A Level · Edexcel · Maths

3 hours · 42 questions

6.3 Modelling with Exponentials & Logarithms

6.3.1 Exponential Growth & Decay / 6.3.2 Using Exps & Logs in Modelling / 6.3.3 Using Log Graphs in Modelling

Easy (8 questions)	/36
Medium (11 questions)	/53
Hard (11 questions)	/55
Very Hard (12 questions)	/62
Total Marks	**/206**

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Easy Questions

1 State whether the following functions could represent exponential growth or exponential decay.

(i) $\mathrm{f}(x) = 5\mathrm{e}^{2x}$

(ii) $\mathrm{f}(\mathrm{t}) = 100\mathrm{e}^{-\mathrm{t}}$

(iii) $\mathrm{f}(\mathrm{a}) = 20\mathrm{e}^{-\mathrm{ka}}, \ \mathrm{k} > 0$

(iv) $\mathrm{f}(\mathrm{t}) = \mathrm{A}\mathrm{e}^{\mathrm{kt}}, \ \mathrm{A}, \mathrm{k} > 0$

(4 marks)

2 Write the following in the form ,e^{kx} where k is a constant and $k > 0$.

(i) $e^{3x} \times e^{2x}$

(ii) 5^{x}

(iii) 2^{x}

(3 marks)

3 Write the following in the form e^{-kx}, where k is a constant and $k > 0$.

(i) $\dfrac{e^{-2x}}{e^{4x}}$

(ii) $\left(\dfrac{1}{5}\right)^x$

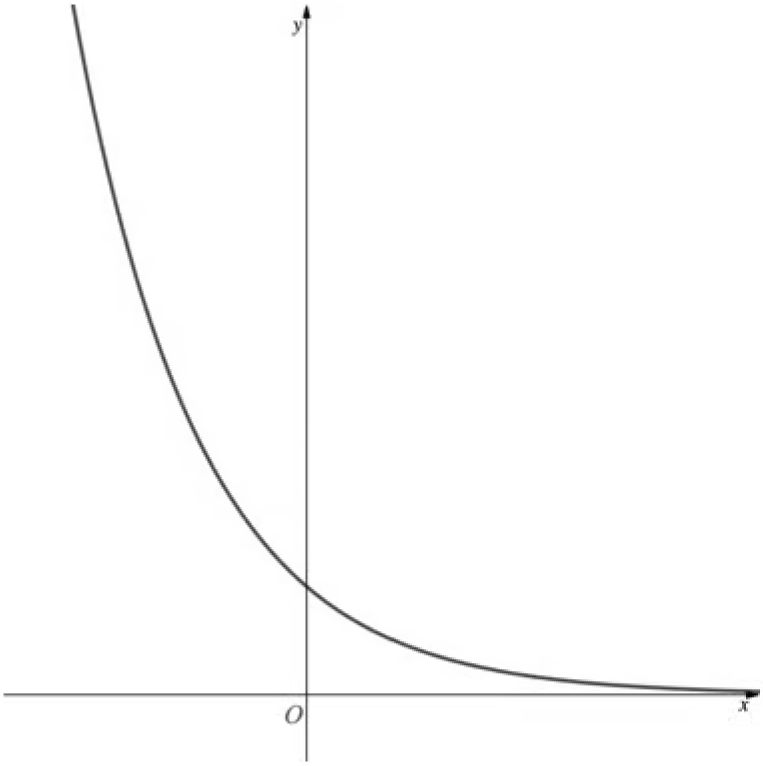
(iii) $\left(\dfrac{1}{2}\right)^x$

(3 marks)

4 The diagram below shows a sketch of the graph of $y = e^{-x}$.

On the diagram, add the graph of $y = e^{-2x}$ labelling the point at which the graph intersects the y-axis.

Write down the equation of any asymptotes on the graph.



(3 marks)

5 (a) By taking logarithms (base e) of both sides show that the equation

$$y = Ae^{kx}$$

can be written in the form $\ln y = kx + \ln A$

(3 marks)

(b) Hence ...

(i) write the equation $y = 2e^{0.01x}$ in the form $\ln y = kx + \ln A$.

(ii) write the equation $\ln y = 0.3x + \ln 5$ in the form $y = Ae^{kx}$.

(4 marks)

6 (a) In an effort to prevent extinction scientists released 24 rare birds into a newly constructed nature reserve.

The population of birds, within the reserve, is modelled by

$$B = Ae^{0.4t}$$

B is the number of birds after t years of being released into the reserve.

A is a constant.

Write down the value of A.

(1 mark)

(b) According to this model, how many birds will be in the reserve after 2 years?

(2 marks)

(c) How many years after release will it take for the population of birds to double?

(2 marks)

7 (a) A simple model for the acceleration of a rocket, $A \text{ ms}^{-2}$, is given as

$$A = 10e^{0.1t}$$

where t is the time in seconds after lift-off.

What is the meaning of the value 10 in the model?

(1 mark)

(b) Find the acceleration of the rocket 15 seconds after lift-off.

(2 marks)

(c) Find how long it takes for the acceleration to reach 100 ms^{-2}.

(3 marks)

8 (a) An exponential growth model for the number of bacteria in an experiment is of the form $N = Ae^{kt}$

N is the number of bacteria and t is the time in hours since the experiment began.

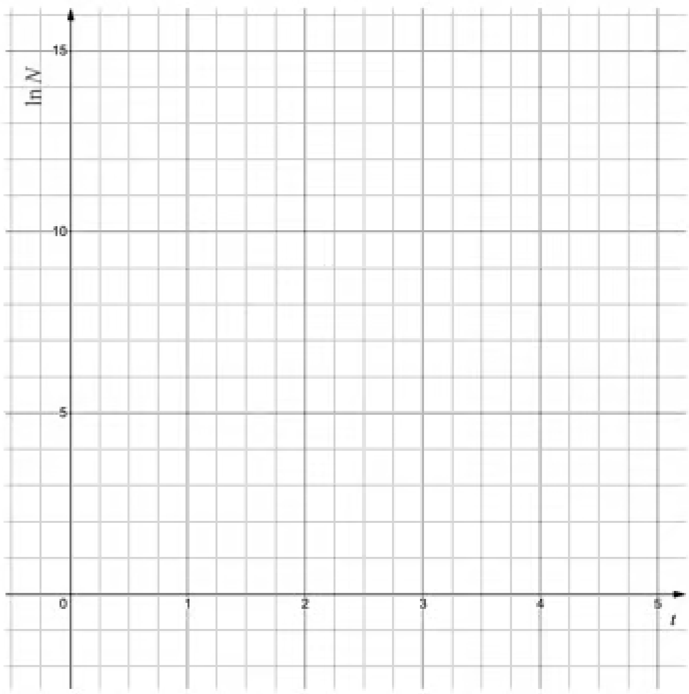
A are k constants.

A scientist records the number of bacteria every hour for 3 hours.

The results are shown in the table below.

t,hours	0	1	2	3	4
N, no. of bacteria	100	210	320	730	1580
$\ln N$ (3SF)	4.61	5.35	5.77	6.59	7.37

Plot the observations on the graph below - plotting $\ln N$ against t.



(1 mark)

(b) Using the points (0, 4.61) and (4, 7.37), find an equation for a line of best fit in the form $\ln N = mt + \ln c$, where m and c are constants to be found.

(2 marks)

(c) Hence estimate the values of A and k.

(2 marks)

Medium Questions

1 (a) Write $\left(\frac{1}{3}\right)^x$ in the form e^{kx}.

(1 mark)

(b) Write $\left(\frac{2}{7}\right)^t$ in the form e^{kt}.

State whether this would represent exponential growth or exponential decay.

(2 marks)

2 (a) Write $\left(\frac{7}{10}\right)^x$ in the form e^{-kx}.

(1 mark)

(b) Sketch the graph of $y = \left(\frac{7}{10}\right)^x$.

State the coordinates of the y-axis intercept.

Write down the equation of the asymptote.

(2 marks)

3 (a) By taking logarithms (base e) of both sides show that the equation

$$y = 5e^{0.1x}$$

can be written as

$$\ln y = 0.1x + \ln 5$$

(1 mark)

(b) Given $y = Ae^{kx}$ and $\ln y = 4.1x + \ln 8$, find the values of A and k.

(2 marks)

4 (a) By taking logarithms (base 10) of both sides show that the equation

$$y = 2x^{3.2}$$

can be written as

$$\log y = 3.2 \log x + \log 2$$

(1 mark)

(b) Given $y = Ax^b$ and $\log y = 1.8 \log x + \log 5$, find the values of A and b.

(2 marks)

5 (a) By taking logarithms (base 2) of both sides show that the equation

$$y = 3 \times 2^{4x}$$

can be written as

$$\log_2 y = 4x + \log_2 3$$

(1 mark)

(b) Given $y = Ab^{kx}$ and $\log_3 y = 5x + \log_3 7$, find the values of A, b and k.

(2 marks)

6 (a) In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.

The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85\,t}$$

B is the number of birds after t years of being released into the reserve.

Write down the number of birds the scientists released into the nature reserve.

(1 mark)

(b) According to this model, how many birds will be in the reserve after 3 years?

(2 marks)

(c) How long will it take for the population of birds within the reserve to reach 500?

(2 marks)

7 (a) A simple model for the acceleration of a rocket, $A\ ms^{-2}$, is given as

$$A = A_0 e^{0.2t}$$

where t is the time in seconds after lift-off. A_0 is a constant.

What does the constant A_0 represent?

(1 mark)

(b) After 10 seconds, the acceleration is $20\ ms^{-2}$.
Find the value of A_0.

(2 marks)

(c) Find how long it takes for the acceleration of the rocket to reach $100\ \mathrm{ms}^{-2}$

(2 marks)

8 (a) Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.

The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_0 and k are constants.

For an object initially containing 100g of carbon-14, write down the value of m_0.

(1 mark)

(b) Briefly explain why, if $m_0 = 100, m$ will equal 50g when $t = 5700$ years.

(2 marks)

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

(2 marks)

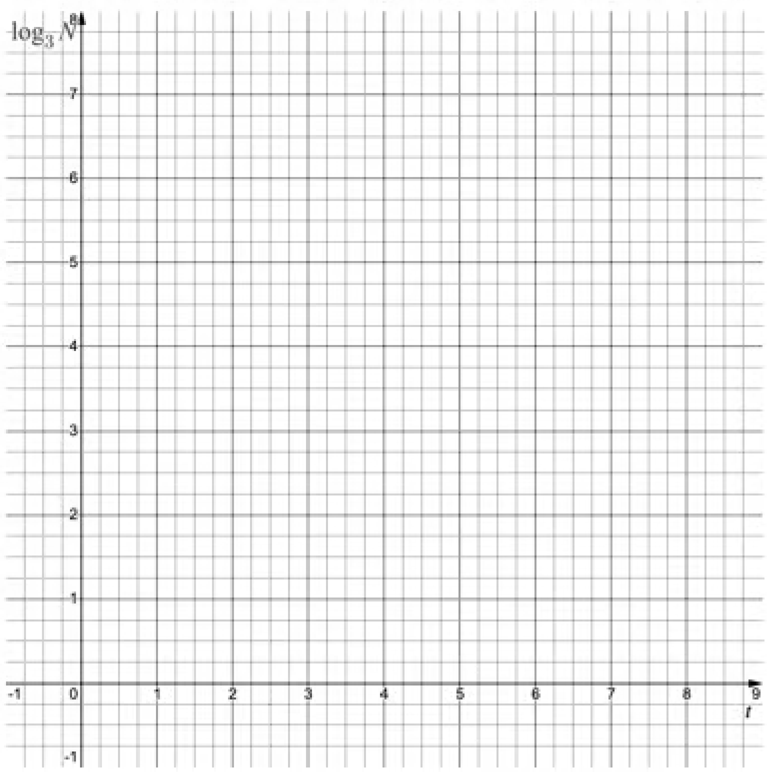
(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

(2 marks)

9 (a) An exponential growth model for the number of bacteria in an experiment is of the form $N = N_0\, a^{kt}$. N is the number of bacteria and t is the time in hours since the experiment began. N_0, a and k are constants. A scientist records the number of bacteria at various points over a six-hour period. The results are shown in the table below.

t, hours	0	2	4	6
N, no. of bacteria	100	180	340	620
$\log_3 N$ (3SF)	4.19	4.73	5.31	5.85

Plot the observations on the graph below - plotting $\log_3 N$ against t.



(1 mark)

(b) Using the points (0, 4.19) and (6, 5.85), find an equation for a line of best fit in the form $\log_3 N = mt + \log_3 c$, where m and c are constants to be found.

(2 marks)

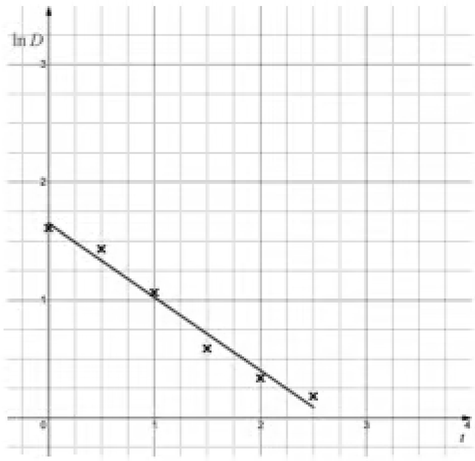
(c) The equation $N = N_0\ a^{kt}$ can be written in the form $\log_a N = kt + \log_a N_0$.

Use your answer to part (b) to estimate the values of , N_0, a , and k.

(2 marks)

10 (a) An exponential model of the form $D = Ae^{-kt}$ is used to model the amount of a pain-relieving drug (D mg/ml) there is in a patient's bloodstream, t hours after the drug was administered by injection. A and k are constants.

The graph below shows values of $\ln D$ plotted against t with a line of best fit drawn.



(i) Use the graph and line of best fit to estimate $\ln D$ at time $t = 0$.

(ii) Work out the gradient of the line of best fit.

(2 marks)

(b) Use your answers to part (a) to write down an equation for the line of best fit in the form $\ln D = mt + \ln c$, where m and c are constants.

(1 mark)

(c) Show that $D = Ae^{-kt}$ can be rearranged to give $\ln D = -kt + \ln A$

(1 mark)

(d) Hence find estimates for the constants A and k.

(2 marks)

(e) Find the time when the amount of the pain-relieving drug in the patient's bloodstream is 1.5 mg/ml.

(2 marks)

11 (a) A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.

£ P is the profit in the y^{th} year of business.

P_0 and k are constants.

Write down two equations connecting P_0 and k.

(2 marks)

(b) Find the values of P_0 and k.

(2 marks)

(c) Find the predicted profit for years 3 and 4.

(2 marks)

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y$$

(2 marks)

Hard Questions

1 (a) Write $\left(\frac{3}{5}\right)^x$ in the form e^{kx}, giving the value of k to three significant figures.

(2 marks)

(b) Write $\left(\frac{4}{7}\right)^{3t}$ in the form e^{kt}, giving the value of k to three significant figures.
State, and justify, whether this would represent exponential growth or decay.

(2 marks)

2 (a) Write $(0.7)^{x+1}$ in the form Ae^{-kx}.

(2 marks)

(b) Sketch the graph of $y = (0.7)^{x+1} - 3$.
State the coordinates of the y-axis intercept.
Write down the equation of the asymptote.

(2 marks)

3 (a) Show that the equation

$$x = 7e^{-0.2t}$$

can be written as

$$\ln x = \ln 7 - 0.2\text{t}$$

(2 marks)

(b) Rewrite the equation $\ln y = 4.1x + \ln 8$ in the form $y = Ae^{kx}$.

(2 marks)

4 (a) Show that the equation

$$y = 2x^{\frac{3}{4}}$$

can be written as

$$\log y = 0.75 \log x + \log 2$$

(2 marks)

(b) Rewrite the equation $\log y = 4.7 \log x + \log 12$ in the form $y = Ax^b$.

(2 marks)

5 (a) Show that the equation

$$y = 0.1 \times 2^{0.01x}$$

can be written as

$$\log_2 y = 0.01x - \log_2 10$$

(2 marks)

(b) Rewrite the equation $\log_3 y = 6.3x + \log_3 4$ in the form $y = Ab^{kx}$.

(2 marks)

6 (a) Scientists introduced a small number of rare breed deer to a large wildlife sanctuary.

The population of deer, within the sanctuary, is modelled by

$$D = 20e^{0.1t}$$

D is the number of deer after t years of first being introduced to the sanctuary.

Write down the number of deer the scientists introduced to the sanctuary.

(1 mark)

(b) How many years does it take for the deer population to double?

(2 marks)

(c) Give one criticism of the model for population growth.

(1 mark)

(d) The scientists suggest that the population of deer are separated after either 25 years or when their population exceeds 400.
Find the earliest time the deer should be separated.

(2 marks)

7 (a) A simple model for the acceleration of a rocket, $A \text{ ms}^{-2}$, is given as

$$A = 5e^{kt}$$

where t is the time in seconds after lift-off. k is a constant.

After 4 seconds the acceleration of the rocket is 10 ms^{-2}.
Find the value of k.

(2 marks)

(b) Find the time at which the acceleration of the rocket has increased by 200%.

(2 marks)

(c) Sketch the graph of the acceleration of the rocket, against time, stating the coordinates of the point that shows the initial acceleration of the rocket.

(2 marks)

8 (a) Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.

The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years

A model for the mass of carbon-14, y g, in an object originally containing 100 g, at time t years is

$$y = 100e^{-kt}$$

where k is a constant.

Find the value of k, giving your answer to three significant figures.

(2 marks)

(b) The object is considered as having no radioactivity once the mass of carbon-14 it contains falls below 0.5 g. Find out how old the object would have to be, to be considered non-radioactive.

(2 marks)

(c) A different object currently contains 25g of carbon-14.
In 500 years' time how much carbon-14 will remain in the object?

(2 marks)

9 (a) An exponential growth model for the number of bacteria in an experiment is of the form

$$N = N_0\, a^{kt}$$

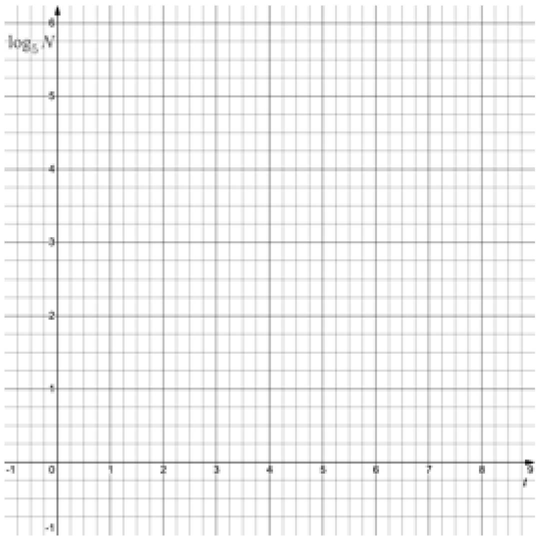
N is the number of bacteria and t is the time in hours since the experiment began. N_0, a and k are constants.

A scientist records the number of bacteria at various points over a six-hour period. The results are in the table below.

t, hours	0	2	4	6
N, no. of bacteria	200	350	600	1100

Plot the observations on the graph below - plotting $\log_5 N$ against t.

Draw a line of best fit.



(2 marks)

(b) Find an equation for your line of best fit in the form $\log_5 N = mt + \log_5 c$.

(2 marks)

(c) Estimate the values of ,N_0 ,a and k.

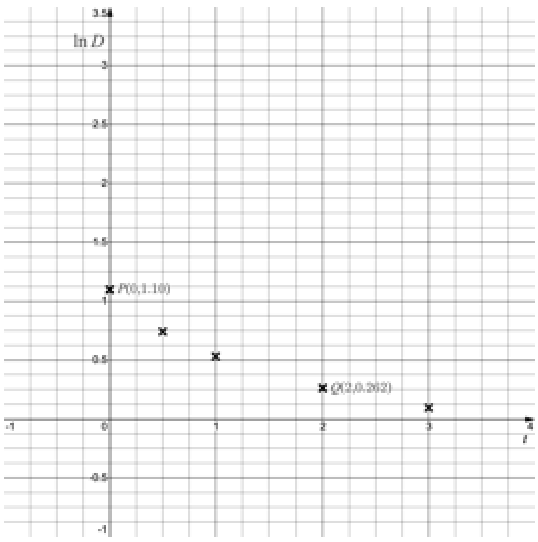
(2 marks)

10 (a) An exponential model of the form

$$D = Ae^{-kt}$$

is used to model the amount of a pain-relieving drug (D mg/ml) there is in a patient's bloodstream, t hours after the drug was administered by injection. A and k are constants.

The graph below shows values of $\ln D$ plotted against t



Using the points marked P and Q, find an equation for the line of best fit, giving your answer in the form $\ln D = mt + \ln c$, where m and c are constants to be found.

(2 marks)

(b) Hence find estimates for the constants A and k

(2 marks)

(c) The patient is allowed a second injection of the drug once the amount of drug in the bloodstream falls below 1% of the initial dose.
Find, to the nearest minute, how long until the patient is allowed a second injection of the drug.

(2 marks)

11 (a) The annual profits, in thousands of pounds, of a small company in the first 4 years of business are given in the table below.

a, years in business	1	2	3	4
P,annual profit	£3100	£4384	£5369	£6200

Using this data the company uses the model

$$P = P_1 a^k$$

to predict future years' profits. P_1 and k are constants.

Use data from the table to find the values of P_1 and k.

(2 marks)

(b) Show that $\log P = k \log a + \log P_1$, where P_1 and k take the values found in part (a).

(2 marks)

(c) State a potential problem with using the model to predict the profit in the company's 12$^{\text{th}}$ year of business.

(1 mark)

Very Hard Questions

1 (a) Write $(0.8)^x$ in the form e^{kx} , giving the value of k to three significant figures.

(1 mark)

(b) (i) Write $\left(\frac{2}{3}\right)^{4t+1}$ in the form Ae^{kt}, giving the values of A and k to three significant figures where necessary.

(ii) State, and justify, whether this would represent exponential growth or decay.

(iii) Write down the initial value of Ae^{kt}.

(2 marks)

2 Sketch the graph of $y=\left(\frac{3}{5}\right)^{2x+1}-4$.

State the coordinates of any points where the graph intercepts the coordinate axes.

Write down the equations of any asymptotes.

(4 marks)

3 (a) Rewrite the equation $\ln x = 2t + \ln 6$ in the form $x = Ae^{kt}$.

(2 marks)

(b) Sketch the graph of $\ln\ = 2t + \ln 6$ by plotting $\ln x$ against t.

(2 marks)

4 (a) Rewrite the equation $y = 3.6x^{-0.4}$ in the form $\log y = \log A - b\log x$

(2 marks)

(b) Sketch the graph of $\log y$ against $\log x$.

(2 marks)

5 (a) Rewrite the equation $y = \frac{2}{3} \times 5^{-0.2x}$ in the form $\log_b y = \log_b p - qx$ where b is an integer and p and q are rational numbers.

(3 marks)

(b) Sketch the graph of $\log_b y$ against x.

(2 marks)

6 (a) Scientists introduced a small number of apes into a previously unpopulated forest.

The population of apes in the forest is modelled by

$$A = 16e^{km}$$

where A is the number of apes after m months of first being introduced to the forest.

State, with a reason, whether you would expect the value of k to be positive or negative.

(2 marks)

(b) After 8 months, the number of apes in the forest has increased by 50%.
Find the value of k.

(2 marks)

(c) Scientists believe the forest cannot sustain a population of apes greater than 3000.
What length of time is the model for the population of the apes reliable for?

(2 marks)

7 (a) A manufacturer claims their flask will keep a hot drink warm for up to 7 hours.

In this sense, warm is considered to be $50°\mathrm{C}$ or higher.

Assuming a hot drink is made at $85°\mathrm{C}$ and its temperature inside the flask is $50°\mathrm{C}$ after exactly 7 hours, find:

(i) a linear model for the temperature of the drink inside the flask of the form $T = a + bt$, and

(ii) an exponential model for the temperature of the drink inside the flask of the form $T = Ae^{-kt}$

where $T°\mathrm{C}$ is the temperature of the drink in the flask after t hours and a, b, A and k are constants.

(4 marks)

(b) Compare the rate of change of the temperature of the drink inside the flask of both models after 3 hours.

(2 marks)

(c) A user of the flask suggests that hot drinks are only kept warm for 5 hours. Suggest a reason why the user's experience may not be up to the claims of the manufacturer.

(1 mark)

8 (a) A simple model for the acceleration of a rocket, $A\ \mathrm{ms}^{-1}$, is given as

$$A = Re^{kt}$$

where t is the time in seconds after lift-off. R and k are constants.

Negative time is often used in rocket launches as a way of counting down until lift off. Despite this the model above is still not suitable for use with negative t values
Briefly explain why.

(2 marks)

(b) After 5 seconds the acceleration of the rocket is $12\ \mathrm{ms}^{-2}$ and after 20 seconds its acceleration is $50\ \mathrm{ms}^{-2}$. Find the values of R and k.

(3 marks)

(c) A space enthusiast suggests that a linear model (of the form $A = R + ct$) would be more suitable.
Using the figures in (b), explain why the enthusiast's model is unrealistic.

(1 mark)

9 (a) Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.

The time it takes carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object, at time t years is

$$m = M_0 e^{-kt}$$

where M_0 and k are constants.

Briefly explain the meaning of the constant M_0.

(1 mark)

(b) Find the value of k, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers to be found.

(3 marks)

(c) An object currently contains 200 g of carbon-14. In 20 000 years' time, how much carbon-14, to the nearest gram, remains in the object?

(2 marks)

(d) The half-life of carbon-14 is believed to only be accurate to ±40 years.
A fossilised bone currently contains 3×10^{-6} g of carbon-14.
It is estimated the bone would have originally contained 1×10^{-2} g of carbon-14.

Find upper and lower estimates for the age of the bone, giving your answers to two significant figures.

(3 marks)

10 (a) An exponential growth model for the number of bacteria in an experiment is of the form

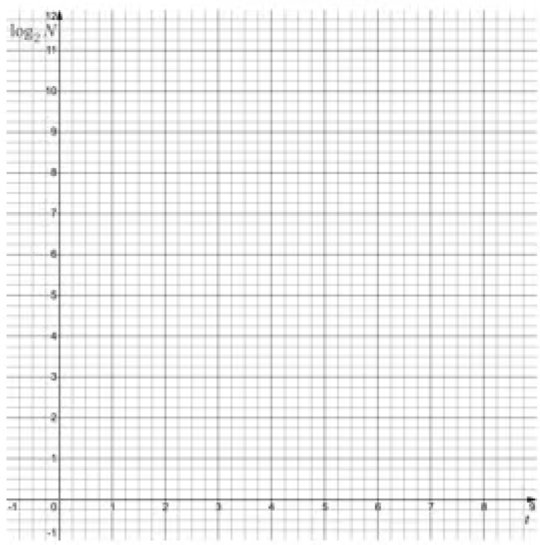
$$N = N_0 a^{kt}$$

N is the number of bacteria and t is the time in hours since the experiment began. N_0, a and k are constants.

A scientist records the number of bacteria at various points over a six-hour period. The results are in the table below.

t, hours	0	1.5	3	4.5	6
N, no. of bacteria	120	190	360	680	1230

By plotting $\log_2 N$ against t, drawing a line of best fit and finding its equation, estimate the values of N_0, a, and k.



(3 marks)

(b) What does the model predict for the value of N after twelve hours?
Comment on the reliability of this prediction.

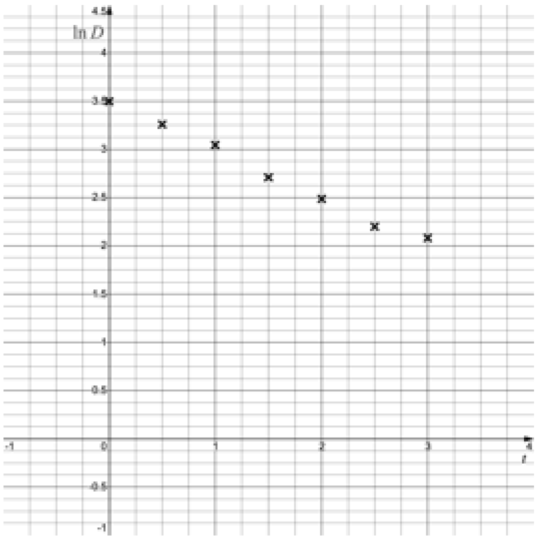
(2 marks)

11 (a) An exponential model of the form

$$D = Ae^{-kt}$$

is used to model the concentration of a pain-relieving drug (D mg/ml) in a patient's bloodstream t hours after the drug was administered by injection. A and k are constants.

The graph below shows values of $\ln D$ plotted against t



Find estimates for the constants A and k.

(3 marks)

(b) Find the time, to the nearest minute, at which the rate of decrease of the concentration of the drug in the patient's bloodstream is 12 mg/ml/hour.

(2 marks)

12 (a) The annual profits, in thousands of pounds, of a small company in the first 4 years of business are given in the table below.

a, years in business	1	2	3	4
$\log P$(£P is annual profit)	3.74	3.86	3.94	4.01

Using this data the company uses the model

$$P = P_1 a^k$$

to predict future years' profits. P_1 and k are constants.

Use the results in the table to estimate the values of P_1 and k.

(3 marks)

(b) Many new companies make a loss in their first year of business.
Briefly explain why, in such circumstances, a model of the form used above would not be suitable.

(1 mark)