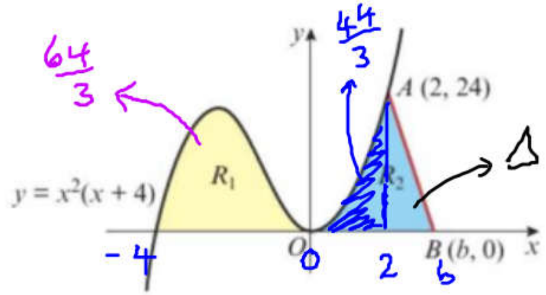
10 The sketch shows part of the curve with equation $y = x^2(x + 4)$. The finite region R_1 is bounded by the curve and the negative x-axis. The finite region R_2 is bounded by the curve, the positive x-axis and AB, where $A(2, 24)$ and $B(b, 0)$.

The area of R_1 = the area of R_2.

a Find the area of R_1.

b Find the value of b.



$\triangle$ $\frac{20}{3}$

Problem-solving

Split R_2 into two areas by drawing a vertical line at $x = 2$.

a)

$$\int_{-4}^{0} x^2(x+4) = \int_{-4}^{0} (x^3 + 4x^2)\,dx$$

$$= \left[\frac{1}{4}x^4 + \frac{4}{3}x^3\right]_{-4}^{0}$$

$$= 0 - \left(\frac{1}{4}(-4)^4 + \frac{4}{3}(-4)^3\right)$$

$$= \frac{64}{3}$$

b) Shaded area $= \left[\frac{1}{4}x^4 + \frac{4}{3}x^3\right]_0^2 = \frac{1}{4}\times 2^4 + \frac{4}{3}\times 2^3$

$$= \frac{44}{3}$$

area of $\triangle = \frac{64}{3} - \frac{44}{3} = \frac{20}{3}$

$$\frac{1}{2}(b-2)24 = \frac{20}{3}$$

$$12(b-2) = \frac{20}{3}$$

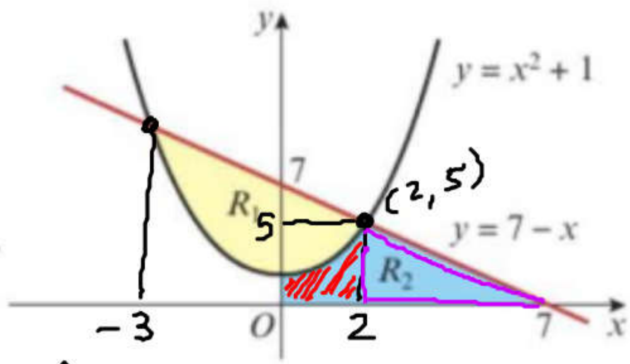
$$b - 2 = \frac{5}{9}$$

$$b = \frac{23}{9}$$

6 The diagram shows a sketch of part of the curve with equation $y = x^2 + 1$ and the line with equation $y = 7 - x$. The finite region, R_1 is bounded by the line and the curve. The finite region, R_2 is below the curve and the line and is bounded by the positive x- and y-axes as shown in the diagram.

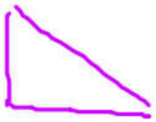
a Find the area of R_1.

b Find the area of R_2.



a) $\int_{-3}^{2} (7 - x) - (x^2 + 1)\, dx$

R_2



$\text{Area} = 5 \times 5 \times \frac{1}{2} = \frac{25}{2}$

$\text{Area} = \int_0^2 (x^2 + 1)\, dx = \left[\frac{1}{3}x^3 + x\right]_0^2$

$= \frac{8}{3} + 2 = \frac{14}{3}$

$\text{Total area} = \frac{25}{2} + \frac{14}{3} = \frac{103}{6}$

5. Given that

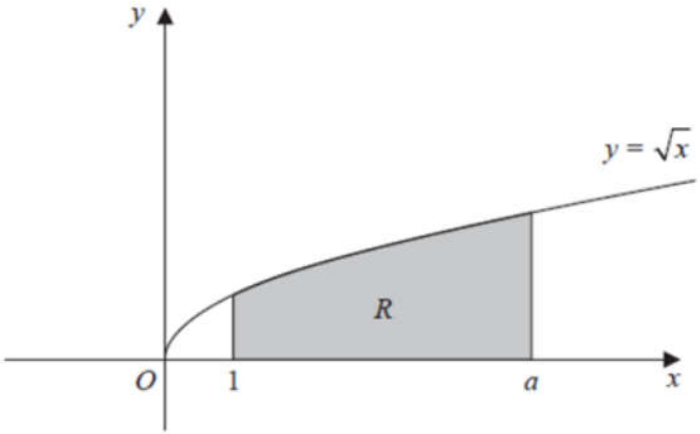
$$\mathrm{f}(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} \mathrm{f}(x)\,\mathrm{d}x = 16 + 3\sqrt{2}$

(5)

Question	Scheme	Marks	AOs
5	$\mathrm{f}(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\frac{12}{x^2}\right)\mathrm{d}x = x^2+3x-\frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\frac{12(\sqrt{2})}{2\times 2}\right)-(-8)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

8.



$\int_1^a \sqrt{x}\, dx = 10$

$\int_1^a \sqrt{8x}\, dx = ?$ $\int_0^a \sqrt{x}\, dx = ?$

Figure 2 shows a sketch of the curve with equation $y = \sqrt{x}$, $x \geq 0$.

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = a$, where a is a constant.

Given that the area of R is 10,

$\int 16x^{3/2}\, dx = 16\int x^{3/2}\, dx$

$16 \times \frac{2}{5} x^{5/2} = 16 \times \frac{2}{5} x^{5/2}$

(*a*) find, in simplest form, the value of

(i) $\int_1^a \sqrt{8x}\ \ dx$,

(ii) $\int_0^a \sqrt{x}\ \ dx$,

(4)

(*b*) show that $a = 2^k$, where k is a rational constant to be found.

(4)

(Total for Question 8 is 8 marks)

Question	Scheme	Marks	AOs
8(a)	(i) $\int_1^a \sqrt{8x}\,\mathrm{d}x = \sqrt{8} \times \int_1^a \sqrt{x}\,\mathrm{d}x = 10\sqrt{8} = 20\sqrt{2}$	M1 A1	2.2a 1.1b
	(ii) $\int_0^a \sqrt{x}\,\mathrm{d}x = \int_0^1 \sqrt{x}\,\mathrm{d}x + \int_1^a \sqrt{x}\,\mathrm{d}x = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 + 10 = \frac{32}{3}$	M1 A1	2.1 1.1b
		(4)	
(b)	$R = \int_1^a \sqrt{x}\,\mathrm{d}x = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^a$	M1 A1	1.1b 1.1b
	$\frac{2}{3}a^{\frac{3}{2}} - \frac{2}{3} = 10 \Rightarrow a^{\frac{3}{2}} = 16 \Rightarrow a = 16^{\frac{2}{3}}$	dM1	3.1a
	$\Rightarrow a = 2^{4\times\frac{2}{3}} = 2^{\frac{8}{3}}$	A1	2.1
		(4)	
			(8 marks)

14.

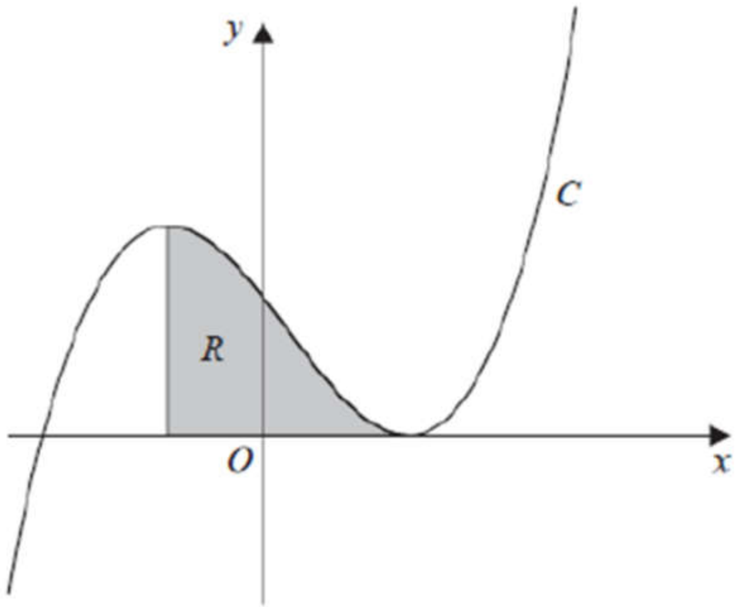


Figure 5

Figure 5 shows a sketch of the curve C with equation $y = (x - 2)^2 (x + 3)$.

The region R, shown shaded in Figure 5, is bounded by C, the vertical line passing through the maximum turning point of C and the x-axis.

Find the exact area of R.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(Total for Question 14 is 9 marks)

Question	Scheme	Marks	AOs
14	$y=(x-2)^2(x+3)=(x^2-4x+4)(x+3)=x^3-1x^2-8x+12$	B1	1.1b
	An attempt to find x coordinate of the maximum point. To score this you must see either • an attempt to expand $(x-2)^2(x+3)$, an attempt to differentiate the result, followed by an attempt at solving $\frac{dy}{dx}=0$ • an attempt to differentiate $(x-2)^2(x+3)$ by the product rule followed by an attempt at solving $\frac{dy}{dx}=0$	M1	3.1a
	$y=x^3-1x^2-8x+12 \Rightarrow \frac{dy}{dx}=3x^2-2x-8$	M1	1.1b
	Maximum point occurs when $\frac{dy}{dx}=0 \Rightarrow (x-2)(3x+4)=0$ $\Rightarrow x=-\frac{4}{3}$	M1 A1	1.1b 1.1b
	An attempt to find the area under $y=(x-2)^2(x+3)$ between two values. To score this you must see an attempt to expand $(x-2)^2(x+3)$ followed by an attempt at using two limits	M1	3.1a
	Area $=\int (x^3-1x^2-8x+12)\,dx=\left[\frac{x^4}{4}-\frac{x^3}{3}-4x^2+12x\right]$	M1	1.1b
	Uses a top limit of 2 and a bottom limit of their $x=-\frac{4}{3}$ $R=\left[\frac{x^4}{4}-\frac{x^3}{3}-4x^2+12x\right]_{-\frac{4}{3}}^{2}$	M1	2.2a
	Uses $=\frac{28}{3}--\frac{1744}{81}=\frac{2500}{81}$	A1	2.1
		(9)	
		(9 marks)	

14.

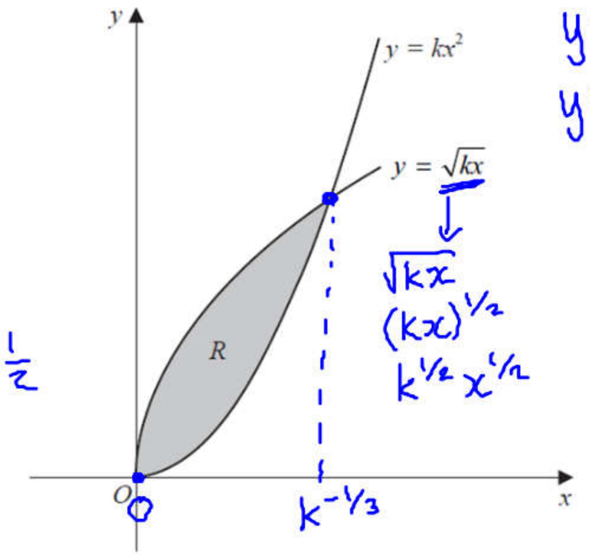


Figure 7

Figure 7 shows the curves with equations

$$y = kx^2 \qquad x \geqslant 0$$

$$y = \sqrt{kx} \qquad x \geqslant 0$$

where k is a positive constant.

The finite region R, shown shaded in Figure 7, is bounded by the two curves.

Show that, for all values of k, the area of R is $\frac{1}{3}$

(5)

$-\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{2} = -\frac{1}{2}$

$\sqrt{kx}$

$(kx)^{1/2}$

$k^{1/2}x^{1/2}$

$y = kx^2$

$y = \sqrt{kx}$

$kx^2 = \sqrt{kx}$

$k^2x^4 = kx$

$k^2x^4 - kx = 0$

$kx(kx^3 - 1) = 0$

$kx = 0$ $\quad$ $kx^3 - 1 = 0$

$x = 0$ $\quad$ $kx^3 = 1$

$x^3 = \frac{1}{k}$

$x = \sqrt[3]{\frac{1}{k}} = k^{-1/3}$

$$\int_0^{k^{-1/3}} \left(k^{1/2}x^{1/2} - kx^2\right) dx$$

$$= \left[k^{1/2} \times \frac{2}{3}x^{3/2} - k \times \frac{1}{3}x^3\right]_0^{k^{-1/3}}$$

$$= \left(k^{1/2} \times \frac{2}{3}\left(k^{-1/3}\right)^{3/2} - k \times \frac{1}{3}\left(k^{-1/3}\right)^3\right)$$

$$= k^{1/2} \times \frac{2}{3} \times k^{-1/2} - k \times \frac{1}{3} \times k^{-1}$$

$$= \frac{2}{3} \times k^{1/2} \times k^{-1/2} - \frac{1}{3} \times k \times k^{-1}$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \underline{\frac{1}{3}}$$

Question	Scheme	Marks	AOs
14	$y = kx^2$ and $y = \sqrt{kx}$, $\quad x \geqslant 0$		
	E.g. • $kx^2 = \sqrt{kx} \Rightarrow k^2x^4 = kx \Rightarrow k^2x^4 - kx = 0 \Rightarrow kx(kx^3 - 1) = 0$ $\{\Rightarrow kx = 0 \Rightarrow x = 0\} \Rightarrow kx^3 - 1 = 0 \Rightarrow x^3 = \dfrac{1}{k} \Rightarrow x = \ldots$ • $kx^2 = \sqrt{kx} \Rightarrow k^2x^4 = kx \Rightarrow kx^3 = 1 \Rightarrow x = \ldots$ • $kx^2 = \sqrt{kx} \Rightarrow k^{\frac{1}{2}}x^{\frac{3}{2}} = 1 \Rightarrow x^{\frac{3}{2}} = k^{-\frac{1}{2}} \Rightarrow x = \ldots$	M1	2.1
	$x = \sqrt[3]{\dfrac{1}{k}}$ or $x = k^{-\frac{1}{3}}$	A1	1.1b
	$\text{Area}(R) = \displaystyle\int_0^{k^{-\frac{1}{3}}} \left(\sqrt{kx} - kx^2\right) \mathrm{d}x = \left[\dfrac{\sqrt{k}\,x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \dfrac{1}{3}kx^3 \right]_0^{k^{-\frac{1}{3}}}$	M1 B1	1.1b 1.1b
	$= \left(\dfrac{2}{3}\sqrt{k}\dfrac{1}{\sqrt{k}} - \dfrac{k}{3}.\dfrac{1}{k}\right) - (0 - 0) = \dfrac{2}{3} - \dfrac{1}{3} = \dfrac{1}{3}$ *	A1*	2.1
		(5)	
		(5 marks)	