QUESTION 5

The graph of $y = x^3$ is translated by vector $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ find the equation of the resulting graph in the form $y = x^3 + ax^2 + bx + c$

WEEK 1

$y = x^3$ $\quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ 1 left. 2 up.

$y = f(x)$

$f(x) = x^3$

$f(x+1) + 2 = (x+1)^3 + 2$

$f(x) + 2$ 2 up.

$f(x+1)$ 1 left.

$f(x+1) + 2$ $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$y = (x+1)^3 + 2$$
$$= x^3 + 3x^2 + 3x + 1 + 2$$
$$= x^3 + 3x^2 + 3x + 3$$

2 A body starts at rest and moves in a straight line. At time t seconds the displacement of the body from its starting point, s m, is given by:

$$s = 4t^3 - t^4, \quad 0 \leqslant t \leqslant 4.$$

a Show that the body returns to its starting position at $t = 4$.

b Explain why s is always non-negative.

c Find the maximum displacement of the body from its starting point.

Hint Write $s = t^3(4 - t)$ and consider the sign of each factor in the range $0 \leqslant t \leqslant 4$.

a) $t=4 \quad s = 4 \times 4^3 - 4^4$

$= 0$

b) $s = 4t^3 - t^4 \quad 0 \leq t \leq 4$

$s = t^3(4-t)$

Because $0 \leq t \leq 4$, t^3 is positive.

$4-t \geq 0$

$t^3(4-t)$ is a positive $\times$ positive which is positive, so s is always non-negative.

$\cancel{s=(t-1)^2 t^2}$

c) $\frac{ds}{dt} = 0$

$\frac{ds}{dt} = 12t^2 - 4t^3$

$0 = 4t^2(3-t)$

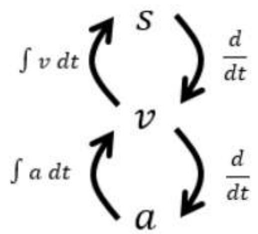
$t=0, \; t=3$

$s=0 \qquad s = \underline{27\,m}$

Using Integration

Differentiating (with respect to time) gets us from displacement to velocity, and from velocity to acceleration.
So naturally, integrating (with respect to time) gets us from acceleration to velocity, and from velocity to displacement.
Also note that the area under a speed-time graph is distance (i.e. integrating velocity gives distance!)
As mentioned earlier, it's helpful to picture the graph on the right, where we move down to differentiate and up to integrate.

Memory Tip: I picture interchanging between s, v, a as differentiating to go downwards and integrating to go upwards:



$t=0, x=5$

A particle is moving on the x-axis. At time $t = 0$, the particle is at the point where $x = 5$.
The velocity of the particle at time t seconds (where $t \geq 0$) is $(6t - t^2)$ ms^{-1}. Find:
(a) An expression for the displacement of the particle from O at time t seconds.
(b) The distance of the particle from its starting point when $t = 6$.

a) $v = 6t - t^2$

$x = \int (6t - t^2)\,dt$

$x = 3t^2 - \frac{1}{3}t^3 + c$

$5 = c$

$x = 3t^2 - \frac{1}{3}t^3 + 5$

b) $t = 6$ $\quad x = 3 \times 6^2 - \frac{1}{3} \times 6^3 + 5$

$x = 41\text{m}$

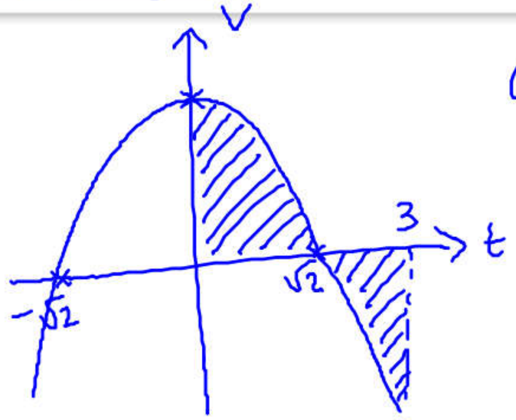
$\text{Dist} = 41 - 5 = \underline{\underline{36\text{m}}}$

Careful with 'negative' areas

A particle travels in a straight line. After t seconds its velocity, v ms^{-1}, is given by $v = 4 - 2t^2$, $t \geq 0$. Find the distance travelled by the particle in the first three seconds of its motion.

area under curve.

$v = 4 - 2t^2$
$0 = 4 - 2t^2$
$2t^2 = 4$
$t^2 = 2$
$t = \pm\sqrt{2}$



$$\int_0^{\sqrt{2}} (4 - 2t^2)\, dt = \left[4t - \frac{2}{3}t^3\right]_0^{\sqrt{2}}$$
$$= 4\sqrt{2} - \frac{2}{3} \times (\sqrt{2})^3$$
$$= 4\sqrt{2} - \frac{4}{3}\sqrt{2}$$
$$= \frac{8}{3}\sqrt{2}$$

$$\int_{\sqrt{2}}^{3} (4 - 2t^2) = \left[4t - \frac{2}{3}t^3\right]_{\sqrt{2}}^{3}$$
$$= \left(4 \times 3 - \frac{2}{3} \times 3^3\right) - \left(\frac{8}{3}\sqrt{2}\right)$$
$$= -6 - \frac{8}{3}\sqrt{2}$$

$$\text{Total dist} = \frac{8}{3}\sqrt{2} + 6 + \frac{8}{3}\sqrt{2}$$
$$= \frac{16}{3}\sqrt{2} + 6$$
$$= \underline{13.5\text{m}} \ (3\text{sf})$$

Edexcel M2 June 2015 Q6

A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)$ m s^{-1}. When $t = 0$, P is 15 m from the origin O.

Find

(*a*) the values of t when P is instantaneously at rest, **(3)**

(*b*) the acceleration of P when $t = 5$, **(3)**

(*c*) the total distance travelled by P in the interval $0 \le t \le 5$. **(5)**

a)

$V = 2t^2 - 9t + 4$

$0 = 2t^2 - 9t + 4$

$t = 0.5, \; t = 4$

b) $a = 4t - 9$

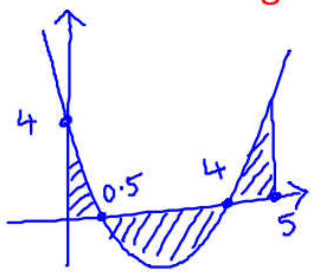
$t = 5$

$a = 4 \times 5 - 9$

$= \underline{\underline{11 \text{ ms}^{-2}}}$

Always be careful for negative areas

c)



Ex 11D

$\int_0^{0.5} (2t^2 - 9t + 4)\, dt = \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_0^{0.5}$

$= \frac{23}{24}$

$\int_{0.5}^{4} v \, dt = \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_{0.5}^{4}$

$= -\frac{40}{3} - \frac{23}{24} = -\frac{343}{24}$

$\int_4^5 v \, dt = \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_4^5$

$= -\frac{55}{6} - \left(-\frac{40}{3}\right)$

$= \frac{25}{6}$

So distance $= \frac{23}{24} + \frac{343}{24} + \frac{25}{6} = \frac{233}{12}$

$= 19.4\text{m}$ (3sf)

Ex 11D

Q7 →