Ex 5B Q12

$y = 3x - 7$

$ax + 4y - 17 = 0$

$P(-3, b)$ (x, y)

$b = 3(-3) - 7$
$= -9 - 7$
$= -16$

$-3a + 4b - 17 = 0$
$-3a - 64 - 17 = 0$
$-81 = 3a$
$a = \underline{-27}$.

Ex 5C Q5

$(a, 4)$ $(3a, 3)$ $x + 6y + c = 0$

$m = \frac{3-4}{3a-a} = -\frac{1}{2a}$

$y - 4 = -\frac{1}{2a}(x - a)$
$2ay - 8a = -(x - a)$
$2ay - 8a = -x + a$
$x + 2ay - 9a = 0$

$2a = 6$
$a = 3$

$c = -9a$
$c = -9 \times 3 = -27$

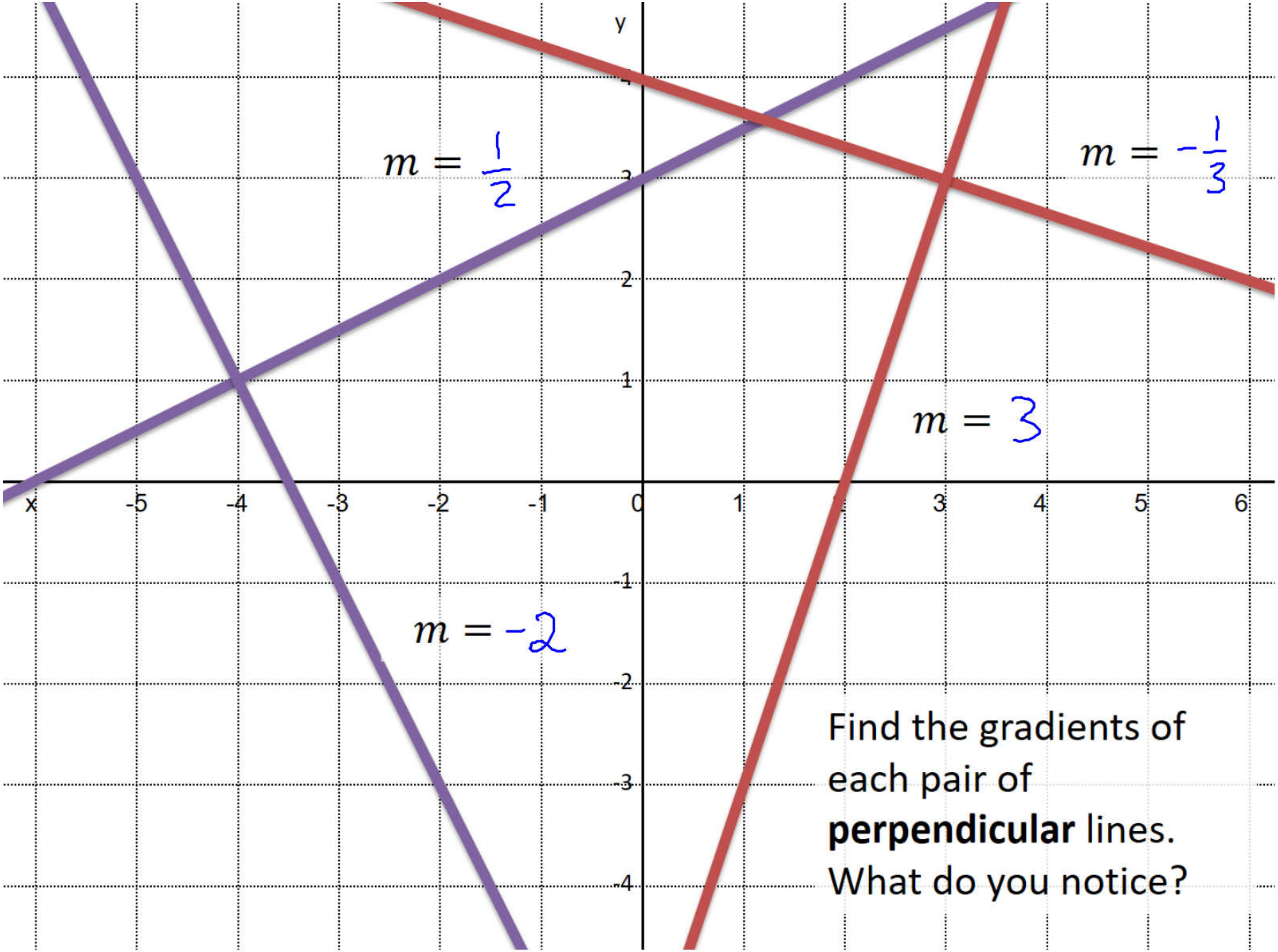
Ex 5D Q16

$y = -2x + 1$ $y = x + 7$ $L(-2, 5)$

$-2x + 1 = x + 7$
$-6 = 3x$
$\underline{\underline{x = -2}}$ $\underline{\underline{y = 5}}$ $m(-3, 1)$

$m_{Lm} = \frac{5-1}{-2--3} = \frac{4}{-2+3} = 4$

$y - 5 = 4(x + 2)$
$y - 5 = 4x + 8$
$y = 4x + 13$



Find the gradients of each pair of **perpendicular** lines. What do you notice?

Perpendicular Lines

The gradients of parallel lines are equal.
If two lines are perpendicular, then the gradient of one is the **negative reciprocal** of the other.

$$m_1 = -\frac{1}{m_2}$$

$$m_1 = -\frac{1}{m_2}$$

$$m_1 m_2 = -1$$

To <u>**show**</u> that two lines are perpendicular:

$$m_1 m_2 = -1$$

Gradient	Gradient of Perpendicular Line
2	$-\frac{1}{2}$
-3	$\frac{1}{3}$
$\frac{1}{4}$	-4
5	$-\frac{1}{5}$
$-\frac{2}{7}$	$\frac{7}{2}$
$\frac{7}{5}$	$-\frac{5}{7}$

1 A line is goes through the point (9,10) and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

$m = 3$ perp $m = -\frac{1}{3}$

$y - 10 = -\frac{1}{3}(x - 9)$

2 A line L_1 goes through the points $A(1,3)$ and $B(3,-1)$. A second line L_2 is perpendicular to L_1 and passes through point B. Where does L_2 cross the x-axis?

$y = 0$

$m_{L_1} = \frac{-1-3}{3-1} = -\frac{4}{2} = -2$

$m_{L_2} = \frac{1}{2}$

$(3,-1)$

L_2 $y + 1 = \frac{1}{2}(x - 3)$

$y = 0$

$1 = \frac{1}{2}(x-3)$

$2 = x - 3$

$x = 5$

$(5,0)$

3 Are the following lines parallel, perpendicular, or neither?

$$y = \frac{1}{2}x \qquad m_1 = \frac{1}{2}$$

$$2x - y + 4 = 0$$

$2x + 4 = y$

$m_2 = 2$

$m_1 m_2 = \frac{1}{2} \times 2 = 1$

Not perp.

$m_1 \neq m_2$ Not parallel.

So neither.

Your Turn

1 A line goes through the point (4,7) and is perpendicular to another line with equation $y = 2x + 2$.
What is the equation of the line?
Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

Perp. gradient is $-\frac{1}{2}$

So $y - 7 = -\frac{1}{2}(x - 4)$ $\} \times 2$

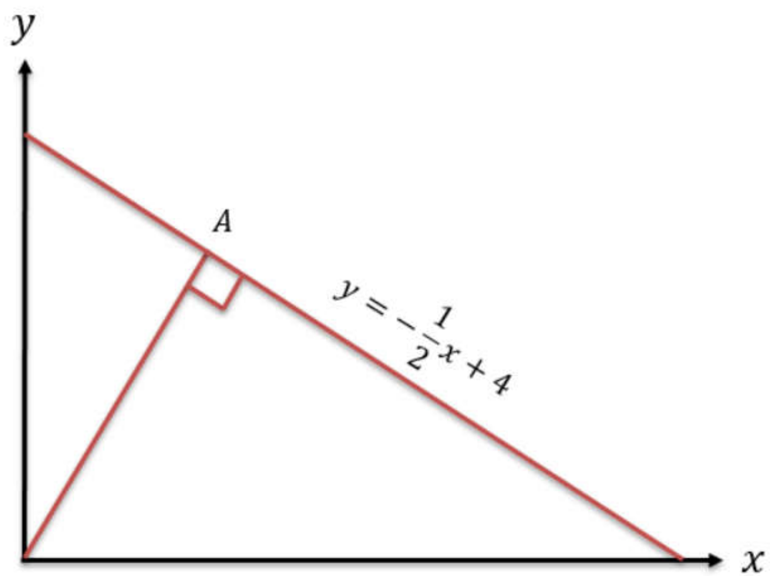
$2y - 14 = -(x - 4)$

$2y - 14 = -x + 4$

$x + 2y - 18 = 0$

Ex 5F
Q11, Q12

2 Determine the point A.
(note that A passes through origin)



Perp. gradient is 2

Passes origin so $y = 2x$

Solve simultaneously

$2x = -\frac{1}{2}x + 4$

$\frac{5}{2}x = 4$

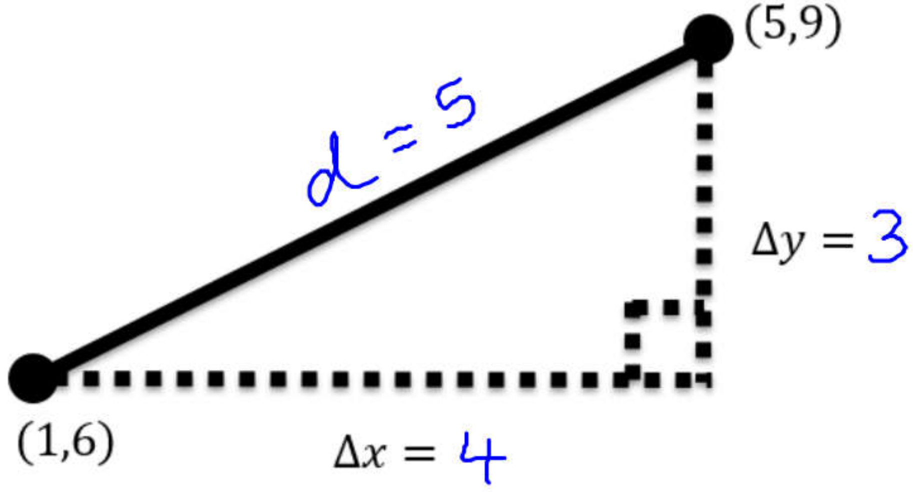
$x = \frac{8}{5}$ $y = 2 \times \frac{8}{5} = \frac{16}{5}$

$A\left(\frac{8}{5}, \frac{16}{5}\right)$

Ex 5E/5F

Distance between 2 points

Recall: Δ (said 'delta') means "change in".



How could we find the **distance** between these two points?

Pythagoras

$d^2 = 4^2 + 3^2$

$d = \sqrt{4^2 + 3^2}$

$d = 5$

✎ Distance between two points:

$$\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Examples:

Distance between:

$(3,4)$ and $(5,7)$

$d = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

$(5,1)$ and $(6,-3)$

$d = \sqrt{1^2 + 4^2}$

$= \sqrt{17}$

$(0,-2)$ and $(-1,3)$

$d = \sqrt{1^2 + 5^2}$

$= \sqrt{26}$

Note: Unlike with gradient, we don't care if the difference is positive or negative (it's being squared to make it positive anyway!)

Your Turn:

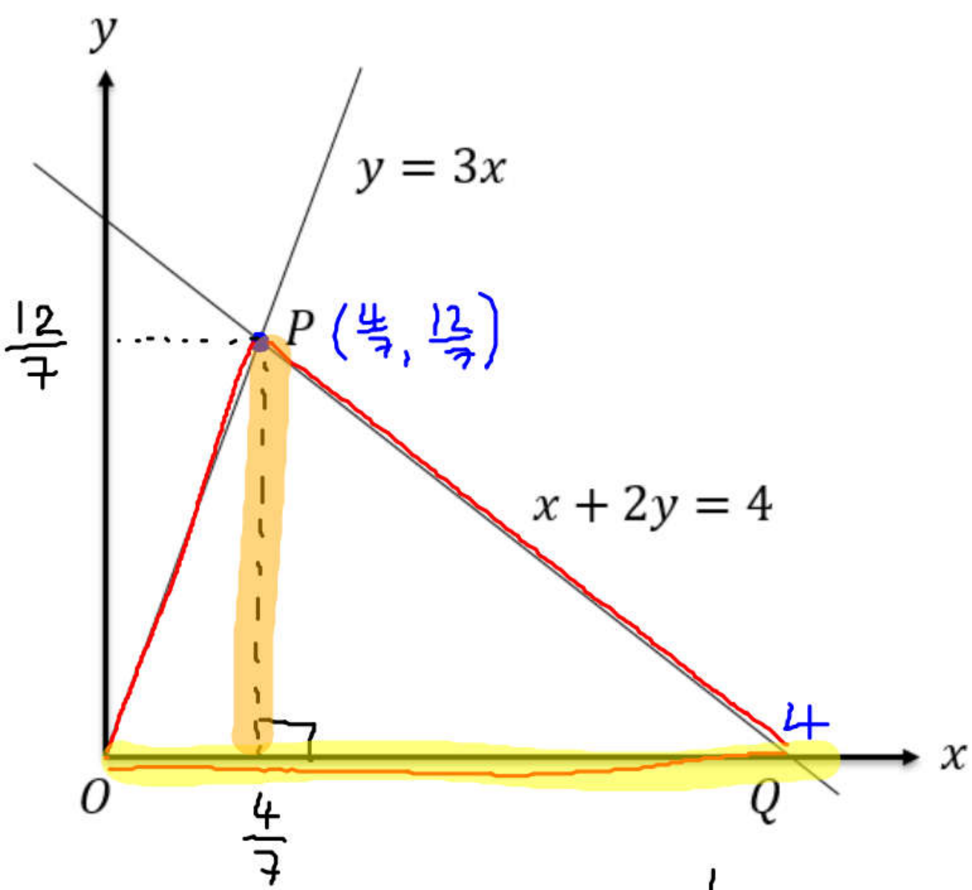
Distance between:

$(1,10)$ and $(4,14)$ $\quad d = \sqrt{3^2 + 4^2} = 5$

$(3,-1)$ and $(0,1)$ $\quad d = \sqrt{3^2 + 2^2} = \sqrt{13}$

$(-4,-2)$ and $(-12,4)$ $\quad d = \sqrt{8^2 + 6^2} = 10$

Area of Shapes



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P.

a) Determine the coordinates of P.

(We did this in a previous lesson)

$\left(\frac{4}{7}, \frac{12}{7}\right)$

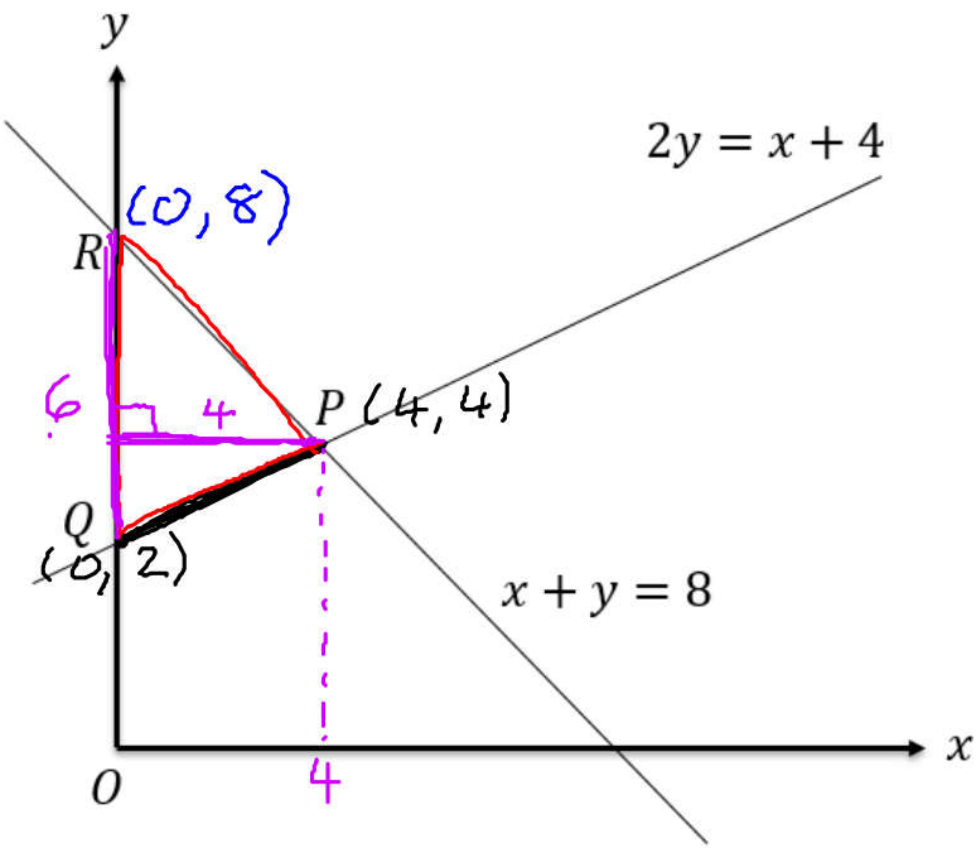
b) The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the area of the triangle OPQ.

$y = 0 \quad x = 4$

base × perp. height × $\frac{1}{2}$

$$\text{Area } \triangle OPQ = 4 \times \frac{12}{7} \times \frac{1}{2}$$

$$= \frac{24}{7} \text{ units}^2$$



Tip: When finding areas of triangles in exam questions, one line is often vertical or horizontal. You should generally choose this to be the 'base' of your triangle.

a. Determine the length of PQ.

Find Q	Find P
$x=0$ $2y=4$ $y=2$ $Q(0,2)$	$2y=x+4$ $x+y=8$ $x=8-y$

$2y = 8 - y + 4$
$3y = 12$
$y = 4 \quad x = 4$
$P(4,4)$

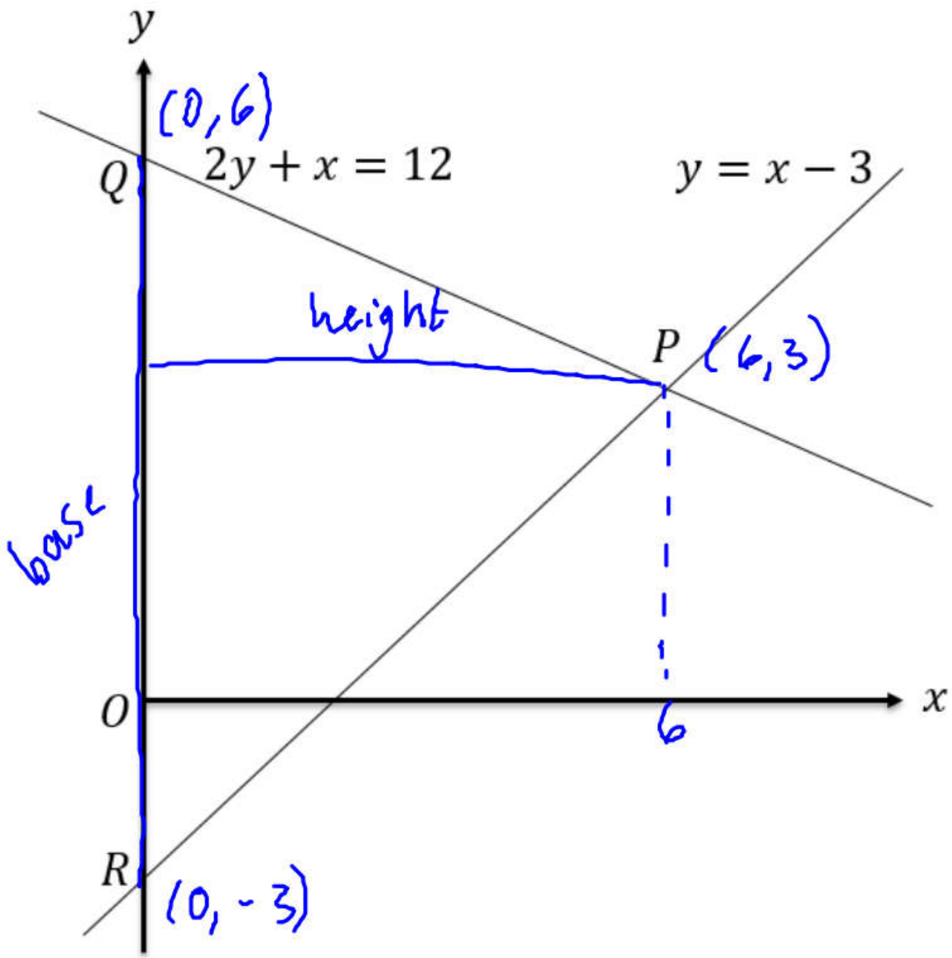
$PQ = \sqrt{4^2 + 2^2}$
$= \sqrt{20} = \underline{\underline{2\sqrt{5}\text{ units}}}$

b. Determine the area PQR.

R is when $x=0$, $y=8$ $(0,8)$

Area $\triangle PQR = 6 \times 4 \times \frac{1}{2} = \underline{12\text{ units}^2}$

Your Turn



a) Determine the coordinate of P.

Simultaneous Equations

$y = x - 3$

$2(x-3) + x = 12$

$2x - 6 + x = 12$

$3x = 18$

$x = 6$

$y = 6 - 3 = 3$

$P(6,3)$

b) Determine the area of PQR.

when $x=0$ $2y=12$, $y=6$ $Q(0,6)$

when $x=0$ $y=-3$ $R(0,-3)$

base $= 9$ height $= 6$ Area $= \frac{9 \times 6}{2} = 27$ units^2

c) Determine the length PQ.

$PQ = \sqrt{6^2 + 3^2}$

$= \sqrt{45} = 3\sqrt{5}$ units

Extension Problem

[MAT 2001 1C]

What is the shortest distance from the origin to the line $3x + 4y = 25$?