OCR A Level Physics

Your notes

Kinetic Theory of Gases

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Kinetic Theory of Gases

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Model of the Kinetic Theory of Gases

- Gases consist of atoms or molecules **randomly** moving around at **high speeds**
- The kinetic theory of gases models the thermodynamic behaviour of gases by linking:
  - The **microscopic** properties of particles i.e. mass and speed
  - The **macroscopic** properties of particles i.e. pressure and volume
- The theory is based on a set of the following assumptions:
  - Molecules of a gas behave as **identical** (or all have the same mass)
  - Molecules of gas are hard, **perfectly elastic** spheres
  - The **volume** of the molecules is **negligible** compared to the volume of the container
  - The **time** of a collision is **negligible** compared to the time between collisions
  - There are **no intermolecular forces** between the molecules (except during impact)
  - The molecules move in **continuous random motion**
  - **Newton's** laws apply
  - There are a **very** large number of molecules
- The number of molecules of gas in a container is very large, therefore the **average** behaviour (eg. speed) is usually considered

Examiner Tips and Tricks

Make sure to memorise **all** the assumptions for your exams, as it is a common exam question to be asked to recall them.

Pressure in the Model of Kinetic Theory of Gases

- A gas is made of a large number of particles
  - Gas particles have **mass** and move **randomly** at high speeds

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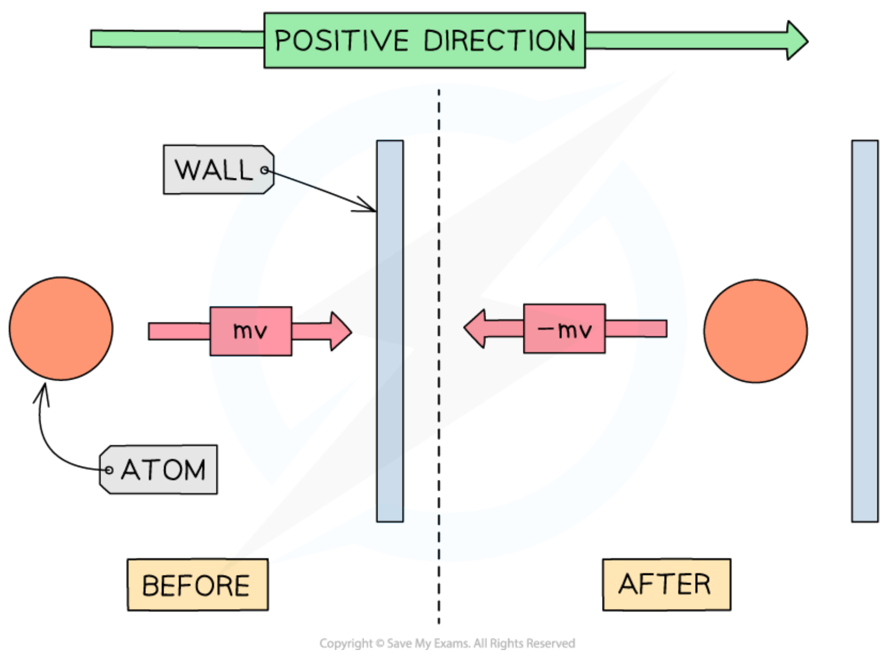
- Pressure in a gas is due to the collisions of the gas particles with the walls of the container that holds the gas
- When a gas particle hits a wall of the container, it undergoes a change in momentum due to the force exerted by the wall on the particle (as stated by Newton's Second Law)
  - Final momentum = $-mv$
  - Initial momentum = mv



- Therefore, the change in momentum Δp can be written as:

$$\Delta p = \text{final momentum} - \text{initial momentum}$$

$$\Delta p = -mv - mv = -2mv$$

$$-F = \frac{\Delta p}{\Delta t} = -\frac{2mv}{\Delta t}$$

- According to Newton's Third Law, there is an **equal** and **opposite** force exerted by the particle on the wall (i.e. $F = \frac{2mv}{\Delta t}$)

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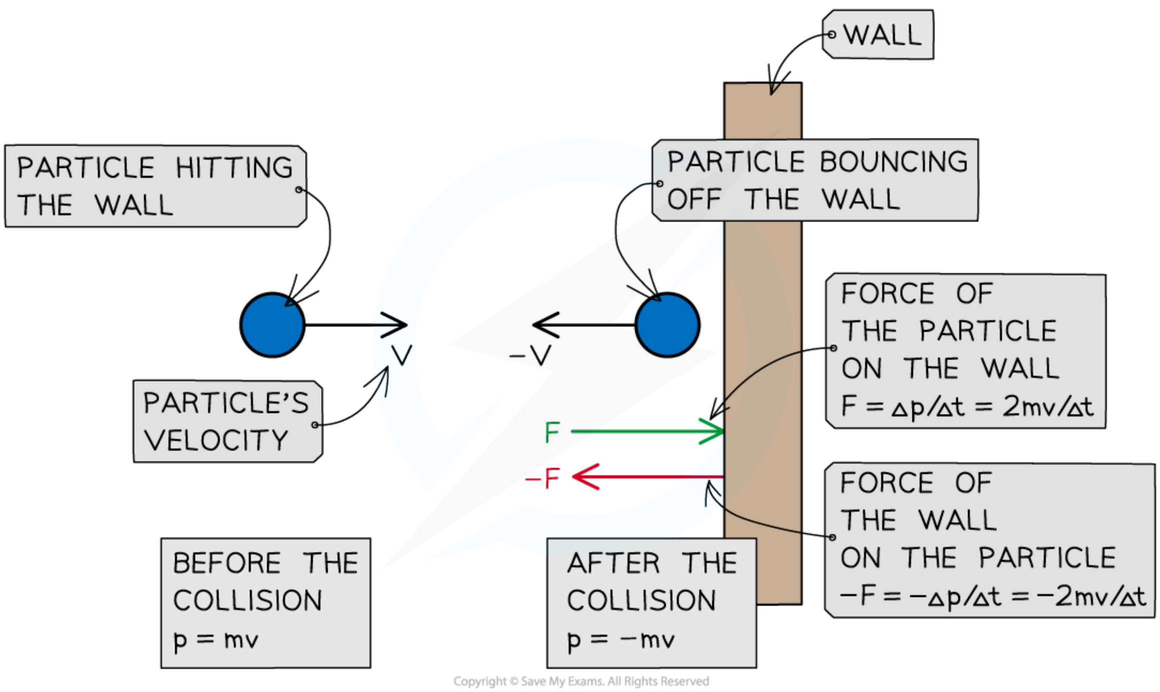

A particle hitting a wall of the container in which the gas is held experiences a force from the wall and a change in momentum. The particle exerts an equal and opposite force on the wall

- Since there is a large number of particles, their collisions with the walls of the container give rise to gas pressure, which is calculated as follows:

$$p = \frac{F}{A}$$

- Where:
  - p = pressure in pascals (Pa)
  - F = force in newtons (N)
  - A = area in metres squared (m^2)

Examiner Tips and Tricks

Momentum is a Mechanics topic that should have been covered in a previous unit. The above derivation of change in momentum and resultant force should have already been studied - if you're not comfortable with it then make sure you go back to revise this!

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Kinetic Theory of Gases Equation

The Root Mean Square Speed

- The **kinetic theory of gases** equation includes the **mean square** speed of the particles:

$$\overline{c^2}$$

- Where
  - c = **average** speed of the gas particles
  - $\overline{c^2}$ has the units $\mathbf{m^2\ s^{-2}}$
- Since particles travel in all directions in 3D space and velocity is a vector, some particles will have a negative direction and others a positive direction
- When there are a large number of particles, the total positive and negative velocity values will cancel out, giving a net zero value overall
- In order to find the pressure of the gas, the **velocities must be squared**
  - This is a more useful method, since a negative or positive number squared is **always positive**
- To calculate the **average speed** of the particles in a gas, take the square root of the mean square speed:

$$\sqrt{\overline{c^2}}$$

- This is known as the **root-mean-square** speed and still has the units of $\mathbf{m\ s^{-1}}$
  - The root-mean-square speed can also have the symbol $\mathbf{c_{r.m.s.}}$
- The mean square speed is **not** the same as the mean speed

Worked Example

A very small group of atoms have the following velocities:

50 m s^{-1}	+80 m s^{-1}	+85 m s^{-1}	–65 m s^{-1}	–90 m s^{-1}

Calculate the mean speed, $\bar{c}$, mean square speed, $\overline{c^2}$, and r.m.s speed, $c_{r.m.s}$, of these atoms.

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Answer:

Step 1: Calculate the mean speed $\bar{c}$:

- Add all the values and divide by the number of values you have to calculate the mean

$$\bar{c} = \frac{50 + 80 + 85 - 65 - 90}{5} = +12 \text{ m s}^{-1}$$

Step 2: Calculate the mean square speed $\overline{c^2}$:

- Square each value, then add them all and divide by the number of values you have to calculate the mean of the squares

$$\overline{c^2} = \frac{50^2 + 80^2 + 85^2 + (-65)^2 + (-90)^2}{5} = +5690 \text{ m}^2 \text{ s}^{-2}$$

- Here the units are also squared, as $(\text{m s}^{-1})^2 = \text{m}^2 \text{ s}^{-2}$

Step 3: Calculate the r.m.s speed $c_{r.m.s}$

- Find the square root of the mean square speed $\overline{c^2}$

$$c_{r.m.s} = \sqrt{\overline{c^2}} = \sqrt{5690 \text{ m}^2\text{s}^{-2}} = 75.4 \text{ m s}^{-1} = \mathbf{75 \text{ m s}^{-1}} \text{ (2 s.f.)}$$

Examiner Tips and Tricks

Make sure you read questions relating to the r.m.s speed carefully! It is easy to get confused between $\bar{c}$, $\overline{c^2}$ and $\sqrt{\overline{c^2}}$

Kinetic Theory of Gases Equation

- The **Kinetic Theory of Gases Equation** is given by:

$$pV = \frac{1}{3} N m \overline{c^2}$$

- Where
  - p = pressure (Pa)
  - V = volume (m^3)
  - N = number of molecules

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- m = mass of one molecule of gas (kg)
- $\overline{c^2}$ = mean square speed of the molecules ($m^2\ s^{-2}$)
- The density, *ρ* of the gas is given by:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{Nm}{V}$$

- Rearranging the equation for pressure, *p* and substituting the density, ρ gives the equation:

$$p = \frac{1}{3}\rho\,\overline{c^2}$$

- Where
  - *p* = pressure (Pa)
  - ρ = density ($kg\ m^{-3}$)
  - $\overline{c^2}$ = mean square speed of the molecules ($m^2\ s^{-2}$)

Worked Example

A gas cylinder contains argon at a pressure of 700 kPa.

The cylinder contains 3.0×10^{25} molecules and each molecule has a mass of 4.7×10^{-26} kg. The r.m.s speed of the molecules is $350\ m\ s^{-1}$.

Calculate the volume of the cylinder.

Answer:

Step 1: List the known quantities

- Number of molecules, $N = 3.0 \times 10^{25}$
  - Mass of each molecule, $m = 4.7 \times 10^{-26}$ kg
  - Root mean square speed, $c_{r.m.s.} = 350\ m\ s^{-1}$
    - Therefore $\overline{c^2} = 350^2\ m^2 s^{-2}$
  - Pressure, $P = 700$ kPa $= 700 \times 10^3$ Pa

Step 2: State the equation

$$pV = \frac{1}{3} N m \overline{c^2}$$

Step 3: Rearrange the equation to make volume, *V*, the subject

$$V = \frac{Nm\overline{c^2}}{3p}$$

Step 3: Substitute the values from the question into the equation and calculate *V*

$$V = \frac{(3.0 \times 10^{25}) \times (4.7 \times 10^{-26}) \times (350)^2}{3 \times (700 \times 10^3)}$$

$V = 0.08225\ m^3$

Step 4: Write the final answer to the correct number of decimal places

- The volume of the cylinder is $0.082\ m^3$ (3 d.p.)

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Worked Example

An ideal gas has a density of $4.5\ kg\ m^{-3}$ at a pressure of 9.3×10^5 Pa.

Determine the root mean square speed, $c_{r.m.s.}$, of the gas atoms at a constant temperature.

Answer:

Step 1: Write out the known quantities

- Pressure, $p = 9.3 \times 10^5$ Pa
- Density, $\rho = 4.5\ kg\ m^{-3}$

Step 2: State the equation linking the pressure of an ideal gas with density

$$p = \frac{1}{3}\rho\overline{c^2}$$

Step 3: Rearrange to make $\overline{c^2}$ the subject

$$\overline{c^2} = \frac{3p}{\rho}$$

Step 4: Take square roots of both sides to give an equation for $c_{r.m.s.}$

$$c_{r.m.s.} = \sqrt{\frac{3p}{\rho}}$$

Step 5: Substitute in known values and calculate $c_{r.m.s.}$

$$c_{r.m.s.} = \sqrt{\frac{3 \times 9.3 \times 10^5}{4.5}} = 787.4 = 790 \text{ m s}^{-1}$$

Examiner Tips and Tricks

You are not expected to know the derivation of the kinetic theory equation. However, a common exam question is to use the kinetic model of a gas and Newton's laws of motion to explain how a gas exerts pressure on the walls of its container.

You will also be expected to explain why the mean square speed of the molecules is used in this equation instead of mean velocity. It is important to note that these quantities are not equivalent. The mean velocity would be zero because particles with opposite velocities cancel out.

Your notes

The Boltzmann Constant

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The Boltzmann Constant

- The Boltzmann constant *k* is used in the ideal gas equation and is defined by the equation:

$$k = \frac{R}{N_A}$$

- Where:
  - R = molar gas constant
  - N_A = Avogadro's constant
- Boltzmann's constant therefore has a value of

$$k = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$$

- The Boltzmann constant relates the properties of microscopic particles (e.g. kinetic energy of gas molecules) to their macroscopic properties (e.g. temperature)
  - This is why the units are $\mathrm{J\,K^{-1}}$
- Its value is very small because the increase in kinetic energy of a molecule is very small for every incremental increase in temperature

Your notes

Average Kinetic Energy of a Molecule

Average Kinetic Energy of a Molecule

- An important property of molecules in a gas is their **average kinetic energy**
- This can be deduced from the ideal gas equations relating pressure, volume, temperature and speed and the equation for kinetic energy
- The ideal gas equation is:

$$pV = NkT$$

- Where
  - p = pressure (Pa)
  - V = volume (m^3)
  - N = number of molecules
  - k = Boltzmann constant, 1.38×10^{-23} ($J\ K^{-1}$)
  - T = Temperature (K)
- The equation linking pressure and mean square speed of the molecules is:

$$pV = \frac{1}{3}Nm\overline{c^2}$$

- Where
  - p = pressure (Pa)
  - V = volume (m^3)
  - N = number of molecules
  - m = mass of one molecule of gas (kg)
  - $\overline{c^2}$ = mean square speed of the molecules ($m^2\ s^{-2}$)
- Since both equations are expressions for pV, we can equate them:

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

- N can be removed from both sides of the equation, giving:

$$\frac{1}{3}m\overline{c^2} = kT$$

- This can be rearranged to give:

$$m\overline{c^2} = 3kT$$

- The equation for kinetic energy is:

$$E = \frac{1}{2}mv^2$$

- Therefore, this can be substituted in to give an equation for **average molecular kinetic energy**:

$$2E = m\overline{c^2} = 3kT$$

$$E = \frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$

- Where:
  - E = Kinetic energy of a molecule (J)
  - m= mass of one molecule (kg)
  - $\overline{c^2}$ = mean square speed of the molecules ($m^2\ s^{-2}$)
  - k = Boltzmann constant, 1.38×10^{-23} ($J\ K^{-1}$)
  - T = Temperature (K)
- **E** is the average kinetic energy for only **one** molecule of the gas
- A key feature of this equation is that the mean kinetic energy of an ideal gas molecule is proportional to its thermodynamic temperature
  - $E \propto T$
- The Boltzmann constant *k* can be replaced with

$$k = \frac{R}{N_A}$$

- Where:
  - R = Molar gas constant, 8.31 ($J\ mol^{-1}\ K^{-1}$)

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- N_A = Avogadro constant, 6.02×10^{23} (mol^{-1})
- Substituting this into the average molecular kinetic energy equation means it can also be written as:

$$E = \frac{1}{2}m\overline{c^2} = \frac{3}{2}kT = \frac{3RT}{2N_A}$$

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Worked Example

Helium can be treated as an ideal gas.

Helium molecules have a root-mean-square (r.m.s.) speed of 730 m s^{-1} at a temperature of 45 °C.

Calculate the r.m.s. speed of the molecules at a temperature of 80 °C.

Answer:

Step 1: Write down the known quantities

- Initial $c_{r.m.s.}$ = 730 m s^{-1}
- Initial temperature = 45 °C = 318 K
- Final temperature = 80 °C = 353 K
- Boltzmann constant, k = 1.38×10^{-23} J K^{-1}

Step 2: Write down the equation for average kinetic energy

$$E = \frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$

Step 3: State the relationship between $\overline{c^2}$ and T

$$\overline{c^2} \propto T$$

Step 4: Determine the relationship between $c_{r.m.s.}$ and T

$$c_{r.m.s.} \propto \sqrt{T}$$

Step 5: Change this into an equation

- Use the letter a as the constant of proportionality to avoid confusion with k, the Boltzmann constant

$$c_{r.m.s.} = a\sqrt{T}$$

Step 6: Rearrange the equation to make 'a' the subject

Your notes

$$a = \frac{c_{r.m.s.}}{\sqrt{T}}$$

Step 7: Substitute in values for initial $c_{r.m.s.}$ and initial T to find an expression for a

$$a = \frac{730}{\sqrt{318}}$$

Step 8: Substitute the expression for 'a' and the final temperature into the equation and calculate $c_{r.m.s.}$ at a temperature of 80 °C

$$c_{r.m.s.} = a\sqrt{T} = \frac{730}{\sqrt{318}} \times \sqrt{353} = 769.12 \text{ m s}^{-1} = 770 \text{ m s}^{-1} \text{ (2 s.f.)}$$

Examiner Tips and Tricks

Keep in mind this particular equation for kinetic energy is only for **one** molecule in the gas. If you want to find the kinetic energy for all the molecules, remember to multiply by ***N***, the total number of molecules. You can remember the equation through the rhyme 'Average K.E is three-halves kT'.

Remember that temperatures must be in Kelvin, which can be obtained by adding 273 to the temperature in °C.

Internal Energy of an Ideal Gas

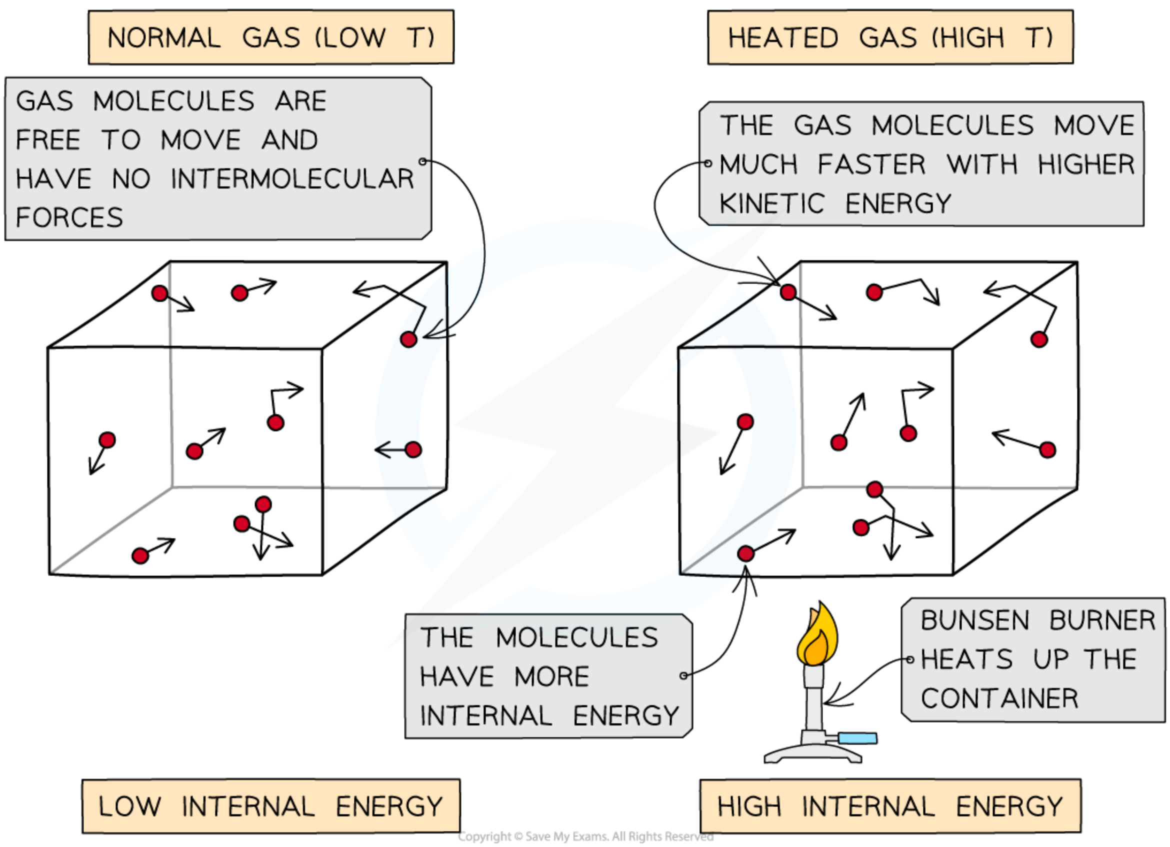
Your notes

Internal Energy of an Ideal Gas

- The internal energy of a gas is defined as:

  The sum of the kinetic and potential energies of the particles inside the gas

- One of the assumptions of an ideal gas states:
  - **Electrostatic forces** between particles in the gas are **negligible** except during collisions
  - So, there is no **electrostatic potential energy** in an ideal gas
- All the internal energy is due to the kinetic energy of the particles



As the container is heated up, the gas molecules move faster with higher kinetic energy and therefore higher internal energy

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- Change in **internal energy,** *ΔU* is equal to the **total kinetic energy,** E_K of **all** the particles

$$\Delta U = E_K = \frac{1}{2}N m \overline{c^2} = \frac{3}{2}Nk\Delta T$$

- Where:
  - E_K = total kinetic energy (J)
  - m = mass of one molecule (kg)
  - $\overline{c^2}$ = mean square speed of a molecule ($m^2\ s^{-2}$)
  - k = Boltzmann constant
  - T = temperature of the gas (K)
  - N = number of molecules
- This equation shows that doubling the **temperature** will also double the **internal energy** of the particles

$$\Delta U = \frac{3}{2}Nk\Delta T$$

$$\frac{3}{2}Nk(2T) = \frac{6}{2}NkT = 3NkT = 2U$$