Circles

1:: Equation of a circle

The diameter of a circle is AB where A and B have the coordinates $(2,5)$ and $(8,13)$. Determine the equation of the circle.

2:: Intersections of lines + circles

Show that the line $y = x - 7$ does not meet the circle $(x+2)^2 + y^2 = 33$

NEW! since GCSE
You should already know the equation $x^2 + y^2 = r^2$ for a circle centred at the origin, but not a circle centred at a specified point.

3:: Chords, tangents and perpendicular bisectors.

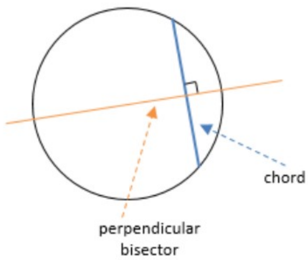
A circle C has the equation $(x-5)^2 + (y+3)^2 = 10$. Find the equations of the two possible tangents whose gradient is -3.

4:: Circumscribing Triangles

Find the equation of the circle that passes through the points $A(-8,1), B(4,5), C(-4,9)$.

Midpoints and Perpendicular Bisectors

Later in the chapter you will need to find the perpendicular bisector of a chord of a circle.



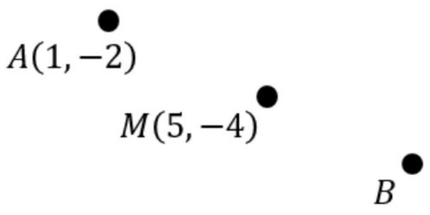
What two properties does a perpendicular bisector of two points A and B have?

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

What is the equation of this perpendicular bisector?

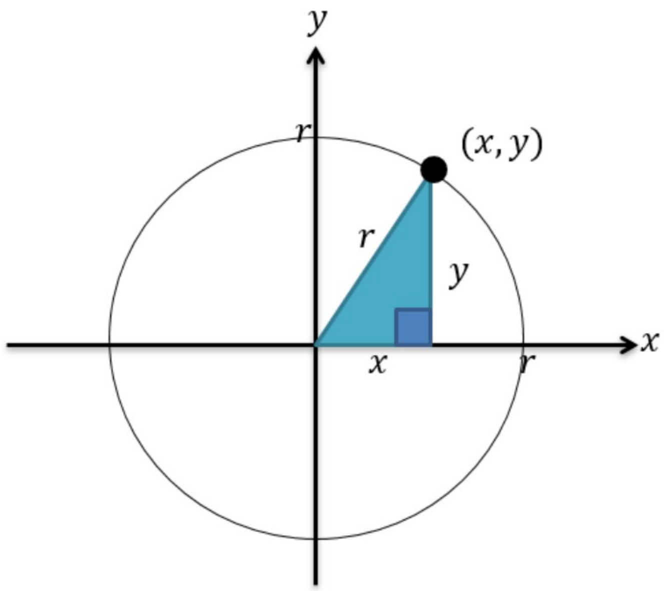
Find the perpendicular bisector of the line AB where A and B have the coordinates:
a) $A(4,7), B(10,17)$

A line segment AB is the diameter of a circle with centre $(5,-4)$. If A has coordinates $(1,-2)$, what are the coordinates of B?

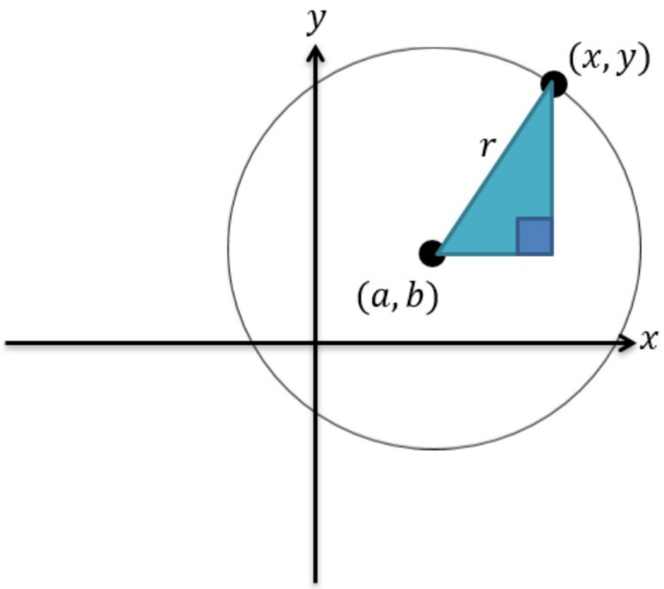


Ex 6A/B

Equation of a circle



Recall that a line can be a set of points (x,y) that satisfy some equation. Suppose we have a point (x,y) on a circle centred at the origin, with radius r. What equation must (x,y) satisfy?



Now suppose we shift the circle so it's now centred at (a, b). What's the equation now?

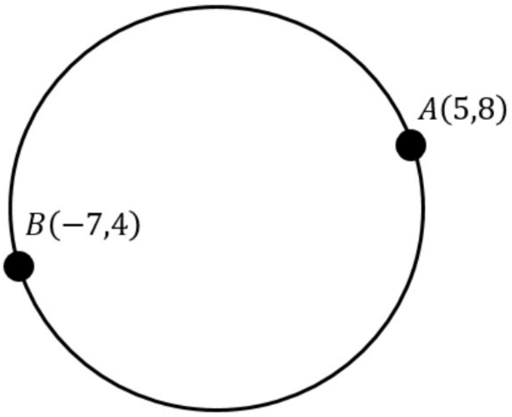
(Hint: What would the sides of this right-angled triangle be now?)

✏ The equation of a circle with centre (a, b) and radius r is:

$$(x-a)^2 + (y-b)^2 = r^2$$

Centre	Radius	Equation
$(0,0)$	5	
$(1,2)$	6	
		$(x+3)^2 + (y-5)^2 = 1$
		$(x+5)^2 + (y-2)^2 = 49$
		$(x+6)^2 + y^2 = 16$
		$(x-1)^2 + (y+1)^2 = 3$
		$(x+2)^2 + (y-3)^2 = 8$

Finding the equation using points



A line segment AB is the diameter of a circle, where A and B have coordinates $(5,8)$ and $(-7,4)$ respectively. Determine the equation of the circle.

Hint: What two things do we need to use the circle formula?

Your Turn

Edexcel C2 Jan 2005 Q2

The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(*a*) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(*b*) find an equation for C.

(4)

Completing the square

When the equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$, we can instantly read off the centre (a,b) and the radius r.
But what if the equation wasn't in this form?

Find the centre and radius of the circle with equation $x^2+y^2-6x+2y-6=0$

Hint: Have we seen a method in a previous chapter that allows us to turn a x^2 term and a x term into a single expression involving x?

Edexcel C2 June 2012 Q3a,b

The circle C with centre T and radius r has equation

$$x^2+y^2-20x-16y+139=0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r=5$

(2)

3. A circle C has equation

Question	Scheme	Marks	AOs
3(a)	Attempts $(x-2)^2 + (y+5)^2 \ldots$	M1	1.1b
	Centre $(2, -5)$	A1	1.1b
		(2)	
(b)	[illegible]	M1	[illegible]
	$\Rightarrow k > -29$	A1ft	1.1b
		(2)	
			(4 marks)

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

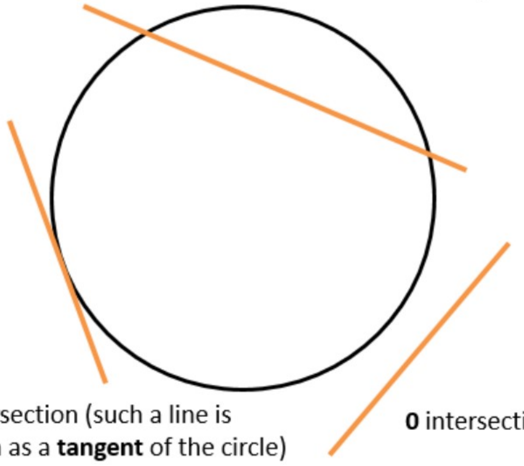
(b) State the range of possible values for k.

(2)

Ex 6C

Intersections of Lines and Circles

Recall that to consider the **intersection of two lines**, we attempt to solve them **simultaneously** by substitution, potentially using the **discriminant** to show that there are no solutions (and hence no points of intersection).

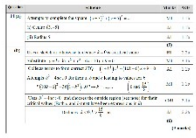


Show that the line $y = x + 3$ never intersects the circle with equation $x^2 + y^2 = 1$.

Find the points of intersection where the line $y = x + 6$ meets $x^2 + (y - 3)^2 = 29$.

Using an algebraic (and not geometric) method, determine the k such that the line $y = x + k$ **touches** the circle with equation $x^2 + y^2 = 1$.

14. The circle C has equation



$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

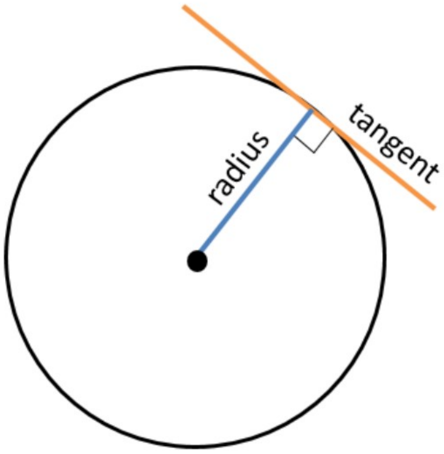
(b) Find the range of values for k.

(6)

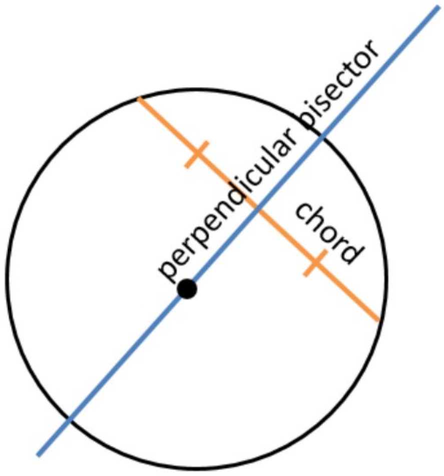
Ex 6D

Tangents, Chords, Perpendicular Bisectors

There are two circle theorems that are of particular relevance to problems in this chapter, the latter you might be less familiar with:



The tangent is perpendicular to the radius (at the point of intersection).



The perpendicular bisector of any chord passes through the centre of the circle.

Why this will help:
If we knew the centre of the circle and the point of intersection, we can easily find the gradient of the radius, and thus the gradient and hence equation of the tangent.

Why this will help:
The first thing we did in this chapter is find the equation of the perpendicular bisector. If we had two chords, and hence found two bisectors, we could find the point of intersection, which would be the centre of the circle.

The circle C has equation
$(x-3)^2 + (y-7)^2 = 100$.

a) Verify the point $P(11,1)$ lies on C.
b) Find an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$

A circle C has equation

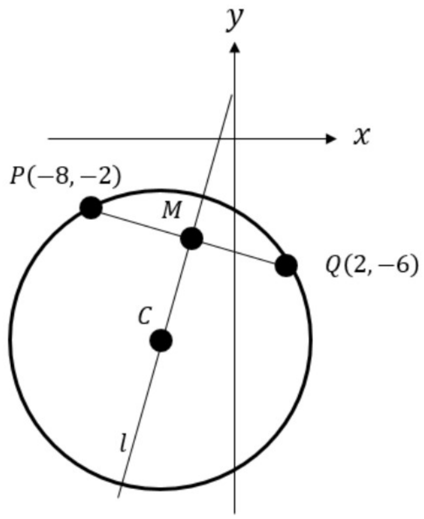
$$(x-4)^2 + (y+4)^2 = 10$$

The line l is a tangent to the circle and has gradient -3. Find two possible equations for l, giving your answers in the form $y = mx + c$.

Determining the Circle Centre

The points P and Q lie on a circle with centre C, as shown in the diagram. The point P has coordinates $(-8, -2)$ and the point Q has coordinates $(2, -6)$. M is the midpoint of the line segment PQ.
The line l passes through the points M and C.

a) Find an equation for l.
b) Given that the y-coordinate of C is -9:
   i) show that the x-coordinate of C is -5.
   ii) find an equation of the circle.

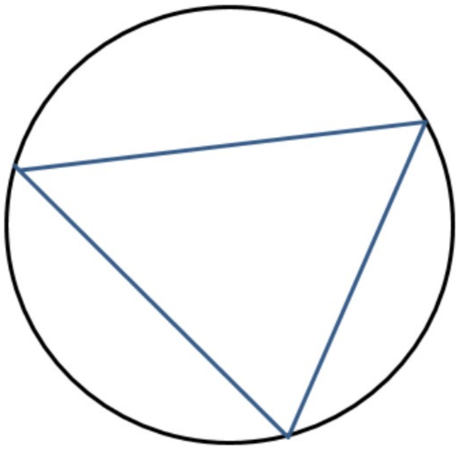


Your Turn

A circle has centre $C(3,5)$, and goes through the point $P(6,9)$. Find the equation of the tangent of the circle at the point P, giving your equation in the form $ax + by + c = 0$ where a, b, c are integers..

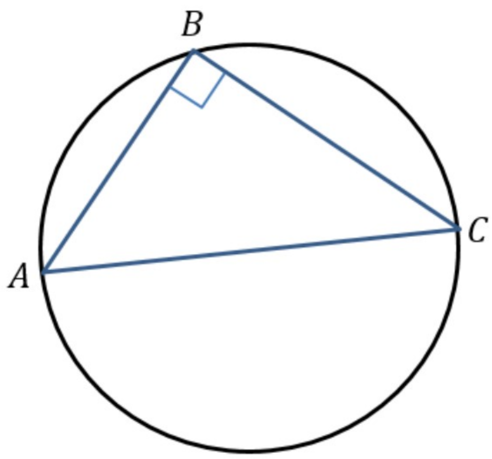
A circle passes through the points $A(0,0)$ and $B(4,2)$. The centre of the circle has x value -1. Determine the equation of the circle.

Triangles in Circles



Some new terminology:

- The triangle **<u>inscribes</u>** the circle.
  (A shape inscribes another if it is inside and its boundaries touch but do not intersect the outer shape)
- The circle **<u>circumscribes</u>** the triangle.
- If the circumscribing shape is a circle, it is known as the **<u>circumcircle</u>** of the triangle.
- The centre of a circumcircle is known as the **<u>circumcentre</u>**.

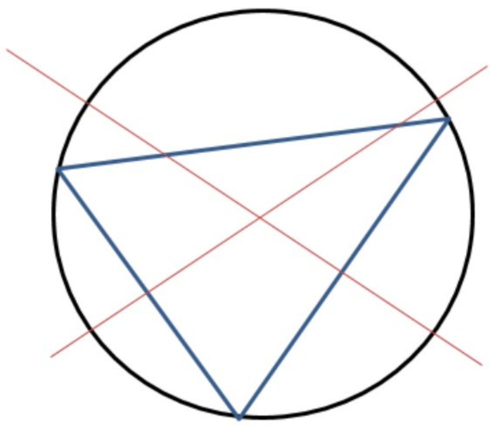


If $\angle ABC = 90°$ then:

- **AC is the diameter of the circumcircle of triangle ABC.**

Similarly if AC is the diameter of a circle:

- **$\angle ABC = 90°$ therefore AB is perpendicular to BC.**
- $\boldsymbol{AB^2 + BC^2 = AC^2}$



Given three points/a triangle we can find the centre of the circumcircle by:

- **Finding the equation of the perpendicular bisectors of two different sides.**
- **Find the point of intersection of the two bisectors.**

The points $A(-8,1), B(4,5), C(-4,9)$ lie on a circle.
a) Show that AB is a diameter of the circle.

Method 1:
Show that $AC^2 + BC^2 = AB^2$

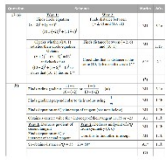
Method 2:
Show that AC is perpendicular to BC.

b) Hence find the equation of the circle.

The points $A(0,2), B(2,0), C(8,18)$ lie on the circumference of a circle. Determine the equation of the circle.

17. A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.



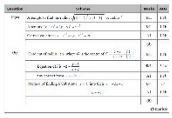
(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

10. A circle C has centre $(2, 5)$. Given that the point $P(-2, 3)$ lies on C.



(a) find an equation for C.

(3)

The line l is the tangent to C at the point P. The point $Q(2, k)$ lies on l.

(b) Find the value of k.

(5)

(Total for Question 10 is 8 marks)

9. A circle with centre $A(3,-1)$ passes through the point $P(-9,8)$ and the point $Q(15,-10)$

(a) Show that PQ is a diameter of the circle.

(2)

(b) Find an equation for the circle.

(3)

A point R also lies on the circle.
Given that the length of the chord PR is 20 units,

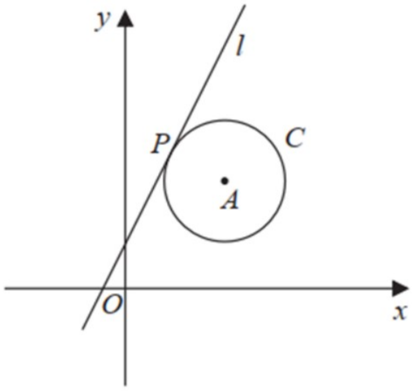
(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.

(2)

(d) Find the size of angle ARQ, giving your answer to the nearest 0.1 of a degree.

(2)

6.



Not to scale

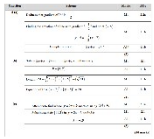


Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $\quad k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)