A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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CHAPTER 5.

Slackish market model

This chapter presents the most basic model of a slackish market. The term "slackish" is used to describe a class of markets where slack is the central mechanism to equalize supply and demand. In these markets the amount of slack—determined by the market tightness—is the key mediating variable, irrespective of whether the market is currently inefficiently slack or tight. Slackish does not imply that the market is always too slack; rather, it refers to the structural mechanism through which the market operates. In these markets, the market tightness, not the market price, adjusts to equilibrate supply and demand and to determine allocations and welfare.

The model presented in this chapter is basic in many ways. It is static, meaning that it lasts only one period. Of course, in reality, the world is dynamic, so in chapter 8 we provide a dynamic extension of the model and show how we can move from a static to a dynamic slackish model. The dynamic extension allows us in particular to introduce long-term relationships. Another simplification in this chapter's model is that the amount of goods for sale is fixed. In chapter 7 we provide an extension in which sellers determine the amount of goods that they bring on the market based on expected market conditions.

5.1. Buyers and sellers

We assume that there are many sellers, each indexed by $i \in [0, 1]$. Each seller is selling a fixed amount of goods k_i. The aggregate amount of goods for sale on the market is $k = \int_0^1 k_i \, di$. The quantity k represents the market capacity.

We assume that there are many buyers as well, each indexed by $j \in [0,1]$. Since the market is slackish and not Walrasian, it is difficult for buyers to find the goods they like. Thus, buyers need to visit various sellers to buy goods. The number of sellers that buyer j visits is denoted by v_j. The aggregate number of visits to sellers on the market is $v = \int_0^1 v_j \, dj$.

5.2. Matching function

First, we examine how sellers are able to actually find buyers for their goods. Based on the number of goods that are available, k, and the number of visits, v, we can determine the number of trades that are realized. Here, a trade is one good being bought by one buyer. The number of trades, which is the output and sales in the model, is denoted by y.

Because we are working with a slackish market, the number of trades is determined by a matching function m, which takes as arguments the aggregate number of goods for sale, k, and the aggregate number of visits to buy goods, v. Hence,

$$y = m(k, v). \tag{5.1}$$

We assume that the matching function satisfies the generic properties listed in chapter 4. In particular, since we are in a static model, the number of trades cannot be more than the minimum of the number of visits and the number of goods available, so the matching function satisfies $y \leq \min(k, v)$.

5.3. Market tightness

As we saw in chapter 4, the trading probabilities are solely determined by the market tightness, θ, which here is the aggregate number of visits divided by the aggregate number of goods for sale:

$$\theta = \frac{v}{k}. \tag{5.2}$$

First, the selling probability satisfies

$$f(\theta) = m(1, \theta). \tag{5.3}$$

From this equation, we recover all the properties of the selling probability. When tightness is 0, the probability to sell a good is also 0. It is impossible to sell if there are no buyers. Next, the selling probability is increasing with tightness. One is more likely to sell in a tighter market, because then buyers are in larger number relative to sellers. Last, the selling probability is concave in tightness. The key takeaway is that, when the market is tighter, it is a good time to be a seller.

We next move to the buying probability, which is the probability that a visit to a seller is successful:

$$q(\theta) = m\left(\frac{1}{\theta}, 1\right). \tag{5.4}$$

Again, we recover the buying probability's usual properties from this equation. When tightness is infinite, the buying probability approaches 0. Indeed, if a buyer has to compete with infinitely many other buyers, the odds that they can purchase the good they want is 0. Then, we see that the buying probability is decreasing in tightness. One is less likely to be able to buy in a tighter market, as there is more competition for available goods. The takeaway here is that, when the market is tighter, it is more difficult to be a buyer.

This is a general property of slackish markets: when the market is tighter, there are more buyers relative to sellers so it is a better time to be a seller. On the other hand, when tightness is low, it is a good time to be a buyer: they are more likely to have successful visits. Figures 5.1A and 5.1B display the selling and buying probabilities and illustrate their properties.

5.4. Rate of slack

Because we are working with a slackish market, some goods always remain unsold. We define the rate of slack as the share of goods that remain unsold. It is easy to compute this slack rate and express it as a function of market tightness.

We previously saw that each good is sold with probability $f(\theta)$. For a seller with k_i goods for sale, omitting randomness at the seller level, the number of goods sold is $y_i = f(\theta)k_i$. The aggregate number of sales on the market is $y = \int_0^1 y_i \, di$, which can be expressed as

$$y = \int_0^1 f(\theta)k_i \, di = f(\theta) \int_0^1 k_i \, di = f(\theta)k.$$

Here we simply recover the matching function from the individual selling probabilities, since $f(\theta)k = m(1, \theta)k = m(k, \theta k) = m(k, v)$.

We define the rate of slack u in the market as the share of goods that remain unsold. In the aggregate, a share $f(\theta) \in [0, 1]$ of goods is sold, so a share

$$u = 1 - f(\theta) \tag{5.5}$$

of goods remain unsold. The properties of the slack rate u directly follow from those of the selling probability $f(\theta)$. The slack rate is 100% when tightness is 0, because $f(0) = 0$: when there are no buyers, nothing gets sold. The slack rate then decreases with tightness, because $f(\theta)$ increases with tightness. And since the selling probability is concave in tightness, the slack rate is convex in tightness.

The rate of slack can be interpreted in different ways depending on the type of goods sold on the market. If labor is sold on the market, then the rate of slack is just the rate of unemployment. If services are sold, then the rate of slack indicates the share of time that sellers are idle. If durable goods are sold, then the rate of slack gives the share of goods that are added to the unsold inventory. If perishable goods are sold, then the rate of slack indicates the share of goods that go to waste.

Figure 5.1C displays the rate of slack as a function of tightness to illustrate its properties. The figure also shows that the basic slackish model produces a Beveridge curve: a negative relationship between slack rate and visit rate. Indeed, in this static model, the visit rate is the number of visits per goods for sale, v/k, so the visit rate coincides with the market tightness. Therefore, the negative relationship between slack rate and market tightness corresponds to a Beveridge curve. In this static model the logic behind the Beveridge curve is trivial: a higher visit rate means a higher market tightness, which implies a higher selling probability, and thus fewer unsold goods and a lower slack rate.

5.5. Market supply

We now compute the market supply, which is the first key relation in analyzing how a slackish market behaves.

5.5.1. Computing the market supply

A key distinction in this model is between the notional supply of goods and the effective supply. On the market, we have k goods that sellers would like to sell, which is the notional market supply. Here the notional supply is fixed, but it could depend on the market price and tightness, as we will see in chapter 7.

However, in the model, we use the effective market supply, which is the amount of goods actually sold given tightness, and which is less than the notional supply. The gap arises because the amount of goods that sellers are actually able to sell is strictly less than the amount of goods that they put on the market, since, in our slackish market, each good is only sold with a probability that is less than one.

From the matching function (5.1) and definition of tightness (5.2), we simply express the market supply:

$$y^s(\theta) = f(\theta)k, \tag{5.6}$$

where $f(\theta)$ is the selling probability and k is the market capacity. The market supply does not depend on the market price at all. To formalize the definition of market supply: it is just the amount of goods sold given the matching structure and the amount of goods

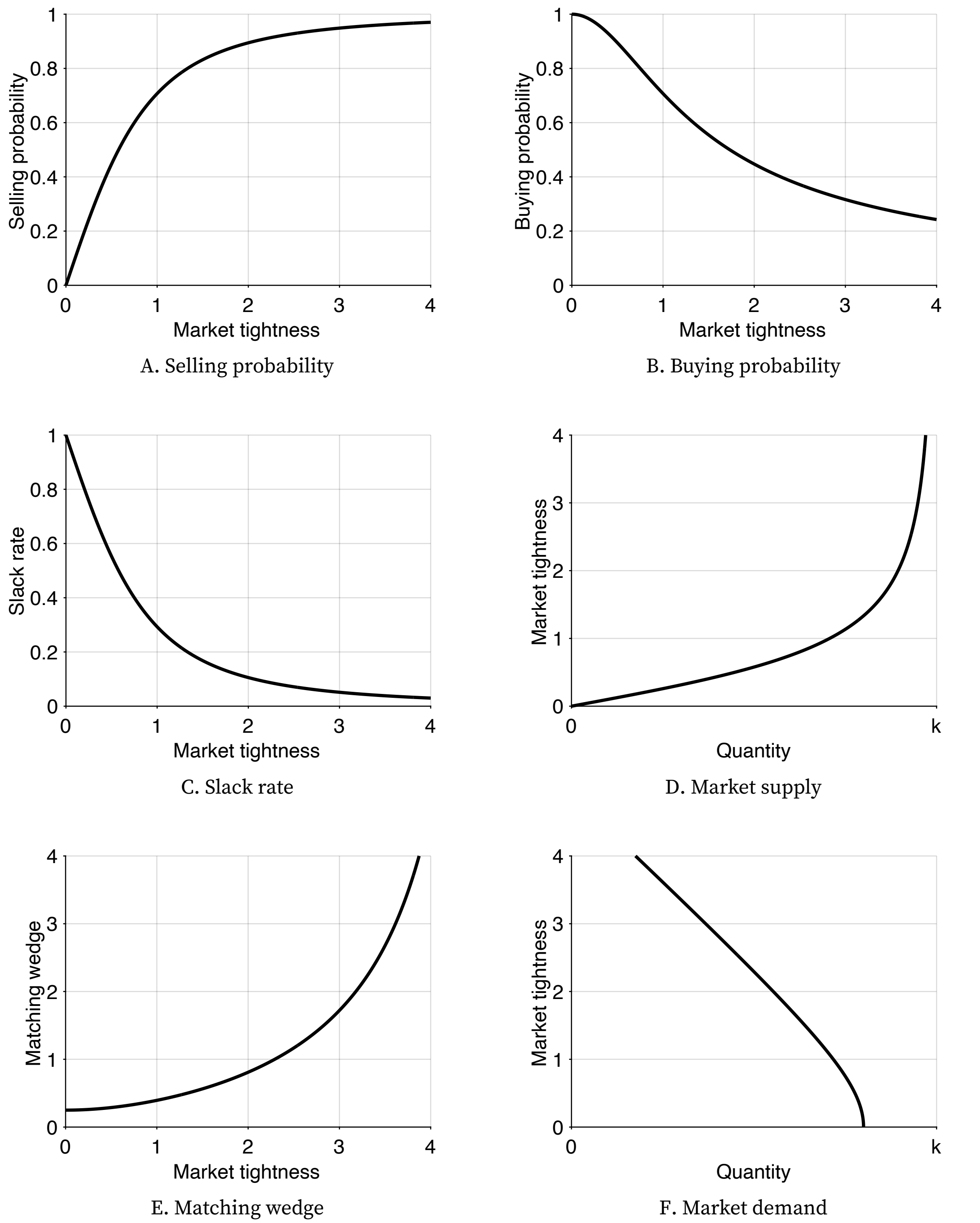


FIGURE 5.1. Numerical illustration of the slackish market model

The selling probability is given by (5.3). The buying probability is given by (5.4). The slack rate is given by (5.5). The market supply is given by (5.6). The matching wedge is given by (5.9). The market demand is given by (5.16). The matching function is CES, given by (4.10). Parameters are set to $\gamma = 2$, $\kappa = 0.2$, $k = 1$, $\alpha = 0.5$, $\delta = 2$, and $p = 1$.

supplied to the market by sellers. We have a gap between notional supply and effective supply that we use in our model, because not all goods supplied are traded: $y^s(\theta) < k$.

5.5.2. Properties of the market supply

Let us now look at the properties of the market supply, given by (5.6). It is clear that the properties of the market supply reflect the properties of the selling probability, $f(\theta)$. When market tightness is 0, the supply is 0, because $f(0) = 0$. Then, the supply is increasing and concave in tightness, because the selling probability is increasing and concave in tightness.

We plot the market supply curve in a tightness-quantity plane to visualize its behavior (figure 5.1D). In Walrasian models, supply and demand curves are represented in price-quantity planes. But in slackish models, tightness is the main equilibrating variable, so we conduct most of the analyses in tightness-quantity planes.

Because the function $\theta \mapsto y^s(\theta)$ is concave, but the axes are inverted in the tightness-quantity plane (in the sense that tightness is on the y-axis instead of the x-axis, and quantities on the x-axis instead of the y-axis), then the supply curve appears convex. Because of this convexity, the slackish model is state dependent—it behaves quite differently when tightness is high and when tightness is low—which has a range of implications throughout the book.

5.6. Matching wedge

We have seen that buyers visit sellers to find goods. But what prevents a buyer from visiting every seller instantly? The answer must be a cost. Theoretically, if there was no cost to visiting different sellers to find goods, buyers could just visit infinitely many sellers and always be able to buy what they want immediately. This means that we would never observe buyers on the market, as they could buy goods instantaneously, just like in a Walrasian market. Of course, this is not realistic: we always observe buyers shopping, just like we always observe sellers selling.

Thus, there must be some sort of cost associated with visiting sellers so that we can have both buyers and sellers present on the market at the same time. We denote the cost of each visit by $\kappa \in (0, q(0))$; κ is the number of goods spent on each visit. In the best case in which there are no other buyers, a visit is successful with probability $q(0)$, so in expectation a visit yields $q(0)$ goods. If the cost of the visit is more than $q(0)$ to begin with, no buyer would ever try to buy anything. Thus, we must impose $\kappa < q(0)$.

We make the convenient assumption that the cost of a visit is paid in terms of goods. This means that to pay for the visits, buyers have to purchase an extra amount of goods that covers the cost of the visits. We represent it in this way for a few reasons. First, it is very tractable to have the cost in terms of goods because then, we do not need to introduce an extra good to measure the cost of the visits. We can just focus on one market without thinking about another market where that other good would be traded. The fact that the market is self-contained is helpful to think about welfare and efficiency. Crucially, this choice affects only the accounting of purchases versus consumption, not the underlying behavior of buyers and sellers.

Second, the assumption is quite portable to think about any other slackish markets, for instance firms that want to purchase intermediate goods or labor. In particular, it allows us to have a product market and a labor market that are isomorphic, which is very convenient. In the labor market, one pays for job vacancies in terms of labor and in the product market, we assume that we are paying for the visits to buy production in terms of product.

It is not unrealistic to have our cost of visit in terms of goods—there are a lot of examples in the real world where this is true. If we think about buying a house, which provides housing services, people usually pay for a real estate agent to help with the process, which is paying in terms of services. Many people who are planning a holiday use a travel agent to help with bookings and scheduling, which again is paying in terms of services.

The cost of visits creates a wedge between the quantity of goods a buyer purchases and the quantity they actually consume. Indeed, when a buyer purchases goods, they use them for two purposes. First, they consume some of the goods, which provide utility. We denote buyer j's consumption by c_j. However, not all the goods are consumed—some goods are used for conducting visits and matching with a seller. This means that the number of goods that the buyer consumes is strictly less than the amount of goods that are purchased, which we denote y_j. Thus, there is a wedge between goods purchased and goods consumed. We need to assess how large the wedge is, and in doing so how consumption and purchases are linked.

Suppose buyer j conducts v_j visits with the aim of consuming c_j goods. Then the total number of goods purchased is $y_j = c_j + \kappa v_j$, as each visit requires κ goods. Where do the purchases come from? Each visit leads to a purchase only with probability $q(\theta)$. Thus, to obtain y_j purchases, a buyer needs to make $v_j = y_j / q(\theta)$ visits. Here we assume that there is no uncertainty at the buyer level: exactly $q(\theta)$ of the visits provide a good. This means that output and consumption are related by

$$y_j = c_j + \frac{\kappa y_j}{q(\theta)}. \tag{5.7}$$

Solving for y_j in the equation, we can see that y_j is proportional to c_j:

$$y_j = [1 + \tau(\theta)]\, c_j, \tag{5.8}$$

where the matching wedge $\tau(\theta)$ is given by

$$\tau(\theta) = \frac{\kappa}{q(\theta) - \kappa}. \tag{5.9}$$

The matching wedge is the gap between the amount of goods consumed and the amount of goods purchased. The gap is caused by the matching process: it arises due to the cost of each visit. Overall, to consume one good, a buyer must purchase $1 + \tau(\theta)$ goods. Reciprocally, if a buyer purchases one good, they only consume $1/[1 + \tau(\theta)]$ goods.

A useful result here is the link between number of visits and consumption. Using the facts that $v_j = y_j/q(\theta)$, $y_j = [1 + \tau(\theta)]c_j$, and the expression (5.9), we can see that v_j is proportional to c_j:

$$v_j = \frac{c_j}{q(\theta) - \kappa}. \tag{5.10}$$

This result tells us that visits and consumption are directly linked. Once we have consumption and market tightness, we can calculate the number of visits that buyers make to be able to achieve this level of consumption.

Let us now discuss the properties of the matching wedge $\tau(\theta)$. First, we can see that the wedge is increasing in tightness, since it is a composite of two decreasing functions: $\theta \mapsto q(\theta)$ and $x \mapsto \kappa/(x - \kappa)$. Critically, when $q(\theta) \to \kappa$, $\tau(\theta) \to \infty$. We define $\bar{\theta}$ to be the value of tightness such that $q(\bar{\theta}) = \kappa$:

$$\bar{\theta} = q^{-1}(\kappa).$$

Then, $\lim_{\theta \to \bar{\theta}} \tau(\theta) = \infty$.[1] This divergence reflects the fixed-point logic that appears in (5.7): matching resources must themselves be purchased through additional matching. At the tightness $\bar{\theta}$, the market is so tight that all goods purchased are devoted to matching: no goods are left for consumption.

We plot the matching wedge against tightness to visualize how it behaves (figure 5.1E). With the calibration used in the plot, the vertical asymptote of the wedge occurs at $\bar{\theta} = 4.9$.

5.7. Market demand

The cornerstone of the demand side is the buyer's optimization problem: choosing how much to consume so as to maximize utility subject to a budget constraint. To build the market demand, we first need to specify buyers' utility function, then their budget constraint, before solving the maximization problem and deriving individual demands.

[1]Since q is strictly decreasing from $(0, \infty)$ to $(0, q(0))$, the inverse q^{-1} is well defined on $(0, q(0))$. Hence $\bar{\theta}$ exists as long as $\kappa < q(0)$. For the urn-ball and CES matching functions, $q(0) = 1$, so $\kappa < 1$ is sufficient.

5.7.1. Buyer's utility function

We assume that buyers have utility over two things. First, buyer j has utility over consumption of the goods sold on the market, c_j. Because we want to have a demand for goods, buyers must have the choice between consuming goods and something else. If there is no choice, the concept of demand is not well-defined. Typically, in a dynamic model, buyers have the choice between consuming now and saving (consuming in the future). However, our model is static, so that choice is not available. Instead, we introduce money and assume that buyers derive utility from money balances, which we denote b_j. To further simplify the analysis, we assume that the buyer's utility function is quasilinear.

Specifically, we assume that buyer j has utility over consumption c_j and money balances b_j, as follows:

$$\mathcal{U}(c_j, b_j) = \delta c_j^{1-\alpha} + b_j, \tag{5.11}$$

where $\delta > 0$ governs the taste for goods relative to money and $\alpha \in (0, 1)$ governs the concavity of the utility over goods.

5.7.2. Buyer's budget constraint

Buyer j maximizes their utility, but of course subject to a budget constraint. We assume that to start with, each buyer has an endowment of money B_j, which they use to hold money or buy goods. The budget simply says buyers can spend the endowment on goods or keep some of the endowment in money. In order to consume c_j goods, buyer j must purchase $[1 + \tau(\theta)]c_j$ goods at a price p. Hence, the expenditure to purchase goods is $p[1 + \tau(\theta)]c_j$. The buyer's budget constraint therefore is:

$$b_j + p\left[1 + \tau(\theta)\right] c_j = B_j. \tag{5.12}$$

The matching structure introduces a new term into the buyer's budget constraint, which we wouldn't see in a Walrasian model. The new element appearing here is the matching wedge, $\tau(\theta)$. It appears because we have a cost of visits. The wedge is specific to the slackish world; in the Walrasian world, anyone can buy any quantity at the given market price without any other cost, so there would be no matching wedge.

This new element modifies the buyer's behavior. Nevertheless, we can still treat the problem as we usually do: maximizing utility subject to a budget constraint. It's just that, in this case, the budget constraint is slightly modified.

5.7.3. Buyer's problem

We now need to solve the buyer's problem. This is a crucial element of the model because buyers are making the only behavioral decision here: how many goods to consume. Equivalently, buyers decide how many sellers to visit: since once they have determined the visits given that the probability that a visit is successful, they also determine their consumption.

Buyer j's problem is to maximize their utility (5.11) given the budget constraint (5.12), taking aggregate variables as given: market price, p, and market tightness, θ. The easiest way to solve the problem is to express the buyer's money balances as a function of consumption, by reworking the budget constraint:

$$b_j = B_j - p\left[1 + \tau(\theta)\right] c_j.$$

Then, substituting these money balances into the buyer's utility function, the buyer's problem becomes $\max_{c_j \geq 0} \mathcal{U}(c_j, B_j - p[1 + \tau(\theta)]c_j)$, or

$$\max_{c_j \geq 0} \delta c_j^{1-\alpha} + B_j - p\left[1 + \tau(\theta)\right] c_j. \tag{5.13}$$

Note that we do not require money balances to be positive. It might be that some buyers purchase more than their endowment and end up with negative balances—which could be interpreted as borrowing instead of saving. We do this to avoid the complications that would arise if some poor buyers were up against their borrowing constraint while other, richer buyers were not.[2]

Before we solve this maximization problem, let us briefly verify that it is well behaved. Since $\alpha > 0$, the first term is concave in c_j. The second term is linear in c_j, so concave in c_j. The objective function is the sum of two concave functions, so it is concave in c_j. In addition, the objective function's domain, $[0, \infty)$, is convex. Accordingly, we are solving an unconstrained concave maximization problem—which we can conveniently do via first-order condition.

Indeed, with this type of problem, the first-order condition is sufficient to find the global maximum. We therefore set the derivative of the objective function to 0 to determine the optimal c_j. The first-order condition directly yields

$$(1 - \alpha)\delta c_j^{-\alpha} = p\left[1 + \tau(\theta)\right]. \tag{5.14}$$

This condition says that at the optimum the buyer is indifferent between consuming one additional unit of good (which provides utility $(1 - \alpha)\delta c^{-\alpha}$) and holding an additional $p[1 + \tau(\theta)]$ units of money (which provides utility $p[1 + \tau(\theta)]$), which is the additional

[2] An equivalent assumption is that each individual endowment B_j is large enough that all buyers are left with some money after making their desired purchases.

amount of money available to the buyer if they forego consumption.

Therefore, at the optimum, the buyer cannot improve their utility by consuming slightly more or slightly less: they are doing the best they can. This optimality condition is standard: it equalizes the marginal rate of substitution between consumption and money, $(\partial \mathcal{U}/\partial c)/(\partial \mathcal{U}/\partial m)$, with the price of consumption relative to money. But, since money is the numeraire, and the marginal utility of money is 1, the condition simply equalizes the marginal utility from consumption, $\partial \mathcal{U}/\partial c$, with the price of consumption, $p[1+\tau(\theta)]$.

We now rewrite the first-order condition to get an expression for the buyer's desired consumption of goods as a function of the market price and tightness:

$$c_j = \left[\frac{(1-\alpha)\delta}{p} \cdot \frac{1}{1+\tau(\theta)}\right]^{1/\alpha}.$$

The consumption has intuitive properties. If the market price is higher, so goods are more expensive, the buyer consumes fewer goods. If the market tightness is higher, so goods are more difficult to find, the buyer consumes fewer goods. And finally if the buyer derives lower utility from goods (lower δ), the buyer consumes fewer goods.

From the amount of goods consumed, c_j, it is easy to back out the rest of the buyer's behaviors. To consume c_j the buyer must purchase $y_j = [1+\tau(\theta)]c_j$ goods, and must visit $v_j = c_j/[q(\theta)-\kappa]$ sellers. The buyer spends $p[1+\tau(\theta)]c_j$ in total on goods and therefore holds $B_j - p[1+\tau(\theta)]c_j$ in money.

5.7.4. Computing the market demand

We now aggregate the individual demands for consumption of goods to obtain the market demand for consumption: $c = \int_0^1 c_j\, dj$. We assumed that different buyers have different initial wealth. One might expect this to lead to a complex distribution of consumption choices. However, a crucial property of the quasilinear utility function (5.11) is that it eliminates wealth effects for the choice of c_j. Hence, all buyers choose the exact same amount c_j, regardless of whether they are rich or poor. This means we can aggregate easily. Since there is a mass 1 of buyers, the market demand for consumption $c_d(\theta, p)$ equals the individual demand that we have just derived:

$$c^d(\theta, p) = \left[\frac{(1-\alpha)\delta}{p} \cdot \frac{1}{1+\tau(\theta)}\right]^{1/\alpha}. \tag{5.15}$$

This equation gives the notional market demand. It indicates how much buyers would like to consume for a given market price and tightness.

However, in our model, we use an effective market demand, which is the amount of goods that buyers actually desire to buy for a given market price and tightness, which is more than the notional demand. The gap arises because the amount of goods that buyers are able to consume is less than the amount of goods that they buy, since, in a slackish market, some of the purchased goods are devoted to matching instead of consumption.

Conveniently, the concepts of market demand and supply are consistent—the supply and demand both measure goods that are traded. Hence, when we solve the model, we can use the condition that supply equals demand, as any good that is sold by a seller is a good that is bought by a buyer. By focusing on the number of trades, we have equality of supply and demand, even though the model is not Walrasian.

Formally, the market demand, $y^d(\theta, p)$, is the total amount of goods that all buyers purchase so as to maximize their utility given the market price, p and the market tightness, θ. To compute this market demand, we translate the amount of consumption desired, given by (5.15), into the amount of purchases desired, using the result that consuming one good requires to purchase $1 + \tau(\theta)$ goods. This means that

$$y^d(\theta, p) = [1 + \tau(\theta)]\, c^d(\theta, p),$$

which then gives

$$y^d(\theta, p) = \left[\frac{(1-\alpha)\delta}{p}\right]^{1/\alpha} [1 + \tau(\theta)]^{1-1/\alpha}. \tag{5.16}$$

The market demand $y^d(\theta, p)$ is the number of goods demanded by buyers given market price and tightness.

5.7.5. Properties of the market demand

Let us now look at the properties of the market demand, $y^d(\theta, p)$. First, we see that it is decreasing in the market price. When the price level is higher, purchasing the good becomes less attractive compared to simply holding money, so demand is lower.

Similarly, the market demand is decreasing in market tightness. This property is visible in (5.16) because the matching wedge $\tau(\theta)$ is increasing in θ, while $1/\alpha > 1$. Intuitively, when the market is tighter, it is more difficult to buy goods: each visit is less likely to be successful, so buyers must visit more sellers per purchase, which pushes the matching wedge up. Hence, buyers have to devote a larger share of their purchase to matching instead of consumption, which makes consumption less appealing, and reduces demand.

Recall that $\bar{\theta}$ is the value of tightness at which $\tau(\bar{\theta}) = \infty$. We see that at this value, demand is 0. When tightness is 0, on the other hand, demand is given by

$$y^d(0, p) = \left[\frac{(1-\alpha)\delta}{p}\right]^{1/\alpha} [1 + \tau(0)]^{1-1/\alpha}.$$

To wrap up our analysis of the market demand, we plot the demand curve in a simple graph with quantity on the x-axis and tightness on the y-axis (figure 5.1F). The graph depicts the properties of the market demand that we have just derived. This representation of quantity against tightness is specific to slackish markets. It differs from the Walrasian representation of quantity against price. In slackish markets, we plot quantity against tightness because tightness is the core variable that equilibrates the market.

5.8. Prices

So far, we have talked about buyers: we discussed how many goods they would like to purchase and consume given the market tightness and price. We have talked about sellers: we discussed how many goods they would offer on the market and how many of those goods would actually be sold given the market tightness and price. We have computed the market demand and supply. But something that we haven't discussed is the price at which goods are traded.

Here, we are dealing with a slackish market where trades occur in a decentralized fashion, as described by the matching function. In a Walrasian market goods are sold at auction and the price is set by the auctioneer so as to equalize supply and demand. Here all trade occurs in individual buyer-seller pairs, who are free to negotiate the price as they wish. We cannot assume that a centralized auctioneer set the price, as we do in a Walrasian market. We need to find a way to model how prices are formed in each of the buyer-seller pairs once they have met on the market.

We introduce a price norm that describes how prices are set in all these buyer-seller pairs. The price norm is very general and can take many forms. As we discuss in chapter 6, we may assume bargaining between buyers and sellers; or we may postulate a price that is a rigid function of the model parameters; we can even assume a competitive price that ensures productive efficiency.

For the time being, we simply assume that prices in all trades are identical and are given by a generic price norm $p = p^n(\text{market variables})$. Since all market variables can be expressed as a function of the market tightness, we simplify the expression of the price norm to

$$p = p^n(\theta). \tag{5.17}$$

The goal should be to specify a price norm that reflects how prices are set in practice in the market studied—we want a price norm that is empirically valid. In chapter 6, we review evidence on how prices and wages are set, and we specify price norms that reflect these real-world properties. The price norm has critical implications for the behavior of the model; therefore, we can also look at how the world operates to infer properties of the price norm so our model is consistent with what we see in the real world.

Instead of assuming an auctioneer, our decentralized model gives us the freedom to make assumptions that are as realistic as possible about how prices are set in the real world. This is a vast improvement over the Walrasian model, because the assumption of perfectly competitive prices is probably the least realistic and therefore the most problematic assumption used in modern market models. Furthermore, the assumption of perfectly competitive prices has a raft of implications about welfare and policy, which are all erroneous if the pricing assumption itself is erroneous.

5.9. Surpluses from trade

A key property of matching models is that buyers and sellers are always happy to trade. Anytime there is an exchange, there is a bilateral surplus from the trade so that both the buyer and the seller take something away from the trade—they are strictly better off trading than not trading. This is true in the real world: sellers are usually happy when they can sell their goods while buyers are happy when they can buy what they were looking for. This is different from the Walrasian model, where the marginal buyer and seller are indifferent between buying or selling and not buying or not selling, because the price exactly equals the seller's marginal cost of production and the buyer's marginal utility.

So by representing the market using a matching function, trades always occur in a bilateral monopoly situation. That is, once a seller and buyer have found each other, there is a surplus that is created from this match, meaning that both the seller and the buyer have something to win by proceeding with the match.

Since the solution to such bilateral monopoly problem is indeterminate, it cannot be used to impose any condition on the price.[3] To resolve this theoretical issue, we assume that prices are given by a price norm.

Assuming a price norm is unusual in economics models, and some readers might wonder why we cannot do better—in the sense of imposing a price derived from some economic principles. The reality is that in a slackish model, there is no obvious economic mechanism that can determine the price. Any price that leaves seller and buyer with a positive individual surplus would be acceptable to both parties. There is no deviation from that price that generates a Pareto improvement. Of course, a seller would be better-off with a higher price, but a buyer would be worse-off with a higher price.

In fact, such theoretical indeterminacy might explain why, in the real world, there exists so many different cultural and institutional arrangements to determine prices. In some places, sellers simply set a fixed price. In other countries, people normally bargain over the price of goods. There are places where there is a set price but customers are

[3]The indeterminacy of the solution to the bilateral monopoly problem has been known since Edgeworth (1881). It is discussed by Howitt and McAfee (1987) and Hall (2005) in the context of matching models.

expected to add a tip whose size is dictated by conventions, and might vary by market. Sometimes, prices are regulated or directly set by governments.

The situation of bilateral monopoly arises because the pairing of a buyer and a seller generates a positive bilateral surplus. Intuitively, the reason is that it is difficult for sellers to sell their goods and for buyers to find a good, so there is a surplus when they meet and have the opportunity to trade.

We now formally establish what the bilateral surplus is. We consider the ex-post surplus from trade, evaluated once the buyer has already incurred the matching costs required to reach the seller.[4] The relevant outside option is to walk away after sinking those costs, not to avoid entering the market. To start, we compute the seller's surplus and buyer's surplus: we compare what the seller and buyer take away from the trade, and compare it with what they would have taken away if the trade didn't occur.

5.9.1. Seller's surplus

We assume that just like buyers, sellers have a quasilinear utility function, so their marginal utility of money is 1. This is what the seller gets from one unit of money. The price of a transaction is p. Then, for p units of money, the seller takes away p utils from the trade. If there was no trade, the seller would get nothing. The seller does not benefit from the good that they put for sale because they can't consume their own good. The seller surplus would therefore simply be:

$$\mathcal{S} = p.$$

Critically, the seller surplus is positive for any positive price, so the seller is always happy to trade.

5.9.2. Buyer's surplus

Given that a buyer's utility is given by (5.11), the marginal utility of one good is:

$$\frac{\partial \mathcal{U}}{\partial c_j} = (1-\alpha)\delta c_j^{-\alpha}.$$

This is what the buyer takes away from the trade, given the planned consumption level. If the trade did not occur, however, the buyer would get to keep their p units of money, which provide p utils since the utility is quasilinear. Of course, the buyer does not recover the already-sunk matching expenditures that brought them here. With this information,

[4] Implicitly, the seller has also already incurred the production costs required to reach the buyer. In this chapter the production decision is exogenous—each seller just brings k_i to the market—but in chapter 7 the production will be endogenous.

we compute the buyer surplus from the trade:

$$\mathcal{B} = (1 - \alpha)\delta c_j^{-\alpha} - p. \tag{5.18}$$

However, the amount of consumption is not random. It is given by the solution to the buyer's optimization problem. The buyer maximizes utility subject to a budget constraint, and that tells us what their consumption is. It turns out that consumption is always such that the buyer's surplus is positive. The first-order condition from the buyer's problem, given by (5.14), tells us that the marginal utility from consumption, $(1-\alpha)\delta c_j^{-\alpha}$, equals $[1+\tau(\theta)]p$, where $\tau(\theta)$ is the matching wedge. Combining this result with the expression for the buyer surplus, we simply get:

$$\mathcal{B} = \tau(\theta) p. \tag{5.19}$$

Similar to the seller surplus, we see that the buyer surplus is positive. The buyer surplus has a simple interpretation: it is the monetary cost sunk to match with this seller, since consuming one good requires to purchase $\tau(\theta)$ goods for matching beforehand, each costing a price p. Another interpretation is that the buyer surplus is the monetary cost required to match with a different seller if the present trade fell through. Indeed, matching with a new seller to purchase one good would require to spend again on matching cost. The implication is that the buyer surplus is larger when matching costs are larger, because a larger matching cost implies a larger matching wedge, τ. In any case, given that the buyer decides to shop fully knowing what the price norm is, when they find a good, their surplus is always positive, so they are always happy to buy it.

5.9.3. Total surplus

Since the buyer and seller surpluses are both positive, we conclude that the total surplus from a trade, $\mathcal{T} = \mathcal{S} + \mathcal{B}$, is also positive. The total surplus admits two simple, equivalent expressions, one in terms of the price and one in terms of the marginal utility of consumption:

$$\mathcal{T} = [1+\tau(\theta)]\, p = (1-\alpha)\delta c^{-\alpha}.$$

In the expression for the total surplus, we have replaced the individual consumption c_j by the market-level consumption c since they are equal.

Hence we see that the total surplus is simply the marginal utility of consumption for the buyer. This is quite obvious in retrospect. Without trade, the good is not sold and therefore not consumed by anyone, so one unit of consumption is lost, which is valued at the buyer's marginal utility. In the slackish model, therefore, all trades that occur generate a positive total surplus, irrespective of the price norm.

So any price norm respects bilateral efficiency. To respect bilateral efficiency, a price must be at such level that when a trade generates a positive total surplus, the price also leaves the buyer and seller with positive individual surpluses. In that way, any time a trade creates some bilateral gains also creates individual gains and be acceptable to both trading parties. This ensures that any trade that creates bilateral gains is conducted. Here we have seen that for any price norm, when buyers shop rationally given the price norm, any possible trade generates both individual and bilateral surpluses, so bilateral efficiency is always respected.

5.9.4. Why are prices not restricted to a price band?

A famous result due to Hall (2005) is that in a DMP model, fixed wages are theoretically legitimate. They do not create any bilateral inefficiencies, as long as they remain within a certain band. Why is the price norm not restricted to a price band here? The reason is that Hall works with a linear model, which artificially created the band.[5] In our context, with a linear utility function, the marginal utility of consumption is fixed. Thus, prices must be below that marginal utility to ensure that the buyer receives a positive surplus from the trade, and accepts to trade (as shown by (5.18)) .

Here our utility function satisfies the Inada condition that it has infinite marginal utility when consumption falls to 0: $\lim_{c_j \to 0} \partial \mathcal{U}/\partial c_j = \infty$. This ensures that any price would be acceptable to buyers when their consumption is low enough. Given the buyers take the price norm as given when they decide to shop, they only plan to purchase goods that deliver at least as much utility as the price they pay for it. This is visible in surplus (5.19), which is always positive once we take into account buyer's decision to shop or not. Given the Inada condition there is no limit on how wide the price band can be, which means that there is no restriction on the price norm: the price band becomes infinitely wide when consumption becomes arbitrarily low.

5.10. Solution of the model

Finally, we present a strategy to solve the model. The main step in solving the model is to determine the market tightness, which we do using a supply-equals-demand condition.

5.10.1. Structure of the solution

Now that we have introduced all the elements of the model and made all the assumptions we need, we are finally in a position to actually solve the model. Before we do so, let us

[5] Hall (2005) works with a model of the labor market, so what he really assumes is a linear production function. But this assumption is isomorphic to assuming a linear utility function in this market model.

TABLE 5.1. Structure of the slackish market model

Variable	Name	Condition	Interpretation	Reference
c	Consumption of goods	$c = c^d(\theta, p)$	Utility maximization	(5.15)
p	Price of goods	$p = p^n(\theta)$	Price norm	(5.17)
y	Sales of goods	$y = [1 + \tau(\theta)]c$	Matching wedge	(5.8)
v	Visits	$q(\theta)v = y$	Matching function	(5.1)
θ	Market tightness	$\theta = v/k$	Definition of tightness	(5.2)
u	Rate of slack	$u = 1 - f(\theta)$	Definition of slack	(5.5)

first discuss the structure of the solution: the elements that we are looking for and what we can use to find them.

Formally, solving a model means figuring out the values of all the variables in the model as a function of the parameters of the model. Here, this is actually pretty simple. In our slackish market model, we have to find 6 aggregate variables using 6 independent equations, as shown in table 5.1. There are also individual variables in the model, such as consumption c_j, expenditure y_j, money balances b_j, and visits v_j by any individual buyer j, but these can easily be determined from the aggregate variables.

Now that we have the structure of our solution, the challenge is to find a simple way to compute all the variables in the model.

Before we do that, it is useful to realize that the description of the model in table 5.1 is not unique: it is one of many equivalent representations. Indeed, it is possible to reshuffle the independent equations into other equations, which offer alternative ways to solve the model. Different representations might be useful in different situations, depending on the question and property of interest.

The 6 equations form one valid representation of the model. We can algebraically combine and rearrange them to form a different but equivalent set of 6 independent equations. The key is that any valid system yields the same solution. For instance, if we divide $y = q(\theta)v$ by $\theta = v/k$, we get $y/\theta = q(\theta)k$, or $y = \theta q(\theta)k$, which is just $y = f(\theta)k = y^s(\theta)$. This manipulation is telling us that we could determine sales from the market supply. However, then we would need to replace either the condition for v or θ, since otherwise the 3 equations for y, v, and θ would not be independent. Conceptually, if we did not replace the equations for v or θ, we would lose the information about the matching wedge and matching cost in the model solution—the information that used to be conveyed by the condition that $y = [1 + \tau(\theta)]c$.[6]

[6]For instance, we could determine the number of visits by $y = c + \kappa v$, or $v = (y - c)/\kappa$. That condition would capture properly how the matching cost enters into the model. And this condition contains the same information as the previous equations $y = [1 + \tau(\theta)]c$ and $q(\theta)v = y$, so no information would be lost by replacing the equations for y and v in table 5.1 by $y = y^s(\theta)$ and $v = (y - c)/\kappa$. Indeed, $y = [1 + \tau(\theta)]c$ implies $y = q(\theta)c/[q(\theta) - \kappa]$, which gives $y[q(\theta) - \kappa] = q(\theta)c$, then $q(\theta)(y - c) = \kappa y$, and finally $(y - c)/\kappa = y/q(\theta)$. But then $q(\theta)v = y$ tells us that $v = y/q(\theta)$, so combining both equations we learn that $v = (y - c)/\kappa$, which is

5.10.2. Strategy to solve the model

To solve the model, we need to figure out the values of the 6 variables that are determined by 6 independent equations (table 5.1). We follow a sequential strategy for solving the model: we first determine the market tightness, θ; then, we compute the values of the 5 remaining variables. We will compute the market tightness in the next section. Here, we describe how the 5 other variables can easily be determined once we know tightness.

First, we know via the price norm that $p = p^n(\theta)$ so we can easily compute p given θ. Similarly, the rate of slack is directly determined by tightness: $u = 1 - f(\theta)$.

For consumption, we simply use the notional market demand: $c = c^d(\theta, p)$. Since the price p is itself a function of tightness, we can determine c if we know θ.

Then, to compute sales, we simply use that $y = [1 + \tau(\theta)]c$. Since c can be determined from θ, we can compute y given θ.

Last, to calculate the number of visits, we have that $v = y/q(\theta)$. Again, since y is a function of tightness θ, we can compute v given as soon as we have θ.

We have shown that we can easily determine the values of the remaining 5 variables once we know market tightness. Tightness is the most critical variable and the next step is to figure out how to compute it.

5.10.3. Computing market tightness from demand and supply

We are now ready to compute the market tightness, θ, in slackish market model. From it we can determine the values of all the other variables in the model.

First, we need to think about the key decisions in our model. The first is the buyers' purchasing decision given market tightness, which is characterized by the market demand:

$$y = y^d(\theta, p^n(\theta)). \tag{5.20}$$

The market demand gives the amount of output demanded by buyers.

The second important decision is buyers' shopping decision, where the number of visits is chosen such that buyers purchase the desired amount of goods given market tightness:

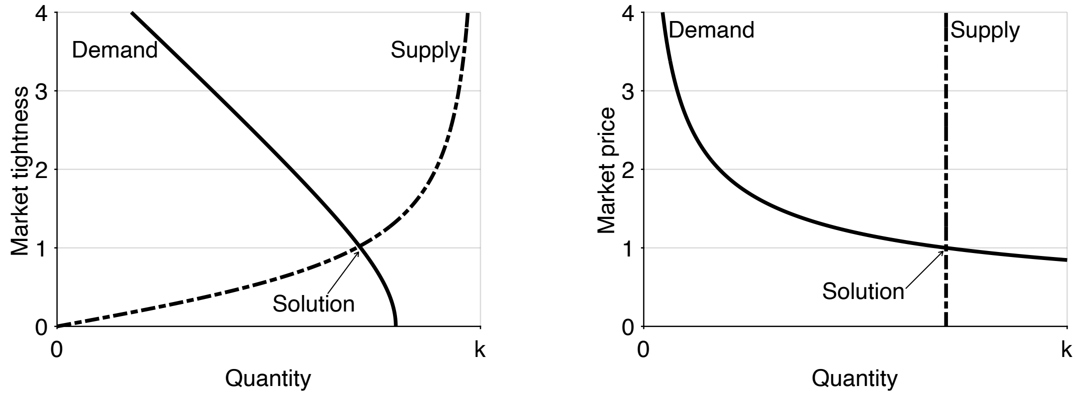
$$y = vq(\theta).$$

By definition of tightness, we know that

$$v = \theta k.$$

So, if we know the number of goods that are supplied to the market, k, and tightness, θ,

just the new equation used to determine v.



A. Solution in tightness-quantity plane

B. Solution in price-quantity plane

FIGURE 5.2. Numerical solution of the slackish market model

The market supply is given by (5.6). The market demand is given by (5.16). The matching function is CES, given by (4.10). Parameters are set to $\gamma = 2$, $\kappa = 0.2$, $k = 1$, $\alpha = 0.5$, and $\delta = 2$. In panel A, the price is set to $p = 1$ and tightness varies. In panel B, tightness is set to $\theta = 1.02$, which is obtained from the intersection of the supply and demand curves in panel A, and p varies.

we can calculate the aggregate number of visits. Then,

$$y = \theta q(\theta) k.$$

Recall that $\theta q(\theta) = f(\theta)$ and $f(\theta)k = y^s(\theta)$, so

$$y = y^s(\theta), \tag{5.21}$$

which just says that output is given by the market supply too.

Thus, in our model, output is given by both market demand and supply (equations (5.20) and (5.21)). The solution of the model must therefore ensure that market demand equals market supply—otherwise how could output be equal to both market demand and market supply. Accordingly, market tightness must equalize market demand and supply:

$$y^d(\theta, p^n(\theta)) = y^s(\theta). \tag{5.22}$$

Thus, the value for market tightness that solves the model can be found at the intersection of the market supply and demand curves. The market supply curve captures the number of trades, for any tightness, given the market capacity and the matching process. The market demand curve captures the number of trades, for any tightness, that maximize the utility of buyers given their budget constraints. Thus, at the solution, the matching process is respected, as well as the utility maximization process.

Next, we illustrate numerically the solution of the model (figure 5.2A). For simplicity, we set the price norm to 1: $p = 1$. To represent the equality of supply and demand, given by (5.22), we plot the market supply and demand curves in the tightness-quantity plane. The tightness that solves the model is found at the intersection of the two curves. In this numerical example, the solution is $\theta = 1.02$.

Of course we can also represent the supply-equals-demand condition in a Walrasian, price-quantity plane (figure 5.2B). In this graph we set the tightness to its solution value: $\theta = 1.02$. The supply curve is just vertical in this plane, because it does not depend on the price. The demand curve is downward sloping. The intersection between supply and demand curves occurs at the price norm, $p = 1$, because we set tightness to the value such that $y^s(\theta) = y^d(\theta, 1)$. Thus, when $p = 1$, supply equals demand. But this graph does not bring new information. It is just repeating what we saw in the tightness-quantity graph: that at $\theta = 1.02$ and $p = 1$, supply equals demand. Furthermore, unlike tightness, the price is not determined by market forces: it is given by the price norm. So the price-quantity graph is not meaningful in a slackish market.

5.10.4. Brief discussion of the solution

To conclude, let us discuss a few things about the solution of the model.

First, just like there are alternative ways to describe the structure of the model, as we saw in table 5.1, there are alternative ways to get the market tightness. An equivalent approach, followed by Michaillat and Saez (2015), is to formulate the market supply and demand in terms of consumption instead of output. Then the market demand $c^d(\theta, p)$, given by (5.15), indicates the number of goods that buyers want to consume. The market supply $c^s(\theta)$ indicates the number of goods traded given the matching process and the amount placed for sale by sellers. This alternative market supply is given by $c^s(\theta) = y^s(\theta)/[1 + \tau(\theta)]$. Condition (5.22) is equivalent to $y^d(\theta, p)/[1 + \tau(\theta)] = y^s(\theta)/[1 + \tau(\theta)]$ and so to $c^d(\theta, p) = c^s(\theta)$. The consumption-based solution makes it easier to think about welfare, but is otherwise slightly less natural, so we follow the output-based approach in this book.

Second, in many models with matching functions, the key decision by buyers is the effort that they put into searching for goods. For instance, in a DMP model, this effort is the vacancies posted by firms. In these search-and-matching models, the solution of the model is organized around the search effort. As we show in appendix C, it is possible to reframe the solution of the model in terms of the number of visits to sellers—which is the search effort in the model. The two solutions are equivalent, but the visit-based approach is less convenient analytically.

Third, a natural question is about the realism of the solution concept. When we solve the model, we assume that buyers maximize their utility given market tightness. But how can buyers determine what market tightness is on the market? This is a typical challenge in markets in which buyers and sellers must figure out the values of market variables on which they base their decisions. It is quite a challenging task for buyers to consider the entire market around them and assess what the market tightness will be once all buyers and sellers enter the market.

We discuss these issues further in appendix D and propose a simple resolution. We assume that a statistical agency announces tightness in advance, and that buyers act based on the tightness that has been announced. Then, if the agency announces the tightness equalizing market supply and market demand, the prevailing tightness is the announced tightness. So the agency is correct, and the supply-equals-demand tightness prevails, just like in this chapter.

In fact, even if the statistical agency is unable to build market demand and supply, or unable to equalize both, the agency may find the tightness solving the model via a tatonnement process. Under certain conditions, the model behaves well enough that the tatonnement process converges to the model solution, at which point the tightness quoted by the agency is the same as the tightness realized on the market.

We have now assembled the basic machinery of a slackish market. The core equation of the model is the supply-equals-demand condition, given by (5.22), which determines market tightness. For the time being, we cannot solve the equation explicitly or perform comparative statics because the solution depends on the price norm, $p^n(\theta)$. In the next chapter, we will look at different assumptions for the price norm and solve what the market tightness is in each case and determine how the model responds to different shocks for each norm.

5.11. Model with fixed prices

To complete this chapter, we analyze a slackish model in which the price norm gives a completely fixed price. The fixed-price model is not only simple but also quite useful because any price that's not fully flexible produces the same qualitative results as a fixed price—but of course with different quantitative results. Furthermore, the model with a fixed price is the most striking example of a slackish model: all adjustments to shocks occur through market tightness, and none through prices. Of course in reality prices are not entirely fixed—they are somewhat flexible. Chapter 6 builds a model with somewhat flexible prices and studies its properties.

5.11.1. Solution with fixed prices

Let us now solve the slackish market model when prices are fixed. We assume that the price norm imposes a fixed price $p > 0$ for all goods.

To solve the model, we need to find the tightness θ that equalizes market demand and supply:

$$y^d(\theta, p) = y^s(\theta). \tag{5.23}$$

We first ensure that the tightness indeed exists and is unique. We know that $y^d(\theta, p)$ is decreasing from $y^d(0, p) > 0$ when $\theta = 0$ to 0 when $\theta = \bar{\theta}$. At the same time, $y^s(\theta)$ is increasing in tightness, starting from 0 when $\theta = 0$. Moreover, both y^d and y^s are continuous in tightness. Thus, we can be sure that equation (5.23) always has a solution and this solution is unique, since we can be sure that the curves $y^d(\theta, p)$ and $y^s(\theta)$ cross once for $\theta \in (0, \bar{\theta})$. The crossing point is the solution of the model. This graphical argument is illustrated in figure 5.3A.

We can also establish the existence and uniqueness of the solution maybe slightly more formally by introducing the excess demand function:

$$z(\theta, p) = y^d(\theta, p) - y^s(\theta). \tag{5.24}$$

With the excess demand function, the solution of the model simply is

$$z(\theta, p) = 0.$$

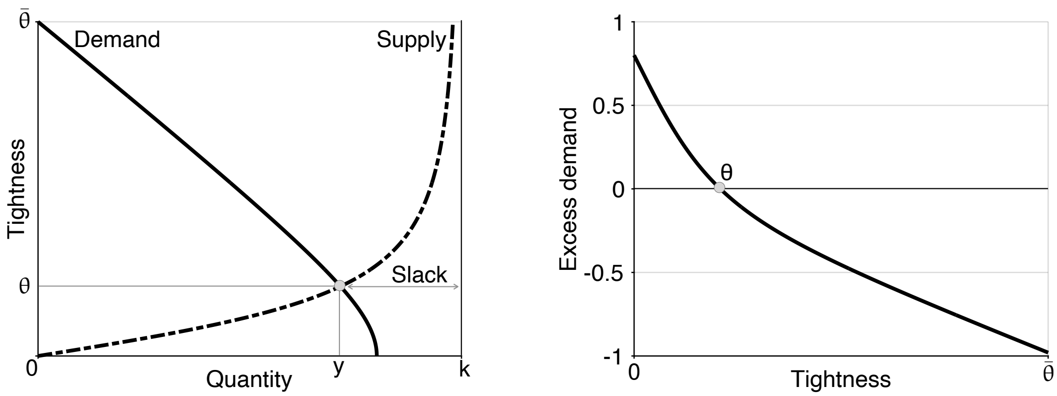
The function z is continuous and decreasing in θ, and it declines from $z(0) = y^d(0, p) > 0$ to $z(\bar{\theta}) = -y^s(\bar{\theta}) < 0$ when θ increases from 0 to $\bar{\theta}$. Thus, by the intermediate-value theorem, the equation $z(\theta, p) = 0$ has a unique solution on $(0, \bar{\theta})$. This argument is illustrated in figure 5.3B.

Comparing figure 5.3A to figure 5.3B, we see that the supply-demand representation conveys more information than the excess-demand representation, which is why we use it throughout the book. The supply-demand representation provides not only the tightness θ that solves the model, but also the corresponding output y, and the amount of slack in the market, $k - y = u(\theta)k$. In particular, we see that in a slackish market, although supply equals demand, there always is some slack: there always are some goods that are available for sale but not sold.

5.11.2. Comparative statics with fixed prices

Now that we have solved the model, we can look at comparative statics: how the model responds to shocks to various parameters.

Let us start by looking at a negative demand shock: a decrease in preference for goods, δ. We start with a high preference, and then reduce the parameter value to a low preference in order to see how the solution of the model changes. It is best to use our graphical representation to visualize the behavior of the model under the negative demand



A. Supply-demand representation

B. Excess-demand representation

FIGURE 5.3. Solution of the slackish market model with fixed prices

The market supply is given by (5.6). The market demand is given by (5.16). The excess demand is given by (5.24). The matching function is CES, given by (4.10). Parameters are set to $\gamma = 2$, $\kappa = 0.2$, $k = 1$, $\alpha = 0.5$, $\delta = 2$, and $p = 1$.

shock.

From figure 5.4A, we see that a negative demand shock brings the demand curve inward, which results in a lower value of both tightness θ and sales y. This was to be expected: as buyers value goods less, they aim to consume and therefore purchase fewer goods. They therefore visit fewer sellers, which reduces tightness and the amount of goods sold. The amount of slack in the market increases as it becomes harder for sellers to sell their goods. Note that at this stage we cannot say what happens to consumption. Although purchases decline, the matching wedge $\tau(\theta)$ also decreases, so a larger share of purchases is devoted to consumption. That is, buyers purchase fewer goods but devote a larger fraction of them to consumption. The reason is that with lower demand, visits are more likely to be successful, so fewer visits are required per purchase. Thus, the response of consumption depends on the specific situation, as we clarify in chapter 9.

Another way to be convinced that tightness must fall after a negative demand shock is to assume that tightness remains the same and realize that we reach a contradiction. Since tightness and price remain the same, but the parameter δ is lower, we know that buyers want to purchase fewer goods. However, through the matching process, if tightness is the same, the number of goods sold remains the same. So here's the contradiction: if tightness remains the same, it is not possible that both buyers maximize utility and the matching process is satisfied. In other words, keeping the same tightness as before is not a solution of the model.

Next we turn to a negative supply shock: a decrease in the market capacity, k. We start with a high capacity, and then change the parameter value to a lower capacity in order to see how the model solution changes. In figure 5.4B, we see that a decrease in capacity

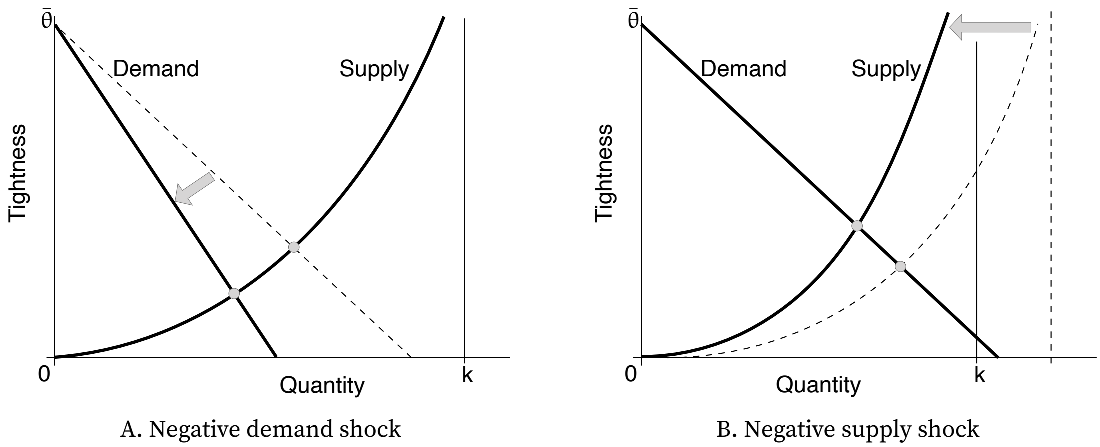


FIGURE 5.4. Comparative statics in the slackish market model with fixed prices

The market supply is given by (5.6). The market demand is given by (5.16). The price is fixed to p. A negative demand shock is a reduction in utility parameter δ. A negative supply shock is a reduction in market capacity k.

brings the supply curve inward. What happens to the solution of our model? The new value of sales y is lower than it was before, while the new value of tightness θ is higher. This makes sense because if fewer goods are supplied, through the matching process, fewer goods are bound to be purchased. And because there is more competition among buyers, tightness is bound to increase. Hence slack falls in the market, both in the sense that the share of goods that remain unsold, $u(\theta)$ is lower, and in the sense that the number of goods that remain unsold, $u(\theta)k$ is lower. Since tightness is higher, the matching wedge $\tau(\theta)$ also increases, so a larger share of fewer purchases is devoted to matching. In that case, we know that consumption decreases. What we do not know here is what happens to the number of visits. Tightness is higher so visits are less likely to be successful, which means mechanically that buyers would have to visit more sellers to keep the amount of purchases constant, but which also means that purchases become more complicated and thus less desirable. As we see in appendix C, visits may go up or down after a negative supply shock.

To summarize, both negative demand and supply shocks lead to lower sales; the key difference is that a negative demand shocks lead to lower tightness while negative supply shocks lead to higher tightness. After a negative demand shock, it becomes harder to sell goods, while after a negative supply shock, the fewer goods that are for sale are sold more easily. The price does not respond at all to the shocks: it is tightness that absorbs all the shocks and re-equilibrates the market.

5.12. Relation with Walrasian markets

To finish this chapter, we discuss the similarities and differences between the slackish model of markets and the most standard of model of markets: the Walrasian model.[7] The two models share a fairly similar formalism. But the introduction of slack in the slackish model completely alters the behavior of prices and therefore the efficiency properties of the model. Most importantly, slackish markets are generally inefficient.

5.12.1. Quoting prices

Walrasian theory makes the institutional assumption that a price is quoted on the market by an auctioneer. Here we do not have an auctioneer, but we also assume that tightness and price are known by sellers and buyers. We assume that the price is given by a price norm, which is known to everyone by definition. Regarding tightness, we make the common assumption that buyers and sellers have all required information about the market and can determine what others do and what the tightness is. An alternative assumption we explore in appendix D, which is entirely equivalent and requires less rationality from market participants, is that a statistical agency announces the market tightness.

5.12.2. Taking prices as given

As in Walrasian theory we make the behavioral assumption that sellers and buyers take prices and tightness as given. It is natural for them to take tightness as given because the tightness is the ratio of aggregate number of visits to aggregate number of goods for sale, and each market participant is small relative to the size of the market. The issue is more complicated for the price since a buyer and a seller could bargain the transaction price once they have matched.

However, the actual transaction price has no influence on anyone's decisions because the decisions are made before the match is realized. What matters is the price at which buyers and sellers expect to trade. This price determines the number of visits that buyers conduct. When capacity is endogenous as in chapter 7, this price also determines whether sellers enter the market or not. The bargained price is only a transfer from buyer to seller: it has no effect on market quantities, since bargaining would never lead to the destruction of a match. For instance if a price is a little lower than expected, the seller takes home a little less income, which means that they hold a little less money, while the buyer spends a little less, which means that they hold a little more money. But the market quantities—visits, sales, consumption, tightness, slack—would be unchanged.

[7] For a standard treatment of the Walrasian market model, see Mas-Colell, Whinston, and Green (1995, chapter 10).

We could generalize the model by allowing the transaction price to be the price norm plus some white noise. This would not change anything, so to keep things simple, we assume that actual prices are just the price norm.

5.12.3. Behaving optimally

As in a Walrasian theory, market participants behave optimally given price and tightness. The difference with Walrasian theory is that buyers and sellers cannot choose the quantities that they trade. These quantities are constrained by the matching function. Buyers only choose how many sellers to visit, knowing that the purchasing probability is $q(\theta)$ and that the purchase of one good yields $1/[1+\tau(\theta)]$ units of consumption. Sellers only choose how many goods to bring to the market, knowing that the selling probability is $f(\theta)$. In a way, market participants only control the inputs into the matching function (visits and goods), not the output (purchases and sales).

5.12.4. Clearing the market

The supply-equals-demand condition (5.22) is the equivalent of the market-clearing condition of the Walrasian equilibrium. The Walrasian market-clearing condition imposes that at the market price, the quantity that buyers desire to buy equals the quantity that sellers desire to sell. This condition is required to ensure the consistency of the Walrasian model because sellers and buyers make their decisions expecting to be able buy and sell any quantity at the quoted price. If supply did not equal demand, one side of the market would be rationed and would not be able to transact as planned, so the model would be inconsistent: participants behaved as if there was no rationing but in fact faced rationing. This inconsistency is avoided when the price clears the market.

Similarly, as we saw, condition (5.22) is required to guarantee the consistency of our model. In this chapter, we saw that supply equals demand ensures that the number of goods purchased by buyers during their visits to sellers equals the number of goods that they desired to maximize their utility. In appendix D, we also see that the tightness at the intersection of the supply and demand curves is the only tightness that a statistical agency could announce without making an error. If that tightness is announced and participants act optimally based on it, then that tightness is realized. Any other announced tightness would not be realized.

There is a last way to connect the supply-equals-demand condition in the Walrasian and slackish models. In the Walrasian model, the condition ensures that the trading probabilities expected by participants (trade with probability 1) are realized. Similarly, in the slackish model, the condition ensures that the trading probabilities $f(\theta)$ and $q(\theta)$ expected by participants, on which they based their decisions, match the actual trading probabilities, realized once buyers have started visiting shops and sellers have started selling goods.[8]

5.12.5. Analysis via supply and demand

Just as a Walrasian market, a slackish market can be conveniently analyzed via supply and demand curves in a market diagram. For instance, figure 5.2A plots the market supply and demand curves for this chapter's slackish market. The key difference with a Walrasian market is that it is the market tightness, not the market price, that equilibrates supply and demand. That's why the supply and demand curves are plotted in a tightness-quantity plane, not in the Walrasian price-quantity plane. The tightness that solves the slackish model is found at the intersection of the two curves, just like the price that solves the Walrasian model is found at the intersection of the Walrasian supply and demand curves.

5.12.6. Efficiency and inefficiency

It is a little early to discuss all the efficiency properties of the slackish market and compare them with the those of the Walrasian model. Yet, a few central properties are already visible. Tightness determines all quantities in the slackish model, so it determines social welfare. If the welfare function has a unique maximum, then there is one tightness that is socially efficient. Then there must be one single price that is socially efficient, and that ensures that the efficient tightness prevails in the market.

But price norms can pick any price: efficient but also inefficiently high or low. So there is no guarantee that slackish markets are efficient. This is a core difference with the Walrasian world, which promises efficient markets. By contrast, in a slackish world, markets are generally inefficient. In chapters 9 and 15 we characterize the efficient tightness and show that it does not seem to prevail in the US economy, with especially large departures from efficiency in recessions.

5.12.7. Not nonclearing markets

Walrasian theory has also been generalized to situations where markets might not clear because of price rigidities.[9] These rigidities might be caused by regulation (minimum wage, price ceilings, price floors) or by norms (downward nominal wage rigidity).

Nonclearing Walrasian markets share similarities with slackish markets. First, they experience slack in situations of excess supply. Second, they are generally inefficient—

[8] This interpretation is developed in Michaillat and Saez (2015) but also follows from the analysis in appendix D.

[9] Benassy (1993) offers a wonderful survey of that literature.

whenever there is excess supply or excess demand.[10]

But there are also important differences between these models. First, in a nonclearing Walrasian market, there is not always slack: only in situations of excess supply. In a slackish model, there is always some slack—some unsold goods. Second, when there is slack in a nonclearing Walrasian market, buyers can buy anything they want at the market price, so only sellers face trading probability below 1. Conversely when there is no slack (excess demand), sellers can sell everything they want, and only buyers face a trading probability below 1. By contrast, in a slackish market, all trading probabilities are always below 1.

On both counts, the slackish market is more realistic, since there seems to always be some slack on all markets, and some buyers and sellers who are unable to trade (as we saw in chapter 3). For instance on the labor market, there is always some unemployment, and it always takes time for job seekers and vacant jobs to find a match.

Beside these differences in how the models describe the world, there are some theoretical differences too. First, the nonclearing Walrasian market is binary in that either its behavior is determined by the market supply (in excess demand) or by the market demand (in excess supply). Demand and supply never matter at the same time. The slackish market offers a more balanced perspective. Market demand and supply both matter at all times, although because the market is highly nonlinear, demand takes a much more important role in slack times and supply in tight times. But the relative importance in supply and demand evolves continuously with the state of the market, instead of jumping from all demand to all supply when the nonclearing Walrasian market moves from excess supply to excess demand.

Second, the nonclearing Walrasian model faces tricky theoretical challenges to deal with the rationing that sellers or buyers face. Consider a situation of excess demand: the buyers expect to be able to buy anything they want at the market price, they plan accordingly, but when they get to the market they face rationing and are actually unable to buy what they planned to buy. The model must then explain which rationing is implemented (first-come-first-served? lottery? rationing based on willingness to pay?). The

[10]The nonclearing Walrasian model was initially described as a disequilibrium model, because the nonclearing prices and quantity rationing were seen as temporary (Edwards 1966; Grossman 1971; Barro and Grossman 1971; Bénassy 1975; Benassy 1976). The presumption was that prices would adjust toward their market-clearing levels, and sellers and buyers would adjust their behavior in the face of rationing, so that the market would slowly converge towards its Walrasian equilibrium, where markets clear. As the literature evolved and the models became more sophisticated, they incorporated the adjustment process from the initial situation to the Walrasian equilibrium. These nonclearing models then moved from situations of "disequilibrium" to "non-Walrasian equilibrium" (Benassy 1993). The notion of "disequilibrium" fell out of favor after that and stopped being used. As the concept of "disequilibrium" vanished, the word "equilibrium" became somewhat meaningless, especially when used as an adjective: an "equilibrium model" is just a model; an "equilibrium condition" is just a condition; "equilibrium dynamics" are just dynamics; and so on. When people define an equilibrium and look for an equilibrium, they really just define the solution of their model and look for it. In this book I use the word "solution" instead of "equilibrium", as I find it more transparent and understandable.

model must also explain how buyers might anticipate this situation in the first place, and maybe alter their behavior. A buyer who anticipates being rationed does not behave the same as someone who does not anticipate it. In the slackish model, all these issues are addressed with the matching function. The matching function dictates what the trading probabilities are, and everyone takes them as given when they make decisions. So nobody is surprised at any point, and everyone anticipates the market conditions that are realized.

5.13. Summary

In this chapter we introduce the basic model of a slackish market: a market in which slack is the central mechanism for equalizing supply and demand. Unlike a Walrasian market, where the price adjusts to equilibrate supply and demand, in a slackish market, the market tightness—here the ratio of buyers' visits to goods for sale—is the variable that equilibrates the market.

The model is built around sellers with a fixed capacity of goods and buyers who must visit sellers to make purchases. The number of trades is determined by a matching function, which captures the complex, decentralized process of buyers finding sellers. Due to the properties of this function, the probability of selling and the probability of buying depend solely on the market tightness. A tighter market favors sellers but harms buyers. The rate of slack, which is the share of goods that remain unsold, is a decreasing function of tightness. The matching wedge, which represents the additional goods buyers must purchase to cover the costs of visiting sellers, is an increasing function of tightness. Importantly, the matching wedge creates a gap between what is purchased and what is consumed.

Buyers maximize their utility, which yields a downward-sloping market demand curve when quantities are plotted against tightness. The market supply is upward-sloping in a tightness-quantity plane because more tightness means a higher probability of selling goods.

The model is solved by finding the market tightness that equalizes market demand and supply. This tightness then determines all other market outcomes, including output, consumption, and the rate of slack. The price at which goods are traded is set by a price norm, which can take various forms.

In the simple case with fixed prices, tightness adjustments absorb all shocks. Negative demand shocks reduce tightness and then sales because of increased slack. By contrast, negative supply shocks increase tightness so they reduce sales despite lower slack.

Regardless of the price, any trade generates a positive surplus for both the buyer and the seller, a key distinction from Walrasian models where marginal traders are indifferent. The model also differs from monopolistic models, in which sellers enjoy a surplus but the marginal buyer is indifferent between buying and not buying; and from monopsonistic models, in which buyers enjoy a surplus but the marginal seller is indifferent between selling and not selling. With price norms giving all bargaining power to either seller or buyer, the slackish model can mimic a monopolistic or monopsonistic model. But it is much more general than these models: it can feature any price between the monopolistic and monopsonistic extremes—in general, both traders derive some surplus from any trade.

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