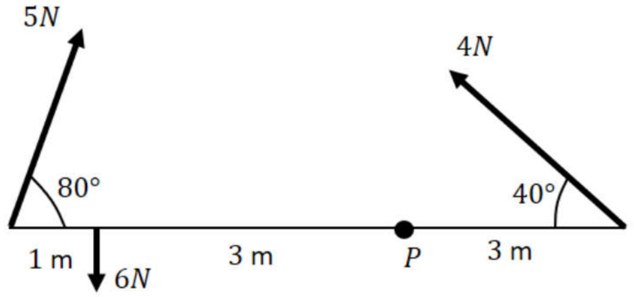
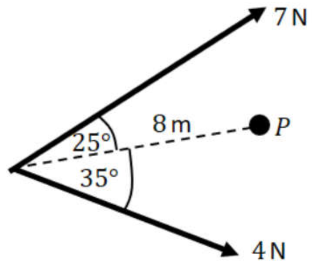
Rigid Bodies - Part 2: pegs, ladders, hinges

Calculating with Angled Forces

The diagram shows a set of forces acting on a light rod. Calculate the resultant moment about the point P.

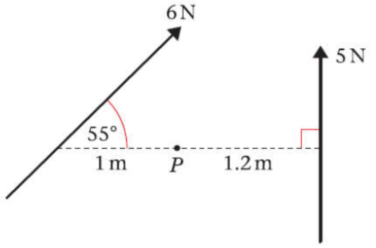


The diagram shows two forces acting on a lamina.
Calculate the resultant moment about the point P.



Find the sum of the moments about P of the forces shown in the following questions.

1



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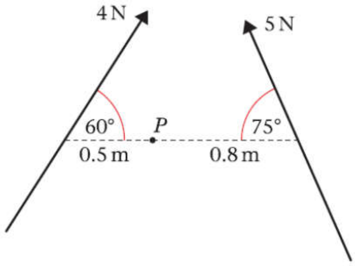

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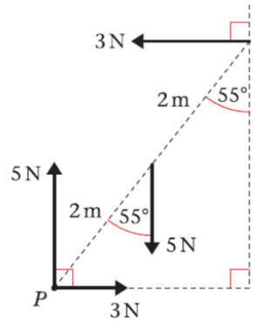

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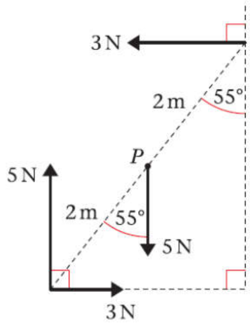

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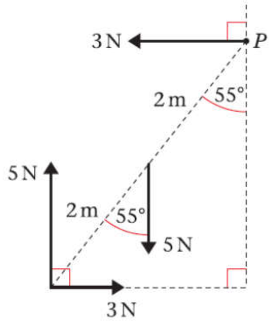

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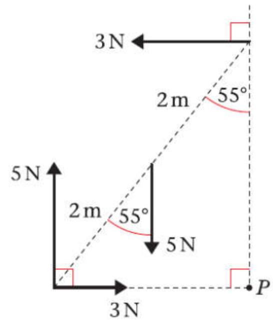


(Sum of moment means resultant moment)

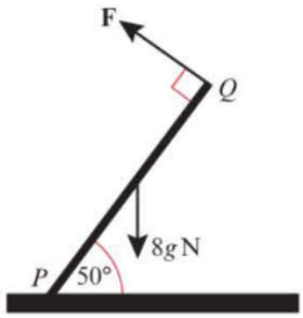
Exercise 5A

1	1.09 Nm anticlockwise	**2**	2.13 Nm anticlockwise
3	1.31 Nm clockwise	**4**	1.31 Nm clockwise
5	1.31 Nm anticlockwise	**6**	1.31 Nm clockwise

Angled Rods/Beams

Example 8

A uniform rod PQ is hinged at the point P, and is held in equilibrium at an angle of 50° to the horizontal by a force of magnitude **F** acting perpendicular to the rod at Q. Given that the rod has a length of 3 m and a mass of 8 kg, find the value of **F**.



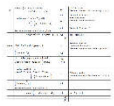
Static Rigid Bodies - resting on pegs/floor

A uniform rod AB of mass 40kg and length 10m rests with the end A on rough horizontal ground.
The rod rests against a smooth peg C where AC = 8m.
The rod is in limiting equilibrium at an angle of 15° to the horizontal.

Find:
a) the magnitude of the reaction at C
b) the coefficient of friction between the rod and the ground

Hints:
- Resolve vertically
- Resolve horizontally
- Take moments about A (usually the floor)
- Remember to find perpendicular distances!

A uniform rod AB, of mass 5 kg and length 8 m, has its end B resting on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a rope attached to the rod at C. The distance $AC = 1$ m. The rope is in the same vertical plane as the rod. The angle between the rope and the rod is β and the tension in the rope is T newtons, as shown in Figure 3. The coefficient of friction between the rod and the ground is $\frac{2}{3}$. The vertical component of the force exerted on the rod at B by the ground is R newtons.



(a) Find the value of R.

(6)

(b) Find the size of angle β.

(5)

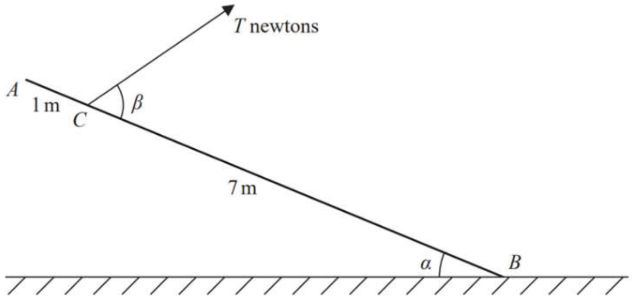
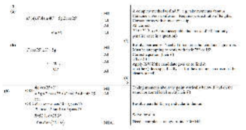


Figure 3

Your Turn



A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

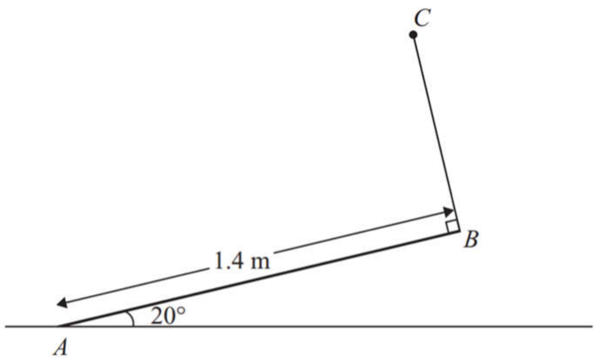


Figure 2

Your Turn

A uniform rod AB, of mass 20 kg and length 4 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a force acting at B, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at A.

(7)

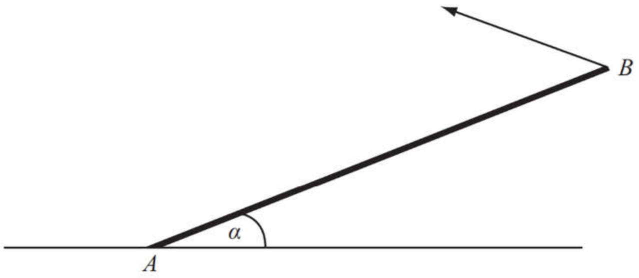


Figure 2

Your Turn

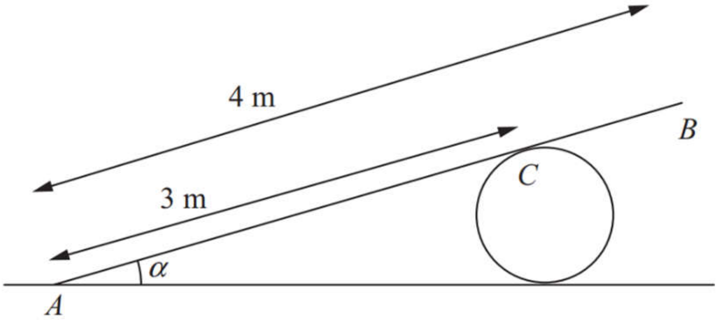
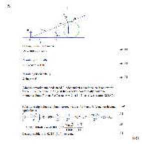


Figure 4



A uniform plank AB, of weight 100 N and length 4 m, rests in equilibrium with the end A on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is C, where $AC = 3$ m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin \alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ. Modelling the plank as a rod, find the least possible value of μ.

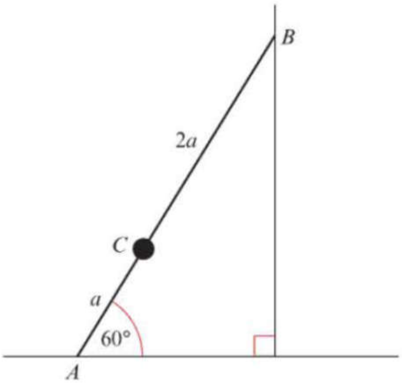
(10)

Static Rigid Bodies - ladders

Hints:

- Resolve vertically
- Resolve horizontally
- Take moments about A (usually the floor)
- Remember to find perpendicular distances!

A ladder AB, of mass m and length 3a, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass 2m is fixed on the ladder at the point C, where AC = a. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium at an angle of 60° with the ground.



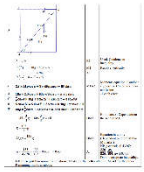
a) Find the coefficient of friction between the ladder and the ground.

b) State how you have used the assumption that the ladder is uniform in your calculations.

A ladder AB, of weight W and length $2l$, has one end A resting on rough horizontal ground. The other end B rests against a rough vertical wall. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ. Friction is limiting at both A and B. The ladder is at an angle θ to the ground, where $\tan\theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

Find the value of μ.

(9)



Your Turn

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point C, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

(9)

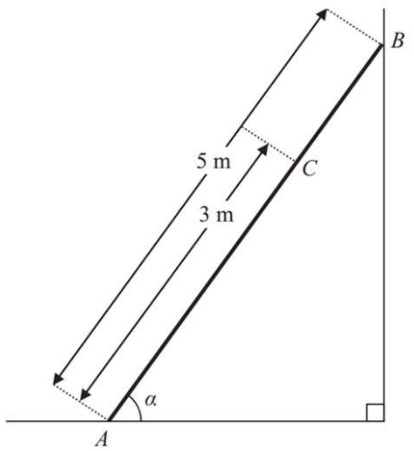
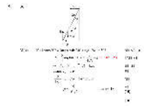


Figure 1

Your Turn



A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

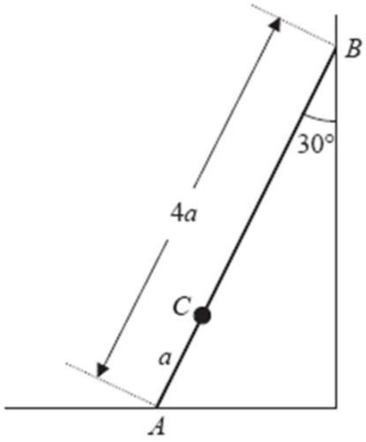


Figure 2

Your Turn

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

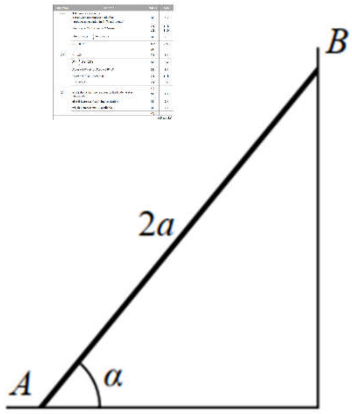


Figure 1

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

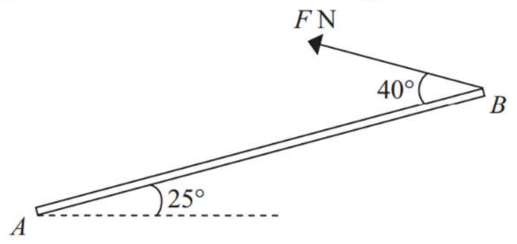
Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

Static Rigid Bodies - hinges

The reaction at the hinge will have a horizontal (X) and vertical (Y) component.



Hints:
- Resolve vertically
- Resolve horizontally
- Take moments about A (usually the floor)
- Remember to find perpendicular distances!

Figure 1

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(a) Find the value of F.

(4)

(b) Find the magnitude and direction of the vertical component of the force acting on the rod at A.

(4)

A beam AB, of mass 20 kg and length 3 m, is smoothly hinged to a vertical wall at one end A.

The beam is held in equilibrium in a horizontal position by a rope of length 1 m. One end of the rope is fixed to a point C on the wall which is vertically above A. The other end of the rope is fixed to the point D on the beam so that angle ACD is 30°, as shown in Figure 2. The beam is modelled as a uniform rod and the rope is modelled as a light inextensible string.

Using the model, find

(a) the tension in the rope,

(4)

(b) the direction of the force exerted by the wall on the beam at A.

(6)

(c) If the rope were not modelled as being light, state how this would affect the tension in the rope, explaining your answer carefully.

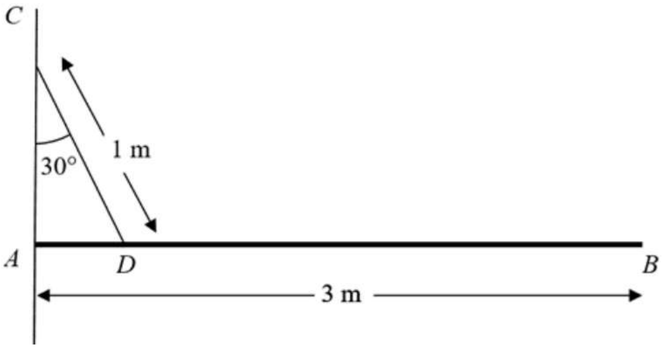
(2)

The rope is now removed and replaced by a longer rope which is still attached to the wall at C but is now attached to the beam at G, where G is the midpoint of AB. The beam AB remains in equilibrium in a horizontal position.

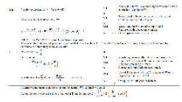
(d) Show that the force exerted by the wall on the beam at A now acts horizontally.

(2)

(Total 14 marks)



Your Turn



A uniform rod AB of weight W has its end A freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle θ to the horizontal, by a force of magnitude P. The force acts perpendicular to the rod at B and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at A is Y.

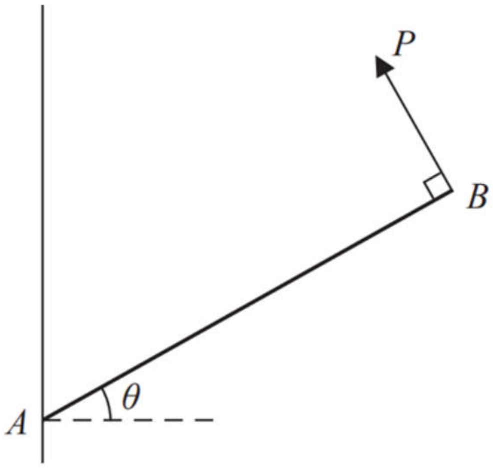
(a) Show that $Y = \frac{W}{2}(2 - \cos^2\theta)$.

(6)

Given that $\theta = 45°$

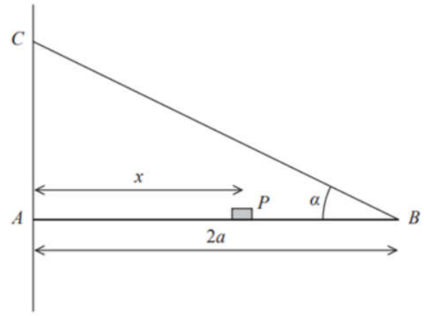
(b) find the magnitude of the force exerted on the rod by the wall at A, giving your answer in terms of W.

(6)



Your Turn

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.



A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

Your Turn

Even though not hinged, this will behave in the same way – there will be a reaction force and frictional force instead.

A uniform rod AB, of mass $3m$ and length $4a$, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A, where $AD = 3a$. A particle of mass $3m$ is attached to the rod at C, where $AC = x$. The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is $\dfrac{25}{4}mg$.

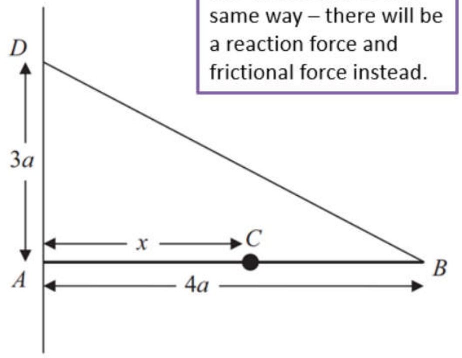


Figure 3

Show that

(a) $x = 3a$,

(5)

(b) the horizontal component of the force exerted by the wall on the rod has magnitude $5mg$.

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is about to slip,

(c) find the value of μ.

(5)

Extra Questions:

Ex 4C
Hinges/ladders: Q10, 11

Ex 7D
Pegs/floor: Q1, 5, 7, 10
Ladders: Q2, 3, 4, 6, 8, 9, 11
Hinges: none in the textbook...

Mixed Exercise 7 has many more ladder questions