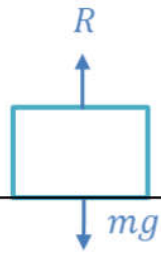
Friction

Scenario 1: A block is on a horizontal rough surface with no forces (other than gravity) acting on it.



T = 0

Comment regarding friction:

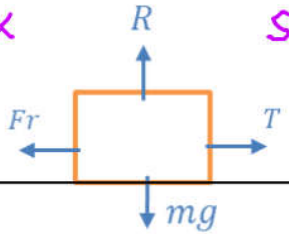
No friction

Equilibrium?

Yes, it is not moving

Scenario 2: A cable is attached to the block and a force applied. The block doesn't move.

$T < Fr_{MAX}$ small T



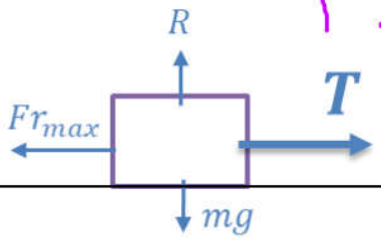
Comment regarding friction:

Friction is the same magnitude as the opposing force.

Equilibrium?

Yes, it is not moving

Scenario 3: The tension is increased ~~until~~ so the block ~~starts to move~~ accelerates. $T > Fr_{MAX}$



Comment regarding friction:

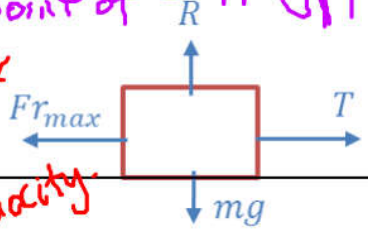
Friction has reached its maximum limit

Equilibrium?

No, the forces are imbalanced, so it accelerates.

F=ma

Scenario 4a/4b: The tension is increased until the block ~~starts to move~~ is on the point of slipping $T = F_{MAX}$



4b) The box moves with constant velocity.

Comment regarding friction:

Friction is limiting - it has reached its max.

Friction is at its max.

a) Equilibrium?

Yes, this is called limiting equilibrium

b) Equilibrium?

Yes, it has constant speed/ no acceleration.

The maximum friction between two surfaces:

$$Fr_{max} = \mu R$$

where μ is the coefficient of friction and R is the normal reaction between two surfaces.

This 'maximum friction' depends on two things:

- How **rough** the surface is (i.e. the rougher the surface, the more force required before the block starts moving).
- How hard the block is pressing against the surface (and more formally, by application of Newton's 3rd Law, how large the **reaction force $\boldsymbol{R}$** is).

Maximum friction = coefficient of friction (μ) × normal reaction (R)

Maximum $F_r = \mu R$

Note, the coefficient of friction is always greater than 0 and usually less than about 1.5.
The coefficient of friction is specific for the particle and the surface it is on.
A box on sandpaper would have a high coefficient of friction.
A box on ice would have a low coefficient of friction.
A box on a smooth surface would have a coefficient of friction = 0 (i.e. there is no friction)

Friction acts in the **opposite direction to its motion** (obviously)
Friction can be less than this maximum value **if the particle is not moving**.

Materials	Coeff. of Static Friction μ_s
Steel on Steel	0.74
Aluminum on Steel	0.61
Copper on Steel	0.53
Rubber on Concrete	1.0
Wood on Wood	0.25-0.5
Glass on Glass	0.94
Waxed wood on Wet snow	0.14
Waxed wood on Dry snow	-
Metal on Metal (lubricated)	
Ice on Ice	0.1
Teflon on Teflon	0.04
Synovial joints in humans	0.01

R = 15

μ = 0.3

$F_{r\,MAX} = \mu R$

$= 0.3 \times 15$

$= 4.5N$

What happens to the box...

... if a force of 10N is applied? It accelerates, $10 > 4.5$

... if a force of 3N is applied? It doesn't move, $3 < 4.5$

... if a force of 4.5N is applied? It is on the point of moving $4.5 = 4.5$

What is the value of the friction...

... if an opposing force of 2N is applied? $F_r = 2N$

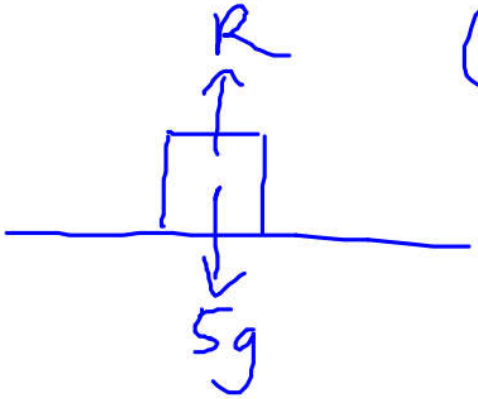
... if an opposing force of 4.5N is applied? $F_r = 4.5N$

... if an opposing force of 14N is applied? $F_r = 4.5N$

A block of mass 5kg lies at rest on rough horizontal ground. The coefficient of friction between the block and the ground is 0.4. A horizontal force P is applied to the block. Find the magnitude of the friction force acting on the block when the magnitude of P is

a) 10N
b) 19.6N
c) 30N

$\mu = 0.4$



$(R\uparrow)\quad R = 5g$

$F_{r\,MAX} = 0.4 \times 5g$

$= 2g$

$= 19.6N$

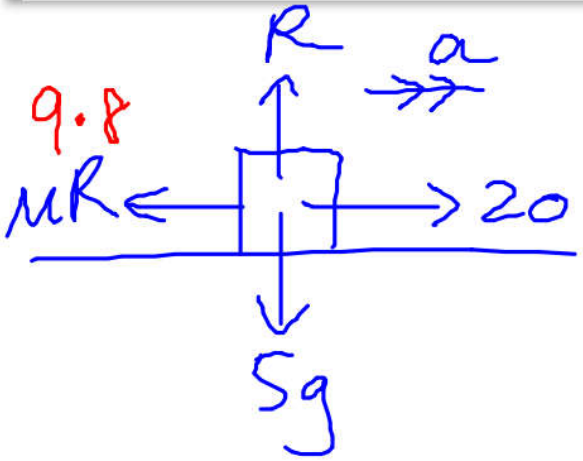
a) $10 \leftarrow F \rightarrow 10$ $\quad F_r = 10$

b) $19.6 \leftarrow F \rightarrow 19.6$ $\quad F_r = 19.6$

c) $19.6 \leftarrow F \rightarrow 30$ $\quad F_r = 19.6$

A particle of mass 5kg is pulled along a rough horizontal surface by a horizontal force of magnitude 20N. The coefficient of friction between the particle and the floor is 0.2.
Calculate:
(a) the magnitude of frictional force
(b) the acceleration of the particle.

$\mu = 0.2$



a) $(R\updownarrow)$ $R = 5g$

$F_r = 5g \times 0.2 = g = 9.8\,N$.

b) $F = ma \rightarrow$

$20 - 9.8 = 5a$

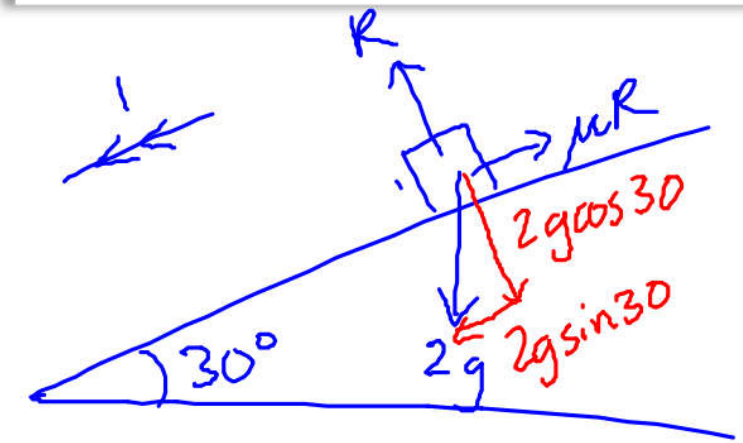
$10.2 = 5a$

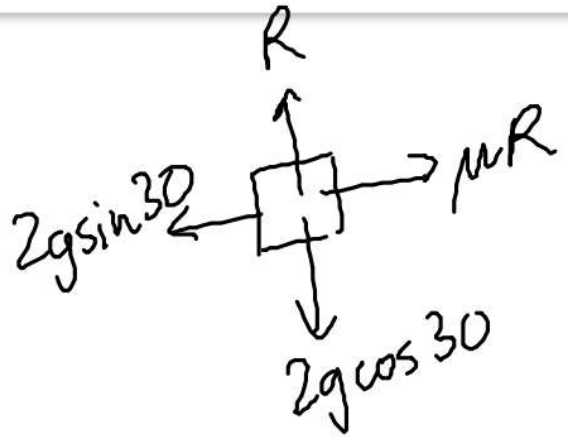
$a = 2.04\,ms^{-2}$

Friction - slopes

A particle of mass 2kg is sliding down a rough slope that is inclined at 30° to the horizontal. Given that the acceleration of the particle is $1ms^{-2}$, find the coefficient of friction μ between the particle and the slope.

Recall that friction acts in opposite direction to motion.





$(R\uparrow)$ $R = 2g\cos 30$

$F = ma \rightarrow$ $2g\sin 30 - \mu 2g\cos 30 = 2 \times 1$

$2g\sin 30 - 2 = \mu 2g\cos 30$

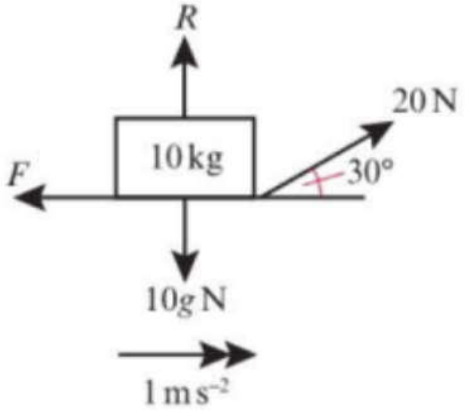
$\frac{2g\sin 30 - 2}{2g\cos 30} = \mu$

$\mu = 0.46$ (2sf)

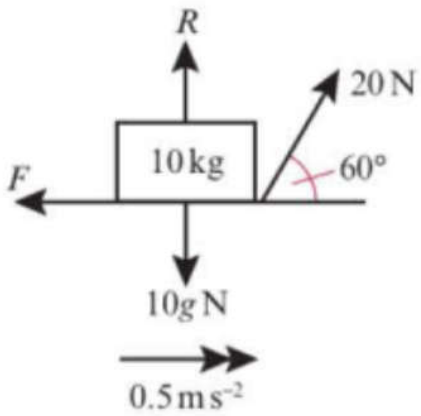
Your Turn

2 In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. R is the normal reaction and F is the frictional force. Find the normal reaction and the coefficient of friction in each case.

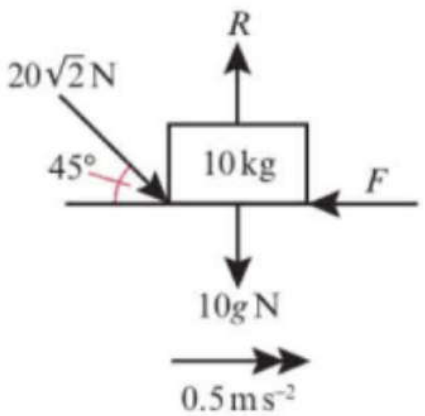
a



b



c



3 A particle of mass 0.5 kg is sliding down a rough slope that is angled at 15° to the horizontal. The acceleration of the particle is 0.25 m s^{-2}. Calculate the coefficient of friction between the particle and the slope. **(3 marks)**

4 A particle of mass 2 kg is sliding down a rough slope that is angled at 20° to the horizontal. A force of magnitude P acts parallel to the slope and is attempting to pull the particle up the slope. The acceleration of the particle is 0.2 m s^{-2} down the slope and the coefficient of friction between the particle and the slope is 0.3. Find the value of P. **(4 marks)**

5 A particle of mass 5 kg is being pushed up a rough slope that is angled at 30° to the horizontal by a horizontal force P. Given that the coefficient of friction is 0.2 and the acceleration of the particle is 2 m s^{-2} calculate the value of P.

6 A sled of mass 10 kg is being pulled along a rough horizontal plane by a force P that acts at an angle of 45° to the horizontal. The coefficient of friction between the sled and the plane is 0.1. Given that the sled accelerates at $0.3\,\text{m s}^{-2}$, find the value of P.

(7 marks)

7 A train of mass m kg is travelling at $20\,\text{m s}^{-1}$ when it applies its brakes, causing the wheels to lock up. The train decelerates at a constant rate, coming to a complete stop in 30 seconds.

a By modelling the train as a particle, show that the coefficient of friction between the railway track and the wheels of the train is $\mu = \frac{2}{3g}$.

(6 marks)

Problem-solving

Use the formulae for constant acceleration.

← Year 1, Chapter 9

The train is no longer modelled as a particle, so that the effects of air resistance can be taken into account.

b State, with a reason, whether the coefficient of friction between the track and the wheels will increase or decrease in this revised model. **(2 marks)**

Challenge

A particle of mass m kg is sliding down a rough slope that is angled at α to the horizontal. The coefficient of friction between the particle and the slope is μ. Show that the acceleration of the particle is independent of its mass.

2 **a** $R = 88\,\text{N}$, $\mu = 0.083$ (2 s.f.)
b $R = 80.679\,\text{N}$, $\mu = 0.062$ (2 s.f.)
c $R = 118\,\text{N}$, $\mu = 0.13$ (2 s.f.)
3 0.242 (3 s.f.)
4 0.778 N (3 s.f.)
5 56.1 N
6 16.5 N (3 s.f.)
7 **a** Use $v = u + at$ to find $a = -\frac{2}{3}\text{ms}^{-2}$
R(→): $-\mu mg = -\frac{2}{3}m$
$\mu = \frac{2}{3g}$
b The coefficient of friction remains unchanged. The air resistance has no effect on the coefficient of friction, which is dependent on the properties of the wheels and the rails.

Challenge
R(↙): $mg\sin\alpha - \mu mg\cos\alpha = ma$
$g\sin\alpha - \mu g\cos\alpha = a$