Chapter 9: Constant Acceleration

Distance-time graphs

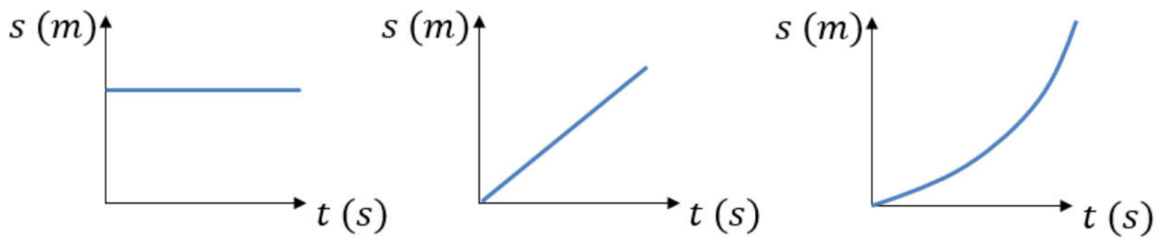
Speed-time graphs

SUVAT formulae

Vertical motion under gravity

Displacement-Time Graphs

Describe the motion of each object:



Velocity is the rate of change of displacement
(i.e. gradient of displacement-time graph)

$$\textbf{Average Velocity} = \frac{\textbf{Displacement from starting point}}{\textbf{Time taken}}$$

$$\textbf{Average Speed} = \frac{\textbf{Total distance travelled}}{\textbf{Time taken}}$$

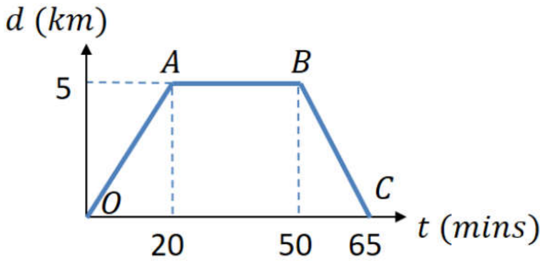
The distinction is important. If you went out then some time later travelled back home, your average velocity is 0 because your eventual displacement is 0!

A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes. This is a displacement-time graph for her journey.

(a) Work out the average velocity for each stage of the journey in km h^{-1}.

(b) Write down the average velocity for the whole journey.

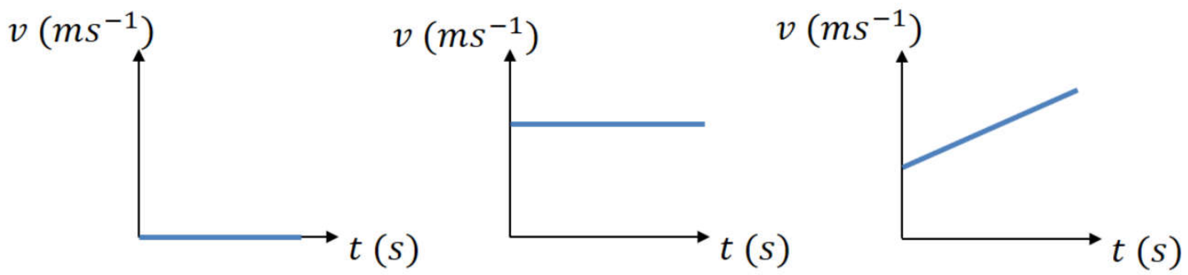
(c) Work out average speed for the whole journey.



Ex 9A

Velocity-Time Graphs

Describe the motion of each object:



Acceleration is the rate of change of velocity
(i.e. <u>gradient</u> of velocity-time graph)

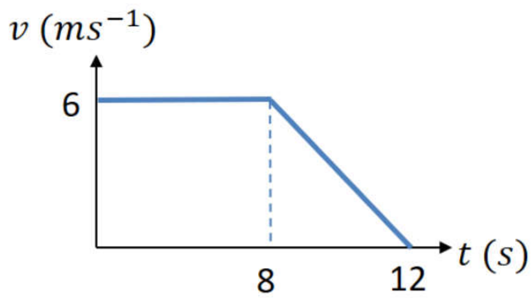
The **area** under a velocity-time graph gives the **distance travelled.**

Note: We'll see later in Chapter 11 that when we differentiate displacement we get velocity, and therefore integrating velocity gives displacement. But we know that integrating finds the area under the graph.

The figure shows a velocity-time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds.

(a) Find the displacement from the starting point of the cyclist after this 12 second period.

(b) Work out the rate at which the cyclist decelerates.



In case you've forgotten:

Area of trapezium
= average of parallel sides
× height between them

$$\frac{1}{2}(a+b)h$$

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 ms^{-1} in T seconds. The particle then travels at a constant velocity of 8 ms^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

(a) Sketch a velocity-time graph to illustrate the motion of the particle.

Give then the total displacement of the particle is 600m.

(b) find the value of T.

Tip: Sometimes it's easier to indicate the period of time that has passed (using arrows) rather than the time at the end of the interval.

6. A car travels along a straight horizontal road between two sets of traffic lights. The distance between the two sets of traffic lights is 1500 m.

In a model of the journey, the car leaves the first set of traffic lights, accelerating uniformly from rest until it reaches a speed of V m s^{-1}, then immediately decelerates uniformly until it comes to rest at the second set of traffic lights.

The car completes the journey between the two sets of lights in 120 s.

(a) Sketch, on the diagram below, a velocity-time graph which represents the above model of the journey of the car between the two sets of traffic lights.

(2)

(b) Using the model, find the value of V.

(2)

It is given that the car accelerates uniformly for T seconds.

(c) Explain why there is a range of possible values for T which satisfy the requirements of the model.

(2)

(d) Suggest one improvement to the model that would make it more realistic.

(1)

Ex 9B

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A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1}. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(2)

(b) Find the value of T.

(3)

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a m s^{-2}, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A.

(4)

(d) Find the value of a.

You won't likely have the knowledge for (d) yet...

(2)

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A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B.

(a) Sketch a speed-time graph to represent the motion of the car from A to B.

(3)

(b) Find the time for which the car is decelerating.

(2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B.

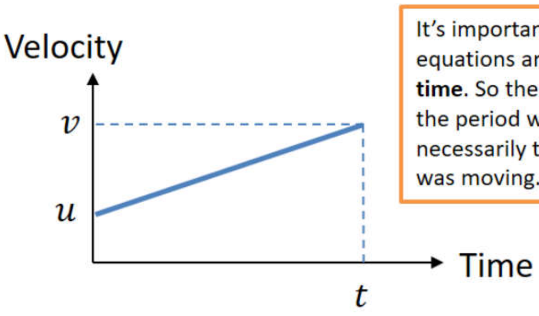
(8)

For (b), it may be helpful to know that:
final velocity = initial velocity
+ (time × acceleration)

SUVAT Equations (Part 1)

When there is **constant acceleration**, there are a variety of formulae which relate the following 5 quantities:

s: displacement
u: initial velocity
v: final velocity
a: acceleration
t: time



It's important you recognise these equations are for **a specific interval of time.** So the time t is the duration of the period we're considering, not necessarily the time since the object was moving.

Each SUVAT equation we will see involves 4 of the 5 quantities. Typically in a problem we'll know 3 of the quantities and we wish to find an unknown 4th quantity. We therefore select the appropriate equation.

Using the gradient of the graph (which we know is acceleration):
$a = \frac{v-u}{t} \quad \rightarrow \quad v = u + at$

You are expected to be able to **prove** each SUVAT question using the above graph.

Using the area under the graph (which we know gives distance):
$s = \left(\frac{u+v}{2}\right) t$

Memorisation Tip: This formula is effectively "distance = average speed × time" which you knew from GCSE.

A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of 4 ms^{-1} to a velocity of 7.5 ms^{-1} in 40 seconds. Find:
(a) the distance she travels in these 40 seconds
(b) her acceleration in these 40 seconds.

A particle moves in a straight line from a point A to a point B with a constant deceleration 1.5 ms^{-2}. The velocity of the particle at A is 8 ms^{-1} and the velocity of the particle at B is 2 ms^{-1}. Find:

(a) the time taken for the particle to move from A to B.

(b) the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration 1.5 ms^{-2}. The particle is at the point C 6 seconds after passing through the point A. Find:

(c) the velocity of the particle at C.

(d) The distance from A to C.

As stated before, think about what period of time we're considering.

Your Turn

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 second later the car passes a speed-trap where it is registered as travelling at 45 km h^{-1}. Find:

(a) the acceleration of the car

(b) the distance between the traffic lights and the speed-trap.

SUVAT equations (Part 2)

The other SUVAT equations can be derived using $\mathrm{v} = \mathrm{u} + \mathrm{at}$ and $\mathrm{s} = \left(\frac{\mathrm{u}+\mathrm{v}}{2}\right)\mathrm{t}$.

Eliminating t:

$$\mathrm{v}^2 = \mathrm{u}^2 + 2\mathrm{as}$$

Eliminating v:

$$\mathrm{s} = \mathrm{ut} + \frac{1}{2}\mathrm{at}^2$$

Note: Because this is quadratic in t, we typically end up with two different possible times.

Eliminating u:

$$\mathrm{s} = \mathrm{vt} - \frac{1}{2}\mathrm{at}^2$$

A particle is moving along a straight line from A to B with constant acceleration 5 ms^{-2}. The velocity of the particle is 3 ms^{-1} in the direction $\overrightarrow{AB}$. The velocity of the particle at B is 18 ms^{-1} in the same direction. Find the distance from A to B.

- $\mathrm{v} = \mathrm{u} + \mathrm{at}$
- $\mathrm{s} = \left(\frac{\mathrm{u}+\mathrm{v}}{2}\right)\mathrm{t}$
- $\mathrm{v}^2 = \mathrm{u}^2 + 2\mathrm{as}$
- $\mathrm{s} = \mathrm{ut} + \frac{1}{2}\mathrm{at}^2$
- $\mathrm{s} = \mathrm{vt} - \frac{1}{2}\mathrm{at}^2$

A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2}. At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1} travelling towards a point A, where $OA = 20$ m. Find:

(a) the times when the particle passes through A

(b) the value of t when the particle returns to O.

Further Example - exam style

7. A car is moving along a straight horizontal road with constant acceleration.

There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m.

The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)

Your Turn

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A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed u m s^{-1}, ($u < 34$), and 10 seconds later passes a point B with speed 34 m s^{-1}. Given that $AB = 240$ m, find

(a) the value of u,

(3)

(b) the time taken for the lorry to move from A to the mid-point of AB.

(6)

Ex 9D
Evens

Using simultaneous equations

14 Two particles P and Q are moving along the same straight horizontal line with constant accelerations $2\,\text{m s}^{-2}$ and $3.6\,\text{m s}^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $4\,\text{m s}^{-1}$. One second later Q passes through A with speed $3\,\text{m s}^{-1}$, moving in the same direction as P.

a Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A. **(2 marks)**

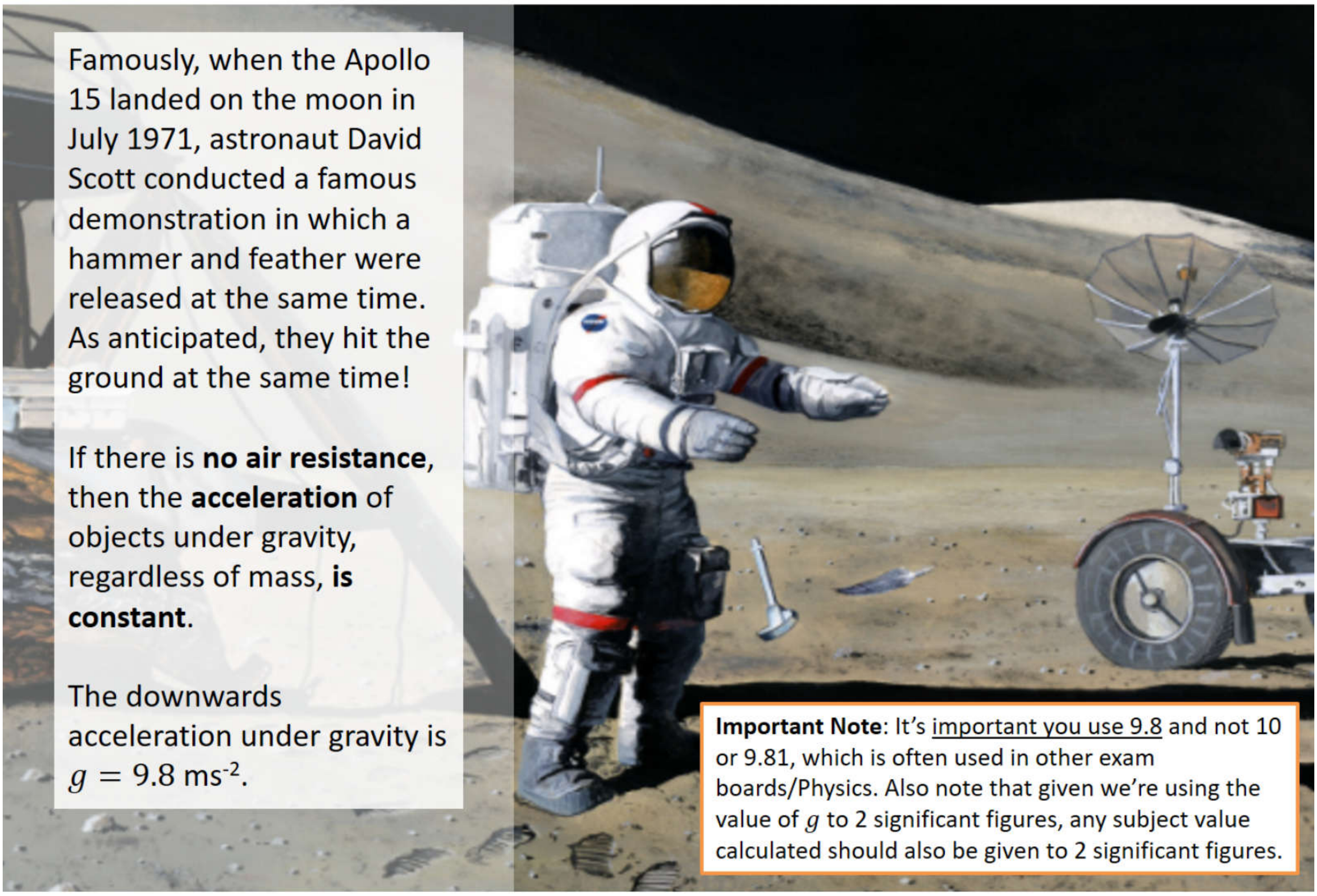
b Find the value of t where the particles meet. **(3 marks)**

c Find the distance of A from the point where the particles meet. **(3 marks)**

Problem-solving

When P and Q meet, their displacements from A are equal.

Vertical Motion Under Gravity



Famously, when the Apollo 15 landed on the moon in July 1971, astronaut David Scott conducted a famous demonstration in which a hammer and feather were released at the same time. As anticipated, they hit the ground at the same time!

If there is **no air resistance**, then the **acceleration** of objects under gravity, regardless of mass, **is constant**.

The downwards acceleration under gravity is $g = 9.8$ ms^{-2}.

Important Note: It's important you use 9.8 and not 10 or 9.81, which is often used in other exam boards/Physics. Also note that given we're using the value of g to 2 significant figures, any subject value calculated should also be given to 2 significant figures.

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:

(a) the time the book takes to reach the floor,

(b) the speed with which the book strikes the floor.

It's VERY important you consider **what direction is considered as 'positive'**, and mark it next to your suvat values. If 'up' was positive, then $a = -9.8$. If 'down' is positive, then $a = +9.8$. Which way you pick does not matter provided that you are consistent with each letter of SUVAT.

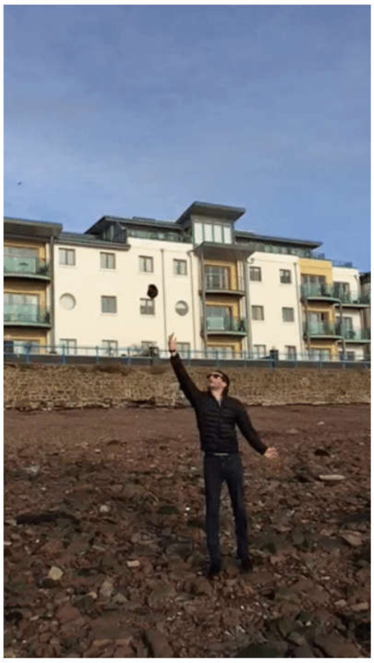
As per previous slide, quote only to 2 significant figures. You may be penalised if you quote more!

A ball is projected vertically upwards, from a point X which is 7m above the ground, with speed 21 ms^{-1}. Find

(a) the greatest height above the ground reached by the ball,

(b) the time of flight of the ball

At maximum height, speed is 0



Displacement is a vector – it has direction!

It is the distance from its start point to its end point. Imagine taking a photo of it at the start of the 'journey' and then at the end. Compare them to find the displacement

A ball is projected vertically upwards from ground level at a speed of 20 ms^{-1}. Determine the amount of time the ball is at least 10m above ground level.

Ex 9E
Evens

Using simultaneous equations

A ball *A* falls vertically from rest from the top of a tower 63m high. At the same time as *A* begins to fall, another ball *B* is projected vertically upwards from the bottom of the tower with speed 21 ms^{-1}. The balls collide.
Find the distance of the point where the balls collide from the bottom of the tower.

Try Q12

Edexcel M1 May 2013(R) Q4

At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1}. At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T,

(5)

(b) the value of h.

(2)

More practice!
Mixed Exercise
Exam Questions

5. A small ball is projected vertically upwards from a point A which is 19.6 m above the ground. The ball strikes the ground, for the first time, 4 s later.

The motion of the ball is modelled as that of a particle moving freely under gravity.

(a) Use the model to find the speed of the ball as it hits the ground for the first time.

(3)

The ball rebounds from the ground with a vertical speed of 14.7 m s^{-1} and next comes to instantaneous rest at the point B.

(b) Use the model to find the height of B above the ground.

(2)

In a refined model of the motion of the ball, the effect of air resistance is included and this refined model is now used to find the speed of the ball as it hits the ground for the first time

(c) How would this new value of the speed of the ball as it hits the ground for the first time compare with the value found using the initial model in part (a)?

(1)