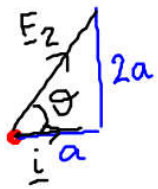
9 A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by $\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j})$ N and $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$ N, where a is a positive constant.

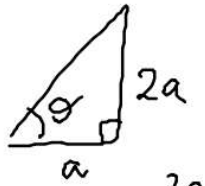
a Find the angle between $\mathbf{F}_2$ and $\mathbf{i}$. **(2 marks)**

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.

b Given that $\mathbf{R}$ is parallel to $13\mathbf{i} + 10\mathbf{j}$, find the value of a. **(4 marks)**

a) $\underline{F}_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\underline{F}_2 = \begin{pmatrix} a \\ 2a \end{pmatrix}$





$\tan\theta = \frac{2a}{a} = 2$

$\theta = \tan^{-1}(2)$

$\theta = 63.4^\circ$.

b) $\underline{R} = \underline{F}_1 + \underline{F}_2$

$\underline{R} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} a \\ 2a \end{pmatrix} = \begin{pmatrix} 3+a \\ -2+2a \end{pmatrix}$

$\begin{pmatrix} 3+a \\ -2+2a \end{pmatrix} = k \begin{pmatrix} 13 \\ 10 \end{pmatrix}$ ← $13\mathbf{i} + 10\mathbf{j}$

resultant force is parallel to

$\begin{pmatrix} 3+a \\ -2+2a \end{pmatrix} = \begin{pmatrix} 13k \\ 10k \end{pmatrix}$

i: $3 + a = 13k$

i: $1a - 13k = -3$

$-2 + 2a = 10k$

j: $12a - 10k = 2$

$k = \frac{1}{2}$ $a = 3.5$

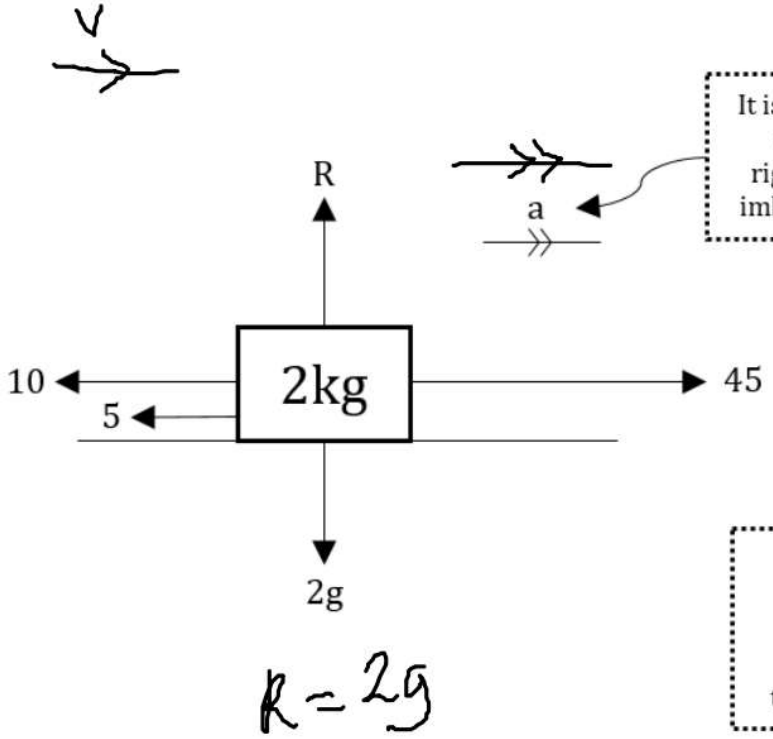
$\underline{R} = \begin{pmatrix} 6.5 \\ 5 \end{pmatrix}$ → is this parallel to $\begin{pmatrix} 13 \\ 10 \end{pmatrix}$

Forces and Acceleration

→ not just the force → RESULTANT FORCE.

Newton's 2nd Law of Motion: $F = ma$
(where the **resultant** force F and acceleration a are in the same direction)

This 'feels' right: if we doubled the force, we double the rate at which it accelerates. Similarly, if we have an object of twice the mass, we'd require twice the force to make it accelerate at the same rate.



It is clearly going to accelerate to the right as there is an imbalance of forces

Notice how we find the resultant force by doing the forces to the right minus the forces to the left

Work out the acceleration for the particle.

Using F = ma

$45 - 10 - 5 = 2a$

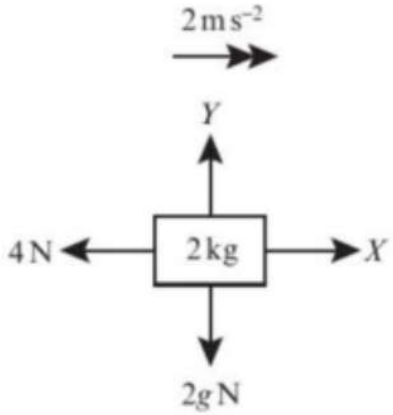
$30 = 2a$

$15 = a$

So the particle will accelerate at $15 ms^{-2}$ to the right

In each of these diagrams the body is accelerating as shown. Find the magnitudes of the unknown forces X and Y.

a



$R\updownarrow \quad Y = 2g$

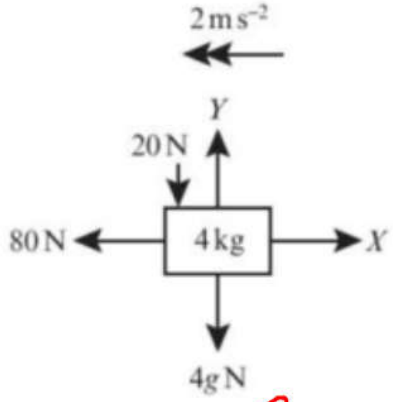
$R\rightarrow, \quad F = ma$

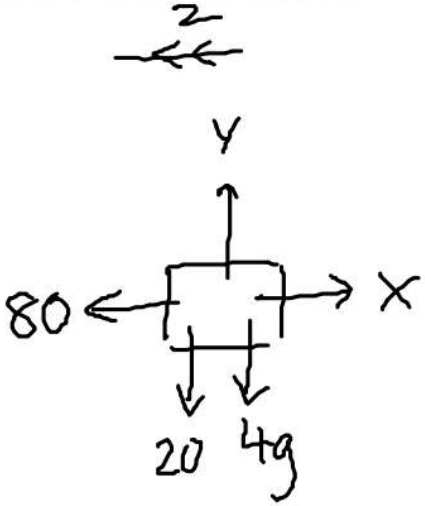
$X - 4 = 2 \times 2$

$X - 4 = 4$

$X = 8$

b





$R\updownarrow \quad Y = 20 + 4g$

$Y = 59.2\ N$

$R\rightarrow, \quad F = ma$

$X - 80 = 4 \times (-2)$

$X - 80 = -8$

$X = 72$

$R\leftarrow, \quad F = ma$

$80 - X = 4 \times 2$

$80 - X = 8$

$X = 72$

A car of 2000kg has a driving force of 800N and forces of 200N resisting its motion. Determine its acceleration.

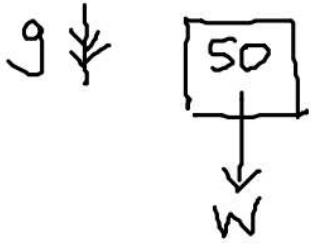
$R\rightarrow$, $F=ma$

$800-200=2000a$

$\frac{600}{2000}=a$

$a=0.3\ ms^{-2}$

A child has a mass of 50kg. What is the gravitational force acting on the child? (i.e. its weight)



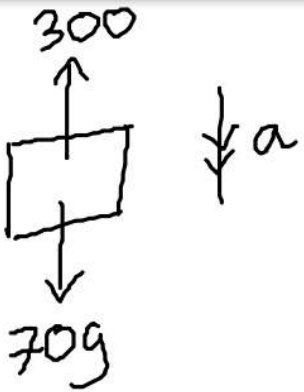
$F=ma$

$W=50g$

$W=mg$

$F=ma$

A falling sheep of mass 70kg experiences air resistance of 300 N. Determine the sheep's acceleration as it plummets towards the ground.



$R\downarrow$, $F=ma$

$70g-300=70a$

$\frac{386}{70}=a$

$a=5.51\ ms^{-2}$ (3sf)

Combining F=ma with SUVAT equations

Since $F = ma$ involves both force and acceleration, it allows us to connect calculations involving forces with a calculations involving $suvat$ values.

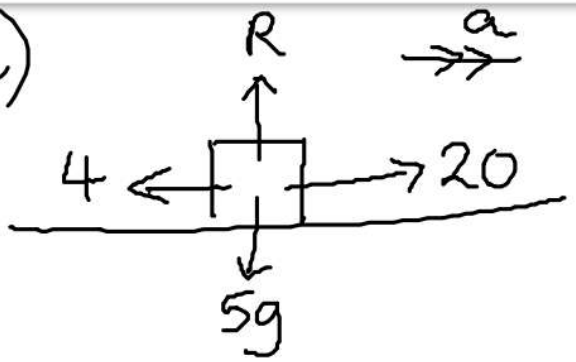
A body of mass 5kg is pulled along a rough horizontal table by a horizontal force of magnitude 20N against a constant friction force of magnitude 4N. Given that the body is initially at rest, find:

(a) the acceleration of the body ✓

(b) the distance travelled by the body in the first 4 seconds ✓

(c) the magnitude of the normal reaction between the body and the table

a)



$R \rightarrow, F = ma$

$20 - 4 = 5a$

$\frac{16}{5} = a$

$a = 3.2\ ms^{-2}$

$m = 5$

$W = 5g$

b) $t = 4$

$a = 3.2$

$u = 0$

$s = ?$

$s = ut + \frac{1}{2}at^2$

$s = 0 \times 4 + \frac{1}{2} \times 3.2 \times 4^2$

$= 25.6m$

c) $(R\updownarrow)$

$R = 5g$

$= 49\ N$

Q.5, 7, 9, 11

Ex 10C

A lift of mass 500kg is lowered or raised by means of a metal cable attached to its top. The lift contains passengers whose total mass is 300kg.
The lift starts from rest and accelerates at a constant rate, reaching a speed of $3ms^{-1}$, after moving a distance of 5m. Find:
a) the acceleration of the lift
b) the tension in the cable if the lift is moving down
c) if the lift is moving up

down < up

a) $u=0$, $v=3$, $s=5$, $a=?$

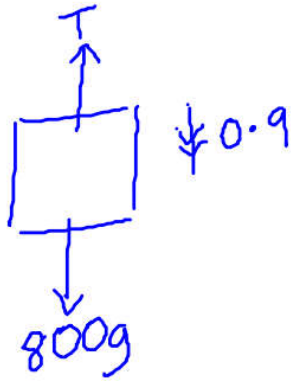
$v^2 = u^2 + 2as$

$3^2 = 0^2 + 2 \times a \times 5$

$9 = 10a$

$a = 0.9$

b)

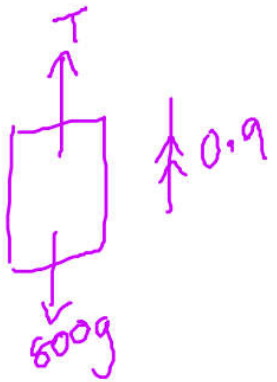


$R\downarrow, F=ma$

$800g - T = 800 \times 0.9$

$7840 - T = 720$

$T = 7120\ N$



$R\uparrow, F=ma$

$T - 800g = 800 \times 0.9$

$T - 7840 = 720$

$T = 8560\ N$

Edexcel M1 May 2012 Q5 (abridged)

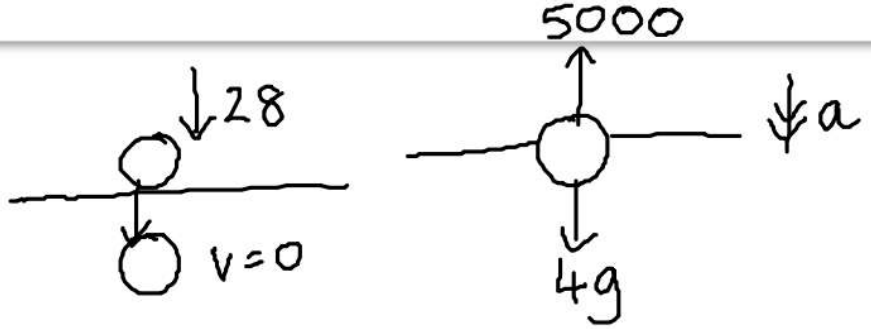
~~A particle P is projected vertically upwards from a point A with speed u m s^{-1}. The point A is 17.5 m above horizontal ground.~~ The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1}.

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)

28 ↓ ○



↓ +

$a = -1240.2$

$u = 28$

$v = 0$

$s = ?$

$R\downarrow, F = ma$

$4g - 5000 = 4a$

$\frac{4 \times 9.8 - 5000}{4} = a$

$a = -1240.2 \text{ ms}^{-2}$

↓ deceleration

$v^2 = u^2 + 2as$

$0 = 28^2 + 2(-1240.2)s$

$2480.4s = 784$

$s = 0.316 \text{ m (3sf)}$ 31.6cm.