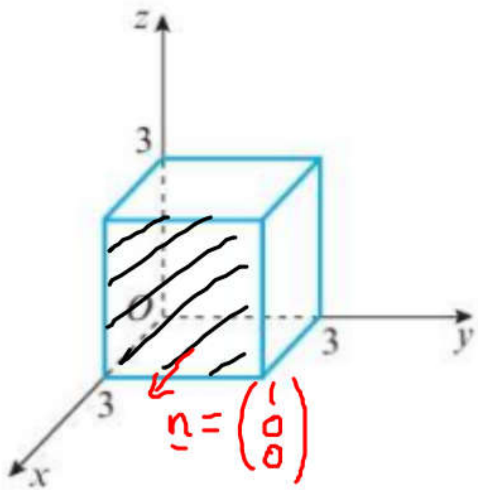
The diagram shows a cube with a vertex at the origin and sides of length 3.
Find a Cartesian equation for each face of the cube.



$x = 3 \qquad 0 \le y \le 3$

$0 \le z \le 3$

$1x + 0y + 0z = c$

$c = 3$

$x = 3$

Scalar Dot Product

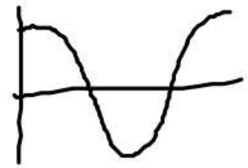
This is the scalar product of two vectors (not to be confused with the cross product, which gives a non-scalar answer)

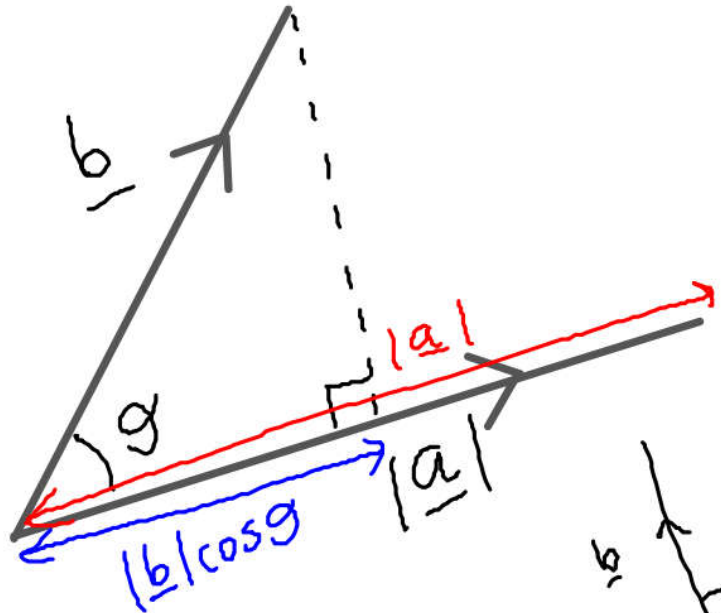
The scalar dot product is defined as:

$$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

$$\cos\theta = \frac{x}{|\underline{b}|}$$

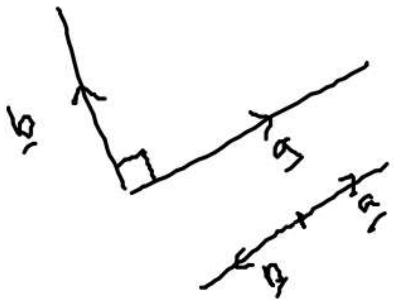
$$x = |\underline{b}|\cos\theta$$





$$\underline{a}\cdot\underline{b} = |\underline{a}|\underbrace{|\underline{b}|\cos\theta}$$

found the amount of $\underline{b}$ going in the same direction as $\underline{a}$.



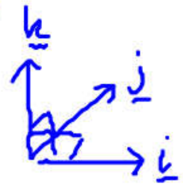
$\cos \boxed{90°} = 0$ $\cos \boxed{0°} = 1$

So, if the angle between the 2 vectors is 90 (ie. the 2 vectors are perpendicular), the dot product is 0.

If the angle between the 2 vectors is 0 (ie. the vectors are parallel), the dot product is the same as the modulus of each vector multiplied together

$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$

Using this idea, what are the following equal to?



Hint: think about the angles between i, j, and k

i.**i** = $|\underline{i}||\underline{i}|\cos 0 = 1$

j.**i** = $|\underline{j}||\underline{i}|\cos 90° = 0$

k.**i** = 0

2**i**.4**i** = $|2\underline{i}||4\underline{i}|\cos 0 = 8$

i.**j** = 0

j.**j** = 1

k.**j** = 0

3**i**.7**k** = 0

i.**k** = 0

j.**k** = 0

k.**k** = 1

2**j**.4**j** = 8

The scalar/dot product $\boldsymbol{a} \cdot \boldsymbol{b}$ of two vectors is the sum of the products of the components.

$$\boldsymbol{a} \cdot \boldsymbol{b} = \sum a_i b_i$$

$$-2\underline{j} \cdot 3\underline{j} = |-2\underline{j}||3\underline{j}| \cos 180$$
$$= 2 \times 3 \times -1$$
$$= -6$$

180

$$\begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} = (5\underline{i} - 2\underline{j} + 0\underline{k}) \cdot (3\underline{i} + 3\underline{j} + \underline{k})$$
$$= 5 \times 3 - 2 \times 3 + 0 \times 1$$
$$= 15 - 6 + 0$$
$$= 9$$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = 3 \times 4 + 2 \times 0 + 1 \times 1$$
$$= 13$$

$$\begin{pmatrix} a \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ b \\ 10 \end{pmatrix} = 3a + 5b + 10.$$

$$\begin{pmatrix} -2 \\ -4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix} = 0 - 16 - 15$$
$$= -31$$

Angle between vectors:

$$\cos\theta = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$$

$$\underset{\sim}{a}\cdot\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$$

$$\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \cos\theta$$

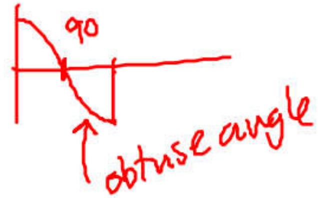
Find the acute angle between the vectors $\boldsymbol{a} = \begin{pmatrix}5\\3\\1\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}1\\0\\5\end{pmatrix}$.

If obtuse, to get acute, $180-\theta$ or. we change the formula $\cos\theta = \left|\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}\right|$

$$\cos\theta = \frac{\begin{pmatrix}5\\3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\5\end{pmatrix}}{\sqrt{5^2+3^2+1^2}\times\sqrt{1^2+5^2}} = \frac{5\times1+0+1\times5}{\sqrt{35}\,\sqrt{26}} = \frac{10}{\sqrt{35}\sqrt{26}}$$

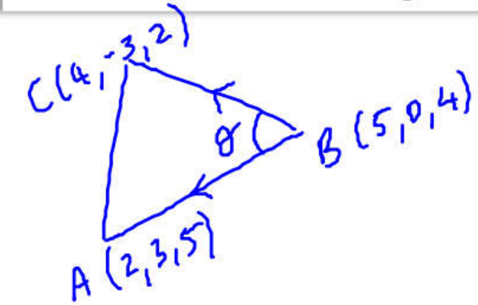
$$\theta = \cos^{-1}\left(\frac{10}{\sqrt{35}\sqrt{26}}\right)$$

$$= \underline{\underline{70.6^\circ}}\ (1\text{dp})$$



If $A(2,3,5)$, $B(5,0,4)$ and $C(4,-3,2)$, determine the angle ABC.

Hence find the area of triangle ABC.



$$\vec{BC} = \underset{\sim}{c}-\underset{\sim}{b} = \begin{pmatrix}4-5\\-3\\2-4\end{pmatrix} = \begin{pmatrix}-1\\-3\\-2\end{pmatrix}$$

$$\vec{BA} = \underset{\sim}{a}-\underset{\sim}{b} = \begin{pmatrix}2-5\\3\\5-4\end{pmatrix} = \begin{pmatrix}-3\\3\\1\end{pmatrix}$$

$$\cos\theta = \frac{\begin{pmatrix}-1\\-3\\-2\end{pmatrix}\cdot\begin{pmatrix}-3\\3\\1\end{pmatrix}}{\sqrt{1^2+3^2+2^2}\sqrt{3^2+3^2+1^2}} = \frac{3-9-2}{\sqrt{14}\sqrt{19}} = \frac{-8}{\sqrt{14}\sqrt{19}}$$

$$\theta = \cos^{-1}\left(\frac{-8}{\sqrt{14}\sqrt{19}}\right)$$

$$= 119.4^\circ$$

Area ABC = "$\frac{1}{2}ab\sin C$" $= \frac{1}{2}\times\sqrt{14}\times\sqrt{19}\times\sin 119.4^\circ$

$= 7.11$ units^2 (2dp)

If two vectors are perpendicular then:

$$\boldsymbol{a} \cdot \boldsymbol{b} = 0$$

Show that $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ are perpendicular.

$$\underline{a} \cdot \underline{b} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = 2 + 0 - 2$$
$$= 0$$

So $\underline{a}$ and $\underline{b}$ are perpendicular

without a calc?

Let $z = 1$

$\cancel{z = 0}$

$-2x + 5y = 4$

$4x - 8y = -5$

$x = \frac{7}{4} \quad y = \frac{3}{2}$

Given that $\boldsymbol{a} = -2\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

$\underline{a} = \begin{pmatrix} -2 \\ 5 \\ -4 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 4 \\ -8 \\ 5 \end{pmatrix} \quad \underline{c} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\underline{a} \cdot \underline{c} = 0 \qquad -2x + 5y - 4z = 0$

$\underline{b} \cdot \underline{c} = 0 \qquad 4x - 8y + 5z = 0$

$z = 1, \; x = \frac{7}{4}, \; y = \frac{3}{2}$

$$\underline{c} = \begin{pmatrix} 7/4 \\ 3/2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix}$$

Ex 9C

Q3, 5, 7, 9, 11, 13, 15

[June 2008 Q6] 6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$
$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(*b*) Show that l_1 and l_2 are perpendicular to each other. **(2)**

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

$r = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ $|\overrightarrow{AD}| = \sqrt{1}$

$|\overrightarrow{AB}| = \sqrt{43}$

$\overrightarrow{AD} \cdot \overrightarrow{AB} = -8$

$\theta = \cos^{-1}\left(-\frac{8}{\sqrt{14}\sqrt{43}}\right)$

$= 109.03°$

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{AD}$

$= \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix}$

$\text{Area} = 2 \times \frac{1}{2} \times \sqrt{14} \times \sqrt{43} \times \sin(109.03)$

$= 23.2$

[Jan 2012 Q7] 7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,

the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,

and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$. **(2)**

(*b*) Find a vector equation for the line l. **(2)**

(*c*) Show that the size of the angle BAD is 109°, to the nearest degree. **(4)**

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(*d*) Find the position vector of C. **(2)**

(*e*) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures. **(3)**