Vectors - Year 1

A

Whereas a **coordinate** represents a **position** in space, a **vector** represents a **displacement** in space.

A vector has 2 properties:

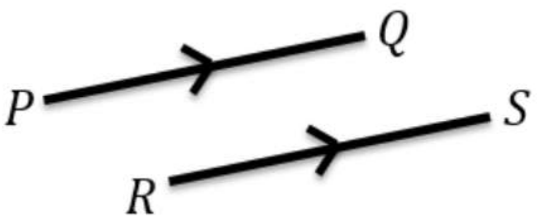
- <u>Direction</u>
- <u>Magnitude</u> (i.e. length)

If P and Q are points then $\overrightarrow{PQ}$ is the vector between them.

B

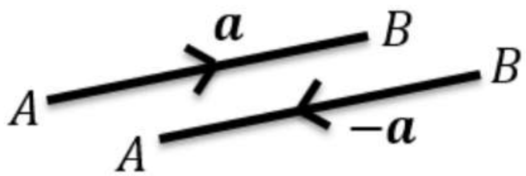
If two vectors $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have the same magnitude and direction, **they're the same vector** and are **parallel**.

This might seem obvious, but students sometimes think the vector is different because the movement occurred at a different point in space. Nope!



C

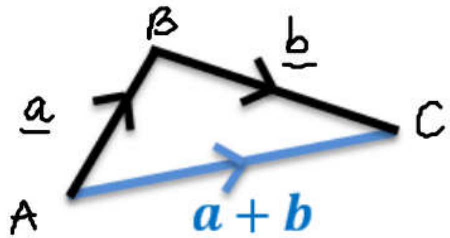
$\overrightarrow{AB} = -\overrightarrow{BA}$ and the two vectors are parallel, equal in magnitude but in **opposite directions**.



D

Triangle Law for vector addition:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

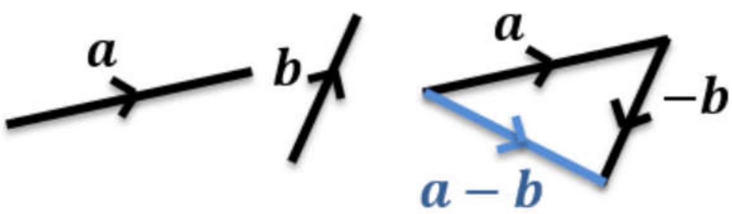


The vector of multiple vectors is known as the **resultant vector.**

(you will encounter this term in Mechanics)

E Vector **subtraction** is defined using vector addition and negation:

$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$$



F The zero vector $\mathbf{0}$ (a bold 0), represents no movement.

$$\overrightarrow{PQ} + \overrightarrow{QP} = \mathbf{0}$$

In 2D: $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

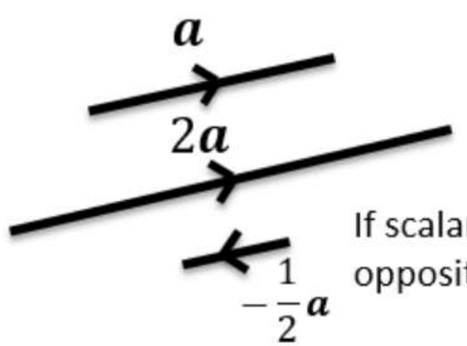
Q ×

P ×

$3\underline{a} + 6\underline{b} = \frac{3}{2}(2\underline{a} + 4\underline{b})$

G A **scalar** is a normal number, which can be used to 'scale' a vector.

- The **direction** will be the **same**.
- But the **magnitude** will be **different** (unless the scalar is 1).



If scalar is negative, opposite direction.

H Any vector parallel to the vector $\boldsymbol{a}$ can be written as $\lambda\boldsymbol{a}$, where λ is a scalar.

The implication is that if we can write one vector **as a multiple of** another, then we can show they are parallel.

"Show $2\boldsymbol{a} + 4\boldsymbol{b}$ and $3\boldsymbol{a} + 6\boldsymbol{b}$ are parallel".

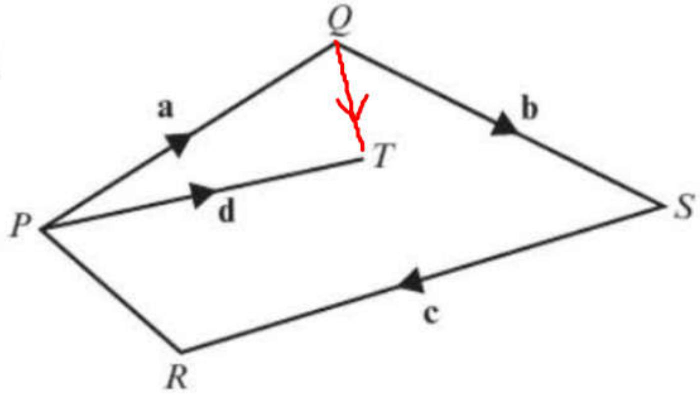
$3\boldsymbol{a} + 6\boldsymbol{b} =$ 3 [redacted] $(\boldsymbol{a} + 2\boldsymbol{b})$ ∴ parallel

$2\underline{a} + 4\underline{b} = 2(\underline{a} + 2\underline{b})$

In the diagram, $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QS} = \mathbf{b}$, $\overrightarrow{SR} = \mathbf{c}$ and $\overrightarrow{PT} = \mathbf{d}$.
Find in terms of **a**, **b**, **c** and **d**:

a $\overrightarrow{QT}$ **b** $\overrightarrow{PR}$

c $\overrightarrow{TS}$ **d** $\overrightarrow{TR}$



a) $\overrightarrow{QT} = \overrightarrow{QP} + \overrightarrow{PT}$

$= -\underline{a} + \underline{d}$

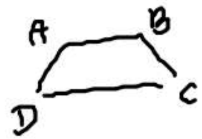
$= \underline{d} - \underline{a}$

b) $\overrightarrow{PR} = \underline{a} + \underline{b} + \underline{c}$

c) $\overrightarrow{TS} = -\underline{d} + \underline{a} + \underline{b} = \underline{a} + \underline{b} - \underline{d}$

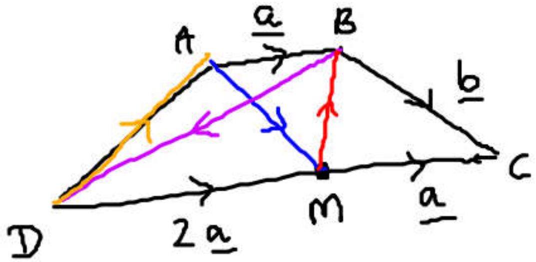
d) $\overrightarrow{TR} = -\underline{d} + \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} - \underline{d}$

ABCD is a trapezium with *AB* parallel to *DC* and $DC = 3AB$.
M divides *DC* such that $DM : MC = 2 : 1$. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.



Find, in terms of **a** and **b**:

a $\overrightarrow{AM}$ **b** $\overrightarrow{BD}$ **c** $\overrightarrow{MB}$ **d** $\overrightarrow{DA}$



a) $\overrightarrow{AM} = \underline{a} + \underline{b} - \underline{a}$

$= \underline{b}$

c) $\overrightarrow{MB} = \underline{a} - \underline{b}$

d) $\overrightarrow{DA} = 2\underline{a} - \underline{b}$

b) $\overrightarrow{BD} = \underline{b} - 3\underline{a}$

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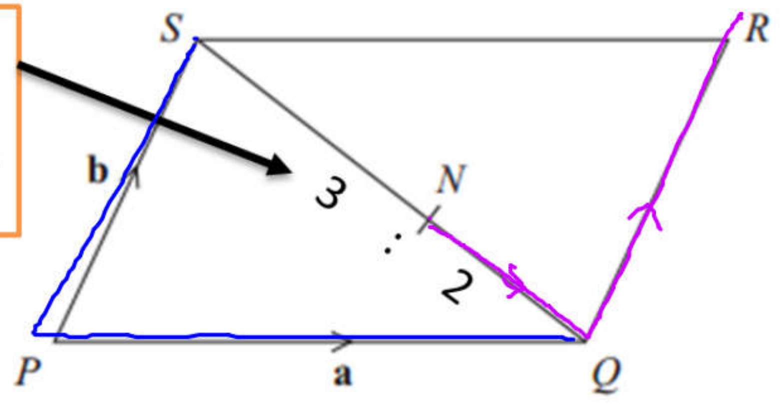
Tip: This ratio wasn't in the original diagram. I like to add the ratio as a visual aid.



$PQRS$ is a parallelogram.
N is the point on SQ such that $SN : NQ = 3 : 2$

$\overrightarrow{PQ} = \mathbf{a}$ $\quad \overrightarrow{PS} = \mathbf{b}$

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

$$\overrightarrow{SQ} = -\underline{b} + \underline{a}$$
$$= \underline{a} - \underline{b}$$

$$\overrightarrow{NR} = \overrightarrow{NQ} + \overrightarrow{QR}$$
$$= \frac{2}{5}\overrightarrow{SQ} + \overrightarrow{QR}$$
$$= \frac{2}{5}(\underline{a} - \underline{b}) + \underline{b}$$
$$= \frac{2}{5}\underline{a} - \frac{2}{5}\underline{b} + \underline{b}$$
$$= \frac{2}{5}\underline{a} + \frac{3}{5}b$$
$$= \frac{1}{5}(2\underline{a} + 3\underline{b})$$

Your Turn

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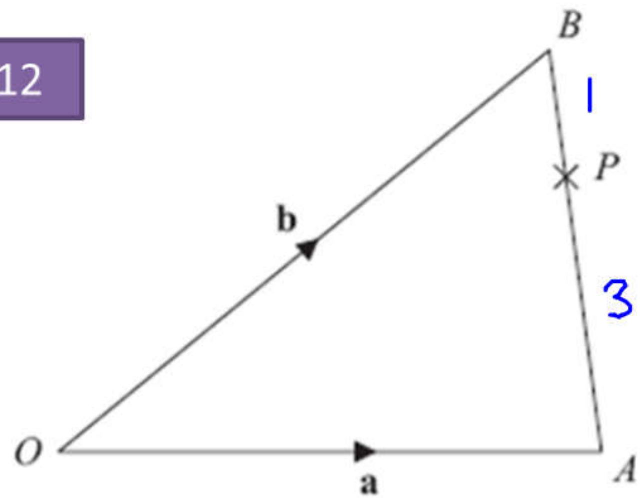

Diagram **NOT** accurately drawn

$\underline{a} - 5\underline{b}$

$-2\underline{a} + 10\underline{b}$

$-2\underline{a} + 10\underline{b} = -2(\underline{a} - 5\underline{b})$

parallel.

Is $2\underline{a} + 3\underline{b}$ parallel to $6\underline{a} + 8\underline{b}$

$6\underline{a} + 8\underline{b} \neq 3(2\underline{a} + 3\underline{b})$

OAB is a triangle.

$\overline{OA} = \mathbf{a}$

$\overline{OB} = \mathbf{b}$

(a) Find $\overline{AB}$ in terms of **a** and **b**.

$\underline{b} - \underline{a}$

....

(1)

P is the point on *AB* such that *AP* : *PB* = 3 : 1

(b) Find $\overline{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\vec{OP} = \vec{OA} + \frac{3}{4}\vec{AB}$

$= \underline{a} + \frac{3}{4}(\underline{b} - \underline{a})$

$= \underline{a} - \frac{3}{4}\underline{a} + \frac{3}{4}\underline{b} = \frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$

$= \frac{1}{4}(\underline{a} + 3\underline{b}).$

Ex 11A
Q7-11