

# 5.3 Trigonometric Equations

5.3.1 Trigonometry - Simple Identities / 5.3.2 Linear Trigonometric Equations / 5.3.3 Quadratic Trigonometric Equations / 5.3.4 Strategy for Trigonometric Equations

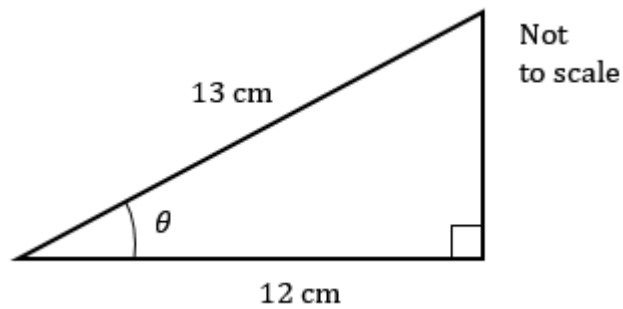
|                         |             |
|-------------------------|-------------|
| Easy (8 questions)      | /29         |
| Medium (8 questions)    | /40         |
| Hard (8 questions)      | /40         |
| Very Hard (8 questions) | /40         |
| <b>Total Marks</b>      | <b>/149</b> |

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# Easy Questions

1 (a) Work out the length of the missing side in the following right-angled triangle.



(2 marks)

(b) Using your answer from part (a) to help, write down the values of the following:

(i)  $\sin \theta$

(ii)  $\cos \theta$

(iii)  $\tan \theta$

(3 marks)

2 Show that

$$\frac{1 - \cos^2 x}{\tan^2 x} \equiv \cos^2 x$$

(2 marks)

3 Solve the equation

$$\sin x = \frac{1}{2}, \quad 0^\circ \leq x \leq 360^\circ$$

(3 marks)

**4 (a)** Solve the equation  $x^2 + x - 2 = 0$ .

**(2 marks)**

**(b)** Hence, or otherwise, solve the equation  $\cos^2 x + \cos x - 2 = 0$  for  $0^\circ \leq x \leq 720^\circ$ .

**(2 marks)**

**5** Solve the equation  $\tan 2\theta = 0.3$  for  $-180^\circ \leq \theta \leq 180^\circ$ , giving your answers to one decimal place.

**(3 marks)**

**6 (a)** Sketch the graph of  $y = \cos 2x$  for  $0^\circ \leq x \leq 360^\circ$ .

**(2 marks)**

**(b)** Solve the equation  $\cos 2x = 0.5$  for  $0^\circ \leq x \leq 360^\circ$ .

**(2 marks)**

**7** Solve the equation  $2(1 - \cos^2 \theta) = 1$  for  $-180^\circ \leq \theta \leq 180^\circ$ .

**(4 marks)**

**8** Solve the equation  $4 - 4\sin^2 \theta = 3$  for  $0^\circ \leq \theta \leq 180^\circ$ .

**(4 marks)**

# Medium Questions

- 1 Solve the equation  $2 \sin 2\theta = 1$  for  $0^\circ \leq \theta \leq 360^\circ$ .

(3 marks)

- 2 (a)** Show that the equation  $2 \sin^2 x + 3 \cos x = 0$  can be written in the form  $a \cos^2 x + b \cos x + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers to be found.

**(2 marks)**

- (b)** Hence, or otherwise, solve the equation  $2 \sin^2 x + 3 \cos x = 0$  for  $-180^\circ \leq x \leq 180^\circ$ .

**(3 marks)**

- 3** Given that  $\sin \theta = \frac{3}{5}$  find the possible values of  $\cos \theta$  and  $\tan \theta$ .

**(3 marks)**

- 4** Solve the equation  $2 \sin x = \frac{1}{\sin x}$  for  $0^\circ \leq x \leq 360^\circ$ .

**(5 marks)**

5 Solve the equation  $2 \sin x \cos x = \cos x$  for  $-180^\circ \leq x \leq 180^\circ$ .

(5 marks)

6 A right-angled triangle has hypotenuse 8 cm. One of its other sides is 5 cm.

Find exact values for  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ , where  $\theta$  is the smallest angle in the triangle.

(6 marks)



**7 (a)** Show that  $(x+1)(x-2)(x-3) \equiv x^3 - 4x^2 + x + 6$ .

**(2 marks)**

**(b)** Hence, or otherwise, solve the equation  $\tan^3 x - 4 \tan^2 x + \tan x + 6 = 0$  for  $0^\circ \leq x \leq 360^\circ$ , giving your answers to 1 decimal place where appropriate.

**(5 marks)**

**8 (a)** A seagull sits on the surface of the sea and moves up and down as waves pass.

Its height,  $h$  metres, above its position in calm water is modelled by the function  $h = \frac{1}{2} \sin(180t)^\circ$  where  $t$  is the time in seconds after timing commences.

Sketch a graph of  $h$  against  $t$  for  $0 \leq t \leq 10$  showing the coordinates of the points of intersection with the  $t$  axis.

**(2 marks)**

**(b)** How many times in the first minute after timing commences is the seagull 0.25 metres above its calm water position?

**(1 mark)**

**(c)** Find the time at which the seagull is first 0.25m above its calm water position **and moving downwards**. Give your answer to 3 significant figures.

**(3 marks)**

# Hard Questions

- 1 Solve the equation  $2 \sin \theta = 3 \cos \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ , giving your answers to 1 decimal place.

(3 marks)

- 2 Solve the equation  $2 \sin^2 \theta = \cos \theta + 1$  for  $-180^\circ \leq \theta \leq 180^\circ$

(5 marks)

- 3 Given that the angle  $\theta$  is obtuse and that  $\sin \theta = \frac{3}{4}$ , find the exact value of  $\cos \theta$ .

(3 marks)

- 4 Solve the equation  $\tan 2x = \frac{3}{\tan 2x}$  for  $-180^\circ \leq x \leq 180^\circ$ .

(5 marks)

- 5 Solve the equation  $2 \tan x - \sin x = 0$  for  $-180^\circ \leq x \leq 180^\circ$ .

(5 marks)

- 6 An isosceles triangle has sides 8 cm, 8 cm and 4 cm and equal base angles  $\theta$ .

Find exact values for  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ .

(6 marks)

**7 (a)** Show that  $x = \frac{1}{2}$  satisfies the equation  $8x^3 - 4x^2 - 6x + 3 = 0$ .

**(1 mark)**

**(b)** Hence solve the equation  $8 \cos^3 x - 4 \cos^2 x - 6 \cos x + 3 = 0$  for  $0^\circ \leq x \leq 360^\circ$ .

**(6 marks)**

**8 (a)** A seagull sits on the surface of the sea and moves up and down as waves pass.

Its height,  $h$  metres, above its position in calm water is modelled by the function  $h = \frac{2}{5} \sin(180t)^\circ$  where  $t$  is the time in seconds after timing commenced.

Find the first time the seagull is 0.3 metres above its calm water position.  
Give your answer to 2 decimal places.

**(4 marks)**

**(b)** How many times in the first minute after timing commences is the seagull 0.3 metres above its calm water position?

**(2 marks)**

# Very Hard Questions

- 1 Solve the equation  $3 \sin 3\theta = 4 \cos 3\theta$  for  $0^\circ \leq \theta \leq 180^\circ$ , giving your answers to 1 decimal place.

(3 marks)

- 2 Solve the equation  $6 \cos^2 2\theta = \sin 2\theta + 5$  for  $-180^\circ \leq \theta \leq 180^\circ$ , giving your answers to 1 decimal place where appropriate.

(5 marks)

- 3 Given that the angle  $\theta$  is reflex and that  $\cos \theta = \frac{1}{3}$ , find the exact value of  $\tan \theta$ .

(3 marks)

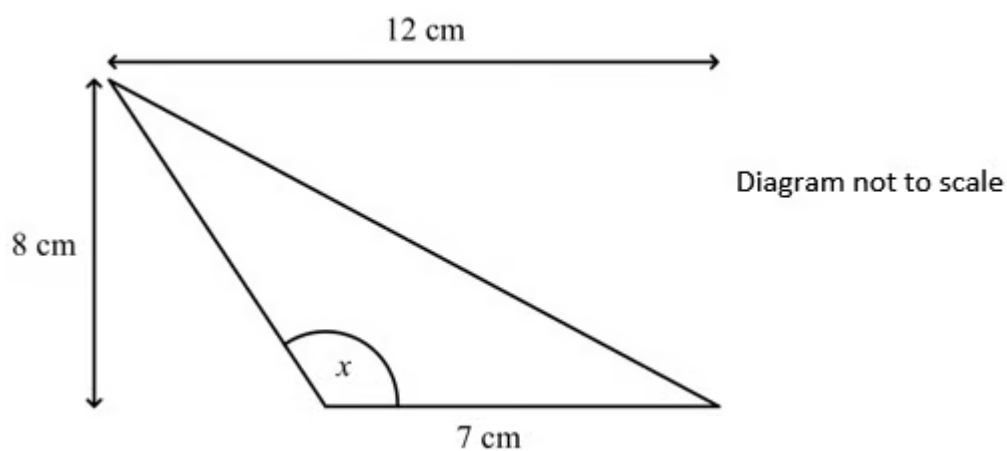
- 4 Solve the equation  $2 \sin^2 3x = 1$  for  $-90^\circ \leq x \leq 90^\circ$ .

(5 marks)

- 5 Solve the equation  $3 \sin(2x + 30^\circ) = \tan(2x + 30^\circ)$  for  $-180^\circ \leq x \leq 180^\circ$ , giving your answers to 1 decimal place where appropriate.

(5 marks)

- 6 For the triangle in the diagram find exact values for  $\sin x$ ,  $\cos x$  and  $\tan x$ .





(6 marks)

- 7 Find all the values of  $x$  in the range  $0^\circ \leq x \leq 180^\circ$  which satisfy the equation  $6 \tan^3 2x - 7 \tan^2 2x - \tan 2x + 2 = 0$ , giving your answers to 1 decimal place.

(6 marks)

- 8 A seagull sits on the surface of the sea and moves up and down as waves pass.

Its height,  $h$  metres, above its position in calm water is modelled by the function

$$h = \frac{3}{5} \sin(90t)^\circ \text{ where } t \text{ is the time in seconds after timing commences.}$$

Find the amount of time the seagull is more than 0.5 metres above its calm water position in the first 20 seconds after timing commences.

Give your answer correct to 3 significant figures.

**(7 marks)**