OCR A Level Physics

Your notes

Capacitors in Circuits

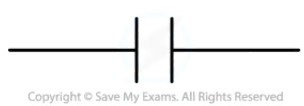
Contents

Capacitance

Your notes

Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- Capacitors do this by storing electric **charge**, which creates a build up of electric **potential energy**
- They are made in the form of two conductive **metal plates** connected to a voltage supply (parallel plate capacitor)
    - There is commonly a **dielectric** in between the plates, to ensure charge does not flow across them
- The capacitor circuit symbol is:



The capacitor circuit symbol is two parallel lines

- Capacitors are marked with a value of their **capacitance**
- Capacitance is defined as:

  The charge stored per unit potential difference (between the plates)

- The greater the **capacitance**, the greater the **charge stored** in the capacitor
- The capacitance of a capacitor is defined by the equation:

$$C = \frac{Q}{V}$$

- Where:
    - C = capacitance (F)
    - Q = charge stored (C)
    - *V* = potential difference across the capacitor plates (V)

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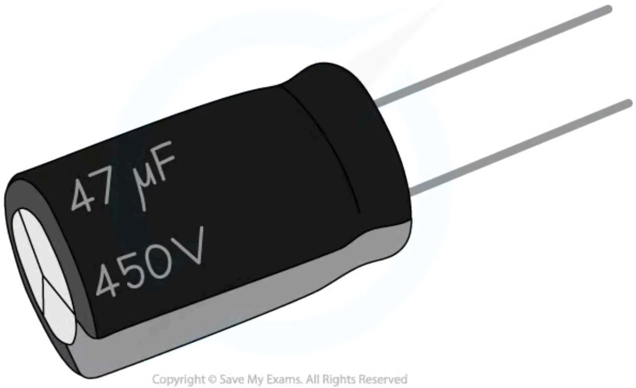


A capacitor used in small circuits

- Capacitance is measured in the unit **Farad (F)**
  - In practice, 1 F is a very large unit
  - Often it will be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)
- If the capacitor is made of parallel plates, Q is the charge on the plates and *V* is the potential difference across the capacitor
  - The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates
- This capacitance equation shows that an object's capacitance is the **ratio of the charge stored by the capacitor to the potential difference between the plates**

Worked Example

A parallel plate capacitor has a capacitance of 1 nF and is connected to a voltage supply of 0.3 kV.

Calculate the charge on the plates.

Answer:

Step 1: Write down the known quantities

- Capacitance, $C = 1\text{ nF} = 1 \times 10^{-9}\text{ F}$
- Potential difference, $V = 0.3\text{ kV} = 0.3 \times 10^{3}\text{ V}$

Step 2: Write out the equation for capacitance

$$C = \frac{Q}{V}$$

Step 3: Rearrange for charge Q

$$Q = CV$$

Step 4: Substitute in values and calculate the final answer

$$Q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\ \text{C} = \mathbf{300\ nC}$$

Examiner Tips and Tricks

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The letter 'C' is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

Uses of Capacitors

- Capacitors are useful because they store **electric potential energy**
- They have a wide variety of applications, such as:
  - in cameras, i.e. a bright flash of light as the capacitor discharges
  - in smoothing currents
  - in electronic timing circuits
  - to use as a back up power supply during unexpected power cuts
  - to power devices with memory to store information when they are switched off e.g. calculators

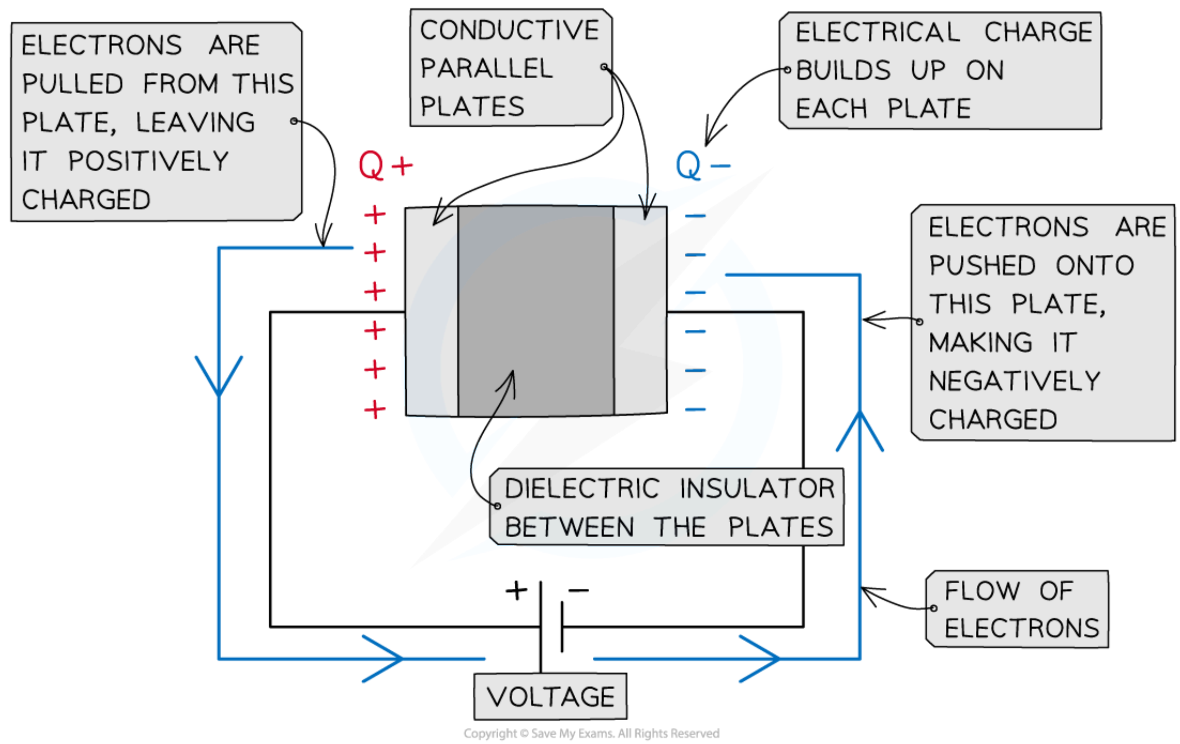
Electron Flow in Charging & Discharging

Your notes

Electron Flow in Charging & Discharging

Charging

- Capacitors are charged by a **power supply** (e.g., a battery)
- One plate of the capacitor is connected to the **positive terminal** of the power supply
  - The positive terminal **'pulls' electrons** from this plate
  - Hence the plate nearest the positive terminal becomes **positively charged**
- These electrons travel around the circuit and are pushed from the **negative terminal** of the power supply
  - The negative terminal **'pushes' electrons** onto the other plate
  - Hence the plate nearest the negative terminal becomes **negatively charged**
- As the negative charge builds up, fewer electrons are pushed onto the plate due to **electrostatic repulsion** from the electrons already on the plate
  - When no more electrons can be pushed onto the negative plate, the charging stops



Electron flow for a charging capacitor. Electrons are pulled from one plate and pushed on to the other

- At the start of charging, the current is **large** and gradually falls to **zero** as the electrons stop flowing through the circuit
  - The current decreases **exponentially**
  - This means the rate at which the current decreases is inversely proportional to the amount of charge on the plate
- Since an equal but opposite charge builds up on each plate, the **potential difference** across the plates slowly increases until it is the **same** as that of the **power supply**
- Similarly, the charge of the plates slowly increases until it is at its maximum charge defined by the capacitance of the capacitor

Discharging

- Capacitors are **discharged** through a resistor with **no** power supply present
- The electrons now flow back from the negative plate to the positive plate until there are equal numbers on each plate and no **potential difference** between them

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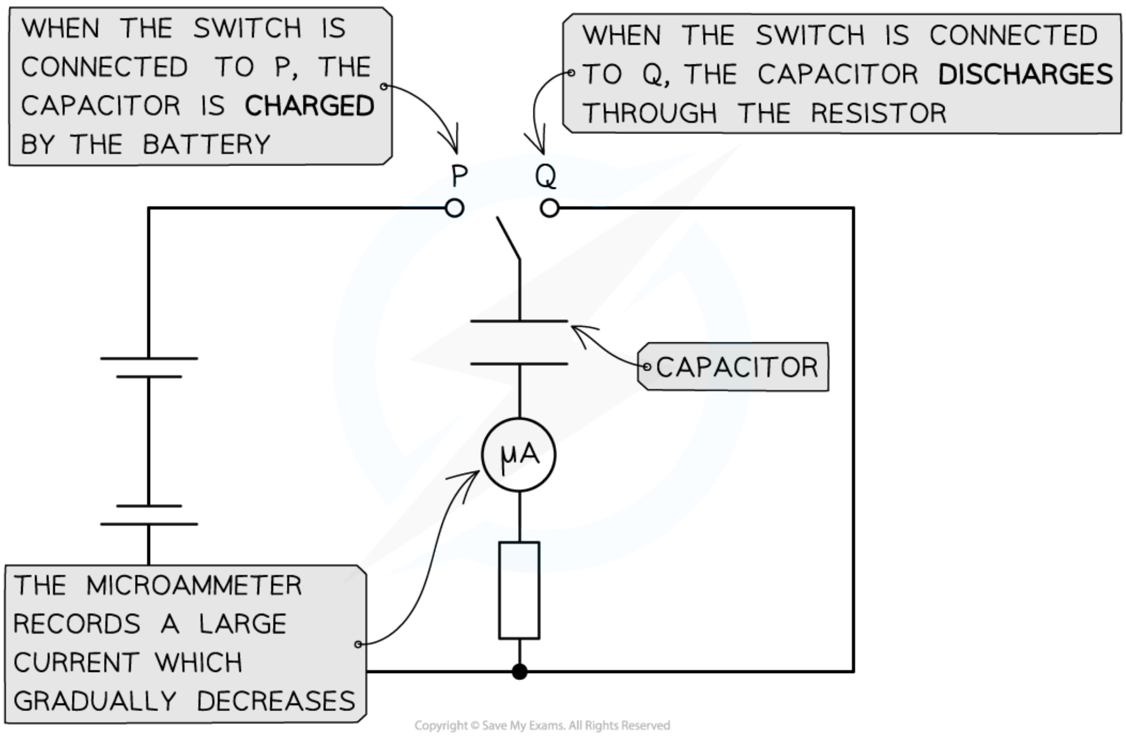
- Charging and discharging is commonly achieved by moving a switch that connects the capacitor between a power supply and a resistor



The capacitor charges when connected to terminal P and discharges when connected to terminal Q

- At the start of discharge, the current is **large** (but in the opposite direction to when it was charging) and gradually falls to zero
- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
  - This means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left
- The graphs of the variation with time of current, p.d and charge are all identical and follow a pattern of **exponential decay**

Examiner Tips and Tricks

Describing the motion of electrons (**electron flow**) for a charging or discharging capacitor is a common exam question. Ensure you are able to specify from which plate electrons are 'pulled' (the plate connected to the **positive terminal** of the power supply) and to which plate electrons are 'pushed' (the plate connected to the **negative terminal**).

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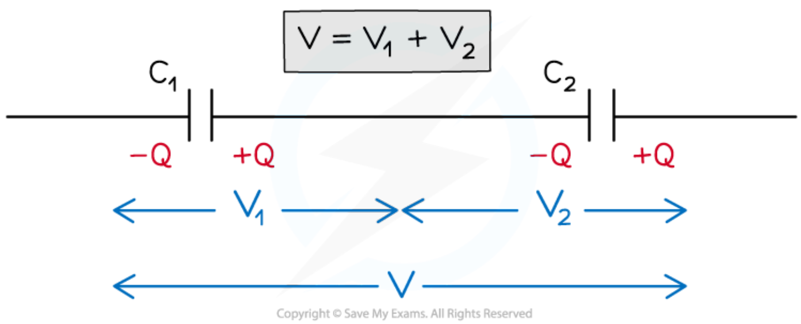
Capacitors in Series & Parallel Circuits

Your notes

Capacitors in Series

- Consider two parallel plate capacitors C_1 and C_2 connected in series, with a potential difference (p.d.) V across them



Capacitors connected in series have different p.d. across them but have the same charge

- In a series circuit, p.d. is **shared** between all the components in the circuit
  - Therefore, if the capacitors store the same charge on their plates but have different p.ds, the p.d. across C_1 is V_1 and across C_2 is V_2
- The total potential difference V is the sum of V_1 and V_2

$$V = V_1 + V_2$$

- Rearranging the capacitance equation for the p.d., V, means V_1 and V_2 can be written as:

$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

- Where the total p.d. V is defined by the total capacitance

$$V = \frac{Q}{C_{total}}$$

- Substituting these into the equation $V = V_1 + V_2$ equals:

$$\frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

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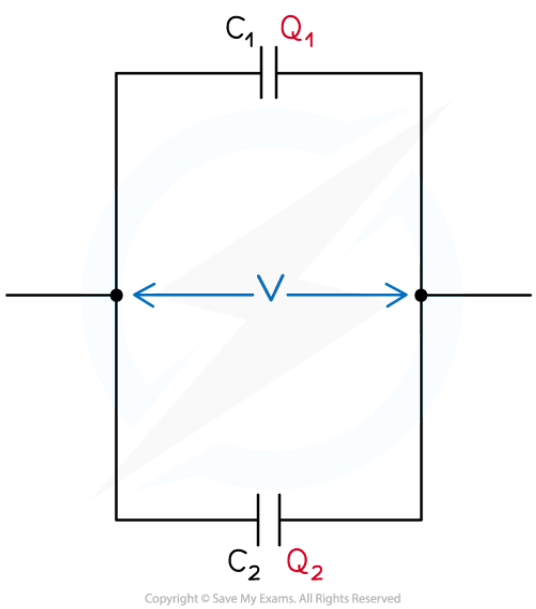
- Since the current is the **same** through all components in a series circuit, the charge Q is the same through each capacitor and cancels out
- Therefore, the equation for combined capacitance of capacitors in **series** is:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ...$$

Capacitors in Parallel

- Consider two parallel plate capacitors C_1 and C_2 connected in parallel, each with p.d V



Capacitors connected in parallel have the same p.d across them, but different charge

- Since the current is **split** across each junction in a parallel circuit, the charge stored on each capacitor is **different**
  - Therefore, the charge on capacitor C_1 is Q_1 and on C_2 is Q_2

- The total charge Q is the sum of Q_1 and Q_2

$$Q = Q_1 + Q_2$$

- Rearranging the capacitance equation for the charge Q means Q_1 and Q_2 can be written as:

$$Q_1 = C_1V \quad \text{and} \quad Q_2 = C_2V$$

- Where the total charge Q is defined by the total capacitance:

$$Q = C_{total}V$$

- Substituting these into the $Q = Q_1 + Q_2$ equals:

$$C_{total}V = C_1V + C_2V = (C_1 + C_2)\,V$$

- Since the p.d is the **same** through all components in each branch of a parallel circuit, the p.d *V* cancels out
- Therefore, the equation for combined capacitance of capacitors in **parallel** is:

$$C_{total} = C_1 + C_2 + C_3 \ldots$$

Solving Problems with Capacitors in Series & Parallel

- For capacitors in series:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

- For capacitors in parallel:

$$C_{total} = C_1 + C_2 + C_3 \ldots$$

Worked Example

Three capacitors with capacitance of 23 μF, 35 μF and 40 μF are connected as shown below.

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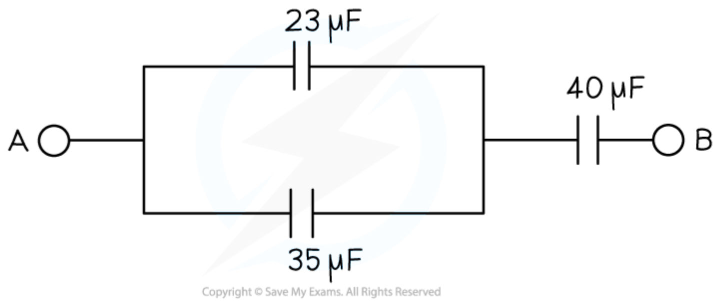

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Calculate the total capacitance between points **A** and **B**.

Answer:

Step 1: Calculate the combined capacitance of the two capacitors in parallel

Capacitors in parallel: $C_{total} = C_1 + C_2 + C_3 ...$

$$C_{parallel} = 23 + 35 = 58\ \mu F$$

Step 2: Connect this combined capacitance with the final capacitor in series

$$\text{Capacitors in series: } \frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ...$$

$$\frac{1}{C_{total}} = \frac{1}{58} + \frac{1}{40} = \frac{49}{1160}$$

Step 3: Rearrange for the total capacitance

$$C_{total} = \frac{1160}{49} = 23.673... = \mathbf{24\ \mu F} \text{ (2 s.f)}$$

Examiner Tips and Tricks

Both the combined capacitance equations look similar to the equations for combined resistance in series and parallel circuits. However, take note that they are the **opposite way** around to each other!

Circuits Containing Capacitors & Resistors

Your notes

Circuits Containing Capacitors & Resistors

- Rearrange the capacitor equation to make charge, Q the subject:

$$Q = CV$$

- The **capacitance** C of a capacitor is **fixed**
  - It is determined during the manufacturing process
- Hence, charge Q is directly proportional to potential difference *V*

Investigation with a test circuit

- The relationship between the **potential difference** across a capacitor and the **charge stored** on it can be investigated experimentally by **charging** a capacitor using a **constant current**
- A suitable test circuit contains:
  - a parallel plate capacitor
  - a switch
  - a battery
  - an ammeter connected in series with the capacitor
  - a variable resistor
  - a voltmeter connected in parallel with the capacitor

Test circuit to charge a capacitor

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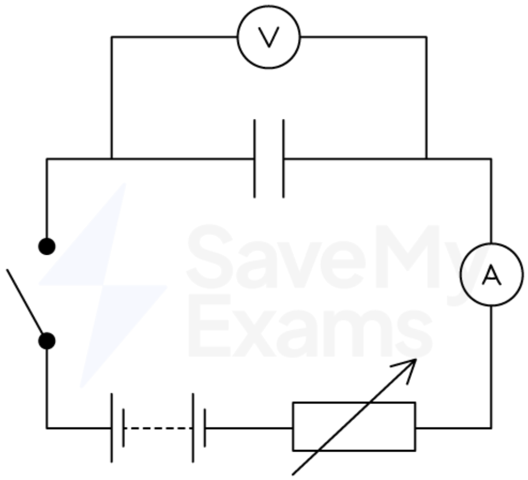
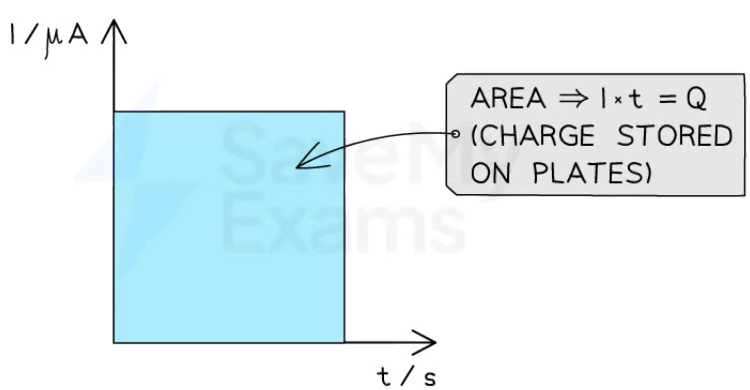


The potential difference across a capacitor and the charge stored on a capacitor is investigated using this test circuit

- Close the switch and constantly adjust the variable resistor to keep the charging current at a constant value for as long as possible
  - This will be impossible when the capacitor is close to fully charged
- Record the potential difference across the capacitor at regular time intervals until it equals the potential difference of the power supply
- Plot a graph of charging current and time taken to charge
  - Once the capacitor is fully charged the current passing through it drops to zero

Graph of charging current and time using test circuit



The current-time graph of the capacitor in the test circuit whilst constantly adjusting the variable resistor

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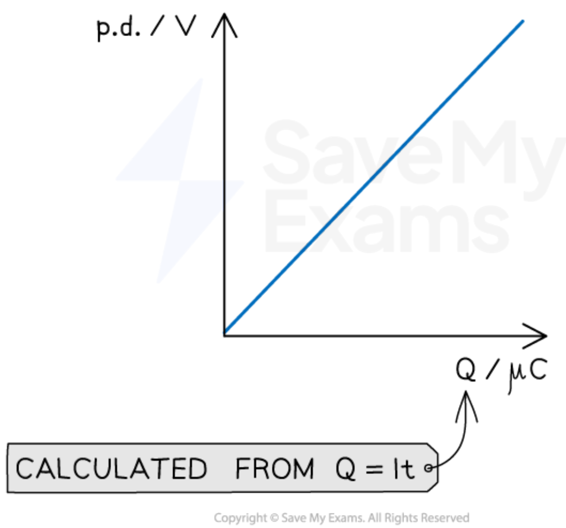
- Recall the equation for charge, current and time:

$$Q = It$$

- Use it to calculate the charge stored on a capacitor at a given time
- Then plot a graph of the charge stored Q against the potential difference at each recorded time interval

Graph of potential difference and charge stored



The charge-potential difference graph of a capacitor is a straight line through the origin

- The calculated charge-potential difference graph is a straight line through the origin
  - Hence, Q and *V* are directly proportional
  - The gradient of the graph $\frac{Q}{V}$ is constant and equal to the given capacitance of the capacitor, C
  - So, $C = \frac{Q}{V}$

Investigating Capacitors in Series & Parallel

Your notes

Aim of the Experiment

- The aim of this experiment is to determine the capacitance of capacitors connected in series and parallel combinations

Variables

- Independent variable = potential difference, *V*
- Dependent variable = charge, *Q*
- Control variables:
  - Current in the circuit
  - E.m.f. of the supply
  - Capacitance of each capacitor

Equipment List

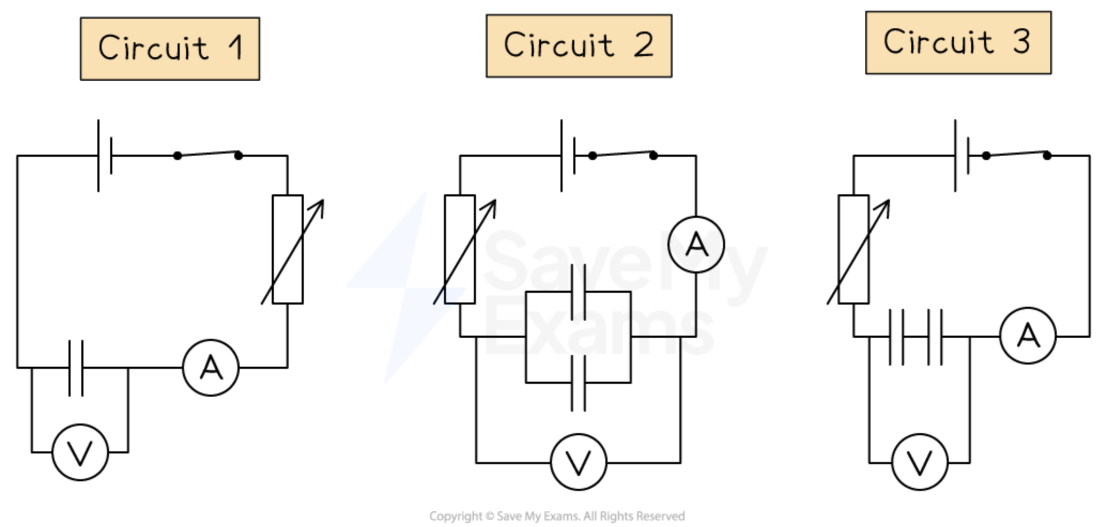
Apparatus	Purpose
Battery pack (power supply)	To provide the e.m.f. to the circuit
Two capacitors	To provide the capacitance and to arrange into series and parallel combinations
Switch	To control the charging and discharging of the capacitors
Ammeter	To measure the current in the capacitors
Voltmeter	To measure the potential difference across the capacitors
Variable resistor	To adjust the resistance to keep the charging current constant
Stopwatch	To measure the time taken for the capacitors to charge

- **Resolution** of measuring equipment:
  - Voltmeter = 0.1 V
  - Ammeter = 0.1 A

- Stopwatch = 0.01 s

Method

- Set up three circuits:
  - Circuit 1: single capacitor
  - Circuit 2: capacitors in parallel
  - Circuit 3: capacitors in series



Circuits for investigating capacitors in series and parallel

1. Close the switch to charge the capacitor and start the stopwatch
2. As the capacitor charges, record the value of the fixed current and adjust the variable resistor to ensure the current remains constant
3. Record the potential difference and the time since closing the switch in a table
4. Repeat the procedure for circuits 2 and 3

- An example table might look like this:

Time t / s	Current I / A	Charge Q / C Q = I × t	Potential difference V / V
5.0			

10.0			
15.0			
20.0			
25.0			
30.0			

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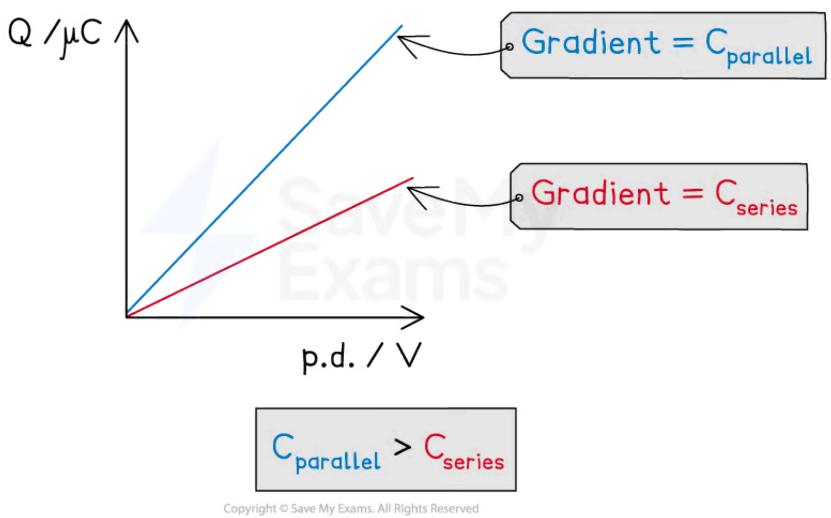
Analysing the Results

- The relationship between charge, potential difference and capacitance is

$$Q = CV$$

- Where:
  - Q = charge across the capacitor (C)
  - V = potential difference across the capacitor (V)
  - C = capacitance of the capacitor (F)
- The total capacitance of each combination of capacitors can be found by
  - plotting a graph of Q against V
  - drawing a line of best fit
  - calculating the gradient, which is equal to:

$$gradient = \frac{\Delta Q}{\Delta V} = C$$



The gradient of the parallel combination is greater than the gradient of the series combination

Your notes

- The expected results are:

1. The total capacitance of the parallel combination is greater than the capacitance of the series combination
2. If the capacitors have the same capacitance, the combined capacitance of the parallel combination is double the capacitance of one
   - $C_{parallel} = C_1 + C_2 + ... + C_n$
   - $C_{parallel} = C + C = 2C$
3. If the capacitors have the same capacitance, then the combined capacitance of the series combination is half the capacitance of one capacitor
   - $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$
   - $C_{series} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \frac{C}{2}$

Evaluating the Experiment

Systematic Errors:

- If a digital voltmeter is used, wait until the reading is settled on a value if it is switching between two

- If an analogue voltmeter is used, reduce parallax error by reading the p.d. at eye level to the meter
- Before closing the switch, check that the voltmeter and ammeter readings start at zero to avoid a zero error

Random Errors:

- Use a data logger to record the potential difference and current. This will allow for calculations of charge in real-time and for graphs of charge against p.d. to be plotted in real-time
- When plotting graphs, only use the values for which the current is approximately constant as it will be difficult to keep the current constant once the capacitor is fully charged

Safety Considerations

- Keep water or any fluids away from the electrical equipment
- Make sure no wires or connections are damaged
- Capacitors can still retain charge after the power supply is removed which could cause an electric shock
    - These should be fully discharged and removed after a few minutes

Your notes

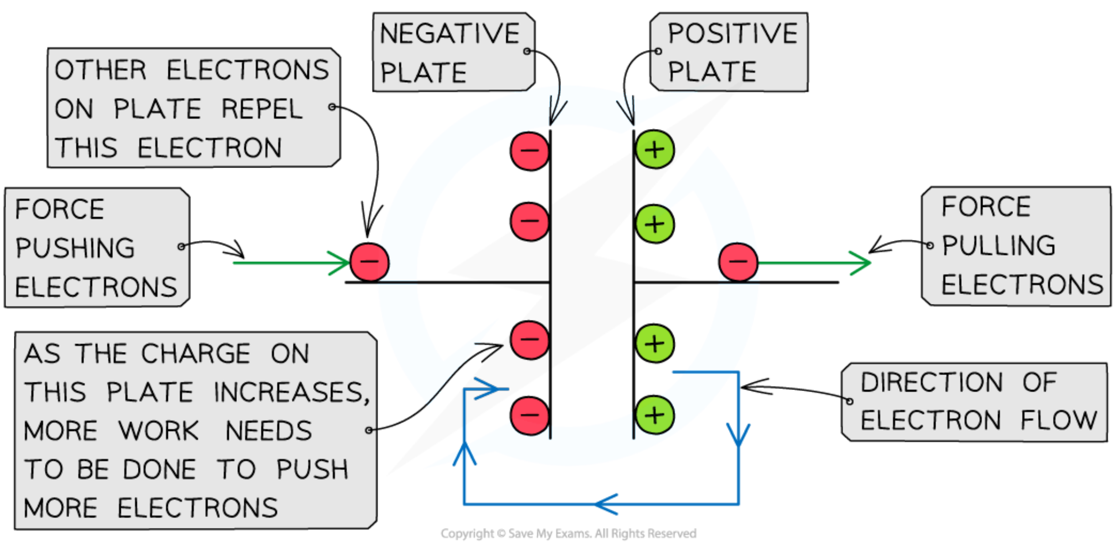
Your notes

Energy Stored by a Capacitor

Area Under a Potential–Charge Graph

- When charging a capacitor, the power supply pushes electrons from the positive to the negative plate
  - It therefore does **work** on the electrons and **electrical energy** becomes stored on the plates
- At first, a small amount of charge is pushed from the positive to the negative plate, then gradually, this builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion
- As the charge of the negative plate increases ie. becomes more negatively charged, the force of repulsion between the electrons on the plate and the new electrons being pushed onto it increases
- This means a greater amount of work must be done to increase the charge on the negative plate or in other words:

  The potential difference across the capacitor increases as the amount of charge increases



As the charge on the negative plate builds up, more work needs to be done to add more charge

- The charge Q on the capacitor is **directly proportional** to its potential difference *V*
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The electrical (potential) energy stored in the capacitor can be determined from the **area under the potential-charge graph** which is equal to the **area** of a right-angled triangle:

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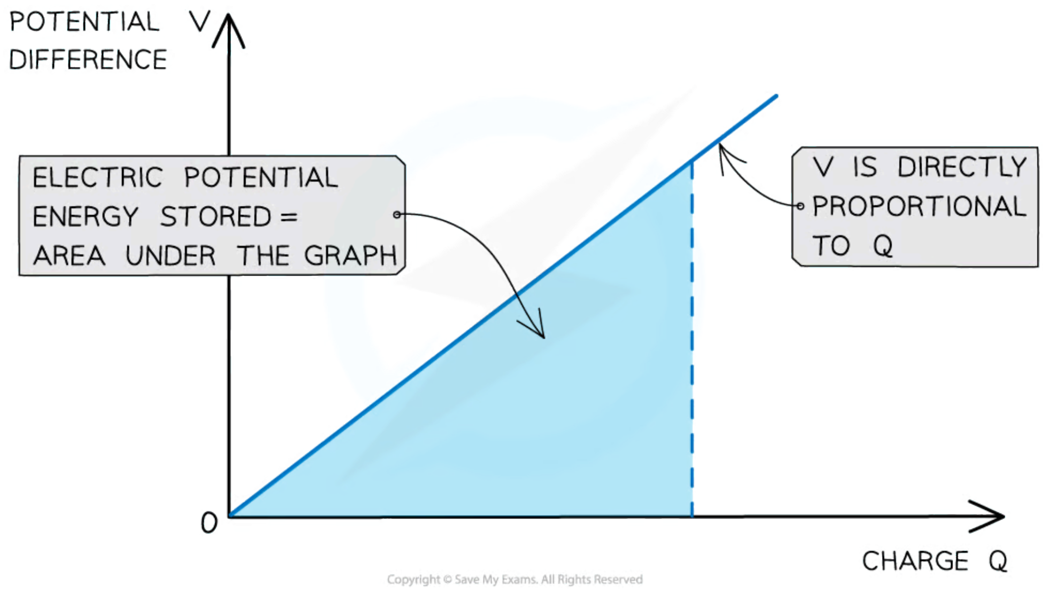
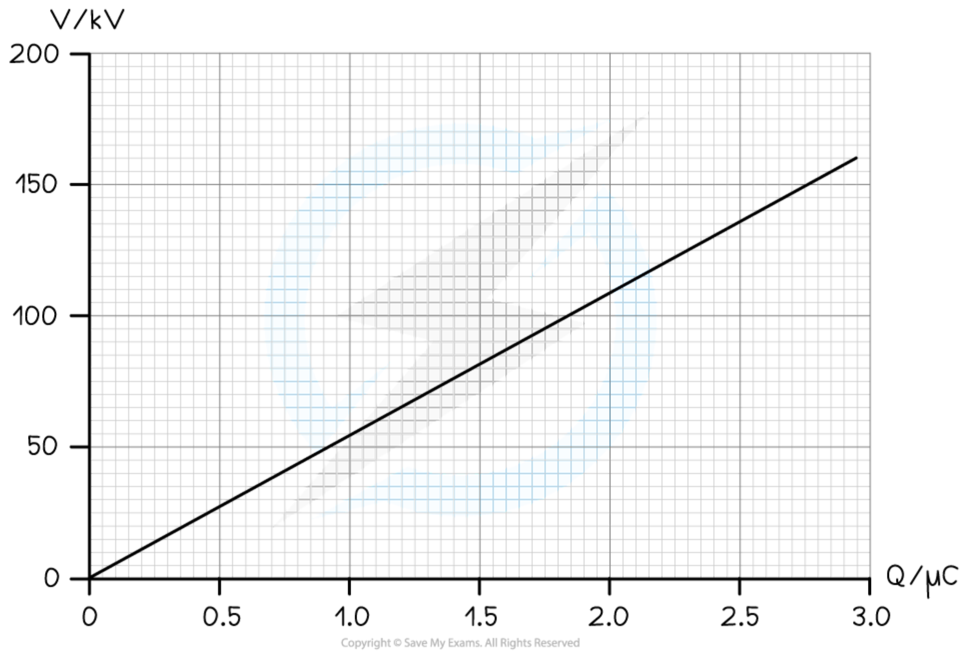
Area = 0.5 × base × height



The electric energy stored in the capacitor is the area under the potential-charge graph

Worked Example

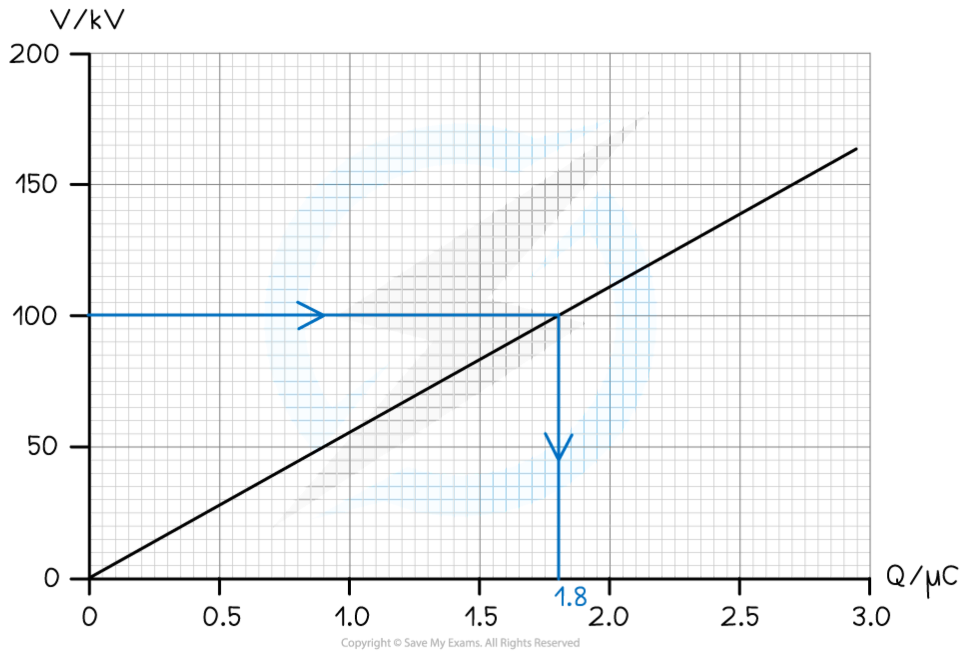
The variation of the potential *V* of a charged isolated metal sphere with surface charge Q is shown on the graph below.



Using the graph, determine the electric potential energy stored on the sphere when charged to a potential of 100 kV.

Answer:

Step 1: Determine the charge on the sphere at the potential of 100 kV



- From the graph, the charge on the sphere at 100 kV is **1.8 μC**

Step 2: Calculate the electric potential energy stored

- The energy stored is equal to the area under the graph at 100 kV
- The area is equal to a right-angled triangle, so, can be calculated with the equation:

$$\text{Area} = 0.5 \times \text{base} \times \text{height}$$

$$\text{Area} = 0.5 \times 1.8\ \mu\text{C} \times 100\ \text{kV}$$

$$\text{Energy } E = 0.5 \times (1.8 \times 10^{-6}) \times (100 \times 10^{3}) = \mathbf{0.09\ J}$$

Energy Stored by a Capacitor

- The work done, or **energy stored** in a capacitor is defined by the equation:

$$E = \frac{1}{2} QV$$

- Where:
  - E = work done or energy stored (J)
  - Q = charge (C)

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- V = potential difference (V)
- Substituting the charge with the **capacitance** equation **Q = CV**, the energy stored can also be defined as:

$$E = \frac{1}{2}CV^2$$

- By substituting the potential V, the energy stored can also be defined in terms of just the charge, Q and the capacitance, C:

$$E = \frac{Q^2}{2C}$$

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Worked Example

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 10 V to 30 V.

Answer:

Step 1: Write down the equation for energy stored in terms of capacitance C and p.d V

$$W = \frac{1}{2}CV^2$$

Step 2: The change in energy stored is proportional to the change in p.d

$$\Delta W = \frac{1}{2}CV_2^2 - \frac{1}{2}CV_1^2 = \frac{1}{2}C\left(V_2^2 - V_1^2\right)$$

Step 3: Substitute in values

$$\Delta W = \frac{1}{2} \times \left(1500 \times 10^{-6}\right) \times \left(30^2 - 10^2\right)$$

$$\Delta W = 0.6 \text{ J}$$