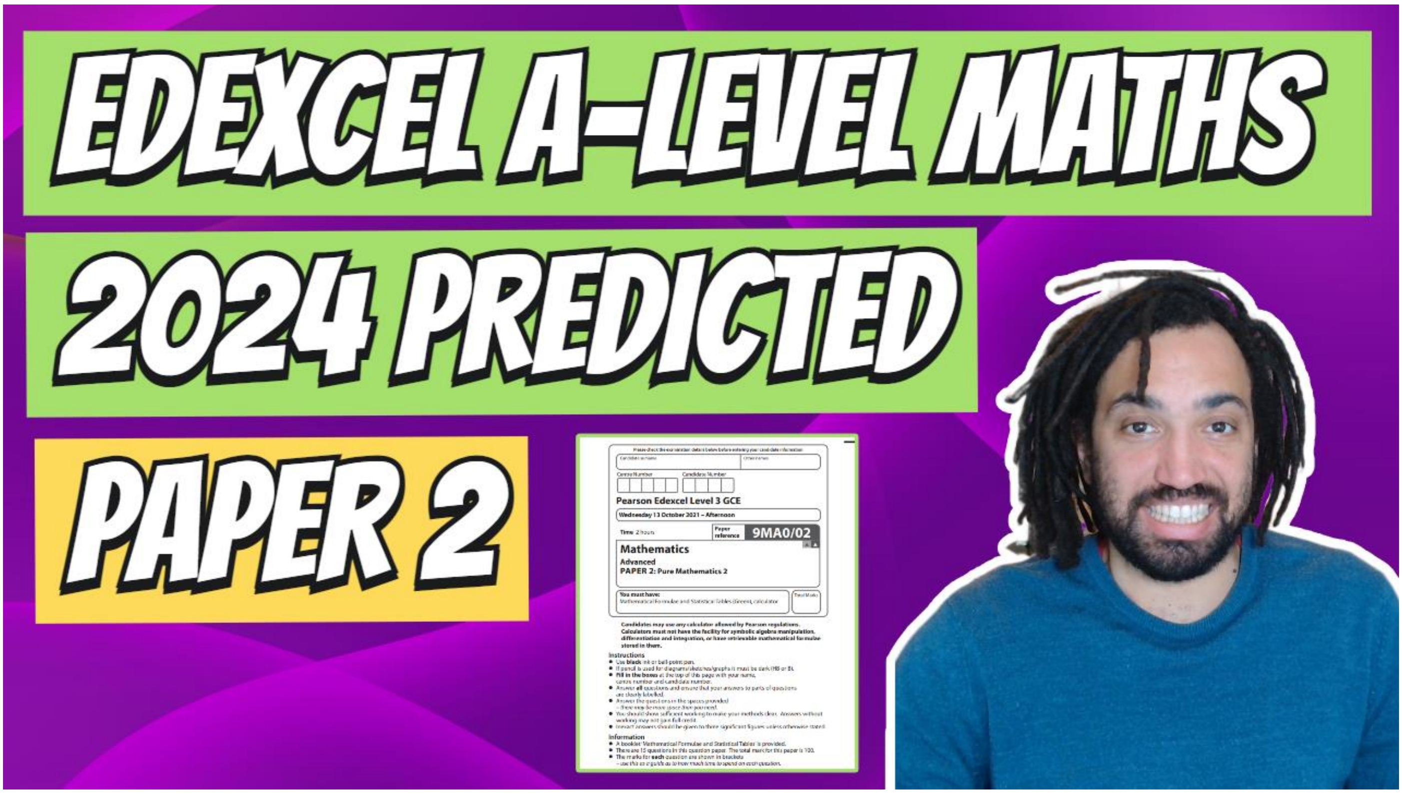

DISCLAIMER:

THERE IS NO GUARANTEE THESE HIGHLIGHTED TOPICS WILL COME UP ON PAPER 2 OR THAT THE UNHIGHLIGHTED ONES WON'T COME UP AGAIN! USE THIS PAPER AS PRACTISE ON TOP OF THOROUGH REVISION.

YOUTUBE SOLUTIONS	LIVE REVISION SESSIONS	SUPPORT MR ASTBURY

19 Questions – 150 Marks

Topic
Circles
Vectors
Exponential Modelling
Logarithmic Modelling
Log Equations
Proof
Factor Theorem
Functions
Modulus
Graph Transformations
Binomial Expansion
Geometric Sequences
Arithmetic Sequences
Recursive Sequences
Trigonometric Identities and Equations
Trigonometric Modelling, $Rsin(\theta + \alpha)$
Radian Measure, Arcs and Sectors
Differentiation Modelling - Year 1
1st Principles
Differentiation (product, chain and quotient)
Implicit Differentiation
Iteration
Newton Raphson
Integration Year 1
Integration Substitution
Integration Trigonometric Identities
Integration Parts
Integration Partial Fractions
Trapezium Rule
Differential Equations
Parametic Equations
Parametric Differentiation
Parametric Integration

N.B. There are other smaller/easier topics that have not made this list. To name a few: Sine and cosine rules, quadratics, equations and inequalities, rates of change, integration as a limit sum, small angle approximation, modelling with linear or quadratic graphs.

Q1. Circles

The circle C has centre X (3, 5) and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

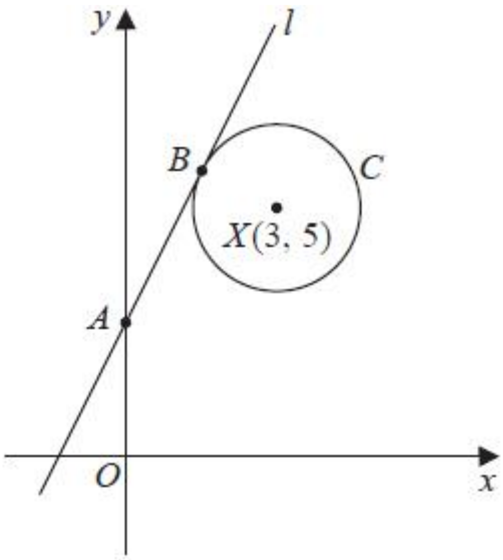

Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total for question = 12 marks)

Q2. Vectors

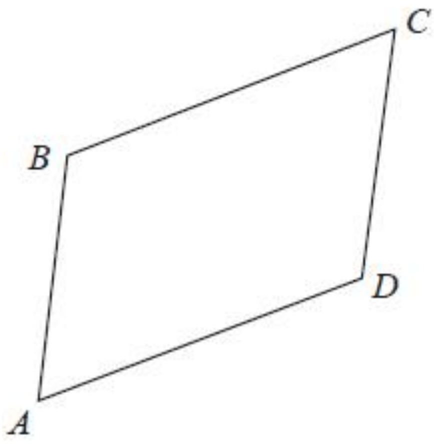

Figure 1

Figure 1 shows a sketch of parallelogram *ABCD*.

Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle *ABC*, giving your answer in degrees, to 2 decimal places.

(3)

(b) Find the area of parallelogram *ABCD*, giving your answer to one decimal place.

(2)

(Total for question = 5 marks)

Q3. Exponential Modelling

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A population of fruit flies is being studied.

The number of fruit flies, F, in the population, t days after the start of the study, is modelled by the equation

$$F = \frac{350e^{kt}}{9 + e^{kt}}$$

where k is a constant.

Use the equation of the model to answer parts (a), (b) and (c).

(a) Find the number of fruit flies in the population at the start of the study.

(1)

Given that there are 200 fruit flies in the population 15 days after the start of the study,

(b) show that $k = \frac{1}{15}\ln 12$

(3)

Given also that, when $t = T$, the number of fruit flies in the population is increasing at a rate of 10 per day,

(c) find the possible values of T, giving your answers to one decimal place.

(5)

(Total for question = 9 marks)

Q4. Logarithmic Modelling

In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$N = aT^b$ where a and b are constants

(a) Show that this relationship can be expressed in the form

$$\log_{10}N = m\log_{10}T + c$$

giving m and c in terms of the constants a and/or b.

(2)

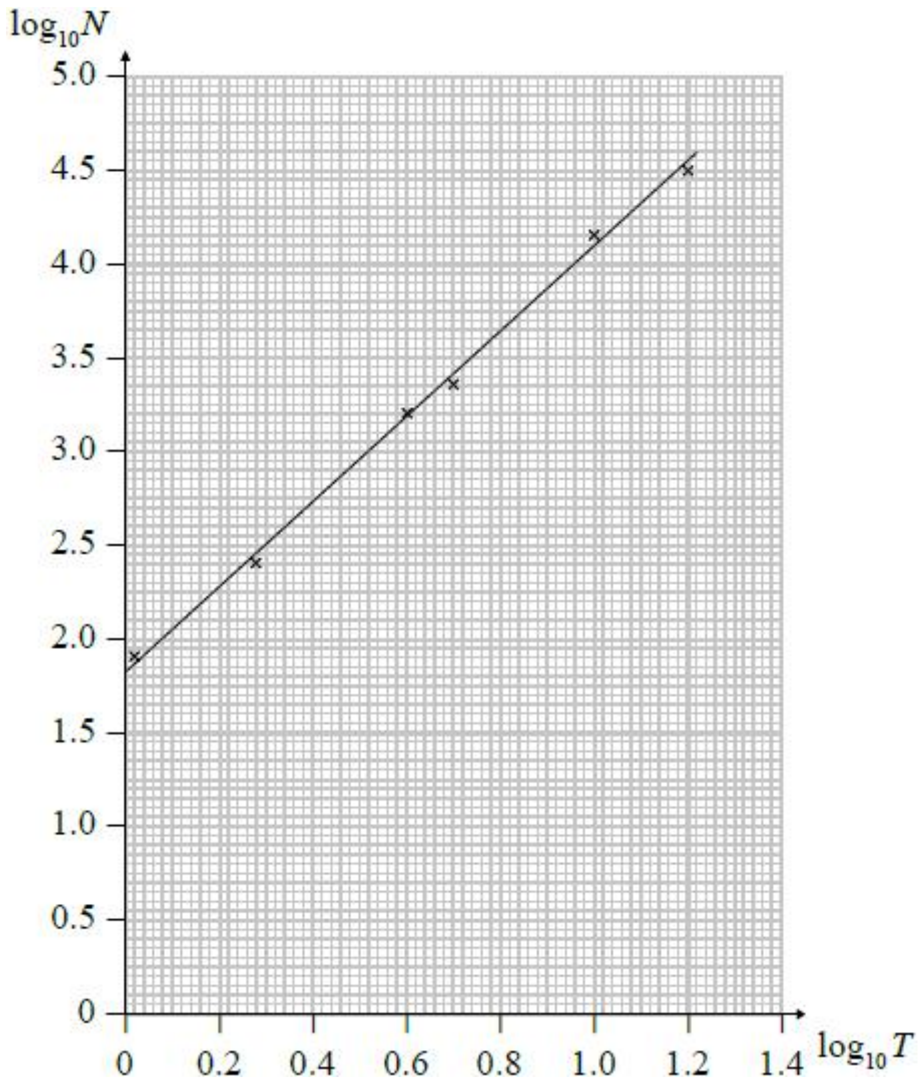

Figure 2

Figure 2 shows the line of best fit for values of $\log_{10}N$ plotted against values of $\log_{10}T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) With reference to the model, interpret the value of the constant a.

(1)

(Total for question = 7 marks)

Q5. Log Equations

Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(Total for question = 3 marks)

Q6. Graph Transformations

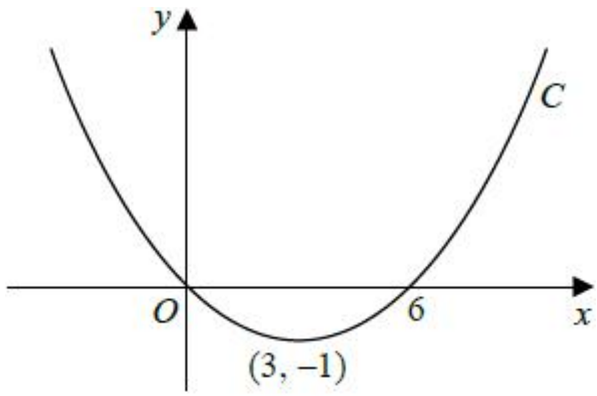

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$

The curve C passes through the origin and through (6, 0)

The curve C has a minimum at the point (3, −1)

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$

(3)

(b) $y = f(x + p)$, where p is a constant and $0 < p < 3$

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

(Total for question = 7 marks)

Q7. Arithmetic Sequences

A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

(a) Find the mass of silver extracted in year 14

(2)

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year N,

(b) show that

$$N^2 - 65N + 1036 = 0$$

(3)

(c) Hence, state the value of N.

(1)

(Total for question = 6 marks)

Q8. Recursive Sequences

A sequence a_1, a_2, a_3, ... is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that an $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

(Total for question = 7 marks)

Q9. Trigonometric Identities and Equation

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

(Total for question = 9 marks)

Q10. Differentiation Modelling

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

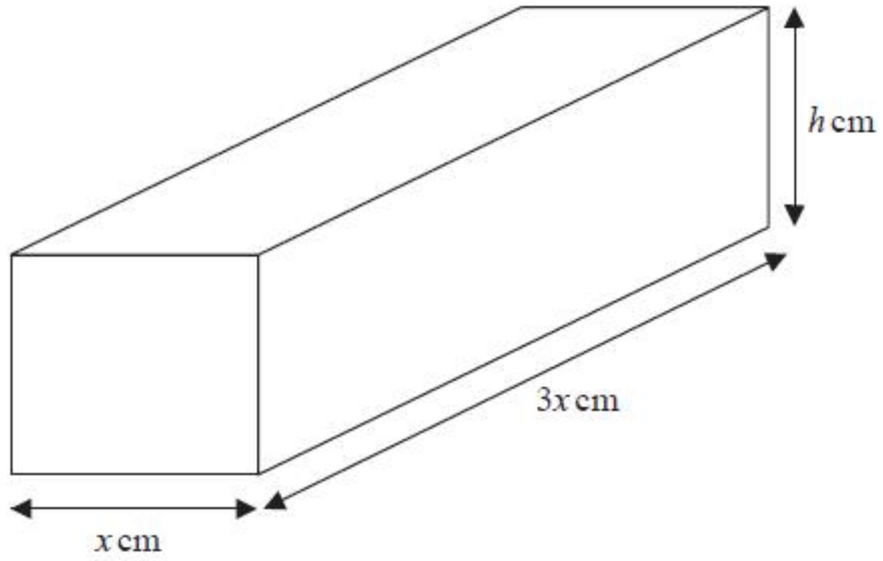

Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is 972 cm^3

(a) Show that the surface area of the brick, S cm^2, is given by

$$S = 6x^2 + \frac{2592}{x}$$

(3)

(b) Find $\frac{dS}{dx}$

(1)

(c) Hence find the value of x for which S is stationary.

(2)

(d) Find $\frac{d^2S}{dx^2}$ and hence show that the value of x found in part (c) gives the minimum value of S.

(2)

(e) Hence find the minimum surface area of the brick.

(1)

(Total for question = 9 marks)

Q11. Differentiation – Product and Chain Rules

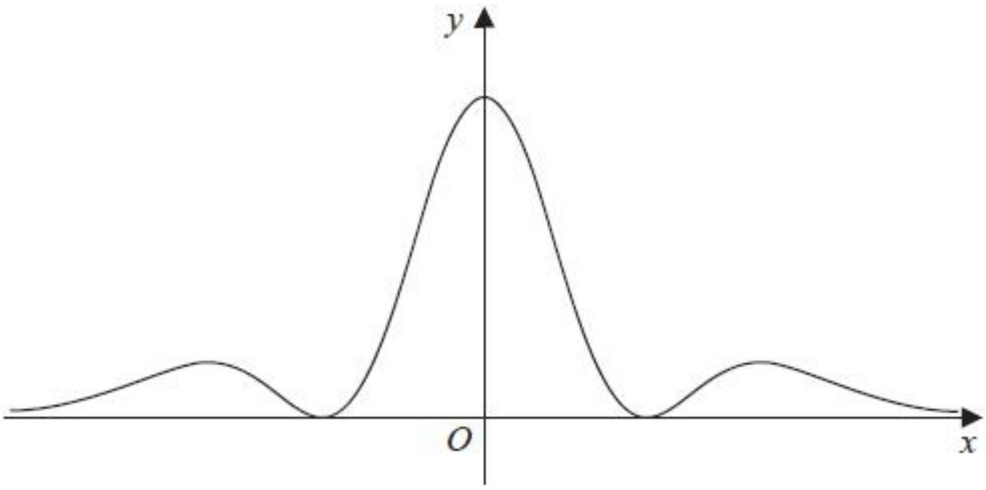

Figure 1

Figure 1 shows a sketch of the curve *C* with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \mathrm{e}^{-x^2}\left(2x^2 - 3\right)^2$$

(a) Find the range of f

(2)

(b) Show that

$$\mathrm{f}'(x) = 2x\left(2x^2 - 3\right)\mathrm{e}^{-x^2}\left(A - Bx^2\right)$$

where *A* and *B* are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

(Total for question = 10 marks)

Q12. Implicit Differentiation

A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

(Total for question = 7 marks)

Q13. Iteration

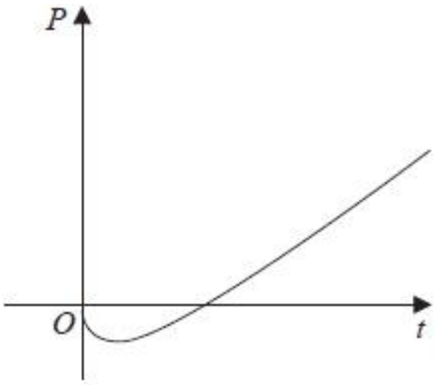

Figure 2

The profit made by a company, £ P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4}\ln\left[\frac{t+1}{(2t+1)^2}\right]$$

The graph of P against t is shown in Figure 2.

According to the model,

(a) show that exactly one year after it started trading, the company had made a loss of approximately £ 830 000

(2)

A manager of the company wants to know the value of t for which $P = 0$

(b) Show that this value of t occurs in the interval [6, 7]

(2)

(c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t+1)^2}{t+1}\right]$$

(2)

(d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t_n+1)^2}{t_n+1}\right] \quad \text{with} \quad t_1 = 6$$

find the value of t_2 and the value of t_6 , giving your answers to 3 decimal places.

(3)

(e) Hence find, according to the model, how many months it takes in total, from when the company started trading, for it to make a profit.

(2)

(Total for question = 11 marks)

Q14. Integration - Partial Fractions

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

(Total for question = 10 marks)

Q15. Integration - Parts

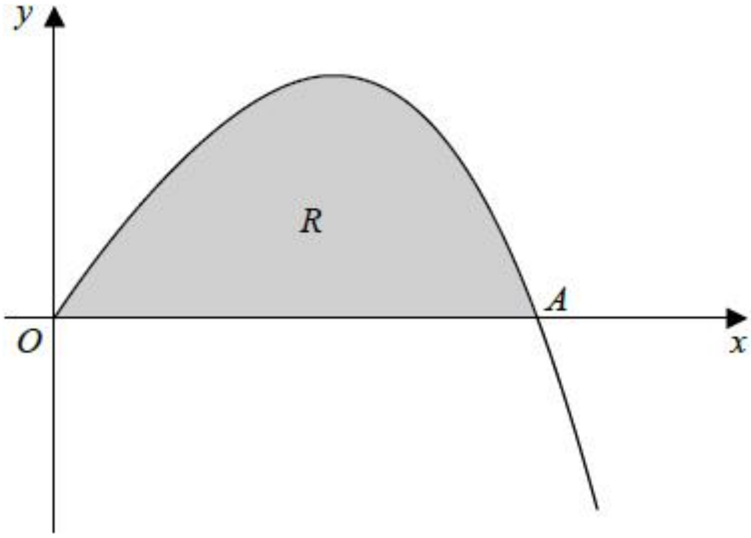

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of ln 2, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\,dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation

$y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of ln 2

(3)

(Total for question = 8 marks)

Q16. Trapezium Rule

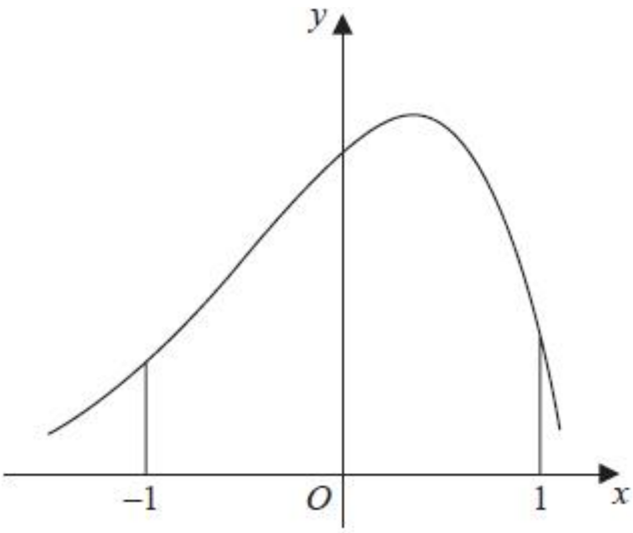

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$

The table below shows some corresponding values of x and y for this curve.

The values of y are given to 3 decimal places.

x	−1	−0.5	0	0.5	1
y	2.287	4.470	6.719	7.291	2.834

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_{-1}^{1} f(x)\,dx$$

giving your answer to 2 decimal places.

(3)

(b) Use your answer to part (a) to estimate

(i) $\displaystyle\int_{-1}^{1} \big(f(x) - 2\big)\,dx$

(ii) $\displaystyle\int_{1}^{3} f(x-2)\,dx$

(3)

(Total for question = 6 marks)

Q17. Parametric Equations

A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

(Total for question = 6 marks)

Q18. Parametric Differentiation

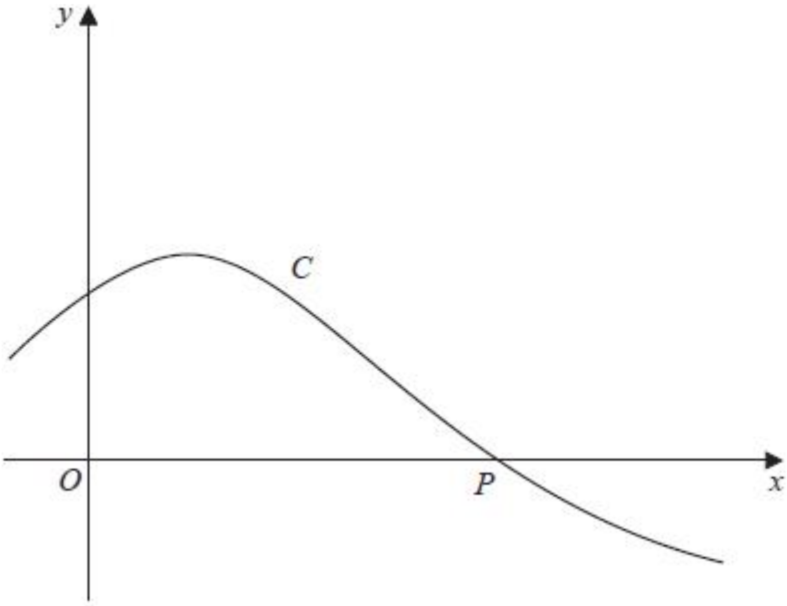

Figure 3

Figure 3 shows a sketch of the curve *C* with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the *x*-axis at point *P*, as shown in Figure 3.

(a) Find the equation of the tangent to *C* at *P*, writing your answer in the form *y* = *mx* + *c*, where *m* and *c* are constants to be found.

(5)

The curve C has equation *y* = f (*x*), where f is a function with domain $\left[k, 1 + 3\sqrt{3}\right]$

(b) Find the exact value of the constant *k*.

(1)

(c) Find the range of f.

(2)

(Total for question = 8 marks)

Q19. Parametric Integration and Reverse Chain Rule

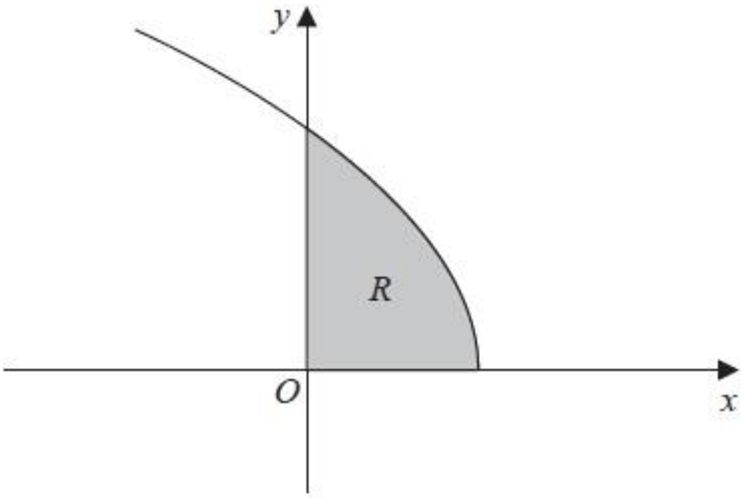

Figure 3

Figure 3 shows a sketch of the curve *C* with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region *R*, shown shaded in Figure 3, is bounded by the curve, the *x*-axis and the *y*-axis.

(a) (i) Show, making your working clear, that the area of R = $\int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t \, dt$

(ii) Hence find, by algebraic integration, the exact value of the area of *R*.

(6)

(b) Show that all points on *C* satisfy $y = \sqrt{ax + b}$, where *a* and *b* are constants to be found.

(3)

The curve *C* has equation *y* = f (*x*) where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and *a* and *b* are the constants found in part (b).

(c) State the range of f.

(1)

(Total for question = 10 marks)