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A Level · Edexcel · Maths

2 hours 25 questions

1.2 Proof by Contradiction (A Level only)

Easy (6 questions)	/20
Medium (6 questions)	/26
Hard (6 questions)	/27
Very Hard (7 questions)	/34
Total Marks	**/107**

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Easy Questions

1 Find the prime factorisation of the following numbers

(i) 100
(ii) 120

(4 marks)

2 State whether the following are rational or irrational quantities.

For those that are rational, write them in the form $\frac{a}{b}$, where a and b are integers and $\frac{a}{b}$ is in its simplest terms.

(i) $\sqrt{2}$
(ii) $\ln 3$
(iii) $\frac{4\sqrt{2}}{\sqrt{18}}$
(iv) $\frac{3 \ln 2}{\ln 32}$

(4 marks)

3 Prove by contradiction that the sum of two consecutive integers is odd.

(3 marks)

4 Prove by contradiction that the product of two odd numbers is odd.

(3 marks)

5 Prove by contradiction that if x is even, then x^2 must be even.

(3 marks)

6 Prove by contradiction that there is an infinite number of multiples of 10.

(3 marks)

Medium Questions

1 Prove by contradiction that if x^2 is odd, then x must be odd.

(3 marks)

2 (a) When a number is rational, it can be written in the form $\frac{a}{b}$.

(i) Write down the condition that a and b must satisfy.

(ii) Write down a further condition on b.

(2 marks)

(b) Two numbers can be written in the form $\frac{p}{q}$ and $\frac{r}{s}$ such that p, q, r and s meet the necessary conditions so that the two numbers are rational.

Prove that the product of these numbers is also rational.

(3 marks)

3 Prove by contradiction that there are an infinite number of even numbers.

(4 marks)

4 (a) A student is attempting to answer the following exam question:

"Prove by contradiction that $\sqrt{2}$ is an irrational number. You may use without proof the fact that if a number n^2 is even, then n must also be even."

The student's proof proceeds as follows:

Line 1: Assume $\sqrt{2}$ is a rational number. Therefore, it can be written in the form $\sqrt{2} = \frac{a}{b}$, where a and b are integers with $b \neq 0$, and where a and b may be assumed to have no common factors.

Line 2: Squaring both sides: $4 = \frac{a^2}{b^2}$

Line 3: $\therefore\ a^2 = 2b^2$

Line 4:

Line 5: Therefore $a = 2m$, for some integer m

Line 6: Then, $a^2 = (2m)^2 = 4m^2$

Line 7: $\therefore\ 2b^2 = 4m^2$

Line 8: $b^2 = 2m^2$

Line 9: So b^2 is even and therefore b is also even.

Line 10: It has been shown that both a and b are even, so they share a common factor of 2.

Line 11: This is a contradiction of the assumption that a and b have no common factors.

Line 12: Therefore, $\sqrt{2}$ is irrational.

There is an error within the first three lines of the proof.

State what the error is and write the correct line down.

(2 marks)

(b) Line 4 of the proof is missing.

Write down the missing line of the proof.

(2 marks)

5 (a) (i) How many distinct factors does a prime number have?

(ii) What can you say about the number of distinct factors a square number has?

(iii) If N is the square of a prime number, then excluding N itself, write down, in terms of N, the largest factor of N.

(3 marks)

(b) A composite number can be written uniquely as the product of its prime factors.i.e., any composite number N can be written uniquely as $N = p_1 \times p_2 \times p_3 \times \ldots$, where $p_1,\ p_2, p_3, \ldots$ are the prime factors of N.

Show that a composite number N may be written in the form $N = pq$, where q is an integer and p is a prime factor of N.

By expressing q in terms of the prime factors of N, be sure to explain why q must be an integer.

(2 marks)

6 Prove by contradiction that a triangle cannot have more than one obtuse angle.

(5 marks)

Hard Questions

1 Prove by contradiction that if x^3 is odd, then x must be odd.

(4 marks)

2 Prove that the product of two rational numbers is rational.

(4 marks)

3 Prove by contradiction that there are an infinite number of powers of 2.

(4 marks)

4 Prove by contradiction that $\sqrt{11}$ is an irrational number. You may use without proof the fact that if n^2 is a multiple of 11, then n is a multiple of 11.

(6 marks)

5 Below is a proof by contradiction that there is no largest multiple of 7.

Line 1: Assume there is a number, S, say, that is the largest multiple of 7.

Line 2: $S = 7k$

Line 3: Consider the number $S + 7$.

Line 4: $S + 7 = 7k + 7$

Line 5: $\therefore S + 7 = 7(k + 1)$

Line 6: So $S + 7$ is a multiple of 7.

Line 7: This is a contradiction to the assumption that S is the largest multiple of 7.

Line 8: Therefore, there is no largest multiple of 7.

The proof contains two omissions in its argument.

Identify both omissions and correct them.

(4 marks)

6 If a positive integer greater than 1 is not a prime number, then it is called a composite number. Prove by contradiction that any composite integer N has a prime factor less than or equal to $\sqrt{N}$.

(5 marks)

Very Hard Questions

1 Prove by contradiction that if x^n is odd, where $n \geq 2$ is a positive integer, then x must be odd.

(4 marks)

2 Prove that the difference between two rational numbers is rational.

(4 marks)

3 Prove by contradiction that there are an infinite number of prime numbers.

(5 marks)

4 Prove by contradiction that $\sqrt{k}$, where k is a prime number, is an irrational number. You may use without proof the fact that any positive integer may be written uniquely as a product of its prime factors.

(6 marks)

5 Below is a proof by contradiction that $\log_2 7$ is irrational.

Line 1: Assume $\log_2 7$ is a rational number. Therefore it can be written in the form $\log_2 7 = \frac{a}{b}$, where a and b are integers, and $b \neq 0$. As $\log_2 4 = 2$ and $\log_2 8 = 3$, we may assume as well that $a > b > 0$.

Line 2: $\therefore 2^{\frac{a}{b}} = 7$

Line 3: $\left(2^{\frac{a}{b}}\right)^b = 7^b$

Line 4: $2^b = 7^a$

Line 5: No power of 2 (all even) is equal to a power of 7 (all odd).

Line 6: $\therefore 2^a \neq 7^b$ unless $a = b = 0$ but this is a contradiction of the original assumption.

Line 7: $\therefore \log_2 7$ is irrational

The proof contains one mathematical error and one logical error.

Identify both errors and correct them.

(4 marks)

6 Prove by contradiction that the solutions to the equation $3x^2 + 10x - 8 = 0$ cannot be written in the form $\frac{a}{b}$ where a and b are both odd integers.

(5 marks)

7 Prove by contradiction that, if p, q, r and s are rational numbers and c is a positive non-square integer, then

$$p + q\sqrt{c} = r + s\sqrt{c}$$

implies that $p = r$ and $q = s$. You may use without proof the fact that for any positive non-square integer n, $\sqrt{n}$ is irrational.

(6 marks)