Constant acceleration formulae

It is also possible to derive all of the constant acceleration formulae using integration, provided that we consider that **acceleration is constant**.

Given a body has constant acceleration a, $\boxed{\text{initial velocity } u}$ and its initial displacement is 0 m, prove that:

(a) Final velocity: $v = u + at$

(b) Displacement: $s = ut + \frac{1}{2}at^2$

$$s \underset{\text{int}}{\overset{\text{diff}}{\rightleftarrows}} v \underset{\text{int}}{\overset{\text{diff}}{\rightleftarrows}} a$$

a)

$v = \int a \, dt$

$v = at + c$

$v = u, t = 0$

$u = a \times 0 + c$

$c = u$

$v = at + u$

$\underline{\underline{v = u + at}}$

b)

$s = \int v \, dt$

$s = \int (u + at) \, dt$

$s = ut + \frac{a}{2}t^2 + c$

$s = 0, t = 0$

$0 = u \times 0 + \frac{a}{2} \times 0^2 + c$

$c = 0$

$\underline{\underline{s = ut + \frac{1}{2}at^2}}$

$\int 3t \, dt$

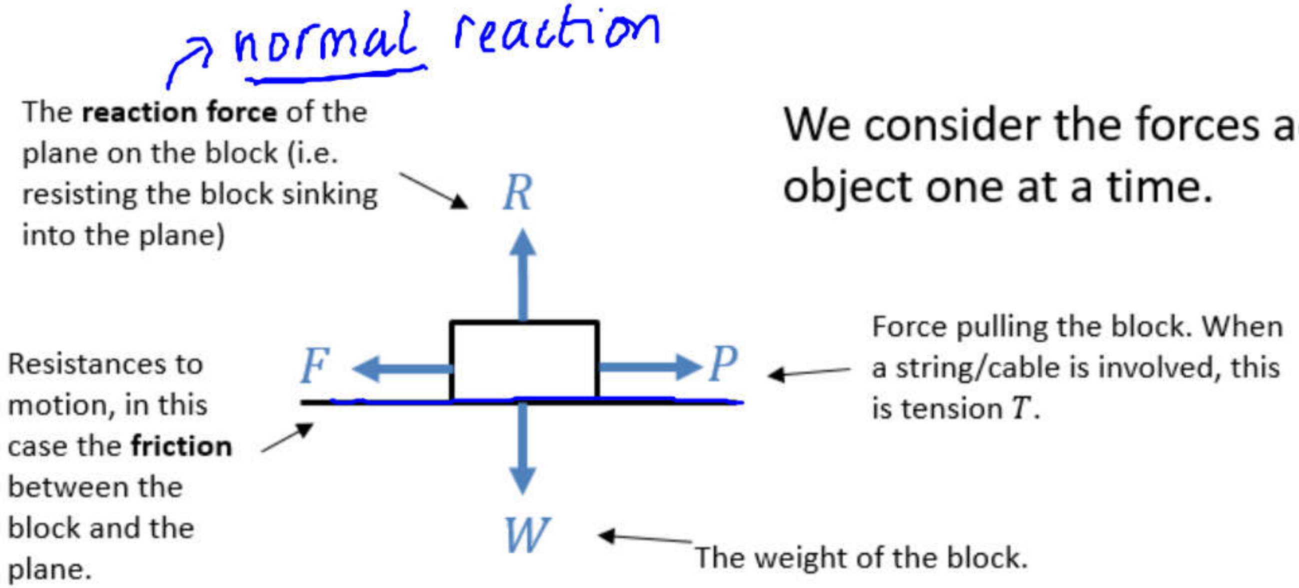
$\frac{3}{2}t^2$

Ex 11E

Forces (Year 1)

Force Diagrams and Common Forces

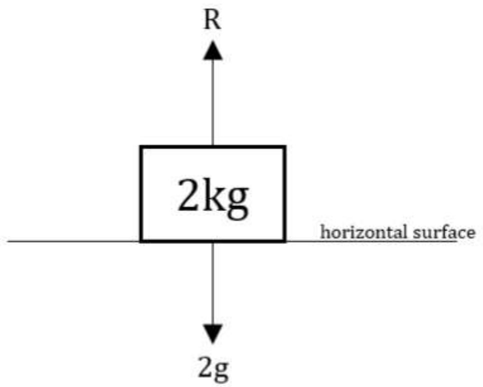
We consider the forces acting on each object one at a time.

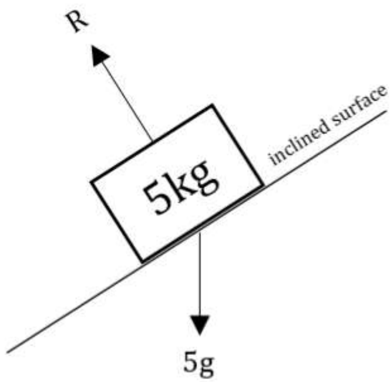


Recall that we often model an object as a **particle**, i.e. a point with negligible dimensions.

weight = mass × g (where g is the acceleration due to gravity, $g = 9.8ms^{-2}$
$W = mg$
Weight acts vertically downwards (obviously)

The normal reaction (sometimes called the contact force) is the force which acts on a box/particle from the surface that it is on.
It is called a **normal** reaction because it acts normal (perpendicular) to the surface.
It is called a normal **reaction** because it has reacted to the forces in the opposing direction.
For example, when you are sat on a chair, your weight acts down, but the chair (surface) has a reaction force upwards which stops you falling to the floor. This is the normal reaction.
We use the letter R for the normal reaction.





Note that the weight acts vertically downwards, but the normal reaction is perpendicular to the slope

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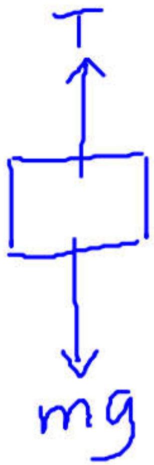
Let’s consider the forces acting on a body suspended from a light inextensible string

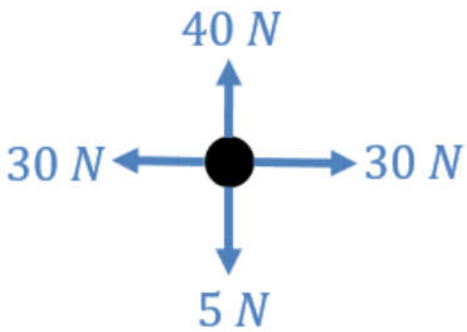


Newton's 1st Law of Motion states than an object at rest will stay at rest and that an object moving with constant velocity will remain at that velocity unless an unbalanced force acts on the object.

In other words, if the object is not accelerating, the **forces are balanced in every direction**, e.g. forces up = forces down and forces left = forces right.

We use $R(\;)$ to 'resolve' the forces in a particular direction. **This is standard notation expected in exams.**

The '**resultant force**' is the overall force acting on the object. An object will accelerate in the direction of the resultant force.

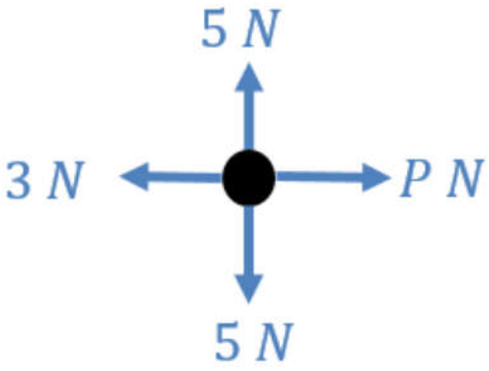


$R(\uparrow)$: $40 - 5 = 35$

$R(\rightarrow)$: $30 - 30 = 0$

so, the particle will accelerate upwards

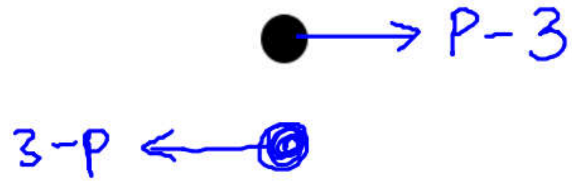
Finding Resultant Forces



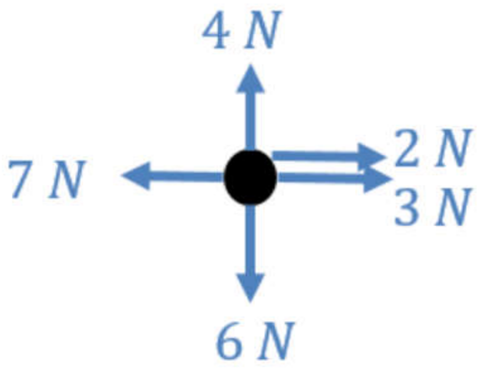
$R(\uparrow)$: $5-5=0$

$R(\rightarrow)$: $P-3$

$R(\leftarrow)$: $3-P$



I do not usually write the 'N' on diagrams



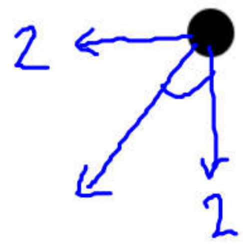
$R(\downarrow)$: $6-4=2$

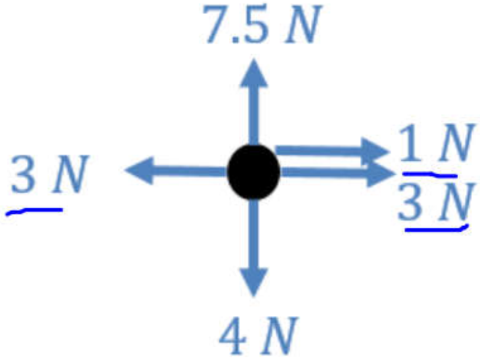
$R(\leftarrow)$: $7-2-3=2$

or

$R(\uparrow)$: $4-6=-2$

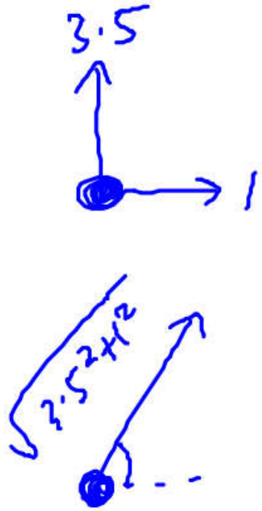
$R(\rightarrow)$: $2+3-7=-2$

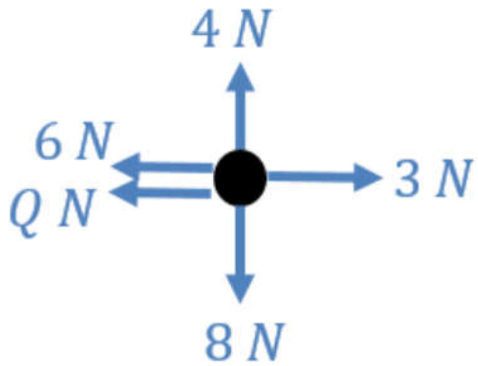




$R(\rightarrow)\quad 3+1-3=1$

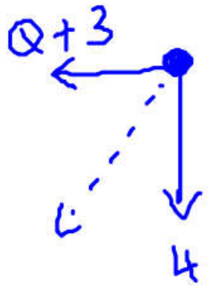
$R(\uparrow)\quad 7.5-4=3.5$





$R(\downarrow)\quad 8-4=4$

$R(\leftarrow)\quad Q+6-3=Q+3$

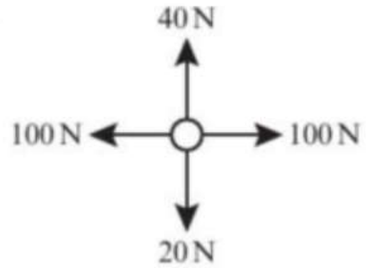


12 Each diagram shows the forces acting on a particle.

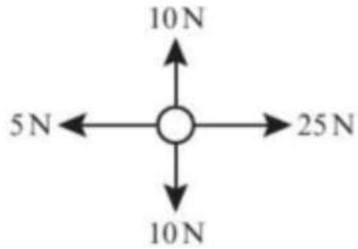
i Work out the size and direction of the resultant force.

ii Describe the motion of the particle.

a



b



R(↑) 20

R(→)

$\bullet \longrightarrow 20$

13 A truck is moving along a horizontal level road. The truck's engine provides a forward thrust of 10 000 N. The total resistance is modelled as a constant force of magnitude 1600 N.

a Modelling the truck as a particle, draw a force diagram to show the forces acting on the truck.

b Calculate the resultant force acting on the truck.

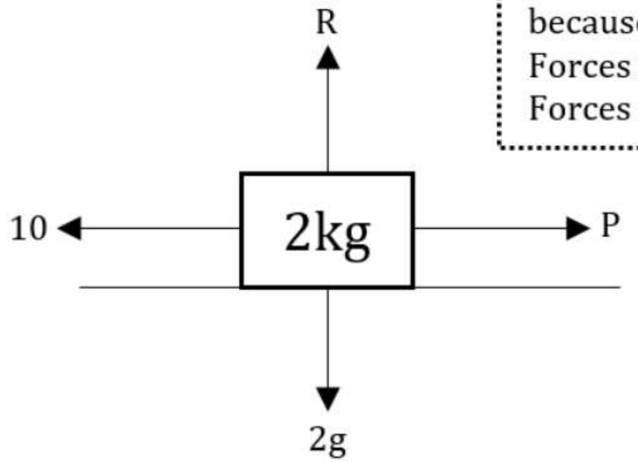
$1600 \longleftarrow \square \longrightarrow 10000$

R(→) 10000 − 1600

= <u>8400 N</u>.

Particles in equilibrium - at rest/constant velocity

If the particle is at rest or moving at constant velocity, there is no resultant force.
Left = right and up = down

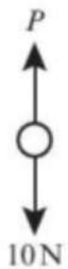


Given that this particle is rest, work out the value of P and R.
Clearly, R = 2g and P = 10, because it is in equilibrium.
Forces left = forces right.
Forces up = forces down

equilibrium

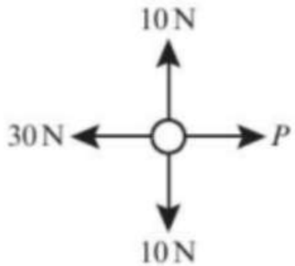
8 Given that each of the particles is stationary, work out the value of P:

a



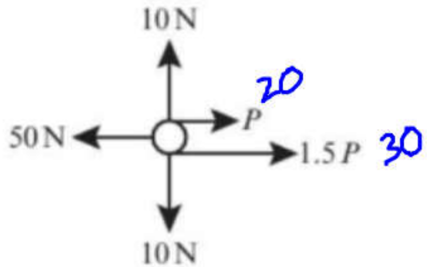
$P = 10$

b



$P = 30$

c

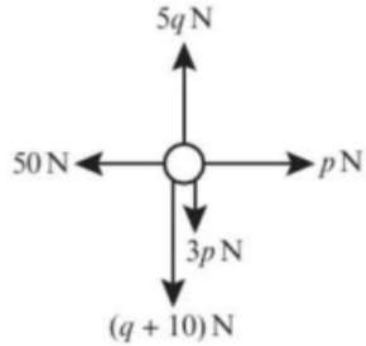


$50 = P + 1.5P$

$50 = 2.5P$

$P = \frac{50}{2.5} = \underline{\underline{20}}$

10 The diagram shows a particle acted on by a set of forces. Given that the particle is at rest, find the value of p and the value of q.



R(↔)

$p = 50$

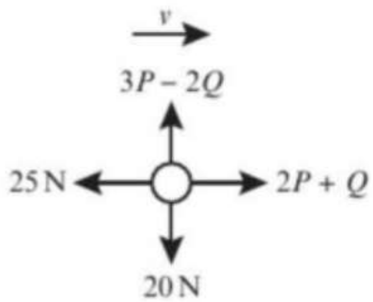
R(↕) $5q = q + 10 + 3p$

$4q = 10 + 150$

$4q = 160$

$q = 40$

11 Given that the particle in this diagram is moving with constant velocity, v, find the values of P and Q.



Problem-solving

Set up two simultaneous equations.

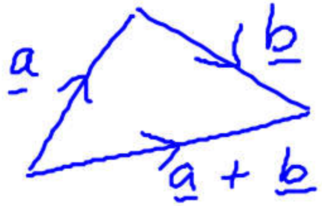
$2P + Q = 25$

$3P - 2Q = 20$

$P = 10$. $Q = 5$.

Forces as Vectors

Forces have direction, and therefore we can naturally write them as vectors, either in ***i*-*j*** notation or as column vectors.



✏ You can find the resultant of two or more forces given as vectors by adding the vectors.

The forces $2\boldsymbol{i} + 3\boldsymbol{j}$, $4\boldsymbol{i} - \boldsymbol{j}$, $-3\boldsymbol{i} + 2\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

$\underline{F}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\quad \underline{F}_2 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $\quad \underline{F}_3 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$\underline{F}_4 = \begin{pmatrix} a \\ b \end{pmatrix}$

Equilibrium, $\underline{R} = 0$

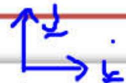
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$2 + 4 - 3 + a = 0$

$3 + a = 0 \qquad \underline{\underline{a = -3}}$

$3 - 1 + 2 + b = 0 \qquad \underline{\underline{b = -4}}$

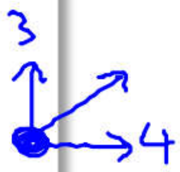
$4 + b = 0$



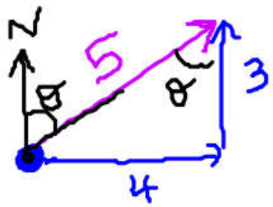
The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

(b) Work out the magnitude and bearing of the resultant force.



$$\underline{R} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{N}}}$$



$|\underline{R}| = \sqrt{3^2 + 4^2} = 5\text{ N}$

$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 53.1°$ $\qquad \underline{\underline{053°}}$

Edexcel M1 May 2009 Q2

A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$. **(2)**

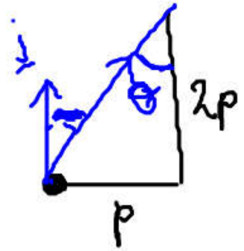
The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$, $+2\mathbf{j}$

(b) find the value of p. **(4)**

Tip: If a vector is parallel to say $\begin{pmatrix}1\\2\end{pmatrix}$, then it could be any multiple of it, i.e. $\begin{pmatrix}k\\2k\end{pmatrix}$

a)

$$\underline{F}_2 = \begin{pmatrix}p\\2p\end{pmatrix}$$



$$\theta = \tan^{-1}\left(\frac{p}{2p}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$$

b)

$$\underline{R} = \underline{F}_1 + \underline{F}_2 = \begin{pmatrix}1\\-3\end{pmatrix} + \begin{pmatrix}p\\2p\end{pmatrix} = \begin{pmatrix}1+p\\-3+2p\end{pmatrix}$$

$\underline{R}$ is parallel to $\underline{i}$

$$\underline{R} = k\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}k\\0\end{pmatrix} \qquad \begin{pmatrix}k\\0\end{pmatrix} = \begin{pmatrix}1+p\\-3+2p\end{pmatrix}$$

$$0 = -3 + 2p$$

$$\frac{3}{2} = p$$

$$k\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}1+p\\-3+2p\end{pmatrix}$$

$$\begin{pmatrix}k\\2k\end{pmatrix} = \begin{pmatrix}1+p\\-3+2p\end{pmatrix}$$

Ex 10B Q9