Using Partial Fractions

Partial fractions allows us to split up a fraction into ones we can then find the binomial expansion of.

a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions.

b) Hence show that the cubic approximation of $\frac{4-5x}{(1+x)(2-x)}$ is $2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$

c) State the range of values of x for which the expansion is valid.

a) $\frac{4-5x}{(1+x)(2-x)} = \frac{A}{1+x} + \frac{B}{2-x}$

$4-5x = A(2-x) + B(1+x)$

$x=2$ $\quad$ $-6 = 3B$ $\quad$ $B=-2$

$x=-1$ $\quad$ $9 = 3A$ $\quad$ $A=3$

$\frac{4-5x}{(1+x)(2-x)} = \frac{3}{1+x} - \boxed{\frac{2}{2-x}}$

b) $3(1+x)^{-1} = 3\left(1 - x + \frac{(-1)(-2)}{2!}(x)^2 + \frac{(-1)(-2)(-3)}{3!}(x)^3\right)$

$n=-1$

$x=x$

$= 3(1 - x + x^2 - x^3)$

$= 3 - 3x + 3x^2 - 3x^3$

$|x| < 1$

$$2(2-x)^{-1} = 2\times 2^{-1}\left(1-\frac{x}{2}\right)^{-1}$$

$\left|-\frac{x}{2}\right| < 1$

$$= \left(1-\frac{x}{2}\right)^{-1}$$

$n = -1$

"x" $= -\frac{x}{2}$

$|x| < 2$

$$= 1 + (-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-2)}{2!}\left(-\frac{x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{3!}\left(-\frac{x}{2}\right)^3$$

$$= 1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8}$$

$$\frac{4-5x}{(1+x)(2-x)} = 3 - 3x + 3x^2 - 3x^3 - 1 - \frac{x}{2} - \frac{x^2}{4} - \frac{x^3}{8}$$

$$= 2 - \frac{7}{2}x + \frac{11}{4}x^2 - \frac{25}{8}x^3$$

c) valid for $|x| < 1$

[C4 June 2010 Q5]

10.
$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(a) Find the values of the constants A, B and C. **(4)**

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction. **(7)**

a) $\frac{2x^2+5x-10}{(x-1)(x+2)} = A + \frac{B}{x-1} + \frac{C}{x+2}$

$2x^2+5x-10 = A(x-1)(x+2) + B(x+2) + C(x-1)$

compare x^2 $\quad \underline{2 = A}$

$x \quad 5 = A + B + C$

$3 = B + C \quad (1)$

const. $\quad -10 = -2A + 2B - C$

$-10 = -4 + 2B - C$

$-6 = 2B - C \quad (2)$

$(1) + (2)$

$-3 = 3B$

$\underline{B = -1}$

$(1) \quad 3 = -1 + C$

$\underline{C = 4}$

Ex 4C

a) $2 - \frac{1}{x-1}^{x-1}_{x-1} + \frac{4}{x+2} = 2 - (x-1)^{-1} + 4(x+2)^{-1}$

(a) $A = 2$	B1	
$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$		
$x \to 1 \qquad -3 = 3B \Rightarrow B = -1$	M1 A1	
$x \to -2 \qquad -12 = -3C \Rightarrow C = 4$	A1	**(4)**
(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$	M1	
$(1-x)^{-1} = 1 + x + x^2 + \ldots$	B1	
$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ldots$	B1	
$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ldots$	M1	
$= 5 + \ldots$ ft their $A - B + \frac{1}{2}C$	A1 ft	
$= \ldots + \dfrac{3}{2}x^2 + \ldots$ $0x$ stated or implied	A1 A1	**(7)**
		[11]