A Level · Edexcel · Maths

3 hours · 30 questions

4.1 Moments (A Level only)

Easy (6 questions)	/44
Medium (8 questions)	/52
Hard (8 questions)	/55
Very Hard (8 questions)	/58
Total Marks	**/209**

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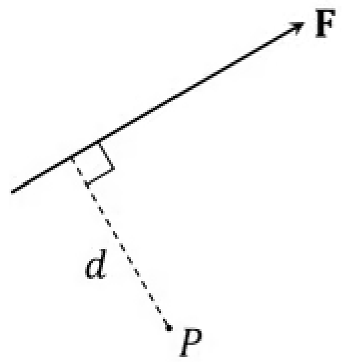
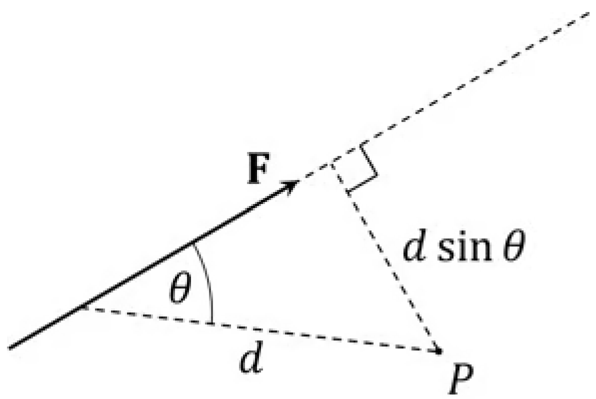
Easy Questions

1 The *moment* of a force about a given point is found by multiplying the magnitude of the force by the perpendicular distance from the point to the line of action of the force:



$$\text{Clockwise moment of } \mathbf{F} \text{ about } P = |\mathbf{F}| \times d$$

If the distance known is not the perpendicular distance, then trigonometry may be used to find the perpendicular distance and calculate the moment:

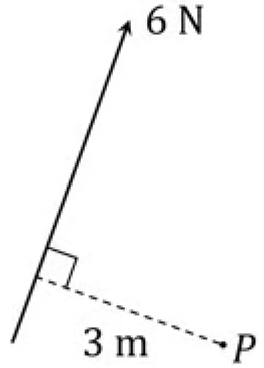


$$\text{Clockwise moment of } \mathbf{F} \text{ about } P = |\mathbf{F}| \times d \sin \theta$$

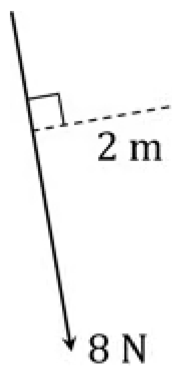
Note that the direction, clockwise or anticlockwise, must be specified when talking about a moment. The standard units for moments are newton metres $(\mathrm{N\ m})$.

Calculate the moment about P of the forces indicated in each of the following diagrams:

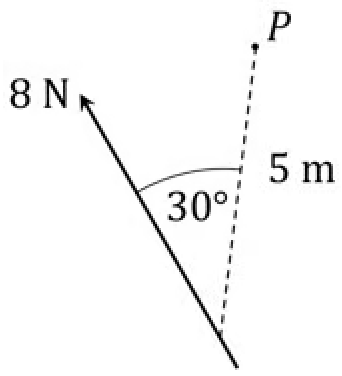
(i)



(ii)



(iii)



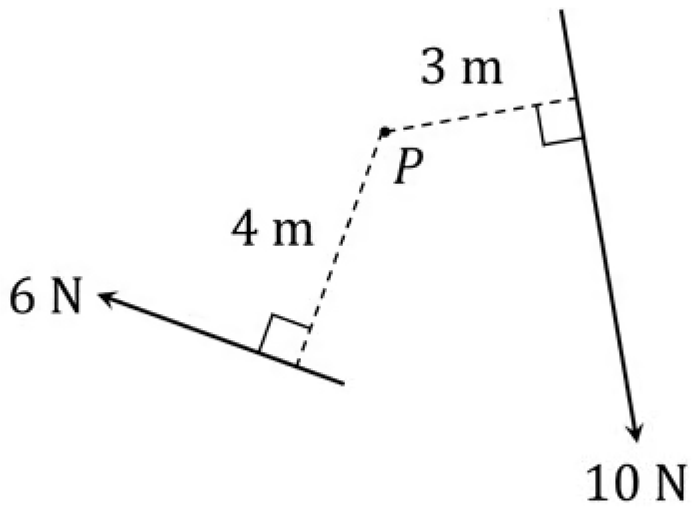
(7 marks)

2 If a number of forces acting on a body all act in the same plane, it is possible to calculate the *resultant moment* of the forces about a given point. he resultant of a number of moments about a point is the total moment about that point.

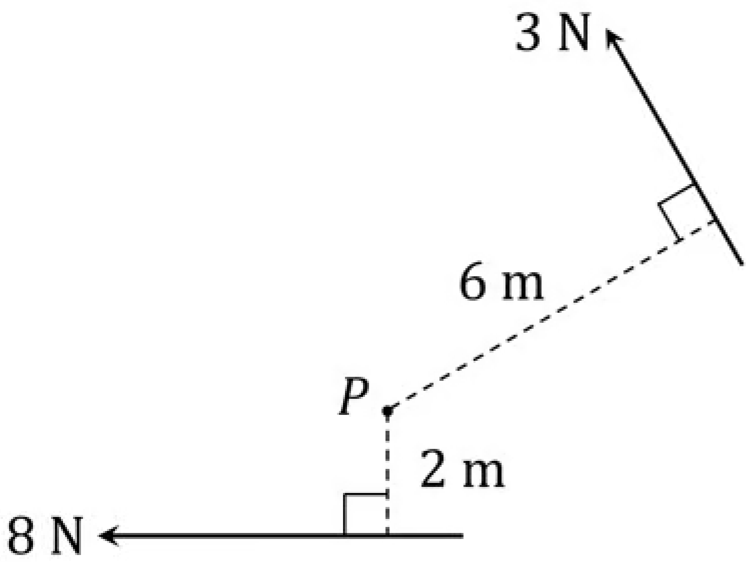
To calculate the resultant moment, a 'positive' direction – clockwise or anticlockwise – must first be chosen. If clockwise is the positive direction, then the resultant moment in the clockwise direction is the sum of all the clockwise moments minus the sum of all the anticlockwise moments. If anticlockwise is the positive direction, then the resultant moment in the anticlockwise direction is the sum of all the anticlockwise moments minus the sum of all the clockwise moments. A negative result means that the resultant moment is in the 'negative' direction.

Calculate the resultant moment about P of the forces indicated in each of the following diagrams:

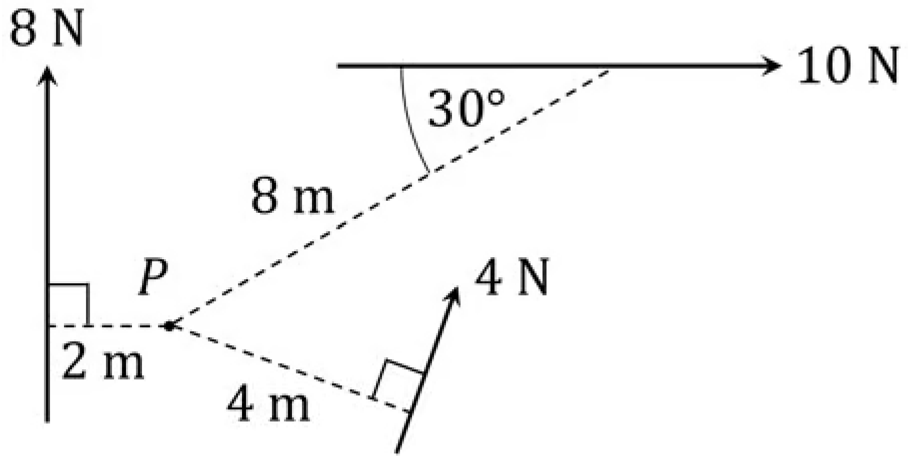
(i)



(ii)



(iii)

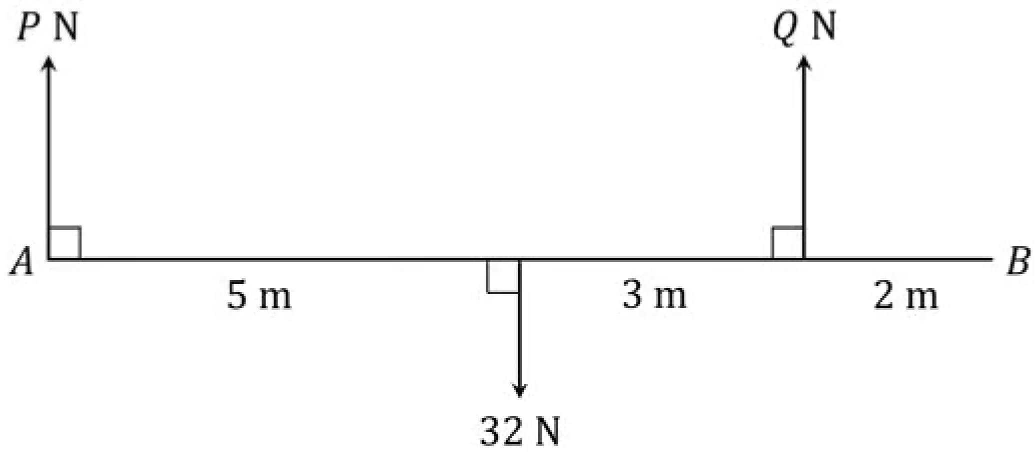


(7 marks)

3 (a) A rigid body is said to be *in equilibrium* when the total force in any direction is zero and the total moment about any point is zero.

In problems involving rigid bodies, a judicious choice of which point to take the moments about can often simplify the problem.

In the following diagram AB is a light rod held in equilibrium by the three forces indicated:



(i) By considering the total force perpendicular to AB and the total moment about point B, show that the following simultaneous equations must hold:

$$P + Q = 32$$
$$10P + 2Q = 160$$

(ii) Solve the simultaneous equations in part (i) to find the values of P and Q.

(4 marks)

(b) Solve to find the values of P and Q by instead considering the total moment about point A.

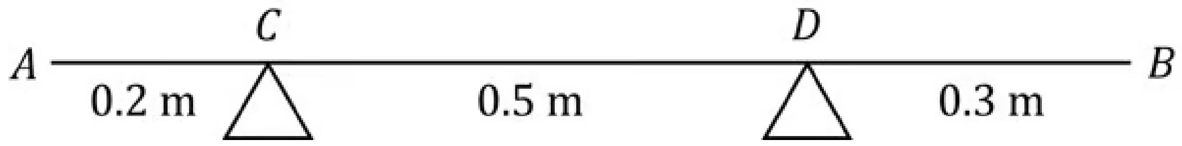
(3 marks)

(c) By comparing the methods used in parts (a) and (b), explain why it was more convenient to choose A as the point to take the moments about.

(1 mark)

4 (a) In rigid body problems involving rods, the weight of the rod may always be represented by a single force vector acting vertically downwards at the *centre of mass* of the rod. If the rod is a *uniform rod* then the centre of mass is at the midpoint. For a non-uniform rod, however, the centre of mass may lie anywhere along the length of the rod.

The following diagram depicts a rod AB of length 1 m and weight 30 N held horizontally in equilibrium by two supports at points C and D:



Besides the weight of the rod, the only forces acting on the rod are the reaction forces from the supports acting vertically upwards at points C and D.

For the case that AB is a uniform rod, calculate the magnitude of the reaction forces at points C and D.

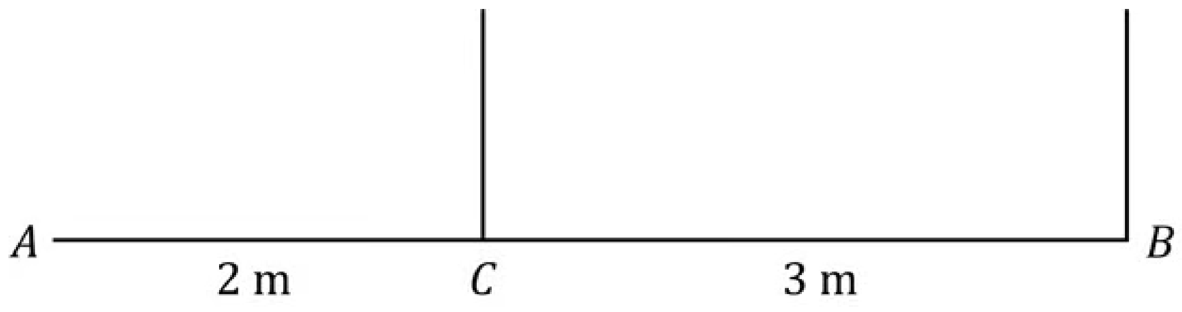
(3 marks)

(b) For the case that AB is a non-uniform rod with its centre of mass 0.1 m to the right of point C, calculate the magnitude of the reaction forces at points C and D.

(3 marks)

5 (a) When a rigid body is on the point of tilting or rotating about a pivot point, it means that the reaction force at any other support, or the tension in any other supporting wire or string, is zero.

In the diagram below AB is a uniform rod of length 5 m and weight 120 N. AB is held horizontally in equilibrium by two wires, one of which is attached at point B and the other of which is attached at point C where $AC = 2\text{m}$ as shown:



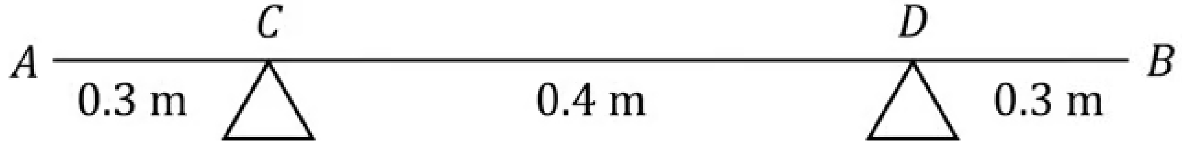
A particle of weight 30 N is attached to the rod at point A, and the rod remains horizontally in equilibrium.

(i) By considering moments around point C, show that the rod is on the point of tilting about C.

(ii) Write down the tension in the wire attached at point C.

(4 marks)

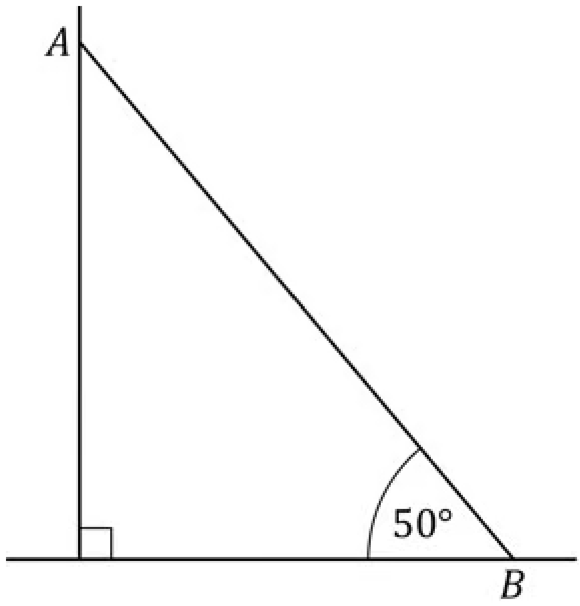
(b) In the diagram below AB is a uniform rod of length 1 m which rests horizontally on supports placed 0.3 m from either end at points C and D as shown:



A particle of weight 24 N is placed at point B, and the rod is then at the point of rotating about D. By considering the moments about D, determine the weight of rod AB.

(3 marks)

6 (a) In the following diagram AB is a ladder of length 10 m and mass 34 kg. End A of the ladder rests on a smooth vertical wall, while end B rests on rough horizontal ground. The ladder rests in limiting equilibrium at an angle of $50°$ with the ground, as shown below:



The ladder is modelled as a uniform rod lying in a vertical plane perpendicular to the wall. This means there are four forces acting on the ladder that need to be considered:

- the normal reaction force R_A exerted by the wall on the ladder at point A.
- the weight of the ladder W acting at the centre of mass
- the normal reaction force R_B exerted by the ground on the ladder at point B.
- the force of friction F_f between the ground and the ladder at point B.

The coefficient of friction between the ground and the ladder is μ.

By considering forces acting in the horizontal and vertical directions separately, show that $R_A = F_f$ and $R_B = W$.

(2 marks)

(b) By considering the moments about B, show that $10R_A \sin 50 = 5W \cos 50$.

(4 marks)

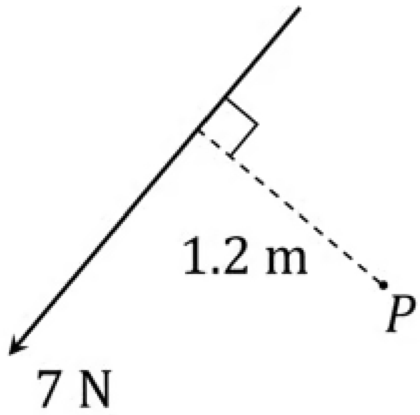
(c) Use your answers to parts (a) and (b) to work out the value of μ.

(3 marks)

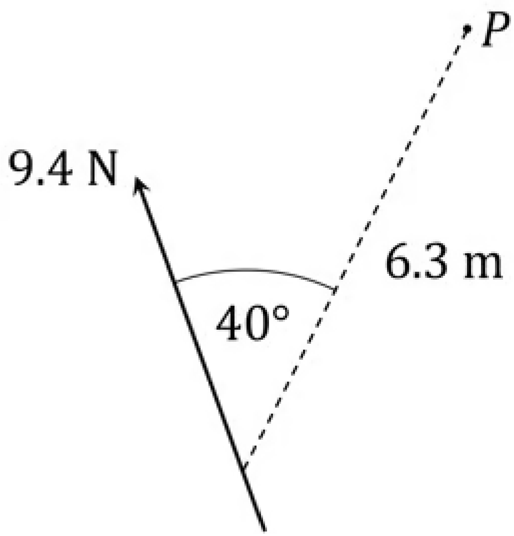
Medium Questions

1 In each of the following examples the force indicated is acting on a lamina. Calculate the moment about the point P in each case.

(i)

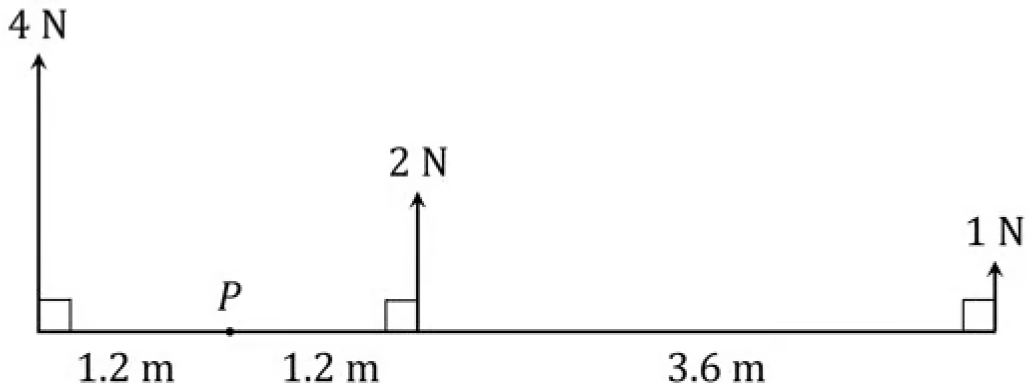


(ii)



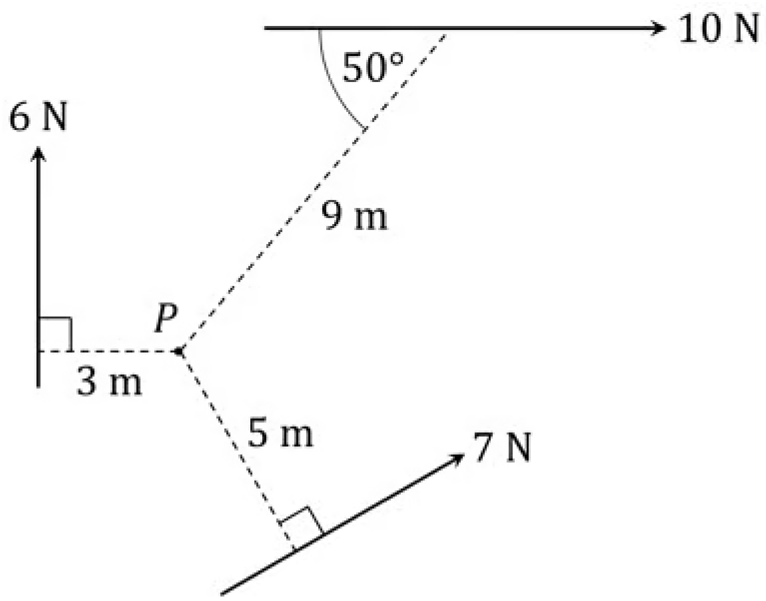
(5 marks)

2 (a) The diagram below shows a set of forces acting on a light rod. Calculate the resultant moment about the point P.



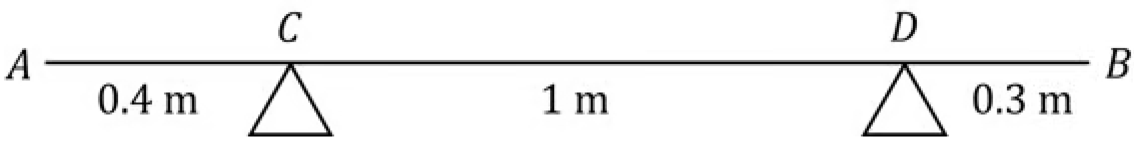
(3 marks)

(b) The diagram below shows a set of forces acting on a lamina. Calculate the resultant moment about the point P.



(4 marks)

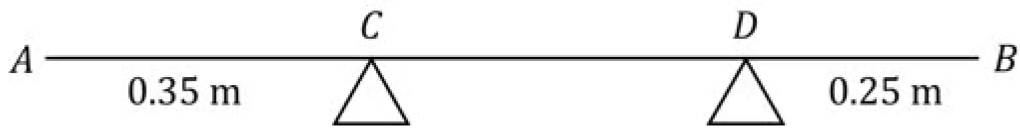
3 AB is a uniform rod of length 1.7 m and weight 50 N. AB rests horizontally on supports placed at points C and D, with $AC = 0.4$ m, $CD = 1$ m and $DB = 0.3$ m as shown in the diagram below:



Calculate the magnitude of the reaction force at each of the support points.

(4 marks)

4 (a) AB is a non-uniform rod of length 1 m and weight 30 N. AB rests horizontally on supports placed at points C and D, with $AC = 0.35 \text{ m}$ and $DB = 0.25 \text{ m}$ as shown in the diagram below:



Given that the centre of mass of AB is 0.45 m from point A, calculate the reaction force at each of the support points.

(3 marks)

(b) Given that the reaction force at C is 9 N, find:

(i) the location of the centre of mass of AB

(ii) the reaction force at D.

(3 marks)

5 (a) In the diagram below AB is a uniform plank of length 8 m. It rests horizontally on two supports, one of which is placed at point B and the other of which is placed 2.4 m from point A as shown:

A man with a weight of 728 N stands on the plank at point C and begins to walk towards point A. When he has gone a distance of 0.6 m, the plank is on the point of tilting.

By modelling the plank as a uniform rod and the man as a particle, use the information above to calculate the weight of the plank.

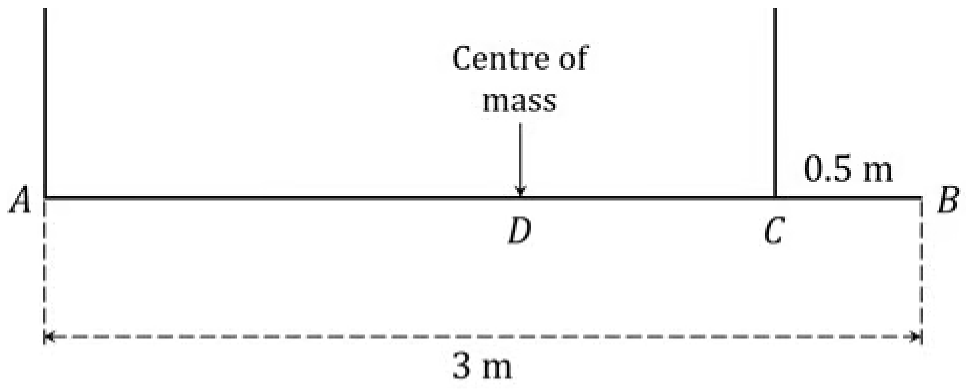
(4 marks)

(b) The man would like to be able to stand at point A without the plank tilting. In order to allow him to do this, he decides to place a large rock on the plank at point B.

Given that the rock may also be modelled as a particle, find the minimum weight of the rock that the man would need.

(3 marks)

6 (a) AB is a non-uniform rod of mass 10 kg and length 3 m, with a load of mass 26 kg attached at point B. AB is held horizontally in equilibrium by two vertical wires attached at points A and C, such that $CB = 0.5 \text{ m}$ as shown in the diagram below:



The position of the centre of mass of the rod is indicated by point D. The load at B may be modelled as a particle.

Given that the rod is on the point of tilting about C, determine the location of the centre of mass of the rod. Give your answer as the value of AD, the distance of the centre of mass from point A.

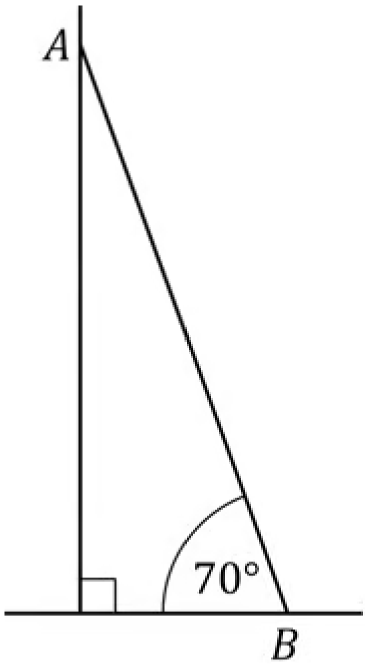
(3 marks)

(b) The load is then removed from point B, and the rod is left suspended in horizontal equilibrium from the two wires.

Determine the tensions in the wires at B and C after the load is removed, giving your answer in terms of the gravitational constant of acceleration g.

(4 marks)

7 In the following diagram AB is a ladder of length 10 m and mass 34 kg. End A of the ladder is resting against a smooth vertical wall, while end B rests on rough horizontal ground so that the ladder makes an angle of $70°$ with the ground as shown below:

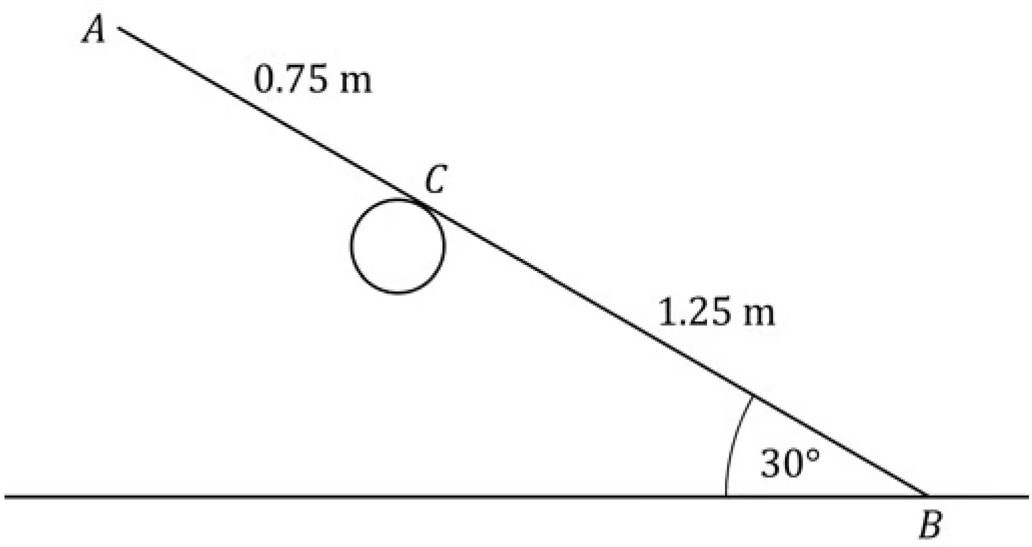


The ladder is modelled as a uniform rod lying in a vertical plane perpendicular to the wall. The coefficient of friction between the ground and the ladder is μ.

Given that the ladder is at rest in limiting equilibrium, calculate the value of μ.

(7 marks)

8 (a) AB is a uniform rod of length 2 m and weight 60 N. End B of the rod is in contact with rough horizontal ground. The rod also rests against a smooth cylindrical peg that contacts the rod at point C such that $AC = 0.75$ m and $CB = 1.25$ m. The rod remains stationary in this configuration, making an angle of $30°$ with the ground as shown in the diagram below:



The magnitude of the normal reaction force exerted by the peg on the rod at point C is denoted by R_C. It is given that R_C acts in a direction perpendicular to AB.

By considering the moments around B show that $R_C = 24\sqrt{3}$ N.

(3 marks)

(b) The magnitude of the normal reaction force exerted by the ground on the rod at point B is denoted by R_B, and the magnitude of the frictional force between the ground and the rod at point B is denoted by F_B.

By considering separately the forces acting in the horizontal and vertical directions, find the values of R_B and F_B.

(4 marks)

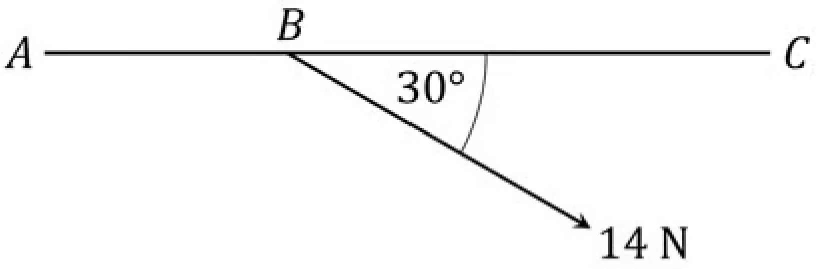
(c) The coefficient of friction between the ground and the rod is denoted by μ.

Given that the rod is about to slip, find the value of μ.

(2 marks)

Hard Questions

1 AC is a light rod, and B is the point on AC such that $AB : BC = 1 : 2$. A force of 14 N is applied to the rod at point B, with the line of action of the force making an angle of $30°$ with AC as shown in the diagram below:



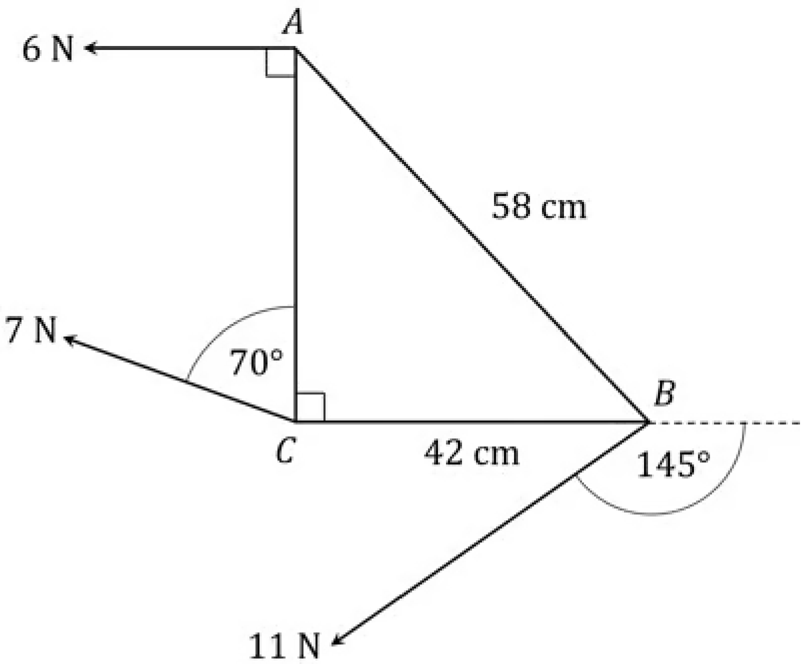
Given that the moment of the force about point A is 2.24 clockwise, find:

(i) the length of rod AC

(ii) the moment of the force about point C.

(6 marks)

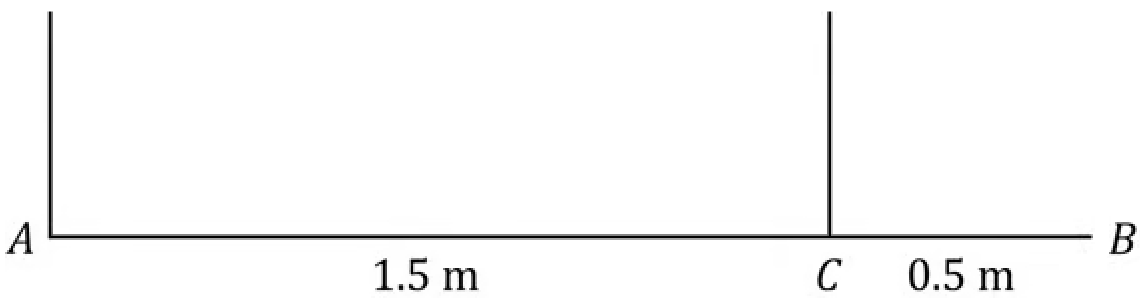
2 ABC is a triangular lamina in which angle ACB is a right angle, and the lengths of sides AB and BC are 58 cm and 42 cm respectively. Three forces are applied to the lamina at points A, B and C as shown in the diagram below:



Calculate the resultant moment of the three forces about point C.

(5 marks)

3 AB is a uniform rod of mass 6 kg and length 2 m, with a load of mass m_B kg attached at point B. AB is held horizontally in equilibrium by two vertical wires attached at points A and C, such that $AC = 1.5\ \mathrm{m}$ as shown in the diagram below:



The tension in the wire at C is found to be eight times the tension in the wire at A. By modelling the load at B as a particle, find:

(i) the value of m_B

(ii) the tensions in the wires at A and C.

Your answers to (ii) should be given as multiples of the gravitational constant of acceleration g.

(6 marks)

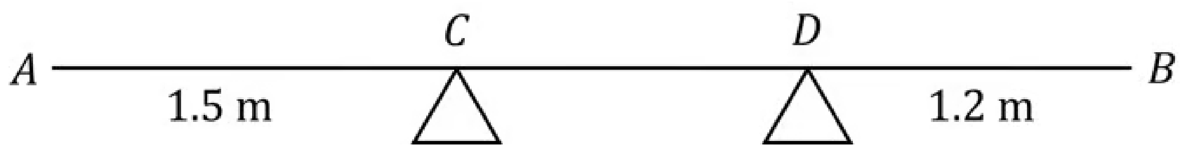
4 AB is a non-uniform rod of mass 10 kg and length 3 m. AB rests horizontally on two supports placed at points C and D, where $AC = DB = 0.6\text{ m}$ as shown in the diagram below:

Esmerelda attaches a 9 kg mass to the rod at a point 0.4 m to the left of D, and measures the reaction force at C. She then removes the mass and reattaches it to the rod at a point 0.4 m to the right of D, and again measures the reaction force at C. She finds that in her second measurement the reaction force at C is only one third the size of the reaction force at C in her first measurement.

By modelling the attached mass as a particle, use the above information to determine the position of the centre of mass of rod AB. Give the position in your answer as the distance of the centre of mass from point A.

(7 marks)

5 In the diagram below AB is a uniform beam of length 4 m. It rests horizontally on two supports placed at points C and D, such that $AC = 1.5\text{ m}$ and $DB = 1.2\text{ m}$ as shown:

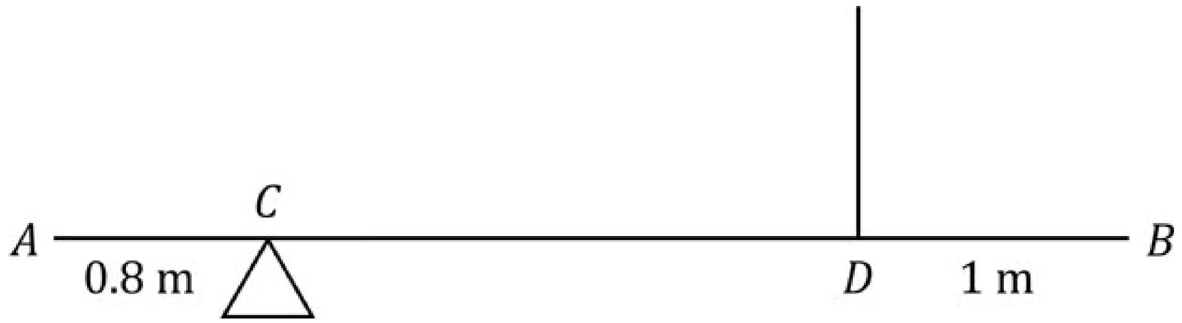


A stone of mass 10 kg is placed at point B and the beam is on the point of tilting. That stone is removed, and another stone of mass m_A kg is placed at point A which causes the beam to begin tilting.

Given that the stones may be modelled as particles, show that $m_A > k$, where k is the largest possible constant for which that inequality must be true.

(6 marks)

6 (a) AB is a non-uniform rod of mass 12 kg and length 4 m. AB is held horizontally in equilibrium by a support placed at point C and a vertical wire attached to point D such that $AC = 0.8\ m$ and $DB = 1\ m$ as shown in the diagram below:



A weight of mass 15 kg is attached to the rod at point B and the rod is at the point of tilting about point D. The weight is then removed.

Find the ratio of the reaction force at C to the tension in the wire at D when there are no external weights attached to the rod. Give your answer in the form $p : q$ where p and q are integers with no common factors other than 1.

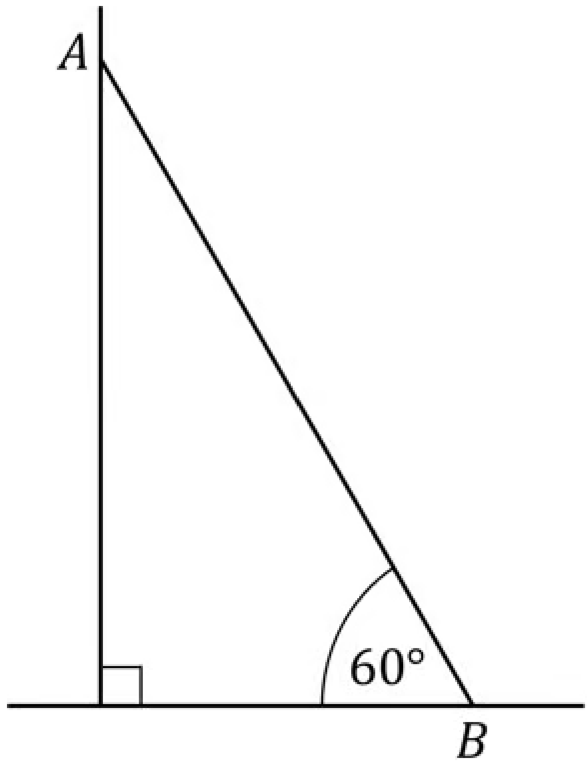
(6 marks)

(b) The 15 kg weight is then attached to the rod between points A and C.

Find the greatest distance to the left of point C that the weight can be attached without the rod beginning to tilt.

(3 marks)

7 In the following diagram AB is a ladder of length 10 m and mass 34 kg. End A of the ladder is resting against a smooth vertical wall, while end B rests on rough horizontal ground so that the ladder makes an angle of $60°$ with the ground as shown below:

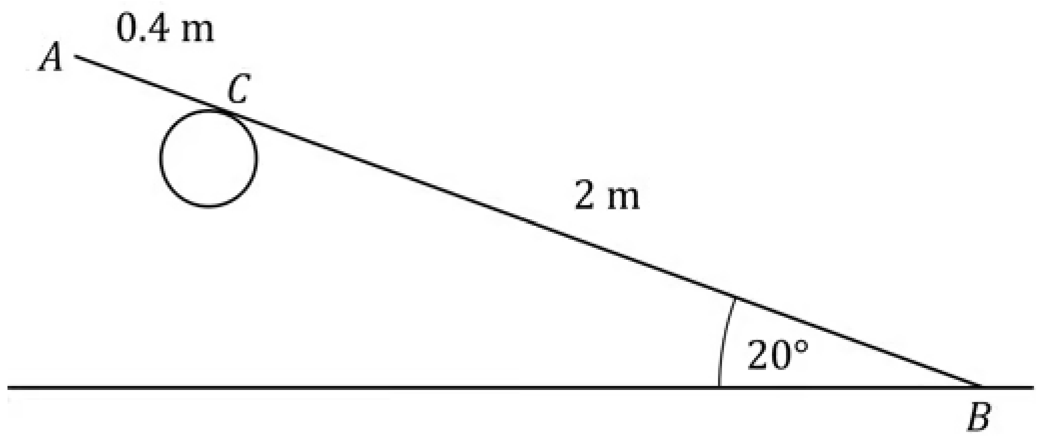


A housepainter with a mass of 75 kg has decided to climb up the ladder without taking any additional precautions to prevent the bottom of the ladder from slipping. The ladder may be modelled as a uniform rod lying in a vertical plane perpendicular to the wall, and the housepainter may be modelled as a particle. The coefficient of friction between the ground and the ladder is 0.4.

Luckily, the housepainter's partner convinces him not to climb up the ladder without providing some additional support at the bottom to prevent slipping. If the housepainter had continued with his original plan, however, how far above the ground would he have been when the ladder began to slip?

(8 marks)

8 (a) AB is a uniform rod of length 2.4 m and weight 72 N. End B of the rod is in contact with rough horizontal ground. The rod also rests against a smooth cylindrical peg that contacts the rod at point C such that $AC = 0.4\text{ m}$ and $CB = 2\text{ m}$. The rod remains stationary in this configuration, making an angle of $20°$ with the ground as shown in the diagram below:



The coefficient of friction between the ground and the rod is μ. It is given that the normal reaction force exerted by the peg on the rod at point C acts in a direction perpendicular to AB.

Find the magnitude of the normal reaction force at C.

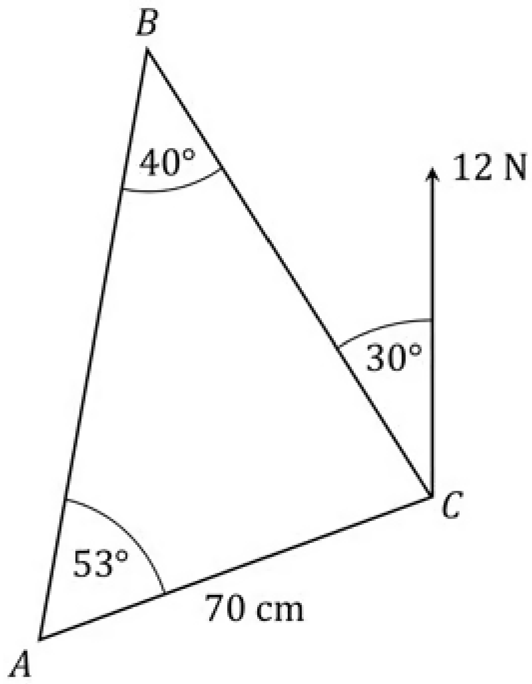
(2 marks)

(b) Given that the rod is on the point of slipping, calculate the value of μ.

(6 marks)

Very Hard Questions

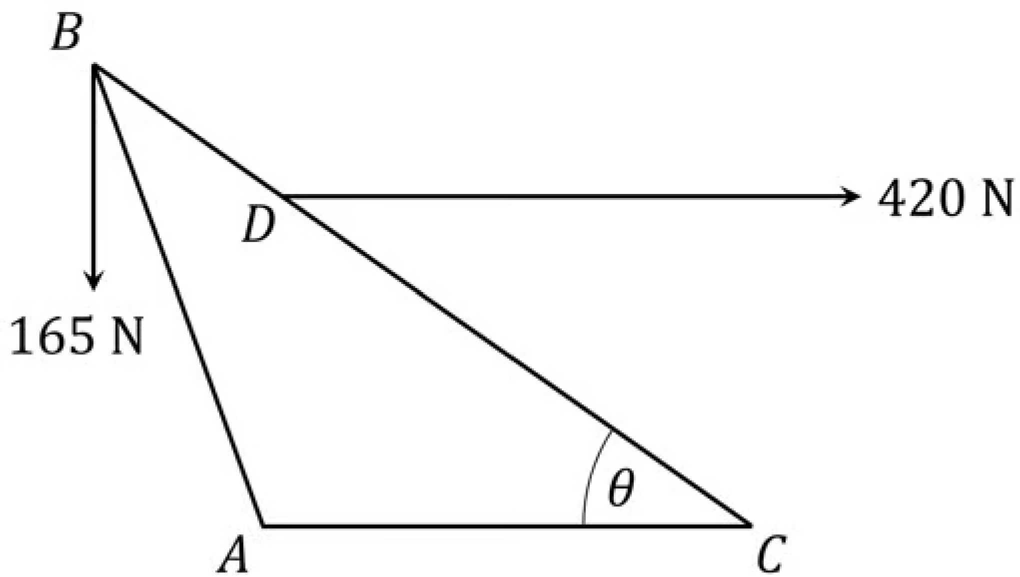
1 ABC is a triangular lamina, in which side AC has a length of 70 cm and angles ABC and BAC are $40°$ and $53°$ respectively. A force of 12 N is applied to point C, with the line of action of the force making an angle of $30°$ with side BC as shown in the diagram below:



Calculate the moment of the force about each of the points A, B and C.

(7 marks)

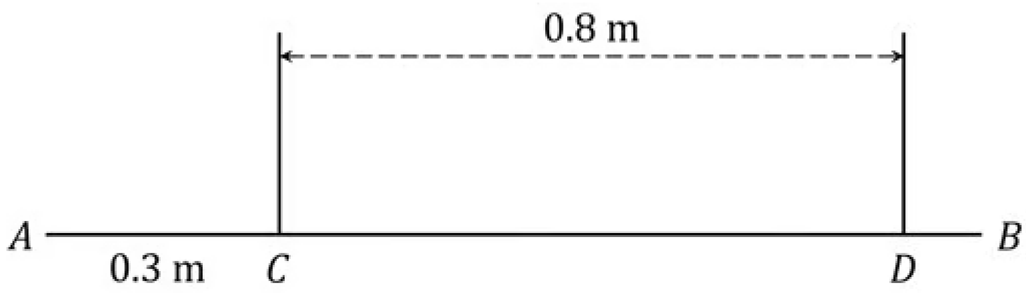
2 ABC is a triangular lamina in which the size of angle ACB is indicated by θ. D is the point on BC such that $BD:DC = 2:5$. A 165 N force acts on point B in a direction perpendicular to AC and a 420 N force acts on point D in a direction parallel to AC, as shown in the diagram below:



Given that the resultant force about point C is in the clockwise direction, show that $\tan\theta > \dfrac{11}{20}$.

(5 marks)

3 AB is a uniform rod of mass 4 kg and length 1.2 m. AB is held horizontally in equilibrium by two vertical wires attached 0.8 m apart at points A and C, where C is 0.3 m from A as shown in the diagram below.

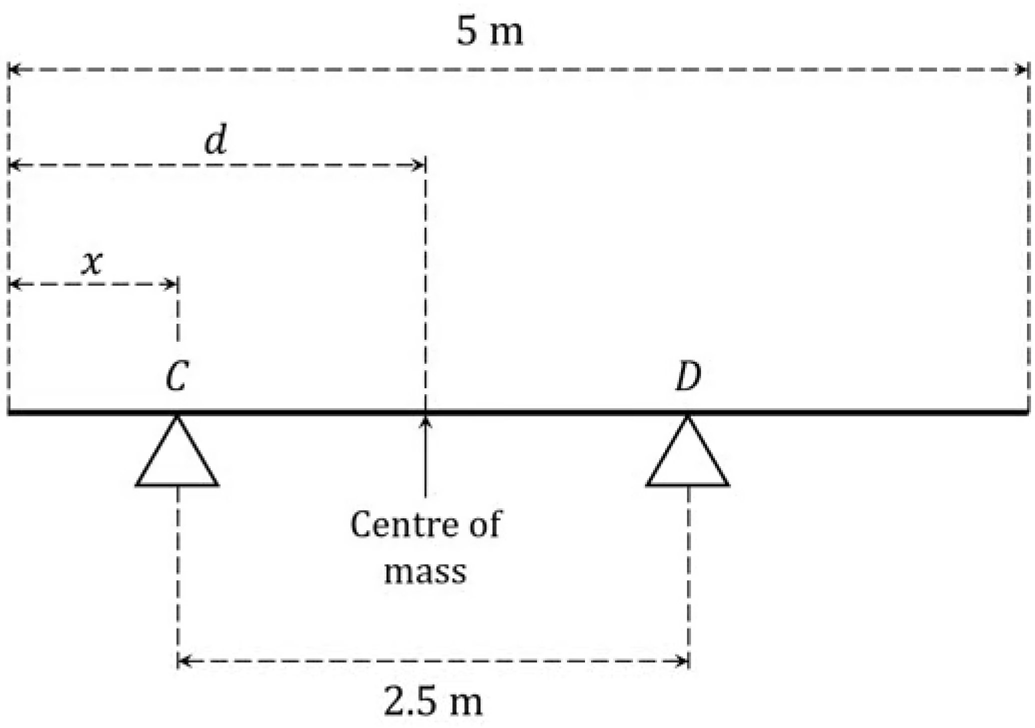


A particle of mass m_E kg is attached to AB at the point E, such that AB remains in horizontal equilibrium and the tensions in the wires at C and D are equal.

Given that point E is in between points D and B, show that $0.8 < m_E < 1$.

(7 marks)

4 (a) A manager at the company Rods-We-Are has invented a device for locating the centre of mass of the 5 metre long barge poles that the company sells. He has connected force metres to two smooth supports located 2.5 m apart at the same horizontal level. A barge pole is placed on the supports so that it is held horizontally in equilibrium, with the supports located at points C and D as indicated in the diagram below:



The pole is then slid back and forth on the supports until a buzzer sounds, which indicates that the reaction force at C is exactly forty-nine times the reaction force at D. The distance x from the left end of the pole in the diagram to C is then measured. Finally, by modelling the barge pole as a rod, the value of x is used to calculate the distance d between the left end of the pole in the diagram and the centre of mass of the pole.

A barge pole is placed on the device described above, and the buzzer sounds when $x = 2.34\text{ m}$. By first finding an expression for d in terms of x, determine the location of the centre of mass of the barge pole.

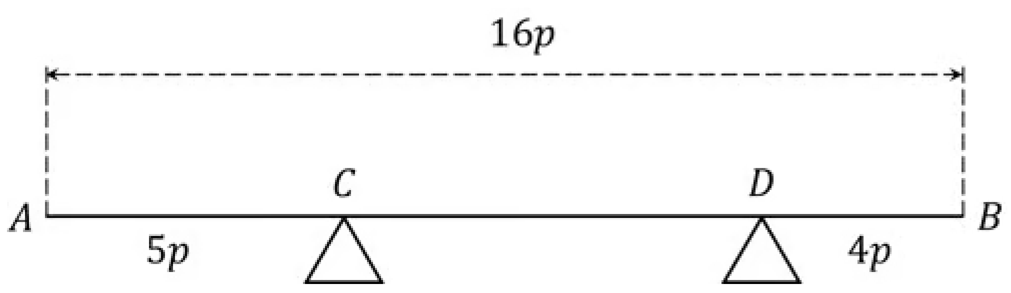
(6 marks)

(b) The manager claims that although he has seen many non-uniform 5 metre barge poles, he has never found one for which the centre of mass could not be determined using his device. A new trainee manager claims that her calculations show that there could be 5 metre barge poles for which the device will not be able to determine the centre of mass.

Explain why both the manager and the trainee manager could be correct, supporting your answer with precise mathematical reasoning.

(3 marks)

5 AB is a non-uniform rod of length $16p$ and mass m. A particle of mass $\frac{3}{44}m$ is attached to the rod at point B, and the rod is then set to rest horizontally on two supports placed at points C and D, with $AC = 5p$ and $DB = 4p$ as shown in the diagram below:

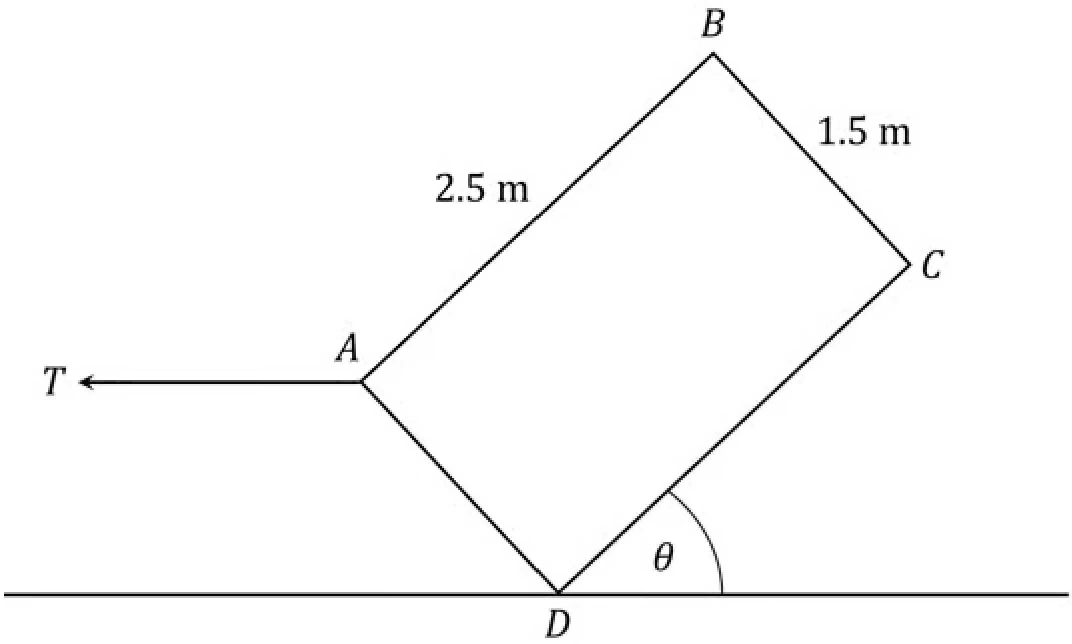


Given that the rod is at the point of tilting, find the two possible locations of the centre of mass of the rod. In your answers give the distance of the centre of mass from point A, with the values given in terms of p.

(5 marks)

6 After laying one of the stone blocks for his new pyramid, the architect Hemiunu realises that his wife's favourite scarab pendant has been left on the ground underneath the block. Therefore he decides to tilt the block up on one of its edges so that the pendant may be retrieved.

The block is a cuboid with weight 98 kN, but it may be modelled as a rectangular lamina $ABCD$ with side lengths $AB = CD = 2.5 \text{ m}$ *and* $BC = AD = 1.5 \text{ m}$. The centre of mass may be assumed to be at the intersection of the diagonals AC and BD. The block is tilted by means of a horizontal rope attached at point A, with tension in the rope causing the block to pivot around point D. As the block is being tilted side DC makes an angle θ of with the ground as shown in the diagram below:



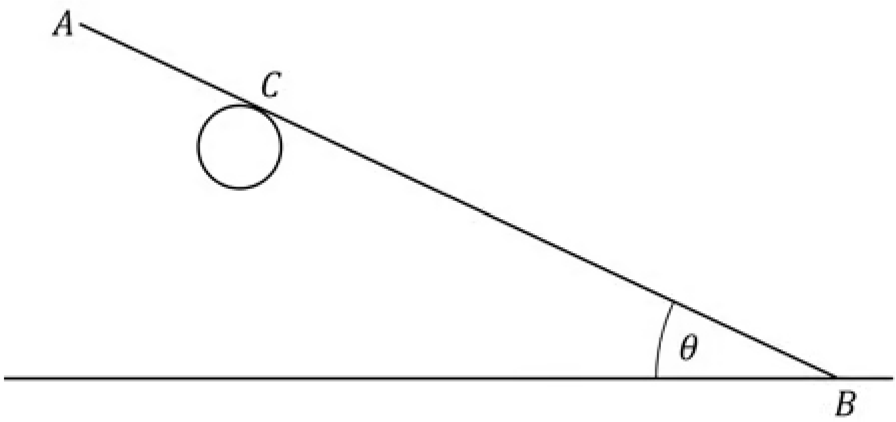
The frictional force between the block and the ground at D is at all times sufficient to prevent the block from slipping.

The block is raised until point C is a vertical distance of 1.7 m from the ground. The rope is then used to hold the block stationary while the pendant is retrieved.

Given that the rope remains horizontal, find the tension T in the rope while the block is being held stationary.

(7 marks)

7 AB is a uniform rod of length $2a$ and mass m. End B of the rod is in contact with rough horizontal ground. The rod also rests against a smooth cylindrical peg that contacts the rod at point C such that the distance from point C to point B is d, with $d \geq a$. The vertical plane containing the rod is perpendicular to the peg. The rod remains stationary in this configuration, making an angle of θ with the ground as shown in the diagram below:



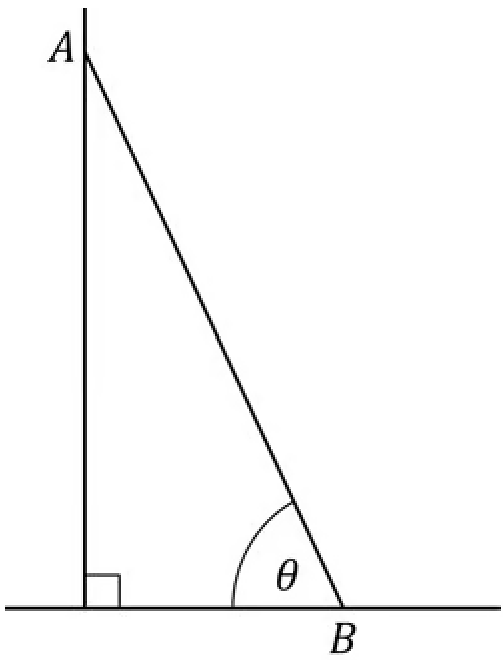
The coefficient of friction between the ground and the rod is indicated by μ. It may be assumed that $0 < \theta < 90°$.

Show that

$$\mu \geq \frac{\frac{a}{d} \sin\theta \cos\theta}{1 - \frac{a}{d} \cos^2\theta}$$

(7 marks)

8 (a) In the following diagram AB is a ladder of length $2a$ and mass m_l. End A of the ladder is resting against a rough vertical wall, while end B rests on rough horizontal ground so that the ladder makes an angle of θ with the ground as shown below:



A person with mass m_p is standing on the ladder a distance d from end B. The ladder may be modelled as a uniform rod lying in a vertical plane which is perpendicular to the wall, and the person may be modelled as a particle. The coefficient of friction between the wall and the ladder is μ_A, and the coefficient of friction between the ground and the ladder is μ_B. It may be assumed that $0 < \theta < 90°$.

Given that the ladder is at rest in limiting equilibrium, show that

$$R_B = \frac{am_l + dm_p}{2a\mu_B(\mu_A + \tan\theta)}\,g$$

where R_B is the normal reaction force exerted by the ground on the ladder at point B and where g is the constant of acceleration due to gravity.

(9 marks)

(b) Hence find an equivalent expression for R_A, the normal reaction force exerted by the wall on the ladder at point A when the ladder is at rest in limiting equilibrium.

(2 marks)