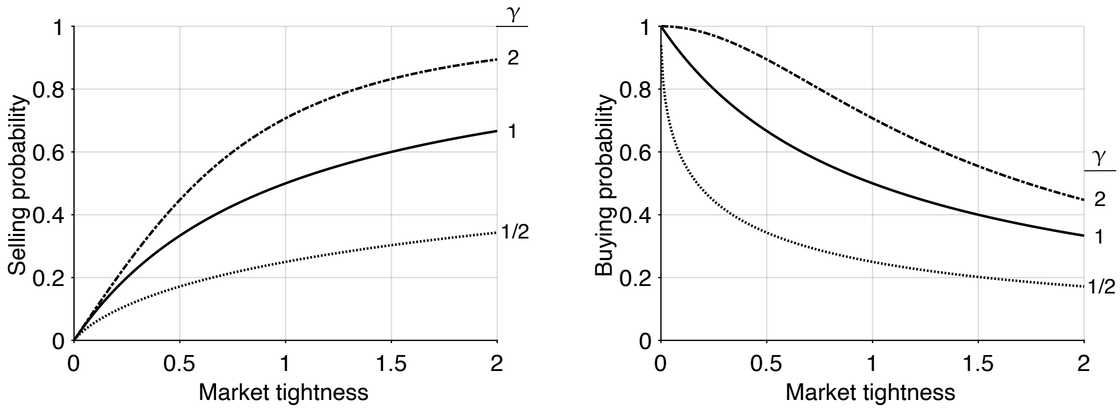


FIGURE 4.2. Selling and buying probabilities from different CES matching functions

The CES matching functions are given by (4.10) with $\gamma = 0.5$, $\gamma = 1$, and $\gamma = 2$. The selling and buying probabilities are computed from the matching functions with (4.3) and (4.4).

matching function is completely symmetric in b and s, so it is also concave in the number of sellers.

Lastly, we verify that the CES matching function is less than the minimum of its two arguments. Since $b^{-\gamma} > 0$, then $s^{-\gamma} + b^{-\gamma} > s^{-\gamma}$. Hence,

$$m(s, b) = \left(s^{-\gamma} + b^{-\gamma}\right)^{-1/\gamma} < \left(s^{-\gamma}\right)^{-1/\gamma} = s.$$

Similarly, we find $m(s, b) < b$, which overall tells us that $m(s, b) \leq \min(s, b)$.

The trading probabilities given by the CES matching function are illustrated in figure 4.2.

4.5.3. Matching elasticity

The matching elasticity is the elasticity of the matching function with respect to the number of sellers. With the CES matching function (4.10), the elasticity varies with the tightness of the market.

We compute the matching elasticity using the elasticity results from appendix B. We can express the matching elasticity as a function of market tightness:

$$\eta(\theta) = \frac{-1}{\gamma} \cdot \frac{s^{-\gamma}}{s^{-\gamma} + b^{-\gamma}} \cdot (-\gamma)$$

so that

$$\eta(\theta) = \frac{1}{1 + \theta^{-\gamma}}.$$

The matching elasticity is an increasing function of tightness, growing from 0 when tightness is 0, to 1/2 when tightness is 1, to 1 when tightness is infinite. Just like in the case of the urn-ball matching function, the behavior of the matching elasticity implies that when there are no buyers on the market, adding more sellers does not lead to any more trades, while when there are almost no sellers, numbers of sellers and trades grow proportionally.

With the CES matching function, it is easy to see what happens when there are very few sellers and buyers. We can rewrite the matching function (4.10) as

$$m(s, b) = b \cdot [1 + \theta^{\gamma}]^{-1/\gamma},$$

where $\theta = b/s$ is the market tightness. When tightness is close to 0, θ^{γ} is negligible compared to 1, so the matching function approximates the number of buyers: $m(s, b) \approx b$. This means that when the number of buyers is small, the number of buyers gives the number of trades, irrespective of the number of sellers.

Since the matching function (4.10) is symmetric in b and s, we can also rewrite it as

$$m(s, b) = s \cdot \left[1 + \theta^{-\gamma}\right]^{-1/\gamma},$$

When tightness is very large, $\theta^{-\gamma}$ is negligible compared to 1, so the matching function approximates the number of sellers: $m(s, b) \approx s$. This means that when the number of sellers is small, the number of sellers gives the number of trades, irrespective of the number of buyers.

4.6. Cobb-Douglas matching function

Maybe the most popular of all matching functions is the Cobb-Douglas matching function. It is appealing because it is even easier to manipulate than the CES matching function, and it describes the US labor market well (Blanchard and Diamond 1989; Petrongolo and Pissarides 2001). There are no standard microfoundations for the Cobb-Douglas matching function, but just like the CES matching function, it is possible to obtain it from a Poisson queuing process or an Erdos-Renyi network (Stevens 2007; Angelis and Bramoulle 2023). The main drawback of the Cobb-Douglas matching function is that it does not guarantee that $m(s, b) \leq \min(s, b)$, so it must be truncated in discrete-time models to avoid trading probabilities greater than 1.

4.6.1. Expression

The Cobb-Douglas matching function takes the following form:

$$m(s, b) = \mu \cdot s^{\gamma} \cdot b^{1-\gamma}. \tag{4.11}$$

The parameter $\mu > 0$ governs the efficacy of the matching process. The parameter $\gamma \in (0, 1)$ governs the shape of the matching function.

4.6.2. Properties

We easily check that the Cobb-Douglas function satisfies the typical properties of a matching function. The function is clearly positive. If there are no sellers or no buyers, the function is clearly 0. The function is also increasing and concave in both arguments—since any function of the form $x \mapsto ax^b$ with $a > 0$ and $b \in (0, 1)$ is increasing and concave in x.

Next, we verify that the Cobb-Douglas matching function has constant returns to scale:

$$m(zs, zb) = \mu(zs)^{\gamma}(zb)^{1-\gamma} = zm(s, b).$$

The Cobb-Douglas matching function has one main limitation: it is not always less than the minimum of its two arguments. In a continuous-time model, where the matching function describes the flow of trades at any point in time, this is not an issue. But in a discrete-time model, where the matching function describes the number of trades in a period, this is an issue because it might lead to trading probabilities that are greater than 1. To avoid trading probabilities that are greater than 1, the matching function must be truncated by imposing

$$m(s, b) = \min\left(\mu \cdot s^{\gamma} \cdot b^{1-\gamma}, s, b\right). \tag{4.12}$$

The truncation in turn might create difficulties in numerical and theoretical work (den Haan, Ramey, and Watson 2000).

The trading probabilities given by the Cobb-Douglas matching function are illustrated in figure 4.3.

4.6.3. Matching elasticity

A reason why the Cobb-Douglas matching function is particularly convenient is that its matching elasticity is constant: $\eta = \gamma$. This property often greatly simplifies the analysis.

Relatedly, the trading probabilities that it produces are really easy to deal with. We can obtain them directly using (4.3) and (4.4):

$$f(\theta) = m(1, \theta) = \mu\theta^{1-\gamma}, \quad q(\theta) = m\left(\frac{1}{\theta}, 1\right) = \mu\theta^{-\gamma}.$$

In particular, the trading probabilities have constant elasticity with respect to tightness: $\epsilon^{f}_{\theta} = 1 - \gamma$ and $\epsilon^{q}_{\theta} = -\gamma$.[3]

[3]These expressions show that $f(\theta)$ is larger than 1 when θ is high enough, and $q(\theta)$ is larger than 1 when

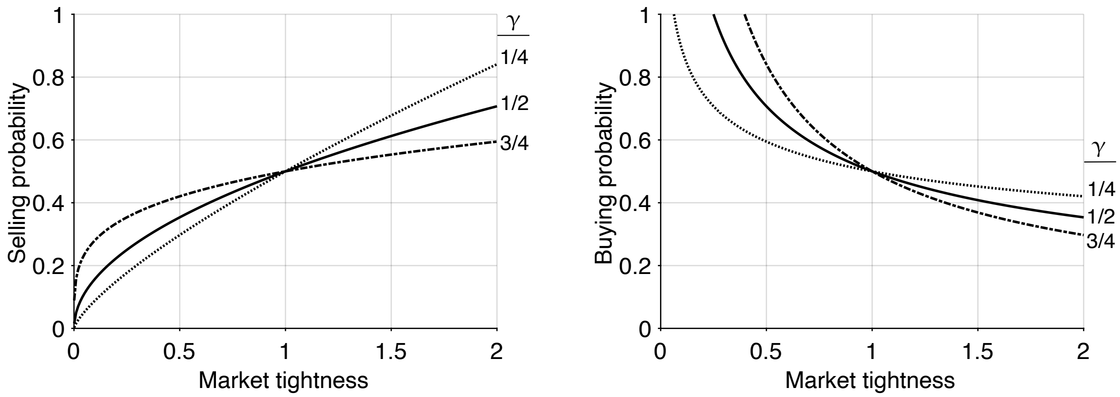


FIGURE 4.3. Selling and buying probabilities from different Cobb-Douglas matching functions

The Cobb-Douglas matching functions are given by (4.11) with $\mu = 0.5$ and $\gamma = 0.25$, $\gamma = 0.5$, $\gamma = 0.75$. The selling and buying probabilities are computed from the matching functions with (4.3) and (4.4).

4.7. Penalized Cobb-Douglas matching function

The Cobb-Douglas matching function is very popular, but it does not produce an isoelastic Beveridge curve. This property is inconsistent with the US Beveridge curve, which appears to have a constant elasticity (chapter 3). The non-isoelastic Beveridge curve also complicates some of the welfare and policy analysis (chapter 9). However, in continuous-time dynamic models, it is easy to generate an isoelastic Beveridge curve by adding a small penalty to the Cobb-Douglas function, as proposed by Michaillat and Saez (2024).

4.7.1. Expression

The penalized Cobb-Douglas matching function takes the following form:

$$m(s, b) = \mu \cdot s^{\gamma} \cdot b^{1-\gamma} - \sigma \cdot s. \tag{4.13}$$

The novelty relative to the standard Cobb-Douglas matching function is the small penalty $-\sigma \cdot s$ added to the function. The parameter $\sigma > 0$ dictates the size of the penalty; it is small relative to μ in practice.

4.7.2. Properties

The penalized Cobb-Douglas function satisfies the standard properties of a matching function, but it requires that the arguments remain within a slightly reduced range (in essence that tightness is not too low).

θ is low enough. This is why the Cobb-Douglas matching function must be truncated in discrete-time models.

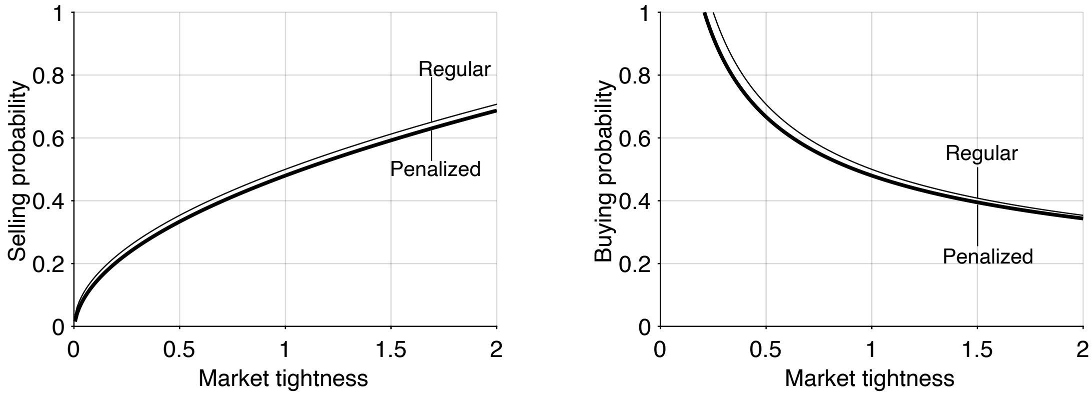


FIGURE 4.4. Selling and buying probabilities from penalized Cobb-Douglas matching function

The penalized Cobb-Douglas matching function is given by (4.13) with $\mu = 0.5$, $\gamma = 0.5$ and $\sigma = 0.05$. The selling and buying probabilities are computed from the matching function with (4.3) and (4.4). The figure also compares the penalized Cobb-Douglas matching function to a regular Cobb-Douglas matching function calibrated with $\mu = 0.5$ and $\gamma = 0.5$.

First, the matching function has constant returns to scale:

$$m(zs, zb) = \mu(zs)^{\gamma}(zb)^{1-\gamma} - \sigma \cdot (zs) = zm(s, b).$$

Second, the function is increasing and concave in b, just like the standard Cobb-Douglas function.

Third, the function is concave in s, as it is the sum of two functions that are themselves concave in s: the Cobb-Douglas function $s \mapsto \mu s^{\gamma} b^{1-\gamma}$ and the linear function $s \mapsto -\sigma s$.

Fourth, the function is increasing in s, as long as tightness $\theta = b/s$ is not too low. Taking the partial derivative of the matching function, we see that the partial derivative is actually a function of tightness only:

$$\frac{\partial m}{\partial s} = \mu\gamma\theta^{1-\gamma} - \sigma.$$

Hence the derivative is positive for any tightness above the lower bound $\underline{\theta}$, given by

$$\underline{\theta} = \left(\frac{\sigma}{\mu\gamma}\right)^{\frac{1}{1-\gamma}}.$$

Fifth, the function is positive, as long as tightness is not too low. Dividing (4.13) by s, we find that the function is positive as long as $\mu\theta^{1-\gamma} - \sigma \geq 0$, which requires tightness to be high enough:

$$\theta \geq \left(\frac{\sigma}{\mu}\right)^{\frac{1}{1-\gamma}}.$$

This bound on tightness is lower than $\underline{\theta}$, since $\gamma < 1$.

Sixth, the function is 0 when $s = 0$. When $b = 0$, the function would be negative, which is why we impose a lower bound on tightness: so b is large enough that the matching function is always positive.

Overall, for any tightness in $(\underline{\theta}, \infty)$, the penalized Cobb-Douglas matching function is positive, has constant returns to scale, is increasing and concave in both arguments. The penalty parameter σ is generally much lower than the efficacy parameter μ, so the lower bound on tightness $\underline{\theta}$ is close to 0.

The trading probabilities given by the penalized Cobb-Douglas matching function are illustrated in figure 4.4. As we can see on the figure, the trading probabilities are very close to those for a regular Cobb-Douglas matching function when σ/μ is small. As an illustration, we set $\sigma/\mu = 0.1$ in the figure. The implied lower bound on tightness is $\underline{\theta} = 0.04$.

4.7.3. Matching elasticity

The matching elasticity is the elasticity of the matching function with respect to the number of sellers. With the penalized Cobb-Douglas matching function (4.13), the matching elasticity is almost constant.

We compute the matching elasticity using the results from appendix B. For $\theta > \underline{\theta}$, it is given by

$$\eta(\theta) = \gamma - (1 - \gamma) \cdot \frac{\sigma s}{m} = \gamma - (1 - \gamma) \cdot \frac{\sigma}{f(\theta)}.$$

The second expression shows that unlike for the Cobb-Douglas matching function, the matching elasticity is not constant: it is an increasing function of tightness, since the selling probability $f(\theta)$ is increasing in tightness. But in practice these fluctuations are bound to be small. The first expression shows that the matching elasticity is only slightly below the parameter γ since the penalty term σs is small relative to the matching function m. Hence, the matching elasticity of the penalized Cobb-Douglas function is close to that of the underlying Cobb-Douglas function when the penalty term is small: $\eta(\theta) \approx \gamma$.

4.8. Empirical properties of the matching function

To conclude this chapter, we briefly review the empirical properties of the matching function to convince ourselves that our theoretical assumptions give us a matching function that describes the real world well. When matching functions were first developed, researchers examined the data to ensure that they built functions that were realistic (Blanchard and Diamond 1989; Petrongolo and Pissarides 2001). Here we review modern US data to do the same.

Data on sellers and buyers are most readily available on the labor market, where we have good counts of who sells labor (job seekers) and who buys labor (vacant jobs). Thus

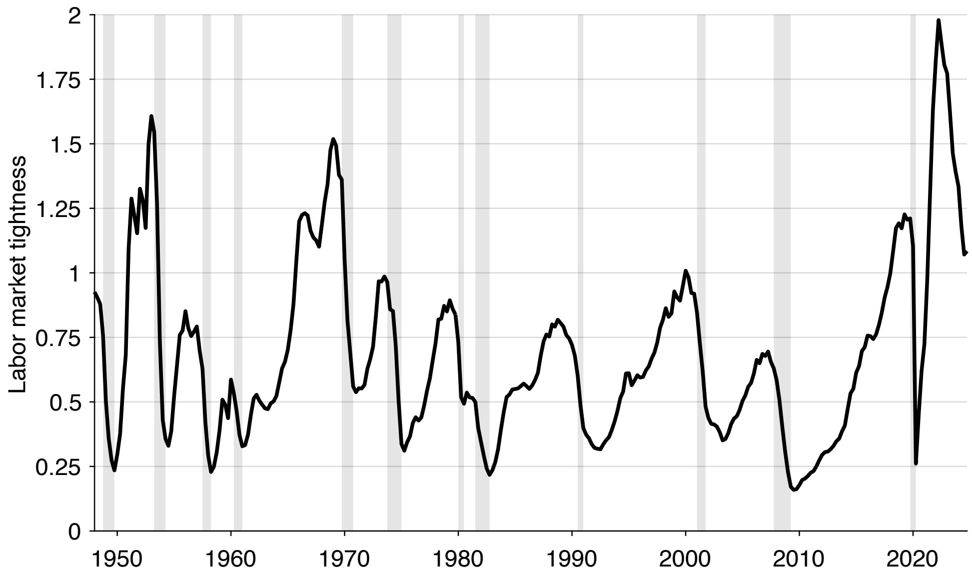


FIGURE 4.5. Labor market tightness in the United States, 1948–2024

Labor market tightness is the ratio of vacancy rate to unemployment rate. The unemployment rate comes from figure 3.1 while the vacancy rate comes from figure 3.5. Shaded areas indicate recessions dated by the NBER (2023).

we focus on the matching function for the labor market.

A central assumption is that the matching function has constant returns to scale. The main implication is that the market tightness determines the rates at which sellers and buyers trade on the market. On the labor market, this means that labor market tightness determines the rate at which job seekers find jobs and the rate at which vacant jobs are filled. We only have data to compute the job-finding rate over a long period of time, so we concentrate on the link between job-finding rate and labor market tightness here.

For reference, we plot the US labor market tightness between 1948 and 2024. Labor market tightness is computed as the number of job vacancies per job seeker, or equivalently, the ratio of vacancy rate to unemployment rate.[4] We start in 1948 because that is when the BLS started to collect the data that we need to compute the job-finding rate (CPS data). We see that labor market tightness is sharply procyclical. It averages 0.69 between 1948 and 2024. Over that period, tightness reached its highest level, 1.98, during the recovery from the pandemic (2022:Q2) and its lowest level, 0.16, during the Great Recession (2009:Q3).

We start with the rate at which job seekers find jobs, and correlate this rate with labor market tightness. To compute the job-finding rate, we follow Shimer (2012). In month

[4] In practice some vacancies are filled not by unemployed workers but by employed workers moving between jobs. The present empirical approach is valid as long as the number of vacancies captures well the demand for unemployed labor.

t, $u(t)$ workers are looking for jobs. Some of them find a job in the month, and some do not. Those who do not find a job remain unemployed, so $u(t) - u(t+1)$ seems to indicate the number of workers who have been able to find a job. There is a complication, however. Some workers who were employed lose their jobs in month t and join the pool of unemployed workers. We need to account for those when we compute the number of job seekers who found a job. We denote by $u^s(t+1)$ the number of workers who were previously employed and who have joined unemployment between months t and $t+1$. The number of workers who have found a job within month t is $u(t) - [u(t+1) - u^s(t+1)]$.[5] Dividing this number of job finders by the number of job seekers in month t, we obtain the probability of finding a job in month t:

$$F(t) = 1 - \frac{u(t+1) - u^s(t+1)}{u(t)}.$$

To calculate $F(t)$, we measure $u(t)$ as the number of unemployed persons in month t computed by the BLS (2025b), and $u^s(t)$ as the number of persons who have been unemployed for less than 5 weeks in month t computed by the BLS (2025a).[6]

Assuming that unemployed workers find a job according to a Poisson process with monthly arrival rate $f(t)$, the probability that they have not found a job after a month is $\exp(-f(t))$, so the probability that they have found a job within one month is $F(t) = 1 - \exp(-f(t))$. Hence, we infer the job-finding rate from the job-finding probability:

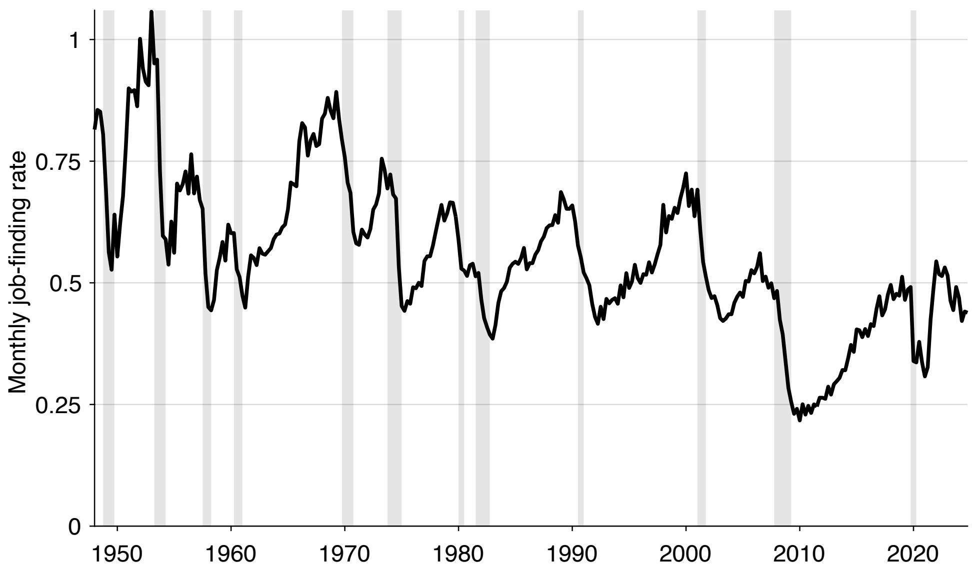
$$f(t) = -\ln(1 - F(t)). \tag{4.14}$$

We thus obtain the monthly job-finding rate in the United States (figure 4.6A). Over 1948–2024, the job-finding rate is highly procyclical and averages 0.56 per month. This means that on average, a US job seeker takes 1/0.56 = 1.8 months to find a job.

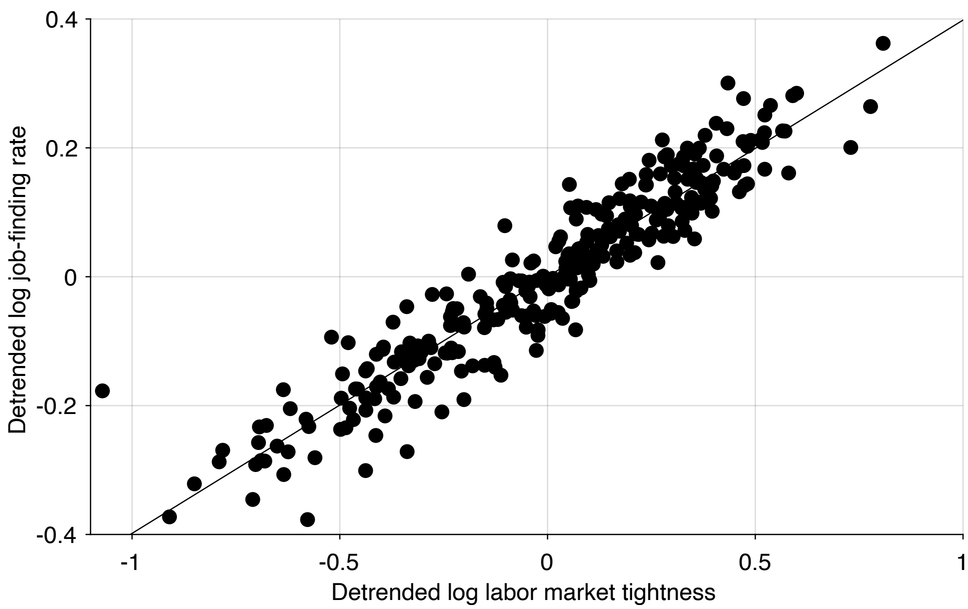
Our objective is to assess whether labor market tightness determines the job-finding rate. Just like in figure 3.8, we focus on the 1948–2019 period, which is when the Beveridge curve and matching function were quite stable. To uncover the relationship between tightness and job-finding rate, we plot the logarithm of the job-finding rate against the logarithm of tightness. To remove slow changes in the matching function, we detrend both series using an HP filter with smoothing parameter 10,000. We find that log job-finding rate is largely determined by log labor market tightness: the least-squares regression gives $R^2 = 0.89$, with a coefficient of 0.40 (figure 4.6B). This result indicates that the job-finding rate is well described as an isoelastic function of tightness, with an elasticity of 0.40.

[5]This calculation assumes that workers only transition between employment and unemployment. In reality, some workers move in and out of the labor force. But the approach is valid as long as the numbers of job seekers who exit and enter the labor force are similar.

[6]Following Shimer (2012, appendix A), we multiply the series for $u^s(t)$ by 1.1 after January 1994 to correct for a change in the design of the CPS.



A. Monthly job-finding rate in the United States, 1948–2024



B. Detrended logarithm of tightness and job-finding rate, 1948–2019

FIGURE 4.6. The US job-finding rate is an isoelastic function of labor market tightness

The monthly job-finding rate is computed from (4.14). Labor market tightness comes from figure 4.5. In panel A, shaded areas indicate recessions dated by the NBER (2023). In panel B, the series are detrended by applying a HP filter with smoothing parameter of 10,000.

Overall, we find that the job-finding rate is a stable function of tightness, which supports the assumption that the matching function has constant returns to scale. In addition, we have seen that the elasticity of the job-finding rate with respect to tightness appears constant, which suggests that the matching elasticity is constant. This finding suggests that a Cobb-Douglas matching function, which has a constant matching elasticity, describes the US labor market better than the urn-ball or CES matching functions, which have matching elasticities that respond strongly to tightness. Furthermore, the matching elasticity that appears from the regression is $\eta = 0.60$, using (4.7).

The empirical findings in this section are in line with findings in the literature on the matching function. Early aggregate studies of the US labor market find that the matching function involves the stock of unemployed workers and job vacancies with constant returns to scale (Petrongolo and Pissarides 2001). The studies converge on a Cobb-Douglas form with a matching elasticity between 0.5 and 0.7. More recent studies obtain comparable results. Shimer (2005, table 2) estimates the matching elasticity at 0.72; Rogerson and Shimer (2011, p. 638) obtain an estimate of 0.58; and Borowczyk-Martins, Jolivet, and Postel-Vinay (2013, table 1) report a lower estimate of 0.30.

4.9. Summary

In this chapter we introduce the matching function as a key tool for modeling slackish markets, where trade is not instantaneous and requires time and effort from both buyers and sellers. Unlike Walrasian markets where trades are seamless, real-world markets are characterized by complexities that make it difficult for buyers and sellers to find each other. The matching function is an aggregate tool, similar to a production function, that captures the outcome of this complex trading process without modeling the micro-level details explicitly.

The chapter presents four specific types of matching functions: urn-ball, CES, Cobb-Douglas, and penalized Cobb-Douglas, which each have their advantages and drawbacks. We discussed the advantages and drawbacks of each functional form. For instance, the CES function is useful in theoretical work, while the penalized Cobb-Douglas function is useful to produce an isoelastic Beveridge curve.

Finally, the chapter reviews empirical evidence on matching from the US labor market, which supports the use of a matching function with constant returns to scale. Furthermore, this evidence suggests a Cobb-Douglas matching function is a good approximation for the US labor market, because the matching elasticity appears roughly constant.

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