Finding Areas

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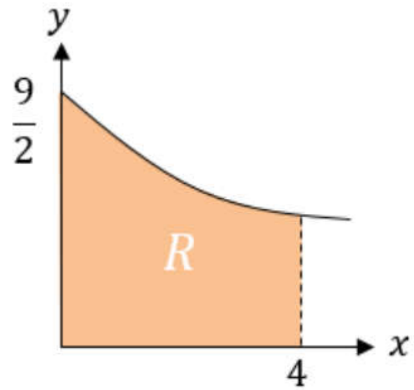
You're already familiar with the idea that definite integration gives you the (signed) area bound between the curve and the x-axis.
Given your expanded integration skills, you can now find the area under a greater variety of curves.

> The diagram shows part of the curve $y = \frac{9}{\sqrt{4+3x}}$
> The region R is bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$, as shown in the diagram. Use integration to find the area of R.



$$\int_0^4 \frac{9}{\sqrt{4+3x}}\,dx = \int_0^4 9(4+3x)^{-1/2}\,dx$$

$$= \left[9 \times \frac{2}{3}(4+3x)^{1/2}\right]_0^4$$

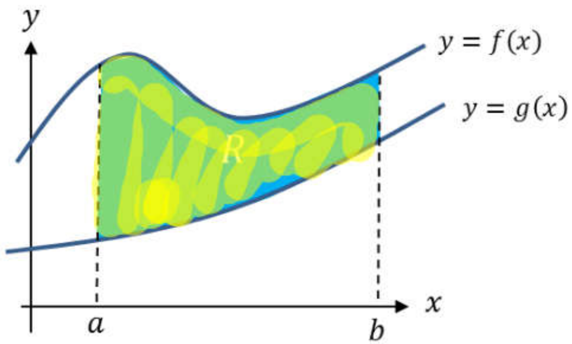
$$= \left[6(4+3x)^{1/2}\right]_0^4$$

$$= 6 \times 16^{1/2} - 6 \times 4^{1/2}$$

$$= 6 \times 4 - 6 \times 2$$

$$= \underline{\underline{12}}$$

Skill #9: Area between two curves

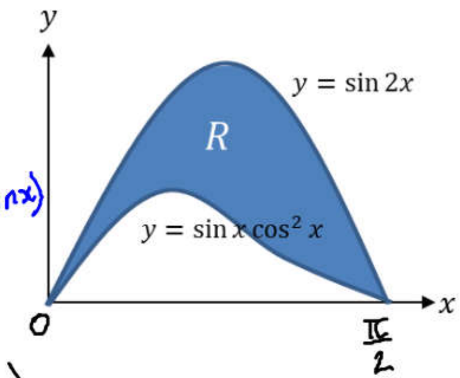


The areas under the two curves are $\int_a^b f(x)\,dx$ and $\int_a^b g(x)\,dx$. It therefore follows the area between them (provided the curves don't overlap) is:

$$R = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$
$$= \int_a^b \big(f(x) - g(x)\big)\,dx$$

Ensure you have top curve minus bottom curve.

The diagram shows part of the curves $y = \sin 2x$ and $y = \sin x \cos^2 x$ where $0 \le x \le \frac{\pi}{2}$. The region R is bounded by the two curves. Use integration to find the area of R.



$$\int_0^{\frac{\pi}{2}} \left(\sin 2x - \sin x \cos^2 x\right) dx$$

consider $\cos^3 x$
diff. $3\cos^2 x(-\sin x)$
scale $\frac{1}{3}$

$$= \left[-\frac{1}{2}\cos 2x + \frac{1}{3}\cos^3 x\right]_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{2}\cos \pi + \frac{1}{3}\cos^3\left(\frac{\pi}{2}\right) - \left(-\frac{1}{2}\cos 0 + \frac{1}{3}\cos^3 0\right)$$

$$= \frac{1}{2} + 0 + \frac{1}{2} - \frac{1}{3} = \underline{\underline{\frac{2}{3}}}$$

Reverse Chain Rule.
Trig identities
Partial Fractions
etc.

Q5–9 Ex11H.