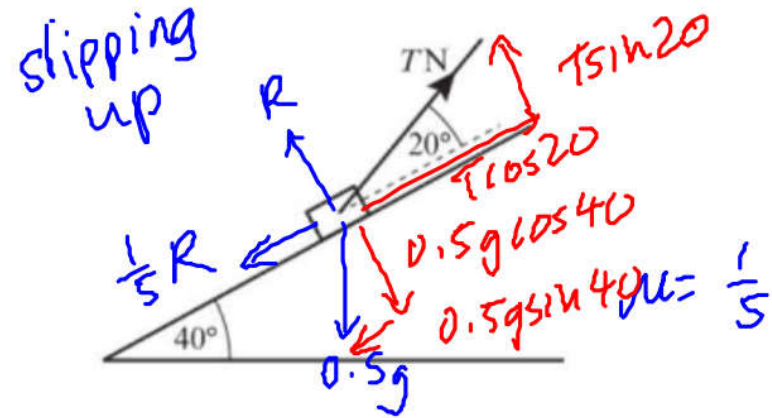


- 11 A box of mass 0.5 kg is placed on a plane which is inclined at an angle of 40° to the horizontal. The coefficient of friction between the box and the plane is $\frac{1}{5}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram. The box is in ~~limiting~~ equilibrium and may be modelled as a particle. The tension in the string is $T \text{ N}$.



(8 marks)

slipping up

limiting eq. $\left. \begin{matrix} (R \uparrow) \\ (R \downarrow) \end{matrix} \right\}$

$$T = \underline{3.87 \text{ N}}$$

$$2.75 \leq T \leq 3.87$$

slipping down

$\left. \begin{matrix} (R \uparrow) \\ (R \downarrow) \end{matrix} \right\}$

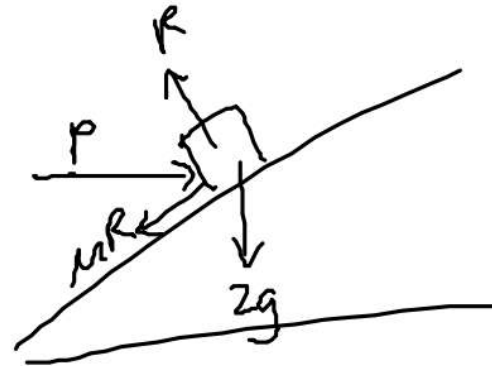
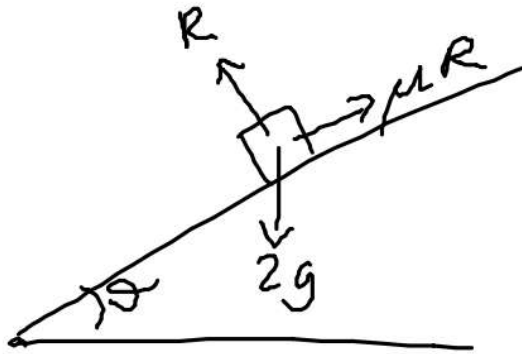
$$T = \underline{2.75 \text{ N}}$$

- 13 A particle of mass 2 kg rests in limiting equilibrium on a rough plane angled at θ above the horizontal where $\tan \theta = \frac{3}{4}$.

$$\mu = ?$$

A horizontal force of magnitude P N acting into the plane is applied to the box. Given that the box remains in equilibrium, find the maximum possible value of P .

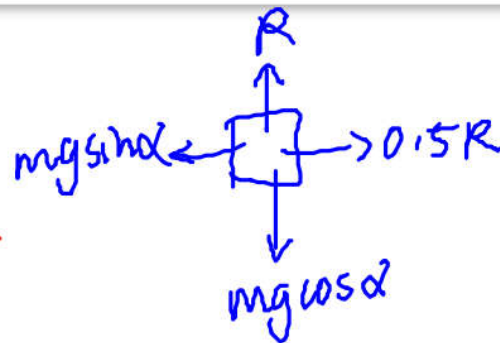
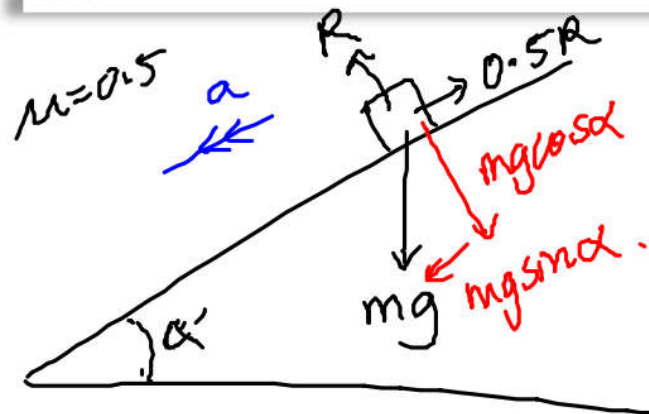
(8 marks)



Dynamics - including friction

A particle is held at rest on a rough plane which is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.5. The particle is released and slides down the plane. Find:

- the acceleration of the particle.
- the distance it slides in the first 2 seconds.



$$\begin{aligned} a) (R \uparrow) \quad R &= mg \cos \alpha \\ R &= \frac{4}{5} mg \end{aligned}$$

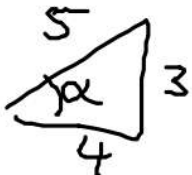
$$F = ma \quad \checkmark$$

$$mg \sin \alpha - 0.5R = ma$$

$$\frac{3}{5} mg - \frac{2}{5} mg = ma$$

$$\frac{1}{5} g = a \quad a = \underline{\underline{1.96 \text{ ms}^{-2}}}$$

$$\begin{aligned} \tan \alpha &= \frac{3}{4} \\ \sin \alpha &= \frac{3}{5} \\ \cos \alpha &= \frac{4}{5} \end{aligned}$$



$$\begin{aligned} b) \quad a &= 1.96 \\ t &= 2 \\ u &= 0 \\ s &= ? \end{aligned}$$

$$\begin{aligned} s &= ut + \frac{1}{2} at^2 \\ &= \frac{1}{2} \times 1.96 \times 2^2 \\ &= \underline{\underline{3.92 \text{ m}}} \end{aligned}$$

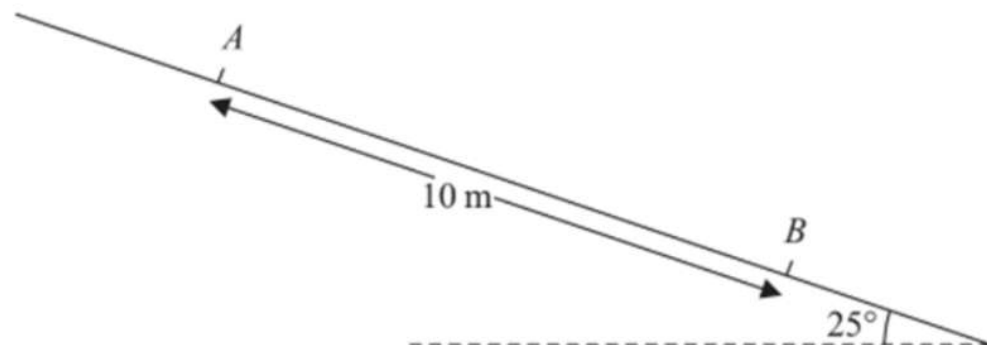


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B , where $AB = 10 \text{ m}$, as shown in Figure 3. The speed of P at A is 2 m s^{-1} . The particle P takes 3.5 s to move from A to B . Find

- the speed of P at B ,
- the acceleration of P ,
- the coefficient of friction between P and the plane.

(a)

$$s = \frac{u+v}{2}t \quad 10 = \frac{2+v}{2} \times 3.5$$

$$v = \frac{20}{3.5} - 2 = \frac{26}{7} = 3.71 \text{ (m s}^{-1}\text{)}$$

M1A1

A1

(b)

$$a = \frac{v-u}{t} = \frac{\frac{26}{7} - 2}{3.5} = \frac{24}{49} = 0.490 \text{ (m s}^{-2}\text{)}$$

M1A1

(c)

$$\text{Normal reaction : } R = 0.6g \cos 25^\circ$$

$$\text{Resolve parallel to the slope : } 0.6g \sin 25^\circ - \mu \times R = 0.6 \times a$$

$$\mu = 0.41 \text{ or } 0.411$$

B1

M1A2

A1

Jun 2014 M1

A rough plane is inclined at 40° to the horizontal. Two points A and B are 3 metres apart and lie on a line of greatest slope of the inclined plane, with A above B , as shown in Figure 2. A particle P of mass m kg is held at rest on the plane at A . The coefficient of friction between P and the plane is $\frac{1}{2}$. The particle is released.

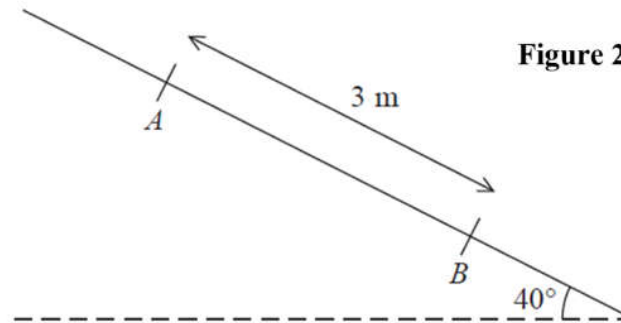
Hint:
Try drawing the
diagram reflected!

(a) Find the acceleration of P down the plane.

(5)

(b) Find the speed of P at B .

(2)



Jan 2013 M1

A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

(9)

Jun 2013 M1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α , where $\tan \alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)

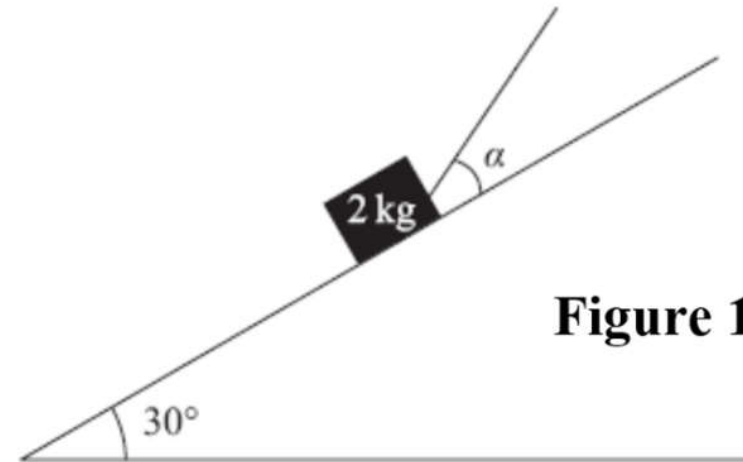


Figure 1

7. A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$.

$$\sin \alpha = \frac{3}{5} \quad \cos \alpha = \frac{4}{5}$$

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ .

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

$$a = -\frac{4}{5}g$$

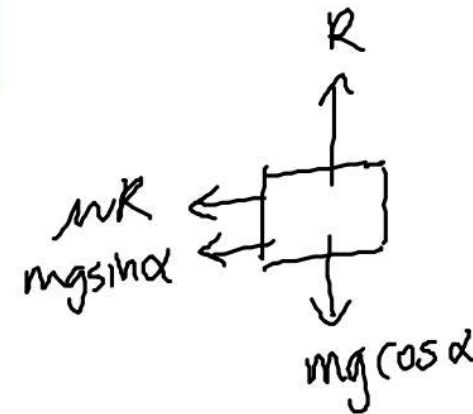
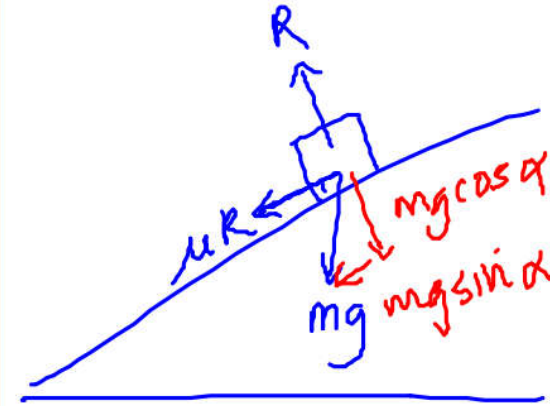
(a) Find the value of μ .

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A , carefully justifying your answer.

(2)



$$\begin{aligned} R &\uparrow \\ \text{a) } R &= mg \cos \alpha \quad \checkmark \\ R &= \frac{4}{5}mg \end{aligned}$$

$$\begin{aligned} F &= ma \quad \checkmark \\ -\mu R - mg \sin \alpha &= m \left(-\frac{4}{5}g \right) \quad \checkmark \checkmark \\ +\mu \left(\frac{4}{5}mg \right) + \frac{3}{5}mg &= +\frac{4}{5}mg \quad \checkmark \end{aligned}$$

$$\begin{aligned} \mu \frac{4}{5}mg &= \frac{1}{5}mg \\ \mu &= \frac{1}{4} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{b) } \quad & \text{Diagram showing forces on the particle at point A: } mg \sin \alpha \text{ down the slope, } \frac{3}{5}mg \text{ (circled), and } F_f \text{ up the slope.} \\ & F_{\text{MAX}} = \mu R \\ & \quad = \frac{1}{4} \times \frac{4}{5}mg \\ & \quad = \frac{1}{5}mg \quad \text{(circled)} \end{aligned}$$

Because $\frac{3}{5}mg > \frac{1}{5}mg$,
it will slide down from A .