A Level · Edexcel · Further Maths

1 hour 5 questions

8.2 Second Order Differential Equations

8.2.1 Solving Second Order Differential Equations / 8.2.2 Coupled First Order Linear Equations

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Total Marks **/78**

1 (a) At the start of the year 2000, a survey began of the number of foxes and rabbits on an island.

At time t years after the survey began, the number of foxes, f, and the number of rabbits, r, on the island are modelled by the differential equations

$$\frac{\mathrm{d}f}{\mathrm{d}t} = 0.2f + 0.1r$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -0.2f + 0.4r$$

(a) Show that $\frac{\mathrm{d}^2 f}{\mathrm{d}t^2} - 0.6\frac{\mathrm{d}f}{\mathrm{d}t} + 0.1f = 0$

(3 marks)

(b) (b) Find a general solution for the number of foxes on the island at time t years.

(4 marks)

(c) (c) Hence find a general solution for the number of rabbits on the island at time t years.

(3 marks)

(d) At the start of the year 2000 there were 6 foxes and 20 rabbits on the island.

(d) (i) According to this model, in which year are the rabbits predicted to die out?

(ii) According to this model, how many foxes will be on the island when the rabbits die out?

(iii) Use your answers to parts (i) and (ii) to comment on the model.

(7 marks)

2 (a) Two compounds, X and Y, are involved in a chemical reaction. The amounts in grams of these compounds, t minutes after the reaction starts, are x and y respectively and are modelled by the differential equations

$$\frac{dx}{dt} = -5x + 10y - 30$$

$$\frac{dy}{dt} = -2x + 3y - 4$$

(a) Show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 50$$

(3 marks)

(b) (b) Find, according to the model, a general solution for the amount in grams of compound X present at time t minutes.

(6 marks)

(c) (c) Find, according to the model, a general solution for the amount in grams of compound Y present at time t minutes.

(3 marks)

(d) Given that x = 2 and y = 5 when t = 0

(d) find

(i) the particular solution for x,

(ii) the particular solution for y.

(4 marks)

(e) A scientist thinks that the chemical reaction will have stopped after 8 minutes.

(e) Explain whether this is supported by the model.

(1 mark)

3 (a) A scientist is investigating the concentration of antibodies in the bloodstream of a patient following a vaccination.

The concentration of antibodies, x, measured in micrograms (μg) per millilitre (ml) of blood, is modelled by the differential equation

$$100\frac{d^2x}{dt^2} + 60\frac{dx}{dt} + 13x = 26$$

where t is the number of weeks since the vaccination was given.

(a) Find a general solution of the differential equation.

(4 marks)

(b) Initially,

- there are no antibodies in the bloodstream of the patient
- the concentration of antibodies is estimated to be increasing at 10 $\mu g / ml$ per week

(b) Find, according to the model, the maximum concentration of antibodies in the bloodstream of the patient after the vaccination.

(8 marks)

(c) A second dose of the vaccine has to be given to try to ensure that it is fully effective. It is only safe to give the second dose if the concentration of antibodies in the bloodstream of the patient is less than 5 $\mu g / ml$.

(c) Determine whether, according to the model, it is safe to give the second dose of the vaccine to the patient exactly 10 weeks after the first dose.

(2 marks)

4 (a) Scientist is studying the effect of introducing a population of white-clawed crayfish into a population of signal crayfish.

At time t years, the number of white-clawed crayfish, w, and the number of signal crayfish, s, are modelled by the differential equations

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{5}{2}(w - s)$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{2}{5}w - 90\mathrm{e}^{-t}$$

(a) Show that

$$2\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} - 5\frac{\mathrm{d}w}{\mathrm{d}t} + 2w = 450\mathrm{e}^{-t}$$

(3 marks)

(b) (b) Find a general solution for the number of white-clawed crayfish at time t years.

(6 marks)

(c) (c) Find a general solution for the number of signal crayfish at time t years.

(2 marks)

(d) The model predicts that, at time T years, the population of white-clawed crayfish will have died out.

Given that w = 65 and s = 85 when t = 0

(d) find the value of T, giving your answer to 3 decimal

(6 marks)

(e) (e) Suggest a limitation of the model.

(1 mark)

5 (a) An engineer is investigating the motion of a sprung diving board at a swimming pool.
Let E be the position of the end of the diving board when it is at rest in its equilibrium position and when there is no diver standing on the diving board.
A diver jumps from the diving board.
The vertical displacement, h cm, of the end of the diving board above E is modelled by the differential equation

$$4\frac{\mathrm{d}^2h}{\mathrm{d}t^2} + 4\frac{\mathrm{d}h}{\mathrm{d}t} + 37h = 0$$

where t seconds is the time after the diver jumps.

(a) Find a general solution of the differential equation.

(2 marks)

(b) When t = 0, the end of the diving board is 20 cm below E and is moving upwards with a speed of 55 cm s^{-1}

(b) Find, according to the model, the maximum vertical displacement of the end of the diving board above E.

(8 marks)

(c) (c) Comment on the suitability of the model for large values of t.

(2 marks)