Application to Mechanics

Out of displacement, ~~speed~~ velocity, acceleration, force, mass and time, all but mass and time are vectors. Clearly these can act in 3D space.

	Vector	Scalar
Force	$\begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} N$	magnitude → $\sqrt{3^2+4^2+1^2} = \sqrt{26}$ N $= 5.10$ N
Acceleration	$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} ms^{-2}$	→ $\sqrt{1^2+1^2} = \sqrt{2} = 1.41\ ms^{-2}$
Displacement	$\begin{pmatrix} 12 \\ 3 \\ 4 \end{pmatrix} m$	→ distance $= \sqrt{12^2+3^2+4^2} = 13$ m
Velocity	$\begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} ms^{-1}$	→ speed $= \sqrt{4^2+3^2} = 5\ ms^{-1}$

A particle of mass 0.5 kg is acted on by three forces.

$m = 0.5$

$$F_1 = (2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})\ N$$
$$F_2 = (-\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})\ N$$
$$F_3 = (4\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k})\ N$$

a. Find the resultant force R acting on the particle.

b. Find the acceleration of the particle, giving your answer in the form $(p\boldsymbol{i} + q\boldsymbol{j} + r\boldsymbol{k})$ ms^{-2}.

c. Find the magnitude of the acceleration.

Given that the particle starts at rest,

d. Find the distance travelled by the particle in the first 6 seconds of its motion.

a) $\underline{R} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$

$$= \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}$$

$$\underline{R} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} N$$

b) $\underline{F} = m\underline{a}$

$$\begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} = 0.5\,\underline{a}$$

$$\underline{a} = \begin{pmatrix} 10 \\ -2 \\ -6 \end{pmatrix} = 10\underline{i} - 2\underline{j} - 6\underline{k}$$

c) $|\underline{a}| = \sqrt{10^2 + 2^2 + 6^2} = 2\sqrt{35} = 11.8 ms^{-2}$

d) $u = 0$, $a = 2\sqrt{35}$, $t = 6$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$= \frac{1}{2} \times 2\sqrt{35} \times 36$

$= 213$ m (3sf)

Ex12D

Mixed Exercise 12