A Level · Edexcel · Maths

4 hours 32 questions

8.3 Differential Equations (A Level only)

Easy (8 questions)	/55
Medium (8 questions)	/58
Hard (8 questions)	/60
Very Hard (8 questions)	/65
Total Marks	**/238**

Scan here to return to the course
or visit savemyexams.com

© 2024 Save My Exams, Ltd.

Easy Questions

1 (a) Find the general solution to the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 9t^2 + 4t.$$

(2 marks)

(b) Find the particular solution to the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}x} = 4e^{2x}$$

given that the graph of S against x passes through the point (0 , 5).

(3 marks)

© 2024 Save My Exams, Ltd.

2 (a) By separating the variables, show that the solution to the differential equation

$$\frac{dy}{dx} = 2xy, \;\; y > 0$$

can be found by solving

$$\int \frac{1}{y}\, dy = \int 2x \; dx.$$

(2 marks)

(b) Show that the general solution to the differential equation in part (a) is

$$y = e^{x^2 + c}$$

where c is a constant.

(3 marks)

(c) By letting $A = e^c$, show that the general solution to the differential equation in part (a) can be written in the form

$$y = Ae^{x^2}.$$

(2 marks)

3 (a) The velocity of a particle is given by the differential equation

$$\frac{\mathrm{d}s}{\mathrm{d}t} = 8t + 1$$

where s is the displacement of the particle in metres from a fixed point O at time t seconds. At time $t = 0$ the particle is located at point O.

Write down the initial velocity of the particle.

(1 mark)

(b) Use the method of separating variables to find an expression for the displacement of the particle from O after t seconds.

(2 marks)

(c) Find $\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ and hence explain why the acceleration of the particle is constant.

(2 marks)

© 2024 Save My Exams, Ltd.

4 (a) The differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -kV,$$

is used to model the rate at which water is leaking from a container.

V is the volume of water in the container at time t seconds.
k is a constant.

Explain the use of a negative sign on the right-hand side of the differential equation, and the impact this has on the value of the constant k.

(1 mark)

(b) Show that

$$\int \frac{1}{V}\,\mathrm{d}V = k\int \mathrm{d}t$$

and hence find the general solution to the differential equation.

(3 marks)

(c) Given that $k = 0.02$ and the initial volume of the container is 300 litres, find the particular solution to the differential equation, giving your answer in the form

$$V = Ae^{-kt}.$$

(2 marks)

© 2024 Save My Exams, Ltd. | Get more and ace your exams at savemyexams.com

5 (a) Given that $y > 1$, find the general solution to the differential equation

$$\frac{1}{y-1}\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2$$

giving your answer in the form

$$y = Ae^{\mathrm{f}(x)} + 1.$$

(3 marks)

(b) Given that $y > -2$, find the general solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 9(y+2)x^{\frac{1}{2}}$$

giving your answer in the form

$$y = Ae^{\mathrm{f}(x)} - 2.$$

(3 marks)

© 2024 Save My Exams, Ltd. | Get more and ace your exams at savemyexams.com

6 (a) Use the given boundary condition to find the particular solution to the following differential equations:

$$\frac{dy}{dx} = \sec^2 x \qquad x = \frac{\pi}{3}, \quad y = 2\sqrt{3},$$

(4 marks)

(b)

$$\sec x \frac{dy}{dx} = \operatorname{cosec} y \qquad x = \frac{\pi}{2}, \quad y = 0,$$

giving your answer in the form $\cos y = f(x)$.

(5 marks)

7 (a) A large weather balloon is being inflated.

The rate of change of its volume, $\frac{dV}{dt}$, where V m^3 is the volume of the balloon t minutes after inflation commenced, is inversely proportional to its volume.

(i) Form a differential equation to describe the relationship between V and t as the weather balloon is inflated.

(ii) The rate of inflation of the balloon is 10 $\text{m}^3\,\text{min}^{-1}$ when its volume is 20 m^3.
Use this information to find the constant of proportionality.

(3 marks)

(b) Show that the general solution to the differential equation found in part (a) is

$$V^2 = 400t + c$$

where c is a constant.

(3 marks)

(c) (i) When not in use, the weather balloon is stored flat and so can initially be considered to have a volume of 0 m^3.
Use this information to find the particular solution to the differential equation.

(ii) What is the volume of the balloon after 25 minutes?

(3 marks)

8 (a) A tree disease is spreading throughout a large forested area.

The differential equation

$$e^{-kt}\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{2}{5}$$

where k is a positive constant, is used to model the number of infected trees, N, at a time t days after the disease was first discovered.

Show that

$$\int 5 \ \mathrm{d}N = \int 2e^{kt} \ \mathrm{d}t.$$

(2 marks)

(b) Hence show that

$$N = \frac{2e^{kt}}{5k} + c$$

where c is a constant.

(2 marks)

(c) Given that $k = 0.1$ and that when the disease was first discovered, four trees were infected.

Find the particular solution of the differential equation and use it to estimate the number of infected trees after 30 days.

(4 marks)

Medium Questions

1 (a) Find the general solution to the differential equation

$$9t^2 - 4 + \frac{\mathrm{d}x}{\mathrm{d}t} = 0$$

(2 marks)

(b) Find the particular solution to the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}x} - 4 = 2e^x$$

given that the graph of V against x passes through the point (0, 3).

(3 marks)

2 (a) By separating the variables, show that the general solution to the differential equation

$$\frac{dy}{dx} = 4xy, \qquad y > 0$$

can be written as

$$y = e^{2x^2 + c}$$

where c is the constant of integration.

(3 marks)

(b) (i) By renaming the constant e^c as A, show that the general solution from part (a) can be written in the form

$$y = Ae^{2x^2}$$

(ii) Explain the significance of the value of A in that form of the general solution, and suggest what it might represent if the equation were being used to model a real- life problem.

(2 marks)

3 The velocity of a particle is given by the differential equation

$$\frac{ds}{dt} = 6t^2 - 2t + 5$$

where s is the displacement of the particle in metres from a fixed point O, and t is the time in seconds. At time $t = 0$ the particle is located at point O.

(i) Write down the initial velocity of the particle.

(ii) Find an expression for the displacement, s m, of the particle after t seconds.

(iii) Find an expression for the acceleration, a m s^{-2}, of the particle after t seconds.

(4 marks)

4 (a) A large container of water is leaking at a rate directly proportional to the volume of water in the container.

Using the variables V, for the volume of water in the container, and t, for time, write down a differential equation involving the term $\frac{\mathrm{d}V}{\mathrm{d}t}$, for the volume of water in the container.

(2 marks)

(b) The general solution to the differential equation in part (a) can be written in the form

$$V = Ae^{-kt}$$

where k is a positive constant.

(i) State, in the context of the question, the significance of the constant A.

(ii) Briefly explain where the negative sign in the solution comes from in the context of the question.

(2 marks)

5 (a) Given that $y > 2$, find the general solution to the differential equation

$$\frac{dy}{dx} = x^2(y-2)$$

(4 marks)

(b) Find the general solution to the differential equation

$$\frac{dy}{dx} = \sin^2 2y$$

giving your answer in the form $x = f(y)$.

(3 marks)

6 (a) Find particular solutions to the following differential equations, using the given boundary conditions.

$$\sin^2 x \frac{\mathrm{d}y}{\mathrm{d}x} = \cos^2 y \qquad x = \frac{\pi}{4}, \quad y = 0$$

(6 marks)

(b)

$$e^{-3x} \frac{\mathrm{d}y}{\mathrm{d}x} = 2e^{y} \qquad x = 0, \quad y = 0$$

(6 marks)

© 2024 Save My Exams, Ltd. Get more and ace your exams at savemyexams.com

7 (a) A large weather balloon is being inflated at a rate that is inversely proportional to its volume.

(i) Using the variables $V\ \mathrm{m}^3$ for the volume of the balloon and t seconds for the time since inflation began, write down a differential equation to describe the relationship between V and t as the weather balloon is inflated.

(ii) The rate of inflation of the balloon is 5 $\mathrm{m}^3\ \mathrm{s}^{-1}$ when its volume is 48 m^3.
Use this information to find the constant of proportionality.

(3 marks)

(b) Find the general solution to the differential equation found in part (a).

(4 marks)

(c) (i) When not in use, the weather balloon is stored flat and so can initially be considered to have a volume of 0 m^3. Use this information to find the particular solution to the differential equation found in part (a).

(ii) What is the volume of the balloon after 50 minutes?

(3 marks)

8 (a) A tree disease is spreading throughout a large forested area.

When the disease was first discovered, three trees were infected.

Ten days later ten trees were infected.

The differential equation

$$\frac{1}{t}\frac{\mathrm{d}N}{\mathrm{d}t} = kN$$

where k is a positive constant, is used to model the number of infected trees, N, at a time t days after the disease was first discovered.

Find the particular solution to the differential equation.

(7 marks)

(b) Scientists believe the majority of the forest can be saved from infection if action is taken before 30 trees are infected.

Measured from the time when the disease was first discovered, how many days does the model predict the scientists have to take action in order to save the majority of the forest from infection?

(4 marks)

© 2024 Save My Exams, Ltd.

Hard Questions

1 (a) Find the general solution to the differential equation

$$5 - \sin 2t + \frac{\mathrm{d}x}{\mathrm{d}t} = 0$$

(2 marks)

(b) Find the particular solution to the differential equation

$$3e^{4x} - \frac{\mathrm{d}V}{\mathrm{d}x} = 2$$

where the graph of V against x passes through the point (0, -4).

(3 marks)

2 (a) Show that the general solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 y, \qquad y \neq 0$$

is

$$y = A\mathrm{e}^{x^3}$$

where A is a constant.

(5 marks)

(b) On the same set of axes sketch the graphs of the solution for the instances where

(i) the constant A is greater than 0

(ii) the constant A is less than 0

In each case be sure to state where the graph intercepts the y-axis.

(3 marks)

3 The acceleration of a particle is given by the differential equation

$$\frac{d^2s}{dt^2} = 18t - 4$$

where s is the displacement of the particle in metres from a fixed reference point O, and t is the time in seconds.

The particle starts its journey at point O at $t = 0$, and has an initial velocity of 1 $\mathrm{m\ s^{-1}}$.

Find an expression for the displacement of the particle in terms of t.

(4 marks)

© 2024 Save My Exams, Ltd.

4 (a) A large container of water is leaking at a rate directly proportional to the volume of water in the container.

Defining any variables, write down a differential equation that describes how the volume of water in the container varies with time.

(2 marks)

(b) By separating the variables, find the general solution to your differential equation from part (a).

(3 marks)

© 2024 Save My Exams, Ltd.

5 (a) Find the general solution to the differential equation

$$\frac{2y-1}{3}\frac{\mathrm{d}y}{\mathrm{d}x}=x^2y^2-x^2y, \qquad y>1$$

(4 marks)

(b) Find the general solution to the differential equation

$$3\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\operatorname{cosec} y^3}{y^2}$$

giving your answer in the form $x=\mathrm{f}(y)$.

(4 marks)

© 2024 Save My Exams, Ltd.

6 **(a)** Show that the general solution to the differential equation

$$y\cot x \frac{dy}{dx} = y^2 + 3$$

can be written in the form

$$y^2 + 3 = A \sec^2 x$$

where A is a constant.

(5 marks)

(b) Find the particular solution to the following differential equation, using the given boundary condition

$$e^{x^2} \frac{dy}{dx} = 2x \operatorname{cosec} 3y \qquad x = 0, \quad y = \frac{\pi}{3}$$

(6 marks)

7 (a) A large weather balloon is being inflated at a rate that is inversely proportional to the square of its volume.

Defining variables for the volume of the balloon (m^3) and time (seconds) write down a differential equation to describe the relationship between volume and time as the weather balloon is inflated.

(2 marks)

(b) Given that initially the balloon may be considered to have a volume of zero, and that after 400 seconds of inflating its volume is 600 m^3, find the particular solution to your differential equation.

(6 marks)

(c) Although it can be inflated further, the balloon is considered ready for release when its volume reaches 1250 $\mathrm{m}^{3.}$ If the balloon needs to be ready for a midday release, what is the latest time that it can start being inflated?

(2 marks)

© 2024 Save My Exams, Ltd.

8 (a) A bar of soap in the shape of a cuboid is placed in a bowl of warm water and its volume is recorded at regular intervals. The water is maintained at a constant temperature.

Before being placed in the water the soap measures 3 cm by 6 cm by 10 cm.

Two minutes later the bar of soap measures 2.85 cm by 5.7 cm by 9.5 cm.

The rate of decrease in volume of the bar of soap is modelled as being directly proportional to its volume.

Defining any variables you use, find and solve a differential equation linking the volume of the bar of soap and time.

(7 marks)

(b) What happens to the volume of the bar of soap for large values of t? Briefly explain why this could be considered a criticism of the model.

(2 marks)

Very Hard Questions

1 (a) Find the general solution to the differential equation

$$\frac{1}{2}\sec^2 3t + 2\frac{\mathrm{d}x}{\mathrm{d}t} = 0$$

(2 marks)

(b) Find the particular solution to the differential equation

$$2xe^{4x} - 3\frac{\mathrm{d}V}{\mathrm{d}x} = 1$$

where the graph of V against x passes through the point (0,2).

(6 marks)

2 (a) Show that the general solution to the differential equation

$$2x\frac{\mathrm{d}y}{\mathrm{d}x} = 3kx^3y, \qquad y \neq 0$$

is

$$y = Ae^{\frac{1}{2}kx^3}$$

where A and k are constants.

(5 marks)

(b) On separate diagrams sketch a graph of the solution for $x \geq 0$ in the instances when

(i) the constant k is greater than 0,
(ii) the constant k is less than 0.

On both diagrams state where the graph intercepts the y-axis.
You may assume $A > 0$ in both cases.

(3 marks)

3 The acceleration of a particle moving in a straight line is given by the differential equation

$$\frac{d^2s}{dt^2} = 18\sin 3t - 2$$

where s m is the displacement of the particle relative to a fixed point O, and t is the elapsed time in seconds. The particle starts its journey with a velocity of $-6\ \mathrm{m\ s^{-1}}$, from a point 50 m in the positive direction from the point O.

Find an expression for the displacement, s, of the particle in terms of t.

(4 marks)

4 A large container of water is leaking at a rate directly proportional to the square of the volume of water in the container.

(i) Given that the initial volume of water in the container is 4000 litres and that after 10 minutes the volume of water in the container has dropped by 30%, write down and solve a differential equation connecting the volume, V, of water in the container to the time, t.

(ii) What does your solution predict will happen to the volume of water in the container after a very long time?

(8 marks)

5 (a) Newton's Law of Cooling states that the rate of cooling of an object is directly proportional to the difference between the object's temperature and the ambient temperature (temperature of the object's surroundings).

By setting up and solving an appropriate differential equation, show that

$$T = T_{amb} + Ae^{-kt}$$

where T °C is the temperature of the object, T_{amb} °C is the ambient temperature, t is time, and $k > 0$ and A are both constants. You may assume in working out your solution that the ambient temperature is constant, and that the temperature of the object is greater than the ambient temperature.

(6 marks)

(b) A meat processing factory must store its products at a temperature below -1 °C.

Due to the production process, products, before cooling, typically have a temperature between 5 °C and 10 °C.

The company therefore has a policy that any products failing to cool to below -1 °C within 6 minutes of being processed must be discarded.

The factory stores its products in a freezer with a constant ambient temperature of -4 °C.

A product that has just finished being processed has a temperature of 7 °C and is immediately placed in the freezer. One minute later its temperature has dropped to 4.7 °C. Determine whether or not this product will need to be discarded.

© 2024 Save My Exams, Ltd.

(4 marks)

6 Find the general solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy + 2x - y - 1, \quad y > -1$$

(5 marks)

© 2024 Save My Exams, Ltd.

7 (a) Using the standard integral result

$$\int \sec k\theta \ \mathrm{d}\theta = \frac{1}{k}\ln\left|\tan\left(\frac{k\theta}{2}+\frac{\pi}{4}\right)\right| + c$$

(where k is a constant, and c is a constant of integration), show that the solution to the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} = \cos y$$

with boundary condition $x = 0,\ y = \pi,$ may be written in the form

$$\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

(4 marks)

(b) (i) Show that the relationship between x and y in $\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$ may also be expressed by the set of all equations of the forms

$$y = x + 2n\pi \qquad or \qquad y = -x + (2n-1)\pi$$

where n is an integer.

(ii) Hence deduce that the particular solution to the differential equation in part (a), with the given boundary condition, is $y = \pi - x$.

© 2024 Save My Exams, Ltd.

(6 marks)

8 (a) A tree disease is spreading throughout a large forested area.

The rate of increase in the number of infected trees is modelled by the differential equation

$$\frac{dN}{dt} = kN(N-1), \quad N > 1$$

where N is the number of infected trees, t is the time in days since the disease was first identified and k is a positive constant.

Solve the differential equation above, and show that the general solution can be written in the form

$$N = \frac{1}{1 - Ae^{kt}}$$

where A is a positive constant.

(6 marks)

(b) Initially two trees were identified as diseased.
A fortnight later, 4 trees were infected.
Using this information, find the values of the constants A and k.

(3 marks)

© 2024 Save My Exams, Ltd.

(c) By considering the solution to the differential equation along with the values of A and k found in part (b), suggest a range of values of t for which the model might be considered reliable.

(3 marks)