A Level · Edexcel · Maths

4 hours 40 questions

5.4 Radian Measure (A Level only)

Easy (10 questions)	/39
Medium (11 questions)	/59
Hard (10 questions)	/58
Very Hard (9 questions)	/57
Total Marks	**/213**

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Easy Questions

1 Without using a calculator, write down:

(i) 60° in radians,

(ii) sin $45°$

(iii) tan π

(iv) $\frac{5\pi}{6}$ radians in degrees

(v) cos 0

(5 marks)

2 A sector of a circle, OPQ , is such that it has radius 4 cm and the angle at its centre, O, is $\frac{\pi}{4}$ radians.

(i) Find the length of the arc PQ.

(ii) Find the area of the sector OPQ.

(4 marks)

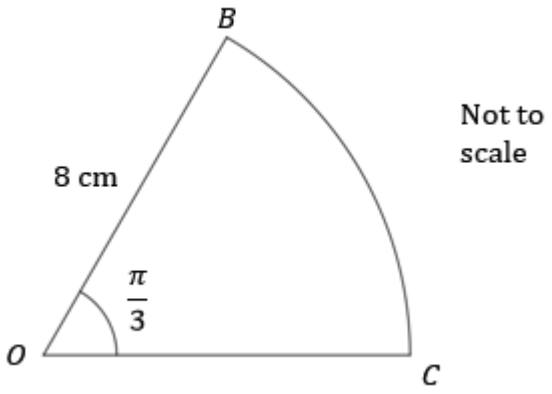
3 Given that θ is small, write an approximation, in terms of θ for

(i) $\sin\theta + \cos\theta$

(ii) $2\tan^2\theta - \cos\theta$

(4 marks)

4 The sector of a circle is shown below.



Find the perimeter of the sector.

(3 marks)

5 The area of a quarter circle is 18π cm^2.

Find the radius of the circle.

(3 marks)

6 Find all the solutions to the equation $2 \sin \theta = \sqrt{3}$ for $-2\pi \leq \theta \leq 2\pi$.

(4 marks)

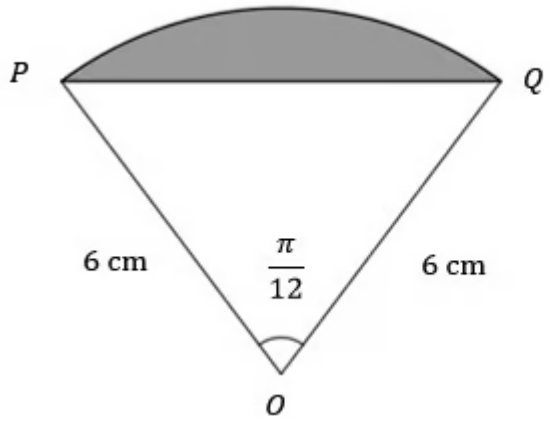
7 The arc length of the sector of a circle is 5 cm.

The radius is twice the length of the arc.

Find the angle at the centre of the circle.

(2 marks)

8 (a) The diagram below shows the sector of a circle, OPQ, radius 6 cm and centre angle $\frac{\pi}{12}$ radians.



Not to scale

Use the formula $\frac{1}{2}$ ab sin C to find the area of the triangle OPQ.

(2 marks)

(b) Find the area of the sector OPQ.

(2 marks)

(c) Hence or otherwise find the area of the shaded segment.

(2 marks)

9 The area of a sector is 8π cm^2.

The arc length of the sector is 4π cm.

Find the radius and the angle at the centre of the circle.

(4 marks)

10 Given that θ is small, show that $\sin 4\theta \approx 2 \sin 2\theta \cos 2\theta$.

(4 marks)

Medium Questions

1 Complete the table.

Degrees	Radians	sin	cos	tan
	$\frac{\pi}{6}$		$\frac{\sqrt{3}}{2}$	
45^{o}			$\frac{1}{\sqrt{2}}$	
60^{o}	$\frac{\pi}{3}$			
	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$		
270^{o}				n/a

(5 marks)

2 A sector of a circle, OPQ, is such that it has radius 3.4 cm and the angle at its centre, O, is $\frac{3\pi}{4}$ radians.

(i) Find the length of the arc PQ.

(ii) Find the area of the sector OPQ.

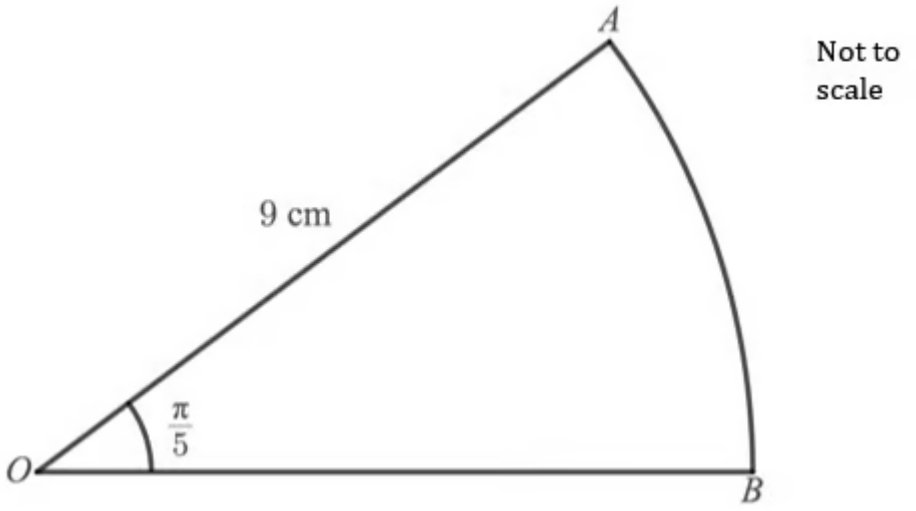
(4 marks)

3 Given that θ is small, write an approximation in terms of θ for

$$2\sin\theta+\cos\theta-\tan^2\theta$$

(3 marks)

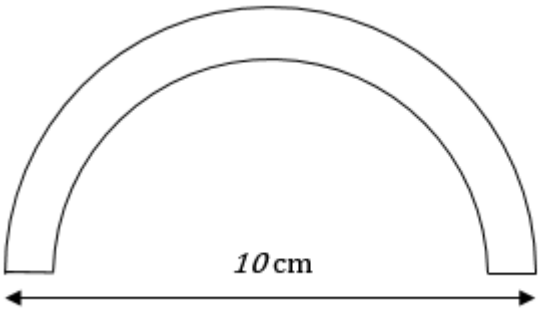
4 The sector of a circle is shown below.



Find the area and the perimeter of the sector.

(5 marks)

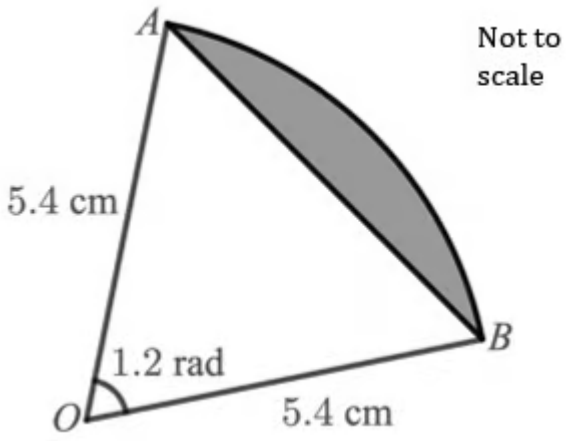
5 A rainbow-shaped logo is formed from two semicircles as shown below.
The radius of the smaller semicircle is 1 cm less than the radius of the larger semicircle.



Find the area of the logo.

(4 marks)

6 (a) The diagram below shows the sector of a circle OAB.



(i) Find the area of the sector OAB, giving your answer to 3 significant figures.

(ii) Find the area of the triangle OAB, giving your answer to 3 significant figures.

(iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

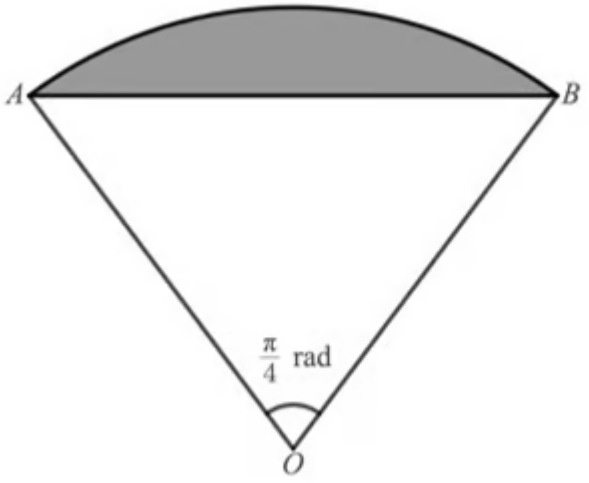
(5 marks)

(b) (i) Find the length of the arc AB.

(ii) Find the perimeter of the sector OAB.

(3 marks)

7 The canopy of a parachute and the outermost connecting cords form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O.



Not to scale

The area of the sector OAB is $\frac{81\pi}{200}$ m^2.

Find the length of one of the connecting cords on the parachute.

(3 marks)

8 (a) Given that x is small, show that

$$16-7\sin x+2\tan 2x-18\cos x\approx 9x^2-3x-2$$

(3 marks)

(b) Hence, or otherwise, find approximate solution(s) to the equation

$$16-7\sin x+2\tan 2x-18\cos x=0$$

(3 marks)

9 (a) Find all solutions to the equation $\cos\theta = \frac{1}{2}$ in the interval $-2\pi \leq \theta \leq 2\pi$, giving your answers in radians as multiples of π.

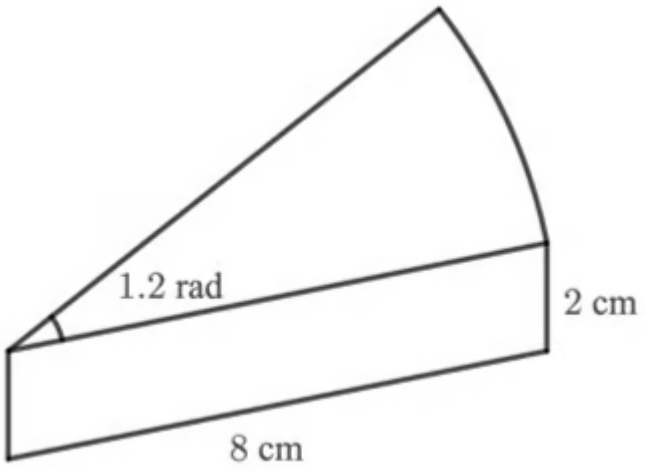
(4 marks)

(b) Find all solutions to the equation $5\sin 3x = 1$ in the interval $0 \leq x \leq \pi$, giving your answer in radians to three significant figures.

(6 marks)

10 A plastic puzzle piece is in the form of a prism with a cross-section that is the sector of a circle, as shown in the diagram below. The radius of the sector is 8 cm, and the angle at the centre is 1.2 radians.

The height of the puzzle piece is 2 cm.



(i) Work out the area of the cross-section.

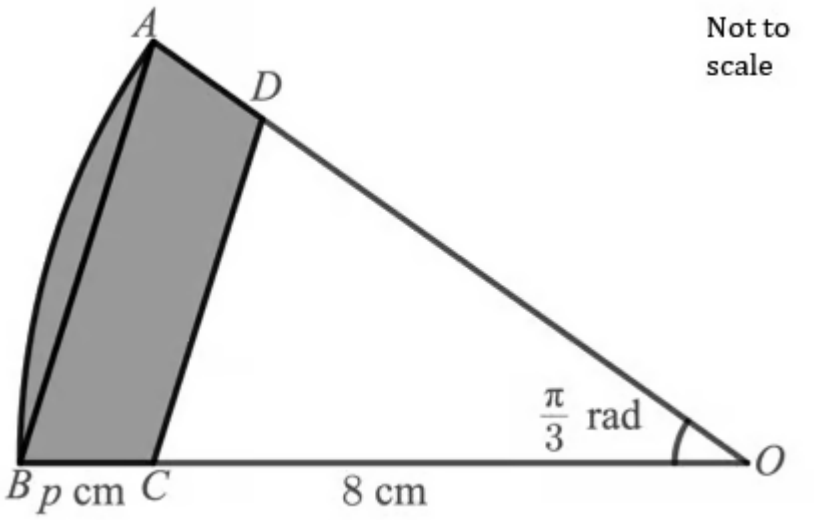
(ii) Hence, or otherwise, work out the volume of the puzzle piece.

(3 marks)

11 (a) The circle sector OAB is shown in the diagram below.

The angle at the centre is $\frac{\pi}{3}$ radians , and the line segments OC and BC have lengths of 8 cm and $p$cm respectively.

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

(2 marks)

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

(2 marks)

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2. find the value of p.

(4 marks)

Hard Questions

1 Complete the table.

Degrees	Radians	sin	cos	tan
	$\frac{\pi}{6}$		$\frac{\sqrt{3}}{2}$	
135^{o}		$\frac{1}{\sqrt{2}}$		
180^{o}				0
		$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	

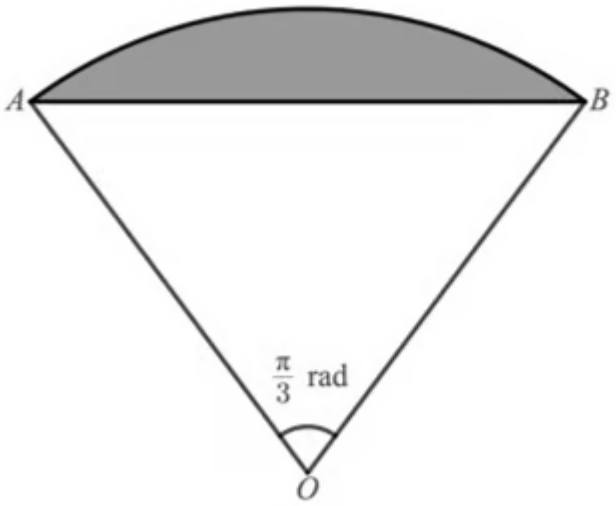
(4 marks)

2 Given that θ is small, write an approximation in terms of θ for

$$\frac{\sin^2\theta + \cos\theta}{\tan\theta + \sin\theta}$$

(3 marks)

3 The canopy of a parachute and the outermost suspension lines (cords) form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O.



Not to scale

The area of the sector OAB is $\frac{49\pi}{150}$ m^2.

The parachute is made with 40 equal length suspension lines.

Disregarding any additional lengths of cord that might be required for knots, fastenings, etc., find the total length of the suspension lines required to make the parachute.

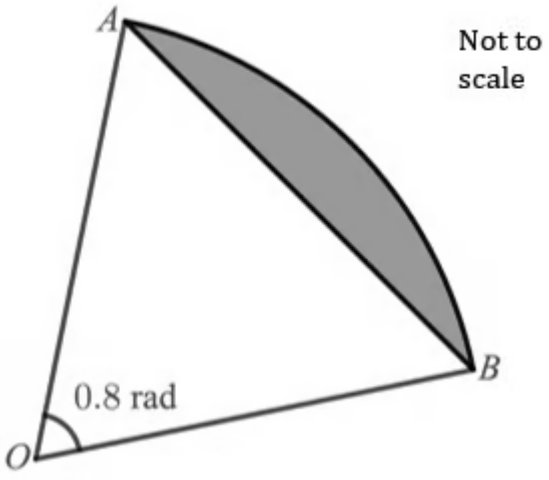
(4 marks)

4 A sector of a circle, OPQ , is such that it has radius a cm and the angle at its centre, O, is $a^2\pi$ radians.

Given that the area of the sector OPQ in cm^2 is three times the length of the arc PQ in cm, find the value of a.

(4 marks)

5 (a) The diagram below shows the sector of a circle OAB.



Given that the area of triangle $OAB = 5.64$ cm^2, find the area of the shaded segment. Give your answer correct to 3 significant figures.

(5 marks)

(b) Find the perimeter of the sector OAB, giving your answer correct to 3 significant figures.

(3 marks)

6 (a) When x is small, show that the equation

$$\cos^2 3x + \frac{1}{4}\tan^4 3x = 1$$

can be written as

$$\frac{81}{2}x^4 - 9x^2 = 0$$

(4 marks)

(b) Hence find approximate solutions to the equation

$$\cos^2 3x + \frac{1}{4}\tan^4 3x = 1$$

(3 marks)

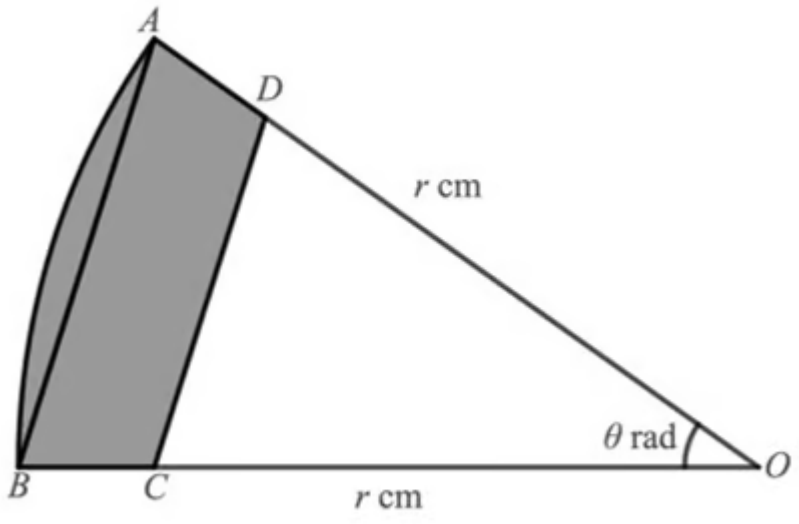
(c) Comment on the validity of your solutions.

(2 marks)

7 (a) The circle sector OAB is shown in the diagram below.

The angle at the centre is θ radians , and the radii OA and OB are each equal to r cm

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



In the case when $AD = BC = 1$ cm, show that the area of the shaded shape $ABCD$ is given by $\frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2 \sin\theta$.

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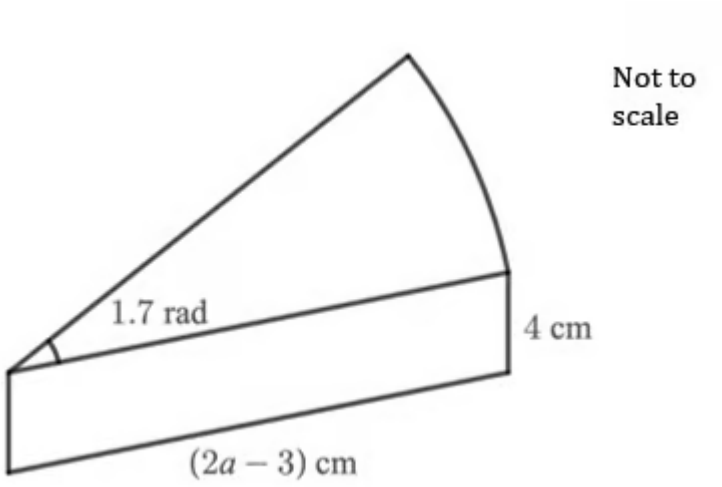
(4 marks)

(b) Show that for small values of θ, the area of $ABCD$ is approximately $\frac{1}{2}\theta(2r-1)$.

(3 marks)



8 The diagram shows a prism with cross-section in the shape of a sector of a circle.

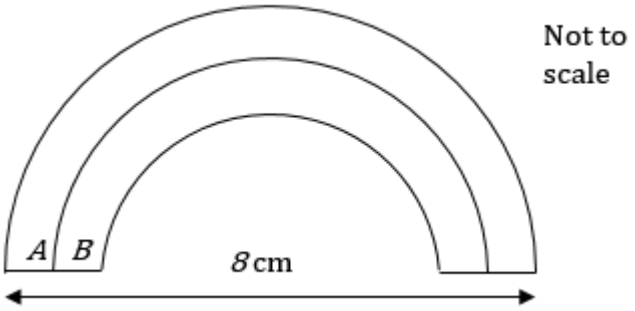
The radius of the sector is $(2a-3)$ cm, and the angle at the centre is 1.7 radians.

The height of the prism is 4 cm.

Given that the volume of the prism is 7.65 cm^3, find the possible value(s) of a.

(5 marks)

9 A rainbow-shaped logo is formed from three semicircles as shown below.
Each of the inner semicircles has a radius that is 1 cm less than that of the next semicircle further out.



Find, in simplest terms, the ratio of the outer area A of the logo, to the inner area B.

(5 marks)

10 (a) Find all the solutions to the equation $\sqrt{3}\tan 2\theta = -1$ in the interval $-\pi \leq \theta \leq \pi$, giving your answers in radians as multiples of π.

(4 marks)

(b) Find all the solutions to the equation $6\sin^2 x + 7\sin x - 3 = 0$ in the interval $0 \leq x \leq 2\pi$, giving your answers in radians to three significant figures.

(5 marks)

Very Hard Questions

1 Complete the table.

Degrees	Radians	sin	cos	tan
45^{o}	$\frac{\pi}{4}$			
150^{o}		$\frac{1}{2}$		
	$\frac{7\pi}{4}$			-1

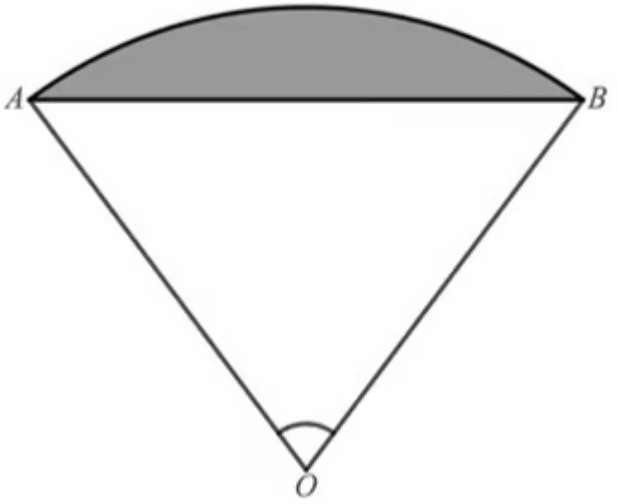
(3 marks)

2 Given that θ is small, find an approximate expression in terms of θ for

$$\frac{\sin^2\theta + 2\cos^2 2\theta - \tan 3\theta}{\sin 4\theta}$$

(3 marks)

3 The canopy of a parachute and the outermost suspension lines (cords) form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O



The area of the sector OAB is $\frac{125\pi}{144}$ m^2 .

The length of the arc AB is $\frac{25\pi}{36}$ m .

Find the length of one suspension line and the angle AOB that the parachutist makes with the two outermost suspension lines.

(5 marks)

4 (a) Given that θ is small, find approximate solution(s) to the equation

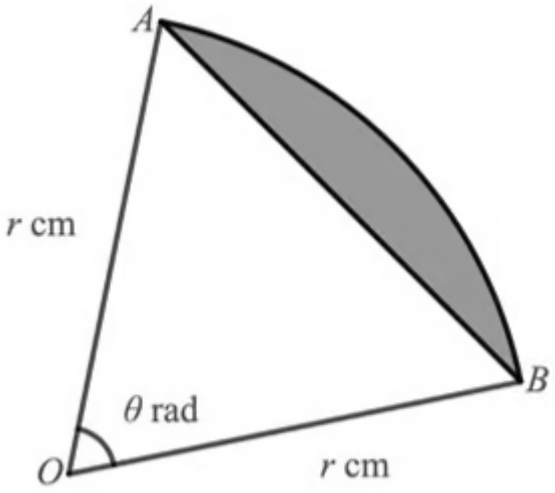
$$\frac{4\cos^2 4\theta - \sin(2\theta)^2}{\tan^2 \theta} = 12$$

(7 marks)

(b) Comment on the relative reliability of the solution(s) you found in part (a).

(2 marks)

5 (a) The diagram below shows the sector of a circle OAB.



Show that the area of the shaded segment is given by $\frac{1}{2}r^2(\theta - \sin\theta)$ cm^2.

(3 marks)

(b) Find, in terms of θ, the percentage of the sector that the segment occupies.

(2 marks)

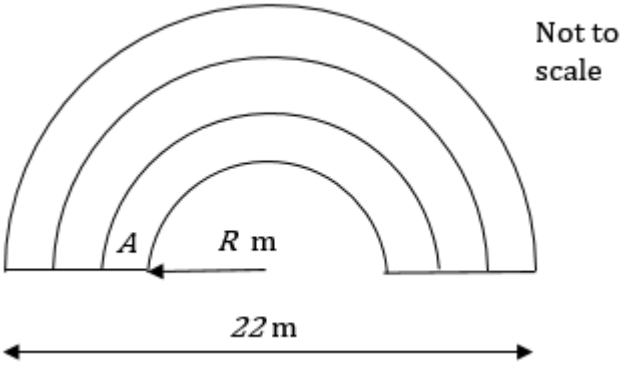
6 An evil wizard has captured a unicorn, and is threatening to kill it unless you can answer the following question:

"I wish to create a rainbow-shaped mosaic for the floor of the throne room in my castle.

The mosaic is to be formed from four semicircles as shown below.

The innermost semicircle is to have a radius of R metres, and each of the outer semicircles must have a radius that is a constant k metres greater than the radius of the next semicircle further in.

The rainbow mosaic must be exactly 22 metres across.



Moreover, I wish the inner part of the area (labelled in the diagram) to take up exactly one quarter of the total area of the rainbow.

Find me the required values of R and k, or the unicorn dies. Bwah-ha-ha-ha-ha!"

Solve the wizard's problem and save the unicorn.

(7 marks)

7 (a) A sector of a circle, OST, is such that it has radius r cm and the angle at its centre, O, is θ radians. The chord ST has length a cm.

Show that $a^2 = 2r^2(1 - \cos\theta)$.

(2 marks)

(b) Given that $r = 4\theta$ and that the area of the sector is $\frac{8\pi^3}{27}$ cm^2, find the value of a.

(5 marks)

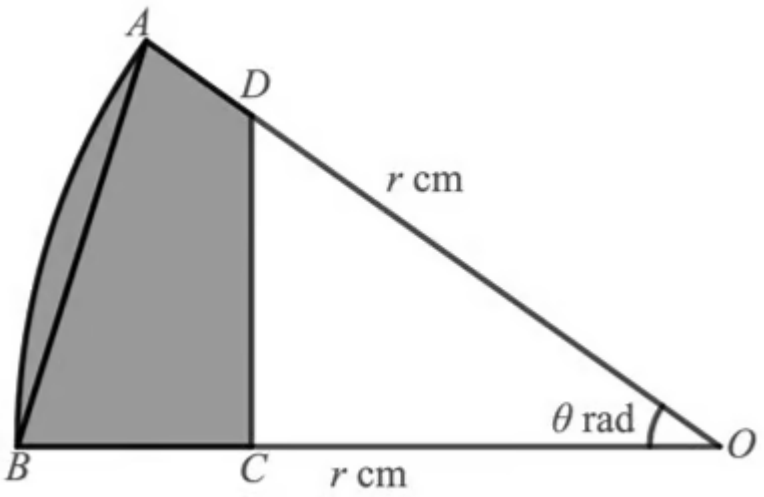
8 (a) Find all the solutions to the equation $2\cos 2\theta = 4\sin 2\theta \cos 2\theta$ in the interval $0 \leq \theta \leq 2\pi$, giving your answers in radians as multiples of π.

(6 marks)

(b) Find all the solutions to the equation $3\cos^2 4x + 13\cos 4x - 10 = 0$ in the interval $0 \leq x \leq \pi$, giving your answers in radians to three significant figures.

(6 marks)

9 The diagram below shows the sector of a circle with centre O. The radii OA and OB are each equal to r cm, and the angle at the centre, AOB , is equal to θ radians. The line DC is perpendicular to the line OB.



Given that $BC:CO=2:3$, show that the area of the shaded shape $ABCD$ is given by

$$\frac{1}{50}r^2(25\theta-9\tan\theta)\text{cm}^2.$$

(6 marks)