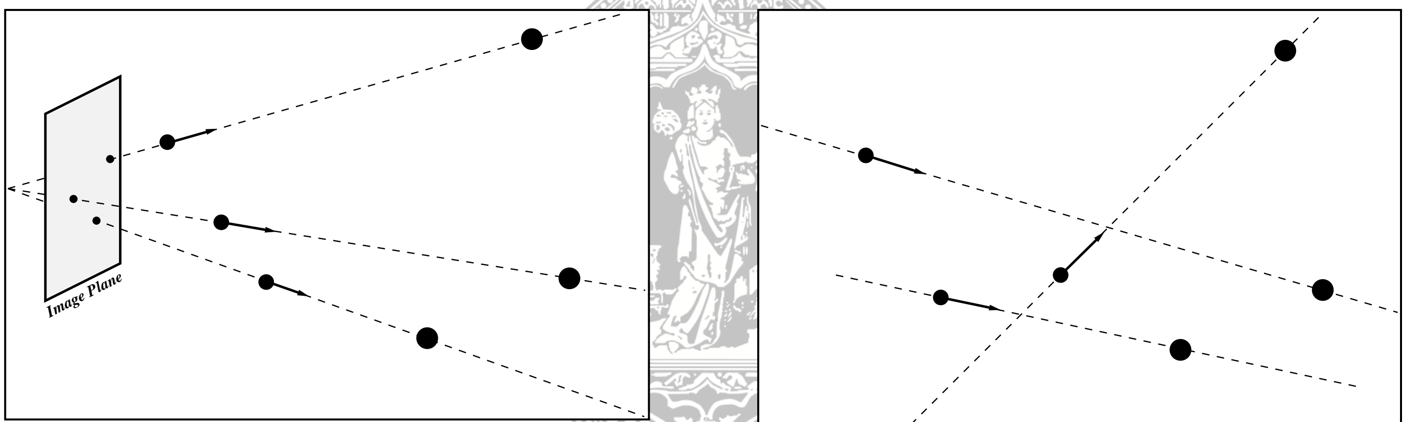
A Simple and Robust Solution to Minimal General Pose Estimation



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Problem Definition

- Generalized Camera Models can be used to define any imaging device -- **central or non-central**.
- The absolute pose is defined by the rigid transformation that defines the map between the world and camera coordinate systems.
- Minimal solutions use **coordinates of three known 3D points** in the world.

GOAL: In this paper we address the minimal 3-point pose estimation under the framework of generalized camera models.

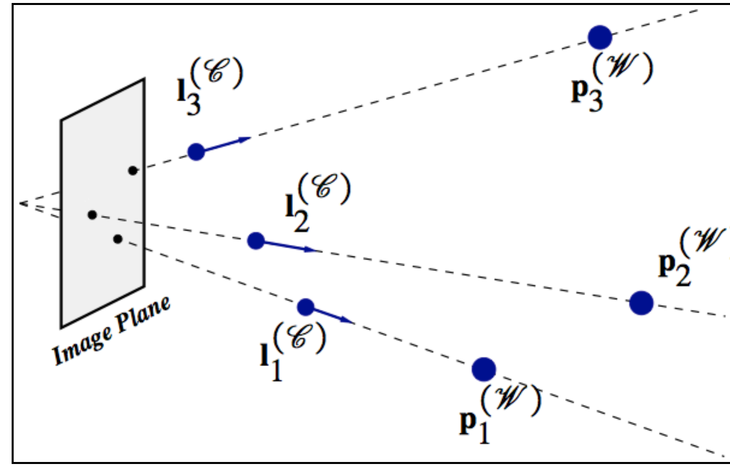

Fig. 1: Minimal Central 3-Point Pose Problem.

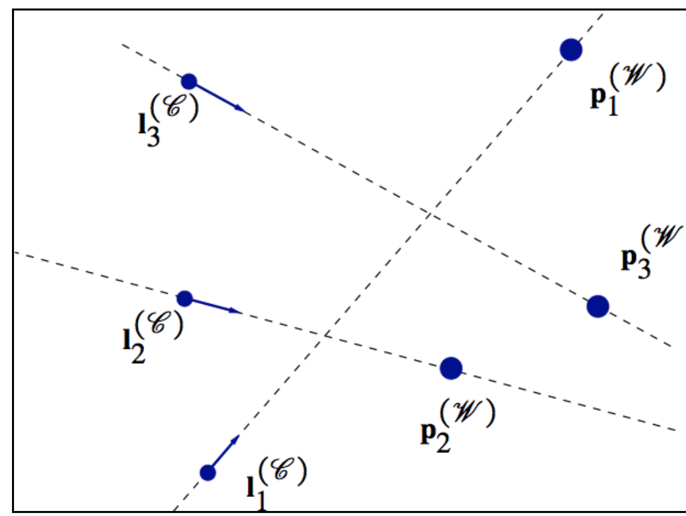

Fig. 2: Minimal Non-Central 3-Point Pose Problem.

Previous Approaches

Chen & Chang's Method

- The goal of the approach is to compute the coordinates of the 3D points in the camera coordinate system.

- After the application of any rigid transformation, the distance between 3D points is preserved.

$$\left|\mathbf{p}_i^{(\mathcal{W})} - \mathbf{p}_j^{(\mathcal{W})}\right| = \left|\mathbf{p}_i^{(\mathcal{C})} - \mathbf{p}_j^{(\mathcal{C})}\right| = \alpha_{i,j} \text{ and } \mathbf{p}_i^{(\mathcal{C})} = \mathbf{q}_i^{(\mathcal{C})} + t_i \mathbf{d}_i^{(\mathcal{C})}$$

- For a set of predefined transformations to the date-set we define:

$$t_2 = t_1 \cos(\theta_{1,2}) \pm \sqrt{\alpha_{1,2}^2 - d_{1,2}^2 - t_1^2 \sin(\theta_{1,2})^2}$$

$$t_3 = (t_1 - d_1)\cos(\theta_{1,3}) \pm \sqrt{\alpha_{1,3}^2 - d_{1,3}^2 - (t_1 - d_1)^2 \sin(\theta_{1,3})^2}$$

- And the main computational step [the computation of t_1] is given by the solutions for the eight degree polynomial equation:

$$\Phi_2^2 + \Phi_1^2\Gamma_2 - \Gamma_1^2\Omega_2 - \Gamma_2\Omega_2 = 2\left(\pm\Gamma_1\Omega_2 \pm \Phi_2\Phi_1\right)\sqrt{\Omega_2}$$

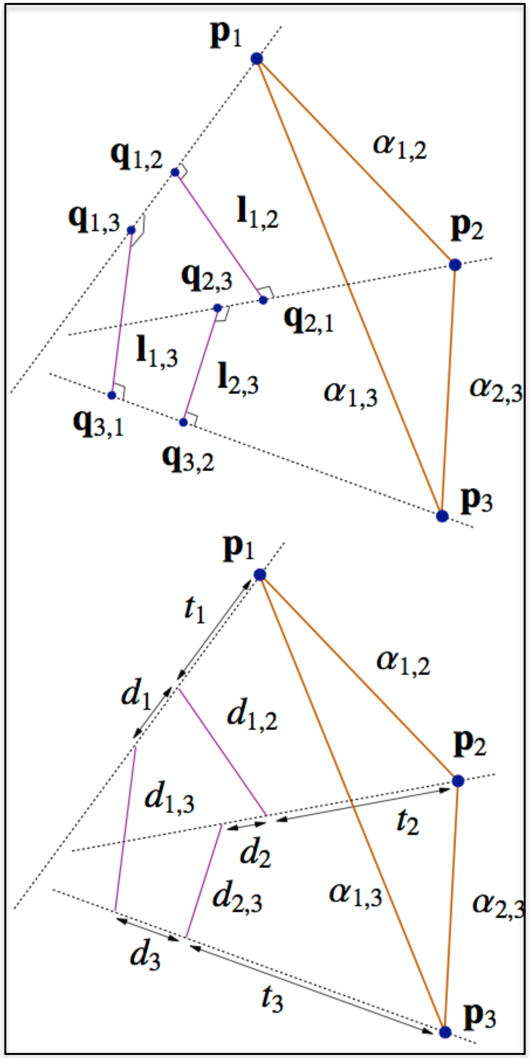

Fig. 3: Chen & Chang's coordinate system.

Previous Approaches

Nister and Stewenius Mathod (1)

- Computes the pose Directly.
- First, they analyze what happens when two of the three 3D lines meet the respective two 3D world points.
- For a valid transformation, the third point is mapped such that:

$$\mathbf{p}_3^{(C)} = \begin{bmatrix} x - ky \\ k(x - D) + sD + y \\ uz \\ u \end{bmatrix}, \text{ where } u = \frac{D}{d}$$

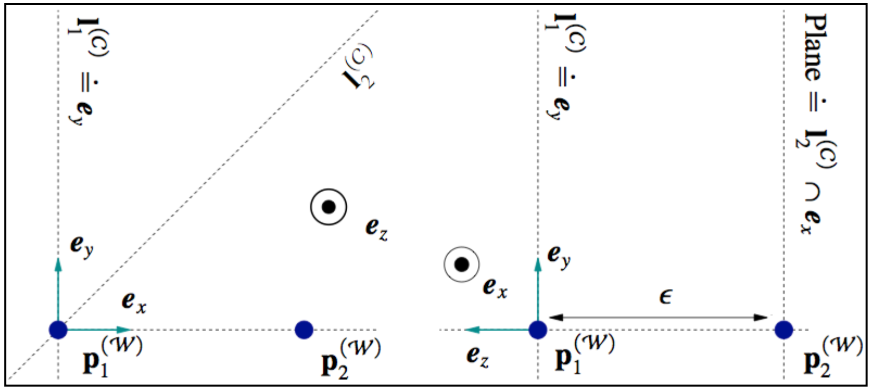

Fig. 4: Nister and Stewenius's Coordinate Systems.

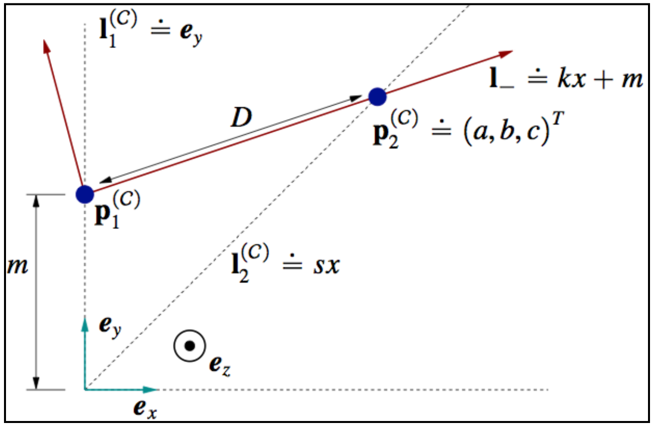

Fig. 5:. When two 3D points verify the incident constraints.

Previous Approaches

Nister and Stewenius Mathod (2)

- **[CIRCLE]** We are free to rotate the third 3D point around an axis defined by the two first 3D points. Thus:

$$x^2 + z^2 + \left(-z\frac{e}{D} + \frac{\left|\mathbf{p}_{\mathbf{2}}^{(\mathcal{W})} - \mathbf{p}_1^{(\mathcal{W})}\right| \mathbf{p}_3^{(\mathcal{W})\,T} \mathbf{d}_6}{D} \right) = \left| p_3^{(\mathcal{W})} \right|^2$$

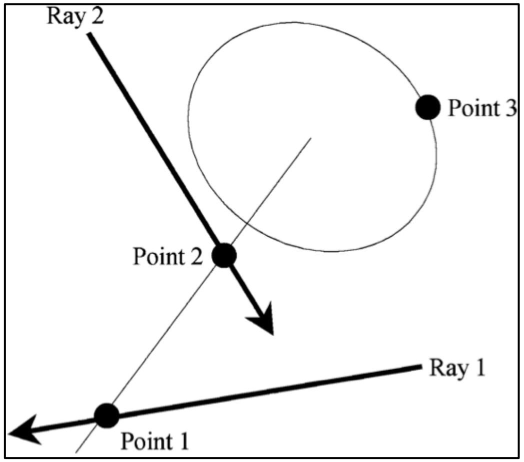

Fig. 6: Representation of the first constraint [**this image was taken from Nister and Steweniu's paper**].

- **[Ruled Quartic]** From the incidence relationship between the third 3D line and its respective 3D point, we get:

$$(\alpha_{1,2}\alpha_{2,1} - \alpha_{2,2}\alpha_{1,1})^2 = (\alpha_{2,2}\alpha_{1,3} - \alpha_{1,2}\alpha_{2,3})^2 + (\alpha_{1,3}\alpha_{2,1} - \alpha_{2,3}\alpha_{1,1})^2$$

where:

$$\alpha_{i,1} = xu_{i,1} + yu_{i,2} + sDu_{i,2}$$
$$\alpha_{i,2} = xu_{i,2} - yu_{i,1} - Du_{i,2}$$
$$\alpha_{i,3} = zu_{i,3} + u_{i,4}$$

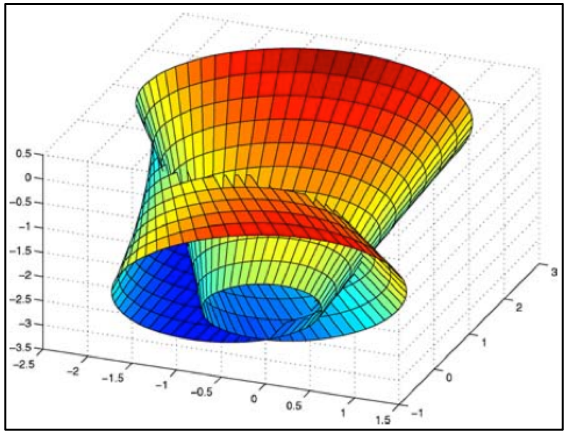

Fig. 7: Representation of the second constraint [**this image was taken from Nister and Steweniu's paper**].

Our Approach
Representation of the pose

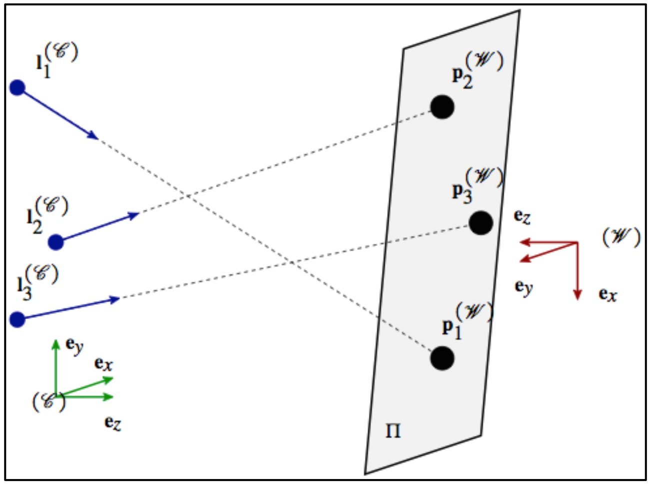

Fig. 8: Representation of the problem using the plane defined by the three 3D points in the world.

- The formulation of the problem is:

$$\text{find: } \mathbf{R} \in \mathcal{SO}(3) \text{ and } \mathbf{t}, \text{such that } \mathbf{p}^{(\mathcal{C})} = \mathbf{R}\mathbf{p}^{(\mathcal{W})} + \mathbf{t}$$

- **Three points define a plane**:

$$\mathbf{\Pi}^{(\mathcal{W})} \doteq \mathbf{p}_1^{(\mathcal{W})} \cup \mathbf{p}_2^{(\mathcal{W})} \cup \mathbf{p}_3^{(\mathcal{W})}$$

- Thus, we can write:

$$\mathbf{p}^{(\mathcal{C})} = \underbrace{\left(\mathbf{R} + \frac{1}{\zeta^{(\mathcal{W})}}\mathbf{t}\ \boldsymbol{\pi}^{(\mathcal{W})\,T}\right)}_{\mathbf{H}} \mathbf{p}^{(\mathcal{W})}, \text{where } \mathbf{H} \text{ is known as the homography matrix.}$$

- Without loss of generality, we can apply a predefined transformation: $\widetilde{\mathbf{p}}^{(\mathcal{W})} = \widetilde{\mathbf{R}}\mathbf{p}^{(\mathcal{W})} + \widetilde{\mathbf{t}}$ such that

$$\mathbf{p}^{(\mathcal{C})} = \underbrace{\left(\mathbf{R} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \frac{1}{\widetilde{\zeta}^{(\mathcal{W})}}\mathbf{t} \end{bmatrix}\right)}_{\mathbf{H}} \widetilde{\mathbf{p}}^{(\mathcal{W})}$$

Our Approach

Line definition and problem formulation (1)

- Representation of 3D lines using *Plucker Coordinates*:

$$\mathbf{l}^{(\mathcal{C})}\mathbb{R} \doteq \left(\mathbf{d}^{(\mathcal{C})}, \mathbf{m}^{(\mathcal{C})}\right) \text{ where } \left\langle \mathbf{d}^{(\mathcal{C})}, \mathbf{m}^{(\mathcal{C})} \right\rangle = 0$$

- Incidence relationship between a 3D line and a 3D point:

$$\mathbf{d}^{(\mathcal{C})} \times \mathbf{p}^{(\mathcal{C})} = \widehat{\mathbf{d}}^{(\mathcal{C})}\mathbf{p}^{(\mathcal{C})} = \mathbf{m}^{(\mathcal{C})}$$

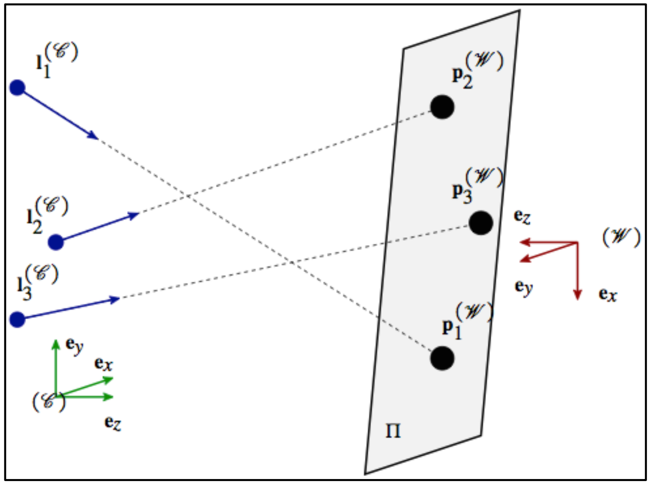

Fig. 8: Representation of the problem using the plane defined by the three 3D points in the world.

- If we consider the pose matrix (here represented by the homography marix) one has:

$$\widehat{\mathbf{d}}^{(\mathcal{C})}\mathbf{H}\widetilde{\mathbf{p}}^{(\mathcal{W})} = \mathbf{m}^{(\mathcal{C})}, \text{ which can be rewritten as } \left(\widetilde{\mathbf{p}}^{(\mathcal{W})^T} \otimes \widehat{\mathbf{d}}^{(\mathcal{C})}\right) \operatorname{vec}(\mathbf{H}) = \mathbf{m}^{(\mathcal{C})}.$$

Our Approach

Problem formulation (2)

- For our problem, the following algebraic relation can be defined:

$$\underbrace{\begin{bmatrix} \widetilde{\mathbf{p}}_1^{(\mathcal{W})\,T} \otimes \widehat{\mathbf{d}}_1^{(\mathcal{C})} \\ \widetilde{\mathbf{p}}_2^{(\mathcal{W})\,T} \otimes \widehat{\mathbf{d}}_2^{(\mathcal{C})} \\ \widetilde{\mathbf{p}}_3^{(\mathcal{W})\,T} \otimes \widehat{\mathbf{d}}_3^{(\mathcal{C})} \end{bmatrix}}_{\mathbf{M}} \operatorname{vec}(\mathbf{H}) = \underbrace{\begin{bmatrix} \mathbf{m}_1^{(\mathcal{C})} \\ \mathbf{m}_2^{(\mathcal{C})} \\ \mathbf{m}_3^{(\mathcal{C})} \end{bmatrix}}_{\mathbf{w}}$$

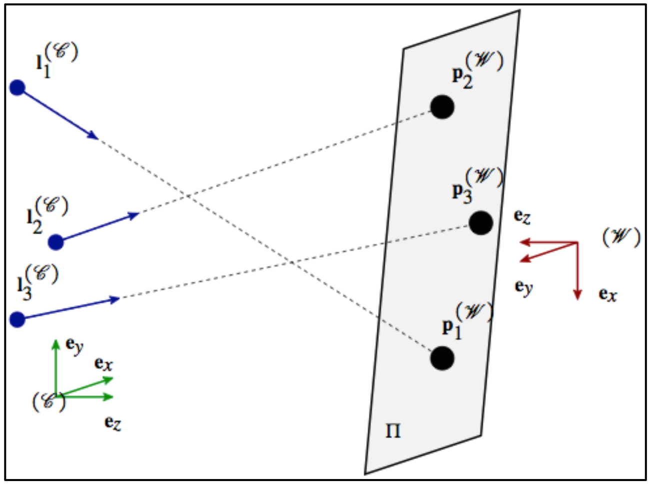

Fig. 8: Representation of the problem using the plane defined by the three 3D points in the world.

Theorem: *For three 3D points defined in the world coordinate system and their respective 3D straight lines in the camera coordinate system, the matrix M has rank equal to six.*

- Thus, we have the following solution:

$$\operatorname{vec}(\mathbf{H}) \doteq \{a\widetilde{\mathbf{e}}_1 + b\widetilde{\mathbf{e}}_2 + c\widetilde{\mathbf{e}}_3 + \widetilde{\mathbf{e}}_4 : a, b, c \in \mathbb{R}\}$$

Our Approach

Problem formulation (3)

- Un-stacking the previous space of solutions, we have:

$$\mathbf{H} = a\,\widetilde{\mathbf{E}}_1 + b\,\widetilde{\mathbf{E}}_2 + c\,\widetilde{\mathbf{E}}_3 + \widetilde{\mathbf{E}}_4$$

- Where (a, b, c) are the remaining unknowns.

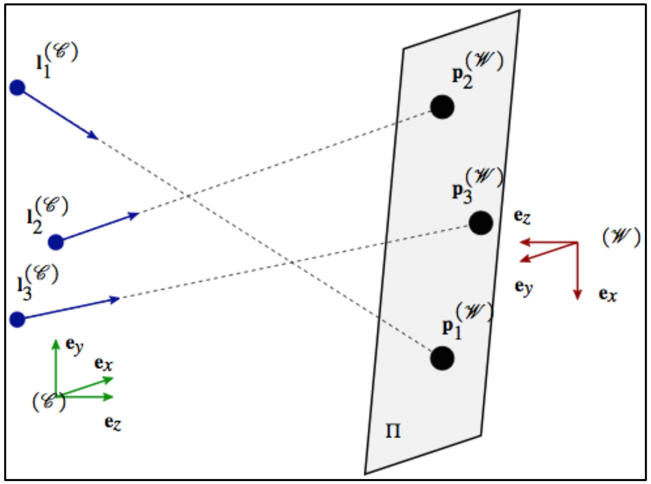

Fig. 8: Representation of the problem using the plane defined by the three 3D points in the world.

- Notice that our goal is a Homography matrix such that:

$$\mathbf{H} \doteq \left(\mathbf{R} + \left[\begin{array}{ccc} \mathbf{0} & \mathbf{0} & \frac{1}{\widetilde{\zeta}^{(\mathcal{W})}}\mathbf{t} \end{array}\right]\right)$$

- Three constraints can be defined:

$$\mathbf{r}_1^T\mathbf{r}_1 = 1, \quad \text{which implies} \quad \mathbf{h}_1^T\mathbf{h}_1 = 1$$

$$\mathbf{r}_2^T\mathbf{r}_2 = 1, \quad \text{which implies} \quad \mathbf{h}_2^T\mathbf{h}_2 = 1$$

$$\mathbf{r}_1^T\mathbf{r}_2 = 0, \quad \text{which implies} \quad \mathbf{h}_1^T\mathbf{h}_2 = 0$$

Our Approach
Problem formulation (4)

- The three constraints applied to $\mathbf{H}$ gives:

$$Q_{i,j}\left(a,b,c\right) = a^2\kappa_1^{(i,j)} + b^2\kappa_2^{(i,j)} + c^2\kappa_3^{(i,j)} + ab\kappa_4^{(i,j)} + \ldots$$
$$\ldots + ac\kappa_5^{(i,j)} + bc\kappa_6^{(i,j)} + a\kappa_7^{(i,j)} + b\kappa_8^{(i,j)} + c\kappa_9^{(i,j)} + \kappa_{10}^{(i,j)}$$

- We get the estimated solutions for $\mathbf{H}$ using:

$$Q_{1,1}\left(a,b,c\right) = Q_{2,2}\left(a,b,c\right) = Q_{1,2}\left(a,b,c\right) = 0,$$

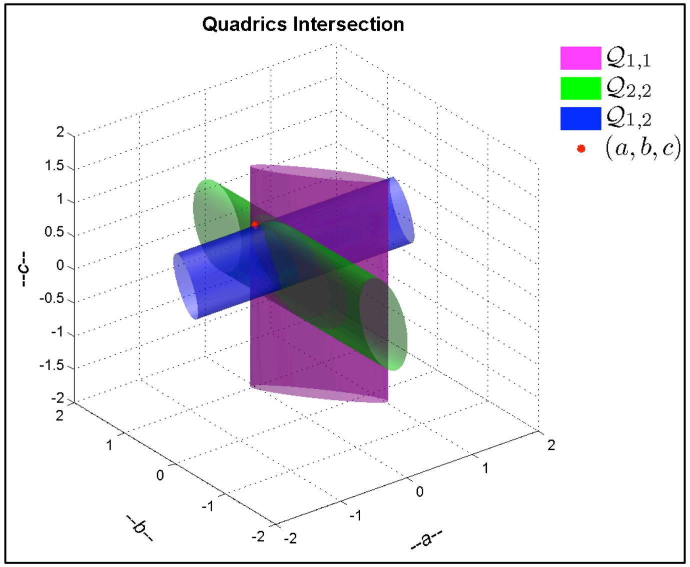

Fig. 9: Example of the quadrics intersection.

- Decompose $\mathbf{H}$ into $\mathbf{R}$ and $\mathbf{t}$:

$$\mathbf{R} = \left[\begin{array}{ccc} \mathbf{h}_1 & \mathbf{h}_2 & (\mathbf{h}_1 \times \mathbf{h}_2) \end{array}\right]$$

$$\mathbf{t} = \widetilde{\zeta}^{(\mathcal{W})}\left(\mathbf{h}_3 - \mathbf{h}_1 \times \mathbf{h}_2\right)$$

- And, to conclude, undo the predefined transformation:

$$\mathbf{R}_{\text{out}} = \mathbf{R}\widetilde{\mathbf{R}}$$

$$\mathbf{t}_{\text{out}} = \mathbf{R}\widetilde{\mathbf{t}} + \mathbf{t}$$

Experimental Results
Computation time

- Our: denotes our method with iterative estimation for the basis of $\mathbf{H}$;
- Our – CfN: is the same for analytical estimation of the basis;
- Nister-Stewenius: is the direct application of their method;
- Chen & Chang: denotes their method for iterative SVD;
- Chen & Chang – CfSVD: is the same for analytical SVD.

Algorithms	**Our**	**Our - CfN**	**Nister & Stewenius**	**Chen & Chang**	**Chen & Chang - CfSVD**
Pre-computation	*40 us*	*0 us*	*0 us*	*0 us*	*0 us*
Solver	*43 us*	*43 us*	*43 us*	*43 us*	*43 us*
Pos-computation	*0 us*	*0 us*	*0 us*	*19 us*	*0 us*

Tab. 1: We only considered the main computational steps - N is the number of possible solutions.

Experimental Results

Numerical Results and Number of Solutions

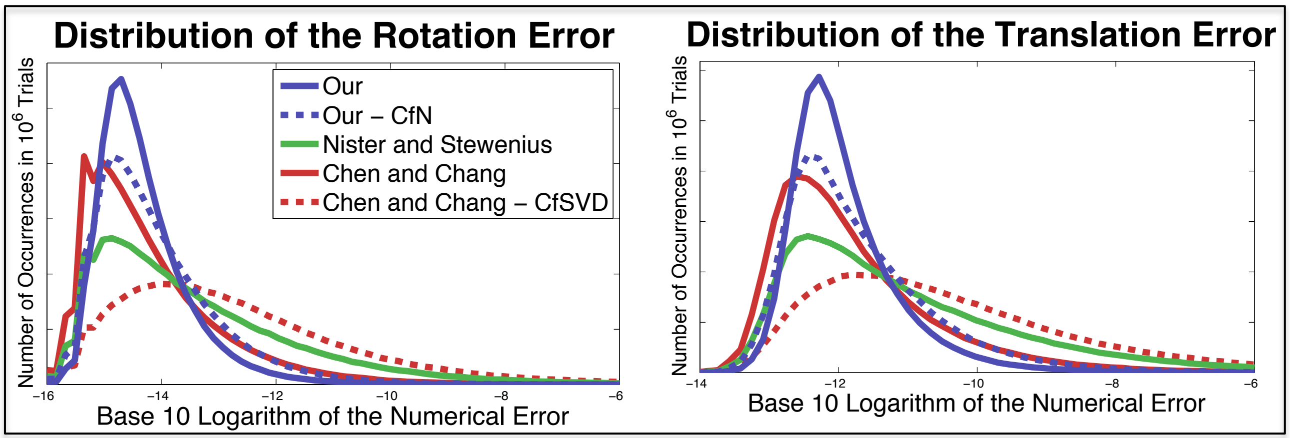

Fig. 10: Evaluation of the numerical accuracy.

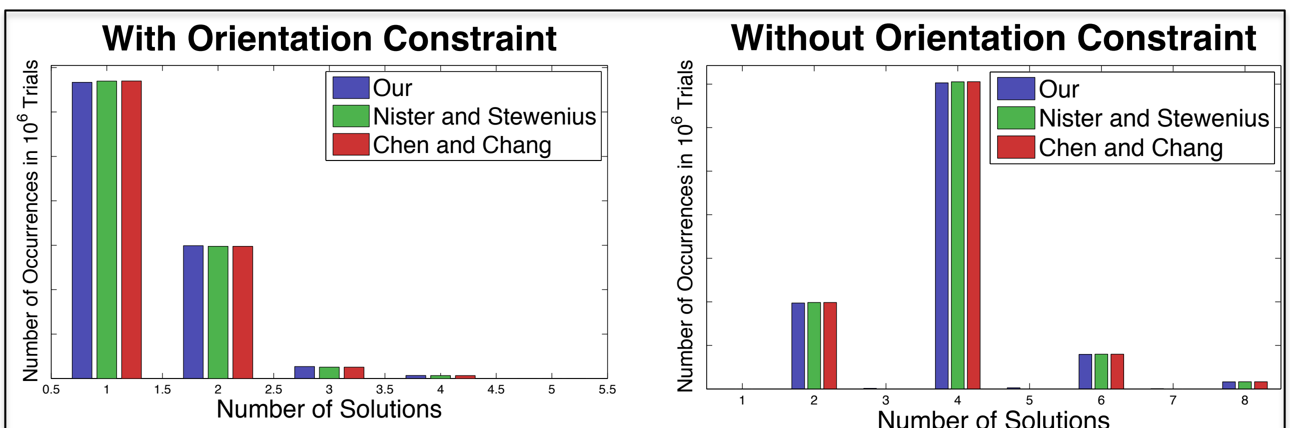

Fig. 11: Results for the number of solutions.

Experimental Results

Critical Configurations (1)

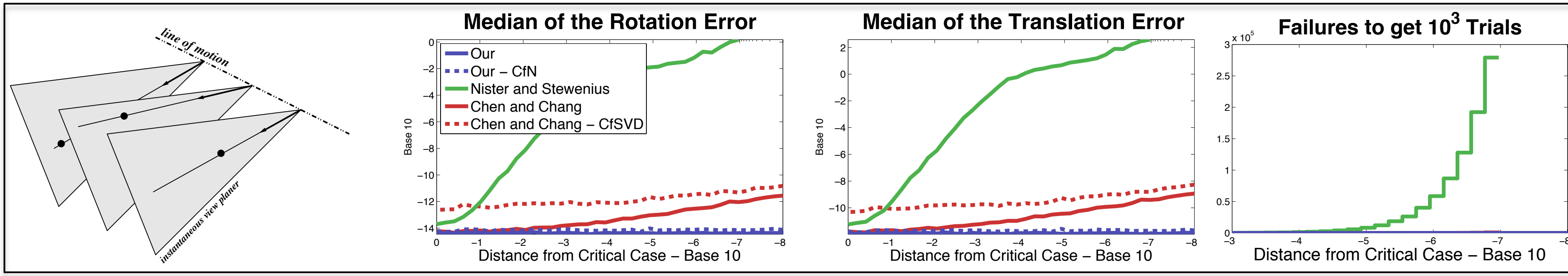

Fig. 12: Robustness and number of failures to configurations close to *pushbroom* cameras.

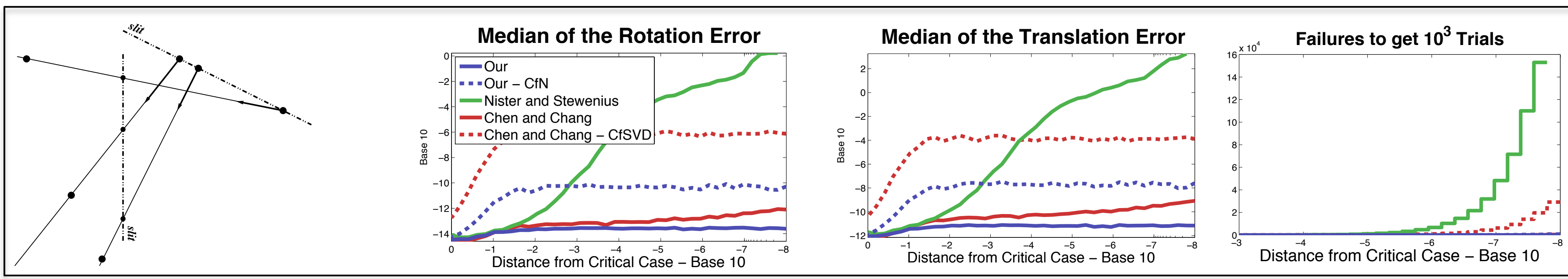

Fig. 13: Robustness and number of failures to configurations close to *X-slit* cameras.

Experimental Results
Critical Configurations (2)

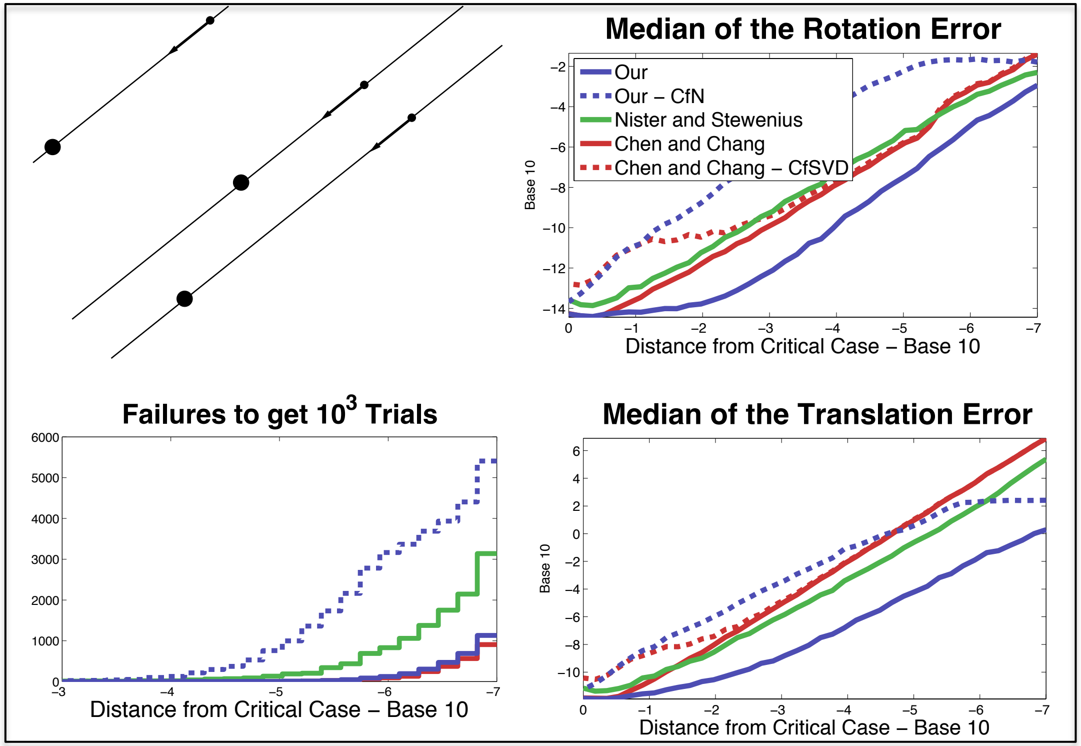

Fig. 14: Robustness and number of failures to configurations close to the degenerative orthogonal configuration.

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Thank You

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