Set A

Please check the examination details below before entering your candidate information

Candidate surname	Other names
KHAMBHAITA	PRAVEET MINAL H

MME Edexcel
Level 3 GCE

Centre Number					Candidate Number			
#	7	7	2	1	-	-	-	-

MME Edexcel Practice Papers

Morning (Time: 2 hours)	Paper Reference **3MME**

Mathematics

Worked Solutions

Advanced

Paper 3: Statistics and Mechanics

You must have:
Mathematical Formulae and Statistical Tables, Calculator

Total Marks

Candidates may use any approved calculator.
Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Stats: 50

Information

- The total mark for this paper is 100. Each section is worth 50 marks.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

MME

Paper 31: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. A meteorologist records the daily mean air temperature in degrees Celsius for Jacksonville over the course of six months.

A sample of this data taken from a spreadsheet is shown below.

Date	Temperature °C
15/05/2015	24.2
15/06/2015	27.2
15/07/2015	28.1
15/08/2015	25.9
15/09/2015	23.3
15/10/2015	19.1

(a) Suggest, with a reason, whether the sample is random or systematic. Give an advantage and disadvantage for each of these methods.

(3)

(b) Calculate the mean, variance and standard deviation of the data set.

(4)

(c) Based on this data, the meteorologist uses the values calculated in part (b) to model the temperature in Jacksonville in 2015 using a normal distribution.

Suggest, with a reason, whether his plan is suitable.

(1)

a) Systematic → data taken at same point over 6 months, each month
Random sampling eliminates some bias
Systematic sampling gives better coverage.

b) $\bar{x} = \frac{\sum x}{n} = \frac{147.8}{6} \approx 24.6$

$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{3693.6}{6} - \left(\frac{147.8}{6}\right)^2 \approx 8.80$

$\sigma = \sqrt{\sigma^2} \approx 2.97$

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Question 1 continued

c) Not suitable → cannot extrapolate unknown months

(Total for Question 1 is 8 marks)

Turn over ▶

2. The discrete random variable X has binomial distribution $B(n, p)$.

Given that, $n \times p = \sqrt{np(1-p)} = 0.9$

Find the value of n.

(4)

$np = 0.9$

$\sqrt{np(1-p)} = 0.9 \longrightarrow \sqrt{0.9(1-p)} = 0.9$

$0.9(1-p) = 0.81$

$1-p = 0.9$

$p = 0.1$

$np = 0.9$

$n \times 0.1 = 0.9 \qquad n = \frac{\cancel{0}.9}{\cancel{0}.1} = 9$

$n = 9$

(Total for Question 2 is 4 marks)

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3. In a survey of maths students, it was found that 75% use Instaphoto, 60% use Snaptalk and 10% use neither.

(a) Find the probability that a student uses both Instaphoto and Snaptalk.

(3)

(b) Draw a Venn diagram to show this information.

(2)

(c) Determine whether the probability of using Instaphoto is independent from the probability of using Snaptalk.

(3)

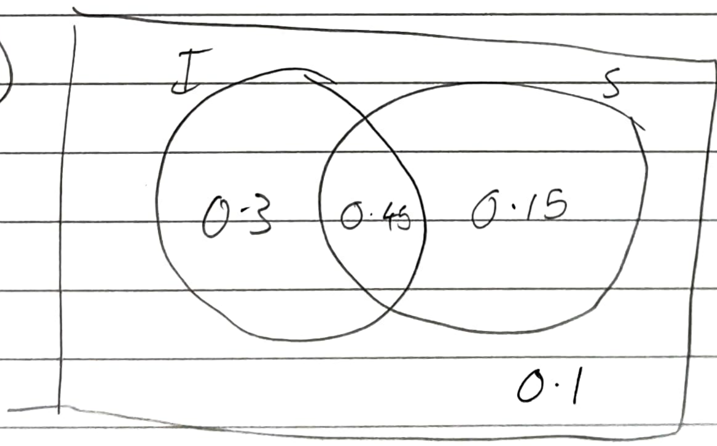
(d) Find $P([I' \cup S'] \cap [I \cup S])$.

(2)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$0.75 + 0.6 - (1 - 0.1) = \underline{0.45}$

b)



c) $P(I) \times P(S) = P(I \cap S)$ if independent

$0.75 \times 0.6 = 0.45 = P(I \cap S)$

$\therefore$ independent

Question 3 continued

d) $0.30 + 0.15$

$= \underline{\underline{0.45}}$

(Total for Question 3 is 10 marks)

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4. Bruce grows championship pumpkins and is testing a new compost.

Over 5 years he has found that there is a 0.25 chance of a pumpkin weighing more than 20 kg.

He hopes that a new compost will increase pumpkin size and wants to test this.

He takes a sample of 10 pumpkins and measures their weight after 6 months growth in the new compost.

(a) State suitable hypotheses for testing the new composts affects.

(1)

(b) Using a 5% significance level find the critical region and write down the actual significance level of this test.

(5)

(c) Given that Bruce finds 4 out of 10 pumpkins sampled weigh more than 20 kg after 6 months of growth with the new compost, state your conclusions from the hypothesis test clearly.

(2)

a)

$H_0: p = 0.25$

$H_1: p > 0.25$

b) $X \sim B(10, 0.25)$

$P(X \geq 6) = 0.0197 < 0.05$

$\boxed{X \geq 6}$ Sig. level = 1.97%

c) $4 < 6$ $\therefore$ accept H_0

Insufficient evidence to suggest new compost will grow ~~pumpkins~~ pumpkins > 20kg

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5. A pump's lifetime, D, follows a normal distribution.

$\mu = 1500$
$\sigma^2 = 1500$

A pumping station needs 5 working pumps, without which it fails.

(a) Find the probability that a pump will fail before 1400 hours of use.

(2)

(b) 5 new pumps were installed at the same time. They have already run for 1400 hours.

Find the probability that the pumping station will fail before the pumps have worked for 1500 hours.

(5)

(c) After 1500 hours the 5 new pumps are all still working.
The engineers believe the mean lifetime is greater than 1500 hours.

A sample of 5 pumps are tested and have a mean lifetime of 1550 hours.
Clearly stating hypotheses, test the engineers' claim at the 5% significance level.

(5)

a) $P(D < 1400) = \underline{\underline{0.0049}}$

b) $P(D > 1500 \mid D > 1400) = \left(\frac{P(D > 1500)}{P(D > 1400)}\right)$

$= 0.502462...$

$(0.502462...)^5 = 0.032027...$ (Successful)

Failure $= 1 -$ Success $\simeq \underline{\underline{0.968}}$

Question 5 continued

c) $H_0 : \mu = 1500$

$H_1 : \mu > 1500$

$\bar{D} \sim N\left(1500, \left(\frac{10\sqrt{15}}{\sqrt{5}}\right)^2\right)$

$P(\bar{D} > 1550) = 0.019... < 0.05$

$\boxed{\text{Reject } H_0}$ There is sufficent evidence to suggest mean life time > 1500hrs

(Total for Question 5 is 12 marks)

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6. A discrete random variable X satisfies

$$X \sim B(n, p)$$

Given that $P(X = 4) = P(X = 5)$, show that the expected number of successes is $5 - p$

(8)

$P(X=4) = {}^{n}C_4\, p^4 (1-p)^{n-4}$

$P(X=5) = {}^{n}C_5\, p^5 (1-p)^{n-5}$

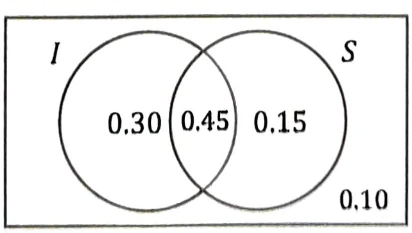
$p^4(1-p)^{n-4}\left(\frac{n(n-1)(n-2)(n-3)}{24}\right) = \frac{n(n-1)(n-2)(n-3)(n-4)}{120} p^5(1-p)^{n-5}$

$\frac{1}{24}(1-p) = \frac{1}{20}(n-4)p$

$5(1-p) = (n-4)p$

$5 - 5p = np - 4p$

$\boxed{5 - p = np}$ as required

Question	Paper 31- Statistics Mark Scheme	Marks
1(a)	The data sample is systematic. This is because it is taken by selecting the same date every month. The advantage of random sampling is a lower risk of introducing artificial patterns in the data; however systematic sampling is more likely to give better coverage of the entire period when few data points are used.	B1 B1 B1 **(3)**
1(b)	Mean: $\bar{x} = \frac{\sum x}{n} = \frac{147.8}{6} = 24.63 \ldots$ Attempts to calculate the variance e.g. Variance $= \sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{3693.6}{6} - 24.63\ldots^2$ Variance $= \sigma^2 = 8.798 \ldots$ Standard Deviation: $\sigma = \sqrt{8.798\ldots} = 2.966 \ldots$	A1 M1 A1 A1 **(4)**
1(c)	Temperature data is suitable to be modelled by a normal distribution with a larger data set. It is inappropriate to extrapolate from 6 months of data to model temperatures all year round given seasonal variation.	B1 **(1)**
2	$Var(X) = \sigma^2 = 0.9^2$ $0.9(1-p) = 0.9^2$ $p = 0.1$ $0.9 \div 0.1 = 9 = n$	M1 M1 B1 B1 **(4)**
3(a)	States or attempts to use $P[A \cap B] = P(A) + P(B) - P(A \cup B)$ $0.75 + 0.6 - (1 - 0.1)$ 0.45	M1 M1 B1 **(3)**
3(b)		B1 At least 2 values placed correctly B2 Fully correct **(2)**

Question	Paper 31- Statistics Mark Scheme	Marks
3(c)	$P(I) \times P(S) = 0.75 \times 0.6 = 0.45$ $P(I \cap S) = 0.45$ Hence, I and S are independent.	M1 M1 A1 **(3)**
3(d)	$P([I' \cup S'] \cap [I \cup S]) = 0.30 + 0.15$ 0.45	M1 B1 **(2)**
4(a)	$H_0 : p = 0.25, H_1 : p > 0.25$	B1 **(1)**
4(b)	$H_0 : X \sim B(10, 0.25)$ $P(X \geq 6) = 0.0197 < 0.05$ $P(X \geq 5) = 0.0781 > 0.05$ Critical region = $\{6, 7, 8, 9, 10\}$ Actual significance level = 1.97%	M1 M1 M1 A1 A1 **(5)**
4(c)	4 does not lie in the critical region Bruce should not reject H_0 as there is insufficient evidence to suggest that the chance of growing over 20 kg is changed by the new compost.	B1 B1 **(2)**
5(a)	$P(D < 1400) = P\left(Z < \frac{1400 - \mu}{\sigma}\right) = P(Z < -2.58)$ $= 1 - P(Z \leq 2.58) = 1 - 0.9951 = 0.0049$	M1 B1 **(2)**

Question	Paper 31- Statistics Mark Scheme	Marks
5(b)	P(Failure) $=$ P(any one or more fail before 1500 given none failed before 1400) $= 1 -$ P(all are still working after 1500 given none failed before 1400) $= 1 - \left(\frac{P(D > 1500)}{P(D > 1400)}\right)^5$	M1
	$P(D > 1500) = 0.5$	M1
	$P(D > 1400) = 1 - 0.0049 = 0.9951$	M1
	$1 - \left(\frac{0.5}{0.9951}\right)^5$	M1
	P(Failure) $= 0.968$	B1
		(5)
5(c)	H_0: $\mu = 1500, H_1$: $\mu > 1500$	M1
	$X \sim N\left(1500, \frac{1500}{5}\right)$	M1
	$P(X > 1550) = 0.019 < 5\%$	M1
	Reject H_0 and accept H_1	A1
	The mean lifetime is $>$ 1500 hours	A1
		(5)
6	$P(X = 4) = \binom{n}{4} p^4(1-p)^{n-4}$	M1
	$P(X = 5) = \binom{n}{5} p^5(1-p)^{n-5}$	M1
	$\binom{n}{4} p^4(1-p)^{n-4} = \binom{n}{5} p^5(1-p)^{n-5}$	M1
	$\frac{n(n-1)(n-2)(n-3)}{1 \times 2 \times 3 \times 4} p^4(1-p)^{n-4} = \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \times 2 \times 3 \times 4 \times 5} p^5(1-p)^{n-5}$	M2 1 mark for each correct side
	Cancel common factors e.g. $\frac{1}{24}(1-p) = \frac{1}{120}(n-4)p$	M1
	Start of process to rearrange e.g. $5(1-p) = (n-4)p$	M1
	Recognises the expected number of successes is np and rearranges for given form $5 - p = np$	M1
		(8)