The line l has equation $\boldsymbol{r} = \begin{pmatrix}-2\\1\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\1\end{pmatrix}$, and the point P has position vector $\begin{pmatrix}2\\1\\3\end{pmatrix}$.

(a) Show that P does not lie on l.

Given that a circle, centre P, intersects l at points A and B, and that A has position vector $\begin{pmatrix}0\\-3\\6\end{pmatrix}$,

(b) find the position vector of B.

Q14, 15, 16, 17

a) $\begin{pmatrix}2\\1\\3\end{pmatrix} = \begin{pmatrix}-2+\lambda\\1-2\lambda\\4+\lambda\end{pmatrix}$

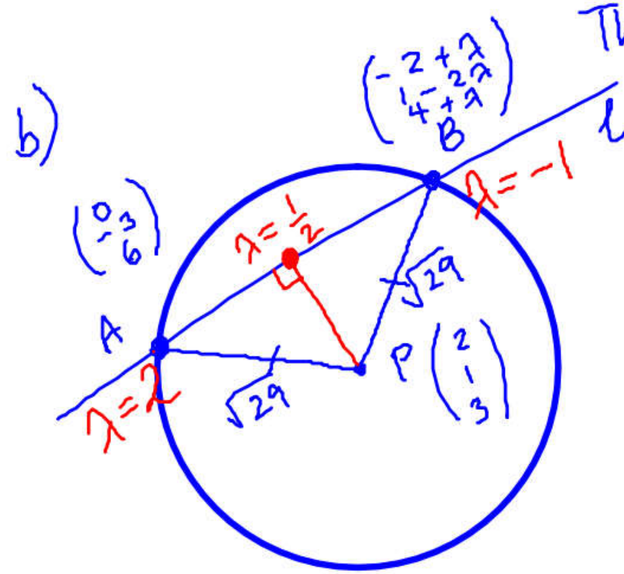
$\underline{i}$: $2 = -2+\lambda$, $4 = \lambda$

$\underline{j}$: $1 = 1-2\lambda$, $\lambda = 0$

$\underline{k}$: $3 = 4+\lambda$, $\lambda = -1$

There is no value for λ such that P lies on l.

b)



$\vec{AP} = \begin{pmatrix}2\\-4\\-3\end{pmatrix}$

$AP = PB$

$AP = \sqrt{2^2+4^2+3^2} = \sqrt{29}$

$\vec{PB} = \underline{b} - \underline{p} = \begin{pmatrix}-2+\lambda & -2\\1-2\lambda & -1\\4+\lambda & -3\end{pmatrix} = \begin{pmatrix}-4+\lambda\\-2\lambda\\1+\lambda\end{pmatrix}$

$|\vec{PB}| = PB = \sqrt{29}$

$(-4+\lambda)^2 + (-2\lambda)^2 + (1+\lambda)^2 = 29$

$\lambda^2 - 8\lambda + 16 + 4\lambda^2 + \lambda^2 + 2\lambda + 1 - 29 = 0$

$6\lambda^2 - 6\lambda - 12 = 0$

$\lambda^2 - \lambda - 2 = 0$

$(\lambda-2)(\lambda+1) = 0$

$\lambda = 2 \quad \lambda = -1$

$\downarrow$ A

When $\lambda = -1$

$\underline{b} = \begin{pmatrix}-2-1\\1-2\times-1\\4-1\end{pmatrix} = \begin{pmatrix}-3\\3\\3\end{pmatrix}$

16 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} -4 \\ 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$. A and B are the points on l_1 with $\lambda = 2$ and $\lambda = 5$ respectively.

a Find the position vectors of A and B. **(2 marks)**

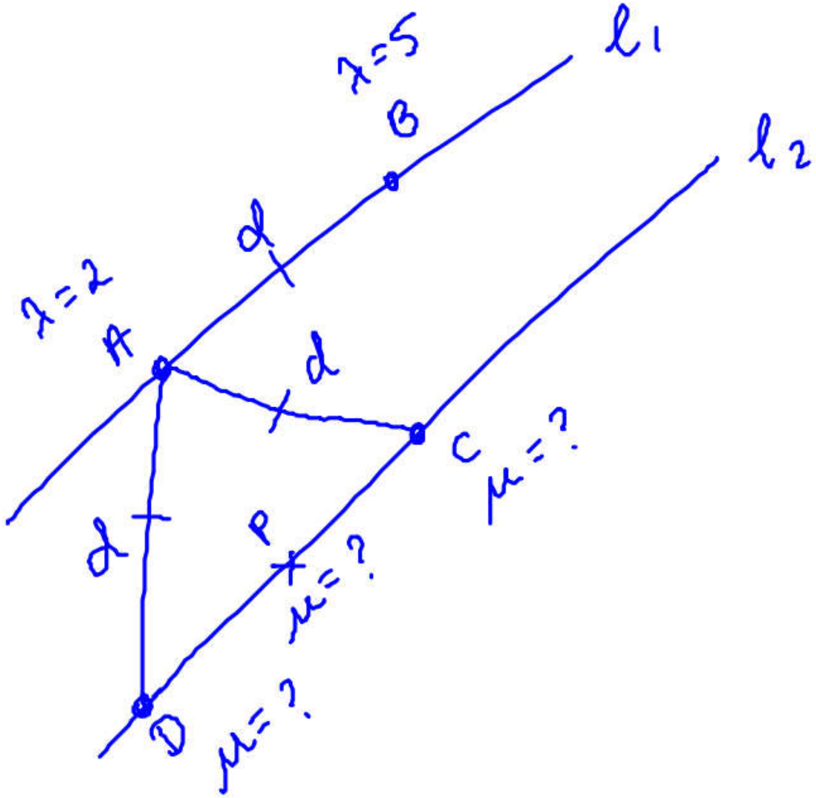
The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$.

The line l_2 passes through the point P and is parallel to the line l_1.

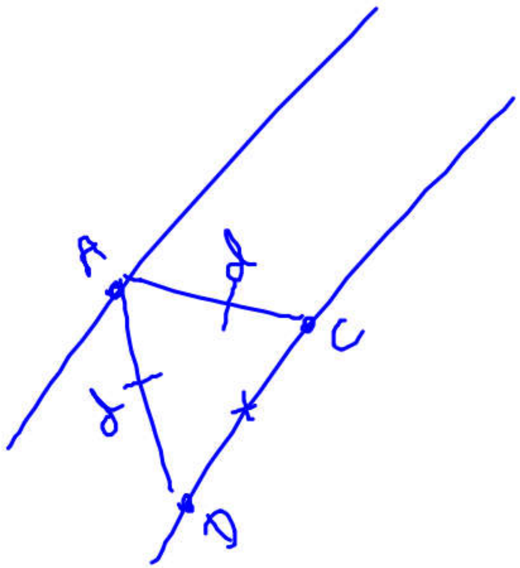
b Find a vector equation of the line l_2. **(2 marks)**

The points C and D both lie on line l_2 such that $AB = AC = AD$.

c Show that P is the midpoint of CD. **(7 marks)**

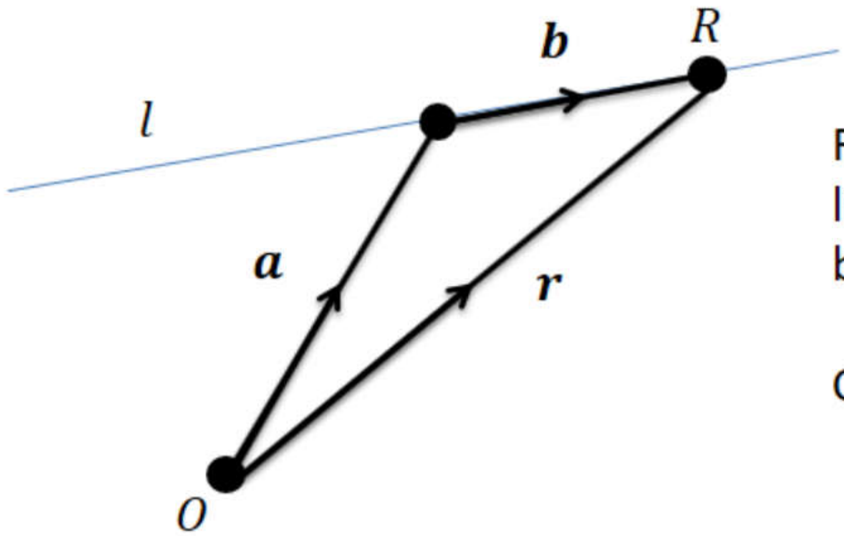


Find dist $AB = \overrightarrow{AB}$



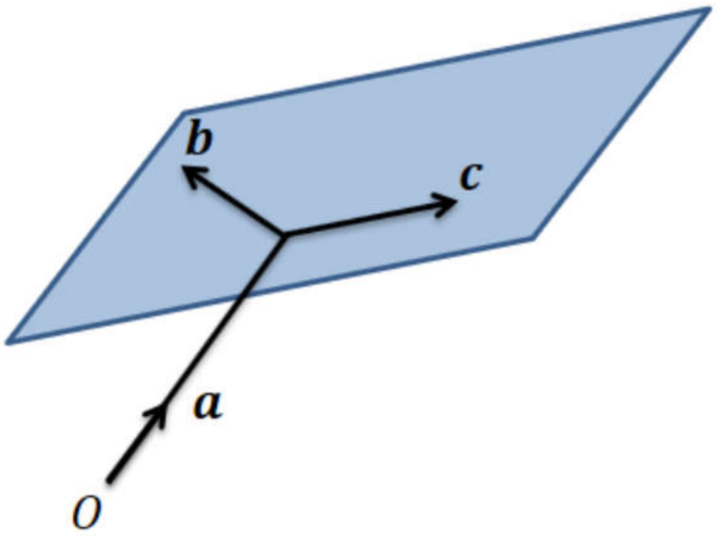
Equation of a plane - parametric vector form

$\boldsymbol{a}$ is the position vector of a point on a plane and $\boldsymbol{b}$ and $\boldsymbol{c}$ are non-parallel vectors on the plane, how could we write the equation of the plane in vector form?



Recall the we could get to a generic point $\boldsymbol{r}$ on a line by first getting to the line using $\boldsymbol{a}$, followed by some amount of b, i.e. $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$.

Could we do a similar thing with a plane?



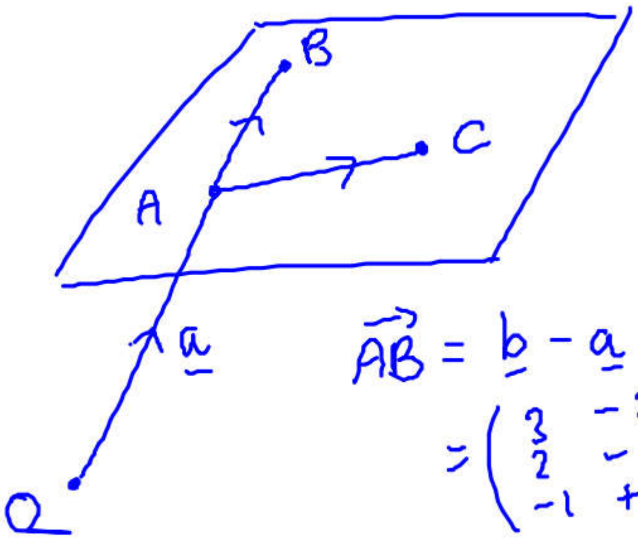
Once on the plane using $\boldsymbol{a}$, we could get to any other point on the plane using 'some amount' of $\boldsymbol{b}$ and 'some amount' of $\boldsymbol{c}$, i.e.

$$\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b} + \mu \boldsymbol{c}$$

λ and μ are parameters

Π

A plane Π passes through the points $A(2,2-1), B(3,2,-1), C(4,3,5)$
Find the equation of the plane Π in the form $\boldsymbol{a} + \lambda \boldsymbol{b} + \mu \boldsymbol{c}$



$$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 3-2 \\ 2-2 \\ -1+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{AC} = \underline{c} - \underline{a} = \begin{pmatrix} 4-2 \\ 3-2 \\ 5+1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$$

$$\underline{r} = \underline{a} + \lambda \vec{AB} + \mu \vec{AC}$$

$$\underline{r} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$$

Verify that the point P with position vector $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ lies in the plane with vector equation

$$r = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 + 2\lambda + \mu \\ 4 + \lambda - \mu \\ -2 + \lambda + 2\mu \end{pmatrix}$$

$\underline{i}$ $\quad 2 = 3 + 2\lambda + \mu$

$\boxed{-1 = 2\lambda + \mu}$

$\underline{j}$ $\quad 2 = 4 + \lambda - \mu$

$\boxed{-2 = \lambda - \mu}$

$-3 = 3\lambda$

$\underline{\lambda = -1}$

$-1 = -2 + \mu$

$\underline{\mu = 1}$

$\underline{k}$ $\quad -1 = -2 + \lambda + 2\mu$

$= -2 - 1 + 2$

$= \underline{-1}$ So P lies on the plane.

[June 2015 Q5] The points A, B and C have position vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ respectively.

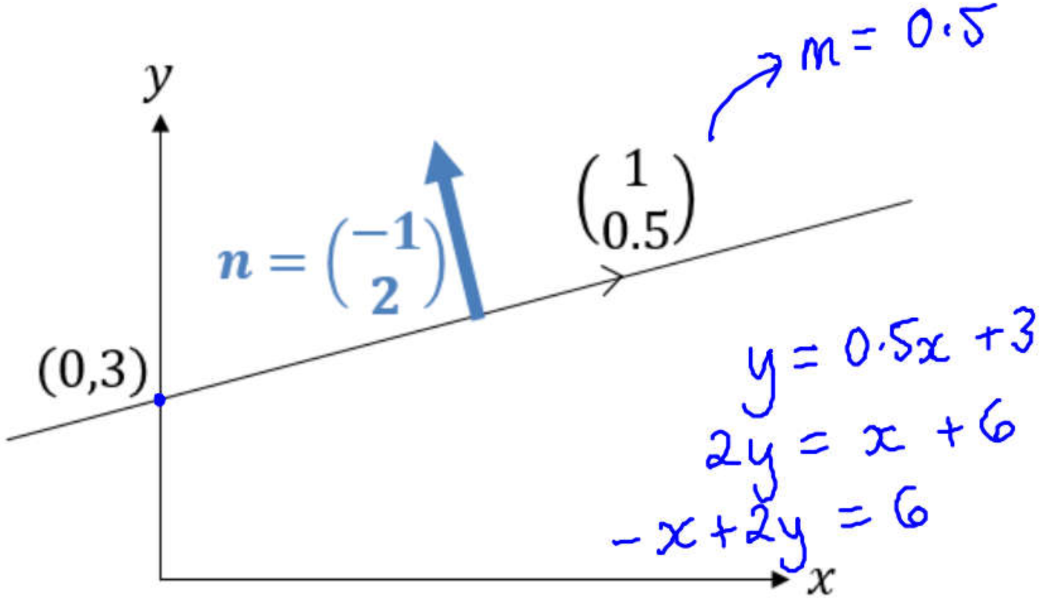
The plane Π contains the points A, B and C.

(c) Find a vector equation of Π **(4)**

Ex 9B
1ac
3ac
7

Equation of a plane - Cartesian form

An alternative approach to find the Cartesian equation of a straight line is to find a vector perpendicular to the line (known as the **normal vector $\boldsymbol{n}$**)



If $\boldsymbol{n} = \binom{n_1}{n_2}$, then the equation of the turns out to be $n_1 x + n_2 y = c$ where c is a constant to be found.

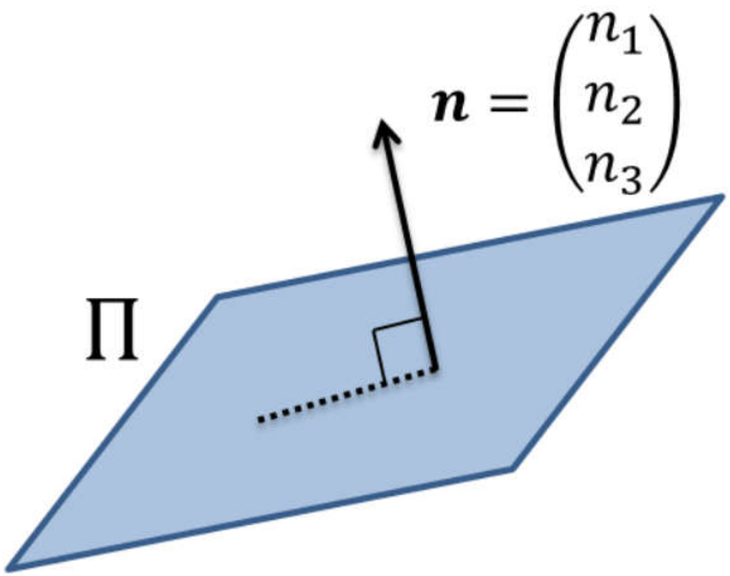
This is one reason we might want the equation of a straight line equation in the form $ax + by = c$: the **normal** to the line will be $\binom{a}{b}$.

For example above: $n = \binom{-1}{2}$ (by observation)

$$\therefore -1x + 2y = c$$

As (0,3) is on the line: $-1(0) + 2(3) = c \quad \rightarrow \quad c = 6$

$$-x + 2y = 6$$



This extends to planes:

Trust me

Equation of plane:

$$n_1 x + n_2 y + n_3 z = c$$

The plane Π is perpendicular to the normal $\boldsymbol{n} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and passes through the point P with position vector $8\boldsymbol{i} + 4\boldsymbol{j} - 7\boldsymbol{k}$.
Find the Cartesian equation of Π.

$(8, 4, -7)$
$\;x \quad y \quad z$

$3x - 2y + z = c$

$3\times 8 - 2\times 4 - 7 = c$

$24 - 8 - 7 = c$

$\underline{\underline{c = 9}}$

$\underline{\underline{\Pi}}\quad \underline{\underline{3x - 2y + z = 9}}$

Ex 9B
2, 4, 5, 6, 9