

# 8.1 Integration

8.1.1 Fundamental Theorem of Calculus / 8.1.2 Integrating Powers of  $x$  / 8.1.3 Definite Integration / 8.1.4 Area Under a Curve / 8.1.5 Area between a curve and a line

Easy (11 questions)	/46
Medium (10 questions)	/44
Hard (10 questions)	/64
Very Hard (10 questions)	/64
<b>Total Marks</b>	<b>/218</b>

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# Easy Questions

## 1 Integrate

- (i)  $2x$ ,
- (ii)  $6x^2$ ,
- (iii)  $\frac{1}{2}x^{\frac{1}{2}}$ .

(3 marks)

## 2 Evaluate

- (i)  $\int_1^2 4x \, dx$ ,
- (ii)  $\int_0^3 (9x^2 + 4x) \, dx$ .

(4 marks)

## 3 Use calculus to find

$$\int \left( 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx.$$

**(3 marks)**

**4 (a)** Show that

$$\frac{3x^3 + 4x^6}{x^2}$$

can be written as  $3x^a + 4x^b$ , where  $a$  and  $b$  are constants to be found.

**(2 marks)**

**(b)** Hence find

$$\int \left( \frac{3x^3 + 4x^6}{x^2} \right) dx.$$

**(3 marks)**

5 (a) Integrate  $5x^4 + 6x^2 + 2x + 3$ .

(3 marks)

(b) Given that  $f(x) = \int (5x^4 + 6x^2 + 2x + 3) \, dx$  and that the graph of  $y = f(x)$  passes through the point  $(1, 10)$ , find an expression for  $f(x)$  in terms of  $x$  only.

(3 marks)

6 Evaluate

$$\int_{-3}^4 (2kx + 3kx^2) \, dx$$

giving your answer in terms of  $k$ .

(4 marks)

7 The curve  $C$ , described by the integral

$$y = \int (2x^3 - x) \, dx,$$

passes through the point  $(2, -2)$ .

Show that  $2y = x^4 - x^2 - 16$ .

**(4 marks)**

- 8 (a)** The area bounded by the curve with equation  $y = 9 - x^2$ , the  $x$ -axis and the vertical lines with equations  $x = 1$  and  $x = 2$  is to be found.

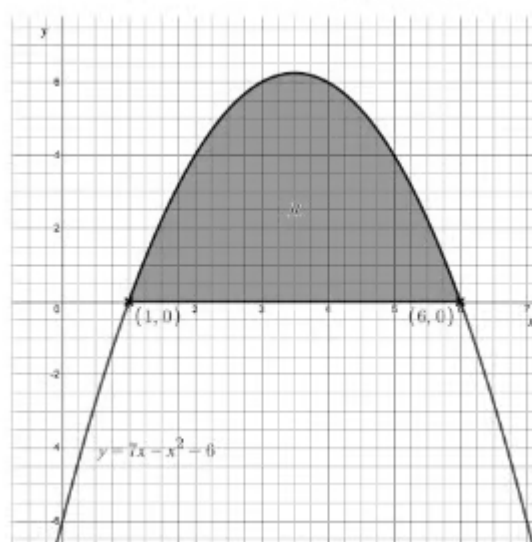
Write down an integral that would find this area.

**(2 marks)**

- (b)** Evaluate your integral from part (a) and hence find the area described above.

**(3 marks)**

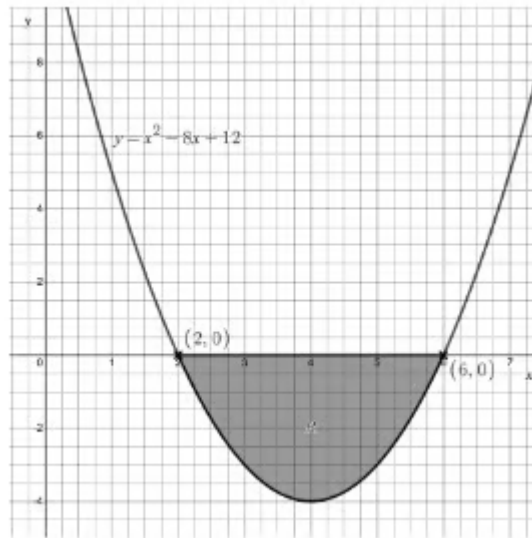
- 9** The diagram below shows the graph of  $y = 7x - x^2 - 6$ .



Find the shaded area, giving your answer as a fraction in its simplest terms.

(3 marks)

10 The diagram below shows the graph of  $y = x^2 - 8x + 12$ .



Find the shaded area marked  $R$ , giving your answer as a fraction in its simplest form.

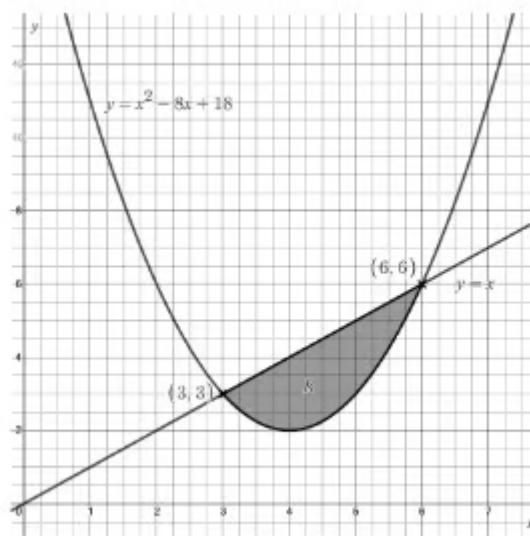
(4 marks)



11 (a) Simplify  $x - (x^2 - 8x + 18)$ .

(1 mark)

(b) The diagram below shows the graphs of  $y = x^2 - 8x + 18$  and  $y = x$ .



Find the shaded area marked  $R$ , giving your answer as a fraction in its simplest terms.

(4 marks)

# Medium Questions

1 Use calculus to find

$$\int (3x^2 + 5x + 3) \, dx$$

(2 marks)

2 Evaluate

$$\int_1^5 (4x + 6x^2) \, dx$$

(2 marks)

3 Find the equation of the curve passing through the point (4, 64) and given by

$$y = \int (3 - 2x + 6x^2) \, dx$$

(3 marks)

**4 (a)** Show that

$$(3 - 2x)^2 = 9 - 12x + 4x^2$$

**(2 marks)**

**(b)** Hence, or otherwise, work out

$$\int (3 - 2x)^2 \, dx$$

**(2 marks)**

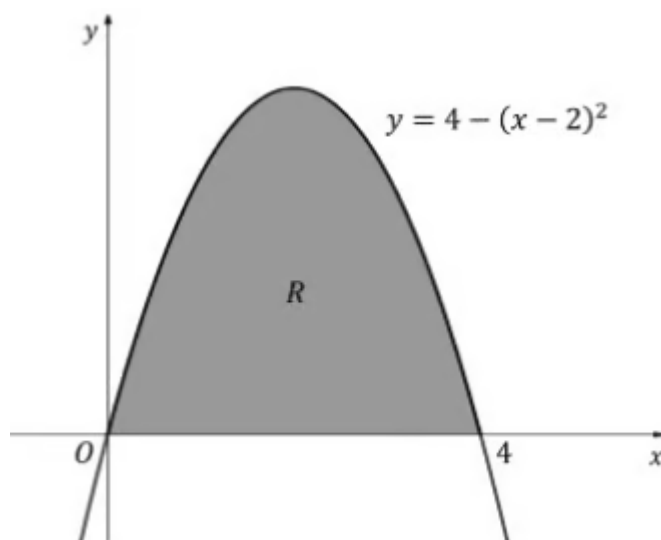
**5** Given

$$\int_k^5 (2x - 1) \, dx = 20$$

find the value of the positive constant  $k$ .

**(4 marks)**

**6 (a)** The diagram below shows part of the graph of  $y = 4 - (x - 2)^2$ .



Write down the values of  $x$  where  $y = 0$ .

**(1 mark)**

**(b)** Show that

$$4 - (x - 2)^2 = 4x - x^2$$

**(1 mark)**

**(c)** Evaluate

$$\int_0^4 (4x - x^2) \, dx$$

**(2 marks)**

(d) Write down the area of the region labelled  $R$ .

(1 mark)

**7 (a)** Given  $\frac{d^2y}{dx^2} = 30x$  and that when  $x = 1$ ,  $\frac{dy}{dx} = 17$ , show that

$$\frac{dy}{dx} = 15x^2 + 2$$

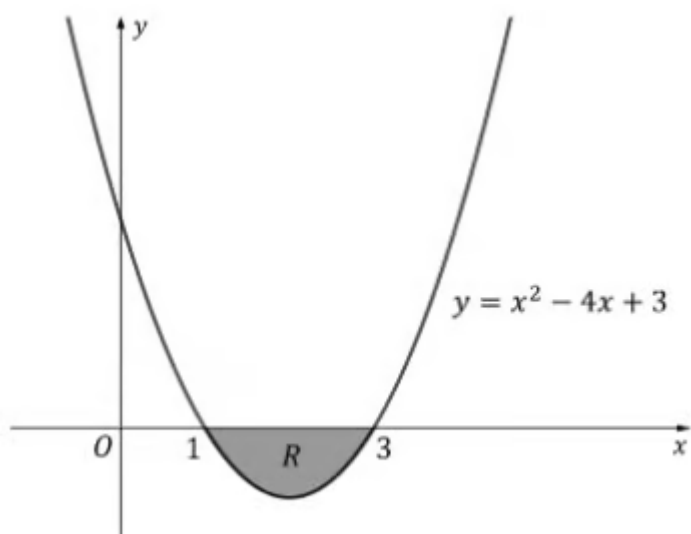
**(4 marks)**

**(b)** Find an equation for  $y$  in terms of  $x$ , given that when  $x = 2$ ,  $y = 40$ .

**(3 marks)**

**8** The diagram below shows part of the graph of  $y = x^2 - 4x + 3$ .

Find the area of the shaded region labelled  $R$ .



(3 marks)

- 9 (a) Find the  $x$ -coordinates of the intercepts of the line with equation  $y=2$  and the curve with equation  $y=x^2-4x+5$ .

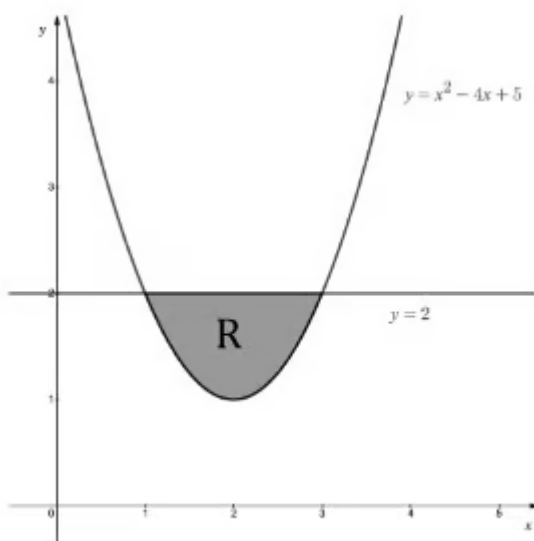
(2 marks)

- (b) Evaluate

$$\int_1^3 (x^2 - 4x + 5) \, dx$$

(2 marks)

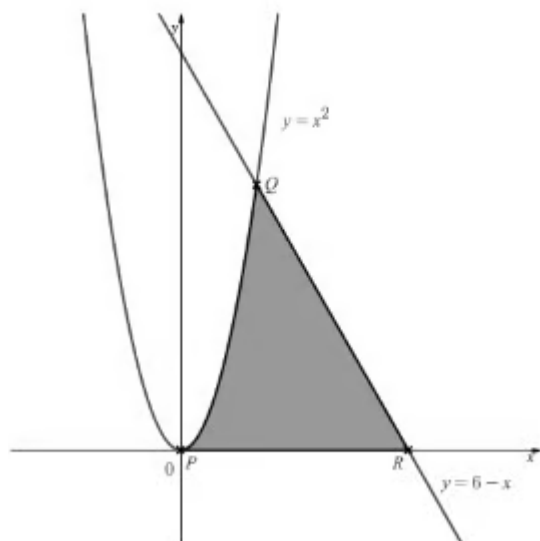
- (c) The diagram below shows the graphs of  $y=2$  and  $y=x^2-4x+5$ . Find the exact area of the shaded region  $R$ .





(4 marks)

**10 (a)** The diagram below shows the graphs of the line  $y = 6 - x$  and the curve  $y = x^2$ .



Work out the  $x$ -coordinates of the points labelled  $P$ ,  $Q$  and  $R$ .

**(2 marks)**

**(b)** Work out the area of the shaded region.

**(4 marks)**

# Hard Questions

1 (a) Use calculus to find

$$\int \left( x^2 + 5 - \frac{3}{x^2} \right) dx$$

(3 marks)

(b) Write down

$$\int 3e^{3x} dx$$

(2 marks)

2 Use calculus to find the value of

$$\int_4^9 \frac{x^2 + 1}{\sqrt{x}} dx$$

(5 marks)

- 3 Find the equation of the curve passing through the point  $(-2, 3)$  and given by

$$y = \int \left( 3 - 2x + \frac{4}{x^2} \right) dx$$

**(4 marks)**

**4 (a)** Using the binomial expansion, or otherwise, show that

$$(2 - x)^3 = 8 - 12x + 6x^2 - x^3$$

**(3 marks)**

**(b)** Hence, or otherwise, work out

$$\int (2 - x)^3 \, dx$$

**(3 marks)**

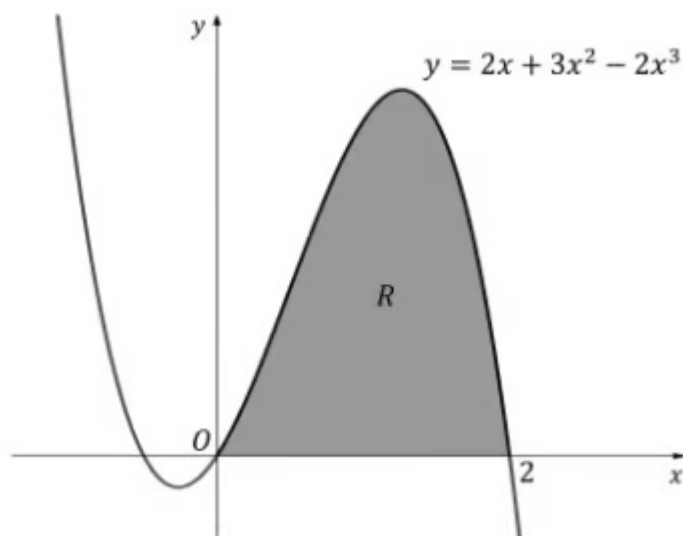
**5** Given

$$\int_1^p \left(1 + \frac{1}{x^2}\right) \, dx = \frac{15}{4}$$

find the value of the constant  $p$ , where  $p > 0$ .

**(5 marks)**

- 6 The diagram below shows part of the graph of  $y = 2x + 3x^2 - 2x^3$ .



Find the area of the shaded region labelled  $R$ .

(4 marks)

- 7 A function,  $f(x)$ , has second derivative given by

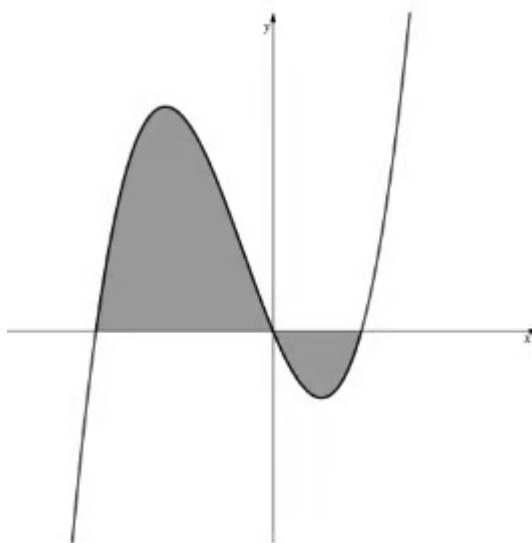
$$f''(x) = 6(x - 2).$$

Given that  $f(3) = 20$ , and  $f'(2) = 8$ , find  $f(x)$ .

(5 marks)

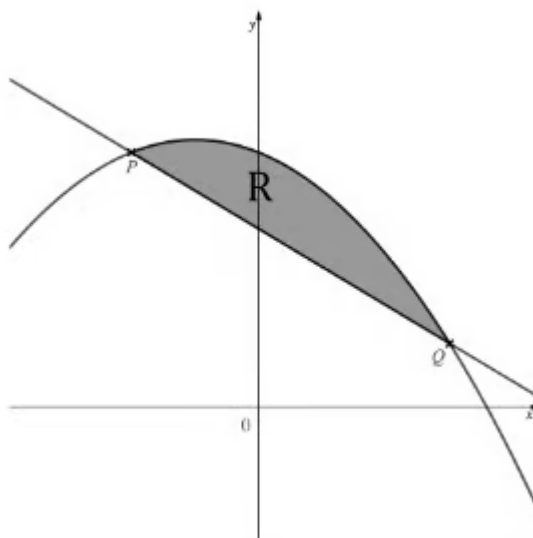
8 The diagram below shows part of the graph of  $y = x(x - 1)(x + 2)$ .

Find the total area of the two shaded regions.



(8 marks)

- 9 (a)** The line with equation  $5y = 14 - 3x$  cuts the curve with equation  $5y = 20 - 2x - x^2$  at the points  $P$  and  $Q$ , as shown.



Find the  $x$ - and  $y$ -coordinates of the points  $P$  and  $Q$ .

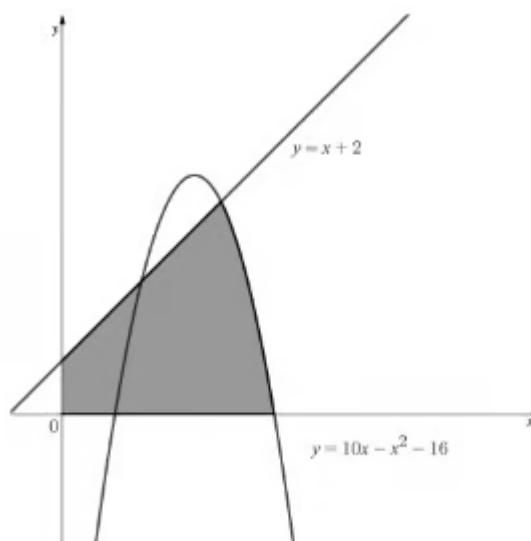
**(5 marks)**

- (b)** Find the exact area of the region labelled  $R$ , giving your answer in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers to be found.



(6 marks)

- 10 The diagram below shows the graphs of  $y = x + 2$  and  $y = 10x - x^2 - 16$ .



Find the exact area of the shaded region.

**(11 marks)**

# Very Hard Questions

1 Use calculus to find

$$\int (2\sqrt{x} + 5x^{\frac{1}{3}}) dx$$

(3 marks)

2 Use calculus to find the value of

$$\int_2^4 \frac{x^3 + \sqrt[3]{x}}{2\sqrt{x}} dx$$

giving your answer correct to 3 significant figures.

(5 marks)

3 Find the equation of the curve passing through the point (4, -8) and given by

$$y = \int \left( \frac{2}{\sqrt{x}} - x - 3 \right) dx$$

**(4 marks)**

**4 (a)** Show that

$$\left(3 - \frac{1}{2}x\right)^3 = 27 - \frac{27}{2}x + \frac{9}{4}x^2 - \frac{1}{8}x^3$$

**(3 marks)**

**(b)** Hence, or otherwise, work out

$$\int \left(2\left(3 - \frac{1}{2}x\right)\right)^3 dx$$

**(3 marks)**

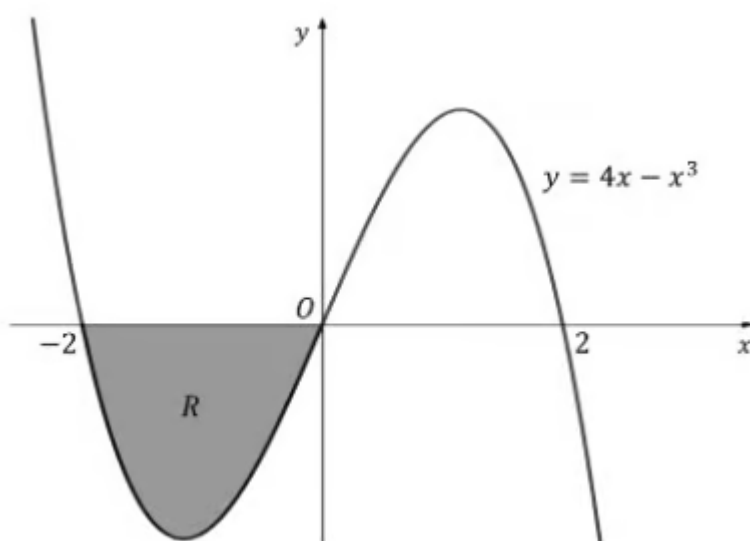
**5** Given

$$\int_q^{4q} 5x\sqrt{x} dx = 15\,066$$

find the value of the constant  $q$ .

(5 marks)

**6 (a)** The diagram below shows part of the graph of  $y = 4x - x^3$ .



Find the area of the shaded region labelled  $R$ .

**(4 marks)**

**(b)** Without doing any additional calculation, explain why  $\int_{-2}^2 (4x - x^3) dx$  must be equal to zero.

**(1 mark)**

**7** A function,  $f(x)$ , has second derivative given by

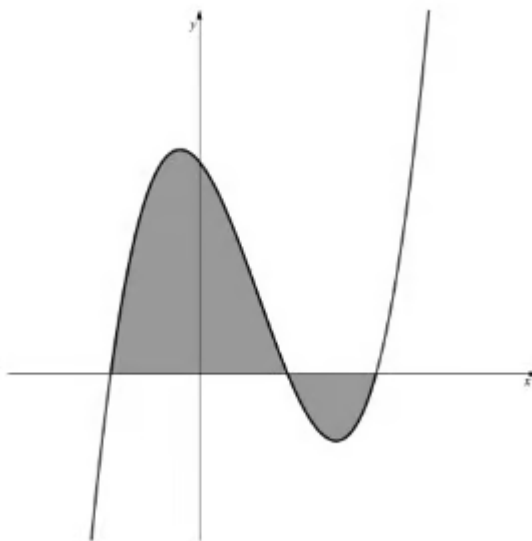
$$f''(x) = 2(18x - 5).$$

Given that  $(2x - 1)$  and  $(3x + 2)$  are factors of  $f(x)$ , find  $f(x)$ .

(6 marks)

- 8 The diagram below shows part of the graph of  $y = x^3 - 2x^2 - x + 2$ .

Find the total area of the two shaded regions.

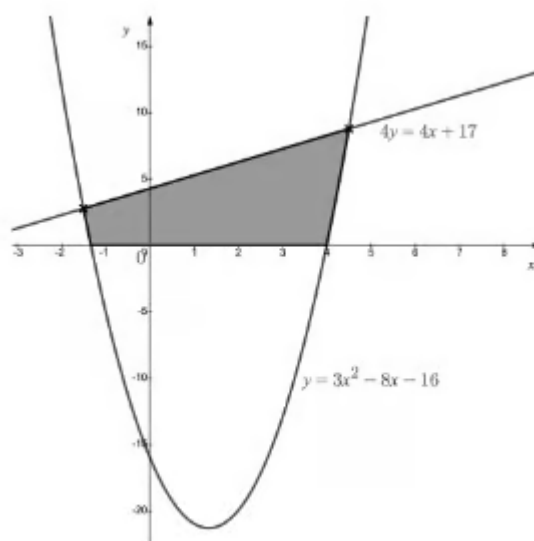




(8 marks)

- 9 The diagram below shows the graphs of  $4y = 4x + 17$  and  $y = 3x^2 - 8x - 16$ .

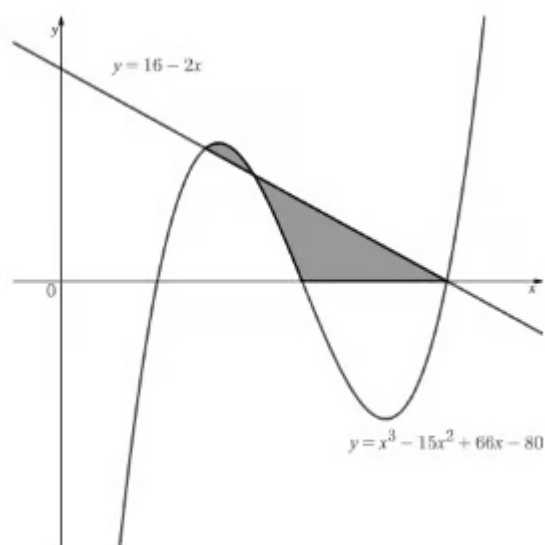
Find the exact area of the shaded region.



(11 marks)

10 The diagram below shows the graphs of  $y = 16 - 2x$  and  $y = x^3 - 15x^2 + 66x - 80$ .

Find the total area of the shaded regions.



**(11 marks)**