Further Kinematics

This chapter concerns how can use **vectors to represent motion**. In the case of constant acceleration, can we still use our 'suvat' equations? And what if we have variable acceleration with expressions in terms of t?

1:: Vector equations for motion.

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$ **(3)**

(b) Find the bearing on which P is moving when $t = 2$ **(2)**

(c) Find the value of t when P is moving

(i) parallel to $\mathbf{j}$,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$. **(6)**

2:: Variable acceleration with vectors.

"A particle P of mass 0.8kg is acted on by a single force $\boldsymbol{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$

Find (a) the speed of P when $t = 4$

(b) The acceleration of P as a vector when $t = 2$

(c) $\boldsymbol{F}$ when $t = 2$."

3:: Integration with vectors to find velocity/displacement

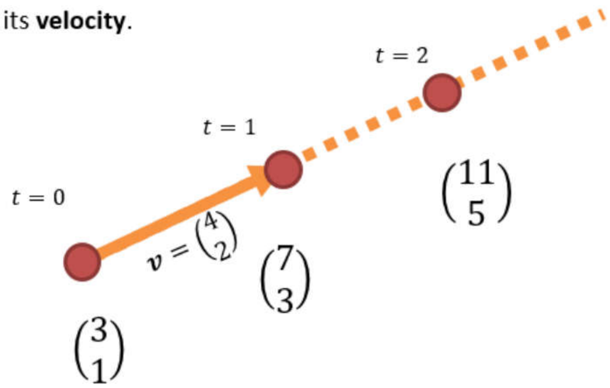
"A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by $\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}$, $t \geq 0$

When $t = 0$, the position vector of P with respect to a fixed origin O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector of P at time t seconds."

Vector motion

Initially, a particle is at the position vector $\binom{3}{1}$.

Each second, it moves $\binom{4}{2}$, i.e. its **velocity**.



So in general, where would the particle be after t seconds, in terms of t?

It'll be $\binom{3}{1}$ with t lots of $\binom{4}{2}$ added on, i.e.:

$$\binom{3}{1} + t\binom{4}{2} \quad \rightarrow \quad \binom{3+4t}{1+2t}$$

Position vector $\boldsymbol{r}$ of particle:

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$$

where r_0 is initial position and $\boldsymbol{v}$ is velocity.

A particle starts from the position vector $(3\boldsymbol{i} + 7\boldsymbol{j})$ m and moves with constant velocity $(2\boldsymbol{i} - \boldsymbol{j})$ ms^{-1}.

(a) Find the position vector of the particle 4 seconds later.

(b) Find the time at which the particle is due east of the origin.

SUVAT with but with vectors

What changes?

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

A particle P has velocity $(-3\boldsymbol{i} + \boldsymbol{j})$ ms^{-1}. The particle moves with constant acceleration $\boldsymbol{a} = (2\boldsymbol{i} + 3\boldsymbol{j})$ ms^{-2}. Find (a) the speed of the particle and (b) the bearing on which it is travelling at time $t = 3$ seconds.

[In this question position vectors are given relative to a fixed origin O.]

6. A particle, P, moves with constant acceleration $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}.

At time $t = 0$ seconds, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j})$ m and is moving with velocity $\mathbf{u}$ m s^{-1}.

At time $t = 3$ seconds, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j})$ m.

Find $\mathbf{u}$.

(4)

Question	Scheme	Marks	AOs
1	[illegible]	B1	[illegible]
	[illegible]	M1	[illegible]
	[illegible]	A1ft	[illegible]
	[illegible]	A1	[illegible]
		(4)	
			(4 marks)

An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is travelling with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})$ ms^{-1}.

At time $t = 20$ s the skater is travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})$ ms^{-1}.

Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds.

Modelling the ice skater as a particle with constant acceleration, find:

(a) The acceleration of the ice skater

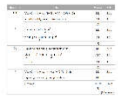
(b) An expression for $\boldsymbol{s}$ in terms of t

(c) The time at which the skater is directly north-east of O.

A second skater travels so that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}$ m relative to O at time t.

(d) Show that the two skaters will meet.

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]



A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is $\mathbf{r}$ metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j}) \text{ m s}^{-1}$.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$.
(2)

(b) Find $\mathbf{r}$ in terms of t.
(2)

(c) Find the value of t when the boat is north-east of O.
(3)

(d) Find the value of t when the boat is moving in a north-east direction.
(3)

Ex 8A Evens

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[In this question **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.]*

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours. **(2)**

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n. **(5)**

(c) Find the distance OP. **(4)**

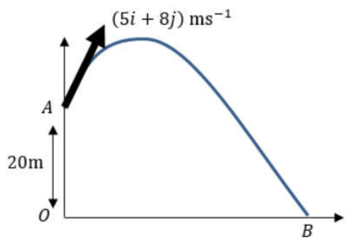
Vector methods for projectiles

A ball is struck by a racket from a point A which has position vector $20j$ m relative to a fixed origin O. Immediately after being struck, the ball has velocity $(5i + 8j)$ ms^{-1}, where i and j are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point B.

(a) Find the speed of the ball 1.5 seconds after being struck.

(b) Find an expression for the position vector, r, of the ball relative to O at time t seconds.

(c) Hence determine the distance OB.



Ex 8B evens

4. [*In this question the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upward.*]

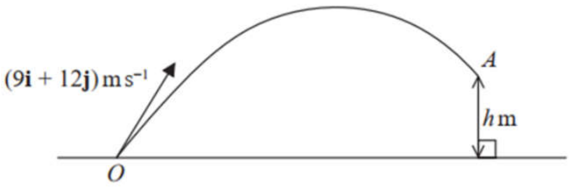


Figure 2

A small ball is projected from the fixed point O on horizontal ground with velocity $(9\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-1}}$

The ball passes through the point A which is h metres vertically above the level of O, as shown in Figure 2.

The velocity of the ball at the instant it passes through the point A is $\lambda(\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$, where λ is a positive constant.

The ball is modelled as a particle moving freely under gravity.

(a) Find the value of h.

(4)

(b) State the minimum speed of the ball as it moves from O to A.

(1)

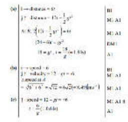
(c) Find the length of time for which the speed of the ball is less than $12\,\mathrm{m\,s^{-1}}$

(4)

The model could be refined by considering air resistance.

(d) Suggest one other refinement to the model that would make it more realistic.

(1)



[*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

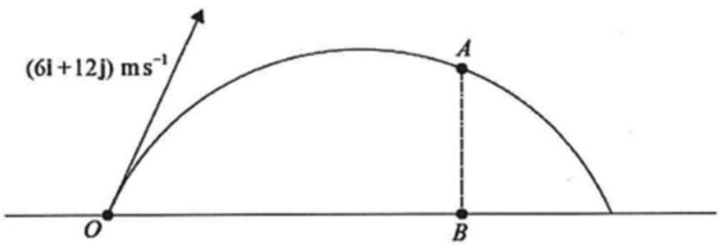


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(*a*) the value of t,
(7)

(*b*) the speed, V m s^{-1}, of the ball at the instant when it passes through A.
(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(*c*) Find the time taken for the ball to travel from O to C.
(3)

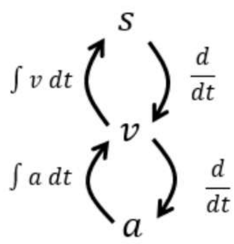
Variable Acceleration in One Dimension - more complex functions

A particle is moving in a straight line with acceleration at time t seconds given by

$$a = \cos 2\pi t \text{ ms}^{-2}, \qquad t \geq 0$$

The velocity of the particle at time $t = 0$ is $\frac{1}{2\pi}$ ms^{-1}. Find:

(a) an expression for the velocity at time t seconds
(b) the maximum speed
(c) the distance travelled in the first 3 seconds.



A particle of mass 6kg is moving on the positive x-axis. At time t seconds the displacement, s, of the particle from the origin is given by

$$s = 2t^{\frac{3}{2}} + \frac{e^{-2t}}{3} \text{ m}, \qquad t \geq 0$$

(a) Find the velocity of the particle when $t = 1.5$.
Given that the particle is acted on by a single force of variable magnitude F N which acts in the direction of the positive x-axis,
(b) Find the value of F when $t = 2$

Ex 8C evens

Differentiating Vectors

Suppose that $\boldsymbol{v} = \begin{pmatrix} t^2 \\ \sin t \end{pmatrix}$. What would be the acceleration?

✎ If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ then $\boldsymbol{v} = \frac{dr}{dt} = \dot{\boldsymbol{r}} = \dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j}$

and $\boldsymbol{a} = \frac{dv}{dt} = \frac{d^2r}{dt} = \ddot{\boldsymbol{r}} = \ddot{x}\boldsymbol{i} + \ddot{y}\boldsymbol{j}$

A particle P of mass 0.8kg is acted on by a single force $\mathbf{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$

Find:

(a) the speed of P when $t = 4$

(b) the acceleration of P as a vector when $t = 2$

(c) $\mathbf{F}$ when $t = 2$.

Integrating Vectors

A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by

$$\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}, \qquad t \geq 0$$

When $t = 0$, the position vector of P with respect to a fixed O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector of P at time t seconds.

A particle P is moving in a plane so that, at time t seconds, its acceleration is $(4\boldsymbol{i} - 2t\boldsymbol{j})$ ms^{-2}. When $t = 3$, the velocity of P is $6\boldsymbol{i}$ ms^{-1} and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find:

(a) the angle between the direction of motion of P and $\boldsymbol{i}$ when $t = 2$

(b) the distance of P from O when $t = 0$.

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At time t seconds the velocity of a particle P is $[(4t - 5)\mathbf{i} + 3\mathbf{j}]$ m s^{-1}. When $t = 0$, the position vector of P is $(2\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin O.

(*a*) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(*b*) Find an expression for the position vector of P at time t seconds.

(4)

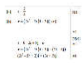
A second particle Q moves with constant velocity $(-2\mathbf{i} + c\mathbf{j})$ m s^{-1}. When $t = 0$, the position vector of Q is $(11\mathbf{i} + 2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i} + 14\mathbf{j})$ m.

(*c*) Find

(i) the value of c,

(ii) the value of d.

(5)



Exam Questions

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

7. A particle, P, moves under the action of a single force in such a way that at time t seconds, where $t \geq 0$, its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = (t^2 - 3t)\,\mathbf{i} - 12t\,\mathbf{j}$$

The mass of P is 0.5 kg.

Find the time at which the magnitude of the force acting on P is 6.5 N.

(7)

3. [*In this question position vectors are given relative to a fixed origin O*]

A particle P moves under the action of a single force $\mathbf{F}$ newtons.

At time t seconds, where $t \geqslant 0$, the position vector of P, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = (t^3 - 5t)\mathbf{i} + (5t^2 + 6t)\mathbf{j}$$

The mass of P is 0.5 kg.

At time T seconds, P is moving in the direction of the vector $(\mathbf{i} + 2\mathbf{j})$.

(a) Find the value of T.

(5)

(b) Find the magnitude of $\mathbf{F}$ when $t = 2$

(4)