NUMBER 5

2019 Pure Paper 2
Vectors, Year 1

33%

10.

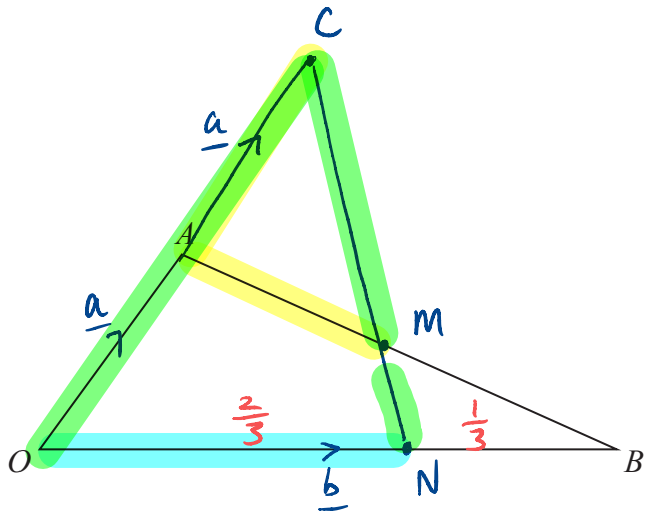

Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON : NB = 2 : 1$

(2)

$\overrightarrow{CN} = \lambda\overrightarrow{CM}$

$\overrightarrow{ON} = \mu\overrightarrow{OB}$

→ find $\overrightarrow{ON}$ in 2 different pathways
→ then compare coefficients.

a) $\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM}$

$= \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$

$= -\underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= -\underline{a} + \frac{1}{2}\underline{b} - \frac{1}{2}\underline{a}$

$= -\frac{3}{2}\underline{a} + \frac{1}{2}\underline{b}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

b) $\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$

$= \overrightarrow{OC} + \lambda\overrightarrow{CM}$

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$= 2\underline{a} + \lambda\left(-\frac{3}{2}\underline{a} + \frac{1}{2}\underline{b}\right)$

$= 2\underline{a} + -\frac{3}{2}\lambda\underline{a} + \frac{1}{2}\lambda\underline{b}$

$= \left(2 - \frac{3}{2}\lambda\right)\underline{a} + \frac{1}{2}\lambda\underline{b}$

c) $\overrightarrow{ON} = \mu\overrightarrow{OB}$ $\quad \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\underline{a} + \frac{1}{2}\lambda\underline{b}$

$= \mu\underline{b}$

$\overrightarrow{ON} = \overrightarrow{ON}$

$\mu\underline{b} = \left(2 - \frac{3}{2}\lambda\right)\underline{a} + \frac{1}{2}\lambda\underline{b}$

$\mu = \frac{1}{2}\lambda$ $\quad 0 = 2 - \frac{3}{2}\lambda$

$\mu = \frac{1}{2} \times \frac{4}{3}$ $\quad 2 = \frac{3}{2}\lambda$

$\mu = \frac{4}{6} = \frac{2}{3}$ $\quad \frac{4}{3} = \lambda$

$\overrightarrow{ON} : \overrightarrow{NB}$

$\frac{2}{3}\underline{b} : \frac{1}{3}\underline{b}$

$ON : NB$

$2 : 1$

NUMBER 4

2019 Pure Paper 1
Proof/Functions

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

29%

10i) Proof by Exhaustion

Investigate if n is even / odd

If n is even then $n = 2k$ where k is an integer.

$n^2 + 2 = (2k)^2 + 2$
$= 4k^2 + 2$
$= 4(k^2) + 2$

So n^2+2 is 2 more than a multiple of 4, so is not divisible by 4.

If n is odd then $n = 2k+1$ where k is an integer

$n^2 + 2 = (2k+1)^2 + 2$
$= 4k^2 + 4k + 1 + 2$
$= 4k^2 + 4k + 3$
$= 4(k^2 + k) + 3$

So n^2+2 is 3 more than a multiple of 4, so is not divisible by 4.

Because n^2+2 is not divisible by 4 when n is odd and when n is even, n^2+2 is not divisible by 4 for $n \in \mathbb{N}$

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Question 10 continued

ii) $|3x - 28| \geq x - 9$

$|3x - 28| = x - 9$

$3x - 28 = x - 9$
$2x = 19$
$x = 9.5$

$-(3x - 28) = x - 9$
$-3x + 28 = x - 9$
$37 = 4x$
$x = 9.25$

$9.25 < x < 9.5$ Try something inside the range.

Let $x = 9.3$ $|3 \times 9.3 - 28| = 0.1$ LHS
$9.3 - 9 = 0.3$ RHS.

LHS < RHS, so statement is false.

Let $x = 0$ $|3 \times 0 - 28| = 28$ LHS
$0 - 9 = -9$ RHS

LHS > RHS, so statement is true.

Hence statement is sometimes true.

(Total for Question 10 is 6 marks)

NUMBER 3

2019 Pure Paper 2
??!?

28%

5.

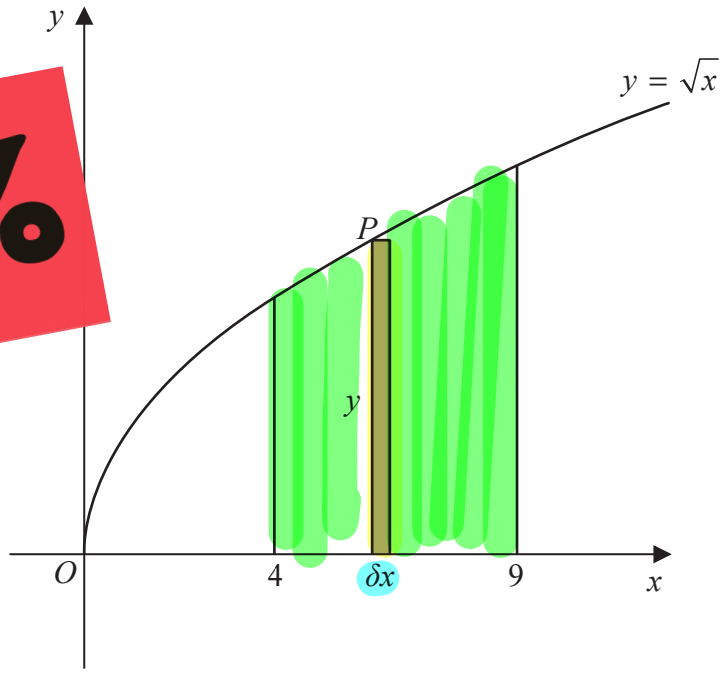

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x = \int_4^9 \sqrt{x}\,dx$$

(3)

$$\int_4^9 x^{1/2}\,dx = \left[\frac{2}{3}x^{3/2}\right]_4^9$$

$$= \frac{2}{3}(9)^{3/2} - \frac{2}{3}(4)^{3/2}$$

$$= \frac{2}{3} \times 27 - \frac{2}{3} \times 8$$

$$= \frac{38}{3}$$

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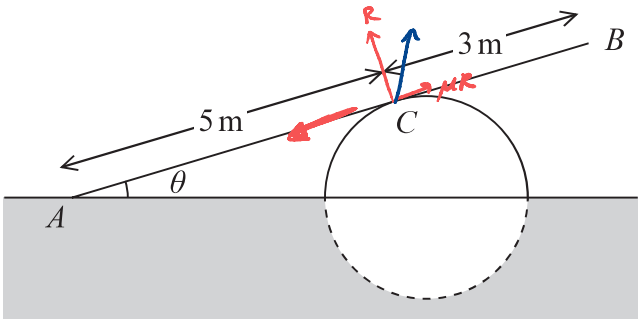
4.



25%

Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

Because the drum is smooth, the reaction force is only from the normal reaction, and hence is perpendicular. **(1)**

(b) Find the magnitude of the resultant force acting on the ramp at A. **(9)**

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

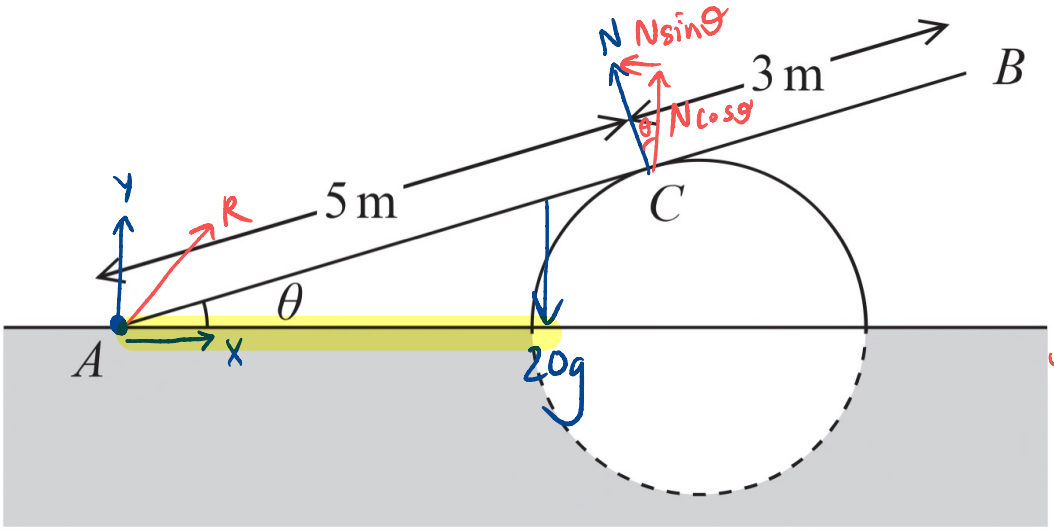
Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

c) This will decrease the magnitude of the normal reaction at C.

Question 4 continued



$m = 20kg$

✓Resolve ↕
✓Resolve ↔
Take moments.

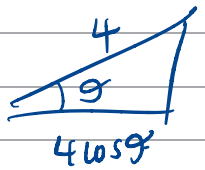
$\tan\theta = \frac{7}{24}$ $\sin\theta = \frac{7}{25}$ $\cos\theta = \frac{24}{25}$

Resolving ↔ $N\sin\theta = X$

$\frac{7}{25}N = X$

Resolving ↕ $Y + N\cos\theta = 20g$

$Y + \frac{24}{25}N = 20g$



Take moments about A $N \times 5 = 20g \times 4\cos\theta$

$5N = 20g \times 4 \times \frac{24}{25}$

$5N = 752.64$

$N = 150.528$

$X = \frac{7}{25}N$

$= \frac{7}{25} \times 150.528$

$= 42.147\ldots$

$Y = 20g - \frac{24}{25}N$

$Y = 51.493\ldots$

$R = \sqrt{X^2 + Y^2}$

$= 66.5N$ (3sf)

NUMBER 1

2018 Statistics Large Data Set

SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. Helen believes that the random variable C, representing cloud cover from the large data set, can be modelled by a discrete uniform distribution.

↳ oktas 0 – 8

(a) Write down the probability distribution for C.

(2)

(b) Using this model, find the probability that cloud cover is less than 50%

(1)

Helen used all the data from the large data set for Hurn in 2015 and found that the proportion of days with cloud cover of less than 50% was 0.315

(c) Comment on the suitability of Helen's model in the light of this information.

(1)

(d) Suggest an appropriate refinement to Helen's model.

(1)

↗ 50% of sky is cloudy

1a)

c	0	1	2	3	4	5	6	7	8
$P(C=c)$	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9

19%

1b) $P(C < 4) = P(C \leq 3)$

$= \frac{4}{9} \approx 0.444$

1c) Helen's model predicted 44% of days with less than 50% of cover cover.

This is much higher than the reality of 31.5%, so Helen's model does not seem suitable.

1d) Because cloud coverage will vary by month, Helen should not use a uniform distribution, and instead vary the probabilities.

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Question Number	Year	Paper	Average Score	Max Score	Average Percentage
Question 1	2018	Stats	0.97	5	19%
Question 4	2019	Mechanics	2.72	11	25%
Question 5	2019	Pure 2	0.83	3	28%
Question 10	2019	Pure 1	1.72	6	29%
Question 10	2019	Pure 2	1.99	6	33%
Question 14	2019	Pure 1	2.36	7	34%
Question 14	2019	Pure 2	5.11	15	34%
Question 8	2019	Pure 2	2.08	6	35%
Question 4	2018	Pure 1	1.39	4	35%
Question 2	2019	Pure 2	1.39	4	35%
Question 5	2018	Stats	5.3	14	38%
Question 5	2019	Mechanics	5.15	13	40%
Question 14	2018	Pure 2	4.04	10	40%
Question 12	2019	Pure 1	4.15	10	42%
Question 3	2018	Stats	4.63	11	42%
Question 2	2019	Mechanics	3.37	8	42%
Question 5	2019	Stats	5.64	13	43%
Question 14	2018	Pure 1	4.36	10	44%
Question 11	2019	Pure 1	3.11	7	44%
Question 1	2019	Pure 2	1.37	3	46%
Question 13	2019	Pure 2	4.79	10	48%
Question 11	2019	Pure 2	5.37	11	49%
Question 2	2019	Pure 1	2.45	5	49%
Question 8	2019	Pure 1	4.93	10	49%
Question 13	2019	Pure 1	5.52	11	50%
Question 12	2019	Pure 2	3.53	7	50%
Question 9	2019	Pure 2	4.56	9	51%
Question 9	2019	Pure 1	2.55	5	51%
Question 8	2018	Pure 2	3.58	7	51%
Question 9	2018	Mechanics	6.79	13	52%
Question 4	2019	Pure 1	3.25	6	54%
Question 3	2019	Mechanics	6.54	12	55%
Question 6	2019	Pure 1	4.38	8	55%
Question 7	2019	Pure 2	3.86	7	55%
Question 3	2019	Pure 1	2.79	5	56%
Question 2	2018	Stats	3.93	7	56%
Question 2	2019	Stats	6.18	11	56%
Question 6	2019	Pure 2	5.63	10	56%
Question 10	2018	Pure 2	4.52	8	57%
Question 4	2018	Stats	7.36	13	57%
Question 4	2019	Stats	5.23	9	58%
Question 7	2019	Pure 1	4.21	7	60%
Question 12	2018	Pure 2	5.44	9	60%
Question 9	2018	Pure 1	6.22	10	62%
Question 10	2018	Mechanics	9.34	15	62%
Question 5	2019	Pure 1	6.3	10	63%
Question 3	2019	Stats	5.81	9	65%
Question 3	2018	Pure 2	3.28	5	66%
Question 10	2018	Pure 1	5.29	8	66%
Question 4	2019	Pure 2	3.97	6	66%
Question 8	2018	Mechanics	5.35	8	67%
Question 5	2018	Pure 2	4.05	6	68%
Question 11	2018	Pure 2	4.84	7	69%
Question 1	2019	Mechanics	4.17	6	70%
Question 13	2018	Pure 2	6.99	10	70%
Question 7	2018	Pure 1	4.99	7	71%
Question 11	2018	Pure 1	7.13	10	71%
Question 7	2018	Pure 2	6.47	9	72%
Question 1	2018	Pure 1	2.18	3	73%
Question 1	2018	Pure 2	4.36	6	73%
Question 4	2018	Pure 2	5.12	7	73%
Question 6	2018	Pure 2	4.4	6	73%
Question 8	2018	Pure 1	3.68	5	74%
Question 6	2018	Mechanics	4.49	6	75%
Question 1	2019	Stats	6.04	8	76%
Question 12	2018	Pure 1	7.56	10	76%
Question 13	2018	Pure 1	5.3	7	76%
Question 5	2018	Pure 1	3.84	5	77%
Question 9	2018	Pure 2	3.88	5	78%
Question 6	2018	Pure 1	8.07	10	81%
Question 2	2018	Pure 2	4.1	5	82%
Question 7	2018	Mechanics	6.64	8	83%
Question 3	2019	Pure 2	2.53	3	84%
Question 1	2019	Pure 1	2.58	3	86%
Question 3	2018	Pure 1	3.46	4	87%
Question 2	2018	Pure 1	6.4	7	91%