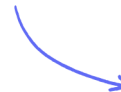


3.2 Circles

3.2.1 Equation of a Circle / 3.2.2 Finding the Centre & Radius / 3.2.3 Bisection of Chords / 3.2.4 Angle in a Semicircle / 3.2.5 Radius & Tangent

Easy (8 questions)	/30
Medium (8 questions)	/43
Hard (8 questions)	/45
Very Hard (8 questions)	/54
Total Marks	/172

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Easy Questions

1 Write down the equations of the circles with the following centres and radii

- (i) Centre: $(0, 0)$ Radius: $r = 4$,
- (ii) Centre: $(3, -4)$ Radius: $r = 2$,
- (iii) Centre: $(-5, 0)$ Radius: $r = 5$.

(3 marks)

2 Write down the centre and the radius for each of the following circles

- (i) $x^2 + y^2 = 5^2$,
- (ii) $(x + 3)^2 + (y - 2)^2 = 49$,
- (iii) $x^2 + (y + 4)^2 = 144$.

(3 marks)

3 On separate diagrams sketch the circles with the following equations

- (i) $x^2 + y^2 = 9$
- (ii) $(x - 4)^2 + (y - 3)^2 = 4^2$

(4 marks)

- 4 (a)** (i) Complete the square of $x^2 + 4x$.
(ii) Complete the square of $y^2 - 6y$.

(2 marks)

- (b)** (i) Use your answers to part (a) to show that the equation $x^2 + y^2 + 4x - 6y + 4 = 0$ can be written in the form $(x + 2)^2 + (y - 3)^2 = 9$.
(ii) Hence, write down the centre and the radius of the circle with equation $x^2 + y^2 + 4x - 6y + 4 = 0$.

(4 marks)

- 5** The line segment connecting the two points (1 , 0) and (9 , 4) is the diameter of a circle.

Find the centre and radius of the circle.

(4 marks)

- 6** Determine if the circles with equations

$$(x + 4)^2 + y^2 = 9 \text{ and } (x - 2)^2 + y^2 = 9$$

intersect once, twice or not at all. Fully explain your answer.

(3 marks)

- 7 On a sketch show how a circle and a line can either have 0, 1 or 2 intersections.

(2 marks)

- 8 The line with equation $y = x - 1$ intersects the circle with equation $(x - 5)^2 + (y - 4)^2 = 18$ at two distinct points.
Find the coordinates of the two points of intersection.

(5 marks)

Medium Questions

- 1 A circle has centre $(6, -5)$ and goes through the point $(1, 7)$. Find the equation of the circle.

(4 marks)

- 2 (a)** Show that $x^2 + y^2 + 2x - 6y + 9 = 0$ can be written in the form $(x - a)^2 + (y - b)^2 = r^2$, where a , b and r are integers to be found.

(2 marks)

- (b)** Hence write down the centre and radius of the circle with equation $x^2 + y^2 + 2x - 6y + 9 = 0$.

(2 marks)

- 3** The line $x + y = -7$ meets the circle with equation $(x - 1)^2 + (y - 2)^2 = 50$.

- (i) Show that the line and circle meet at one point only.
- (ii) Find the coordinates of the point of intersection.

(4 marks)

- 4** The line $7x + y = -6$ intersects the circle $(x - 2)^2 + (y - 5)^2 = 25$ at the points A and B . Find the coordinates of A and B .

(4 marks)

5 (a) A circle C has centre $(-4, 1)$ and passes through the point $P(0, 3)$.

Find an equation for the circle C .

(4 marks)

(b) Find an equation for the tangent to the circle at P .

(3 marks)

6 (a) The points $A(3, 5)$, $B(5, 3)$ and $C(9, 7)$ lie on a circle.

Show that triangle ABC is a right-angle triangle.

(2 marks)

(b) Explain why the line segment AC must be the diameter of the circle.

(1 mark)

(c) Hence find the equation of the circle.

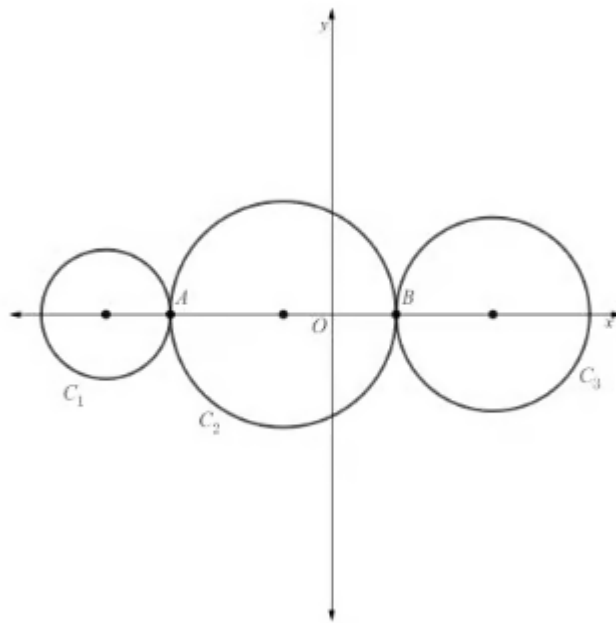
(4 marks)

7 Circles C_1 , C_2 and C_3 all have their centres on the x -axis.

Circle C_1 has equation $(x + 7)^2 + y^2 = 4$.

Circle C_3 has equation $x^2 + y^2 - 10x + 16 = 0$.

Circles C_1 and C_2 touch at point A , and circles C_2 and C_3 touch at point B .

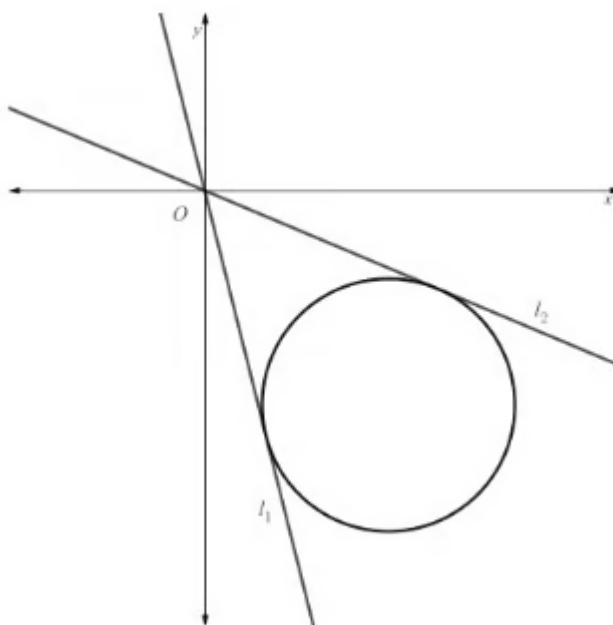


Find the coordinates of the centre of circle C_2 .

(6 marks)

8 (a) A circle has equation $x^2 + y^2 - 12x + 14y = -68$.

The lines l_1 and l_2 are both tangents to the circle, and they intersect at the origin.



Explain why the equations for l_1 and l_2 must each be in the form $y = mx$, where m is the gradient of the line.

(1 mark)

(b) Show that the gradients of l_1 and l_2 must be the solutions to the equation

$$19m^2 + 84m + 32 = 0.$$

(4 marks)

(c) Hence find the equations of l_1 and l_2 , giving your answers in the form $y = mx..$

(2 marks)

Hard Questions

- 1 The points $A(-3, 1)$ and $B(3, -7)$ are the two endpoints of the diameter AB of a circle. Find the equation of the circle.

(5 marks)

- 2 (a)** Show that $x^2 + y^2 + 5x - 2y - 5 = 0$ can be written in the form $(x - a)^2 + (y - b)^2 = r^2$, where a , b and r are constants to be found.

(2 marks)

- (b)** Hence write down the centre and radius of the circle with equation $x^2 + y^2 + 5x - 2y - 5 = 0$.

(2 marks)

- 3** The line $y + 2x = 11$ meets the circle with equation $x^2 + y^2 + 6x - 14y = -38$.

- (i) Show that the line and circle meet at one point only.
- (ii) Find the coordinates of the point of intersection.

(4 marks)

- 4** The line $x + 5y + 22 = 0$ intersects the circle $x^2 + y^2 + 4x + 8y - 6 = 0$ at the points A and B . Find the coordinates of A and B

(4 marks)

5 (a) A circle C has centre $(-2, 3)$ and passes through the point $P(6, -3)$.

Find an equation for the circle C .

(4 marks)

(b) Find an equation for the tangent to the circle at P .

(3 marks)

6 (a) The points $A(-3, 6)$, $B(5, -4)$ and $C(6, 5)$ lie on a circle.

Show that $\angle ACB = 90^\circ$.

(2 marks)

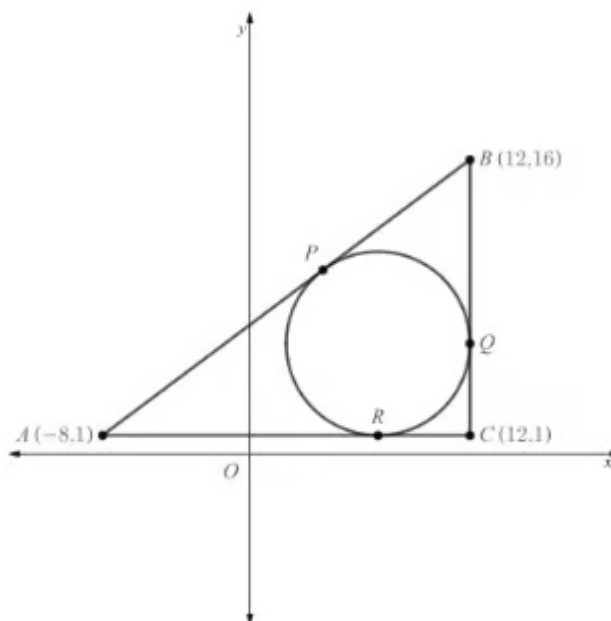
(b) Deduce a geometrical property of the line segment AB .

(1 mark)

(c) Hence find the equation of the circle.

(4 marks)

- 7 (a)** Triangle ABC has vertices $A(-8, 1)$, $B(12, 16)$ and $C(12, 1)$. A circle with equation $(x - 7)^2 + (y - 6)^2 = 25$ touches Triangle ABC at the three points P , Q and R , as shown in the diagram below:



Write down the coordinates of points R and Q .

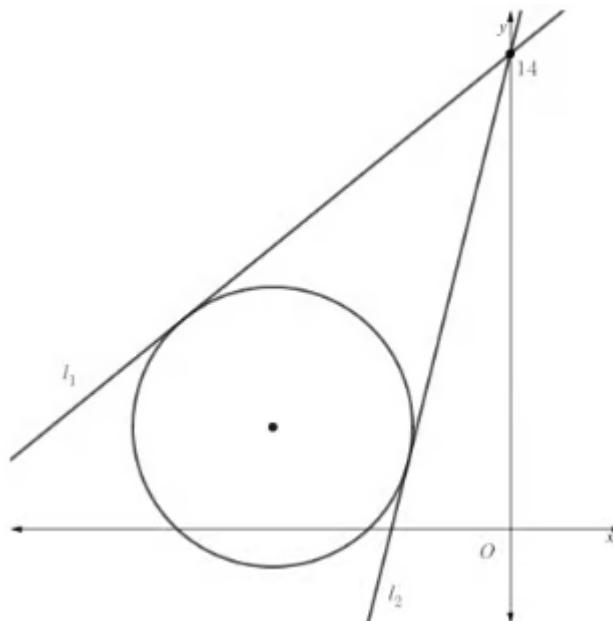
(2 marks)

- (b)** Find the coordinates of point P .

(5 marks)

8 A circle has equation $x^2 + y^2 + 14x - 6y = -41$.

The lines l_1 and l_2 are both tangents to the circle, and they intersect at the point $(0, 14)$.



Find the equations of l_1 and l_2 , giving your answers in the form $y = mx + c$.

(7 marks)

Very Hard Questions

- 1 The points $A(2, -21)$ and $B(-5, 3)$ are the two endpoints of the diameter AB of a circle. Find the equation of the circle in the form $ax^2 + ay^2 + bx + cy + d = 0$, where a, b, c and d are integers to be found.

(6 marks)

- 2 Find the centre and radius of the circle with equation $x^2 + y^2 + x - 3y + 2 = 0$.

(4 marks)

- 3 The line $x + y = c$ intersects the circle $x^2 + y^2 - 6x + 10y - 16 = 0$ at exactly two points. Find the range of possible values of c .

(7 marks)

- 4 The points $A(-2, 3)$, $B(0, 6)$ and $C(k, -1)$ lie on a circle, where BC is the diameter of the circle.

Find the value of k .

(4 marks)

- 5 A circle C has equation $x^2 + y^2 - 10x - 4y + 19 = 0$. Point P lies on the circle, and the tangent to the circle at point P has a gradient of -3 . Find the two possible sets of coordinates for point P .

(7 marks)

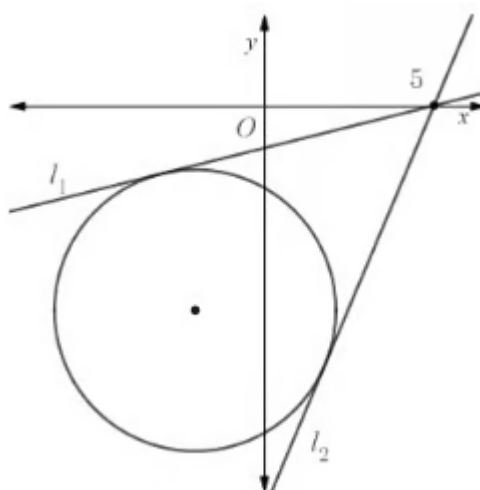
- 6 The points $A(4, 6)$, $B(7, 2)$ and $C(12, 12)$ lie on a circle.

Find the equation of the circle.

(7 marks)

- 7 A circle has equation $x^2 + y^2 + 4x + 12y = -23$.

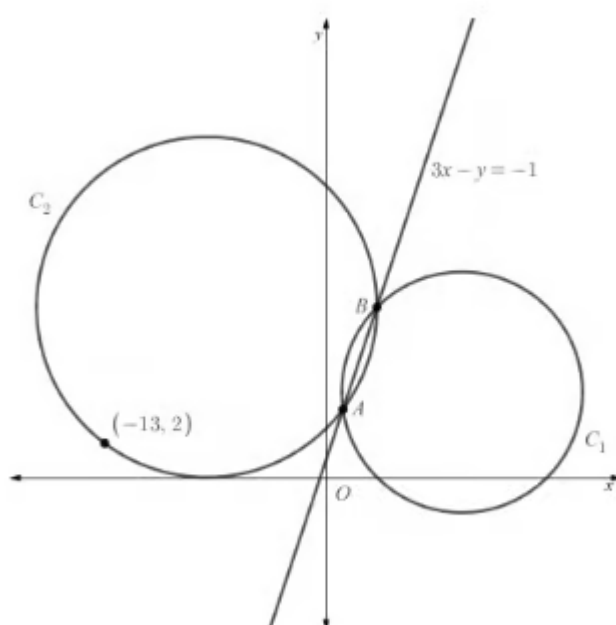
The lines l_1 and l_2 are both tangents to the circle, and they intersect at the point $(5, 0)$.



Find the equations of l_1 and l_2 , giving your answers in the form $y = mx + c$.

(8 marks)

- 8 The diagram below shows circles C_1 and C_2 which intersect at the two points A and B . Circle C_1 has equation $x^2 + y^2 - 16x - 10y + 39 = 0$, and points A and B lie along the line with equation $3x - y = -1$. Circle C_2 also passes through the point $(-13, 2)$.



Find an equation of circle C_2 .

(11 marks)