2. A mechanic carried out a survey on the defects of cars he was servicing. He found that the probability of a car needing a new tyre is 0.33 and that a car needing a new tyre has a probability of 0.7 of needing tracking. A car not needing a new tyre has a probability of 0.04 of needing tracking.

(a) Draw a tree diagram to represent this information.

**(3)**

(b) Find the probability that a randomly chosen car has exactly one of the two defects, needing a new tyre or needing tracking.

**(2)**

The mechanic also finds that cars need new brake pads with probability 0.35 and that this is independent of needing new tyres or tracking. A car is chosen at random.

(c) Find the probability that the car has at least one of these three defects.

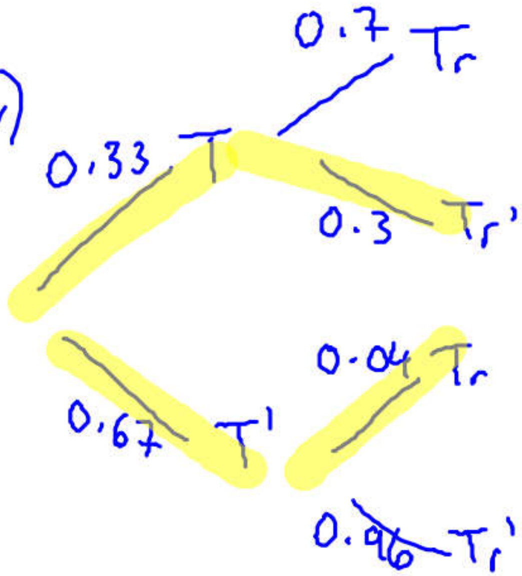
**(2)**

(d) What advice would you give to motorists?

**(1)**

**(Total 8 marks)**

a)




b) $0.33 \times 0.3 + 0.67 \times 0.04 = 0.1258.$

c) No defects at all $= 0.67 \times 0.96 \times 0.65$

$1 - 0.41808 = \underline{0.58192}.$

**7.** Two identical 5 m light see-saws are joined at their ends. Robert, who weighs 80 kg, stands on top of the joint. The distance between Robert and each of the pivots is 2 m. Poppy and Quentin stand on the two remaining ends of the see-saws, as shown in Figure 4. Poppy weighs $p$ kg and Quentin weighs $q$ kg. The system is in equilibrium.

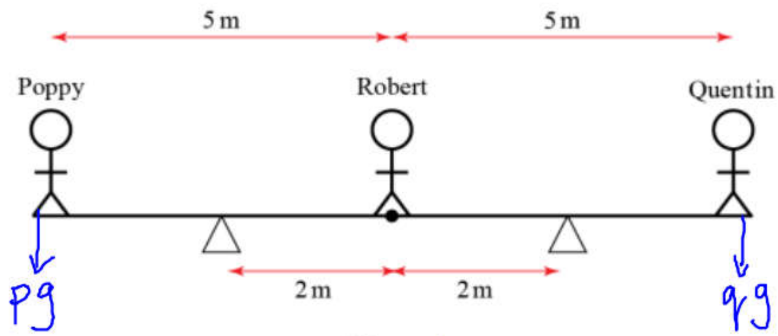
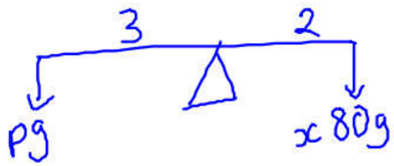



**Figure 4**

Show that, to the nearest whole number, $p + q = 53$.

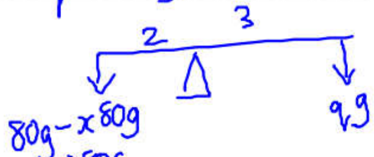
**(Total 8 marks)**

$p + q = \frac{160}{3}$




$3pg = 2x\,80g$

$3p = 160x$



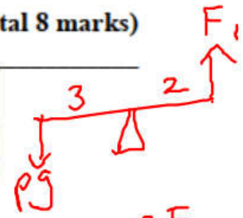

$2(1-x)80\cancel{g} = 3q\cancel{g}$

$160 - 160x = 3q$

$160 - 3p = 3q$

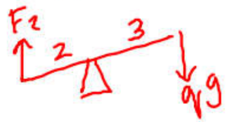
$160 = 3p + 3q$

$53 = p + q$




$3pg = 2F_1$

$F_1 = \frac{3}{2}pg$




$3qg = 2F_2$

$F_2 = \frac{3}{2}qg$

$F_1 + F_2 = 80g$

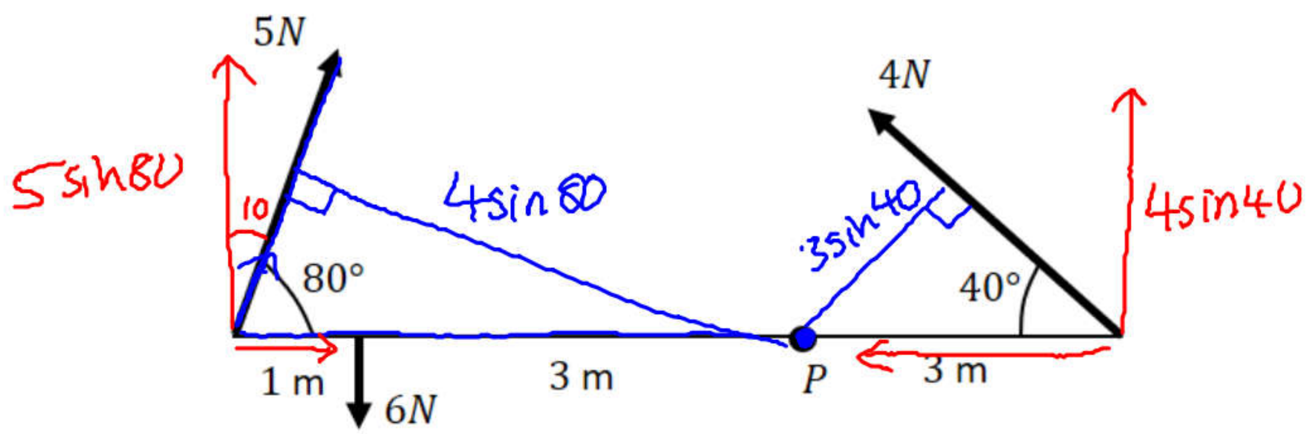
$\frac{3}{2}qg + \frac{3}{2}pg = 80g$

$3q + 3p = 160$

# Rigid Bodies - Part 2: pegs, ladders, hinges

## Calculating with Angled Forces

The diagram shows a set of forces acting on a light rod. Calculate the resultant moment about the point $P$.




moment = force × perp. dist.

$\circlearrowright$ $4 \times 3\sin 40 + 6 \times 3 = 25.7$ N

$\circlearrowleft$ $5 \times 4\sin 80 = 19.7$ N

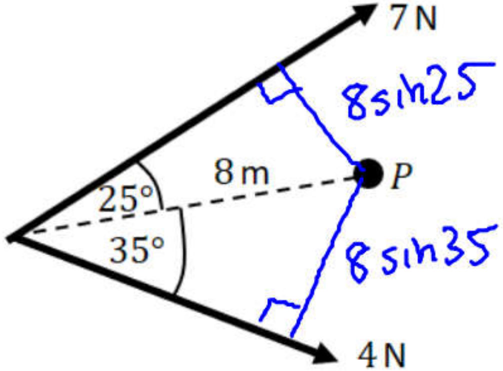
Resultant $= 25.7 - 19.7 = 6$ N $\circlearrowright$

dist × perp. force

$3 \times 4\sin 30 + 6 \times 3$

$4 \times 5\sin 80$

The diagram shows two forces acting on a lamina.
Calculate the resultant moment about the point $P$.



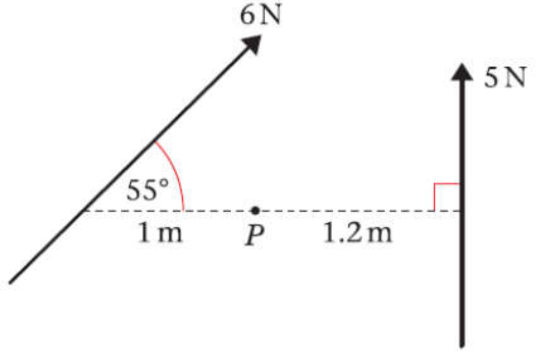

$\curvearrowright$ $7 \times 8\sin 25 = 23.7$

$\curvearrowleft$ $4 \times 8\sin 35 = 18.4$

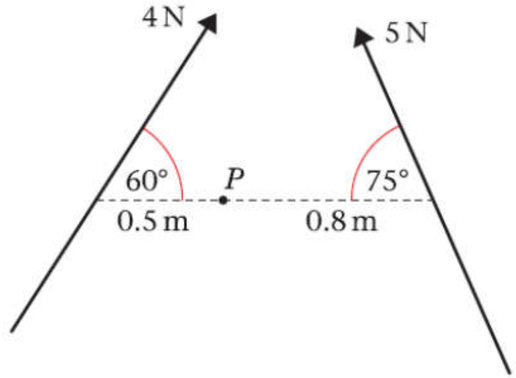
$5.3$ Nm $\curvearrowright$

Find the sum of the moments about $P$ of the forces shown in the following questions.

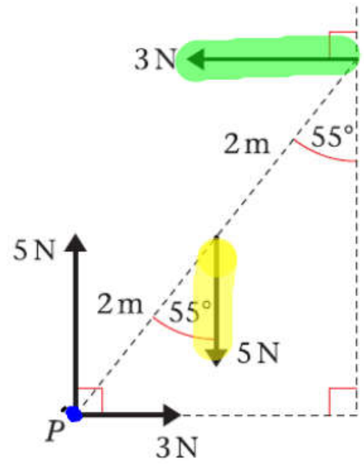
**1**



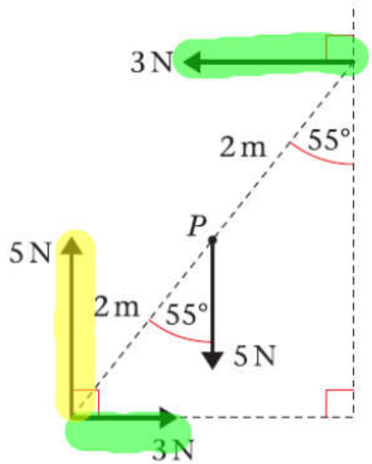

**2**



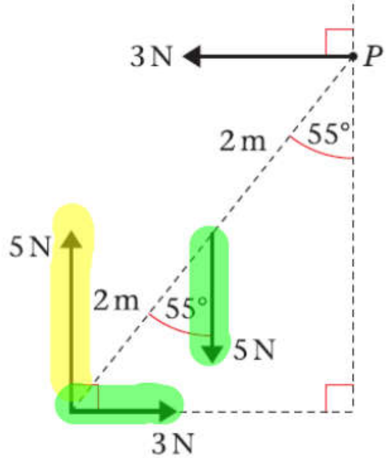

**3**



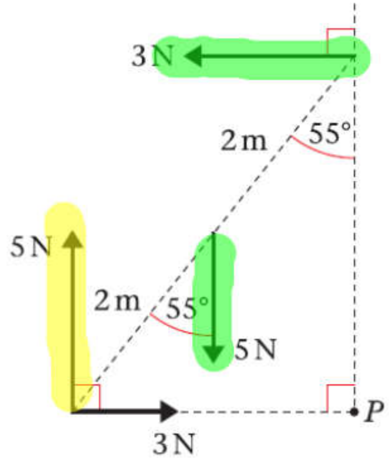

**4**




**5**




**6**

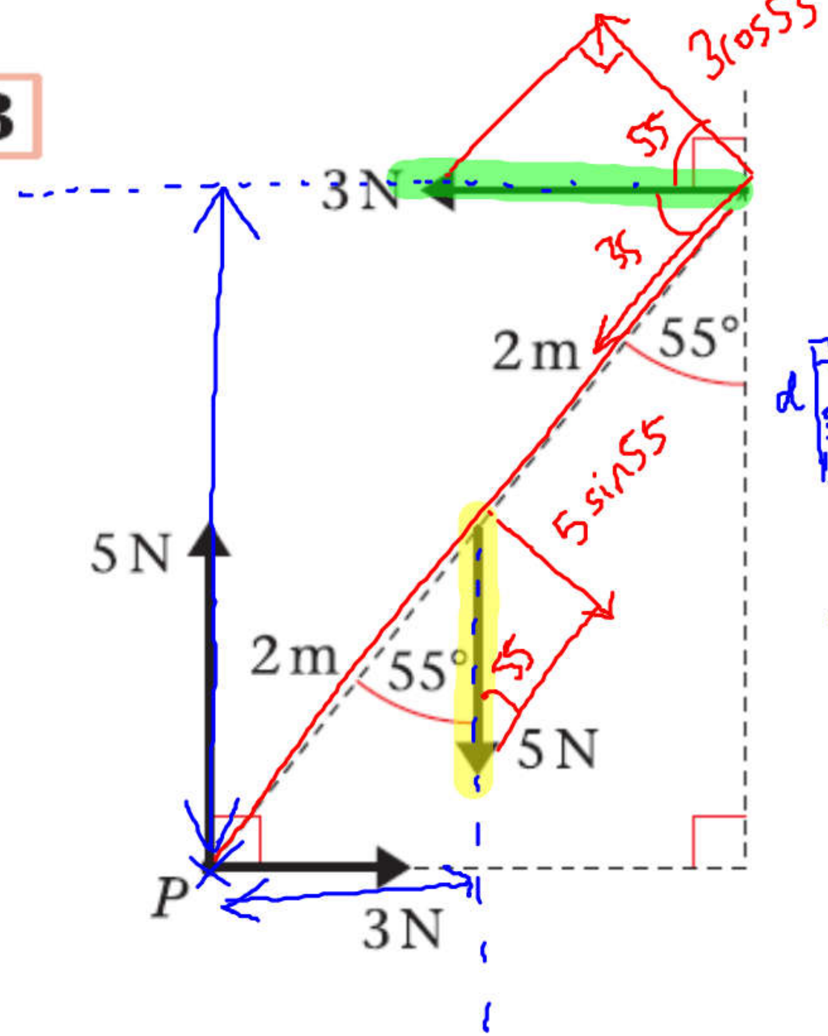



## Exercise 5A

**1** 1.09 Nm anticlockwise  **2** 2.13 Nm anticlockwise
**3** 1.31 Nm clockwise  **4** 1.31 Nm clockwise
**5** 1.31 Nm anticlockwise  **6** 1.31 Nm clockwise

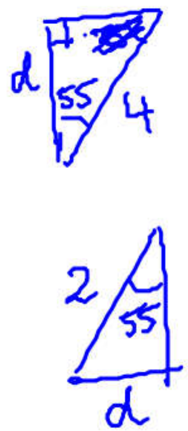
(Sum of moment means resultant moment)

**3**




force × perp. dist.

$\circlearrowleft$ $3 \times 4\cos 55 = 6.9$



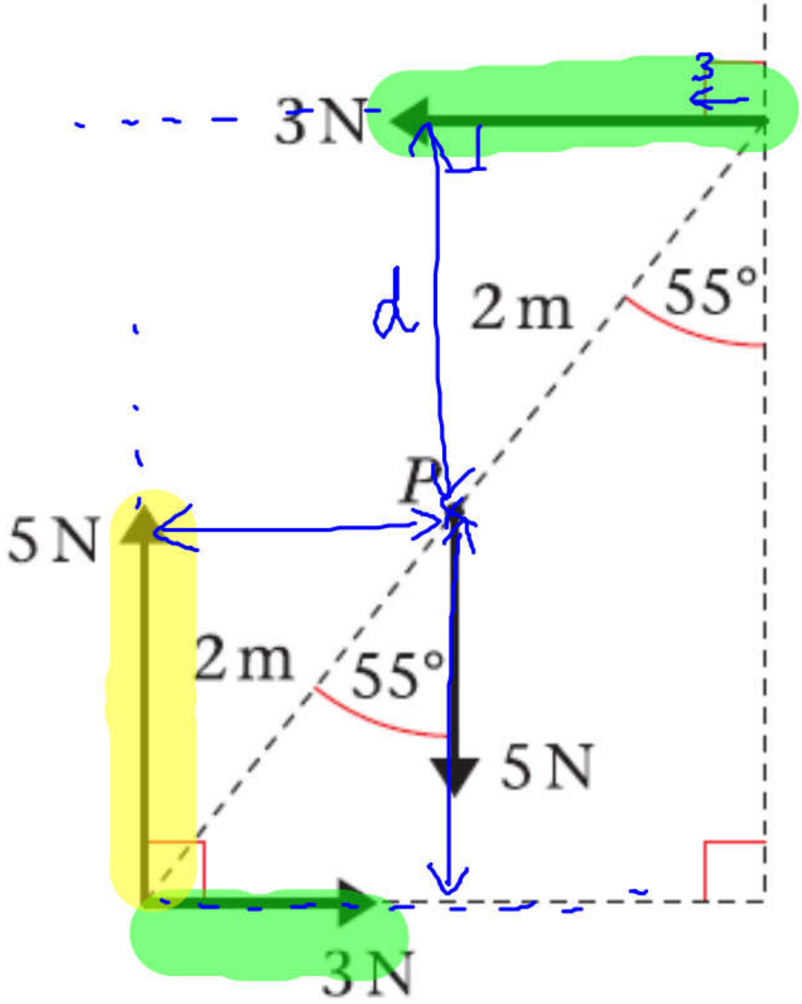

$\circlearrowright$ $5 \times 2\sin 55 = 8.2$

$\circlearrowright$ 1.3 Nm

$4 \times 3\cos 55$

$2 \times 5\sin 55$

**4**

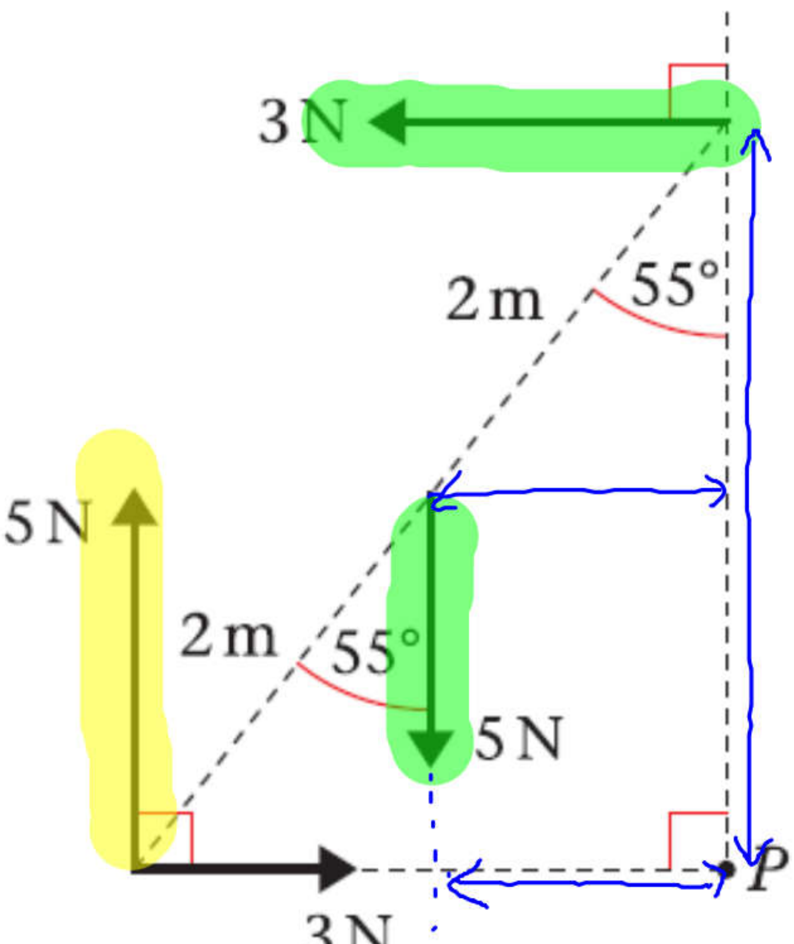



F x perp. dist

$3 \times 2\cos 55 + 3 \times 2\cos 55$


$5 \times 2\sin 55$

**6**



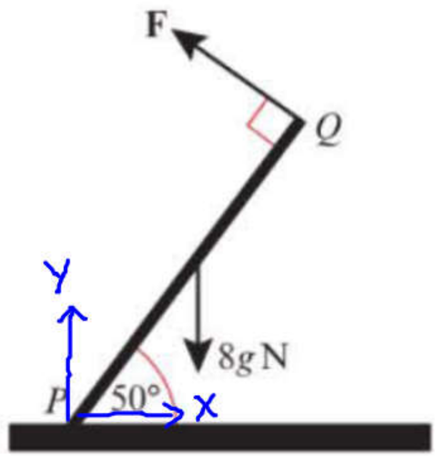

$3 \times 4\cos 55 + 5 \times 2\sin 55$

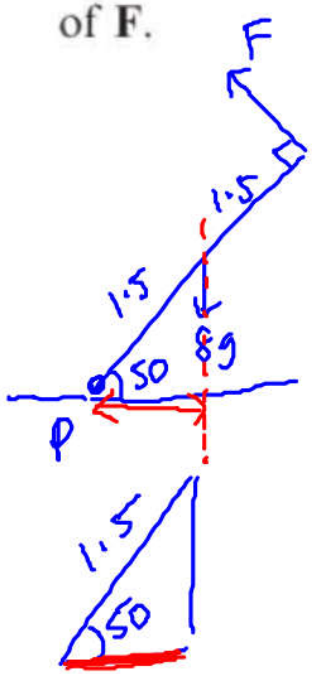
$5 \times 4\sin 55$

# Angled Rods/Beams

**Example 8**

A uniform rod $PQ$ is hinged at the point $P$, and is held in equilibrium at an angle of 50° to the horizontal by a force of magnitude $\mathbf{F}$ acting perpendicular to the rod at $Q$. Given that the rod has a length of 3 m and a mass of 8 kg, find the value of $\mathbf{F}$.




M(P)

$$F \times 3 = 8g \times 1.5\cos 50$$

$$F = \frac{8g \times 1.5\cos 50}{3}$$

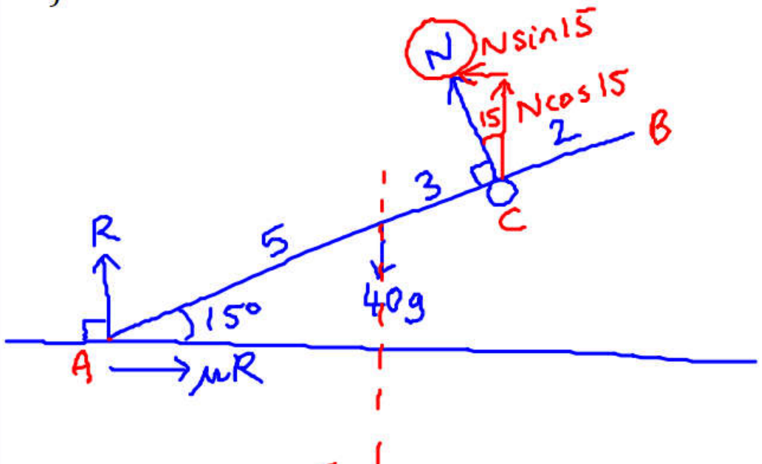
$$= \underline{\underline{25.2\,\text{N}}}.$$

# Static Rigid Bodies - resting on pegs/floor

A uniform rod AB of mass 40kg and length 10m rests with the end A on rough horizontal ground.
The rod rests against a smooth peg C where AC = 8m.
The rod is in limiting equilibrium at an angle of 15° to the horizontal.

Find:
a) the magnitude of the reaction at C
b) the coefficient of friction between the rod and the ground

Hints:
- Resolve vertically
- Resolve horizontally
- Take moments about A (usually the floor)
- Remember to find perpendicular distances!





a) M(A)

$$8 \times N = 40g \times 5\cos 15$$

$$N = \frac{40g \times 5\cos 15}{8} = 236.6518$$

$$= \underline{\underline{237\,N}}$$

b) $(R\updownarrow)$

$$40g = R + N\cos 15$$

$$R = 40g - N\cos 15$$

$$R = 163.4118$$

$R(\leftrightarrow)$ $\mu R = N\sin 15$

$$\mu = \frac{N\sin 15}{R} \qquad \mu = 0.3748$$

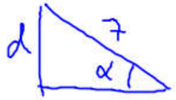
$$= \underline{\underline{0.37}}\ (2sf)$$

A uniform rod $AB$, of mass 5 kg and length 8 m, has its end $B$ resting on rough horizontal ground. The rod is held in limiting equilibrium at an angle $\alpha$ to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a rope attached to the rod at $C$. The distance $AC = 1$ m. The rope is in the same vertical plane as the rod. The angle between the rope and the rod is $\beta$ and the tension in the rope is $T$ newtons, as shown in Figure 3. The coefficient of friction between the rod and the ground is $\frac{2}{3}$. The vertical component of the force exerted on the rod at $B$ by the ground is $R$ newtons.

$\tan\alpha = \frac{3}{4}$ O A

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$




(a) Find the value of $R$. **(6)**

(b) Find the size of angle $\beta$. **(5)**

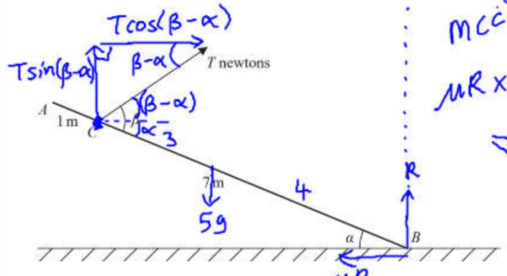
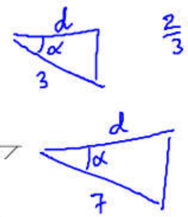



Figure 3




M(C)

$\mu R \times 7\sin\alpha + 5g \times 3\cos\alpha = R \times 7\cos\alpha$

$\frac{2}{3}R \times 7 \times \frac{3}{5} + 5g \times 3 \times \frac{4}{5} = R \times 7 \times \frac{4}{5}$

$\frac{14}{5}R + 12g = \frac{28}{5}R$

$12g = \frac{14}{5}R$

$R = 42$

b) $(R\updownarrow)$ $R + T\sin(\beta-\alpha) = 5g$

$T\sin(\beta-\alpha) = 7$ ①

$(R\leftrightarrow)$ $\mu R = T\cos(\beta-\alpha)$

$T\cos(\beta-\alpha) = 28$ ②

① ÷ ②

$\tan(\beta-\alpha) = \frac{1}{4}$

$\frac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha} = \frac{1}{4}$

$4\tan\beta - 4\tan\alpha = 1 + \tan\beta\tan\alpha$

$4\tan\beta - 3 = 1 + \frac{3}{4}\tan\beta$

$\frac{13}{4}\tan\beta = 4$

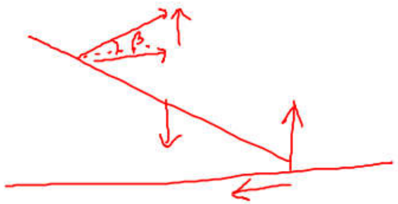
$\tan\beta = \frac{16}{13}$

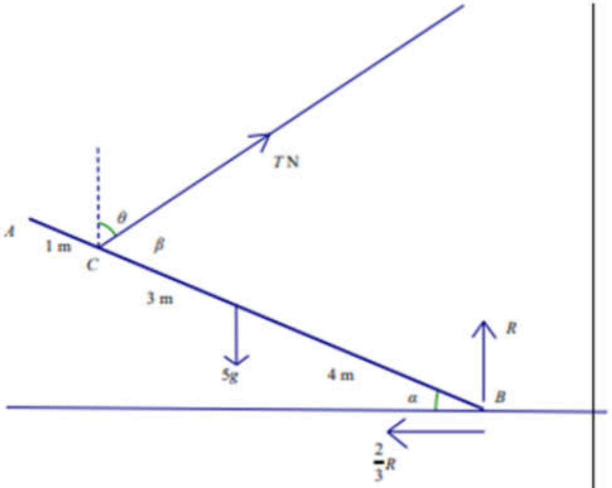
$\beta = 50.9°$

$\beta - \alpha = \tan^{-1}\left(\frac{1}{4}\right)$

$\beta = \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$= 50.9°$



| | | | |
|---|---|---|---|
| **5a** |  | | |
| | $F=\frac{2}{3}R$ seen or implied | B1 | Use of $F=\mu R$ . Could be on diagram. Allow in (b) if not seen before |
| | M($C$): $5g\times 3\cos\alpha+F\times 7\sin\alpha=7\cos\alpha\times R$ | M1 | Moments about $C$ or alternative complete method to find equation in $F$ and $R$ or $R$ only. Dimensionally correct and all terms needed. Condone sin/cos confusion and sign error(s). |
| | | A1 | At most one error |
| | | A1 | Correct unsimplified equation |
| | $15g\cos\alpha=R\left(7\cos\alpha-\frac{14}{3}\sin\alpha\right)$ | | |
| | $15g\times\frac{4}{5}=R\left(7\times\frac{4}{5}-\frac{14}{3}\times\frac{3}{5}\right)=\frac{14}{5}R$ | dM1 | Substitute for $F$ and trig and solve for $R$ Dependent on previous M1 |
| | $R=\frac{30}{7}g=42\text{ (N)}$ | A1 | |
| | | (6) | |
| | | | |
| | e.g. of alternative for M1A1A1: | | |
| | M($A$): $T\sin\beta+8R\cos\alpha=8F\sin\alpha+20g\cos\alpha$ **and** M($B$): $7T\sin\beta=20g\cos\alpha$ | (M1) | |
| | | (A1) | At most 1 error |
| | $\frac{20g}{7}\cos\alpha+8R\cos\alpha=8F\sin\alpha+20g\cos\alpha$ | (A1) | Correct unsimplified equation in $F$ and $R$ or $R$ only |

| | | | |
|---|---|---|---|
| **5b** | Resolve $\updownarrow$: $T\cos\theta + R = 5g$<br>$R + T\sin(\beta-\alpha) = 5g$ | M1 | Need all terms.<br>Condone sin/cos confusion and sign error(s). |
| | | A1 | Correct in $R$ or *their* $R$ |
| | Resolve $\leftrightarrow$: $T\sin\theta = F(=28)$<br>$F\left(=\frac{2}{3}R\right) = T\cos(\beta-\alpha)$ | M1 | Need both terms.<br>Condone sin/cos confusion |
| | | A1 | Correct in $R$ or *their* $R$ |
| | Solve simultaneous equations for $\beta-\alpha$ | | |
| | $\tan(\beta-\alpha) = 4$, $\beta = 50.9°$ $(51°)$ | A1 | cso . Max 3 s.f. |
| | | (5) | |
| | | | |
| **Alt 5b** | M($B$): $7\times T\sin\beta = 5g\cos\alpha\times 4$ | M1 | Moments equation.<br>Dimensionally correct.<br>Condone sin/cos confusion and sign error(s). |
| | $\left(T\sin\beta = \frac{16}{7}g\right)$ | A1 | |
| | OR: resolve perpendicular to the rod:<br>$T\sin\beta + R\cos\alpha = 5g\cos\alpha + \frac{2}{3}R\sin\alpha$ | (M1)<br>(A1) | |
| | | | |
| | Resolve parallel to rod:<br>$T\cos\beta + 5g\sin\alpha = F\cos\alpha + R\sin\alpha$<br>$\left(=\frac{2}{3}R\cos\alpha + R\sin\alpha\right)$ | M1 | All terms needed.<br>Condone sin/cos confusion and sign error(s). |
| | $\left(T\cos\beta = \frac{13}{7}g\right)$ | A1 | |
| | Solve simultaneous equations for $\beta$ | | |
| | $\tan\beta = \frac{16}{13}$ , $\beta = 50.9°$ $(51°)$ | A1 | cso. Max 3 s.f. |
| | | (5) | |
| | | [11] | |

# Your Turn

A uniform rod $AB$ has mass 4 kg and length 1.4 m. The end $A$ is resting on rough horizontal ground. A light string $BC$ has one end attached to $B$ and the other end attached to a fixed point $C$. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

**(4)**

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

**(7)**

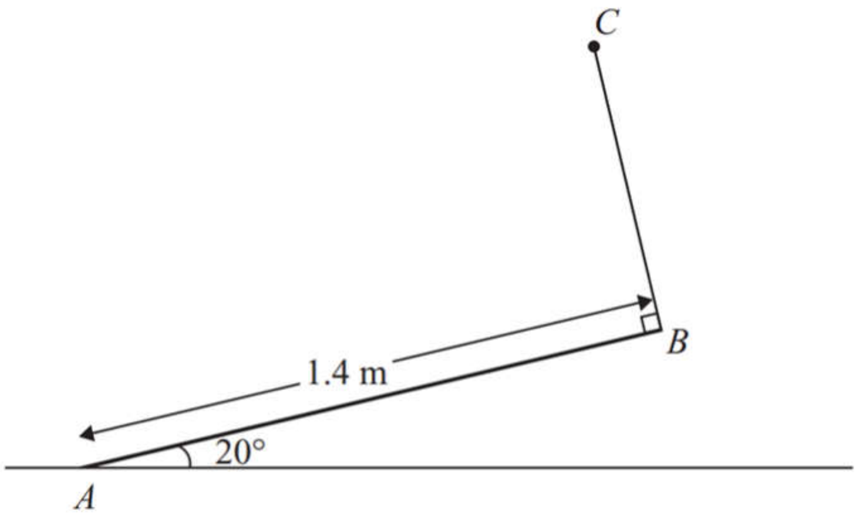



**Figure 2**

| | | | |
|---|---|---|---|
| **3** **(a)** | $M(A), F.4\sin 40^o = 5g.2\cos 25^o$ | M1 | A complete method to find $F$, e.g. take moments about $A$. Condone sin/cos confusion. Requires correct ratio of lengths. |
| | | A1 | Correct terms with at most one slip |
| | | A1 | All correct |
| | $F = 35$ | A1 | 35 or 34.5 (>3sf not acceptable due to use of 9.8, but only penalise once in a question) |
| | | (4) | |
| **(b)** | $F\cos 75^o \pm Y = 5g$ | M1 | Resolve vertically. Need all three terms but condone sign errors. Must be attempting to work with their 75° or 15°. |
| | | A1 | Correct equation (their $F$) |
| | $Y = 40$ ; | A1 | 40 or 40.1 Apply ISW if the candidate goes on to find $R$. |
| | UP | A1 | cso (the Q does specifically ask for the direction, so this must be clearly stated) |
| | | (4) **8** | |
| **(b)** | OR1: $4m\cos 25\times Y = 5g\times 2m\cos 25 + F\cos 15\times 4m\sin 25$ etc. | M1 A1 | Taking moments about the point vertically below $B$ and on the same horizontal level as $A$.(Their $F$) |
| | OR2: $R\cos\alpha = F\cos 40 + 5g\cos 65$<br>$R\sin\alpha + F\sin 40 = 5g\cos 25$ | | Resolve parallel & perpendicular to the rod |
| | $R$=52.1, $\alpha$=25.3° | | Solve for $R$, $\alpha$ |
| | $Y = R\sin(25+\alpha)$ | M1A1 | Need a complete strategy to find $Y$ for M1. |

# Your Turn

A uniform rod $AB$, of mass 20 kg and length 4 m, rests with one end $A$ on rough horizontal ground. The rod is held in limiting equilibrium at an angle $\alpha$ to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a force acting at $B$, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at $A$.

**(7)**

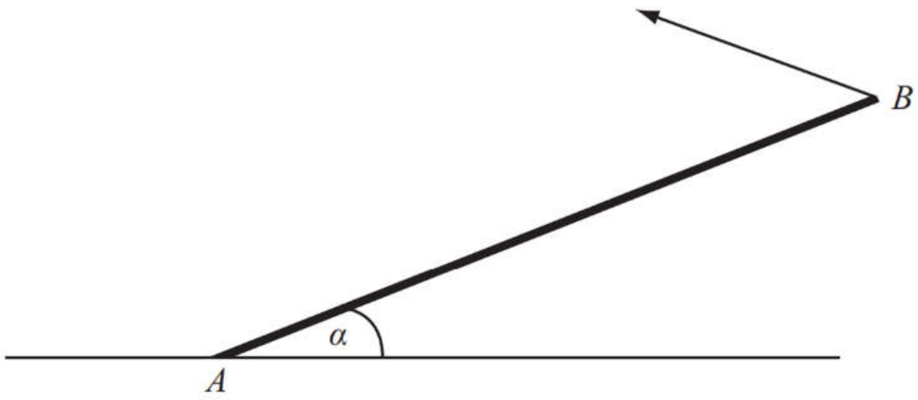



**Figure 2**

| | | |
|---|---|---|
| Q6. | $m(B): R \times 4\cos\alpha = F \times 4\sin\alpha + 20g \times 2\cos\alpha$ | M1 A2 |
| | Use of $F = \frac{1}{2}R$ | M1 |
| | Use of correct trig ratios | B1 |
| | R = 160N or 157N | DM1 A1 |
| | | [7] |

## Your Turn

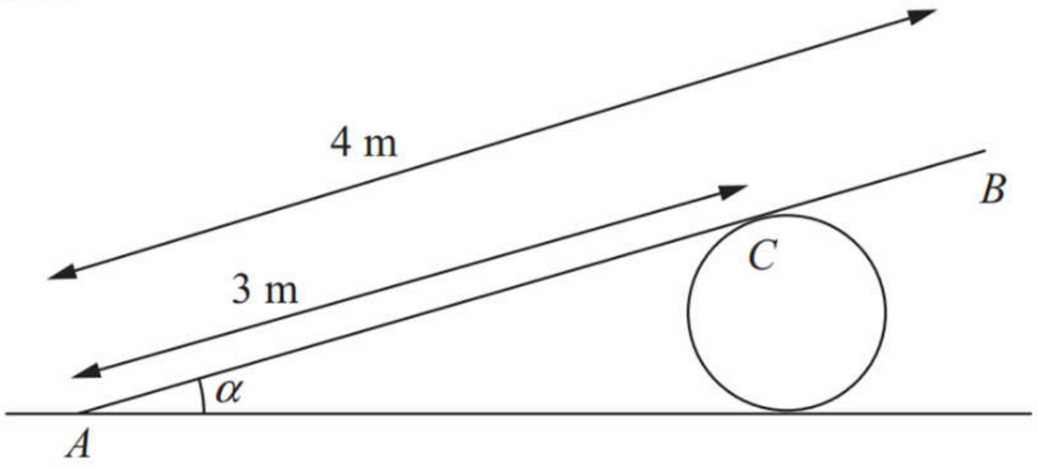
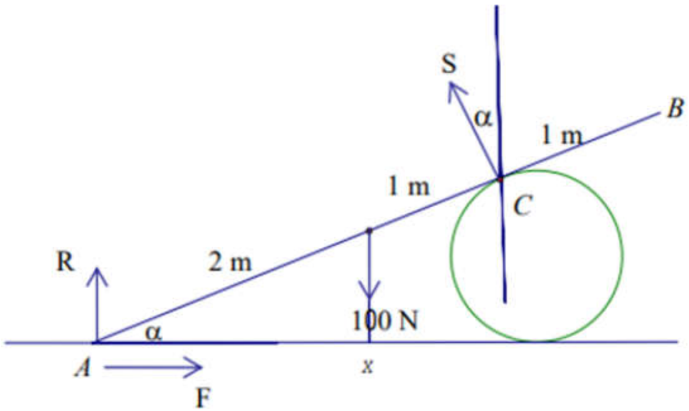



**Figure 4**

A uniform plank $AB$, of weight 100 N and length 4 m, rests in equilibrium with the end $A$ on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is $C$, where $AC = 3$ m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle $\alpha$ to the horizontal, where $\sin\alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is $\mu$. Modelling the plank as a rod, find the least possible value of $\mu$.

**(10)**

7.




| | |
|---|---|
| Taking moments about A: $3S = 100 \times 2 \times \cos\alpha$ | M1 A1 |
| Resolving vertically: $R + S\cos\alpha = 100$ | M1 A1 |
| Resolving horizontally: $S\sin\alpha = F$ | M1 A1 |
| (Most alternative methods need 3 independent equations, each one worth M1A1. Can be done in 2 e.g. if they resolve horizontally and take moments about $X$ then $R \times 2 \times \cos\alpha = S \times (3 - 2 \times \cos^2\alpha)$ scores M2A2) | |
| Substitute trig values to obtain correct values for F and R (exact or decimal equivalent). | DM1 |
| $\left(S = \dfrac{200\sqrt{8}}{9}\right)$, $R = 100 - \dfrac{1600}{27} = \dfrac{1100}{27} \approx 40.74$ , $F = \dfrac{200\sqrt{8}}{27} \approx 20.95...$ | A1 |
| $F \le \mu R,\ 200\sqrt{8} \le \mu \times 1100,\ \ \mu \ge \dfrac{200\sqrt{8}}{1100} = \dfrac{2\sqrt{8}}{11}$. | M1 |
| Least possible $\mu$ is 0.514 $(3\text{sf})$, or exact. | A1 |
| | [10] |