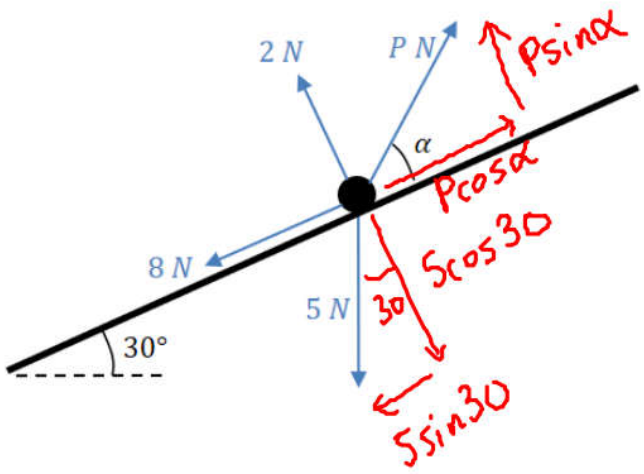
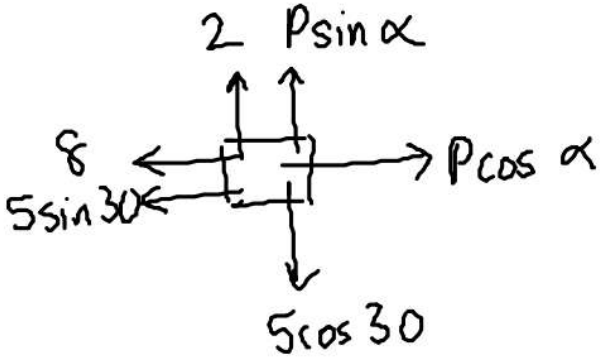
Statics - revisited

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force P and the size of the angle α.





$(R\downarrow)$ $2 + P\sin\alpha = 5\cos 30$

$P\sin\alpha = 5\cos 30 - 2$ (1)

$(R\leftarrow)$ $P\cos\alpha = 8 + 5\sin 30$ (2)

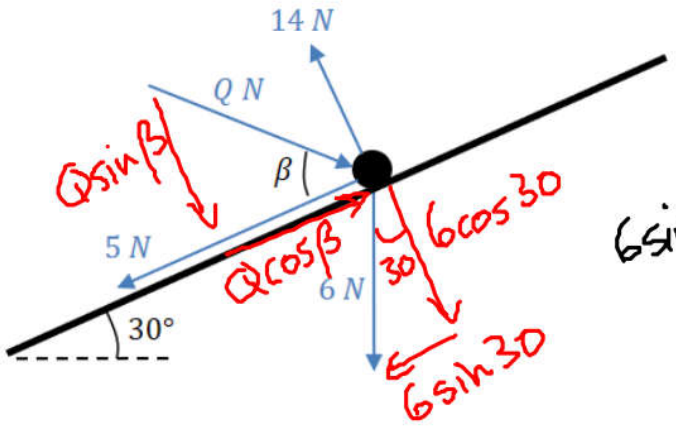
(1) ÷ (2) $\tan\alpha = \dfrac{5\cos 30 - 2}{8 + 5\sin 30}$

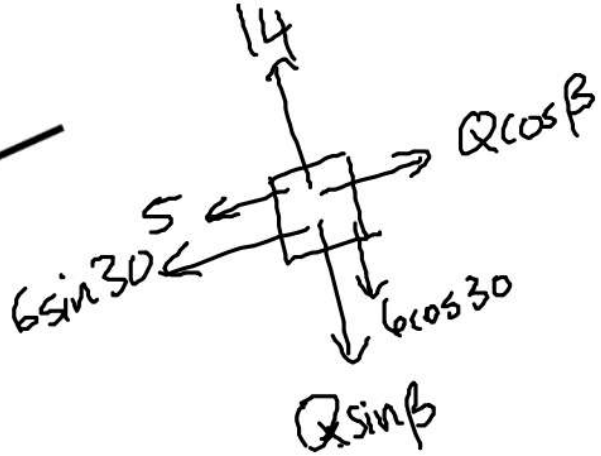
$\alpha = \underline{12.5^\circ}$ (3sf).

Using (1) $P = \dfrac{5\cos 30 - 2}{\sin\alpha} = \underline{10.8\ N}$ (3sf)

Your Turn

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force Q and the size of the angle β.





$(R\downarrow)$ $Q\sin\beta + 6\cos 30 = 14$

$Q\sin\beta = 14 - 6\cos 30$ (1)

$(R\leftrightarrow)$ $Q\cos\beta = 6\sin 30 + 5$ (2)

(1) ÷ (2)

$\tan\beta = \dfrac{14 - 6\cos 30}{6\sin 30 + 5}$

$\beta = 47.7°$

$Q = \dfrac{6\sin 30 + 5}{\cos\beta}$

$= 11.9\,N$

Statics revisted - included connected particles

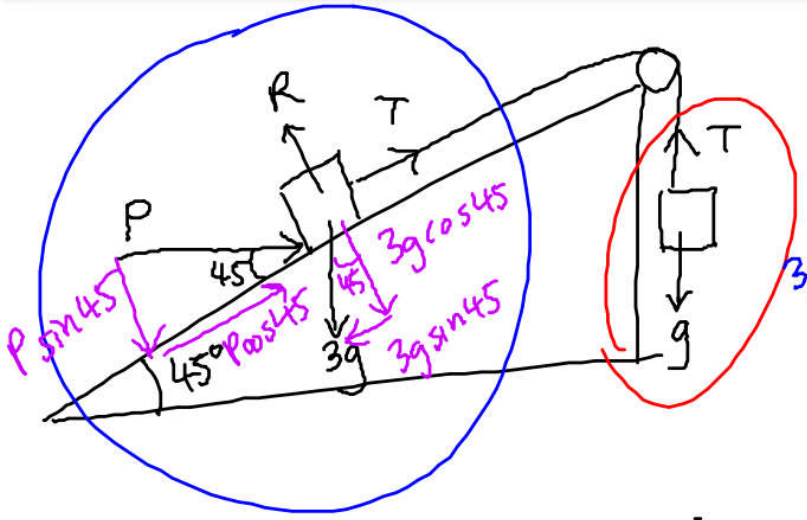
A mass of 3kg rests on the surface of a smooth plane which is inclined at an angle of 45° to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1kg freely suspended at the other end. The masses are modelled as particles, and the cable as a light inextensible string. There is a force of P N acting horizontally on the 3kg mass and the system is in equilibrium.

Calculate

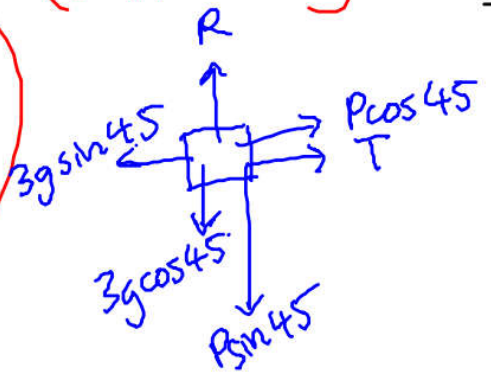
(a) the magnitude of P

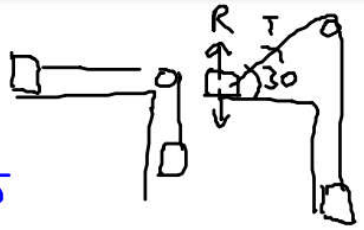
(b) the normal reaction between the mass and the plane

(c) State how you have used the assumption that the pulley is smooth in your calculations.



(R↕) T = g





a) (R↔) $3g\sin 45 = P\cos 45 + g$

$P = \dfrac{3g\sin 45 - g}{\cos 45} = 15.5\text{N}$ (3sf)

Ex 7B

~~Q6~~ Q8, Q11

b) (R↕) $R = 3g\cos 45 + P\sin 45$

$= 31.8\text{N}$ (3sf)

c) Tension on both sides of pulley is equal.

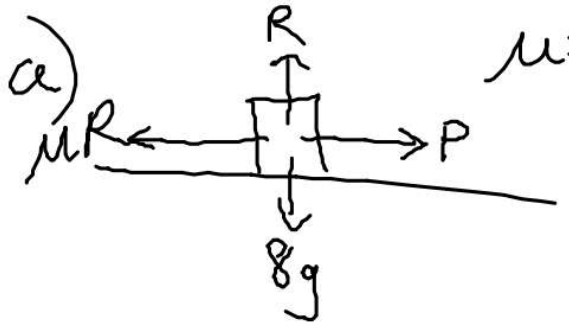
Statics - including friction

Earlier we saw that the frictional force $F \leq \mu R$, where $F = \mu R$ if the object on the plane is moving. Where the object is not moving, we saw that the **force of friction acts in a direction opposite** to that which it would be moving if the frictional force wasn't there.

A box of mass 8kg rests on a rough horizontal plane. The coefficient of friction between the mass and the plane is 0.5. Find the magnitude of the maximum force *P* N which acts on this mass without causing it to move if: → limiting equilibrium

(a) The force *P* is horizontal

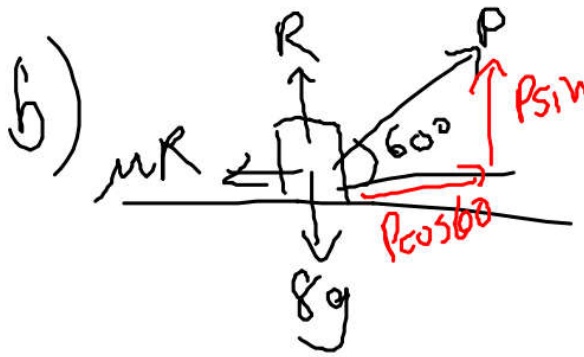
(b) The force *P* acts at an angle 60° above the horizontal

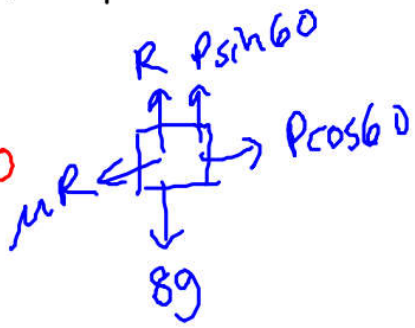


$\mu = 0.5$

$R = 8g$

$P = \mu R = 0.5 \times 8g = 4g$.





$R = 8g - P\sin 60$.

$P\cos 60 = 0.5R$

$P\cos 60 = 0.5(8g - P\sin 60)$

$P\cos 60 = 4g - 0.5P\sin 60$

$P\cos 60 + 0.5P\sin 60 = 4g$

$P(\cos 60 + 0.5\sin 60) = 4g$

$P = \frac{4g}{\cos 60 + 0.5\sin 60} = 42.0$ N

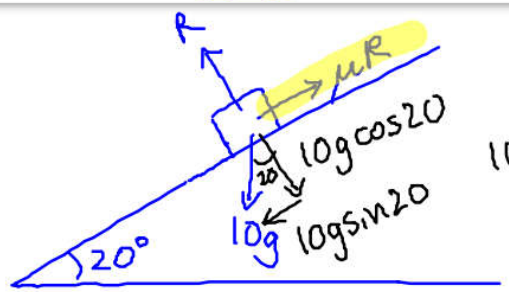
A box of mass 10kg rests in limiting equilibrium on a rough plane inclined at 20° above the horizontal.

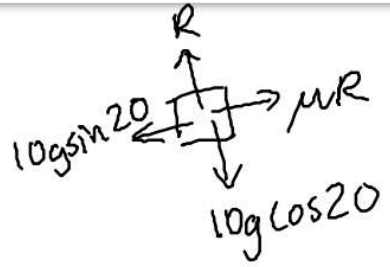
$F_r = \mu R$

(a) Find the coefficient of friction between the box and the plane.

A horizontal force of magnitude P N is applied to the box. Given that the box remains in equilibrium,

(b) find the maximum possible value of P.





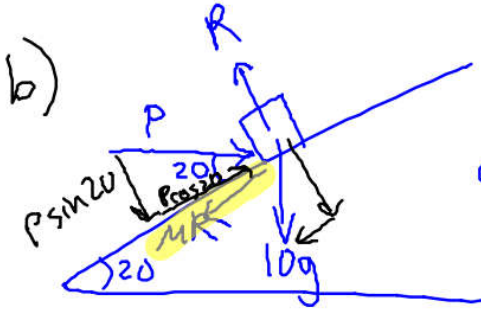
$(R\updownarrow)\ R = 10g\cos 20$

Ex 7C odd

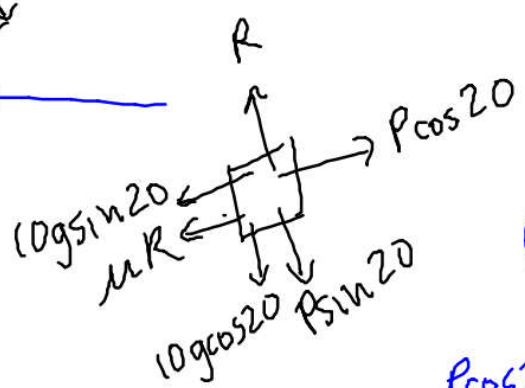
a) $(R\leftrightarrow)\ 10g\sin 20 = \mu 10g\cos 20$

$\mu = \tan 20$

$= 0.364$ (3sf)



Friction changes direction because it will be on point of slipping up.



$(R\updownarrow)\ R = 10g\cos 20 + P\sin 20$

$(R\nearrow)\ P\cos 20 = 10g\sin 20 + \mu R$

$P\cos 20 = 10g\sin 20 + \tan 20(10g\cos 20 + P\sin 20)$

$P\cos 20 = 10g\sin 20 + 10g\sin 20 + P\sin 20\tan 20$

$P(\cos 20 - \sin 20\tan 20) = 20g\sin 20$

$P = 82.2$ N (3sf)