

# 11.2 Vectors in 3 Dimensions (A Level only)

Easy (9 questions)	/44
Medium (8 questions)	/44
Hard (8 questions)	/51
Very Hard (8 questions)	/58
<b>Total Marks</b>	<b>/197</b>

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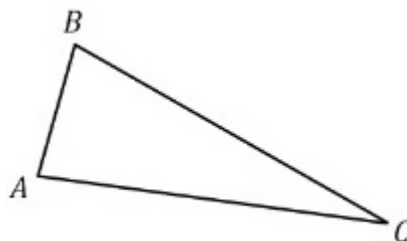


# Easy Questions

- 1 The coordinates of two points  $A$  and  $B$  are  $(3, -4, 2)$  and  $(-5, 2, -8)$  respectively. Find the distance from  $A$  to  $B$  giving your answer in the form  $a\sqrt{b}$ , where  $a$  and  $b$  are integers to be found.

(3 marks)

- 2 In the triangle  $ABC$ ,  $\overrightarrow{AB} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$  and  $\overrightarrow{AC} = 6\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}$ .



- (i) Find the vector  $\overrightarrow{BC}$ .
- (ii) Hence, or otherwise, find the distance  $BC$ , giving your answer to three significant figures.

(4 marks)

**3 (a)** Point  $P$  has coordinates  $(2, -1, -5)$  and point  $Q$  has coordinates  $(-6, -12, 11)$ .

(i) Find the vector  $\vec{PQ}$ .

(ii) Find the distance  $PQ$ .

**(3 marks)**

**(b)** Find the unit vector in the direction of  $\vec{PQ}$ .

**(2 marks)**

**4 (a)** Point  $A$  has coordinates  $(-3, 4, 9)$ .

The vector  $\vec{AB} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}$ , and the vector  $\vec{CD} = \begin{pmatrix} -2 \\ 2.5 \\ -1 \end{pmatrix}$ .

Find the coordinates of the point  $B$ .

**(1 mark)**

**(b)** State, with a reason, whether or not the vectors  $\vec{AB}$  and  $\vec{CD}$  are parallel.

**(2 marks)**

**5** The triangle  $PQR$  has vertices  $P(4, -3, 12)$ ,  $Q(3, -7, 9)$  and  $R(7, -9, 15)$ .

Find the distances  $PQ$ ,  $PR$  and  $QR$  and thus determine whether triangle  $PQR$  is scalene, isosceles or equilateral.

**(4 marks)**

**6** Vectors  $\mathbf{p}$  and  $\mathbf{q}$  are defined by

$$\mathbf{p} = 14\mathbf{i} + (a + b)\mathbf{j} + (c - b + 1)\mathbf{k}$$

$$\mathbf{q} = a\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$$

Given that  $\mathbf{p} = 2\mathbf{q}$ , find the values of  $a$ ,  $b$  and  $c$ .

**(4 marks)**

**7 (a)** A particle of mass 0.5 kg is acted upon by a force of  $(3\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$  N.

Use Newton's Second Law of Motion to find:

- (i) the acceleration of the particle while the force acts,
- (ii) the magnitude of the acceleration to 3 significant figures.

**(3 marks)**

**(b)** A second force now acts on the particle such that the resultant of the two forces is  $(4\mathbf{i} + 2\mathbf{k})$  N.

Find the second force in the form  $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$  N.

**(2 marks)**

**8 (a)** Two forces act upon a particle with a mass of 10 kg:

$$\mathbf{F}_1 = (2\mathbf{i} + p\mathbf{j} - 8\mathbf{k}) \text{ N}$$

$$\mathbf{F}_2 = (q\mathbf{i} + 3q\mathbf{j} + (p - q)\mathbf{k}) \text{ N}$$

Under the action of these two forces, the particle is in equilibrium.

- (i) Find the values of  $p$ ,  $q$ .
- (ii) Explain how you can verify your answer to part (i).

**(3 marks)**

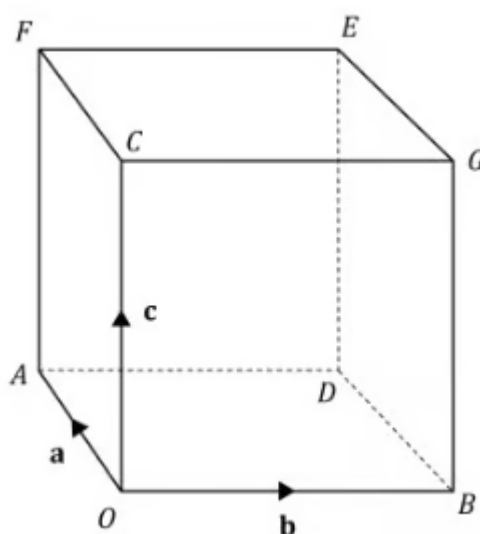
**(b)** A third force,  $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j} + pq\mathbf{k}) \text{ N}$ , is applied to the particle.

Work out:

- (i) the resultant force  $\mathbf{R}$  now acting on the particle,
- (ii) the acceleration of the particle under the action of  $\mathbf{R}$ ,
- (iii) the magnitude of the acceleration under the action of  $\mathbf{R}$ , giving your answer to three significant figures.

**(5 marks)**

9 (a) The diagram below shows a cube whose vertices are  $O, A, B, C, D, E, F$  and  $G$ .



$\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are the vectors  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$  and  $\overrightarrow{OC}$  respectively.

Find vectors  $\overrightarrow{OF}$  and  $\overrightarrow{OG}$ .

(2 marks)

(b) (i) Explain why vector  $\overrightarrow{GE} = \mathbf{a}$ .

(ii) Show that  $\overrightarrow{OF} + \overrightarrow{FE} = \overrightarrow{OG} + \overrightarrow{GE}$  and thus, state the vector  $\overrightarrow{OE}$ .

(3 marks)

(c)  $A$  has the coordinates  $(5, 0, 0)$ .

Find the exact distance of  $OE$ .



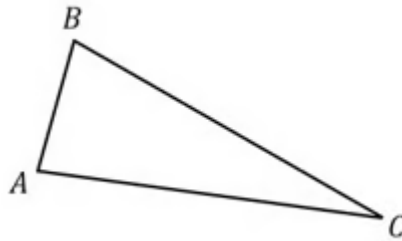
**(3 marks)**

# Medium Questions

- 1 The coordinates of points  $A$  and  $B$  are  $(-2, 5, -7)$  and  $(-7, k, 3)$  respectively. Given that the distance from  $A$  to  $B$  is  $5\sqrt{14}$  units, find the possible values of  $k$ .

(3 marks)

- 2 In the triangle  $ABC$ ,  $\vec{AB} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$  and  $\vec{AC} = -5\mathbf{i} - 4\mathbf{j} - 7\mathbf{k}$ .



Show that  $\angle BAC = 81.9^\circ$  to 1 d.p.

(5 marks)

**3 (a)** Point  $R$  has coordinates  $(-1, 5, 14)$  and point  $S$  has coordinates  $(7, -2, 12)$ .

Find:

- (i) the vector  $\overrightarrow{RS}$ ,
- (ii) the unit vector in the direction of  $\overrightarrow{RS}$ .

**(3 marks)**

**(b)** Find the angle  $\overrightarrow{RS}$  makes with the positive  $y$ -axis.

**(2 marks)**

**(c)** The vector  $\overrightarrow{TU} = -24\mathbf{i} + 21\mathbf{j} + 6\mathbf{k}$ .

Explain, giving a reason for your answer, whether the vectors  $\overrightarrow{RS}$  and  $\overrightarrow{TU}$  are parallel.

**(2 marks)**

**4** The triangle  $PQR$  has vertices  $P(12, 3, -3)$ ,  $Q(7, -8, k)$  and  $R(3, 3, -12)$ . Given that  $PQR$  is an equilateral triangle, find the value of  $k$ .

(4 marks)

5 Vectors  $\mathbf{a}$  and  $\mathbf{b}$  are defined by

$$\mathbf{a} = -12\mathbf{i} - 7\mathbf{j} + 15\mathbf{k}$$

$$\mathbf{b} = 4p\mathbf{i} + (pqr + 2qr - p)\mathbf{j} - pq\mathbf{k}$$

Given that  $\mathbf{a} = \mathbf{b}$ , find the values of  $p$ ,  $q$  and  $r$ .

(3 marks)

**6 (a)** A particle of mass 0.4 kg is acted upon by a force of  $(-2\mathbf{i} + 6\mathbf{j} + 10\mathbf{k})$  N.

Find:

- (i) the acceleration of the particle while the force acts,
- (ii) the magnitude of the acceleration to 3 s.f.

**(3 marks)**

**(b)** A second force with a magnitude of  $10\sqrt{3}$  N now acts on the particle. The resultant of the two forces is parallel to the vector  $\mathbf{j}$  and has a magnitude of 8 N.

Find this second force, giving your answer in the form  $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$  N.

**(4 marks)**

**7 (a)** Three forces act upon a particle with a mass of 5 kg:

$$\mathbf{F}_1 = (3\mathbf{i} - 7\mathbf{j} + p\mathbf{k}) \text{ N}$$

$$\mathbf{F}_2 = (q\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \text{ N}$$

$$\mathbf{F}_3 = (-2\mathbf{i} + r\mathbf{j} - 5\mathbf{k}) \text{ N}$$

Under the action of those three forces, the particle is in equilibrium.

Find the values of  $p$ ,  $q$  and  $r$ .

**(2 marks)**

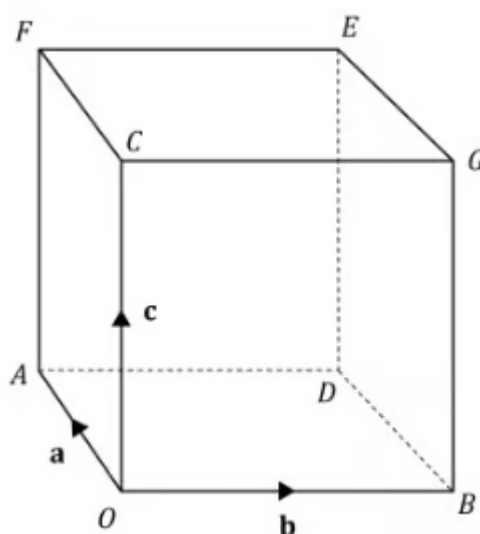
**(b)** The third force is doubled, so that the three forces now acting on the particle are  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $2\mathbf{F}_3$ .

Work out:

- (i) the resultant force  $\mathbf{R}$  now acting on the particle.
- (ii) the acceleration of the particle under the action of  $\mathbf{R}$ .
- (iii) the magnitude of the acceleration found in (ii), giving your answer as an exact value.

**(5 marks)**

**8 (a)** The diagram below shows a cube whose vertices are  $O, A, B, C, D, E, F$  and  $G$ .



$\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are the vectors  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$  and  $\overrightarrow{OC}$  respectively.

Find vectors  $\overrightarrow{OE}$  and  $\overrightarrow{AG}$ .

**(2 marks)**

**(b)** Let  $P$  be a point on  $OE$ , and let  $Q$  be a point on  $AG$ .

Explain why the vectors  $\overrightarrow{OP}$  and  $\overrightarrow{OQ}$  can be expressed in the forms

$$\begin{aligned}\overrightarrow{OP} &= \lambda \overrightarrow{OE} \\ \overrightarrow{OQ} &= \mathbf{a} + \mu \overrightarrow{AG}\end{aligned}$$

where  $\lambda$  and  $\mu$  are constants with  $0 \leq \lambda \leq 1$  and  $0 \leq \mu \leq 1$

**(2 marks)**

- (c) By solving the equation  $\overrightarrow{OP} = \overrightarrow{OQ}$ , using your results from (a) and (b), show that the diagonals  $OE$  and  $AG$  intersect each other, and determine the ratios into which they are cut by the point of intersection.

(4 marks)

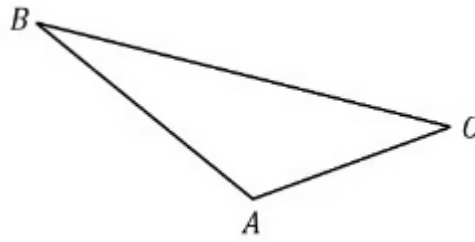


# Hard Questions

- 1 The coordinates of points  $A$  and  $B$  are  $(-1, 3, 14)$  and  $(2k, 3k, 13)$  respectively. Given that the distance from  $A$  to  $B$  is  $\sqrt{163}$  units, and that  $k$  is an integer, find the value of  $k$ .

(3 marks)

2 (a) In the triangle  $ABC$ ,  $\vec{AB} = 7\mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\vec{AC} = -2\mathbf{i} + 5\mathbf{k}$ .



Show that  $\angle BAC = 119.6^\circ$  to 1 d.p.

(5 marks)

(b) Hence find the area of triangle  $ABC$ , giving your area to 3 s.f.

(2 marks)

**3 (a)** Point  $R$  has position vector  $\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$  and point  $S$  has position vector  $10\mathbf{i} + 13\mathbf{k}$ .

Find the unit vector in the direction of  $\overrightarrow{RS}$ .

**(3 marks)**

**(b)** Find the angle  $\overrightarrow{RS}$  makes with the negative  $z$ -axis.

**(2 marks)**

**(c)** The vector  $\overrightarrow{TU} = -12\mathbf{i} + 8\mathbf{j} - 20\mathbf{k}$ .

Explain, giving a reason for your answer, whether the vectors  $\overrightarrow{RS}$  and  $\overrightarrow{TU}$  are parallel.

**(2 marks)**

**4** The triangle  $PQR$  has vertices  $P(2, -3, 1)$ ,  $Q(-1, -4, 3)$  and  $R(k, 0, 3)$ . Given that  $PQR$  is isosceles, and that  $k > 1$ , find the value of  $k$ .

**(4 marks)**

5 Vectors  $\mathbf{a}$  and  $\mathbf{b}$  are defined by

$$\mathbf{a} = (p + 1)\mathbf{i} - 7\mathbf{j} + (q - 3p)\mathbf{k}$$

$$\mathbf{b} = 5\mathbf{i} + (14q + r)\mathbf{j} + (1 - 2r)\mathbf{k}$$

Given that  $\mathbf{a} = \mathbf{b}$ , find the values of  $p$ ,  $q$  and  $r$ .

(4 marks)

**6 (a)** A particle of mass  $0.5 \text{ kg}$  is acted upon by a force of  $(12\mathbf{i} - 4\mathbf{j} + p\mathbf{k}) \text{ N}$ .

Given that the magnitude of the acceleration experienced by the particle under the influence of this force is  $26 \text{ m s}^{-2}$ , find the possible values of  $p$ .

**(3 marks)**

**(b)** An additional second force with a magnitude of  $q\mathbf{k} \text{ N}$  now acts on the particle, where  $q < 0$ . Under the influence of the resultant of the two forces, the particle now experiences an acceleration of magnitude  $8\sqrt{10} \text{ m s}^{-2}$ .

Explain why this additional information shows that  $p > 0$ .

**(2 marks)**

**(c)** Hence find the value of  $q$ .

**(3 marks)**

**7 (a)** Three forces act upon a particle with a mass of 5 kg:

$$\mathbf{F}_1 = (r\mathbf{i} + 5\mathbf{j} + (r-p)\mathbf{k}) \text{ N}$$

$$\mathbf{F}_2 = ((p+q)\mathbf{i} - 3p\mathbf{j} + 7\mathbf{k}) \text{ N}$$

$$\mathbf{F}_3 = (-\mathbf{i} + r\mathbf{j} - 2q\mathbf{k}) \text{ N}$$

Under the action of those three forces, the particle is in equilibrium.

Find the values of  $p$ ,  $q$  and  $r$ .

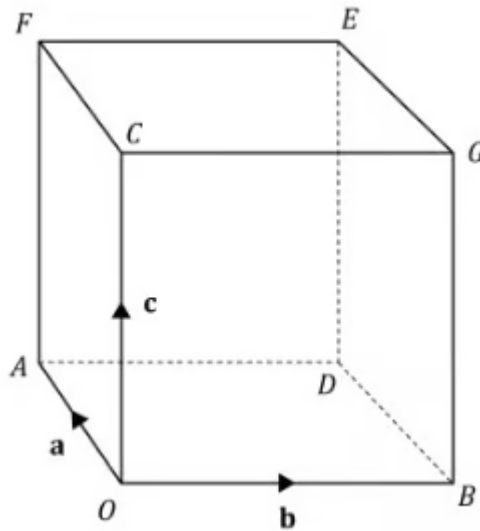
**(4 marks)**

**(b)** A new force,  $\mathbf{F}_4$ , is added. Under the combined action of the four forces, the particle experiences an acceleration with a magnitude of  $2.2 \text{ m s}^{-2}$ , in the same direction as the vector  $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ .

Find  $\mathbf{F}_4$ , giving your answer in the form  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  where  $x$ ,  $y$  and  $z$  are given as exact values. Be sure to include the correct units in your answer.

**(5 marks)**

**8** The diagram below shows a cube whose vertices are  $O$ ,  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$  and  $G$ .



$\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are the vectors  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$  and  $\overrightarrow{OC}$  respectively.

Using vector methods, prove that the diagonals  $CD$  and  $BF$  bisect each other.

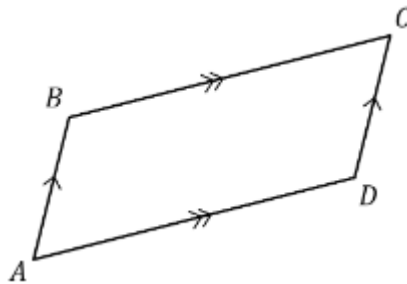
**(9 marks)**

# Very Hard Questions

- 1 The coordinates of points  $A$  and  $B$  are  $(5, 0, -1)$  and  $(k, -2, 3k)$  respectively. Given that the distance from  $A$  to  $B$  is  $6\sqrt{k}$  units, and that point  $B$  lies at a distance of  $\sqrt{14}$  units from the origin, find the value of  $k$ .

(3 marks)

- 2 In the parallelogram  $ABCD$ ,  $AB$  is parallel to  $CD$ , and  $BC$  is parallel to  $AD$ .



Given that  $\vec{AB} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$  and  $\vec{AD} = 7\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ , find the area of the parallelogram. Give your answer correct to 3 s.f.



(8 marks)

**3 (a)** The vector  $\vec{RS} = x\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$  makes an angle with the positive  $x$ -axis.

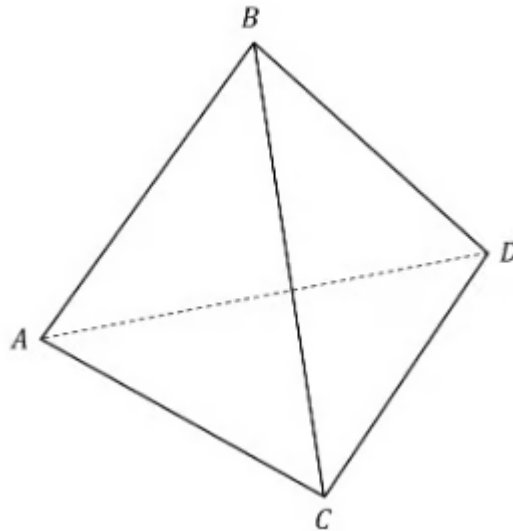
Show that  $x^2 = \frac{90}{\tan^2 \theta}$ .

**(5 marks)**

**(b)** Given that  $\theta$  is acute and that  $\cos \theta = \frac{4}{5}$ , find a unit vector in the direction of  $\vec{RS}$ .

**(4 marks)**

**4**  $A, B, C$  and  $D$  are the vertices of a regular tetrahedron.



The coordinates of points  $A$ ,  $B$ ,  $C$  and  $D$  are  $(1, 1, 1)$ ,  $(-8, 10, 1)$ ,  $(-3, 6, -10)$  and  $(-2k, 13, k)$  respectively. Find the coordinates of point  $D$ .

(6 marks)

5 Vectors  $\mathbf{a}$  and  $\mathbf{b}$  are defined by

$$\mathbf{a} = pq\mathbf{i} - 24\mathbf{j} + 9r\mathbf{k}$$

$$\mathbf{b} = 6\mathbf{i} + p - r\mathbf{j} + p - 3q\mathbf{k}$$

Given that  $\mathbf{a} = 3\mathbf{b}$ , and that  $r > 0$ , find the values of  $p$ ,  $q$  and  $r$ .

**(5 marks)**

**6 (a)** A particle of mass  $0.2 \text{ kg}$  is acted upon by a force of  $(-3\mathbf{i} + p\mathbf{j} + 4\mathbf{k}) \text{ N}$ .

Given that the magnitude of the acceleration experienced by the particle under the influence of this force is  $65 \text{ m s}^{-2}$ , find the possible values of  $p$ .

**(3 marks)**

**(b)** An additional second force with a magnitude of  $q \text{ N}$  now acts on the particle, where  $q > 0$ . Under the influence of the resultant of the two forces, the particle now experiences an acceleration of magnitude  $\frac{5\sqrt{221}}{2} \text{ m s}^{-2}$ .

Find the possible values of  $q$ .

**(5 marks)**

- 7 (a)** For this question, the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  point in the directions east and north respectively, while the unit vector  $\mathbf{k}$  points vertically upwards.

A robotic submarine has a mass of 750 kg and is originally moving in a level direction such that the  $\mathbf{k}$  component of its velocity is zero. In addition to gravity, the forces acting on the submarine are the combined thrust and lift  $\mathbf{T}$  from its propeller and manoeuvring planes, its buoyancy  $\mathbf{B}$ , and the water resistance  $\mathbf{W}$ . These forces, in newtons, are given by:

$$\mathbf{T} = 600\mathbf{i} - 750\mathbf{j} - 120\mathbf{k}$$

$$\mathbf{B} = 7360\mathbf{k}$$

$$\mathbf{W} = -500\mathbf{i} + 600\mathbf{j} + 50\mathbf{k}$$

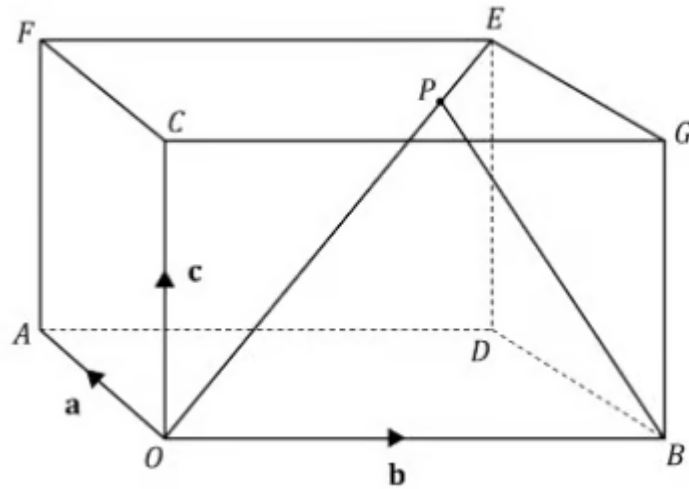
Taking the acceleration due to gravity to be  $g = 9.8 \text{ ms}^{-2}$ , find the magnitude of the acceleration of the submarine. Give your answer accurate to 3 s.f.

**(5 marks)**

- (b)** Determine whether the submarine is rising or sinking in the water and determine the angle its acceleration makes with the vector  $\mathbf{k}$ . Give your answer accurate to 1 d.p.

**(3 marks)**

- 8** The diagram below shows a cuboid whose vertices are  $O, A, B, C, D, E, F$  and  $G$ .



$P$  is a point on the diagonal  $OE$  that divides  $OE$  in the ratio  $a : b$ , where  $a > b$ .

Show that if line segment  $BP$  is extended it will intersect  $FE$ , and show that  $FE$  is divided in the ratio  $(a - b) : b$  by the point of intersection.

(11 marks)