Chapter 7a: Algebraic Methods, Factor Theorem

1:: Algebraic Fractions

Simplify $\frac{2x^2-5x-3}{2x^2-9x+9}$

2:: Dividing Polynomials

Divide $x^3+2x^2-17x+6$ by $(x-2)$

3:: The Factor Theorem

Given that $(x-1)$ is a factor of $5x^3-9x^2+2x+a$, find the value of a.

Simplifying Algebraic Fractions

$$\frac{7x^4-2x^3+6x}{x}=$$

$$\frac{x^2-1}{x^2+x}=$$

$$\frac{x^2+3x+2}{x+1}=$$

$$\frac{2x^2 + 11x + 12}{x^2 + 9x + 20} =$$

$$\frac{4 - x^2}{x^2 + 2x - 8} =$$

Ex 7A

Normal Long Division

$$423 \div 11$$

$$11 \div 4 = 2\, rem\ 3$$

dividend
(the thing we're dividing)

divisor
(the thing we're dividing by)

quotient

remainder

$1735 \div 15$

$25168 \div 9$

Algebraic Long Division

$(6x^3 + 28x^2 - 7x + 15) \div (x + 5)$

Divide
Multiply
Subtract
Bring Down

Use different columns for different powers.

You only need to look at the highest power term in the divisor when dividing.

Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $x - 4$.

Divide
Multiply
Subtract
Bring Down

Find the remainder when $3x^3 - 2x + 4$ is divided by $x - 1$.

Divide
Multiply
Subtract
Bring Down

Let $f(x) = 8x^3$. By dividing $8x^3 - 1$ by $2x - 1$, write $f(x)$ in the form $(2x - 1)(ax^2 + bx + c)$

Divide
Multiply
Subtract
Bring Down

Write $25x^4 + 75x^3 + 6x^2 - 28x - 6$ in the form $(5x + 3)(ax^3 + bx^2 + cx + d)$

Divide
Multiply
Subtract
Bring Down

The Factor Theorem

$$x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

We can see that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.
What would happen if x is 2?

✏ The Factor Theorem states that if $f(x)$ is a polynomial then:

- If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$.
- Conversely, if $(x - p)$ is a factor of $f(x)$, then $f(p) = 0$.

Show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.

Let $f(x) = x^3 + x^2 - 4x - 4$

Writing $f(x) = \cdots$ gives us appropriate notation, i.e. $f(2)$, to show we're substituting 2 into the polynomial on the next line.

4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

Fully factorise $2x^3 + x^2 - 18x - 9$.

You can use the 'Table' mode on your calculator to try lots of values in a range

Using Factor Theorem to find unknown coefficients

Given that $2x + 1$ is a factor of $6x^3 + ax^2 + 1$, determine the value of a.

Given that $3x - 1$ is a factor of $3x^3 + 11x^2 + ax + 1$, determine the value of a.

Given that $(x + 1)$ and $(x - 2)$ are factors of $ax^3 + bx^2 - 9x - 10$, determine the values of a and b.

Exam Questions

5. $$f(x) = x^3 + 3x^2 - 4x - 12.$$

(*a*) Using the factor theorem, explain why f(x) is divisible by $(x + 3)$.

(2)

(*b*) Hence fully factorise f(x).

(3)

(*c*) Show that $\dfrac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x}$ can be written in the form $A + \dfrac{B}{x}$, where A and B are integers to be found.

(3)

(Total for Question 5 is 8 marks)

9.
$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

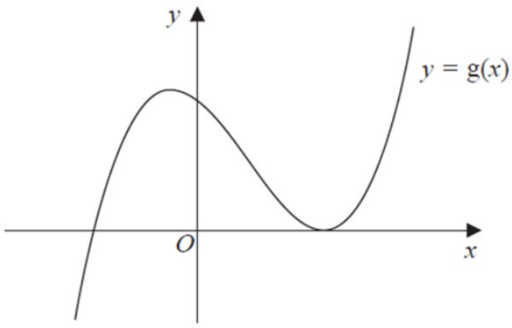

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate $f(2)$

(ii) Write $f(x)$ as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

5. $f(x) = x^3 + ax^2 - ax + 48$, where a is a constant

Given that $f(-6) = 0$

(a) (i) show that $a = 4$

(ii) express $f(x)$ as a product of two algebraic factors.

(4)