Edexcel C4 Jan 2012 Q6c

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant. **(5)**

Hint: You might want to use your double angle formula first.

$u = 1 + \cos x$

$\frac{du}{dx} = -\sin x$

$-\frac{1}{\sin x}\,du = dx$

$\cos x = u - 1$

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = \int \frac{4\sin x\cos x}{(1+\cos x)}\,dx$$

$$= \int \frac{4\cancel{\sin x}(u-1)}{u} \times -\frac{1}{\cancel{\sin x}}\,du$$

$$= -4\int \frac{u-1}{u}\,du$$

$$= -4\int \left(1 - \frac{1}{u}\right) du$$

$$= -4\left(u - \ln|u|\right) + c$$

$$= -4\left(1+\cos x - \ln|1+\cos x|\right) + c$$

$$= -4 - 4\cos x + 4\ln(1+\cos x) + c$$

$$= 4\ln(1+\cos x) - 4\cos x + k \quad \text{where } k = c - 4$$

$\frac{du}{dx} = -\sin x \quad \rightarrow \quad dx = -\frac{1}{\sin x}du$

$\cos x = u - 1$ (As before $x = \arccos(u-1)$ is going to be messy)

$\int \frac{2\sin 2x}{1+\cos x}dx = \int \frac{4\sin x\cos x}{1+\cos x}dx = \int -\frac{4\sin x(u-1)}{u}\frac{1}{\sin x}du$

$= -4\int \frac{u-1}{u}du = -4\int 1 - \frac{1}{u}du$

$= -4(u - \ln|u|) + c = -4(1+\cos x - \ln|\cos x + 1|) + c = \cdots$

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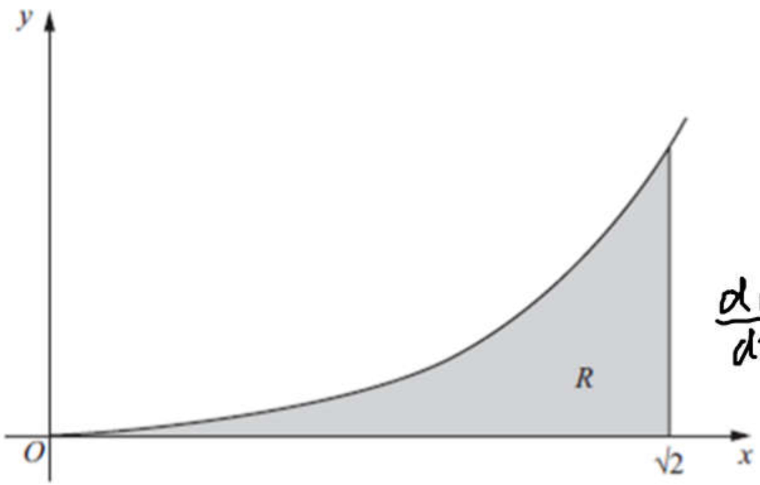

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u-2)\ln u \ du.$$

$$y = x^3 \ln(x^2+2)$$

$$\int_0^{\sqrt{2}} x^3 \ln(x^2+2) \, dx$$

$$u = x^2 + 2$$

$$\frac{du}{dx} = 2x$$

$$\frac{1}{2x} du = dx$$

x	u
$\sqrt{2}$	4
0	2

$$\int_0^{\sqrt{2}} x^3 \ln(x^2+2) \, dx$$

$$= \int_2^4 x^{\cancel{3}\,2} \ln u \, \frac{1}{2\cancel{x}} \, du$$

$$\int_0^{\sqrt{2}} x^2 x \ln(x^2+2) \, dx$$

$$= \int_2^4 (u-2)\cancel{x} \ln u \, \frac{1}{2\cancel{x}} \, du$$

$$= \frac{1}{2}\int_2^4 (u-2)\ln u \, du$$

SKILL #6: Integration by Parts

$$\int x \cos x \; dx = ???$$

Just as the Product Rule was used to **differentiate the product** of two expressions, we can often use 'Integration by Parts' to **integrate a product.**

To integrate by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} \; dx$$

$$\int uv' \; dx = uv - \int vu' \; dx$$

Product Rule

$$\frac{d}{dx} uv = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$(uv)' = uv' + vu'$$

$$\frac{d}{dx} uv - v \frac{du}{dx} = u \frac{dv}{dx}$$

$$u \frac{dv}{dx} = \frac{d}{dx} uv - v \frac{du}{dx}$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} \; dx$$

"L-I-A-T-E" Choose 'u' to be the function that comes first in this list:

L: Logrithmic Function

I: Inverse Trig Function

A: Algebraic Function

T: Trig Function

E: Exponential Function

$$\int uv' \, dx = uv - \int v u' \, dx$$

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx$$

$$= x \sin x - - \cos x + c$$

$$= x \sin x + \cos x + c$$

$$\boxed{\begin{array}{ll} u = x & v = \sin x \\ u' = 1 & v' = \cos x \end{array}}$$

Check to see it works

Let $y = x \sin x + \cos x + c$

$$\frac{dy}{dx} = \sin x + x \cos x - \sin x$$

$$\frac{dy}{dx} = x \cos x$$

"L-I-A-T-E" Choose 'u' to be the function that comes first in this list:

L: Logrithmic Function

~~I: Inverse Trig Function~~ ~~sec x~~ arccos x,

A: Algebraic Function

~~T: Trig Function~~

~~E: Exponential Function~~

Find $\int \underset{v'}{x^2} \underset{u}{\ln x}\ dx$

$u = \ln x \quad v = \frac{1}{3}x^3$

$u' = \frac{1}{x} \quad v' = x^2$

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{x} \times \frac{1}{3}x^3 \, dx$$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 \, dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$$

Checking/Showing it works!

$y = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$

$u = \frac{1}{3}x^3 \quad v = \ln x$

$u' = x^2 \quad v' = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{1}{3}x^3 \times \frac{1}{x} + x^2 \ln x - \frac{1}{3}x^2$$

$$= \frac{1}{3}x^2 + x^2 \ln x - \frac{1}{3}x^2$$

$$= x^2 \ln x$$

You will need the following standard results (given in your formula booklet) for the main exercise. We'll prove them later.

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C \qquad \int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x + \cot x| + C$$

Ex 11F
Q1
Q2 (NOT d)

Find $\int x^2 e^x \, dx$

$$\int x^2 e^x \, dx = x^2 e^x - \underbrace{\int 2x e^x}_{\text{requires IBP again.}} \, dx$$

$u = x^2 \quad v = e^x$

$u' = 2x \quad v' = e^x$

Work it out separately

$u = 2x \quad v = e^x$

$u' = 2 \quad v' = e^x$

$$\int 2xe^x \, dx = 2xe^x - \int 2e^x \, dx$$

$$= 2xe^x - 2e^x$$

$$\int x^2 e^x \, dx = x^2 e^x - (2xe^x - 2e^x)$$

$$= \underline{\underline{x^2 e^x - 2xe^x + 2e^x + C}}.$$

$$= e^x(x^2 - 2x + 2) + C$$

Your Turn

Find $\int x^2 \sin x \; dx$

$u = x^2 \quad v = -\cos x$
$u' = 2x \quad v' = \sin x$

$$\int x^2 \sin x \, dx = -x^2 \cos x - \int -2x \cos x \, dx$$
$$= -x^2 \cos x + \int 2x \cos x \, dx$$

$u = 2x \quad v = \sin x$
$u' = 2 \quad v' = \cos x$

$$\int 2x \cos x \, dx = 2x \sin x - \int 2 \sin x \, dx$$
$$= 2x \sin x - -2\cos x$$
$$= 2x \sin x + 2\cos x$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + c$$

3 Find the following integrals.

a $\int x^2 e^{-x} \, dx$ **b** $\int x^2 \cos x \, dx$ **c** $\int 12x^2(3 + 2x)^5 \, dx$

d $\int 2x^2 \sin 2x \, dx$ **e** $\int 2x^2 \sec^2 x \tan x \, dx$

Ex 11F
Q3