STATISTICS YEAR 1 RECAP:
Distributions and Hypothesis Testing

Probability Distributions

There are a few ways to show the mapping from outcomes to probabilities:

The random variable X represents the **number of heads when three coins are tossed**.

Underlying Sample Space

{ HHH,
HHT,
HTT,
HTH,
THH,
THT,
TTH,
TTT }

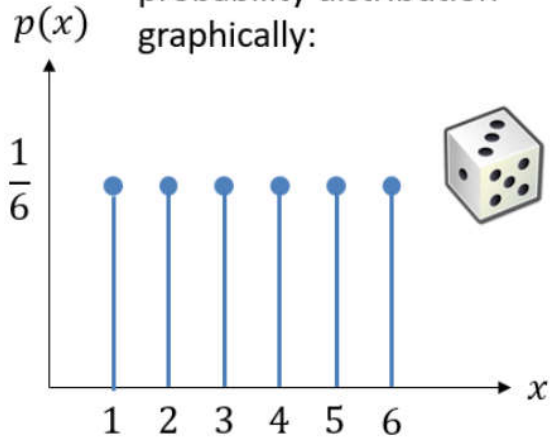
Distribution as a Table

Num heads x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Distribution as a Function

$$P(X = x) = \begin{cases} \frac{1}{8} & x = 0,3 \\ \frac{3}{8} & x = 1,2 \\ 0 & otherwise \end{cases}$$

We can also represent a probability distribution graphically:



The throw of a die is an example of a **discrete uniform distribution** because the probability of each outcome is the same.

The Binomial Distribution

You can model a random variable X with a binomial distribution $B(n, p)$ if

- there are a fixed number of trials, n, F
- there are two possible outcomes: 'success' and 'failure', T
- there is a fixed probability of success, p C
- the trials are independent of each other I

If $X \sim B(n, p)$ then:

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

← In the example below, 'success' was 'left handed'.

← r is the number of successes out of n.

← "$\sim$" means "has the distribution"

On a table of 8 family members, 6 people are left handed.

a) Suggest a suitable model for a random variable X: the number of left-handed people in a group of 8, where the probability of being left-handed is 0.1.

b) Find the probability 6 people are left handed.

c) Suggest why the chosen model may not have been appropriate.

a) $X \sim B(8, 0.1)$

b) $P(X=6) = \binom{8}{6} 0.1^6 \times 0.9^2$

$= 0.00002268$

c) because there may be a genetic link in left handedness, so trials may not be independent.

Your Turn

1 $X \sim B(6, 0.2)$

What is $P(X = 2)$? $P(X=2) = \binom{6}{2} 0.2^2 \times 0.8^4 = 0.24576$

What is $P(X \geq 5)$? $P(X \geqslant 5) = P(X=5) + P(X=6)$

$= \binom{6}{5} 0.2^5 \times 0.8 + 0.2^6$

$= \underline{0.0016}$

2 I have a bag of 2 red and 8 white balls. X represents the number of red balls I chose after 5 selections (with replacement).

a How is X distributed? $X \sim B(5, 0.2)$

b Determine the probability that I chose 3 red balls.

$P(X=3) = \binom{5}{3} \times 0.2^3 \times 0.8^2$

$= \underline{0.0512}$