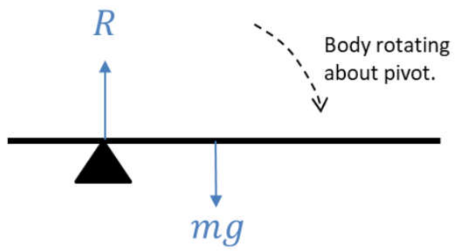
Rigid Bodies

We previously dealt with particles, where each object was modelled just as a single point, and considered forces acting on each point separately.



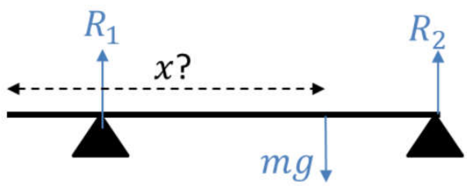
In this chapter we consider rigid bodies (in this case **rods**), which takes into account the size of the object. This means we can consider other properties, e.g. **rotation** of the body.

1:: Moments in equilibrium

Clockwise moment = Anticlockwise moment

2:: Centre of Mass

For a 'non-uniform' rod we can't model its weight as acting at the centre.



3:: On the point of Tilting

"A non uniform wooden plank of mass M kg rests horizontally on supports at A and B, as shown. When a bucket of water of mass 18kg is placed at point C, the plank is in equilibrium, and is **on the point of tilting** about B. Find the value of M and the magnitude of the reaction at B."

Key Questions

a) What does it mean if a rod/beam is uniform?

b) What does it mean if a rod/beam is non-uniform?

c) What does it mean if a rod/beam is on the point of tilting about A?

d) Why might you take moments from different points on the beam?

e)There are only 2 things you can do in a moments question. What are they?

Moments

The ‘moment’ of a force measures the **turning effect** of the force on the body on which it is acting.

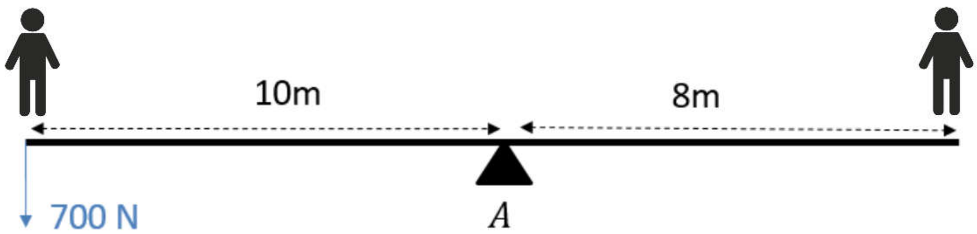
$$moment\ of\ force = force \times distance$$

about a point

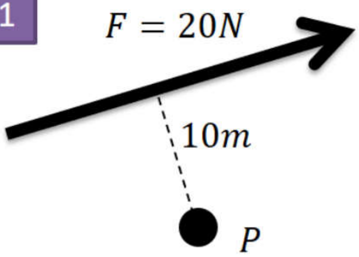
perpendicular distance

What happens?

Tom has a mass of 75kg.

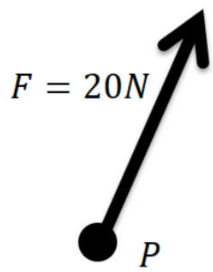


1



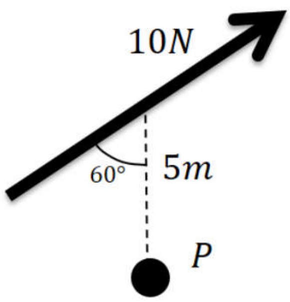
Moment of force F about point P

2



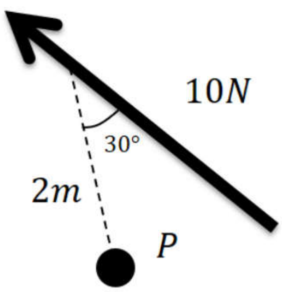
Moment of force F about point P

3



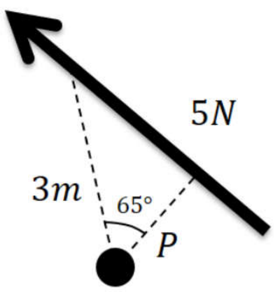
Moment of force F about point P

4



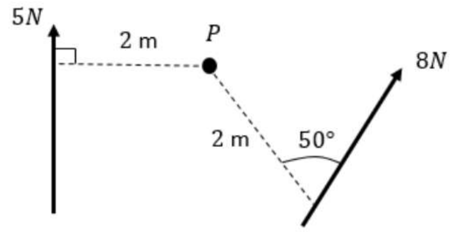
Moment:

5



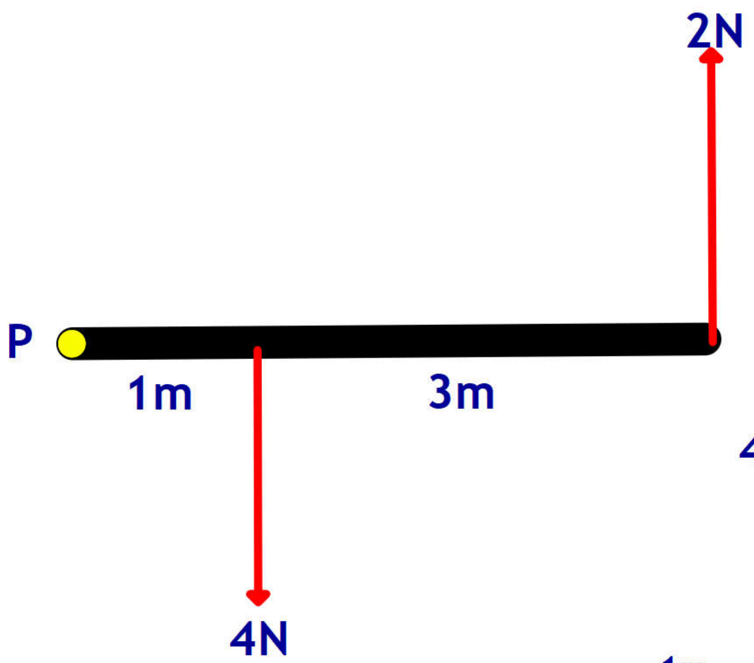
Moment:

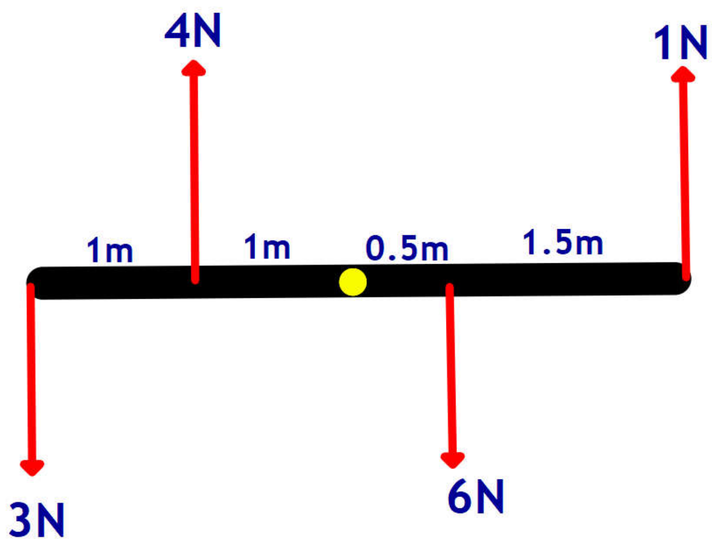
The diagram shows two forces acting on a lamina. Find the moment of each of the forces about the point P.



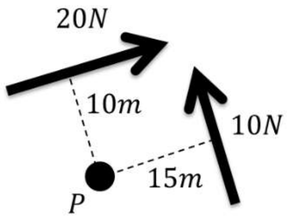
Terminology: A *lamina* is a 2D object whose thickness can be ignored.

Resultant Moments





1



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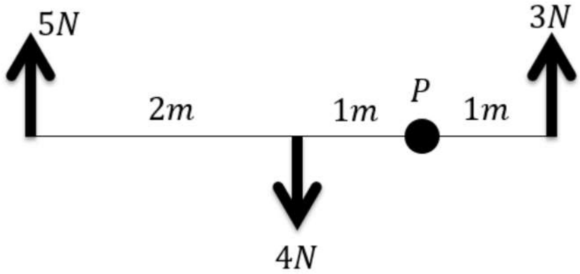

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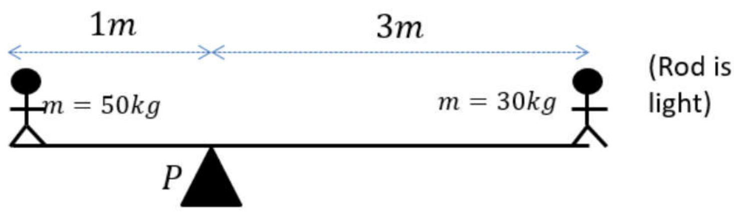
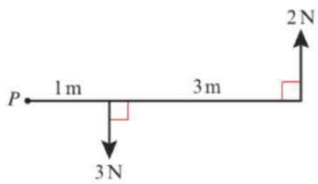


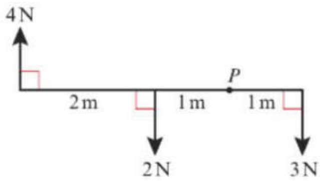
Exercise 4B

1 These diagrams show sets of forces acting on a light rod. In each case, calculate the resultant moment about P.

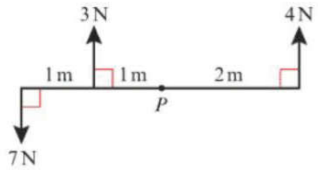
a



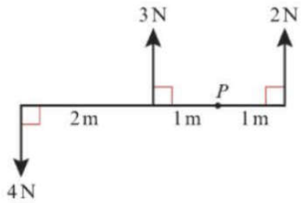
b



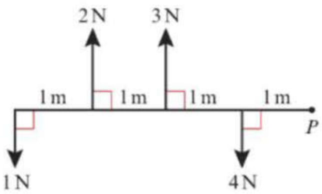
c



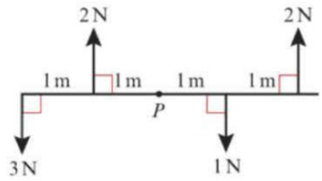
d



e

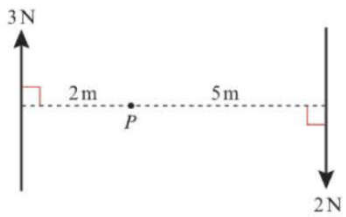


f

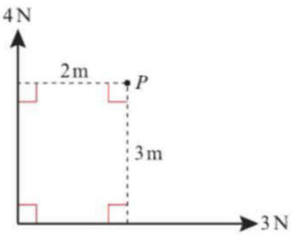


2 These diagrams show forces acting on a lamina. In each case, find the resultant moment about P.

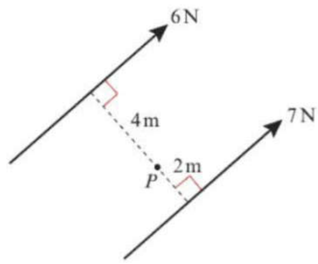
a



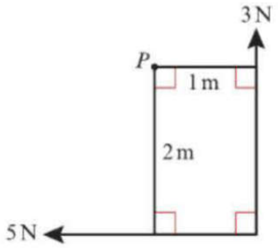
b



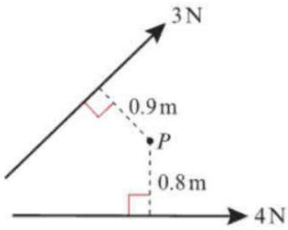
c



d



e



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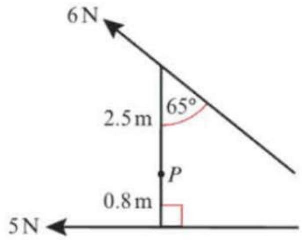
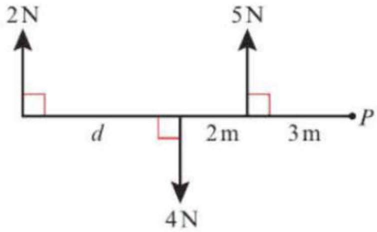
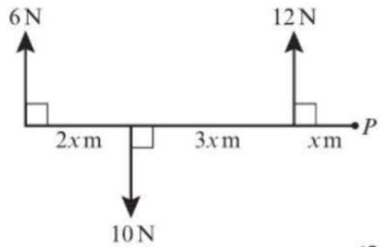


(P) **3** The diagram shows a set of forces acting on a light rod.
The resultant moment about P is 17 N m clockwise.
Find the length, d.



(E/P) **4** The diagram shows a set of forces acting on a light rod.
The resultant moment about P is 12.8 N m clockwise.
Find the value of x.



(3 marks)

Exercise 4B

1 a 5 Nm anticlockwise b 13 Nm clockwise
c 19 Nm anticlockwise d 11 Nm anticlockwise
e 4 Nm clockwise f 7 Nm anticlockwise
2 a 16 Nm clockwise b 1 Nm anticlockwise
c 10 Nm clockwise d 7 Nm clockwise
e 0.5 Nm anticlockwise f 9.59 Nm anticlockwise
3 6 m
4 1.6

This entire topic summarised…

If a rigid body is in **equilibrium** then:

a The resultant force in any direction is 0.
b The resultant moment about any point is 0.

i.e. Forces up = forces down, as per Year 1

In other words, clockwise moments = anticlockwise moments

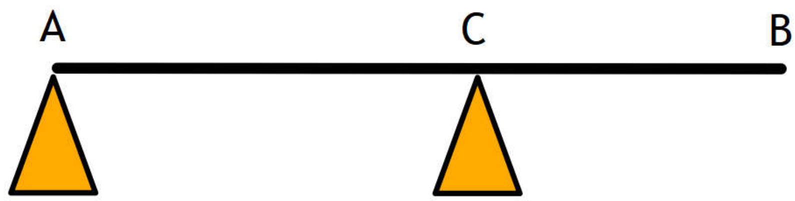
You will typically use <u>**both**</u> these properties to solve exam questions.

Rigid Bodies in Equilibrium

Where does weight act on a body?

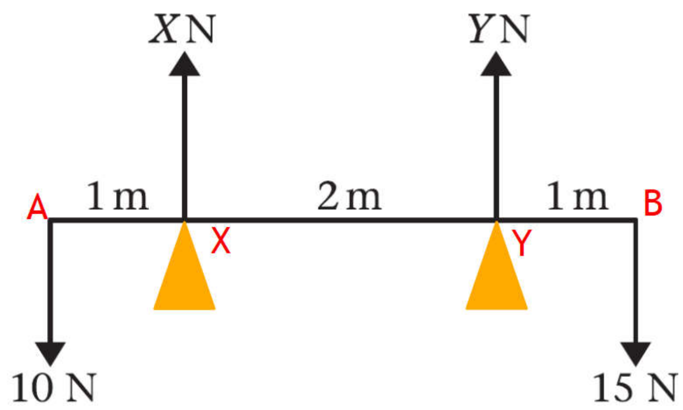
A uniform rod AB of length 3m and weight 20N rests horizontally on supports at A and C, where AC = 2m. (see diagram)

Calculate the magnitude of the reaction at each of the supports.



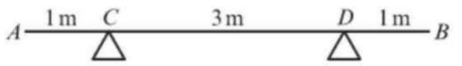
Weights and reactions
Moments
Resolving vertically

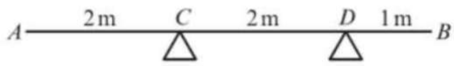
Given that the rod is in equilibrium, find the values of X and Y

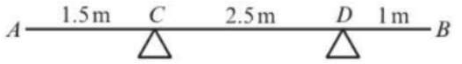


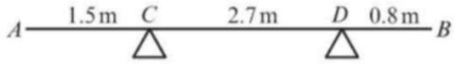
Exercise 4C

1 AB is a uniform rod of length 5 m and weight 20 N. In these diagrams AB is resting in a horizontal position on supports at C and D. In each case, find the magnitudes of the reactions at C and D.

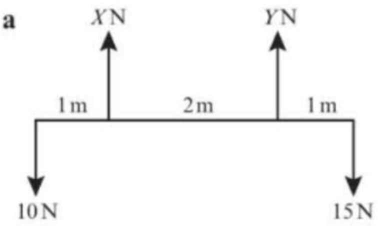
a


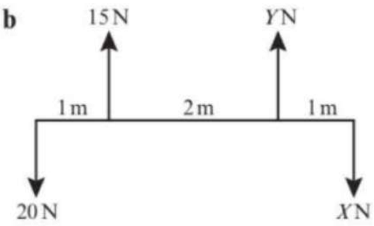
b


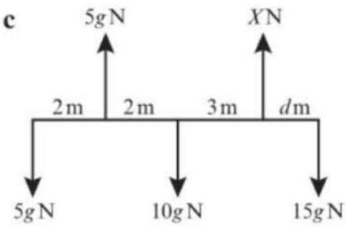
c


d


2 Each of these diagrams shows a light rod in equilibrium in a horizontal position under the action of a set of forces. Find the values of the unknown forces and distances.

a


b


c


3 Jack and Jill are playing on a seesaw made from a uniform plank AB, of length 5 m pivoted at M, the midpoint of AB. Jack has mass 35 kg and Jill has mass 28 kg. Jill sits at A and Jack sits at a distance x m from B. The plank is in equilibrium. Find the value of x.

Exercise 4C

1 **a** 10 N, 10 N **b** 15 N, 5 N
c 12 N, 8 N **d** 12.6 N, 7.4 N
2 **a** 7.5, 17.5 **b** 30, 35 **c** 245, $2\frac{2}{3}$
3 0.5 m

Rayhan and Luke are having fun on a **uniform** seesaw of mass 20kg. Fouad weighs 70kg and is 10m from the pivot. Luke is 8m from the pivot. The seesaw remains horizontal.

a) Determine the reaction force at the pivot of the seesaw.

b) Determine Luke's mass.

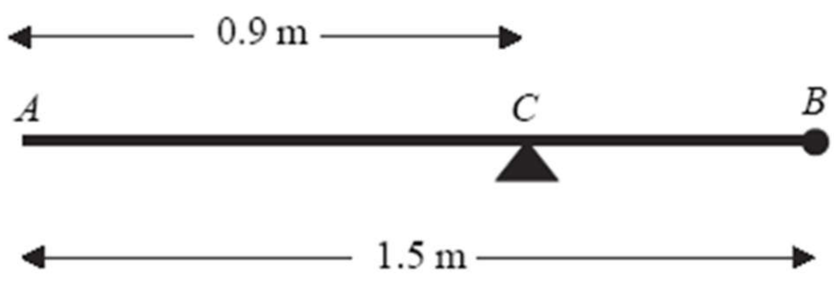
Weights and reactions
Moments
Resolving vertically

e.g.

A uniform beam AB, of mass 40kg and length 5m, rests horizontally on supports at C and D, where AC=DB=1m.
When a man of 80kg stands on the beam at E the magnitude of the reaction at D is twice the magnitude of the reaction at C.
Find the distance AE.

Weights and reactions
Moments
Resolving vertically

Your Turn



A uniform rod AB has length 1.5 m and mass 8 kg. A particle of mass m kg is attached to the rod at B. The rod is supported at the point C, where $AC = 0.9$ m, and the system is in equilibrium with AB horizontal, as shown in Figure 2.

(*a*) Show that $m = 2$.

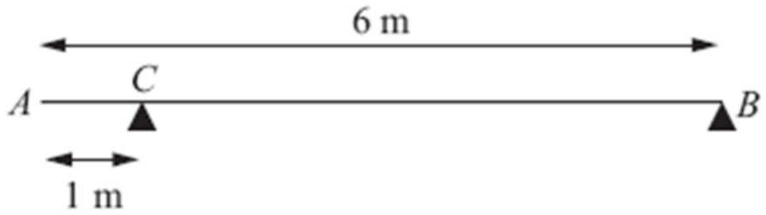
(4)

A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

(*b*) Find the distance AD.

(5)

Your Turn



A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in Figure 1. The beam is modelled as a rod.

(*a*) Find the magnitudes of the reactions on the beam at B and at C.

(5)

A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

(*b*) Find the distance AD.

(5)

A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

(*a*) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

(10)

(*b*) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.

(1)

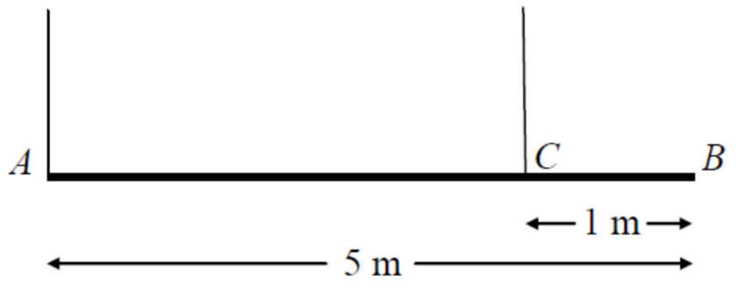
Your Turn

A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D, where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B.

(7)

Hanging Rods/Beams

A uniform rod AB of length 4 m and weight 20 N is suspended horizontally by two vertical strings attached at A and at B. A particle of weight 10 N is attached to the rod at point C, where $AC = 1.5$ m. Find the magnitudes of the tensions in the two strings.



A beam AB has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A, the other to the point C on the beam, where $BC = 1$ m, as shown in Figure 2. The beam is modelled as a uniform rod, and the ropes as light strings.

(a) Find

(i) the tension in the rope at C,

(ii) the tension in the rope at A.

(5)

A small load of mass 16 kg is attached to the beam at a point which is y metres from A. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(b) find, in terms of y, an expression for the tension in the rope at C.

(3)

The rope at C will break if its tension exceeds 98 N. The rope at A cannot break.

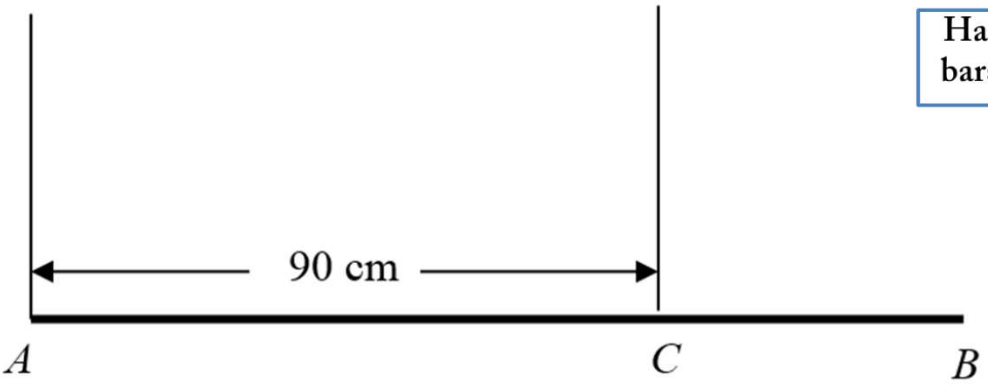
(c) Find the range of possible positions on the beam where the load can be attached without the rope at C breaking.

(3)

June 2006

Your Turn

Hanging bars/rods



A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other able is attached to the point C on the girder, where $AC = 90$ cm, as shown in Figure 3. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(a) the tension in the cable at A,

(2)

(b) show that $AB = 120$ cm.

(4)

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

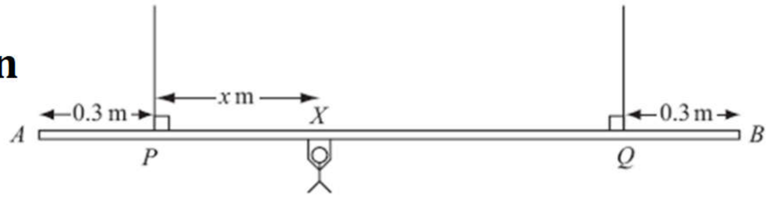
(c) Find the value of W.

(7)

May 2009

Your Turn

Hanging bars/rods



A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(*a*) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(*b*) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(*c*) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

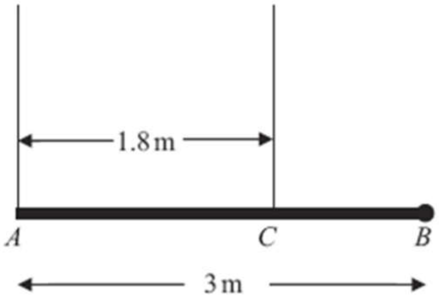
Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(*d*) find the value of x.

(3)

Jan 2010

Your Turn



Hanging bars/rods

A pole AB has length 3 m and weight W newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points A and C where $AC = 1.8$ m, as shown in Figure 2. A load of weight 20 N is attached to the rod at B. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(*a*) Show that the tension in the rope attached to the pole at C is $\left(\frac{5}{6}W + \frac{100}{3}\right)$ N.

(4)

(*b*) Find, in terms of W, the tension in the rope attached to the pole at A.

(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

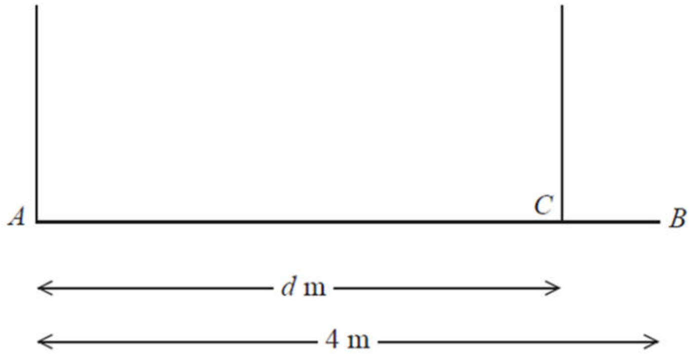
(*c*) find the value of W.

(3)

June 2014

Your Turn

Hanging bars/rods



A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(*a*) Find the value of d.

(6)

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(*b*) Find the value of k.

(6)

Non-uniform Rods

What if the rod is non-uniform? We cannot assume that its centre of mass lies at the centre! We must have its weight acting at a different point...

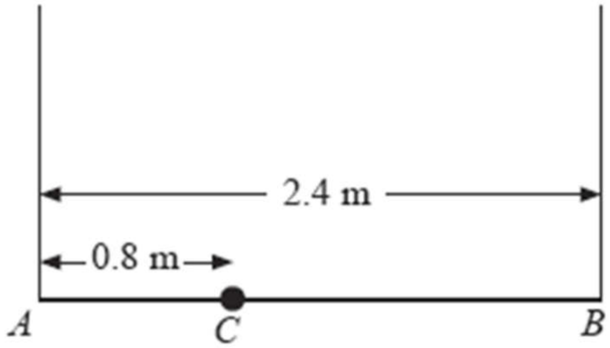
e.g. A rod AB is 3m long and has weight 20N. It is in a horizontal position resting on supports at points C and D, where AC = 1m and AD = 2.5m. The magnitude of the reaction at C is three times the magnitude of the reaction at D. Find the distance of the centre of mass of the rod from A.

Hint: take moments about the point where you have an unknown

May 2008

Your Turn

Non-uniform



A plank AB has mass 12 kg and length 2.4 m. A load of mass 8 kg is attached to the plank at the point C, where $AC = 0.8$ m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in Figure 2. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.

(*a*) Find the tension in the rope attached at B.

(4)

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 10 N greater than the tension in the rope attached at B.

(*b*) Find the distance of the centre of mass of the plank from A.

(6)

Jan 2012 **Your Turn**

Non-uniform

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(*a*) Show that $GD = \frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(*b*) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

On the Point of Tilting/Tipping

A uniform rod AB of length 4m and mass 12kg is resting in a horizontal position on supports at C and D, with AC=DB=0.5m.
When a particle of mass *m*kg is placed on the rod at point B the rod is on the point of turning about D. Find the value of *m.*

✎ When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or tension in any other wire/string) is zero.

8 A uniform rod *AB* has length 4 m and mass 8 kg. It is resting in a horizontal position on supports at points *C* and *D* were *AC* = 1 m and *AD* = 5 m. A particle of mass *m* kg is placed at point *E* where *AE* = 3.3 m. Given that the rod is about to tilt about *D*, calculate the value of *m*.

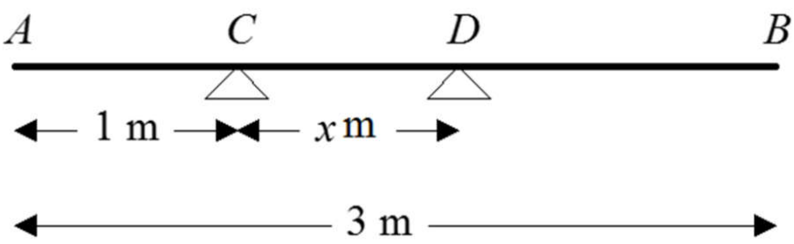
9 A uniform bar *AB* of length 6 m and weight 40 N is resting in a horizontal position on supports at points *C* and *D* where *AC* = 2 m and *AD* = 2.5 m. When a particle of weight 30 N is attached to the bar at point *E* the bar is on the point of tilting about *C*. Calculate the distance *AE*.

10 A plank *AB* of mass 12 kg and length 3 m is in equilibrium in a horizontal position resting on supports at *C* and *D* where *AC* = 0.7 m and *DB* = 1.1 m. A boy of mass 32 kg stands on the plank at point *E*. The plank is about to tilt about *D*. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance *AE*.

5, 2/3, 2.05

Jan 2007

Your Turn



On the point of tipping

A uniform plank AB has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $CD = x$ m, as shown in Figure 2. The reaction of the support on the plank at D has magnitude 80 N. Modelling the plank as a rod,

(*a*) show that $x = 0.75$.

(3)

A rock is now placed at B and the plank is on the point of tilting about D. Modelling the rock as a particle, find

(*b*) the weight of the rock,

(4)

(*c*) the magnitude of the reaction of the support on the plank at D.

(2)

(*d*) State how you have used the model of the rock as a particle.

(1)

Your Turn

6. A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S.

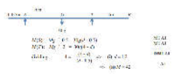
When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T.

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i) the value of d,

(ii) the value of M.

(7)



4.

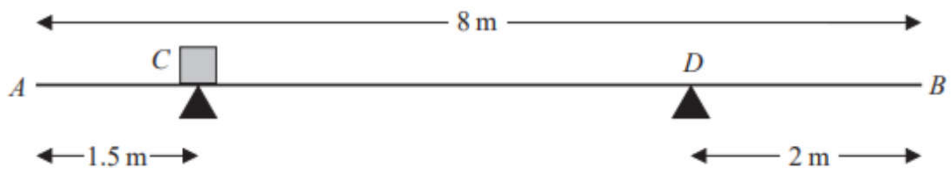


Figure 2

A plank AB of mass 20 kg and length 8 m is resting in a horizontal position on two supports at C and D, where $AC = 1.5$ m and $DB = 2$ m. A package of mass 8 kg is placed on the plank at C, as shown in Figure 2. The plank remains horizontal and in equilibrium. The plank is modelled as a uniform rod and the package is modelled as a particle.

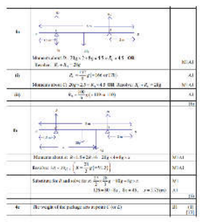
(a) Find the magnitude of the normal reaction

(i) between the plank and the support at C,

(ii) between the plank and the support at D.

(6)

The package is now moved along the plank to the point E. When the package is at E, the magnitude of the normal reaction between the plank and the support at C is R newtons and the magnitude of the normal reaction between the plank and the support at D is $2R$ newtons.



(b) Find the distance AE.

(6)

(c) State how you have used the fact that the package is modelled as a particle.

(1)