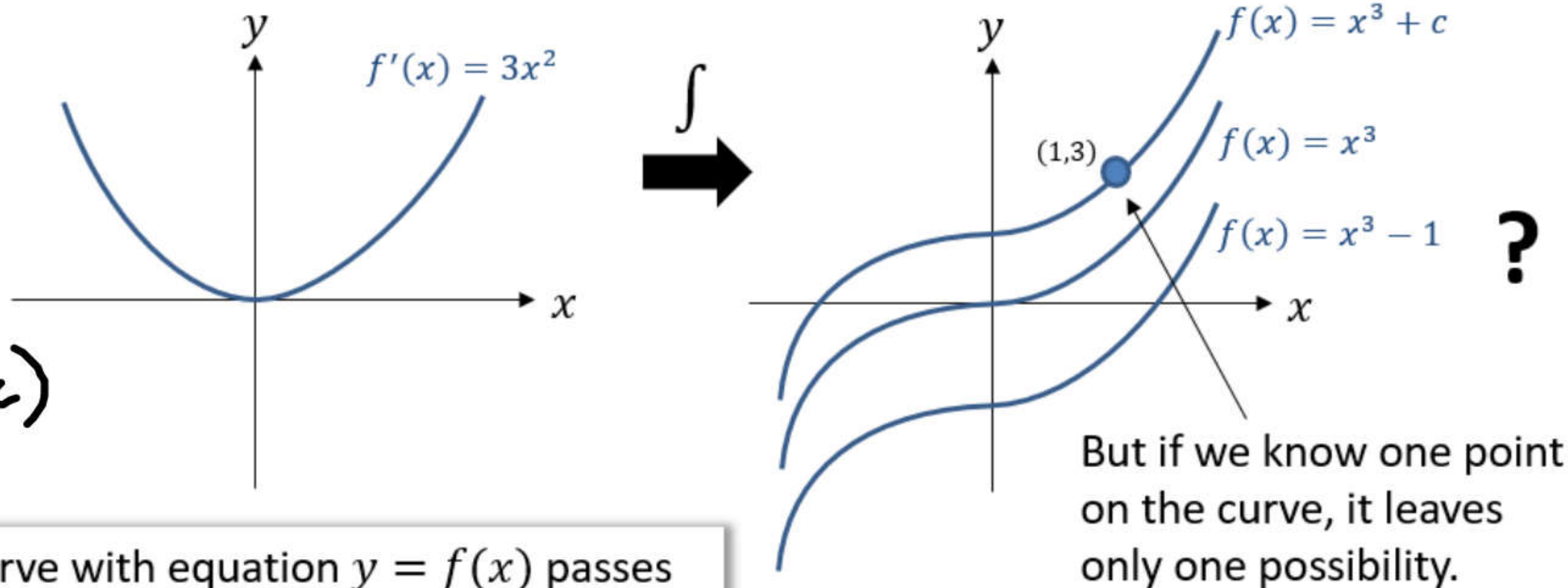
Finding constant of integration

Recall that when we integrate, we get a constant of integration, which could be any real value. This means **we don't know what the exact original function was**.



$f'(x)$

The curve with equation $y = f(x)$ passes through $(1,3)$. Given that $f'(x) = 3x^2$, find the equation of the curve.

$f'(x) = 3x^2$

$f(x) = x^3 + c$

$3 = 1^3 + c$

$c = 2$

$(1, 3)$

$x = 1$

$f(x) = 3$

$f(1) = 3$

$f(x) = x^3 + 2$

Edexcel C1 May 2014 Q10

A curve with equation $y = f(x)$ passes through the point (4, 25).

Given that

$-10x^{-1/2}$

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

$\frac{dy}{dx} = \frac{3}{8}x^2 - 10x^{-1/2} + 1$

(a) find f(x), simplifying each term.

Find y

(5)

(b) Find an equation of the normal to the curve at the point (4, 25).

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

To keep you occupied if you finish (a) quickly!

$f'(x) = \frac{3}{8}x^2 - 10x^{-1/2} + 1$

Integrate to find $f(x)$

$f(x) = \frac{1}{8}x^3 - 20x^{1/2} + x + c$

Sub in values from (4, 25) to find c

$25 = \frac{1}{8}(4)^3 - 20(4)^{1/2} + 4 + c$

$25 = 8 - 40 + 4 + c$

$53 = c$

$f(x) = \frac{1}{8}x^3 - 20x^{1/2} + x + 53$

$y - y_1 = m(x - x_1)$

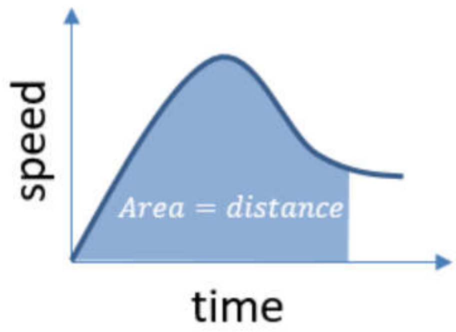
Ex 13C

1ace, 3, 6

Definite Integration

So far we've seen integration as '**the opposite of differentiation**', allowing us to find $y = f(x)$ when we know the gradient function $y = f'(x)$.

In practical settings however the most useful use of integration is that **it finds the area under a graph**. Remember at GCSE for example when you estimated the area under a speed-time graph, using trapeziums, to get the distance?
If you knew the equation of the curve, you could get the exact area!



Before we do this, we need to understand how to find a **definite integral**:

These are known as **limits**, which give the values of x we're finding the area between.

We integrate as normal, but put expression in **square brackets**, meaning **we still need to evaluate the integrated expression using the limits**.

$$\int_1^5 4x^3\,dx = \left[x^4\right]_1^5$$

$$= (5^4) - (1^4)$$

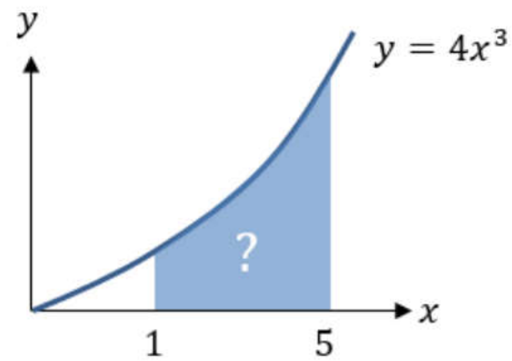
$$= 625 - 1$$

$$= \underline{\underline{624}}.$$

$$\left[x^4 + c\right]_1^5$$

$$= (5^4 + c) - (1^4 + c)$$

$$= 5^4 + \cancel{c} - 1^4 - \cancel{c}$$



Write (...) − (...) and evaluate the expression for each of the limits, top one first.

$$\int_{-3}^{3}(x^2+1)dx = \left[\frac{1}{3}x^3 + x\right]_{-3}^{3}$$

$$= \left(\frac{1}{3}(3)^3 + 3\right) - \left(\frac{1}{3}(-3)^3 + (-3)\right)$$

$$= 12 - (-9 - 3)$$

$$= 12 - (-12)$$

$$= \underline{\underline{24}}.$$

We **DON'T have a constant of integration** when doing definite integration.

Write out you working EXACTLY as seen here. The (…) − (…) brackets are particularly crucial as you'll otherwise likely make a sign error.

You can use the $\left[\!\left[\int_b^a \square\right]\!\right]$ button on your calculator to evaluate definite integrals.
<u>But only use it to check your answer.</u>

Problem Solving

Given that P is a constant and $\int_1^5 (2Px + 7)\, dx = 4P^2$, show that there are two possible values for P and find these values.

$$\int_1^5 (2Px+7)\,dx = \left[Px^2 + 7x\right]_1^5$$

$$= \left(P(5)^2 + 7\times 5\right) - \left(P(1)^2 + 7\times 1\right)$$

$$= 25P + 35 - (P + 7)$$

$$= 25P + 35 - P - 7$$

$$= 24P + 28$$

$$24P + 28 = 4P^2$$

$$0 = 4P^2 - 24P - 28$$

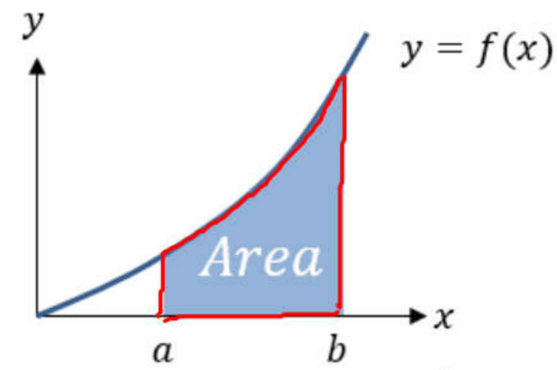
$$P = 7, \quad P = -1$$

Ex 13D

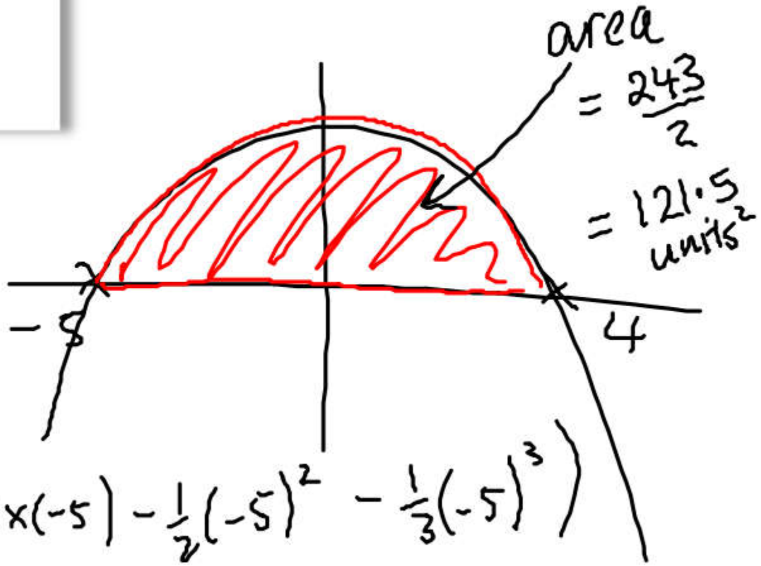
Q 3, 4

Areas under curves

Earlier we saw that the definite integral $\int_b^a \boldsymbol{f(x)}\ \boldsymbol{dx}$ gives the **area** between a positive curve $y = f(x)$, the $\boldsymbol{x}$**-axis**, and the lines $x = a$ and $x = b$.
(We'll see why this works in a sec)



Find the area of the finite region between the curve with equation $y = 20 - x - x^2$ and the x-axis.



$$\int_{-5}^{4} (20 - x - x^2)\ dx$$

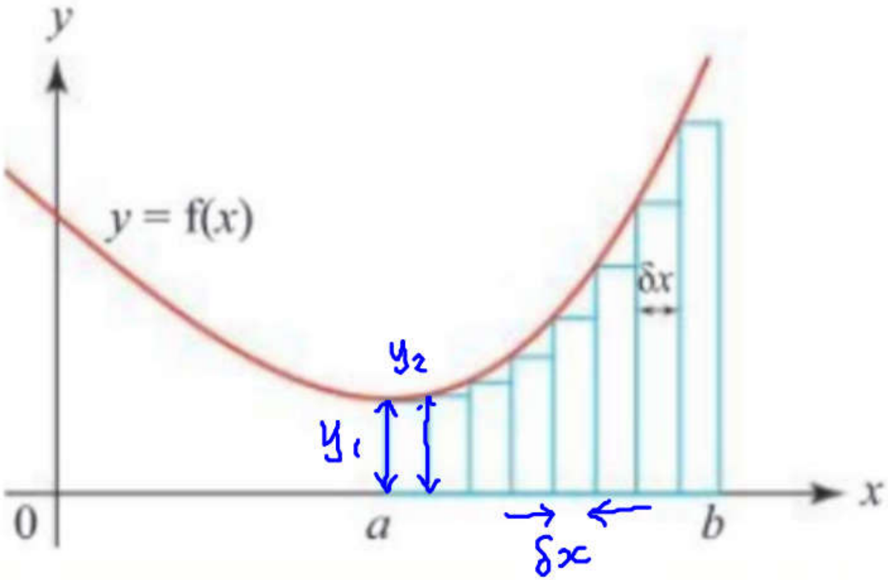
$$= \left[20x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_{-5}^{4}$$

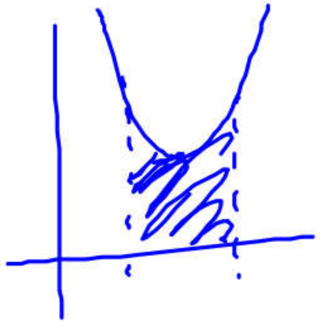
$$= \left(20\times 4 - \tfrac{1}{2}(4)^2 - \tfrac{1}{3}(4)^3\right) - \left(20\times(-5) - \tfrac{1}{2}(-5)^2 - \tfrac{1}{3}(-5)^3\right)$$

$$= 80 - 8 - \frac{64}{3} - \left(-100 - \frac{25}{2} + \frac{125}{3}\right)$$

$$= 80 - 8 - \frac{64}{3} + 100 + \frac{25}{2} - \frac{125}{3} = \frac{243}{2}$$

Why does integration give the area under the curve?





Ex13E

$\delta x \to dx$

$$A = \lim_{\delta x \to 0} \sum_{i=1}^{n} y_i \, \delta x$$

$$A \approx y_1\delta x + y_2\delta x + \ldots + y_n\delta x$$

approx.

$$A \approx \sum_{i=1}^{n} y_i \delta x$$

sum of

$$\sum_{i=1}^{n} y_i \, \delta x$$

$$\boxed{A = \int_a^b y \, dx}$$

$$= \int_0^b y \, dx - \int_0^a y \, dx$$