Your notes

OCR A Level Physics

Gravitational Potential & Energy

Contents

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Gravitational Potential

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Gravitational Potential

Near the Earth's Surface

- The gravitational potential energy (G.P.E) is the energy an object has when lifted off the ground given by the familiar equation:

$$\text{G.P.E} = mg\Delta h$$

- When using this equation, the G.P.E on the surface of the Earth is taken to be zero
  - This means **work is done** to lift the object
- This equation can **only** be used for objects that are **near the Earth's surface**
  - This is because, near Earth's surface, the gravitational field is approximated to be **uniform**
  - Far away from the Earth's surface, the gravitational field is **radial** because the Earth is a **sphere**

In a Radial Field

- In a radial field, G.P.E is defined as the energy an object possesses due to its position in a gravitational field
- The gravitational potential at a point is the **gravitational potential energy per unit of mass** for an object at that point
- Gravity is always attractive, so work must be done on a mass to move it away to a point infinitely far away from every other mass
  - '**Infinity**' is the point at which the gravitational potential is **zero**
  - Therefore, since the potential energy of all masses **increases** as work is done on them to move them infinitely far away, the value of the potential is always **negative**
- Gravitational potential is formally defined as:

The work done per unit of mass in bringing a mass from infinity to a defined point

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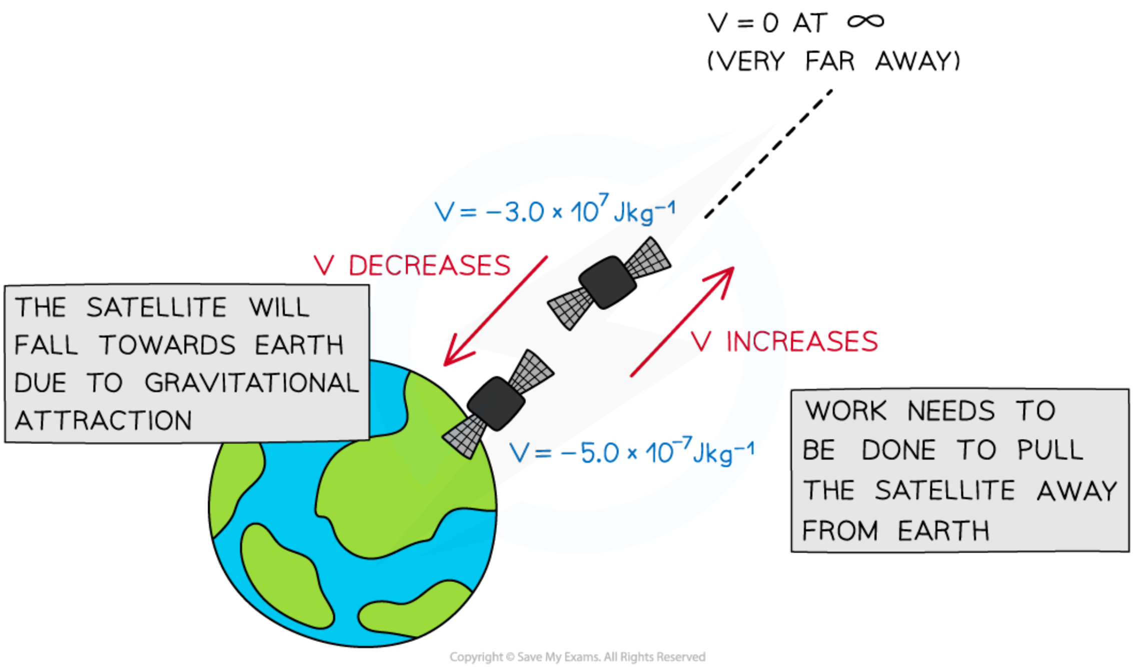

Gravitational potential decreases as the satellite moves closer to the Earth. It increases if it moves further away, towards infinity, where gravitational potential is zero

Examiner Tips and Tricks

A common exam question requires you to explain the 'negative sign' for values of gravitational potential. Remember the two key facts:

- Gravitational fields are always **attractive**
- It requires **work** to move a mass to infinity, where potential is **defined as zero**

Since the potential energy of a mass therefore **increases** as it moves toward infinity (where $V = 0$), the value of the potential everywhere else must be **negative**.

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Calculating Gravitational Potential

Calculating Gravitational Potential

Calculating Gravitational Potential

- Gravitational potential V_g can be calculated at a distance r from a point mass M using the equation:

$$V_g = -\frac{GM}{r}$$

- Where:
  - V_g = gravitational potential (J kg^{-1})
  - G = Newton's gravitational constant (N m^2 kg^{-2})
  - M = mass of the body causing the gravitational field (kg)
  - r = distance from the centre of mass of M to the point in the field (m)
- This means that the gravitational potential is negative on the surface of a mass (such as a planet), and **increases** with distance from that mass (becomes less negative toward zero)
- Work has to be done **against** the gravitational pull of the planet to take a unit mass **away** from the planet
- The gravitational potential at a point depends on the mass of the object producing the gravitational field and the distance the point is from that mass

Changes in Gravitational Potential

- Two points at different distances from a mass will have different gravitational potentials
  - This is because the gravitational potential **increases** with distance from a mass
- Therefore, there will be a **gravitational potential difference** between the two points
  - This is represented by the symbol ΔV
- ΔV can therefore be expressed as the difference between the 'final' gravitational V_f potential and the 'initial' gravitational potential V_i

$$\Delta V = V_f - V_i$$

- Therefore, the change in potential between two points a distance r_1 and r_2 from some mass M is given by:

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$$\Delta V = -\frac{GM}{r_2} - \left(-\frac{GM}{r_1}\right)$$

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- This simplifies to:

$$\Delta V = GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Where:
  - ΔV = change in potential (J kg^{-1})
  - G = Newton's gravitational constant (N m^2 kg^{-2})
  - M = mass causing the gravitational field (kg)
  - r_1 = **initial** distance from mass M (m)
  - r_2 = **final** distance from mass M (m)

Worked Example

Calculate gravitational potential at the surface of Mars.

Radius of Mars = 3400 km

Mass of Mars = 6.4×10^{23} kg

Answer:

Step 1: Write the gravitational potential equation

$$V_g = -\frac{GM}{r}$$

Step 2: Substitute known quantities

$$V_g = -\frac{(6.67 \times 10^{-11}) \times (6.4 \times 10^{23})}{3400 \times 10^3} = -1.3 \times 10^7 \text{ J kg}^{-1}$$

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Examiner Tips and Tricks

The equation for gravitational potential in a radial field looks very similar to the equation for gravitational field strength in a radial field, but there is a **very important difference**! Remember, for gravitational potential:

$$V_g = -\frac{GM}{r} \text{ so } V_g \propto -\frac{1}{r}$$

However, for gravitational field strength:

$$g = -\frac{GM}{r^2} \text{ so } g \propto -\frac{1}{r^2}$$

Additionally, remember that both V_g and g are measured from the **centre of the mass *M*** causing the field!

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Force-Distance Graph

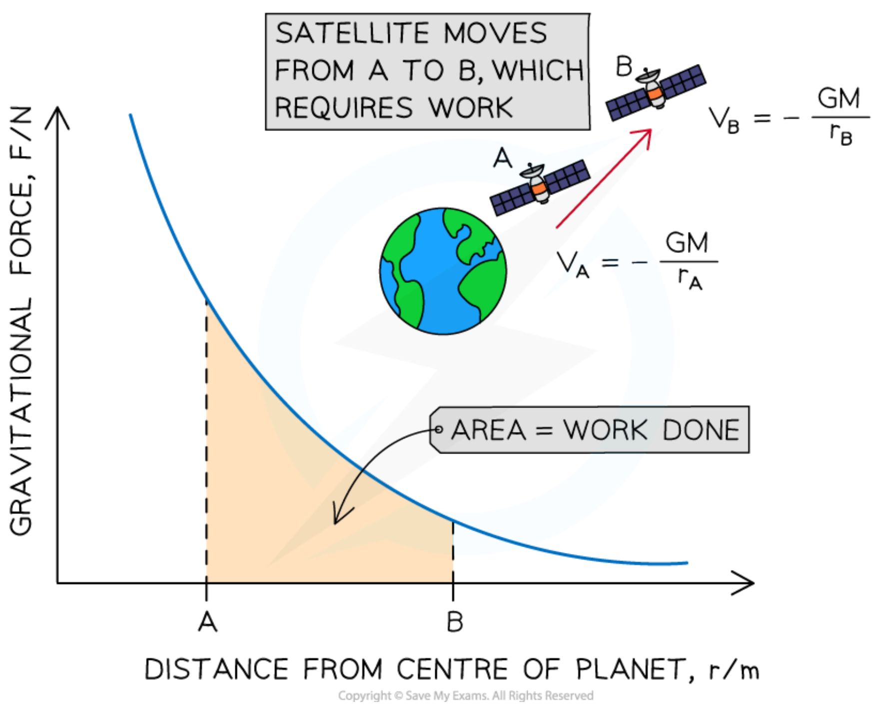
Force Distance Graphs for Point or Spherical Masses

- Recall that Newton's Law of Gravitation says the magnitude of the force F between a mass M and a mass m is given by the equation:

$$F = \frac{GMm}{r^2}$$

- Therefore, a **force-distance** graph would be a curve, because F is **inversely proportional** to r^2, or:

$$F \propto \frac{1}{r^2}$$



Work is done on the satellite of mass m to move it from A to B, because gravity is attractive. The area under the curve represents the magnitude of energy transferred

- The product of **force** and **distance** is equal to **work done** (or energy transferred)

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- Therefore, the **area** under the **force-distance** graph for gravitational fields is equal to the **work done**
  - In the case of a mass m moving further away from a mass M, the potential **increases**
  - Since gravity is attractive, this requires **work to be done** on the mass m
  - The area between two points under the **force-distance** curve therefore gives the change in **gravitational potential energy** of mass m

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Examiner Tips and Tricks

You should be able to interpret areas under curves by thinking about what the **product** of the quantities on the axes would represent. Since, in this case, **force × distance = work done**, then it follows that the **area** under the curve represents the change in **energy** between two points. Specifically, this would be a change in **gravitational potential energy**!

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Gravitational Potential Energy

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Gravitational Potential Energy

- The **gravitational potential** V_g at a point is given by:

$$V_g = \frac{-GM}{r}$$

- Recall that potential V is defined as the **energy required** to bring **unit mass** $\boldsymbol{m}$ from infinity to a defined point in the field
  - Recall that the gravitational field is usually **caused by 'big mass'**, M
- Therefore, the potential energy E is given by:

$$E = mV_g$$

- Substituting the equation for gravitational potential V_g gives the equation for the **gravitational potential energy** $\boldsymbol{E}$ between two masses M and m:

$$E = \frac{-GMm}{r}$$

- Where:
  - G = Universal Gravitational Constant (N m^2 kg^{-2})
  - M = mass causing the field (kg)
  - m = mass moving within the field of M (kg)
  - r = distance between the centre of m and M (m)

Calculating Changes in Gravitational Potential Energy

- When a mass is moved against the force of gravity, work is required
  - This is because gravity is **attractive**, therefore, energy is needed to work against this attractive force
- The work done (or energy transferred) ΔW to move a mass m between two different points in a gravitational potential ΔV is given by:

$$\Delta W = m\Delta V$$

- Where:

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- ΔW = work done or energy transferred (J)
- m = mass (kg)
- ΔV = change in gravitational potential (J kg^{-1})

- This work done, or energy transferred, is the change in **gravitational potential energy** (G.P.E) of the mass
  - When ΔV = 0, then the the change in G.P.E = 0
- The change in G.P.E, or work done, for an object of mass m at a distance r_1 from the centre of a larger mass M, to a distance of r_2 further away can be written as:

$$\Delta G.P.E = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right) = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Where:
  - M = mass that is producing the gravitational field (e.g., a planet) (kg)
  - m = mass that is moving in the gravitational field (e.g., a satellite) (kg)
  - r_1 = **initial** distance of m from the centre of M (m)
  - r_2 = **final** distance of m from the centre of M (m)
- Work is done when an object in a planet's gravitational field moves **against** the gravitational field lines, i.e., away from the planet
  - This is, again, because gravity is **attractive**
  - Therefore, energy is required to move **against** gravitational field lines

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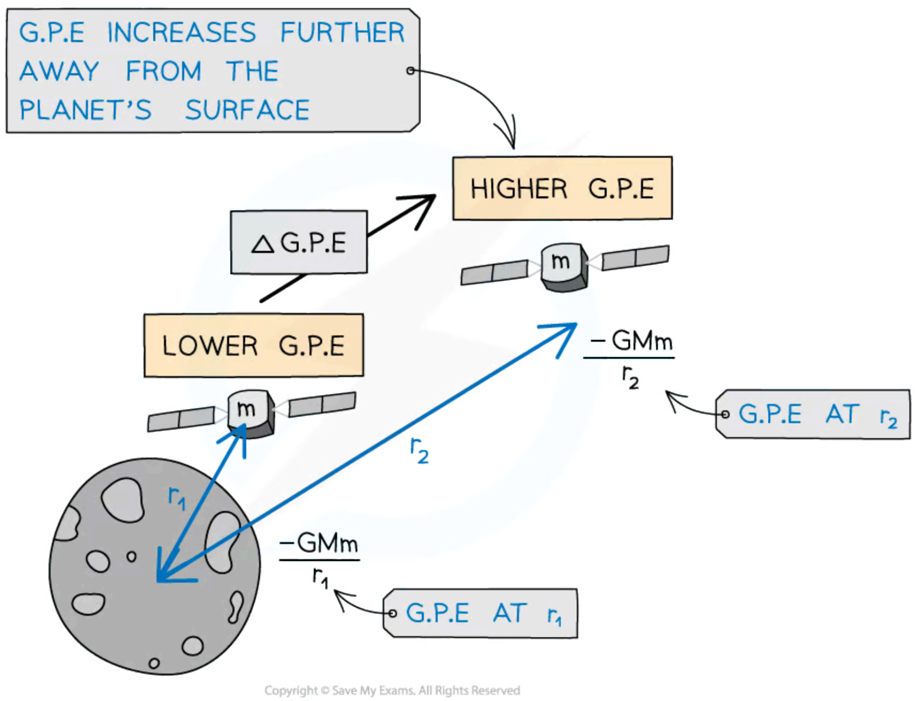

Gravitational potential energy increases as a satellite leaves the surface of the Moon

Worked Example

A spacecraft of mass 300 kg leaves the surface of Mars to an altitude of 700 km. Calculate the work done by the spacecraft.

Radius of Mars = 3400 km

Mass of Mars = 6.40×10^{23} kg

Answer:

Step 1: Write down the work done (or change in G.P.E) equation

$$\Delta G.P.E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Step 2: Determine values for r_1 and r_2

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r_1 is the radius of Mars = 3400 km = 3400 × 10^3 m

r_2 is the radius + altitude = 3400 + 700 = 4100 km = 4100 × 10^3 m

Step 3: Substitute in values

$$\Delta G.P.E = (6.67 \times 10^{-11}) \times (6.40 \times 10^{23}) \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3}\right)$$

$$\Delta G.P.E = 643.076 \times 10^6 = 640 \text{ MJ (2 s.f.)}$$

Examiner Tips and Tricks

Make sure to not confuse the ΔG.P.E equation with

ΔG.P.E = mgΔh

The above equation is only relevant for an object lifted in a uniform gravitational field (close to the Earth's surface). The new equation for G.P.E will not include *g*, because this varies for different planets and is no longer a constant (decreases by $1/r^2$) outside the surface of a planet.

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Escape Velocity

Escape Velocity

- To escape a gravitational field, a mass must travel at the **escape velocity**
- This is dependent on the **mass** and **radius** of the object creating the gravitational field, such as a planet, a moon or a black hole
- Escape velocity is defined as:

  The minimum speed that will allow an object to escape a gravitational field with no further energy input

- It is the **same** for **all masses** in the **same gravitational field** i.e., the escape velocity of a rocket from Earth is **the same** as a tennis ball
- An object reaches escape velocity when all its **kinetic energy** has been transferred to **gravitational potential energy**
- Mathematically, equating the kinetic energy to gravitational potential energy gives:

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

- Where:
  - m = mass of the object in the gravitational field of mass M (kg)
  - v = escape velocity of the object (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object to be escaped from, causing the gravitational field (i.e., a planet) (kg)
  - r = distance from the centre of mass of M (m)

- Since mass m is the same on both sides of the equation, it can be cancelled
  - This is the reason why the escape velocity is the same for any object in the gravitational field of M
  - Therefore, the equation simplifies to give:

$$\frac{1}{2}v^2 = \frac{GM}{r}$$

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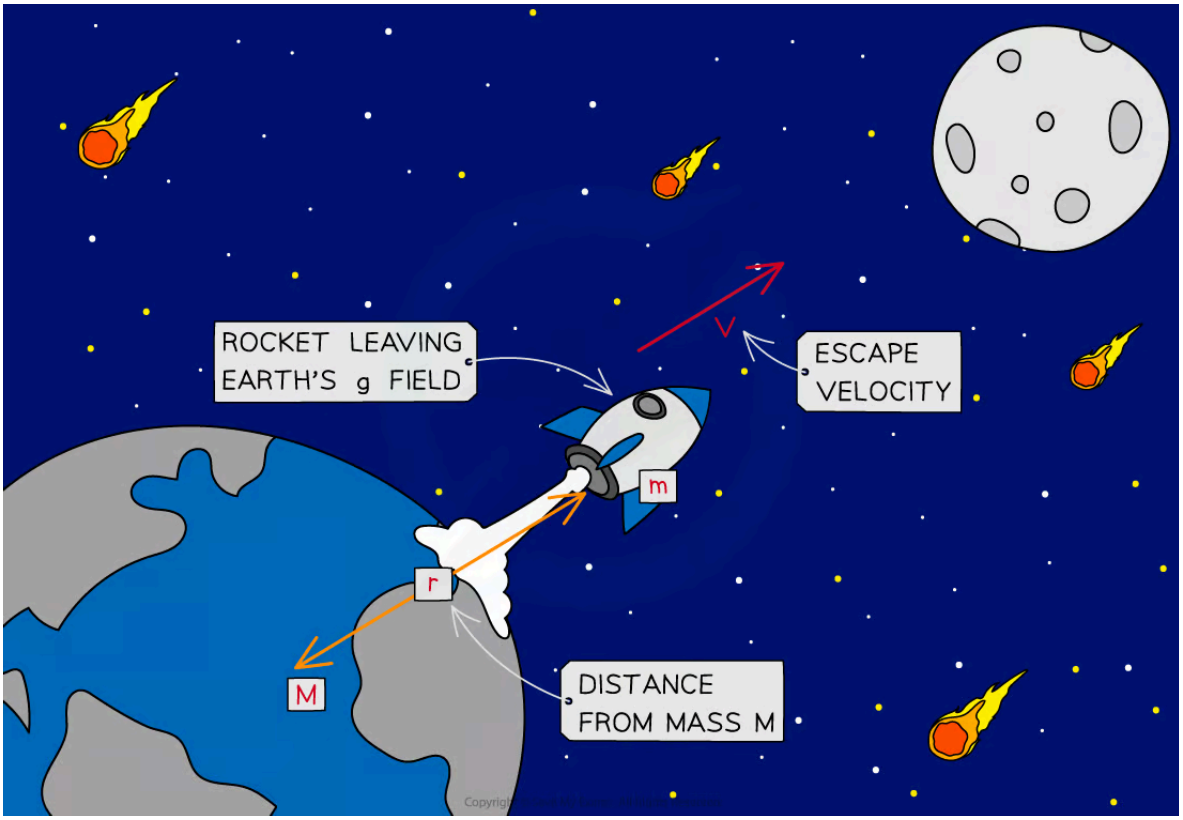
- Rearranging this equation gives the **escape velocity**, *v*:

$$v = \sqrt{\frac{2GM}{r}}$$

- This equation is **not** given: be sure to memorise how to derive it



In order to escape Earth's gravitational field, objects have to travel greater than Earth's escape velocity. However, rockets burn fuel continuously, so they are given sufficient energy to escape while travelling significantly less than escape velocity

- Rockets launched from the Earth's surface do **not** need to achieve escape velocity to reach their orbit around the Earth
- This is because:
  - They are given energy through fuel **continuously** to provide **thrust**
  - **Less energy is needed** to achieve **orbit** than to **escape** from Earth's gravitational field

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- The escape velocity is **not** the velocity needed to escape the planet but to escape the planet's **gravitational field** altogether
  - This could be quite a large distance away from the planet

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Worked Example

Calculate the escape velocity at the surface of the Moon given that its density is 3340 kg m^{-3} and has a mass of 7.35×10^{22} kg.

Newton's Gravitational Constant = 6.67×10^{-11} N m^2 kg^{-2}

Answer:

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Step 1: Rearrange the density equation for radius

$$\rho = \frac{M}{V} \;\&\; V = \frac{4}{3}\pi r^3$$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

$$r = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

Step 2: Calculate the radius by substituting in the values

$M = 7.35 \times 10^{22}$ kg

$\rho = 3340$ kg m^{-3}

$$r = \sqrt[3]{\frac{3 \times (7.35 \times 10^{22})}{4\pi \times 3340}} = 1.7384 \times 10^6 \text{ m}$$

Step 3: Substitute *r* into escape velocity equation

$$v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times \left(6.67 \times 10^{-11}\right) \times (7.35 \times 10^{22})}{1.7384 \times 10^6}}$$

$$\mathbf{v = 2.37 \text{ km s}^{-1}}$$

Examiner Tips and Tricks

When writing the definition of **escape velocity**, avoid terms such as 'gravity' or the 'gravitational pull / attraction' of the planet. It is best to refer to its gravitational field.

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