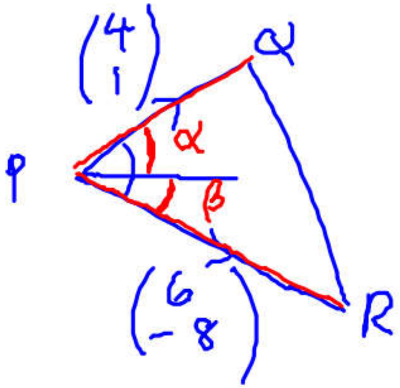
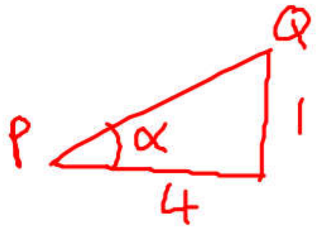
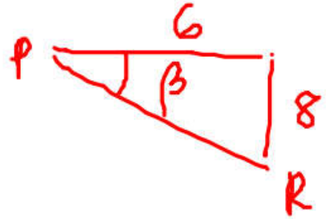
Q11 Ex11C





$\alpha = \tan^{-1}\left(\frac{1}{4}\right) = 14.03$

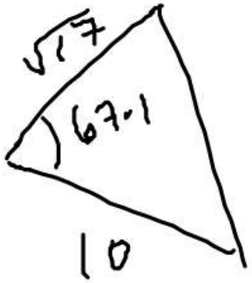


$\beta = \tan^{-1}\left(\frac{8}{6}\right) = 53.1^\circ$

$\angle PQR = \alpha + \beta = 14.03 + 53.1$

$= \underline{\underline{67.1^\circ}}$ (1dp).

"$\frac{1}{2}ab\sin C$"



$PQ = \sqrt{4^2+1^2} = \sqrt{17}$

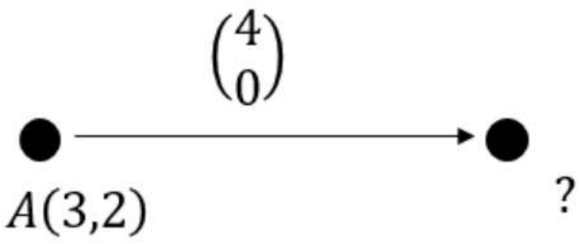
$PR = \sqrt{6^2+8^2} = 10$

$\text{Area} = \frac{1}{2} \times 10 \times \sqrt{17} \times \sin 67.1$

$= \underline{\underline{19.0 \text{ units}^2}}$.

Position Vectors

Suppose we started at a point $(3,2)$ and translated by the vector $\binom{4}{0}$:

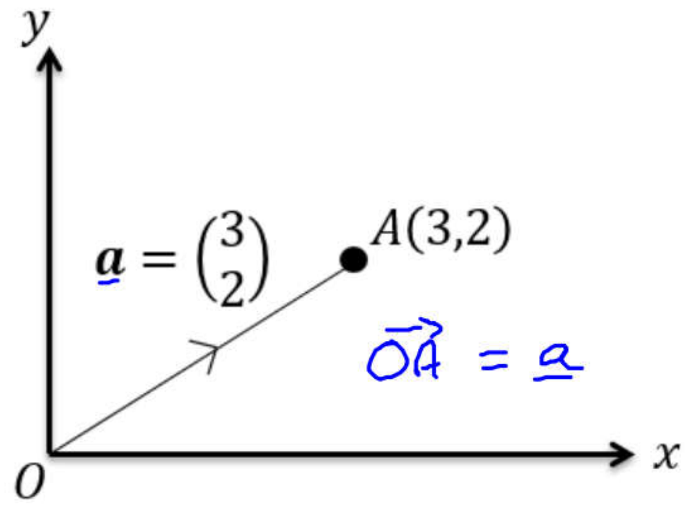


You might think we can do something like:

$$(3,2) + \binom{4}{0} = (7,2)$$

But only vectors can be added to other vectors. **If we treated the point $(3,2)$ as a vector**, then this solves the problem:

$$\binom{3}{2} + \binom{4}{0} = \binom{7}{2}$$



A vector used to represent a position is unsurprisingly known as a **position vector**.
A position can be thought of as a translation from the origin, as per above. It enables us to use positions in all sorts of vector (and matrix!) calculations.

✎ The position vector of a point A is the vector $\overrightarrow{OA}$, where O is the origin. $\overrightarrow{OA}$ is usually written as $\boldsymbol{a}$.

The points A and B have coordinates $(3,4)$ and $(11,2)$ respectively.
Find, in terms of i and j:

a) The position vector of A $3\underline{i} + 4\underline{j}$ $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

b) The position vector of B $11\underline{i} + 2\underline{j}$ $\begin{pmatrix} 11 \\ 2 \end{pmatrix}$

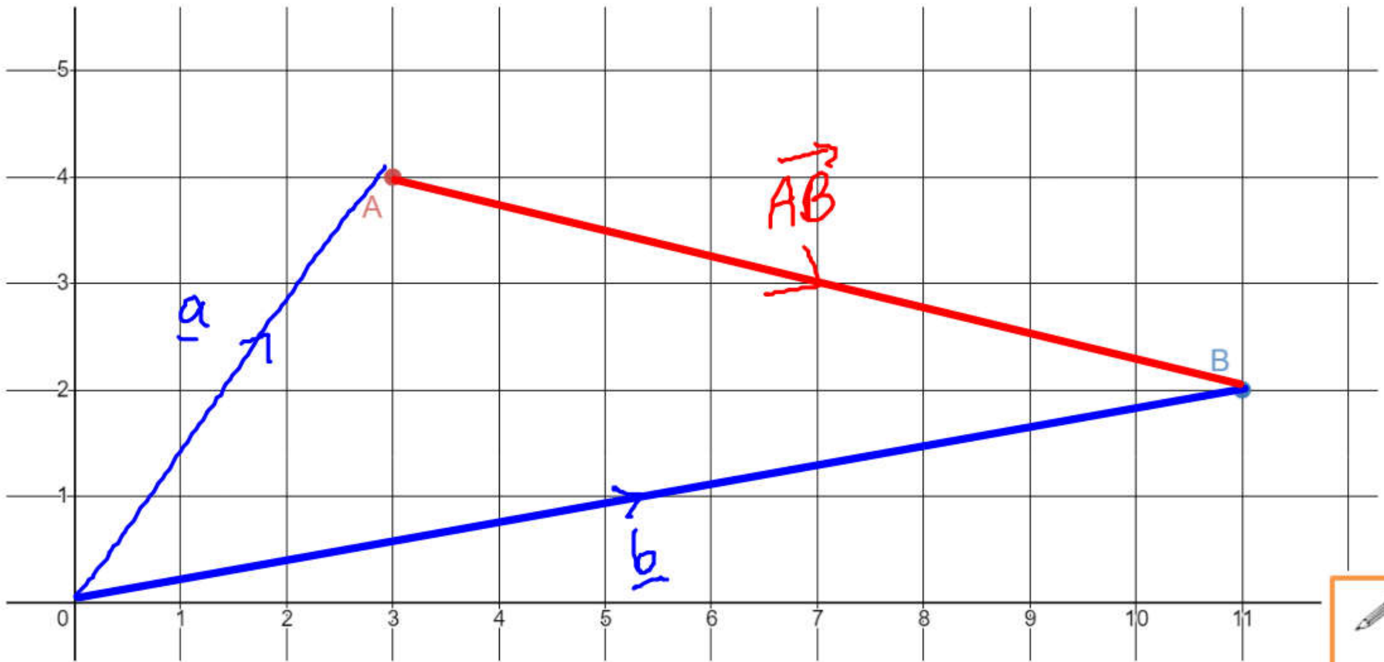
c) The vector $\overrightarrow{AB}$

$\overrightarrow{AB} = -\underline{a} + \underline{b}$

$\boxed{\overrightarrow{AB} = \underline{b} - \underline{a}}$

$\overrightarrow{AB} = \begin{pmatrix} 11 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$



For position vectors a and b:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$$

The points A, B and C have coordinates $(3, -1)$, $(4, 5)$ and $(-2, 6)$ respectively, and O is the origin.

Find, in terms of **i** and **j**:

a **i** the position vectors of A, B and C **ii** $\overrightarrow{AB}$ **iii** $\overrightarrow{AC}$

b Find, in surd form: **i** $|\overrightarrow{OC}|$ **ii** $|\overrightarrow{AB}|$ **iii** $|\overrightarrow{AC}|$

a) $\underline{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\underline{c} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$= \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$\overrightarrow{AC} = \underline{c} - \underline{a}$

$= \begin{pmatrix} -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

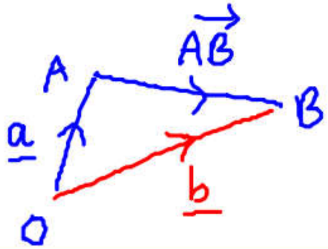
$= \begin{pmatrix} -5 \\ 7 \end{pmatrix}$

b) $|\overrightarrow{OC}| = |\underline{c}| = \sqrt{2^2+6^2} = \sqrt{40}$

$= 2\sqrt{10}$

$|\overrightarrow{AB}| = AB = \sqrt{1^2+6^2} = \sqrt{37}$

$|\overrightarrow{AC}| = \sqrt{5^2+7^2} = \sqrt{74}$



$\underline{b} = \underline{a} + \overrightarrow{AB}$

$\overrightarrow{OA} = 5i - 2j$ and $\overrightarrow{AB} = 3i + 4j$. Find:

a) The position vector of B.

b) The exact value of $|\overrightarrow{OB}|$ in simplified surd form.

$\underline{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \underline{b} - \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \underline{b}$

$\underline{b} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

2 **a** $-\mathbf{i} + 5\mathbf{j}$ or $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

b **i** 5 **ii** $\sqrt{13}$ **iii** $\sqrt{26}$

3 **a** $-\mathbf{i} - 9\mathbf{j}$ or $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$

b **i** $\sqrt{82}$ **ii** 5 **iii** $\sqrt{61}$

5 $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

6 **a** $2\mathbf{i} + 8\mathbf{j}$ **b** $2\sqrt{17}$

2 $\overrightarrow{OP} = 4\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{OQ} = 3\mathbf{i} + 2\mathbf{j}$

a Find $\overrightarrow{PQ}$

b Find, in surd form: **i** $|\overrightarrow{OP}|$ **ii** $|\overrightarrow{OQ}|$ **iii** $|\overrightarrow{PQ}|$

3 $\overrightarrow{OQ} = 4\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{PQ} = 5\mathbf{i} + 6\mathbf{j}$

a Find $\overrightarrow{OP}$

b Find, in surd form: **i** $|\overrightarrow{OP}|$ **ii** $|\overrightarrow{OQ}|$ **iii** $|\overrightarrow{PQ}|$

2 **a** $-\mathbf{i} + 5\mathbf{j}$ or $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

b **i** 5 **ii** $\sqrt{13}$ **iii** $\sqrt{26}$

3 **a** $-\mathbf{i} - 9\mathbf{j}$ or $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$

b **i** $\sqrt{82}$ **ii** 5 **iii** $\sqrt{61}$

5 $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

6 **a** $2\mathbf{i} + 8\mathbf{j}$ **b** $2\sqrt{17}$

5 The position vectors of 3 vertices of a parallelogram are $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$.
Find the possible position vectors of the fourth vertex.

more challenging

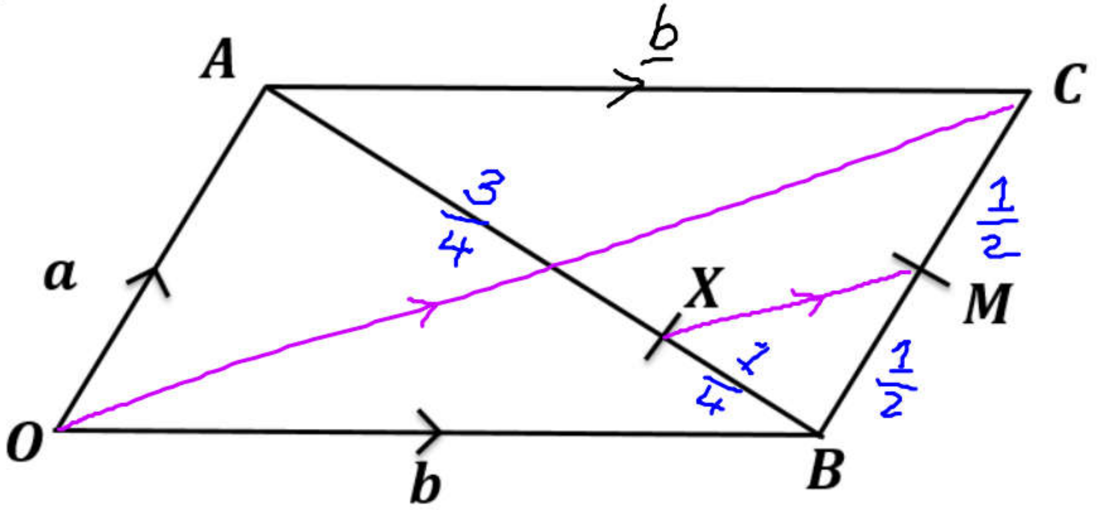
6 Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $6\mathbf{i} + 3\mathbf{j}$,

a find the vector $\overrightarrow{AB}$. **(2 marks)**

b find $|\overrightarrow{AB}|$ giving your answer as a simplified surd. **(2 marks)**

Skip
Ex 11D

Solving Geometric Problems



X is a point on AB such that $AX{:}XB = 3{:}1$. M is the midpoint of BC.
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

↳ multiples of each other.

We hope to show that $\overrightarrow{OC} = k\overrightarrow{XM}$ where k is a constant.

$\overrightarrow{OC} = \underline{a} + \underline{b}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\overrightarrow{XM} = \frac{1}{4}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

$= \frac{1}{4}(\underline{b} - \underline{a}) + \frac{1}{2}\underline{a}$

$= \frac{1}{4}\underline{b} - \frac{1}{4}\underline{a} + \frac{1}{2}\underline{a}$

$\overrightarrow{XM} = \frac{1}{4}\underline{a} + \frac{1}{4}\underline{b}$

$\underline{a} + \underline{b} = 4\left(\frac{1}{4}\underline{a} + \frac{1}{4}\underline{b}\right)$

$\overrightarrow{OC} = 4\overrightarrow{XM}$ or $\frac{1}{4}\overrightarrow{OC} = \overrightarrow{XM}$

So, they are parallel.

OC : XM
4 : 1

Extra work

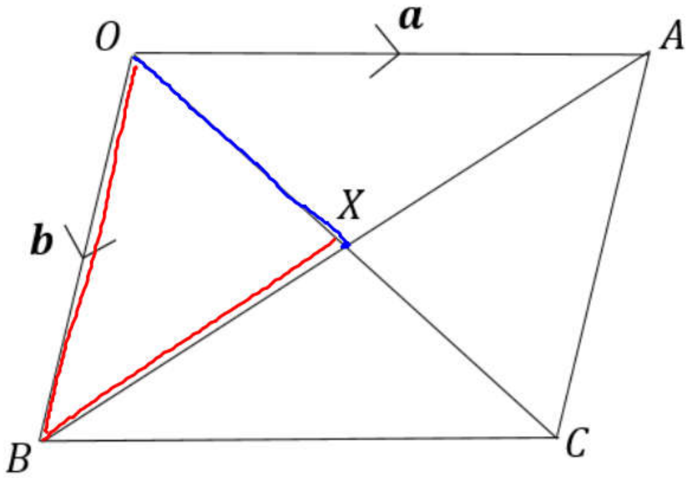
Introducing Scalars and Comparing Coefficients

Remember when we had **identities** like:

$$ax^2 + 3x \equiv 2x^2 + bx$$

we could **compare coefficients**, so that $a = 2$ and $3 = b$.

We can do the same with (non-parallel) vectors!



$OACB$ is a parallelogram, where $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$.
The diagonals OC and AB intersect at a point X.
Prove that the diagonals bisect each other.
cut in half
(Hint: Perhaps find $\overrightarrow{OX}$ in two different ways?)

'lambda' λ
'mu' μ

$\boxed{\overrightarrow{OX} = \lambda \overrightarrow{OC}}$ (where $0 \leq \lambda \leq 1$)

$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX}$

$\overrightarrow{OX} = \underline{b} + \mu \overrightarrow{BA}$ (where $0 \leq \mu \leq 1$)

$\overrightarrow{BA} = -\underline{b} + \underline{a}$
$= \underline{a} - \underline{b}$

$\overrightarrow{OX} = \underline{b} + \mu(\underline{a} - \underline{b})$

$\overrightarrow{OX} = \underline{b} + \mu\underline{a} - \mu\underline{b}$

$\overrightarrow{OX} = \mu\underline{a} + \underline{b} - \mu\underline{b}$

$\boxed{\overrightarrow{OX} = \mu\underline{a} + (1-\mu)\underline{b}}$

$\overrightarrow{OX} = \lambda(\underline{a} + \underline{b})$

$\boxed{\overrightarrow{OX} = \lambda\underline{a} + \lambda\underline{b}}$

sim. equations

$\boxed{\begin{aligned} \mu &= \lambda \\ 1-\mu &= \lambda \end{aligned}}$ (a) (b)

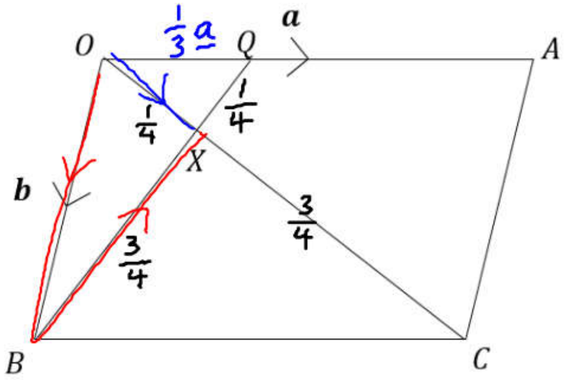
$1 - \mu = \mu$

$1 = 2\mu$

$\frac{1}{2} = \mu \qquad \frac{1}{2} = \lambda$

So, the diagonals bisect.

Your Turn



In the above diagram, $\overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OQ} = \frac{1}{3}\boldsymbol{a}$. We wish to find the ratio $OX : XC$.

a) If $\overrightarrow{OX} = \lambda\,\overrightarrow{OC}$, find an expression for $\overrightarrow{OX}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and λ.

b) If $\overrightarrow{BX} = \mu\,\overrightarrow{BQ}$, find an expression for $\overrightarrow{OX}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and μ.

c) By comparing coefficients or otherwise, determine the value of λ, and hence the ratio $OX : XC$.

a) $\overrightarrow{OX} = \lambda\overrightarrow{OC}$ $\overrightarrow{OC} = \underline{a} + \underline{b}$

$\overrightarrow{OX} = \lambda(\underline{a} + \underline{b})$

☆ $\overrightarrow{OX} = \lambda\underline{a} + \lambda\underline{b}$

b) $\overrightarrow{BX} = \mu\overrightarrow{BQ}$ $\overrightarrow{BQ} = -\underline{b} + \frac{1}{3}\underline{a}$

$\overrightarrow{BX} = \mu(-\underline{b} + \frac{1}{3}\underline{a})$

$\overrightarrow{BX} = -\mu\underline{b} + \frac{1}{3}\mu\underline{a}$

$\overrightarrow{OX} = \underline{b} + \overrightarrow{BX}$

$\overrightarrow{OX} = \underline{b} - \mu\underline{b} + \frac{1}{3}\mu\underline{a}$

☆ $\overrightarrow{OX} = \frac{1}{3}\mu\underline{a} + (1-\mu)\underline{b}$

compare coefficents.

$\lambda = \frac{1}{3}\mu$

$\lambda = 1 - \mu$

sim eq. $\frac{1}{3}\mu = 1 - \mu$

$\frac{4}{3}\mu = 1$

$\mu = \frac{3}{4}$ $\lambda = \frac{1}{4}$

Ratio

$OX : XC$

$\frac{1}{4} : \frac{3}{4}$

$\underline{\underline{1 : 3}}$.