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A Level · Edexcel · Maths

🕐 4 hours   ❓ 44 questions

5.7 Further Trigonometric Equations (A Level only)

Easy (11 questions)	/42
Medium (11 questions)	/56
Hard (11 questions)	/56
Very Hard (11 questions)	/63
Total Marks	**/217**

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Easy Questions

1 Solve the equation $\sec \theta = 1$ for $0° \le \theta \le 360°$.

(3 marks)

2 Given that

$$\tan(A° - 30°) = \frac{\sqrt{3}}{3}$$
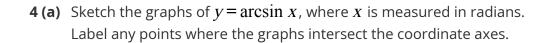
find the values of A such that $-180° \le A° \le 180°$.

(3 marks)

3 Solve the equation

$$\frac{1}{\sec x} = \frac{\sqrt{2}}{2}, \qquad -\pi \le x \le \pi$$

(3 marks)

4 (a) Sketch the graphs of $y = \arcsin x$, where x is measured in radians.
Label any points where the graphs intersect the coordinate axes.

(3 marks)

(b) Find the only solution to $\arcsin x = \dfrac{\pi}{4}$.

(2 marks)

5 Use the identity

$$R \cos(A - B) \equiv R \cos A \cos B + R \sin A \sin B$$

to show that

$$8 \cos \theta + 6 \sin \theta$$

can be written as

$$10\cos(\theta - \alpha) \qquad \text{where } \alpha = 0.644 \text{ to three significant figures.}$$

(4 marks)

6 (a) Show that the equation $\csc^2 x = 2\csc x - 1$ can be written as

$$(\csc x - 1)^2 = 0$$

(b) Hence, or otherwise, solve the equation

$$\csc^2 x = 2\csc x - 1, \qquad -2\pi \leq x \leq 2\pi$$

7 Solve the equation

$$\cos 2\theta = \frac{1}{2}, \qquad -\pi \leq \theta \leq \pi$$

State your answers as multiples of π.

8 (a) Write down the domain and range for the function

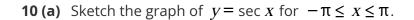$$f(x) = \arccos x$$

(2 marks)

(b) Solve the equation

$$f(x) = \frac{\pi}{6}$$

(2 marks)

9 (a) Use a small angle approximation to estimate the solution to the equation

$$4 \cot \theta - 2 = 3$$

(b) Solve the equation $4 \cot \theta - 2 = 3,$ $\qquad 0 < \theta < \frac{\pi}{2}$.

Give your answer to three significant figures.

10 (a) Sketch the graph of $y = \sec x$ for $-\pi \leq x \leq \pi$.

(3 marks)

(b) (i) Add a line to your graph demonstrate how the equation

$$\sec x = k \qquad -\pi \leq x \leq \pi$$

where k is a constant could have no real solutions.

(ii) For which values of k does this equation have no real solutions?

(2 marks)

11 Solve the equation

$$\cot^2 \theta - \cos \theta \, \text{cosec}^2 \, \theta = 0, \qquad\qquad 0 < \theta < 2\pi$$

(3 marks)

Medium Questions

1 Solve the equation cosec $2\theta = 2$ for $0° \leq \theta \leq 180°$.

(3 marks)

2 Given that

$$\tan(3A° - 30°) = -\frac{\sqrt{3}}{3}$$

find the values of A such that $-120° \leq A° \leq 120°$.

(3 marks)

3 Solve the equation

$$\frac{\sin x}{\sec x} = \frac{1}{4}, \qquad -\pi \leq x \leq \pi$$

(4 marks)

4 (a) Use the identity

$$R\sin(A + B) \equiv R \cos B \sin A + R\sin B \cos A$$

to show that

$$3 \sin \theta + 4 \cos \theta$$

can be written as

$$5\sin (\theta + \alpha), \qquad \text{where } \alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

(4 marks)

(b) Hence, or otherwise, solve the equation $3 \sin \theta + 4 \cos \theta = 1$ for $0 \leq \theta \leq \pi$.
Give your answers to three significant figures.

(3 marks)

(c) Write down the maximum value of $3 \sin \theta + 4 \cos \theta$ and state the first positive value of θ for which it occurs. Give your value of θ to three significant figures.

(2 marks)

5 (a) On the same axes, sketch the graphs of $y = \arcsin x$ and $y = \arccos x$, where x is measured in radians.
Label any points where the graphs intersect the coordinate axes and each other.

(4 marks)

(b) Hence, or otherwise find the only solution to $\cos(\arcsin x) = x$.

(2 marks)

6 (a) Show that the equation $3 \tan^2 x = 18 - 2 \sec x$ can be written as

$$3 \sec^2 x + 2 \sec x - 21 = 0$$

(2 marks)

(b) Hence, or otherwise, solve the equation

$$3 \tan^2 x = 18 - 2 \sec x, \qquad -\pi \leq x \leq \pi$$

Give your answers to three significant figures.

(4 marks)

7 Solve the equation

$$\cos 2\theta = \cos \theta - 1 \qquad -\pi \leq \theta \leq \pi$$

State your answers as multiples of π.

(5 marks)

8 (a) Write down the domain and range for the function

$$f(x) = \arcsin x$$

(2 marks)

(b) Solve the equation

$$[f(x)]^2 = \frac{\pi^2}{16}$$

(3 marks)

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9 (a) Use a small angle approximation to estimate the solution to the equation

$$\operatorname{cosec} \theta = 10$$

(2 marks)

(b) Solve the equation $\operatorname{cosec} \theta = 10$ $\quad 0 < \theta < \dfrac{\pi}{2}$.

Give your answer to six decimal places.

(2 marks)

(c) Find the percentage error, to two significant figures, in the approximation from part (a) compared to your answer in part (b).

(2 marks)

10 (a) Sketch the graph of $y = \cot^2 \theta$ for $-2\pi \leq \theta \leq 2\pi$.

(3 marks)

(b) By adding three lines to your graph demonstrate how the equation

$$\cot^2 \theta = k \qquad -2\pi \leq \theta \leq 2\pi$$

where k is a constant has either 0, 4 or 8 real solutions.

(2 marks)

11 Solve the equation

$$\cot^2 \theta = \sec^2 \theta - 1, \qquad 0° \leq \theta \leq 360°$$

(4 marks)

Hard Questions

1 Solve the equation $\sec^2 2x = 1 + \tan 2x$ for $0° \leq x \leq 180°$.

(4 marks)

2 Given that

$$\sin(2A° - B°) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

and that

$$3A = 4B \text{ and } 60° < B° < A° < 300°$$

find the values of A and B.

(4 marks)

3 Solve the equation

$$\frac{\cos x}{\csc x} - \cot x = 0, \qquad -2\pi \leq x \leq 2\pi$$

(4 marks)

4 (a) Find the value of …

(i) $\quad \arcsin\left(\sin\dfrac{7\pi}{6}\right),$

(ii) $\quad \arccos\left(\cos\left(-\dfrac{2\pi}{3}\right)\right)$

(3 marks)

(b) Briefly explain why $\arctan(\tan \pi) \neq \pi$.

(2 marks)

5 (a) Show that $6 \cos \theta - 8 \sin \theta$ can be written in the form $R \cos(\theta + \alpha)$, where $R > 0$ and α is an acute angle measured in radians.

(3 marks)

(b) Hence, or otherwise, solve the equation $3 \cos \theta - 4 \sin \theta - 2 = 0$, for $0 \leq x \leq 2\pi$
Give your answers to three significant figures.

(3 marks)

(c) Write down the minimum value of $6 \cos \theta - 8 \sin \theta$ and the smallest positive value of θ for which it occurs. Give your value of θ to three significant figures.

(2 marks)

6 Solve the equation

$$2 \cot^2 x = 8 - \operatorname{cosec} x, \qquad -\pi \leq x \leq \pi$$

giving your answers to three significant figures where appropriate.

7 Solve the equation

$$8 \cos^4 \theta - 5 \cos 2\theta - 2 = 0 \qquad 0 \leq \theta \leq \pi$$

State your answers as multiples of π.

(5 marks)

8 (a) Write down the domain and range for the function

$$f(x) = \cos(\arcsin x)$$

(2 marks)

(b) Solve the equation

$$2[f(x)]^2 - 3f(x) + 1 = 0$$

(4 marks)

9 (a) Use a small angle approximation to estimate the positive solution to the equation

$$\sec 2\theta = 1.05$$

Give your answer to six decimal places.

(3 marks)

(b) Solve the equation $\sec 2\theta = 1.05$ $\quad 0 < \theta < \dfrac{\pi}{2}$.

Give your answer to six decimal places.

(3 marks)

(c) Find the percentage error, to two significant figures, in the approximation from part (a) compared to your answer in part (b).

(2 marks)

10 Determine the values of the constant k for which the equation

$$\operatorname{cosec} \theta = k, \quad -\pi \le \theta \le 2\pi$$

has   (i) no real solutions,

      (ii) 1 real solution,

      (iii) 2 real solutions,

      (iv) 4 real solutions

(4 marks)

11 Solve the equation

$$\cot^2 \theta = 15 - 6\operatorname{cosec} \theta, \qquad -180° \leq \theta \leq 180°$$

Give your answers to one decimal place where appropriate.

(4 marks)

Very Hard Questions

1 Solve the equation $\sin^3 3\theta - \sin 3\theta \cos^2 3\theta = 0$ for $0° \leq \theta < 180°$.

(4 marks)

2 Given that

$$\cos(A° - B°) = -\frac{\sqrt{3}}{2} \text{ and } \tan\left(\frac{1}{2}A° - B°\right) = \sqrt{3}$$
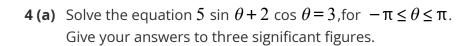
and that

$$0 \leq 2B° < A° \leq 360°$$

find the possible values of A and B.

(4 marks)

3 (a) Sketch the graph of $y = \pi + \arctan(2x - 1)$ for $x \in \mathbb{R}$.

Label the point of intercept with the y-axis and state the equations of any asymptotes.

(4 marks)

(b) Hence, or otherwise, write down the intercepts between the graph of

$y = \pi + \arctan(1 - 2x), x \in \mathbb{R}$ and

(i)   the y-axis,

(ii)   the graph of $y = \pi + \arctan(2x - 1), x \in \mathbb{R}$.

(2 marks)

4 (a) Solve the equation $5 \sin \theta + 2 \cos \theta = 3$, for $-\pi \leq \theta \leq \pi$.
Give your answers to three significant figures.

(4 marks)

(b) Write down the maximum value of $5 \sin \theta + 2 \cos \theta$ and the second positive value of θ for which it occurs. Give your value of θ to three significant figures.

(2 marks)

5 Solve the equation

$$3 \sec^4 \theta + 16 = 16 + 16 \tan^2 \theta, \qquad -\pi \leq \theta \leq \pi$$

giving your answers to three significant figures where appropriate.

(4 marks)

6 Solve the equation

$$\text{cosec}^2 x - 2 \frac{\text{cosec } x}{\sec x} = 9 \qquad 0 \leq x \leq 2\pi$$

Give your answers to three significant figures.

(5 marks)

7 (a) Write down the domain and range for the function

$$f(x) = \tan(\arccos x)$$

(2 marks)

(b) Solve the equation

$$[f(x)]^3 - f(x) = 0$$

(4 marks)

8 (a) Use a small angle approximation to estimate the positive solution to the equation

$$\cot 2\theta - 5\cos\theta = 0 \qquad 0 < \theta < 1$$

Give your answer to six decimal places.

(4 marks)

(b) Solve the equation $\cot 2\theta - 5\cos\theta = 0, \qquad 0 < \theta < 1$.
Give your answer to six decimal places.

(4 marks)

(c) Find the percentage error, to two significant figures, in the approximation from part (a) compared to your answer in part (b).

(2 marks)

9 Solve the equation

$$8\sin^4 2\theta = 2 - 5\cos 4\theta \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

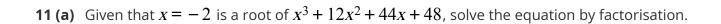State your answers as multiples of π.

(5 marks)

10 The number of real solutions to the equation

$$|\sec x - 2| = k, \qquad -2\pi \le x \le 2\pi$$

is determined by the value of the constant k.

Find the number of real solutions for all values of k, given that $k \in \mathbb{R}$.

(5 marks)

11 (a) Given that $x = -2$ is a root of $x^3 + 12x^2 + 44x + 48$, solve the equation by factorisation.

(4 marks)

(b) Solve the equation

$$\sec \theta(\sec^2 \theta + 44) + 12(\tan^2 \theta + 5) = 0, \qquad 0° \leq \theta \leq 180°$$

Give your answers to one decimal place where appropriate.

(4 marks)