A Level · Edexcel · Maths

3 hours 38 questions

2.8 Functions (A Level Only)

Easy (8 questions)	/39
Medium (10 questions)	/53
Hard (9 questions)	/50
Very Hard (11 questions)	/63
Total Marks	**/205**

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Easy Questions

1 State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto 4x - 2$

(ii) $\mathrm{f}: x \mapsto x^2$

(iii) $\mathrm{f}: x \mapsto \frac{x}{4}$

(iv) $\mathrm{f}: x \mapsto \sqrt{x}$

(4 marks)

2 State the largest possible domains for the following functions.

(i) $\mathrm{f}: x \mapsto \sqrt{x}$

(ii) $\mathrm{f}: x \mapsto \ln(x - 2)$

(iii) $\mathrm{f}: x \mapsto \arcsin x$

(3 marks)

3 State the range for the following functions based on the given domains.

(i) $f: x \mapsto e^x \qquad x \in \mathbb{R}$

(ii) $f: x \mapsto x^2 + 1 \qquad x \in \mathbb{R}$

(iii) $f: x \mapsto \frac{1}{x} \qquad x \in \mathbb{R}$

(3 marks)

4 (a) The function $f(x)$ is defined as

$$f(x) = x^2 - 8x - 20 \qquad x \in \mathbb{R}$$

Sketch the graph of $y = f(x)$, giving the coordinates of any points where the graph intersects the coordinate axes.

(3 marks)

(b) The minimum point on the graph of $y = f(x)$ has x-coordinate 4.

Find the range of $f(x)$.

(2 marks)

5 The function $f(x)$ is defined as

$$f(x) = x^2 - 9 \qquad x \geq 3$$

(i) Work out the range of $f(x)$.

(ii) If the domain of $f(x)$ is changed to $x \geq 0$, what would be the new range of $f(x)$?

(3 marks)

6 (a) The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = 3x + 5 \qquad x \in \mathbb{R}$$

$$g(x) = -2x \qquad x \in \mathbb{R}$$

Find

(i) $fg(x)$

(ii) $gf(x)$

(4 marks)

(b) Solve the equation $f(x) = g(x)$.

(2 marks)

7 (a) The function $f(x)$ is defined by

$$f(x) = 3x^2 + 1 \qquad x \in \mathbb{R}$$

Find the inverse of $f(x)$, $f^{-1}(x)$.

(3 marks)

(b) Find the domain and range for $f^{-1}(x)$.

(2 marks)

8 (a) Solve the equation $|6-2x| = 4$.

(3 marks)

(b) On the same diagram, sketch the graphs of $y = |6-2x|$ and $y = 4$.
Label the coordinates of the points where the two graphs intersect each other and the coordinate axes.

(4 marks)

(c) Consider the graphs of $y = |6-2x|$ and $y = k$, where k is a constant.
For which values of k...

(i) ... will the two graphs have no points of intersection?

(ii) ... will the two graphs have one point of intersection?

(iii) ... will the two graphs have two points of intersection?

(3 marks)

Medium Questions

1 State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto x^2$

(ii) $\mathrm{f}: x \mapsto 3x + 1$

(iii) $\mathrm{f}: x \mapsto (x + 1)^3$

(iv) $\mathrm{f}: x \mapsto \pm\sqrt{x}$

(4 marks)

2 (a) The function $f(x)$ is defined as

$$f(x) = x^2 + 2x - 3 \qquad x \in \mathbb{R}$$

Sketch the graph of $y = f(x)$, giving the coordinates of any points where the graph intercepts the coordinate axes and the coordinates of the turning point.

(3 marks)

(b) Write down the range of $f(x)$.

(1 mark)

3 (a) The function $f(x)$ is defined as

$$f(x) = x^2 - 4 \qquad x \geq 0$$

Work out the range of $f(x)$.

(2 marks)

(b) If the domain of $f(x)$ is changed to $x \leq 0$, what is the range of $f(x)$?

(1 mark)

4 (a) The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = x^2 \qquad x \in \mathbb{R}$$

$$\mathrm{g}(x) = 4x - 3 \qquad x \in \mathbb{R}$$

Write down the range of $\mathrm{f}(x)$.

(1 mark)

(b) Find

(i) $(f \circ g)(x)$

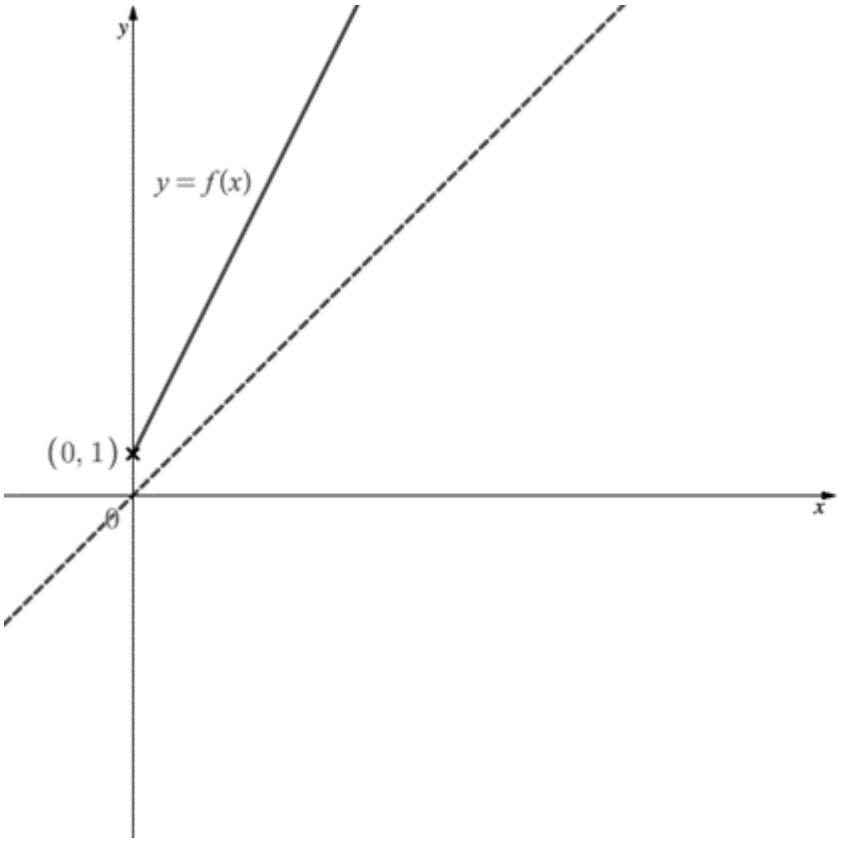
(ii) $(g \circ f)(x)$

(4 marks)

(c) Solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$.

(2 marks)

5 (a) The graph of $y = \mathrm{f}(x)$ is shown below.



(i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.

(ii) Given that the point (1, 1) lies on the dotted line, write down the equation of the line.

(3 marks)

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

(2 marks)

6 (a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |g(x)|$ where

$$\mathrm{f}(x) = (x + 2)^2 \qquad x \in \mathbb{R}$$

$$g(x) = 2x + 4 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

(3 marks)

(b) Solve the equation $\mathrm{f}(x) = |g(x)|$.

(2 marks)

7 (a) The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R}, x \neq 0$$

Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

(2 marks)

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

(2 marks)

(c) The domain of $f(x)$ is changed to $x > 0$.

Find an expression for $f^{-1}(x)$ and state its domain and range.

(4 marks)

8 Solve the equation $|4x + 2| = 5$.

(3 marks)

9 (a) The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

Find

(i) $\mathrm{f}g(x)$
(ii) $g\mathrm{f}(x)$

(3 marks)

(b) Write down $\mathrm{f}^{-1}(x)$ and state its domain and range.

(3 marks)

10 (a) The functions $f(x)$, $g(x)$ are defined as follows

$$f(x) = |x - 9| \qquad x \in \mathbb{R}$$

$$g(x) = x^2 \qquad x \in \mathbb{R}$$

Sketch the graph of $y = fg(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

(4 marks)

(b) How many solutions are there to the equation $fg(x) = 5$?

How many solutions are there to the equation $fg(x) = 9$?

(2 marks)

(c) Write down the solutions to the equation $fg(x) = 0$.

(2 marks)

Hard Questions

1 State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto 2 - x^3$

(ii) $\mathrm{f}: x \mapsto \sin x$

(iii) $\mathrm{f}: x \mapsto \dfrac{1}{x^2}$

(iv) $\mathrm{f}: x \mapsto \ln x$

(4 marks)

2 (a) It is given

$$f(x) = \frac{2}{x}$$

Write down the domain of the function $f(x)$

(1 mark)

(b) Sketch the graph of $y = f(x)$, stating the coordinates of any intersections with the coordinate axes and the equations of any asymptotes.

(3 marks)

(c) Write down the range of $f(x)$.

(1 mark)

3 (a) The function $f(x)$ is defined as

$$f(x) = x(x + 3)^2 + 1 \qquad x \geq 0$$

Work out the range of $f(x)$.

(1 mark)

(b) If the domain of $f(x)$ is changed to $x \leq 0$, what is the range of $f(x)$?

(2 marks)

4 (a) The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = 3x^2 + 2 \qquad x \in \mathbb{R}$$

$$g(x) = 1 - 3x \qquad x \in \mathbb{R}$$

Write down the range of $f(x)$.

(1 mark)

(b) Find

(i) $fg(x)$

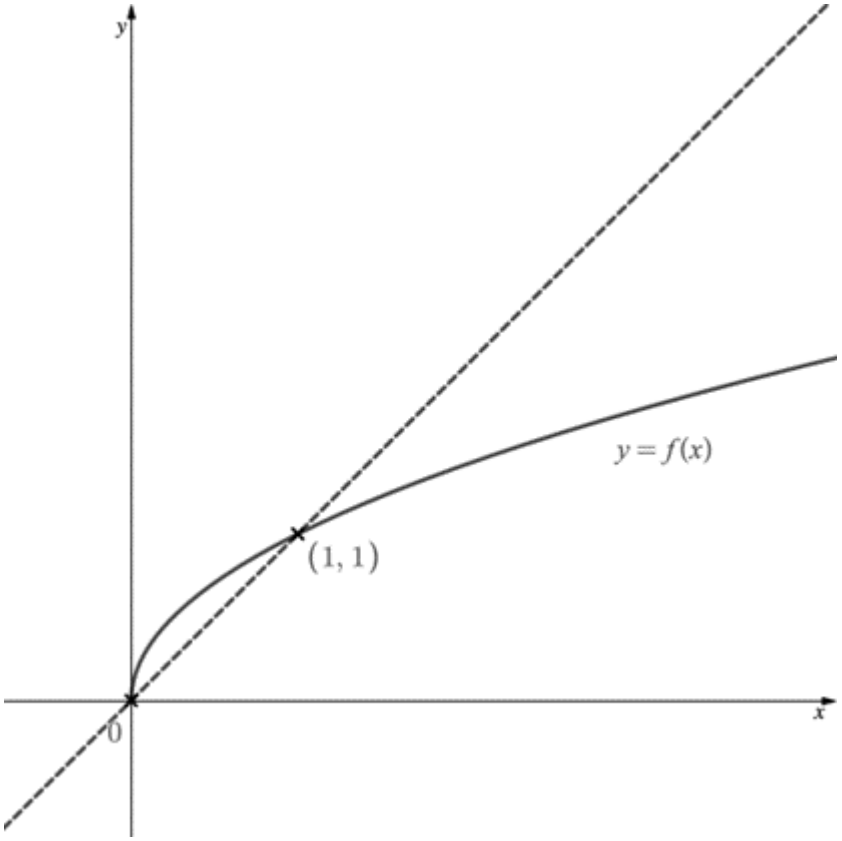
(ii) $gf(x)$

(4 marks)

(c) Solve the equation $f(x) = g(x) + 1$

(2 marks)

5 (a) The graph of $y = \mathrm{f}(x)$ is shown below.



(i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.

(ii) Write down the equation of the dotted line on the graph.

(3 marks)

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

(2 marks)

6 (a) On the same axes, sketch the graphs of $y = |f(x)|$ and $y = |g(x)|$ where

$$f(x) = 3x - 1 \qquad x \in \mathbb{R}$$

$$g(x) = 2x + 2 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

(3 marks)

(b) Solve the equation $|f(x)| = |g(x)|$.

(3 marks)

(c) Which of the solutions to $|f(x)| = |g(x)|$ is also a solution to $f(x) = g(x)$?

(1 mark)

7 (a) The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = e^{x-2}$ $\quad x \in \mathbb{R}$

$g(x) = 2 + \ln x$ $\quad x \in \mathbb{R}, x > 0$

Find

(i) $fg(x)$

(ii) $gf(x)$

(3 marks)

(b) Write down $f^{-1}(x)$ and state its domain and range.

(2 marks)

(c) The graphs of $f(x)$ and $f^{-1}(x)$ are drawn on the same axes.
Describe the transformation that would map one graph onto the other.

(2 marks)

8 (a) The functions $f(x)$, $g(x)$ are defined as follows

$$f(x) = |x-2| - 5 \qquad x \in \mathbb{R}$$

$$g(x) = |x| \qquad x \in \mathbb{R}$$

Sketch the graph of $y = gf(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

(4 marks)

(b) (i) How many solutions are there to the equation $gf(x) = 1$?

(ii) How many solutions are there to the equation $gf(x) = 10$?

(2 marks)

(c) Solve the equation $gf(x) = 2$.

(3 marks)

9 Solve the equation $|x^2 - 4| = 3$, giving your answers in exact form.

(3 marks)

Very Hard Questions

1 State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto \tan x$

(ii) $\mathrm{f}: x \mapsto \left|\frac{1}{x}\right|$

(iii) $\mathrm{f}: x \mapsto \sqrt{x^2}$

(iv) $\mathrm{f}: x \mapsto \pm\sqrt{25-x^2}$

(4 marks)

2 (a) It is given

$$f(x) = 4x^3 + 4x^2 - 7x + 2$$

Write down the domain and range of the function $f(x)$.

(2 marks)

(b) Sketch the graph of $y = f(x)$, stating the coordinates of any intersections with the coordinate axes. (You do not need to give the coordinates of any turning points.)

(3 marks)

3 (a) The function $f(x)$ is defined as

$$f(x) = (x - 3)^2(x - 4)^2 \qquad 2 \leq x \leq 5$$

Work out the range of $f(x)$.

(2 marks)

(b) If the domain of $f(x)$ is changed to $x \leq 2$, what is the range of $f(x)$?

(1 mark)

(c) State another domain for $f(x)$ that would have the same effect as that in part (b).

(1 mark)

4 (a) The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 - 2 \qquad x \in \mathbb{R}$$

$$g(x) = 1 - \frac{2}{x} \qquad x \in \mathbb{R}, x \neq 0$$

Write down the range of $f(x)$.

(1 mark)

(b) Leaving your answers as single fractions, find

(i) $fg(x)$

(ii) $gf(x)$

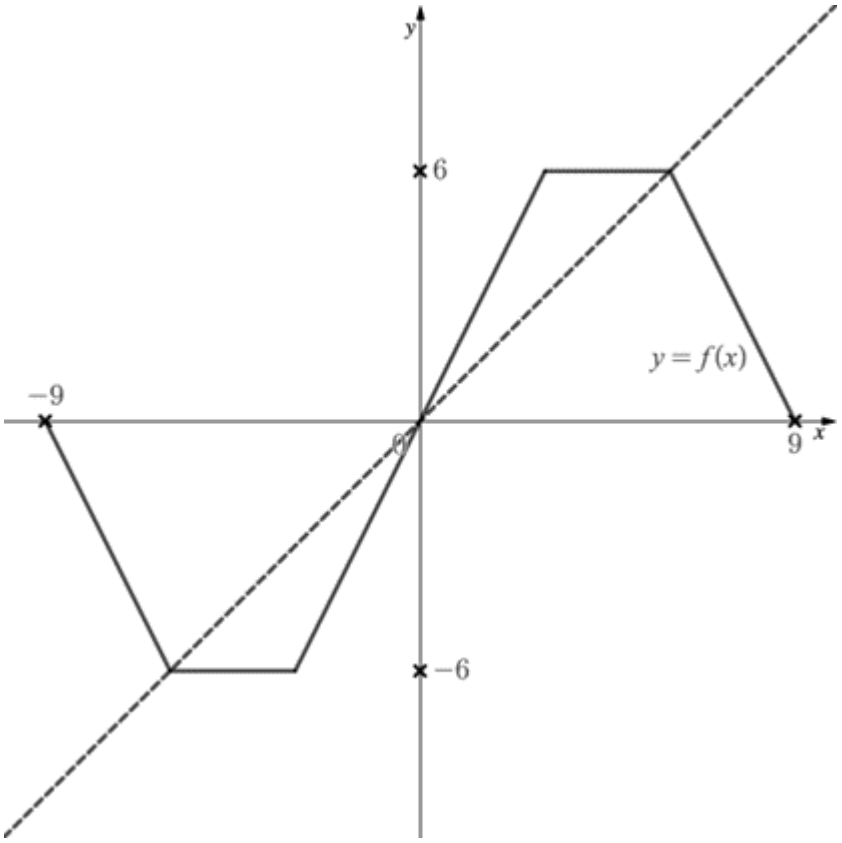
(3 marks)

(c) Solve the equation $f(x) = g(x)$

(2 marks)

5 (a) The graphs of $y = f(x)$ and $y = x$ (dotted line) are shown in the diagram below.

$f(x)$ has rotational symmetry about the origin and for $x > 0$,there is a vertical line line of symmetry at $x = 4.5$.



(i) Use the graph to write down the domain and range of $f(x)$.

(ii) On the diagram above sketch the reflection of $f(x)$ in the line $y = x$ and explain why this cannot be the graph of $f^{-1}(x)$.

(3 marks)

(b) (i) Given that the maximum solution to $f(x) = 6$. is $x = 6$ state the restriction on the domain of $f(x)$ such that $f^{-1}(x)$ exists.

(ii) Hence, or otherwise, write down the domain and range of $f^{-1}(x)$.

(2 marks)

6 (a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |g(x)|$ where

$$\mathrm{f}(x) = \sqrt{x} \qquad x \geq 0$$

$$g(x) = 2x - 3 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

(3 marks)

(b) Solve the equation $\mathrm{f}(x) = |g(x)|$.

(3 marks)

(c) Which of the solutions to $\mathrm{f}(x) = |g(x)|$ is ***not*** a solution to $\mathrm{f}(x) = \mathrm{g}(x)$?

(1 mark)

7 (a) The function $f(x)$ is defined as

$$f: x \mapsto \sqrt{(25 - x^2)} \qquad x \in \mathbb{R}, -5 \leq x \leq 5$$

Explain why the inverse of $f(x)$ does not exist.

(1 mark)

(b) Suggest an adaption to the domain of $f(x)$ so the following conditions are met:

- the inverse of $f(x)$ exists,
- the graph of $y = f(x)$ lies in the first quadrant only, and,
- the domain of $f(x)$ is as large as possible.

State the range for your adapted $f(x)$.

(2 marks)

(c) The domain of $f(x)$ is changed to $-5 \leq x \leq 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

(3 marks)

8 (a) The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = (x - 1)^2 - 4 \qquad x \in \mathbb{R}, x \geq 1$$

$$g(x) = 1 + \sqrt{(x+4)} \qquad x \in \mathbb{R}, x \geq -4$$

Find

(i) $\mathrm{fg}(x)$

(ii) $\mathrm{gf}(x)$

(3 marks)

(b) Write down $\mathrm{f}^{-1}(x)$ and state its domain and range.

(2 marks)

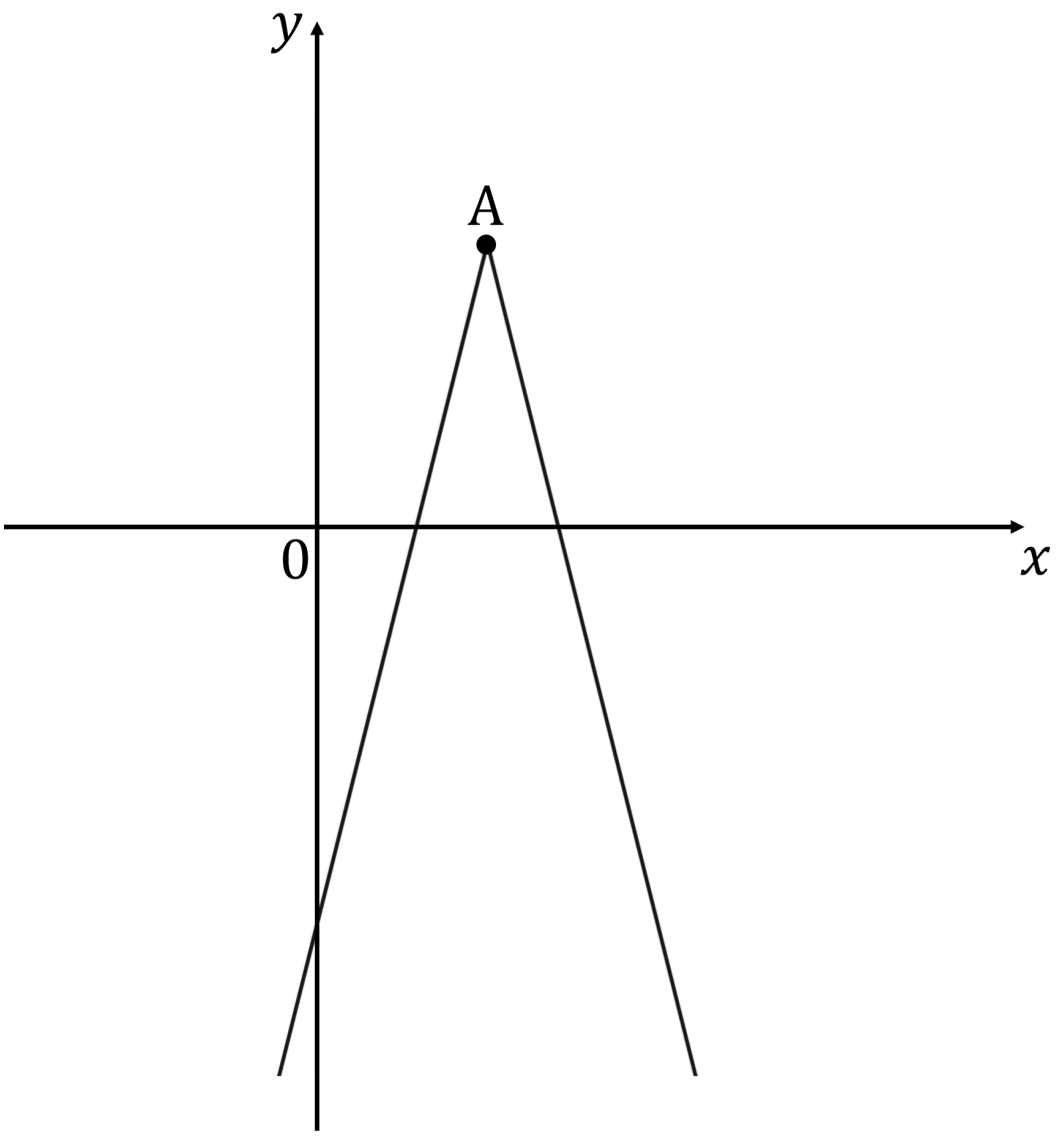
(c) The graphs of $\mathrm{f}(x)$ and $\mathrm{f}^{-1}(x)$ are drawn on the same axes.
Describe the transformation that would map one graph onto the other.

(2 marks)

(d) Find the coordinates of the point where the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ meet.

(2 marks)

9 (a) The graph of the function $f(x) = a|x + p| + q$ is shown below, where $a, p, q \in \mathbb{R}$. The graph has a local maximum at the point A(3, 5) and intersects the y-axis at (0,-7).



Find the values of a, p and q. Hence solve $f(x) = 0$.

(4 marks)

(b) Find the solutions to the equation $f(x) = |7 - 2x|$.

(4 marks)

10 (a) The functions $f(x)$, $g(x)$ are defined as follows

$$f(x) = |x^3 - 8| \qquad x \in \mathbb{R}$$

$$g(x) = |x| \qquad x \in \mathbb{R}$$

Sketch the graph of $y = fg(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

(3 marks)

(b) There are between 0 and 4 solutions to the equation $fg(x) = c$, where c is a real number. Determine the values of c that produce each number of solutions.

(3 marks)

11 Solve the equation $|x^2 - 9| = 6 - 0.25x^2$, giving your answers in exact form.

(3 marks)