Edexcel A Level Further Maths: Further Mechanics 1

Your notes

Elastic Collisions in 1D

Contents

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Newton's Law of Restitution

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Newton's Law of Restitution

What is Newton's law of restitution?

- Newton's law of restitution (also known as **Newton's Experimental Law**) concerns the **ratio** of the **relative speed of separation** and the **relative speed of approach** when two objects collide
  - Essentially this just means the speed of separation divided by the speed of approach
- This ratio can be written as a coefficient; the "coefficient of restitution"
  - It is **denoted by e**
  - e is dimensionless as it is a ratio
  - The value of e in a particular situation will depend on the materials that the two particles are made from
- e can take the values in the range **$0 \le e \le 1$**
  - **e =1**: These are called **perfectly elastic** collisions and in these collisions there is **no loss in kinetic energy**
  - **e =0**: These are called **perfectly inelastic** collisions and in these collisions the objects **coalesce** (merge to form one object)

How do I calculate the coefficient of restitution between two objects?

- $e = \dfrac{\text{Speed of separation of the objects}}{\text{Speed of approach of the objects}}$
- The speed of approach depends on whether the objects are travelling towards each other or in the same direction. For example, if the speeds of the two objects **before** the collision are 5 m s^{-1} and 2 m s^{-1} then the **speed of approach** is:
  - 3 m s^{-1} if they are moving in the **same direction** (each second the objects approach each other by a further 3 metres)
  - 7 m s^{-1} if they are moving in **opposite directions** (each second the objects approach each other by a further 7 metres)
- The speed of separation depends on whether the objects are travelling away from each other or in the same direction. For example, if the speeds of the two objects **after** the collision are 5 m s^{-1} and 2 m s^{-1} then the **speed of separation** is:
  - 3 m s^{-1} if they are moving in the **same direction** (each second the objects separate by a further 3 metres)
  - 7 m s^{-1} if they are moving in **opposite directions** (each second the objects separate by a further 7 metres)
- If the velocities of the two objects before the collision are u_1 m s^{-1} and u_2 m s^{-1} and the velocities after the collision are v_1m s^{-1} and v_2 m s^{-1} then:
  - $e = \dfrac{v_2 - v_1}{u_1 - u_2}$
  - Note that **velocities can be negative** so be careful with signs

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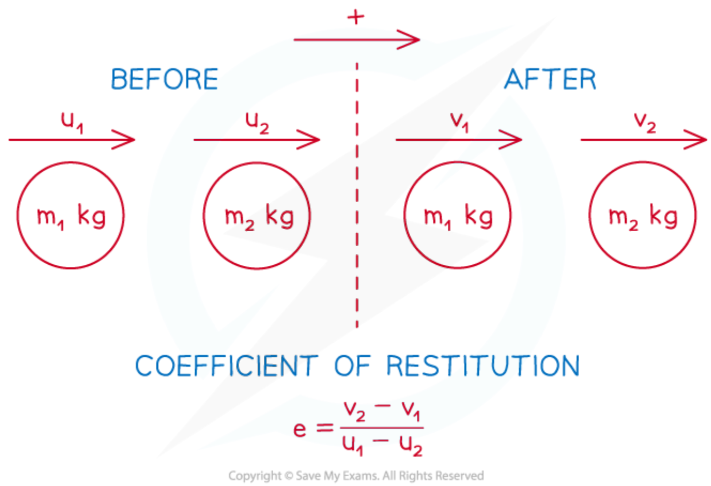

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How do I solve collision problems involving the coefficient of restitution?

- **STEP 1**: **Draw** a before/after diagram and label the **positive direction**
  - There may be multiple diagrams if there are multiple collisions
- **STEP 2**: Form an equation using the **coefficient of restitution**
  - The unknown(s) could be the coefficient of restitution or any of the speeds or directions
- **STEP 3**: Form an equation using the principle of **conservation of momentum**
  - In the case of a collision with a wall you may be given the impulse or some other information instead
- **STEP 4**: **Solve** and give your answer in **context**
  - You may have to solve simultaneous equations
  - You may have to solve an inequality
  - You may have to form an inequality using $0 \le e \le 1$ or using the fact that velocity is positive (or negative) if the object is going forwards (or backwards)

Examiner Tip

- Exam questions refer to spheres as having equal radii, this just means the objects are the same size so that dimensions don't affect the collision. Exam questions often leave velocities in terms of e. If you know the direction of the object then you know whether the velocity is positive or negative. This can be used to form an inequality for the range of possible values of e for that scenario.

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Worked example

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Two balls A and B are of equal radius and have masses 2 kg and 5 kg respectively. A and B collide directly. Immediately before the collision, A and B are moving in the same direction along a straight line on a smooth horizontal surface with speeds 9 $\mathrm{m\ s^{-1}}$ and $1\ \mathrm{m\ s^{-1}}$ respectively. Immediately after the collision, the direction of motion of A is reversed. The coefficient of restitution between A and B is $\frac{3}{4}$.

Find the speeds of A and B immediately after the collision.

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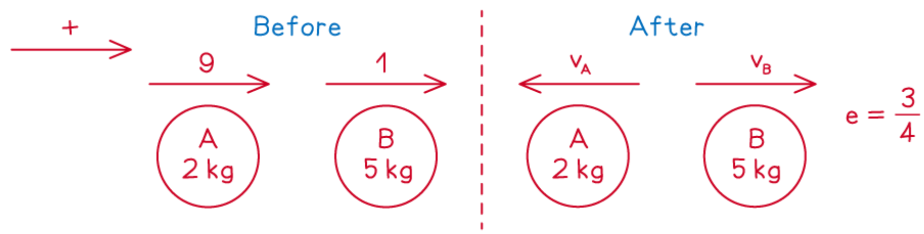
Step 1: Draw a before/after diagram



Step 2: Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

v_A is in opposite direction

$$\frac{3}{4} = \frac{v_B - (-v_A)}{9 - 1} = \frac{v_B + v_A}{8}$$

$$6 = v_B + v_A \quad (1)$$

Step 3: Conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$2(9) + 5(1) = 2(-v_A) + 5(v_B)$$

$$23 = 5v_B - 2v_A \quad (2)$$

Step 4: Solve

Rearrange (1) $v_A = 6 - v_B$

Substitute into (2) $23 = 5v_B - 2(6 - v_B)$

$$23 = 7v_B - 12$$

$$\therefore v_B = 5$$

Substitute back into (1) $v_A = 6 - 5$

$$\therefore v_A = 1$$

After collision:

Speed of A is $1\ \text{ms}^{-1}$

Speed of B is $5\ \text{ms}^{-1}$

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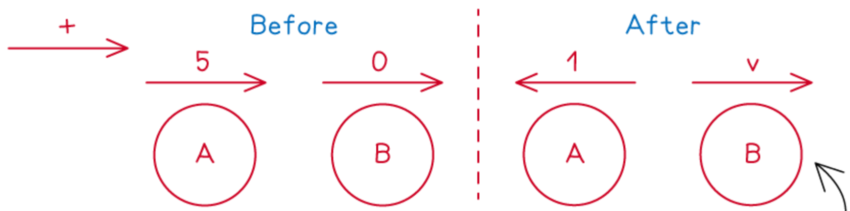
Worked example

A sphere A is projected with speed 5 m s^{-1} along a straight line towards another sphere B, of equal radius, which is at rest on a smooth horizontal surface. After the collision, B moves with speed $v \text{ m s}^{-1}$ and A moves in the opposite direction with speed 1 m s^{-1}. The coefficient of restitution between A and B is e.

a) Show that $v = 5e - 1$.

b) Hence, find the range of possible values of e.

Step 1: Draw a before/after diagram



We don't have information about the masses

Step 2: Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{v - (-1)}{5 - 0}$$ 1 is in opposite direction

$$e = \frac{v + 1}{5}$$

We now have enough information to do part (a)

a) $v + 1 = 5e$

$v = 5e - 1$

b) After the collision B moves forward so $v > 0$

$5e - 1 > 0$

$e > \frac{1}{5}$

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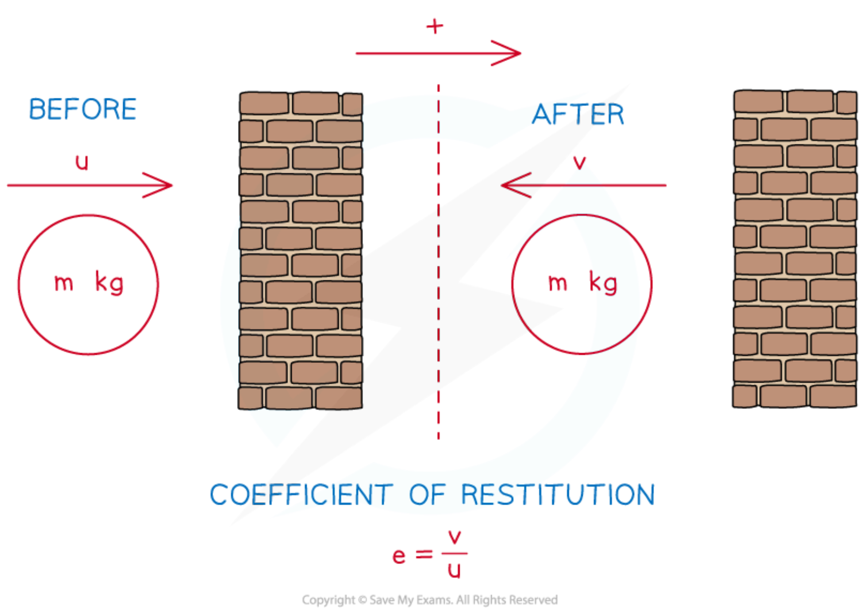
Collisions with Planes in 1D

How do I calculate the coefficient of restitution between an object and a wall?

- Instead of a sphere colliding with another sphere, it may collide with a vertical or horizontal plane, usually characterised as a wall (vertical) or floor (horizontal)
- As the wall (or floor) does not have any velocity
  - $e = \dfrac{\text{Speed of rebound of the object}}{\text{Speed of approach of the object}}$
- If the **speed** of the object **before** the hitting the wall is u m s^{-1} and the **speed after** is v m s^{-1} then the formula above simplifies to:
  - $e = \dfrac{0 - (-v)}{u - 0}$
  - $e = \dfrac{v}{u}$



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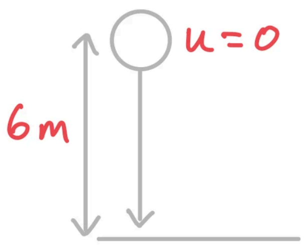
Worked example

A particle falls from rest from a height of 6 metres onto a smooth horizontal plane, and rebounds. After the rebound, the particle reaches a maximum height halfway between the plane and the height it was originally dropped from.

a) Find the exact value of the coefficient of restitution.



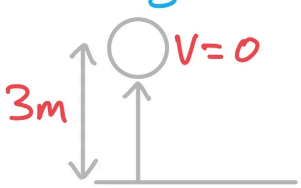
To find e, we need to find the speed it strikes, and leaves the plane at.

On way down

$s = 6$
$u = 0$
$v = v$
$a = 9.8$

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2 \times 9.8 \times 6$
$v^2 = 117.6$
$v = \frac{14\sqrt{15}}{5}$

On way up



$s = 3$
$u = u$
$v = 0$
$a = -9.8$

$v^2 = u^2 + 2as$
$0^2 = u^2 + 2 \times -9.8 \times 3$
$u^2 = 58.8$
$u = \frac{7\sqrt{30}}{5}$

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{7\sqrt{30}}{5} \div \frac{14\sqrt{15}}{5}$$

$$\boxed{e = \frac{\sqrt{2}}{2}}$$

b)

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Explain how the maximum height reached after the rebound would change if the coefficient of restitution was smaller.

As $e = \frac{\text{speed of separation}}{\text{speed of approach}}$ and the speed of approach would not change, the speed of separation (rebound) would reduce. Hence the maximum height reached after the rebound would be lower.

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Energy in 1D Collisions

Energy in 1D Collisions

How might energy be involved in collision problems?

- A question may require the change in kinetic energy to be calculated due to a collision, or to an impulse being applied
- Remember that although **total energy** will be conserved, there may be a change in the **kinetic energy** of the objects involved in the collision
- The kinetic energy of an object can be calculated using $\frac{1}{2}mv^2$
- If a particle with mass m_1 and velocity u_1 collides with a particle of mass m_2 and velocity u_2, then the loss in kinetic energy would be:
    - $\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$
    - This is essentially the difference between the total kinetic energy before the collision, and the total kinetic energy after the collision

When is kinetic energy conserved in collisions?

- When $e = 1$, which would be a **perfectly elastic collision**, kinetic energy will be **conserved**, and no energy is lost due to the impact
- When $e < 1$, some energy will be **lost** due to the collision
    - In reality, all collisions will have a coefficient of restitution of less than 1, but we may still choose to model some scenarios as perfectly elastic
    - It is also important to understand that energy is not "lost", it is simply transferred to other forms such as heat and sound
- There can also be situations where the kinetic energy of a system increases
    - For example when a cannon is fired, the cannonball and cannon itself start with zero velocity, and hence zero kinetic energy
    - When fired, the cannon ball moves forward, and the cannon recoils backwards, so they now both have velocities, and hence kinetic energy
    - In this scenario, the energy has been converted from chemical energy stored in the gunpowder

Examiner Tip

- As v^2 is used when finding kinetic energy, it will always be positive (and hence a scalar), so you do not need to enter the negative signs in your calculator when finding the kinetic energy

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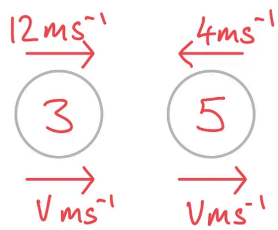
Worked example

A small smooth sphere *A* of mass 3 kg moves at 12 ms^{-1} on a smooth horizontal table. It collides directly with a second small smooth sphere *B* of mass 5 kg, which is moving in the opposite direction with a speed of 4 ms^{-1}. The spheres coalesce and move with velocity V after the collision.

Find the loss of kinetic energy due to the impact.



Conservation of momentum

$$(\rightarrow)\quad 3\times12 + 5\times -4 = 3v + 5v$$

$$16 = 8v$$

$$V = 2\text{ ms}^{-1}$$

Kinetic Energy at start $\left(\frac{1}{2}mv^2\right)$

$$\left.\begin{aligned}\tfrac{1}{2}\times3\times12^2 &= 216\text{ J}\\ \tfrac{1}{2}\times5\times4^2 &= 40\text{ J}\end{aligned}\right]\ 256\text{ J}$$

Kinetic Energy at end

$$\left.\begin{aligned}\tfrac{1}{2}\times3\times2^2 &= 6\text{ J}\\ \tfrac{1}{2}\times5\times2^2 &= 10\text{ J}\end{aligned}\right]\ 16\text{ J}$$

$$\text{Loss in KE} = 256 - 16$$

$$\boxed{240\text{ J}}$$

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Successive Collisions in 1D

Successive Collisions in 1D

Can there be successive/multiple collisions?

- After **two objects collide** it is possible that one (or both) of them **collides with something else** such as
  - A **third object**
  - A **wall** (perpendicular to the motion)
- Deal with **each collision separately** and use the steps for direct collisions
  - Drawing a separate diagram for each collision can help
  - Be clear and unambiguous with labelling and variables
    - e.g. V is usually the speed after the first collision, so W may be used for the speed after the second collision

Can there be a second collision between the original two objects?

- Let A, B and C be three objects travelling in the **same straight line** and suppose **A and B collide directly** and subsequently **B and C collide directly**
- After the collisions between **A and B** and **B and C** there will be a **second collision** between **A and B if**:
  - **One** is **stationary** and the **other** is **travelling towards** it
  - Both are **travelling in opposite** directions **towards** each other
  - Both are **travelling in the same** direction and the **one in front is slower** than the one behind
- The process is **similar** if object **C is a wall**
  - After **B collides with the wall** its **direction will be reversed** so it will be travelling **towards A**
  - B will **collide with A** again if its **velocity in that direction** is **greater** than the velocity of A in that direction (or if A was also travelling towards the wall after the first collision!)
  - To help you work out the speed of B after hitting the wall you will be given extra information such as the **impulse** exerted by the wall or the **coefficient of restitution**

How do I solve collisions questions involving distance and time?

- As momentum problems always deal with velocities, there may also be questions that involve distances, and times
- In between the collisions in collisions questions we are dealing with constant speed, so there is no need to use the *suvat* equations
  - We can instead make use of $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$
  - or its rearrangements of $\text{Speed} \times \text{Time} = \text{Distance}$ and $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$
- In these types of questions, the distances and/or times will usually be algebraic
  - e.g. "The collision between *A* and *B* takes place a distance *d* from the wall"
  - A calculator can still be used to help with any complicated fractions, just remember to account for the algebraic term afterwards

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- e.g. X can be found for $\frac{2}{5} = \frac{x}{\frac{5}{17}d}$ by finding $\frac{2}{5} \times \frac{5}{17} = \frac{2}{17}$ on your calculator, and then the answer would be $x = \frac{2}{17}d$
- Keeping track of where the objects are, and when, in these problems is key
  - Draw a diagram for each stage
  - Use different letters for the final velocities of each stage e.g. *v*, and then *w*
  - Label distances on the diagrams
- A common mistake after two objects collide, is forgetting that they are **both** moving
  - e.g. If *A* and *B* collide, and then *B* goes on to collide with a wall, and rebounds to strike *A* again
  - *A* will no longer be in the location where *A* and *B* collided; it will have moved whilst B was travelling to the wall
    - To find how far A has moved, the time for B to reach the wall would need to be found first
    - This could then be used in conjunction with A's speed, to find how far it has moved in this time
- A common scenario in a part of these problems is when two objects at speeds V_A and V_B are approaching each other, a distance X apart
  - They will cover distance X, moving closer to each other at a rate of $(V_A + V_B)\ \mathrm{ms}^{-1}$, this can be used to find a time for when they collide
  - The distance between the objects will reduce in proportion to their speeds
    - e.g. If $V_A = 3$ and $V_B = 4$ then the ratio of distance covered will be $3:4$
    - So they will meet at a point $\frac{3}{7}X$ from *A*'s starting point

Examiner Tip

- These questions can be difficult to visualise in your head so draw simple diagrams to show each collision.
- Use common sense, and think how many possible (or impossible) ways there are for objects to move after the first collision. You will often have to consider the speed of one or more objects to decide if a second or third collision is possible.
- These questions can involve lots of algebra, negatives and inequalities so do not rush them as you might make a silly mistake which can affect subsequent parts.

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Worked example

Three uniform balls A, B and C of equal radius, and mass 0.1 kg, 0.2 kg and 0.3 kg respectively, can move along the same straight line on a smooth horizontal table with B in the middle of A and C. A and B are projected towards each other in opposite directions with speed 10 m s^{-1} and 2 m s^{-1} respectively while C is at rest. A and B collide directly which does not change the direction of motion of A and subsequently A moves with speed 1 m s^{-1}.

a) Show that the speed of B immediately after it collides with A is 2.5 m s^{-1}.

b) In the subsequent motion, B collides directly with C. Immediately after this collision, C moves with speed 1.5 m s^{-1}. Determine if there will be a second collision between A and B.

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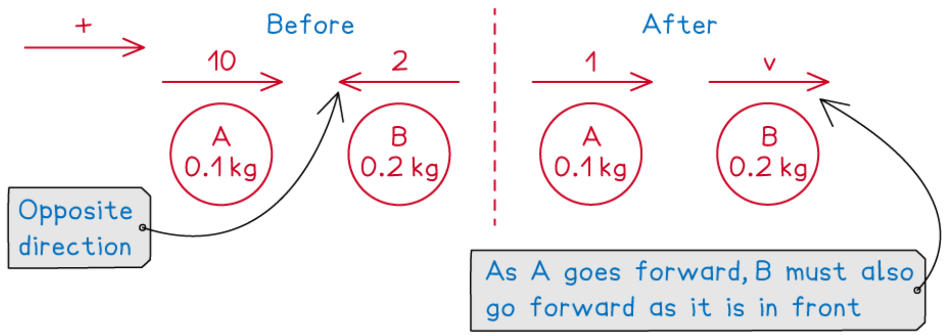
a) Look at the collision between A and B

Step 1: Choose positive direction

Let the initial direction of A be positive

Step 2: Draw a before/after diagram



Step 3: Use conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0.1(10) + 0.2(-2) = 0.1(1) + 0.2(v)$$

$$0.6 = 0.1 + 0.2v$$

Step 4: Solve

$v = 2.5$

Speed of B = $2.5\ \mathrm{m\ s^{-1}}$

b) Look at the collision between B and C

Step 1: Choose positive direction

Step 2: Draw a before/after diagram

Step 3: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$0.2(2.5) + 0.3(0) = 0.2(u) + 0.3(1.5)$$

$$0.5 = 0.2u + 0.45$$

Step 4: $u = 0.25$

Look at velocities of A and B after collisions

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A and B are traveling in the same direction
A's speed is greater than B's speed
There will be another collision between A and B

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Worked example

Particles P and Q have masses 2kg and 3kg respectively. Initially they are moving in the same direction along the same straight line, such that the speed of particle P is 4 ms^{-1} and the speed of particle Q is 1 ms^{-1}. The two particles collide when they are at a distance d from a smooth fixed vertical wall, which is perpendicular to their direction of motion. After the collision with P, particle Q collides directly with the wall and rebounds so that there is a second collision between P and Q.
This second collision takes place at a distance x from the wall.

Given that the coefficient of restitution between P and Q is 1, and the coefficient of restitution between Q and the wall is 0.5, find x in terms of d.

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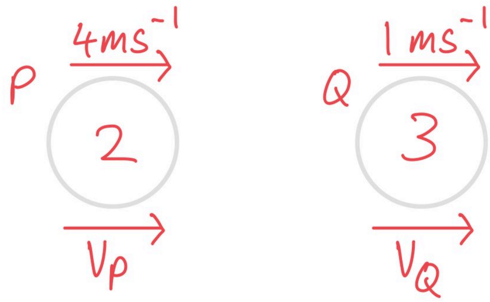
Find velocities after 1st collision



Conservation of momentum $(\rightarrow)$ $2 \times 4 + 3 \times 1 = 2V_P + 3V_Q$

① $11 = 2V_P + 3V_Q$

$e = \dfrac{\text{separation}}{\text{approach}}$ $\quad 1 = \dfrac{V_Q - V_P}{4 - 1}$

② $3 = V_Q - V_P$

Solving ① and ② $\quad V_P = \dfrac{2}{5}\text{ ms}^{-1} \rightarrow$

$V_Q = \dfrac{17}{5}\text{ ms}^{-1} \rightarrow$

What speed does Q have after colliding with the wall?

$e = \dfrac{\text{separation}}{\text{approach}}$ $\quad 0.5 = \dfrac{W_Q}{\frac{17}{5}}$

$W_Q = \dfrac{17}{10}\text{ ms}^{-1} \leftarrow$

How long does it take Q to reach the wall?

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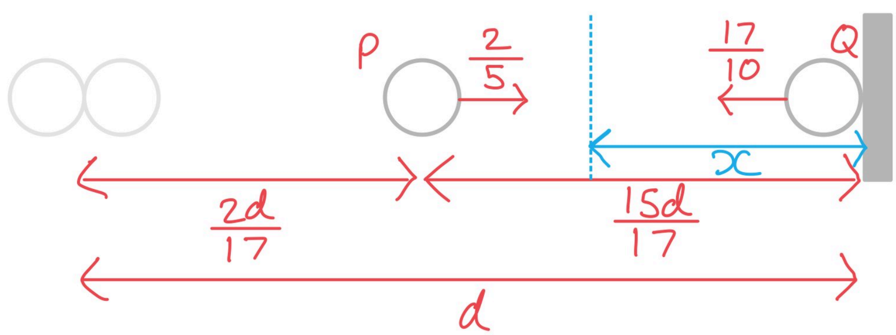
$$\text{Speed} = \frac{\text{distance}}{\text{time}} \qquad \frac{17}{5} = \frac{d}{t}$$

$$t = \frac{5d}{17} \text{ seconds}$$

How far does P travel in this time?

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \qquad \frac{2}{5} = \frac{\text{dist}}{\frac{5d}{17}}$$

$$\text{dist} = \frac{2d}{17} \text{ m}$$



Split the distance in the ratio of the speeds

$$\frac{2}{5} : \frac{17}{10}$$

$$4 : 17$$

We want the right-hand distance (from the wall)

$$\left(\frac{15d}{17} \div (4 + 17)\right) \times 17$$

$$x = \frac{5d}{\ldots}$$

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