Aiming for A*: ‘in between’ Pure 1 and Pure 2 [Edexcel]

Bicen Maths

Note:
- I have used questions from the international A-Level, old specification A-Level, and other exam boards – I only used ‘recent’ years of the old specification, as this is when examiners were gearing up for the new style of questions, so they’re a good fit!
- This is to ensure the questions are all ‘fresh’, as at this stage I know that exam question fatigue is real.
- I’ve used my knowledge to select ones that match Edexcel’s style as much as possible.

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OCR Pure 1 2020

Try these questions whilst we wait for everyone to arrive

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1 **(a)** For a small angle θ, where θ is in radians, show that $2\cos\theta + (1-\tan\theta)^2 \approx 3 - 2\theta$. **[3]**

(b) Hence determine an approximate solution to $2\cos\theta + (1-\tan\theta)^2 = 28\sin\theta$. **[2]**

OCR Pure/Mech 3 2020

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2 A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. **[3]**

OCR Pure 1 2018

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4 Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n. **[4]**

OCR MEI Pure 1 2021

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1 Beth states that for all real numbers p and q, if $p^2 > q^2$ then $p > q$.

Prove that Beth is **not** correct. **[2]**

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What came up in Paper 1?

- factor theorem
- binomial expansion
- numerical methods, Newton-Raphson
- differentiation first principles
- quotient rule/decreasing functions
- modulus graphs
- simple differential equation (tank)
- functions - composite, inverse, domain, range
- geometric series (included indices)
- area integration/differentiation problem
- sector/segment areas
- harmonic identity (Rcos Rsin)
- integration by substitution
- differential equation - set up and solve (balloon)
- proof, inc. proof by contradiction

Key
- Doesn't usually appear in both
- Could potentially appear again
- Could easily appear again

What *should* come up in Paper 2?

- Small angle approximations
- Graph transformations
- Proof (again!)
- Sequences + series – arithmetic and/or sigma notation
- Circles
- Vectors
- Trigonometric identities equations
- Implicit differentiation, dx/dy
- Connected rates of change + optimisation problems
- More integration - limit of a sum, by parts, partial fractions
- Parametric differentiation + integration
- Exponential + logarithmic equations
- Modelling - quadratics, linear, exp & logs
- Trapezium rule
- Inequalities? Regions?
- Numerical methods?
- Surds?
- Pseudo-quadratics?

Key
- Not covered in this session
- Covered in the 'starter' – use textbook and Exam Qs PDF for more practice questions
- Covered in the main session or Your Turn questions

After the session, make sure you:
- Review anything you didn't understand
- Complete the Your Turn questions
- Review the red and blue topics
- Look over your notes for topics which have already come up, *just in case*

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Sequences and Series

- They usually have at least 2 questions on this topic

- $S_n = \frac{n}{2}(2a + (n-1)d)$
- $u_n = a + (n-1)d$

Arithmetic series

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

- $S_n = \frac{a(1-r^n)}{1-r}$
- $u_n = ar^{n-1}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

OCR Pure/Mech 3 2021

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3 An arithmetic progression has first term 2 and common difference d, where $d \neq 0$. The first, third and thirteenth terms of this progression are also the first, second and third terms, respectively, of a geometric progression.

By determining d, show that the arithmetic progression is an increasing sequence. **[5]**

OCR MEI Pure 3 2020

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3 A particular phone battery will last 10 hours when it is first used. Every time it is recharged, it will only last 98% of its previous time.

Find the maximum total length of use for the battery. **[3]**

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2. A sequence $u_1, u_2, u_3 \ldots$ is defined by

$$u_1 = 20$$

$$u_{n+1} = u_n + 5\sin\left(\frac{n\pi}{2}\right) - 3(-1)^n$$

(a) (i) Show that $u_2 = 28$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

7. A sequence is defined by

$$u_1 = 3$$
$$u_{n+1} = u_n - 5, \qquad n \geqslant 1$$

Find the values of

(a) u_2, u_3 and u_4

(2)

(b) u_{100}

(3)

(c) $\sum_{i=1}^{100} u_i$

(3)

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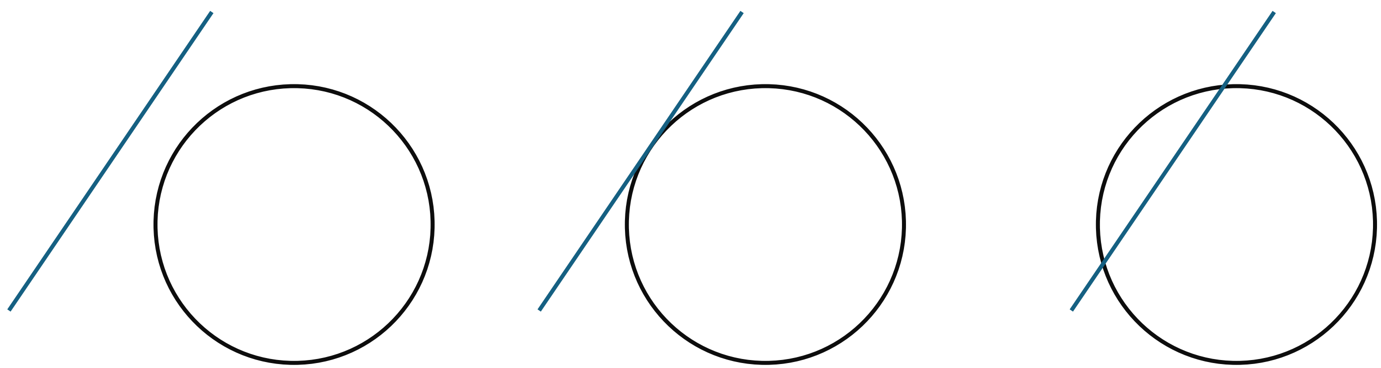
Circles

Tips:

- Use completing the square to get to standard form
- Use discriminant to decide how many intersections



14. The circle C has equation

$$x^2 + y^2 + 16y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

Given that the radius of C is 10

(b) find the value of k.

(2)

The point $A(a, -16)$, where $a > 0$, lies on the circle C. The tangent to C at the point A crosses the x-axis at the point D and crosses the y-axis at the point E.

(c) Find the exact area of triangle ODE.

(7)

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Your Turn

2 A circle with centre C has equation $x^2+y^2-6x+4y+4=0$.

(a) Find

(i) the coordinates of C, **[2]**

(ii) the radius of the circle. **[1]**

(b) Determine the set of values of k for which the line $y=kx-3$ does not intersect or touch the circle. **[5]**

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Vectors

- I'm going to concentrate on **geometric** problems here – *but of course, you should learn the basics for magnitude, angles with axes, etc.*
- They've not asked one of these style of questions for a long time, so I think they're worth revisiting

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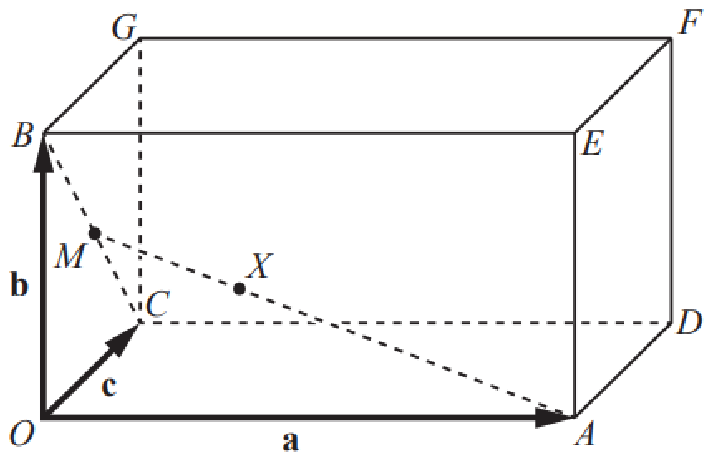
9 Points A, B and C have position vectors **a**, **b** and **c** relative to an origin O in 3-dimensional space. Rectangles $OADC$ and $BEFG$ are the base and top surface of a cuboid.



- The point M is the midpoint of BC.
- The point X lies on AM such that $AX = 2XM$.

(a) Find $\overrightarrow{OX}$ in terms of **a**, **b** and **c**, simplifying your answer. **[4]**

(b) Hence show that the lines OF and AM intersect. **[2]**

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Your Turn

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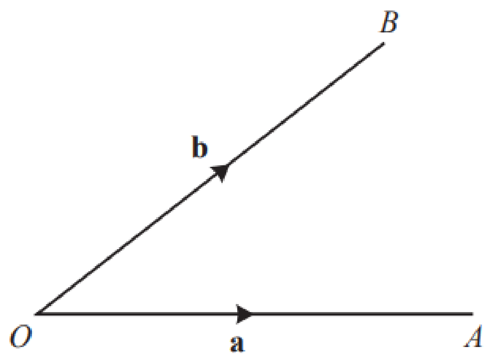

The diagram shows points A and B, which have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to an origin O.
P is the point on OB such that $OP : PB = 3:1$ and Q is the midpoint of AB.

(a) Find $\overrightarrow{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **[2]**

The line OA is extended to a point R, so that PQR is a straight line.

(b) Explain why $\overrightarrow{PR} = k\,(2\mathbf{a} - \mathbf{b})$, where k is a constant. **[2]**

(c) Hence determine the ratio $OA : AR$. **[4]**

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Trigonometry

- In my view, the following are essential to know by heart:

Simple:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

Pythagorean:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

Double angle:

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

- Use the shortcuts for finding the solutions:

$\sin x = \sin(180 - x)$ then ± 360

$\cos x = \cos(360 - x)$ then ± 360

$\tan x = \tan(x + 180)$

- Work on the 'messy' side, or one where can add fractions

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9. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \qquad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3 \tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

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Hint: (a) is in tan and cot… keep it that way!

8. (a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leqslant x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

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Your Turn

Hint: Can $1 = \sin^2 x + \cos^2 x$ help us at some point?

8. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \; n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

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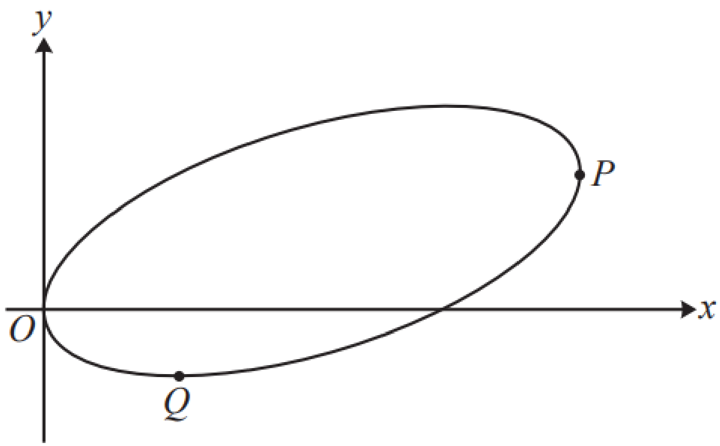
Implicit Differentiation

- If you differentiate y with respect to x, it becomes $\frac{dy}{dx}$
- If you differentiate a function in y with respect to x, do as you expect, then multiply by $\frac{dy}{dx}$
- I recommend writing out the product rule carefully for any product expressions including a y
  - e.g. $4x^2y^3$
- If you have $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, then $f(x, y) = 0$ for zero (horizontal) gradient, and $g(x, y) = 0$ for undefined (vertical) gradient

6 **In this question you must show detailed reasoning.**



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

I've pre-done some of (b) so as not to waste your time in the session

Your Turn

6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

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(ii) Given $x = \sin^2 2y$, $0 < y < \dfrac{\pi}{4}$, find $\dfrac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

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5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

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Your Turn

8. Given that

$$y = 8\tan(2x), \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

show that

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{A}{B + y^2}$$

where A and B are integers to be found.

(4)

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Modelling with differentiation

- Connected rates of change
  - Write out what you are looking for, then split the derivative up:
    - $\frac{dh}{dt} = \frac{dh}{\quad} \times \frac{\quad}{dt}$
- Optimisation problems
  - Setup by eliminating variables, differentiate and set to 0, then solve

4.

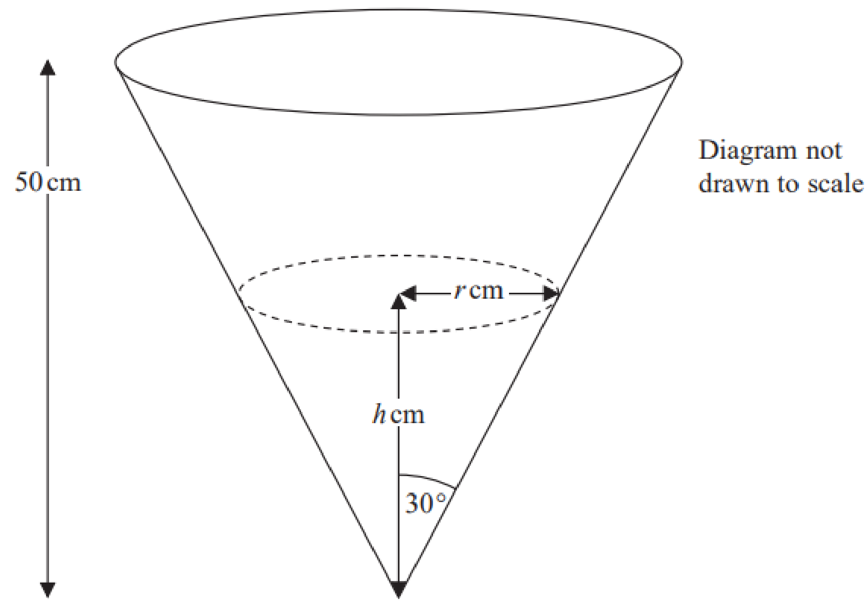

Figure 1

A water container is made in the shape of a hollow inverted right circular cone with semi-vertical angle of 30°, as shown in Figure 1. The height of the container is 50 cm.

When the depth of the water in the container is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(2)

Given that the volume of water in the container increases at a constant rate of 200 cm^3 s^{-1},

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 15$

Give your answer in its simplest form in terms of π.

(4)

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Your Turn

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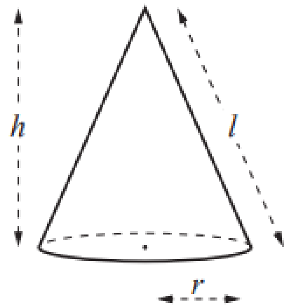

For a cone with base radius r, height h and slant height l, the following formulae are given.

Curved surface area, $S = \pi r l$

Volume, $V = \frac{1}{3}\pi r^2 h$

A container is to be designed in the shape of an inverted cone with no lid. The base radius is r m and the volume is $V\,\text{m}^3$.

The area of the material to be used for the cone is $4\pi\,\text{m}^2$.

(a) Show that $V = \frac{1}{3}\pi\sqrt{16r^2 - r^6}$. **[4]**

(b) **In this question you must show detailed reasoning.**

It is given that V has a maximum value for a certain value of r.

Find the maximum value of V, giving your answer correct to 3 significant figures. **[5]**

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Integration

- By parts
- Partial fractions

- … more in the parametric section…

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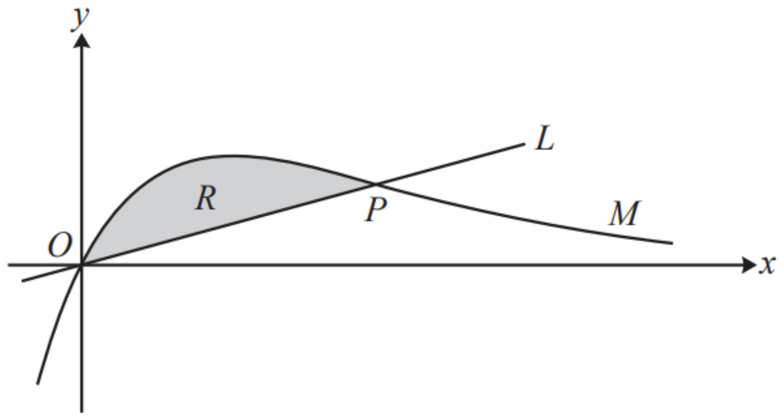
8



The diagram shows the curve M with equation $y = x\mathrm{e}^{-2x}$.

(a) Show that M has a point of inflection at the point P where $x = 1$. **[5]**

The line L passes through the origin O and the point P. The shaded region R is enclosed by the curve M and the line L.

(b) Show that the area of R is given by

$\frac{1}{4}(a + b\mathrm{e}^{-2})$,

where a and b are integers to be determined. **[6]**

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This one is a little weird – so I hope by showing you, you'll be ready for it in the exam!
It's where you have an exponential and trig product integral.

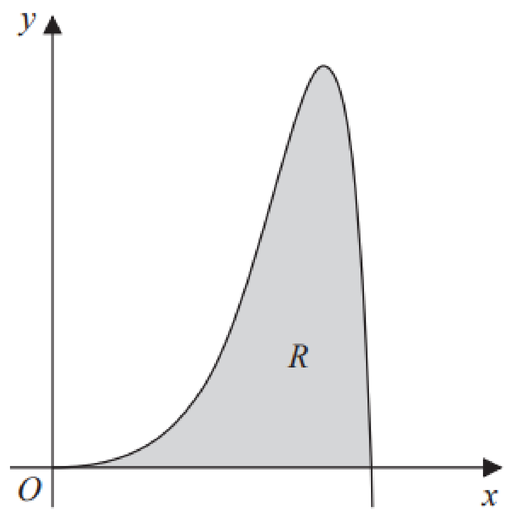

Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \qquad x \geqslant 0$$

The finite region R is bounded by the curve and the x-axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\dfrac{e^{2\pi} + 1}{5}$

(*Solutions relying on calculator technology are not acceptable.*)

(2)

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Your Turn

5. (a) Express $\dfrac{9(4+x)}{16-9x^2}$ in partial fractions.

(3)

Given that

$$f(x) = \frac{9(4+x)}{16-9x^2}, \qquad x \in \mathbb{R}, \qquad -\frac{4}{3} < x < \frac{4}{3}$$

(b) express $\int f(x)\,dx$ in the form $\ln(g(x))$, where $g(x)$ is a rational function.

(4)

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Parametrics

- 'Parametric world' is easier to work in than 'Cartesian world' – i.e. only create a Cartesian equation if they ask for one

- Differentiation: $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$
- Integration: $\int y \frac{dx}{dt}\, dt$ *Make sure the limits are in t*

- Be prepared to use a lot of different techniques in these questions – differentiation, integration, exponentials, logarithms, trigonometric identities…

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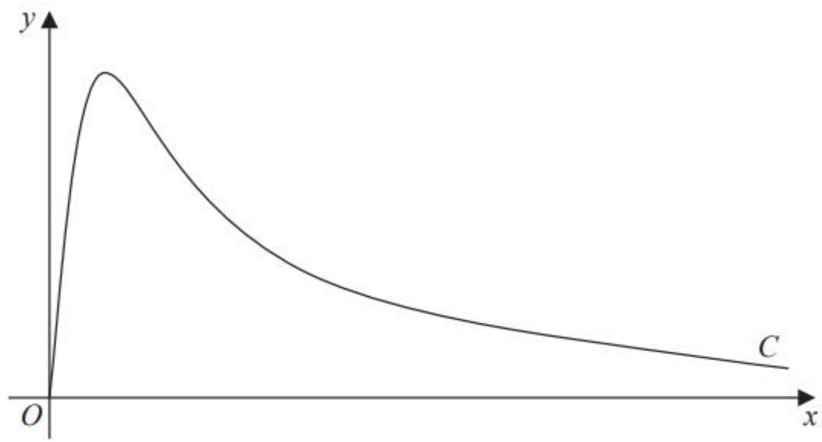

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(b) Find the exact coordinates of the point Q.

(2)

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Your Turn

The curve C shown in Figure 3 has parametric equations

$$x = 3\cos t, \qquad y = 9\sin 2t, \qquad 0 \leqslant t \leqslant 2\pi$$

The curve C meets the x-axis at the origin and at the points A and B, as shown in Figure 3.

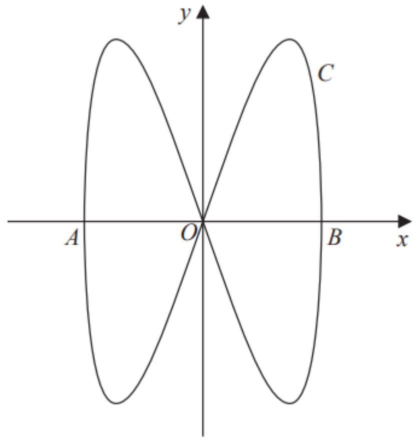

Figure 3

(a) Write down the coordinates of A and B.

(2)

(b) Find the values of t at which the curve passes through the origin.

(2)

(c) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, and hence find the gradient of the curve when $t = \frac{\pi}{6}$

(4)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y^2 = ax^2(b - x^2)$$

where a and b are integers to be determined.

(4)

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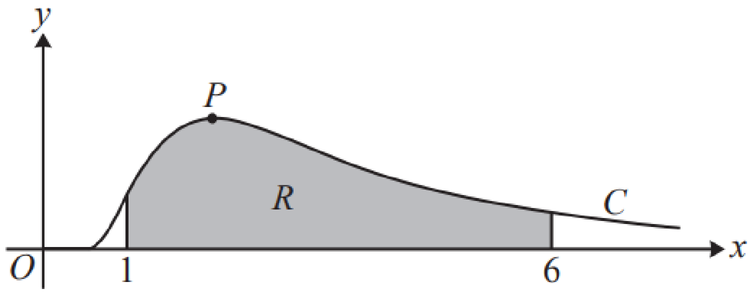
5



The diagram shows the curve C with parametric equations

$x = \frac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. **[4]**

The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

(b) Show that the area of R is given by

$$\int_a^b 3te^{-2t}\,dt,$$

where a and b are constants to be determined. **[3]**

(c) Hence determine the exact area of R. **[5]**

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Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta \sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

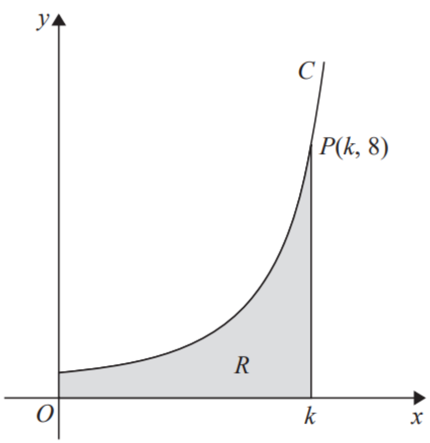

Figure 4

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda \int_{\alpha}^{\beta} \left(\theta \sec^2\theta + \tan\theta \sec^2\theta\right) \mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

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Exponentials and Logarithms - modelling

- Remember – modelling can only ask you to substitute, solve, or comment – don't reinvent maths just because it is in context!

- If it asks for rate, differentiate

- For non-linear to linear, take logs of both sides – I've done loads of these before, so only covered in a Your Turn question.

9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

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6 A mobile phone company records their annual sales on 31st December every year.

Paul thinks that the annual sales, S million, can be modelled by the equation $S = ab^t$, where a and b are both positive constants and t is the number of years since 31st December 2015.

Paul tests his theory by using the annual sales figures from 31st December 2015 to 31st December 2019. He plots these results on a graph, with t on the horizontal axis and $\log_{10} S$ on the vertical axis.

(a) Explain why, if Paul's model is correct, the results should lie on a straight line of best fit on his graph. **[3]**

The results lie on a straight line of best fit which has a gradient of 0.146 and an intercept on the vertical axis of 0.583.

(b) Use these values to obtain estimates for a and b, correct to 2 significant figures. **[2]**

(c) Use this model to predict the year in which, on the 31st December, the annual sales would first be recorded as greater than 200 million. **[3]**

(d) Give a reason why this prediction may not be reliable. **[1]**

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Sorry if another rabbit population question is a little triggering...

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1 + 3e^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island

A sketch of the graph of P against t is shown in Figure 3.

Figure 3

(a) Calculate the number of rabbits that were introduced onto the island.

(1)

(b) Find $\frac{dP}{dt}$

(3)

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(c) Using your answer from part (b), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below k, where k is a positive constant.

(d) Use the model to state the maximum value of k.

(1)

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5. A bath is filled with hot water. The temperature, θ °C, of the water in the bath, t minutes after the bath has been filled, is given by

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the water in the bath is initially 38 °C,

(a) find the value of A.

(2)

The temperature of the water in the bath 16 minutes after the bath has been filled is 24.5 °C.

(b) Show that $k = \frac{1}{8}\ln 2$

(4)

Using the values for k and A,

(c) find the temperature of the water 40 minutes after the bath has been filled, giving your answer to 3 significant figures.

(2)

(d) Explain why the temperature of the water in the bath cannot fall to 19 °C.

(1)

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3. The value of a car is modelled by the formula

$$V = 16\,000\mathrm{e}^{-kt} + A, \qquad t \geqslant 0,\ t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

(a) find the value of A,

(1)

(b) show that $k = \ln\left(\dfrac{2}{\sqrt{3}}\right)$

(4)

(c) Find the age of the car, in years, when the value of the car is £6000

Give your answer to 2 decimal places.

(4)

Your Turn

11 In a science experiment a substance is decaying exponentially. Its mass, M grams, at time t minutes is given by $M = 300e^{-0.05t}$.

(i) Find the time taken for the mass to decrease to half of its original value. **[3]**

A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

(ii) Find the time at which both substances are decaying at the same rate. **[8]**

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Exponentials and Logarithms - equations

- Follow the log laws! You should know these by now…
  - Adding logs… multiply the input
  - Subtracting logs… divide the input
  - Factor of a log… becomes input’s power
    - And vice versa!

- Note: check your answers, your input cannot be negative for a logarithm

2. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

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Your Turn

(ii) Given that

$$2\log_4(3x+5) = \log_4(3x+8) + 1, \qquad x > -\frac{5}{3}$$

(a) show that

$$9x^2 + 18x - 7 = 0$$

(4)

(b) Hence solve the equation

$$2\log_4(3x+5) = \log_4(3x+8) + 1, \qquad x > -\frac{5}{3}$$

(2)

Your Turn

7. (i) $$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

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