Look back at your notes to remind yourself about how to differentiate implicitly.

Think to yourself... what do I have to do to differentiate a term in y, but with respect to x?

9.

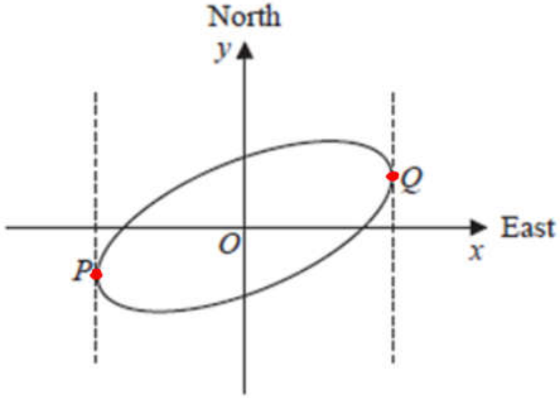


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y-x}{3y-x}$.

(4)

$$x^2 \underline{- 2xy} + 3y^2 = 50$$

$$u = -2x \qquad v = y$$

$$u' = -2 \qquad v' = \frac{dy}{dx}$$

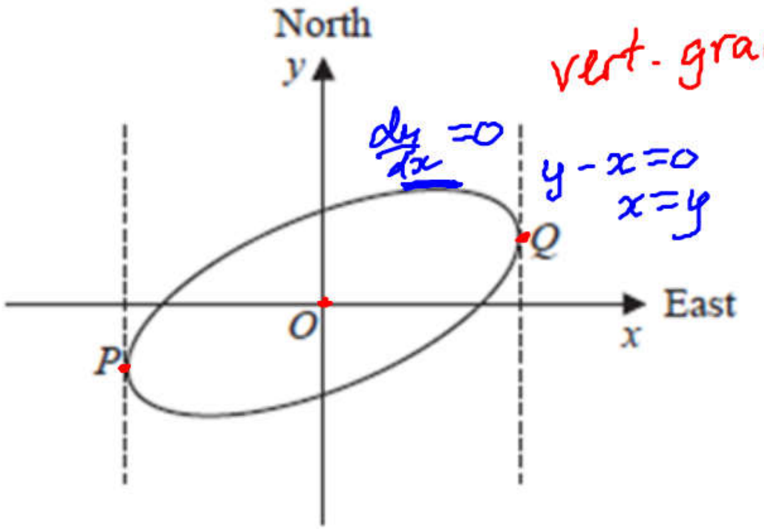
$$2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx} = 0$$

$$(6y - 2x)\frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y-2x}{6y-2x}$$

$$\frac{dy}{dx} = \frac{y-x}{3y-x}$$

$$\frac{dy}{dx} = \frac{x-y}{x-3y} \times \frac{-1}{-1} = \frac{y-x}{3y-x}$$



vert. gradient is undefined.

$$x^2 - 2xy + 3y^2 = 50$$

$$\frac{dy}{dx} = \boxed{\frac{y-x}{3y-x}}$$

undefined.

when $3y - x = 0$

$x = 3y$

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

$(3y)^2 - 2(3y)y + 3y^2 = 50$

$9y^2 - 6y^2 + 3y^2 = 50$

Using part (a),

$6y^2 = 50$

(b) find the exact coordinates of the point P.

$y^2 = \frac{50}{6} = \frac{25}{3}$

$y = \pm\frac{5\sqrt{3}}{3}$

$x = 3y$

$= 3 \times -\frac{5\sqrt{3}}{3}$

$= -5\sqrt{3}$

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

$P\left(-5\sqrt{3}, -\frac{5\sqrt{3}}{3}\right)$

At $(2, -3)$, with $x = 2$ and $y = -3$

$$3^x = y - 2xy$$

$$\ln 3 \times 3^x = \frac{dy}{dx}$$

$u = -2x \qquad v = y$

$u' = -2 \qquad v' = \frac{dy}{dx}$

$$\longrightarrow \ln 3 \times 3^x = \frac{dy}{dx} - 2x\frac{dy}{dx} - 2y$$

$$\ln 3 \times 3^2 = \frac{dy}{dx} - 4\frac{dy}{dx} + 6$$

$$9\ln 3 = \cancel{0} - 3\frac{dy}{dx} + 6$$

$$3\frac{dy}{dx} = 6 - 9\ln 3$$

$$\frac{dy}{dx} = 2 - 3\ln 3$$

$\sin x + \cos y = 0.5$ $\quad -\pi < x < \pi$

$-\pi < y < \pi.$

$\cos x - \sin y \dfrac{dy}{dx} = 0$

$\cos x = \dfrac{dy}{dx} \sin \cancel{x} y$

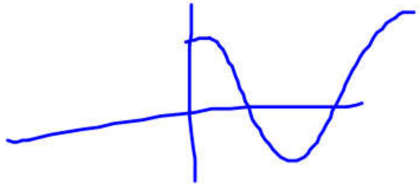
$\dfrac{dy}{dx} = \dfrac{\cos x}{\sin y}$

$\cos x \rightarrow = 0$ zero gradient

$\sin y \rightarrow = 0$ vertical gradient.

$x =$ $\quad x =$

Stat. pt. $\dfrac{dy}{dx} = 0$ $\quad \cos x = 0 \quad \dfrac{\pi}{2}, -\dfrac{\pi}{2}.$



$x = \dfrac{\pi}{2}$ $\quad 1 + \cos y = 0.5$

$\cos y = -\dfrac{1}{2}$

$y = \dfrac{2\pi}{3}, -\dfrac{2\pi}{3}$

$x = -\dfrac{\pi}{2}$ $\quad \sin(-\dfrac{\pi}{2}) + \cos y = 0.5$

$-1 + \cos y = 0.5$

$\cos y = 1.5$

No solutions

$\left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and $\left(\dfrac{\pi}{2}, -\dfrac{2\pi}{3}\right)$

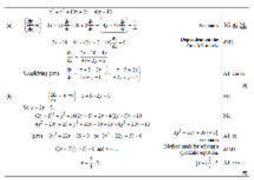
C4 June 2014(R) Q3

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(*a*) Find $\dfrac{dy}{dx}$ in terms of x and y, fully simplifying your answer. **(5)**

(*b*) Find the values of y for which $\dfrac{dy}{dx} = 0$. **(5)**

Hint for (b): Solve simultaneously with original equation.



a) $x^2 + y^2 + 10x + 2y - 4xy = 10$

$2x + 2y\dfrac{dy}{dx} + 10 + 2\dfrac{dy}{dx} - 4y - 4x\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(2y + 2 - 4x) = 4y - 2x - 10$

$\dfrac{dy}{dx} = \dfrac{4y - 2x - 10}{2y + 2 - 4x} = \dfrac{2y - x - 5}{y + 1 - 2x}$

b) It wants y values, so make x subject.

$2y - x - 5 = 0$

$2y - 5 = x$

$(2y-5)^2 + y^2 + 10(2y-5) + 2y - 4(2y-5)y = 10$

$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$

$-3y^2 + 22y - 35 = 0$

$y = 5$ or $y = \dfrac{7}{3}$

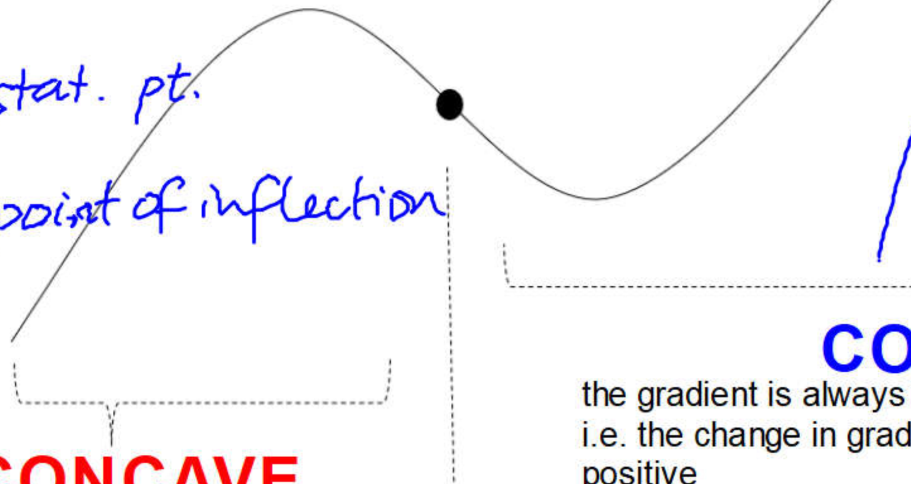
Using the second derivative

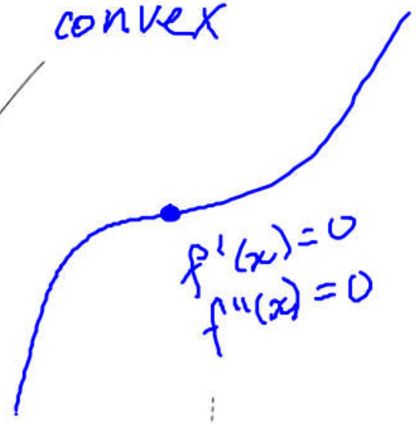
$f'(x) < 0$ decreasing function
$f''(x) \leq 0$ concave function

$f'(x) > 0$ increasing
$f''(x) \geq 0$ convex

$f'(x) = 0$ stat. pt.

$f''(x) = 0$ point of inflection





CONVEX

the gradient is always increasing
i.e. the change in gradient is positive

CONCAVE

the gradient is always decreasing
i.e. the change in gradient is negative

At point of inflection, the gradient is not changing, i.e. the gradient of the gradient is 0. Note that points of inflection (as with this example) are not necessarily stationary points.

✎ $f(x)$ convex when $f''(x) \geq 0$

✎ $f(x)$ concave when $f''(x) \leq 0$

✎ Point of inflection when $f''(x) = 0$

https://www.youtube.com/watch?v=7mufIG2LFlQ