Modelling with Linear Equations

We saw in Chapter 2 that lots of things in real life have a 'quadratic' relationship, e.g. vertical height with time. Lots of real life variables have a 'linear' relationship, i.e. **there is a fixed increase/decrease in one variable each time the other variable goes up by 1 unit.**

Examples

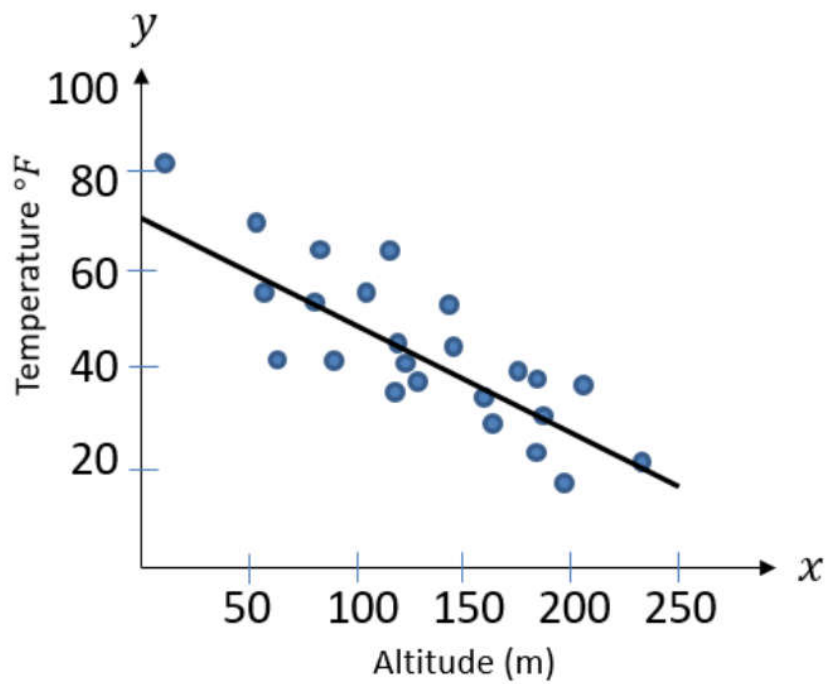
Car sales made and take home pay.

The relationship between Celsius and Fahrenheit.

Temperature and altitude (in a particular location)

(And a pure maths one:)
The nth term of an arithmetic series.

3, 5, 8, 11, 14, ...



The temperature y at different points on a mountain is recorded at different altitudes x.
Suppose we were to use a linear model $y = mx + c$.

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a Determine m and c (you can assume the line goes through $(0,70)$ and $(250,20)$).

$m = \frac{20-70}{250} = \frac{-50}{250} = -\frac{1}{5}$ $\quad c = 70$ $\quad y = -\frac{1}{5}x + 70$

b **Interpret** the meaning of m and c **in this context**.

For every 1m the altitude increases, the temperature decreases by 0.2°F
At sea level, the temperature is predicted as 70°F.

c Predict at what altitude the temperature reaches $0°F$.

$y = 0$ $\quad -\frac{1}{5}x + 70 = 0$

$\frac{1}{5}x = 70$

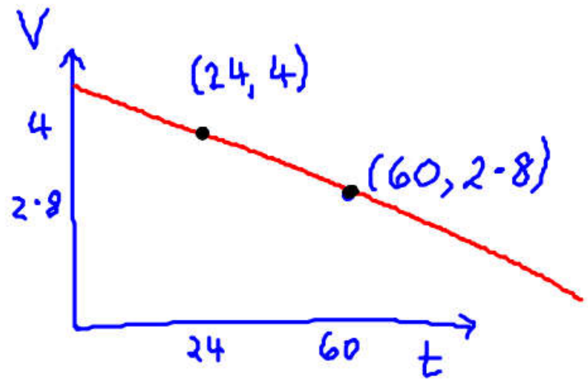
$x = 350$ m

3. A tank, which contained water, started to leak from a hole in its base.

The volume of water in the tank 24 minutes after the leak started was 4 m^3.

The volume of water in the tank 60 minutes after the leak started was 2.8 m^3.

The volume of water, V m^3, in the tank t minutes after the leak started, can be described by a linear model between V and t.



(*a*) Find an equation linking V with t.

(4)

Use this model to find

(*b*) (i) the initial volume of water in the tank, $t = 0$ $\quad V = \frac{24}{5} = 4 \cdot 8 \text{ m}^3$

(ii) the time taken for the tank to empty. $V = 0$ $\quad 0 = -\frac{1}{30}t + \frac{24}{5}$

(3)

$\frac{1}{30}t = \frac{24}{5}$

$t = 144$ minutes

(*c*) Suggest a reason why this linear model may not be suitable.

(1)

(Total for Question 3 is 8 marks)

a) $\underset{t_1 \ V_1}{(24, 4)}$ and $(60, 2 \cdot 8)$

$$m = \frac{2 \cdot 8 - 4}{60 - 24} = \frac{-1 \cdot 2}{36} = -\frac{1}{30}$$

$$y - y_1 = m(x - x_1)$$

$$V - V_1 = m(t - t_1)$$

$$V - 4 = -\frac{1}{30}(t - 24)$$

$$V - 4 = -\frac{1}{30}t + \frac{4}{5}$$

$$V = -\frac{1}{30}t + \frac{24}{5} \qquad V = -0.033t + 4 \cdot 8$$

c) Because water is unlikely to leak out at a constant rate. It will leak faster at the start.

Your Turn

The height, H metres, of a plant was measured t years after planting.

Exactly 2 years after planting, the height of the plant was 1.43 metres.
Exactly 5 years after planting, the height of the plant was 3.23 metres.

Using a linear model,

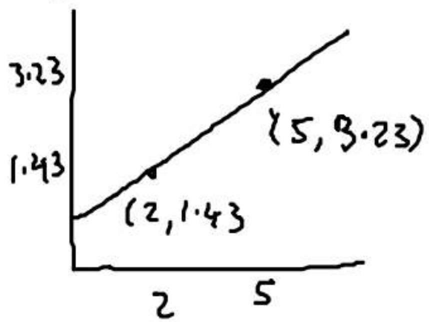
(a) find an equation linking H with t.

0.55

The height of the plant was approximately 55cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a)

a)



(2, 1.43)
(5, 3.23)

$$m = \frac{3.23 - 1.43}{5 - 2} = 0.6$$

$$H - 1.43 = 0.6(t - 2)$$
$$H - 1.43 = 0.6t - 1.2$$
$$\underline{\underline{H = 0.6t + 0.23}}$$

b) $t = 0$, $H = 0.23m$

0.23m is not close to 55cm, so the linear model is not supported.

Ex 5H Q5-9