

2.2 Quadratics

2.2.1 Quadratic Graphs / 2.2.2 Discriminants / 2.2.3 Completing the Square / 2.2.4 Solving Quadratic Equations / 2.2.5 Further Solving Quadratic Equations (Hidden Quadratics)

Easy (12 questions)	/40
Medium (11 questions)	/46
Hard (11 questions)	/54
Very Hard (12 questions)	/57
Total Marks	/197

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Easy Questions

1 Expand and simplify

(i) $(x + 4)(2x - 3)$

(ii) $(3x - 4)(3x + 4)$

(iii) $(2x + 1)^2$

(3 marks)

2 Factorise

(i) $x^2 + 5x - 14$

(ii) $25x^2 - 36$

(iii) $2x^2 + 9x + 9$

(3 marks)

3 Complete the square for

(i) $x^2 + 8x - 4$

(ii) $2x^2 + 12x - 5$

(iii) $5x^2 - 3x + 2$

(3 marks)

4 Solve

(i) $x^2 + 8x - 9 = 0$

(ii) $3x^2 - 13x + 4 = 0$

(iii) $4x^2 - 6x - 5 = 0$

(3 marks)

5 Write down the value of the discriminant of

(i) $x^2 - 3x + 4$

(ii) $4x + 3 - 2x^2$

(iii) $5 - 8x + 2x^2$

(3 marks)

6 (a) (i) Write down the y -axis intercept on the graph of $y = 2x^2 + 5x - 3$.

(ii) Find the roots of $y = 2x^2 + 5x - 3$.

(3 marks)

(b) Sketch the graph of $y = 2x^2 + 5x - 3$, labelling all points where the graph crosses the coordinate axes.

(3 marks)

7 The function $f(x) = x^2 + kx + 3$ has no real roots.

Show that $k^2 < 12$.

(2 marks)

8 (a) Write $x^2 + 10x + 24$ in the form $(x + a)^2 + b$, where a and b are constants to be found.

(2 marks)

(b) Hence write down the minimum point on the graph of $y = x^2 + 10x + 24$.

(1 mark)

9 The function $f(x) = kx^2 + 2kx - 3$ has two distinct real roots.

Show that $4k(k + 3) > 0$.

(2 marks)

10 (a) (i) Find the roots of the function $g(x) = 12 + 4x - x^2$.

(ii) Write down the y -axis intercept on the graph of $y = g(x)$.

(3 marks)

(b) (i) Write $g(x)$ in the form $a - (x - b)^2$, where a and b are constants to be found.

(ii) Hence write down the coordinates of the turning point on the graph of $y = g(x)$.

(3 marks)

(c) Sketch the graph of $y = g(x)$, labelling all points where the graph intercepts the coordinate axes and the turning point.

(2 marks)

11 Sketch the graph of $y = (2x - 5)^2$, labelling any points where the graph intercepts the coordinate axes.

(3 marks)

- 12** Without showing it algebraically, explain how you know that the function $f(x) = (ax - b)^2$ has a discriminant of zero.

(1 mark)

Medium Questions

1 (a) The curve C has equation $y = x^2 - 3x + 2$.

Find the coordinates of any points where C intersects the coordinate axes.

(3 marks)

(b) Sketch the graph of C , showing clearly all points of intersection with the coordinate axes.

(3 marks)

- 2 (a)** Write the quadratic function $y = x^2 + 8x - 9$ in the form $y = a(x + b)^2 + c$ where a , b and c are integers to be found.

(2 marks)

- (b)** Write down the minimum point on the graph of $y = x^2 + 8x - 9$.

(1 mark)

- (c)** Sketch the graph of $y = x^2 + 8x - 9$, clearly labelling the minimum point and any point where the graph intersects the coordinate axes.

(3 marks)

3 (a) Solve the equation $2x^2 + x - 6 = 0$.

(2 marks)

(b) Find the coordinates of the turning point on the graph of $y = 2x^2 + x - 6$.

(3 marks)

(c) Sketch the graph of $y = 2x^2 + x - 6$, labelling the turning point and any points where the graph crosses the coordinate axes.

(2 marks)

4 (a) Find the minimum value of the function $f(x) = x^2 + 4x + 5$.

(3 marks)

(b) Hence, or otherwise, prove that the function $f(x) = x^2 + 4x + 5$ has no real roots.

(2 marks)

5 The function $f(x) = kx^2 + 2kx - 3$ has two distinct real roots.

Show that $k < -3$ or $k > 0$.

(3 marks)

6 The equation $2x^2 - 4x + 3 - 2k = 0$ has real roots.

Find the possible values of k .

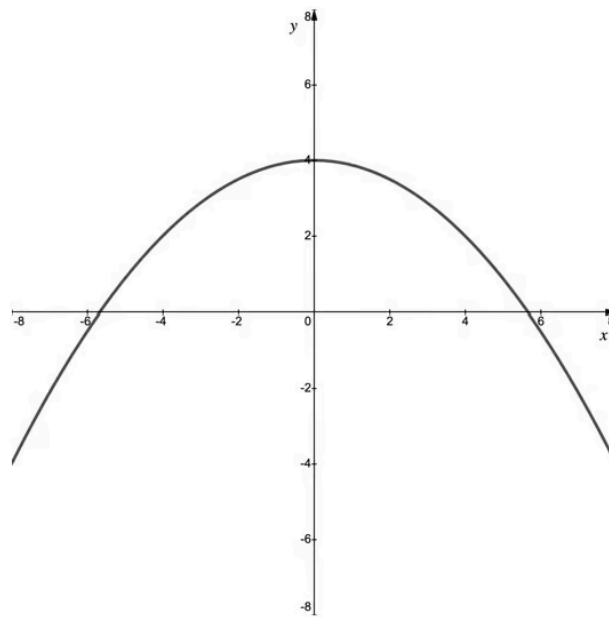
(3 marks)

7 The equation $y = x^2 + px + q$ has no real roots. Show that $p^2 < 4q$.

(2 marks)

- 8 (a)** The graph below shows the curve $f(x) = 4 - \frac{x^2}{8}$.

The curve is to be used as the model for the arch on a bridge where the water level under the bridge is represented by the x -axis. All measurements are in meters.



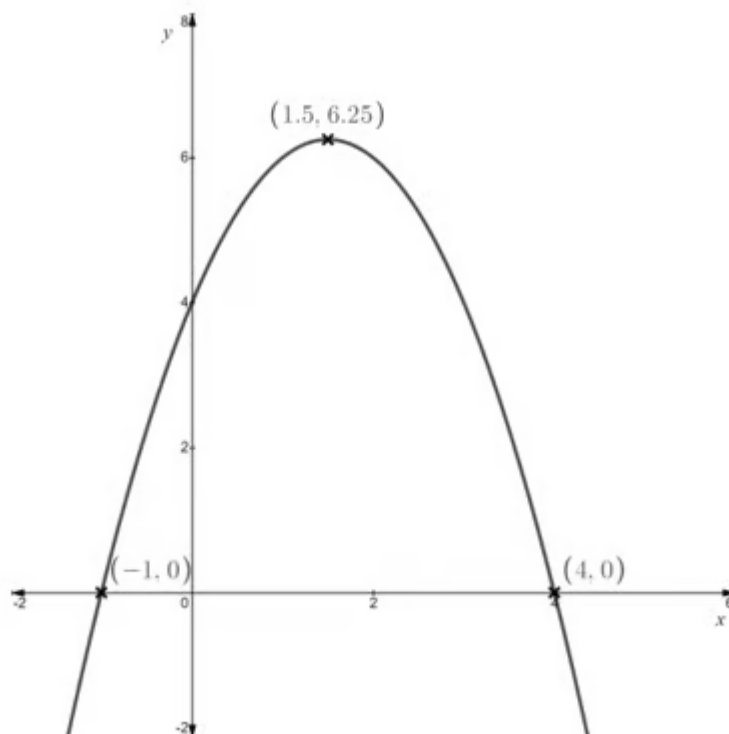
Write down the maximum height of the bridge above the water.

(1 mark)

- (b)** Is the bridge wide enough to span a river of width 11 m?

(3 marks)

- 9** The diagram below shows the graph of $y = f(x)$, where $f(x)$ is a quadratic function. The intercepts with the x -axis and the turning point have been labelled.



Sketch the graph of $y = f(x + 2)$, stating the coordinates of any points that intersect the x -axis and the coordinates of the turning point.

(3 marks)

10 Solve the equation $x^4 - 13x^2 + 36 = 0$.

(3 marks)

11 Solve $x^{\frac{2}{5}} + x^{\frac{1}{5}} = 6$.

(4 marks)

Hard Questions

- 1 (a) Write the quadratic function $y = 4x^2 + 8x - 5$ in the form $y = a(x + b)^2 + c$ where a , b and c are integers to be found.

(2 marks)

- (b) Write down the minimum point on the graph of $y = 4x^2 + 8x - 5$.

(1 mark)

- (c) Sketch the graph of $y = 4x^2 + 8x - 5$, clearly labelling the minimum point and any point where the graph intersects the coordinate axes.

(3 marks)

2 (a) The curve C has equation $y = x^2 - 3x + 2$. The line l has equation $y = 3x - 7$.

Find any points of intersection between C and l .

(3 marks)

(b) Sketch the graphs of C and l , showing clearly any points of intersection with the coordinate axes for both graphs, the minimum point of C and any points of intersection found between C and l .

(3 marks)

3 (a) The equation $y = 3x^2 + 2px + 4q$ has no real roots. Show that $p^2 < 12q$.

(2 marks)

(b) Given that the curve $y = 3x^2 + 2px + 4q$ passes through $(-2, 6)$ and $(2, 6)$ find the values of p and q .

(3 marks)

- 4 (a)** The equation $2k - 3kx - x^2 = 0$ has two distinct real roots.
 k is a negative constant.

Find the possible values of k .

(2 marks)

- (b)** In the case $k = -1$ sketch the graph of $y = 2k - 3kx - x^2$, labelling all points where the graph crosses the coordinate axes.

(3 marks)

- 5 (a)** Find the minimum value of the function $f(x) = x^2 + 4x + c$, giving your answer in terms of c .

(2 marks)

- (b)** Given that $c = 5$, hence, or otherwise, show that the function $f(x) = x^2 + 4x + c$ has no real roots.

(2 marks)

- 6** Sketch the graph of $y = 12x^2 - 5x - 72$, labelling any points where the graph crosses the coordinate axes. (You do not need to label the turning point.)

(3 marks)

- 7** The function $f(x) = kx^2 + 2kx - 3$ has two distinct real roots.

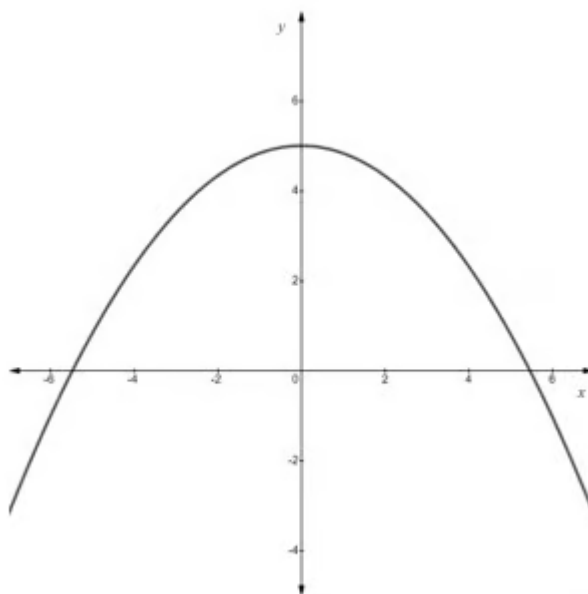
The function $g(x) = kx^2 + 4kx - 16$ has no real roots.

Find the possible values of k .

(3 marks)

- 8 (a)** The graph below shows the curve $y = f(x)$ where $f(x) = 5 - \frac{x^2}{6}$.

The curve is to be used as the model for the arch on a bridge where the water level under the bridge is represented by the x -axis. All measurements are in meters.



Write down the maximum height of the bridge above the water.

(1 mark)

- (b)** Is the bridge wide enough to span a river of width 11 m?

(2 marks)

- (c)** A second bridge is modelled by the curve $y = f(x)$ where $f(x) = 4 - \frac{x^2}{8}$. To support the bridge the arch will continue 2 m under the water (ground) level.

Find the distance between the base of the arch on either side of the river.

(2 marks)

9 (a) Solve the equation $5\sqrt{x} + 3 = 2x$.

(3 marks)

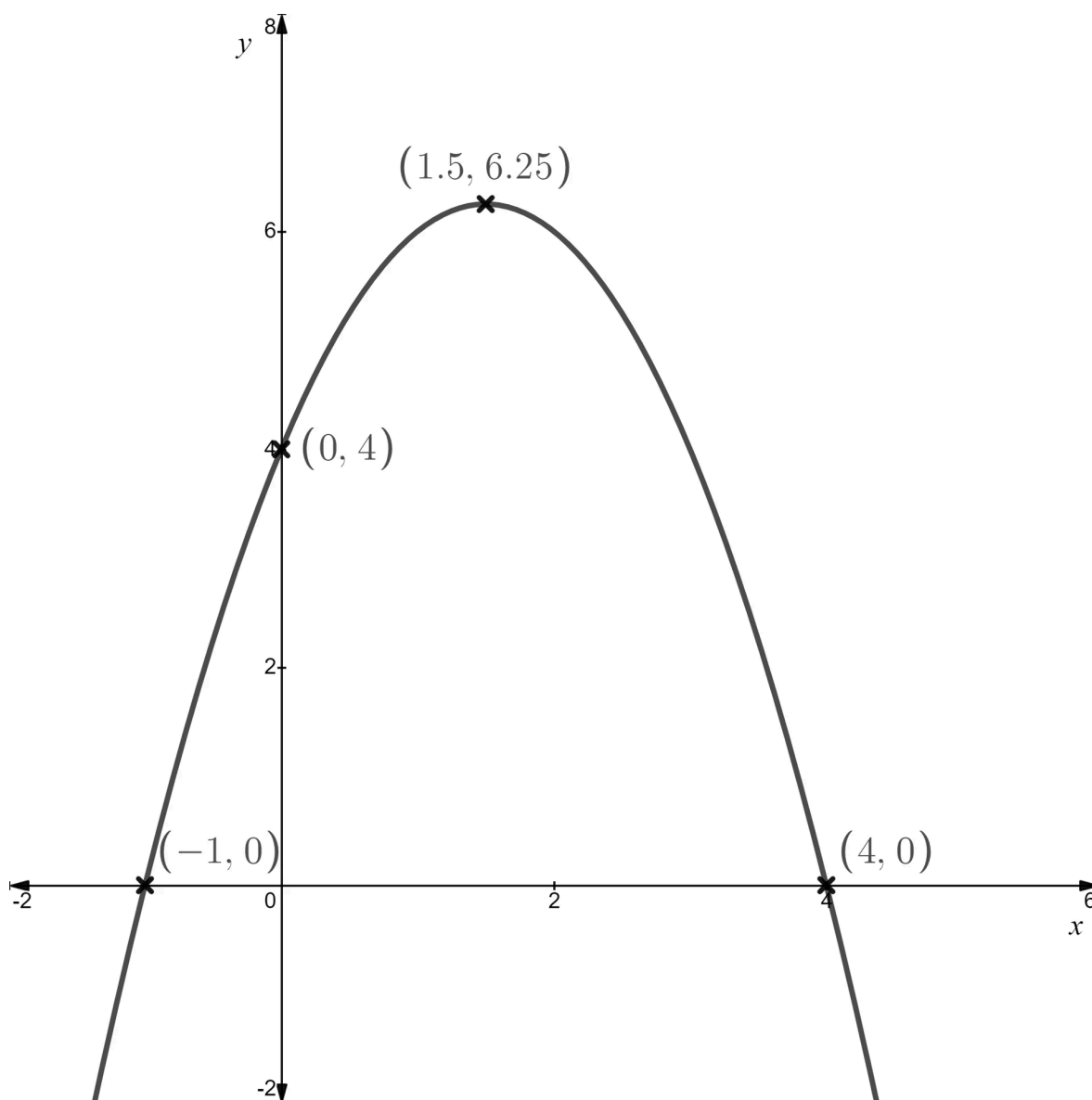
(b) Solve $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} = 8$.

(3 marks)

(c) Solve the equation $2^{2x} + 64 = 20(2^x)$.

(3 marks)

10 The diagram below shows the graph of $y = f(x)$, where $f(x)$ is a quadratic function. The intercepts with the coordinate axes and the turning point have been labelled.



Sketch the graph of $y = f(x + 3)$, stating the coordinates of any points that intersect the coordinate axes and the turning point.

(3 marks)

- 11 (a)** A stone is thrown vertically upwards from the top of a cliff. The path of the stone is modelled by the quadratic function $h(t) = 24 + 2t - 0.5t^2$, $t \geq 0$, where h is the height, in meters, of the stone above the sea and t is the time in seconds since the stone was thrown.

Write down the height of the cliff from which the stone was thrown.

(1 mark)

- (b)** Find the maximum height the stone reaches above the sea.

(2 marks)

- (c)** How long does it take for the stone to hit the sea?

(2 marks)

Very Hard Questions

- 1 (a) Write the quadratic function $y = -6x^2 + 8x - 5$ in the form $y = a - b(x + c)^2$ where a , b and c are constants to be found.

(2 marks)

- (b) Write down the maximum point on the graph of $y = -6x^2 + 8x - 5$.

(1 mark)

- (c) Sketch the graph of $y = -6x^2 + 8x - 5$, clearly labelling the maximum point and any point where the graph intersects the coordinate axes.

(3 marks)

- 2 (a)** The equation $y = x^2 + px + q$ has no real roots. Show that $p^2 < 4q$ and explain why q must be a positive value.

(2 marks)

- (b)** Given that the minimum point on the graph of $y = x^2 + px + q$ is $(3, 1)$ find the values of p and q .

(2 marks)

- 3** The equation $k^2x^2 - 4x + 5 = k^2$ has two distinct real roots.

Find the possible values of k .

(2 marks)

- 4 (a)** The equation $4k - 6kx - x^2 = 0$ has two distinct real roots, α and β .
 k is a negative constant and $0 < \alpha < \beta$.

Sketch the graph of $y = 4k - 6kx - x^2$, labelling the points where the graph crosses the coordinate axes.

(2 marks)

- (b)** Find the possible values of k .

(3 marks)

- 5 (a)** Find the minimum value of the function $f(x) = x^2 + 8x + c$, giving your answer in terms of c .

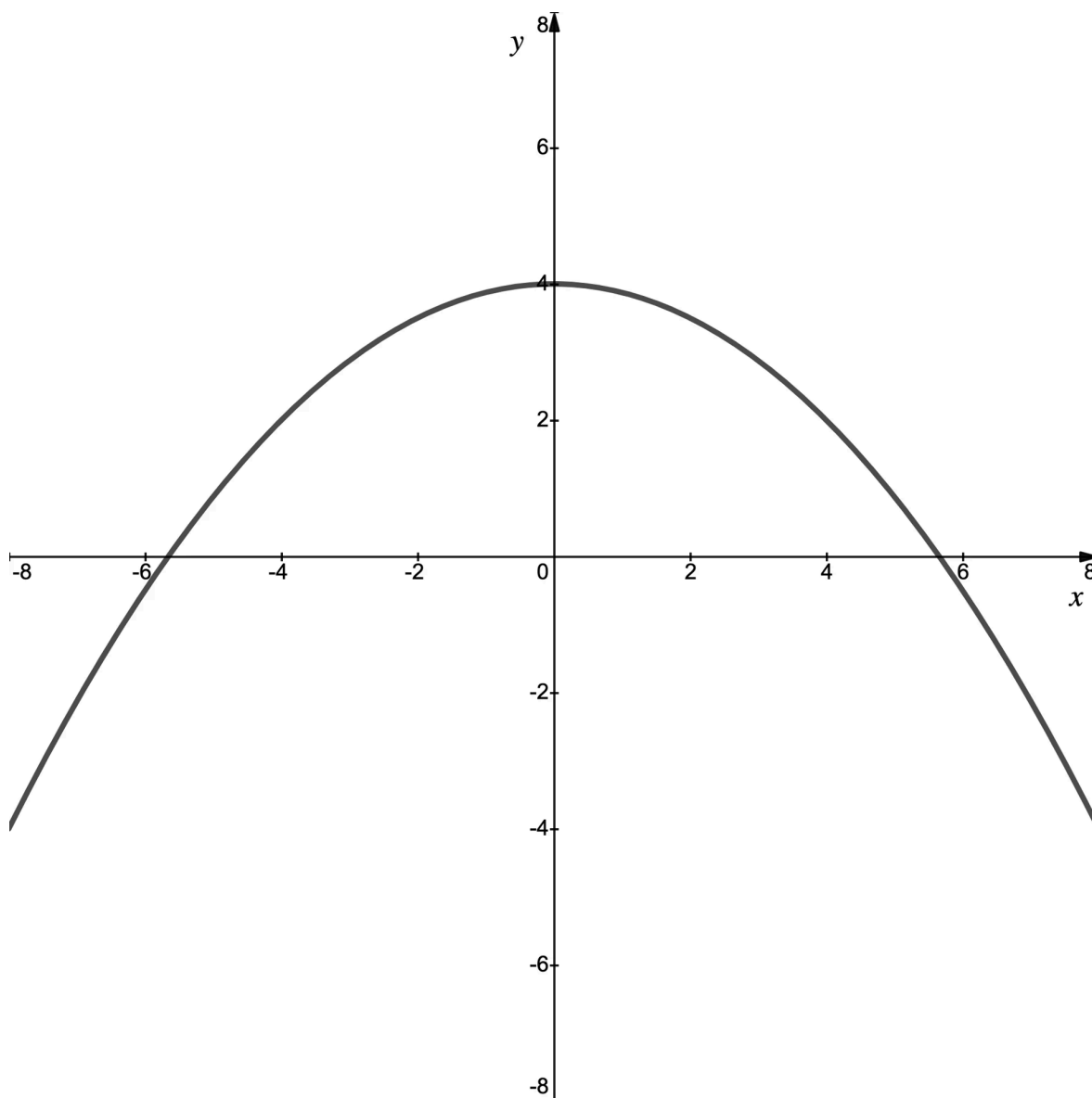
(2 marks)

- (b)** Find the values of c for which the function $f(x) = x^2 + 8x + c$ has no real roots.

(2 marks)

6 (a) The graph below shows the curve $y = f(x)$ where $f(x) = 4 - \frac{x^2}{8}$.

The curve is to be used as the model for the arch on a bridge where the water level under the bridge is represented by the x -axis. All measurements are in meters.



Depending on rainfall throughout the year, the water level can rise by up to 0.5 m, determine whether the bridge is still wide enough to span a river of width 11 m when it is at its peak height.

(2 marks)

- (b)** A barge in the shape of a cuboid (above water level) has a cross-section measuring 6 m wide by 2.5 m tall. The barge regularly travels along the river where the bridge is to be built. Justifying your answer, determine if the barge will fit underneath the bridge or not.

(2 marks)

- (c)** To support the bridge the arch will continue 2.5 m under the water (ground) level.

Find the exact distance between the base of the arch on either side of the river.

(2 marks)

7 (a) Show that the equation $ax^2 + bx + c = 0$ can be written in the form

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2 - 4ac)}{4a} = 0$$

(2 marks)

(b) Hence show that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(2 marks)

8 (a) The function $f(x)$ is defined by $f(x) = (k - 1)x^2 - (k - 2)x - 2k$, $x \in \mathbb{R}$.

The function $g(x)$ is defined by $g(x) = (k - 1)x^2 - 3kx + k + 1$, $x \in \mathbb{R}$.

k is a non-zero constant and $k \neq 1$.

The graphs of $y = f(x)$ and $y = g(x)$ intersect once.

Find the x -coordinate of the intersection, giving your answer in terms of k .

(3 marks)

(b) In the case when $k = 3$, find the coordinates of the point of intersection of $y = f(x)$ and $y = g(x)$.

(2 marks)

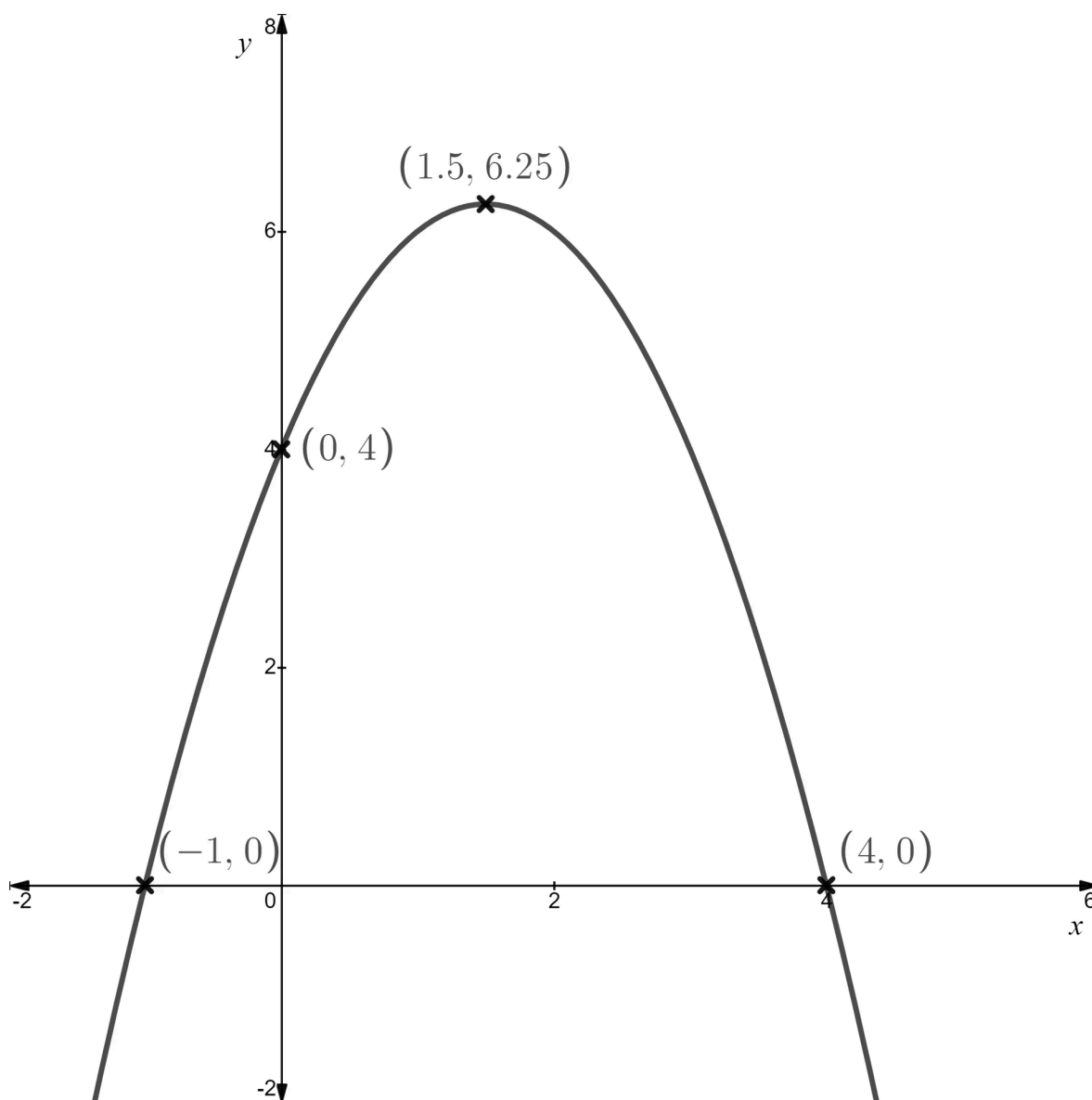
9 (a) Solve the equation $8\sqrt{x} = 48 - x$.

(3 marks)

(b) Solve the equation $2^{4x} + 64 = 20(2^{2x})$.

(3 marks)

10 The diagram below shows the graph of $y = f(x)$. The intercepts with the coordinate axes and the turning point have been labelled.



The graph is transformed by the function $y = f(x) + 6$. One of the new x -axis intercepts is $(-2, 0)$.

Sketch the graph of $y = f(x) + 6$, stating the coordinates of any points that intersect the coordinate axes and the turning point.

(3 marks)

- 11 (a)** A stone is thrown vertically upwards from the top of a cliff. The path of the stone is modelled by the quadratic function $h(t) = 52 + 3t - 0.5t^2$, $t \geq 0$, where h is the height, in meters, of the stone above the sea and t is the time in seconds since the stone was thrown.

Write down the height of the cliff from which the stone was thrown.

(1 mark)

- (b)** Find the maximum height the stone reaches above the sea.

(2 marks)

- (c)** How long does it take for the stone to hit the sea?

(2 marks)

- (d)** How long does the stone stay above its starting height for?

(2 marks)

12 (a) Factorise $x^2 + 6x + 9$

(1 mark)

(b) Factorise $x^2 + 6xy + 9y^2$

(2 marks)

(c) Find a relationship between x and y such that $x^2 + 6xy + 9y^2 = 0$

(2 marks)