Sigma Notation

What does each bit of this expression mean?

The Greek letter, capital sigma, means 'sum'.

$$\sum_{r=1}^{5}(2r+1)$$

The numbers top and bottom tells us what r varies between. **<u>It goes up by 1 each time.</u>**

We work out this expression for each value of r (between 1 and 5), and add them together.

$$= \underset{r=1}{3} \quad \underset{r=2}{+5} \quad \underset{r=3}{+7} \quad \underset{r=4}{+9} \quad \underset{r=5}{+11} \quad = 35$$

$a = 3$

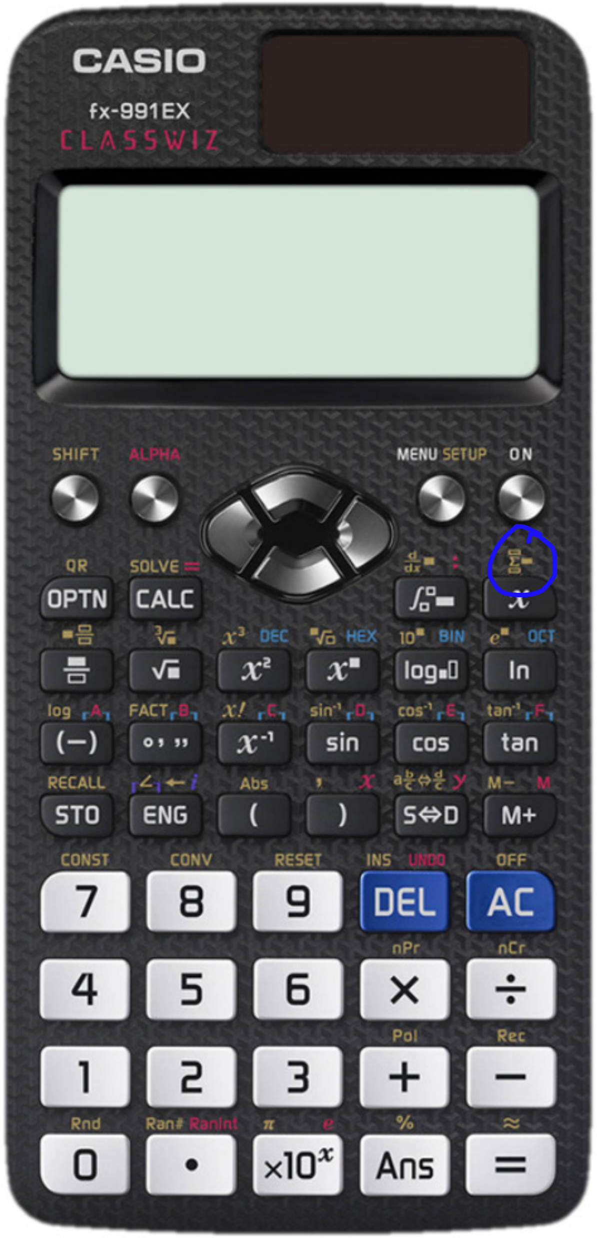
$d = 2$

$n = 5$

If the expression being summed (in this case $2r+1$) is **linear**, we get an **arithmetic series**. We can therefore apply our usual approach of establishing a, d and n before applying the S_n formula.

	First few terms?	Values of a, n, d or r?	Final result?
$\sum_{n=1}^{7} 3n$	$n=1 \quad n=2 \quad n=3$ $3 + 6 + 9 + 12$ $+15 + 18 + 21$	$a = 3$ $n = 7$ $d = 3$	$S_n = \frac{n}{2}(2a + (n-1)d)$ $S_n = \frac{n}{2}(a + L)$ $S_7 = \frac{7}{2}(3 + 21) = \underline{\underline{84}}$
$\sum_{k=5}^{15} (10 - 2k)$	$k=5 \quad k=6 \quad k=7$ $0 + (-2) + (-4)$ $0 - 2 - 4 - 6 - 8 \ldots$	$a = 0$ $d = -2$ $n = 11$ 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	$S_{11} = \frac{11}{2}(0 - 20)$ $= \underline{\underline{-110}}$ $\sum_{k=5}^{15}(10 - 2k) = \underline{\underline{-110}}$
$\sum_{k=1}^{12} 5 \times 3^{k-1}$	$k=1 \quad k=2 \quad k=3$ $5 \times 3^0 \quad 5 \times 3^1 \quad 5 \times 3^2$ $5 \xrightarrow{\times 3} 15 + 45$	$a = 5$ $r = 3$ $n = 12$	$S_n = \frac{a(1 - r^n)}{1 - r}$ $S_{12} = \frac{5(1 - 3^{12})}{1 - 3}$ $= \underline{1,328,600}$

On your calculator



$$\sum_{k=5}^{12} 2 \times 3^k$$

$$\sum_{x=5}^{12}(2 \times 3^x) = 1,594,080$$

Given that $\sum_{r=1}^{k} 2 \times 3^r = 59\,046$,

a show that $k = \dfrac{\log 19\,683}{\log 3}$ $= 9$

b For this value of k, calculate $\sum_{r=k+1}^{13} 2 \times 3^r$.

$\hookrightarrow 9$

a) $2 \times 3^1 = 6$

$2 \times 3^2 = 18$

$2 \times 3^3 = 54$

$\underbrace{6 + 18 + 54 + \dots}_{\text{how many terms?}} = 59{,}046$

$\underline{\underline{n}} = k$

$a = 6$

$n = k$

$r = 3$

$S_k = 59046$

$59046 = \dfrac{6(1 - 3^k)}{1 - 3}$

$59046 = -3(1 - 3^k)$

$-19682 = 1 - 3^k$

$3^k = 19683$

$k \log 3 = \log 19683$

$k = \dfrac{\log 19683}{\log 3}$

b) $\sum_{r=10}^{13} 2 \times 3^r = \sum_{r=1}^{13} (2 \times 3^r) - \sum_{r=1}^{9} (2 \times 3^r)$

$= 4{,}723{,}920$

Ex 3F

Exam Question - challenging!

13. Given that p is a positive constant,

(a) show that

$$\sum_{n=1}^{11} \ln(p^n) = k \ln p$$

where k is a constant to be found,

(2)

(b) show that

$$\sum_{n=1}^{11} \ln(8p^n) = 33\ln(2p^2)$$

(2)

(c) Hence find the set of values of p for which

$$\sum_{n=1}^{11} \ln(8p^n) < 0$$

giving your answer in set notation.

(2)

$a = 1$
$d = 1$
$n = 11$

$S_{11} = \frac{11}{2}(1+11)$
$= 66$

a) $\sum_{n=1}^{11} \ln p^n = \ln p + \ln p^2 + \ln p^3 + \dots + \ln p^{11}$

$= \ln p + 2\ln p + 3\ln p + \dots + 11\ln p$

$= \ln p(1+2+3+4+5+6+7+8+9+10+11)$

$= 66\ln p$

b) $\sum_{n=1}^{11} \ln 8p^n = \ln 8p + \ln 8p^2 + \ln 8p^3 + \dots + \ln 8p^{11}$

$= \ln 8 + \ln p + \ln 8 + 2\ln p + \dots + \ln 8 + 11\ln p$

$= 11\ln 8 + 66\ln p$

$= 11\ln 2^3 + 66\ln p$

$= 33\ln 2 + 33\ln p^2$

$= 33(\ln 2 + \ln p^2) = \underline{\underline{33\ln 2p^2}}.$

c) $33\ln 2p^2 < 0$

$\ln 2p^2 < 0$

$\{p : 0 < p < \frac{\sqrt{2}}{2}\}$

$\ln 2p^2 = 0$

$e^0 = 2p^2$

$1 = 2p^2$

$\frac{1}{2} = p^2$

$\frac{\sqrt{2}}{2} = p$

Question	Scheme	Marks	AOs
13 (a)	$\sum_{n=1}^{11} \ln(p^n) = \ln p + \ln p^2 + \ln p^3 + \ldots + \ln p^{11}$ $= \ln p + 2\ln p + 3\ln p + \ldots + 11\ln p$ $= \frac{11}{2}(2\ln p + (11-1)\ln p)$ or $\frac{1}{2}(11)(12)\ln p$	M1	3.1a
	$= 66\ln p \quad \{k = 66\}$	A1	1.1b
		(2)	
(b)	$S = \sum_{n=1}^{11} \ln(8p^n) = \ln 8p + \ln 8p^2 + \ln 8p^3 + \ldots + \ln 8p^{11}$ $= 11\ln 8 + 66\ln p$	M1	1.1b
	e.g. • $11\ln 8 + 66\ln p = 11\ln 2^3 + 66\ln p = 33\ln 2 + 66\ln p$ $= 33(\ln 2 + 2\ln p) = 33(\ln 2 + \ln p^2) = 33\ln(2p^2)$ * • $11\ln 8 + 66\ln p = 11\ln 2^3 + 66\ln p = 33\ln 2 + 66\ln p$ $= \ln(2^{33}p^{66}) = \ln\left((2p^2)^{33}\right) = 33\ln(2p^2)$ *	A1*	2.1
		(2)	
(c)	$S < 0 \Rightarrow 33\ln(2p^2) < 0 \Rightarrow \ln(2p^2) < 0$		
	so either $0 < 2p^2 < 1$ or $2p^2 < 1$	M1	2.2a
	$\Rightarrow p^2 < \frac{1}{2}$ and $p > 0 \Rightarrow 0 < p < \frac{1}{\sqrt{2}}$		
	In set notation, e.g. $\left\{p : 0 < p < \frac{1}{\sqrt{2}}\right\}$	A1	2.5
		(2)	