Edexcel A Level Further Maths: Further Mechanics 1

Your notes

Work, Energy & Power

Contents

Work Done

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Work Done

What does the term work mean in Mechanics?

- In Mechanics the word work refers to the **work done** by a force when it causes an object to move
  - Mechanical work happens when both a **force** is applied and the object **moves**
  - A force that is holding an object stationary is not producing any mechanical work
  - The object **gains energy** due to the work done by forces acting in the **direction of motion**
  - The object **loses energy** due to the work done **against resistive forces**
- The **line of action** of a force refers to the **point of application** of the force and the **direction** the force was applied in
- Work is a **scalar quantity,** it has size without direction

How do we calculate work done by a force?

- If the **point of application** of a force of magnitude ***F* N**, which moves an object a distance, ***d* metres**, is in the same direction as the **line of action** of the force, then the **work done**, W , by the force is

$$W = Fd$$

(sometimes s is used in place of d)

  - If the line of action of the force is at an **angle** to the direction of motion, then the **component** of the force in the direction travelled is multiplied by the distance instead
  - If the object moves **vertically upwards** then the work is done against gravity and this calculation becomes

$$W = mgh$$

- The units for the work done by a force are **Newton metres (Nm)**, but it is more common to use **Joules**
  - **1 Joule = 1 N m**
  - 1 Joule is equal to the amount of work done by a force of 1 Newton moving an object 1 metre along the line of action of the force
  - **1 Kilojoule** is equal to **1000 Joules** (**1 kJ = 1000 J**)
- If the line of action of the force is different to the direction of motion, start by **resolving** the force into components **parallel** and **perpendicular** to the direction of motion
  - When a force of ***F* N** is acting at an angle of $\theta°$ to the direction of motion, the component of F acting in the direction of motion will be $F\cos\theta$ and the **work done** by the force will be $Fd\cos\theta$
  - The **perpendicular** component of the force will produce no work
- The **net work done** on an object will be equal to the work done by the force that moves the object forwards, in addition to any work done against resistive forces

How do we use work done with N2L (*F = ma*)?

- If the **work done by a force** is known it can often be used along with Newton's second law **(N2L)** to find one of the components in the formula **F=ma**

- Often the object will be moving at constant speed so the **acceleration** of the object will be zero, so the forces acting on the object in the **direction of motion** will be in equilibrium

  STEP 1: Draw a diagram or add all the forces to the diagram given in the question

  STEP 2: If the line of action of the force is at an angle to the direction of motion, find the component of the force parallel to the direction of motion

  STEP 3: If there is more than one force, resolve the forces to find the resultant component acting in the direction of motion

  STEP 4: Use ***W = Fs*** to find the work done by the force on the object

- If the problem involves **friction**, you may have to resolve perpendicular to the plane to find the value of the **normal reaction force, *R***, and then use $F \leq \mu R$
  - If the force overcomes friction to cause an object to move, it is said to do **work done against friction**
  - The object will be moving, so **friction will be limiting** and $F_{MAX} = \mu R$

How do we use work done on an inclined plane?

- The object will be said to travel up the **line of greatest slope** on an **inclined plane**
- If the **coefficient of friction** is involved then the weight will need to be resolved perpendicular to the slope, to find the value of the **normal reaction force, *R***
  - There is no work done by the normal reaction force as it acts perpendicular to the direction of motion
- The **work done against gravity** can be found by using right – angled trigonometry to find the change in **vertical** height of the object
  - Note that this is the vertical height, not the distance up the plane

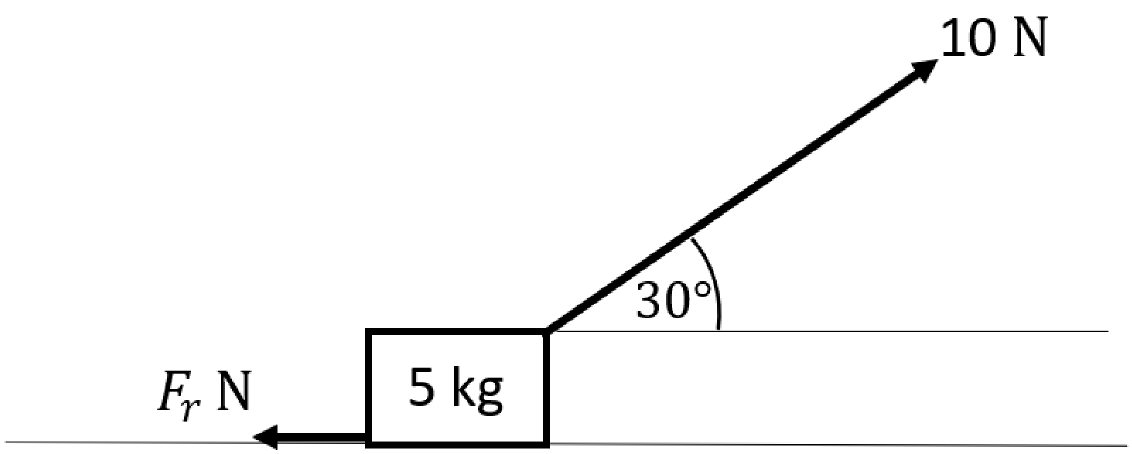
Examiner Tip

- Read the question carefully to decide if the point of application of the force is acting in the direction of motion. Always draw a diagram or add to the diagram given in the question. Check to see if there are any resistive forces.

Your notes

Worked example

A child pulls a box of mass 5 kg along a rough horizontal surface at a constant speed by a force of magnitude 10 N inclined at 30° to the horizontal. The only resistive force is from friction, F_r N as shown in the diagram below.

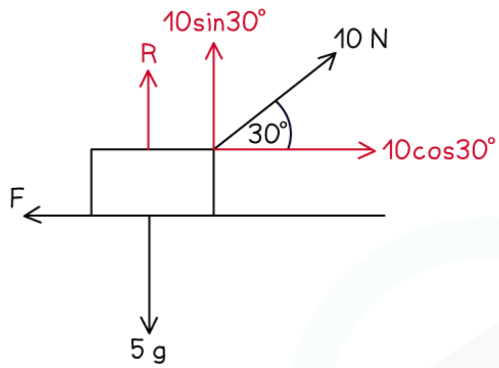


Calculate the work done against friction as the child pulls the box 5 metres along the floor.

Your notes

Your notes

Step 1: Draw a diagram and add the forces



Step 2: Resolve the angled force into components parallel and perpendicular to the direction of motion

Step 3: Acceleration is zero so forces in the horizontal direction balance

$$F = 10\cos 30° = 5\sqrt{3}\ \text{N}$$

Step 4: Use $W = Fs$ to find the work done

Work done against friction = Work done by the force

$$= (10\cos 30)(5)$$

$$= 25\sqrt{3}\ \text{N m}$$

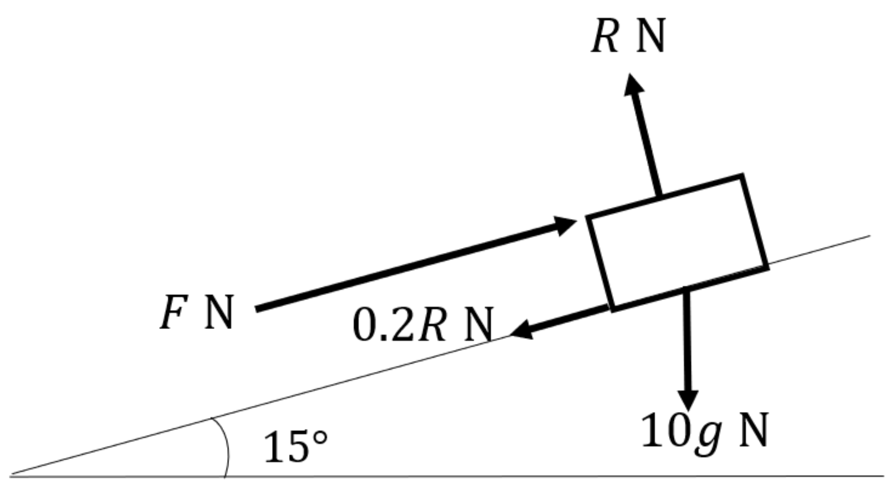
$$= 43.30...\ \text{J}$$

WD = 43.3 J (3sF)

Your notes

Worked example

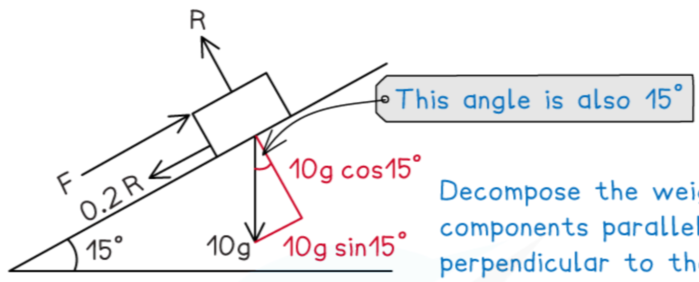
A crate of mass 10 kg is pushed 6 metres up a rough ramp inclined at 15° to the horizontal by a force F N. The crate moves with constant speed along the line of greatest slope. The coefficient of friction between the container and the ramp is 0.2 as shown in the diagram below.



i) Calculate the work done against friction.
ii) Calculate the work done against gravity.

Your notes

(i)



Decompose the weight into components parallel and perpendicular to the slope.

Find the value of R: $R = 10g\cos 15°$

$= (10)(10)(\cos 15) = 96.592...\text{ N}$

Use $Fr = \mu R$ as the crate is moving

$Fr = 0.2 \times 96.592...$

$= 19.318...$

Use $W = Fs$ to find the work done.

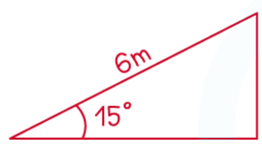
Work done against friction $= Fs = (19.318)(6)$

$= 115.911...\text{ Nm}$

Work done against friction = 116 J (3.s.f.)

(ii)

Use right-angled trigonometry to find the gain in vertical height



$h = 6\sin 15° = 1.5529...\text{ m}$

Use work done against gravity = mgh

work done against gravity $= (10)(10)(1.5529...)$

$= 155.291...\text{ Nm}$

Work done against gravity = 155J (3 s.f.)

Kinetic & Potential Energy

Your notes

Kinetic Energy

There are many different forms of energy including, but not limited to, heat energy, light energy, chemical energy and nuclear energy. Two forms of energy which are of particular interest in mechanics are **kinetic energy** (**KE**) and **potential energy** (or gravitational potential energy, **GPE**). Elastic potential energy is also considered when dealing with springs and strings.

What is kinetic energy?

- A particle has **kinetic energy** when it is moving
- Kinetic energy is a **scalar quantity**, it cannot be negative
- The **work done** by a resultant force that acts to move an object in a particular direction will be equal to the **change in kinetic energy** of the object

How is kinetic energy calculated?

- A particle can only have kinetic energy when it is moving
- If a particle with mass, *m* kg is moving with speed v m s^{-1} then its kinetic energy can be calculated using the formula

$$KE = \frac{1}{2}mv^2$$

- If the particle is moving in two dimensions with the velocity vector **v** then kinetic energy can be calculated in two ways
  - Using the formula on each component individually and finding the sum of the KE in each component
  - Finding the magnitude of the velocity to get the speed and then using the formula for KE
- Kinetic energy is measured in **joules (J)**
  - 1 Kilojoule = 1000 joules (1 kJ = 1000 J)

How can we link the work done to a change in kinetic energy?

- The **work done** by the resultant force is equal to the **change in kinetic energy**
  - This is the **final kinetic energy** minus the **initial kinetic energy**
  - The formula for the **change in kinetic energy** is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

where ***u*** is the initial velocity and ***v*** is the final velocity

  - This is often written as $\frac{1}{2}\mathrm{m}(\mathrm{v}^2 - \mathrm{u}^2)$

- Newton's Second Law ($F = ma$) and the suvat equation $v^2 = u^2 + 2as$ can be used to show why this is equivalent to the work done

NEWTON'S SECOND LAW: $F = ma$ (1)

'SUVAT' EQUATION: $v^2 = u^2 + 2as$ (2)

REARRANGE EQUATION (2) TO MAKE 'a' THE SUBJECT:

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

SUBSTITUTE 'a' INTO EQUATION (1):

$$F = m\left(\frac{v^2 - u^2}{2s}\right)$$

$$= \frac{m(v^2 - u^2)}{2s}$$

MULTIPLY BOTH SIDES BY 's'

$$Fs = \frac{m(v^2 - u^2)}{2}$$

$$\therefore \ WD = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$\therefore$ WORK DONE = CHANGE IN KINETIC ENERGY

Examiner Tip

- Always double check the units are in kg for mass and m s^{-1} for velocity before carrying out any calculations
- Be careful not to make the mistake of using the difference between the velocities with the equation, remember it should be the difference between the squares of the speeds

Your notes

Worked example

A jogger increases her speed from 2 m s^{-1} to 3 m s^{-1} and her change in kinetic energy is 150 J, find the mass of the jogger.

Final kinetic energy

Initial kinetic energy

Change in kinetic energy $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2}m(v^2 - u^2)$$

$$u = 2\text{ms}^{-1}$$

$$v = 3\text{ms}^{-1}$$

Substitute values into equation:

$$\frac{1}{2}m(3^2 - 2^2) = 150$$

$$m(9 - 4) = 300$$

$$5m = 300$$

$$m = 60$$

Jogger's mass is 60 kg

Your notes

Potential Energy

Your notes

What is potential energy?

- Potential energy is the energy stored in a **stationary** object
- **Gravitational potential energy (GPE)** is the energy a particle possesses when it is at a **fixed height** and **gravity** is acting on it
- There are other types of potential energy such as elastic potential energy, however it is usually gravitational potential energy which is being referred to by "potential energy"
- GPE will change as the vertical height of an object changes
  - The **work done against gravity** on a particle as it moves **upwards** is equal to its **increase** in **GPE**
  - The **work done by gravity** on a particle as it moves **downwards** is equal to its **decrease** in **GPE**

How is gravitational potential energy calculated?

- Gravitational potential energy is equal to the **product** of the **weight** of an object and its vertical **height,** ***h*** , above a fixed point

$$GPE = mgh$$

  - If the object is sitting on the ground or the point chosen as the fixed base level, the object will have no gravitational potential energy
  - As the object moves upwards, its GPE will increase
  - As the object moves downwards again, its GPE will decrease
- When **mass** is measured in **kg**, **acceleration due to gravity** is measured in **m s^{-2}**, and height is in metres, m, gravitational potential energy is measured in **joules (J)**
  - 1 kilojoule = 1000 joules (1 kJ = 1000 J)

Examiner Tip

- Always double check the units are in **kg** for mass and **m** for height before carrying out any calculations.
- Remember that it is the **vertical height** that must be used within the calculations for **GPE**. If you are given the distance up a slope for example, you must use trigonometry to find the vertical height first.

Your notes

Worked example

A ball of mass 400 grams is thrown vertically upwards from a height of 1 metre above the ground. It reaches a maximum height of 4 metres before falling to the ground. Stating clearly whether is represents a gain or a loss, write down the change in the gravitational potential energy of the ball

(i) between the instant it is thrown and the instant it reaches its maximum height,

(ii) between the instant it is thrown and the instant it hits the ground.

Convert any non-SI units into SI units

400 g = 0.4 kg

Take the ground to be the base level, then

At start: $m = 0.4,\ g = 9.8,\ h = 1$

$GPE = mgh = (0.4)(9.8)(1)$

$= 3.92$ J

At maximum height: $m = 0.4,\ g = 9.8,\ h = 4$

$GPE = mgh = (0.4)(9.8)(4)$

$= 15.68$ J

(i) Change in GPE = 15.68 − 3.92 = 11.76 J

The ball's height has increased, so its GPE has increased

Gain in GPE = 11.8 J (3sf)

(ii) When the ball hits the ground it is at base level so has no GPE

Change in GPE = 3.92 − 0 = 3.92 J

The ball's height has decreased, so its GPE has decreased

Loss in GPE = 3.92 J (3sf)

Your notes

Work-Energy Principle

Work-Energy Principle

What is the Work-Energy Principle?

- The **Work-Energy Principle** has many forms, but is in essence an **energy balance**
  - the **final** amount is the **initial** amount plus any energy put in (or minus any taken out)
  - it's a bit like money in a bank account!
- The principle can be written as:
  - **total final energy = total initial energy ± work done** by **non-gravitational** forces
    - or, using subscripts for final and initial, $E_f = E_i \pm WD$
  - "total energy" here means the **sum of Gravitational Potential Energy and Kinetic Energy**
    - e.g. $E_i = GPE_i + KE_i = mgh_i + \frac{1}{2}mv_i^2$
  - **Non-gravitational** forces are any external forces that are not related to gravity
    - e.g. frictions, tensions, driving forces, etc
    - but **wouldn't** include weight, mg, because work done against gravity has **already** been considered in the **GPE** part
  - **Use +** for work done by forces that **"help"** the object to move forwards
    - e.g. **tension** in a string pulling **forwards**, a **driving** force, etc
  - **Use -** for work done by forces that **"hinder"** (resist) the object from moving forwards
    - e.g. **friction**, **tension** in a string pulling **backwards**, a **resistance** force, air resistance, etc
- Some situations may have **more than one** form of work done
  - add or subtract each one, depending whether they help or hinder

How else can the Work-Energy Principle be written?

- You can write the Work-Energy Principle in terms of **gains** or **losses** in KE and GPE
  - but this method can cause a lot of **sign errors**!
- Write out each term in the original version, "**total final energy = total initial energy ± work done** by **non-gravitational** forces"
  - use subscripts for **final** and **initial**
    - $mgh_f + \frac{1}{2}mv_f^2 = mgh_i + \frac{1}{2}mv_i^2 \pm WD$
  - **group** together KEs and GPEs as an overall **change** ("final - initial")
    - $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\left(mgh_f - mgh_i\right) \pm WD$
  - **change in KE = - change in GPE ± WD**
    - you can read this as **gain** in KE = **loss** in GPE ± WD
    - but some situations **lose** KE, so the gain is **negative** (you need to be really careful with the **signs** and what "loss" means!)

- The first method (**energy balance**) works for **every situation**, but this "**gain-loss**" method needs **adapting** for **each** situation

Can I use the Work-Energy Principle on an inclined plane?

- Yes, and remember to calculate **work done** as "the **component** of force in the **direction of motion**" × "distance moved in direction of motion"
  - in this case, the direction of motion is **parallel** to the **slope**
- You may also need to calculate the **final vertical height** for GPE
  - e.g. using trigonometry, with the distance in the direction of the slope as the hypotenuse

Examiner Tip

- If a question asks you to find something "using the work-energy principle", don't use Newton's 2nd Law with SUVAT!

Your notes

Your notes

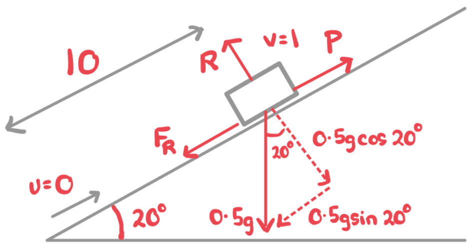
Worked example

A dog uses a constant force of P N to push its toy of mass 0.5 kg up a rough slope inclined at 20° to the horizontal. The force, P, acts parallel to the slope. The toy starts from rest and, after moving 10 metres up the line of greatest slope, it is travelling at 1 ms^{-1}. The coefficient of friction between the toy and the slope is 0.1.

Use the work-energy principle to find P.

Your notes

Draw a diagram and include important information



$\mu = 0{\cdot}1$

First find friction, $F_R = \mu R$
Resolving perpendicular to the plane

$R = 0{\cdot}5g\cos 20°$
$\therefore F_R = 0{\cdot}1(0{\cdot}5g\cos 20°) = 0{\cdot}05g\cos 20°$

Work-energy principle ("$E_f = E_i \pm WD$")
Work done, P 'helps' toy move, friction (F_R) 'hinders' it
("$WD = Fd$")

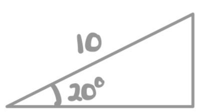
$\therefore WD = (P - F_R) \times 10 = 10P - 0{\cdot}5g\cos 20°$

Consider kinetic energy ("$KE = \frac{1}{2}mv^2$")

$KE_i = 0$ (initial)
$KE_f = \frac{1}{2} \times 0{\cdot}5 \times 1^2 = 0{\cdot}25$ (final)

Gravitational potential energy ("mgh")

$GPE_i = 0$
$GPE_f = 0{\cdot}5g(10\sin 20°)$
$= 5g\sin 20°$



$10\sin 20°$

Now apply $E_f = E_i + WD$

$0.25 + 5g\sin 20° = 10P - 0.5g\cos 20°$

Using $g = 9.8$

$P = 2.161\ 348...$

$\boxed{\therefore P = 2.2\text{ N} \quad (2\text{ s.f.})}$

2 s.f. is appropriate since we used $g = 9.8$ which is 2 s.f.

Your notes

Conservation of Energy

What is Conservation of Energy?

- **Conservation of Energy** is a **special case** of the **Work-Energy Principle** when there is **no work done** by **non-gravitational forces**
  - so **total final energy = total initial energy**
  - total energy here means **GPE + KE**
  - or, using subscripts for **final** and **initial**:
    - $mgh_f + \frac{1}{2}mv_f^2 = mgh_i + \frac{1}{2}mv_i^2$
- This could be because there are **no non-gravitational forces**
  - e.g. a particle falling **freely under gravity** from a fixed height above the ground
- Or this could be because all non-gravitational forces are always **perpendicular** to the direction of motion
  - e.g. the vertical reaction force on an object moving along a horizontal surface contributes no work done (the force has no horizontal component)
    - recall work done is the "**component of force** in the **direction of motion**" × "distance moved in direction of motion"

Examiner Tip

- In practice, you can apply the Work-Energy Principle as before, but it'll be slightly easier as there's no work done by external forces

Your notes

Your notes

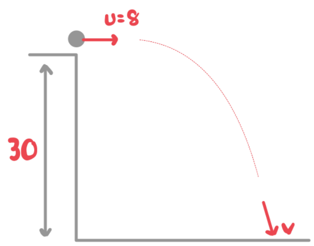
Worked example

A particle of mass m kg is fired horizontally off a vertical cliff at 8 ms^{-1} and travels freely under gravity. The cliff is 30 metres high and the ground below is horizontal.

Use energy considerations to find the speed of impact of the particle with the horizontal ground below.

Your notes

Draw a diagram



The work-energy principle is $E_f = E_i \pm WD$
There are no non-gravitational forces, so energy is conserved

$$E_f = E_i$$

Kinetic energy ("$\frac{1}{2}mv^2$"): $KE_i = \frac{1}{2}m(8)^2 = 32m$
$KE_f = \frac{1}{2}mv^2$

Gravitational potential energy ("mgh") $GPE_i = mg(30) = 30mg$
$GPE_f = 0$

Applying $E_f = E_i$

$$\frac{1}{2}\cancel{m}v^2 = 32\cancel{m} + 30\cancel{m}g$$

Using $g = 9.8$

$$v^2 = 652$$
$$v = 25.534\ldots$$

$\boxed{\therefore \text{ Speed of impact is } 26 \text{ m s}^{-1} \text{ (2 s.f.)}}$

Your notes

Problem Solving with Energy

Problem Solving with Energy

How do I include air resistance in the Work-Energy Principle?

- The **work done** by a **constant air resistance** / drag force, D Newtons, when moving x metres is Dx Joules
- Air resistance hinders (slows down) the particle, so is negative in the **Work-Energy Principle**
  - total final energy = total initial energy - work done by air resistance
- This can work for particles moving **horizontally** or **vertically**
  - sometimes the air resistance experienced upwards has a **different value** to that experienced downwards
- Air resistances, in reality, are often **proportional** to the **speed** (or square of the speed) of the particle
  - but this makes it a non-constant force
    - and the work done formula only works for constant forces

How do I use the Work-Energy Principle on curved surfaces?

- The **Work-Energy Principle** can be used in **new situations** that aren't always inclined planes!
- e.g. skateboarding down a **curving** slope
  - the skater may put in their own work done (e.g. using their legs) which "helps" to go faster (+ work done)
  - but there may be a constant resistive force acting against them throughout (- work done)
    - assume that the **resistances** are always **parallel** to the curved slope at any given time (and **reactions** are always **perpendicular**)

How do I apply the Work-Energy Principle to connected particles?

- You can still use the Work-Energy Principle with connected particles by considering it all as one object
  - **total final energy = total initial energy ± work done**
- The total energies will be the **sum** of the GPEs and KEs of **all particles**
- There will be a combination of "work done" terms with + or - depending on whether it's helping or hindering its respective particle
  - e.g. for a driving car pulling a trailer, the terms look like:
  - + WD(by driving force on car) - WD(by tension in towbar on car) - WD(by resistances on car) + WD(by tension from towbar on trailer) - WD(resistances on trailer)
    - Notice that the work done by the tensions will **cancel each other out**

How do I apply the Work-Energy Principle to collisions?

- Some questions use the **Work-Energy Principle** and the theory of **collisions**
- There may be a particle projected into a **perpendicular wall**
  - Use the Work-Energy Principle to find the **speed** with which it impacts the wall
    - You can find the speed by making the **kinetic energy** the subject

Your notes

    - This gives the speed of **impact**
    - To find the speed of **rebound**, calculate **"e" ×** the speed of impact
        - "e" is the **coefficient of restitution**
- Other questions may have two spheres colliding on a horizontal table, then one falling off
    - Use **conservation of momentum** and **Newton's Law of Restitution** to find velocities after the collision
    - When the sphere rolls off the table, it becomes a **projectile** (projected horizontally with it's new velocity)
        - If you know the height of the table, you can use the Work-Energy Principle to find the **speed of impact** with the ground

Examiner Tip

- It is common for harder energy questions to be fully algebraic
    - look out for masses, m, cancelling in the working

Your notes

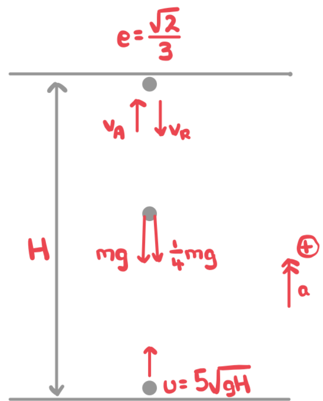
Worked example

A particle of mass m kg is projected vertically upwards from ground level at a speed of $5\sqrt{gH}$ ms^{-1}, where H is the vertical height in metres between the ground and the ceiling. The particle is subjected to a constant air resistance force of $\frac{1}{4}mg$ N, opposing its motion. The coefficient of restitution between the particle and the ceiling is $\frac{\sqrt{2}}{3}$.

Find, in terms of g and H, the exact speed of the ball immediately after rebounding with the ceiling.

Your notes

Draw a diagram



Newton's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$\therefore \frac{\sqrt{2}}{3} = \frac{v_R}{v_A}$$

Kinetic energy ("$\frac{1}{2}mv^2$"):

$$KE_i = \frac{1}{2}m\left(5\sqrt{gH}\right)^2 = \frac{25}{2}mgH$$

$$KE_f = \frac{1}{2}m{v_A}^2$$

Gravitational potential energy ("mgh"):

$$GPE_i = 0$$

$$GPE_f = mgH$$

Work done ("WD = Fd"): ...against air resistance

(non-gravitational forces only - i.e. not weight)

$$WD = \frac{1}{4}mgH$$

Applying the work-energy principle ("$E_f = E_i \pm WD$")

$$\frac{1}{2}\cancel{m}{v_A}^2 + \cancel{m}gH = \frac{25}{2}\cancel{m}gH - \frac{1}{4}\cancel{m}gH$$

negative as air resistance hinders motion

$$v_A^2 = 22.5gH$$

$$v_A = \frac{3\sqrt{10gH}}{2}$$

Using Newton's law of restitution

$$v_R = \frac{\sqrt{2}}{3}\left(\frac{3\sqrt{10gH}}{2}\right)$$

∴ Speed of ball immediately after rebounding with the ceiling is $\sqrt{5gH}$ m s^{-1}

Your notes

Power

Your notes

Power

What is power?

- Power is developed as an engine or machine does **work**
- Power is the **rate of doing work**, usually by the driving force of an engine
  - It is the same as **work done per second**
  - The work done is converting fuel (a type of chemical energy) into a driving force
- Power is a **scalar** quantity and can only take a positive value

How is power calculated?

- Power is the rate of doing work and can be calculated by the formula

$$P = \frac{WD}{t}$$

Where *WD* is the **work done** by the driving force in **Joules** and *t* is the **time** in **seconds**

- If a **driving force**, *F* N, is acting on an object moving with **velocity**, *v* m s^{-1} then power can be calculated using the formula

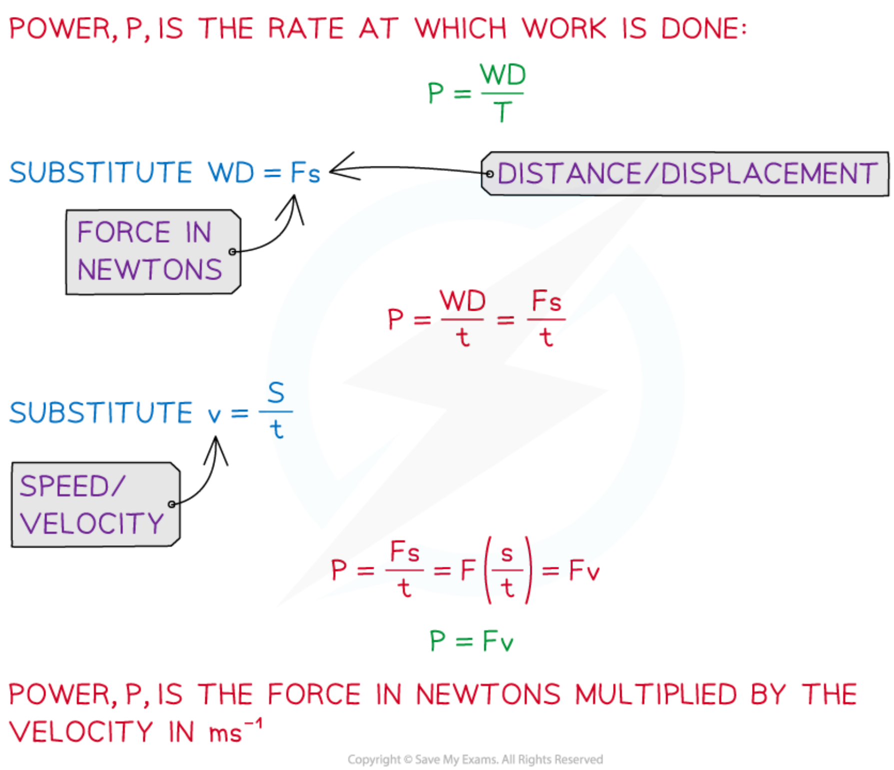
$$P = Fv$$

  - The velocity must be in the same direction as the force
  - When labelling forces on a diagram, the rearrangement $F = \frac{P}{v}$ can be useful to write the force generated by a particular power, at a particular speed
  - For a constant velocity and a constant force, the above formula can be derived by recalling the formula

$$v = \frac{s}{t}$$

where v is speed or **velocity** and s is distance or **displacement**.

Your notes



- The derivation can also be shown for a constant force, *F* N and considering the rate of doing work over a very small interval of time
  - $P = F\dfrac{\delta s}{\delta t}$
  - As the time δt gets smaller and approaches $\dfrac{ds}{dt}$ then
  - $P = F\dfrac{ds}{dt} = Fv$
- The units for power are **watts (W)** or **kilowatts (kW)**
  - **1 watt = 1 joule** per **second**
  - **1 kilowatt = 1000 watts = 1000 joules** per **second**

How is power used in calculating maximum speed?

- If the power is constant, then as a vehicle gains speed (*v* increases), the driving force, *F*, must decrease
  - This can be observed by considering $P = Fv$
- If the vehicle maintains maximum power, its driving force will decrease as its speed increases and it will eventually reach a point where its **resultant force** is zero (the driving force will balance the resistance forces)
  - When the resultant force becomes zero, by Newton's Second Law, acceleration will also become zero and the vehicle will be travelling at **maximum speed**

- The **maximum power output** of a vehicle can be found when the vehicle is moving at its maximum speed and the acceleration is zero
- If the maximum power output is known, then maximum speed can be found when the vehicle is travelling at its maximum power with zero acceleration and the resultant forces are balanced

Your notes

Examiner Tip

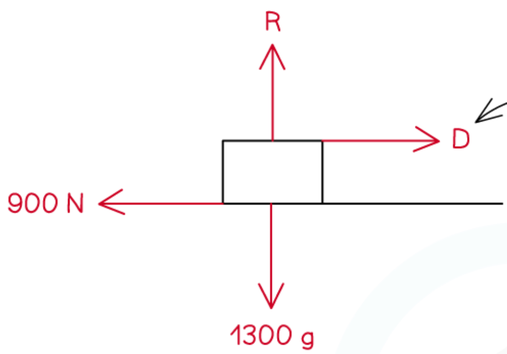
- Make sure you are using the correct force in your calculation, power is **only** generated by the **driving force** of an engine and so only this force should be used in the formula.
- Always draw a diagram and add the forces. If the question involves an inclined slope remember to resolve the weight into components parallel and perpendicular to the slope first.
- Remember to check the units carefully, power questions could be given in watts or kilowatts. It is also important to give your answer in the correct units, or if not specified, choose the most appropriate units for the question.
- Exam questions can say "an engine **works at a rate** of..." to mean "an engine has a power of"

Your notes

Worked example

A car of mass 1300 kg, including the driver, moves forwards on a straight horizontal road. There is a constant resistive force of 900 N acting on the car. Its maximum possible speed is 40 m s^{-1} Calculate the maximum power that the engine of the car can produce.

Start with a diagram:



It's a good idea to use D for driving force so as not to get confused with the resultant force or if friction is involved

Maximum power will occur when the car is moving at maximum speed and the resultant force is zero

At maximum speed $a = 0$ so the resultant force is zero

$$D - 900 = 0$$
$$D = 900$$

Power = Driving Force × speed

$$P_{max} = v_{max} D$$
$$= 40 \times 900$$
$$= 36000$$

Maximum power = 36000 W = 36 kW

Your notes

Worked example

A car of mass 1 tonne is moving at a constant speed of 9 ms^{-1} up a straight road inclined at 9° to the horizontal. The engine is working at a rate of 22 kW. Find the magnitude of the non-gravitational resistance to motion, to 3 significant figures.

Your notes

Convert into standard units (1 tonne = 1000 kg, 1 kW = 1000 W)

1 tonne = 1000 kg , 22 kW = 22000 W

The car has an engine, so introduce a driving force

driving force, D N

The question asks about resistance, so introduce a resistance force

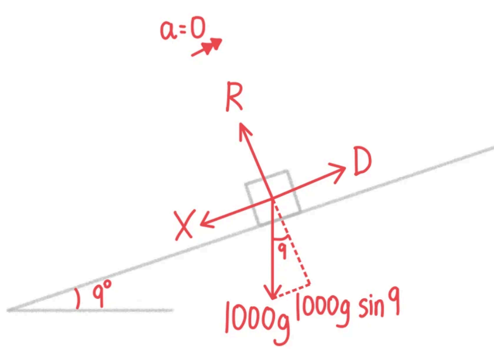
non-gravitational resistance, X N

The speed is constant, so acceleration is zero

$v = 9, \quad a = 0$

Draw a force diagram

Find the component of weight down the slope



You need to find X

Form an equation using Newton's 2nd Law parallel to the slope

$(\nearrow) \quad D - X - 1000g\sin 9 = 0$ (*)

Your notes

You need to find D

Use "Power = Driving Force × Speed"

$$22000 = D \times 9$$

Rearrange to find D

$$D = \frac{22000}{9}$$

Substitute this value of D back into (*) and solve to find X

$$\frac{22000}{9} - X - 1000g \sin 9 = 0$$

$$\frac{22000}{9} - 1000g \sin 9 = X$$

Let $g = 9.8$

$$X = 911.3866...$$

The question asks for 3 s.f.

$$\boxed{911 \text{ N to 3 s.f.}}$$