$$\int \cos x \sin x\,(1+\sin x)^3\ dx$$

STEP 1: Using substitution, work out x and dx **(or variant)**

$u = 1 + \sin x$

$u - 1 = \sin x$

$u = 1 + \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{\cos x} = dx$

$\frac{1}{\cos x}\,du = dx$

$$\int \cos x \sin x(1+\sin x)^3\,dx = \int \cancel{\cos x}\,(u-1)\,u^3 \times \frac{1}{\cancel{\cos x}}\,du$$

STEP 2: Substitute these into expression.

$$= \int (u-1)u^3\,du$$

$$= \int (u^4 - u^3)\,du$$

STEP 3: Integrate simplified expression.

$$= \frac{1}{5}u^5 - \frac{1}{4}u^4 + C$$

STEP 4: Write answer in terms of x.

$$= \frac{1}{5}(1+\sin x)^5 - \frac{1}{4}(1+\sin x)^4 + C$$

Definite Integration with Substitution

Calculate $\int_0^{\frac{\pi}{2}} \cos x\sqrt{1+\sin x}\ dx$

Note: $\int_a^b f(x)\,dx = \int_b^a -f(x)\,dx$

Limits match the "dx" part at the end

$u = 1+\sin x$

$\frac{du}{dx} = \cos x$

$\frac{1}{\cos x}\,du = dx$

x	u
$\frac{\pi}{2}$	2
0	1

Change limits

$u = 1+\sin\frac{\pi}{2}$
$= 2$

$u = 1+\sin 0$
$= 1$

$$\int_0^{\frac{\pi}{2}} \cos x\sqrt{1+\sin x}\ dx$$

$$= \int_1^2 \cancel{\cos x}\, u^{1/2}\, \frac{1}{\cancel{\cos x}}\, du$$

$$= \int_1^2 u^{1/2}\, du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_1^2$$

$$= \frac{2}{3}\times 2^{3/2} - \frac{2}{3}\times 1^{3/2}$$

$$= \frac{2}{3}\times 2\sqrt{2} - \frac{2}{3}$$

$$= \frac{2}{3}\left(2\sqrt{2}-1\right)$$

1 Use the substitutions given to find:

~~**a** $\int x\sqrt{1+x}\,\mathrm{d}x$; $u = 1 + x$~~

~~**b** $\int \frac{1+\sin x}{\cos x}\,\mathrm{d}x$; $u = \sin x$~~

c $\int \sin^3 x\,\mathrm{d}x$; $u = \cos x$

~~**d** $\int \frac{2}{\sqrt{x}(x-4)}\,\mathrm{d}x$; $u = \sqrt{x}$~~

e $\int \sec^2 x \tan x\sqrt{1+\tan x}\,\mathrm{d}x$; $u^2 = 1 + \tan x$

f $\int \sec^4 x\,\mathrm{d}x$; $u = \tan x$

2 Use the substitutions given to find the exact values of:

a $\int_0^5 x\sqrt{x+4}\,\mathrm{d}x$; $u = x + 4$

b $\int_0^2 x(2+x)^3\,\mathrm{d}x$; $u = 2 + x$

c $\int_0^{\frac{\pi}{2}} \sin x\sqrt{3\cos x + 1}\,\mathrm{d}x$; $u = \cos x$

Hint First apply a trigonometric identity.

d $\int_0^{\frac{\pi}{3}} \sec x \tan x\sqrt{\sec x + 2}\,\mathrm{d}x$; $u = \sec x$

~~**e** $\int_1^4 \frac{1}{\sqrt{x}(4x-1)}\,\mathrm{d}x$; $u = \sqrt{x}$~~

could you do these without subst?

5 Using the substitution $u^2 = 4x + 1$, or otherwise, find the exact value of $\int_6^{20} \frac{8x}{\sqrt{4x+1}}\,\mathrm{d}x$

6 Use the substitution $u^2 = \mathrm{e}^x - 2$ to show that $\int_{\ln 3}^{\ln 4} \frac{\mathrm{e}^{4x}}{\mathrm{e}^x - 2}\,\mathrm{d}x = \frac{a}{b} + c\ln d$, where a, b, c and d are integers to be found.

8 Use the substitution $u = \cos x$ to show

$$\int_0^{\frac{\pi}{3}} \sin^3 x\cos^2 x\,\mathrm{d}x = \frac{47}{480}$$

(7 marks)

6 Use the substitution $u^2 = e^x - 2$ to show that $\int_{\ln 3}^{\ln 4} \frac{e^{4x}}{e^x - 2}\,dx = \frac{a}{b} + c\ln d$, where a, b, c and d are integers to be found.

$$\int_{\ln 3}^{\ln 4} \frac{e^{4x}}{e^x - 2}\,dx$$

$$u^2 = e^x - 2$$
$$u^2 + 2 = e^x$$
$$(u^2+2)^4 = e^{4x}$$

$$u^2 = e^x - 2$$
$$2u\frac{du}{dx} = e^x$$
$$\frac{2u}{e^x}\,du = dx$$

x	u
$\ln 3$	1
$\ln 4$	$\sqrt{2}$

$$u^2 = e^{\ln 3} - 2$$
$$u^2 = 3 - 2$$
$$u = 1$$
$$u^2 = e^{\ln 4} - 2$$
$$u^2 = 4 - 2$$
$$u = \sqrt{2}$$

Cubic binomial

1 3 3 1

Side Note

$$(a+b)^3$$
$$a^3 + 3a^2b + 3ab^2 + b^3$$

$$\int_{\ln 3}^{\ln 4} \frac{e^{4x}}{e^x - 2}\,dx = \int_1^{\sqrt{2}} \frac{(u^2+2)^4}{u^2}\,\frac{2u}{e^x}\,du$$

$$= \int_1^{\sqrt{2}} \frac{(u^2+2)^{\cancel{4}\,3}}{\cancel{u^2}\,u}\,\frac{2\cancel{u}}{\cancel{(u^2+2)}}\,du$$

$$= 2\int_1^{\sqrt{2}} \frac{(u^2+2)^3}{u}\,du$$

$$= 2\int_1^{\sqrt{2}} \frac{(u^2)^3 + 3(u^2)^2 2 + 3(u^2)2^2 + 2^3}{u}\,du$$

$$= 2\int_1^{\sqrt{2}} \frac{u^6 + 6u^4 + 12u^2 + 8}{u}\,du$$

$$= 2\int_1^{\sqrt{2}} \left(u^5 + 6u^3 + 12u + \frac{8}{u}\right)du$$

$$= 2\left[\frac{1}{6}u^6 + \frac{3}{2}u^4 + 6u^2 + 8\ln u\right]_1^{\sqrt{2}}$$

$$= 2\left[\left(\frac{1}{6}\times 8 + \frac{3}{2}\times 4 + 6\times 2 + 8\ln\sqrt{2}\right) - \left(\frac{1}{6} + \frac{3}{2} + 6 + 8\ln 1\right)\right]$$

$$0$$

$$= 2\left(\frac{58}{3} + 8\ln\sqrt{2} - \frac{23}{3}\right)$$

$$= 2\left(\frac{35}{3} + 8\ln\sqrt{2}\right)$$

$$= \frac{70}{3} + 16\ln\sqrt{2}$$

$$= \frac{70}{3} + 16\ln 2^{1/2}$$

$$= \frac{70}{3} + 8\ln 2$$

$a = 70$ $c = 8$
$b = 3$ $d = 2$

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(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant. **(5)**

Hint: You might want to use your double angle formula first.

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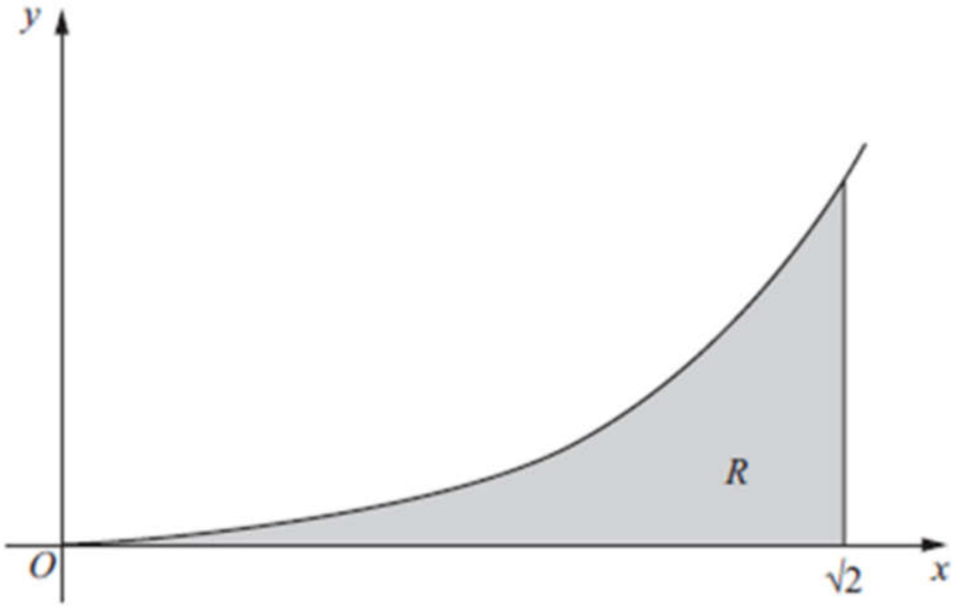

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln (x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

(*c*) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_{2}^{4} (u - 2)\ln u \ \mathrm{d}u.$$