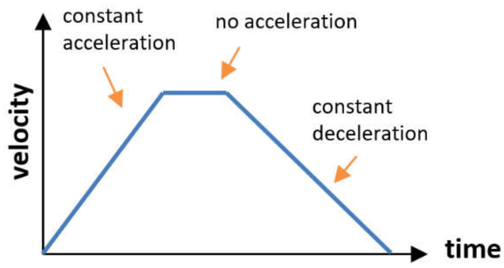
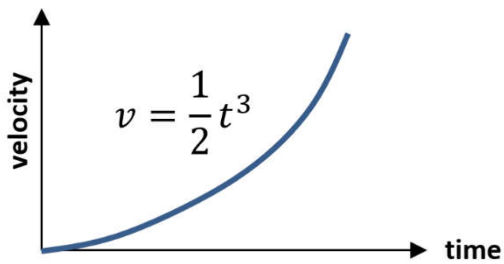
Variable Acceleration (Year 1)

Functions of time

Up to now, the acceleration has always been constant in any particular period of time… but this isn't realistic, as there should be a **smooth change** from acceleration to constant velocities.



It is possible to specify either the displacement, velocity or acceleration as any function of time (i.e. an expression in terms of t). This allows the acceleration to constantly change.



The velocity-time graph of a body is shown above, where $v = \frac{1}{2}t^3$.

(a) What is the velocity after 4 seconds have elapsed?

(b) How many seconds have elapsed when the velocity of the body is 108 ms^{-1}?

A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 2t^2 - 16t + 24$. Find

(a) The initial velocity

(b) The values of t when the body is instantaneously at rest.

(c) The value of t when the velocity is 64 ms^{-1}.

(d) The greatest speed of the body in the interval $0 \leq t \leq 5$.

Ex 11A

Using Differentiation

Earlier on, we saw that velocity v is the rate of change of displacement s (i.e. the gradient). But in Pure, we know that we can use differentiation to find the gradient function:

$$v = \frac{ds}{dt}$$

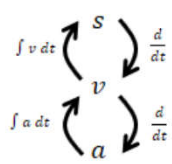
velocity is the rate of change of displacement

and similarly…

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

acceleration is the rate of change of velocity

Memory Tip: I picture interchanging between s, v, a as differentiating to go downwards and integrating to go upwards:



(We will do integration a bit later)

A particle P is moving on the x-axis. At time t seconds, the displacement x metres from O is given by $x = t^4 - 32t + 14$. Find:

(a) the velocity of P when $t = 3$

(b) The value of t when P is instantaneously at rest

(c) The acceleration of P when $t = 1.5$

A cat's displacement from a house, in metres, is $t^3 - \frac{3}{2}t^2 - 36t$ where t is in seconds.
(a) Determine the velocity of the cat when $t = 2$.
(b) At what time will the cat be instantaneously at rest?
(c) What is the cat's acceleration after 5 seconds?

Ex 11B

Maxima and Minima Problems

Recall from Pure that at minimum/maximum points, the gradient is 0. We could therefore for example find where the velocity is minimum/maximum by finding when $\frac{dv}{dt} = 0$ (i.e. when the acceleration is 0).

A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight lien before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 0.6t + 0.4t^2 - 0.2t^3, \qquad 0 \leq t \leq 3$$

(a) Justify the restriction $0 \leq t \leq 3$
(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

A dolphin escapes from Seaworld and its velocity as it speeds away from the park, is $t^3 - 9t^2 - 48t + 500$ (in ms^{-1}), until it reaches its maximum velocity, and then subsequently remains at this velocity.

(a) When does the dolphin reach its maximum velocity?

(b) What is this maximum velocity?

Your Turn

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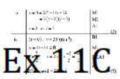
A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(*a*) the times when P is instantaneously at rest, **(3)**

(*b*) the greatest speed of P in the interval $0 \leq t \leq 4$, **(5)**

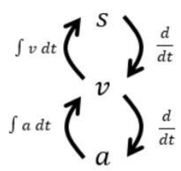


Ex 11C

Using Integration

Differentiating (with respect to time) gets us from displacement to velocity, and from velocity to acceleration.
So naturally, integrating (with respect to time) gets us from acceleration to velocity, and from velocity to displacement.
Also note that the area under a speed-time graph is distance (i.e. integrating velocity gives distance!)
As mentioned earlier, it's helpful to picture the graph on the right, where we move down to differentiate and up to integrate.

Memory Tip: I picture interchanging between s, v, a as differentiating to go downwards and integrating to go upwards:



A particle is moving on the x-axis. At time $t = 0$, the particle is at the point where $x = 5$. The velocity of the particle at time t seconds (where $t \geq 0$) is $(6t - t^2)$ ms^{-1}. Find:

(a) An expression for the displacement of the particle from O at time t seconds.

(b) The distance of the particle from its starting point when $t = 6$.

Careful with 'negative' areas

A particle travels in a straight line. After t seconds its velocity, v ms^{-1}, is given by $v = 4 - 2t^2, t \geq 0$. Find the distance travelled by the particle in the first three seconds of its motion.

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A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)$ m s^{-1}. When $t = 0$, P is 15 m from the origin O.

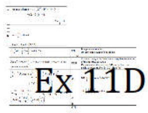
Find

(a) the values of t when P is instantaneously at rest, **(3)**

(b) the acceleration of P when $t = 5$, **(3)**

(c) the total distance travelled by P in the interval $0 \leq t \leq 5$. **(5)**

Always be careful for negative areas



Ex 11D

Constant acceleration formulae

It is also possible to derive all of the constant acceleration formulae using integration, provided that we consider that **acceleration is constant**.

Given a body has constant acceleration a, initial velocity u and its initial displacement is 0 m, prove that:
(a) Final velocity: $v = u + at$
(b) Displacement: $s = ut + \frac{1}{2}at^2$

Ex 11E