A Level · Edexcel · Maths

3 hours · 32 questions

7.3 Further Differentiation (A Level only)

Easy (8 questions)	/44
Medium (8 questions)	/47
Hard (8 questions)	/40
Very Hard (8 questions)	/45
Total Marks	**/176**

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Easy Questions

1 (a) Given that $f(x) = x^2$

Use differentiation from first principles to show that

$$f'(x) = \lim_{h \to 0}\left(\frac{x^2 + 2hx + h^2 - x^2}{h}\right).$$

(2 marks)

(b) Hence prove that

$$f'(x) = 2x.$$

(3 marks)

2 (a) A curve has the equation $y = 5e^{-2x}$.

Find an expression for $\frac{dy}{dx}$.

(2 marks)

(b) (i) Find the gradient of the tangent at the point where $x = 1$, giving your answer in the form $-ae^{-2}$ where a is a positive integer to be found.

(ii) Hence show that the gradient of the normal to the curve at the point where $x = 1$ is $\frac{1}{10}e^2$.

(3 marks)

3 Find $\frac{dy}{dx}$ for

(i) $y = \sin(3x^2)$,
(ii) $y = 2\ln(x^3)$.

(4 marks)

4 The curve with equation $y = e^{x^2 - 9}$ passes through the point with coordinates (-3 , 1).

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Find the equation of the tangent to the curve at the point (-3 , 1).

(4 marks)

5 (a) Differentiate $(x^3 - 2x)\ln x$ with respect to x.

(3 marks)

(b) Differentiate $e^x \cos 2x$ with respect to x.

(3 marks)

6 (a) Differentiate $\dfrac{\cos x}{\sin x}$ with respect to x

(3 marks)

(b) Differentiate $\dfrac{2x^2 - 3x + 4}{\sin 3x}$ with respect to x.

(3 marks)

7 Write down $\dfrac{dy}{dx}$ when

(i) $y = \sec 5x$,

(ii) $y = \operatorname{cosec} 3x$.

(2 marks)

8 (a) The function $f(x)$ is defined as

$$f(x) = (x^2 - 4x + 4)\ln(x), \qquad x > 0$$

Show that the graph of $y = f(x)$ intercepts the x-axis at the points (1 , 0) and (2 , 0).

(4 marks)

(b) Find $f'(x)$.

(4 marks)

(c) Find the gradient of the tangent at the point (1 , 0).

(2 marks)

(d) Hence find the equation of the tangent at the point (1 , 0), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(2 marks)

Medium Questions

1 (a) Given that $f(x) = \sin x$

Show that

$$f'(x) = \lim_{h \to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right)$$

(4 marks)

(b) Hence prove that $f'(x) = \cos x$.

(3 marks)

2 A curve has the equation $y = e^{-3x} + \ln x\ ,\ \ x > 0.$

Find the gradient of the normal to the curve at the point $(1,\ e^{-3})$, giving your answer correct to 3 decimal places.

(4 marks)

3 (a) Find $\frac{dy}{dx}$ for each of the following:

$$y = \cos(x^2 - 3x + 7) + \sin(e^x)$$

(4 marks)

(b) Find $\frac{dy}{dx}$ for each of the following:

$$y = \ln(2x^3)$$

(3 marks)

4 Find the equation of the tangent to the curve $y = e^{3x^2 + 5x - 2}$ at the point $(-2,\ 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4 marks)

5 (a) Differentiate with respect to x, simplifying your answers as far as possible:

$(4\cos x - 3\sin x)\, e^{3x-5}$

(3 marks)

(b) $(x^3 - 4x^2 + 7)\ln x$

(3 marks)

6 Differentiate $\dfrac{5x^7}{\sin 2x}$ with respect to x.

(4 marks)

7 (a) Show that if $y = \text{cosec}2x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\text{cosec}\,2x\cot 2x$$

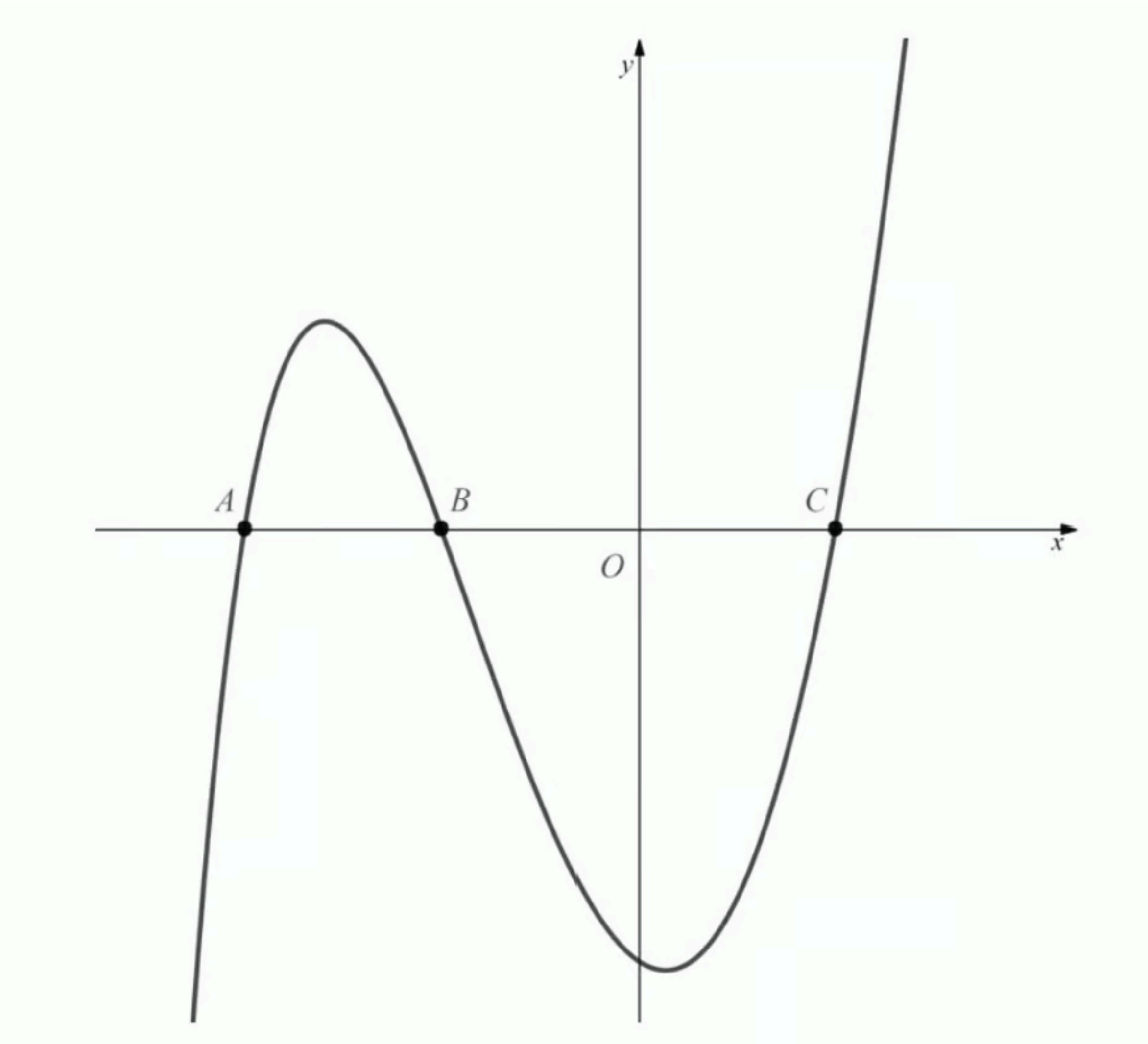
(5 marks)

(b) Hence find the gradient of the tangent to the curve $y = \text{cosec}\,2x$ at the point with coordinates $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

(1 mark)

8 (a) The diagram below shows part of the graph of $y = f(x)$, where $f(x)$ is the function defined by

$$f(x) = (x^2 - 1)\ln(x + 3), \quad x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

Find $f'(x)$.

(4 marks)

(b) Show that the coordinates of point A are (-2, 0).

(2 marks)

(c) Find the equation of the tangent to the curve at point A.

(3 marks)

Hard Questions

1 Show from first principles that the derivative of $\cos x$ is $-\sin x$.

(7 marks)

2 A curve has the equation $y = e^{-3x} + \ln x$, $x > 0$.

Show that the equation of the tangent to the curve at the point with x-coordinate 1 is

$$y = \left(\frac{e^3 - 3}{e^3}\right)x + \frac{4 - e^3}{e^3}$$

(6 marks)

3 For $y = \ln(ax^n)$, where $a > 0$ is a real number and $n \geq 1$ is an integer, show that

$$\frac{dy}{dx} = \frac{n}{x}$$

(3 marks)

4 Find the gradient of the normal to the curve $y = 5\cos\left(e^x - \frac{\pi}{2}\right)$ at the point with x-coordinate 0. Give your answer correct to 3 decimal places.

(4 marks)

5 (a) Differentiate with respect to x, simplifying your answers as far as possible:

$(2\sin 3x - \cos 3x)\, e^{6-x}$

(3 marks)

(b) $(x^2 - x)^2 \ln 5x$

(3 marks)

6 By writing $y = \dfrac{f(x)}{g(x)}$ as $y = f(x)[g(x)]^{-1}$ and then using the product and chain rules, show that

$$\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

(3 marks)

7 (a) Given that $x = \sec 7y$,

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y

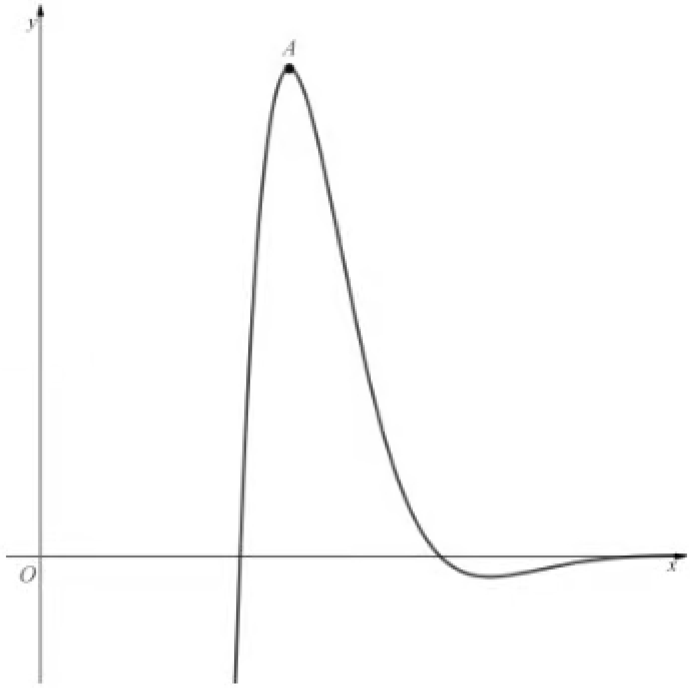
(2 marks)

(b) Hence find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

(4 marks)

8 The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = \frac{\sin x}{1 - e^x}, \quad x > 0$$



Point A is a maximum point on the graph.

Show that the x-coordinate of A is a solution to the equation

$$\frac{\cos x + e^{x}(\sin x - \cos x)}{e^{2x} - 2e^{x} + 1} = 0$$

(5 marks)

Very Hard Questions

1 Show from first principles that the derivative of $\tan 3x$ is $3\sec^2 3x$.

(9 marks)

2 A curve has the equation $y = 3^x + 2^{-x}$.

Show that the gradient of the normal to the curve at the point $\left(1, \frac{7}{2}\right)$ is

$$\frac{2}{\ln 2 - 6\ln 3}$$

(4 marks)

3 Find the derivative of the function $\mathrm{f}(x)=\sin\left(\cos\left(\ln\frac{1}{x}\right)\right),\ x>0.$

(4 marks)

4 (a) Show that the derivative $y = 4^{-x^4}$ is

$$\frac{dy}{dx} = -(\ln 4)\, x^3 4^{1-x^4}$$

(4 marks)

(b) Hence find the equation of the tangent to the curve at the point $\left(1, \frac{1}{4}\right)$, giving your answer in the form $y = ax + b$, where a and b are to be given as exact values.

(2 marks)

5 (a) Differentiate with respect to x, simplifying your answers where possible:

$$(5+\sin^2 3x)\, e^{x^2-3x+2}$$

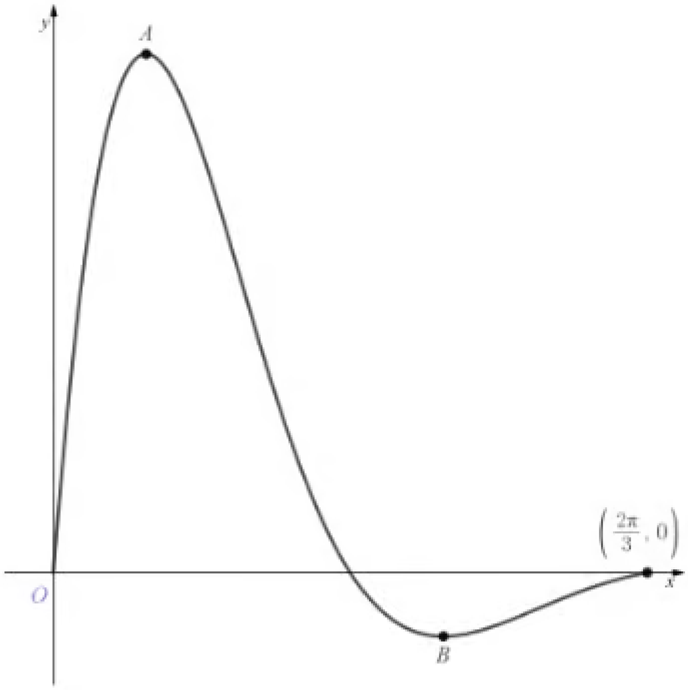
(3 marks)

(b) $3\sqrt{x}\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)$

(3 marks)

6 The diagram below shows the graph of $y=\mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x)=\frac{\sin 3x}{e^{2x-3}}, \qquad 0 \le x \le \frac{2\pi}{3}$$



The points A and B are maximum and minimum points, respectively.

Find the range of $\mathrm{f}(x)$, giving your answer correct to 3 decimal places.

(6 marks)

7 A is the point on the graph of $y = \arctan x$ such that the tangent to the graph at A passes through the point $\left(0, \frac{1}{2}\right)$. Show that the x-coordinate of A satisfies the equation

$$x - \tan\left(\frac{(1+x)^2}{2(1+x^2)}\right) = 0$$

(5 marks)

8 A sequence of functions is defined by the recurrence relation

$$u_{k+1}(x) = \frac{\mathrm{d}}{\mathrm{d}x} u_k(x), \quad u_1(x) = \sin\left(x\sqrt{2}\right)$$

Based on that sequence, the function $\mathrm{f}_n(x)$ is defined by

$$\mathrm{f}_n(x) = \sum_{r=1}^{n} u_r(x)$$

Calculate the value of $\mathrm{f}_{41}\left(\frac{\pi\sqrt{2}}{4}\right)$

(5 marks)