# Using simultaneous equations

**14** Two particles $P$ and $Q$ are moving along the same straight horizontal line with constant accelerations $2\,\text{m s}^{-2}$ and $3.6\,\text{m s}^{-2}$ respectively. At time $t = 0$, $P$ passes through a point $A$ with speed $4\,\text{m s}^{-1}$. One second later $Q$ passes through $A$ with speed $3\,\text{m s}^{-1}$, moving in the same direction as $P$.

**a** Write down expressions for the displacements of $P$ and $Q$ from $A$, in terms of $t$, where $t$ seconds is the time after $P$ has passed through $A$. **(2 marks)**

**b** Find the value of $t$ where the particles meet. **(3 marks)**

**c** Find the distance of $A$ from the point where the particles meet. **(3 marks)**

**Problem-solving**

When $P$ and $Q$ meet, their displacements from $A$ are equal.

a)

P

$a = 2$

$u = 4$

"$t$" $= t$

$s = ?$

Q

$a = 3.6$

$u = 3$

"$t$" $= t - 1$

$s = ?$

$s = ut + \frac{1}{2}at^2$

P $s = 4t + t^2$

Q $s = 3(t-1) + 1.8(t-1)^2$

b) when they meet, displacement P = displacement Q.

$4t + t^2 = 3(t-1) + 1.8(t-1)^2$

$4t + t^2 = 3t - 3 + 1.8(t^2 - 2t + 1)$

$4t + t^2 = 3t - 3 + 1.8t^2 - 3.6t + 1.8$

$0 = 0.8t^2 - 4.6t - 1.2$

$t = -0.25,\ t = 6$

$t > 0$, so $t = 6$ secs.

c) $t = 6$

$s = 4t + t^2$

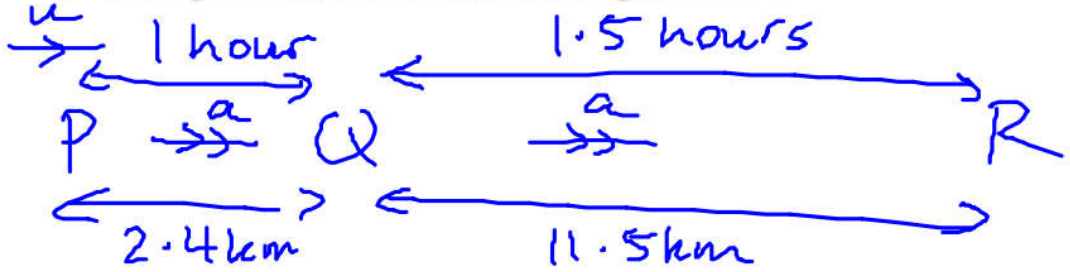
$= 4 \times 6 + 6^2$

$= 60\text{m}$

**15** In an orienteering competition, a competitor moves in a straight line past three checkpoints, $P$, $Q$ and $R$, where $PQ = 2.4$ km and $QR = 11.5$ km. The competitor is modelled as a particle moving with constant acceleration. She takes 1 hour to travel from $P$ to $Q$ and 1.5 hours to travel from $Q$ to $R$. Find:

**a** the acceleration of the competitor

**b** her speed at the instant she passes $P$. **(7 marks)**




$\overrightarrow{PQ}$

$t = 1$

$s = 2.4$

$a = a$

$u = u$

$\overrightarrow{PR}$

$t = 2.5$

$s = 13.9$

$a = a$

$u = u$

$s = ut + \frac{1}{2}at^2$

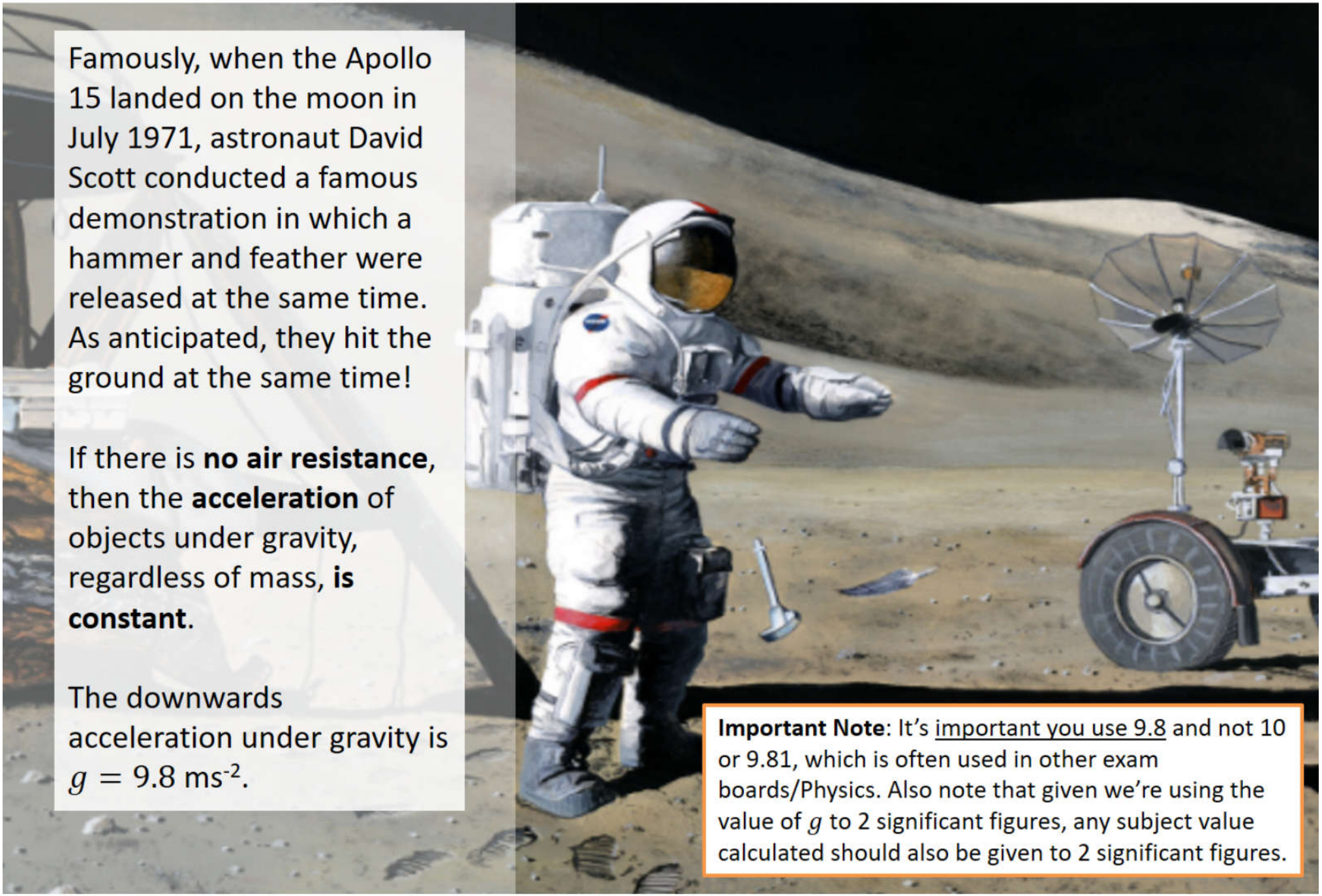
$\overrightarrow{PQ}$ $\quad 2.4 = u + \frac{1}{2}a$

$\overrightarrow{PR}$ $\quad 13.9 = 2.5u + 3.125a$

$u = 0.293 \text{ km h}^{-1}$

$a = 4.21 \text{ km h}^{-2}$

# Vertical Motion Under Gravity



Famously, when the Apollo 15 landed on the moon in July 1971, astronaut David Scott conducted a famous demonstration in which a hammer and feather were released at the same time. As anticipated, they hit the ground at the same time!

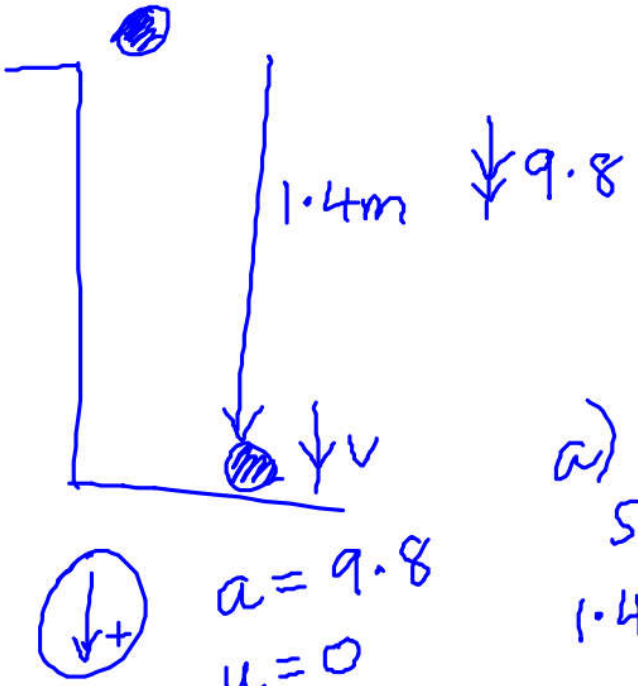
If there is **no air resistance**, then the **acceleration** of objects under gravity, regardless of mass, **is constant**.

The downwards acceleration under gravity is $g = 9.8\ \text{ms}^{-2}$.

**Important Note**: It's important you use 9.8 and not 10 or 9.81, which is often used in other exam boards/Physics. Also note that given we're using the value of $g$ to 2 significant figures, any subject value calculated should also be given to 2 significant figures.

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor.
Find:
(a) the time the book takes to reach the floor,
(b) the speed with which the book strikes the floor.

It's VERY important you consider **what direction is considered as 'positive'**, and mark it next to your suvat values. If 'up' was positive, then $a = -9.8$. If 'down' is positive, then $a = +9.8$. Which way you pick does not matter provided that you are consistent with each letter of SUVAT.




$a = 9.8$
$u = 0$
$s = 1.4$
$t = ?$
$v = ?$

a)
$s = ut + \frac{1}{2}at^2$
$1.4 = \frac{1}{2} \times 9.8 \times t^2$
$1.4 = 4.9t^2$
$\sqrt{\frac{1.4}{4.9}} = t$
$t = 0.53$ secs (2sf)

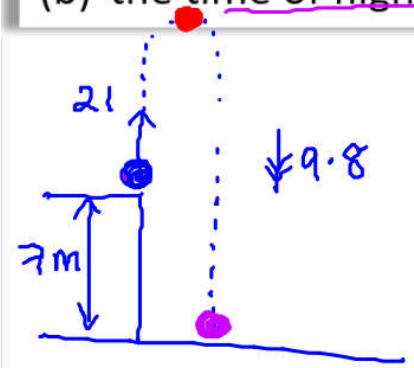
b) $v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 9.8 \times 1.4$
$v = 5.2\ ms^{-1}$ (2sf)

As per previous slide, quote only to 2 significant figures. You may be penalised if you quote more!

A ball is projected vertically upwards, from a point $X$ which is 7m above the ground, with speed 21 $ms^{-1}$. Find

(a) the greatest height above the ground reached by the ball,

(b) the time of flight of the ball




a) ↓+

$u = -21$

$a = 9.8$

$v = 0$

$s = ?$

$v^2 = u^2 + 2as$

$0 = (-21)^2 + 2 \times 9.8 \times s$

$0 = 441 + 19.6s$

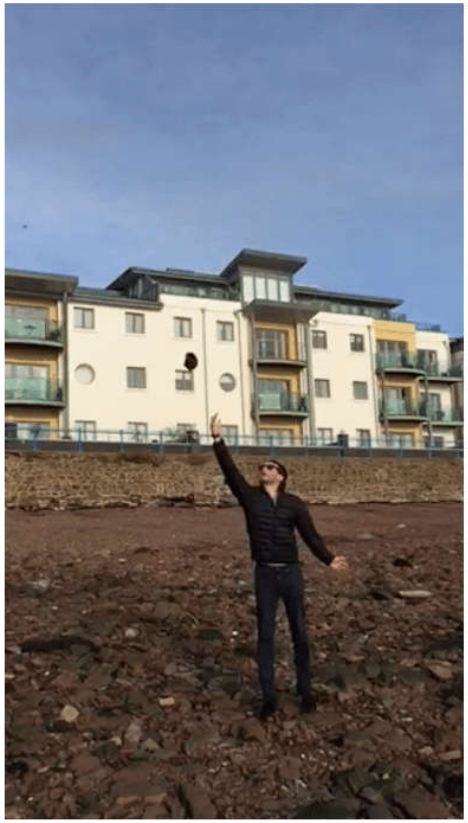
$s = -22.5\text{m}$

Distance above ground $= 7 + 22.5$

$= 29.5\text{m}$

$= 30\text{m}$ (2sf)

At maximum height, speed is 0



b) ↓+

$a = 9.8$

$u = -21$

$s = 7$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$7 = -21t + 4.9t^2$

$0 = 4.9t^2 - 21t - 7$

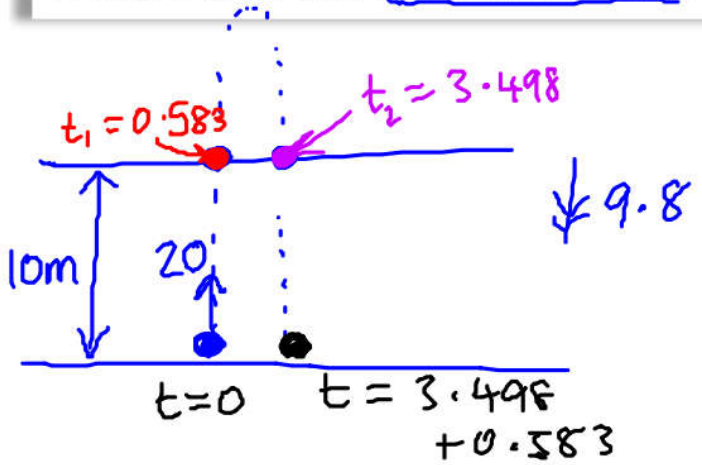
$t = 4.6$ (2sf) $t = -0.31$ (2sf)

$t > 0$, $t = 4.6$ secs

Displacement is a vector – it has direction!

It is the distance from its start point to its end point. Imagine taking a photo of it at the start of the 'journey' and then at the end. Compare them to find the displacement

A ball is projected vertically upwards from ground level at a speed of 20 $ms^{-1}$.
Determine the amount of time the ball is at least 10m above ground level.




How long 'til it comes back to the ground?

$u = 20$ (↑+)
$s = 0$
$a = -9.8$
$t = ?$ $s = ut + \frac{1}{2}at^2$

$0 = 20t - 4.9t^2$

$t = 0$ $t = 4.081\ldots$

$3.498 + 0.583$

(↑+)

$s = 10$
$u = 20$
$a = -9.8$
$t = ?$

$s = ut + \frac{1}{2}at^2$

$10 = 20t - 4.9t^2$

$0 = -4.9t^2 + 20t - 10$

$t_2 = 3.498$ $t_1 = 0.583$

time above 10m is $t_2 - t_1 = 2.915\ldots$
$= 2.9$ secs (2sf).

Ex 9E
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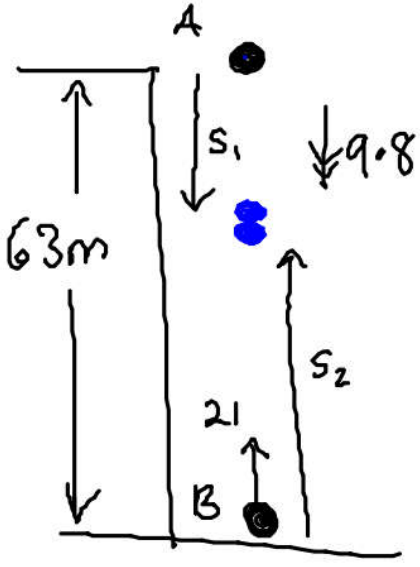
# Using simultaneous equations

A ball *A* falls vertically from rest from the top of a tower 63m high. At the same time as *A* begins to fall, another ball *B* is projected vertically upwards from the bottom of the tower with speed 21 $ms^{-1}$. The balls collide. Same time, same place
Find the distance of the point where the balls collide from the bottom of the tower.




dist A + dist B = 63 $\longrightarrow$ $s_1 + s_2 = 63$

A (↓+)

$a = 9.8$
$u = 0$
$t = t$
$s = s_1$
$s = ut + \frac{1}{2}at^2$
$s_1 = 4.9t^2$

B (↑+)

$a = -9.8$
$u = 21$
$t = t$
$s = s_2$
$s_2 = 21t - 4.9t^2$

$4.9t^2 + 21t - 4.9t^2 = 63$
$21t = 63$
$t = 3$

We want $s_2$
$s_2 = 21 \times 3 - 4.9 \times 3^2$
$= 18.9m$
$= 19m$ (2sf)

Try Q12

# Edexcel M1 May 2013(R) Q4

At time $t = 0$, two balls $A$ and $B$ are projected vertically upwards. The ball $A$ is projected vertically upwards with speed 2 m $s^{-1}$ from a point 50 m above the horizontal ground. The ball $B$ is projected vertically upwards from the ground with speed 20 m $s^{-1}$. At time $t = T$ seconds, the two balls are at the same vertical height, $h$ metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of $T$,

**(5)**

(b) the value of $h$.

**(2)**