A Level · Edexcel · Maths

3 hours 32 questions

4.3 Arithmetic Sequences & Series (A Level only)

Easy (8 questions)	/28
Medium (8 questions)	/42
Hard (8 questions)	/43
Very Hard (8 questions)	/40
Total Marks	**/153**

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Easy Questions

1 Write down the next three terms in these arithmetic sequences

(i) 30, 18, 6, ...

(ii) $\frac{1}{4}, \frac{5}{12}, \frac{7}{12}, \frac{3}{4}, \ldots\ldots$

(2 marks)

2 Find the sum of the first four terms in the sequence defined by $u_n = 2n + 3$.

Justify why this sequence is an arithmetic sequence.

(2 marks)

3 Write down a formula for the n^{th} term of each of the following arithmetic sequences

(i) 16, 20, 24, ...

(ii) First term: $a = 3$

Common difference: $d = -3$

(iii) $a = 2,\ d = 6$

(3 marks)

4 Find the 10^{th} and 20^{th} terms in each of the following arithmetic sequences

(i) $u_n = 4 + 5n$

(ii) $u_n = \frac{1}{2} - \frac{1}{4}n$

(iii) $u_n = 50 - 5n$

(3 marks)

5 (a) The 4^{th} and 8^{th} terms of an arithmetic sequence are 20 and 64 respectively.
Find the first term and the common difference.

(3 marks)

(b) The 12^{th} and 16^{th} terms of an arithmetic sequence differ by 20.
Find the possible values of the common difference.

(2 marks)

6 Find the sum of the first 20 terms of the arithmetic series that has first term 3 and common difference 4.

(2 marks)

7 (a) The first term of an arithmetic sequence is 3.
The 10^{th} term of the sequence is 30.
The sum of the first terms is 630.

Find the common difference.

(2 marks)

(b) Show that $n^2 + n - 420 = 0$..

(2 marks)

(c) Hence find the value of n.

(2 marks)

8 (a) An arithmetic series is given by

$$k + 2k + 3k + 4k + \cdots$$

where k is a constant.

Write down a formula for the n^{th} term of the series, in terms of k.

(1 mark)

(b) Show that the sum of the first n terms is $\frac{kn}{2}(n+1)$.

(2 marks)

(c) The sum of the first 12 terms is 39.
Find the value of k.

(2 marks)

Medium Questions

1 The first three terms in an arithmetic sequence are $(p-9), -7, (9-3p), ...$

Find the value of p.

(3 marks)

2 The first three terms in an arithmetic sequence are $-1, (q+1), q^2, ...$

Find the possible values of q.

(3 marks)

3 An arithmetic sequence has first term r^2 and common difference $3r$, where $r > 0$. The fifth term of the sequence is 85.

Find:

(i) the value of r

(ii) the ninth term in the sequence.

(4 marks)

4 (a) The third term of an arithmetic series is 2. The twelfth term is 65. The sum of the first n terms is 390.

Show that $7n^2 - 31n - 780 = 0$..

(4 marks)

(b) Hence find the value of n.

(1 mark)

5 The sum of the first ten terms in an arithmetic series is 40. The sum of the first twenty terms in the same series is 280. Find the first term, a, and the common difference, d, of the series.

(3 marks)

6 (a) The sum of the first n terms of an arithmetic series is

$$S_n = (k+7) + (2k+18) + (3k+29) + ... + 36$$

Show that $n = \dfrac{40}{k+11}$.

(2 marks)

(b) Hence show that the sum of the first n terms is $\dfrac{20k+860}{k+11}$.

(3 marks)

(c) Given that $S_n = 180$, find the value of k.

(1 mark)

7 (a) The fifth term of an arithmetic series is k, where k is a constant, and the sum of the first eight terms of the series is $2k$.

Show that the first term, a, of the series is $-5k$.

(3 marks)

(b) Find an expression for the common difference, d, of the series in terms of k.

(2 marks)

(c) Given that the ninth term of the sequence is 14, calculate:

the value of k

(2 marks)

(d) The sum of the first 30 terms of the series.

(2 marks)

8 (a) Calculate the sum of all the odd numbers between 0 and 150,

$$1+3+5+\ ...+149$$

(3 marks)

(b) An arithmetic series is defined by

$$k+2k+3k+\ ...+360$$

where k is an integer and a positive factor of 360.

(i) In terms of k, find an expression for the number of terms in this series.

(ii) Show that the sum of this series is $180+\frac{64800}{k}$.

(4 marks)

(c) In terms of q, find the 100th term of the arithmetic sequence defined by

$$(3q-7),(5q-4),(7q-1),...$$

Give your answer in simplest form.

(2 marks)

Hard Questions

1 The first three terms in an arithmetic sequence are $(q-2)$, q^2, $(4q+5)$,...

Find the possible values of q.

(3 marks)

2 The first two terms in an arithmetic sequence are $(p+15)$ and 3. The fourth term is $(3p-16)$.

Find the value of p.

(3 marks)

3 An arithmetic sequence has first term r^2 and common difference $2r$, where $r>0$. The fourth term in the sequence is 4.

Find the value of r, giving your answer as an exact value.

(4 marks)

4 The third term of an arithmetic series is 32. The eleventh term is 0. The sum of the first n terms is -44.

Find the value of n.

(5 marks)

5 The sum of the first twelve terms in an arithmetic series is 654. The sum of the first twenty terms in the same series is 530. Find the 21^{st} term.

(4 marks)

6 (a) Prove that the sum of the first n odd numbers is a square number for any value of $n \geq 1$.

(3 marks)

(b) An arithmetic series is defined by

$$7k + 14k + 21k + \ ... + 1008$$

where k is an integer.

(i) In terms of k, find an expression for the number of terms in this series.

(ii) In addition to being an integer, what other two conditions must k satisfy for this to be a valid arithmetic series?

(iii) Show that the sum of this series is $504 + \frac{72576}{k}$.

(5 marks)

7 (a) The seventh term of an arithmetic series is $3k$, where k is a constant, and the sum of the first nine terms of the series is $4k - 3$.

In terms of k, find expressions for the first term and common difference of the series.

(5 marks)

(b) Given that the nineteenth term of the sequence is 57, find the sum of the first 25 terms of the series.

(4 marks)

8 (a) An arithmetic series is defined by

$$S_n = (k+13) + (2k+9) + (3k+5) + ... + u_n + ...$$

Find an expression for n in terms of u_n and k.

(2 marks)

(b) For a particular value of n, $u_n = -16$ and $S_n = -11$.

Find the value of k.

(5 marks)

Very Hard Questions

1 The first four terms in an arithmetic sequence are $(2q-p), 1, (p-q), (3p-4q),...$

Find the values of p and q.

(4 marks)

2 The first three terms in an arithmetic sequence are $2, 3q^2, q,...$

Given that the first three terms in the sequence are all positive, find the fortieth term in the sequence.

(4 marks)

3 An arithmetic sequence has first term r^2 and common difference r, where $r>0$. The ninth term in the sequence is 7.

S is the set of all numbers that are in the sequence. T is the set of all rational numbers.

Determine the elements of the set $S \cap T$.

(6 marks)

4 The k^{th} term of an arithmetic series is 0. The sum of the first n terms is also 0.

Find the value of n in terms of k, giving clear reasons for your answer.

(3 marks)

5 The sum of the first 20 terms in an arithmetic series is 290. The sum of the first 24 terms in the same series is -180. In general, the sum of the first n terms is S_n. Find the greatest value attained by S_n for any $n \geq 1$.

(6 marks)

6 The sum of the first 24 terms in an arithmetic series is nine times the sum of the first two terms in the series.

Find the sum of the first 90 terms in the series.

(4 marks)

7 The nth term in an arithmetic sequence can be expressed in the form

$$u_n = (n + \frac{1}{2}) \ln 4$$

Show that the sum of the first terms of the sequence is

$$S_n = \ln 2^{\mathrm{f}(n)}$$

where $\mathrm{f}(n)$ is a polynomial in n to be found.

(7 marks)

8 (a) An arithmetic sequence is defined by

$$(-21+\ln 3),(-20+\ln 27),(-19+\ln 243),...$$

Find, in terms of n and $\ln 3$, an expression for the sum of the first n terms of the sequence.

(4 marks)

(b) Hence find the smallest value of n for which the sum of the first n terms of the sequence is greater than zero.

(2 marks)