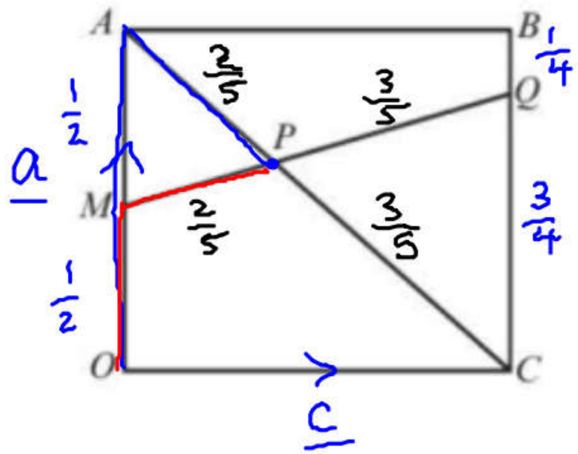
Ex 11E Vectors

4 $OABC$ is a square. M is the midpoint of OA, and Q divides BC in the ratio $1:3$.
AC and MQ meet at P.

a If $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$, express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

b Show that P divides AC in the ratio $2:3$.



$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$\overrightarrow{OP} = \underline{a} + \lambda \overrightarrow{AC}$

$(\overrightarrow{AC} = \underline{c} - \underline{a})$

$\overrightarrow{OP} = \underline{a} + \lambda(\underline{c} - \underline{a})$

$= \underline{a} + \lambda \underline{c} - \lambda \underline{a}$

$\overrightarrow{OP} = (1-\lambda)\underline{a} + \lambda \underline{c}$

$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP}$

$\overrightarrow{OP} = \frac{1}{2}\underline{a} + \mu \overrightarrow{MQ}$

$(\overrightarrow{MQ} = \frac{1}{2}\underline{a} + \underline{c} - \frac{1}{4}\underline{a}$

$= \frac{1}{4}\underline{a} + \underline{c})$

$\overrightarrow{OP} = \frac{1}{2}\underline{a} + \mu(\frac{1}{4}\underline{a} + \underline{c})$

$\overrightarrow{OP} = \frac{1}{2}\underline{a} + \frac{1}{4}\mu \underline{a} + \mu \underline{c}$

$\overrightarrow{OP} = (\frac{1}{2} + \frac{1}{4}\mu)\underline{a} + \mu \underline{c}$

$(1-\lambda)\underline{a} + \lambda \underline{c} = (\frac{1}{2} + \frac{1}{4}\mu)\underline{a} + \mu \underline{c}$

$1 - \lambda = \frac{1}{2} + \frac{1}{4}\mu$

$\lambda = \mu$

$1 - \lambda = \frac{1}{2} + \frac{1}{4}\lambda$

$\frac{1}{2} = \frac{5}{4}\lambda$

$\frac{2}{5} = \lambda$

$AP : PC$

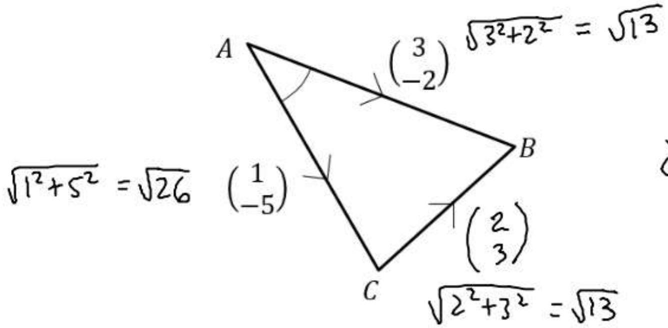
$\frac{2}{5} : \frac{3}{5}$

$2 : 3$

Area of a Triangle

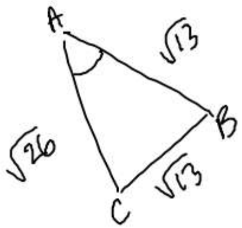
$\overrightarrow{AB} = 3\boldsymbol{i} - 2\boldsymbol{j}$ and $\overrightarrow{AC} = \boldsymbol{i} - 5\boldsymbol{j}$. Determine $\angle BAC$.

Strategy: Find 3 lengths of triangle then use cosine rule to find angle.



Cosine Rule:
$a^2 = b^2 + c^2 - 2bc\cos A$

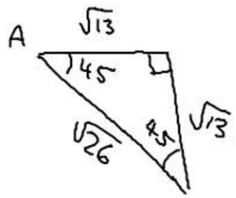
$$\overrightarrow{CB} = -\begin{pmatrix}1\\-5\end{pmatrix} + \begin{pmatrix}3\\-2\end{pmatrix}$$
$$= \begin{pmatrix}-1\\5\end{pmatrix} + \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$13 = 26 + 13 - 2\sqrt{13}\sqrt{26}\cos A$$
$$-26 = -26\sqrt{2}\cos A$$
$$\frac{-26}{-26\sqrt{2}} = \cos A$$

$$\cos A = \frac{\sqrt{2}}{2}$$
$$A = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$
$$\underline{\underline{A = 45°}}$$

Ex 11E
1, 3, 4, 5, (6)



Area? "$\frac{1}{2}ab\sin C$" $= \frac{1}{2} \times \sqrt{13} \times \sqrt{26} \times \sin 45$
$= \frac{13}{2}$ units^2

Area? $\frac{1}{2}bh = \frac{1}{2} \times \sqrt{13} \times \sqrt{13} = \frac{13}{2}$ units^2

Ex 11E

5 In triangle ABC the position vectors of the vertices A, B and C are $\begin{pmatrix}5\\8\end{pmatrix}$, $\begin{pmatrix}4\\3\end{pmatrix}$ and $\begin{pmatrix}7\\6\end{pmatrix}$. Find:

a $|\overrightarrow{AB}|$ **b** $|\overrightarrow{AC}|$ **c** $|\overrightarrow{BC}|$

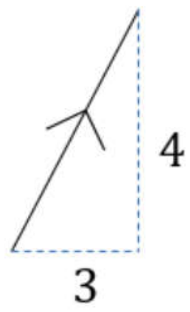
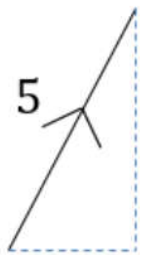
d the size of $\angle BAC$, [redacted] to the nearest degree.

Mixed Exercise Q7.

Modelling

In Mechanics, you will see certain things can be represented as a simple number (without direction), or as a vector (with direction):

> Remember a ‘scalar’ just means a normal number (in the context of vectors). It can be obtained using the **magnitude** of the vector.

Vector Quantity	**Equivalent Scalar Quantity**
Velocity e.g. $\begin{pmatrix}3\\4\end{pmatrix}\ km/h$  	Speed $= 5\ km/h$ 
 Displacement e.g. $\begin{pmatrix}-5\\12\end{pmatrix}\ km$	Distance $= 13\ km$

Find the distance moved by a particle which travels for:

scalar

a 5 hours at velocity $(8\mathbf{i} + 6\mathbf{j})$ km h^{-1}

b 10 seconds at velocity $(5\mathbf{i} - \mathbf{j})$ m s^{-1}

c 45 minutes at velocity $(6\mathbf{i} + 2\mathbf{j})$ km h^{-1}

d 2 minutes at velocity $(-4\mathbf{i} - 7\mathbf{j})$ cm s^{-1}.

a) $\underline{v} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ $\quad |\underline{v}| = \sqrt{8^2+6^2} = 10 \text{ kmh}^{-1}$

$d = |\underline{v}| \times t$

$d = 10 \times 5 = 50\text{km}$

b) $|\underline{v}| = \sqrt{5^2+1^2} = \sqrt{26}$ $\quad d = \sqrt{26} \times 10 = 10\sqrt{26} = \underline{\underline{51.0 \text{ m}}}$

c) $|\underline{v}| = \sqrt{6^2+2^2} = 2\sqrt{10}$ $\quad d = 2\sqrt{10} \times \frac{3}{4} = \underline{\underline{4.74 \text{ km}}}$

d) $|\underline{v}| = \sqrt{4^2+7^2} = \sqrt{65}$ $\quad d = \sqrt{65} \times 120 = \underline{\underline{967\text{cm}}}$

All answers to 3sf.