A Level · Edexcel · Further Maths

34 mins 4 questions

1.2 Exponential Form & de Moivre's Theorem

1.2.1 Exponential Form / 1.2.2 de Moivre's Theorem / 1.2.3 Applications of de Moivre's Theorem / 1.2.4 Roots of Complex Numbers

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Total Marks **/34**

1 (a) A complex number z has modulus 1 and argument θ.

(a) Show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta, \quad n \in \mathbb{Z}^+$$

(2 marks)

(b) (b) Hence, show that

$$\cos^4\theta = \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)$$

(5 marks)

2 (a) (a) Use de Moivre's theorem to prove that

$$\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$$

(5 marks)

(b) (b) Hence find the distinct roots of the equation

$$1 + 7x - 56x^3 + 112x^5 - 64x^7 = 0$$

giving your answer to 3 decimal places where appropriate.

(5 marks)

3 (a) The infinite series C and S are defined by

$$C = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \frac{1}{8}\cos 13\theta + \dots$$

$$S = \sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \frac{1}{8}\sin 13\theta + \dots$$

Given that the series C and S are both convergent,

a) show that

$$C + \mathrm{i}S = \frac{2\mathrm{e}^{\mathrm{i}\theta}}{2 - \mathrm{e}^{4\mathrm{i}\theta}}$$

(4 marks)

(b) (b) Hence show that

$$S = \frac{4\sin\theta + 2\sin 3\theta}{5 - 4\cos 4\theta}$$

(4 marks)

4 (a) In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6+2\mathrm{i}$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x+\mathrm{i}y$, where x and y are real and exact.

(6 marks)

(b) The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3 marks)