A Level · Edexcel · Maths

3 hours · 32 questions

4.2 Binomial Distribution

4.2.1 The Binomial Distribution / 4.2.2 Calculating Binomial Probabilities

Easy (8 questions)	/43
Medium (8 questions)	/40
Hard (8 questions)	/46
Very Hard (8 questions)	/52
Total Marks	**/181**

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Easy Questions

1 (a) A random variable $X \sim B(20, 0.15)$

(i) Write down the name of this distribution

(ii) Write down the number of trials, n

(iii) Write down the probability of success, p.

(3 marks)

(b) Find:

(i) $\mathrm{P}(X=4)$

(ii) $\mathrm{P}(X \leq 1)$

(iii) $\mathrm{P}(X \geq 8)$

(4 marks)

2 (a) A biased coin has probability 0.8 of landing on heads. Sunita and Mark model the probabilities of obtaining X heads when the coin is tossed 10 times using the random variable $X \sim B(10, p_1)$..

(i) Explain why $p_1 = 0.8$ in this case.

(ii) Sunita decides to use her calculator to determine any probabilities. She determines $\mathrm{P}(X = 4)$ using the calculation

$$ {}^{10}C_4 \times (0.8)^4 (1 - 0.8)^{10-4} $$

Use Sunita's calculation to find $\mathrm{P}(X = 4)$ to four decimal places.

(3 marks)

(b) Mark decides to use statistical tables to determine any probabilities.

(i) Explain why Mark will not be able to use the random variable $X \sim B(10, 0.8)$ with statistical tables.

(ii) Mark says that instead of considering the number of heads obtained he will consider the number of tails obtained, Y, instead. He will use the random variable $Y \sim B(10, p_2)$. Find the value of p_2 and explain how you found it.

(3 marks)

(c) Sunita and Mark use their methods to calculate the probability that the coin lands on heads at least 9 times.

(i) Mark will use tables to find $P(Y \leq y)$. State the value of y and find $P(Y \leq y)$ from statistical tables, writing down all four decimal places given.

(ii) Sunita will use her calculator to find $P(X \geq 9)$. Using either a calculation similar to the one given in part (a)(ii) or the statistical features of your calculator, find $P(X \geq 9)$, to four decimal places.

(3 marks)

(d) Clearly stating the probabilities to be found, use both Sunita's and Mark's methods to find the probability that no more than 5 heads are obtained from the coin being tossed 10 times. Give both answers to four decimal places.

(2 marks)

3 A random variable $X \sim B(9, 0.6)$

Use either calculations of the form $\binom{n}{x} p^x(1-p)^{n-x}$, the statistical features on your calculator or statistical tables to find:

(i) $P(X = 5)$

(ii) $P(X \leq 1)$

(iii) $P(X \geq 8)$

Give your answers to four decimal places.

(4 marks)

4 (a) A snowboarder is trying to perform the Poptart trick.
The snowboarder has a success rate of 25% of completing the trick.

The snowboarder will model the number of times they can expect to successfully complete the Poptart trick, out of their next 12 attempts, using the random variable $X \sim B(12, 0.25)$.

(i) Give a reason why the model is suitable in this case.

(ii) Suggest a reason why the model may not be suitable in this case.

(2 marks)

(b) Using the model, find the probability that the snowboarder

(i) successfully completes the Poptart trick more than 3 times in their next 12 attempts

(ii) fails to successfully complete the trick on any of their next 12 attempts.

(2 marks)

5 A random variable $X \sim B(50, 0.05)$.
Use either the statistical features on your calculator or statistical tables to find:

(i) $\mathrm{P}(X = 4)$

(ii) $\mathrm{P}(X \leq 8)$

(iii) $\mathrm{P}(X \geq 7)$

Give your answers to four decimal places.

(4 marks)

6 A random variable $Y \sim B(25, 0.55)$..

Find:

(i) $\mathrm{P}(Y = 13)$

(ii) $\mathrm{P}(Y \leq 8)$

(iii) $\mathrm{P}(Y \geq 20)$

Give your answers to four decimal places.

(4 marks)

7 (a) A company manufacturing energy-saving light bulbs claims the mean lifetime of a bulb is 8000 hours. It is known from past quality assurance procedures that the probability of any particular light bulb having a lifetime of less than 5000 hours is 0.1.

A random sample of 30 light bulbs is taken.
The random variable $X \sim B(n, p)$ is used to model the probability that light bulbs in the sample last less than 5000 hours.

(i) Write down the values of n and p.

(ii) State how the situation meets the criterion "a fixed sample size" for a binomial distribution model.

(2 marks)

(b) Find the probability that

(i) exactly one light bulb

(ii) no more than three light bulbs

last less than 5000 hours.

(2 marks)

8 (a) Farmer Kate rears a herd of 50 alpacas. She takes a random sample of 8 alpacas and tests them for the disease Tuberculosis (TB). From previous testing of the herd Farmer Kate knows that any individual alpaca has a 95% chance of testing negative for Tuberculosis.

Let N represent the number of alpacas in Farmer Kate's sample that test negative for Tuberculosis.

(i) Write down the probability distribution that describes N.

(ii) Write down an alternative probability distribution that describes P, where P represents the number of alpacas in Farmer Kate's sample that test positive for Tuberculosis.

(2 marks)

(b) Find the probability that

(i) zero

(ii) more than 2

alpacas in Farmer Kate's sample test **positive** for Tuberculosis.

(3 marks)

Medium Questions

1 (a) A fair coin is tossed 20 times and the number of times it lands heads up is recorded.

Define a suitable distribution to model the number of times the coin lands heads up, and justify your choice.

(2 marks)

(b) Find the probability that the coin lands heads up 15 times.

(2 marks)

2 (a) For a jellyfish population in a certain area of the ocean, there is a 95% chance that any given jellyfish contains microplastic particles in its body.

State any assumptions that are required to model the number of jellyfish containing microplastic particles in their bodies in a sample of size n as a binomial distribution.

(2 marks)

(b) Using this model, for a sample size of 40, find the probability of

(i) exactly 38 jellyfish

(ii) all the jellyfish

having microplastic particles in their bodies.

(3 marks)

3 Giovanni is rolling a biased dice, for which the probability of landing on a two is 0.25. He rolls the dice 10 times and records the number of times that it lands on a two. Find the probability that

(i) the dice lands on a two 4 times

(ii) the dice lands on a two 4 times, with the fourth two occurring on the final roll.

(4 marks)

4 (a) For cans of a particular brand of soft drink labelled as containing 330 ml, the actual volume of soft drink in a can varies. Although the company's quality control assures that the mean volume of soft drink in the cans remains at 330 ml, it is known from experience that the probability of any particular can of the soft drink containing less than 320 ml is 0.0296.

Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans in the pack that contain less than 320 ml of soft drink.

Write down the probability distribution that describes L.

(2 marks)

(b) Find the probability that

(i) none of the cans

(ii) exactly two of the cans

(iii) at least two of the cans

contain less than 320 ml of soft drink.

(4 marks)

5 The random variable $X \sim B(40, 0.15)$. Find:

(i) $\mathrm{P}(X < 10)$

(ii)

$P(X > 7)$

(iii) $P(3 \leq X < 14)$

(iv) $P(5 < X < 12)$

(4 marks)

6 The random variable $X \sim B(40, 0.25)$. Find:

(i) the largest value of k such that $P(X < k) < 0.10$

(ii) the smallest value of r such that $P(X \geq r) < 0.05$

(iii) the largest value of s such that $P(X > s) > 0.95$.

(5 marks)

7 In an experiment, the number of specimens testing positive for a certain characteristic is modelled by the random variable $X \sim B(50, 0.35)$. Find the probability of

(i) fewer than 20

(ii) no more than 20

(iii) at least 20

(iv) at most 20

(v) more than 20

of the specimens testing positive for the characteristic.

(5 marks)

8 (a) In the town of Wooster, Ohio, it is known that 90% of the residents prefer the locally produced Woostershire brand sauce when preparing a Caesar salad. The other 10% of residents prefer another well-known brand.

30 residents are chosen at random by a pollster. Let the random variable X represent the number of those 30 residents that prefer Woostershire brand sauce.

Suggest a suitable distribution for X and comment on any necessary assumptions.

(2 marks)

(b) Find the probability that

(i) 90% or more of the residents chosen prefer Woostershire brand sauce

(ii) none of the residents chosen prefer the other well-known brand.

(3 marks)

(c) The pollster knows that there is a greater than 97% chance of at least k of the 30 residents preferring Woostershire brand sauce, where k is the largest possible value that makes that statement true.

Find the value of k.

(2 marks)

Hard Questions

1 (a) A fair dice is rolled 24 times and the number of times it lands on a 4 is recorded.

Define a suitable distribution to model the number of times the dice lands on a 4, and justify your choice.

(2 marks)

(b) Find the probability that the dice lands on a '4' four times.

(2 marks)

2 (a) For a population of squirrels in a certain area of woodland, there is a 92% chance that any given squirrel was born in that area of woodland. Squirrels born in that area of woodland are referred to by researchers as being 'local'.

State any assumptions that are required to model the number of local squirrels in a sample of size n as a binomial distribution.

(2 marks)

(b) Using this model, for a sample size of 50, find the probability of

(i) exactly 45 squirrels

(ii) all but one of the squirrels

being local.

(4 marks)

3 Guglielma is rolling a biased dice, for which the probability of landing on a 5 is $\frac{2}{11}$. She rolls the dice twenty times and records the number of times that it lands on a 5. Find the probability that

(i) the dice lands on a '5' four times

(ii) the dice lands on a '5' four times, but the final '5' does not occur on the final roll.

(4 marks)

4 (a) For bars of a particular brand of chocolate labelled as weighing , the actual weight of the bars varies. Although the company's quality control assures that the mean weight of the bars remains at 300 g, it is known from experience that the probability of any particular bar of the chocolate weighing between 297 g and 303 g is 0.9596. For bars outside that range, the proportion of underweight bars is equal to the proportion of overweight bars.

Millie buys 25 bars of this chocolate to hand out as snacks at her weekly Chocophiles club meeting. It may be assumed that those 25 bars represent a random sample. Let U represent the number of bars out of those 25 that weigh less than 297 g.

Write down the probability distribution that describes U.

(3 marks)

(b) The chocolate fanaticism of the club members means that no bars weighing less than 297 g can be handed out as snacks at their meetings.

Given that 24 people (including Millie) will be attending the meeting, find the probability that there will be enough bars to hand out to

(i) all

(ii) all but one, but not all

of the attendees.

(3 marks)

(c) After an incident where there were not enough chocolate bars weighing 297 g or more to hand out to all of a meeting's attendees, Millie decides to reorganise the way she runs the meetings. She will still only buy 25 of the chocolate bars each week, but she wants to reduce the number of attendees to make sure that she will have a certainty of at least 99.9% of being able to hand out a chocolate bar to every single attendee (including herself).

Work out the greatest number of attendees that a meeting will be able to have under this new system.

(2 marks)

5 The random variable $X \sim B(50, 0.3)$. Find:

(i) $P(X > 20)$

(ii) $P(7 \leq X < 16)$

(iii) $P(23 > X > 5)$

(iv) $P(X < 8 \ \ or \ \ X > 16)$

(4 marks)

6 The random variable $X \sim B(50, 0.85)$. Find:

(i) the largest value of q such that $P(X < q) < 0.16$

(ii) the largest value of r such that $P(X \geq r) > 0.977$

(iii) the smallest value of s such that $P(X > s) < 0.025$

(6 marks)

7 (a) Abner, an American baseball fanatic, has just moved to a town in which it is known that 25% of the residents are familiar with the rules of the game.

Abner takes a random sample of 40 residents of the town. Find the probability that

(i) fewer than 13

(ii) no more than 13

(iii) more than 13

(iv) at most 13 but at least 5

of the residents in Abner's sample are familiar with the rules of baseball.

(4 marks)

(b) Abner would like to field a town team for an upcoming regional baseball tournament. Abner is intending to play himself, but he needs to find enough other players to fill up the team. As he does not yet know anyone in the town, he decides to take another random sample of residents in hopes of finding enough other players. Only people familiar with the rules of baseball are able to be included in the team, but it may be assumed that anyone in the town familiar with the rules would also be willing to join Abner's team.

Given that an American baseball team must have a minimum of nine players, find the smallest number of people that Abner should include in his sample in order to have at least a 90% chance of filling up his team.

(3 marks)

8 (a) In the town of Edinboro, Pennsylvania, a festival of trimmed below the forehead hairstyles is held every year, known as the Edinboro Fringe Festival. It is known that 70% of the residents of the town are in favour of the festival because of the tourism revenue it brings in. The other 30% of residents oppose the festival because of the sometimes hostile reactions of the large number of tourists who arrive every year thinking they had actually made bookings to attend another well-known fringe festival.

25 residents are chosen at random by a local newspaper reporter. Let the random variable X represent the number of those 25 residents that are in favour of the festival.

Suggest a suitable distribution for and comment on any necessary assumptions.

(2 marks)

(b) Find the probability that

(i) 76% or more of the residents chosen are in favour of the festival

(ii) more of the residents chosen oppose the festival than are in favour of it.

(3 marks)

(c) The reporter knows that the chance of k or more of the 25 residents being opposed to the festival is less than 0.5%, where k is the smallest possible value that makes that statement true.

Find the value of k.

(2 marks)

Very Hard Questions

1 (a) Two fair dice are rolled and the numbers showing on the dice are added together. This is done 18 times and the number of times the sum is *not* equal to 7 or 11 is recorded.

Define a suitable distribution to model the number of times the sum is *not* equal to 7 or 11, and justify your choice.

(3 marks)

(b) Find the probability that the sum of the two dice is *not* equal to 7 or 11 exactly fourteen times.

(2 marks)

2 (a) Researchers studying malaria in a certain geographical region know that there is an 80% chance of any given female mosquito in the region carrying the malaria parasite.

State any assumptions that are required to model the number of female mosquitoes that carry the malaria parasite in a sample of n female mosquitoes as a binomial distribution.

(2 marks)

(b) Male mosquitoes do not bite humans and therefore are unable to transmit the malaria parasite to a human. A female mosquito is only able to transmit the malaria parasite to a human if it is carrying the malaria parasite itself.

Given that 50% of the mosquitoes in the region are male, find the probability that in a random sample of six mosquitoes none of them are able to transmit the malaria parasite to a human. Give your answer as an exact value.

(5 marks)

3 Maifreda is rolling a biased dice, for which the probability of landing on a prime number is $\frac{1}{2}$ and the probability of landing on a square number is $\frac{5}{16}$. She rolls the dice twenty times and records the number of times that it lands on a 6. Find the probability that

(i) the dice lands on a '6' four times

(ii) the dice lands on a '6' four times, but all of those sixes occur within the first k rolls (where $4 \leq k \leq 20$).

Your answer for (ii) should be given in terms of k, in the form

$$\binom{a}{b}\left(\frac{p}{16}\right)^q\left(\frac{r}{16}\right)^s$$

where $\binom{a}{b}=\frac{a!}{b!(a-b)!}$ is a binomial coefficient, and a,b,p,q,r and s are constants to be found.

(6 marks)

4 (a) Although a particular manufacturer of academic gowns advertises the material of their gowns as being 93% silk, the actual silk content of the gowns varies. Although the manufacturer's quality control protocols assure that the mean percentage of silk in the gowns remains at 93%, it is known from experience that the probability of the silk content of any particular gown being between 90% and 95% is 0.9805. For gowns falling outside that range, the probability that a gown contains less than 90% silk is exactly half the probability that a gown contains more than 95% silk.

Camford University has received an order of 100 gowns from the manufacturer. It may be assumed that those gowns represent a random sample. Let W represent the number of gowns out of those 100 that have a silk content greater than 95%.

Write down the probability distribution that describes W.

(3 marks)

(b) At an upcoming ceremony the university's Department of Obfuscation is going to be awarding honorary degrees to four government statisticians. The university prefers whenever possible to provide the recipients of such degrees with gowns containing more than 95% silk.

Out of the order of 100 gowns, find the probability that there will be enough gowns containing more than 95% silk to provide

(i) all

(ii) all but one (but not all)

(iii) less than half

of the honorary degree recipients with such a gown.

(4 marks)

(c) Due to a mix-up at the ceremony, the four honorary degree recipients are simply handed gowns at random from the order of 100 gowns. It had previously been determined that exactly one of the 100 gowns in the order contained less than 90% silk, and the university is worried that if one of the honorary degree recipients received that gown then the university's government grant funding will be cut.

Work out the probability that one of the honorary degree recipients received the gown containing less than 90% silk.

(3 marks)

5 The random variable $X \sim B(50, 0.75)$. Find:

(i) $\mathrm{P}(40 > X \geq 30)$

(ii) $\mathrm{P}(X < 29 \ \ or \ \ X > 38)$

(iii) $\mathrm{P}(X \leq 52 \ \ and \ \ X > 31)$

(3 marks)

6 The table below contains part of the cumulative distribution function for the random variable $X \sim B(30, 0.45)$:

x	5	6	7	8	9	10	11	12
$\mathrm{P}(X \leq x)$	0.0011	0.0040	0.0121	0.0312	0.0694	0.1350	0.2327	0.3592

13	14	15	16	17	18	19	20	21
0.5025	0.6448	0.7691	0.8644	0.9286	0.9666	0.9862	0.9950	0.9984

The random variable Y is defined in terms of X as $Y = 30 - X$, while the random variable $Z \sim B(30, 0.55)$.

Using the table above, and showing your working, find:

(i) the smallest value of q such that $\mathrm{P}(X > q) < 0.21$

(ii) the largest value of r such that $\mathrm{P}(Y > r) > 0.93$

(iii) the smallest value of s such that $\mathrm{P}(Z < s) > 0.988.$.

(6 marks)

7 (a) After falling asleep while reading his A Level mathematics textbook, Gwion awakens to find that he has been transported in his sleep to the magical kingdom of Statistica. As every statistics student knows, the kingdom of Statistica has a very large population (it has been reputed to be nearly infinite), and the chance that any given resident of the kingdom will welcome a newcomer with tea and cakes is 97%.

Gwion takes a random sample of 50 residents of the kingdom. Find the probability that of those 50 residents

(i) all 50

(ii) no more than 46

(iii) more than 25 but at most 49

(iv) at least 40 but fewer than 46

will welcome Gwion (who is a newcomer) with tea and cakes.

(4 marks)

(b) Luckily for Gwion, all 50 residents in his first sample welcome him with tea and cakes. From one of them, however, Gwion learns an unsettling fact: those residents of the kingdom who will not welcome a newcomer with tea and cakes, will instead insist on making the newcomer sit a five-hour statistics mock exam paper. Even worse, each such resident will insist on making the newcomer sit a *different* five-hour statistics mock exam paper

Gwion wants to take another random sample of residents of the kingdom, in hopes that one of them will be able to advise him on how to get home. However he does not want to have to sit more than one five-hour statistics mock exam paper.

Work out the size of the largest random sample that Gwion can take such that he will have a greater than 95% chance of not having to sit more than one five-hour statistics mock exam paper.

(3 marks)

8 (a) In Surry County, North Carolina, local farmers and agricultural equipment suppliers gather each year to celebrate at the Surry Slurry Fest. It is known that 80% of the residents of the county are opposed to the Slurry Fest because of the mess it leaves behind on local roads, fields and government buildings. The other 20% of residents are in favour of the Slurry Fest because it is (according to them) "one heck of a good ol' time".

An organiser of the rival Surry ♥ Curry Not Slurry food festival is attempting to gather evidence to support his campaign to have the Surry Slurry Fest banned. He selects 25 county residents at random in order to poll them about their opinions on the Slurry Fest. Let the random variable represent the number of those 25 residents that are opposed to the Slurry Fest.

Suggest a suitable distribution for X and comment on any necessary assumptions.

(2 marks)

(b) Find the probability that

(i) 90% or more of the residents chosen are opposed to the Slurry Fest

(ii) a majority of the residents chosen are in favour of the Slurry Fest.

(3 marks)

(c) Before he is able to conduct his poll of the selected residents, the food festival organiser is interviewed by a local newspaper. He would like to be able to predict with at least 90% certainty that not more than a given percentage of the 25 residents selected for the poll will be in favour of the Slurry Fest.

Given that the organiser would like his prediction to support his anti-Slurry Fest campaign in the strongest manner possible, determine the 'given percentage' that he should quote to the newspaper.

(3 marks)