

11.1 Vectors in 2 Dimensions

11.1.1 Basic Vectors / 11.1.2 Magnitude & Direction / 11.1.3 Vector Addition / 11.1.4 Position Vectors / 11.1.5 Problem Solving using Vectors

Easy (10 questions)	/34
Medium (9 questions)	/41
Hard (9 questions)	/48
Very Hard (8 questions)	/52
Total Marks	/175

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Easy Questions

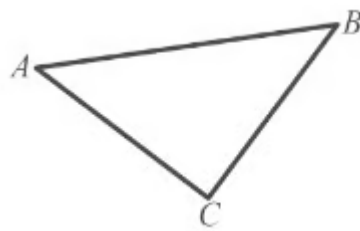
- 1 The vectors \mathbf{a} and \mathbf{b} are given by $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j}$.

Find:

- (i) $\mathbf{a} + \mathbf{b}$
- (ii) $5\mathbf{a}$,
- (iii) $3\mathbf{a} - 2\mathbf{b}$
- (iv) $\mathbf{a} - t\mathbf{b}$,

(5 marks)

- 2 In triangle ABC , $\overrightarrow{AB} = 5\mathbf{i} + \mathbf{j}$ and $\overrightarrow{AC} = 3\mathbf{i} - 2\mathbf{j}$.



- (i) Find \overrightarrow{BC} in terms of \mathbf{i} and \mathbf{j} .
- (ii) Calculate $|\overrightarrow{BC}|$.

(4 marks)

3 (a) The vectors **a**, **b**, **c** and **d** are given by

$$\mathbf{a} = 2\mathbf{i} + 4\mathbf{j}, \quad \mathbf{b} = 3\mathbf{i} + p\mathbf{j}, \quad \mathbf{c} = q\mathbf{i} - 2\mathbf{j}, \quad \mathbf{d} = 6\mathbf{i} - 2\mathbf{j}$$

Given that $\mathbf{a} - 2\mathbf{b} = 3\mathbf{c}$, find the values of the constants p and q .

(3 marks)

(b) Find $|\mathbf{d}|$.

(2 marks)

4 On the same diagram sketch the following position vectors

- (i) $3\mathbf{i} + 4\mathbf{j}$,
- (ii) $-5\mathbf{i}$,
- (iii) $-8\mathbf{i} - 6\mathbf{j}$.

(3 marks)

5 The vectors **a**, **b** and **c** are given as

$$\mathbf{a} = \begin{pmatrix} 3 \\ -p \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} p \\ 4 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$$

Given that $\mathbf{a} + \mathbf{b}$ is parallel to \mathbf{c} find the value of p .

(3 marks)

6 The position vector \vec{AB} is given by $\vec{AB} = 6\mathbf{i} + 3\mathbf{j}$.

- (i) Find the magnitude of \vec{AB} , giving your answer in the form $p\sqrt{q}$ where p and q are integers to be found.
- (ii) Find the angle between \vec{AB} and the positive x -axis, giving your answer in degrees to one decimal place.

(4 marks)

7 The vector \vec{OA} is given by $\vec{OA} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Find a unit vector in the direction of \vec{OA} .

(2 marks)

8 Starting at the origin, a ship sails on a bearing of 060° for 400 km, until it reaches point P . Find the position vector of P relative to the origin.

Giving your answer in the form $(x\mathbf{i} + y\mathbf{j})$ km, where x and y are exact values.

(4 marks)

- 9 The points A and B have position vectors $\mathbf{a} = 3\mathbf{i} - 7\mathbf{j}$ and $\mathbf{b} = -3\mathbf{i} + \mathbf{j}$ respectively.

Find the distance AB .

(2 marks)

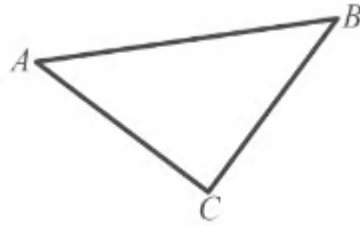
- 10 A force \mathbf{F} acts on a particle, where $\mathbf{F} = p\mathbf{i} + 2p\mathbf{j}$ N.

Calculate the magnitude of the force \mathbf{F} , giving your answer in terms of p .

(2 marks)

Medium Questions

1 (a) In triangle ABC , $\overrightarrow{AB} = 7\mathbf{i} + \mathbf{j}$ and $\overrightarrow{AC} = 4\mathbf{i} - 3\mathbf{j}$



(i) Write down \overrightarrow{CA} in terms of \mathbf{i} and \mathbf{j} .

(ii) Find \overrightarrow{BC} .

(3 marks)

(b) Calculate $|\overrightarrow{BC}|$.

(1 mark)

2 (a) $\mathbf{a} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} m \\ -3 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 5 \\ n \end{pmatrix}$

Given that $\mathbf{a} + 2\mathbf{b} = \mathbf{c}$, find the values of m and n .

(3 marks)

(b) $\mathbf{d} = \begin{pmatrix} -5 \\ k \end{pmatrix}$

Given that $|\mathbf{d}| = 15$, find two possible values for k . Give your answer as an exact value.

(2 marks)

- 3 The point A lies on the line with equation $y = 3x + 5$. The position vector of A is $\vec{OA} = 2k\mathbf{i} + 7k\mathbf{j}$. Find the value of k , and hence determine the coordinates of A .

(3 marks)

- 4 The vectors \mathbf{a} , \mathbf{b} and \mathbf{c} are given as

$$\mathbf{a} = \begin{pmatrix} -5 \\ 17 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} k \\ 5 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 9 \\ -29 \end{pmatrix}$$

Given that $\mathbf{a} - \mathbf{b}$ is parallel to $\mathbf{b} + \mathbf{c}$ find the value of k .

(4 marks)

5 (a) $\vec{AB} = 11\mathbf{i} - 2\mathbf{j}$

Find

- (i) the magnitude of \vec{AB} , giving your answer as an exact value
- (ii) the angle between \vec{AB} and the positive x -axis, giving your answer in degrees correct to two decimal places.

(3 marks)

(b) Find a unit vector in the direction of \vec{AB} .

(2 marks)

- 6 A ship leaves its starting position O in port and travels 300 km on a bearing of 120° . It then travels 500 km due south before dropping anchor at point A . Given that the position vector of A relative to O is $(x\mathbf{i} + y\mathbf{j})\text{ km}$, find the exact values of x and y .

(4 marks)

- 7 (a) Two forces \mathbf{F}_1 and \mathbf{F}_2 act on a particle, where $\mathbf{F}_1 = 7\mathbf{i} - 2\mathbf{j}$ newtons and $\mathbf{F}_2 = -12\mathbf{i} - 10\mathbf{j}$ newtons.

The resultant force \mathbf{R} acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

Calculate the magnitude of \mathbf{R} in newtons.

(3 marks)

- (b) A third force $\mathbf{F}_3 = k\mathbf{j}$ newtons is to be applied to the particle. The constant k is to be selected so that the line of action of the new resultant force $\mathbf{R}_{\text{new}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ is at an angle of 45 degrees to the vector \mathbf{j} , measured anticlockwise.

Find the value of k .

(3 marks)

- 8 (a)** Points A , B and C have position vectors $\vec{OA} = -4\mathbf{i} - 7\mathbf{j}$, $\vec{OB} = 3\mathbf{j}$ and $\vec{OC} = 6\mathbf{i} + 18\mathbf{j}$, respectively.

Find \vec{AB} and \vec{AC} .

(3 marks)

- (b)** Show that \vec{AB} and \vec{AC} are parallel, and state what this tells you about the points A , B and C .

(2 marks)

9 (a) In triangle ABC , $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$. Point P divides BC in the ratio 3:2.

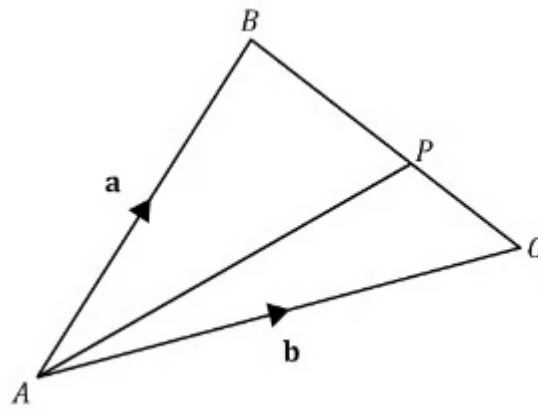


Diagram not
to scale

- i) Write down vector \overrightarrow{BC} in terms of \mathbf{a} and \mathbf{b} .
- ii) Find \overrightarrow{BP} in terms of \mathbf{a} and \mathbf{b} .

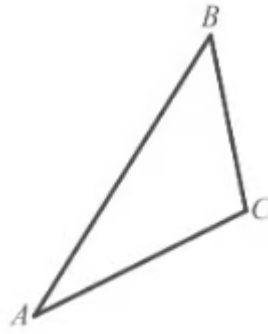
(3 marks)

(b) Given that $\mathbf{a} = 7\mathbf{i} + 8\mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + 3\mathbf{j}$, find \overrightarrow{BP} in terms of \mathbf{i} and \mathbf{j} .

(2 marks)

Hard Questions

- 1 (a) In triangle ABC , $\vec{AB} = 5\mathbf{i} + 8\mathbf{j}$ and $\vec{BC} = \mathbf{i} - 5\mathbf{j}$



Explain why $\vec{AB} + \vec{BC} + \vec{CA} = \mathbf{0}$.

(1 mark)

- (b) Find \vec{CA} and calculate its magnitude.

(3 marks)

2 (a) $\mathbf{a} = \begin{pmatrix} -1 \\ n \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} m \\ 6 \end{pmatrix}$

Given that the resultant of \mathbf{a} , \mathbf{b} and \mathbf{c} is the zero vector, find the values of m and n .

(2 marks)

(b) $\mathbf{d} = \begin{pmatrix} -3k \\ k \end{pmatrix}$

Given that $|\mathbf{d}| = 2\sqrt{15}$, find two possible values for k . Give your answer as an exact value.

(2 marks)

- 3 The point A lies on the curve with equation $y = x^2 - 2$. The position vector of A is $\overrightarrow{OA} = 3k\mathbf{i} - 17k\mathbf{j}$, where k is a positive constant. Find the value of k , and hence determine the coordinates of A .

(4 marks)

- 4 The vectors \mathbf{a} , \mathbf{b} and \mathbf{c} are given as

$$\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -3k \\ k \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

Given that $\mathbf{a} - \mathbf{b}$ is parallel to $\mathbf{a} + \mathbf{c}$ find the value of k .

(4 marks)

- 5 (a)** Vector \vec{AB} has a magnitude of $6\sqrt{3}$ and makes an angle of 150° with the positive x -axis.

Find \vec{AB} in the form $x\mathbf{i} + y\mathbf{j}$, where both x and y are given as exact values.

(3 marks)

- (b)** Find a unit vector in the direction of \vec{AB} .

(2 marks)

- 6** In the enchanted kingdom of Vectoria, a magical flying unicorn takes off from the wizard's palace at the point known as O and travels 30 km on a bearing of 300° . Chased by an evil dragon, it then travels an unknown distance of k km due north before reaching the enchanted grove at point P . Given that the position vector of P relative to O is $(x\mathbf{i} + y\mathbf{j})$ km, and that the straight-line distance between the grove and the palace is known to be $30\sqrt{3}$ km, find the exact values of x and y .

(6 marks)

- 7 (a)** Two forces \mathbf{F}_1 and \mathbf{F}_2 act on a particle, where $\mathbf{F}_1 = 5\mathbf{i} - 3\mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons.

The resultant force \mathbf{R} acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$, and acts in a direction parallel to the vector $(-\mathbf{i} - 3\mathbf{j})$.

Find the angle between \mathbf{R} and the vector \mathbf{j} , giving your answer in degrees correct to 2 decimal places.

(2 marks)

- (b)** Show that $3x - y = -18$.

(3 marks)

- (c)** Given that $y = -3$, find the magnitude of \mathbf{R} .

(3 marks)

- 8** Points A , B and C have position vectors $\overrightarrow{OA} = -9\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OB} = -6\mathbf{i}$ and $\overrightarrow{OC} = 3\mathbf{i} - 12\mathbf{j}$, respectively.

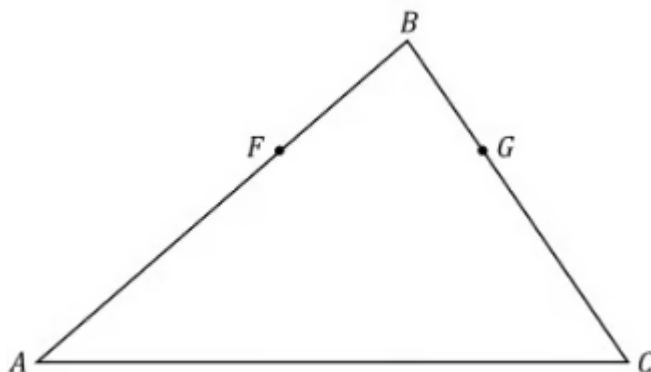
Use a vector method to show that points A , B and C lie on the same straight line.

(5 marks)

9 (a) In triangle ABC , point F lies on AB and point G lies on BC .

F divides AB in the ratio $m:n$.

The line segment FG is parallel to AC .



Explain why $\vec{BG} = \lambda \vec{BC}$ for some constant λ , where $0 \leq \lambda \leq 1$.

(1 mark)

(b) Given that $\vec{AB} = \mathbf{a}$ and $\vec{AC} = \mathbf{b}$, show that

$$\vec{FG} = \left(\frac{n}{m+n} - \lambda \right) \mathbf{a} + \lambda \mathbf{b}$$

(4 marks)

(c) Using your result from (b), prove that G divides BC in the ratio $n:m$.

(3 marks)

Very Hard Questions

- 1 (a) A , B and C are the three vertices of a triangle. $\overrightarrow{AC} = 5\mathbf{i} - 2\mathbf{j}$ and $\overrightarrow{BC} = -3\mathbf{i} + k\mathbf{j}$, where k is a constant.

Find \overrightarrow{AB} in terms of \mathbf{i} , \mathbf{j} and \mathbf{k} .

(2 marks)

- (b) Given that $|\overrightarrow{AB}| = \sqrt{89}$, find the two possible values of k .

(3 marks)

2 (a) $\mathbf{a} = \begin{pmatrix} 8 \\ m \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} n \\ -2 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} m \\ n \end{pmatrix}$

Given that $\mathbf{a} + \mathbf{b} = \mathbf{c} - 2\mathbf{b}$, find the values of m and n .

(3 marks)

(b) $\mathbf{d} = \begin{pmatrix} 2k+1 \\ 2k-1 \end{pmatrix}$

Given that $|\mathbf{d}| = 3k\sqrt{2}$, find two possible values for k . Give your answer as an exact value.

(2 marks)

- 3 (a)** The point A lies on the circle with equation $(x - 11)^2 + (y - 7)^2 = 34$. A has position vector $\vec{OA} = 3k\mathbf{i} + 5k\mathbf{j}$, where k is a constant.

Find the value of k , and hence determine the coordinates of A .

(4 marks)

- (b)** Explain why a line passing through O and A must be a tangent to the circle.

(2 marks)

- 4** Points A , B and C have position vectors $\vec{OA} = -6\mathbf{i} - 2\mathbf{j}$, $\vec{OB} = \mathbf{i} + m\mathbf{j}$ and $\vec{OC} = 3\mathbf{i} - 8\mathbf{j}$, respectively.

Given that A , B and C lie on the same straight line, use a vector method to find the value of m .

(5 marks)

- 5 (a) Vector \overrightarrow{AB} has a magnitude of $2\sqrt{6}$ and makes an angle of 165° with the positive y -axis (measuring anticlockwise from the positive y -axis).

Find \overrightarrow{AB} in the form $a\mathbf{i} + b\mathbf{j}$, where both a and b are given as exact values.

(3 marks)

- (b) Find a unit vector in the direction of \overrightarrow{AB} .

(2 marks)

- 6 (a)** A ship is searching for a radio buoy whose transmitter has ceased functioning. The ship sets out from point O and heads in the approximate direction of the buoy, travelling at a constant speed of 40 km/h in a direction parallel to the vector $\mathbf{i} + 3\mathbf{j}$. After travelling for ninety minutes the ship has reached point P . At that time, the ship receives a brief transmission from the buoy indicating that the buoy is at a bearing of 210° from the ship. The ship heads on that bearing at the same constant speed, and reaches the buoy at point Q in another 45 minutes. Given that vector $\overrightarrow{OQ} = x\mathbf{i} + y\mathbf{j}$ km, find the exact values of x and y .

Given that vector $\overrightarrow{OQ} = x\mathbf{i} + y\mathbf{j}$ km, find the exact values of x and y .

(7 marks)

- (b)** How far was the buoy from the ship, and at what bearing, at the time the ship initially left point O ? Give the distance in kilometers, and give your answers correct to 1 decimal place.

(3 marks)

- 7 (a) In an experiment, three forces are acting on a particle. $\mathbf{F}_1 = 7\mathbf{i} - \mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons are both constant forces, although the values of x and y are initially unknown. The third force is $\mathbf{F}_3 = k\mathbf{i} + k\sqrt{3}\mathbf{j}$ newtons, where $k \geq 0$ is a parameter that can be varied by the experimenters. The resultant force \mathbf{R} acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

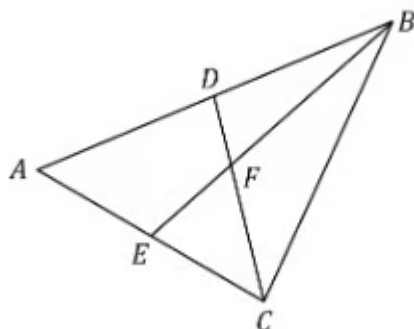
Given that $\mathbf{R} = \mathbf{0}$ when the magnitude of \mathbf{F}_3 is 10 newtons, find the exact values of x and y .

(4 marks)

- (b) Find the magnitude of \mathbf{F}_2 and the angle it makes with the vector \mathbf{i} . Give your answers correct to 1 decimal place.

(3 marks)

- 8 (a)** In triangle ABC , D is the midpoint of AB and E is the midpoint of AC . BE and CD intersect at point F .



Given that $\vec{AB} = 2\mathbf{a}$ and $\vec{AC} = 2\mathbf{b}$, write the vectors \vec{BC} , \vec{BE} and \vec{CD} in terms of \mathbf{a} and \mathbf{b} .

(3 marks)

- (b)** By setting up and solving suitable vector equations, prove that each of BE and CD divides the other in the ratio 1:2.

(6 marks)