A Level · Edexcel · Maths

4 hours · 32 questions

3.2 Further Probability (A Level only)

3.2.1 Set Notation & Conditional Probability / 3.2.2 Further Venn Diagrams / 3.2.3 Further Tree Diagrams / 3.2.4 Probability Formulae

Easy (8 questions)	/45
Medium (8 questions)	/53
Hard (8 questions)	/59
Very Hard (8 questions)	/61
Total Marks	**/218**

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Easy Questions

1 (a) Write the following probability statements in words.
The first one has been done for you.

(i) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$ " The probability that *A* has happened and *B* has happened.”

(ii) $\mathrm{P}(\mathrm{A} \cup \mathrm{B})$

(iii) $P(A' \cap B')$

(iv) $\mathrm{P}((\mathrm{A} \cup \mathrm{B})')$

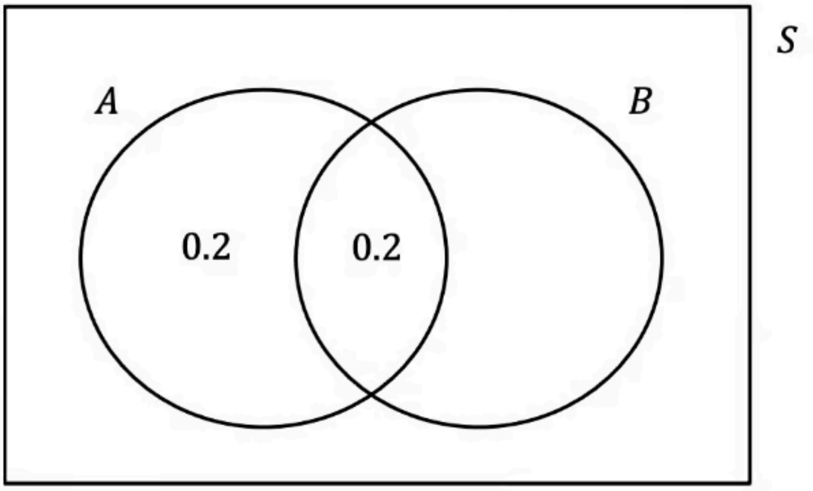
(3 marks)

(b) For each of the probability statements from part (a), including part (i), draw a Venn diagram and use shading to illustrate the probability.
(You may assume that and are not mutually exclusive.)

(3 marks)

2 (a) A and B are two events such that $\mathrm{P}(A)=0.4$, $\mathrm{P}(B)=0.7$ and $P(A \cap B)=0.2$.

Complete the two missing probabilities in the Venn diagram below.



(2 marks)

(b) Use the Venn diagram, or otherwise, to find

(i) $\mathrm{P}(A \cup B)$

(ii) $\mathrm{P}(A')$

(iii) $\mathrm{P}((A \cap B)')$

(3 marks)

3 (a) 100 children were asked whether they liked football and cricket. 84 said they liked football (F). 58 said they liked cricket (C). Of those who did not like football, 10 also said they did not like cricket.

Complete the two-way table illustrating this information.

	Like Football ($\mathbf{F}$)	**Dislike Football ($\mathbf{F}'$)**	**Total**
Like Cricket (C)			58
Dislike Cricket (C′)		10	
Total	84		100

(2 marks)

(b) One of the children is selected at random. Find the probability that

(i) they like cricket

(ii) they like both football and cricket

(iii) they like football or cricket, but not both.

(3 marks)

4 (a) A and B are two independent events such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.8$

(i) Complete this formula for independent events: $\mathrm{P}(A \cap B) = \text{—} \times \mathrm{P}(\mathrm{B})$

(ii) Use the formula to find $\mathrm{P}(A \cap B)$.

(2 marks)

(b) Another formula for independent events is $\mathrm{P}(A \cap B) = \mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B}|\mathrm{A})$ where $\mathrm{P}(B|A)$ means the probability of B happening given that A has already happened.

(i) Use the formula to find $\mathrm{P}(B|A)$

(ii) Deduce a similar formula involving $\mathrm{P}(A|B)$ and use it to find $\mathrm{P}(A|B)$.

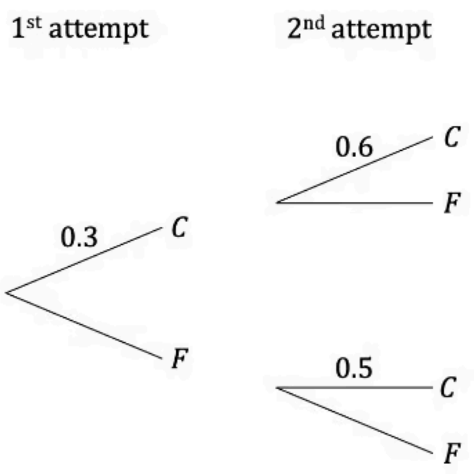
(3 marks)

(c) Briefly explain why, for independent events A and B, $\mathrm{P}(A|B) = \mathrm{P}(\mathrm{A})$.

(1 mark)

5 (a) Kimona is participating in a snowboarding competition whereby participants are given two attempts to complete a particular trick. If a participant completes the trick at the first attempt, they are still allowed a second attempt.
From experience the probability of Kimona completing the trick at the first attempt is 0.3
If Kimona completes the trick at the first attempt, the probability of completing the trick at the second attempt is 0.6. However, if Kimona fails at the first attempt, the probability of completing the trick at the second attempt is 0.5.

Complete the tree diagram below representing Kimona's situation.
(C denotes a completed trick, F denotes a failed trick.)



(2 marks)

(b) Use the tree diagram to find the probability that

(i) Kimona fails both attempts at completing the trick

(ii) Kimona completes the trick at least once.

(2 marks)

6 (a) Two events, A and B are mutually exclusive.

(i) Briefly explain why $\mathrm{P}(A \cap B) = 0$.

(ii) Hence use the formula $\mathrm{P}(A \cup B) = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \cap B)}$ to show that, for mutually exclusive events, $\mathrm{P}(A \cup B) = \mathrm{P(A)} + \mathrm{P(B)}$.

(2 marks)

(b) Two events, D and E are such that $P(D) = 0.2$, $P(E) = 0.4$ and $P(E|D) = 0.7$.

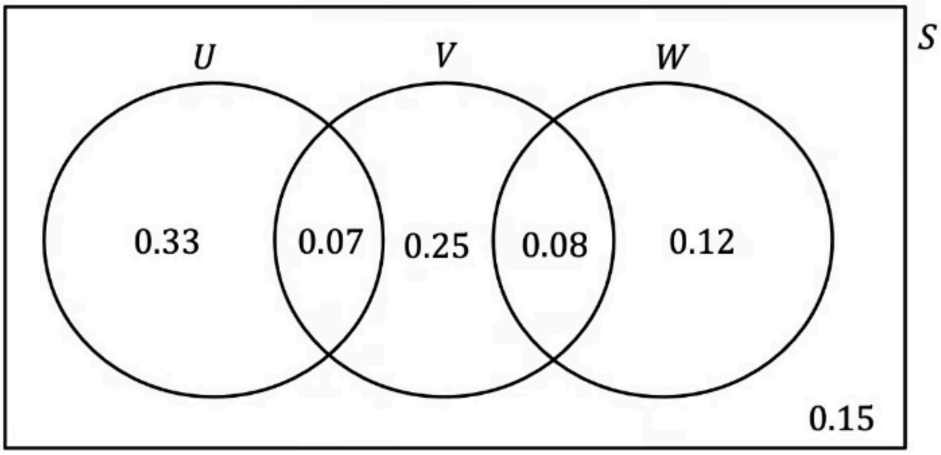
(i) Use the formula $\mathrm{P}(D \cap E) = \mathrm{P}(D) \times \mathrm{P}(E|D)$ to find $\mathrm{P}(D \cap E)$.

(ii) Use the formula $P(D \cap E) = P(D) + P(E) - P(D \cup E)$ to find $\mathrm{P}(D\ U\ E)$.

(iii) Use the formula $P(D \cap E) = P(D) \times P(E)$ to deduce whether the events D and E are independent or not.

(3 marks)

7 (a) The Venn diagram below shows the probabilities associated with three events, U, V and W.



Explain how the Venn diagram shows that events U are W mutually exclusive.

(1 mark)

(b) Use the Venn diagram to find

(i) $\mathrm{P}(U \cup V)$

(ii) $\mathrm{P}(W')$

(iii) $P(U' \cap V')$

(3 marks)

(c) (i) Find $\mathrm{P}(U)$, $\mathrm{P}(V)$ and $\mathrm{P}(W)$

(ii) Use the general formula $P(A \cap B) = P(A) \times P(B)$ to show that V and W are independent but U and V are not.

(4 marks)

8 (a) A box contains 7 blue and 3 red equally sized counters. A counter is taken from the box and its colour is noted, but it is not replaced in the box. A second counter is then taken from the box and its colour noted.

Draw a tree diagram to represent this information.

(3 marks)

(b) Use your tree diagram from part (a) to find the probability that:

(i) both counters are of the same colour

(ii) neither counter is blue.

(2 marks)

(c) Explain why, if four counters were taken one at a time and without replacement, the probability of all of them being red is zero.

(1 mark)

Medium Questions

1 (a) A, B and C are three events with $\mathrm{P}(A) = 0.2$, $\mathrm{P}(B) = 0.25$, $\mathrm{P}(C) = 0.6$ and $\mathrm{P}(B \cap C) = 0.08$. .

Given that events A and C are mutually exclusive, and that events A and B are independent, draw a Venn diagram to illustrate the probabilities.

(4 marks)

(b) Find:

(i) $P(A' \cap C')$

(ii) $P((A \cap B') \cup C)$

(iii) $P(A' \cup (B \cap C)')$

(3 marks)

2 (a) 240 students are surveyed regarding their belief in supernatural creatures. 144 say they believe in unicorns (U). 75 say they believe in vampires (V). Of those who believe in vampires, 27 also believe in unicorns.

Draw a two-way table to show this information.

(2 marks)

(b) One student is chosen at random. Find:

(i) $\mathrm{P}(U')$

(ii) $P(U' \cap V')$

(iii) $\mathrm{P}(U|V)$

(iv) $\mathrm{P}(V|U)$

(4 marks)

3 **(a)** A and B are two events with $\mathrm{P}(A) = 0.47$ and $P(B) = 0.31$. Given that A and B are independent, write down

(I) $\mathrm{P}(A \mid B)$

(ii) $P(B \mid A')$

(2 marks)

(b) A group of middle and senior school students were asked whether they preferred vinegar or ketchup as a topping on their chips. The following two-way table shows the results of the survey:

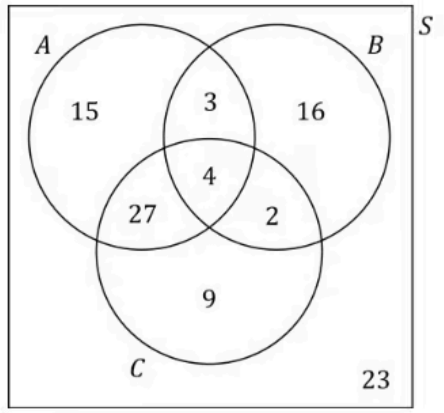
	vinegar	**ketchup**	**total**
middle	49	21	70
senior	63	27	90
total	112	48	160

(i) Find and P(ketchup | middle) and P(middle | ketchup).

(ii) Use your results from part (b)(i) to show that for the students in the sample 'is in middle school' and 'prefers ketchup on chips' are independent events.

(4 marks)

4 The following Venn diagram shows the number of adults in a poll who said they enjoy watching action films (A), Bollywood musicals (B), and crime thrillers (C).



One of the adults who was polled is selected at random. Given that the adult chosen enjoys watching at least one of those three genres of film, find the probability that the adult enjoys watching:

(i) Bollywood musicals

(ii) only one of the three genres of film

(iii) exactly two of the three genres of film.

(3 marks)

5 (a) Three events A, B and C, are such that B and C are mutually exclusive and A and C are independent. $\mathrm{P}(A) = 0.3$, $P(B) = 0.45$ and $\mathrm{P}(C) = 0.1$.

Given that $P((A \cup B \cup C)') = 0.43$, draw a Venn diagram to show the probabilities for events A, B, and C.

(4 marks)

(b) Find:

(i) $P(B|A)$

(ii) $P(A|B')$

(iii) $P(A|(BUC))$

(3 marks)

6 (a) Given that $P(A) = 0.27$, $P(B) = 0.39$ and $P(A \cap B) = 0.21$, find:

(i) $P(A \cup B)$

(ii) $P(B|A)$

(4 marks)

(b) The event C has $P(C) = 0.19$. The events A and C are mutually exclusive.

Given that $P(B \cap C) = 0.04$, find $P(A \cup B \cup C)$.

(2 marks)

7 (a) A bag contains 15 blue tokens and 27 yellow tokens. A token is taken from the bag and its colour is recorded, but it is not replaced in the bag. A second token is then taken from the bag and its colour is recorded.

Draw a tree diagram to represent this information.

(3 marks)

(b) Find the probability that:

(i) the second token selected is blue

(ii) both tokens selected are blue, given that the second token selected is blue.

(4 marks)

8 (a) Ichabod is a keen chess player who plays one game of chess online every night before going to bed. In any one of those games, the probabilities of Ichabod winning, drawing, or losing are 0.4, 0.27 and 0.33 respectively. Following each game, the probabilities of Ichabod sleeping well after winning, drawing or losing are 0.7, 0.9 and 0.2 respectively.

Draw a tree diagram to represent this information.

(3 marks)

(b) Find the probability that on a randomly chosen night

(i) Ichabod loses his chess game and sleeps well

(ii) Ichabod sleeps well.

(4 marks)

(c) Given that Ichabod sleeps well, find the probability that his chess game did not end in a draw.

(4 marks)

Hard Questions

1 (a) A, B and C are three events with $\mathrm{P}(B) = 0.3$, $\mathrm{P}(A \cap B) = 0.01$, $P(B \cap C) = 0.12$ and $\mathrm{P}((A \cup B \cup C)') = 0.13$.

Given that events A and C are mutually exclusive, and that events B and C are independent, draw a Venn diagram to illustrate the probabilities.

(4 marks)

(b) Find:

(i) $\mathrm{P}(A \cap B')$

(ii) $\mathrm{P}((A' \cap C') \cup A)$

(iii) $\mathrm{P}(((A \cap B') \cup (B' \cap C))')$

(3 marks)

2 (a) Some attendees at a pizza fandom convention are surveyed regarding their opinions about anchovies and bananas as pizza toppings. 144 of them say they do not like anchovies. 320 of them say they do not like bananas. 28 of them say they like bananas but not anchovies. Only 12 of them like both toppings.

Let A and B be the events 'likes anchovies as a pizza topping' and 'likes bananas as a pizza topping' respectively.

Draw a two-way table to show this information.

(2 marks)

(b) One of the attendees is chosen at random. Find:

(i) $\mathrm{P}(B)$

(ii) $\mathrm{P}(A' \cap B')$

(iii) $\mathrm{P}(B|A)$

(iv) $\mathrm{P}(A|B')$

(4 marks)

3 (a) A and B are two events with $\mathrm{P}(A) = x$ and $\mathrm{P}(B) = \mathrm{y}$, where $x \neq 0$. Given that A and B are independent, find the following probabilities in terms of x and y:

(i) $\mathrm{P}(B | A')$

(ii) $\mathrm{P}(A \cap B | A)$

(3 marks)

(b) A group of 18- to 25-year-olds, and a group of people over 65 years old, were asked whether they would prefer to holiday in Ibiza or Skegness. The following two-way table shows part of the results of the survey:

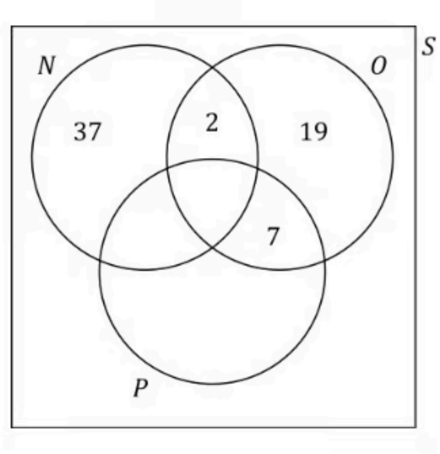
	Ibiza	**Skegness**	**total**
18-25			99
over 65			45
total	64	80	144

(i) Find P(over 65) and P(Ibiza)

(ii) Given that for the people in the sample the events 'is over 65' and 'prefers to holiday in Ibiza' are independent, find the missing values and complete the table of survey results.

(4 marks)

4 (a) The following Venn diagram shows some of the results for the number of chess players in a poll who said they enjoy playing any, all or none of three chess game openings – the Najdorf Sicilian (*N*), the Orangutan (*O*), and the Ponziani countergambit (*P*).



120 chess players were polled in total, of whom one half said they enjoy playing the Najdorf Sicilian, one third said they enjoy playing the Orangutan, and one fourth said they enjoy playing the Ponziani countergambit.

Use the above information to fill in the missing values in the Venn diagram.

(2 marks)

(b) One of the chess players who was polled is selected at random.

Given that the chess player chosen enjoys playing at least one of the three openings, find the probability that the chess player enjoys playing:

(i) the Orangutan

(ii) only one of the three openings

(iii) at least two of the three openings

(3 marks)

5 (a) Three events, A, B and C, are such that A and B are independent B and C and are mutually exclusive. $\mathrm{P}(C) = 0.55$, $P((A \cap B) \cup (A \cap C)) = 0.07$, and the following two relations also hold:

$$8\ \mathrm{P}(A) = 25\ \mathrm{P}(B)$$

$$\mathrm{P}(A' \cap C) = 10\ \mathrm{P}(A \cap C)$$

Using the above information, draw a Venn diagram to show the probabilities for events A, B and C.

(5 marks)

(b) Find:

(i) $\mathrm{P}(C|A)$

(ii) $\mathrm{P}(B|A')$

(iii) $\mathrm{P}((A \cup B \cup C)'|(A \cup B)')$

(3 marks)

6 (a) Given that $\mathrm{P}(A) = 0.34$, $P(A \cup B) = 0.67$ and $\mathrm{P}(A \cap B) = 0.02$, find:

(i) $\mathrm{P}(B)$

(ii) $\mathrm{P}(A \mid B')$

(4 marks)

(b) The event C has $\mathrm{P}(C) = 0.2$. The events B and C are mutually exclusive.

Given that A and C are independent, find $P(A \cup B \cup C)$.

(3 marks)

7 (a) A bag contains 12 orange marbles, 8 purple marbles and 5 red marbles. A marble is taken from the bag and its colour is recorded, but it is not replaced in the bag. A second marble is then taken from the bag and its colour is recorded.

Draw a tree diagram to represent this information.

(4 marks)

(b) Find the probability that:

(i) both marbles are different colours

(ii) the second marble is purple, given that both marbles are different colours.

(4 marks)

8 (a) Rosco is a somewhat inept rural county sheriff who frequently finds himself involved in car chases with well-meaning local entrepreneurs. During any given car chase, Rosco inevitably runs into one of three obstacles – a damaged bridge (with probability 0.47), an oil slick (with probability 0.32), or a pigpen at the end of a dead-end road.

If he encounters a damaged bridge there is a 25% chance that he will make it across safely; otherwise he lands in the river and ends up covered in mud. If he encounters an oil slick there is a 40% chance that his car will spin around and he will end up continuing his hot pursuit in the wrong direction; otherwise he goes off the road into a farm pond and ends up covered in mud. If he encounters a pigpen at the end of a dead-end road there is a 15% chance he will stop his car in time; otherwise he drives into the muddy end of the pigpen while the pigs sit at the other end laughing. If he drives into the muddy end of a pigpen there is a 20% chance he will only end up covered in mud; otherwise he ends up covered in mud and other things that are found in pigpens.

Draw a tree diagram to represent this information.

(3 marks)

(b) Find the probability that in the course of a randomly chosen car chase

(i) Rosco ends up covered in mud

(ii) Rosco ends up covered in mud, but only in mud.

(3 marks)

(c) Given that Rosco ends up covered in mud in the course of a randomly chosen car chase, find the probability that he didn't encounter an oil slick. Give your answer as an exact value.

(3 marks)

(d) In the course of a particular day Rosco finds himself engaged in three separate car chases with well-meaning local entrepreneurs. The car chases may be considered to be independent events.

Determine the probability that on that day Rosco will not end up covered in other things that are found in pigpens.

(2 marks)

Very Hard Questions

1 (a) A, B and C are three events such that $\mathrm{P}(C) = 0.37$, $\mathrm{P}(A' \cap B' \cap C) = 0.18$, $\mathrm{P}(A \cap B \cap C) = 0.09$ and $\mathrm{P}((A \cup B \cup C)') = 0.21$. In addition the following two relations hold:

$$\mathrm{P}(A \cap B \cap C') : \mathrm{P}(A \cap B' \cap C) : \mathrm{P}(A' \cap B \cap C) = 3:4:1$$

$$\mathrm{P}(A) : \mathrm{P}(B) = 9:10$$

Draw a Venn diagram to illustrate the probabilities.

(5 marks)

(b) Find:

(i) $\mathrm{P}(A \cup B' \cup C)$

(ii) $P(A \cup (B \cap C') | A \cup C)$

(iii) $P(B \cup C | (A \cup B \cup C) \cup (A \cup B \cup C)')$

(4 marks)

2 (a) The discrete random variable X has probability function

$$P(X = x) = kx^2, \quad x = 1,2,3,4,5$$

Find the value of k.

(1 mark)

(b) Draw a two-way table of probabilities for the events 'X is odd' (O) and 'X is prime' (P).

(3 marks)

(c) Find:

(i) $\mathrm{P}(O' \cap P')$

(ii) $\mathrm{P}(O' | P)$

(iii) $\mathrm{P}(P | O)$

(3 marks)

3 (a) A and B are two events with $P(A \cap B') = x$, $P(A \cap B) = y$ and $P(A' \cap B) = z$.
Given that A and B are independent, show that

$$y^2 + (x + z - 1)y + xz = 0$$

(3 marks)

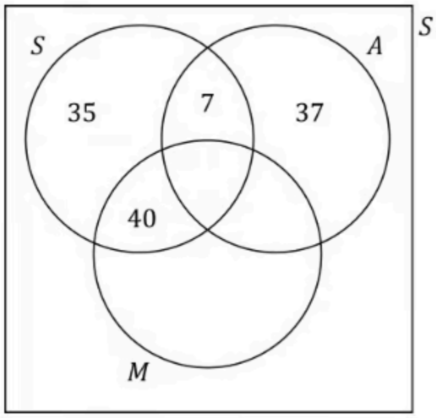
(b) A group of maths students and maths teachers were asked whether they preferred reading Latin poetry or partaking in extreme sports during their free time. The following two-way table shows part of the results of the survey:

	Latin poetry	**extreme sports**	**total**
student	63		
teacher			72
total			180

Given that for the teachers and students in the sample the events 'is a maths student' and 'prefers to read Latin poetry during free time' are independent, find the missing values and complete the table of survey results.

(4 marks)

4 A lifestyle magazine for ancient historians recently conducted a poll of its readership. The following Venn diagram shows some of the results for the numbers of the historians polled who said that they were fans of any, all or none of the following three ancient Near Eastern rulers – the Hittite king Suppiluliuma I (*S*), the Assyrian king Ashurbanipal (*A*), and the Akkadian king Manishtushu (*M*).



180 historians were polled in total. For each historian who was only a fan of Manishtushu, there were 7 who were only fans of Suppiluliuma I. For each two historians who were fans of both Ashurbanipal and Manishtushu but not of Suppiluliuma I, there were nine historians who were fans of all three rulers. Most of the historians were, as expected, fans of at least one of the three rulers, but it was also discovered that being a fan of Ashurbanipal and being a fan of Manishtushu were independent events.

One of the historians who was polled is selected at random. Given that the historian chosen is a fan of at least one of the three rulers, find the exact probability that the historian is a fan of:

(i) Suppiluliuma I

(ii) at least two the three rulers

(iii) exactly two of the three rulers.

(8 marks)

5 (a) Events A and B are such that $\mathrm{P}(A) = 0.23$ and $\mathrm{P}(B) = 0.72$.

Given that $P(A' \cap B) = p$, find the range of possible values of p.

(3 marks)

(b) Additionally, event C is such that $\mathrm{P}(C) = 0.53$ and $P(A \cap B \cap C) = 0.17$.

Determine whether this additional information would change your answer to part (a), and if it would then give the modified range of possible values of p when this additional information is taken into account.

(2 marks)

(c) Given that $P(A' \cap B \cap C) = q$, find the range of possible values of q.

(3 marks)

6 A and B are events such that $P(A \cup B) = x$, $P(A \cap B') = y$ and $P(A' \cap B) = z$, where $x \neq z$ and $y \neq 0$.

Find the following probabilities in terms of x, y and z:

(i) $\mathrm{P}(A \cap B)$

(ii)

$P(B|A)$

(iii) $P(A|B')$

(4 marks)

7 A bag contains tokens that each have a single number written on them. The integers between 1 and 15 are all represented on the tokens. There is one token with a '1' on it, two tokens with a '2' on them, and so on, up until fifteen tokens with a '15' on them.

A token is taken from the bag and the number on it is recorded, but it is not replaced in the bag. A second token is then taken from the bag and the number on it is recorded.

Using a tree diagram, or otherwise, work out the probabilities of the following events:

(i) the numbers on the two tokens are neither both prime numbers nor both square numbers

(ii) the number on one of the tokens is a prime number, given that the numbers on the two tokens are neither both prime numbers nor both square numbers

(iii) the number on the first token is a prime number, given that the numbers on the two tokens are neither both prime numbers nor both square numbers

(7 marks)

8 (a) A crafty coyote spends most of his spare time trying to catch a very fast roadrunner bird. The coyote's schemes always involve one of three items procured from a well-known mail order retailer – a crate of TNT (with probability 0.51), a large boulder (with probability 0.29), or a rocket on wheels.

If the coyote uses a crate of TNT there is a 95% chance it will explode at the wrong time and injure the coyote while the roadrunner escapes; otherwise it will simply not explode at all and the roadrunner will escape. If the coyote uses a large boulder there is an 85% chance it will injure him by landing on his head while the roadrunner escapes; otherwise it will injure the coyote by landing on his foot while the roadrunner escapes. If the coyote uses a rocket on wheels, there is a 60% chance he will injure himself by running into a cliff face while the roadrunner escapes; otherwise the roadrunner will escape after tricking the coyote into riding the rocket off the top of a cliff. If the coyote rides the rocket off the top of a cliff he will either get injured when the rocket explodes in mid-air, or get injured by crashing into a mountain on the other side of the valley, or land safely on the ground after his parachute unexpectedly functions properly. It is twice as likely that the rocket will explode as it is that the coyote will crash into a mountain, and five times as likely that the coyote will crash into a mountain as it is that he will land safely using his parachute.

Draw a tree diagram to represent this information.

(3 marks)

(b) (i) Given that the coyote is injured during one of his schemes, find the exact probability that he is not injured by an explosion.

(ii) Given that the coyote is not injured during one of his schemes, find the exact probability that his scheme involved a rocket on wheels.

(5 marks)

(c) A wandering statistician informs the coyote that if he undertakes n of his schemes to capture the roadrunner, then he will have a better than 50% chance of not being injured during at least one of them.

Find the smallest value of that makes the statistician's statement true.

(3 marks)