A Level · Edexcel · Maths

4 hours 43 questions

2.6 Rational Expressions (A Level only)

Easy (10 questions)	/42
Medium (11 questions)	/53
Hard (11 questions)	/57
Very Hard (11 questions)	/58
Total Marks	**/210**

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Easy Questions

1 Simplify

(i) $\frac{x^2}{x}$

(ii) $\frac{x(x-1)}{x}$

(iii) $\frac{6x+2}{2}$

(3 marks)

2 (i) Factorise $x^2+7x+12$

(ii) Hence simplify $\frac{x^2+7x+12}{2(x+3)}$

(3 marks)

3 (a) Simplify fully $\frac{2x^2 + 10x}{2(x + 5)}$

(2 marks)

(b) Simplify fully $\frac{3x^2}{x + 4} \times \frac{x^2 + 5x + 4}{x}$

(3 marks)

4 (i) Fully factorise $x^3 - 9x^2 + 20x$

(ii) Hence simplify $\frac{x^3 - 9x^2 + 20x}{x^2 - 5x}$

(4 marks)

5 (a) The function is given by $f(x) = 2x^3 + 7x^2 - 4x$.

Show that $f(x) = x(2x - 1)(x + 4)$.

(2 marks)

(b) Hence, or otherwise, write down the real solutions to the equation

$$\frac{f(x)}{x + 1} = 0.$$

(2 marks)

6 (a) The function $f(x)$ is given by

$$f(x) = x^3 - 4x^2 - 7x + 10$$

Work out $f(1)$ and hence write down a factor of $f(x)$.

(2 marks)

(b) Work out $f(x) \div (x + 2)$.

(2 marks)

(c) Write $f(x)$ in the form $(x + a)(x + b)(x + c)$ where a, b and c are integers to be found.

(3 marks)

7 Which one of the following algebraic fractions is improper? Explain your answer.

$$\frac{x^2 + 5x - 1}{x^3 - 2} \qquad \frac{x^2 + 3x + 2}{x^2 - 3x + 2} \qquad \frac{x + 1}{(x - 1)^2}$$

(2 marks)

8 Find the remainder when $x^3 + 2x^2 - 5x + 8$ is divided by $(x - 3)$.

(3 marks)

9 (a) Given that $(x^2 - 8x - 20) \div (x - 2) = Ax + B + \dfrac{C}{x - 2}$

where A, B and C are integer constants.

In terms of A, B and/or C as appropriate

(i) Write down the divisor.
(ii) Write down the quotient.
(iii) Write down the remainder.

(3 marks)

(b) Find the values of A, B and C.

(4 marks)

10 The function $\mathrm{f}(x)$ is given by

$$\mathrm{f}(x) = x^2 + ax + b$$

where a and b are integer constants.

It is also given that $\mathrm{f}(3) = \mathrm{f}(-8) = 0$.

Find the values of a and b.

(4 marks)

Medium Questions

1 Simplify

(i) $\frac{x}{2x}$

(ii) $\frac{x+1}{x(x+1)}$

(iii) $\frac{6x+12}{x^2+2x}$

(4 marks)

2 (a) Simplify fully $\frac{2x^2+6x}{x^3+3x^2}$

(3 marks)

(b) Simplify fully $\frac{x+4}{x^3} \times \frac{x^2+2x}{x+4}$

(3 marks)

(c) Simplify fully $\frac{x^2+4x}{3x+6} \div \frac{2x+8}{x+2}$

(3 marks)

3 (a) The function $f(x)$ is given by

$$f(x) = 3x^3 - 5x^2 - 4x + 4$$

Show that $f\left(\frac{2}{3}\right) = 0$.

(2 marks)

(b) Hence write down a factor of $f(x)$.

(1 mark)

(c) Fully factorise $f(x)$.

(3 marks)

(d) Write down the solutions to the equation $f(x) = 0$.

(2 marks)

4 (a) Show that $(2x-3)$ is a factor of $2x^3-13x^2+23x-12$.

(2 marks)

(b) Fully factorise $2x^3-13x^2+23x-12$.

(2 marks)

(c) Find all the real solutions to $2x^3-13x^2+23x-12=0$.

(2 marks)

5 Given that $(2x-1)$ is a factor of $2x^3+x^2-25x+a$ find the value of a.

(2 marks)

6 (a) Work out $(x^3 + 3x^2 - 2x + 4) \div (x + 1)$.

(3 marks)

(b) Work out $\dfrac{2x^3 - 4x + 3}{x - 2}$.

(3 marks)

7 (a) Given $(x^2+8x-4)\div(x-3)=x+11+\dfrac{29}{x-3}$

(i) Write down the divisor.

(ii) Write down the quotient.

(iii) Write down the remainder.

(3 marks)

(b) (i) Write down the degree of x^2+8x-4.

(ii) Write down the degree of $x-3$.

(iii) Explain why you would expect the quotient to be of degree 1 in this case.

(3 marks)

8 One of the three algebraic fractions below is improper ('top-heavy').

$$\frac{x+2}{x^2+2} \qquad \frac{x}{x+2} \qquad \frac{1}{x+2}$$

Identify which fraction is improper and write it in the form $A+\dfrac{B}{x+2}$, where A and B are integers to be found.

(3 marks)

9 (a) Simplify fully $\dfrac{x^3-2x^2-8x}{x-4}$

(2 marks)

(b) Hence solve the equation $\dfrac{x^3-2x^2-8x}{x-4}=x^2+10x+16.$

(3 marks)

10 It is given that

$$\frac{\mathrm{f}(x)}{x+2}=3x+4-\frac{2}{x+2}$$

Find $\mathrm{f}(x)$.

(2 marks)

11 The result of dividing x^2+ax-5 by $(x+1)$ is $x+3+\dfrac{d}{x+1}$.

Find the values of a and d.

(2 marks)

Hard Questions

1 Simplify fully

(i) $\dfrac{x + 3}{x^2 + 3x}$

(ii) $\dfrac{x^3 + x}{x^4}$

(iii) $\dfrac{x^3 + 3x^2 - 4x}{x^4 - x^3}$

(6 marks)

2 (a) Simplify fully $\frac{x^2+x-2}{x^3+4x^2-4x-1}$

(3 marks)

(b) Simplify fully $\frac{3x+9}{x+2} \times \frac{x^2+6x+8}{x+3}$

(3 marks)

(c) Simplify fully $\frac{x^2+8x-9}{x^2+7x+12} \div \frac{x^2+11x+18}{2x^2+7x-4}$

(3 marks)

3 (a) The function $f(x)$ is given by

$$f(x) = 4x^3 - 7x^2 - 21x + 18$$

Show that $(4x - 3)$ is a factor of $f(x)$.

(2 marks)

(b) Hence, or otherwise, fully factorise $f(x)$.

(4 marks)

(c) Write down the roots of $f(x)$.

(2 marks)

4 Show that $(5x - 2)$ is a factor of $25x^3 + 55x^2 - 56x + 12$.

Hence find all the real solutions to the equation $25x^3 + 55x^2 - 56x + 12 = 0$.

(5 marks)

5 (a) Given that $(4x-5)$ is a factor of $4x^3-9x^2+ax+30$ find the value of a.

(2 marks)

(b) Hence, or otherwise, fully factorise $4x^3-9x^2+ax+30$.

(2 marks)

6 (a) Work out $(x^3 + 5x^2 - 4) \div (x - 5)$.

(2 marks)

(b) Work out $\dfrac{3x^3 + 2x - 5}{x^2 + 1}$.

(2 marks)

7 (i) Find the remainder when $x^3 - 2x^2 + 4x - 3$ is divided by $x - 2$.

(ii) Find the value of $f(2)$ when $f(x) = x^3 - 2x^2 + 4x - 3$.

(iii) Comment on your answers to parts (i) and (ii).

(4 marks)

8 One of the three algebraic fractions below is improper ('top-heavy'):

$$\frac{x^2 - 5x + 1}{x + 1} \qquad \frac{x + 2}{(x + 1)^2} \qquad \frac{x^2 - 5x + 1}{(x + 1)^3}$$

Identify which fraction is improper and write it in the form $Ax + B + \dfrac{C}{x + 1}$, where A, B and C are integers to be found.

(3 marks)

9 (a) Simplify $\frac{x^3 - 7x^2 + 14x - 8}{x - 1}$

(3 marks)

(b) Hence solve $\frac{x^3 - 7x^2 + 14x - 8}{x - 1} = 2x^2 - 5x + 2.$

(3 marks)

10 (a) It is given that

$$\frac{\mathrm{f}(x)}{\mathrm{g}(x)} = 2x + 3 - \frac{4}{x + 1}$$

Why would assuming that $\mathrm{g}(x) = x + 1$ be a logical first step in attempting to determine the precise forms of $\mathrm{f}(x)$ and $\mathrm{g}(x)$?

(1 mark)

(b) By first making the assumption from part (a), find $\mathrm{f}(x)$.

(2 marks)

(c) Explain, with an example, why the forms of $\mathrm{f}(x)$ and $\mathrm{g}(x)$ determined in parts (a) and (b) are not the only possible forms for those functions.

(2 marks)

11 When $x^3 + ax^2 + 4x - 1$ is divided by $x + 2$ the quotient is $x^2 - 4x + 12$ and the remainder is b.

Find the values of a and b.

(3 marks)

Very Hard Questions

1 Simplify fully

(i) $\dfrac{x^2+5x}{x^3+3x^2-10x}$

(ii) $\dfrac{x^2-4}{x^4-16}$

(iii) $\dfrac{(x+2)^2+(x+2)(x+4)}{x^2+5x+6}$

(8 marks)

2 (a) Simplify fully $\dfrac{x^2-5x+4}{x^3-2x^2-11x+12}$

(3 marks)

(b) Simplify fully $\dfrac{2x^2-x-6}{x^2-4} \times \dfrac{5x\ +\ 10}{4x^2-9}$

(3 marks)

(c) Simplify fully $\dfrac{45x^3+90x^2-5x-10}{x^2-25} \div \dfrac{3x^2+7x+2}{2x^2+9x-5}$

(3 marks)

3 Given that $(2x-5)$ is a factor of the function

$$f(x)=2x^3+kx^2-11x-60$$

find the value of k and fully factorise $f(x)$.

(4 marks)

4 Show that $(9x^2 - 4)$ is a factor of $9x^4 - 40x^2 + 16$ and hence find all the real solutions to the equation $9x^4 - 40x^2 + 16 = 0$.

(5 marks)

5 (a) Show that $(ax-2)$ is a factor of $3ax^2+(a-6)x-2$.

(2 marks)

(b) Given that $x=\frac{-1}{a-4}$ is a root of $3ax^2+(a-6)x-2$, find the value of a.

(3 marks)

6 (a) Work out $(3x^4 + 2x^3 - 5x + 2) \div (x - 3)$.

(3 marks)

(b) Work out $\dfrac{x^5}{3x^2 - 2}$

(3 marks)

7 (a) For a polynomial $f(x)$, the Remainder Theorem states that

When $f(x)$ is divided by $(ax - b)$ the remainder is $f\left(\frac{b}{a}\right)$.

Use the Remainder Theorem to find the remainder when $8x^3 + 6x^2 - x - 2$ is divided by $(2x + 1)$.

(2 marks)

(b) Work out the remainder when $6x^2 - x - 2$ is divided by $(2x + 1)$.

(2 marks)

8 One of the three algebraic fractions below is improper ('top-heavy'):

$$\frac{x+4}{(x-4)^2} \qquad \frac{5x-25}{(x-4)(x+4)} \qquad \frac{x^3-2x^2+6x-1}{(x-4)}$$

Identify which fraction is improper and rewrite it as a quotient and a remainder term.

(3 marks)

9 Solve $\frac{8x^4 + 25x^3 + 3x^2 - 32x - 4}{x^2 + x - 2} = 2x^2 + 16x + 3$

(5 marks)

10 It is given that

$$\frac{(x+a)^3}{x+b} = (x^2+7x+13) + \frac{1}{x+b}$$

where a and b are integers.

Find the values of a and b.

(5 marks)

11 When $2x^3+(a+b)x^2+(a-b)x-3$ is divided by $x+4$ the quotient is $2x^2+(2a+3)x+(2b-5)$ and the remainder is c.

Find the values of a, b and c.

(4 marks)