A Level · Edexcel · Maths

4 hours 40 questions

5.6 Compound & Double Angle Formulae (A Level only)

Easy (10 questions)	/45
Medium (10 questions)	/55
Hard (10 questions)	/53
Very Hard (10 questions)	/65
Total Marks	**/218**

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Easy Questions

1 (i) Write down the exact value of cos $60°$.

(ii) Write down the exact value of cos $45°$.

(iii) Use your calculator to find the exact value of cos $105°$.

(iv) Hence show that cos $60° +$ cos $45° \neq$ cos $105°$.

(5 marks)

2 (a) Express sin $15°$ in terms of sin $45°$ and sin $30°$.

(2 marks)

(b) Hence show that

$$\sin 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$$

(3 marks)

3 (a) Starting with the identity

$$\sin(A + B) \equiv \sin A \cos B + \sin B \cos A$$

And using the substitution $B = A$, show that $\sin 2A \equiv 2 \sin A \cos A$.

(2 marks)

(b) Hence show the exact value of $\sin 120° = \frac{\sqrt{3}}{2}$.

(2 marks)

4 (a) Use an appropriate identity to find $\sin(\theta + \alpha)$ in terms of sines and cosines of θ and α.

(2 marks)

(b) Hence show that $R \sin(\theta + \alpha) \equiv R \cos \alpha \sin \theta + \mathrm{R} \sin \alpha \cos \theta$.

(1 mark)

5 (a) Solve the following equations in the given intervals.

$\sin 2\theta = \frac{1}{2}, \quad -\pi \le \theta \le \pi$

(4 marks)

(b) $\cos 2\theta = \frac{\sqrt{3}}{2}, \quad 0 \le \theta \le 2\pi$

(4 marks)

6 Show that

$$5\cos\left(\theta - \frac{\pi}{6}\right) \equiv \frac{5\sqrt{3}}{2}\cos\theta + \frac{5}{2}\sin\theta$$

(4 marks)

7 Show that

$$\cos^2 x + \cos 2x \equiv 3\cos^2 x - 1$$

(2 marks)

8 (a) (i) Show that , $R\sin(\theta+\alpha)\equiv R\ \cos\alpha\sin\theta+R\sin\alpha\cos\theta$ where R and α are constants with $R>0$ and $0<\alpha<\frac{\pi}{2}$.

(ii) Use your result from part (i) to show that $\sqrt{3}\ \sin\theta+\cos\theta\equiv 2\ \sin\left(\theta+\frac{\pi}{6}\right)$.

(4 marks)

(b) Write down the maximum value of $\sqrt{3}\ \sin\theta+\cos\theta$.

(1 mark)

9 Sketch the graph of $y=\tan 2\theta$ for $0\le\theta\le 2\pi$.

Label the points at which the graph intersects the coordinate axes.

(3 marks)

10 (a) Use the difference of two squares to show that

$$\cos^4 x - \sin^4 x \equiv \cos 2x$$

(3 marks)

(b) Hence solve the equation

$$\cos^4 x - \sin^4 x = \frac{\sqrt{2}}{2}$$

for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

(3 marks)

Medium Questions

1 Prove by a counter-example that $\sin(A+B) = \sin A + \sin B$ is **not** true in general.

(2 marks)

2 (a) Express $\tan(210°)$ in terms of $\tan(180°)$ and $\tan(30°)$

(2 marks)

(b) Hence show that $\tan(210°) = \dfrac{\sqrt{3}}{3}$.

(2 marks)

3 (a) Starting with the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

and using the substitution $B = A$, show that $\cos 2A \equiv \cos^2 A - \sin^2 A$

(2 marks)

(b) Hence, or otherwise, show that $\cos 2A \equiv 1 - 2\sin^2 A$.

(2 marks)

4 (a) Using an appropriate trigonometric identity, show that

$$R\sin(\theta + \alpha) \equiv R\cos\alpha\sin\theta + \mathrm{R}\sin\alpha\cos\theta$$

where R and α are constants.

(2 marks)

(b) Hence show that $3\sin\theta + 2\cos\theta = \sqrt{13}\sin(\theta + 0.588)$.

(3 marks)

5 (a) Use appropriate double angle formulae to solve the following equations in the given intervals.

$\cos^2\theta - \sin^2\theta = \frac{1}{2} \quad -\pi \le \theta \le \pi$

(5 marks)

(b) $4\sin x\cos x = -\sqrt{3} \quad 0 \le x \le \pi$

(5 marks)

6 Show that

$$\frac{5\sin 2x}{\tan x} \equiv 10\cos^2 x \qquad x \ne \frac{k\pi}{2}$$

(3 marks)

7 (a) (i) Show that $R\cos(x+\alpha) \equiv R\cos\alpha\ \cos x - R\sin\alpha\sin x$, where R and α are constants.

(ii) Use your result from part (i) to show that $\cos x - \sqrt{3}\sin x \equiv 2\cos\left(x+\frac{\pi}{3}\right)$.

(4 marks)

(b) Hence solve the equation $\cos x - \sqrt{3}\sin x = 1$ for $0 \le x \le 2\pi$.

(3 marks)

8 (a) Using the identities

$$\sin(A+B) \equiv \sin A \cos B + \sin B \cos A \quad \text{and}$$

$$\cos 2A \equiv 1 - 2\sin^2 A$$

show that $\sin 3A \equiv 3\sin A - 4\sin^3 A$

(5 marks)

(b) Hence, or otherwise, solve the equation

$$3\sin\theta - 4\sin^3\theta = \frac{1}{2} \qquad -\pi \leq \theta \leq \pi$$

(4 marks)

9 (a) Show that $5\sin\theta + 12\cos\theta$ can be written in the form $R\sin(\theta + \alpha°)$ where $R > 0$ and $0° < \alpha < 90°$.

(4 marks)

(b) Sketch the graph of $y = 5\sin x + 12\cos x$ for $0° \leq x \leq 360°$.
Label any points where the graph intercepts the coordinate axes.

(4 marks)

10 Show that $2\operatorname{cosec} 2A \equiv \operatorname{cosec} A \sec A$.

(3 marks)

Hard Questions

1 If $A = B$, then

$$\sin(A - B) = \sin(A - A) = \sin(0) = 0 = \sin A - \sin A = \sin A - \sin B$$

By using a suitable counter-example with $A \neq B$, prove that $\sin(A - B) = \sin A - \sin B$ is **not** true in general.

(2 marks)

2 (a) Express $\cos(285°)$ in terms of cosines and sines of 315° and 30°.

(2 marks)

(b) Hence show that $\cos(285°) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$.

(3 marks)

3 Show that

$$\sin 2A \equiv 2 \sin A \cos A$$

(You may use the identity $\sin(A+B) \equiv \sin A \cos B + \cos A \sin B$.)

(2 marks)

4 Show that $2\cos\theta - 5\sin\theta$ can be written in the form $R\cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give R in the form $\sqrt{k}$ where k is an integer, and give α correct to three significant figures.

(5 marks)

5 (a) Solve the equation

$$\sin 2\theta = \sin \theta \qquad -\pi \le \theta \le \pi$$

(6 marks)

(b) Solve the equation

$$\cos 2x + \sin^2 x = 0 \qquad 0 \le x \le 2\pi$$

(4 marks)

6 Show that

$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} \equiv \tan A \qquad \left(A, B \ne \left(k+\frac{1}{2}\right)\pi\right)$$

(4 marks)

7 (a) Show that $2 \sin\theta + 4\cos\theta$ can be written as $2\sqrt{5}\cos(\theta - \alpha)$, where $\alpha = 0.464$ to three significant figures.

(4 marks)

(b) Hence solve the equation

$$2\sin\theta + 4\cos\theta = 3 \qquad -\pi \leq \theta \leq \pi$$

giving your answers correct to 3 significant figures.

(3 marks)

8 (a) By letting $B = 2A$, use the identity for $\tan(A + B)$ to derive an expression for $\tan 3A$ in terms of tan A.

(5 marks)

(b) Hence, or otherwise, solve the equation

$$\frac{6 \tan x - 2 \tan^3 x}{1 - 3 \tan^2 x} = 2 \qquad 0 \leq x \leq \pi$$

(3 marks)

9 Sketch the graph of $y = 2(\sin x - \cos x)$ for $0° \leq x \leq 360°$.

Be sure to label any points where the graph intercepts the coordinate axes, and state the coordinates of any maximum and minimum points.

(7 marks)

10 Show that

$$2 - 2\cot 2A \tan A \equiv \sec^2 A \qquad A \neq k\pi$$

(3 marks)

Very Hard Questions

1 (i) Prove that $\sin(A-B) = \sin A + \sin B$ is **not** true in general.

(ii) Find values for A and B, with $A \neq 0$ and $B \neq 0$, for which $\sin(A-B) = \sin A + \sin B$ **is** true.

(3 marks)

2 (a) Use the identities $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$ and $\cos(A \pm B) \equiv \cos A \cos B \pm \sin A \sin B$ to show that

$$\sin(X + Y - Z) \equiv$$
$$\sin X \cos Y \cos Z + \cos X \sin Y \cos Z - \cos X \cos Y \sin Z + \sin X \sin Y \sin Z$$

(3 marks)

(b) Hence show that $\sin(165°) = \dfrac{\sqrt{6} - \sqrt{2}}{4}$.

(4 marks)

3 Show that

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

State clearly any trigonometric identities you use to show this result.

(4 marks)

4 Given that $a \sin\theta + \mathrm{b} \cos\theta$, where a and b are positive constants, is to be written in the form $R\sin(\theta + \alpha)$, find expressions for:

(i) α in terms of a and b

(ii) R in terms of a and b

(6 marks)

5 (a) Solve the equation

$$\cos 2\theta = \cos\theta \qquad 0 \le \theta < 2\pi$$

(5 marks)

(b) Solve the equation

$$\tan 2x = 3\tan x \qquad -\pi \le x \le \pi$$

(6 marks)

6 Show that

$$\tan 2\theta \tan\theta \equiv \sec 2\theta - 1$$

(5 marks)

7 (a) Show that $5\sin\theta - 3\cos\theta$ can be written in the form $R\sin(\theta - \alpha)$ where $R = \sqrt{34}$, and $\alpha = 0.540$ radians correct to three significant figures.

(4 marks)

(b) Use your result from part (a), and the properties of the sine and cosine functions, to solve the equation

$$3\cos 2x + 5\sin 2x = 0.4 \qquad 0 \le x \le 2\pi$$

(5 marks)

8 (a) Use an identity for cos $2A$ to derive an identity for cos $4A$, in terms of cos A.

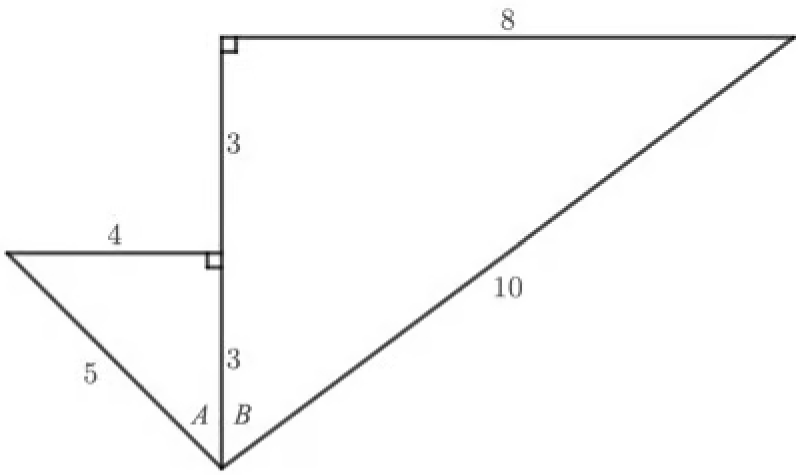
(4 marks)

(b) Hence, or otherwise, solve the equation

$$2\cos 4x = 7\sin^2 x - 2 \qquad 0 \leq x \leq \pi$$

(5 marks)

9 The diagram below shows two right-angled triangles.
Angles A and B have been labelled.



Given that $\alpha = A + B$, find the exact values of sin α,cos α and tan α.

(7 marks)

10 (i) Explain briefly why $\theta = 0$ is **not** a solution to the equation $3\theta \cot 2\theta = 0$.

(ii) By using an appropriate approximation, determine the value of $\lim\limits_{\theta \to 0} 3\theta \cot 2\theta$

(4 marks)