Chapter 3: Equations and Inequalities

1:: Simultaneous Equations

Solve:

$$x + y = 11$$
$$xy = 30$$

2:: Simultaneous Equations using Graphs

Find the points of intersection of $y = 3x^2 - 2x + 4$ and $7x + y + 3 = 0$

NEW! (since GCSE)
You may have to use the discriminant to show that the two graphs have no points of intersection.

3:: Solving Inequalities

Find the set of values of x for which:

$$x^2 - 11x + 24 < 0$$

NEW! (since GCSE, and new to A Level 2017+)
Use set notation to represent solutions to inequalities.

4:: Sketching Inequalities

Sketch the region that satisfies the inequalities:

$$2y + x < 14$$
$$y \geq x^2 - 3x - 4$$

Solutions sets

The solution(s) to an equation may be:

A single value:	$2x + 1 = 5$
Multiple values:	$x^2 + 3x + 2 = 0$
An infinitely large set of values:	$x > 3$
No (real) values!	$x^2 = -1$
Every value!	$x^2 + x = x(x + 1)$

The point is that you shouldn't think of the solution to an equation/inequality as an 'answer', but a **set** of values, which might just be a set of 1 value (known as a singleton set), a set of no values (i.e. the empty set Ø), or an infinite set (in the last example above, this was $\mathbb{R}$)

✏ The solutions to an equation are known as the **solution set**.

For simultaneous equations, the same is true, except each 'solution' in the solution set is an assignment to **multiple** variables.
All equations have to be satisfied **at the same time, i.e. 'simultaneously'**.

Scenario	Example	Solution Set
A single solution:	$x + y = 9$ $x - y = 1$	**Solution 1:** $\boldsymbol{x = 5, y = 4}$ To be precise here, the solution set is of size 1, but this solution is an assignment to multiple variables, i.e. a pair of values.
Two solutions:	$x^2 + y^2 = 10$ $x + y = 4$	**Solution 1:** $\boldsymbol{x = 3, y = 1}$ **Solution 2:** $\boldsymbol{x = 1, y = 3}$ This time we have two solutions, each an x, y pair.
No solutions:	$x + y = 1$ $x + y = 3$	The solution set is empty, i.e. $\emptyset$, as both equation can't be satisfied at the same time.
Infinitely large set of solutions:	$x + y = 1$ $2x + 2y = 2$	**Solution 1:** $\boldsymbol{x = 0, y = 1}$ **Solution 2:** $\boldsymbol{x = 1, y = 0}$ **Solution 3:** $\boldsymbol{x = 2, y = -1}$ **Solution 4:** $\boldsymbol{x = 0.5, y = 0.5}$ **...** Infinite possibilities!

Linear Simultaneous Equations: Elimination

Solve the simultaneous equations:

$$3x + y = 8$$
$$2x - 3y = 9$$

Linear Simultaneous Equations: Substitution

Solve the simultaneous equations:

$$y = 8 + 2x$$
$$5x - 3y = 9$$

Non-Linear Simultaneous Equations: Substitution

Solve the simultaneous equations:

$$x + 2y = 3$$
$$x^2 + 3xy = 10$$

Your Turn

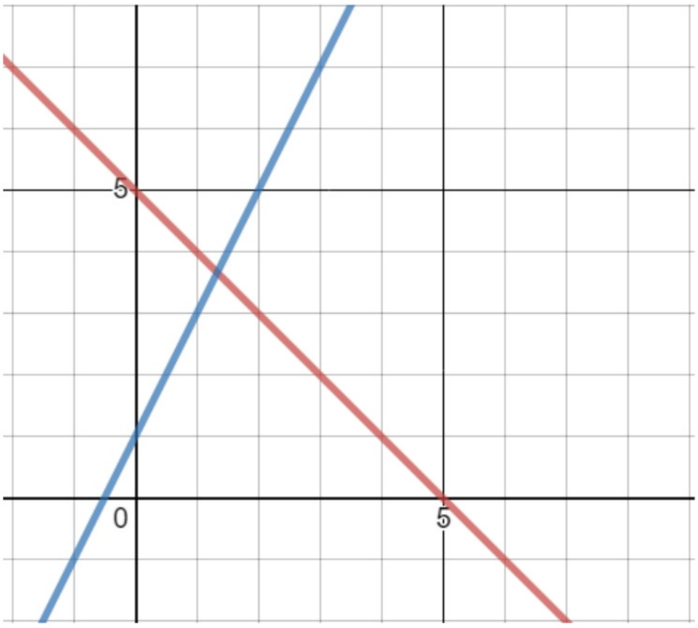
Solve the simultaneous equations:

$$3x^2 + y^2 = 21$$
$$y = x + 1$$

Ex 3A/3B

Simultaneous Equations and Graphs

The point on a graph that satisfies two equation simultaneously is the point of intersection



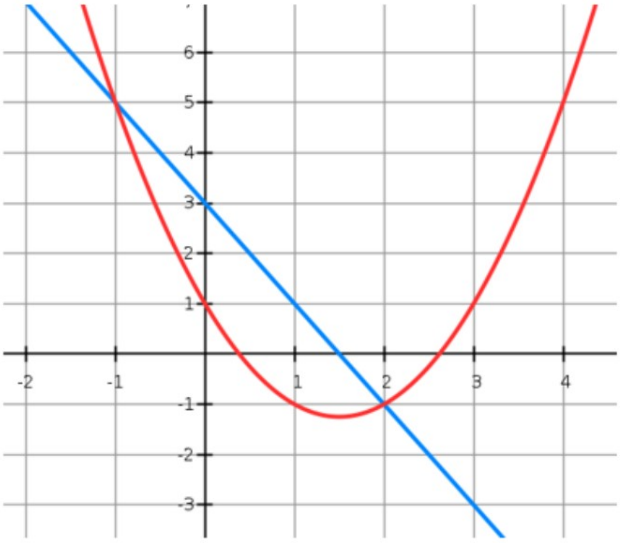
$x + y = 5$

$y = 2x + 1$

Using the Discriminant with Simultaneous Equations

a) On the same axes, draw the graphs of:

$$2x + y = 3$$
$$y = x^2 - 3x + 1$$

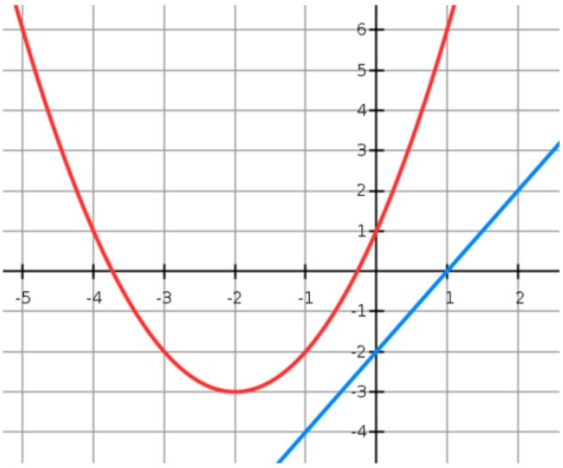


b) Use your graph to write down the solutions to the simultaneous equations.

c) What algebraic method (perhaps thinking about the previous chapter), could we have used to show the graphs would have intersected twice?

a) On the same axes, draw the graphs of:

$$y = 2x - 2$$
$$y = x^2 + 4x + 1$$



b) Prove algebraically that the lines never meet.

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection.

Ex 3C

5. The line l has equation

$$3x - 2y = k$$

where k is a real constant.

Given that the line l intersects the curve with equation

$$y = 2x^2 - 5$$

at two distinct points, find the range of possible values for k.

(5)

Set Builder Notation

Recap from GCSE:

- We use curly braces to list the values in a set, e.g. $A = \{1,4,6,7\}$
- If A and B are sets then $A \cap B$ is the **intersection** of A and B, giving a set which has the elements in A **and** B.
- $A \cup B$ is the **union** of A and B, giving a set which has the elements in A **or** in B.
- $\emptyset$ is the empty set, i.e. the set with nothing in it.
- Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals).
- We write $x \in A$ to mean "x is a member of the set A". So $x \in \mathbb{R}$ would mean "x is a real number".

$$\{1,2,3\} \cap \{3,4,5\} =$$
$$\{1,2,3\} \cup \{3,4,5\} =$$
$$\{1,2\} \cap \{3,4\} =$$

It is possible to construct sets without having to explicitly list its values. We use:

The : means "such that".

$$\boxed{\{expr : \; condition\}}$$

Can you guess what sets the following give?

(In words "All numbers $2x$ such that x is an integer)

$$\{2x : x \in \mathbb{Z}\} =$$
$$\{2^x : x \in \mathbb{N}\} =$$
$$\{xy : x, y \text{ are prime}\} =$$

We previously talked about 'solutions sets', so set builder notation is very useful for specifying the set of solutions!

Can you use set builder notation to specify the following sets?

All odd numbers.

All (real) numbers
greater than 5.

All (real) numbers less
than 5 **or** greater than 7.

All (real) numbers
between 5 and 7 inclusive.

$\{2x + 1 : x \in \mathbb{Z}\}$

$\{x : x > 5\}$

$\{x : x < 5\} \cup \{x : x > 7\}$

$\{x : 5 \leq x \leq 7\}$

Recap of Linear Inequalities

$$2x + 1 > 5$$

$$-x \geq 2$$

$$3(x-5) \geq 5-2(x-8)$$

Combining Inequalities:

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

Ex 3D

Recap of Quadratic Inequalities

Solve $x^2 + 2x - 15 > 0$

Step 1: Get 0 on one side
(already done!)

Step 2: Factorise

Step 3: Sketch and reason

Solve $x^2 + 2x - 15 \leq 0$

Solve $\boldsymbol{x^2 + 5x \geq -4}$

Solve $\boldsymbol{x^2 < 9}$

Find the set of values of x for which

(a) $3(x-2) < 8-2x$,

(2)

(b) $(2x-7)(1+x) < 0$,

(3)

(c) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$.

(1)

(a)	$3x-6<8-2x \rightarrow 5x<14$ (Accept $5x-14<0$ (o.e.))	M1	
	$x<2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone $\leq$)	A1	(2)
(b)	Critical values are $x=\frac{7}{2}$ and -1	B1	
	Choosing "inside" $-1<x<\frac{7}{2}$	M1 A1	(3)
(c)	$-1<x<2.8$	B1ft	(1)
	Accept any exact equivalents to -1, 2.8, 3.5		6

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$.

(2)

(*b*) Hence find the set of possible values of q.

(3)

(*a*)	[No real roots implies $b^2 - 4ac < 0$] $b^2 - 4ac = q^2 - 4 \times 2q \times (-1)$	M1
	So $q^2 - 4 \times 2q \times (-1) < 0$ i.e. $q^2 + 8q < 0$ (*)	A1 cso (2)
(*b*)	$q(q + 8) = 0$ or $(q \pm 4)^2 \pm 16 = 0$	M1
	(q) $= 0$ or -8 (2 cvs)	A1
	$-8 < q < 0$ or $q \in (-8, 0)$ or $q < 0$ and $q > -8$	A1 ft (3)

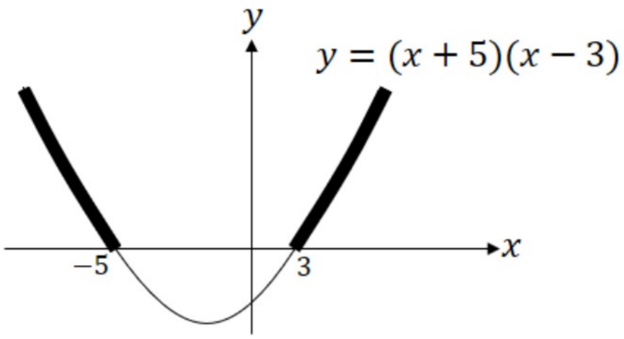
Ex 3E

Dealing with inequalities involving division by x

Find the set of values for which $\frac{6}{x} > 2, \quad x \neq 0$

Ex 3E Q4

Inequalities on Graphs

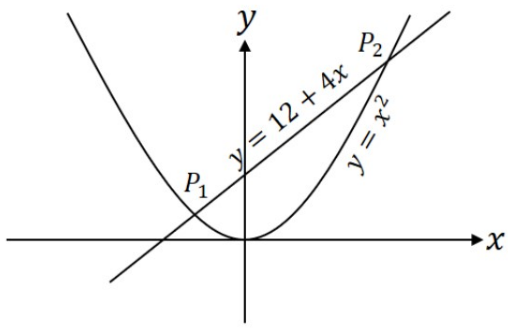


When we solved quadratic inequalities, e.g. $(x+5)(x-3) > 0$ We plotted $y = (x+5)(x-3)$ and observed the values of x for which $y > 0$.

Can we use a similar method when we don't have 0 on one side?

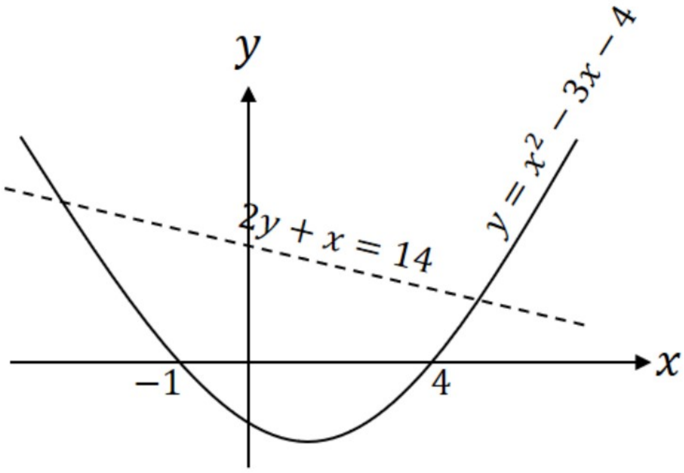
Example: L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.
The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.
b) Hence write down the solution to the inequality $12 + 4x > x^2$.



On a sketch, shade the region that satisfies the inequalities:

$$2y + x < 14$$
$$y \geq x^2 - 3x - 4$$



Pro Tip: To quickly sketch $2y + x = 14$, consider what happens when x is 0 and when y is 0.

Pro Tip: Make sure y is on the side where it is positive.
If y is on the smaller side, you're below the line.
If y is on the greater side, you're above the line.

Ex 3F/G