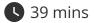
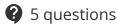


A Level · Edexcel · Further Maths





## 5.2 Methods in **Calculus**

5.2.1 Improper Integrals / 5.2.2 Mean Value of a Function / 5.2.3 Integrating with Partial Fractions / 5.2.4 Calculus involving Inverse Trig / 5.2.5 Integration by Substitution

Scan here to return to the course

or visit savemyexams.com





**Total Marks** 

/39

1 (a) 
$$f(x) = \frac{x+2}{x^2+9}$$

Show that (a)

$$\int f(x) dx = A \ln(x^2 + 9) + B \arctan\left(\frac{x}{3}\right) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4 marks)

Hence show that the mean value of f(x) over the interval [0, 3] is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi$$

(3 marks)

Use the answer to part (b) to find the mean value, over the interval [0, 3], of **(c)** (c)

$$f(x) + \ln k$$

where k is a positive constant, giving your answer in the form  $p+\frac{1}{6}\ln q$ , where p and q are constants and q is in terms of k.

(2 marks)



**2 (a)** Explain why 
$$\int_{1}^{\infty} \frac{1}{x(2x+5)} dx$$
 is an improper integral.

(1 mark)

$$\int_{1}^{\infty} \frac{1}{x(2x+5)} dx = a \ln b$$

where a and b are rational numbers to be determined.

(6 marks)

**3 (a)** (a) 
$$y = \tan^{-1} x$$

Assuming the derivative of tan x, prove that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2}$$

(3 marks)

$$f(x) = x \tan^{-1} 4x$$

Show that (b)

$$\int f(x) dx = Ax^2 \tan^{-1} 4x + Bx + C \tan^{-1} 4x + k$$

where k is an arbitrary constant and A, B and C are constants to be determined.

(5 marks)

(c) Hence find, in exact form, the mean value of 
$$f(x)$$
 over the interval  $\left[0, \frac{\sqrt{3}}{4}\right]$ 

(2 marks)

## 4 Show that

$$\int_0^\infty \frac{8x - 12}{(2x^2 + 3)(x + 1)} dx = \ln k$$

where k is a rational number to be found

(7 marks)

5 (a) 
$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

Using a substitution, that should be stated clearly, show that (a)

$$\int f(x) dx = A \sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4 marks)

Hence find, in exact form in terms of natural logarithms, the mean value of f(x)**(b)** (b) over the interval [0, 3].

(2 marks)