A Level · Edexcel · Maths

4 hours 34 questions

10.2 Modelling involving Numerical Methods (A Level only)

Easy (7 questions)	/43
Medium (9 questions)	/60
Hard (9 questions)	/58
Very Hard (9 questions)	/64
Total Marks	**/225**

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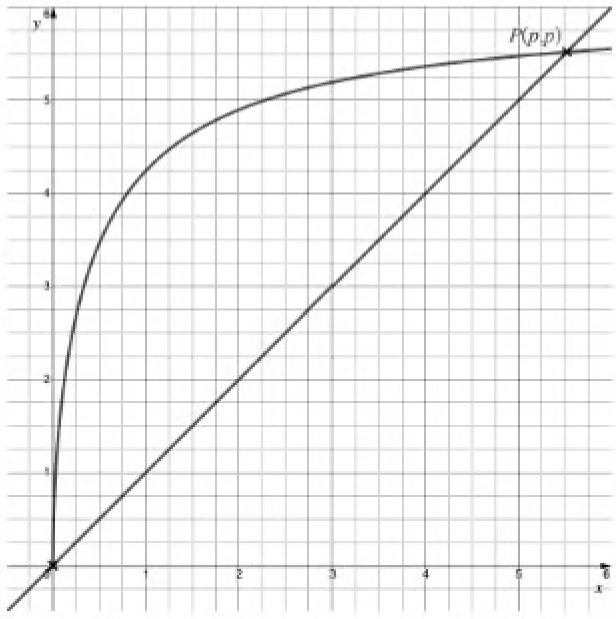
Easy Questions

1 (a) A bypass is to be built around a village.
On the graph below the road through the village is modelled by the line $y = x$.

The bypass is modelled by the equation $y = \sqrt{\dfrac{40x}{x+1}}$



The bypass runs from the origin to the point $P(p, p)$.

Use the iterative formula $x_{n+1} = \sqrt{\dfrac{40x_n}{x_n + 1}}$ with $x_0 = 5$ to find the value of p, correct to three significant figures.

(3 marks)

(b) (i) Calculate $f(5.835)$ and $f(5.845)$ where $f(x) = x - \sqrt{\dfrac{40x}{x+1}}$.

(ii) Hence use the sign change rule to show your answer to part (a) is correct to three significant figures.

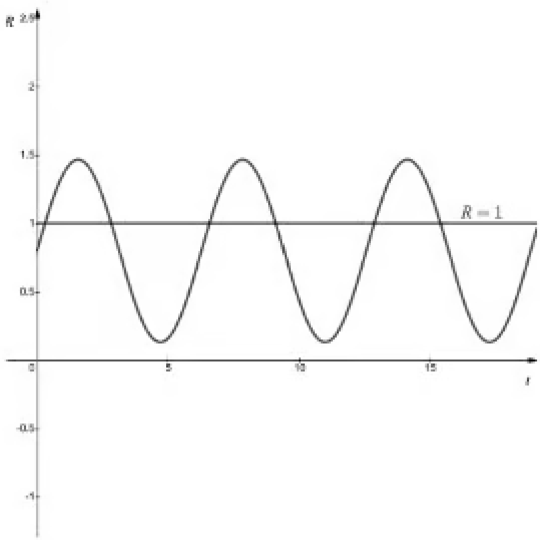
(3 marks)

2 (a) Scientists are analysing the infection rate, R, at which a virus spreads amongst a population. They model R using the function

$$R(t) = 1.1 + \cos t \qquad (t \geq 0)$$

where t is the number of months since the first case of the virus was discovered.

The diagram below shows the graphs of $y = R(t)$ and $y = 1$.



(i) Show that the equation $R(t) = 1$ can be written in the form $\cos t + 0.1 = 0$.

(ii) Find the derivative of $\cos t + 0.1$.

(2 marks)

(b) Use the Newton-Raphson method with $t_0 = 1.5$, along with your answers to part (a), to find the time when R first equals 1. Give your answer to three significant figures.

(3 marks)

(c) (i) Write down the minimum R value the model predicts.

(ii) Approximately, how often (in months) does this minimum infection rate occur?

(3 marks)

3 Following an oil spill into the ocean experts recorded the rate of oil leaking at regular intervals whilst engineers worked to clear up the spillage.

The rate of oil leaking (measured in barrels per week) was recorded every fortnight for eight weeks. The results are shown in the table below.

Time	0	2	4	6	8
Rate of leak	0	14 000	9600	3400	1800

The trapezium rule is to be used to estimate the total amount of oil leaked during the eight weeks. **All** the data in the table is to be used.

(i) Write down the width of each strip (h) that will be used.

(ii) Show that the trapezium rule estimates the total amount of oil spilled in the eight weeks is 56000 barrels to two significant figures.

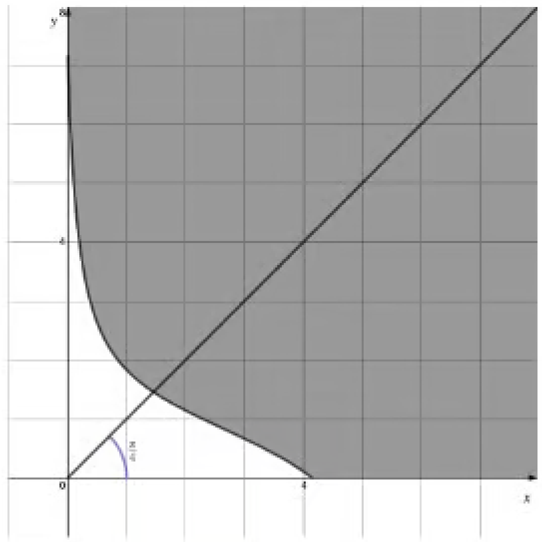
(4 marks)

4 The game of Tanball is played on a flat table.

A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest in the winning zone.

A particular player decides to roll the ball at an angle of $\frac{\pi}{4}$ radians.

This is illustrated by the graph below with the ball being rolled from the origin and the shaded area being the winning zone.



The boundary of the winning zone is given by part of the curve with equation

$$y = 1 - \tan\left(\sqrt{3(x+1)}\right).$$

(i) Using the iterative formula $x_{n+1} = 1 - \tan\left(\sqrt{3(x_n + 1)}\right)$, with initial starting value $x_0 = 1.5$, find the estimates x_1, x_2 and x_3, writing each to five decimal places.

(ii) Continue using the iterative feature on your calculator to find the value of x correct to three significant figures.

(iii) Write down the y-coordinate of the point where this player's ball should cross the winning boundary, give your answer to three significant figures.

(4 marks)

5 (a) Traffic is monitored by three average speed cameras, along a stretch of the road where the speed limit is 30 mph.

A car passes the first camera at time zero.
The car's speed and the time it passes each camera, are recorded.

The results are shown in the table below.

Camera	1	2	3
Time (hours)	0	0.25	0.5
Speed (mph)	32	38	27

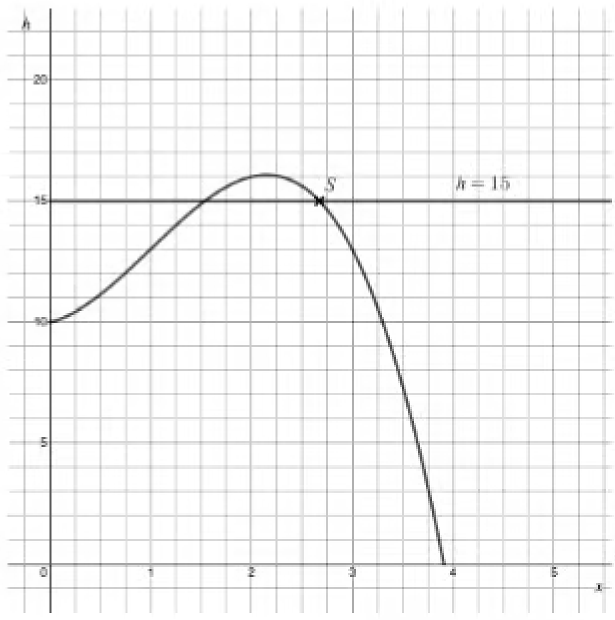
Use the trapezium rule with **all** the data in the table to estimate the distance between the first and last camera. Give your estimate to three significant figures.

(3 marks)

(b) Is the car driving within the speed limit? You must show clearly how you achieve your answer.

(2 marks)

6 (a) The profile of a vertical drop along a rollercoaster track is modelled using part of the function $f(x) = 10 - (x^3 - 3x^2 - x)$.
The graph of $h = f(x)$, $x \geq 0$, $h \geq 0$, is shown below.



h is the height, in metres, above the ground of the front carriage of the rollercoaster.

x is the horizontal distance from the origin, in metres, of the front carriage of the rollercoaster.

(i) Show that the equation $f(x) = 15$ can be rewritten as $x^3 - 3x^2 - x + 5 = 0$.

(ii) Find the derivative of $x^3 - 3x^2 - x + 5 = 0$.

(2 marks)

(b) The rollercoaster briefly stops at the point labelled S on the graph, where $f(x) = 15$.

Use the Newton-Raphson method with initial value $x_0 = 2.6$ to find the x-coordinate of point S. Give your answer to three significant figures.

(4 marks)

(c) State a possible problem if $x_0 = 1.7$ were to be used as the initial value in part (b).

(1 mark)

7 (a) According to legend, unicorn tears can heal an injury almost instantly.

If a unicorn tear is applied to a burn of initial size $B\,\mathrm{mm}^2$ on human skin it will heal according to the model

$$\mathrm{b}(t) = B - t^3 + \sqrt{t} \qquad\qquad t \geq 0$$

where b is the area of the burn, in square millimetres, at time t seconds after the unicorn tear has been applied.

Show that the equation $\mathrm{b}(t) = 0$ can be written as

$$t = \sqrt[3]{B + \sqrt{t}}.$$

(2 marks)

(b) Use the iterative formula $t_{n+1} = \sqrt[3]{40 + \sqrt{t_n}}$, with initial value $t_0 = 3$, to find how many seconds it takes a burn of size 40 mm^2 to heal once a unicorn tear is applied.

Give your final answer to three significant figures.

(4 marks)

(c) An alternative iterative formula is $t_{n+1} = \left(t_n^3 - B\right)^2$.

(i) Using $t_0 = 3$, find t_1, t_2 and t_3, for the same initial burn size as part (b), giving each to three significant figures.

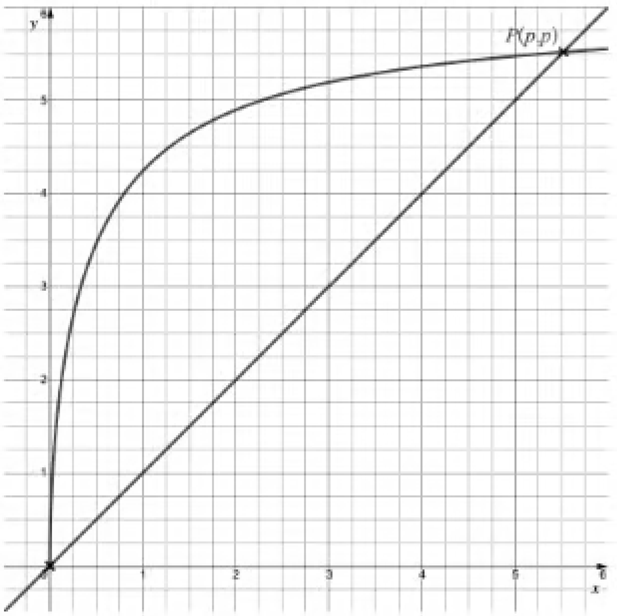
(ii) Explain how you can deduce whether this sequence of estimates is converging or diverging.

(3 marks)

Medium Questions

1 (a) The village of Greendale lies on a straight road, as modelled by the line $y = x$ on the graph below. To ease rush hour congestion, a bypass is to be built around Greendale.

The path of the bypass is modelled by the equation $y = 6\sqrt{1 - \frac{1}{x+1}}$.



The bypass runs from the origin to the point $P(p, p)$.

On the diagram show how using the iterative formula $x_{n+1} = 6\sqrt{1 - \frac{1}{x_n + 1}}$ with $0 < x_0 < p$ will lead to convergence at the point P.

(1 mark)

(b) Use the iterative method $x_{n+1} = 6\sqrt{1 - \frac{1}{x_n + 1}}$ with $x_0 = 5$ to find the value of p, correct to two decimal places.

(3 marks)

(c) Use the sign change rule with the function $f(x) = x - 6\sqrt{1 - \frac{1}{x+1}}$ to show your answer to part (b) is correct to two decimal places.

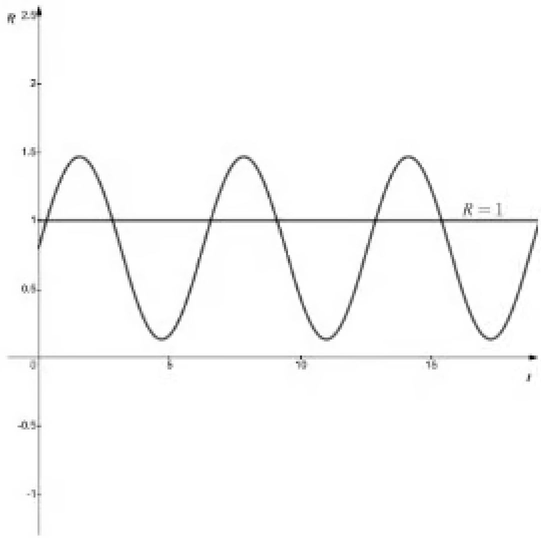
(2 marks)

2 (a) Scientists are analysing the infection rate, R, at which a virus, Pi-flu, spreads amongst a population. They model R using the function

$$R(t) = \frac{4}{5} + \frac{2}{3}\sin t \qquad (t \geq 0)$$

where t is the number of months since the first case of the virus was discovered.

The diagram below shows the graphs of $y = R(t)$ and $y = 1$.



(i) Show that the equation $R(t) = 1$ can be written in the form $\frac{2}{3}\sin t - \frac{1}{5} = 0$.

(ii) Find the derivative of $\frac{2}{3}\sin t - \frac{1}{5}$.

(3 marks)

(b) Use the Newton-Raphson method with $t_0 = 0.5$, along with your answers to part (a), to find the number of months after Pi-flu was discovered that the value of R first equals 1. Give your answer to three significant figures.

(3 marks)

(c) (i) Write down the maximum infection rate value the model predicts.

(ii) Approximately, how often (in months) does this maximum infection rate occur?

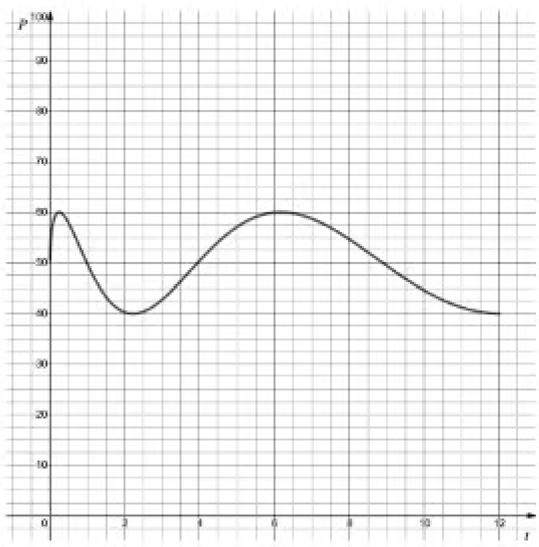
(iii) Suggest a reason why Pi-flu may have an increased infection rate at regular intervals.

(3 marks)

3 (a) The energy company Power^X operate a wind turbine.
Engineers from Power^X model the power output, P kW (kiloWatts), of the wind turbine over a twelve-hour period according to the function

$$P(t) = 50 + 10\sin\sqrt{10t} \qquad 0 \le t \le 12$$

where t is time measured in hours. The graph of $y = P(t)$ is shown below.



Using the trapezium rule, with six strips of width 1, estimate the total power generated by the wind turbine in the first six hours.
You may use the table below to help.

Time (t)	0	1	2	3	4	5	6
Power (P)							

(4 marks)

(b) Briefly explain why it is difficult for the engineers from Power^X to determine whether the total amount of power found, using the method in part (a), is an over- or under-estimate.

(1 mark)

4 (a) Following an explosion on the Longwater drilling rig, oil began to leak into the ocean from a damaged underwater pipe.

It took several months for experts to seal the pipe and stop further oil from leaking.

For the first fifteen weeks of the oil spill, the rate of oil leaking from the pipe (measured in barrels per week) was recorded every three weeks.

The results are shown in the table below.

Time	0	3	6	9	12	15
Rate of leak	0	2500	8000	15 000	25 000	37 500

The trapezium rule is to be used to estimate the total amount of oil leaked during the first fifteen weeks of the spillage. **All** the data in the table is to be used.

(i) Write down the number of strips (n) that will be used.

(ii) Write down the width of each strip (h) that will be used.

(2 marks)

(b) Show that the trapezium rule described in part (a) estimates the amount of oil spilled in the first fifteen weeks is 208 000 barrels to three significant figures.

(3 marks)

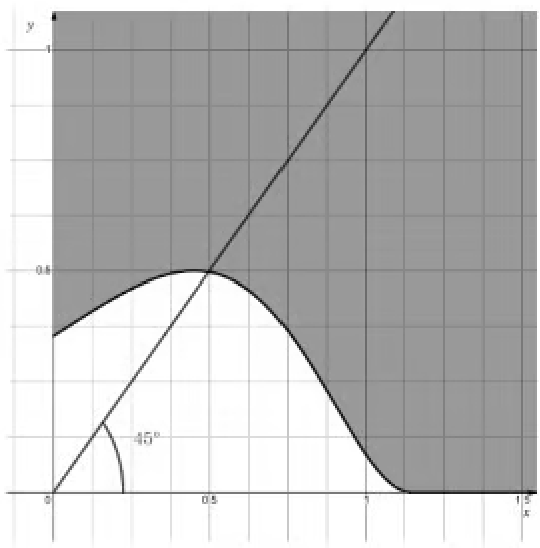
(c) The leakage rate of the oil rose steadily for the first 15 weeks.
State whether the estimate in part (b) is an under- or an over- estimate.

(1 mark)

5 (a) The game of Curveball is played on a flat table.

A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest in the winning zone.

A particular player decides to roll the ball at an angle of 45°.
This is illustrated by the graph below with the ball being rolled from the origin and the shaded area being the winning zone.



The boundary of the winning zone is given by part of the curve with equation

$$y = \frac{1}{2}(e^x).$$

Use the iterative formula $x_{n+1} = \frac{1}{2}(e^x)$ with initial starting value $x_0 = 0.5$, to show that the x-coordinate of the point where this player's ball should cross the winning zone boundary is 0.497 to three significagtnt figures.

(3 marks)

(b) Use your answer to part (a) to find the minimum distance the ball should travel for this player to win Curveball.

(2 marks)

6 (a) Coronation Street has a speed limit of 40 mph and traffic is monitored along one stretch of the road by four average speed cameras.

Vera is driving her car along Coronation Street and passes the first camera at time zero. Vera's speed, measured in miles per hour, and the time she passes each camera, are recorded.

The results are shown in the table below.

Camera	1	2	3	4
Time (hours)	0	0.1	0.2	0.3
Speed (mph)	36	40	38	35

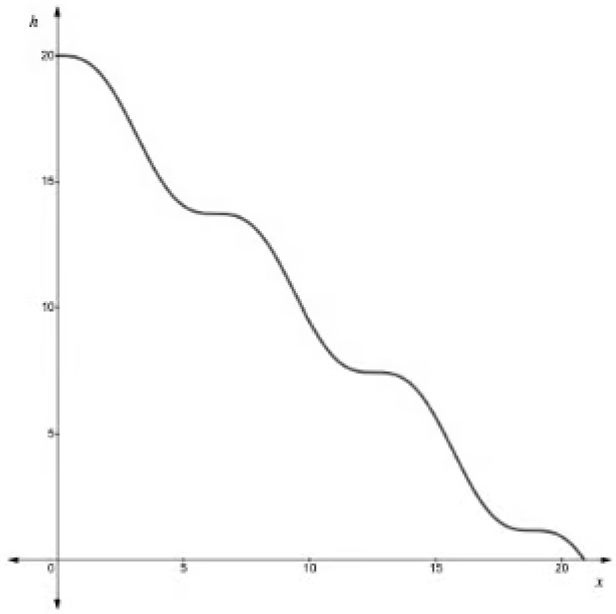
Use the trapezium rule with **all** the data in the table to estimate the distance between the first and last camera.
Give your estimate to three significant figures.

(3 marks)

(b) Vera's car consumes fuel at a rate of 52.6 miles per gallon.
Estimate the amount of fuel, in gallons, Vera's car uses between the first and last camera.

(2 marks)

7 (a) The profile of the Wibbley-Wobbley-Slide is modelled using part of the function $f(x) = 20 - x + \sin x$. The graph of $h = f(x)$, $x \geq 0$, $h \geq 0$, is shown below.



h is the height, in metres, above the ground of a person using the slide.

x is the horizontal distance, in metres, of a person on the slide, measured from the point directly underneath the top of the slide (ie the origin).

Find the height above the ground of a person on the slide when they have travelled 10 m horizontally.

(1 mark)

(b) Show that $f'(x) = \cos x - 1$.

(1 mark)

(c) Use the Newton-Raphson method with initial valueto find the total horizontal distance travelled by a person using the slide.
Give your answer to three significant figures.

(4 marks)

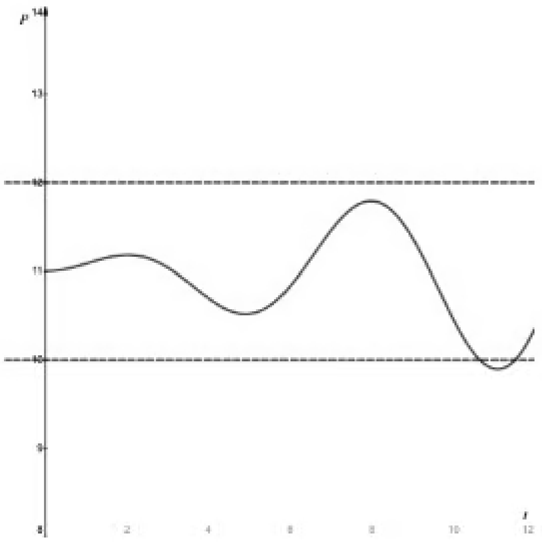
(d) Use the sign change rule to confirm your answer to part (c) is correct to three significant figures.

(2 marks)

8 (a) The air pressure inside a ThetaAir aeroplane is kept between 10 and 12 psi (pounds per square inch). A computer automatically adjusts the air pressure according to the model

$$P(t) = 11 + 0.1t\sin t \qquad\qquad t \geq 0$$

where P is the pressure (psi) and t is the time, in hours, since take off.
The graph of the model is shown below.



Explain why this model is only suitable for flights roughly less than 10 hours long.

(1 mark)

(b) (i) Show that $1 + 0.1t\sin t = 0$ when $P = 10$.

(ii) Show that $\frac{\mathrm{d}P}{\mathrm{d}t} = 0.1(t\cos t + \sin t)$.

(3 marks)

(c) Use the Newton-Raphson method with initial value $t_0 = 10$ to find the time that the air pressure on a ThetaAir aeroplane first goes outside of the range 10 to 12 psi.
Give your answer to three significant figures.

(3 marks)

(d) The air pressure increases back to 10 psi after 11.5 hours. Estimate the length of time, in minutes, that the air pressure was below 10 psi.

(2 marks)

9 (a) According to legend, a unicorn can heal an injury almost instantly by touching it with its horn.

When a unicorn touches a cut in human skin of length L mm, it will heal according to the model

$$\mathrm{f}(t) = Le^{-t} - t \qquad\qquad t \geq 0$$

where f is the length of the cut in millimetres, at time t seconds after the unicorn has touched the injury with its horn.

(i) Write down the value f would be when the cut is completely healed.

(ii) Show that, for a cut in human skin of length 5 mm the equation $\mathrm{f}(t) = 0$ can be rearranged into the form

$$t = -\ln\left(\frac{t}{5}\right)$$

(2 marks)

(b) Use the iterative formula $t_{n+1} = -\ln\left(\frac{t_n}{5}\right)$, with initial value $t_0 = 1$, to find how many seconds it takes a cut of size 5 mm to heal once a unicorn has touched it with its horn.

(i) Write down the values of the estimates t_1, t_2 and t_3 to four decimal places.

(ii) Give your final answer to two significant figures.

(iii) State the number of iterations required for convergence to two significant figures.

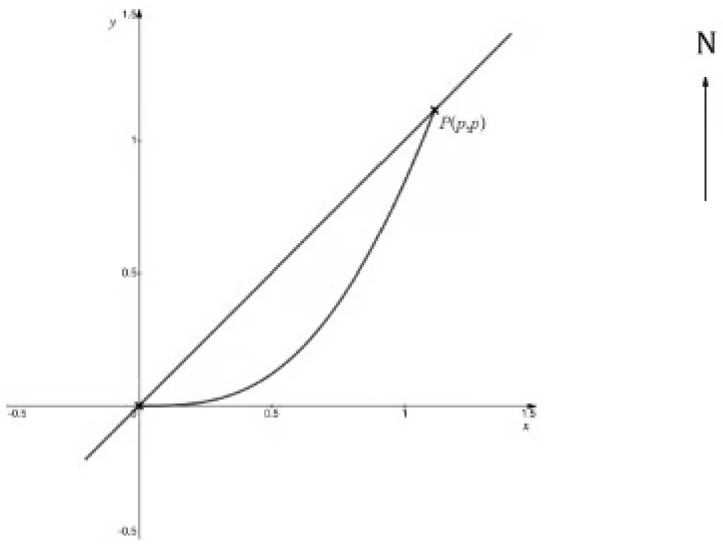
(4 marks)

(c) Briefly explain why the model should also restrict the range of $\mathrm{f}(t)$ to be greater than or equal to zero?

(1 mark)

Hard Questions

1 (a) The village of Crinkley Bottom lies on a straight road, as modelled by the line $y = x$ on the graph below. Rush hour traffic causes much air pollution in the village so to improve the air quality around Crinkley Bottom a bypass is to be built.
The path of the bypass is modelled by part of the equation $y = x^2 \sin x$.



The bypass is to be built with a roundabout south of the village at the origin and a northern roundabout which re-joins the road through Crinkley Bottom at the point $P(p, p)$.

On the diagram show how using the iterative formula $x_{n+1} = x_n^2 \sin x$ with $0 < x_0 < p$ will lead to convergence at the southern roundabout

(1 mark)

(b) Use the alternative iterative method

$$x_{n+1} = \sqrt{\frac{x}{\sin x}}$$

with $x_0 = 1$, to find the position of the roundabout at P to four significant figures.

(3 marks)

(c) Verify that your answer to part (b) is correct to four significant figures.

(2 marks)

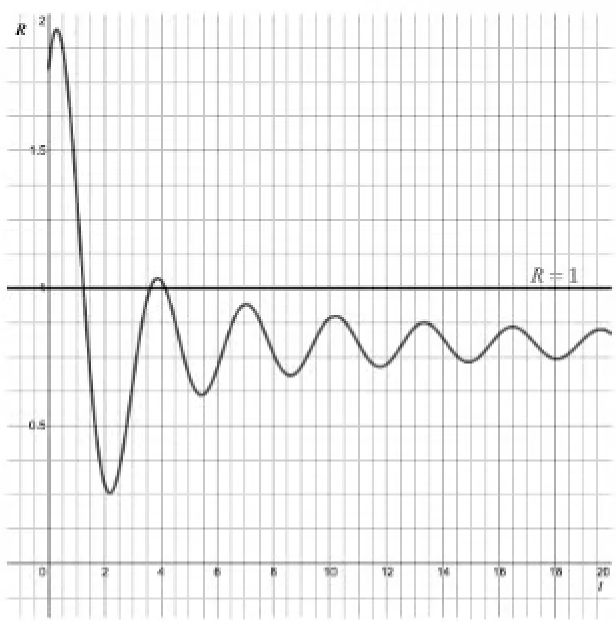
2 (a) Scientists are analysing the infection rate, R, at which a virus, Newrap-20, spreads amongst a population. R is modelled using the function

$$R(t) = 0.6 + \frac{1}{t}\sin(2t) - e^{-t} \qquad (t > 0)$$

where t is the number of months since the first case of the virus was discovered.

To control the spread of Newrap-20, scientists need to keep the value of R below 1.

The diagram below shows the graphs of $y = R(t)$ and y=1.



Using the Newton-Raphson method with a suitable initial value, find the number of months after Newrap-20 was discovered, that the value of R first drops below 1. Give your answer to three significant figures.

(4 marks)

(b) Given that the first two months both have 31 days, find the number of days (to the nearest day) for which the value of R first drops below one.

(2 marks)

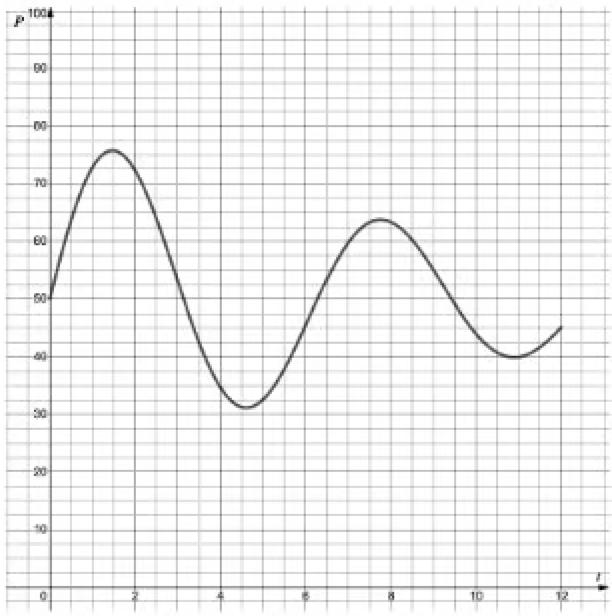
(c) In the long term, the scientists expect the value of R to remain constant.
Write down this value of R.

(1 mark)

3 (a) The energy company Power^{Y} own and maintain a wind farm.
Engineers from Power^{Y} model the power output, P kW (kiloWatts), of a single wind turbine over a twelve-hour period according to the function

$$P(t) = 50 + 30e^{-0.1t}\sin t \qquad\qquad 0 \le t \le 12$$

where t is time measured in hours. The graph of $y = P(t)$ is shown below.



Use the trapezium rule with 5 ordinates to estimate the total power generated by the wind turbine in the last four hours of the twelve-hour period.

(4 marks)

(b) Estimate the average amount of power per minute produced by the wind turbine during these last four hours.

(2 marks)

4 (a) The drilling rig AlphaBeta began leaking oil into the North Sea following a technical fault. It took 14 hours for engineers to trace and repair the fault.

During that time the rate of oil leaking, measured in tonnes per hour, was recorded every 2 hours. The results are shown in the table below.

Time	0	2	4	6	8	10	12	14
Rate of leak	0	8	12	18	26	38	18	0

For safety reasons oil rigs are required to shut down and stop all operations until an inspection is carried out should the total amount of oil leaked during any incident exceed 250 tonnes.

Use all the data in the table to decide if the AlphaBeta rig should be shut down or not.

(3 marks)

(b) Explain why using the trapezium rule to estimate the total amount of oil spilled from AlphaBeta in the first 10 hours would be an over-estimate.

(1 mark)

5 (a) The game of Logball is played on a flat table.

A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest within a winning zone.

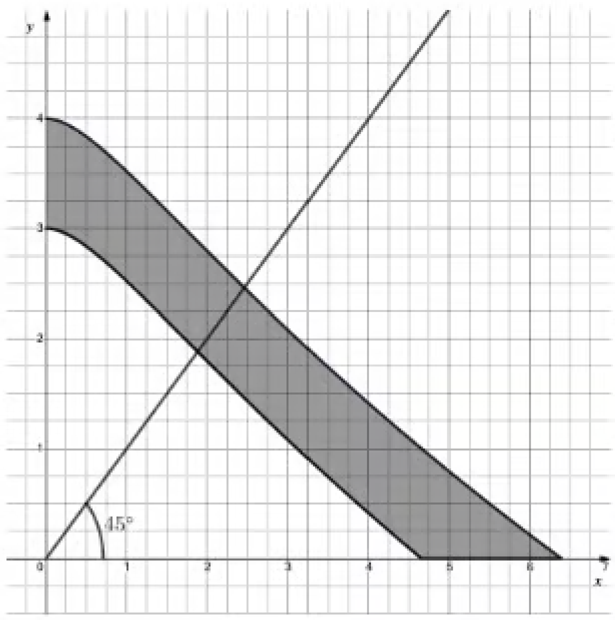
A particular player decides to roll the ball at an angle of 45°, as illustrated in the graph below, with the ball being rolled from the origin and the shaded area being the winning zone.

The lower boundary of the winning zone has equation

$y = 3 - \ln(x+1)^2 \qquad x,\ y \geq 0$

The upper boundary of the winning zone has equation

$y = 4 - \ln(x+1)^2 \qquad x,\ y \geq 0$



Using an appropriate iterative formula with initial value $x_0 = 1.8$, find the minimum distance this player's ball needs to travel to stop within the winning zone. Give your answer to two significant figures.

(3 marks)

(b) Using another iterative formula with initial value $x_0 = 2.5$, find the maximum distance this player's ball can travel yet remain within the winning zone.
Give your answer to two significant figures.

(3 marks)

6 (a) A stretch of road along Equality Street has a speed limit of 70 mph and traffic is monitored by six average speed cameras.

Ricky is driving his car along Equality Street and passes the first camera at time zero. Ricky's speed, measured in miles per hour, and the time he passes each camera, are recorded in the table below.

Camera	1	2	3	4	5	6
Time (hours)	0	0.05	0.1	0.15	0.2	0.25
Speed (mph)	68	72	69	71	70	70

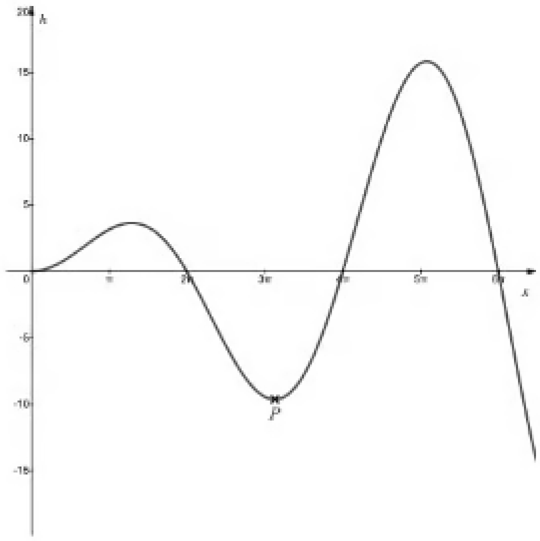
Use **all** the results above to estimate the distance between the first and last camera.

(3 marks)

(b) A driver will receive a speeding ticket if their average speed between the first and last camera exceeds the speed limit.
Should Ricky receive a speeding ticket or not? Justify your answer.

(2 marks)

7 (a) The profile of the first part of the Big Delta rollercoaster is modelled using the function $f(x) = x\sin\left(\frac{1}{2}x\right)$. The graph of $h = f(x), \quad x \geq 0,$ is shown below.



h is the height, in metres of a person sat on the rollercoaster.
$h = 0$ is the height at which passengers board the rollercoaster.

x is the horizontal distance travelled, in metres, measured from the starting point.

Explain why the Newton-Raphson method would fail should it be used to attempt to find the horizontal distance travelled when a passenger on the Big Delta is at the point marked P on the graph.

(1 mark)

(b) Show that $f'(x) = \frac{1}{2}x\cos\left(\frac{1}{2}x\right) + \sin\left(\frac{1}{2}x\right)$.

(3 marks)

(c) Use the Newton-Raphson method with initial value $x_0 = 14$ to find the horizontal distance travelled the first time Big Delta reaches a height of 10 metres above its starting point. Give your answer to three significant figures.

(3 marks)

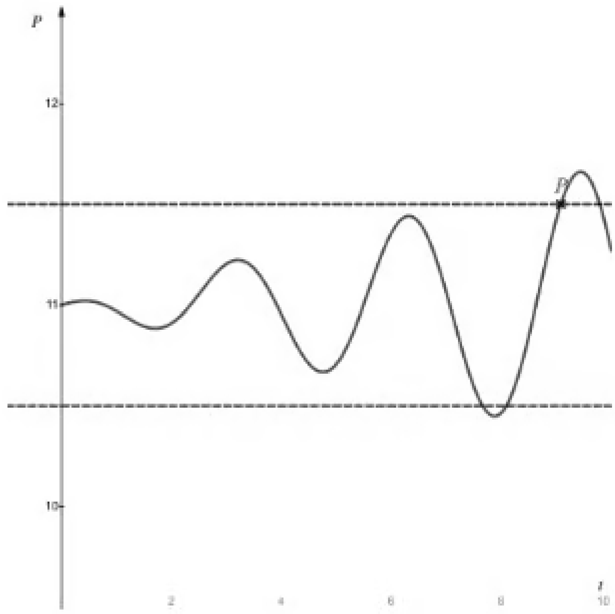
(d) Confirm your answer to part (c) is correct to three significant figures.

(2 marks)

8 (a) The air pressure inside a Flappyjet aeroplane is kept between 10.5 and 11.5 psi (pounds per square inch). A computer makes automatic adjustments according to the model

$$P = 11 + 0.07t\cos 2t \qquad\qquad t \geq 0$$

where P is the pressure (psi) and t is the time of flight, in hours, since take off.
The graph of the model is shown below.



(i) Show that the Newton-Raphson method would use the formula

$$t_{n+1} = t_n - \frac{0.5 + 0.07t_n\cos 2t_n}{0.07(\cos 2t_n - 2t_n\sin 2t_n)}$$

for finding the time at which the air pressure first drops below 10.5 psi.

(ii) Use this Newton-Raphson method with $t_0 = 7$ to find the time at which the air pressure first drops below 10.5 psi to three significant figures.

(4 marks)

(b) Human beings can cope with the air pressure being below 10.5 psi as long as it is for no longer than 30 minutes. Point P has coordinates (9.1 , 11.5).

Given that the air pressure increases to 10.5 psi at $t = 8.10$, determine whether or not the model above can be used for flights up to 9 hours long.

(3 marks)

9 (a) According to legend, unicorn tears have magical healing powers.

When a unicorn tear is applied to a bruise of size A mm^2 it will heal according to the model

$$f(t) = Ae^{-0.15t} - 0.2t \qquad t \geq 0$$

where f is the area of the bruise, in square millimetres, at time t seconds after the unicorn tear is applied.

Show that, for a bruise of initial size 20 mm^2, the equation $f(t) = 0$ can be rearranged into the form

$$t = \frac{20}{3}\ln\left(\frac{100}{t}\right).$$

(3 marks)

(b) Using the equation from part (a) as an iterative formula and initial value $t_0 = 12$, find how many seconds it takes a bruise of size 20 mm^2 to heal once a unicorn tear is applied. Give your answer to three significant figures.

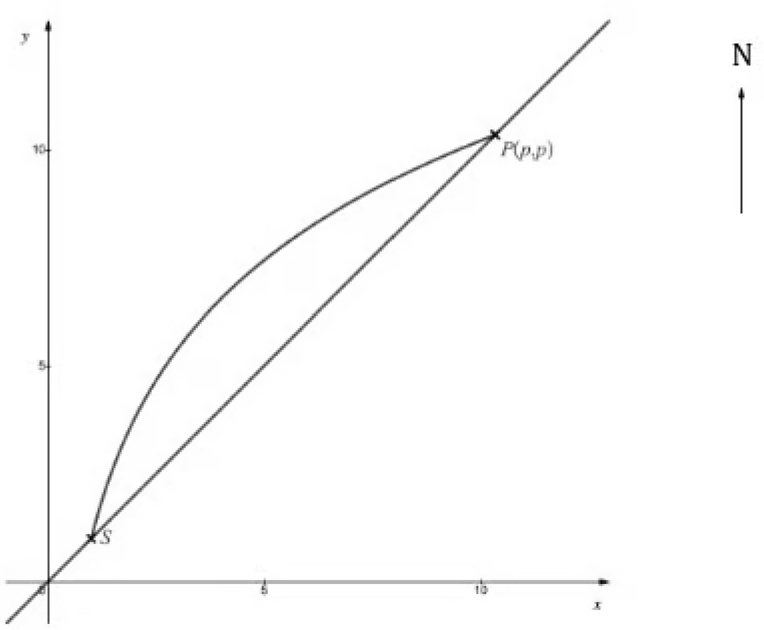
(3 marks)

(c) It is rumoured that a unicorn tear can heal bruises one-hundred-thousand times faster than they would heal naturally. Approximately how many days would it take a bruise of initial size 20 mm^2 to heal without a unicorn tear?

(2 marks)

Very Hard Questions

1 (a) The village of Camberwick Green lies on a straight road, as modelled by the line $y = x$ on the graph below. Rush hour traffic through the village causes both congestion and air pollution.To ease congestion and improve the air quality around Camberwick Green a bypass ,modelled by part of the equation $y = 1 + 2\ln\left(x^2\right)$, is to be built.



The bypass is to be built with a roundabout south of the village at point S and a northern roundabout which re-joins the road through Camberwick Green at the point $P(p, p)$.

Write down the coordinates of the southern roundabout at point S.

(1 mark)

(b) (i) Show on the diagram how an iterative method with a starting value (x_0) in the interval (1 , p) will converge to the northern roundabout at point P.

(ii) Use a suitable iterative formula with an appropriate initial value (x_0) to find the value of p to five significant figures.

(3 marks)

(c) Find the length of road through Camberwick Green that will benefit from the construction of the bypass. Decide on an appropriate unit of measurement.

(2 marks)

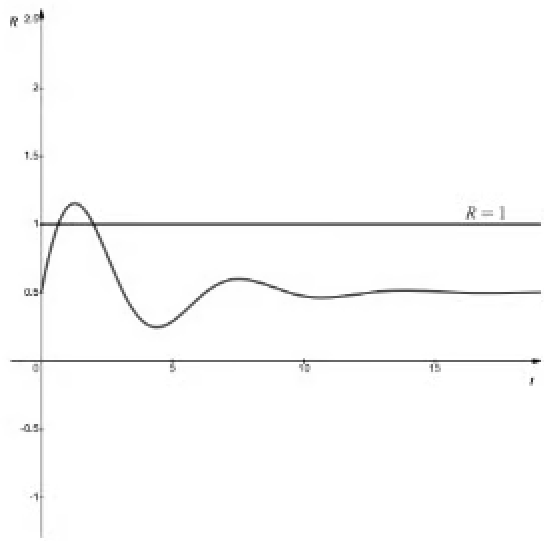
2 (a) Scientists are analysing the infection rate, R, at which a virus Divid-e20 spreads amongst a population.They model R using the function

$$R(t) = 0.5 + e^{-0.3t}\sin(t) \qquad (t > 0)$$

where t is the number of months since the first case of the virus was discovered.

To control the spread of Divid-e20 scientists need to keep the value of R below 1.

The diagram below shows the graphs of $y = R(t)$ and $R = 1$.



Use the Newton-Raphson method, with a suitable initial value, to find the number of months after Divid-e20 was discovered that R first equals 1.
Give your answer to three decimal places.

(4 marks)

(b) Given that the infection rate for Divid-e20 first went above 1 on 1st July, and drops back below 1 roughly 1.997 months (to three decimal places) after Divid-e20 was first discovered, work out the date when the infection rate drops back below 1.

(3 marks)

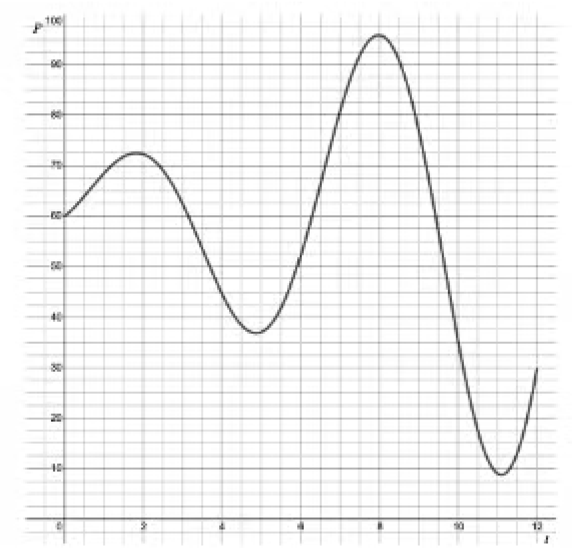
(c) In the long term, scientists expect the value of R to remain roughly constant around 0.5. Briefly explain how you can tell this from the function $R(t)$.

(1 mark)

3 (a) The energy company Power^{Z} own and maintain a wind farm.
Engineers from Power^{Z} model the power output, P kW (kiloWatts), of a single wind turbine over a twelve-hour period according to the function

$$P(t) = 60 + 5e^{0.7\sqrt{t}}\sin t \qquad\qquad 0 \leq t \leq 12$$

where t is the number of hours after 6pm. The graph of $y = P(t)$ is shown below.



Estimate the total power generated by this wind turbine between midnight and 4am.

(4 marks)

(b) Once every twelve hours, Power^{Z} turn each wind turbine off for half an hour for maintenance and safety checks.
Suggest, with a reason, at what time between 6pm and 6am the maintenance and safety checks should be carried out on the wind turbine modelled above.

(2 marks)

4 (a) The XnValdez container ship was carrying 4000 tonnes of crude oil when it ran aground and oil started leaking from its hull into the ocean.

It took 36 days for experts to stop the oil leak.

During that time the leakage rate of the oil, measured in tonnes per day, was recorded every 4 days. The results are shown in the table below.

Time	0	4	8	12	16	20	24	28	32	36
Rate of leak	0	10	20	40	70	110	160	230	150	0

An environmental disaster is declared if the total amount of crude oil that leaks into the ocean exceeds 2500 tonnes.

Use **all** the data in the table to decide if environmentalists were right to declare the XnValdez incident an environmental disaster or not.

(4 marks)

(b) Use the data in the table to find a lower bound and an upper bound estimate to the total amount of crude oil leaked by the XnValdez.

(3 marks)

5 The game of Funcball is played on a flat table. A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest within a winning zone.

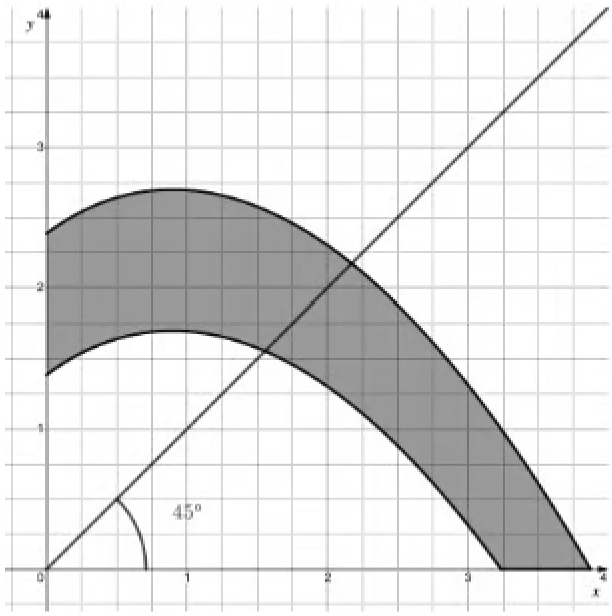
The winning zone is modelled by the function

$$f(x) = \ln(3x + 4) - 0.25x^2.$$

The lower boundary of the winning zone has equation $y = f(x), \quad x, y \geq 0$

The upper boundary of the winning zone has equation $y = f(x) + 1, \; x, y \geq 0$

A particular player decides to roll the ball at an angle of 45°, as illustrated by the graph below, with the ball being rolled from the origin and the shaded area being the winning zone.



Using iterative formulas with initial values $x_0 = 1.5$ and $x_0 = 2.1$ as appropriate, find the exact distances between which the ball must stop for this player to win Funcball.
Give your answers to two significant figures.

(6 marks)

6 (a) A stretch of road along Baker Street has a speed limit of 60 mph and traffic is monitored by eight average speed cameras.

Sherlock is driving along Baker Street and passes the first camera at time zero. Sherlock's speed, measured in miles per hour, and the time he passes each camera, are recorded in the table below.

Camera	1	2	3	4	5	6	7	8
Time (minutes)	0	4	8	12	16	20	54	n/a
Speed (mph)	64	59	61	62	57	58	60	n/a

(i) Suggest a reason why the last camera did not record any data for Sherlock's journey.

(ii) Suggest a reason why there was a longer time gap between the sixth and seventh camera.

(2 marks)

(b) Use the results from the table to estimate the distance between the first and sixth camera.

(3 marks)

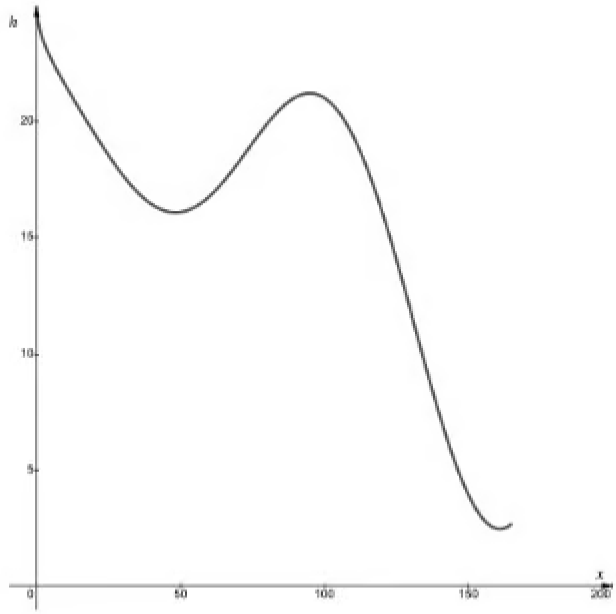
(c) Briefly explain why it would not be suitable to use the trapezium rule for estimating the distance between the first and seventh camera.

(2 marks)

7 **(a)** The top of the slope at the No-Sno Dry Ski Centre is at a height of 25 metres above sea level. The profile of the slope, shown in the graph below, is modelled by the function

$$h(x) = 25 - \sqrt{x} - \frac{1}{16}x\sin\left(\frac{x}{20}\right) \qquad 0 \le x \le 160$$

where x is the horizontal distance travelled, in metres, by a skier using the slope.



On the graph mark any points where the Newton-Raphson method would fail.

(1 mark)

(b) Use the Newton-Raphson method, with a suitable initial value, to find the horizontal distance travelled by a skier when they are at a height of 18 m above sea level and travelling uphill.

(5 marks)

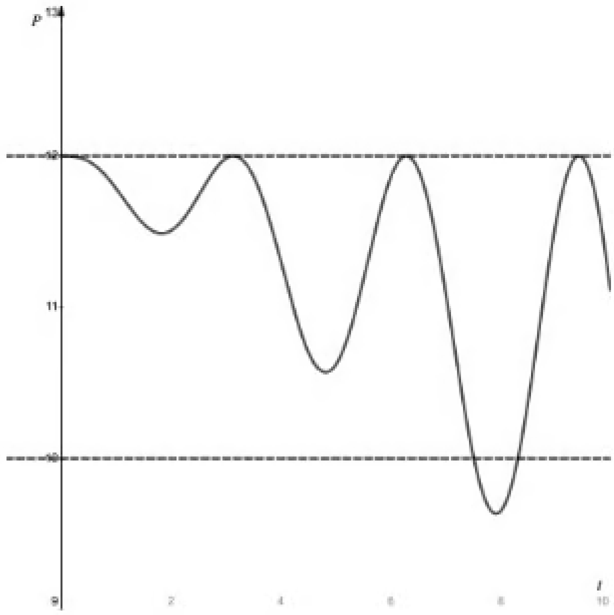
(c) Briefly describe how you could verify your answer to part (b) is correct to a desired degree of accuracy.

(2 marks)

8 (a) The air pressure inside a PiPlane aircraft is kept between 10 and 12 psi (pounds per square inch). A computer adjusts the air pressure automatically, according to the model

$$P = 12 - 0.3t\sin^2 t \qquad\qquad t \geq 0$$

where P is the pressure (psi) and t is the time in hours since take off, as shown in the graph below.



Briefly explain why this model will never lead to an air pressure above 12 psi.

(2 marks)

(b) Use the Newton-Raphson method, with initial value $t_0 = 7.5$, to find the time at which the air pressure first drops below 10 psi, give your answer to three significant figures.

(4 marks)

(c) Human beings can tolerate air pressure below 10 psi for up to an hour, after which there can be an adverse effect on their comfort and behaviour. In the confines of an aircraft this is undesirable for all PiPlane passengers and crew.

Use the Newton-Raphson method again with $t_0 = 8.5$ to find the next time the air pressure is 10 psi and hence show that the model is suitable for flights of at least 10 hours.

(3 marks)

9 (a) According to legend, unicorn tears have magical healing powers.

Without unicorn tears, a bruise of initial size $A\ \mathrm{mm}^2$ will heal according to the model

$$f(t) = Ae^{-0.25t} - 0.1t \qquad t \geq 0$$

where f is the area of the bruise, in square millimetres, at time t days since the bruise first appeared.

Use the iterative formula

$$t_{n+1} = 4\ln\left(\frac{120}{t_n}\right)$$

with $t_0 = 10$ to find how many days it takes a bruise to heal without unicorn tears. Give your answer to three significant figures.

(3 marks)

(b) With unicorn tears, a bruise of the same initial size will heal according to the model

$$u(T) = ATe^{-T} - 0.1T \qquad T \geq 0$$

where time T is measured in seconds.

(i) Find the initial size of the bruise considered in part (a).

(ii) Find how many seconds it takes the same size bruise to heal using unicorn tears.

(3 marks)

(c) Using your answers from parts (a) and (b), work out approximately how many times quicker a bruise heals when unicorn tears are used.

(1 mark)