A Level · Edexcel · Maths

3 hours 26 questions

4.1 Probability Distributions

4.1.1 Discrete Probability Distributions

Easy (8 questions)	/40
Medium (6 questions)	/31
Hard (6 questions)	/39
Very Hard (6 questions)	/42
Total Marks	**/152**

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Easy Questions

1 (a) The discrete random variable, X, is defined as the number of sixes obtained from rolling two fair dice.

(i) Find the probability of obtaining two sixes from rolling two fair dice.

(ii) Complete the following probability distribution table for X:

x	0	1	2
$P(X = x)$	$\frac{25}{36}$		

(3 marks)

(b) Use the table, or otherwise, to find the probability of obtaining at least one six from rolling two fair dice.

(2 marks)

2 (a) The discrete random variable X has the probability function

$$P(X=x) = \begin{cases} \frac{1}{4} & x = 0,1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

Briefly explain why X has a **uniform** probability distribution.

(1 mark)

(b) Find:

(i) $P(1 \le X \le 2)$

(ii) $P(X < 3)$.

(2 marks)

3 (a) The discrete random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 2,3 \\ 0 & \text{otherwise} \end{cases}$$

Use the fact that the sum of all probabilities equals 1 to show that $k = 0.2$.

(2 marks)

(b) Write down:

(i) $P(2 \le X < 3)$

(ii) $P(X = 5)$

(2 marks)

4 (a) A discrete random variable X has the probability distribution shown in the following table:

x	2	4	6	8	10
$\mathrm{P}(X = x)$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{5}$	p	$\frac{1}{10}$

Use the fact that the sum of all probabilities equals 1 to find the value of p.

(2 marks)

(b) Find:

(i) $\mathrm{P}(X \le 4)$

(ii) $\mathrm{P}(X > 7)$

(iii) $\mathrm{P}(2 \le X \le 6)$

(iv) $\mathrm{P}(3 < X < 7)$

(4 marks)

5 (a) The discrete random variable has the probability function

$$P(X = x) = \begin{cases} kx & x = 1,3 \\ \dfrac{kx}{2} & x = 2,4 \\ 0 & \text{otherwise} \end{cases}$$

Use the fact that the sum of all probabilities equals 1 to show that $k = \dfrac{1}{7}$.

(2 marks)

(b) Briefly explain why X has a **non-uniform** probability distribution.

(1 mark)

(c) Show that $P(X \le 2) = P(X = 4)$.

(2 marks)

6 (a) The discrete random variable X has the probability distribution shown in the following table:

x	1	2	3	4	5
$P(X=x)$	$\frac{5}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{1}{12}$

Complete the following **cumulative** probability function table for X:

x	1	2	3	4	5
$P(X=x)$	$\frac{5}{12}$	$\frac{7}{12}$			1

(2 marks)

(b) Use your table from part (a) to find:

(i) $P(X \le 3)$

(ii) $P(X \ge 4)$

(iii) $P(2 \le X \le 4)$

(5 marks)

7 (a) The discrete random variable X has the **cumulative** probability distribution shown in the following table:

x	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$

Complete the following probability distribution table for X:

x	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{5}$	$\frac{1}{5}$			

(3 marks)

(b) Find:

(i) $P(X<0)$

(ii) $P(X>0)$.

(2 marks)

(c) Explain, with a reason, whether X has a **uniform** probability distribution or not.

(2 marks)

8 (a) The discrete random variable has the probability function

$$P(X=x)=\begin{cases} \frac{1}{4} & x=0 \\ \frac{1}{8} & x=1,2 \\ \frac{5}{16} & x=3 \\ p & x=4 \\ 0 & \text{otherwise} \end{cases}$$

Briefly explain how you can deduce that $p=\frac{3}{16}$.

(1 mark)

(b) Find $P(1 \le X \le 2)$.

(2 marks)

Medium Questions

1 (a) Three biased coins are tossed.

Write down all the possible outcomes when the three coins are tossed.

(1 mark)

(b) A random variable, X, is defined as the number of heads when the three coins are tossed.

Given that for each coin the probability of getting heads is $\frac{2}{3}$,

complete the following probability distribution table for X:

x	0	1	2	3
P(X=x)				

(3 marks)

(c) represent the probability distribution for X as a probability mass function.

(2 marks)

2 The random variable X has the probability function

$$\mathrm{P}(X=x)=\begin{cases}\dfrac{1}{k} & x=1,2,3,4,5\\ 0 & \text{otherwise}\end{cases}$$

(i) Show that k = 5.

(ii) Write down the name of this probability distribution.

(3 marks)

3 (a) The random variable X has the probability function

$$P(X=x)=\begin{cases} kx & x=1,3,5,7 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k.

(2 marks)

(b) Find $P(X>3)$.

(2 marks)

(c) State, with a reason, whether or not X is a discrete random variable.

(1 mark)

4 (a) The random variable X has the probability function

$$P(X=x)=\begin{cases} 0.23 & x=-1,4 \\ k & x=0,2 \\ 0.13 & x=1,3 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k.

(2 marks)

(b) Construct a table giving the probability distribution of X.

(2 marks)

(c) Find $P(0 \le X < 3)$.

(1 mark)

5 A discrete random variable X has the probability distribution shown in the following table:

x	0	1	2	3	4
P(X = x)	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$

Find:

(i) $P(X<4)$

(ii)

$\mathrm{P}(X > 1)$

(iii) $\mathrm{P}(2 < X \leq 4)$

(iv) $\mathrm{P}(0 < X < 4)$

(4 marks)

6 (a) Leonardo has constructed a biased spinner with six sectors labelled 0,1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

Find the value of p.

(1 mark)

(b) Leonardo is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

(3 marks)

(c) Complete the following cumulative probability function table for X:

Score x	0	1	2	3	5
$\mathbf{P(X \leq x)}$	$\frac{6}{20}$				1

(2 marks)

(d) Find the probability that X is

(i) no more than 1

(ii) at least 3.

(2 marks)

Hard Questions

1 (a) Three biased coins are tossed.

Write down all the possible outcomes when the three coins are tossed.

(1 mark)

(b) A random variable, X, is defined as the number of heads when the three coins are tossed minus the number of tails.

Given that for each coin the probability of getting heads is $\frac{3}{5}$,

complete the following probability distribution table for X:

x				
$P(X = x)$				

(3 marks)

(c) Represent the probability distribution for X as a probability mass function.

(2 marks)

2 The random variable X has the probability function

$$P(X=x)=\begin{cases}\dfrac{1}{k} & x=1,2,3,5,8,13\\ 0 & \text{otherwise}\end{cases}$$

(i) Find the value of k.

(ii) Write down the name of this probability distribution.

(3 marks)

3 (a) A student claims that a random variable X has a probability distribution defined by the following probability mass function:

$$P(X=x)=\begin{cases} \dfrac{x^2}{30} & x=1,1,3,5 \\ 0 & \text{otherwise} \end{cases}$$

Explain how you know that the student's function does not describe a probability distribution.

(2 marks)

(b) Given that the correct probability mass function is of the form

$$P(X=x)=\begin{cases} \dfrac{x^2}{k} & x=-1,1,3,5 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant,

find the exact value of k.

(2 marks)

(c) Find $\mathrm{P}(X>0)$.

(2 marks)

(d) State, with a reason, whether or not X is a discrete random variable.

(1 mark)

4 (a) The random variable X has the probability function

$$P(X=x)=\begin{cases} 0.21 & x=0,1 \\ kx & x=3,6 \\ 0.11 & x=10,15 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k.

(2 marks)

(b) Construct a table giving the probability distribution of X.

(2 marks)

(c) Find $\mathrm{P}(3 < X \leq 14)$

(1 mark)

5 (a) A discrete random variable X has the probability distribution shown in the following table:

x	-1	1	2
$\mathrm{P}(X = x)$	$\frac{5}{12}$	p	$\frac{1}{4}$

Find the value of p.

(1 mark)

(b) X is sampled twice such that the results of the two experiments are independent of each other, and the outcomes of the two experiments are recorded. A new random variable, Y, is defined as the sum of the two outcomes.

Complete the following probability distribution table for Y:

y	-2	0	1	2	3	4
$\mathrm{P}(Y = y)$						

(5 marks)

(c) Find:

(i) $\mathrm{P}(Y \neq 0)$

(ii) $\mathrm{P}(Y > 1)$

(iii)

$\mathrm{P}(-2 < Y < 2)$

(iv) $\mathrm{P}(Y < 0 \ \ or \ \ Y \geq 2)$

(4 marks)

6 (a) Leonidas is playing a game with a fair six-sided dice on which the faces are numbered 1 to 6. He rolls the dice until either a '6' appears or he has rolled the dice four times. The random variable X is defined as the number of times that the dice is rolled.

Write down the probability distribution of X in table form.

(4 marks)

(b) Complete the following cumulative probability function table for X:

x	1	2	3	4
$\mathrm{P}(X \leq x)$				

(2 marks)

(c) Find the probability that X is

(i) at most 3

(ii) at least 3.

(2 marks)

Very Hard Questions

1 (a) Two biased coins are tossed and a fair spinner with three sectors numbered 1 to 3 is spun.

Write down all the possible outcomes when the two coins are tossed and the spinner is spun.

(1 mark)

(b) A random variable, X, is defined as the number of heads when the two coins are tossed multiplied by the number the spinner lands on when it is spun.

For each coin the probability of getting heads is $\frac{1}{3}$.

Complete the following probability distribution table for X:

x	0	1	2	3	4	6
$P(X = x)$						

(5 marks)

(c) Represent the probability distribution for X as a probability mass function.

(2 marks)

2 The random variable X can take the values $k^2(-1)^k$ for $k = 0, 2, 3, 5, 6.$

Given that X is distributed as a discrete uniform distribution, write down the probability mass function of X.

(3 marks)

3 (a) A student claims that a random variable X has a probability distribution defined by the following probability mass function:

$$P(X=x)=\begin{cases} \dfrac{1}{3x^2} & x=-3,-1 \\ \dfrac{1}{3x^3} & x=1,3 \\ 0 & \text{otherwise} \end{cases}$$

Explain how you know that the student's function does not describe a probability distribution.

(2 marks)

(b) Given that the correct probability mass function is of the form

$$P(X=x)=\begin{cases} \dfrac{k}{x^2} & x=-3,-1 \\ \dfrac{k}{x^3} & x=1,3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant,

Find the exact value of k.

(2 marks)

(c) Find $\mathrm{P}(X<2)$.

(2 marks)

(d) State, with a reason, whether or not X is a discrete random variable.

(1 mark)

4 (a) The random variable X has the probability function

$$P(X = x) = \frac{x^2}{495}, \qquad x = p, 2p, 3p, 4p, 5p$$

where $p > 0$ is a constant.

Construct a table giving the probability distribution of X.

(4 marks)

(b) Complete the following cumulative probability function table for X:

$\boldsymbol{x}$					
$\mathbf{P}(\boldsymbol{X \leq x})$					1

(2 marks)

(c) Find:

(i) $\mathrm{P}(3 < X \leq 12)$

(ii) the probability that X is no more than 10

(iii) the probability that X is at least 10.

(3 marks)

5 The independent random variables X and Y have probability distributions

$$\mathrm{P}(X = x) = p, \qquad x = 1, 2, 3, 5, 8, 11$$

$$\mathrm{P}(Y = y) = \frac{q}{y}, \qquad y = 1, 3, 6$$

where p and q are constants.

Find $\mathrm{P}(X > Y)$.

(6 marks)

6 (a) Leofranc is playing a gambling game with a fair six-sided dice on which the faces are numbered 1 to 6. He must pay £2 to play the game. He then chooses a 'lucky number' between 1 and 6, and rolls the dice until either his lucky number appears or he has rolled the dice four times. If his lucky number appears on the first roll, he receives £5 back. If his lucky number appears on the second, third or fourth rolls, he receives £3, £2 or £1 back respectively. If his lucky number has not appeared by the fourth roll, then the game is over and he receives nothing back.

The random variable W is defined to be Leofranc's profit (i.e., the amount of money he receives back minus the cost of playing the game) when he plays the game one time. Note that a negative profit indicates that Leofranc has lost money on the game.

Write down the probability distribution of W in table form.

(6 marks)

(b) Find the probability that when playing the game one time Leofranc

(i) wins money

(ii) loses money

(iii) breaks even (i.e., does not lose money).

(3 marks)