A Level · Edexcel · Maths

3 hours 40 questions

2.10 Combinations of Transformations (A Level only)

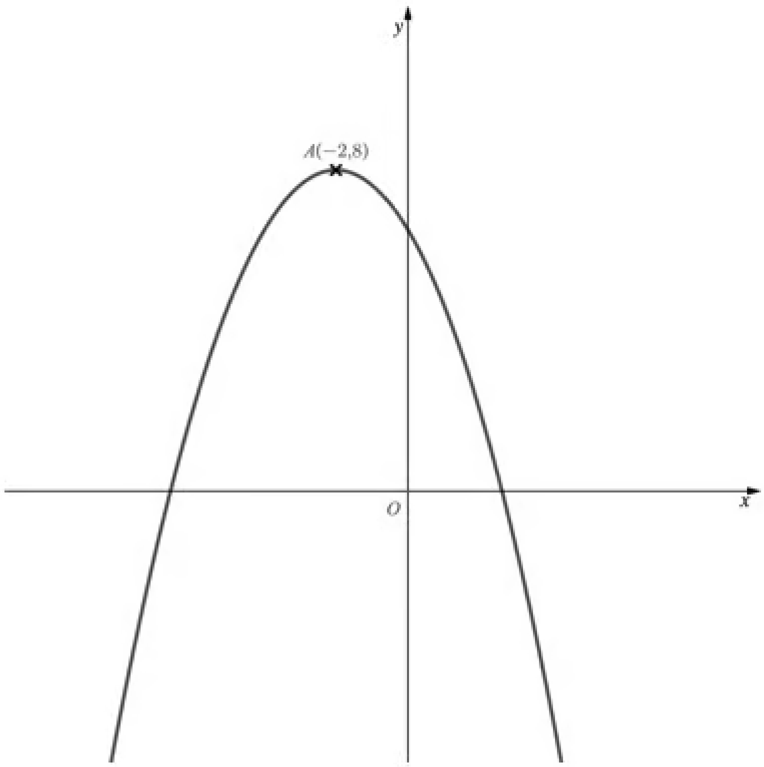
Easy (9 questions)	/40
Medium (12 questions)	/58
Hard (10 questions)	/51
Very Hard (9 questions)	/47
Total Marks	**/196**

Scan here to return to the course
or visit savemyexams.com

Easy Questions

1 The diagram below shows the graph of $y = \mathrm{f}(x)$.

The stationary point $A(-2, 8)$ is marked on the diagram.



On separate diagrams, sketch the following graphs

(i) $y = 2\mathrm{f}(x) + 1$

(ii) $y = \frac{1}{2}\mathrm{f}(x+1)$

On each diagram, state the coordinates of the image of point A under the given transformation.

(4 marks)

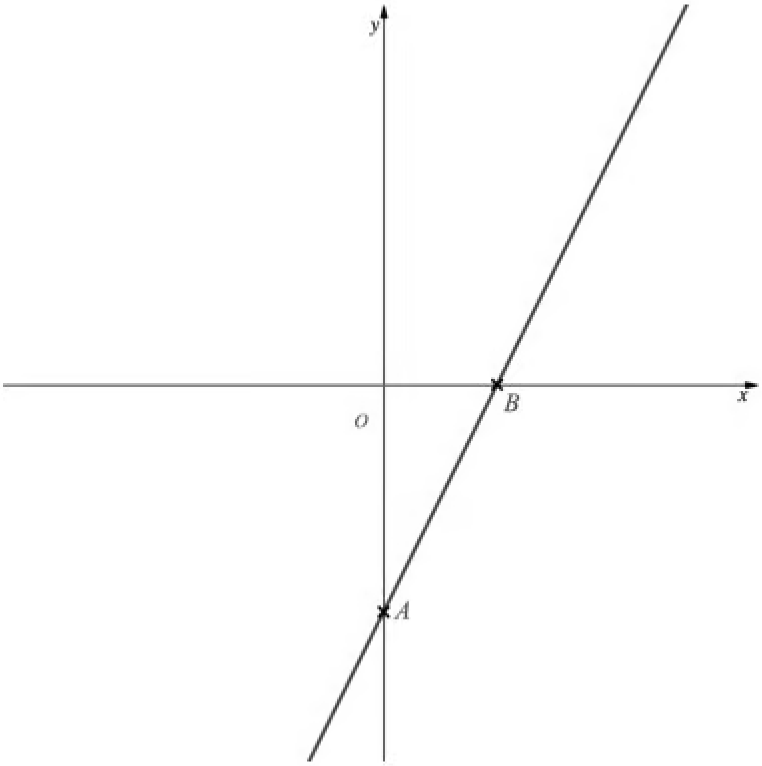
2 Given that $y = \mathrm{f}(x)$, find equations, in terms of $\mathrm{f}(x)$, for the following transformations:

(i) Translation by $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

(ii) Horizontal stretch of scale factor 2 followed by a vertical stretch of scale factor 3

(4 marks)

3 (a) The graph of $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 2x - 3$ is shown below.



Determine the coordinates of the points marked A and B.

(1 mark)

(b) (i) On the diagram above sketch the graph of $y = |\mathrm{f}(x-1)|$.

(ii) Determine the coordinates of the image of the points A and B under the transformation in part (i).

(5 marks)

4 Describe, in order, a sequence of transformations of the graph of $y = \mathrm{f}(x)$ given by the following equations:

(i) $y = 3\mathrm{f}(x) - 1$

(ii) $y = \frac{1}{3}\mathrm{f}(x - 1)$

(3 marks)

5 (a) The function $g(x)$ is given as $g(x) = 2x$.

On the same diagram, sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$
Label the coordinates of the points where each graph crosses the coordinate axes.

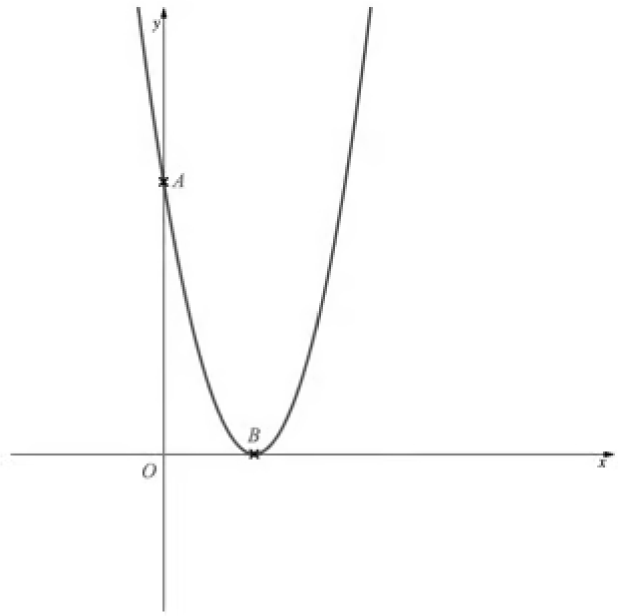
(2 marks)

(b) (i) Write down an expression for $g^{-1}(x)$ in terms of x.

(ii) Find an expression for $g^{-1}(x)$ in terms of $g(x)$ and state the type of transformation this would be.

(4 marks)

6 The equation $y = f(x)$, where $f(x) = (x-2)^2$, is shown below..



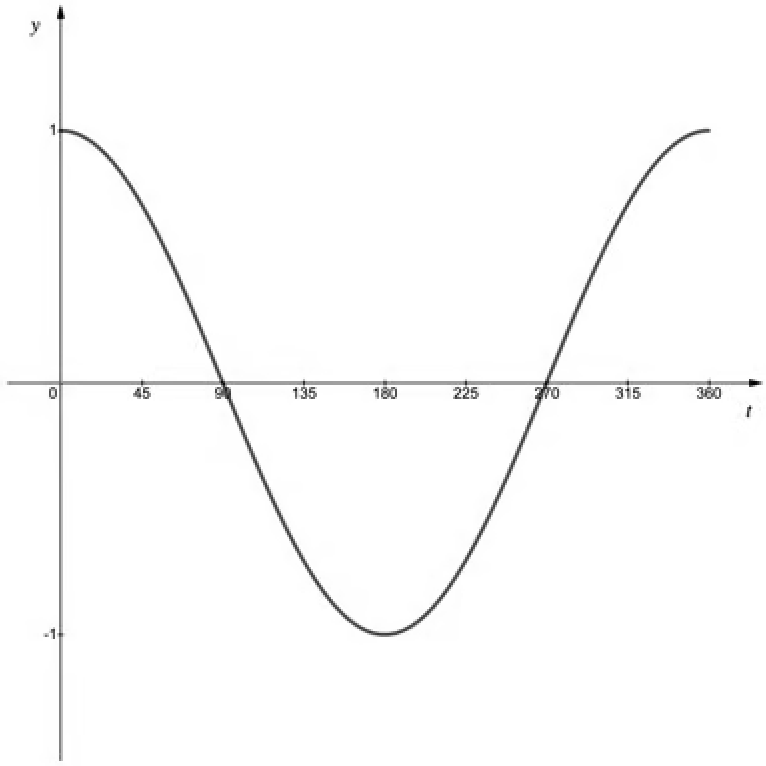
The points A and B are the points where the graph intersects the coordinate axes.

(i) Write down the coordinates of A and B.

(ii) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = -\mathrm{f}(x) - 4$.
Find the coordinates of the images of points A and B under these transformations.

(4 marks)

7 (a) The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \cos t$, $0° \le t \le 360°$.



(i) Write down the minimum value of $y = \mathrm{f}(t)$ in the given domain for t.

(ii) Write down the value of t for which this minimum occurs.

(2 marks)

(b) (i) Write down the minimum value of $y = 3\mathrm{f}(t - 45°)$ in the given domain for t.

(ii) Write down the value of t for which this minimum occurs.

(2 marks)

(c) Find, in terms of $f(t)$, the combination of transformations that would map the graph of $y = f(t)$ onto the graph of $y = 3\cos t + 1$, $0° \le t \le 360°$.

(2 marks)

8 The function $f(x)$ is to be transformed by a sequence of functions, in the order detailed below:

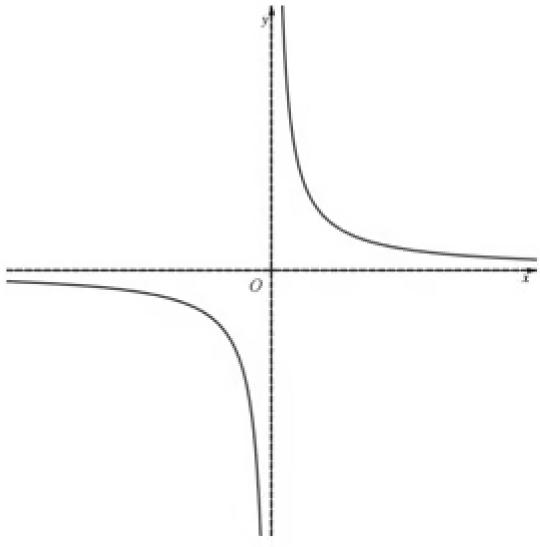
1. A reflection in the y-axis.
2. A vertical stretch of scale factor 4.
3. A translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Write down an expression for the combined transformation in terms of $f(x)$.

(3 marks)

9 The diagram below shows the graph of $y = g(x)$ where

$$g(x) = \frac{1}{x}, \qquad x \neq 0$$



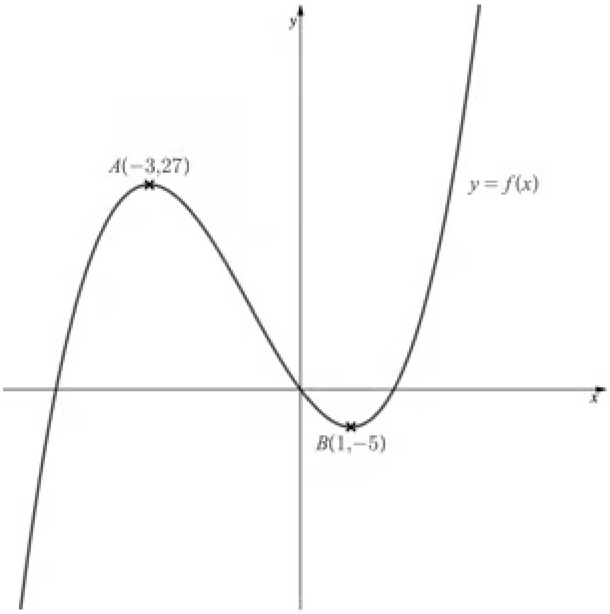
(i) Write down the equations of the two asymptotes.

(ii) Determine the equations of the two asymptotes on the graph of $y = \mathrm{g}(x-1)+5$.

(4 marks)

Medium Questions

1 The diagram below shows the graph of $y = \mathrm{f}(x)$.
The stationary points are marked on the diagram.



On separate diagrams, sketch the graphs with equation

(i) $y = 2\mathrm{f}(x) - 4$

(ii) $y = \mathrm{f}(x+1) + 3$

On each diagram, state the coordinates of the images of the points A and B under the given transformation.

(4 marks)

2 Describe, in order, a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the following graphs:

(i) $y = 3\mathrm{f}(x+2)$

(ii) $y = \mathrm{f}(-x) - 1$

(3 marks)

3 Given that $\mathrm{f}(x) = 3x^2 - 2x$ find an expression for $\mathrm{g}(x)$, where $\mathrm{g}(x)$ is obtained by applying the following sequence of transformations to $\mathrm{f}(x)$.

1. Translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
2. Vertical stretch of scale factor 4
3. Translation by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

(4 marks)

4 (a) (i) Sketch the graph of $y = \text{p}(x)$, where $\text{p}(x) = 3x - 4$.

(ii) On the same set of axes, sketch the graph of $y = p^{-1}(x)$.
Label the coordinates of the points where each graph crosses the coordinate axes

(4 marks)

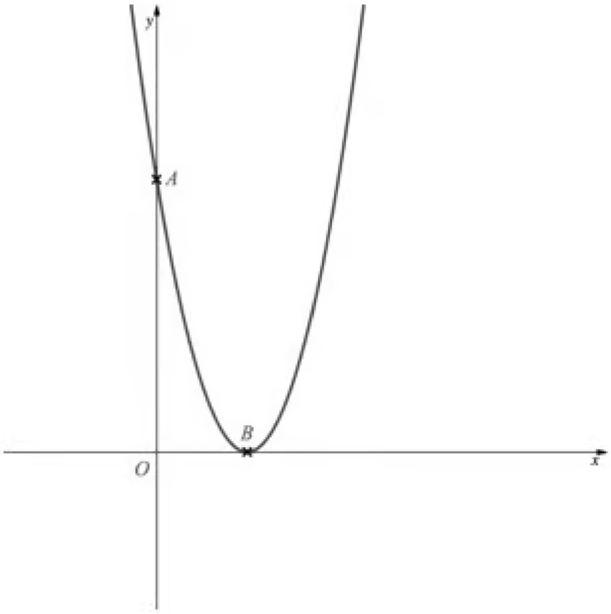
(b) (i) Find an expression for $p^{-1}(x)$.

(ii) Find an expression for $\frac{1}{9}\,[\text{p}(x) + 16]$.

(iii) What can you deduce about the sequence of transformations given by $\frac{1}{9}[\text{p}(x) + 16]$?

(4 marks)

5 (a) The equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - a)^2$, with $a > 1$, is shown below.



The points A and B are the points where the graph intercepts the coordinate axes.

Write down, in terms of a, the coordinates of A and B.

(2 marks)

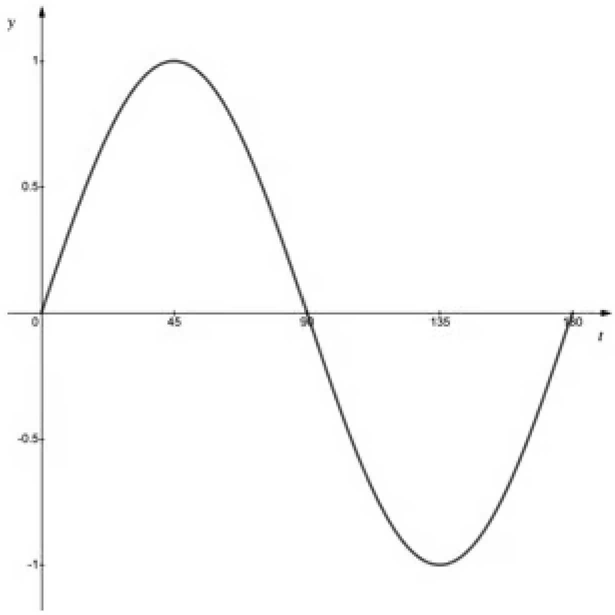
(b) Sketch the graph of $y = -\mathrm{f}(-x)$, labelling the images of the points A and B stating their coordinates in terms of a.

(3 marks)

(c) Write down the value of a such that the point A is three times as far from the origin as the point B.

(1 mark)

6 (a) The diagram shows the graph of $y = f(t)$, where $f(t) = \sin 2t, 0° \le x \le 180°$.



(i) Write down the maximum value of y when $y = 3f(t)$.

(ii) Write down the first value of t for which this maximum occurs.

(2 marks)

(b) (i) Write down the minimum value of when $y = 5f(t + 30°)$.

(ii) Write down the first value of t for which this minimum occurs.

(2 marks)

(c) Find, in terms of $f(t)$, the combination of transformations that would map the graph of $y = f(t)$ onto the graph of $y = 2 + \sin t$, $0° \le x \le 180°$.

(2 marks)

7 The function $\mathrm{f}(x)$ is to be transformed by a sequence of functions, in the order detailed below:

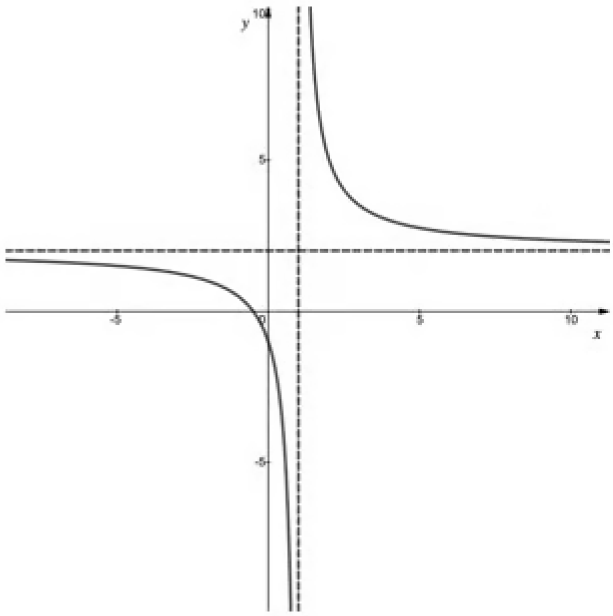
1. A horizontal stretch by scale factor 2
2. A reflection in the x-axis
3. A translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

Write down an expression for the combined transformation in terms of $\mathrm{f}(x)$.

(3 marks)

8 (a) The diagram below shows the graph of $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \frac{2x+1}{x-1}, \qquad x \neq 1$$



Write down the equations of the two asymptotes.

(2 marks)

(b) Determine the equations of the two asymptotes on the graph of $y = \mathrm{g}(2x) - 3$.

(2 marks)

(c) Determine the range of $|\mathrm{g}(3x) - 2|$.

(2 marks)

9 The point with coordinates $(1\ ,-4)$ is a stationary point on the graph with equation $y = \mathrm{h}(x)$.

Determine the coordinates of the stationary point on the graphs with the following equations:

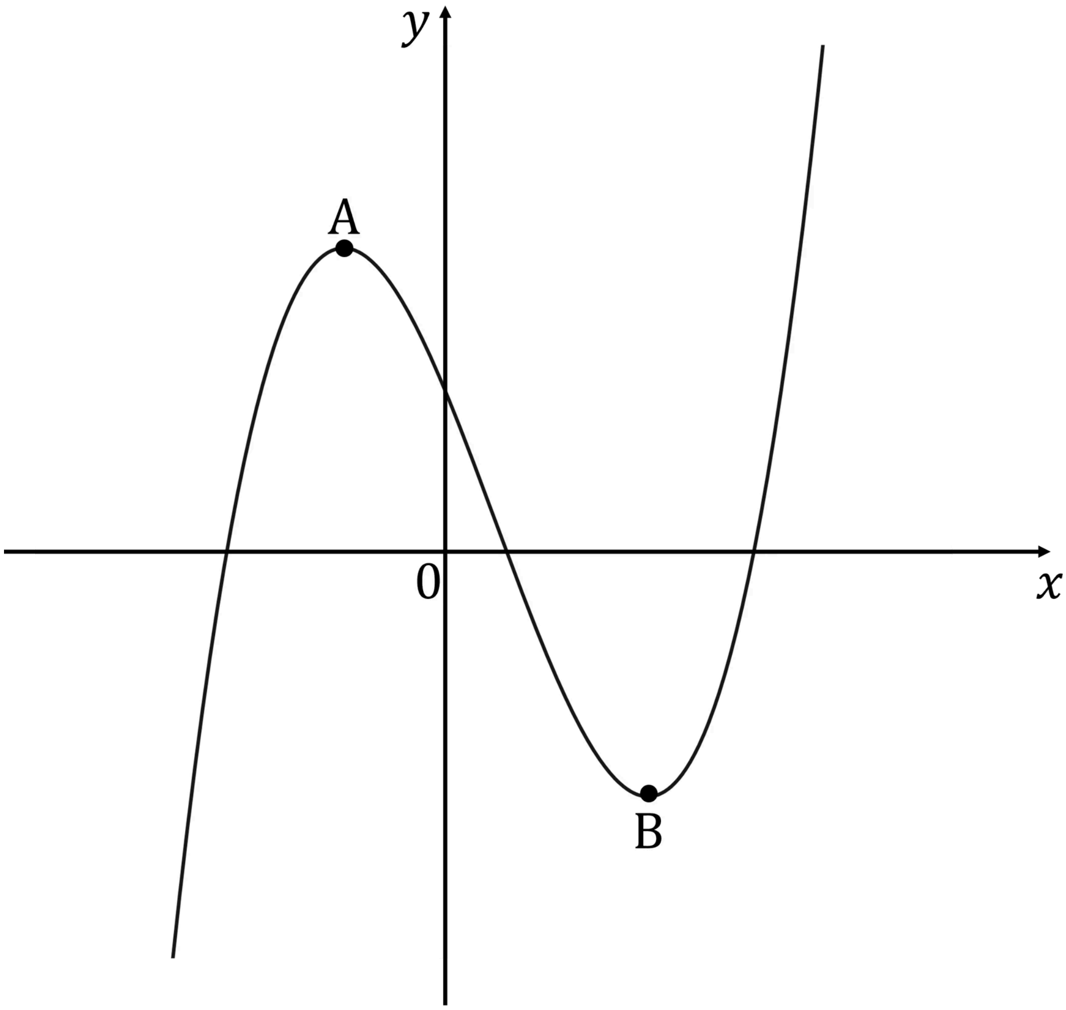
(i) $y = 2\mathrm{h}(x-1)$

(ii) $y = -\mathrm{h}(x+1)+2$

(iii) $y = |\mathrm{h}(3x)+2|$

(3 marks)

10 The graph of f is shown below. The points A(-2, 10) and B(4,-10) lie on the curve.



Sketch the graph of:

(i) $y = \mathrm{f}(2x - 1)$,
(ii) $y = \mathrm{f}(4 - x)$,

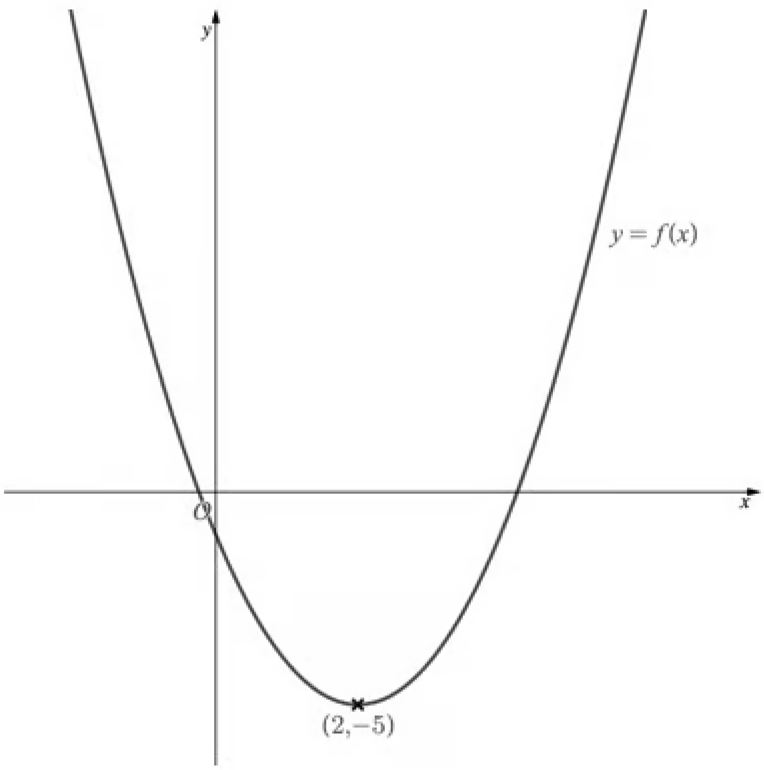
Clearly indicate the new coordinates of the images of the points A and B.

(7 marks)

11 Describe a sequence of transformations that map the graph of $y = \ln x$ onto the graph of $y = 5 + \ln(\frac{1}{2}x + 4)$..

(4 marks)

12 The turning point on the graph of $y = f(x)$ has coordinates $(2, -5)$ as shown on the diagram below.



(i)

On the diagram above sketch the graph of $y = |f(x)| + 1$ and state the coordinates of the turning point.

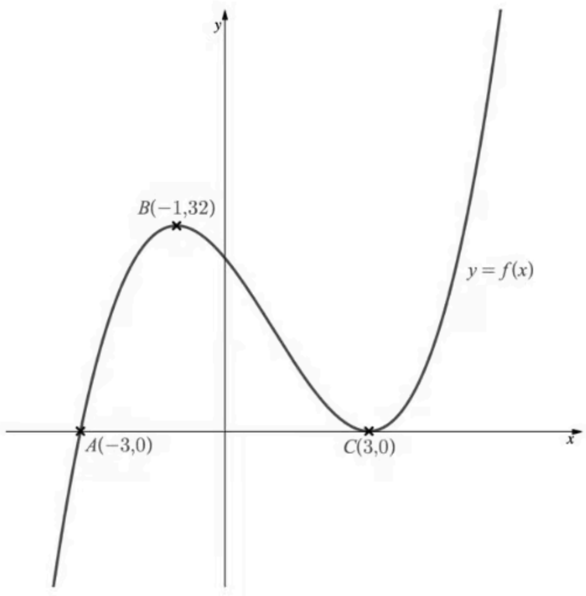
(ii) State the distance between the turning points on the graphs of $y = f(x)$ and $y = |f(x)| + 1$

.

(4 marks)

Hard Questions

1 The diagram below shows the graph of $y = \mathrm{f}(x)$.
The stationary points and intercepts with the x-axis are marked on the diagram.



On separate diagrams, sketch the graphs with equations

(i) $y = \mathrm{f}(\frac{1}{2}x) + 2,$

(ii) $y = -\mathrm{f}(x-1)$

On each diagram, mark the coordinates of the images of the points A, B and C under the given transformation.

(6 marks)

2 Describe, in order, a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the following graphs:

(i) $y = -\mathrm{f}(3x - 1)$

(ii) $y = 2\mathrm{f}(5 - x)$

(4 marks)

3 Given that $\mathrm{f}(x) = \ln(2x + 1)$ find an expression for $\mathrm{g}(x)$, where $\mathrm{g}(x)$ is obtained by applying the following sequence of transformations to $\mathrm{f}(x)$.

1. Translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$,
2. Horizontal stretch by scale factor $\frac{1}{2}$,
3. Reflection in the x-axis.

(4 marks)

4 (a) On the same axes sketch the graphs of $y = \mathrm{p}(x)$ and $y = p^{-1}(x)$, where $\mathrm{p}(x) = |2x|, x \leq 0$.

(3 marks)

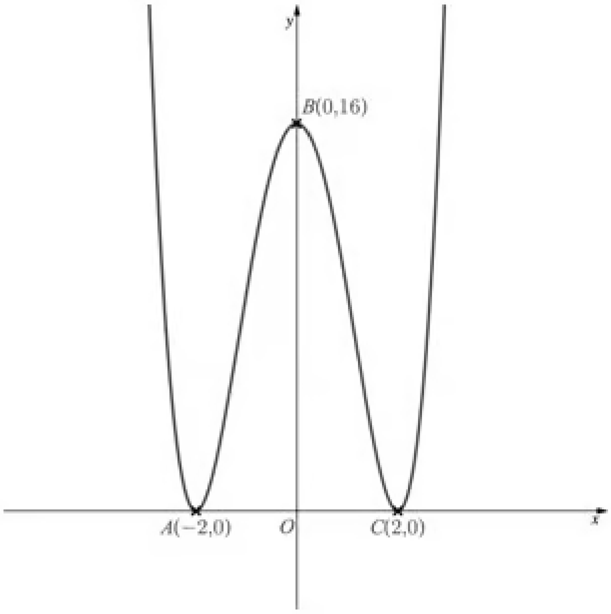
(b) Find an expression for $p^{-1}(x)$ and state its domain.

(3 marks)

(c) Show that $p^{-1}(x) = -\frac{1}{2}p\left(-\frac{1}{2}x\right)$.

(3 marks)

5 (a) A sketch of the graph with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x^2 - 4)^2$ is shown below.



The points A, B and C are the points where the graph intercepts the coordinate axes.

Sketch the graph of $y = -3\mathrm{f}(2x)$, labelling the images of the three points A, B and C

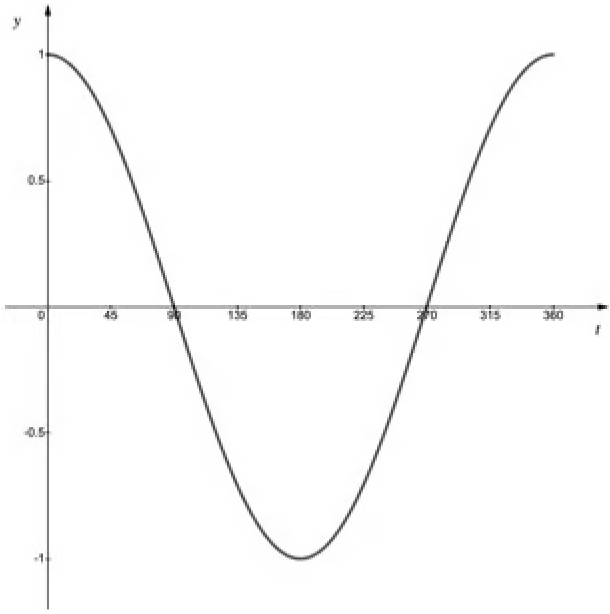
(3 marks)

(b) Suggest a combination of at least two transformations that will transform the points A, B and C such that none of them lie on the coordinate axes.

Give your answer in the form of an expression in terms of $\mathrm{f}(x)$.

(2 marks)

6 (a) The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \cos t$, $0° \leq x \leq 360°$.



(i) Write down the maximum value of y when $y = -2\mathrm{f}(3t)$.

(ii) Write down the value of t for which this maximum occurs.

(2 marks)

(b) Find, in terms of $\mathrm{f}(t)$, the combination of transformations that would map the graph of $y = \mathrm{f}(t)$ onto the graph of $y = 2 - 4\sin t, 0° \leq x \leq 180°$.

(2 marks)

7 The function $\mathrm{f}(x)$ is to be transformed by a sequence of functions, in the order detailed below.

1. A translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

2. A reflection in the y-axis
3. A vertical stretch by scale factor $\frac{2}{3}$
4. A translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

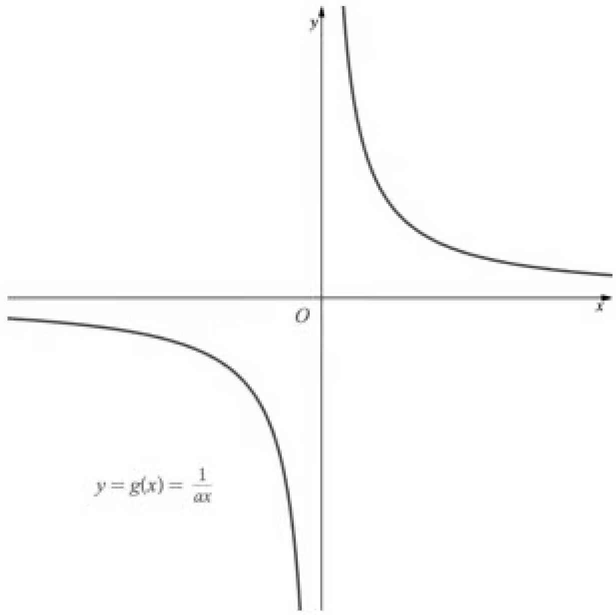
Write down the combined transformation in terms of $\mathrm{f}(x)$.

(3 marks)

8 (a) The diagram below shows the graph of $y = g(x)$ where

$$g(x) = \frac{1}{ax}, \qquad a, x \neq 0$$

where a is a constant.



(i) Write down the equations of the asymptotes on the graph of $y = g(x)$.

(ii) Determine the equations of the asymptotes on the graph of $y = 3g(2x + 1)$.

(5 marks)

(b) Determine the domain and range of the series of transformations to $y = f(x)$ where $f(x) = -2g(\frac{1}{3}x + 3) - 4$.

(3 marks)

9 The point with coordinates $(-3, -5)$ is a stationary point on the graph with equation $y = \mathrm{h}(x)$.

Determine the coordinates of the stationary point on graphs with the following equations:

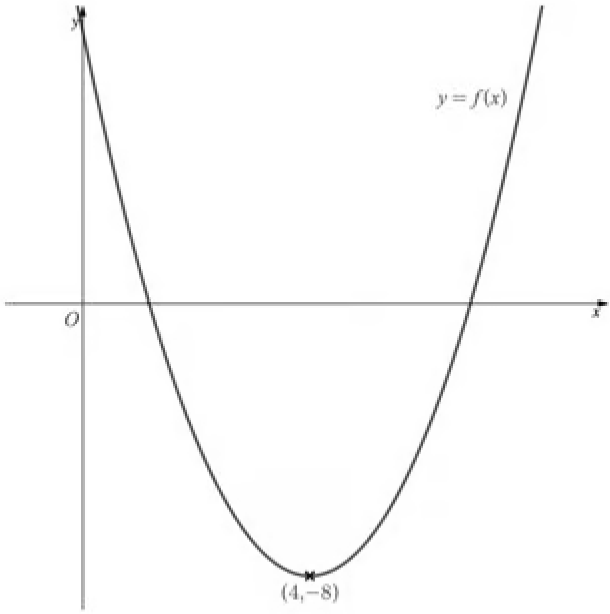
(i) $y = |\mathrm{h}(2x) - 5|$

(ii) $y = \mathrm{h}(\frac{1}{4}x + 1)$

(iii) $y = 2 - \frac{1}{5}\mathrm{h}(\frac{1}{2}x)$

(3 marks)

10 (a) The minimum point on the graph of $y = \mathrm{f}(x)$ has coordinates $(4\ ,-8)$ as shown on the diagram below.



Sketch the graph of $y = |\mathrm{f}(2x)| - 3$ and state the coordinates of the maximum point.

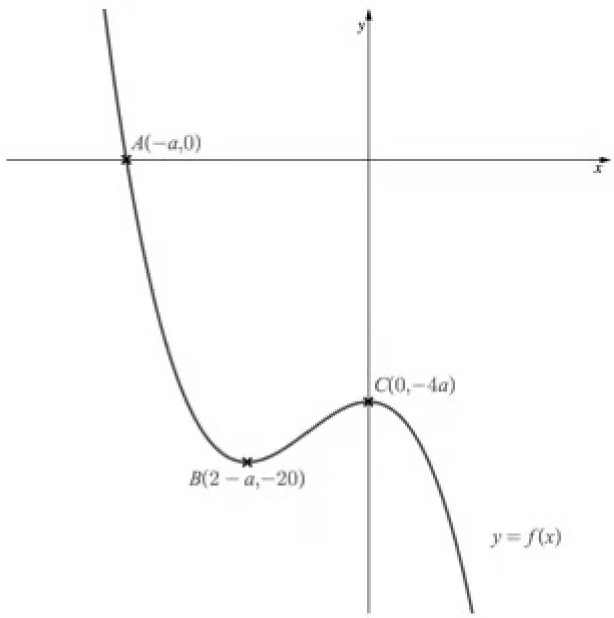
(3 marks)

(b) Find the exact distance between the minimum point on the graph of $y = \mathrm{f}(x)$ and the maximum point on the graph of $y = |\mathrm{f}(2x)| - 3$.

(2 marks)

Very Hard Questions

1 The diagram below shows the graph of $y = \mathrm{f}(x)$.
The stationary points and intercepts with the coordinate axes are marked on the diagram



On separate diagrams, sketch the graphs with the following equations:

(i) $y = 2\mathrm{f}(\frac{1}{3}x - 1)$

(ii) $y = -2\mathrm{f}(x + 1) + 1$

On each diagram, mark the coordinates of the images of the points A, B and C under the given transformation, giving your coordinates in terms of a.

(6 marks)

2 (a) Describe, in order, a sequence of transformations that would map the graph of $y = \mathrm{f}(x)$ onto each of the following graphs:

(i) $y = a\mathrm{f}(x+b)+c$ for the case when $a > 0$.

(ii) $y = -\mathrm{f}(-x)$

(4 marks)

(b) How, if at all, would your answer to part (a) (i) change if $a = 1$ or if $a < 0$?

(2 marks)

3 The function $\mathrm{f}(x) = e^{3x} - x - 6$ is transformed by a sequence of transformations as described below.

1. Horizontal stretch by scale factor 3,
2. The modulus of the function is then taken,
3. Reflection in the y-axis.

Write down the resulting transformation in terms of $\mathrm{f}(x)$ as well as an expression in terms of x

(4 marks)

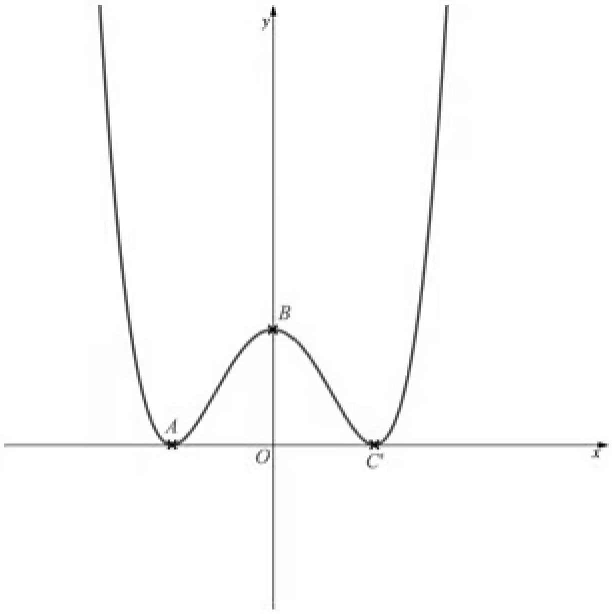
4 Show that the graph of $y = \mathrm{p}(x)$ where $\mathrm{p}(x) = 2x + 1$ maps onto the graph of its inverse under the transformations described by $\frac{1}{2}\mathrm{p}(\frac{1}{2}\ x) - 1$.

(4 marks)

5 Prove, that for a constant $k, k \neq 0$, if $\mathrm{f}(x) = kx$, then $\mathrm{f}^{-1}(x) = \frac{1}{k}\ \mathrm{f}(\frac{1}{k}x)$.

(4 marks)

6 (a) A sketch of the graph with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x^2 - a)^2$, with $a > 1$ is shown below.



The points A, B and C are points where the graph intercepts the coordinate axes.

Write down, in terms of a, the coordinates of A, B and C.

(2 marks)

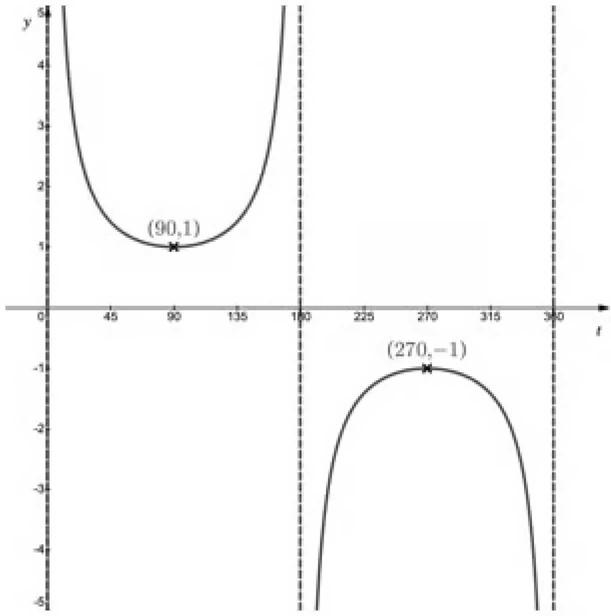
(b) Sketch the graph of $y = -\frac{1}{2}\mathrm{f}(x-1)$, labelling the images of the three points A, B and C and stating their coordinates in terms of a.

(3 marks)

(c) Suggest, in terms of $\mathrm{f}(x)$, a combination of at least two transformations, such that the points A, B and C transform to new positions but remain lying on their respective axes.

(2 marks)

7 (a) The diagram shows the graph of $y = f(t)$, where $f(t) = \text{cosec } t$, $0° \leq x \leq 360°$.



The vertical distance between the minimum point, $(90, 1)$, and the maximum point, $(270, -1)$ is 2. The horizontal distance between them is 180.

Find, in terms of a, the vertical and horizontal distances between the minimum and maximum point on the graph of $y = -\frac{1}{a}f(at), a \neq 0$.

(4 marks)

(b) Hence or otherwise show that the distance between the minimum and maximum point on the graph of $y = -\frac{1}{a}f(at), a \neq 0$

$$\frac{2\sqrt{8101}}{a}$$

(2 marks)

8 The point with coordinates $(-a, a^2)$, where $a>0$, is a stationary point on the graph with equation $y=\mathrm{h}(x)$.

Determine, in terms of a, the coordinates of the stationary point on the graphs with the following equations:

(i) $y=3\mathrm{h}\left(\frac{1}{2}x\right)-2$

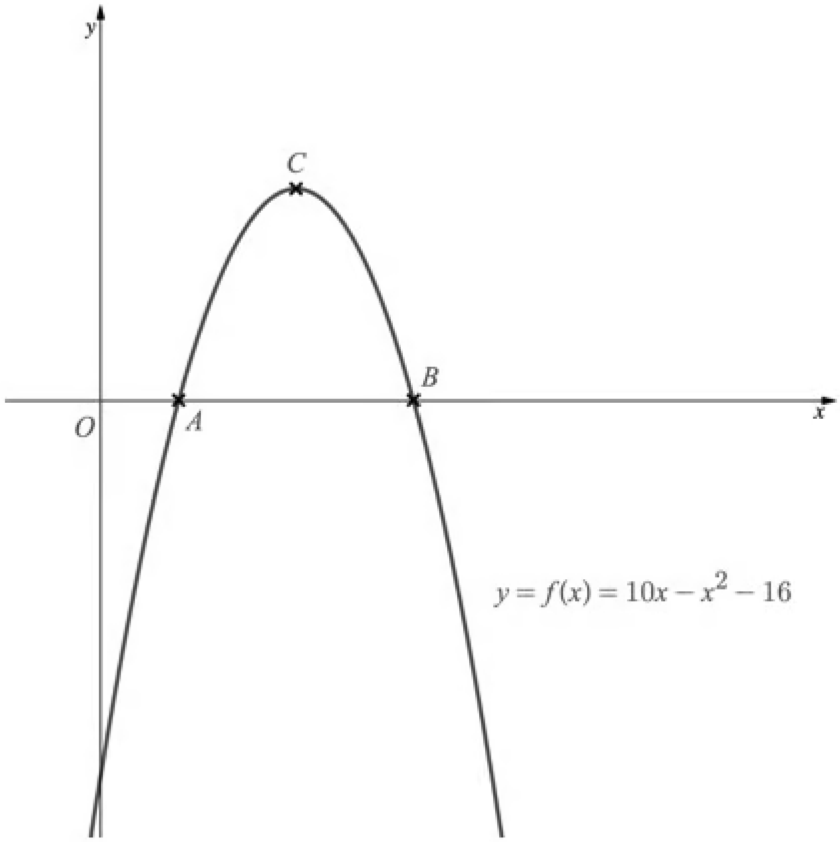
(ii) $y=3-|\mathrm{h}(-x)|$

(iii) $y=-\frac{1}{3}\left[\mathrm{h}\left(\frac{1}{3}x-\frac{1}{3}\right)+1\right]$.

(3 marks)

9 (a) A sketch of the graph with equation $y = f(x)$ where $f(x) = 10x - x^2 - 16$ is shown below.

Points A and B are the x-axis intercepts and point C is the maximum point on the graph.



On the diagram above, sketch the graph of $y = -\left|\frac{1}{4} f\left(\frac{1}{2}x\right)\right|$ labelling the image of the points A, B and C with A', B' and C'.

(3 marks)

(b) Show that the area of ABC is twice the area of triangle $A'B'C'$.

(4 marks)