

2.9 Transformations of Functions

2.9.1 Translations / 2.9.2 Stretches / 2.9.3 Reflections

Easy (7 questions)	/27
Medium (8 questions)	/41
Hard (9 questions)	/44
Very Hard (8 questions)	/48
Total Marks	/160

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Easy Questions

- 1 A curve has equation $y = f(x)$.

Describe the transformation of the curve given by the equations below:

(i) $y = f(x) + 2$,

(ii) $y = f(x - 2)$,

(iii) $y = 3f(x)$,

(iv) $y = f(2x)$.

(4 marks)

- 2 A curve has equation $y = f(x)$.

Write down the equations of the curves, in terms of $f(x)$, given by the following transformations:

(i) Translation by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$,

(ii) Horizontal stretch, scale factor 2,

(iii) Vertical stretch, scale factor $\frac{1}{3}$,

(iv) Reflection in the y -axis.

(4 marks)

- 3** The point $P(2,6)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = f(x) + 1$

(ii) $y = -f(x)$

(iii) $y = f\left(\frac{1}{4}x\right)$.

(3 marks)

- 4** A point $P(-2,8)$, on the graph of $y = f(x)$, is mapped to the point P' under a single transformation.

For the following coordinates of P' write down what the transformation could have been:

(i) $P'(-2,3)$,

(ii) $P'(-4,8)$,

(iii) $P'(-2, -8)$.

(3 marks)

- 5 Point P has coordinates $(3, -4)$ and lies on the curve with equation $y = f(x)$.

Write down the value of a given that:

- (i) On the graph of $y = f(x + a)$, point P is mapped to point $P'(-3, -4)$,
- (ii) On the graph of $y = af(x)$, point P is mapped to point $P'(3, -12)$,
- (iii) On the graph of $y = f(ax)$, point P is mapped to point $P'(-3, -4)$.

(3 marks)

6 (a) The function $f(x)$ is defined as $f(x) = (x - 2)(x - 6)$

Sketch the graph of $y = f(x)$, showing clearly the coordinates of the points where the graph intersects the coordinate axes and the coordinates of the turning point.

(3 marks)

(b) On separate diagrams sketch the graphs of:

(i) $y = f(x - 4)$,

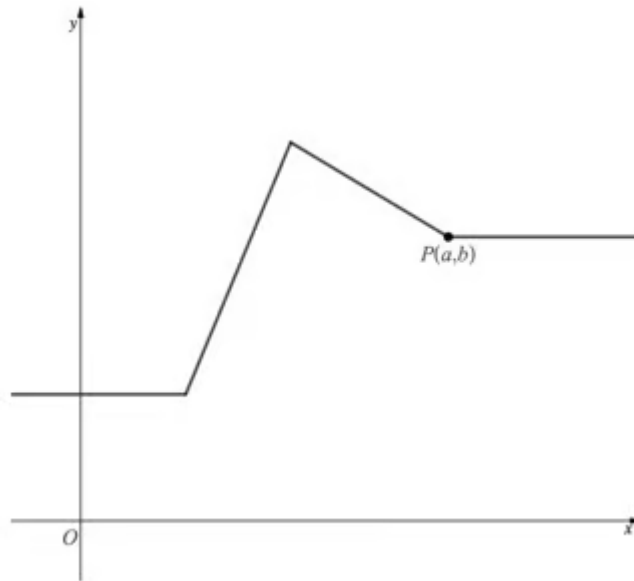
(ii) $y = f(-x)$.

In each case clearly show the coordinates of the points where the graph intersects the coordinate axes and the coordinates of the turning point.

(4 marks)

7 The diagram below shows the graph of $y = f(x)$.

The point P has coordinates (a, b) , where $a, b > 0$.



In terms of a and b write down the coordinates of the image of point P under the following graph transformations:

- (i) $y = f(2x)$,
- (ii) $y = -f(x)$
- (iii) $y = af(x)$

.

(3 marks)

Medium Questions

- 1 The point $P(-1, 4)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = f(x) + 3$

(ii) $y = f(x + 3)$

(iii) $y = 3f(x)$

(iv) $y = f(3x)$

(4 marks)

- 2 The point $P(-3, -4)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = f(-x)$

(ii) $y = -f(x)$

(2 marks)

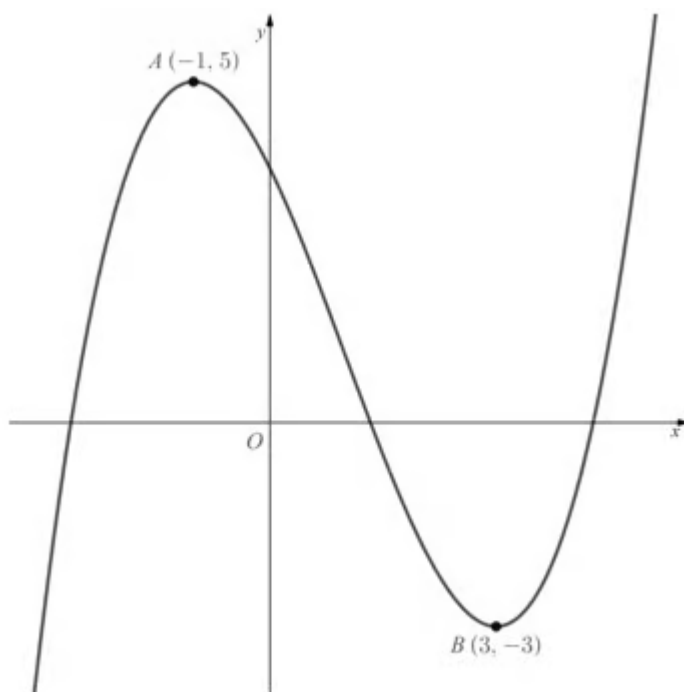
- 3 The point $P(3, 2)$ lies on the curve with equation $y = f(x)$.

(i)

- On the graph of $y = f(x) + a$, where a is a constant, the point P is mapped to the point $(3, -5)$. Determine the value of a .
- (ii) On the graph of $y = f(x + b)$, where b is a constant, the point P is mapped to the point $(-1, 2)$. Determine the value of b .
- (iii) On the graph of $y = cf(x)$, where c is a constant, the point P is mapped to the point $(3, 1)$. Determine the value of c .
- (iv) On the graph of $y = f(dx)$, where d is a constant, the point P is mapped to the point $(1, 2)$. Determine the value of d .

(4 marks)

- 4 (a)** The diagram below shows the graph of $y = f(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.



In separate diagrams, sketch the curves with equation

- (i) $y = f(x - 1)$
- (ii) $y = f(x) + 3$

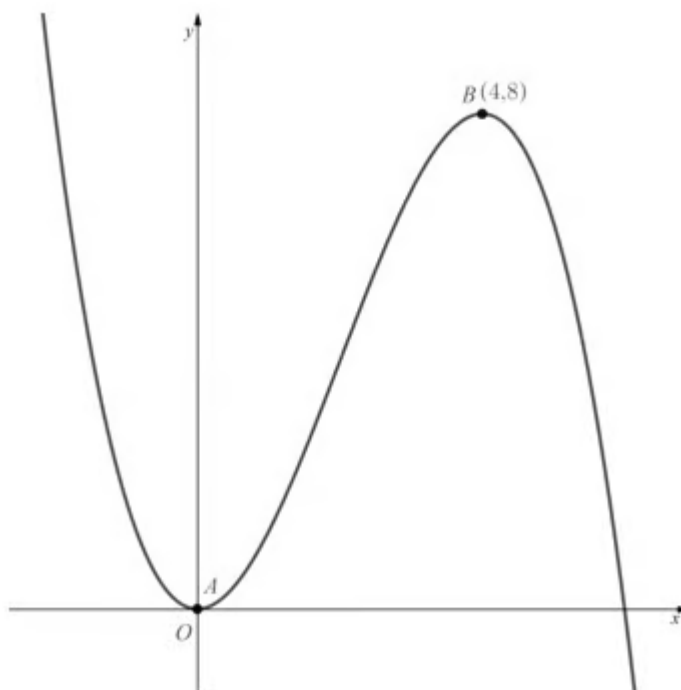
On each diagram, give the coordinates of the images of points A and B under the given transformation.

(4 marks)

- (b)** On the graph of $y = f(x + a)$ the image of one of the two marked points has an x coordinate of 2. Find the two possible values of a .

(2 marks)

- 5 (a)** The diagram below shows the graph of $y = f(x)$. The marked point $B(4,8)$ lies on the graph, and the graph meets the origin at the marked point A .



In separate diagrams, sketch the curves with equation

- (i) $y = -f(x)$
- (ii) $y = f(4x)$

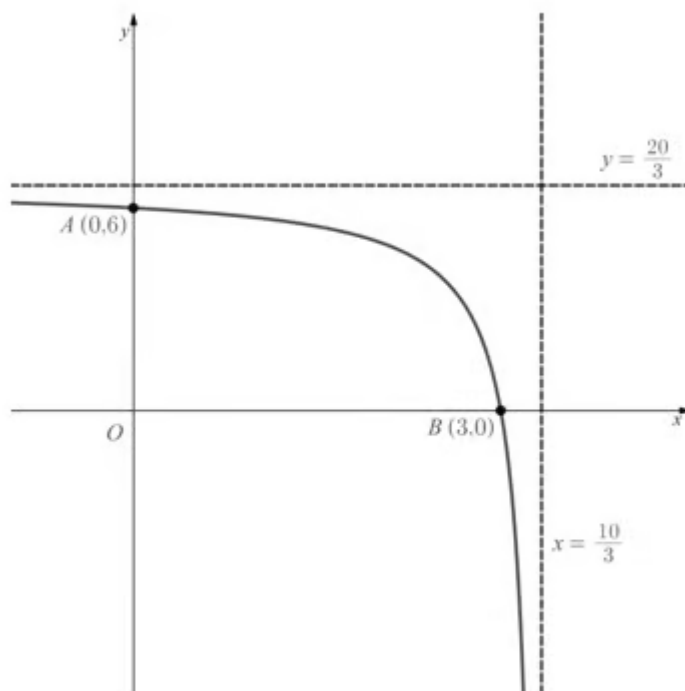
On each diagram, give the coordinates of the images of points A and B under the given transformation.

(4 marks)

- (b)** On the graph of $y = af(x)$ the image of one of the two marked points has a y coordinate of 4. Find the value of a .

(2 marks)

- 6 (a)** The diagram below shows the graph of $y = f(x)$. The graph intersects the coordinate axes at the two marked points $A(0,6)$ and $B(3,0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$



In separate diagrams, sketch the curves with equation

- (i) $y = f(x) - 6$
- (ii) $y = f(-x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

(6 marks)

- (b)** The graph of $y = f(x + a)$ has an asymptote at one of the coordinate axes. Find the value of a .

(2 marks)

- 7 (a)** Sketch the graph of $y = \frac{1}{x} + 3$, showing clearly the points where the curve crosses the coordinate axes and stating the equations of the asymptotes.

(4 marks)

- (b)** The graph of $y = \frac{1}{(x + a)} + 3$ passes through the origin. Find the value of a .

(1 mark)

8 (a) Given that $x^3 - 10x^2 - 24x = x(x + 2)(x - 12)$

Sketch the graph of $y = x^3 - 10x^2 - 24x$, showing clearly the coordinates of the points where the curve crosses the coordinate axes.

(4 marks)

(b) The graph with equation $y = (x + a)^3 - 10(x + a)^2 - 24(x + a)$ passes through the point $(-2, 0)$. Find the three possible values of a .

(2 marks)

Hard Questions

- 1 The point $P(-3, -2)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y - 2 = f(x) - 6$

(ii) $y = f(x - 3)$

(iii) $2y = f(x)$

(iv) $y = f\left(\frac{1}{2}x\right)$

(4 marks)

- 2 The point $P(0, 5)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = f(-x)$

(ii) $-y = f(x)$

(2 marks)

3 (a) The point $P(-12, -9)$ lies on the curve with equation $y = x^2 + 15x + 27$.

The graph is translated so that the point P is mapped to the point $(-12, 3)$. Write down the equation of the transformed function.

(2 marks)

(b) The graph is translated so that the point P is mapped to the point $(-10, -9)$.

Write down the equation of the transformed function in the form

$y = (x + a)^2 + 15(x + a) + 27$, where a is a constant to be found.

(2 marks)

- 4 (a)** The point $P(3, -12)$ lies on the curve with equation $y = x^2 - 12x + 15$.

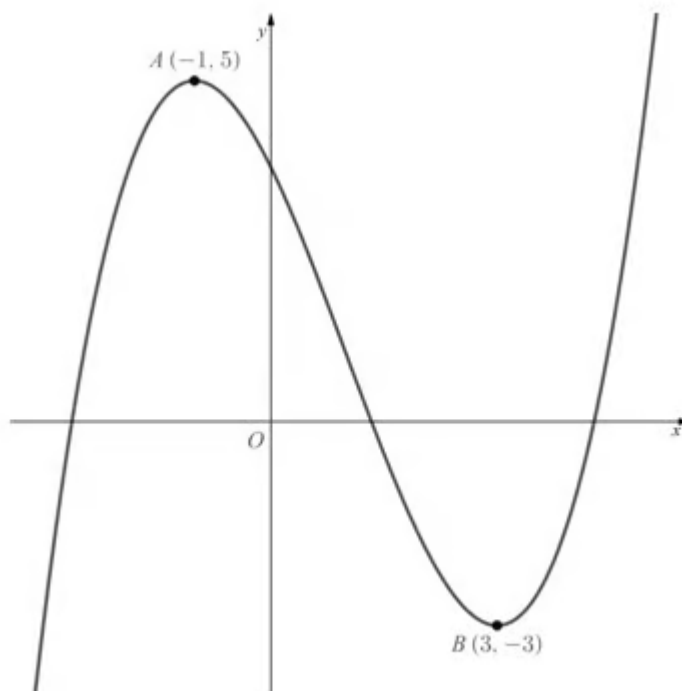
The graph is stretched so that the point P is mapped to the point $(3, -4)$. Write down the equation of the transformed function in the form $y = ax^2 + bx + c$, where a, b and c are constants to be found.

(2 marks)

- (b)** The graph is stretched so that the point P is mapped to the point $(1, -12)$. Write down the equation of the transformed function in the form $y = (dx)^2 - 12(dx) + 15$, where d is a constant to be found.

(2 marks)

- 5 (a)** The diagram below shows the graph of $y = f(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.



In separate diagrams, sketch the curves with equation

- (i) $y = f(-x)$
- (ii) $-y = f(x)$

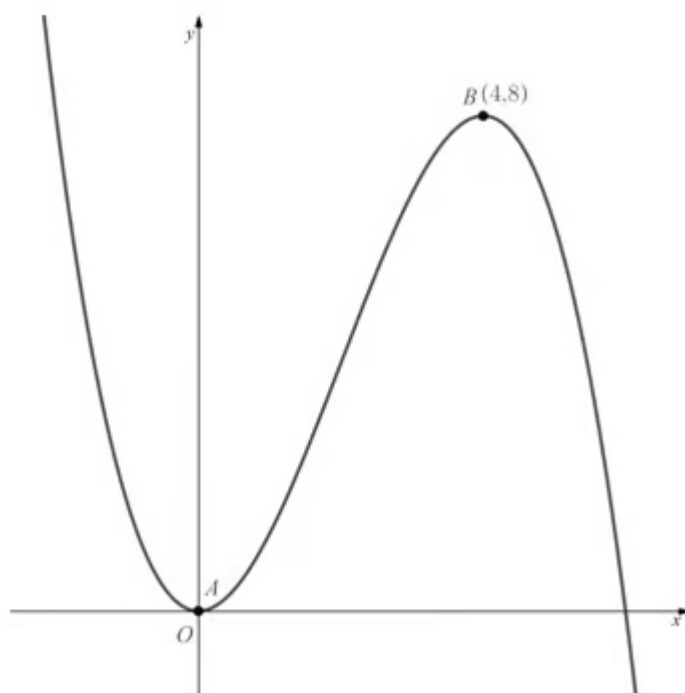
On each diagram, give the coordinates of the images of points A and B under the given transformation.

(4 marks)

- (b)** On the graph of $y = f(x + a)$ the images of the two marked points both lie on the same side of the y -axis. Find the range of possible values of a .

(3 marks)

- 6 The diagram below shows the graph of $y = f(x)$. The marked point $B(4,8)$ lies on the graph, and the graph meets the origin at the marked point A .



In separate diagrams, sketch the curves with equation

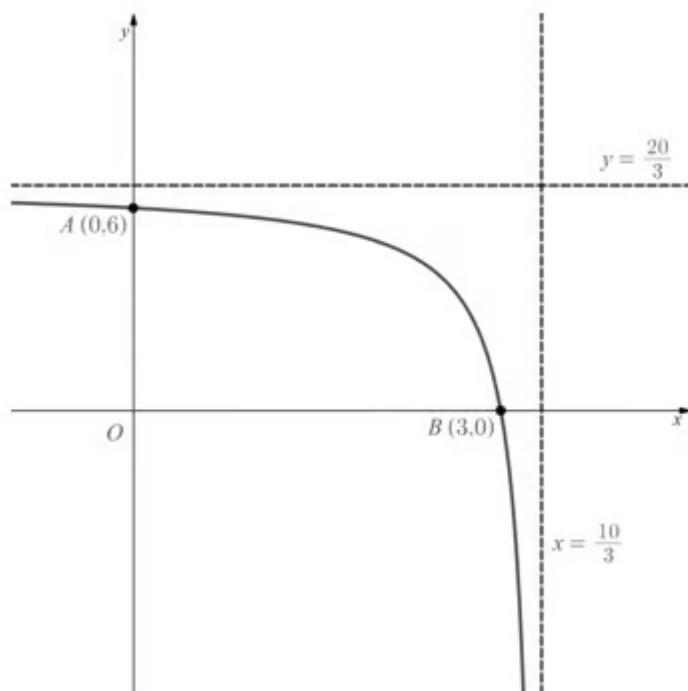
(i) $y = f\left(\frac{1}{3}x\right)$

(ii) $6y = f(x)$

On each diagram, give the coordinates of the images of points A and B under the given

(4 marks)

- 7 (a)** The diagram below shows the graph of $y = f(x)$. The graph intersects the coordinate axes at the two marked points $A(0,6)$ and $B(3,0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$.



In separate diagrams, sketch the curves with equation

- (i) $y = f(5x)$
- (ii) $y = -f(x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

(6 marks)

(b) The graph of $y = af(x)$ has an asymptote with equation $y = 2$. Find the value of a .

(1 mark)

- 8 (a)** Sketch the graph of $y = 2 - \frac{8}{x^2}$, showing clearly the points where the curve crosses the coordinate axes and stating the equations of the asymptotes.

(4 marks)

- (b)** The graph of $y = 2 - \frac{8}{(x+a)^2}$ passes through the origin. Find the two possible values of a .

(2 marks)

9 (a) Given that $x^3 - 8x^2 + 16x = x(x - 4)^2$

Sketch the graph of $y = x^3 - 8x^2 + 16x + 3$, showing clearly the coordinates of the points where the curve crosses the coordinate axes and the co-ordinates of any minimum points. (You do not need to state the co-ordinates of any maximum points).

(4 marks)

(b) The graph with equation $y + a = x^3 - 8x^2 + 16x$ crosses the x -axis three times. Find the range of possible values of a .

(2 marks)

Very Hard Questions

- 1 The curve with equation $y = f(x)$ has two asymptotes, for which the equations are $y = -3$ and $x = 2$.

Give the equations of the asymptotes for the curves with the following equations:

- (i) $y + 3 = f(x)$
- (ii) $y = f(x - 2)$

(4 marks)

- 2 The curve with equation $y = f(x)$ has two asymptotes, for which the equations are $y = 5$ and $x = -4$.

Give the equations of the asymptotes for the curves with the following equations:

- (i) $\frac{1}{3}y = f(x)$
- (ii) $y = f\left(\frac{1}{3}x\right)$

(4 marks)

- 3** The curve with equation $y = f(x)$ has two asymptotes, for which the equations are $y = -1$ and $x = -2$.

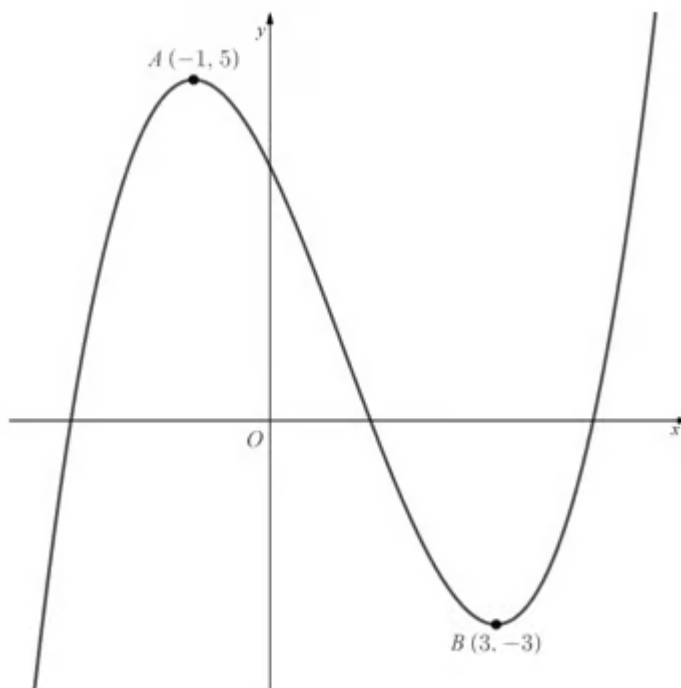
Give the equations of the asymptotes for the curves with the following equations:

(i) $y = f(-x)$

(ii) $-y = f(x)$

(4 marks)

- 4 (a)** The diagram below shows the graph of $y = f(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.



In separate diagrams sketch the curves with equation

(i) $y = f\left(\frac{1}{3}x\right)$

(ii) $5y = f(x)$

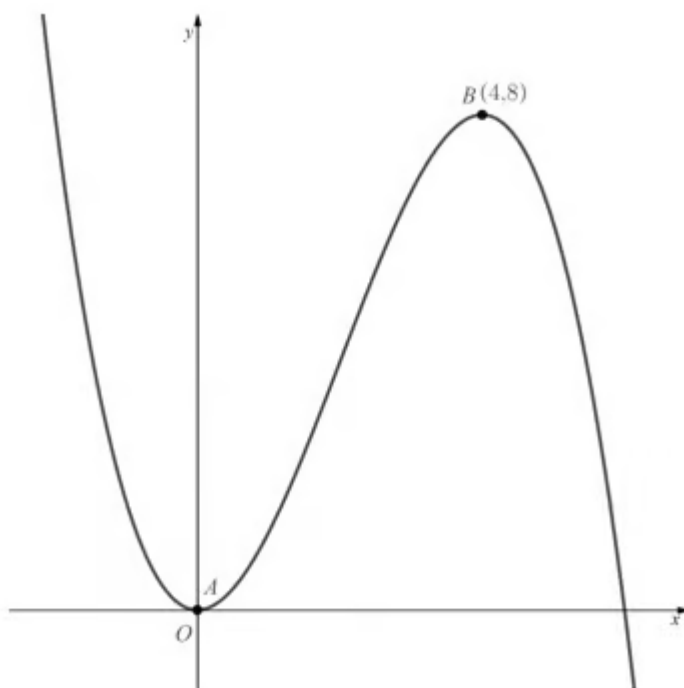
On each diagram, give the coordinates of the images of points A and B under the given transformation.

(4 marks)

- (b) On the graph of $y = f(ax)$ the image of one of the two marked points has an x coordinate of $\frac{5}{3}$. Given that $a > 0$, find the value of a .

(2 marks)

- 5 The diagram below shows the graph of $y = f(x)$. The marked point $B(4,8)$ lies on the graph, and the graph meets the origin at the marked point A .



Consider the three following transformations of the graph

$$y = f(-x) \qquad y = f(ax) \qquad y = f(x) + b$$

where a and b are constants, and $a > 0$

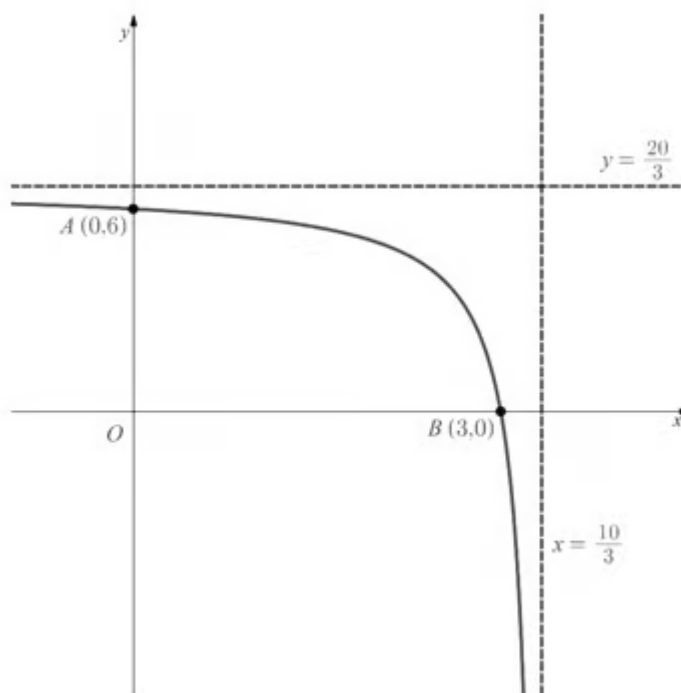
State which of the transformations satisfies each of the following conditions, and determine the range of possible values of the variables a and b where relevant.

- (i) The images of the two marked points under the transformation lie on opposite sides of the x -axis.

- (ii) The image of point B under the transformation has coordinates (x, y) , where $-6 < x < -3$.
- (iii) The image of point B under the transformation has coordinates (x, y) , where $0 < x < 3$.
- .

(5 marks)

- 6 (a)** The diagram below shows the graph of $y = f(x)$. The graph intersects the coordinate axes at the two marked points $A(0,6)$ and $B(3,0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$.



In separate diagrams sketch the curves with equation

(i) $y = f\left(\frac{20}{3}x\right)$

(ii) $5y = 4f(x)$

On each diagram give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

(6 marks)

- (b)** The graph of $y = f(ax)$ has an asymptote with equation $x = k$, where $1 < k < 100$. Find the range of possible values of a .

(2 marks)

7 (a) The function $f(x)$ is defined by the equation

$$f(x) = 9 - \frac{16}{(x-2)^2}$$

Sketch the graph of $y = f(x)$, showing clearly the points where the curve crosses the coordinate axes and stating the equations of the asymptotes.

(6 marks)

(b) The graph of $y = f(x + a)$ is such that, for all points $P(x, y)$ that lie on the graph, if the y coordinate is less than 5 then the x coordinate is less than zero. Find the range of possible values of a .

(2 marks)

8 (a) Given that $f(x) = x^3 - (2\sqrt{3})x^2 + 3x$

Sketch the graph of $y = f(x)$, showing clearly the coordinates of the points where the curve crosses or touches the coordinate axes.

(6 marks)

(b) The functions $g(x)$ and $h(x)$ are defined by the equations

$$g(x) = f(-x)$$

$$h(x) = g(x + a)$$

The graph of $h(x)$ touches the x -axis at the point $(5, 0)$. Find the value of a , giving your answer as an exact value.

(3 marks)