Novel Approaches to Fast Filling of Hydrogen Cylinders

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- 5 Engineering, Faculty of Engineering and the Environment, University of Southampton.

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6 Declaration

- I, Pau Miquel Mir, declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research. I confirm that:
 - 1. This work was done wholly or mainly while in candidature for a degree at this University;
 - 2. Where any part of this thesis has previously been submitted for any other qualification at this University or any other institution, this has been clearly stated;
 - 3. Where I have consulted the published work of others, this is always clearly attributed;
 - 4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
 - 5. I have acknowledged all main sources of help;
 - 6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
 - 7. None of this work has been published before submission.

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24	I also want to thank my parents, for their constant support and for proofreading the paper.			

25 Abstract

This is the abstract.

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- 62 Acronyms
- 63 **CFD** Computational Fluid Dynamics

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65	Reword	1
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77	General stuff:	
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79	- Check for repetition	
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1 Introduction

{sec:introduction}

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Reword

1.1 Purpose of the investigation

Hydrogen is a very promising alternative fuel for the future, mainly due to the absence of greenhouse emissions when burning it. In this regard, it is superior to current petroleum-, and more generally, carbon-based fueling systems. However, air pollution is a negative externality associated with carbon-based fuels, as it is a cost to a third party (in this case, society as a whole) that is not accounted for in the price of the good. By definition, the negative consequences of polluting the air are not accounted for in the price of carbon vehicles, and therefore will have little influence on a consumer's choice. Thus, if hydrogen is to succeed as an alternative to carbon fuels, it is essential that its use be as - if not more - convenient than traditional fuel.

One of the main aspects that currently lags behind traditional fuel is the refueling experience. Given that refueling hydrogen involves its compression, there is a significant rise in temperature, which must be kept below certain standards (358 °K as per SAE J2601). This in turn leads to long refueling times, potentially lasting more than five minutes, which is cumbersome for users. Therefore, it is of prime importance to research and develop systems that enable the faster refueling of hydrogen cylinders. To this end, this project will build upon a model of filling a hydrogen cylinder which has already been developed by members of the department in order to analyse novel methods of improving fill times.

9 1.2 Outline of the investigation

One of the current solutions to improve fill times involves cooling the hydrogen before filling the cylinder as to keep it below he maximum temperature. However, this is quite expensive, both in energy terms and in economic terms. Consequently, the aim of this project is to continue exploring several of the options available to reduce fill times and simultaneously reduce the energy consumption of the process, thus improving both convenience for users and energy efficiency of the fueling stations. Indeed, by building upon the existing cylinder model several options shall be considered, namely: refrigeration, flow regulation, heat sink usage, active cooling, heat pipe usage, and phase change materials. From here, several options are open to further deepen or potentially broaden the investigation. An attempt to further simplify the model can be made, perhaps even reducing it so a simple algebraic relationship. Also, the model would benefit from FEA validation to aid our understanding of the heat transfer in the structure. This would go hand in hand with analyzing the temperature of the structure, and see how close this matches the gas temperature. Indeed, if the structure is at a much lower temperature than the gas, the case can be made that the current regulations are slightly erroneous, as they are meant to protect the materials of the structure, but instead regulate the gas temperature.

Update with actual options considered at end of project

2 Background

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2.1 Vehicles and objectives

We must firstly consider what real-world relevance of this project is. As explained in Section 1, the hydrogen cylinders being considered are for road vehicles. Although still an emerging technology, the basic functioning of a hydrogen fueled cylinders is relatively well established. They store compressed hydrogen in fuel tanks at high pressures, 35 and 70 MPa are the two current standards for pressure. This hydrogen is then used in the fuel cell and converted into electricity,, which can be used directly by the electric motor or stored on onboard batteries. It therefore follows that the objectives of the hydrogen storage system are several:

- High fuel capacity
- Low overall weight
- Low fill times

These have certain implications. Firstly, it is clear that several tradeoffs and compromises must be made. Increasing fuel capacity can be done by either using larger tanks, which leads to higher weight, or using higher pressures, which inevitably leads to higher temperatures. These higher temperatures are problematic due to material constraints as explained below in Section 2.1.1. Indeed, they lead to longer filling times as heat is allowed to dissipate through the tank walls. This leads to the desire to optimize filling patterns and develop other solutions to minimize temperature rise when filling to high pressures.

2.1.1 Material constraints

{sec:materialConst:

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Cite

The main reason that a problem exists, and consequently this paper (and much other work) is being 136 undertaken, is the material limitations that exist in hydrogen cylinders. Indeed, the cylinders that 137 are used for high pressure scenarios such as the one we are presented with, are constructed with a 138 composite material such as carbon fibre or glass/aramid fibre. In addition, they have a liner that is 139 made out of metal, typically aluminum, in Type III cylinders, and out of a thermoplastic material for Type IV cylinders. Of concern is the composite material, as the polymer matrix cannot withstand high temperatures, and as such the material properties of the cylinder will begin to degrade. The 142 specific temperature at which this occurs is usually around the glass transition temperature of the 143 epoxy, where the thermosetting polymer changes from a hard "glassy" state to a more compliant 144 "rubbery" state. 145

cite:

http://www.epote

2.2 Previous work

We must first establish a baseline of previous work that has been conducted in this field, and subsequently analyze the shortcomings that exist in order to direct the research.

49 2.2.1 Modeling Work

A large amount of research has been conducted using multidimensional analysis, especially using
Computational Fluid Dynamics (CFD). Indeed, there is myriad papers describing different methods
and setups, such as Dicken and Mérida's work, which uses the standard k-ε turbulence model and the
Redlich-Kwong real gas equation of state for real gas properties [1]. Some of these models use more
advanced property models, such as the model presented by Zhao et al., which employs REFPROP
(see Section 2.5.3) in order to have more precise real gas properties [2].

experimental_work}

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2.2.2 Experimental work

Several papers describe experimental work that has been conducted regarding the fast-filling of hydrogen cylinders, in particular comparing the results to simulations. Dicken and Mérida's work indicates that the temperature inside the cylinder is rather uniform [3]. This claim is not, however supported by the work of Zheng et al nor that of Woodfield et al [4], wherein large discrepancies among gas temperatures in different regions of the cylinder are found.

Citation

162 **2.2.3 Analysis**

The simulation work that has been conducted can be, in broad terms, divided into complex CFD models and more simplified 0-dimensional models. It is important analyze the difference between these two systems. The main advantage of reduced dimension models is that they are much less computationally expensive. This, in turn, means it can be incorporated as part of a larger analysis, in which it must be run multiple times, such as optimization routines or probabilistic whole station models.

Reduced dimension models assume a constant gas temperature, which, as outlined in Section 2.2.2, is supported by some experimental work, but refuted by others. For this reason, more research should be conducted to validate the uniform gas temperature approximation.

2.3 Cylinder filling models

Several ways of modelling the cylinder can be considered when analysing their filling, with varying complexity and accuracy.

2.3.1 Zonal

A zonal model, also referred to as a 0-dimensional model, refers to models that consider the gas inside the cylinder as the control volume, with homogenous properties. This allows for simpler calculations and much lower computational time.

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2.3.2 Multidimensional

c:multidimensional 179

More complex multidimensional models can be created, either 2D axisymmetric models or full 3D models. 2D models will not include the effects of gravity or buoyancy, but these can be considered

negligible, and the computational expense of full 3D simulations is rarely justifiable as the gains in accuracy are very small.

DRAFT

fill

184 2.4 Heat transfer models

In order to successfully analyse the behaviour of the system as a whole we must consider several local heat transfer methods that occur at different places inside of the cylinder.

187 2.4.1 Impinging jet

The first behaviour that we will consider is that of an impinging jet of fluid onto a surface.

FIGURE 1: Impinging jet

{fig:inpinging_jet]

189 **2.4.2** Pipe flow

A second behaviour that we will consider is that of pipe flow. This behaviour has been the focus of much research, as it is arguably the most common mode of heat transfer that occurs in fluid systems.

192 **2.4.3 Turbulent Jets**

Lastly, we must also consider turbulent jets, as there will be such a situation at the exit of the nozzle.

[5]

195 2.5 Methodology

Several techniques and methods will be used throughout the analysis. These are described in this section.

2.5.1 Optimization

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2.5.2 Non-dimensioning

One of the fundamental principals used throughout this paper, and indeed, throughout engineering, is that of non-dimensioning. By operating using non-dimensional parameters such as the Reynolds number $Re = \frac{\rho uL}{v}$ or the Prandtl number $Pr = \frac{c_p \mu}{k}$ solving problems involving differential equations becomes simplified. Also, the analysis becomes much more general, and can be scaled.

Fill Section once work on optimization starts.

2.5.3 Numerical methods

{sec:num

Integrating ODEs An central part of solving unsteady heat transfer problems involves integrating ODEs, as will be seen in Section 3. One of the simplest methods available, both conceptually and in terms of ease of implementing in code, is forward Euler time integration. It can be informally described as follows: given a function that can be defined by:

$$y'(t) = f(t, y(t)), \quad y(t_0) = y_0$$
 (2.1)

we can compute the approximate shape of the function given the initial point and finding the slope of the curve for small intervals. Indeed, from the initial point, we can find the tangent of the curve at that point, and take a small step along that tangent until arriving at the next point, where the procedure can be repeated. Denoting the step size h, we can express forward Euler time integration as:

$$y_{n+1} = y_n + hf(t_n, y_n)$$
 (2.2)

ec:property_models}

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Property models As the gases that are being treated in this report are at very high pressures, ideal gas approximations are inaccurate, and thus real gas properties must be employed. To this end, property models must be used, which can determine any gas property from two other independent properties.

The property model that will be employed throughout this analysis is REFPROP, a tool developed by the National Institute of Standards and Technology [6]. It uses values of critical and triple points together with equations for the thermodynamic and transport properties to calculate the state points of fluids.

3 Formulation

{sec:formulation}

3.1 Heat transfer across cylinder

The main mode of heat transfer that occurs in the cylinder and that will be used throughout this report is heat conduction through the wall of the cylinder. This is modeled using:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{3.1}$$

where a is the thermal diffusivity, which is defined by :

$$\alpha = \frac{k}{\rho c} \tag{3.2}$$

where k is the material's thermal conductivity, ρ is the density of the material, and c is the specific heat capacity of the material. This differential equation will be solved using forward Euler time integration as described in Section 2.5.3.

230 3.2 Gas flow into cylinder

The nozzle and corresponding gas flow will be modeled using isentropic relations, and then a discharge coefficient relationship will be used to find the approximate real values. Indeed, the isentropic Reynold's number is first calculated as follows:

$$Re_{ideal} = \frac{\rho_{exit} d_{inlet} u_{exit}}{\mu_{exit}}$$
 (3.3)

where ρ_{exit} , d_{inlet} , u_{exit} , and μ_{exit} are determined using real gas models as described in Section 2.5.3. In order to find the real mass flow an empirical discharge coefficient is employed:

$$CD = \frac{\dot{m}_{\text{in}}}{\dot{m}_{\text{ideal}}} = A + B \operatorname{Re}_{\text{ideal}}$$
 (3.4)

A discharge coefficient must be used to account for the formation of a boundary layer inside the inlet tube. The empirical model that was used was obtained from , and uses the following values:

Citation

$$A = 0.938, \quad B = -2.71 \tag{3.5}$$

238 3.3 Hysteresis

The initial work this project builds upon uses the flow at the nozzle to determine the heat transfer at the wall of the cylinder. This assumes that the hydrogen flows instantaneously from the nozzle to the wall, when in reality there of course is a time delay, measured to be approximately 2 seconds for short cylinders. This assumption is acceptable for stable inflows, but for more complex filling patterns and also for improved accuracy it becomes necessary to incorporate hysteresis. Indeed, the heat transfer coefficient at the wall in fact can be said to depend on the nozzle flow seconds prior, or more generally, on the history of the nozzle flow. This can be modeled as follows:

define short and long cylinders at some point

$$\frac{d}{dt}\left(Nu\right) = \frac{Nu_{ss} - Nu}{\tau} \tag{3.6}$$

3.4 Throttling

Throttling is introduced in order to more accurately represent real life conditions. When a maximum temperature is reached, the inflow of hydrogen must be stopped in order to protect the materials of the tank, as detailed in Section 2.1.1. A simple method of throttling, with the flow stopping at the designated maximum temperature, 85 °C, and the flow restarting at a chosen temperature, in this case 75 °C.

Play around with this perhaps?

3.5 Optimization

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General stuff:

Check tense and person

- Check for repetition

- Check for excessive verboseness

54 Appendices

A Filling Code

```
{app:filling_code}
```

```
%Author: Vishagen Ramasamy
    %Date: -
   % Copyright University of Southampton 2017.
   \% No warranty either expressed or implied is given to the results produced
    % by this software. Neither the University, students or its employees
    %accept any responsibility for use of or reliance on results produced by
    %this software.
    %% script that computes the filling of the tank(s)
10
    %% Importing the pressure and temperature profile at the entrance of the delivery pipe from Dicken a
11
12
   % and calulating the stagnation enthalpy and entropy
13
14
    for j = 1:maxt
        time(j+1) = (j)*dt;
                                                                                     % Time as filling proce
15
        for i = 1:tank_number
16
            \% Select inlet pressure / temperature profile based on user input
17
18
            if blnUseStandardData == 1
                % Constant inlet pressure profile
19
                P_inlet(i,j+1) = ConstPressKPa;
20
21
            else
                % Read inlet pressure profile from specified file
22
23
                P_inlet(i, j+1) = interp1(InletPressureData(:,1),InletPressureData(:,3), dt*j);
            end
24
25
            if blnUseStandardData == 1
26
                %Temp_inlet(i,j+1) = interp1(time_in, temp_in, dt*j);
27
                                                                           % Temperature profile after the
                Temp_inlet(i, j+1) = ConstTempK;
28
            else
29
                % Read inlet temperature profile from specified file
30
31
                Temp_inlet(i, j+1) = interp1(InletTempData(:,1), InletTempData(:,3), dt*j);
            end
32
33
            h_inlet(i,j+1) = refpropm('H','T',Temp_inlet(i,j+1),'P',P_inlet(i,j+1),Fluid{i}, refpropdir)
34
            entropy_inlet(i,j+1) = refpropm('S','T',Temp_inlet(i,j+1),'P',P_inlet(i,j+1),Fluid{i}, refpr
35
36
            %% Determine which pressure to use at the exit of the delivery pipe which is dependent upon
37
38
39
            if l_d(i) > 3 \&\& blnOneZone{i} == 0
                                                           % The length-to-diameter ratio of the tank(s) d
                Pressure_exit = P_gas_zone1{i}(j);
                                                       % The pressure at the exit of the delivery pipe is
40
41
42
                Pressure_exit = P_gas(i,j);
                                                       % The pressure at the exit of the delivery pipe is
43
44
            if (P_inlet(i,j+1) > Pressure_exit) % condition for filling of tank(s)
45
                sound_exit(i,j+1) = refpropm('A','P',Pressure_exit,'S',entropy_inlet(i,j+1),Fluid{i}, re
46
                h_static_exit(i,j+1) = refpropm('H','P',Pressure_exit,'S',entropy_inlet(i,j+1),Fluid{i},
47
                visc_exit(i,j+1) = refpropm('V','P',Pressure_exit,'S',entropy_inlet(i,j+1),Fluid{i}, ref
48
                \label{eq:mach_exit} \verb|mach_exit(i,j+1)| = \verb|sqrt(2*(h_inlet(i,j+1)-h_static_exit(i,j+1))/sound_exit(i,j+1)^2); \\
49
50
                %% If Mach number is greater than one, the exit pressure is greater than the pressure wi
51
                \% is incremetally increased and iterated in a while loop until mach number is equal to
52
53
54
                Inlet_entropy = entropy_inlet(i,j+1);
55
                Inlet_stagnation_enthalpy = h_inlet(i,j+1);
56
                P_guess = Pressure_exit;
57
58
                if mach_exit(i,j+1) > 1
                    Pressure_exit = find_exit_pressure(Inlet_stagnation_enthalpy,Inlet_entropy,Fluid{i},
59
                    sound_exit(i,j+1) = refpropm('A','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refpr
60
                    h_static_exit(i,j+1) = refpropm('H','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, re
61
```

```
visc_exit(i,j+1) = refpropm('V','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refpropdir);
   62
                                                                                                    mach_{exit(i,j+1)} = sqrt(2*(h_{inlet(i,j+1)}-h_{static_{exit(i,j+1)}})/sound_{exit(i,j+1)}^2);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       % C
   63
                                                                                 end
   64
   65
   66
                                                                                 %% Calculation of the mass flow rate into the tank(s)
   67
                                                                                 rho_exit(i,j+1) = refpropm('D','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refpropdir);
   68
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   % Calcui
                                                                                 vel_exit(i,j+1) = mach_exit(i,j+1)*sound_exit(i,j+1);
   69
                                                                                 Re_exit_isentropic(i,j+1) = rho_exit(i,j+1) *d_inlet*vel_exit(i,j+1)/visc_exit(i,j+1);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   % Calcu
   70
   71
                                                                                 cd(i,j+1) = I + J/(Re_exit_isentropic(i,j+1)^(0.25));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   % Calcui
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   % Calcu
   72
                                                                                 mfr(i,j+1) = cd(i,j+1)*rho_exit(i,j+1)*vel_exit(i,j+1)*A_inlet;
                                                                                 Re_entrance_actual(i,j+1)= 4*mfr(i,j+1)/(pi*d_inlet*visc_exit(i,j+1));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   % Calcu
   73
   74
                                                                                 dM_inlet = mfr(i,j+1)*dt;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   % Amoun
                                                            end
   75
   76
                                                            %% Heat transfer calculations & caluclations of the thermodynamic properties of the gas in the tank(s)
   77
                                                            if l_d(i) \le 3 \mid blnOneZone\{i\} == 1
   78
   79
                                                                                                                                                  a_1*Re_entrance_actual(i,j+1)^(b_1);
                                                                                 Nus(i,j+1) =
                                                                                                                                                                                                                                                                                                                                                                                                                                       % Nusselt number and Reynolds
   80
                                                                                 k_gas(i,j+1) = refpropm('L','T',Temp_gas(i,j),'P',P_gas(i,j),Fluid{i}, refpropdir); % Thermal condu
   81
   82
                                                                                 heat\_coef\_forced(i,j+1) = Nus(i,j+1)*k\_gas(i,j+1)/d\_tank(i);
                                                                                                                                                                                                                                                                                                                                                                                                                                       % Calculation of the heat tran
   83
                                                                                 if Inner_wall_boundary(i) == 1
   84
                                                                                                    Qsurf(i,j+1) = -dt*surf_area(i)*heat_coef_forced(i,j+1)*(Temp_gas(i,j)-Inner_temp_wall_isothermatical entry) = -dt*surf_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_isothermatical entry) = -dt*surf_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp_wall_area(i,j)*(Temp_gas(i,j)-Inner_temp
   85
   86
                                                                                 else
                                                                                                   87
                                                                                                   Temp_wall\{i\}(1,j+1) = Temp_wall\{i\}(1,j) + CFL_liner(i)*(Temp_wall\{i\}(2,j) - Temp_wall\{i\}(1,j) - Q_i 
   88
   89
   90
                                                                                                    if Outer wall boundary(i)==1
                                                                                                                       Temp_wall{i}(number_of_gridpoints(i),j+1) = Outer_temp_wall_isothermal(i);
   91
   92
                                                                                                                         Temp_wall{i}(number_of_gridpoints(i),j+1) = Temp_wall{i}(number_of_gridpoints(i),j)+2*CFL_1
   93
   94
                                                                                                    \mbox{\ensuremath{\mbox{\%}}} Computation of the temperature of the struture of the tank(s)
   95
   96
                                                                                                    for k=2:number_of_gridpoints(i)-1
                                                                                                                        if (k>=2)&&(k<=int_pt_liner_laminate(i)-1)</pre>
   97
                                                                                                                                           \label{temp_wall_i} Temp\_wall\{i\}(k,j+1) = Temp\_wall\{i\}(k,j) + CFL\_liner(i) * (Temp\_wall\{i\}(k+1,j) - 2*Temp\_wall\{i\}(k+1,j) + (Temp\_wall(k+1,j) - 2*Temp\_wall(k+1,j) + (Temp\_wall(k+1,j) - 2*Temp\_wall(k+1,j) + (Temp\_wall(k+1,j) - 2*Temp\_wall(k+1,j) + (Temp\_wall(k+1,j) - 2*Temp\_wall(k+1,j) + (Temp\_wall(k+1,j) + (Temp\_w
   98
                                                                                                                         elseif (k>=int_pt_liner_laminate(i)+1)&&(k<=number_of_gridpoints(i)-1)
   99
                                                                                                                                           Temp_wall\{i\}(k,j+1) = Temp_wall\{i\}(k,j) + CFL_laminate(i) * (Temp_wall\{i\}(k+1,j) - 2 * Temp_wall\{i\}(k,j+1) = Temp_wall\{i\}(k,j) + CFL_laminate(i) * (Temp_wall\{i\}(k+1,j) - 2 * Temp_wall\{i\}(k,j+1) = Temp_wall\{i\}(k,j) + CFL_laminate(i) * (Temp_wall\{i\}(k+1,j) - 2 * Temp_wall\{i\}(k+1,j) - 2 * Temp_wall(i) - 2 * Temp_wall
100
101
                                                                                                                         else
                                                                                                                                           Temp_wall{i}(k,j+1) = (cond_laminate(i)*(Temp_wall{i}(k+1,j)+CFL_laminate(i)*(Temp_wall-
102
103
                                                                                                                         end
                                                                                                    end
104
105
                                                                                 end
                                                                                 m_{gas}(i,j+1) = m_{gas}(i,j) + dM_{inlet};
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Mass of gas in
106
                                                                                 \label{eq:continuous} Ugas(\texttt{i},\texttt{j}+\texttt{1}) = Ugas(\texttt{i},\texttt{j}) + Qsurf(\texttt{i},\texttt{j}+\texttt{1}) + h_inlet(\texttt{i},\texttt{j}+\texttt{1}) * dM_inlet;
 107
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Internal energy
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Specific energy
                                                                                 u_{gas}(i,j+1) = U_{gas}(i,j+1)/m_{gas}(i,j+1);
108
109
                                                                                 rho_gas(i,j+1)=m_gas(i,j+1)/vol_tank(i);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Density of the
                                                                                P_gas(i,j+1) = refpropm('P','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir);
Temp_gas(i,j+1) = refpropm('T','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 %
110
111
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                %
112
                                                            else
113
                                                                               Re_compression(i,j+1) = Re_entrance_actual(i,j+1)*(d_inlet/d_tank(i));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Reynolds number
114
115
                                                                                 Nus_zone1{i}(j+1) = a_1*Re_entrance_actual(i,j+1)^(b_1);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Nusselt number
116
                                                                                 Nus_{zone2\{i\}(j+1)} = c_1*Re_{compression(i,j+1)^(d_1)};
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        % Nusselt number
117
118
                                                                                 k_gas_zone1{i}(j+1) = refpropm('L','T',Temp_gas_zone1{i}(j),'P',P_gas_zone1{i}(j),Fluid{i}, refprop
119
                                                                                 k\_gas\_zone2\{i\}(j+1) = refpropm('L','T',Temp\_gas\_zone2\{i\}(j),'P',P\_gas\_zone2\{i\}(j),Fluid\{i\}, refpropm('L','T',Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Temp\_gas\_zone2\{i\}(j),Te
120
121
                                                                                \label{eq:heat_coef_forced_zone1} $$ heat_coef_forced_zone1_{i}(j+1) = Nus_zone1_{i}(j+1)*k_gas_zone1_{i}(j+1)/d_tank(i); $$ heat_coef_forced_zone1_{i}(j+1)/d_tank(i); $$ hea
122
                                                                                \label{eq:heat_coef_forced_zone2} \begin{aligned} \text{heat\_coef\_forced\_zone2}\{i\}(j+1) &= \text{Nus\_zone2}\{i\}(j+1) * \text{k\_gas\_zone2}\{i\}(j+1)/1 \text{\_zone2\_total}(i); \end{aligned}
123
124
125
                                                                                 \% Step 1. Equating the respecting values of zone 1 and zone 2 to
                                                                                 % matrices with 'similar' names
126
127
```

193

```
Vgas(1)=volume_zone1(i);
128
                                                                               Vgas(2)=volume_zone2(i);
129
                                                                               Mgas(1)=m_gas_zone1{i}(j);
130
131
                                                                               Mgas(2)=m_gas_zone2{i}(j);
132
                                                                               Ugas_twozone(1)=u_gas_zone1{i}(j)*Mgas(1);
                                                                               Ugas_twozone(2)=u_gas_zone2{i}(j)*Mgas(2);
133
134
                                                                               Asurf(1)=surface_area_zone1(i);
                                                                               Asurf(2)=surface_area_zone2(i);
135
                                                                               Tgas(1)=Temp_gas_zone1{i}(j);
136
                                                                               Tgas(2)=Temp_gas_zone2{i}(j);
137
                                                                               if Inner_wall_boundary(i) == 1
138
                                                                                                   Twall(1,1)=Inner_temp_wall_isothermal(i);
139
140
                                                                                                   Twall(2,1)=Inner_temp_wall_isothermal(i);
                                                                               else
141
142
                                                                                                   Twall(1,1)=Temp_wall_zone1{i}(1,j);
                                                                                                   Twall(2,1)=Temp_wall_zone2{i}(1,j);
143
                                                                               end
144
                                                                               % Step 2: apply the change of internal energy due to heat transfer and mass
145
                                                                               % input through nozzle:
146
147
148
                                                                               dM_inlet = mfr(i,j+1)*dt;
149
                                                                               Qsurf(1)=- dt*Asurf(1)*(heat_coef_forced_zone1{i}(j+1))*(Tgas(1)-Twall(1,1));
150
                                                                               Qsurf(2)=- dt*Asurf(2)*(heat_coef_forced_zone2{i}(j+1))*(Tgas(2)-Twall(2,1));
151
152
                                                                               Ugas_twozone(1)=Ugas_twozone(1)+Qsurf(1)+h_inlet(i,j+1)*dM_inlet;
153
                                                                               Ugas_twozone(2)=Ugas_twozone(2)+Qsurf(2);
154
155
156
                                                                               Mgas(1)=Mgas(1)+dM_inlet;
                                                                               Mgas(2)=Mgas(2);
157
                                                                               for m=1:2
159
                                                                                                  hgas(m)=refpropm('H','D',Mgas(m)/Vgas(m),'U',Ugas_twozone(m)/Mgas(m),Fluid{i}, refpr
160
                                                                               end
161
162
163
                                                                               \% Step 2: Find the amount of mass that needs to be transferred from zone 1
164
                                                                               % to zone 2 to equalise their pressure
165
                                                                               % (we assume forward Euler integration, therefore we use the specific
166
167
                                                                               \% enthalpies h from the start of the timestep. We provide enthalpies for
                                                                               % both zones in case the flow gets reversed).
168
169
                                                                               dM_guess = dM_inlet*(volume_zone2(i)/(volume_zone1(i)+volume_zone2(i))) ;
170
171
                                                                               dM_12 = find_dM_12(hgas,Vgas,Mgas,Ugas_twozone,Fluid{i},dM_guess, refpropdir);
172
173
                                                                               \% Step 3: Apply this change to the mass and update all properties:
174
175
                                                                               m_{gas\_zone1{i}(j+1)=Mgas(1)-dM_12;
                                                                               m_{gas}=20ne2\{i\}(j+1)=Mgas(2)+dM_{12};
176
177
                                                                               u_{gas}=0 u_{g
178
                                                                               u_{gas_{zone}(1)}(j+1)=(U_{gas_{two_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(1)+min(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2))/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{gas_{zone}(2)+max(0,dM_{12})*hgas(2)/m_{g
179
180
                                                                               rho_gas_zone1{i}(j+1)=m_gas_zone1{i}(j+1)/volume_zone1(i);
181
                                                                               182
183
                                                                               \label{temp_gas_zone1} Temp\_gas\_zone1\{i\}(j+1) = refpropm('T', 'D', rho\_gas\_zone1\{i\}(j+1), 'U', u\_gas\_zone1\{i\}(j+1), Farabox{ and } farabox{ also } farabox{ 
184
                                                                               Temp\_gas\_zone2\{i\}(j+1) = refpropm('T','D', rho\_gas\_zone2\{i\}(j+1),'U', u\_gas\_zone2\{i\}(j+1), Full for example in the context of the context o
185
186
                                                                               P_{gas\_zone1\{i\}(j+1)=refpropm('P','D',rho\_gas\_zone1\{i\}(j+1),'U', u\_gas\_zone1\{i\}(j+1),Flui}
187
                                                                               P_gas_zone2{i}(j+1)=refpropm('P','D',rho_gas_zone2{i}(j+1),'U', u_gas_zone2{i}(j+1),Flui
188
189
                                                                               m_{gas}(i,j+1) = m_{gas}zone1\{i\}(j+1) + m_{gas}zone2\{i\}(j+1);
190
191
                                                                               rho_gas(i,j+1) = m_gas(i,j+1)/vol_tank(i);
                                                                                u_{gas}(i,j+1) = (u_{gas\_zone1}\{i\}(j+1)*m_{gas\_zone1}\{i\}(j+1) + u_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)*m_{gas\_zone2}\{i\}(j+1)
192
```

 $Ugas(i,j+1) = u_gas(i,j+1)*m_gas(i,j+1);$

```
Temp_gas(i,j+1) = refpropm('T','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir);
 194
                                                                                                                                                                            P_gas(i,j+1) = refpropm('P','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir);
 195
 196
 197
                                                                                                                                                                            if Inner_wall_boundary(i) == 1
                                                                                                                                                                                                                      198
                                                                                                                                                                                                                      Qsurf_zone2{i}(j+1) = -dt*surface_area_zone2(i)* heat_coef_forced_zone2{i}(j+1)*(Temp_gas_zone2
 199
 200
                                                                                                                                                                            else
                                                                                                                                                                                                                      Qsurf_zone1{i}(j+1) = -dt*surface_area_zone1(i)* heat_coef_forced_zone1{i}(j+1)*(Temp_gas_zone1
 201
                                                                                                                                                                                                                    Qsurf\_zone2\{i\}(j+1) = -dt * surface\_area\_zone2(i) * heat\_coef\_forced\_zone2\{i\}(j+1) * (Temp\_gas\_zone2(i) * forced\_zone2(i) * forced\_zone2
 202
 203
                                                                                                                                                                                                                    Temp\_wall\_zone1\{i\}(1,j+1) = Temp\_wall\_zone1\{i\}(1,j) + CFL\_liner(i)*(Temp\_wall\_zone1\{i\}(2,j) - Temp\_wall\_zone1\{i\}(2,j) + CFL\_liner(i)*(Temp\_wall\_zone1\{i\}(2,j) - Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_wall\_zone1*(i)*(Temp\_w
 204
                                                                                                                                                                                                                      Temp\_wall\_zone2\{i\}(1,j+1) = Temp\_wall\_zone2\{i\}(1,j) + CFL\_liner(i)*(Temp\_wall\_zone2\{i\}(2,j) - Temp\_wall\_zone2\{i\}(2,j) + 
 205
 206
                                                                                                                                                                                                                      if Outer_wall_boundary(i)==1
 207
                                                                                                                                                                                                                                                               Temp_wall_zone1{i}(number_of_gridpoints(i),j+1) = Outer_temp_wall_isothermal(i);
 208
                                                                                                                                                                                                                                                                 Temp_wall_zone2{i}(number_of_gridpoints(i),j+1) = Outer_temp_wall_isothermal(i);
 209
                                                                                                                                                                                                                      else
 210
                                                                                                                                                                                                                                                               \label{temp_wall_zone1} Temp_wall_zone1\{i\} (number_of_gridpoints(i),j+1) = Temp_wall_zone1\{i\} (number_of_g
211
                                                                                                                                                                                                                                                                 Temp_wall_zone2{i}(number_of_gridpoints(i),j+1) = Temp_wall_zone2{i}(number_of_gridpoints(i)
212
                                                                                                                                                                                                                      end
213
 214
                                                                                                                                                                                                                      % Computation of the temperature of the struture of the tank(s)
                                                                                                                                                                                                                      for k=2:number_of_gridpoints(i)-1
215
                                                                                                                                                                                                                                                                 if (k>=2)&&(k<=int_pt_liner_laminate(i)-1)</pre>
                                                                                                                                                                                                                                                                                                          Temp_wall_zone1{i}(k,j+1) = Temp_wall_zone1{i}(k,j)+CFL_liner(i)*(Temp_wall_zone1{i}(k+
217
                                                                                                                                                                                                                                                                                                         \label{temp_wall_zone2} Temp_wall_zone2\{i\}(k,j) + CFL_liner(i)*(Temp_wall_zone2\{i\}(k+1)) + CFL_liner(i)*(Temp_wall_zo
 218
                                                                                                                                                                                                                                                                 elseif (k>=int_pt_liner_laminate(i)+1)&&(k<=number_of_gridpoints(i)-1)
219
                                                                                                                                                                                                                                                                                                         \label{temp_wall_zone1} Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_wall_zone1*(Temp_w
 220
                                                                                                                                                                                                                                                                                                         \label{temp_wall_zone2} Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k,j)+CFL_laminate(i)*(Temp_wall_z
221
 222
                                                                                                                                                                                                                                                                                                         Temp\_wall\_zone1\{i\}(k,j+1) = (cond\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1\{i\}(k+1,j)+CFL\_laminate(i)*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_wall\_zone1*(Temp\_w
223
                                                                                                                                                                                                                                                                                                         Temp_wall_zone2\{i\}(k,j+1) = (cond_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2\{i\}(
 224
                                                                                                                                                                                                                                                                 end
225
 226
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 227
 228
 229
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 230
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                                              end
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6 References

- C. J. B. Dicken and W. Mérida, "Modeling the Transient Temperature Distribution within a Hydrogen Cylinder during Refueling", *Numerical heat transfer, part a: Applications*, vol. 53, no. 7, pp. 685–708, Nov. 2007, ISSN: 1040-7782. DOI: 10.1080/10407780701634383. [Online]. Available: http://www.tandfonline.com/doi/abs/10.1080/10407780701634383
- Y. Zhao, G. Liu, Y. Liu, J. Zheng, Y. Chen, L. Zhao, J. Guo, and Y. He, "Numerical study on fast filling of 70 MPa type III cylinder for hydrogen vehicle", *International journal of hydrogen energy*, vol. 37, no. 22, pp. 17517-17522, 2012, ISSN: 0360-3199. DOI: https://doi.org/10.1016/j.ijhydene.2012.03.046. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0360319912006477.
- C. Dicken and W. Mérida, "Measured effects of filling time and initial mass on the temperature distribution within a hydrogen cylinder during refuelling", *Journal of power sources*,
 vol. 165, no. 1, pp. 324–336, Feb. 2007, ISSN: 03787753. DOI: 10.1016/j.jpowsour.
 269 2006.11.077. [Online]. Available: https://www.engineeringvillage.com/share/
 document.url?mid=cpx%7B%5C_%7D30c221110dfeaa71cM69f92061377553%7B%5C&
 271 %7Ddatabase=cpx.
- P. L. WOODFIELD, M. MONDE, and T. TAKANO, "Heat Transfer Characteristics for Practical Hydrogen Pressure Vessels Being Filled at High Pressure", *Journal of thermal science and technology*, vol. 3, no. 2, pp. 241–253, 2008. DOI: 10.1299/jtst.3.241.
- 275 [5] S. B. Pope, *Turbulent Flows*. Cambridge University Press, 2000, ISBN: 9780521598866. [Online]. Available: https://books.google.co.uk/books?id=HZsTw9SMx-OC.
- E. W. Lemmon, M. L. Huber, and M. O. McLinden, NIST Standard Reference Database 23:

 Reference Fluid Thermodynamic and Transport Properties-REFPROP, Version 9.1, National

 Institute of Standards and Technology, 2013. DOI: http://dx.doi.org/10.18434/T4JS3C.

 [Online]. Available: https://www.nist.gov/srd/refprop.