A Level · Edexcel · Maths

3 hours · 31 questions

7.2 Applications of Differentiation

7.2.1 Gradients, Tangents & Normals / 7.2.2 Increasing & Decreasing Functions / 7.2.3 Second Order Derivatives / 7.2.4 Stationary Points & Turning Points / 7.2.5 Sketching Gradient Functions / 7.2.6 Modelling with Differentiation inc. Optimisation

Easy (8 questions)	/39
Medium (8 questions)	/42
Hard (8 questions)	/46
Very Hard (7 questions)	/55
Total Marks	**/182**

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Easy Questions

1 (a) Find an expression for $\frac{dy}{dx}$ when $y = 3x^2 - 2x$.

(2 marks)

(b) Find the gradient of $y = 3x^2 - 2x$ at the points where

(I) $x = 3$,
(ii) $x = -2$.

(2 marks)

2 (i) Find the gradient of the tangent at the point (2 , 3) on the graph of $y = 2x^3 - 3x^2 - 1$.

(ii) Hence find the equation of the tangent at the point (2 , 3).

(5 marks)

3 (i) Find an expression for $f'(x)$ when $f(x) = x^3 + x^2 - 5x$.
(ii) Solve the equation $3x^2 + 2x - 5 = 0$.

(iii) Hence, or otherwise, find the values of x for which $f(x)$ is a decreasing function.

(6 marks)

4 (a) The curve C has equation $y = 3x^3 + 6x^2 - 5x + 1$.

Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(3 marks)

(b) (i) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = \frac{1}{3}$.

(ii) What does your answer to part (b) tell you about curve C at the point where $x = \frac{1}{3}$?

(4 marks)

5 (a) For the graph with equation $y = 3x - \frac{1}{2}x^2$, find the gradient of the tangent at the point where $x = 5$.

(2 marks)

(b) (i) Find the gradient of the normal at the point where $x = 5$.

(ii) Hence find the equation of the normal at the point where $x = 5$.

(3 marks)

6 Find the values of x for which $f(x) = 2x^2 - 16x$ is an increasing function.

(3 marks)

7 Find the x-coordinates of the stationary points on the curve with equation

$$y = \frac{1}{3}x^3 + \frac{5}{2}x^2 - 6x + 2.$$

(4 marks)

8 Show that the point (2 , 1) is a (local) maximum point on the curve with equation

$$y = 2x^2 - \frac{2}{3}x^3 - \frac{5}{3}.$$

(5 marks)

Medium Questions

1 Find the values of x for which $f(x) = -9x^2 + 5x - 3$ is an increasing function.

(3 marks)

2 Show that the function $f(x) = x^3 - 3x^2 + 6x - 7$ is increasing for all $x \in \mathbb{R}$.

(3 marks)

3 (a) The curve C has equation $y = 2x^3 - 3x^2 + 4x - 3$.

Show that the point $P(2, 9)$ lies on C.

(1 mark)

(b) Show that the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P is 16.

(3 marks)

(c) Find an equation of the tangent to C at P.

(2 marks)

4 (a) The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1,\ 1)$ lies on C.

Find an expression for $\frac{dy}{dx}$.

(2 marks)

(b) Show that an equation of the normal to C at point P is $x + 2y = 3$.

(3 marks)

(c) This normal cuts the x-axis at the point Q.

Find the length of PQ, giving your answer as an exact value.

(2 marks)

5 (a) Given that $y = 2x^3 - 8\sqrt{x}$, find

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

(2 marks)

(b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(2 marks)

6 (a) A curve has the equation $y = x^3 - 12x + 7$.

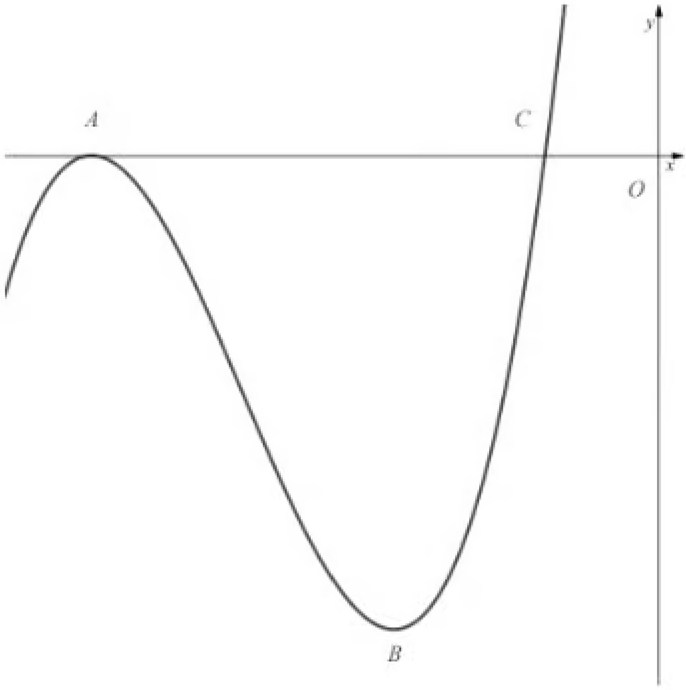
Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(3 marks)

(b) Determine the coordinates of the local minimum of the curve.

(3 marks)

7 (a) The diagram below shows part of the curve with equation $y = x^3 + 11x^2 + 35x + 25$.
The curve touches the x-axis at A and cuts the x-axis at C. The points A and B are stationary points on the curve.



Using calculus, and showing all your working, find the coordinates of A and B.

(5 marks)

(b) Show that (-1, 0) is a point on the curve and explain why those must be the coordinates of point C.

(2 marks)

8 (a) A company manufactures food tins in the shape of cylinders which must have a constant volume of $150\pi \text{ cm}^3$. To lessen material costs the company would like to minimise the surface area of the tins.

By first expressing the height h of the tin in terms of its radius r, show that the surface area of the cylinder is given by $S = 2\pi r^2 + \dfrac{300\pi}{r}$.

(2 marks)

(b) Use calculus to find the minimum value for the surface area of the tins. Give your answer correct to 2 decimal places.

(4 marks)

Hard Questions

1 Find the values of x for which $f(x) = x^3 - 5x^2 + 3x - 2$ is a decreasing function.

(5 marks)

2 Show that the function $f(x) = 7x^2 - 2x(x^2 + 5)$ is decreasing for all $x \in \mathbb{R}$.

(3 marks)

3 The curve C has equation $y = 3x^2 - 6x + \sqrt{2x}$. The point $P(2,\ 2)$ lies on C.

Find an equation of the tangent to C at P.

(5 marks)

4 The curve C has equation $y = \dfrac{9}{\sqrt{3x}} - \dfrac{3}{x}$. The point $P(3, 2)$ lies on C.

The normal to C at P intersects the x-axis at the point Q.

Find the coordinates of Q.

(6 marks)

5 (a) Given that $y = \frac{4}{x} - \sqrt[3]{\frac{27}{x}}$, find

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

(3 marks)

(b) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

(2 marks)

6 A curve has the equation $y = x(x+6)^2 + 4(3x+11)$.

The point $P(x, y)$ is the stationary point of the curve.

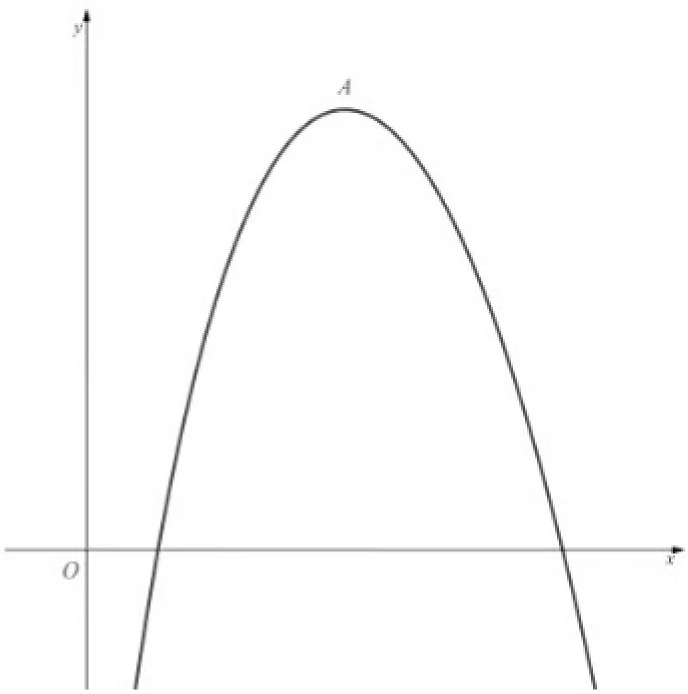
Find the coordinates of P and determine its nature.

(5 marks)

7 (a) The diagram below shows a part of the curve with equation $y = f(x)$, where

$$f(x) = 460 - \frac{x^3}{300} - \frac{8100}{x}, \qquad x > 0$$

Point A is the maximum point of the curve.



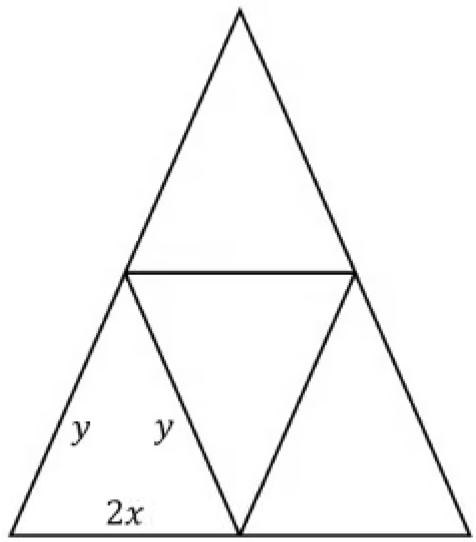
Find $f'(x)$.

(3 marks)

(b) Use your answer to part (a) to find the coordinates of point A.

(4 marks)

8 (a) A garden bed is to be divided by fencing into four identical isosceles triangles, arranged as shown in the diagram below:



The base of each triangle is $2x$ metres, and the equal sides are each y metres in length.

Although x and y can vary, the total amount of fencing to be used is fixed at P metres.

Explain why $0 < x < \dfrac{P}{6}$.

(1 mark)

(b) Show that

$$A^2 = \frac{4}{9}P^2x^2 - \frac{16}{3}Px^3$$

where A is the total area of the garden bed.

(4 marks)

(c) Using your answer to (b) find, in terms of P, the maximum possible area of the garden bed.

(4 marks)

(d) Describe the shape of the bed when the area has its maximum value.

(1 mark)

Very Hard Questions

1 Find the values of x for which $f(x) = 4x + \frac{3}{x}$ is a decreasing function, where $x \neq 0$.

(4 marks)

2 Show that the function $f(x) = \sqrt{x} - \frac{7}{\sqrt{x}}$, $x > 0$, is increasing for all x in its domain.

(4 marks)

3 (a) A curve has equation $y = 5 - (x - 3)^2$.

A is the point on the curve with x coordinate 0, and B is the point on the curve with x coordinate 6.

C is the point of intersection of the tangents to the curve at A and B.

Find the coordinates of point C.

(7 marks)

(b) Calculate the area of triangle ABC.

(2 marks)

4 (a) A curve is described by the equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1}{\sqrt{x}}, \quad x > 0$$

P is the point on the curve such that the normal to the curve at P also passes through the origin.

Find the coordinates of point P. Give your answer in the form $(2^a, 2^b)$, where a and b are rational numbers to be found.

(6 marks)

(b) Write down the equation of the normal to the curve at P.

(1 mark)

(c) Show that an equation of the tangent to the curve at P is

$$\left(2^{\frac{1}{3}}\right)x + \left(2^{\frac{5}{6}}\right)y = 3$$

(4 marks)

5 (a) A curve is described by the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 7 - 2x^2 + \sqrt{x}$, $x \geq 0$.

Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$.

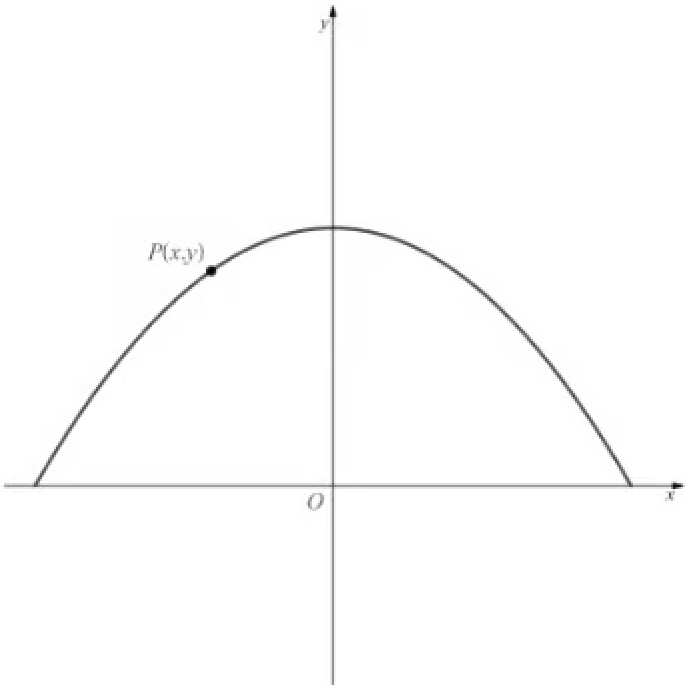
(3 marks)

(b) P is the stationary point on the curve.

Find the coordinates of P and determine its nature.

(4 marks)

6 (a) The diagram below shows the part of the curve with equation $y = 3 - \frac{1}{4}x^2$ for which $y > 0$. The marked point $P(x, y)$ lies on the curve. O is the origin.



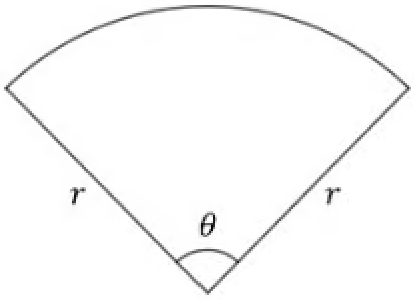
Show that $OP^2 = 9 - \frac{1}{2}x^2 + \frac{1}{16}x^4$.

(3 marks)

(b) Find the minimum distance from O to the curve, using calculus to prove that your answer is indeed a minimum.

(8 marks)

7 (a) The top of a patio table is to be made in the shape of a sector of a circle with radius r and central angle , where $0° < \theta < 360°$.



Although r and θ may be varied, it is necessary that the table have a fixed area of A m^2.

Explain why $r > \sqrt{\dfrac{A}{\pi}}$.

(2 marks)

(b) Show that the perimeter, P, of the table top is given by the formula

$$P = 2r + \frac{2A}{r}$$

(2 marks)

(c) Show that the minimum possible value for P is equal to the perimeter of a square with area A. Be sure to prove that your value is a minimum.

(5 marks)