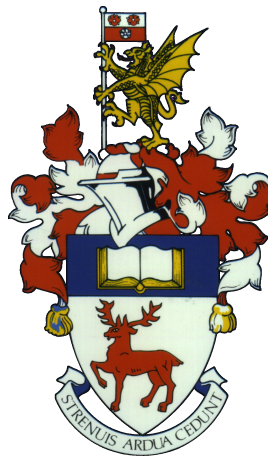


1 **Novel Approaches to Fast Filling of Hydrogen Cylinders**

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Supervisor: Dr. Edward Richardson
Word count: 3558

3 May 2018



4 This report is submitted in partial fulfillment of the requirements for the MEng Mechanical
5 Engineering, Faculty of Engineering and the Environment, University of Southampton.

6

Declaration

7

I, Pau Miquel Mir, declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research. I confirm that:

8

9

1. This work was done wholly or mainly while in candidature for a degree at this University;

10

2. Where any part of this thesis has previously been submitted for any other qualification at this University or any other institution, this has been clearly stated;

11

12

3. Where I have consulted the published work of others, this is always clearly attributed;

13

4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;

14

15

5. I have acknowledged all main sources of help;

16

6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;

17

18

7. None of this work has been published before submission.

19

Acknowledgements

20

I want to thank my advisor, Dr. Edward Richardson, for his time, dedication, patience, and support throughout this project.

21

22

Furthermore, I would like to thank Vishagen Ramasamy for his initial work upon which this paper builds.

23

24

I also want to thank my parents, for their constant support and for proofreading the paper.

25

Abstract

26

This is the abstract.

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⁷⁰ **Acronyms**

⁷¹ **CFD** Computational Fluid Dynamics

⁷² **ODE** Ordinary Differential Equation

73 **Todo list**

| | | |
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| 74 | Reword | 1 |
| 75 | Update with actual options considered at end of project | 1 |
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| 81 | Fill Section once work on optimization starts. | 6 |
| 82 | consistent time derivatives | 7 |
| 83 | Citation | 8 |
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| 87 | explain why it's minimum | 9 |
| 88 | need to explain why? | 9 |
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| 92 | - Check for repetition | |
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1 Introduction

1.1 Purpose of the investigation

Hydrogen is a very promising alternative fuel for the future, mainly due to the absence of greenhouse emissions when burning it. In this regard, it is superior to current petroleum- and, more generally, carbon-based fueling systems currently used by most road vehicles. However, air pollution is a negative externality associated with carbon-based fuels, as it is a cost to a third party (in this case, society as a whole) that is not accounted for in the price of the good. By definition, the negative consequences of polluting the air are not accounted for in the price of carbon vehicles, and therefore will have little influence on a consumer's choice. Thus, if hydrogen is to succeed as an alternative to carbon fuels, it is essential that its use be as, if not more, convenient than traditional fuel.

One of the main aspects where hydrogen currently lags behind traditional fuel is the refueling experience. Given that refueling hydrogen involves its compression, there is a significant rise in temperature, which must be kept below certain standards (358 °K as per SAE J2601). This in turn leads to long refueling times, potentially lasting more than five minutes, which is cumbersome for users. Therefore, it is of prime importance to research and develop systems that enable faster refueling of hydrogen cylinders. To this end, this project builds upon a model of filling a hydrogen cylinder which has already been developed by members of the department to analyse novel methods of improving fill times.

1.2 Outline of the investigation

One of the current solutions to improve fill times involves cooling the hydrogen before filling the cylinder as to keep it below the maximum temperature. However, this is quite expensive, both in energy terms and in economic terms. Consequently, the aim of this project is to continue exploring several of the options available to reduce fill times and simultaneously reduce the energy consumption of the process, thus improving both convenience for users and energy efficiency of the fueling stations. Indeed, by building upon the existing cylinder model several options shall be considered, namely: refrigeration, flow regulation, heat sink usage, active cooling, heat pipe usage, and phase change materials. From here, several options are open to further deepen or potentially broaden the investigation. An attempt to further simplify the model can be made, perhaps even reducing it to a simple algebraic relationship. Also, the model would benefit from FEA validation to aid our understanding of the heat transfer in the structure. This would go hand in hand with analysing the temperature of the structure, and seeing how close this matches the gas temperature. Indeed, if the structure is at a much lower temperature than the gas, the case can be made that the current regulations are slightly erroneous, as they are meant to protect the materials of the structure, but instead regulate the gas temperature.

2 Background

2.1 Objective

We must firstly consider what real-world relevance of this project is. As explained in Section 1, the hydrogen cylinders being considered are for road vehicles. Although still an emerging technology, the basic functioning of a hydrogen fueled vehicle is relatively well established. They store compressed hydrogen in fuel tanks at high pressures, 35 and 70 MPa being the two current standards. This hydrogen is then used in the fuel cell and converted into electricity, which can be used directly by the electric motor or stored on onboard batteries. It therefore follows that the objectives of the hydrogen storage system are several:

- High fuel capacity
- Low overall weight
- Low fill times

These goals have certain implications. Firstly, it is clear that several tradeoffs and compromises must be made. Increasing fuel capacity can be done by either using larger tanks, which leads to higher weight, or using higher pressures, which inevitably leads to higher temperatures. These higher temperatures are problematic due to material constraints as explained below in Section 2.1.1. Indeed, they lead to longer filling times as heat has to be allowed to dissipate through the tank walls. Thus, other solutions must be developed to minimize the temperature rise when filling to high pressures.

2.1.1 Material constraints

The main reason that a problem exists, and consequently this paper (and much other work) is being undertaken, is the material limitations that exist in hydrogen cylinders. Indeed, the cylinders that are used for high pressure scenarios such as the one we are presented with, are constructed with a composite material such as carbon fibre or glass/aramid fibre. In addition, they have a liner that is made out of metal, typically aluminum, in Type III cylinders, and out of a thermoplastic material for Type IV cylinders. Of concern is the composite material, as the polymer matrix cannot withstand high temperatures, and as such the material properties of the cylinder will begin to degrade. The specific temperature at which this occurs is usually around the glass transition temperature of the epoxy, where the thermosetting polymer changes from a hard "glassy" state to a more compliant "rubbery" state.

2.2 Previous work

We must first establish a baseline of previous work that has been conducted in this field, and subsequently analyze the shortcomings that exist in order to direct the research.

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161 2.2.1 Experimental work

162 Several papers describe experimental work that has been conducted regarding the fast-filling of
 163 hydrogen cylinders, in particular comparing the results to simulations. Dicken and Mérida’s work
 164 indicates that the temperature inside the cylinder is rather uniform [1]. This claim is not, however
 165 supported by the work of Zheng et al [2] nor that of Woodfield et al [3], wherein large discrepancies
 166 among gas temperatures in different regions of the cylinder are found.

167 2.2.2 Modeling Work

168 **Computational Fluid Dynamics (CFD)** A large amount of research has been conducted using
 169 multidimensional analysis, especially using CFD. Indeed, there are many papers describing different
 170 methods and setups, such as Dicken and Mérida’s work, which uses the standard k- ϵ turbulence model
 171 and the Redlich-Kwong real gas equation of state for real gas properties [4]. Other investigations use
 172 more advanced property models, such as the one presented by Zhao et al., which employs REFPROP
 173 (see Section 2.5.3) in order to have more precise real gas properties [5].

174 **Reduced order simulations** Less work has been conducted regarding reduced order simulations,
 175 which employ assumptions such as axisymmetry or uniform gas temperature.

176 The work by Woodfield et al [3] presents a reduced order model employing a mass flow and inter-
 177 nal energy balance for the compressed gas and one-dimensional unsteady heat conduction equation
 178 for the wall solved simultaneously. It uses the Lee-Kesler method [6] to find the compressibility and
 179 thus the density and from that the mass flow. The heat transfer coefficient between the gas and the
 180 wall is assumed to be constant: 500 W/(m²K) during filling and 250 W/(m²K) after full.

181 Similarly, the work by Monde et al [7] uses the same assumption regarding heat transfer coeffi-
 182 cient values. They achieve a reasonable fit with experimental data, shown in Fig. 1, even though the
 183 measured heat transfer coefficients were significantly lower, shown in Table 1.

FIGURE 1: Comparison between estimated and measured temperatures by Monde et al [7]

| | | | |
|---|---|---|---|
| ? | ? | ? | ? |
| ? | ? | ? | ? |
| ? | ? | ? | ? |
| ? | ? | ? | ? |
| ? | ? | ? | ? |
| ? | ? | ? | ? |

TABLE 1: Heat transfer coefficients at various conditions by Monde et al [7]

2.2.3 Analysis

The simulation work that has been conducted can be, in broad terms, divided into complex CFD models and more simplified reduced dimensional models. It is important analyze the difference between these two systems.

CFD models have several advantages, as they take into consideration many things which reduced order models must simplify or assume. Firstly, one can represent the geometry of the cylinder more accurately using CFD. Secondly, time effects can be directly simulated without the need of timescales to be derived. However, they are extremely computationally expensive, with run times lasting from hours to weeks. It thus follows that the main advantage of reduced dimension models is that they are much less computationally expensive. This, in turn, means they can be incorporated as a part of larger analyses, in which they must be run multiple times, such as optimization routines or probabilistic whole station models.

Reduced dimension models up to date make assumptions that are difficult to support with evidence, and therefore lead to results not being very accurate. This leads to adjustments being made to the model to match the experimental data from the same experiment, which produces a better fit for the model, but leads to a less predictive model. For this reason, it is important to improve reduced order models by devising ways to calculate quantities previously assumed. This will improve the models' predictive ability, which will increase confidence in results from simulations regarding events which haven't been tried experimentally.

For instance, they assume a constant gas temperature, which, as outlined in Section 2.2.1, is supported by some experimental work, but refuted by others. For this reason, more research should be conducted to validate the uniform gas temperature approximation. Another assumption typically made is a constant heat transfer coefficient derived from time-averaged experimental results, as explained in Section 2.2.2. This report will propose a new method of deriving the heat transfer coefficient numerically to achieve better results.

2.3 Cylinder filling models

Several ways of modelling the cylinder can be considered when analysing their filling, with varying complexity and accuracy.

2.3.1 Zonal

A zonal model, also referred to as a 0-dimensional or reduced order model, refers to models that consider the gas inside the cylinder as the control volume, with homogenous properties. This allows for simpler calculations and much lower computational time.

Fill

2.3.2 Multidimensional

{sec:multidimension

More complex multidimensional models can be created, either 2D axisymmetric models or full 3D models. 2D models will not include the effects of gravity or buoyancy, but these can be considered

negligible, and the computational expense of full 3D simulations is rarely justifiable as the gains in accuracy are very small.

2.4 Heat transfer models

In order to successfully analyse the behaviour of the system as a whole we must consider several local heat transfer methods that occur at different places inside of the cylinder.

2.4.1 Impinging jet

The first behaviour that we will consider is that of an impinging jet of fluid onto a surface.

FIGURE 2: Impinging jet

2.4.2 Pipe flow

A second behaviour that we will consider is that of pipe flow. This behaviour has been the focus of much research, as it is arguably the most common mode of heat transfer that occurs in fluid systems.

2.4.3 Turbulent Jets

We must also consider turbulent jets, as the main driver of flow in the cylinder, which in turn drives convective heat transfer, is the turbulent jet that develops from the end of the nozzle throughout the length of the cylinder. Experimental results can be used to derive expressions for the axial velocity of the jet. Radial profiles of mean axial velocity can be seen in Fig. 3.

FIGURE 3: Radial profiles of mean axial velocity in a turbulent round jet with $Re = 95,000$. Adapted from [8] with the data from [9]

By plotting the inverse of the non-dimensional speed, u_{exit}/u against x/d we get the clearly linear relationship shown in Fig. 4.

FIGURE 4: Jet speed vs distance. Adapted from [8] with the data from [9]

From this experimental result the following relationship is obtained:

$$\frac{u(x)}{u_{exit}} = \frac{B}{(x - x_0)/d_{inlet}} \quad (2.1)$$

where $u(x)$ is the axial velocity at a distance x from the nozzle, x_0 is the position of the virtual origin, and B is an empirical constant.

2.5 Methodology

Several techniques and methods will be used throughout the analysis. These are described in this section.

2.5.1 Optimization

Fill Section
once work on
optimization
starts.

2.5.2 Non-dimensioning

One of the fundamental principles used throughout this paper, and indeed, throughout engineering, is that of non-dimensioning. By operating using non-dimensional parameters such as the Reynolds number $Re = \frac{\rho u L}{\mu}$ or the Prandtl number $Pr = \frac{c_p \mu}{k}$ solving problems involving differential equations becomes simplified. Also, the analysis becomes much more general, and can be scaled.

2.5.3 Numerical methods

{sec:numerical_meth

Integrating Ordinary Differential Equations (ODEs) A central part of solving unsteady heat transfer problems involves integrating ODEs, as will be seen in Section 3. One of the simplest methods available, both conceptually and in terms of ease of implementing in code, is forward Euler time integration. It can be described as follows: given a function that can be defined by:

$$y'(t) = f(t, y(t)), \quad y(t_0) = y_0 \quad (2.2)$$

we can compute the approximate shape of the function given the initial point and finding the slope of the curve for small intervals. Indeed, from the initial point, we can find the tangent of the curve at that point, and take a small step along that tangent until arriving at the next point, where the procedure can be repeated. Denoting the step size as h , we can express forward Euler time integration as:

$$y_{n+1} = y_n + hf(t_n, y_n) \quad (2.3)$$

{sec:property_model

Property models As the gases that are being treated in this report are at very high pressures, ideal gas approximations are inaccurate, and thus real gas properties must be employed. To this end, property models must be used, which can determine any gas property from two other independent properties.

The property model that will be employed throughout this analysis is REFPROP, a tool developed by the National Institute of Standards and Technology [10]. It uses values of critical and triple points together with equations for the thermodynamic and transport properties to calculate the state points of fluids.

3 Formulation

3.1 Governing equation

The governing equation for mass and internal energy of the gas in the tank is given by:

$$\frac{dU_{gas}}{dt} = \frac{d(m_{gas}u_{gas})}{dt} = H_{in}\dot{m}_{in} - \dot{Q}_{out} \quad (3.1)$$

where U_{gas} is the total internal energy of the gas, m_{gas} is the mass of the gas, u_{gas} is the specific internal energy of the gas, H_{in} is the enthalpy of the inlet gas, \dot{m}_{in} the mass flow into the cylinder, and \dot{Q}_{out} is the rate of heat transfer out of the gas to the cylinder.

The change in internal energy of the gas is equal to the difference in energy entering the system in the enthalpy of gas inflow and the energy leaving the system through the walls of the cylinder. The enthalpy of the inlet can be determined from real gas models as a function of the inlet pressure and temperature. Therefore, to obtain the internal energy variation one must calculate the gas mass flow and the heat transferred to the cylinder.

3.2 Gas mass flow into cylinder

The nozzle and corresponding gas flow will be modeled using isentropic relations, and then a discharge coefficient relationship will be used to find the approximate real values. Indeed, the isentropic Reynold's number is first calculated as follows:

$$Re_{ideal} = \frac{\rho_{exit} d_{inlet} u_{exit}}{\mu_{exit}} \quad (3.2)$$

where d_{inlet} is the diameter of the inlet delivery pipe and ρ_{exit} , u_{exit} , and μ_{exit} are determined using real gas models as described in Section 2.5.3. More specifically:

$$\rho_{exit} = f(P_{exit}, S_{in}), \quad u_{exit} = \sqrt{2(H_{in} - H_{static})}, \quad \mu_{exit} = f(P_{exit}, S_{in}) \quad (3.3)$$

where P_{exit} is the pressure at the end of the inlet tube, S_{in} is the entropy at the inlet, H_{in} the stagnation enthalpy at the inlet, H_{static} the static enthalpy at the end of the inlet tube. As the system is treated as isentropic, the inlet entropy can be used to calculate exit properties. We also have:

$$S_{in} = f(P_{in}, T_{in}), \quad H_{in} = f(P_{in}, T_{in}), \quad H_{static} = f(P_{exit}, S_{in}) \quad (3.4)$$

where P_{in} and T_{in} are the inlet pressure and temperature, respectively. The exit pressure P_{exit} is taken to be the pressure inside the gas tank. However, in the case that the inlet diameter is choked, these calculations would yield velocities higher than the speed of sound, which is impossible due to the nature of the inlet pipe. For this reason, if a velocity of Mach 1 or higher is achieved at any given time, an iterative process is used to find the value of P_{exit} that will yield a velocity equal to the speed of sound, and the rest of properties and ultimately the Reynolds number calculated accordingly.

In order to find the real mass flow an empirical discharge coefficient is employed.

$$C_D = \frac{\dot{m}_{in}}{\dot{m}_{ideal}} = A + C \text{Re}_{ideal} \quad (3.5)$$

A discharge coefficient must be used to account for the formation of a boundary layer inside the inlet tube. The empirical model that was used was obtained from , and uses the following values:

Citation

$$A = 0.938, \quad C = -2.71 \quad (3.6)$$

From the real mass flow the actual Reynold's number of the inflow can be calculated and used to find forced convection heat transfer coefficients in Section 3.3.1 and Eq. (3.11) as follows:

avoid forward references

$$\text{Re} = \frac{4\dot{m}_{in}}{\pi\mu d} \quad (3.7)$$

3.3 Heat transfer from gas to cylinder

The heat transferred from the gas to the wall of the tank is given by a simple convection relationship:

$$\dot{Q} = hA(T_{gas} - T_{wall}) \quad (3.8) \quad \{\text{equ:convection}\}$$

where A is the internal surface area of the cylinder, h is the heat transfer coefficient, and T_{gas} and T_{wall} are the temperatures of the gas and the wall, respectively. The heat transfer coefficient is a result of the combination of both forced and natural convection. This is expressed as follows, as per [11] and [12]:

$$h = \sqrt[4]{h_f^4 + h_n^4} \quad (3.9)$$

where h_f is the heat transfer coefficient due to forced convection and h_n is the heat transfer coefficient due to natural convection. Each coefficient can be non-dimensionalised using Nusselt's numbers, expressed as:

$$\text{Nu}_f = \frac{h_f D}{k}, \quad \text{Nu}_n = \frac{h_n D}{k} \quad (3.10)$$

where D is the characteristic length, in this case the diameter of the cylinder, and k , the thermal conductivity of the fluid. The values of the Nusselt's numbers can be determined from empirical correlations, as detailed in Sections 3.3.1 and 3.3.2.

3.3.1 Forced convection

{sec:forcedConvection}

As the magnitude of heat transfer is related to the flow in forced convection, the Nusselt's number can be said to be a function of the Reynolds number, taking the form:

$$\text{Nu}_{f,ss} = E \text{Re}^F \quad (3.11) \quad \{\text{equ:nusseltReynold}\}$$

where $Nu_{f,ss}$ is the steady state Nusselt's number and the empirical constants E and F are given by as:

$$E =, \quad F = \quad (3.12)$$

The initial work this project builds upon uses the flow at the nozzle to determine the heat transfer at the wall of the cylinder. This assumes that the hydrogen flows instantaneously from the nozzle to the wall, when in reality there is of course a time delay. This assumption is acceptable for stable inflows, hence us referring to the steady state Nusselt's number $Nu_{f,ss}$, but for more complex filling patterns and also for improved accuracy it becomes necessary to incorporate hysteresis. Indeed, the heat transfer coefficient at the wall in fact can be said to depend on the nozzle flow seconds prior, or more generally, on the history of the nozzle flow. This can be modeled as follows:

$$\frac{d}{dt}(Nu_f) = \frac{Nu_{f,ss} - Nu_f}{\tau} \quad (3.13)$$

where Nu_f is the forced convection Nusselt's number of the flow, $Nu_{f,ss}$ is the steady state forced convection Nusselt's number, and τ is the time scale. The time constant reflects the amount of time it takes for the flow to recirculate. Two time scales are considered, as two different situations may be present. The flow in the cylinder may be driven by the inflow of mass from the nozzle, in which case we have a time scale of production τ_{prod} . However, if the inflow halts, the flow field in the cylinder will be driven by the dissipation of the existing flow and thus the time scale is denoted τ_{diss} . Therefore the overall time scale is given by the minimum of these two:

$$\tau = \min(\tau_{prod}, \tau_{diss}) \quad (3.14)$$

The production time scale is given by the spatial integral of the axial speed of the jet, which is given in Eq. (2.1) in Section 2.4.3.

$$\begin{aligned} \tau_{prod} &= \int_0^L \frac{1}{u(x)} dx = \frac{1}{Bu_{exit}} \int_0^L (x - x_0) dx \\ &= \frac{\frac{L^2}{2} - x_0 L}{Bu_{exit}} \end{aligned} \quad (3.15)$$

As we have $x_0 \ll L$ we can write:

$$\tau_{prod} = \frac{L^2}{2Bu_{exit}} \quad (3.16)$$

we can rewrite the constant:

$$\tau_{prod} = G \frac{L^2}{u_{exit} d} \quad (3.17)$$

The dissipation time scale can be said to be proportional to the length of the cylinder L and the recirculation velocity u_{recirc} :

$$\tau_{diss} = H \frac{L}{u_{recirc}} \quad (3.18)$$

335 Let us assume that instantaneous Nusselt's number depends on u_{recirc} as it depends on Re^D and
 336 $D \sim \mathcal{O}(1)$. From Eq. (3.11) we know that the

$$\frac{Du_{recirc}}{\nu} = INu_f \quad (3.19)$$

337 We can therefore rewrite:

$$\tau_{diss} = \frac{H}{I} \frac{LD}{\nu Nu_f} \quad (3.20)$$

338 Combining the two constants and absorbing the L/D coefficient:

$$\tau_{diss} = J \frac{L^2}{\nu Nu_f} \quad (3.21)$$

339 We can then make use of the fact that in steady state we know $\tau_{prod} = \tau_{diss}$, which leads to:

$$\tau_{diss} = J \frac{L^2}{\nu Nu_f} = G \frac{L^2}{u_{exit} d} \quad (3.22)$$

340 Substituting the definitions of Nusselt's and Reynolds numbers from Eqs. (3.2) and (3.11):

$$J = G \frac{\nu Nu_f}{u_{exit} d} = GE \quad (3.23)$$

341 Which leads to a final definition of τ_{diss} :

$$\tau_{diss} = GE \frac{L^2}{\nu Nu_f} \quad (3.24)$$

342 Therefore, in a pure dissipation scenario, where $Nu_{f,ss} \rightarrow 0$, we have:

$$\frac{d}{dt} (Nu_f) = -Nu_f^2 \frac{\nu}{GELD} \quad (3.25)$$

343 Thus, when the inflow stops the heat transfer at the wall will decrease quadratically.

344 3.3.2 Natural convection

{sec:naturalConvection}

345 Natural convection is caused by buoyancy driven flow, so the natural Nusselt's number can be found
 346 to be related to the Rayleigh's number (itself a product of Grashof's and Prandtl's numbers) by the
 347 following relationship:

$$Nu_n = KRa^L \quad (3.26)$$

where Rayleigh's number is defined as:

$$Ra = \left| \frac{g\beta(T_{wall} - T_{gas})D^3}{\nu\alpha} \right| \quad (3.27)$$

where g is the acceleration due to gravity, β is the coefficient of thermal expansion, D is the characteristic length, in this case the cylinder diameter, ν is the kinematic viscosity, and α is the thermal diffusivity, as defined by:

$$\alpha = \frac{k}{\rho c} \quad (3.28)$$

where k is the material's thermal conductivity, ρ is the density of the material, and c is the specific heat capacity of the material. The coefficients are given by :

$$K =, \quad L = \quad (3.29)$$

3.4 Heat transfer across cylinder

The main mode of heat transfer that occurs in the cylinder and that will be used throughout this report is heat conduction through the wall of the cylinder. This is modeled using:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (3.30)$$

where α is the thermal diffusivity, as defined in Eq. (3.28). This differential equation will be solved using forward Euler time integration as described in Section 2.5.3. Solving this equation will yield the temperature distribution throughout the thickness of the cylinder wall, and more specifically, the internal wall temperature that is used to calculate the heat transferred out of the gas in Eq. (3.8).

3.4.1 Discretization

3.4.2 Outer Wall

3.5 Throttling

Throttling is introduced in order to more accurately represent real life conditions. When a maximum temperature is reached, the inflow of hydrogen must be stopped in order to protect the materials of the tank, as detailed in Section 2.1.1. A simple method of throttling, with the flow stopping at the designated maximum temperature, 85 °C, and the flow restarting at a chosen temperature. The optimum temperature at which flow restarts was considered in this study, and results are presented in ??

3.6 Optimization

371

General stuff:
- Check tense and person
- Check for repetition
- Check for excessive verbosity - define

372 **4 Results**

{sec:results}

373 **5 Discusion**

{sec:discusion}

374 **6 Conclusion**

{sec:conclusion}

375 This is the conclusion.

376 **Appendices**377 **A Filling Code**

{app:filling_code}

```

1  %Author: Vishagen Ramasamy
2  %Date: -
3
4  % Copyright University of Southampton 2017.
5  % No warranty either expressed or implied is given to the results produced
6  % by this software. Neither the University, students or its employees
7  % accept any responsibility for use of or reliance on results produced by
8  % this software.
9
10 %% script that computes the filling of the tank(s)
11 %% Importing the pressure and temperature profile at the entrance of the delivery pipe from Dicken a
12 % and calculating the stagnation enthalpy and entropy
13
14 for j = 1:maxt
15     time(j+1) = (j)*dt; % Time as filling process
16     for i = 1:tank_number
17         % Select inlet pressure / temperature profile based on user input
18         if blnUseStandardData == 1
19             % Constant inlet pressure profile
20             P_inlet(i,j+1) = ConstPressKPa;
21         else
22             % Read inlet pressure profile from specified file
23             P_inlet(i, j+1) = interp1(InletPressureData(:,1),InletPressureData(:,3), dt*j);
24         end
25
26         if blnUseStandardData == 1
27             %Temp_inlet(i,j+1) = interp1(time_in, temp_in, dt*j); % Temperature profile after the
28             Temp_inlet(i, j+1) = ConstTempK;
29         else
30             % Read inlet temperature profile from specified file
31             Temp_inlet(i, j+1) = interp1(InletTempData(:,1), InletTempData(:,3), dt*j);
32         end
33
34         h_inlet(i,j+1) = refpropm('H','T',Temp_inlet(i,j+1),'P',P_inlet(i,j+1),Fluid{i}, refproppdir)
35         entropy_inlet(i,j+1) = refpropm('S','T',Temp_inlet(i,j+1),'P',P_inlet(i,j+1),Fluid{i}, refproppdir)
36
37         %% Determine which pressure to use at the exit of the delivery pipe which is dependent upon
38
39         if l_d(i) > 3 && blnOneZone{i} == 0 % The length-to-diameter ratio of the tank(s) d
40             Pressure_exit = P_gas_zone1{i}(j); % The pressure at the exit of the delivery pipe is
41         else
42             Pressure_exit = P_gas(i,j); % The pressure at the exit of the delivery pipe is
43         end
44
45         if (P_inlet(i,j+1) > Pressure_exit) % condition for filling of tank(s)
46             sound_exit(i,j+1) = refpropm('A','P',Pressure_exit,'S',entropy_inlet(i,j+1),Fluid{i}, refproppdir)
47             h_static_exit(i,j+1) = refpropm('H','P',Pressure_exit,'S',entropy_inlet(i,j+1),Fluid{i}, refproppdir)
48             visc_exit(i,j+1) = refpropm('V','P',Pressure_exit,'S',entropy_inlet(i,j+1),Fluid{i}, refproppdir)
49             mach_exit(i,j+1) = sqrt(2*(h_inlet(i,j+1)-h_static_exit(i,j+1))/sound_exit(i,j+1)^2);
50
51             %% If Mach number is greater than one, the exit pressure is greater than the pressure wi
52             % is incrementally increased and iterated in a while loop until mach number is equal to
53
54             Inlet_entropy = entropy_inlet(i,j+1);
55             Inlet_stagnation_enthalpy = h_inlet(i,j+1);
56             P_guess = Pressure_exit;
57
58             if mach_exit(i,j+1) > 1
59                 Pressure_exit = find_exit_pressure(Inlet_stagnation_enthalpy,Inlet_entropy,Fluid{i},
60                 sound_exit(i,j+1) = refpropm('A','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refproppdir)
61                 h_static_exit(i,j+1) = refpropm('H','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refproppdir)

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62         visc_exit(i,j+1) = refpropm('V','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refpropdir);
63         mach_exit(i,j+1) = sqrt(2*(h_inlet(i,j+1)-h_static_exit(i,j+1))/sound_exit(i,j+1)^2); % Co
64     end
65
66     %% Calculation of the mass flow rate into the tank(s)
67
68     rho_exit(i,j+1) = refpropm('D','P',Pressure_exit,'S',Inlet_entropy,Fluid{i}, refpropdir);
69     vel_exit(i,j+1) = mach_exit(i,j+1)*sound_exit(i,j+1); % Calcul
70     Re_exit_isentropic(i,j+1) = rho_exit(i,j+1) *d_inlet*vel_exit(i,j+1)/visc_exit(i,j+1); % Calcul
71     cd(i,j+1) = I + J/(Re_exit_isentropic(i,j+1)^(0.25)); % Calcul
72     mfr(i,j+1) = cd(i,j+1)*rho_exit(i,j+1)*vel_exit(i,j+1)*A_inlet; % Calcul
73     Re_entrance_actual(i,j+1) = 4*mfr(i,j+1)/(pi*d_inlet*visc_exit(i,j+1)); % Calcul
74     dM_inlet = mfr(i,j+1)*dt; % Amount
75 end
76
77 %% Heat transfer calculations & caluculations of the thermodynamic properties of the gas in the tank(s) v
78 if l_d(i) <= 3 | blnOneZone{i} == 1
79
80     Nus(i,j+1) = a_1*Re_entrance_actual(i,j+1)^(b_1); % Nusselt number and Reynolds
81     k_gas(i,j+1) = refpropm('L','T',Temp_gas(i,j),'P',P_gas(i,j),Fluid{i}, refpropdir); % Thermal condu
82     heat_coef_forced(i,j+1) = Nus(i,j+1)*k_gas(i,j+1)/d_tank(i); % Calculation of the heat tran
83
84     if Inner_wall_boundary(i) == 1
85         Qsurf(i,j+1) = -dt*surf_area(i)*heat_coef_forced(i,j+1)*(Temp_gas(i,j)-Inner_temp_wall_isotherma
86     else
87         Qsurf(i,j+1) = -dt*surf_area(i)*heat_coef_forced(i,j+1)*(Temp_gas(i,j)-Temp_wall{i}(1,j));
88         Temp_wall{i}(1,j+1) = Temp_wall{i}(1,j) + CFL_liner(i)*(Temp_wall{i}(2,j)- Temp_wall{i}(1,j) - Qs
89
90     if Outer_wall_boundary(i)==1
91         Temp_wall{i}(number_of_gridpoints(i),j+1) = Outer_temp_wall_isothermal(i);
92     else
93         Temp_wall{i}(number_of_gridpoints(i),j+1) = Temp_wall{i}(number_of_gridpoints(i),j)+2*CFL_l
94     end
95     % Computation of the temperature of the struture of the tank(s)
96     for k=2:number_of_gridpoints(i)-1
97         if (k>=2)&&(k<=int_pt_liner_laminate(i)-1)
98             Temp_wall{i}(k,j+1) = Temp_wall{i}(k,j)+CFL_liner(i)*(Temp_wall{i}(k+1,j)-2*Temp_wall{i}
99         elseif (k>=int_pt_liner_laminate(i)+1)&&(k<=number_of_gridpoints(i)-1)
100             Temp_wall{i}(k,j+1) = Temp_wall{i}(k,j)+CFL_laminate(i)*(Temp_wall{i}(k+1,j)-2*Temp_wall
101         else
102             Temp_wall{i}(k,j+1) = (cond_laminate(i)*(Temp_wall{i}(k+1,j)+CFL_laminate(i)*(Temp_wall
103         end
104     end
105 end
106 m_gas(i,j+1) = m_gas(i,j) + dM_inlet; % Mass of gas in
107 Ugas(i,j+1)=Ugas(i,j)+Qsurf(i,j+1)+h_inlet(i,j+1)*dM_inlet; % Internal energy
108 u_gas(i,j+1)= Ugas(i,j+1)/m_gas(i,j+1); % Specific energy
109 rho_gas(i,j+1)=m_gas(i,j+1)/vol_tank(i); % Density of the
110 P_gas(i,j+1) = refpropm('P','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir); %
111 Temp_gas(i,j+1) = refpropm('T','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir); %
112
113 else
114     Re_compression(i,j+1) = Re_entrance_actual(i,j+1)*(d_inlet/d_tank(i)); % Reynolds number
115
116     Nus_zone1{i}(j+1) = a_1*Re_entrance_actual(i,j+1)^(b_1); % Nusselt number
117     Nus_zone2{i}(j+1) = c_1*Re_compression(i,j+1)^(d_1); % Nusselt number
118
119     k_gas_zone1{i}(j+1) = refpropm('L','T',Temp_gas_zone1{i}(j),'P',P_gas_zone1{i}(j),Fluid{i}, refprop
120     k_gas_zone2{i}(j+1) = refpropm('L','T',Temp_gas_zone2{i}(j),'P',P_gas_zone2{i}(j),Fluid{i}, refprop
121
122     heat_coef_forced_zone1{i}(j+1) = Nus_zone1{i}(j+1)*k_gas_zone1{i}(j+1)/d_tank(i); %
123     heat_coef_forced_zone2{i}(j+1) = Nus_zone2{i}(j+1)*k_gas_zone2{i}(j+1)/l_zone2_total(i); %
124
125     % Step 1. Equating the respecting values of zone 1 and zone 2 to
126     % matrices with 'similar' names
127

```



```

128     Vgas(1)=volume_zone1(i);
129     Vgas(2)=volume_zone2(i);
130     Mgas(1)=m_gas_zone1{i}(j);
131     Mgas(2)=m_gas_zone2{i}(j);
132     Ugas_twozone(1)=u_gas_zone1{i}(j)*Mgas(1);
133     Ugas_twozone(2)=u_gas_zone2{i}(j)*Mgas(2);
134     Asurf(1)=surface_area_zone1(i);
135     Asurf(2)=surface_area_zone2(i);
136     Tgas(1)=Temp_gas_zone1{i}(j);
137     Tgas(2)=Temp_gas_zone2{i}(j);
138     if Inner_wall_boundary(i) == 1
139         Twall(1,1)=Inner_temp_wall_isothermal(i);
140         Twall(2,1)=Inner_temp_wall_isothermal(i);
141     else
142         Twall(1,1)=Temp_wall_zone1{i}(1,j);
143         Twall(2,1)=Temp_wall_zone2{i}(1,j);
144     end
145     % Step 2: apply the change of internal energy due to heat transfer and mass
146     % input through nozzle:
147
148     dM_inlet = mfr(i,j+1)*dt;
149
150     Qsurf(1)=- dt*Asurf(1)*(heat_coef_forced_zone1{i}(j+1))*(Tgas(1)-Twall(1,1));
151     Qsurf(2)=- dt*Asurf(2)*(heat_coef_forced_zone2{i}(j+1))*(Tgas(2)-Twall(2,1));
152
153     Ugas_twozone(1)=Ugas_twozone(1)+Qsurf(1)+h_inlet(i,j+1)*dM_inlet;
154     Ugas_twozone(2)=Ugas_twozone(2)+Qsurf(2);
155
156     Mgas(1)=Mgas(1)+dM_inlet;
157     Mgas(2)=Mgas(2);
158
159     for m=1:2
160         hgas(m)=refpropm('H','D',Mgas(m)/Vgas(m),'U',Ugas_twozone(m)/Mgas(m),Fluid{i}, refpropdir);
161     end
162
163
164     % Step 2: Find the amount of mass that needs to be transferred from zone 1
165     % to zone 2 to equalise their pressure
166     % (we assume forward Euler integration, therefore we use the specific
167     % enthalpies h from the start of the timestep. We provide enthalpies for
168     % both zones in case the flow gets reversed).
169
170     dM_guess = dM_inlet*(volume_zone2(i)/(volume_zone1(i)+volume_zone2(i))) ;
171     dM_12 = find_dM_12(hgas,Vgas,Mgas,Ugas_twozone,Fluid{i},dM_guess, refpropdir);
172
173     % Step 3: Apply this change to the mass and update all properties:
174
175     m_gas_zone1{i}(j+1)=Mgas(1)-dM_12;
176     m_gas_zone2{i}(j+1)=Mgas(2)+dM_12;
177
178     u_gas_zone1{i}(j+1)=(Ugas_twozone(1)-max(0,dM_12)*hgas(1)-min(0,dM_12)*hgas(2))/m_gas_zone1{i}(j+1);
179     u_gas_zone2{i}(j+1)=(Ugas_twozone(2)+max(0,dM_12)*hgas(1)+min(0,dM_12)*hgas(2))/m_gas_zone2{i}(j+1);
180
181     rho_gas_zone1{i}(j+1)=m_gas_zone1{i}(j+1)/volume_zone1(i);
182     rho_gas_zone2{i}(j+1)=m_gas_zone2{i}(j+1)/volume_zone2(i);
183
184     Temp_gas_zone1{i}(j+1)=refpropm('T','D',rho_gas_zone1{i}(j+1),'U', u_gas_zone1{i}(j+1),Fluid{i}, refpropdir);
185     Temp_gas_zone2{i}(j+1)=refpropm('T','D',rho_gas_zone2{i}(j+1),'U', u_gas_zone2{i}(j+1),Fluid{i}, refpropdir);
186
187     P_gas_zone1{i}(j+1)=refpropm('P','D',rho_gas_zone1{i}(j+1),'U', u_gas_zone1{i}(j+1),Fluid{i}, refpropdir);
188     P_gas_zone2{i}(j+1)=refpropm('P','D',rho_gas_zone2{i}(j+1),'U', u_gas_zone2{i}(j+1),Fluid{i}, refpropdir);
189
190     m_gas(i,j+1) = m_gas_zone1{i}(j+1) + m_gas_zone2{i}(j+1);
191     rho_gas(i,j+1) = m_gas(i,j+1)/vol_tank(i);
192     u_gas(i,j+1)= (u_gas_zone1{i}(j+1)*m_gas_zone1{i}(j+1)+ u_gas_zone2{i}(j+1)*m_gas_zone2{i}(j+1))/m_gas(i,j+1);
193     Ugas(i,j+1) = u_gas(i,j+1)*m_gas(i,j+1);

```

```

194     Temp_gas(i,j+1) = refpropm('T','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir);
195     P_gas(i,j+1) = refpropm('P','D',rho_gas(i,j+1),'U',u_gas(i,j+1),Fluid{i}, refpropdir);
196
197     if Inner_wall_boundary(i) == 1
198         Qsurf_zone1{i}(j+1) = -dt*surface_area_zone1(i)* heat_coef_forced_zone1{i}(j+1)*(Temp_gas_zone1{i}(j+1)-Temp_gas_zone2{i}(j+1));
199         Qsurf_zone2{i}(j+1) = -dt*surface_area_zone2(i)* heat_coef_forced_zone2{i}(j+1)*(Temp_gas_zone2{i}(j+1)-Temp_gas_zone1{i}(j+1));
200     else
201         Qsurf_zone1{i}(j+1) = -dt*surface_area_zone1(i)* heat_coef_forced_zone1{i}(j+1)*(Temp_gas_zone1{i}(j+1)-Temp_gas_zone2{i}(j+1));
202         Qsurf_zone2{i}(j+1) = -dt*surface_area_zone2(i)* heat_coef_forced_zone2{i}(j+1)*(Temp_gas_zone2{i}(j+1)-Temp_gas_zone1{i}(j+1));
203
204         Temp_wall_zone1{i}(1,j+1) = Temp_wall_zone1{i}(1,j)+ CFL_liner(i)*(Temp_wall_zone1{i}(2,j)-Temp_wall_zone1{i}(1,j));
205         Temp_wall_zone2{i}(1,j+1) = Temp_wall_zone2{i}(1,j)+ CFL_liner(i)*(Temp_wall_zone2{i}(2,j)-Temp_wall_zone2{i}(1,j));
206
207     if Outer_wall_boundary(i)==1
208         Temp_wall_zone1{i}(number_of_gridpoints(i),j+1) = Outer_temp_wall_isothermal(i);
209         Temp_wall_zone2{i}(number_of_gridpoints(i),j+1) = Outer_temp_wall_isothermal(i);
210     else
211         Temp_wall_zone1{i}(number_of_gridpoints(i),j+1) = Temp_wall_zone1{i}(number_of_gridpoints(i),j);
212         Temp_wall_zone2{i}(number_of_gridpoints(i),j+1) = Temp_wall_zone2{i}(number_of_gridpoints(i),j);
213     end
214     % Computation of the temperature of the struture of the tank(s)
215     for k=2:number_of_gridpoints(i)-1
216         if (k>=2)&&(k<=int_pt_liner_laminate(i)-1)
217             Temp_wall_zone1{i}(k,j+1) = Temp_wall_zone1{i}(k,j)+CFL_liner(i)*(Temp_wall_zone1{i}(k+1,j)-Temp_wall_zone1{i}(k,j));
218             Temp_wall_zone2{i}(k,j+1) = Temp_wall_zone2{i}(k,j)+CFL_liner(i)*(Temp_wall_zone2{i}(k+1,j)-Temp_wall_zone2{i}(k,j));
219         elseif (k>=int_pt_liner_laminate(i)+1)&&(k<=number_of_gridpoints(i)-1)
220             Temp_wall_zone1{i}(k,j+1) = Temp_wall_zone1{i}(k,j)+CFL_laminate(i)*(Temp_wall_zone1{i}(k+1,j)-Temp_wall_zone1{i}(k,j));
221             Temp_wall_zone2{i}(k,j+1) = Temp_wall_zone2{i}(k,j)+CFL_laminate(i)*(Temp_wall_zone2{i}(k+1,j)-Temp_wall_zone2{i}(k,j));
222         else
223             Temp_wall_zone1{i}(k,j+1) = (cond_laminate(i)*(Temp_wall_zone1{i}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone1{i}(k+2,j)-Temp_wall_zone1{i}(k+1,j)));
224             Temp_wall_zone2{i}(k,j+1) = (cond_laminate(i)*(Temp_wall_zone2{i}(k+1,j)+CFL_laminate(i)*(Temp_wall_zone2{i}(k+2,j)-Temp_wall_zone2{i}(k+1,j)));
225         end
226     end
227 end
228
229 end
230
231 end
232 end

```

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