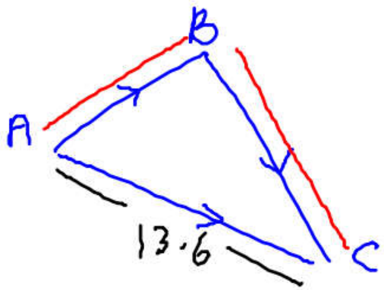
A man walks from A to B and then from B to C.
His displacement from A to B is $6\boldsymbol{i} + 4\boldsymbol{j}$ m.
His displacement from B to C is $5\boldsymbol{i} - 12\boldsymbol{j}$ m.
(a) What is the magnitude of the displacement from A to C?
(b) What is the total distance the man has walked in getting from A to C.



$$\vec{AC} = \vec{AB} + \vec{BC} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} 11 \\ -8 \end{pmatrix}$$

$$|\vec{AC}| = \sqrt{11^2 + 8^2} = \sqrt{185} = 13.6\text{ m}$$
(3sf)

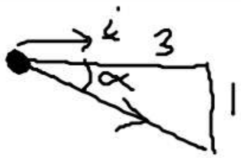
$$|\vec{AB}| = \sqrt{6^2+4^2} = \sqrt{52}$$

$$|\vec{BC}| = \sqrt{5^2+12^2} = 13$$

Total dist $= 13 + \sqrt{52}$
$= \underline{\underline{20.2\text{ m}}}$ (3sf)

A raccoon has a velocity of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ ms^{-1}.
Determine the angle the trajectory of the raccoon makes with the unit vector $\boldsymbol{i}$.

Raccoon



$$\tan\alpha = \frac{1}{3}$$

$$\alpha = \tan^{-1}\left(\frac{1}{3}\right) = \underline{\underline{18.4^\circ}}$$

below the unit vector $\underline{i}$

Ex 8D

Exercise 8D

1	**a** 2.1 m s⁻¹	**b** 500 m		**c** −1.8 m s⁻¹		
	d −2.7 m s⁻¹	**e** −750 m		**f** 2.5 m s⁻¹		
2	**a** 15.6 m s⁻¹		**b** 39.8°			
3	**a** 5 m s⁻²		**b** 143°			
4	**a** 15.3 m		**b** 24.3 m			**c** 78.7°

Constant Acceleration:

Distance-time graphs

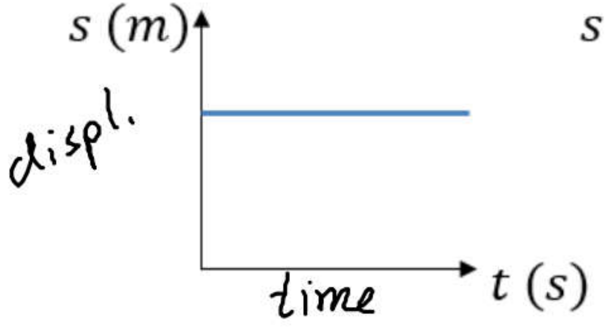
Speed-time graphs

SUVAT formulae

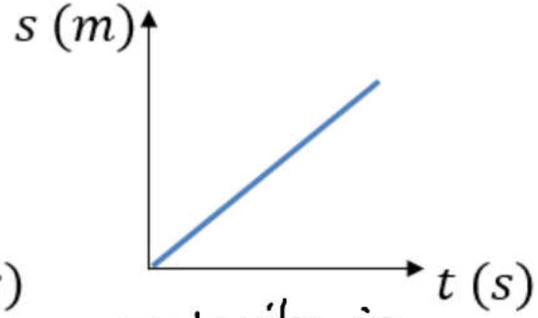
Vertical motion under gravity

Displacement-Time Graphs

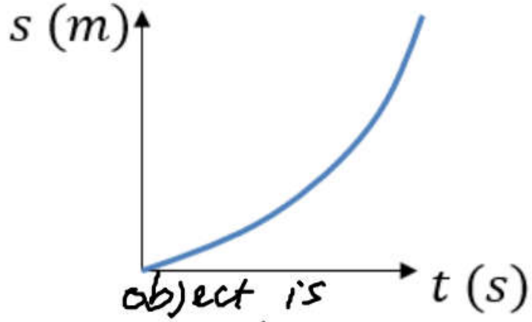
Describe the motion of each object:



object is stationary
$v = 0$
$a = 0$



velocity is constant
$a = 0$



object is accelerating,
its velocity is increasing

Velocity is the rate of change of displacement
(i.e. gradient of displacement-time graph)

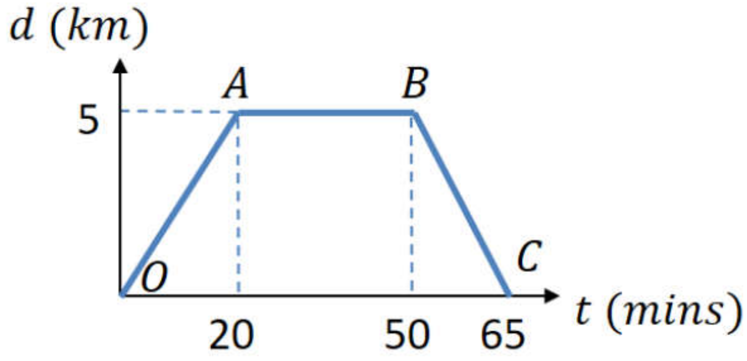
$$\textbf{Average Velocity} = \frac{\textbf{Displacement from starting point}}{\textbf{Time taken}}$$

$$\textbf{Average Speed} = \frac{\textbf{Total distance travelled}}{\textbf{Time taken}}$$

The distinction is important. If you went out then some time later travelled back home, your average velocity is 0 because your eventual displacement is 0!

A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes. This is a displacement-time graph for her journey.

(a) Work out the average velocity for each stage of the journey in km h^{-1}.
(b) Write down the average velocity for the whole journey.
(c) Work out average speed for the whole journey.



a) $\vec{OA}$ average velocity $= \frac{5}{20} = \frac{1}{4}$ km min^{-1}

$= \frac{1}{4} \times 60$ km h^{-1} $= 15$ km h^{-1}

$\vec{AB}$ avg. velo $= 0$

$\vec{BC}$ average velocity $= \frac{5}{15}$ km min^{-1} $= \frac{5}{15} \times 60 = 20$ km h^{-1} back home

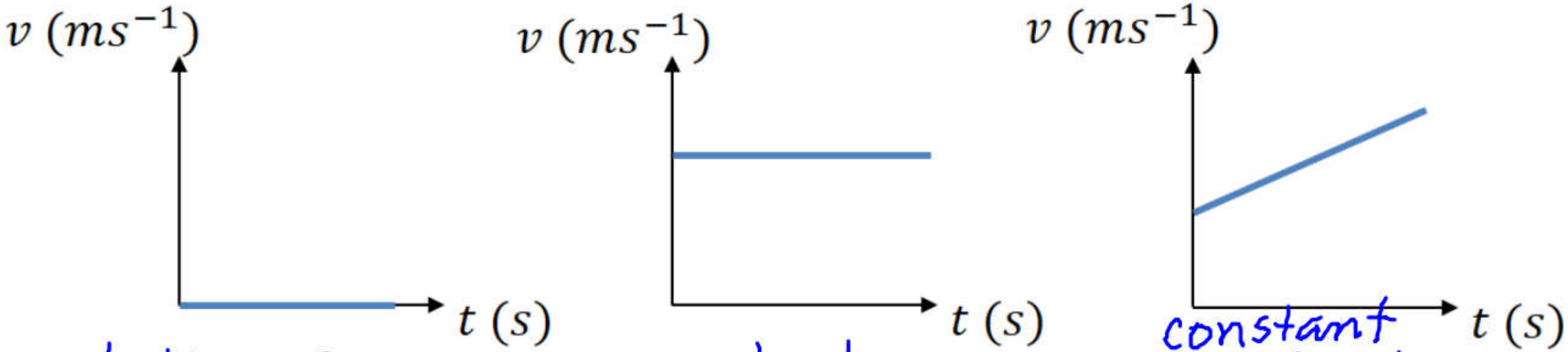
b) average velocity $= \frac{0}{65} = 0$ km h^{-1}

c) average speed $= \frac{10}{65}$ km min^{-1} $= \frac{10}{65} \times 60$ km h^{-1} $= \underline{\underline{9.23 \text{ km h}^{-1}}}$ (3sf).

total distance travelled

Velocity-Time Graphs

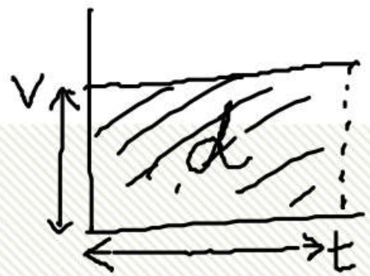
Describe the motion of each object:



stationary, $v = 0$

constant velocity $a = 0$

constant acceleration



$d = v \times t$

Acceleration is the rate of change of velocity (i.e. gradient of velocity-time graph)

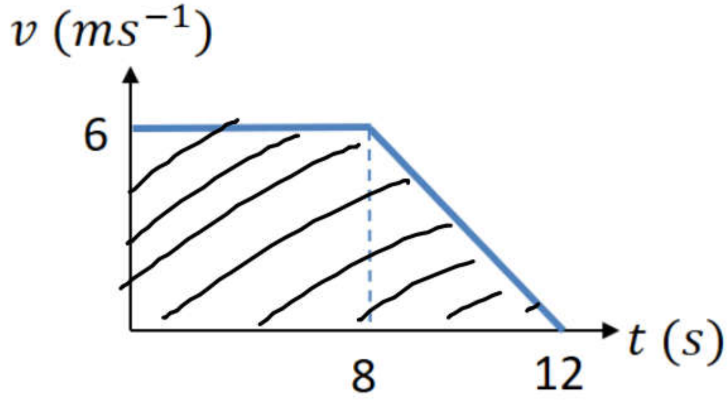
The **area** under a velocity-time graph gives the **distance travelled.**

Note: We'll see later in Chapter 11 that when we differentiate displacement we get velocity, and therefore integrating velocity gives displacement. But we know that integrating finds the area under the graph.

The figure shows a velocity-time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds.

(a) Find the displacement from the starting point of the cyclist after this 12 second period.

(b) Work out the rate at which the cyclist decelerates.



In case you've forgotten:

Area of trapezium

= average of parallel sides
× height between them

$$\frac{1}{2}(a+b)h$$

a) $\text{dist} = \frac{1}{2}(8+12) \times 6$

$= 10 \times 6 = \underline{\underline{60\,m}}$

b) $\text{deceleration} = \frac{6}{4} = \underline{\underline{1.5\,ms^{-2}}}$

$\text{acceleration} = \underline{\underline{-1.5\,ms^{-2}}}$

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 ms^{-1} in T seconds. The particle then travels at a constant velocity of 8 ms^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

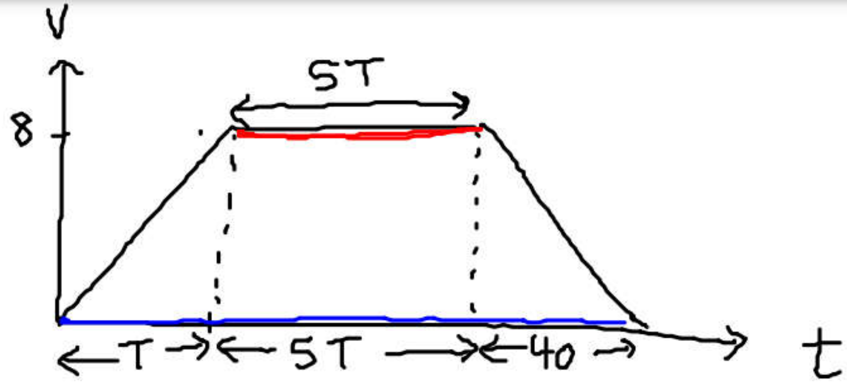
(a) Sketch a velocity-time graph to illustrate the motion of the particle.

Give [redacted]n the total displacement of the particle is 600m.

(b) find the value of T.

Tip: Sometimes it's easier to indicate the period of time that has passed (using arrows) rather than the time at the end of the interval.

a)



b) Area = 600

$600 = \frac{1}{2}(5T + T + 5T + 40) \times 8$

$600 = 4(11T + 40)$

$150 = 11T + 40$

$110 = 11T$

$T = 10$

6. A car travels along a straight horizontal road between two sets of traffic lights.
The distance between the two sets of traffic lights is 1500 m.

In a model of the journey, the car leaves the first set of traffic lights, accelerating uniformly from rest until it reaches a speed of V m s^{-1}, then immediately decelerates uniformly until it comes to rest at the second set of traffic lights.

The car completes the journey between the two sets of lights in 120 s.

(a) Sketch, on the diagram below, a velocity-time graph which represents the above model of the journey of the car between the two sets of traffic lights. **(2)**

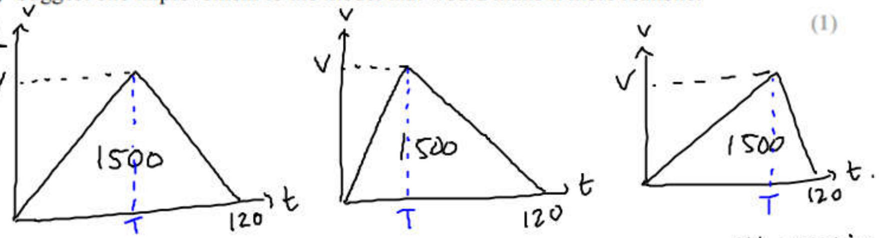
(b) Using the model, find the value of V. **(2)**

It is given that the car accelerates uniformly for T seconds.

(c) Explain why there is a range of possible values for T which satisfy the requirements of the model. **(2)**

(d) Suggest one improvement to the model that would make it more realistic. **(1)**

a)



b) Area = 1500

$1500 = \frac{120 \times V}{2}$

$1500 = 60V$

$V = 25 \text{ ms}^{-1}$

c) The area will remain 1500 for any value of T, with $0 < T < 120$. The base and height remain the same.

d) We could adjust the acceleration so that it was not uniform, as this is unrealistic.

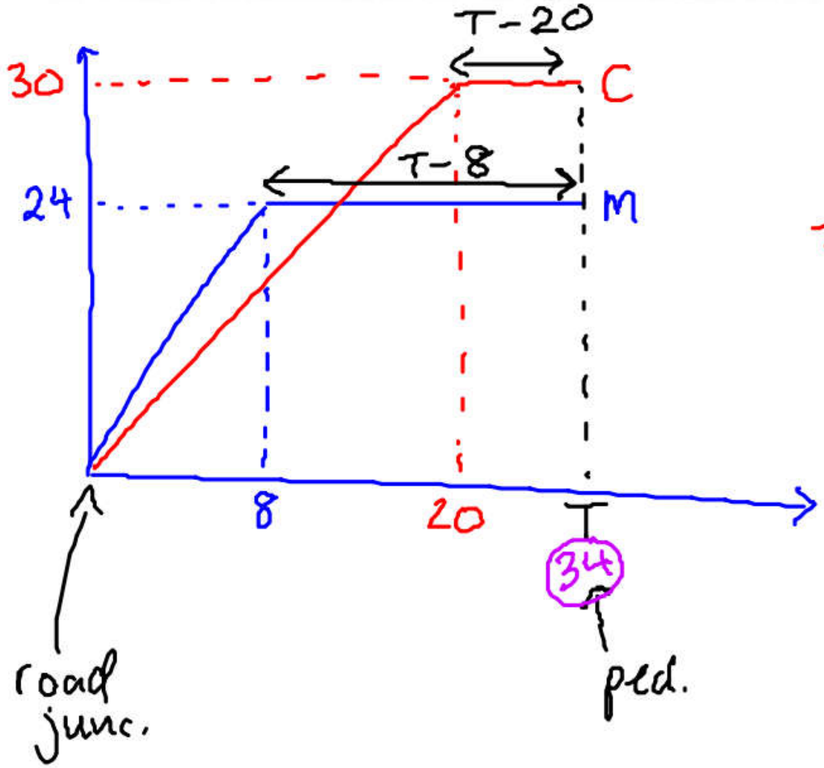
Ex 9B

Even Questions

8 A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates from rest at a rate of 3 m s^{-2} for 8 s and then maintains the velocity she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to 30 m s^{-1} in 20 s and then maintains the velocity of 30 m s^{-1}. C passes M as they both pass a pedestrian. travelled the same distance as each other in the same time period.

a On the same diagram, sketch velocity–time graphs to illustrate the motion of M and C. **(3 marks)**

b Find the distance of the pedestrian from the road junction. **(3 marks)**



Area under C = Area under M

$$\frac{1}{2}(T-20+T)\times 30 = \frac{1}{2}(T-8+T)\times 24$$

$$15(2T-20) = 12(2T-8)$$

$$30T - 300 = 24T - 96$$

$$6T = 204$$

$$T = 34$$

Area under C $= 15(2\times 34 - 20)$

$= 15(68-20)$

$= 15 \times 48$

$= 720\text{m}$

Edexcel M1 May 2013

A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1}. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(2)

(b) Find the value of T.

(3)

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a m s^{-2}, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A.

(4)

(d) Find the value of a.

You won't likely have the knowledge for (d) yet…

(2)