A Level · Edexcel · Maths

3 hours 32 questions

4.5 Sequences & Series (A Level only)

Easy (8 questions)	/31
Medium (8 questions)	/44
Hard (8 questions)	/45
Very Hard (8 questions)	/46
Total Marks	**/166**

Scan here to return to the course
or visit savemyexams.com

Easy Questions

1 (a) Calculate

$$\sum_{r=1}^{5} 2r+1$$

(2 marks)

(b) The sum given in part (a) is an arithmetic series.
Write down the first term and the common difference.

(2 marks)

2 (a) Calculate

$$\sum_{r=1}^{3} 2(3)^r$$

(2 marks)

(b) The sum given in part (a) is a geometric series.
Write down the first term and the common ratio.

(2 marks)

3 (a) It is given that

$$\sum_{r=1}^{4} a(r+2) = 72$$

where a is a positive integer.

(i) Show that $18a = 72$.

(ii) Find the value of a.

(3 marks)

(b) Determine if the series is arithmetic or geometric, justifying your answer.

(1 mark)

4 (a) A Fibonacci sequence can be expressed as the following recurrence relation

$$u_{n+2} = u_{n+1} + u_n, \qquad n \geq 1$$

Write down the first six terms of the Fibonacci sequence with $u_1 = u_2 = 1$.

(2 marks)

(b) Find

$$\sum_{r=1}^{5} u_r$$

with $u_1 = 2,\ u_2 = 4$

(2 marks)

5 (a) A sequence is defined by the recurrence relation $u_{n+1} = 2u_n, \quad u_1 = 5, \quad n \geq 1.$.

Write down the first five terms of the sequence.

(2 marks)

(b) Determine if the sequence is arithmetic or geometric, justifying your answer.

(1 mark)

(c) Find

$$\sum_{r=1}^{5} 2u_n$$

(2 marks)

6 (a) The n^{th} term of an arithmetic series is given by $u_n = 3n + 5$.

Write the sum of the series, up to the n^{th} term, in sigma notation.

(2 marks)

(b) The n^{th} term of a geometric series is given by $u_n = 5 \times 2^{n-1}$.

Write the sum of the series, up to the n^{th} term, in sigma notation.

(2 marks)

7 Given that

$$\sum_{r=1}^{k} r^2 = 55$$

determine the value of k.

(2 marks)

8 (a) A sequence is defined for $n \geq 1$ by the recurrence relation $u_{n+1} = 2u_n - 2$ with $u_1 = 4$.

Calculate

$$\sum_{r=1}^{6} u_r$$

(2 marks)

(b) What value of u_1 would make every term of the sequence equal?

(1 mark)

(c) Find the range of values for u_1 that would ensure every term of the sequence is positive?

(1 mark)

Medium Questions

1 (a) The first k terms of a series are given by $\sum_{r=1}^{k}(7+5r)$.

Show that this is an arithmetic series, and determine its first term and common difference.

(2 marks)

(b) Given that $\sum_{r=1}^{k}(7+5r)=1190$,

(i) Show that $(5k+119)(k-20)=0$.

(ii) Hence find the value of k.

(3 marks)

2 (a) The first k terms of a series are given by $\sum_{r=1}^{k} 5 \times 2^r$.

Show that this is a geometric series, and determine its first term and common ratio.

(2 marks)

(b) Given that $\sum_{r=1}^{k} 5 \times 2^r = 20470$,

Show that $k = \dfrac{\log 2048}{\log 2}$

(3 marks)

(c) For this value of k, calculate $\sum_{r=1}^{k+3} 5 \times 2^r$.

(2 marks)

3 A geometric series is given by $1 + 2x + 4x^2 + ...$

(i) Write down the common ratio, r, of the series.

(ii) Given that the series is convergent, and that $\sum_{n=1}^{\infty} 2x^{n-1} = 19$, calculate the value of x.

(4 marks)

4 An arithmetic series is given by $a+(a+d)+(a+2d)+\ldots$

Given that $\sum_{n=1}^{7}(a+(n-1)d)=91$ and $\sum_{n=1}^{10}(a+(n-1)d)=175$, find the values of a and d.

(4 marks)

5 (a) A sequence is defined for $k \geq 1$ by the recurrence relation $u_{k+1} = u_k - 3, \quad u_1 = 23.$

Calculate

$$\sum_{n=1}^{10} u_n$$

(2 marks)

(b) $$\sum_{n=11}^{15} u_n$$

(3 marks)

6 (a) A sequence is defined for $k \geq 1$ by the recurrence relation $u_{k+1} = \frac{u_k}{3}$, $u_1 = 54$.

Calculate, giving your answers as exact values

$$\sum_{n=1}^{9} u_n$$

(2 marks)

(b) $\sum_{n=10}^{\infty} u_n$

(3 marks)

7 (a) A sequence is defined for $k \geq 1$ by the recurrence relation $u_{k+1} = pu_k - 2, \quad u_1 = 2,$

where p is a constant.

Write down expressions for u_2 and u_3 in terms of p.

(2 marks)

(b) Given that the sequence is periodic with order 2, and given as well that $u_1 \neq u_2$,

Find the value of p.

(4 marks)

(c) For the value of p found in part (b)

Calculate $\sum_{n=1}^{1001} u_n$

(2 marks)

8 (a) The terms of a sequence are defined by $u_k = k^2$ for all $k \geq 1$.

State, with a reason, whether this sequence is increasing, decreasing, or neither.

(1 mark)

(b) It can be shown that, for all $n \geq 1$,

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

Using that formula,

Calculate $\sum_{r=1}^{50} u_r$

(2 marks)

(c) Find the value of $51^2 + 52^2 + 53^2 + ... + 99^2 + 100^2$,i.e. the sum of the squares of all the integers between 51 and 100 inclusive.

(3 marks)

Hard Questions

1 Given that $\sum_{r=1}^{k}(31-6r)=-943$,

(i) Show that $(3k+41)(k-23)=0$

(ii) Hence, find the value of k.

(4 marks)

2 Given that $\sum_{n=1}^{9}(a+(n-1)d)=-279$ and $\sum_{n=1}^{13}(a+(n-1)d)=-585$, find the values of a and d.

(4 marks)

3 (a) Given that $\sum_{r=1}^{k} 7 \times 3^r = 620004$,

Show that $k = \dfrac{\log 59049}{\log 3}$

(4 marks)

(b) For this value of k, calculate $\sum_{r=0}^{k+3} 7 \times 3^r$.

(3 marks)

4 (a) A convergent geometric series is given by $1 - 4x + 16x^2 - 64x^3 + \ldots$

Write down the range of possible values of x.

(3 marks)

(b) Given that $\sum_{n=1}^{\infty} (-4x)^{n-1} = 24$

Calculate the value of x.

(3 marks)

5 (a) A sequence is defined for $k \geq 1$ by the recurrence relation $u_{k+1} = u_k + 7, \quad u_1 = 23.$

Calculate

$$\sum_{n=15}^{25} u_n$$

(3 marks)

(b) $\sum_{n=1}^{25} (u_n - 3)$

(2 marks)

6 (a) A sequence is defined for $k \geq 1$ by the recurrence relation $u_{k+1} = \frac{2u_k}{7}$, $u_1 = 686.$

Calculate, giving your answers as exact values

$$\sum_{n=7}^{\infty} u_n$$

(3 marks)

(b) $$\sum_{n=1}^{\infty} u_{n+4}$$

(2 marks)

7 (a) A sequence is defined for $k \geq 1$ by the recurrence relation

$$u_{k+1} = (p-2)u_k - 2, \qquad u_1 = 3$$

where p is a constant.

Given that the sequence is periodic with order 2, and given as well that $u_1 \neq u_2$,

Find the value of p.

(5 marks)

(b) For the value of p found in part (a),

Calculate $\sum_{n=50}^{900} u_n$

(2 marks)

8 (a) The terms of a sequence are defined, for all $k \geq 1$, by $u_k = (-1)^k \times k^2$.

State, with a reason, whether this sequence is increasing, decreasing, or neither.

(1 mark)

(b) It can be shown that, for all $n \geq 1$,

$$\sum_{r=1}^{n} (2r)^2 = \frac{2n(n+1)(2n+1)}{3} \quad \text{and} \quad \sum_{r=1}^{n} (2r-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

Using those formulas,

Show that $\sum_{r=1}^{100} u_r = \sum_{r=1}^{100} r$.

(6 marks)

Very Hard Questions

1 Given that $\sum_{r=1}^{k}(89-5r) = -35$, find the value of k.

(4 marks)

2 (a) Given that $\sum_{r=1}^{k} 3 \times (-2)^r = -262146$,

(i) show that $\dfrac{k-1}{2} = \dfrac{\log 65536}{\log 4}$

(ii) hence find the value of k.

(5 marks)

(b) For this value of k, calculate $\sum_{r=5}^{k+2} 3 \times (-2)^r$.

(3 marks)

3 Given that $\sum_{n=7}^{12} (a+(n-1)d) = -69$, $\sum_{n=7}^{16} (a+(n-1)d) = -175$ and $\sum_{n=1}^{6} (a+(n-1)d) = -13d$, find the values of a and d.

(5 marks)

4 (a) A convergent geometric series is given by $\sqrt{3} + \sqrt{6x} + 2x\sqrt{3} + \ldots$, , where in all cases the square root symbol indicates the positive square root of the number in question.

Write down the range of possible values of x.

(4 marks)

(b) Given that $\sum_{n=2}^{\infty} \sqrt{3} \times \left(\sqrt{2x}\right)^{n-1} = 3\sqrt{3}$

Calculate the value of x.

(3 marks)

5 A sequence is defined for $k \geq 1$ by $u_k = \sqrt{13} + (-2)^{k-1}$.

Calculate $\sum_{r=11}^{23} u_r$, giving your answer as an exact value.

(5 marks)

6 (a) A sequence is defined for all $k \geq 1$ by

$$u_k = -2k \times (cos(k\pi))^{k+1}$$

Determine, giving reasons for your answer, whether the sequence is increasing, decreasing, or neither.

(3 marks)

(b) A different sequence is defined for all $k \geq 1$ by $v_k = sin(kq\pi)$

where q is a real constant.

Given that the sequence is not periodic,

suggest a possible value for q, giving a reason for your answer.

(2 marks)

7 (a) A sequence is defined for $k \geq 1$ by the recurrence relation

$$u_{k+2} = \frac{u_{k+1}}{u_k}, \qquad u_1 = a, \qquad u_2 = b$$

where a and b are real numbers.

Show that the sequence is periodic, and determine its order.

(3 marks)

(b) Given that $\sum_{r=1}^{44} u_r = -50$ and $\sum_{r=1}^{84} u_r = -92$

determine the possible values of a and b.

(5 marks)

8 Prove that, for all $n \geq 1$,

$$\sum_{r=1}^{n} (2r)^2 - \sum_{r=1}^{n} (2r-1)^2 = \sum_{r=1}^{2n} r$$

(4 marks)