



OCR A Level Computer Science



4.3 Boolean Algebra

Contents

- * Boolean Logic
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Boolean Logic

Your notes

Logic Gates

What is Boolean Logic?

- Boolean logic is used in computer science and electronics to make logical decisions
- Boolean operators are either TRUE or FALSE, often represented as 1 or 0
- Inputs and outputs are given letters to represent them
- To define Boolean logic we use **special symbols** to make **writing expressions** much **easier**

Combination of Boolean operators

- Can be **combined** to form more **complex** expressions
- Use parentheses to clarify the order of operations
 Example: NOT (TRUE AND FALSE) = TRUE

Evaluating Boolean expressions

- There is a specific sequence for evaluating expressions with multiple operators just like in normal maths where BIDMAS applies
- Brackets come first then NOT then AND then OR
- Using Brackets can alter the standard order of operations
- Expressions within parentheses are evaluated first, following the same NOT, AND, OR precedence inside the parentheses
- Example: NOT (TRUE AND FALSE) equals NOT FALSE, which equals TRUE

Logic Gates

- Logic gates are a visual way of representing a Boolean expression
- The logic gates covered in this course are:
 - Conjunction (AND)
 - Disjunction (OR)
 - Negation (NOT)
 - Exclusive disjunction (XOR)

Conjunction (AND)

Operation	Circuit symbol	Notes
A∧B		Returns TRUE only if both inputs are TRUE
A . B		TRUE AND TRUE = TRUE
		Otherwise = FALSE
		Next highest precedence after NOT
		Executes before OR operations



Disjunction (OR)

Explanation
urns TRUE if either input is TRUE
TRUE OR FALSE = TRUE
FALSE OR FALSE = FALSE
orecedence in Boolean expressions
es after NOT and AND operations

Negation (NOT)

Symbol	Circuit symbol	Notes
¬A		Inverts the input value
\overline{A}		NOT TRUE = FALSE
		NOT FALSE = TRUE
		Highest precedence in Boolean expressions
		Executes before AND and OR operations

Exclusive Disjunction (XOR)



Operation	Circuit symbol	Notes
A <u>∨</u> B A ⊕		Outputs TRUE if the inputs are different Outputs FALSE if they are the same
В		





Examiner Tips and Tricks

- Understanding the order of operations is crucial for correctly interpreting complex Boolean expressions
- Misunderstanding the order can lead to incorrect results
- Always use parentheses for clarity when combining multiple Boolean operations

Truth Tables

- A tool used in logic and computer science to **visualise** the **results** of **Boolean expressions**
- They represent all possible inputs and the associated outputs for a given Boolean expression

Conjunction (AND)

Circuit symbol	Truth Table			
	А	В	A ^ B	
	0	0	0	
	0	1	0	
			,	

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1	0	0
1	1	1



Disjunction (OR)

Circuit symbol		Truth Table			
	A	В	AVB		
	0	0	0		
	0	1	1		
	1	0	1		
	1	1	1		

Negation (NOT)

Circuit symbol	Truth Table		
	A	¬A	
	0	1	
	1	0	

Exclusive Disjunction (XOR)



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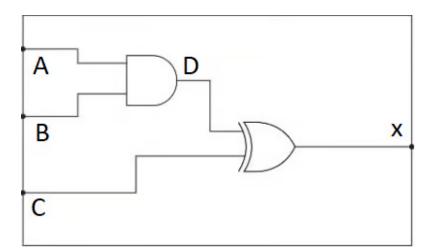
Circuit symbol	Truth Table				
4					
	A	В	AVB		
	0	0	0		
	0	1	1		
	1	0	1		
	1	1	0		





Worked Example

Daniel is an engineer. He has created the following logic circuit as shown



Complete the truth table below for the logic circuit shown

|--|



0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



4 marks

Answer:

Example answer that gets full marks:

Α	В	С	Calculating D	D	Calculating X	Х	Mark
0	0	0	D is the result of	0	X is the result of	0	1 Mark
0	0	1	А AND В	0	D <u>XOR</u> C	1	
0	1	0		0		0	1 Mark
0	1	1		0		1	
1	0	0		0		0	1 Mark
1	0	1		0		1	
1	1	0		1		1	1 Mark
1	1	1		1		0	



Karnaugh Maps

Your notes

Karnaugh Maps

What is a Karnaugh Map?

- This is a tool that is used for simplifying Boolean algebra expressions
- It offers a visual method of grouping together expressions with common factors
- The format of the map makes it easy to identify and eliminate redundant terms
- They are used in digital logic design, such as simplifying the logic of digital circuits

Steps:

- 1. **Create the Map**: Each cell in the grid corresponds to a term in the Boolean expression. Fill cells with 1s and 0s corresponding to the output of that term
- 2. **Grouping**: Group the 1s in the grid. Each group must be a rectangle and the size of the group must be a power of 2. A cell can be part of multiple groups
- 3. **Simplifying**: Write down a simplified Boolean expression for each group. The simplified expression for a group consists of the variables that remain constant in all terms in the group
- 4. **Final Expression**: Combine the simplified expressions from each group using OR operations to get the final simplified Boolean expression

Creating Karnaugh Maps

- A Karnaugh Map (KMap) can be used to simplify a Boolean expression with 2 inputs
- Here is an example for the expression AVB (A ORB)

Step 1

- Add each variable starting with A at the top and B down the side
- Add each possible state for A and B



		Α	Α
		0	1
В	0		
В	1		



Step 2

- Take each expression in turn **separated** by the **V** (**OR**)
- First look at **A** on it's own
- Find all cells where A is 1
- Add 1 to the cell

		Α	Α
		0	1
В	0		1
В	1		1

Step 3

Repeat for B



		Α	Α
		0	1
В	0		1
В	1	1	1



Step 4

■ This is now a **completed KMap** for the expression **AVB** (**AORB**)

		Α	Α
		0	1
В	0		1
В	1	1	1

Simplifying Expressions Using Karnaugh Maps

Simplify $\neg A^{-} \neg B^{-} C \lor \neg A^{-} B^{-} \neg C \lor A^{-} \neg B^{-} C \lor A^{-} B$ using a KMap.

In this example there will be **3 variables** A,B and C so the empty KMap will look like this:



		АВ			
		00	01	11	10
	0				
С	1				



Step 1:

Split this long term at each **OR** giving 4 smaller expressions (subterms) to add to the table:

- ¬A^¬B^C
- ¬A^B^¬C
- A^¬B^C
- A^B

Step 2:

- Take the first subterm ¬A^¬B^C
- Put a 1 in the map for every cell where this term would be TRUE (1)
- So if A and B were 0 and C was 1 this subterm would be 1
- So put a 1 in every cell in the KMap where A is 0, B is 0 and C is 1

		AB			
		00	01	11	10
_	0				
	1	1			

Step 3:

- The next subterm is ¬A^B^¬C
- Put a 1 in the KMap where A is 0, B is 1 and C is 0



		АВ			
		00	01	11	10
_	0		1		
	1	1			



Step 4:

- The next subterm is **A^¬B^C**
- Put a 1 in the KMap where A is 1, B is 0 and C is 1

		AB			
0.		00	01	11	10
_	0		1		
	1	1			1

Step 5:

- The final subterm is **A^B**
- Put a 1 in the KMap where A is 1 and B is 1 (2 cells this time)

		AB			
		00	01	11	10
	0		1	1	
C	1	1		1	1

Making the groups



- Once you have written the 1s and 0s into your KMap, you can then use this to simply the expression
- In order to do this, you need to identify the groups
- This is the **key** to using the Karnaugh map to derive the simplified expression
- The aim of making the groups is to
 - Make rectangular groups
 - Make groups that are as large as possible
 - Make groups that contain either 8,4,2 or 1 ones
 - Groups can overlap (i.e. some ones can be in multiple groups)
 - The Karnaugh map 'grid' wraps round in all directions so the groups can wrap round

Example grouping

Group 1:

So this would be one group.

	AB			
	00	01	11	10
0	1		1	1
1	1		1	1

This group would represent **A** since within this group both C and B change (have zeros and ones) whereas for A all the cells are one.

Group 2:

This would be another group (note the wrapping around).





		AB			
		00	01	11	10
_	0	1		1	1
C	1	1		1	1



This group would represent $\neg \mathbf{B}$ since within this group both A and C change (have zeros and ones) whereas for B all the cells are zero

Final simplified expression

- To get the final simplified expression we **OR** the terms representing the two groups together.
- So the simplified expression is A v ¬B



Examiner Tips and Tricks

In questions where you have to use a Karnaugh Map always show the groups by drawing a box/circle round them.



Worked Example

A Boolean expression is entered into a Karnaugh map.



		АВ			
		00	01	11	10
CD	00	1	1	1	1
	01	1	1	1	1
	11	0	1	1	0
	10	0	1	1	0



Give a simplified version of the expression using the Karnaugh map.

You must show your working

3 marks

How to answer this question:

/		AB					
		00	01	11	10		
CD	00	1	1	1	1		
CD	01	1	1	1	1		
	11	0	1	1	0		
	10	0	1	1	0		

- Looking at the group of 8 in the middle of the map, for all the cells in the group the variable B is always a 1
- The 3 other variables change across the group (i.e. for some cells they are 0 and for others they are 1

So this group is B



■ Looking at the other group of 8, for all the cells in this group, C is always 0 but the other 3 variables can be 1 or 0 in this group So this group simplifies to ¬C

Your notes

Answer:

Answer that gets full marks:

 $Bv\neg C$



Simplifying Boolean Algebra

Your notes

De Morgan's Law

- Complex expressions can be made simpler using the rules of Boolean algebra
- This is a **more powerful simplification** method than Karnaugh maps and can simplify expressions that Karnaugh maps cannot
- There are various different rules that you need to learn and that can then be applied to certain expressions to simplify them
- Combining these rules can mean that complex expressions can be reduced to much simpler ones

General rules

General AND rules

- XAND0=0
- X AND 1 = X
- XANDA=X
- NOTXANDX=0

Note, the value ox X is unknown and it is used as a placeholder. Therefore **X AND 1 = X** means that the output will be whatever the value of X is.

General OR rules

XOR0=X XOR1=1 XORA=X NOTXORX=1

DeMorgan's Law

- This provides a strategy for simplifying expressions that include a negation of a conjunction or disjunction (simplifying by inverting all variable)
- NOT(A AND B) is the same as (NOT A) OR (NOT B)



NOT (A AND B) \neg (A ^ B)		¬(A ^ B)	is the same as	(NOT A) OR (NOT B)			¬AV¬B		
A	В	A AND B	NOT (A AND B)		A	В	NOTA	NOTB	(NOT A) OR (NOT B)
0	0	0			0	0	1	1	
0	1	0		=	0	1	1	0	
1	0	0			1	0	0	1	
1	1	1	0		1	1	0	0	0



■ Step1

■ Change AND to OR (or vice versa) - ¬(A V B)

■ Step 2

■ NOT the terms either side of the operator - ¬(¬A V ¬B)

Step 3

■ NOT everything that has changed - ¬¬(¬AV¬B)

Step 4

■ Get rid of any double negation - (¬A V ¬B)

■ Step 5

■ Remove any unnecessary brackets - ¬AV¬B

NOT(A OR B) is the same as (NOT A) AND (NOT B)

NOT (A OR B) ¬(A V B)		is the same as	(NOT A) AND (NOT B) ¬A ^¬B						
A	В	AORB	NOT (A OR B)		A	В	NOTA	NOTB	(NOT A) AND (NOT B)
0	0	0	1 1		0	0	1	1]
0	1	1	0	≡	0	1	1	0	0
1	0	1	0		1	0	0	1	0
1	1	1	0		1	1	0	0	0

Step 1

■ Change AND to OR (or vice versa) - ¬(A ^ B)



- Step 2
 - NOT the terms either side of the operator ¬(¬A ^ ¬B)
- Step 3
 - NOT everything that has changed ¬¬(¬A ^ ¬B)
- Step 4
 - Get rid of any double negation (¬A ^ ¬B)
- Step 5
 - Remove any unnecessary brackets ¬A ^ ¬B
- Simplifying using this law allows the use of only NAND or NOR gates which makes building microprocessors much easier (i.e. Flash drives)

Distribution

Distributive Law

- This explains how AND and OR interact with each other
- This is a bit like factorising in normal maths
- A AND (B OR C) is the same as (A AND B) OR (A AND C)
- A OR (B AND C) is the same as (A OR B) AND (A OR C

Real-life example

- "You can pick one subject from group A and either one from group B or group C"
- is the same as
- "You can pick one subject from group A and one from group B or one subject from group A and one from group C"

Association

Associative Law

- This explains how variables associate in expressions of more than two variables
- Allows us to remove brackets and regroup variables
- (A AND B) AND C is the same as A AND (B AND C) is the same as A AND B AND C
- (A OR B) OR C is the same as A OR (B OR C) is the same as A OR B OR C





Real-life example

- "Zarmeen and her friends, Zahra and Ella have been chosen to represent the school"
- is the same as
- "Zarmeen and Zahra, and their friend Ella have been chosen to represent the school"
- is the same as
- "Zarmeen, Zahra and Ella have been chosen to the represent the school"

Commutation

Commutative Law

- States that the order of the variables does not change the truth value of the expression
- A AND B is the same as B AND A
- A OR B is the same as B OR A

Real-life example

- "Fynn and George won gold medals"
- is the same as
- George and Fynn won gold medals"

Double Negation

Double Negation Law

- States that the double negation of a variable results in the original variable
- NOT(NOT(A)) = A

Real-life example

- "I don't not want to visit the castle"
- is the same as
- "I do want to visit the castle"



Worked Example



SIMPLIFYING EXPRESSION EXAMPLE



How to answer this question:

Step one - Distribution

This is a bit like multiplying out the brackets in an expression in regular maths. Think of OR being like ADD and AND being like MULTIPLY.

 $(A \lor B) \land (A \lor C)$

becomes

 $(A ^A) \vee (B ^A) \vee (A ^C) \vee (B ^C)$

Step two - General rules

Since $(A ^A)$ is just A we can replace this term in the expression with a simpler one.

 $(A ^A) \vee (B ^A) \vee (A ^C) \vee (B ^C)$

becomes

 $A \lor (B \land A) \lor (A \land C) \lor (B \land C)$

Step three - Commutation

This means the order of the logical operators does not matter so can change (B ^ A) into (A ^ B).

 $A \vee (B \wedge A) \vee (A \wedge C) \vee (B \wedge C)$

becomes

 $A \lor (A \land B) \lor (A \land C) \lor (B \land C)$

Step four - Absorption

This rule says that AAND (AORB) = A.

 $A \vee (A \wedge B) \vee (A \wedge C) \vee (B \wedge C)$

becomes

 $Av(A^C)v(B^C)$

Step five - Another absorption

Again this rule says that A AND (A OR C) = A so

 $Av(A^C)v(B^C)$

becomes

 $Av(B^C)$

Example answer that gets full marks:

 $Av(B^C)$



Flip Flop Circuits

Your notes

D Type Flip Flops

What is a D Type Flip Flop?

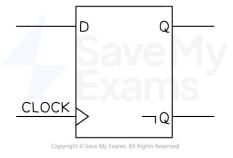
- A fundamental component in digital circuits and computer memory
- Can be referred to as **Positive Edge Triggered**

Key Features

- Contains two stable states, making it a type of bistable circuit
- Used to store the state of 1 bit of data
- Changes state on the edge of the clock pulse

Components

- Data input (**D**)
- Clock input (CLK)
- Two outputs: **Q** and **NOT(Q)**



D Type Flip Flip Components

Operation

- On the rising edge of the clock pulse:
 - If D is high (1), Q goes high and NOT(Q) goes low
 - If D is low (0), Q goes low and NOT(Q) goes high



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• The state of Q holds or "remembers" its value until the next rising clock edge

Use Cases

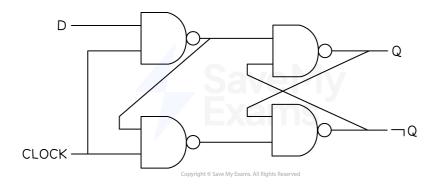
- Forms the basis of most types of flip flops and latches
- Used in **shift registers**, **counters**, and **memory units**
- Helpful in edge-triggered devices, synchronous circuits, and data storage



Examiner Tips and Tricks

You will not be asked to recall the logic gates that make up a D-type flip flop and they can be made from various different gates

They are often built using NAND gates which are AND gates that have their output inverted:



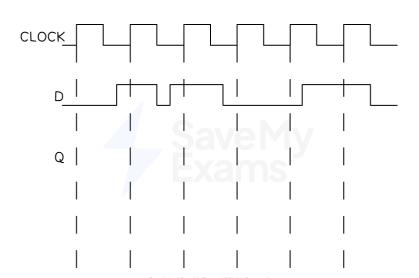
D Type Flip Flip Logic Gates



Worked Example

Draw the output of the Flip Flop on the diagram below

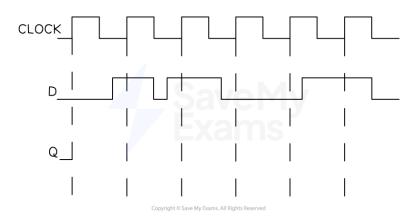






D Type Flip Flop Question

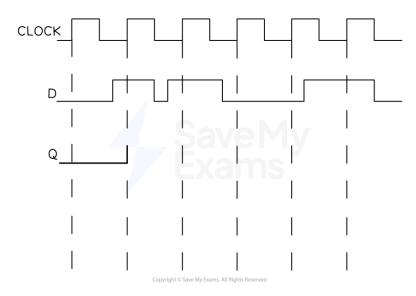
- For a question like this remember that the output line Q that you are asked to draw wants to be the same as the input D BUT it can only change when the clock signal changes from low to high (i.e. at the dotted lines)
- At the start assume Q starts the same as D (low in this case)
- Then draw it along from left to right until you get to a vertical dotted line (in an exam you would draw these lines on to help you if they weren't there
- At each dotted line, Q has the chance to change to whatever D is



D Type Flip Flop Working 1

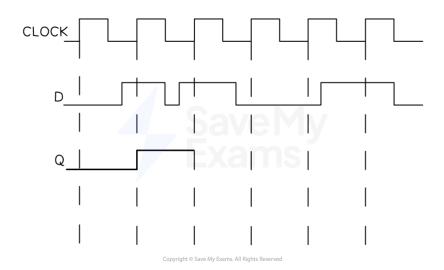
- So at the first line, D is low so Q stays low
- At the second dotted line D has changed to high so now Q can become high





D Type Flip Flop Working 2

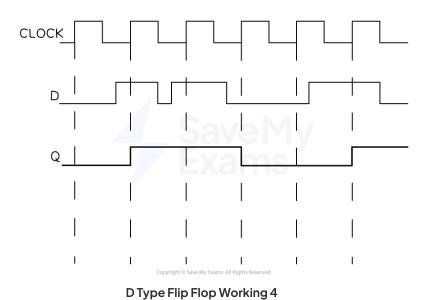
• Q now stays high until the next rising edge of the clock (the next dotted line) where it gets another chance to change, but here D is still high so Q stays high



D Type Flip Flop Working 3

• And you continue to do this until you reach the right hand side of the diagram







Adder Circuits

Your notes

Half Adders

What is a Half Adder Circuit?

- Basic digital circuit used in computation to perform the addition of two single bit numbers.
- Has two inputs, usually labelled as A and B
- Produces two outputs labelled Carry out (C_{out}) and Sum(s)

Α	В	C _{out}	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0
		A AND B	AXORB

- Remember that you are adding together the binary numbers represented by A and B
- Create the C_{out} column first then for each row you can just add A and B together and write the answer in 2 bits in the C_{out} and S columns
 - For example in row 2, A is 0 and B is 1 and 0+1=1, which is 01 in 2 bits (C_{out} 0 and Sum 1)
 - In the last row, A is 1 and B is 1 and 1+1 = 2 which is 10 in 2 bit binary (C_{out} 1 and Sum 0)

Drawing a Half Adder Circuit

- A half adder circuit has two inputs, typically labelled as A and B, and two outputs: the Sum (S) and
 Carry (C_{out}). This circuit can be created using an XOR gate for the Sum output and an AND gate for the
 Carry output
- Label Inputs:
 - Begin by drawing two lines on the left side of your paper or drawing space. Label the top line as 'A' and the bottom line as 'B'. These represent your inputs

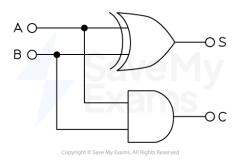


XOR Gate (Sum):

- **Draw an XOR gate** (often a shape like a curved 'D' or a shape similar to an OR gate but with an additional curved line on the input side) in the middle of the paper or drawing space. Connect the A and B lines to the two inputs of the **XOR gate**
- The output from the XOR gate is the 'Sum'. Draw a line from the output of the XOR gate to the right side of your paper and label it as 'S'

AND Gate (Carry):

- Draw an AND gate (typically a D-shaped symbol) above the XOR gate. Again, connect the A and B lines to the two inputs of the AND gate.
- The output from the AND gate is the 'Carry'. Draw a line from the output of the AND gate to the right side of your paper and label it as 'Cout'



Half Adder Logic Gates

Full Adders

- Extends the half adder to handle the addition of three bits
- Has three inputs: A, B, and an input carry (C_{in})
- Produces two outputs: carry (Cout) and sum (S)

A	В	C _{in}	C _{out}	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1





0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



- To easily reproduce this **Truth Table**, remember:
 - The full adder adds up three binary inputs A,B and C
 - So the answer can be 0,1,2 or 3
 - For each row, add up A, B and C and the **write the answer as a 2 bit binary number** in the last 2 columns (C_{out} and Sum)
 - For example in row 4, A=0, B=1 and C=1 0+1+1=2 which is 10 in binary, so Cout is 0 and Sum is 1
 - In the last row, A=1, B=1 and C=1, 1+1+1=3 which is 11 in binary so Cout is 1 and Sum is 1

Operation

- The "Sum" output **provides the XOR** of the inputs A, B, and C_{in}
- The "Carry" output is **TRUE** if at **least two of the inputs** A, B, and C_{in} are **TRUE**

Drawing a Full Adder Circuit

- A full adder circuit consists of three inputs: A, B, and Carry (C_{in}), and two outputs: Sum (S) and Carry (C_{out})
- It can be designed using two half adders and an OR gate.
- Label Inputs:
 - Start by drawing three lines on the left side of your paper or drawing space. Label the top line as 'A', the middle line as 'B', and the bottom line as 'C_{in}'. These represent your inputs
- First Half Adder:
 - Draw a half adder with A and B as inputs. This consists of an XOR gate (for the Sum) and an AND gate (for the Carry). Label the output of the XOR gate as 'Sum1' and the output of the AND gate as 'Carry1'



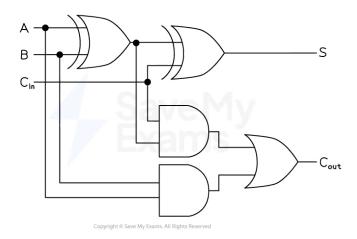
Second Half Adder:

■ Draw a second half adder underneath the first, using Sum1 and C_{in} as inputs. Again, it consists of an XOR gate (for the Sum) and an AND gate (for the Carry). Label the output of the XOR gate as 'S' (final Sum) and the output of the AND gate as 'Carry2'



OR Gate:

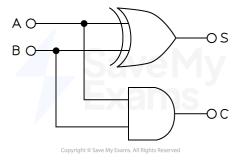
■ **Draw an OR gate** to the right of the half adders. Connect Carryl and Carry2 to the inputs of the OR gate. The output of the OR gate is the final Carry (**C**_{out})



Full Adder Logic Gates



Worked Example



Describe how this logic circuit can be adapted to add together two 4-bit binary numbers.

4 marks



Answer:

Logic circuit adds together 2 binary digits / half adder

S gives sum, C gives carry

Two half adders can be joined together...

...with an OR gate

to form full adder

4 full adders can be used to add two four bit numbers

Carry out on one joined to carry in on next

