AS Topics

Algebraic Expressions	Quadratics	Simultaneous Equations & Inequalities	Graphs & Transformations	Straight Line Graphs
Circles	Algebraic Methods	Binomial Expansion	Trigonometry – Sine and Cosine Rules	Trigonometry – Identities and Equations
Vectors	Differentiation	Integration	Exponentials and Logarithms	

Contains:
AS SAMs
AS 2018
AS 2019
AS 2020
AS 2021
AS 2022

BICEN MATHS

A2 Topics

HOME

A2 Topics

Algebraic Methods – proof, partial fractions, etc.	Functions and Graphs	Sequences and Series	Binomial Expansion	Radians – small angle, arc length, etc.
Trigonometry – sec, cosec, cot, identities	Trigonometry – addition, double angle, harmonic	Parametric Equations	Differentiation	Numerical Methods
Integration	Vectors			

Contains:
A2 SAMs
A2 2018
A2 2019
A2 2020
A2 2021
A2 2022

BICEN MATHS

AS Topics

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Algebraic Expressions

12. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

$$\text{Let} \quad 2^x = y$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

Question	Scheme		Marks	AOs
12(a)	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	**<u>Way 1:</u>** $2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	**<u>Way 2:</u>** $(2x+4)\log 2-\log 9-x\log 2=0$	M1	2.1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer	$x=\dfrac{\log 9}{\log 2}-4$ o.e.	A1	1.1b
			(2)	
			(4 marks)	

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

Question 2 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	$16a^2 = 2\sqrt{a}$ so $\dfrac{16a^2}{2a^{\frac{1}{2}}} = 1$ $8a^{\frac{3}{2}} = 1$ so $a^{\frac{3}{2}} = \dfrac{1}{8}$	M1	This mark is given for a method to find an equation to solve with the terms in a on one side
	$a = \left(\dfrac{1}{8}\right)^{\frac{3}{2}}$	M1	This mark is given for finding a way to deal with the indices when solving the equation
	$a = \dfrac{1}{4}$	A1	This mark is given for finding one correct solution to the equation
	$a = 0$ is also a solution	B1	This mark is given for deducing that $a = 0$ is also a solution
(ii)	$b^4 + 7b^2 - 18 = 0$ factorises to $(b^2 + 9)(b^2 - 2) = 0$	M1	This mark is given for factorising the equation given
	$b^2 = -9, 2$	A1	This mark is given for finding two correct solutions for b^2
	For real solutions, $b^2 = 2$ only	M1	This mark is given for recognising that $b = \sqrt{-9}$ is not a real solution
	$b = \sqrt{2}, -\sqrt{2}$	A1	This mark is given for finding the two real solutions to the equation

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

Question	Scheme	Marks	AOs
3 (i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\frac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\frac{\sqrt{18}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\frac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\frac{3}{2} \Rightarrow x=\ldots$	dM1	1.1b
	$x=\frac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

2. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

(3)

Question	Scheme	Marks	AOs
2	$\frac{9^{x-1}}{3^{y+2}}=81 \Rightarrow \frac{3^{2x-2}}{3^{y+2}}=3^4$ or $\frac{9^{x-1}}{3^{y+2}}=81 \Rightarrow \frac{9^{x-1}}{9^{\frac{1}{2}(y+2)}}=9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4 \Rightarrow y=$ or $\Rightarrow x-1-\frac{1}{2}y-1=2 \Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\frac{9^{x-1}}{3^{y+2}}\right)=\log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3\left(9^{x-1}\right)-(y+2)\log_3\left(3^{y+2}\right)=4$ $\Rightarrow 2(x-1)-y-2=4 \Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3 marks)	

6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

Question	Scheme	Marks	AOs
6 (a)	$3x^3-17x^2-6x=0 \Rightarrow x(3x^2-17x-6)=0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6)=0$	dM1	1.1b
	$\Rightarrow x=0,-\dfrac{1}{3},6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2=n$ where n is any solution $\geqslant 0$ to (a)	M1	2.2a
	Two of $2, 2\pm\sqrt{6}$	A1ft	1.1b
	All three of $2, 2\pm\sqrt{6}$	A1	2.1
		(3)	
			(6 marks)

A2 2019 Paper 2

Algebraic Expressions

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

BICEN MATHS

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A2 2019 Paper 2

Algebraic Expressions

Question 1 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$2^x \times (2^2)^y = 2^{-\frac{3}{2}} \Rightarrow 2^{x+2y} = 2^{-\frac{3}{2}}$	M1	This mark is given for writing all terms in the same base and applying an index law
	$x + 2y = -\frac{3}{2}$	M1	This mark is given for writing an equation to link x and y
	$y = -\frac{1}{2}x - \frac{3}{4}$	A1	This mark is given for rearranging to find a correct expression of y as a function of x

Quadratics

10. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

Question	Scheme	Marks	AOs
10	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

2. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

~~(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."~~

~~State, giving a reason, if the above statement is always true, sometimes true or never true.~~

~~**(2)**~~

Question	Scheme	Marks	AOs
2(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2 \geqslant 0 \Rightarrow (x-4)^2+1 \geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	

6.

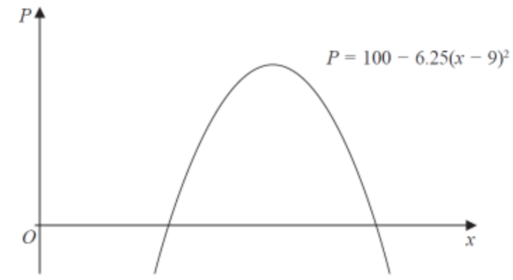


Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

Question	Scheme	Marks	AOs
6 (a)	Attempts $P = 100 - 6.25(15-9)^2$	M1	3.4
	$= -125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P > 80 \Rightarrow (x-9)^2 < 3.2$ or $P = 80 \Rightarrow (x-9)^2 = 3.2$	M1	3.1b
	$\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

Question 9 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$1200 - 3(1 - 20)^2$ $= 1200 - 3(-19)^2$ $= 1200 - 1083$ $= 117$ tonnes	B1	This mark is given for the correct answer
(b)	1200 tonnes	B1	This mark is given for deducing that $(n - 20)^2$ is always positive, and so deducing the maximum value for T Units (tonnes) must be stated
(c)	$[1200 - 3(5 - 20)^2] - [1200 - 3(4 - 20)^2]$ $= 525 - 432$	M1	This mark is given for a method to find the mass of tin that will be mined in 2023
	$= 93$ tonnes	A1	This mark is given for the correct answer (units need not be given)
(d)	$n \leq 20$	B1	This mark is given for an appreciation that the model only predicts the mass of tin mined for the next 20 years
	This model predicts that the mass of tin mined will increase each year	B1	This mark is given for an appreciation that the total mass of tin mined cannot decrease but that for $n > 20$ the value of T decreases as n increases.

11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.

(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

Question	Scheme	Marks	AOs
11(a)	Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $$d=\frac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times-0.002}$$	dM1	1.1b
	Distance = awrt $204(\text{m})$ only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$	M1	1.1b
	$=-0.002\left((d-100)^2-10000\right)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

8.

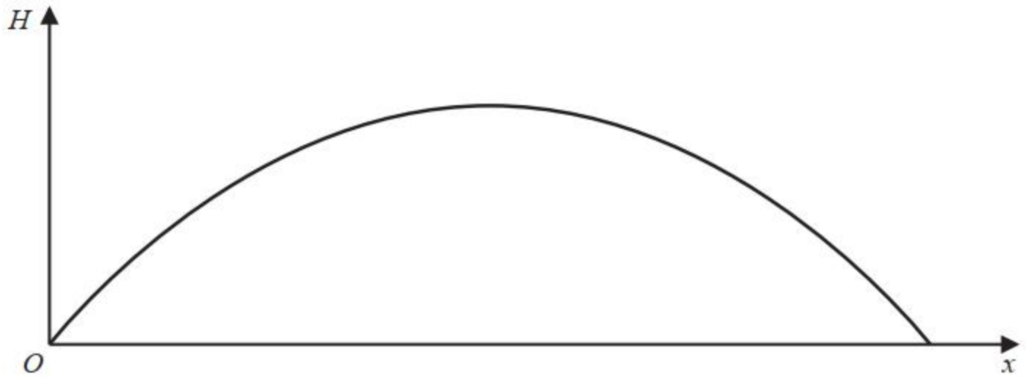


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

A2 2018 Paper 2

Quadratics

(a) Way 3	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x=0,\ H=0 \Rightarrow 0 = 0+0+c \Rightarrow c=0$ **and either** $x=40,\ H=0 \Rightarrow 0 = 1600a + 40b$ **or** $x=20,\ H=12 \Rightarrow 12 = 400a + 20b$ **or** $\frac{-b}{2a} = 20\ \{\Rightarrow b = -40a\}$	M1	3.3
	$b=-40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H=3 \Rightarrow\}\ 3 = \frac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H=3 \Rightarrow\}\ 3 = 12 - \frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\{\text{chooses } 20 + \sqrt{300} \Rightarrow\}$ greatest distance $=$ awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

HOME

5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

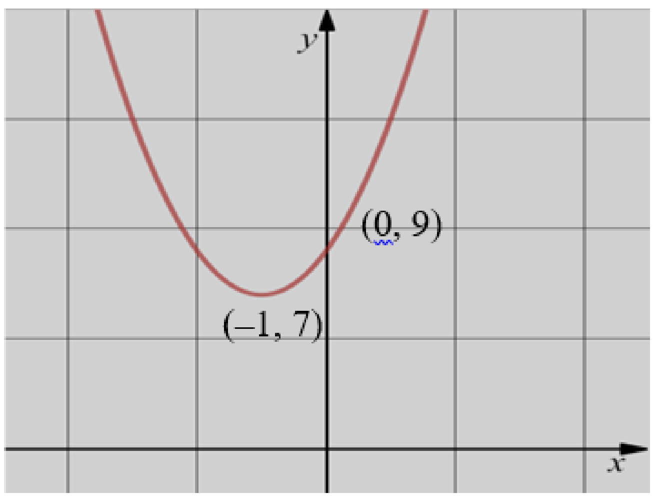
(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

A2 2019 Paper 1

Quadratics

Question 5 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2x^2 + 4x + 9 = 2(x + b)^2 + c$	B1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + c$	M1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$ and $b = 1$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + 7$	A1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$, $b = 1$ and $c = 7$
(b)		B1	This mark is given for a U shaped curve in any position
		B1	This mark is given for a y-intercept shown at (0, 9)
		B1	This mark is given for a minimum shown at (–1, 7)

2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express f(x) in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

A2 2021 Paper 1

Quadratics

Question	Scheme	Marks	AOs
2(a)	$f(x)=(x-2)^2 \pm \ldots$	M1	1.2
	$f(x)=(x-2)^2+1$	A1	1.1b
		(2)	
(b)(i)	$P=(0, 5)$	B1	1.1b
(b)(ii)	$Q=(2, 1)$	B1ft	1.1b
		(2)	
			(4 marks)

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A2 2021 Paper 1

Quadratics

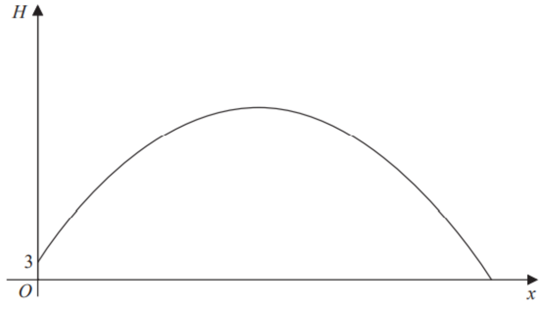

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

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A2 2021 Paper 1

Quadratics

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0,\ H = 3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\text{ m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\text{ (m)}$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
		(9 marks)	

HOME

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.

(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.

(2)

5 (a)	Attempts to use $h^2 = at + b$ with either $t = 2, h = 2.6$ or $t = 10, h = 5.1$	M1
	Correct equations $2a + b = 6.76$ $10a + b = 26.01$	A1
	Solves simultaneously to find values for a and b	dM1
	$h^2 = 2.41t + 1.95$ cao	A1
		(4)
(b)	Substitutes $t = 20$ into their $h^2 = 2.41t + 1.95$ and finds h or h^2 Or substitutes $h = 7$ into their $h^2 = 2.41t + 1.95$ and finds t	M1
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h = 7.08\,(\text{m})$ and states that it is a good model as 7.08 (m) is close to 7 (m) E.g II Finds t =19.5 years and states that the model is accurate as 19.5 (years) $\approx$ 20 (years)	A1
		(2)

Simultaneous Equations & Inequalities

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

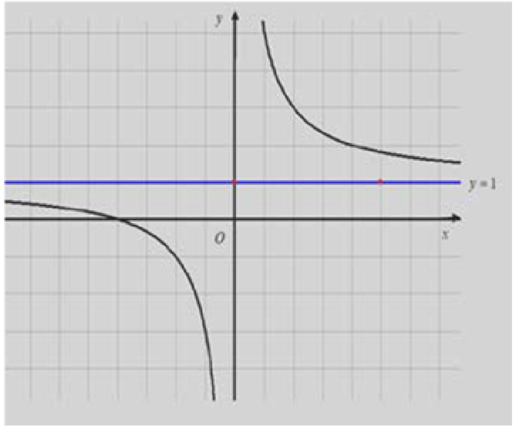
$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

Question 7 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for a graph with shape $\frac{1}{x}$ in the first quadrant
		A1	This mark is given for a fully correct sketch
		B1	This mark is given for the asymptote $y = 1$ correctly shown on the sketch
(b)	$\frac{k^2}{x} + 1 = -2x + 5$	M1	This mark is given for deducing the point of intersection
	$k^2 + x = -2x^2 + 5x$ $-2x^2 + 5x - x - k^2 = 0$ $2x^2 - 4x + k^2 = 0$	A1	This mark is given for correct working to show the result required
(c)	$16 = 4 \times 2 \times k^2$ $16 = 8k^2$	M1	This mark is given for deducing that the equation has a single root and setting $b^2 - 4ac = 0$
		A1	This mark is given for correctly finding $b^2 - 4ac = 0$
	$k^2 = \pm\sqrt{2}$	A1	This mark is given for finding the two exact values of k

1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

Question	Scheme	Marks	AOs
1	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
			(3 marks)

Notes

M1: Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found

M1: Chooses the outside region for their critical values. This may appear in incorrect inequalities such as $5 < x < -4$

A1: Presents in set notation as required $\{x : x < -4\} \cup \{x : x > 5\}$ Accept $\{x < -4 \cup x > 5\}$.

Do not accept $\{x < -4,\ x > 5\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

A2 2019 Paper 1

Sim. Eqns & Inequalities

2.

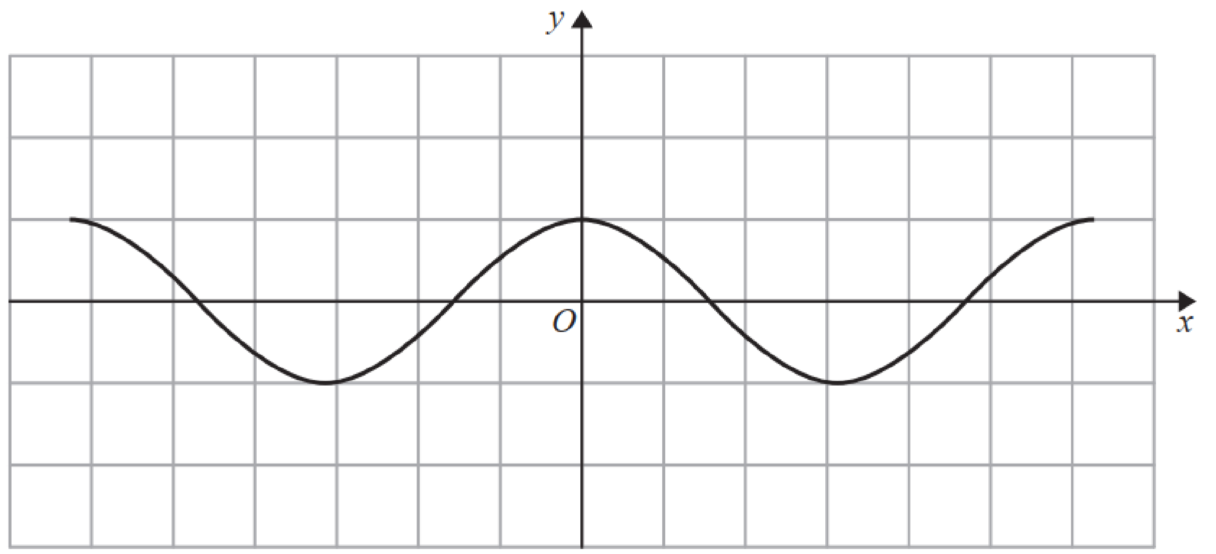


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

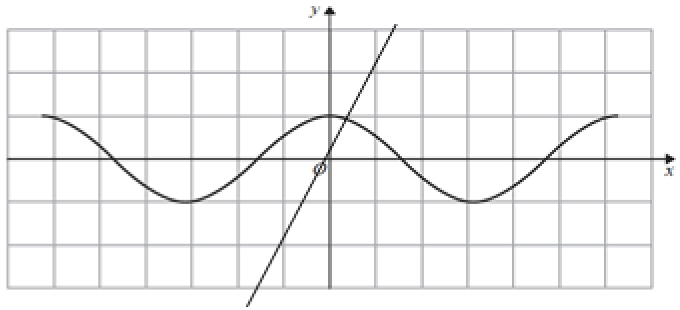
$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

HOME

Question 2 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for plotting the line $y = 2x + \frac{1}{2}$ on the diagram with a correct gradient and intercept
	Only one intersection means that there is one root	A1	This mark is given for a reason why there is only one real root

7.

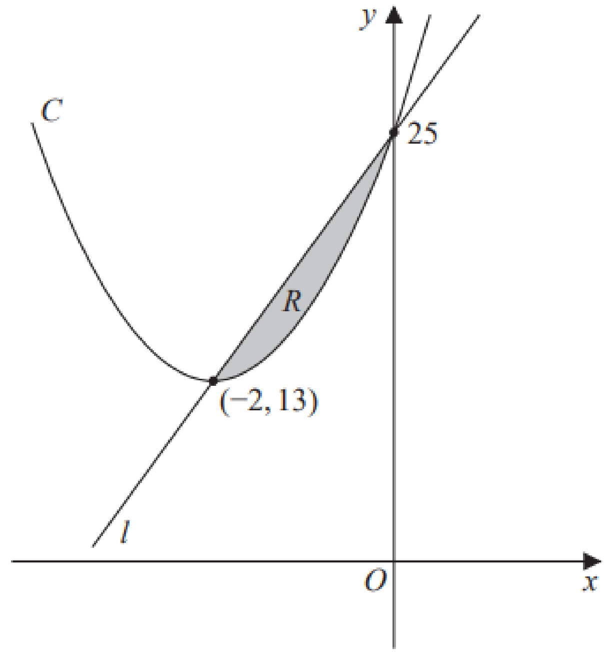


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define R.

(5)

Sim. Eqns & Inequalities

Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2,13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0,25)$ within the equation $y = a(x \pm 2)^2 + 13,$ in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
			(5 marks)

HOME

Graphs & Transformations

13. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

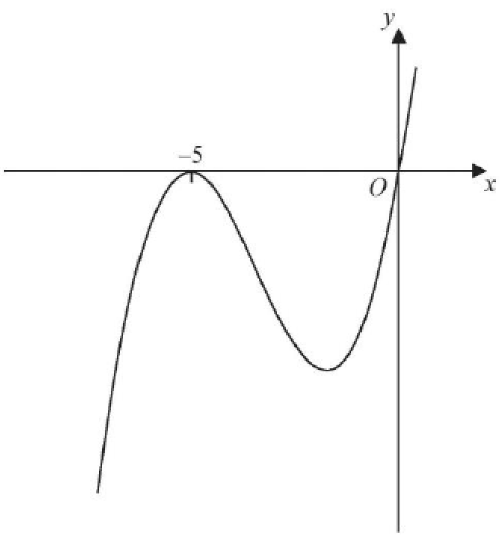
The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

Question	Scheme	Marks	AOs
13(a)	$x^3+10x^2+25x=x(x^2+10x+25)$	M1	1.1b
	$=x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (− 5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	

9.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

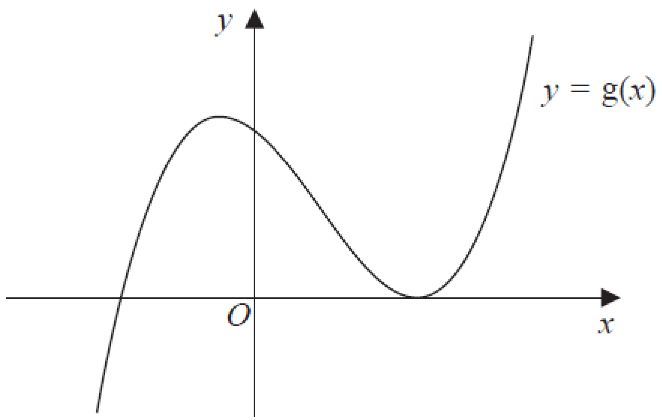


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

AS 2018

Graphs & Transformations

Question	Scheme	Marks	AOs
9(a)	$(g(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$g(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
		(9 marks)	

HOME

11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

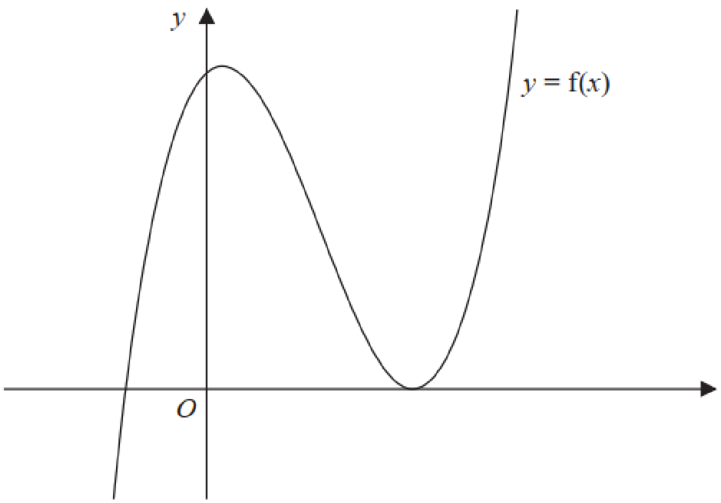


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

Question 11 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$f(4) = (2 \times 4^3) - (13 \times 4^2) + (8 \times 4) + 48$ $= 128 - 208 + 32 + 48$	M1	This mark is given for a method to find f(4)
	$f(4) = 0$ so $(x - 4)$ is a factor	A1	
(b)	$2x^3 - 13x^2 + 8x + 48 =$ $(x - 4)(2x^2 - 5x - 12)$	M1	This mark is given for attempting to factorise the expression ($2x^2$ and -12 seen)
		A1	This mark is given for a fully correct factorisation
	$2x^2 - 5x - 12 = (x - 4)(2x + 3)$	M1	This mark is given for factorising the expression $2x^2 - 5x - 12$
	$f(x) = (x - 4)^2(2x + 3)$ Thus $f(x) =$ only has two roots 4 and $-\frac{3}{2}$	A1	This mark is given for a valid explanation of why the expression only has two roots
(c)	The curve will move two units down	M1	This mark is given for deducing that the curve will be translated by two units
	The curve will cross the axis at three places and so have three roots	A1	This mark is given for deducing that the curve will intersect the x-axis in three places and so the expression will have three roots
(d)	Since $f(x)$ passes through the origin, $f(0) = 0$ so	M1	This mark is given for deducing that $f(0) = 0$ when $f(x + k) = 0$
	$f(x + k) = 0$ when $k = 4,\ -\frac{3}{2}$	A1	This mark is given for the correct two values of k

Part (c) requires differentiation – leave this if you haven't studied it yet

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

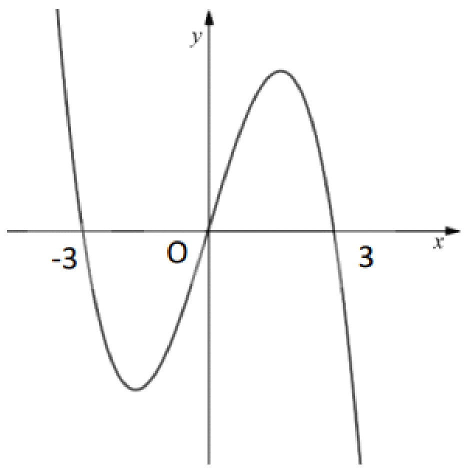
The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

7(a)	$9x - x^3 = x(9 - x^2)$		M1	1.1b
	$9x - x^3 = x(3 - x)(3 + x)$ oe		A1	1.1b
			(2)	
(b)		A cubic with correct orientation	B1	1.1b
		Passes though origin, (3, 0) and (–3, 0)	B1	1.1b
			(2)	
(c)	$y = 9x - x^3 \Rightarrow \dfrac{dy}{dx} = 9 - 3x^2 = 0 \Rightarrow x = (\pm)\sqrt{3} \Rightarrow y = ...$		M1	3.1a
	$y = (\pm)6\sqrt{3}$		A1	1.1b
	$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$ oe		A1ft	2.5
			(3)	
				(7 marks)

A2 2019 Paper 1

Graphs & Transformations

5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x-2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

~~(ii) Find the range of the function~~

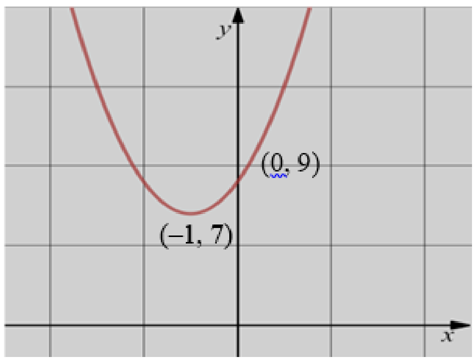
$$\cancel{h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}}$$

~~**(4)**~~

HOME

A2 2019 Paper 1

Graphs & Transformations

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2x^2 + 4x + 9 = 2(x + b)^2 + c$	B1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + c$	M1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$ and $b = 1$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + 7$	A1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$, $b = 1$ and $c = 7$
(b)		B1	This mark is given for a U shaped curve in any position
		B1	This mark is given for a y-intercept shown at (0, 9)
		B1	This mark is given for a minimum shown at (–1, 7)
(c)(i)	$g(x) = 2(x – 2)^2 + 4(x – 2) + 5$	M1	This mark is given for writing g(x) in the form $a(x + b)^2 + c$ and comparing to f(x)
	Translation of $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	This mark is given for deducing the translation of $y = f(x)$ to $y = g(x)$
(c)(ii)	$h(x) = \dfrac{21}{2(x+1)^2 + 7}$ Maximum value = $\dfrac{21}{7}$ (when $x = –1$)	M1	This mark is given for writing h(x) in the form $\dfrac{21}{a(x+b)^2 + c}$ and finding its maximum value
	$0 < h(x) \le 3$	A1	This mark is given for finding the correct range of the function h(x)

HOME

Straight Line Graphs

1. The line l passes through the points A (3, 1) and B (4, −2).

Find an equation for l.

(3)

AS SAMs

Straight Line Graphs

Question	Scheme	Marks	AOs
1 **<u>Way 1</u>**	Uses $y = mx + c$ with both (3, 1) and (4, −2) and attempt to find m or c	M1	1.1b
	$m = -3$	A1	1.1b
	$c = 10$ so $y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or **<u>Way 2</u>**	Uses $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ with both (3, 1) and (4, −2)	M1	1.1b
	Gradient simplified to -3 (may be implied)	A1	1.1b
	$y = -3x + 10$ o.e.	A1	1.1b
		(3)	

4. The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(4)

AS 2018

Straight Line Graphs

Question	Scheme	Marks	AOs
4	States gradient of $4y-3x=10$ is $\dfrac{3}{4}$ oe or rewrites as $y=\dfrac{3}{4}x+\ldots$	B1	1.1b
	Attempts to find gradient of line joining $(5,-1)$ and $(-1,8)$	M1	1.1b
	$=\dfrac{-1-8}{5-(-1)}=-\dfrac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
			(4 marks)

HOME

1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

Question 1 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = -\frac{1}{2}x + \frac{3}{4}$	M1	This mark is given for a method to rearrange to find an equation for l_1 in terms of $y =$
	$m = 2$	A1	This mark is given for deducing the gradient of the perpendicular line l_2
(b)	Substituting $y = 2x + 7$ into $2x + 4y - 3 = 0$ gives $2x + 4(2x + 7) - 3 = 0$	M1	This mark is given for a method to substitute to form and solve an equation in a single variable.
	$10x + 25$ $x = -2.5$	A1	This mark is given for solving to find the value of the x-coordinate of the point P.

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

Question 4 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2.35 = 3m + b$ $3.28 = 6m + b$	M1	This mark is given for using the information to create a model of the form $H = mt + b$ where m is the rate of growth and b is the original height of the tree
	$0.93 = 3m$, $m = 0.31$	M1	This mark is given for finding a value for m
	$H = 0.31m$ so $b = 1.42$ $H = 0.31m + 1.42$	A1	This mark is given for finding a value for b
(b)	b represents the original height of the tree	B1	This mark is given for recognising what b represents
	140 cm = 1.4 m, very close to 1.42 m so supports the use of a linear model	B1	This mark is given for a valid statement to show the use of a linear model is justified

4. In 1997 the average CO_2 emissions of new cars in the UK was 190 g/km.

In 2005 the average CO_2 emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO_2 emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n.

(3)

In 2016 the average CO_2 emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

Question	Scheme	Marks	AOs
4 (a)	Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ Or attempts gradient eg $m = \pm\frac{190-169}{8}(=-2.625)$	M1	3.3
	Full method to find a linear equation linking A with n E.g. Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously	dM1	3.1b
	$A = -2.625n + 190$	A1	1.1b
		(3)	
(b)	Attempts $A = -2.625 \times 19 + 190 = \ldots$	M1	3.4
	$A = 140.125 \text{ g km}^{-1}$	A1	1.1b
	It is predicting a much higher value and so is not suitable	B1ft	3.5a
		(3)	
			(6 marks)

8.

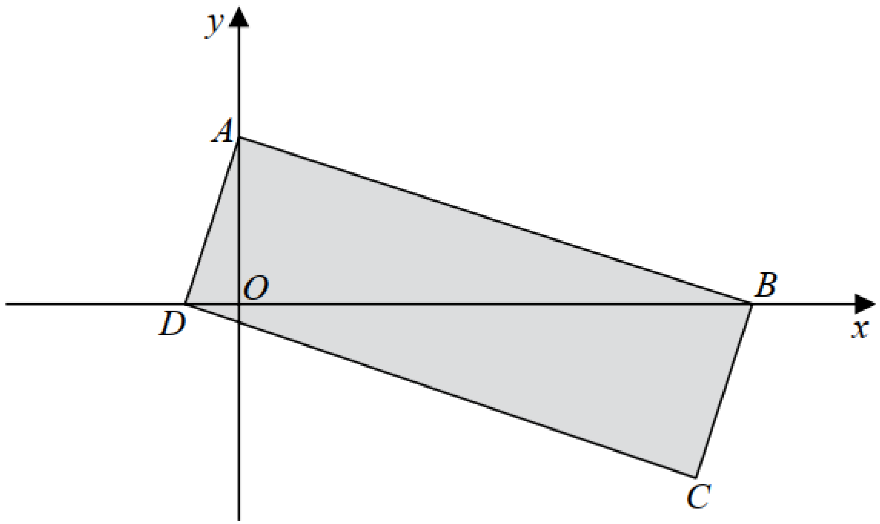


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point A lies on the y-axis and the points B and D lie on the x-axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

A2 SAMs Paper 2

Straight Line Graphs

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
			(7 marks)

HOME

7. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

A2 2019 Paper 2

Straight Line Graphs

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = a + kx$, where a and k are constants	B1	This mark is given for stating a correct general equation
(b)	$500 = 800 \times 2 - (a + 800k)$ $-80 = 300 \times 2 - (a + 300k)$	M1	This mark is given for modelling the profit on the two days when bars of soap are sold for £2
	$a + 800k = 1100$ $a + 300k = 680$	M1	This mark is given for forming a pair of simultaneous equations to find values for a and k
	$500k = 420 \Rightarrow k = \frac{420}{500} = 0.84$ $a + (800 \times 0.84) = 1100$ $a = 1100 - 672 = 428$ Thus $y = 0.84x + 428$	A1	This mark is given for finding the values of a and k to show $y = 0.84x + 428$ as required
(c)	0.84 represents the cost of making one extra bar of soap in £s (i.e. 84p)	B1	This mark is given for a valid interpretation of the significance of 0.84
(d)	For n bars of soap $2n - (428 + 0.84n) > 0$	M1	This mark is given for a method to find the number of bars of soap to be made
	$1.16n - 428 > 0$ $n - \frac{428}{1.16} > 0$ $n = 369$ bars of soap	A1	This mark is given for correctly finding the number of bars of soap to be made

Circles

17. A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.

(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

Question	Scheme		Marks	AOs
17 (a)	**<u>Way 1:</u>** Finds circle equation $(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$	**<u>Way 2:</u>** Finds distance between $(-2, 6)$ and $(10, 11)$	M1	3.1a
	Checks whether $(10, 1)$ satisfies their circle equation	Finds distance between $(-2, 6)$ and $(10, 1)$	M1	1.1b
	Obtains $(x+2)^2 + (y-6)^2 = 13^2$ and checks that $(10+2)^2 + (1-6)^2 = 13^2$ so states that $(10, 1)$ lies on C *	Concludes that as distance is the same $(10, 1)$ lies on the circle C *	A1*	2.1
			(3)	
(b)	Finds radius gradient $\dfrac{11-6}{10-(-2)}$ or $\dfrac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\dfrac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or –23		A1	1.1b
	<u>Way 1</u>: Deduces gradient of second tangent	**<u>Way 2</u>:** Deduces midpoint of PQ from symmetry $(0, 6)$	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ = 35 + 23= 58*		A1*	1.1b
			(7)	

14. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

(6)

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = ...$	M1	1.1b
	(i) Centre $(3, -5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Substitute $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2 + (10k-6)x + 9 = 0$	A1	1.1b
	Attempts $b^2 - 4ac ... 0$ for their a, b and c leading to values for k $"(10k-6)^2 - 36(1+k^2) ... 0" \rightarrow k = ..., ...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2 - 4ac > 0$ and chooses the outside region (see note) for **their** critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k < 0, k > \frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

10. A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

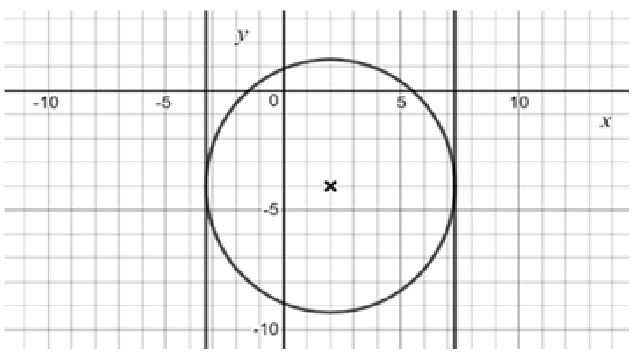
(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

(2)

Question 10 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(x-2)^2+(y+4)^2-4-16-8=0$	M1	This mark is given for a method to complete the square
	Centre at $x=2$ and $y=-4$, $(2, -4)$	A1	This mark is given for finding the correct coordinates of the centre of the circle
	$(x-2)^2+(y+4)^2-28=0$ Radius $=\sqrt{28}=2\sqrt{7}$	A1	This mark is given for finding the exact radius of the circle
(b)	 Tangent of $x=k$ touches circle at $2+\sqrt{28}$ and $2-\sqrt{28}$	M1	This mark is given for adding or subtracting the length of the radius of the circle from 2
		A1	This mark is given for deducing both values of k

11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

(4)

Question	Scheme	Marks	AOs
11. (i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" = $\frac{7-1}{-5+9}=\left(\frac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\frac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<$ 4 $\Rightarrow$ "$52-k$" $<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
			(9 marks)

15.

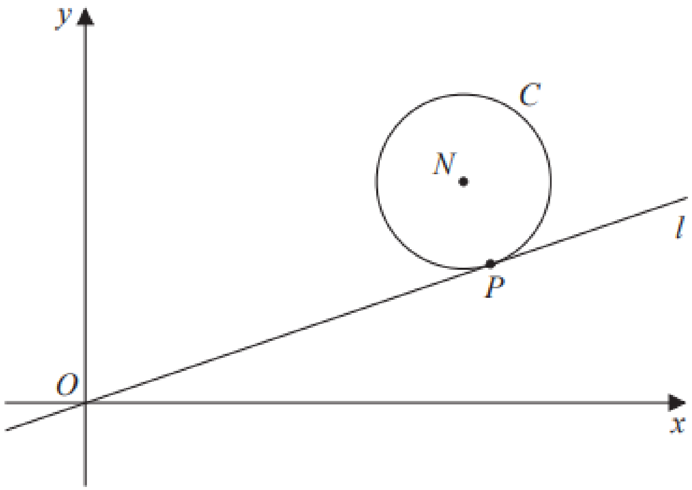


Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

(3)

Question	Scheme	Marks	AOs
15 (a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)

11.

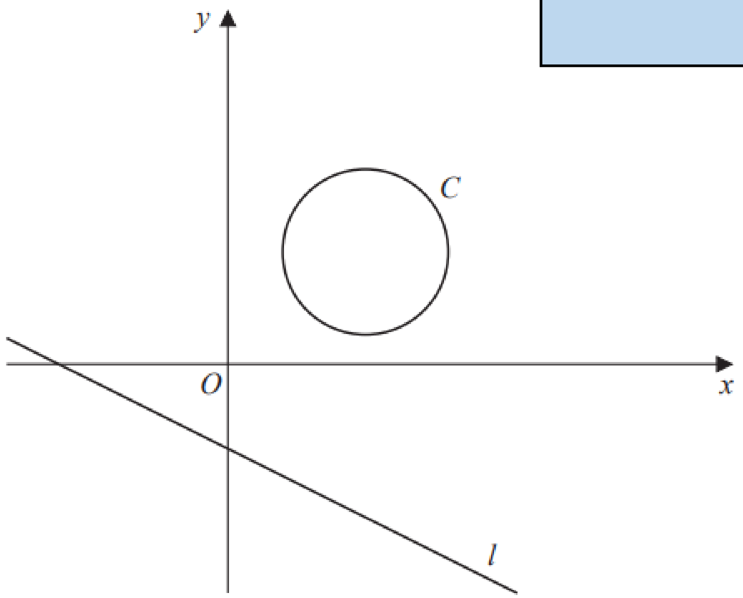


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

11(a)	$(x\pm5)^2+(y\pm4)^2$	M1
	(i) Centre is (5, 4)	A1
	(ii) Radius is 3	A1
		(3)
(b)	$2y+x+6=0\Rightarrow y=-\frac{1}{2}x+\ldots\Rightarrow -\frac{1}{2}\to 2$	B1
	$m_N=2\Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0\Rightarrow x=\ldots,\ y=\ldots$	M1
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left(\text{"}5\text{"}-\text{"}\frac{6}{5}\text{"}\right)^2+\left(\text{"}4\text{"}+\text{"}\frac{18}{5}\text{"}\right)^2}-\text{"}3\text{"}$	dM1
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1
		(5)

3. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

A2 SAMs Paper 1

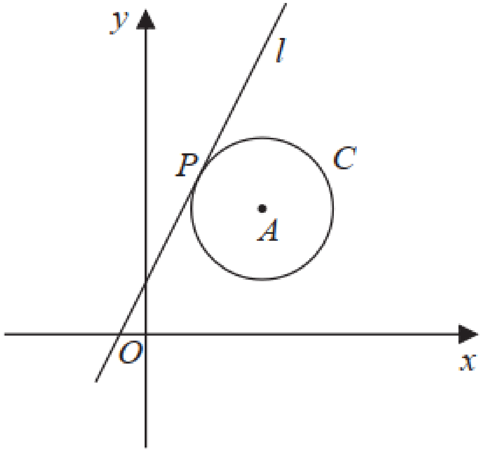
Circles

Question	Scheme	Marks	AOs
3(a)	Attempts $(x-2)^2+(y+5)^2=....$	M1	1.1b
	Centre $(2, -5)$	A1	1.1b
		(2)	
(b)	Sets $k+2^2+5^2>0$	M1	2.2a
	$\Rightarrow k>-29$	A1ft	1.1b
		(2)	
			(4 marks)

BICEN MATHS

HOME

6.



Not to scale

Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

A2 2018 Paper 1

Circles

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)

HOME

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

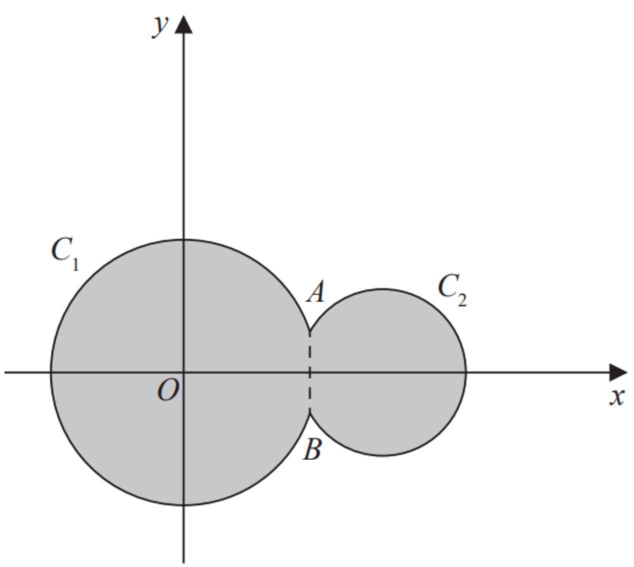


Figure 3

A2 2020 Paper 1

Circles

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\frac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\frac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2\times\arccos\left(\frac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ $= 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\frac{15-"9.5"}{\sqrt{40}} \Rightarrow \beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
			(8 marks)

HOME

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

A2 2020 Paper 2

Circles

14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or $y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$	M1	1.1b
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow(2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow(r-9)^2-81+36=0\Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

HOME

7. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

A2 2021 Paper 1

Circles

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2+(y+2)^2=\ldots$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

HOME

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

3 (a)	(i) $x^2 + y^2 - 10x + 16y = 80 \Rightarrow (x-5)^2 + (y+8)^2 = \ldots$	M1
	Centre $(5, -8)$	A1
	(ii) Radius 13	A1
		(3)
(b)	Attempts $\sqrt{"5"^2 + "8"^2} + "13"$	M1
	$13 + \sqrt{89}$ but ft on their centre and radius	A1ft
		(2)

Algebraic Methods – algebraic fractions, polynomial division, factor theorem, proof

4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

Question	Scheme	Marks	AOs
4(a)	States or uses $f(+3) = 0$	M1	1.1b
	$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so x $4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + \ldots)$	M1	2.1
	$4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + 2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x So $x = 3$ is the only real root of $f(x) = 0$ *	A1*	2.4
		(4)	
		(6 marks)	

11. (a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x + y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

Question	Scheme	Marks	AOs
11 (a) Way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x = -3$ and $y = -5$ then LHS $= \sqrt{15}$ and RHS$= -4$ so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
			(3 marks)

2. (i) ~~Show that $x^2 - 8x + 17 > 0$ for all real values of x~~

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(ii)	For an explanation that it may not always be true Tests say $x=-5$ $\quad (-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $\quad (5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $\quad (-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

9. $$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

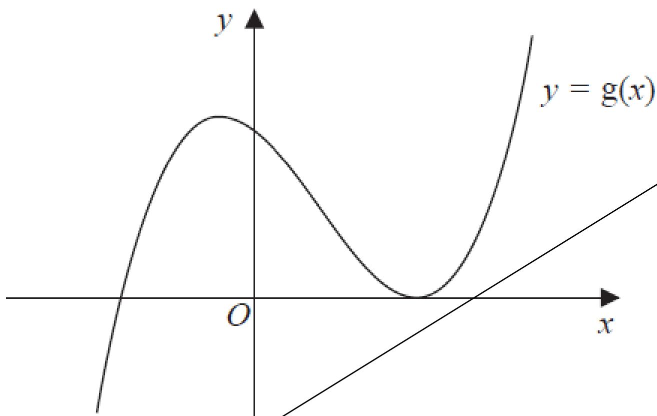


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

AS 2018

Algebraic Methods

Question	Scheme	Marks	AOs
9(a)	$(\mathrm{g}(-2)) = 4\times-8-12\times4-15\times-2+50$	M1	1.1b
	$\mathrm{g}(-2)=0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3-12x^2-15x+50=(x+2)(4x^2-20x+25)$	M1 A1	1.1b 1.1b
	$=(x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x=2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x=-1,\ x=1.25$	B1ft	2.2a
		(3)	
			(9 marks)

HOME

11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

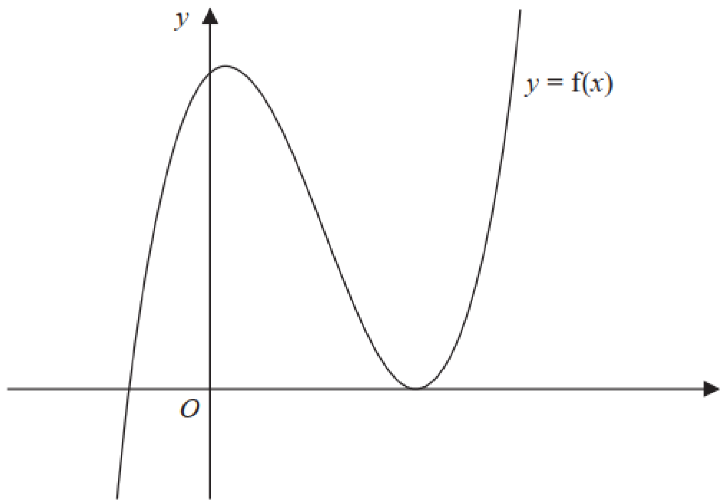


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

HOME

Question 11 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$f(4) = (2 \times 4^3) - (13 \times 4^2) + (8 \times 4) + 48$ $= 128 - 208 + 32 + 48$	M1	This mark is given for a method to find f(4)
	$f(4) = 0$ so $(x - 4)$ is a factor	A1	
(b)	$2x^3 - 13x^2 + 8x + 48 =$ $(x - 4)(2x^2 - 5x - 12)$	M1	This mark is given for attempting to factorise the expression ($2x^2$ and -12 seen)
		A1	This mark is given for a fully correct factorisation
	$2x^2 - 5x - 12 = (x - 4)(2x + 3)$	M1	This mark is given for factorising the expression $2x^2 - 5x - 12$
	$f(x) = (x - 4)^2(2x + 3)$ Thus $f(x) =$ only has two roots 4 and $-\frac{3}{2}$	A1	This mark is given for a valid explanation of why the expression only has two roots
(c)	The curve will move two units down	M1	This mark is given for deducing that the curve will be translated by two units
	The curve will cross the axis at three places and so have three roots	A1	This mark is given for deducing that the curve will intersect the x-axis in three places and so the expression will have three roots
(d)	Since $f(x)$ passes through the origin, $f(0) = 0$ so	M1	This mark is given for deducing that $f(0) = 0$ when $f(x + k) = 0$
	$f(x + k) = 0$ when $k = 4,\ -\frac{3}{2}$	A1	This mark is given for the correct two values of k

15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

Question 15 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	If n is even, $n = 2k$ and $n^3 + 2 = (2k)^3 + 2$ $= 8k^3 + 2$	M1	This mark is given for finding expressions for n and $n^3 + 2$ when n is even
	$8k^3 + 2$ is two more than a multiple of 8 and so not divisible by 8	A1	This mark is given for a correct conclusion following correct working
	If n is odd, $n = 2k + 1$ and $n^3 + 2 = (2k + 1)^3 + 2$ $= 8k^3 + 12k^2 + 6k + 3$	M1	This mark is given for finding expressions for n and $n^3 + 2$ when n is odd
	$8k^3 + 12k^2 + 6k + 3$ is an odd number and so not divisible by 8	A1	This mark is given for a correct conclusion following correct working

10. $$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

Question	Scheme	Marks	AOs
10 (a)	$g(5) = 2\times5^3 + 5^2 - 41\times5 - 70 = ...$	M1	1.1a
	$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3 + x^2 - 41x - 70 = (x-5)(2x^2 ... x \pm 14)$	M1	1.1b
	$= (x-5)(2x^2 + 11x + 14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x)) = (x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5} g(x)dx$ $-\frac{1525}{3} - \frac{190}{3}$	M1	2.2a
	Area = $571\frac{2}{3}$	A1	2.1
		(4)	
			(10 marks)

13. (a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b.

(1)

Question	Scheme	Marks	AOs
13 (a)	States $(2a-b)^2 .. 0$	M1	2.1
	$4a^2+b^2 .. 4ab$	A1	1.1b
	(As $a>0, b>0$) $\dfrac{4a^2}{ab}+\dfrac{b^2}{ab} ... \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b}+\dfrac{b}{a} ... 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \dfrac{4a}{b}+\dfrac{b}{a}=-20-\dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

10. A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

Question	Scheme	Marks	AOs
10(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	..........and factorise $8k^3 \pm 12k^2 \pm 4k = 4k(2k^2 \pm 3k \pm 1) =$	dM1	1.1b
	Correct work with statement $4\times$.. is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
		(5 marks)	
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	
Notes:			

2.

$$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

2(a)	$\mathrm{f}(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$ $= -54 + 45 - 6 + 15$	M1
	$\mathrm{f}(-3) = 0 \Rightarrow (x + 3)$ is a factor	A1
		(2)
(b)	At least 2 of: $a = 2,\ b = -1,\ c = 5$	M1
	All of: $a = 2,\ b = -1,\ c = 5$	A1
		(2)
(c)	$b^2 - 4ac = (-1)^2 - 4(2)(5)$	M1
	$b^2 - 4ac = -39$ which is < 0 so the quadratic has no real roots so $\mathrm{f}(x) = 0$ has only 1 real root	A1
		(2)
(d)	$(x =)\ 2$	B1
		(1)

14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

14(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1
		(1)
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n\left(n+1\right)\left(n+2\right)$	M1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1
		(3)

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

A2 SAMs Paper 2

Algebraic Methods

Question	Scheme	Marks	AOs
1	Sets $f(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
			(3 marks)

HOME

6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. (2)				
(ii) If $ax > b$ then $x > \frac{b}{a}$ (2)				
(iii) The difference between consecutive square numbers is odd. (2)				

(Total for Question 6 is 6 marks)

HOME

A2 SAMs Paper 2

Algebraic Methods

Question	Scheme	Marks	AOs
6(i)	$x^2 - 6x + 10 = (x-3)^2 + 1$	M1	2.1
	Deduces "always true" as $(x-3)^2 \geqslant 0 \Rightarrow (x-3)^2 + 1 \geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a < 0$ then $ax > b \Rightarrow x < \dfrac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a > 0$ then $ax > b \Rightarrow x > \dfrac{b}{a}$ if $a < 0$ then $ax > b \Rightarrow x < \dfrac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $= (n+1)^2 - n^2 = 2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

A2 2018 Paper 2

Algebraic Methods

Question	Scheme	Marks	AOs
6	(a) $\mathrm{f}(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{\mathrm{f}(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ \mathrm{f}(2) = 0$	B1	1.1b
	(ii) $\{\mathrm{f}(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \le \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

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1.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

BICEN MATHS

A2 2019 Paper 1

Algebraic Methods

Question 1 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$f(-3) = 3 \times (-3)^2 + 2a \times (-3)^2 - 4 \times -3 + 5a = 0$	M1	This mark is given for a method to set $f(-3) = 0$
	$f(-3) = 23a - 69 = 0$ $23a = 69$	M1	This mark is given for finding an equation to solve for a
	$a = 3$	A1	This mark is given for finding the correct value of a

A2 2019 Paper 1

Algebraic Methods

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

~~(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.~~

~~**(2)**~~

HOME

A2 2019 Paper 1

Algebraic Methods

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	For even numbers $n = 2m$, $n^2 + 2 = 4m^2 + 2$	M1	This mark is given for showing the case for all even numbers
	This is a multiple of 4 with 2 added, so cannot be divisible by 4	A1	This mark is given for a correct conclusion with a reason why $n^2 + 2$ is not divisible by 4 for all even numbers
	For odd numbers $n = 2m + 1$, $n^2 + 2 = (2m + 1)^2 + 2 = 4m^2 + 4m + 3$ $= 4(m^2 + m) + 3$	M1	This mark is given for showing the case for all odd numbers
	This is a multiple of 4 with 3 added, so cannot be divisible by 4 Hence, for all $n \in \mathbb{N}$, $n + 2$ is not divisible by 4	A1	This mark is given for a correct conclusion with a reason why $n^2 + 2$ is not divisible by 4 for all odd numbers and a full concluding statement that for all $n \in \mathbb{N}$, $n + 2$ is not divisible by 4

16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b
	$(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

1.

$$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

A2 2021 Paper 1

Algebraic Methods

Question	Scheme	Marks	AOs
1	$\mathrm{f}(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = \ldots$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)
Notes			

Main method seen:

M1: Attempts $\mathrm{f}(1) = 0$ to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$

Condone a slip but attempting $\mathrm{f}(-1) = 0$ is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set $\mathrm{f}(\pm 1) = 0$

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: $a = 3$ (following correct work)

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2. $$f(x) = (x - 4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of k.

(3)

2	Sets $\mathrm{f}(-2)=0 \Rightarrow (-2-4)\left((-2)^2-3\times -2+k\right)-42=0$	M1
	$-6(k+10)=42 \Rightarrow k=\ldots$	M1
	$k=-17$	A1
		(3)

11.

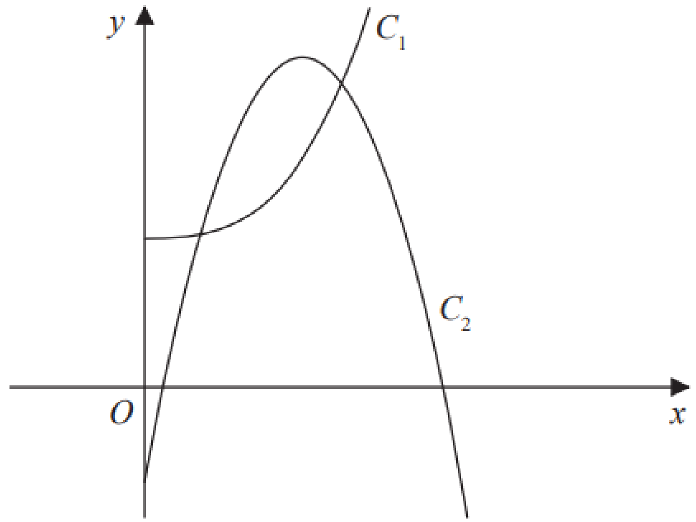

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

Algebraic Methods

11 (a)	Substitutes $x=\frac{1}{2}$ into $y=2x^3+10$ **and** $y=42x-15x^2-7$ and finds the y values for both	M1
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*
		(2)
(b)	Sets $42x-15x^2-7=2x^3+10 \Rightarrow 2x^3+15x^2-42x+17=0$	M1
	Deduces that $(2x-1)$ is a factor and attempts to divide	dM1
	$2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$	A1
	Solves their $x^2+8x-17=0$ using suitable method	M1
	Deduces $x=-4+\sqrt{33}$ (see note)	A1
		(5)

11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

(4)

11	$n\left(n^2+5\right)$	
	Attempts even **or** odd numbers Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	M1
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **Or** achieves $(2k+1)\left(4k^2+4k+6\right)=2(2k+1)\left(2k^2+2k+3\right)$ (for $n=2k+1$) and states "even" Or e.g. achieves $(2k-1)\left(4k^2-4k+6\right)=2(2k-1)\left(2k^2-2k+3\right)$ (for $n=2k-1$) and states "even"	A1
	Attempts even **and** odd numbers Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	dM1
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **and** achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ (for $n=2k\pm1$) and states "even" Correct work and states even for both WITH a final conclusion showing that true for all $n(\in\mathbb{N})$ or e.g. true for all even and odd numbers.	A1
		(4)

Binomial Expansion

7. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2 - \frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

AS SAMs

Binomial Expansion

Question	Scheme	Marks	AOs
7(a)	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right) + \binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2 + \ldots$	M1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = 128 + \ldots$	B1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = \ldots -224x + \ldots$	A1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = \ldots + \ldots + 168x^2 \ (+ \ldots)$	A1	1.1b
		(4)	
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1	2.4
		(1)	
		(5 m	

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11. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

Question	Scheme	Marks	AOs
11(a)	$\left(2-\frac{x}{16}\right)^9 = 2^9 + \binom{9}{1}2^8.\left(-\frac{x}{16}\right) + \binom{9}{2}2^7.\left(-\frac{x}{16}\right)^2 + \ldots$	M1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = 512 + \ldots$	B1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = \ldots -144x + \ldots$	A1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = \ldots + \ldots + 18x^2 \ (+\ldots)$	A1	1.1b
		(4)	
(b)	Sets $'512'a = 128 \Rightarrow a = \ldots$	M1	1.1b
	$(a =)\frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets $'512'b + '-144'a = 36 \Rightarrow b = \ldots$	M1	2.2a
	$(b =)\frac{9}{64}$ oe	A1	1.1b
		(2)	
			(8 marks)

8. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6

You do not need to perform the calculation.

(1)

Question 8 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2^6 = 64$	B1	This mark is given for finding the first term of the expansion
	$\left(2+\frac{3x}{4}\right)^6 =$ $2^6 + {}^6C_1\ 2^5 \left(\frac{3x}{4}\right)^1 + {}^6C_2\ 2^4 \left(\frac{3x}{4}\right)^2 + \ldots$	M1	This mark is given for a method to write out the binomial expansion
	$= 64 + (6 \times 32 \times \frac{3x}{4}) + (15 \times 16 \times \frac{9x^2}{16}) + \ldots$	A1	This mark is given for a correct binomial expansion up to the second and third terms
	$= 64 + 144x + 135x^2 + \ldots$	A1	This mark is given for a fully correct binomial expansion
(b)	$2 + \frac{3x}{4} = 1.925$ $\frac{3x}{4} = -0.075$ so $x = -0.1$ So find the value of $64 + 144x + 135x^2 + \ldots$ with $x = -0.1$	B1	This mark is given for a correct explanation of how the expansion could be used to find an estimate for 1.925^6

6. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

Question	Scheme	Marks	AOs
6 (a)	$(1+kx)^{10} = 1 + \binom{10}{1}(kx)^1 + \binom{10}{2}(kx)^2 + \binom{10}{3}(kx)^3 \ldots$	M1 A1	1.1b 1.1b
	$= 1 + 10kx + 45k^2x^2 + 120k^3x^3 \ldots$	A1	1.1b
		(3)	
(b)	Sets $120k^3 = 3 \times 10k$	B1	1.2
	$4k^2 = 1 \Rightarrow k = \ldots$	M1	1.1b
	$k = \pm\frac{1}{2}$	A1	1.1b
		(3)	
			(6 marks)

8. $$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

AS 2021

Binomial Expansion

Question	Scheme	Marks	AOs
8 (a)	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \dfrac{243}{32}$	M1	1.1b
	$\Rightarrow a = \dfrac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	$= 256 + 5670 = 5926$	A1	1.1b
		(3)	
			(7 marks)

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

AS 2022

Binomial Expansion

6(a)	3^8 or 6561 as the constant term	B1
	$\left(3-\frac{2x}{9}\right)^8 = ...+{}^8C_1(3)^7\left(-\frac{2x}{9}\right)+{}^8C_2(3)^6\left(-\frac{2x}{9}\right)^2+{}^8C_3(3)^5\left(-\frac{2x}{9}\right)^3+...$ $=...+8\times(3)^7\left(-\frac{2x}{9}\right)+28\times(3)^6\left(-\frac{2x}{9}\right)^2+56(3)^5\left(-\frac{2x}{9}\right)^3$	M1 A1
	$=6561-3888x+1008x^2-\frac{448}{3}x^3+...$	A1
		(4)
(b)	Coefficient of x^2 is $\frac{1}{2}\times\text{"}1008\text{"}-\frac{1}{2}\times\text{"}-\frac{448}{3}\text{"}$	M1
	$=\frac{1736}{3}$ (or $578\frac{2}{3}$)	A1
		(2)

BICEN MATHS

HOME

4. In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

A2 2020 Paper 2

Binomial Expansion

Question	Scheme	Marks	AOs
4	${}^7\mathrm{C}_4 a^3 (2x)^4$	M1	1.1b
	$\dfrac{7!}{4!3!} a^3 \times 2^4 = 15120 \Rightarrow a = \ldots$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

HOME

Trigonometry – Sine and Cosine Rules

8.

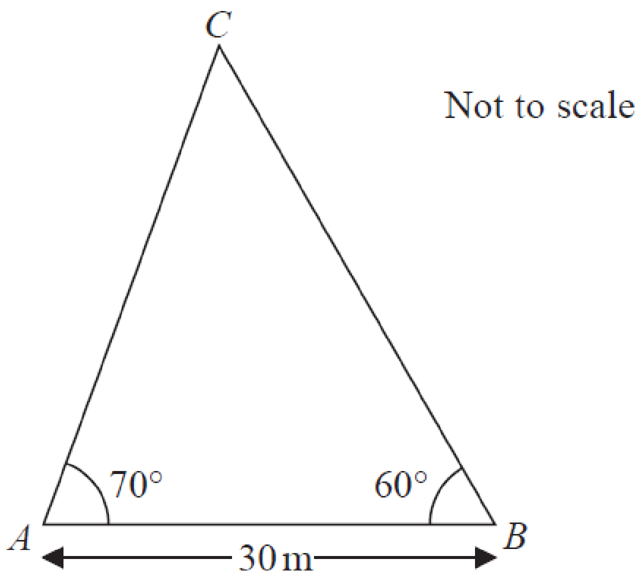


Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

Question	Scheme		Marks	AOs
8(a)	**Way 1** Finds third angle of triangle and uses or states $\dfrac{x}{\sin 60°} = \dfrac{30}{\sin"50°"}$	**Way 2** Finds third angle of triangle and uses or states $\dfrac{y}{\sin 70°} = \dfrac{30}{\sin"50°"}$	M1	2.1
	So $x = \dfrac{30\sin 60°}{\sin 50°}$ $(= 33.9)$	So $y = \dfrac{30\sin 70°}{\sin 50°}$ $(= 36.8)$	A1	1.1b
	Area $= \frac{1}{2} \times 30 \times x \times \sin 70°$ or $\frac{1}{2} \times 30 \times y \times \sin 60$		M1	3.1a
	$= 478\ \text{m}^2$		A1ft	1.1b
			(4)	
(b)	Plausible reason e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat		B1	3.2b
			(1)	
			(5 marks)	

7. In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

Question	Scheme	Marks	AOs
7 (a)	Uses $15=\frac{1}{2}\times 5\times 10\times \sin\theta$	M1	1.1b
	$\sin\theta=\frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta=1-\sin^2\theta$	M1	2.1
	$\cos\theta=\pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2=10^2+5^2-2\times 10\times 5\times "-\frac{4}{5}"$	M1	3.1a
	$BC=\sqrt{205}$	A1	1.1b
		(2)	
			(6 marks)

6.

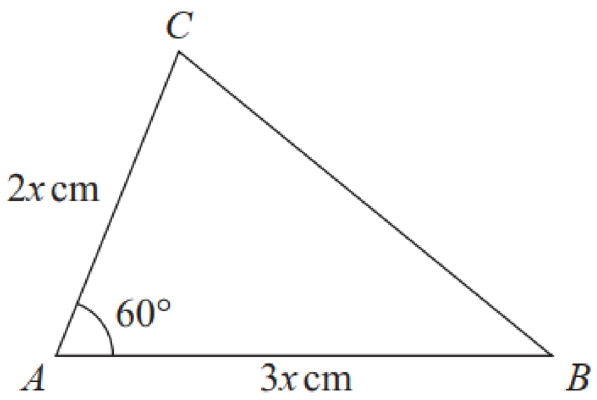


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

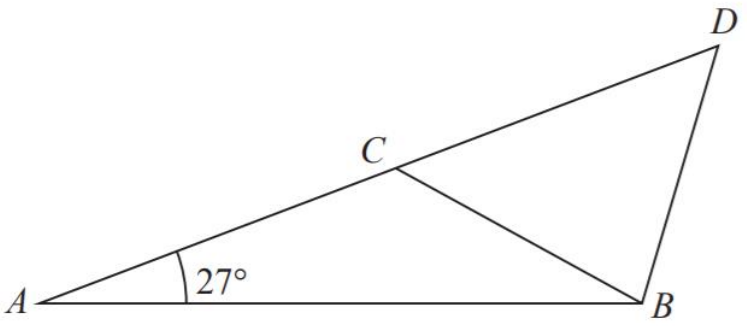
(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

Question 6 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$18\sqrt{3} = \frac{1}{2} \times 2x \times 3x \times \sin 60°$	M1	This mark is given for use of the formula $A = \frac{1}{2}ab \sin C$ for the area of the triangle
	$18\sqrt{3} = 3x^2 \times \frac{\sqrt{3}}{2}$ $x^2 = 12$	M1	This mark is given for using a value of sin 60° to find a value for x^2
	$x = \sqrt{12}$ $= \sqrt{(4 \times 3)}$ $= 2\sqrt{3}$	A1	This mark is given for a full solution to show that $x = 2\sqrt{3}$
(b)	$BC^2 =$ $(6\sqrt{3})^2 + (4\sqrt{3})^2 - 2 \times 6\sqrt{3} \times 4\sqrt{3} \times \cos 60°$	M1	This mark is given for using the cosine rule to start to find the length BC
	$BC^2 = 84$	A1	This mark is given for finding a value for BC^2
	$BC = 2\sqrt{21}$	A1	This mark is given for a correct answer presented as a simplified surd

5.



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = 12$ m, $BC = BD = 7$m and angle $BAC = 27°$

(a) find, to one decimal place, the size of angle ACB.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

Question	Scheme	Marks	AOs
5 (a)	States $\dfrac{\sin\theta}{12}=\dfrac{\sin 27}{7}$	M1	1.1b
	Finds $\theta=$ awrt $51°$ or awrt $129°$	A1	1.1b
	$=$ awrt $128.9°$	A1	1.1b
		(3)	
(b)	Attempts to find part or all of AD Eg $AD^2=7^2+12^2-2\times12\times7\cos101.9=(AD=15.09)$ Eg $(AC)^2=7^2+12^2-2\times12\times7\cos(180-\text{"}128.9\text{"}-27)$ Eg $12\cos27$ or $7\cos\text{"}51\text{"}$	M1	1.1b
	Full method for the total length $=12+7+7+\text{"}15.09\text{"}=$	dM1	3.1a
	$=42$ m	A1	3.2a
		(3)	
			(6 marks)

7. A parallelogram $PQRS$ has area $50\,\text{cm}^2$

Given

- PQ has length 14 cm
- QR has length 7 cm
- angle SPQ is obtuse

find

(a) the size of angle SPQ, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal SQ, in cm, to one decimal place.

(2)

AS 2021

Trigonometry - Rules

Question	Scheme	Marks	AOs
7 (a)	Sets $50 = 7\times14\sin(SPQ)$ oe	B1	1.2
	Finds $180° - \arcsin\left("\frac{50}{98}"\right)$	M1	1.1b
	$= 149.32°$	A1	1.1b
		(3)	
(b)	Method of finding SQ $SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$	M1	1.1b
	$= 20.3$ cm	A1	1.1b
		(2)	
			(5 marks)
Alt(a)	States or uses $14h = 50$ or $7h_1 = 50$	B1	1.2
	Full method to find obtuse $\angle SPQ$. In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$	M1	1.1b
	awrt $149.32°$	A1	1.1b

HOME

4.

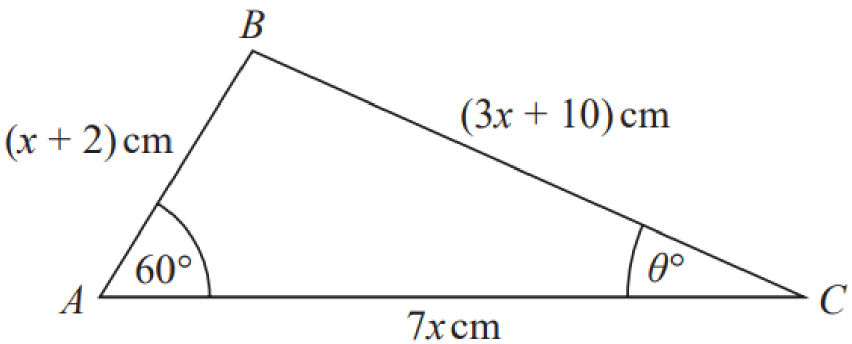


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

4(a)(i)	$(3x+10)^2=(x+2)^2+(7x)^2-2(x+2)(7x)\cos 60°$ oe	M1
	Uses cos 60° = ½, expands the brackets and proceeds to a 3 term quadratic equation	dM1
	$17x^2-35x-48=0$ *	A1*
		(3)
(ii)	$x=3$	B1
		(1)
(b)	$\frac{5}{\sin ACB}=\frac{19}{\sin 60°}\Rightarrow \sin ACB=...\left(\frac{5\sqrt{3}}{38}\right)$ or e.g. $5^2=21^2+19^2-2\times19\times21\cos ACB\Rightarrow \cos ACB=...\left(\frac{37}{38}\right)$	M1
	$\theta=$ awrt 13.2	A1
		(2)

Trigonometry – Identities and Equations

9. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question	Scheme	Marks	AOs
9	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x = 430.5°,\ 435.5°$	A1	1.1b
		(5 marks)	

12. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

AS 2018

Question	Scheme	Marks	AOs
12 (a)	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \dfrac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1 - \cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \dfrac{2}{3}, -\dfrac{1}{2}$	B1	1.1b
	$x = \dfrac{1}{3}\arccos\left(\dfrac{2}{3}\right)$ or $\dfrac{1}{3}\arccos\left(-\dfrac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
			(8 marks)

12. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

AS 2019

Question 12 (Total 7 marks)

Part	Working or answer an examiner might expect to see		
(a)	$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta}$ $\equiv \frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$	M1	This mark is given for using the identity sin $\theta = 1 - \cos^2\theta$ in the fraction
	$\equiv \frac{12 - 7\cos\theta - 10\cos^2\theta}{3 + 2\cos\theta}$	A1	This mark is given for finding a simplified expression in terms of cos θ only
	$\equiv \frac{(3 + 2\cos\theta)(4 - 5\cos\theta)}{3 + 2\cos\theta}$	M1	This mark is given for factorising the numerator of the expression
	$\equiv 4 - 5\cos\theta$	A1	This mark is given for a fully correct proof with correct notation and no errors.
(b)	$4 - 5\cos x = 4 + 3\sin x$ $\tan x = -\frac{5}{3}$	M1	This mark is given for substituting for the fraction and rearranging the equation, using $\frac{\sin x}{\cos x} = \tan x$
	$x = 121°$	A1	This mark is given for one correct value of x
	$x = 301°$	A1	This mark is given for the other correct value of x

AS 2020

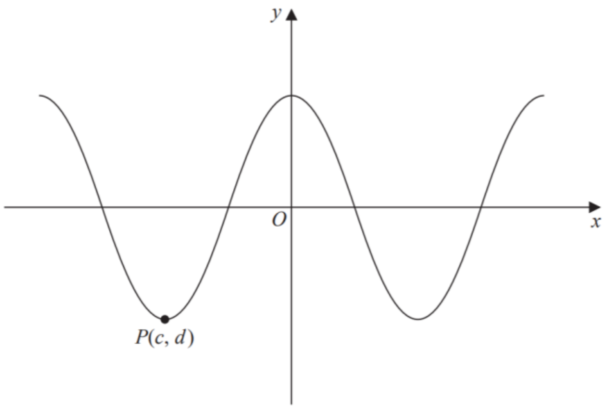


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\dfrac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

Question	Scheme	Marks	AOs
9 (a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \frac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

12. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

(5)

12 (i)	Uses $\cos^2\theta = 1 - \sin^2\theta$ $5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin\theta = \dfrac{-3+\sqrt{34}}{5} \Rightarrow \theta = \ldots$	dM1	3.1a
	$\Rightarrow \theta = 34.5°, 145.5°, 394.5°$	A1 A1	1.1b 1.1b
		(5)	

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are α_1, α_2, α_3 and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

(ii) (a)	One of • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40° = 720° - 53.1°$	M1	3.1a
	$\alpha = 125°$	A1	1.1b
		(2)	

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1 - \sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

13(a)	$\dfrac{1}{\cos\theta}+\tan\theta=\dfrac{1+\sin\theta}{\cos\theta}$ or $\dfrac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$		M1
	$=\dfrac{1+\sin\theta}{\cos\theta}\times\dfrac{1-\sin\theta}{1-\sin\theta}=\dfrac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\dfrac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ or $\dfrac{(1+\sin\theta)\cos\theta}{\cos^2\theta}=\dfrac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta}=\dfrac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)}$		dM1
	$=\dfrac{\cos\theta}{1-\sin\theta}$ *		A1*
			(3)
(b)	$\dfrac{1}{\cos 2x}+\tan 2x=3\cos 2x$ $\Rightarrow 1+\sin 2x=3\cos^2 2x=3\left(1-\sin^2 2x\right)$	$\dfrac{\cos 2x}{1-\sin 2x}=3\cos 2x$ $\Rightarrow \cos 2x=3\cos 2x(1-\sin 2x)$	M1
	$\Rightarrow 3\sin^2 2x+\sin 2x-2=0$	$\Rightarrow \cos 2x(2-3\sin 2x)=0$	A1
	$\sin 2x=\dfrac{2}{3},\ (-1)\Rightarrow 2x=\ldots\Rightarrow x=\ldots$		M1
	$x=20.9°,\ 69.1°$		A1 A1
			(5)

A2 SAMs Paper 2

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

A2 SAMs Paper 2

Trigonometry – Identities and Equations

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\dfrac{\cos\theta}{\sin\theta} = \tan\theta$ It should be $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°) \neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
			(3 marks)

HOME

A2 SAMs Paper 2

Trigonometry – Identities and Equations

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

A2 SAMs Paper 2

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
		(8 marks)	

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

A2 2018 Paper 1

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30 \times 6.5)^\circ =$ awrt 4.48 m with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)^\circ \Rightarrow \sin(30t)^\circ = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Vectors

3. Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

Question	Scheme	Marks	AOs
3(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB\rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB\rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
			(4 marks)

3. Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

Question	Scheme	Marks	AOs
3(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\lvert AB \rvert = 3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

16. (i) Two non-zero vectors, $\mathbf{a}$ and $\mathbf{b}$, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

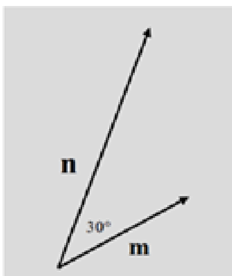
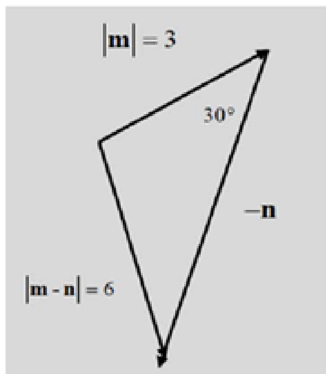
(ii) Two different vectors, $\mathbf{m}$ and $\mathbf{n}$, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector $\mathbf{m}$ and vector $\mathbf{n}$ is $30°$

Find the angle between vector $\mathbf{m}$ and vector $\mathbf{m} - \mathbf{n}$, giving your answer, in degrees, to one decimal place.

(4)

HOME

Question 16 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	**a** and **b** lie in the same direction	B1	This mark is given for a valid explanation
(ii)	 	M1	This mark is given for showing the vector problem graphically (may be implied)
	$\dfrac{\sin 30°}{6} = \dfrac{\sin \theta}{3}$ $\sin\ \theta = \dfrac{1.5}{6} = \dfrac{1}{4}$	M1	This mark is given for using the sine rule as a method to find the angle between −**n** and **n** – **m**
	$\theta = 14.5°$	A1	This mark is given for finding the the angle between −**n** and **n** – **m**
	Angle between **m** and **m** – **n** = (180 – 30 – 14.5) = 135.5°	A1	This mark is given for the angle between vector **m** and vector **m** – **n**

2. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O.

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O.

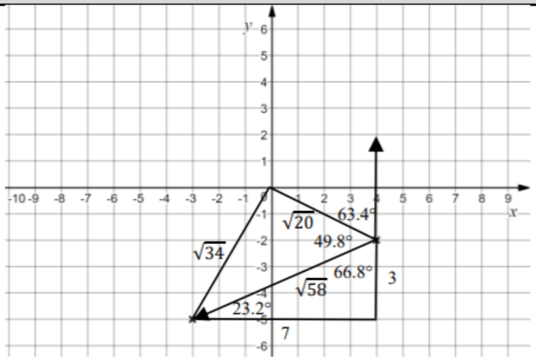
The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

Question	Scheme	Marks	AOs
2(a)			
	Attempts to find an "allowable" angle Eg $\tan\theta = \frac{7}{3}$	M1	1.1b
	A full attempt to find the bearing Eg $180° + "67°"$	dM1	3.1b
	Bearing = awrt $246.8°$	A1	1.1b
		(3)	
(b)	Attempts to find the distance travelled = $\sqrt{(4--3)^2+(-2+5)^2} = \left(\sqrt{58}\right)$	M1	1.1b
	Attempts to find the speed = $\frac{\sqrt{58}}{2.75}$	dM1	3.1b
	= awrt 2.77 km h^{-1}	A1	1.1b
		(3)	
			(6 marks)

4. [*In this question the unit vectors* $\mathbf{i}$ *and* $\mathbf{j}$ *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

Question	Scheme	Marks	AOs
4(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i}-10\mathbf{j}) = -2\times(12\mathbf{i}+5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point O.	A1	
		(2)	
(b)	Attempts distance $AB = \sqrt{(12+24)^2+(10+5)^2}$	M1	
	Attempts speed $= \dfrac{\sqrt{(12+24)^2+(10+5)^2}}{4}$	dM1	
	Speed $=$ 9.75 ms^{-1}	A1	
		(3)	
			(5
Alt(a)	Attempts to find the equation of the line which passes through A and B E.g. $y-5=\dfrac{5+10}{12+24}(x-12)$ $\quad \left(y=\dfrac{5}{12}x\right)$	M1	
	Shows that when $x=0,\ y=0$ and concludes the stone passes through the point O.	A1	

Notes

(a)

M1: Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ either way around.

E.g. States that $(-24\mathbf{i}-10\mathbf{j}) = -2\times(12\mathbf{i}+5\mathbf{j})$

Alternatively, allow an attempt finding the gradient using any two of AO, OB or AB

Alternatively attempts to find the equation of the line through A and B proceeding as far as $y = \ldots x$ Condone sign slips.

A1: States that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ or as AO is parallel to OB (and the stone is travelling in a straight line) the stone passes through the point O. Alternatively, shows that the point $(0,0)$ is on the line and concludes (the stone) passes through the point O.

(b)

M1: Attempts to find the distance AB using a correct method.

Condone slips but expect to see an attempt at $\sqrt{a^2+b^2}$ where a or b is correct

dM1: Dependent upon the previous mark. Look for an attempt at $\dfrac{\text{distance } AB}{4}$

A1: 9.75 ms^{-1} Requires units

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

(b) Hence find $\left|\overrightarrow{QR}\right|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS : SR = 3 : 2$

(c) Find $\overrightarrow{PS}$

(2)

3(a)	$\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ} = 13\mathbf{i} - 15\mathbf{j} - (3\mathbf{i} + 5\mathbf{j})$	M1
	$= 10\mathbf{i} - 20\mathbf{j}$	A1
		(2)
(b)	$\left\lvert \overrightarrow{QR} \right\rvert = \sqrt{"10"^2 + "(-20)"^2}$	M1
	$= 10\sqrt{5}$	A1ft
		(2)
(c)	$\overrightarrow{PS} = \overrightarrow{PQ} + \frac{3}{5}\overrightarrow{QR} = 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5}("10\mathbf{i} - 20\mathbf{j}") = \ldots$ or $\overrightarrow{PS} = \overrightarrow{PR} + \frac{2}{5}\overrightarrow{RQ} = 13\mathbf{i} - 15\mathbf{j} + \frac{2}{5}("-10\mathbf{i} + 20\mathbf{j}") = \ldots$	M1
	$= 9\mathbf{i} - 7\mathbf{j}$	A1
		(2)

10.

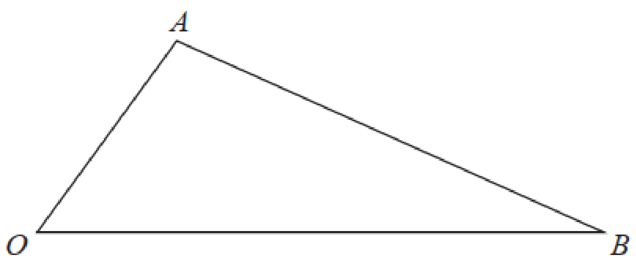


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

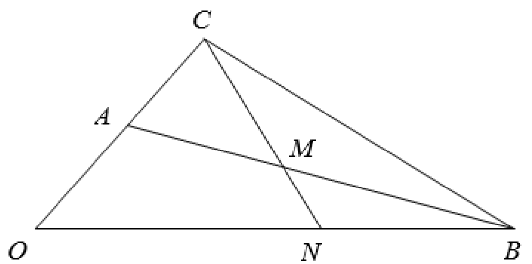
(2)

(c) Hence prove that $ON : NB = 2 : 1$

(2)

A2 2019 Paper 2

Vectors

Part	Working or answer an examiner might expect to see	Mark	Notes
			
(a)	$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$	M1	This mark is given for a method to find an expression for $\overrightarrow{CM}$
	$\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{CM}$ in terms of **a** and **b**
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	This mark is given for a method to find an expression for $\overrightarrow{ON}$
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $= \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2}\lambda\right)\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{ON}$ in terms of **a** and **b**
(c)	$\left(2 - \frac{3}{2}\lambda\right) = 0$ so $\lambda = \frac{4}{3}$	M1	This mark is given for deducing that the coefficient of **a** = 0 and finding a value for λ
	$\overrightarrow{ON} = 0 \times \mathbf{a} + \frac{2}{3}\mathbf{b}$ Hence $ON{:}NB = \frac{2}{3} : \frac{1}{3} = 2{:}1$	A1	This mark is given for finding $\overrightarrow{ON}$ and giving a valid conclusion

HOME

2. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)

Question Number	Scheme	Marks	AO's
2	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$ $\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p}\Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})$ *	A1*	2.1
		(3)	
			(3 marks)

HOME

Differentiation

2. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question	Scheme	Marks	AOs
2	Attempt to differentiate	M1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	M1	1.1b
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 8$	A1ft	1.1b
			(4 marks)

6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

Question	Scheme	Marks	AOs
6	Considers $\dfrac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
	So gradient $=\dfrac{6xh+3h^2}{h}=6x+3h$ or $\dfrac{6x\delta x+3(\delta x)^2}{\delta x}=6x+3\delta x$	A1	1.1b
	States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $=6x$ *	A1*	2.5
			(4 marks)

15.

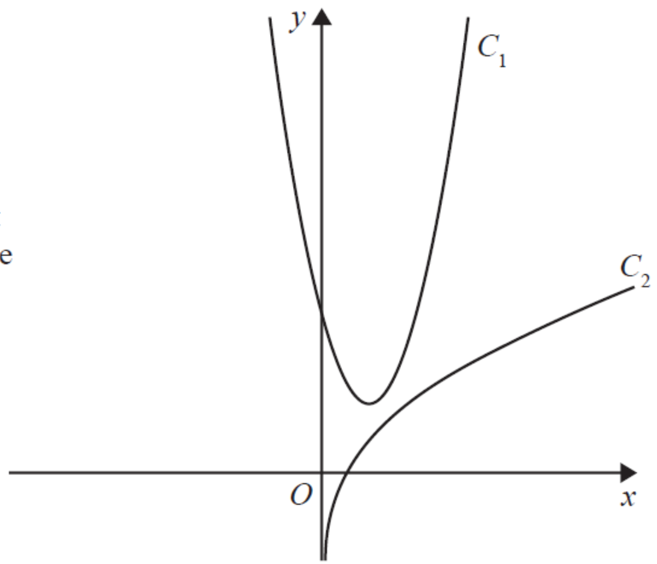


Diagram not drawn to scale

Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

Question	Scheme	Marks	AOs
15	Finds $\frac{dy}{dx} = 8x - 6$	M1	3.1a
	Gradient of curve at P is -2	M1	1.1b
	Normal gradient is $-\frac{1}{m} = \frac{1}{2}$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
			(8 marks)

16.

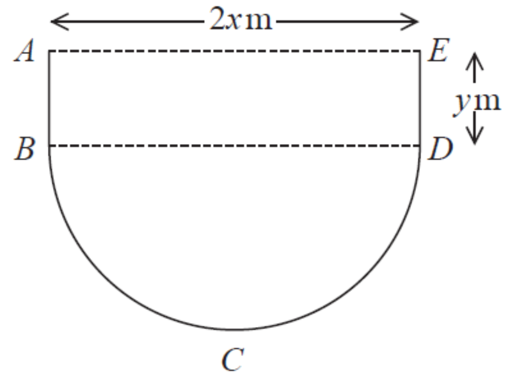


Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

Question	Scheme	Marks	AOs
16(a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{\mathrm{d}P}{\mathrm{d}x}$; $x^{-1}\to x^{-2}$	M1	3.4
	$\frac{\mathrm{d}P}{\mathrm{d}x}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{\mathrm{d}P}{\mathrm{d}x}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8 M	A1	1.1b
		(4)	
		(10 marks)	

HOME

8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

(b) Prove by using $\dfrac{\mathrm{d}^2C}{\mathrm{d}v^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

Question	Scheme	Marks	AOs
8 (a)(i)	$C=\frac{1500}{v}+\frac{2v}{11}+60 \Rightarrow \frac{\mathrm{d}C}{\mathrm{d}v}=-\frac{1500}{v^2}+\frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0 \Rightarrow v^2=8250$	M1	1.1b
	$\Rightarrow v=\sqrt{8250} \Rightarrow v=90.8\,(\mathrm{km\,h^{-1}})$	A1	1.1b
(ii)	For substituting their $v=90.8$ in $C=\frac{1500}{v}+\frac{2v}{11}+60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=+\frac{3000}{v^3}$ at $v=90.8$	M1	1.1b
	$\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=(+0.004)>0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

10. Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

Question	Scheme	Marks	AOs
10	Considers $\dfrac{(x+h)^3 - x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1	1.1b
	so gradient (of chord) $= \dfrac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$	A1	1.1b
	States as $h \to 0$, $3x^2 + 3xh + h^2 \to 3x^2$ so derivative $= 3x^2$ *	A1*	2.5

(4 marks)

5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

AS 2019

Differentiation

Question 5 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for recognising that x^n becomes x^{n-1} when differentiating
	$\frac{dy}{dx} = 6x - \frac{24}{x^2}$	A1	This mark is given for one of the two terms $6x$ or $-\frac{24}{x^2}$ given correctly
		A1	This mark is given for $\frac{dy}{dx}$ given fully correct
(b)	$6x - \frac{24}{x^2} > 0$	M1	This mark is given for setting $\frac{dy}{dx}$ greater than 0 (allow $\geq$)
	$6x^3 - 24 > 0$ $x^3 - 4 > 0$ $x > \sqrt[3]{4}$	A1	This mark is given for the exact range of values of x for which the curve is increasing (allow $\geq$)

1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

Question	Scheme	Marks	AOs
1	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

14. A curve has equation $y = \mathrm{g}(x)$.

Given that

- $\mathrm{g}(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = \mathrm{g}(x)$ passes through the origin
- the curve with equation $y = \mathrm{g}(x)$ has a stationary point at $(2, 9)$

(a) find $\mathrm{g}(x)$,

(7)

(b) prove that the stationary point at $(2, 9)$ is a maximum.

(2)

Question	Scheme	Marks	AOs
14 (a)	Deduces $g(x) = ax^3 + bx^2 + ax$	B1	2.2a
	Uses $(2,9) \Rightarrow 9 = 8a + 4b + 2a$ $\Rightarrow 10a + 4b = 9$	M1 A1	2.1 1.1b
	Uses $g'(2) = 0 \Rightarrow 0 = 12a + 4b + a$ $\Rightarrow 13a + 4b = 0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a, b$	dM1	1.1b
	$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$	M1	1.1b
	$g''(2) = -\frac{33}{2} < 0$ hence maximum	A1	2.4
		(2)	
			(9 marks)

5.

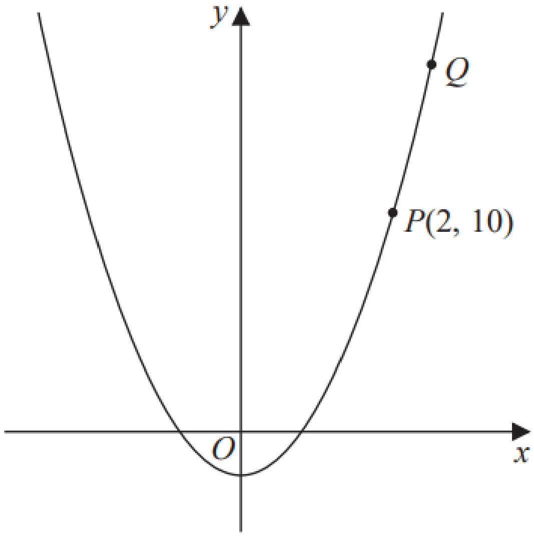


Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

Question	Scheme	Marks	AOs
5(a)	Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=2$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2-2-10}{(2+h)-2}$ oe	B1	1.1b
	$=\frac{3(2+h)^2-12}{(2+h)-2}=\frac{12h+3h^2}{h}$	M1	1.1b
	$=12+3h$	A1	2.1
		(3)	
(c)	Explains that as $h\to 0$, $12+3h\to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

***Note**: this question tests many skills from several different chapters, not just differentiation*

16. The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is -3

(a) (i) show that the value of a is -2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

Question	Scheme	Marks	AOs
16 (a) (i)	Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=-3$ at $x=2 \Rightarrow\ 12a+60-39=-3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a+60-39=-3 \Rightarrow 12a=-24 \Rightarrow a=-2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $\quad 10=8a+60-78+b$	M1	3.1a
	Subs $a=-2$ into $10=8a+60-78+b \Rightarrow b=44$	A1	1.1b
		(4)	
(b)	$\mathrm{f}(x)=-2x^3+15x^2-39x+44 \Rightarrow \mathrm{f}'(x)=-6x^2+30x-39$	B1	1.1b
	Attempts to show that $-6x^2+30x-39$ has no roots Eg. calculates $b^2-4ac=30^2-4\times-6\times-39=-36$	M1	3.1a
	States that as $\mathrm{f}'(x)\neq 0 \Rightarrow$ hence $\mathrm{f}(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3+15x^2-39x+44 \equiv (x-4)\left(-2x^2+7x-11\right)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	

(11 marks)

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is $355\,\text{cm}^3$

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

AS 2022

Differentiation

12(a)	$V=\pi r^2 h=355 \Rightarrow h=\dfrac{355}{\pi r^2}$ $\left(\text{or } rh=\dfrac{355}{\pi r} \text{ or } \pi rh=\dfrac{355}{r}\right)$	B1
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\dfrac{355}{\pi r^2}\right)$	dM1
	$C=0.13\pi r^2+\dfrac{28.4}{r}*$	A1*
		(4)
(b)	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0.26\pi r-\dfrac{28.4}{r^2}$	M1 A1
	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0 \Rightarrow r^3=\dfrac{28.4}{0.26\pi} \Rightarrow r=\ldots$	M1
	$r=\sqrt[3]{\dfrac{1420}{13\pi}}=3.26\ldots$	A1
		(4)
(c)	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ 0.26\pi+\dfrac{56.8}{r^3}=0.26\pi+\dfrac{56.8}{\text{"}3.26\text{"}^3}$	M1
	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1
		(2)
(d)	$C=0.13\pi(\text{"}3.26\text{"})^2+\dfrac{28.4}{\text{"}3.26\text{"}}$	M1
	$(C=)13$	A1
		(2)

HOME

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Question	Scheme	Marks	AOs
1(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12\times 2^3 - 24\times 2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36\times 2^2 - 48\times 2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

A2 SAMs Paper 2

Differentiation

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant	M1	2.1
	Radius $= 4.30$ cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2} \Rightarrow$ Height $= 8.60$ cm	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
		9 marks	

HOME

2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

A2 2018 Paper 1

Differentiation

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = \ldots$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

HOME

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

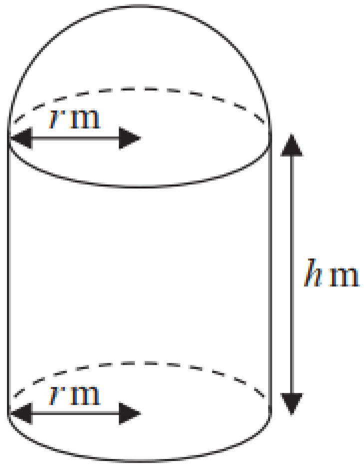


Figure 9

A2 2018 Paper 2

Differentiation

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	This mark is given for a method to find the volume of the cylinder and the semi-hemisphere
	$A = 3\pi r^2 + 2\pi\left(\dfrac{6 - \frac{2}{3}\pi r^3}{\pi r}\right)$	M1	This mark is given for a method to find the surface area of the tank
		A1	This mark is given for finding an expression for the surface area of the tank
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4\pi r^2}{3} = \frac{12}{r} + \frac{5\pi r^2}{3}$	A1	This mark is given for a fully correct proof to show the surface area of the tank as required
(b)	$A = \frac{12}{r} + \frac{5\pi r^2}{3} \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r} = -\frac{12}{r^2} + \frac{10\pi r}{3}$	M1	This mark is given for a method to differentiate to find r
		A1	This mark is given for accurately differentiating to find r
	When $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$, $-\frac{12}{r^2} + \frac{10\pi r}{3} = 0$ $r^3 = \frac{18}{5\pi}$	M1	This mark is given for a method to set $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ to find a value for r
	$r = 1.046$	A1	This mark is given for finding the radius for which the surface area is a minimum
(c)	$A = \frac{12}{1.046} + \frac{5\pi(1.046)^2}{3}$	M1	This mark is given for a method to substitute a value for r
	$A = 17\ \mathrm{m}^2$	A1	This mark is given for correctly finding the minimum surface area of the tank (to the nearest integer)

HOME

6.

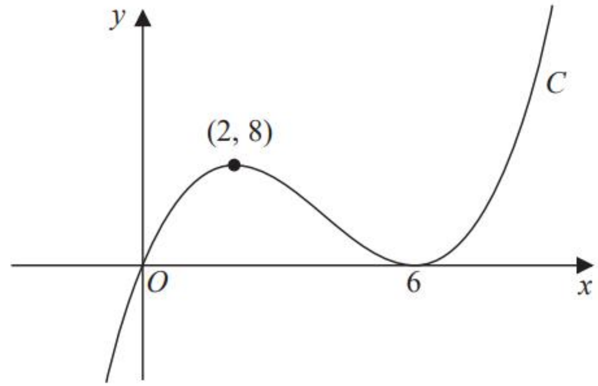


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

6 (a)	$2 < x < 6$	B1
		(1)
(b)	States either $k > 8$ or $k < 0$	M1
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1
		(2)
(c)	**Please see notes for alternatives**	
	States $y = ax(x-6)^2$ or $\mathrm{f}(x) = ax(x-6)^2$	M1
	Substitutes $(2,8)$ into $y = ax(x-6)^2$ and attempts to find a	dM1
	$y = \frac{1}{4}x(x-6)^2$ or $\mathrm{f}(x) = \frac{1}{4}x(x-6)^2$ o.e	A1
		(3)

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

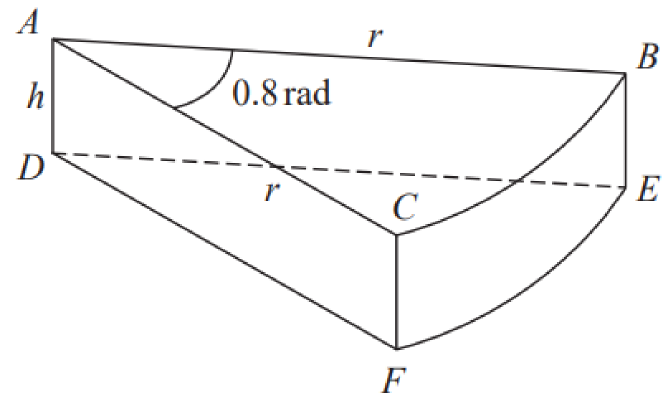


Figure 5

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

15 (a)	Sets up an allowable equation using volume $=240$ E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S=)\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*
		(4)
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r =$ awrt 10.2	dM1 A1
		(4)
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\left.\frac{d^2S}{dr^2}\right\|_{r=10.2} = 5 > 0$ proving a minimum value of S	A1
		(2)

Integration

5. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\displaystyle\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

Question	Scheme	Marks	AOs
5	$f(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\frac{12}{x^2}\right)dx = x^2+3x-\frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\frac{12(\sqrt{2})}{2\times 2}\right)-(-8)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

AS 2018

Integration

1. Find

$$\int \left(\frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) \mathrm{d}x$$

giving your answer in its simplest form.

(4)

HOME

Question	Scheme	Marks	AOs
1	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x=\frac{2}{3}\times\frac{x^4}{4}+\ldots+x$	A1	1.1b
	$=\ldots-6\frac{x^{\frac{3}{2}}}{3/2}+\ldots.$	A1	1.1b
	$=\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$	A1	1.1b
			(4 marks)

15.

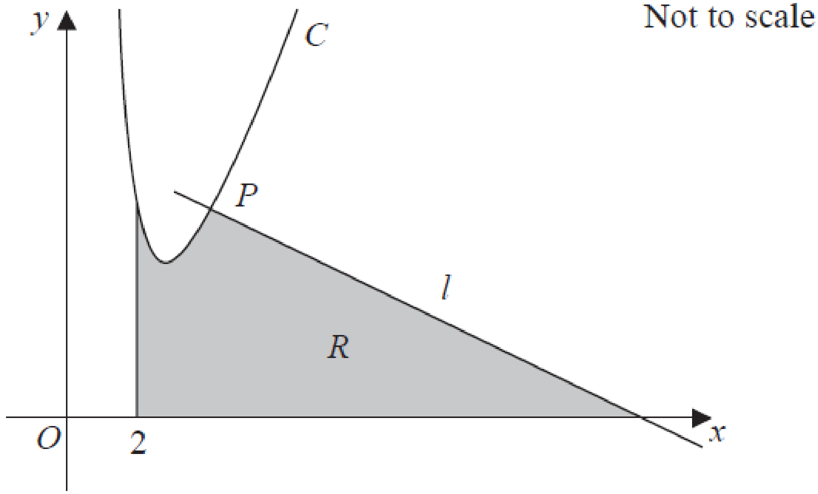


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

Question	Scheme	Marks	AOs
15.	For the complete strategy of finding where the normal cuts the x-axis. Key points that must be seen are • Attempt at differentiation • Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis	M1	3.1a
	$y=\frac{32}{x^2}+3x-8 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{64}{x^3}+3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6="-\frac{1}{2}"(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6="-\frac{1}{2}"(x-4) \Rightarrow x=...$ or an attempt using just gradients $"-\frac{1}{2}"=\frac{6}{a-4} \Rightarrow a=...$	dM1	
	Normal cuts the x-axis at $x=16$	A1	

For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{"16"} "\left(-\frac{1}{2}x+8\right)" \mathrm{d}x$ The "16" cannot have just been made up.	M
$\int \frac{32}{x^2}+3x-8 \, \mathrm{d}x=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M A
Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM
Total area $=10+\ 36\ \ =46$ *	A1
	(10

HOME

3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

Question 3 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\int 4x^{-3} + kx \, \mathrm{d}x = -2x^{-2} + \frac{1}{2}kx$	M1	This mark is given for recognising that x^n becomes x^{n+1} when integrating
		A1	This mark is given for two correctly integrated terms (without c)
	$-\frac{2}{x^2} + \frac{kx^2}{2} + c$	A1	This mark is given for a full answer with a constant (in any correct form)
(b)	$\left[-\frac{2}{x^2} + \frac{k}{2}x^2\right]_{0.5}^{2} =$ $\left(-\frac{2}{2^2} + \frac{4k}{2}\right) - \left(-\frac{2}{0.5^2} + \frac{0.5^2 k}{2}\right) = 8$	M1	This mark is given for substituting the limits 2 and 0.5 and setting equal to 8
	$\left(-\frac{1}{2} + 2k\right) - \left(-8 + \frac{k}{8}\right) = 8$ $7.5 + \frac{15}{8}k = 8$	M1	This mark is given for a method to solve a linear equation in k
	$k = \frac{4}{15}$	A1	This mark is given for finding a correct value for k

13.

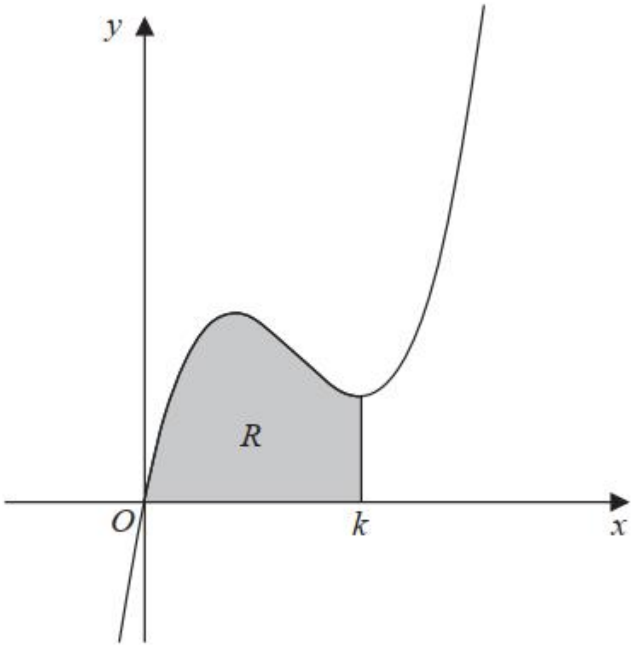


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Question 13 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\frac{dy}{dx} = 6x^2 - 34x + 40$	B1	This mark is given for the equation correctly differentiated
	$\frac{dy}{dx} = 0$ when $6x^2 - 34x + 40 = 0$	M1	This mark is given for setting the equation equal to zero
	$2(3x - 5)(x - 4) = 0$	M1	This mark is given for factorising the expression
	$(x = \frac{5}{3}), x = 4,$	A1	This mark is given for finding two solutions and choosing $x = 4$ as the upper limit of the integral
	$R = \int_0^4 2x^3 - 17x^2 + 40x \ \mathrm{d}x$ $= \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]_0^4$	B1	This mark is given for integrating the expression from 0 to 4
	$R = (\frac{1}{2} \times 4^4) - (\frac{17}{3} \times 4^3) + (20 \times 4^2)$	M1	This mark is given for a calculation for find the area
	$R = 127 - \frac{1088}{3} + 320 = \frac{256}{3}$	A1	This mark is given for a full proof with correct notation and no errors

7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) \mathrm{d}x = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) \mathrm{d}x = 4$$

(4)

HOME

AS 2020

Integration

Question	Scheme	Marks	AOs
7 (a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9},\cancel{9}$	A1	2.3
		(4)	
			(8 marks)

HOME

3. Find

$$\int \frac{3x^4 - 4}{2x^3}\,\mathrm{d}x$$

writing your answer in simplest form.

(4)

Question	Scheme	Marks	AOs
3	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e	A1	1.1b
		(4)	
		(4 marks)	

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.

Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

9. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = 20$$

(4)

Question	Scheme	Marks	AOs
9	$\displaystyle\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36 - 12\sqrt{k} = 20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36 - 12\sqrt{k} = 20 \Rightarrow k =$	dM1	3.1a
	$\Rightarrow k = \dfrac{16}{9}$ oe	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36 - 12\sqrt{k} = 20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both

limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}} < 0$

A1: $k = \dfrac{16}{9}$

14. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

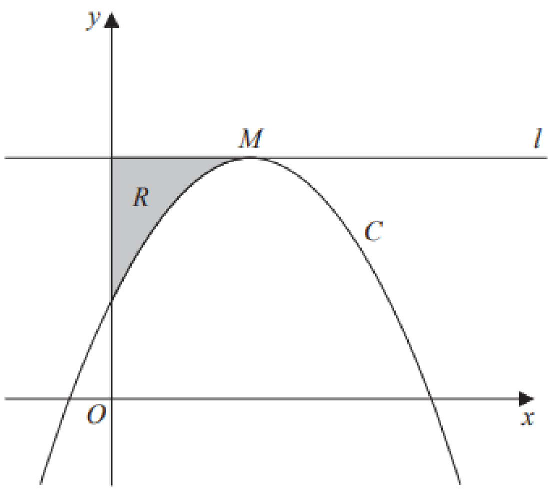


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

HOME

Question	Scheme	Marks	AOs
14 (a)	$\mathrm{f}(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + \ldots$	M1	1.1b
	$= -3(x-2)^2 + \ldots$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \,\mathrm{d}x = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 \left(-3x^2 + 12x + 8\right) \mathrm{d}x$	M1	3.1a
	$R = 40 - \left[-2^3 + 24 + 16\right]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	

(10 marks)

1. Find

$$\int\left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right)\mathrm{d}x$$

giving your answer in simplest form.

(4)

Question	Scheme	Marks	AOs
1	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) \mathrm{d}x = \frac{8x^4}{4} \ldots + 5x$	A1	1.1b
	$= \ldots - 2 \times \frac{3}{2} x^{\frac{1}{2}} + \ldots$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	

(4 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

8(a)	$(k =)\ 0.8$	B1
		(1)
(b)	$1 = 0.8 + 1.4\mathrm{e}^{-0.5t} \Rightarrow 1.4\mathrm{e}^{-0.5t} = 0.2$	M1
	$-0.5t = \ln\left(\frac{0.2}{1.4}\right) \Rightarrow t = \ldots$	M1
	awrt 3.9 minutes	A1
		(3)
(c)	$\left(\frac{\mathrm{d}P}{\mathrm{d}t} =\right) -0.7\mathrm{e}^{-0.5t}$ $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=2} = -0.7\mathrm{e}^{-0.5\times 2}$	M1
	$=$ awrt 0.258 (kg/cm^2 per minute)	A1
		(2)

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\dfrac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

9(a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=p-2$	B1
(ii)	$\log_3\left(\sqrt{x}\right)=\frac{1}{2}p$	B1
		(2)
(b)	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow 2p-4+\frac{3}{2}p=-11\Rightarrow p=\ldots$	M1
	$p=-2$	A1
	$\log_3 x=-2\Rightarrow x=3^{-2}$	M1
	$x=\frac{1}{9}$	A1
		(4)

AS 2022

Integration

10.

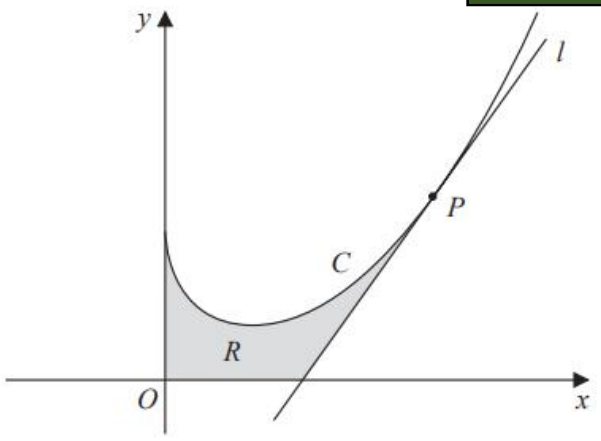


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

HOME

10(a)	$y=\frac{1}{3}x^2-2\sqrt{x}+3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}x-x^{-\frac{1}{2}}$	M1 A1
	$x=4 \Rightarrow y=\frac{13}{3}$	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=4}=\frac{2}{3}\times 4-4^{-\frac{1}{2}}\left(=\frac{13}{6}\right) \quad \therefore y-\frac{13}{3}=\frac{13}{6}(x-4)$	M1
	$13x-6y-26=0$ *	A1*
		(5)
(b)	$\int\left(\frac{x^2}{3}-2\sqrt{x}+3\right)\mathrm{d}x=\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x(+c)$	M1 A1
	$y=0 \Rightarrow x=2$	B1
	Area of R is $\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^4-\frac{1}{2}\times(4-\text{"}2\text{"})\times\text{"}\frac{13}{3}\text{"}=\frac{76}{9}-\frac{13}{3}$	M1
	$=\frac{37}{9}$	A1
		(5)

15.

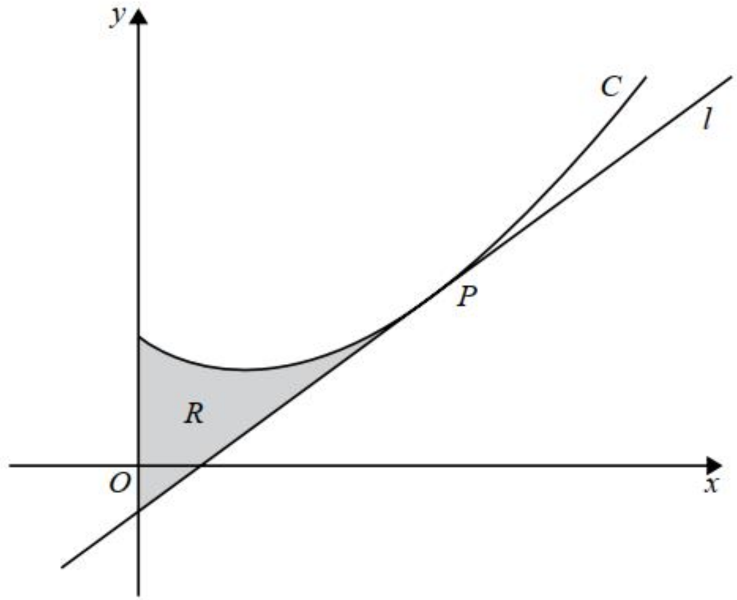


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \ x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

A2 SAMs Paper 2

Integration

Question	Scheme	Marks	AOs
15	$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \frac{dy}{dx} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x-4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x-9)\,dx$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\,(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\right]_0^4 = 2\times 4^{\frac{5}{2}} - \frac{15}{2}\times 4^2 + 20\times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
		(10 marks)	

HOME

A2 2019 Paper 1

Integration

Figure 2 shows a sketch of part of the curve with equation $y = x(x+2)(x-4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\dfrac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b+2)^2(3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

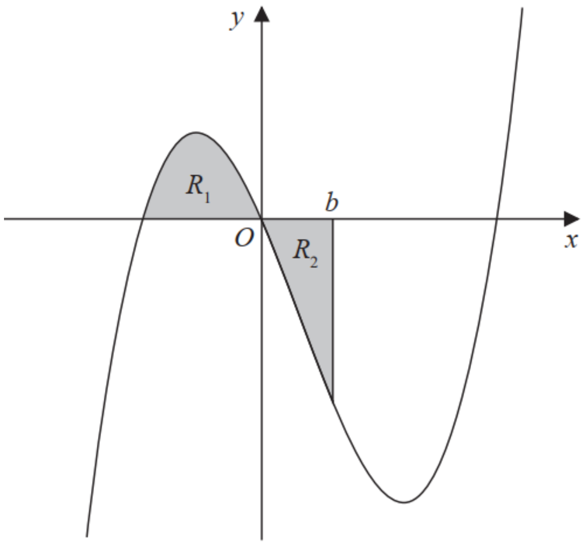
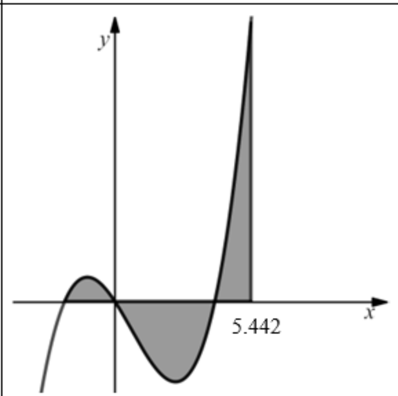


Figure 2

HOME

A2 2019 Paper 1

Integration

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	This mark is given for expanding brackets as a first step to a solution
	$\int_{-2}^{0} x^3 - 2x^2 - 8x \ \mathrm{d}x$	M1	This mark is given for a method to find the exact are of R_1
	$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$	M1	This mark is given for a method to evaluate the integral
	$= 0 - (4 - \frac{-16}{3} - 16) = \frac{20}{3}$	A1	This mark is given for a full method to show the exact value of R_1
(b)	$\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = -\frac{20}{3}$	M1	This mark is given for deducing the area of $R_2 = -\frac{20}{3}$
	$3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	This mark is given for rearranging the equation to a quartic
	$(b+2)^2(3b^2 - 20b + 20)$ $= (b^2 + 4b + 4)(3b^2 - 20b + 20)$	M1	This mark is given for expanding the equation given
	$= 3b^4 - 8b^3 - 48b^2 + 80 = 0$ The two equations are the same, so verified	A1	This mark is for showing, and stating, that the equations are the same
(c)		B1	This mark is given for a sketch of the curve with $b = 5.442$ shown
	Between $x = -2$ and $b = 5.442$, the area above the x-axis is the same as the area below the x-axis	B1	This mark is given for a valid explanation of the significance of the root 5.442

HOME

5.

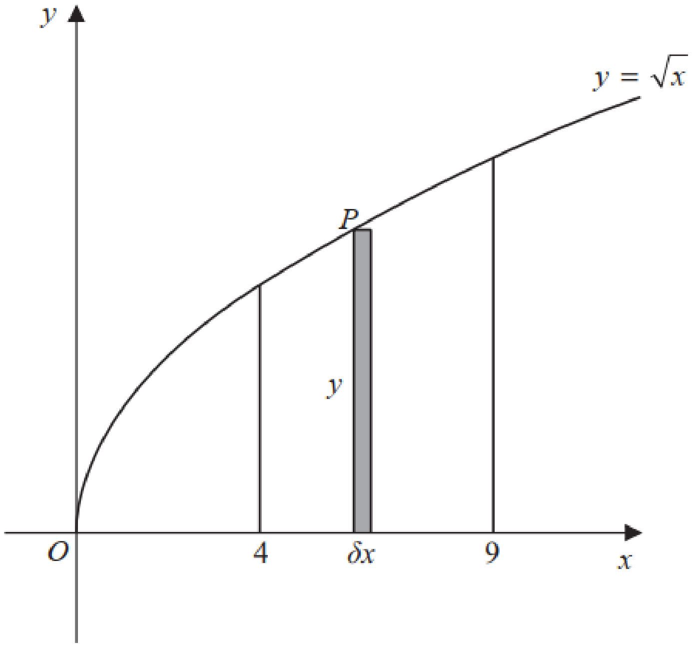


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x$$

(3)

A2 2019 Paper 2

Integration

Question 5 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x = \int_4^9 \sqrt{x}\ \mathrm{d}x$	B1	This mark is given for writing the expression for a sum as an integral
	$\left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9 = \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}}$	M1	This mark is given for a method to evaluate the integral
	$= \frac{38}{3}$	A1	This mark is given for a correct evaluation of the integral

HOME

8. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x+4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

8	$f'(x) = 6x^2 + ax - 23 \Rightarrow f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	$"c" = -12$	B1	2.2a
	$f(-4) = 0 \Rightarrow 2\times(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \ldots\ (6)$	dM1	1.1b
	$(f(x)=)2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(f(x)=)(x+4)(2x^2-5x-3)$ $(f(x)=)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

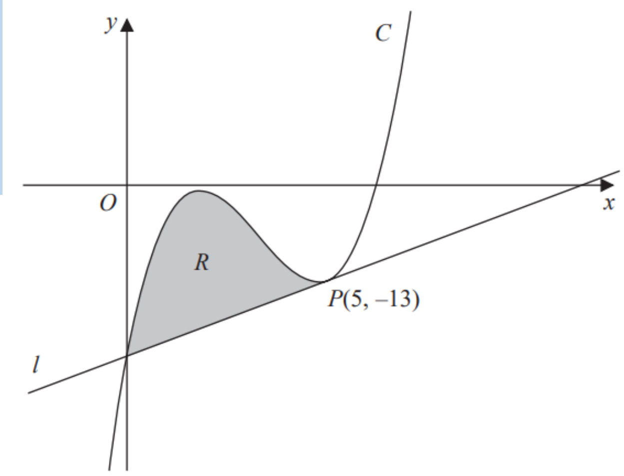


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

HOME

Question	Scheme	Marks	AOs
7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3\times 5^2 - 20\times 5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both C and l pass through (0, –23) and so C meets l again on the y-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

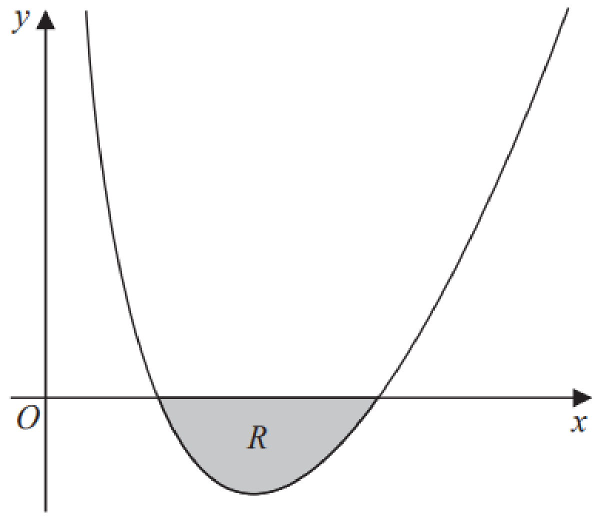


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

A2 2022 Paper 2

Integration

8	$y=\dfrac{(x-2)(x-4)}{4\sqrt{x}}=\dfrac{x^2-6x+8}{4\sqrt{x}}=\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1
	$\displaystyle\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\,\mathrm{d}x=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$	dM1 A1
	Deduces limits of integral are 2 and 4 and applies to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1
	$\left(\dfrac{32}{10}-8+8\right)-\left(\dfrac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}$ Area $R=\dfrac{12}{5}\sqrt{2}-\dfrac{16}{5}$ $\left(\text{or } \dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}\right)$	A1
		(6)

HOME

Exponentials and Logarithms

14.

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

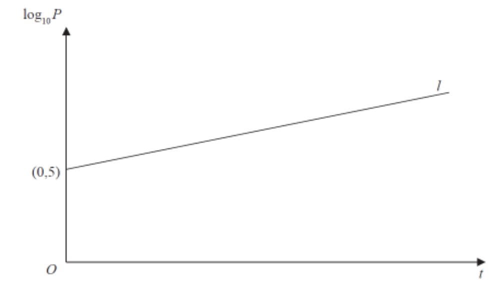


Figure 2

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

Question	Scheme	Marks	AOs
14(a)	$\log_{10} P = mt + c$	M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$	A1	1.1b
		(2)	
(b)	**Way 1:** As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$ **Way 2:** As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	Way 1: $\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$ **Way 2:** $a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	So a = 100 000 **or** b = 1.0116	A1	1.1b
	Both a = 100 000 **and** b = 1.0116 (awrt 1.01)	A1	1.1b
		(4)	
(c)(i)	The initial population	B1	3.4
(c)(ii)	The proportional increase of population each year	B1	3.4
		(2)	
(d)(i)	300000 to nearest hundred thousand	B1	3.4
(d)(ii)	Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give $t =$	M1	3.4
	60.2 years to 3sf	A1ft	1.1b
		(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease		

HOME

5. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$

$2\log_2 \left(\dfrac{x}{\sqrt{x}}\right) = 3$ using the subtraction law for logs

$2\log_2 \left(\sqrt{x}\right) = 3$ simplifying

$\log_2 x = 3$ using the power law for logs

$x = 3^2 = 9$ using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

Question	Scheme		Marks	AOs
5 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\dfrac{x^2}{\sqrt{x}}\right) = 3$	$\dfrac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\dfrac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
			(5 marks)	

AS 2018

13.

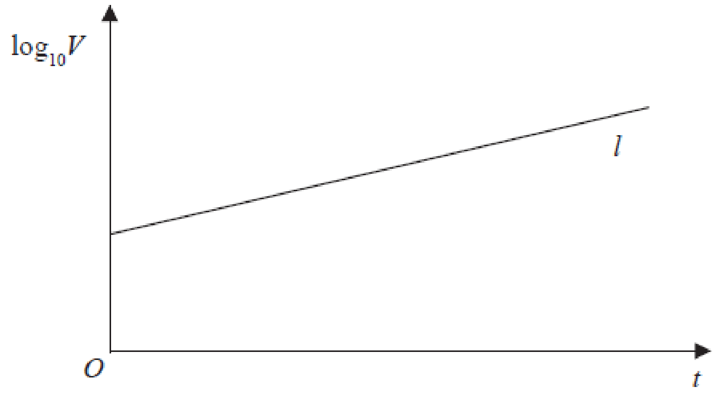


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10}V$ since 1st January 1980.

The equation of line l is $\log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $\quad p = 10^{4.8}$ or $q = 10^{0.05}$	M1	1.1b
	For $p =$ awrt 63100 or $q =$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p = 10^{4.8}$ and $q = 10^{0.05}$	dM1	3.1a
	For $p =$ awrt 63100 and $q =$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V = 63100 \times 1.122^{30}$ or $\log V = 0.05 \times 30 + 4.8$ leading to $V =$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

14. The value of a car, £V, can be modelled by the equation

$$V = 15\,700\mathrm{e}^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925\mathrm{e}^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

Question 14 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(15\,700 \times e^0) + 2300 = 18\,000$	B1	This mark is given for a correct value for the initial value of the car
(b)(i)	$\frac{dV}{dt} = (-0.25 \times 15\,700)\, e^{-0.25t}$ $= -3925e^{-0.25t}$	M1	This mark is given for making the link between gradient and rate of change and finding $\frac{dV}{dt} = ke^{-0.25t}$
		A1	This mark is given for a fully correct expression for $\frac{dV}{dt}$
	$-3925e^{-0.25T} = -500$ thus $3925e^{-0.25T} = 500$	A1	This mark is given fully correct working to show that $3925e^{-0.25T} = 500$
(b)(ii)	$e^{-0.25T} = \frac{500}{3925}$ $-0.25T = \ln \frac{500}{3925}$	M1	This mark is given for the start of a method to find the age of the car using logarithms
	$T = \frac{\ln 0.127\ldots}{-0.25} = \frac{-2.0605\ldots}{-0.25} = 8.24\ldots$	A1	This mark is given for rearranging and solving for T
	8 years and 3 months	A1	This mark is given for finding the answer in years and months to the nearest month
(c)	£2300	B1	This mark is given for deducing from the original equation that as $e^{-0.25t}$ tends to zero, V tends to 2300
(d)	Other factors can affect the price such as mileage or condition The price may rise as the car becomes rare	B1	This mark is given for any valid limitation to the model stated

8. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

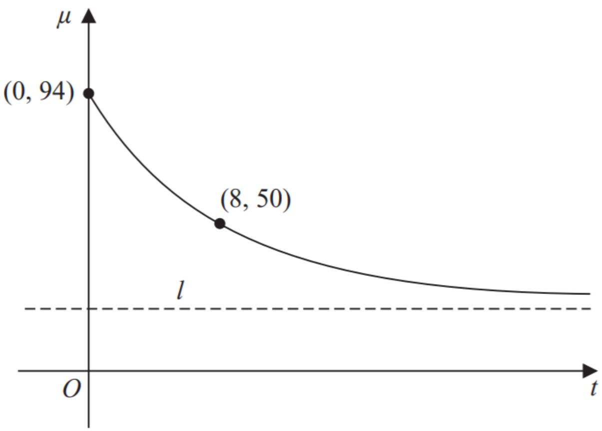


Figure 2

The temperature, μ °C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

Question	Scheme		Marks	AOs
8 (a)	Temperature = 83°C		B1	3.4
			(1)	
(b)	$18+65e^{-\frac{t}{8}}=35 \Rightarrow 65e^{-\frac{t}{8}}=17$		M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$	$\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$		A1	1.1b
			(3)	
(c)	States a suitable reason • As $t\to\infty, \theta\to 18$ from above. • The minimum temperature is 18°C		B1	2.4
			(1)	
(d)	$A+B=94$ or $A+Be^{-1}=50$		M1	3.4
	$A+B=94$ and $A+Be^{-1}=50$		A1	1.1b
	Full method to find at least a value for A		dM1	2.1
	Deduces $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4		A1	2.2a
			(4)	
				(9 marks)

12. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30,\ t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live. Give your answer to 2 significant figures.

(2)

Question	Scheme		Marks	AOs
12 (a)	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
(b)	The value of *ab* is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
(c)	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
			(7 marks)	

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

Question	Scheme	Marks	AOs
11 (a)	$35\ (\text{km}^2)$	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \dfrac{1}{14}\ln\left(\dfrac{20}{45}\right) = \cdots -0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
			(6 marks)

13.

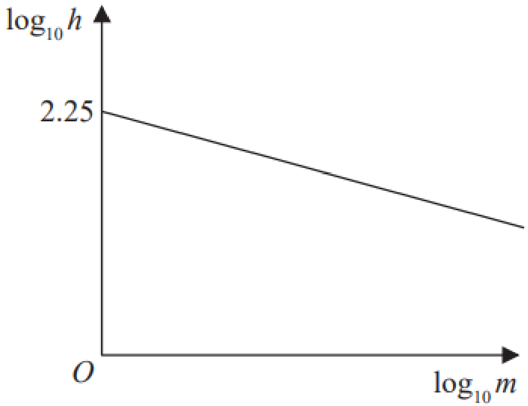


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

Question	Scheme		Marks	AOs
13 (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

HOME

5(a)	$p=10^{0.5}$ (or $\log_{10} p = 0.5$) or $q=10^{0.03}$ (or $\log_{10} q = 0.03$)	M1
	$p =$ awrt 3.162 or $q =$ awrt 1.072	A1
	$p=10^{0.5}$ (or $\log_{10} p = 0.5$) and $q=10^{0.03}$ (or $\log_{10} q = 0.03$)	dM1
	$A = 3.162 \times 1.072^t$	A1
		(4)
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1
(b)(ii)	The ratio of algae from one week to the next.	B1
		(2)
(c)(i)	5.5 kg	B1
(c)(ii)	$4 = "3.162" \times "1.072"^t$ or $\log_{10} 4 = 0.03t + 0.5$	M1
	awrt 3.4 (weeks)	A1
		(3)
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1
		(1)

A2 SAMs Paper 1

Exponentials and Logs

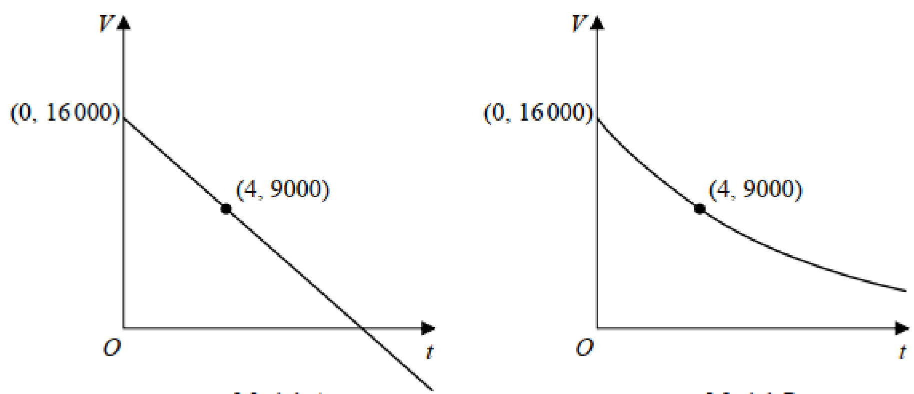
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



Model A Model B

(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

HOME

Question	Scheme	Marks	AOs
6 (a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t = 10, V = -1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t = 3 \Rightarrow V =$ awrt 10 400 barrels	B1ft	3.4
		(5)	
			(7 marks)

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

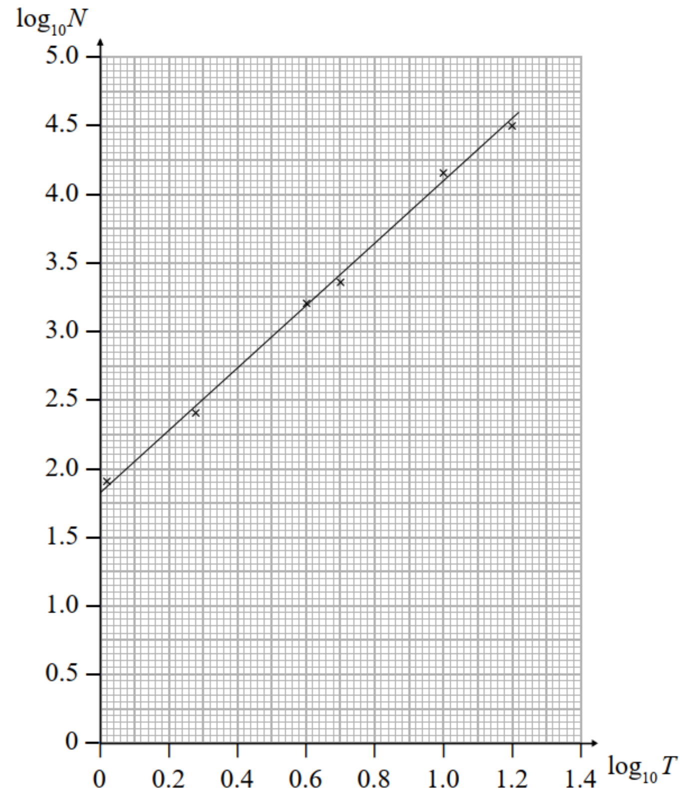


Figure 3

A2 SAMs Paper 1

Exponentials and Logs

Question	Scheme	Marks	AOs
12 (a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
		(9 marks)	

HOME

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\dfrac{dm}{dt} = km$, where k is a constant to be found.

(2)

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{d}{dt}\left(e^{-0.05t}\right) = \pm C\,e^{-0.05t}$	M1	2.1
	$\frac{dm}{dt} = -0.05\times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
		(4 marks)	

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

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Exponentials and Logs

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $$p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = \ldots$$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $$A = \frac{32000}{'1.0658'^4} \text{ or } A = \frac{50000}{'1.0658'^{11}}$$	M1	1.1b
	$A = 24795 \rightarrow 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

HOME

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

A2 2019 Paper 1

Exponentials and Logs

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$V = Ae^{-kt}$	M1	This mark is given for suggesting a suitable exponential model for V in terms of t
	When $t = 0$ and $V = 20\,000$, $A = 20\,000$	M1	This mark is given for using the model to show the initial value for A is £20 000
	When $t = 1$ and $V = 16\,000$, $16\,000 = 20\,000e^{-1k}$ $k = \ln 0.8 = -0.223$	M1	This mark is given for using the value of the car after one year to find a value for k
	$V = 20\,000e^{-0.223t}$	A1	This mark is given for finding a fully correct exponential model
(b)	When $t = 10$, $V =$ £2150	M1	This mark is given for finding a value for V when $t = 10$
	This model is reliable since the value £2150 is close to £2000	A1	This mark is given for a valid statement comparing the two possible values of the car after 10 years
(c)	For example: The value of k should be increased (e.g. $V = 20\,000e^{-0.1t}$) A constant should be added (e.g. $V = 20\,000e^{-0.223t} + 2000$)	B1	This mark is given for a statement suggesting a valid adaptation

HOME

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

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Exponentials and Logs

Question 9 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\log a - \log b = \log \dfrac{a}{b}$	B1	This mark is given for restating the log equation using $\log \dfrac{a}{b}$
	$a - b = \dfrac{a}{b}$ $ab - b^2 = a$ $ab - a = b^2$	M1	This mark is given for rearranging so that terms in a are on one side of the equation
	$a(b-1) = b^2$ $a = \dfrac{b^2}{(b-1)}$	A1	This mark is for rearranging to show the result required
(b)	$b \neq 1$	B1	This mark is given for deducing that $b \neq 1$
	Since $a > 0$, $\dfrac{b^2}{(b-1)} > 0$ $b > 1$ since b^2 is positive	B1	This mark is given for stating that $b > 1$ and explaining the reason for the restriction

HOME

9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V\,\text{km}\,\text{h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

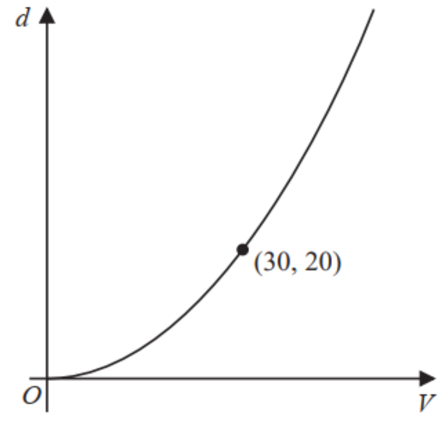


Figure 5

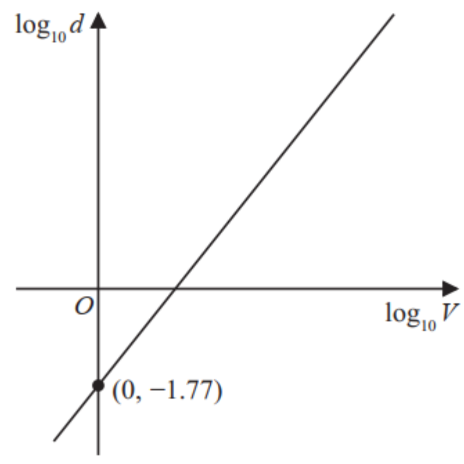


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at $60\,\text{km}\,\text{h}^{-1}$ in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

A2 2019 Paper 2

Exponentials and Logs

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	If $d = kV^n$, then $\log_{10} d = \log_{10} k + n \log_{10} V$	M1	This mark is given for
	Plotting $\log_{10} d$ against $\log_{10} V$ will result in a straight line with gradient n and intercept $\log_{10} k$	A1	This mark is given for an explanation of why the second graph shows that $d = kV^n$
	$\log_{10} k = -1.77$ $k = 10^{-1.77} = 0.01698\ldots \approx 0.017$	A1	This mark is for showing fully that $k \approx 0.017$
(b)	$d = kV^n$ When $V = 30$, $d = 20$ and $k = 0.17$ then $20 = 0.017 \times 30^n$	M1	This mark is given for substituting in the formula as a method to find the value of n
	$n \log 30 = \log\left(\dfrac{20}{0.017}\right)$	M1	This mark is given for a correct expression for n
	$n = 2.08$ to 3 significant figures $d = 0.017 \times V^{2.08}$	A1	This mark is given for finding a correct value of n to 3 significant figures and writing a complete equation for the model
(c)	$\dfrac{60}{3600} \times 0.8 \times 1000 = 13.33$ m	M1	This mark is given for a method to find the distance, in metres, covered in the reaction time of 0.8 seconds
	$d = 0.017 \times 60^{2.08} = 84.92$ m	M1	This mark is given for a method to use the formula to find the stopping distance
	13.33 m + 84.92 m = 98.25 m Sean will be able to stop before reaching the puddle	A1	This mark is given for finding a correct value of the total stopping distance and giving a valid conclusion

HOME

A2 2020 Paper 1

Exponentials and Logs

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

HOME

Question	Scheme	Marks	AOs
2	$4^{3p-1} = 5^{210} \Rightarrow (3p-1)\log 4 = 210\log 5$	M1	1.1b
	$\Rightarrow 3p = \dfrac{210\log 5}{\log 4} + 1 \Rightarrow p = \ldots$	dM1	2.1
	$p = \text{awrt } 81.6$	A1	1.1b
		(3)	
			(3 marks)

3. (a) Given that

$$2\log(4-x)=\log(x+8)$$

show that

$$x^2-9x+8=0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2-9x+8=0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x)=\log(x+8)$$

giving a reason for your answer.

(2)

3(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) $(x=)$ 1, 8	B1	1.1b
	(ii) 8 is **not** a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

Question	Scheme	Marks	AOs
5	$15-2^{x+1}=3\times 2^x$	B1	1.1b
	$\Rightarrow 15-2\times 2^x=3\times 2^x \Rightarrow 2^x=3$ or e.g. $\Rightarrow \dfrac{15}{2^x}-2=3 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=\ldots$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y=3\times 2^x \Rightarrow 2^x=\dfrac{y}{3} \Rightarrow y=15-2\times\dfrac{y}{3}$	B1	1.1b
	$3y+2y=45 \Rightarrow y=9 \Rightarrow 3\times 2^x=9 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=\ldots$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
			(4 marks)

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

A2 2020 Paper 2

Exponentials and Logs

Question	Scheme	Marks	AOs
9(a)	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ **or** $t = 10,\ \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$	M1	3.1b
	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ **and** $t = 10,\ \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$ **and** $\Rightarrow A = \ldots,\ B = \ldots$	M1	3.1a
	At least one of: $A = 69.6$, $B = 51.6$ but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta = 69.6 - 51.6e^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is “69.6”(°C) (according to the model) (The model has an) upper limit of “69.6”(°C) (The model suggests that) the boiling point is “69.6”(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

HOME

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

HOME

A2 2021 Paper 1

Exponentials and Logs

Question	Scheme	Marks	AOs
8(a)	$A = 1000$	B1	3.4
	$2000 = 1000e^{5k}$ or $e^{5k} = 2$	M1	1.1b
	$e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = ...$	M1	2.1
	$N = 1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt} = 1000 \times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times 0.139e^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4\times\left(\frac{1}{5}\ln 2\right)T} = 1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4\times\text{"}0.139\text{"}t} = 1000e^{\text{"}0.139\text{"}t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4 \times \text{"}0.339\text{"}T = \ln 2 + \text{"}0.339\text{"}t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
		(9 marks)	

BICEN MATHS

HOME

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\dfrac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\dfrac{12y+5}{1-3y}=2 \Rightarrow \dfrac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\ldots$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=\ldots$	M1	2.1
	$y=\dfrac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

A2 2021 Paper 2

Exponentials and Logs

10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data. The straight line passes through the points $(-0.7, 0)$ and $(0.21, 0.45)$

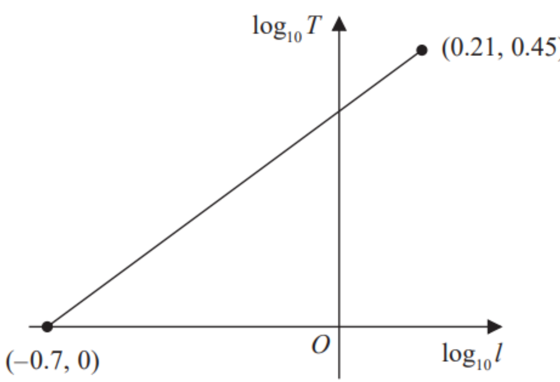


Figure 3

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

HOME

A2 2021 Paper 2

Exponentials and Logs

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \dfrac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

HOME

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

10 (a)	265 thousand	B1
		(1)
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*
		(3)
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1
	$T = 12.08$	A1
		(4)

2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

(2)

(b) Solve

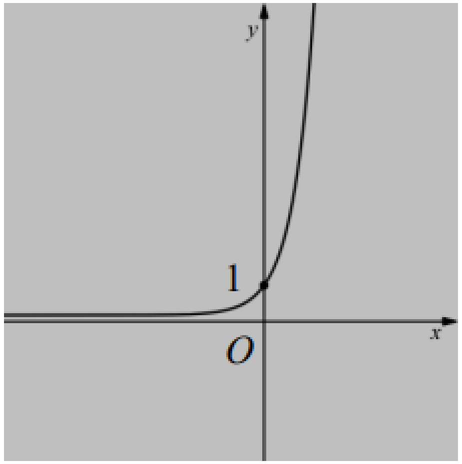
$$4^x = 100$$

giving your answer to 2 decimal places.

(2)

A2 2022 Paper 2

Exponentials and Logs

2(a)		Correct shape or correct intercept – see notes	B1
		Fully correct – see notes	B1
			(2)
(b)	$4^x = 100 \Rightarrow x = \log_4 100$ or e.g. $x\log 4 = \log 100 \Rightarrow x = \dfrac{\log 100}{\log 4}$		M1
	$\Rightarrow (x =)$ awrt 3.32		A1
			(2)

HOME

Algebraic Methods – proof, partial fractions, etc.

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

Question	Scheme	Marks	AOs
16(a)	Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P = 0$ or $P = \dfrac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A \text{ or } B$	M1	1.1b
	$\dfrac{1}{P(11-2P)} = \dfrac{1/11}{P} + \dfrac{2/11}{(11-2P)}$	A1	1.1b
		(3)	

A2 2018 Paper 2

Algebraic Methods

3. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

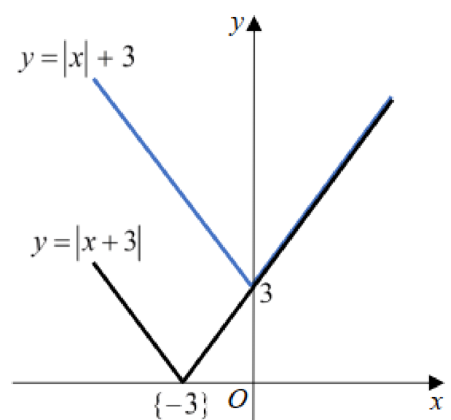
(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

(3)

HOME

A2 2018 Paper 2

Algebraic Methods

Question	Scheme		Marks	AOs
3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."			
(a)	E.g. $m = \sqrt{3}, n = \sqrt{12}$		M1	1.1b
	$\{mn =\}$ $(\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational		A1	2.4
			(2)	
(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1	1.1b
		Superimposes the graph of $y = \|x+3\|$ on top of the graph of $y = \|x\| + 3$	M1	3.1a
	the graph of $y = \|x\| + 3$ is either the same or above the graph of $y = \|x+3\|$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \|x\| + 3$ is above the graph of $y = \|x+3\|$		A1	2.4
			(3)	
(b)(ii) Way 2	<u>**Reason 1**</u> When $x \geq 0$, $\|x\| + 3 = \|x+3\|$ <u>**Reason 2**</u> When $x < 0$, $\|x\| + 3 > \|x+3\|$	Any one of Reason 1 or Reason 2	M1	3.1a
		Both Reason 1 and Reason 2	A1	2.4
			(5 marks)	

HOME

A2 2018 Paper 2

Algebraic Methods

11.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants A, B and C.

(4)

HOME

A2 2018 Paper 2

Algebraic Methods

Question	Scheme	Marks	AOs
11	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) **Way 1**	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=\ldots,\ C=\ldots$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=\ldots$ or $C=\ldots$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) **Way 2**	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=\ldots,\ C=\ldots$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=\ldots$ or $C=\ldots$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	

HOME

A2 2020 Paper 1

Algebraic Methods

16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

HOME

A2 2020 Paper 1

Algebraic Methods

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ or $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
			(4 marks)

HOME

6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \;\; x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2}\,\mathrm{d}x$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=\ldots, B=\ldots, C=\ldots$ **or** $\begin{array}{r} x+6 \\ x+2\overline{)x^2+8x-3} \\ \underline{x^2+2x} \\ 6x-3 \\ \underline{6x+12} \\ -15 \end{array}$	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

A2 2021 Paper 1

Algebraic Methods

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B=\ldots$ or $C=\ldots$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0 \Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8 \Rightarrow A=\ldots$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	

HOME

15. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leqslant 4$

$$(n+1)^3 > 3^n$$

(2)

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

A2 2021 Paper 1

Algebraic Methods

Question	Scheme	Marks	AOs
15(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n\leqslant 4, n\in\mathbb{N}$ then $(n+1)^3>3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=\ldots$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

HOME

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

A2 2022 Paper 1

Algebraic Methods

7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1
	For example, sets $p = 2m+1$ and $q = 2n+1$ and then attempts $pq = (2m+1)(2n+1) = \ldots$	M1
	Obtains $pq = (2m+1)(2n+1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so " if pq is even, then at least one of p and q is even" *	A1*
		(3)
(ii)		
	$(x+y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*
		(2)

HOME

Functions and Graphs

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

Question	Scheme	Marks	AOs
4 (a)	$\mathrm{gf}(x) = 3\ln \mathrm{e}^x$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\mathrm{gf}(x) = \mathrm{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

11.

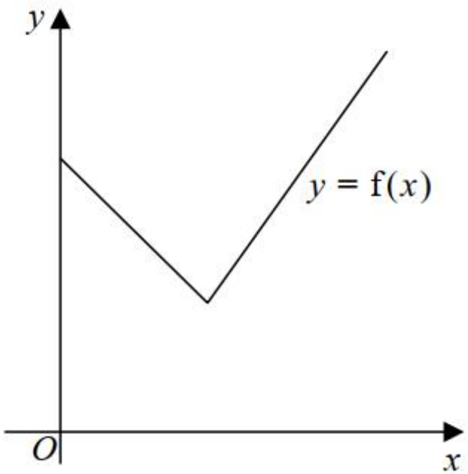


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

A2 SAMs Paper 2

Functions and Graphs

Question	Scheme	Marks	AOs
11 (a)	$f(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$	A1	2.5
		(2)	
		(6 marks)	

HOME

1.

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

A2 2018 Paper 2

Functions and Graphs

Question	Scheme	Marks	AOs
1	$\mathrm{g}(x)=\dfrac{2x+5}{x-3},\ x\geq 5$		
(a) **Way 1**	$\mathrm{g}(5)=\dfrac{2(5)+5}{5-3}=7.5 \Rightarrow \mathrm{gg}(5)=\dfrac{2(\text{"}7.5\text{"})+5}{\text{"}7.5\text{"}-3}$	M1	1.1b
	$\mathrm{gg}(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) **Way 2**	$\mathrm{gg}(x)=\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3} \Rightarrow \mathrm{gg}(5)=\dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$\mathrm{gg}(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	$\{\text{Range:}\}\quad 2<y\leq\dfrac{15}{2}$	B1	1.1b
		(1)	
(c) **Way 1**	$y=\dfrac{2x+5}{x-3} \Rightarrow yx-3y=2x+5 \Rightarrow yx-2x=3y+5$	M1	1.1b
	$x(y-2)=3y+5 \Rightarrow x=\dfrac{3y+5}{y-2}\quad \left\{\text{or } y=\dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x)=\dfrac{3x+5}{x-2},\quad 2<x\leq\dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) **Way 2**	$y=\dfrac{2x-6+11}{x-3} \Rightarrow y=2+\dfrac{11}{x-3} \Rightarrow y-2=\dfrac{11}{x-3}$	M1	1.1b
	$x-3=\dfrac{11}{y-2} \Rightarrow x=\dfrac{11}{y-2}+3\quad \left\{\text{or } y=\dfrac{11}{x-2}+3\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x)=\dfrac{11}{x-2}+3,\quad 2<x\leq\dfrac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

HOME

5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

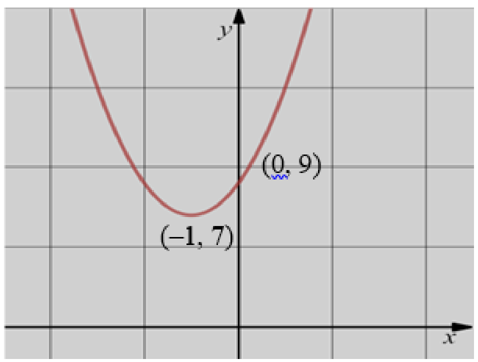
(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

A2 2019 Paper 1

Functions and Graphs

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2x^2 + 4x + 9 = 2(x + b)^2 + c$	B1	This mark is given for writing $f(x)$ in the form $a(x + b)^2 + c$ with $a = 2$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + c$	M1	This mark is given for writing $f(x)$ in the form $a(x + b)^2 + c$ with $a = 2$ and $b = 1$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + 7$	A1	This mark is given for writing $f(x)$ in the form $a(x + b)^2 + c$ with $a = 2$, $b = 1$ and $c = 7$
(b)		B1	This mark is given for a U shaped curve in any position
		B1	This mark is given for a y-intercept shown at (0, 9)
		B1	This mark is given for a minimum shown at (–1, 7)
(c)(i)	$g(x) = 2(x – 2)^2 + 4(x – 2) + 5$	M1	This mark is given for writing $g(x)$ in the form $a(x + b)^2 + c$ and comparing to $f(x)$
	Translation of $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	This mark is given for deducing the translation of $y = f(x)$ to $y = g(x)$
(c)(ii)	$h(x) = \dfrac{21}{2(x+1)^2+7}$ Maximum value $= \dfrac{21}{7}$ (when $x = –1$)	M1	This mark is given for writing $h(x)$ in the form $\dfrac{21}{a(x+b)^2+c}$ and finding its maximum value
	$0 < h(x) \leq 3$	A1	This mark is given for finding the correct range of the function $h(x)$

10. (i) ~~Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4~~

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(ii)	For example, for $x = 9.4$ $\lvert 3x - 28 \rvert = 0.2$ and $(x - 9) = 0.4$	M1	This mark is given for showing that the statement is not true for $9.25 < x < 9.5$
	The statement is sometimes true; For example, for $x = 12$ $\lvert 3x - 28 \rvert = 8$ and $(x - 9) = 3$	A1	This mark is given for a correct statement and an example where the statement is true

A2 2019 Paper 2

Functions and Graphs

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

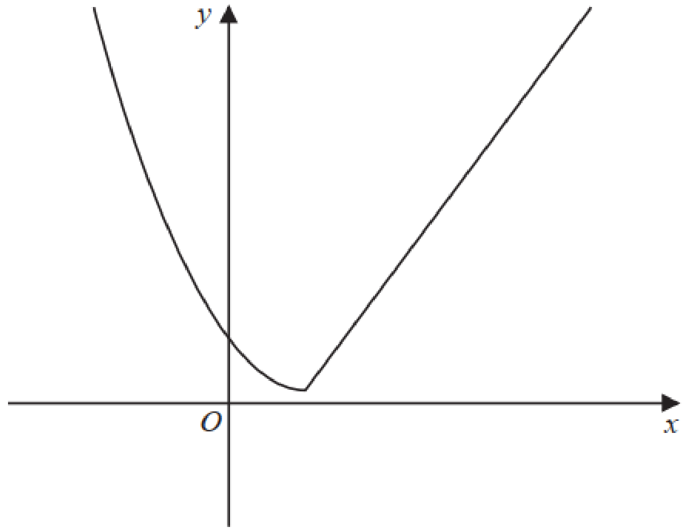


Figure 4

(a) Find the value of $\mathrm{gg}(0)$.

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

HOME

A2 2019 Paper 2

Functions and Graphs

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$g(0) = 5$	M1	This mark is given for a method to find $g(0)$
	$gg(0) = g(5) = 13$	A1	This mark is given for a correct value for $gg(0)$
(b)	$(x-2)^2 + 1 > 28$ $(x-2)^2 > 27$ $x - 2 > 3\sqrt{3}$	M1	This mark is given for a method to solve $g(x) > 28$ when $x \leq 2$
	$x < 2 - 3\sqrt{3}$	A1	
	$4x - 7 > 28$ $4x > 35$ $x > \frac{35}{4}$	M1	This mark is given for a solving $g(x) > 28$ when $x > 2$
	$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$	A1	This mark is given for a correct range of values of x for which $g(x) > 28$ stated
(c)	h^{-1} exists since h is a one-to-one function; g^{-1} does not exists since g is a many-to-one function	B1	This mark is given for a valid explanation
(d)	$h^{-1}(x) = 2 - \sqrt{(x-1)}$	B1	This mark is given for finding an expression for $h^{-1}(x)$
	$2 \pm \sqrt{(x-1)} = -\frac{1}{2}$	M1	This mark is given for a method to rearrange to find a value for x
	$x = 7.25$	A1	This mark is given for a correct value of x

HOME

4. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \dfrac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

Functions and Graphs

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\dfrac{3x-7}{x-2}=7 \Rightarrow x=\ldots$ Or attempts $\mathrm{f}^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\dfrac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $\mathrm{ff}(x)=\dfrac{3\times\left(\dfrac{3x-7}{x-2}\right)-7}{\left(\dfrac{3x-7}{x-2}\right)-2}=\dfrac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\dfrac{2x-7}{x-3}$	A1	2.1
		(3)	
			(5 marks)

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

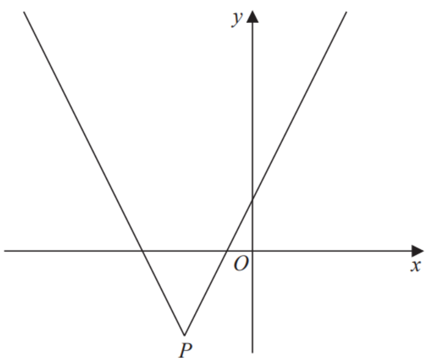


Figure 2

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

A2 2020 Paper 2

Functions and Graphs

Question	Scheme	Marks	AOs
11(a)	$x=-4$ **or** $y=-5$	B1	1.1b
	$P(-4,-5)$	B1	2.2a
		(2)	
(b)	$3x+40=-2(x+4)-5 \Rightarrow x=\ldots$	M1	1.1b
	$x=-10.6$	A1	2.1
		(2)	
(c)	$a>2$	B1	2.2a
	$y=ax \Rightarrow -5=-4a \Rightarrow a=\dfrac{5}{4}$	M1	3.1a
	$\{a : a \leqslant 1.25\} \cup \{a : a>2\}$	A1	2.5
		(3)	
			(7 marks)

HOME

2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf(1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

Functions and Graphs

Question	Scheme	Marks	AOs
2(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\mathrm{f}(1.8) = 7 - 2\times 1.8^2 = 0.52 \Rightarrow \mathrm{gf}(1.8) = \mathrm{g}(0.52) = \dfrac{3\times 0.52}{5\times 0.52 - 1} = \ldots$	M1	1.1b
	$\mathrm{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\mathrm{g}^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
			(5 marks)

A2 2021 Paper 2

Functions and Graphs

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

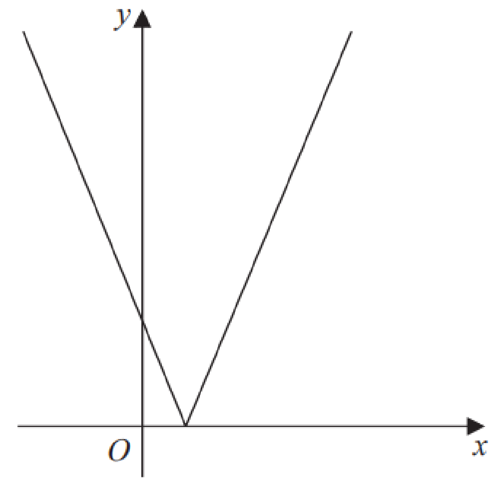


Figure 4

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

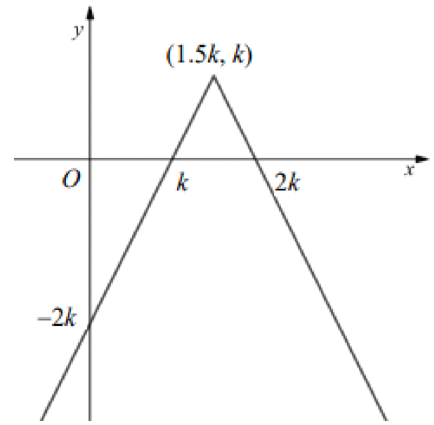
$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

HOME

A2 2021 Paper 2

Functions and Graphs

Question	Scheme	Marks	AOs
11(a)			
	$\wedge$ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a $\wedge$ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-(2x-3k)=x-k \Rightarrow x = ...$	M1	3.1a
	$x=\frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x<\frac{5k}{3}\right\} \cap \{x : x>k\}$	A1	2.5
		(4)	
(c)	$x=3k$ **or** $y=3-5k$	B1ft	2.2a
	$x=3k$ **and** $y=3-5k$	B1ft	2.2a
		(2)	
			(10 marks)

HOME

1. The point $P(-2, -5)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(1)

(b) $y = |\mathrm{f}(x)|$

(1)

(c) $y = 3\mathrm{f}(x - 2) + 2$

(2)

A2 2022 Paper 1

Functions and Graphs

1 (a)	$(-2, -3)$	B1
		(1)
(b)	$(-2, 5)$	B1
		(1)
(c)	Either $x = 0$ or $y = -13$	M1
	$(0, -13)$	A1
		(2)

BICEN MATHS

HOME

A2 2022 Paper 2

Functions and Graphs

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

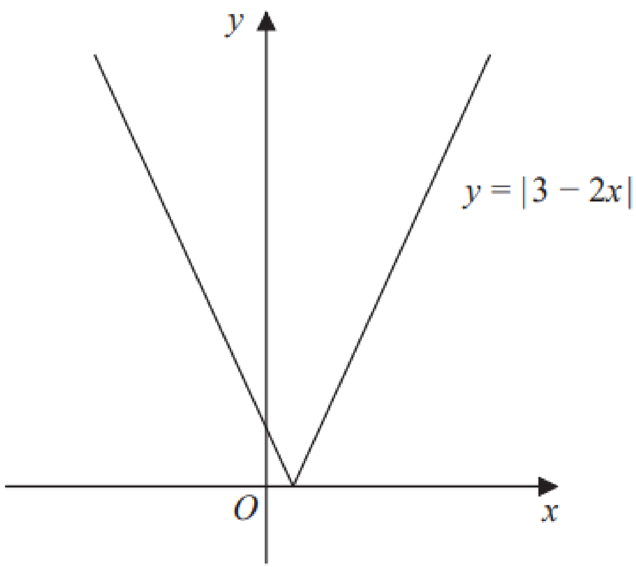


Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

HOME

1	For an attempt to solve **Either** $3-2x=7+x \Rightarrow x=\ldots$ **or** $2x-3=7+x \Rightarrow x=\ldots$	M1
	Either $x=-\frac{4}{3}$ **or** $x=10$	A1
	For an attempt to solve **Both** $3-2x=7+x \Rightarrow x=\ldots$ **and** $2x-3=7+x \Rightarrow x=\ldots$	dM1
	For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions	A1
		(4)

10. The function f is defined by

$$f(x) = \frac{8x+5}{2x+3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$ **(2)**

(b) Show that

$$f(x) = A + \frac{B}{2x+3}$$

where A and B are constants to be found. **(2)**

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1} **(1)**

(d) Find the range of fg^{-1} **(3)**

A2 2022 Paper 2

Functions and Graphs

10(a)	Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ Or substitutes $x=\frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1
	$\left(\mathrm{f}^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$	A1
		(2)
(b)	$\left(\frac{8x+5}{2x+3}=\right)4\pm\frac{...}{2x+3}$	M1
	$\left(\frac{8x+5}{2x+3}=\right)4-\frac{7}{2x+3}$	A1
		(2)
(c)	$0\leqslant \mathrm{g}^{-1}(x)\leqslant 4$	B1
		(1)
(d)	Attempts either boundary $\mathrm{f}(0)=\frac{8\times0+5}{2\times0+3}$ **or** $\mathrm{f}(4)=\frac{8\times4+5}{2\times4+3}$	M1
	Attempts both boundaries $\mathrm{f}(0)=\frac{8\times0+5}{2\times0+3}$ **and** $\mathrm{f}(4)=\frac{8\times4+5}{2\times4+3}$	dM1
	Range $\frac{5}{3}\leqslant \mathrm{fg}^{-1}(x)\leqslant\frac{37}{11}$	A1
		(3)
	Alternative by attempting $\mathrm{fg}^{-1}(x)$	
	$\mathrm{g}^{-1}(x)=\sqrt{16-x}\Rightarrow \mathrm{fg}^{-1}(x)=\frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $\mathrm{fg}^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **or** $\mathrm{fg}^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	M1
	$\mathrm{fg}^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **and** $\mathrm{fg}^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	dM1
	Range $\frac{5}{3}\leqslant \mathrm{fg}^{-1}(x)\leqslant\frac{37}{11}$	A1
		(3)

HOME

Sequences and Series

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

A2 SAMs Paper 2

Sequences and Series

Question	Scheme	Marks	AOs
10	Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
			(4 marks)

4. (i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

A2 2018 Paper 2

Sequences and Series

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}(3+5r+2^r)=131\,798$; (ii) $u_1, u_2, u_3, \ldots : u_{n+1}=\frac{1}{u_n},\ u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}(3+5r+2^r)=\right\}\ \sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=728+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}(3+5r+2^r)=\right\}\ \sum_{r=1}^{16}3\ +\ \sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=48+680+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum = $10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*	3.1a 1.1b 1.1b 2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\},\ u_2=\frac{3}{2},\ u_3=\frac{2}{3},\ldots$ ***(can be implied by later working)***	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}\ \left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

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11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds, **(2)**

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race. **(4)**

A2 2019 Paper 1

Sequences and Series

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$24 + (6 \times 1.05) + (6 \times 1.05^2)$ minutes	M1	This mark is for a method to find the time taken for the competitor to run 6 km
	$= 96.915$ minutes $= 36$ minutes 55 seconds	A1	This mark is given for finding the total time as required
(b)	For example, 5th km $= 6 \times 1.05^1$ 6th km $= 6 \times 1.05^2$ 7th km $= 6 \times 1.05^3$ … rth km $= 6 \times 1.05^{r-4}$	B1	This mark is given for showing the time taken to run the rth km, as required
(c)	$24 + \sum_{r=5}^{20} 6 \times 1.05^{r-4}$	M1	This mark is given for showing the total time to run the race is the time taken for the first 4 km added to the time taken from 5th to 20th km
	$= 24 + 6.3 \times \frac{(1.05^{16} - 1)}{1.05 - 1}$	M1	This mark is given for using $s = a\left(\frac{1-r^n}{1-r}\right)$ where $a = 6 \times 1.05 = 6.3$, $r = 1.05$ and $n = 20 - 4 = 16$
	$= 24 + 149.04$	A1	This mark is given for a correct total time (represented decimally)
	$= 173$ minutes and 3 seconds	A1	This mark is given for finding a correct total time given in minutes in seconds

BICEN MATHS

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A2 2019 Paper 2

Sequences and Series

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

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A2 2019 Paper 2

Sequences and Series

Question 8 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$ $= \sum_{r=1}^{\infty} 20 \times \left(\frac{1}{2}\right)^r - (10 + 5 + 2.5)$	M1	This mark is given for a method to find the sum to infinity of a GP
	$= \frac{10}{1-\frac{1}{2}} - (10 + 5 + 2.5)$	M1	This mark is given for a method to use a correct sum formula with a correct first term
	$= 2.5$	A1	This mark is given for a correct value for the sum
(ii)	$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right)$ $= \log_5 \frac{50}{49} + \log_5 \frac{49}{48} + \ldots + \log_5 \frac{4}{3} + \log_5 \frac{3}{2}$	M1	This mark is given for writing out at least four terms of the sum, including the first two and the last two
	$= \log_5 \frac{3 \times 4 \times \ldots \times 48 \times 49 \times 50}{2 \times 3 \times 4 \times \ldots \times 48 \times 49} = \log_5 \frac{50}{2}$	M1	This mark is given for using the rules of logs and cancelling terms
	$= 2$	A1	This mark is given for a full proof to show the expression is equal to 2 as required

HOME

5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

A2 2020 Paper 1

Sequences and Series

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2 \times "17.4" = \ldots$	M1	3.4
	$= 62.8$ (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28 \times "1.3265^4" = \ldots$ or $\dfrac{115}{"1.3265"}$	M1	3.4
	$= 86.7$ (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)

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A2 2020 Paper 1

Sequences and Series

13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

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A2 2020 Paper 1

Sequences and Series

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \dfrac{k(a_n+2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k+1, a_4 = \dfrac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\dfrac{k(k+3)}{k+1} = 2 \Rightarrow k^2+3k = 2k+2 \Rightarrow k^2+k-2=0$ *	A1*	2.1
		(3)	
(b)	States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26\times(2+-4+-1)+2+-4$	M1	3.1a
	$=-80$	A1	1.1b
		(3)	
			(7 marks)

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

A2 2020 Paper 2

Sequences and Series

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + ... + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + ... + ar^n \Rightarrow S_n - rS_n = ...$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4 \times \dfrac{a(1-r^5)}{1-r}$ or $4 \times \dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1 - r$)	M1	3.1a
	$1 - r^{10} = 4(1 - r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5 - 1)(r^5 - 3) = 0 \Rightarrow r^5 = ...$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = ...$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

HOME

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

Question	Scheme	Marks	AOs
3(a)	$u_2 = k - 12,\ u_3 = k - \dfrac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \dfrac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \dfrac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0$ *	A1*	2.1
		(3)	
(b)	$k = 6, \left(\dfrac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =) 10$	B1	2.2a
		(1)	
			(6 marks)

5. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

A2 2021 Paper 1

Sequences and Series

Question	Scheme	Marks	AOs
5(a)	$u_3 = £20\,000 \times 1.08^2 = (£)\,23\,328$*	B1*	1.1b
		(1)	
(b)	$20\,000 \times 1.08^{n-1} > 65\,000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20\,000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)

HOME

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

A2 2021 Paper 2

Sequences and Series

Question	Scheme	Marks	AOs
1(a)	$16+(21-1)\times d = 24 \Rightarrow d = \ldots$	M1	1.1b
	$d = 0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n = \frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500} = \frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$= 57\,900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n = \frac{1}{2}n\{a+l\}$		
	$l = 16+(500-1)\times "0.4" = 215.6 \Rightarrow S_{500} = \frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$= 57\,900$	A1	1.1b
		(4 marks)	

HOME

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

A2 2021 Paper 2

Sequences and Series

Question	Scheme	Marks	AOs
9	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)} = \ldots$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3)	

HOME

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

A2 2022 Paper 1

Sequences and Series

13 (i)	States that $S = a + (a+d) + \ldots\ldots\ldots\ldots\ldots\ldots + (a+(n-1)d)$	B1
	$S = \quad a + \quad . \quad (a+d) + \quad (a+(n-1)d)$ $S = (a+(n-1)d) + (a+(n-2)d) + \ldots\ldots\ldots\ldots\ldots\ldots + a$	M1
	Reaches $2S = n \times (2a+(n-1)d)$ And so proves that $S = \frac{n}{2}[2a+(n-1)d]$ *	A1*
		(3)
(ii)	(a) $S = 10 + 9.20 + 8.40 + \ldots$	
	$64 = \frac{n}{2}(20 - 0.8(n-1))$ o.e	M1
	$128 = 20n - 0.8n^2 + 0.8n$ $0.8n^2 - 20.8n + 128 = 0$ $n^2 - 26n + 160 = 0$ *	A1*
		(2)
	(b) $n = 10, 16$	B1
		(1)
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1
		(1)

HOME

3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

A2 2022 Paper 2

Sequences and Series

3(a)(i)	$a_1 = 3,\ a_2 = 5,\ a_3 = 3\ \ldots$	B1
(ii)	2	B1
		(2)
(b)	$\sum_{n=1}^{85} a_n = 42 \times (3+5) + 3$ o.e.	M1
	$= 339$	A1
		(2)

HOME

15. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k\left(1 - \sqrt{3}\right)$$

where k is a constant to be found.

(5)

A2 2022 Paper 2

Sequences and Series

15(a)	Uses the common ratio $\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*
		(3)
(a) Alt	**(a) Alternative example:**	
	Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ $\Rightarrow 12\cos\theta\left(\dfrac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$	M1
	Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$	dM1
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*
		(3)
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\dfrac{1}{2}\left(,\dfrac{25}{2}\right)$	M1
	$\theta=\dfrac{5\pi}{6}$	A1
		(2)
(c)	Attempts a value for either a or r e.g. $a=12\cos\theta=\ 12\times-\dfrac{\sqrt{3}}{2}$ **or** $r=\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1
	$"a"=-6\sqrt{3}$ **and** $"r"=-\dfrac{1}{\sqrt{3}}$ o.e.	A1
	Uses $S_\infty=\dfrac{a}{1-r}=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1
	Rationalises denominator $S_\infty=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\dfrac{-18}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1
	$(S_\infty=)9(1-\sqrt{3})$	A1
		(5)

HOME

Binomial Expansion

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

BICEN MATHS

A2 SAMs Paper 2

Binomial Expansion

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^{2}+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^{2}+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^{2}+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used	B1	2.4
		(1)	
		(5 marks)	

HOME

A2 2018 Paper 1

Binomial Expansion

11. (a) Use binomial expansions to show that $\sqrt{\dfrac{1+4x}{1-x}} \approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$

(6)

A student substitutes $x = \dfrac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \dfrac{1}{2}$

(1)

(c) Substitute $x = \dfrac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

HOME

A2 2018 Paper 1

Binomial Expansion

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{0.5 \times -0.5}{2} \times (4x)^2$	M1	1.1b
	$(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5)\times(-1.5)}{2}(-x)^2$	M1	1.1b
	$(1+4x)^{0.5} = 1 + 2x - 2x^2$ and $(1-x)^{-0.5} = 1 + 0.5x + 0.375x^2$ oe	A1	1.1b
	$(1+4x)^{0.5} \times (1-x)^{-0.5} = (1 + 2x - 2x^2 \ldots\ldots) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 \ldots\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2 + \ldots$ $= A + Bx + Cx^2$	dM1	2.1
	$= 1 + \frac{5}{2}x - \frac{5}{8}x^2 \ldots\ldots$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x \rvert < \frac{1}{4}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}} = \frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

HOME

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

Question 4 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$	M1	This mark is given for rearranging $\frac{1}{\sqrt{4-x}}$ to attempt a binomial expansion
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} =$	M1	This mark is given for an attempt at a binomial expansion
	$1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)$	A1	This mark is given for a fully correct binomial expansion
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	This mark is given for a fully correct expansion with the first three terms
(b)(i)	$x = -14$, since the expansion is only valid for $\lvert x\rvert < 4$	B1	This mark is given for the correct value chosen with a correct reason
(b)(ii)	$x = -\frac{1}{2}$, since the smaller value will give the more accurate approximation	B1	This mark is given for the correct value chosen with a correct reason

A2 2020 Paper 1

Binomial Expansion

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1+8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

HOME

A2 2020 Paper 1

Binomial Expansion

Question	Scheme	Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+....$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)

HOME

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

A2 2021 Paper 1

Binomial Expansion

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B=\ldots$ or $C=\ldots$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0 \Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8 \Rightarrow A=\ldots$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+\ldots$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2}=1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\frac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$=\frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert<\frac{2}{5}$	B1	2.2a
		(7)	
			(11 marks)

HOME

Binomial Expansion

7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

A2 2022 Paper 2

Binomial Expansion

7(a)	$\sqrt{4-9x} = 2(1 \pm \ldots)^{\frac{1}{2}}$	B1
	$\left(1-\text{"}\frac{9x}{4}\text{"}\right)^{\frac{1}{2}} = \ldots + \dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^2}{2!}$ or $\ldots + \dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^3}{3!}$	M1
	$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$	A1
	$\sqrt{4-9x} = 2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$	A1
		(4)
(b)	States that the approximation will be an **<u>overestimate</u>** since all terms (after the first one) in the expansion are negative (since $x > 0$)	B1
		(1)

BICEN MATHS

HOME

Radians – small angle, arc length, etc.

2.

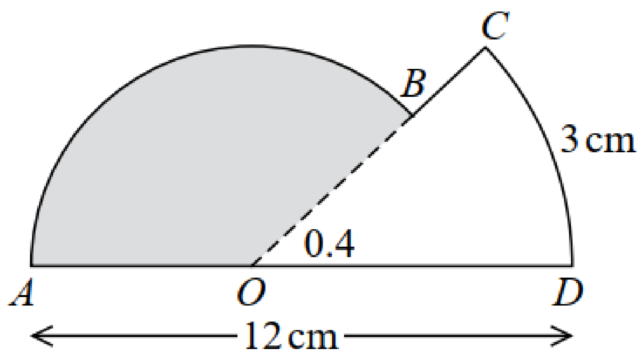

Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length $CD = 3$ cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

Question	Scheme	Marks	AOs
2(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5$ cm	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
			(5 marks)

A2 2018 Paper 1

Radians

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

HOME

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \to \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

3.

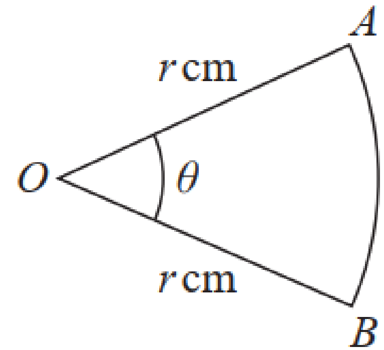

Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is 11 cm^2

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

A2 2018 Paper 1

Radians

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = \ldots$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

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A2 2019 Paper 1

Radians

2.

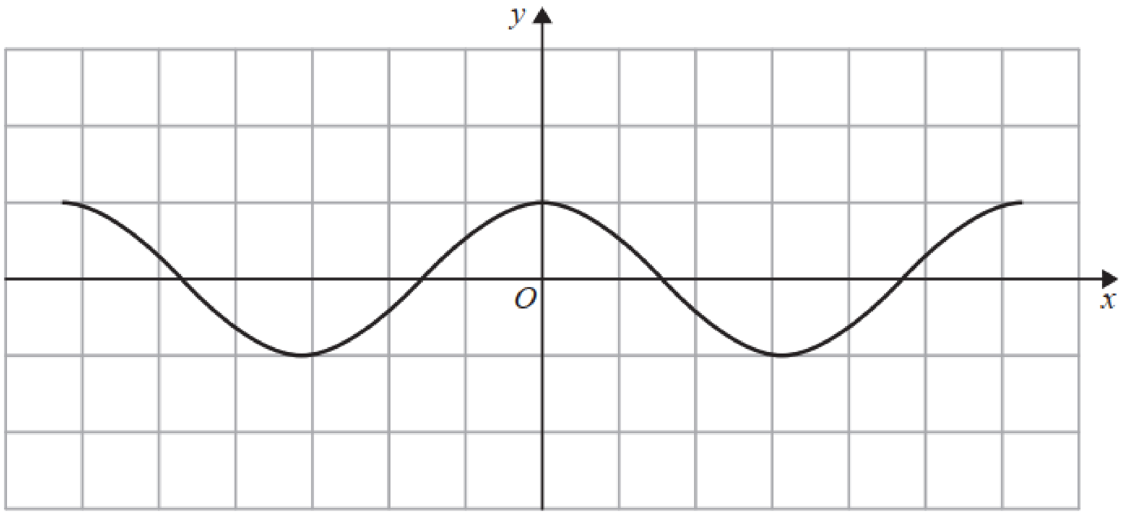

Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians
Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

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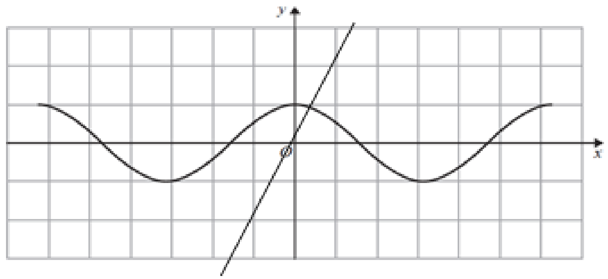
A2 2019 Paper 1

Radians

Question 2 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for plotting the line $y = 2x + \frac{1}{2}$ on the diagram with a correct gradient and intercept
	Only one intersection means that there is one root	A1	This mark is given for a reason why there is only one real root
(b)	$1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	This mark is given for using the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation
	$x^2 + 4x - 1 = 0$	M1	This mark is given for rearranging to find a quadratic equation to solve
	0.236 or $-2 + \sqrt{5}$	A1	This mark is given for finding the correct (positive) solution for x

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3.

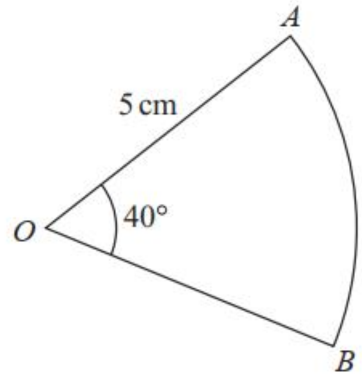

Figure 1

Figure 1 shows a sector AOB of a circle with centre O, radius 5 cm and angle $AOB = 40°$

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500\ \text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

A2 2019 Paper 2

Radians

Question 3 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The formula is only valid when the angle *AOB* is given in radians	B1	This mark is given for a correct explanation
(b)	$\frac{40}{360} \times \pi \times 5^2$	M1	This mark is given for a correct method to find the area of the sector
	$\frac{25\pi}{9}$ cm^2	A1	This mark is given for a correct value for the area of the sector

4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

A2 2021 Paper 2

Radians

Question	Scheme	Marks	AOs
4	**Examples:** $4\sin\frac{\theta}{2}\approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta\approx 3\left(1-\frac{\theta^2}{2}\right)^2$ $3\cos^2\theta = 3\left(1-\sin^2\theta\right)\approx 3\left(1-\theta^2\right)$ $3\cos^2\theta = 3\frac{\left(\cos 2\theta+1\right)}{2}\approx\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	M1	1.1a
	Examples: $4\sin\frac{\theta}{2}+3\cos^2\theta\approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right)\approx 2\theta+3\left(1-\theta^2\right)$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\sin\frac{\theta}{2}+3\frac{\left(\cos 2\theta+1\right)}{2}\approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	dM1	1.1b
	$=2\theta+3\left(1-\theta^2+\ldots\right)=3+2\theta-3\theta^2$	A1	2.1
		(3)	
			(3 marks)

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The shape $OABCDEFO$ shown in Figure 1 is a design for a logo.

In the design

- OAB is a sector of a circle centre O and radius r
- sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius $2r$
- AOF is a straight line

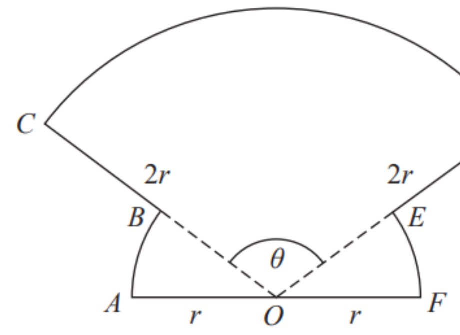


Figure 1

Given that the size of angle COD is θ radians,

(a) write down, in terms of θ, the size of angle AOB

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

Radians

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \dfrac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area $= 2\times\dfrac{1}{2}r^2\left(\dfrac{\pi-\theta}{2}\right)+\dfrac{1}{2}(2r)^2\theta$	M1	2.1
	$=\dfrac{1}{2}r^2\pi-\dfrac{1}{2}r^2\theta+2r^2\theta=\dfrac{3}{2}r^2\theta+\dfrac{1}{2}r^2\pi=\dfrac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter $= 4r+2r\left(\dfrac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

Trigonometry – sec, cosec, cot, identities

A2 2018 Paper 2

Trig – sec, cosec, cot

7. (i) Solve, for $0 \leqslant x < \dfrac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

HOME

A2 2018 Paper 2

Trig – sec, cosec, cot

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	

HOME

12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec}x - \sin x = \cos x\cot(3x - 50^\circ)$$

(5)

A2 2020 Paper 1

Trig – sec, cosec, cot

Question	Scheme	Marks	AOs
12 (a)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50°)$		
	$\cot x = \cot(3x - 50°) \Rightarrow x = 3x - 50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x - 50°) \Rightarrow x + 180° = 3x - 50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)

HOME

Trigonometry – addition, double angle, harmonic

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

A2 SAMs Paper 1

Trig – addition, double, etc.

Question	Scheme	Marks	AOs
9(a)	$\tan\theta + \cot\theta \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\operatorname{cosec} 2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta + \cot\theta = 1 \Rightarrow \sin 2\theta = 2$ AND no real solutions as $-1 \leqslant \sin 2\theta \leqslant 1$	B1	2.4
		(1)	
			(5 marks)

HOME

A2 SAMs Paper 2

Trig – addition, double, etc.

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

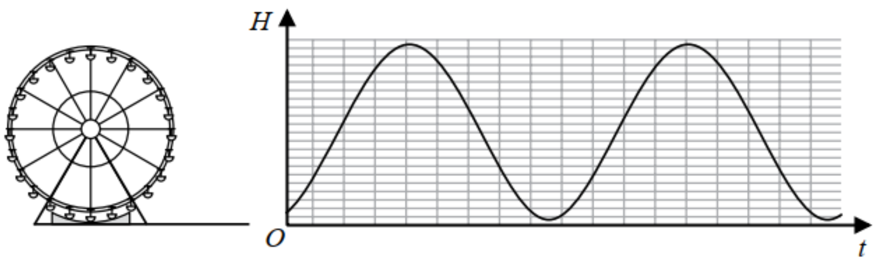


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

A2 SAMs Paper 2

Trig – addition, double, etc.

Question	Scheme	Marks	AOs
13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\dfrac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos\left(\theta+16.70°\right)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos\left(80t+16.70\right)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\dfrac{540-"16.70"}{80}=\left(6.54\right)$	M1	1.1b
	$t=6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the ‘80’ in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
		(9 marks)	

HOME

A2 2018 Paper 2

Trig – addition, double, etc.

7. (i) Solve, for $0 \leqslant x < \dfrac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

BICEN MATHS

HOME

A2 2018 Paper 2

Trig – addition, double, etc.

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) **Way 1**	For $\sec x = \dfrac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \dfrac{1}{2}$	M1	3.1a
	$x = \dfrac{1}{2}\arcsin\left(\dfrac{1}{2}\right)$ or $\dfrac{1}{2}\left(\pi - \arcsin\left(\dfrac{1}{2}\right)\right) \Rightarrow x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta - \alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \dfrac{2}{\sqrt{50}}$ $\sin 2\theta = \dfrac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\dfrac{2}{\sqrt{50}}\right) + 45°$ e.g. $\theta = \dfrac{1}{2}\left(\arcsin\left(\dfrac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
		(9 marks)	

HOME

12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

A2 2018 Paper 2

Trig – addition, double, etc.

Question	Scheme	Marks	AOs
12	$1-\cos 2\theta \equiv \tan\theta\sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\quad -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x = 0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x-5 = 3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = \ldots$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
		(9 marks)	

HOME

A2 2019 Paper 1

Trig – addition, double, etc.

6. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

HOME

A2 2019 Paper 1

Trig – addition, double, etc.

Question 6 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$5\sin 2\theta = 9\tan\theta \Rightarrow$ $10\sin\theta\cos\theta = 9 \times \dfrac{\sin\theta}{\cos\theta}$	M1	This mark is given for a method to substitute terms to form an equation in terms of $\cos\theta$
	$10\cos^2\theta = 9$	M1	This mark is given for a correct equation in terms of $\cos\theta$
	$\theta = \arccos \pm\sqrt{\dfrac{9}{10}}$	M1	This mark is given for finding a value for θ in terms of arccos
	$\theta = \pm 18.4°, \pm 161.6°$	A1	This mark is given for any one value of 18.4° or 161.6° found.
		A1	This mark is given for four values of θ found correctly
	$\theta = \pm 0°, 180°$	B1	This mark is given for the deduction of the two other solutions for θ
(b)	$10\cos^2(x - 25) = 9$ x has smallest positive value when $x - 25° = -18.4°$	M1	This mark is given for finding an equation to solve for x
	$x = 6.6°$	A1	This mark is given for correctly finding the smallest positive solution to the equation

HOME

12. (a) Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4$$

giving any solutions to one decimal place.

(3)

A2 2019 Paper 2

Trig – addition, double, etc.

Question 12 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = \frac{\cos 3\theta\cos\theta + \sin 3\theta}{\sin\theta\cos\theta}$	M1	This mark is given for a method to form a single fraction
	$= \frac{\cos(3\theta - \theta)}{\sin\theta\cos\theta}$	M1	This mark is given for a method to use a compound angle formula on the numerator
	$= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$	M1	This mark is given for a method to use a compound angle formula on the denominator
	$= 2\cot 2\theta$	A1	This mark is given for a fully correct proof to show the answer required
(b)	$\tan 2\theta = \frac{1}{2}$	M1	This mark is given for deducing that the value of $\tan 2\theta$
	$180° + 26.6°$	M1	This mark is given for finding the solution in the third quadrant for arctan $\frac{1}{2}$
	$\theta = 103.3°$	A1	This mark is given for finding a correct value for θ

HOME

6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

Question	Scheme	Marks	AOs
6 (a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = \ldots$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2} \Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)

Trig – addition, double, etc.

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

HOME

A2 2020 Paper 2

Trig – addition, double, etc.

Question	Scheme	Marks	AOs
10 (a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$= \left(2\cos^2 A - 1\right)\cos A - \left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$= \left(2\cos^2 A - 1\right)\cos A - 2\cos A\left(1 - \cos^2 A\right)$	ddM1	2.1
	$= 4\cos^3 A - 3\cos A$ *	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = \ldots$	dM1	3.1a
	Two of $-90°$, 0, $90°$, awrt $139°$	A1	1.1b
	All four of $-90°$, 0, $90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

HOME

A2 2021 Paper 1

Trig – addition, double, etc.

10. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

HOME

A2 2021 Paper 1

Trig – addition, double, etc.

Question	Scheme	Marks	AOs
10(a)	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-(1-2\sin^2\theta)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\cos 2\theta+\sin 2\theta}{1+(2\cos^2\theta-1)+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-(1-2\sin^2\theta)+2\sin\theta\cos\theta}{1+(2\cos^2\theta-1)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\frac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$	dM1	2.1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow\sin 2x(1-3\cos 2x)=0$ $\Rightarrow(\sin 2x=0,)\ \cos 2x=\frac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)

HOME

A2 2021 Paper 2

Trig – addition, double, etc.

15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

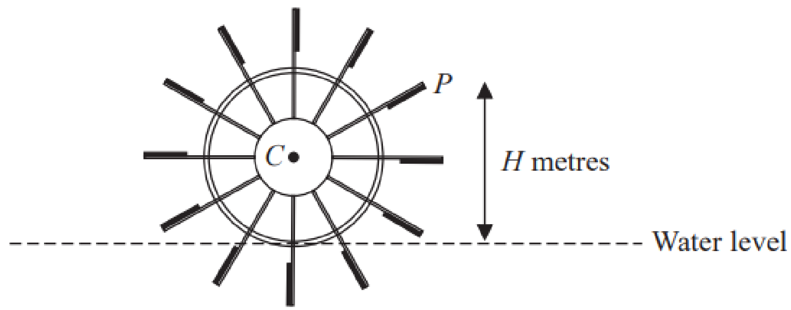


Figure 6

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

HOME

Question	Scheme	Marks	AOs
15(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
		(3)	
(b)(i)	$3+2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$ $\Rightarrow t=...$	M1	3.4
	$t=11.6$	A1	1.1b
		(3)	
(c)	$3+2\sqrt{5}\cos(0.5t+0.464)=0$ $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977...-0.464)-2(2.306...-0.464)$	dM1	3.1b
	$=3.34$	A1	1.1b
		(4)	
(d)	e.g. the "3" would need to vary	B1	3.5c
		(1)	
		(11 marks)	

HOME

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

14(a)	Attempts to use both $\sin(x-60°)=\pm\sin x\cos 60°\pm\cos x\sin 60°$ $\cos(x-30°)=\pm\cos x\cos 30°\pm\sin x\sin 30°$	M1
	Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$	A1
	Either uses $\frac{\sin x}{\cos x}=\tan x$ and attempts to make $\tan x$ the subject E.g. $(2\cos 60°-\sin 30°)\tan x=\cos 30°+2\sin 60°$ Or attempts $\sin 30°$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2\times\frac{1}{2}-\frac{1}{2}\right)\sin x=\left(2\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\cos x$,	M1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2}\tan x=\frac{3\sqrt{3}}{2}\Rightarrow\tan x=3\sqrt{3}$ *	A1*
		(4)
(b)	Deduces that $x=2\theta+60°$	B1
	$\tan(2\theta+60°)=3\sqrt{3}\Rightarrow 2\theta+60°=79.1°,259.1°,.....$	M1
	Correct method to find one value of θ E.g $\theta=\frac{79.1°-60°}{2}$	dM1
	$\theta=$ awrt $9.6°, 99.6°$ (See note)	A1
		(4)

HOME

A2 2022 Paper 2

Trig – addition, double, etc.

9.

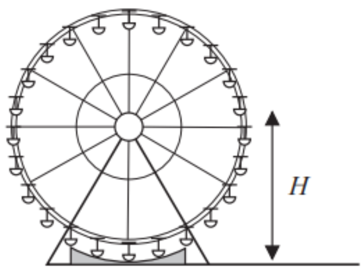


Figure 4

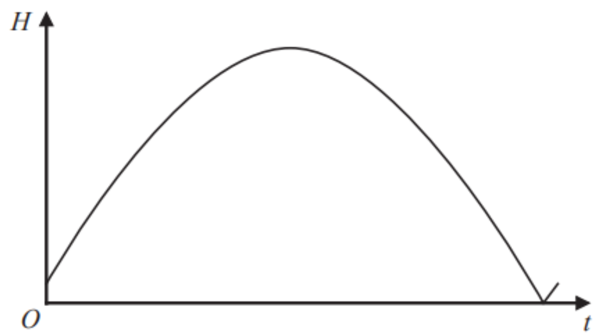


Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A\sin(bt + \alpha)°|$$

where A, b and α are constants.

Figure 5 shows a sketch of the graph of H against t, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = |A\sin(bt + \alpha)°| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

HOME

A2 2022 Paper 2

Trig – addition, double, etc.

9(a)	Deduces that $A = \pm 50$ **or** $b = \frac{1}{4}$	B1
	Deduces that $A = \pm 50$ **and** $b = \frac{1}{4}$	B1
	Uses $t = 0, H = 1 \Rightarrow \alpha = ...$ E.g. $1 = "50"\sin(\alpha)^\circ \Rightarrow \alpha = ...$	M1
	$H = \left\| \pm 50\sin\left(\frac{1}{4}t + 1.15\right)^\circ \right\|$	A1
		(4)
(b)	E.g. the minimum height above the ground of the passenger on the original model was 0 m or Adding "d" means the passenger does not touch the ground.	B1
		(1)

HOME

Parametric Equations

5. A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1$$

where a and b are integers to be found.

(3)

A2 SAMs Paper 1

Parametric Equations

Question	Scheme	Marks	AOs
5	Attempts to substitute $=\dfrac{x+1}{2}$ into $y \Rightarrow y = 4\left(\dfrac{x+1}{2}\right)-7 + \dfrac{6}{(x+1)}$	M1	2.1
	Attempts to write as a single fraction $y = \dfrac{(2x-5)(x+1)+6}{(x+1)}$	M1	2.1
	$y = \dfrac{2x^2-3x+1}{x+1} \qquad a=-3, b=1$	A1	1.1b
		(3 marks)	

HOME

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

(3)

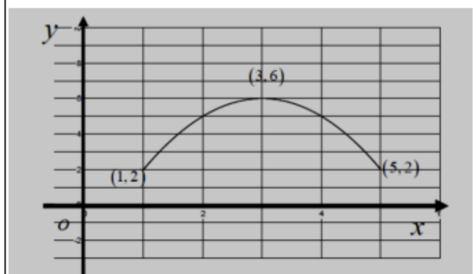
The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

A2 2018 Paper 1

Parametric Equations

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \dfrac{y-4}{2} = 1 - 2\left(\dfrac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \dfrac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \dfrac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow\ 49 - 4\times 1\times(k+3) = 0 \Rightarrow k = \left(\dfrac{37}{4}\right)$ or $(7-2k)^2 - 4\times 1\times(k^2 - 6k + 3) = 0 \Rightarrow k = \left(\dfrac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \dfrac{37}{4}\right\}$	A1	2.5
		(5)	

HOME

4.

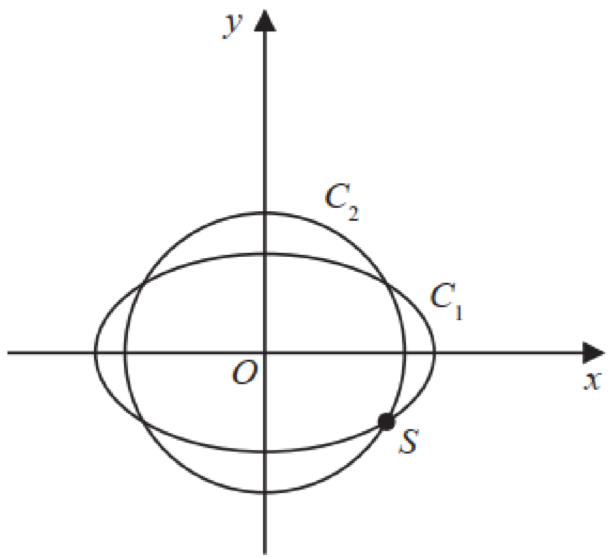


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

A2 2019 Paper 2

Parametric Equations

Part	Working or answer an examiner might expect to see	Mark	Notes
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	This mark is given for combining the two equations to show where the curve and circle meet
	$100(\cos t)^2 + 32(1 - \cos t)^2 = 66$	M1	This mark is given for forming an equation in $\cos t$ only
	$68\cos^2 t = 34$	A1	This mark is given for simplifying to find an equation in terms of $\cos t$
	$\cos t = \pm\frac{1}{\sqrt{2}} \quad \Rightarrow \quad t = \frac{\pi}{4}$	M1	This mark is given for finding a value for t
	$x = 10 \times \frac{1}{\sqrt{2}}$ $y = 4\sqrt{2} \times -\sin\frac{\pi}{4} = 4\sqrt{2} \times -\frac{1}{\sqrt{2}}$	M1	This mark is given for a method to substitute back into the original equations to find value for x and y
	$S = (5\sqrt{2}, -4)$	A1	This mark is given for the correct coordinates of S

HOME

13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(3)

A2 2021 Paper 1

Parametric Equations

Question	Scheme	Marks	AOs
13	$(x-3)^2+y^2=\left(\dfrac{t^2+5}{t^2+1}-3\right)^2+\left(\dfrac{4t}{t^2+1}\right)^2$	M1	3.1a
	$=\dfrac{(2-2t^2)^2+16t^2}{(t^2+1)^2}=\dfrac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\dfrac{4(t^4+2t^2+1)}{(t^2+1)^2}=\dfrac{4(t^2+1)^2}{(t^2+1)^2}=4$*	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.

dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.

A1*: Fully correct proof showing all key steps

BICEN MATHS

HOME

Differentiation

10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

Question	Scheme	Marks	AOs
10	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \dfrac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

A2 SAMs Paper 1

Differentiation

Question	Scheme	Marks	AOs
13(a)	Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx}=\frac{\sqrt{3}\sin 2t}{\sin t}\quad\left(=2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t=\frac{2\pi}{3}$ in $\frac{dy}{dx}=\frac{\sqrt{3}\sin 2t}{\sin t}=\left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal $=-\frac{1}{dy/dx}=\left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P=\left(-1,-\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y+\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x-2\sqrt{3}y-1=0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$	M1	3.1a
	Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t-4\cos t-5=0$	A1	1.1b
	Finds $\cos t=\frac{5}{6},\cancel{-\frac{1}{2}}$	M1	2.4
	Substitutes their $\cos t=\frac{5}{6}$ into $x=2\cos t,\ \ y=\sqrt{3}\cos 2t$,	M1	1.1b
	$Q=\left(\frac{5}{3},\frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

BICEN MATHS

HOME

A2 SAMs Paper 1

15.

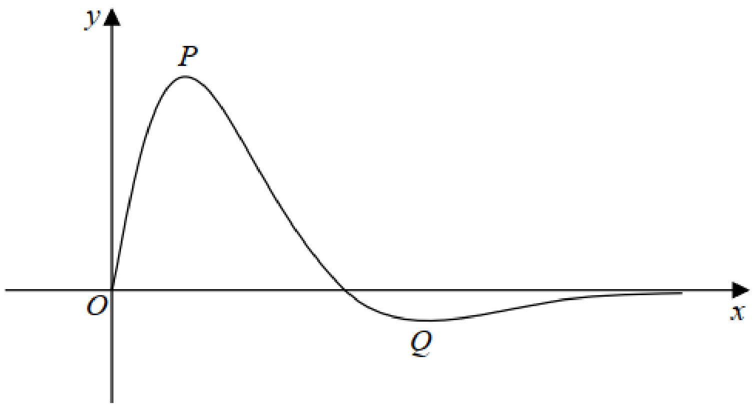


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

A2 SAMs Paper 1

Differentiation

Question	Scheme	Marks	AOs
15(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$f'(x) = \dfrac{e^{\sqrt{2}x-1} \times 8\cos 2x - 4\sin 2x \times \sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $f'(x) = 0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x} = \dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x = \sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x = \sqrt{2}$ and attempts to find the 2nd solution	M1	3.1a
	$x = 1.02$	A1	1.1b
	(ii) Solves $\tan 2x = \sqrt{2}$ and attempts to find the 1st solution	M1	3.1a
	$x = 0.478$	A1	1.1b
		(4)	
		(8 marks)	

HOME

3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where n, A and B are constants to be found.

(4)

A2 SAMs Paper 2

Differentiation

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3 \{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
			(4 marks)

HOME

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

A2 2018 Paper 1

Differentiation

Question	Scheme	Marks	AOs
5	$\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{\dots}{\dots\dots\dots C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
	(5 marks)		

HOME

9.

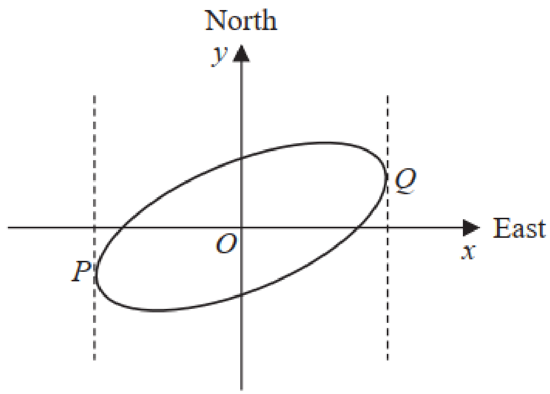


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

A2 2018 Paper 1

Differentiation

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \rightarrow Ay\dfrac{dy}{dx}$ or $2xy \rightarrow 2x\dfrac{dy}{dx}+2y$	M1	2.1
	$2x-2x\dfrac{dy}{dx}-2y+6y\dfrac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\dfrac{dy}{dx}=2y-2x$	M1	2.1
	$\dfrac{dy}{dx}=\dfrac{2y-2x}{6y-2x}=\dfrac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \dfrac{dy}{dx}\rightarrow\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\dfrac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\dfrac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\dfrac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\dfrac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

A2 2018 Paper 2

Differentiation

Question	Scheme	Marks	AOs
9	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h)-\cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1 A1	1.1b 1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$ so $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ *	dM1 A1*	2.1 2.5
		(5)	
			(5 marks)

BICEN MATHS

HOME

11.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$\mathrm{f}(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that $\mathrm{f}(x)$ is a decreasing function.

(3)

A2 2018 Paper 2

Differentiation

Question	Scheme	Marks	AOs
11	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) **Way 1**	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) **Way 2**	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$\mathrm{f}(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$\mathrm{f}'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $\mathrm{f}'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $\mathrm{f}'(x)=-(+\text{ve})-(+\text{ve})<0$, so $\mathrm{f}(x)$ is a decreasing function	A1	2.4
		(3)	
		(7 marks)	

HOME

14. A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

A2 2018 Paper 2

Differentiation

Question	Scheme	Marks	AOs
14	$N = \dfrac{900}{3+7e^{-0.25t}} = 900(3+7e^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\ \dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 2	$\dfrac{dN}{dt} = -900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{= \dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200} = \dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300-\dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS} = \dfrac{1575e^{-0..25t}}{(3+7e^{-0.25t})^2}$ o.e., $\text{RHS} = \dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2} = \dfrac{1575e^{-0..25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N = 150$ (can be implied)	B1	2.2a
	so $150 = \dfrac{900}{3+7e^{-0.25T}} \Rightarrow e^{-0.25T} = \dfrac{3}{7}$	M1	3.4
	$T = -4\ln\left(\dfrac{3}{7}\right)$ or $T =$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
		(10 marks)	

HOME

A2 2018 Paper 1

Differentiation

3. $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\dfrac{dy}{dx} < 0$

(1)

HOME

A2 2018 Paper 1

Differentiation

Question 3 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dy}{dx} = \frac{(x+1)^2 \times (10x+10) - (5x^2+10x) \times 2(x+1)}{(x+1)^4}$	M1	This mark si given for an attempt to differentiate the expression for y
		A1	This mark is given for correctly differentiating the expression for y
	$\frac{dy}{dx} = \frac{(x+1) \times (10x+10) - (5x^2+10x) \times 2}{(x+1)^3}$	M1	This mark sis given for cancelling the expression through by $(x + 1)$
	$\frac{dy}{dx} = \frac{10}{(x+1)^3}$	A1	This mark is given for a fully correct expression for $\frac{dy}{dx}$
(b)	If $A > 0$ and $n = 1, 3$ then $x < -1$	B1	This mark is given for deducing that $\frac{dy}{dx} < 0 \Rightarrow x < -1$.

HOME

12. $$f(x) = 10e^{-0.25x} \sin x, \qquad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

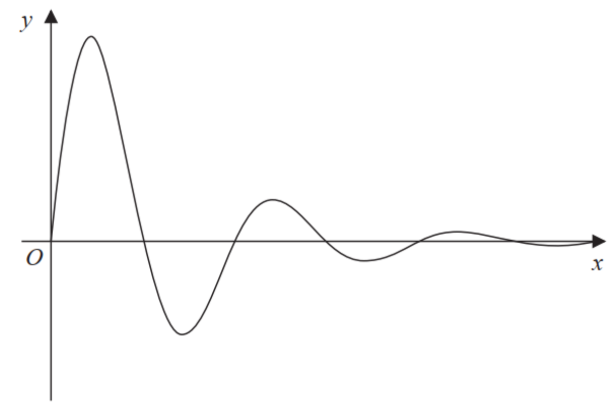


Figure 3

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t} \sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

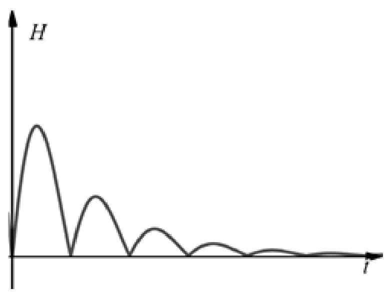
(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

A2 2019 Paper 1

Differentiation

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$f'(x) = -2.5e^{-0.25x}\sin x + 10e^{-0.25x}\cos x$	M1	This mark is given for a method to differentiate to find an expression for $f'(x)$
		A1	This mark is given for correctly differentiating to find an expression for $f'(x)$
	$f'(x) = 0$ $\Rightarrow e^{-0.25x}(-2.5\sin x + 10\cos x) = 0$ $\Rightarrow (-2.5\sin x + 10\cos x) = 0$	M1	This mark is given for setting $f'(x) = 0$ and finding as method to solve for $\tan x$
	$\frac{\sin x}{\cos x} = \frac{10}{2.5}$ $\tan x = 4$	A1	This mark is given for showing that $\tan x = 4$ as required.
(b)		M1	This mark is given for a graph with a correct shape
		A1	This mark is given for a graph with heights > 0
(c)	$\tan x = 4$, $x = 1.326$ $t = \pi + 1.326 = 4.47$	M1	This method is given for finding a value for t between the first and second bounce
	$H(4.47) = \lvert 10e^{-0.25\times 4.47}\sin 4.47\rvert$	M1	This mark is given for substituting the value of $t = \pi + \arctan 4$ into $H(t)$
	$= \lvert 3.27 \times -0.97\rvert$ $= 3.17$ metres	A1	This mark is given for finding the maximum height of the ball
(d)	The time between each bounce should not stay the same when the heights of each bounce are getting smaller	B1	This mark is given for a valid explanation of why the model should not be used the predict the time of each bounce

HOME

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

A2 2019 Paper 1

Differentiation

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	This mark is given for differentiating and inverting
	At (0, 0), $\frac{dy}{dx} = \frac{1}{8}$	A1	This mark is given for finding $\frac{dy}{dx}$ when $y = 0$
(b)(i)	$\sin 2y \approx 2y \Rightarrow x \approx 8y$	B1	This mark is given for finding an approximation for x
(b)(ii)	When x and y are small, $x = 4\sin 2y$ approximates to the line $x = 8y$	B1	This mark is given for a valid explanation of the relationship between x and y when both are small
(c)	$\sin^2 2y + \cos^2 2y = 1$ $\Rightarrow \cos^2 2y = 1 - \sin^2 2y$ $x = 4\sin 2y \Rightarrow \sin^2 2y = \left(\frac{x}{4}\right)^2$	M1	This mark is given for a method to use find an expression for $\sin^2 2y$ in terms of x
	$\frac{dy}{dx} = \frac{1}{8\cos 2y} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$	A1	This mark is given for an unsimplified expression for $\frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	This mark is given for a fully correct answer with $a = 2$ and $b = 16$

HOME

A2 2019 Paper 2

Differentiation

11.

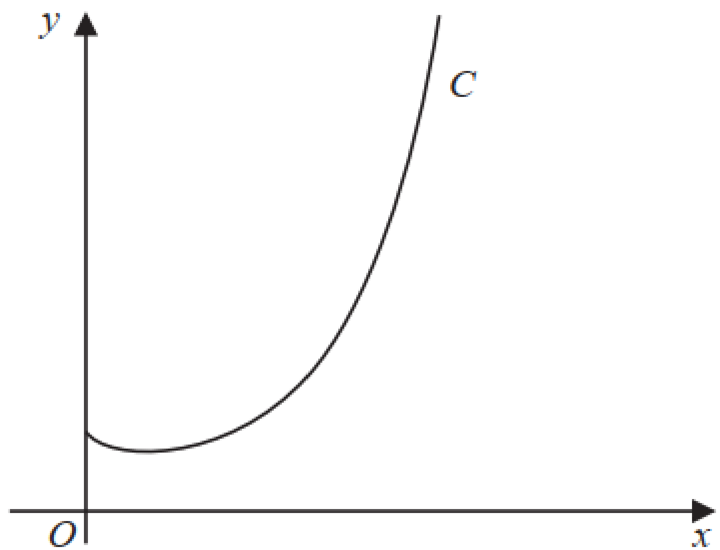


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x, \; x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

HOME

A2 2019 Paper 2

Differentiation

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x^x \Rightarrow \ln y = x \ln x$	M1	This mark is for a method to find the x-coordinate of the turning point of C by taking logarithms
	$\ln y = x \ln x \Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \ln x + 1$	M1	This mark is given for a method using implicit differentiation
		A1	This mark is given for a correct expression for $\dfrac{1}{y}\dfrac{dy}{dx}$
	Setting $\dfrac{dy}{dx} = 0$, $\ln x + 1 = 0$	M1	This mark is given for a method for finding the turning point of C by setting $\dfrac{dy}{dx} = 0$
	$x = e^{-1}$	A1	This mark is given for correctly finding a value for the x-coordinate of the turning point of C

HOME

A2 2020 Paper 1

Differentiation

Figure 2 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = 4(x^2 - 2)e^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $f'(x) = 8(2 + x - x^2)e^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

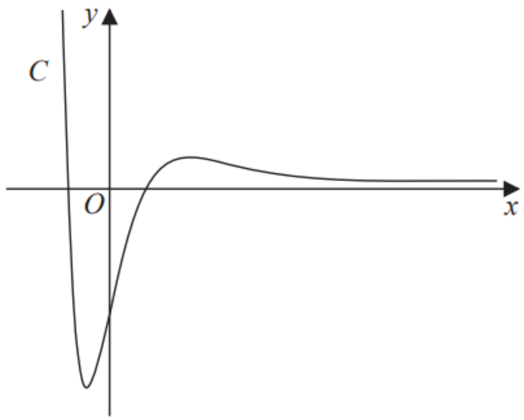


Figure 2

The function g and the function h are defined by

$$g(x) = 2f(x) \qquad x \in \mathbb{R}$$

$$h(x) = 2f(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

HOME

Question	Scheme	Marks	AOs
9(a)	$f(x)=4(x^2-2)e^{-2x}$		
	Differentiates to $e^{-2x}\times 8x+4(x^2-2)\times -2e^{-2x}$	M1 A1	1.1b 1.1b
	$f'(x)=8e^{-2x}\{x-(x^2-2)\}=8(2+x-x^2)e^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $f'(x)=0$ $x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $(-1,-4e^2)$ and $(2,8e^{-4})$	A1	1.1b
		(3)	
(c)	(i) Range $[-8e^2,\infty)$ o.e. such as $g(x)\geqslant -8e^2$	B1ft	2.5
	(ii) For • Either attempting to find $2f(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times "8e^{-4}"-3$ and identifying this as the **upper bound**	M1	3.1a
	Range $[-19,16e^{-4}-3]$	A1	1.1b
		(3)	
			(9 marks)

HOME

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

A2 2020 Paper 1

Differentiation

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x\tan y + x^2 \sec^2 y \dfrac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-2x \times \dfrac{9}{x^2}}{x^2\left(1 + \dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4 + 81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-18 \times (x^4+81) - (-18x)(4x^3)}{(x^4+81)^2} = \dfrac{54(x^4-27)}{(x^4+81)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)

HOME

A2 2020 Paper 2

Differentiation

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

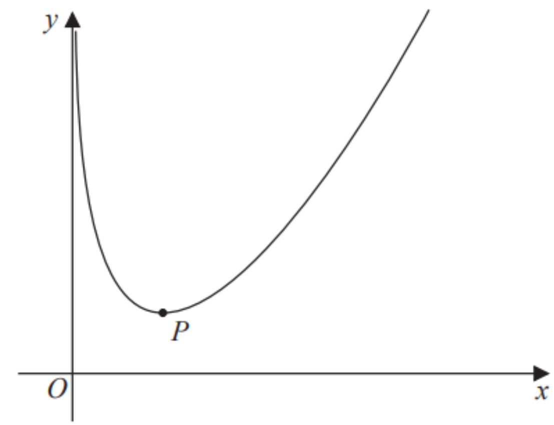


Figure 1

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

HOME

A2 2020 Paper 2

Differentiation

7(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0\Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	$x_2=$ awrt 1.13894	A1	1.1b
	$x=1.15650$	A1	2.2a
		(3)	
			(10 marks)

HOME

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

A2 2020 Paper 2

Differentiation

Question	Scheme	Marks	AOs
13(a)	$k=\mathrm{e}^2$ **or** $x\neq \mathrm{e}^2$	B1	2.2a
		(1)	
(b)	$$g'(x)=\frac{(\ln x-2)\times\frac{3}{x}-(3\ln x-7)\times\frac{1}{x}}{(\ln x-2)^2}=\frac{1}{x(\ln x-2)^2}$$ or $$g'(x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(3-(\ln(x)-2)^{-1}\right)=(\ln x-2)^{-2}\times\frac{1}{x}=\frac{1}{x(\ln x-2)^2}$$ or $$g'(x)=(\ln x-2)^{-1}\times\frac{3}{x}-(3\ln x-7)(\ln x-2)^{-2}\times\frac{1}{x}=\frac{1}{x(\ln x-2)^2}$$	M1 A1	1.1b 2.1
	As $x>0$ (or $1/x>0$) **AND** $\ln x-2$ is squared so $g'(x)>0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x-7 \ldots 0$ or $\ln x-2 \ldots 0$ or $3\ln a-7 \ldots 0$ or $\ln a-2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x>\ldots$ or $a>\ldots$	M1	3.1a
	$0<a<\mathrm{e}^2,\quad a>\mathrm{e}^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

HOME

14. Given that

$$y = \frac{x-4}{2+\sqrt{x}} \qquad x > 0$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(4)

A2 2021 Paper 1

Differentiation

Question	Scheme	Marks	AOs
14	$y=\dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{dy}{dx}=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$=\dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded

Product: $\alpha\left(2+\sqrt{x}\right)^{-1}+\beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t=\sqrt{x}$, $y=\dfrac{t^2-4}{2+t}\Rightarrow\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}=\dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t=\sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

A2 2021 Paper 2

Differentiation

Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = ...$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = ...$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0,$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = ...$ $\left(\frac{dy}{dx}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0,$ $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a

8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

A2 2021 Paper 2

Differentiation

Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(3y^2) = 6y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}(qxy) = qx\frac{\mathrm{d}y}{\mathrm{d}x} + qy$	M1	2.1
	$3px^2 + qx\frac{\mathrm{d}y}{\mathrm{d}x} + qy + 6y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1	1.1b
	$(qx+6y)\frac{\mathrm{d}y}{\mathrm{d}x} = -3px^2 - qy \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	dM1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-3px^2 - qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3 + q(-1)(-4) + 3(-4)^2 = 26$	M1	1.1b
	$19x + 26y + 123 = 0 \Rightarrow m = -\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2 - q(-4)}{q(-1)+6(-4)} = \frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2 + q(-4)} = -\frac{19}{26}$	M1	3.1a
	$p - 4q = 22,\ 57p - 102q = 624 \Rightarrow p = \ldots, q = \ldots$	dM1	1.1b
	$p = 2,\ q = -5$	A1	1.1b
		(5)	
			(9 marks)

BICEN MATHS

HOME

13. The curve C has parametric equations

$$x = \sin 2\theta \qquad y = \operatorname{cosec}^3 \theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

A2 2021 Paper 2

Differentiation

Question	Scheme	Marks	AOs
13(a)	$y = \text{cosec}^3\theta \Rightarrow \dfrac{dy}{d\theta} = -3\text{cosec}^2\theta\text{cosec}\theta\cot\theta$	B1	1.1b
	$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \text{cosec}^3\theta = 8 \Rightarrow \sin^3\theta = \dfrac{1}{8} \Rightarrow \sin\theta = \dfrac{1}{2}$	M1	3.1a
	$\theta = \dfrac{\pi}{6} \Rightarrow \dfrac{dy}{dx} = \dfrac{-3\text{cosec}^3\left(\dfrac{\pi}{6}\right)\cot\left(\dfrac{\pi}{6}\right)}{2\cos\left(\dfrac{2\pi}{6}\right)} = \ldots$ **or** $\sin\theta = \dfrac{1}{2} \Rightarrow \dfrac{dy}{dx} = \dfrac{\dfrac{-3}{\sin^3\theta} \times \dfrac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)} = \dfrac{-3\times 8\times\dfrac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\dfrac{1}{4}\right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

HOME

4. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

(3)

A2 2022 Paper 2

Differentiation

4	$\dfrac{2(x+h)^2 - 2x^2}{h} = \ldots$	M1	2.1
	$\dfrac{2(x+h)^2 - 2x^2}{h} = \dfrac{4xh + 2h^2}{h}$	A1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lim\limits_{h \to 0} \dfrac{4xh + 2h^2}{h} = \lim\limits_{h \to 0}(4x + 2h) = 4x\,*$	A1*	2.5
		(3)	
			(3 marks)

BICEN MATHS

HOME

12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k.

(3)

12(a)	$\mathrm{f}(x)=\dfrac{\mathrm{e}^{3x}}{4x^2+k}\Rightarrow \mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $\mathrm{f}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}\Rightarrow \mathrm{f}'(x)=3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$	M1 A1
	$\mathrm{f}'(x)=\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$	A1
		(3)
(b)	If $y=\mathrm{f}(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root	B1
	Applies $b^2-4ac\ (\geqslant)\ 0$ with $a=12, b=-8, c=3k$	M1
	$0<k\leqslant\dfrac{4}{9}$	A1
		(3)

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2\tan t + 1 \qquad y = 2\sec^2 t + 3 \qquad -\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{3}$$

The line l is the normal to C at the point P where $t = \frac{\pi}{4}$

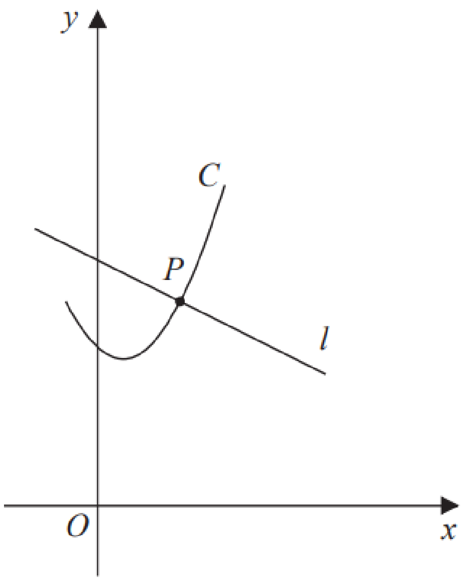


Figure 6

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2}$$

(5)

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2}(x-1)^2 + 5$$

(2)

The straight line with equation

$$y = -\frac{1}{2}x + k \qquad \text{where } k \text{ is a constant}$$

intersects C at two distinct points.

(c) Find the range of possible values for k.

(5)

A2 2022 Paper 2

Differentiation

16(a)	Attempts $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\sec^2 t \tan t}{2\sec^2 t} (= 2\tan t)$	M1 A1
	At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = 2, x = 3, y = 7$	M1
	Attempts equation of normal $y - 7 = -\frac{1}{2}(x-3)$	M1
	$y = -\frac{1}{2}x + \frac{17}{2}$ *	A1*
		(5)
(b)	Attempts to use $\sec^2 t = 1 + \tan^2 t \Rightarrow \frac{y-3}{2} = 1 + \left(\frac{x-1}{2}\right)^2$	M1
	$\Rightarrow y - 3 = 2 + \frac{(x-1)^2}{2} \Rightarrow y = \frac{1}{2}(x-1)^2 + 5$ *	A1*
		(2)
(c)	**Attempts the lower limit for k:** $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11-2k) = 0$ $b^2 - 4ac = 1 - 4(11-2k) = 0 \Rightarrow k = ...$	M1
	$(k =)\frac{43}{8}$	A1
	Attempts the upper limit for k: $(x,y)_{t=-\frac{\pi}{4}} : t = -\frac{\pi}{4} \Rightarrow x = 2\tan\left(-\frac{\pi}{4}\right) + 1 = -1,\ y = 2\sec^2\left(-\frac{\pi}{4}\right) + 3 = 7$ $(-1, 7), y = -\frac{1}{2}x + k \Rightarrow 7 = \frac{1}{2} + k \Rightarrow k = ...$	M1
	$(k =)\frac{13}{2}$	A1
	$\frac{43}{8} < k \leqslant \frac{13}{2}$	A1
		(5)

HOME

Numerical Methods

8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval $[3.5, 4]$

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

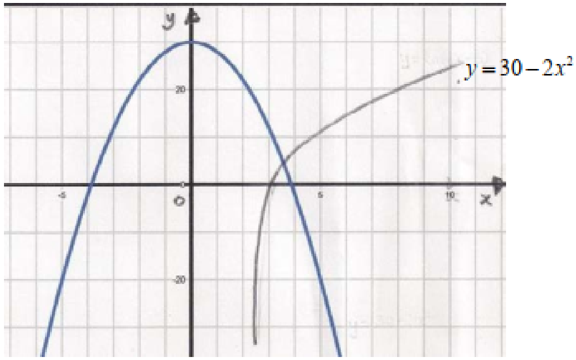
(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

A2 SAMs Paper 1

Numerical Methods

Question	Scheme	Marks	AOs
8 (a)	$f(3.5) = -4.8,\ f(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} \Rightarrow \quad x_1 = 4 - \dfrac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	**(2)**	
(c)	 Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow f(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

HOME

14.

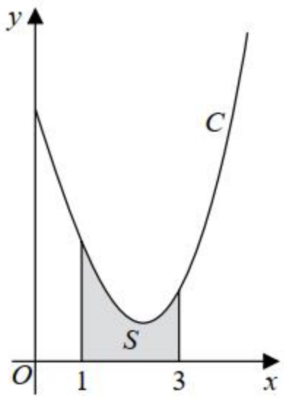


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

HOME

A2 SAMs Paper 1

Numerical Methods

Question	Scheme	Marks	AOs
14(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	

HOME

A2 2018 Paper 1

Numerical Methods

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

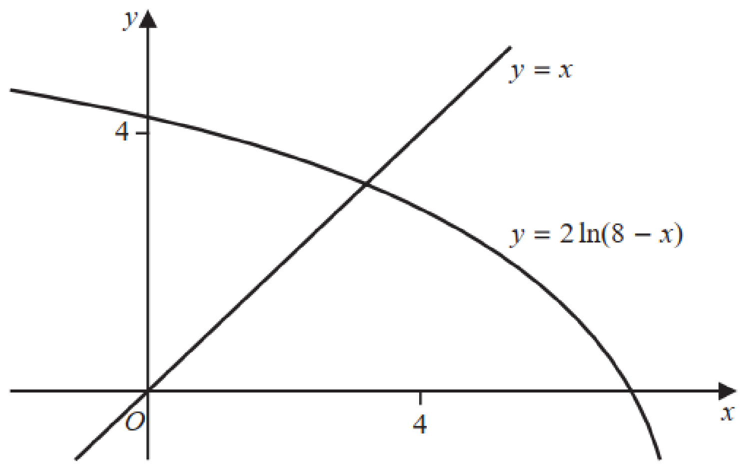


Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

HOME

A2 2018 Paper 1

Numerical Methods

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3)=$ and $f(4)=$ where $f(x)=\pm\left(2\ln(8-x)-x\right)$	M1	2.1
	$f(3)=\left(2\ln(5)-x\right)=(+)0.22$ and $f(4)=\left(2\ln(4)-4\right)=-1.23$ Change of sign and function continuous in interval $[3,4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

HOME

A2 2018 Paper 2

Numerical Methods

5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

A2 2018 Paper 2

Numerical Methods

Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\}$ $f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \Rightarrow\right\}$ $\{x_{n+1}\} = x_n - \dfrac{2x_n^{\ 3} + x_n^{\ 2} - 1}{6x_n^{\ 2} + 2x_n}$	M1	1.1b
	$= \dfrac{x_n(6x_n^{\ 2} + 2x_n) - (2x_n^{\ 3} + x_n^{\ 2} - 1)}{6x_n^{\ 2} + 2x_n} \Rightarrow x_{n+1} = \dfrac{4x_n^{\ 3} + x_n^{\ 2} + 1}{6x_n^{\ 2} + 2x_n}$ *	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}$ $x_2 = \dfrac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \dfrac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \dfrac{3}{4},\ x_3 = \dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

HOME

2. The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in $\mathrm{m\,s^{-1}}$.

Time (s)	0	5	10	15	20	25
Speed ($\mathrm{m\,s^{-1}}$)	2	5	10	18	28	42

Using all of this information,

(a) estimate the length of runway used by the jet to take off.

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(b) explain whether your answer to part (a) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(1)

A2 2019 Paper 2

Numerical Methods

Question 2 (Total 4 marks)

Part	**Working or answer an examiner might expect to see**	**Mark**	**Notes**
(a)	$\left(\frac{1}{2} \times 5\right)(42 + 2 + 2 \times (5 + 10 + 18 + 28))$	M1	This mark is given for a method to use the trapezium rule as an approximation to the area under the curve
		M1	This mark is given for a correct terms used for the trapezium rule
	415 m	A1	This mark is given for a correct estimate of the length of the runway
(b)	An overestimate since the area of the five trapezia is greater than the area under the curve	B1	This mark is given for a valid explanation

A2 2019 Paper 2

Numerical Methods

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

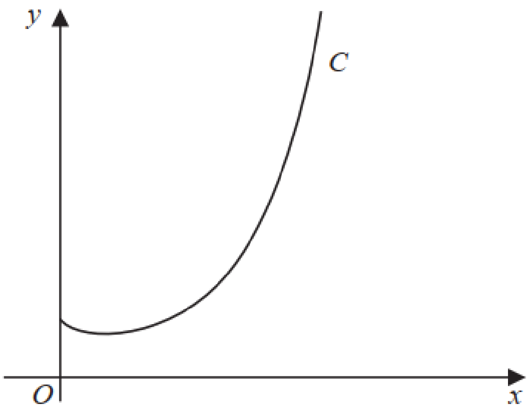


Figure 8

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{\,1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

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A2 2019 Paper 2

Numerical Methods

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x^x \Rightarrow \ln y = x \ln x$	M1	This mark is for a method to find the x-coordinate of the turning point of C by taking logarithms
	$\ln y = x \ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln x + 1$	M1	This mark is given for a method using implicit differentiation
		A1	This mark is given for a correct expression for $\frac{1}{y}\frac{dy}{dx}$
	Setting $\frac{dy}{dx} = 0$, $\ln x + 1 = 0$	M1	This mark is given for a method for finding the turning point of C by setting $\frac{dy}{dx} = 0$
	$x = e^{-1}$	A1	This mark is given for correctly finding a value for the x-coordinate of the turning point of C
(b)	$1.5^{1.5} = 1.837\ldots$, $1.6^{1.6} = 2.121\ldots$	M1	This mark is given for substituting 1.5 and 1.6 into $y = x^x$
	The curve C contains the points (1.5, 1.8) and (1.6, 2.1). At P, $y = 2$ Since C is continuous, $1.5 < \alpha < 1.6$	A1	This mark is given for a valid explanation that C contains the points (1.5, 1.8) and (1.6, 2.1) and is continuous
(c)	$x_1 = 1.5$ $x_2 = 2 \times 1.5^{-0.5} = 1.633$	M1	This mark is given for finding a correct value for x_2
	$x_3 = 2 \times 1.633^{-0.633} = 1.466$ $x_4 = 2 \times 1.466^{-0.466} = 1.673$	A1	This mark is given for finding a correct value for x_4
(d)	For example: x_n oscillates is periodic is non-convergent	B1	This mark is given for a valid statement about the long-term behaviour of x_n
	between 1 and 2	B1	This mark is given for stating that the behaviour is between 1 and 2

HOME

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

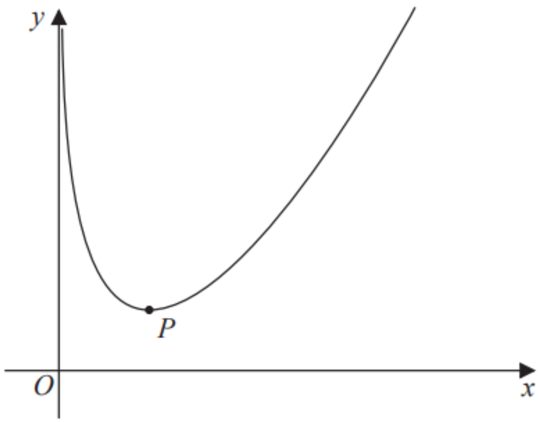

Figure 1

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

A2 2020 Paper 2

Numerical Methods

7(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0\Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	x = 1.15650	A1	2.2a
		(3)	
			(10 marks)

BICEN MATHS

HOME

4. The curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\ 2} - 2x_n^{\ 3})$$

is used to find an approximate value for α.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

A2 2021 Paper 1

Numerical Methods

Question	Scheme	Marks	AOs
4(a)	$\mathrm{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x\left(2x^2-4x+5\right)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$ *	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$\mathrm{h}(x) = 2x^3-4x^2+7x-2$ $\mathrm{h}(0.3415) = 0.00366...$ $\mathrm{h}(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $\mathrm{f}'(x)$ is continuous • $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
			(9 marks)

HOME

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

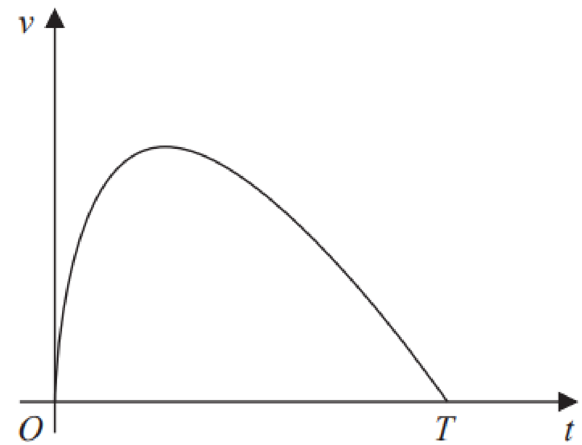

Figure 2

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

HOME

A2 2022 Paper 1

Numerical Methods

8 (a)	25	B1
		(1)
(b)	Attempts to differentiate using the product rule $\frac{dv}{dt} = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1
	Sets their $\frac{dv}{dt} = 0 \Rightarrow \frac{(10-0.4\boldsymbol{t})}{(\boldsymbol{t}+1)} = 0.4\ln(t+1)$ **and** then makes progress towards making "***t***" the subject (See notes for this)	dM1
	$t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ $t = \frac{26}{1+\ln(t+1)} - 1$ *	A1*
		(4)
(c)	(i) Attempts $t_2 = \frac{26}{1+\ln 8} - 1$	M1
	awrt 7.298	A1
	(ii) awrt 7.33 seconds	A1
		(3)

HOME

5. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\displaystyle\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\displaystyle\int_3^9 \log_3 18x \, dx$

(3)

5(a)	States or uses $h = 1.5$	B1
	Full attempt at the trapezium rule $= \frac{\dots}{2}\{1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)\}$	M1
	$=$ awrt 13.3 or $\frac{531}{40}$	A1
		(3)
(b)(i)	$\int_3^9 \log_3(2x)^{10}\,dx = 10\times\text{"13.3"} = \text{awrt}\,133$ or e.g. $\frac{531}{4}$	B1ft
(ii)	$\int_3^9 \log_3 18x\,dx = \int_3^9 \log_3(9\times 2x)\,dx = \int_3^9 2 + \log_3 2x\,dx$ $= [2x]_3^9 + \int_3^9 \log_3 2x\,dx = 18 - 6 + \int_3^9 \log_3 2x\,dx = \dots$	M1
	Awrt 25.3 or $\frac{1011}{40}$	A1ft
		(3)

A2 2022 Paper 2

Numerical Methods

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)

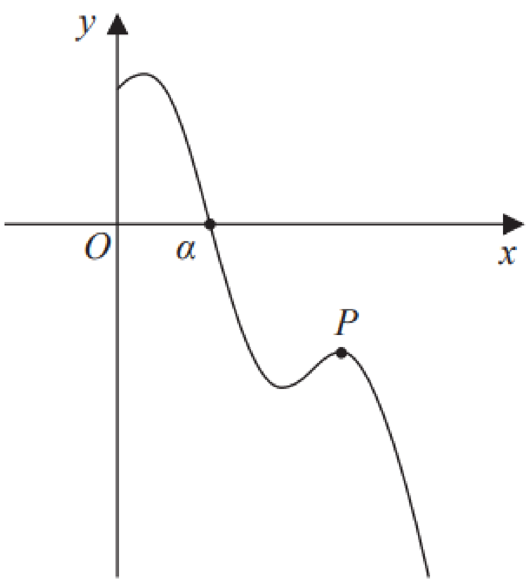

Figure 2

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A2 2022 Paper 2

Numerical Methods

6(a)	$\left(f'(x)=\right)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1
	Sets $f'(x)=4\cos\left(\frac{1}{2}x\right)-3=0 \Rightarrow x=$	dM1
	$x=14.0$ Cao	A1
		(4)
(b)	Explains that $f(4)>0$, $f(5)<0$ **and** the function is continuous	B1
		(1)
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB $f(5)=-1.212\ldots$ and $f'(5)=-6.204\ldots$)	M1
	$x_1=$ awrt 4.80	A1
		(2)

HOME

Integration

4. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

Question	Scheme	Marks	AOs
4	Writes $\int \frac{t+1}{t}\,\mathrm{d}t = \int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate	M1	2.1
	$= t + \ln t \; (+c)$	M1	1.1b
	$(2a + \ln 2a) - (a + \ln a) = \ln 7$	M1	1.1b
	$a = \ln\frac{7}{2}$ with $k = \frac{7}{2}$	A1	1.1b
			(4 marks)

14.

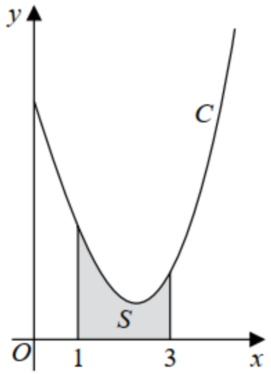

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

A2 SAMs Paper 1

Integration

Question	Scheme	Marks	AOs
14(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \,\mathrm{d}x$	M1	2.1
	$=\frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,\mathrm{d}x$	A1	1.1b
	$\int -2x+5\,\mathrm{d}x = -x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S = \int_1^3 \frac{x^2 \ln x}{3} - 2x + 5\,\mathrm{d}x = \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S = \frac{28}{27} + \ln 27 \quad (a = 28, b = 27, c = 27)$	A1	1.1b
		(6)	
	(10 marks)		

HOME

9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">9</td><td colspan="2">$\int\left(3x^{0.5}+A\right)\mathrm{d}x = 2x^{1.5}+Ax(+c)$</td><td>M1
A1</td><td>3.1a
1.1b</td></tr>
<tr><td colspan="2">Uses limits and sets $= 2A^2 \Rightarrow (2\times 8+4A)-(2\times 1+A)=2A^2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Sets up quadratic and attempts to solve</td><td>Sets up quadratic and attempts b^2-4ac</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\Rightarrow A=-2, \frac{7}{2}$ and states that there are two roots</td><td>States $b^2-4ac=121>0$ and hence there are two roots</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="5" align="right">(5 marks)</td></tr>
</table>

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{22}P(11-2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + C\mathrm{e}^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

A2 SAMs Paper 2

Integration

Question	Scheme	Marks	AOs
16(a)	Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\dfrac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A \text{ or } B$	M1	1.1b
	$\dfrac{1}{P(11-2P)} = \dfrac{\frac{1}{11}}{P} + \dfrac{\frac{2}{11}}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$	B1	3.1a
	Uses (a) and attempts to integrate $\displaystyle\int \frac{2}{P} + \frac{4}{(11-2P)}\,\mathrm{d}P = t + c$	M1	1.1b
	$2\ln P - 2\ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c = (-2\ln 9)$	M1	3.1a
	Substitutes $P = 2 \Rightarrow t = 2\ln 2 + 2\ln 9 - 2\ln 7$	M1	3.1a
	Time $= 1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $2\ln P - 2\ln(11-2P) = t - 2\ln 9$ $\Rightarrow \ln\left(\dfrac{9P}{11-2P}\right) = \dfrac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\dfrac{9P}{11-2P}\right) = e^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P = f\left(e^{\frac{1}{2}t}\right)$ or $\Rightarrow P = f\left(e^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \dfrac{11}{2+9e^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$	A1	1.1b
		(3)	
		(12 marks)	

HOME

A2 2018 Paper 1

Integration

7. Given that $k \in \mathbb{Z}^+$

(a) show that $\displaystyle\int_k^{3k} \frac{2}{(3x-k)}\,\mathrm{d}x$ is independent of k,

(4)

(b) show that $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\,\mathrm{d}x$ is inversely proportional to k.

(3)

BICEN MATHS

HOME

A2 2018 Paper 1

Integration

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

HOME

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T.

(2)

A2 2018 Paper 1

Integration

Question	Scheme	Marks	AOs
10(a)	$\frac{dH}{dt} = \frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}\,dH = \int \frac{\cos 0.25t}{40}\,dt$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t\,(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t = 0, H = 5 \Rightarrow c = \ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10}\sin 0.25t \Rightarrow H = 5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1} = 5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

HOME

13. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

A2 2018 Paper 1

Integration

Question	Scheme for Substitution		Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • Using a valid substitution $u =\ldots$, changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u^2 \pm 2\right)u^2\mathrm{d}u$	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u \pm 2\right)\sqrt{u}\,\mathrm{d}u$	M1	1.1b
	$=Pu^5 \pm Qu^3$	$=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$ *		A1*	2.1
			(7)	
				(7 marks)

HOME

10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)

(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.

(2)

(c) Suggest a limitation of the model.

(1)

A2 2018 Paper 2

Integration

Question	Scheme		Marks	AOs
10 (a)	$\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$ (for k or a numerical k)		M1	3.3
	$\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t \Rightarrow \ldots$ (for k or a numerical k)		M1	2.1
	$\frac{1}{3}r^3 = \pm kt \ \{+c\}$		A1	1.1b
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth	$t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1	3.1a 1.1b
			(5)	
(b)	$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = \ldots$		M1	3.4
	time = 5 minutes 6 seconds		A1	1.1b
			(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth		B1	3.5b
			(1)	
			(8 marks)	

BICEN MATHS

HOME

13.

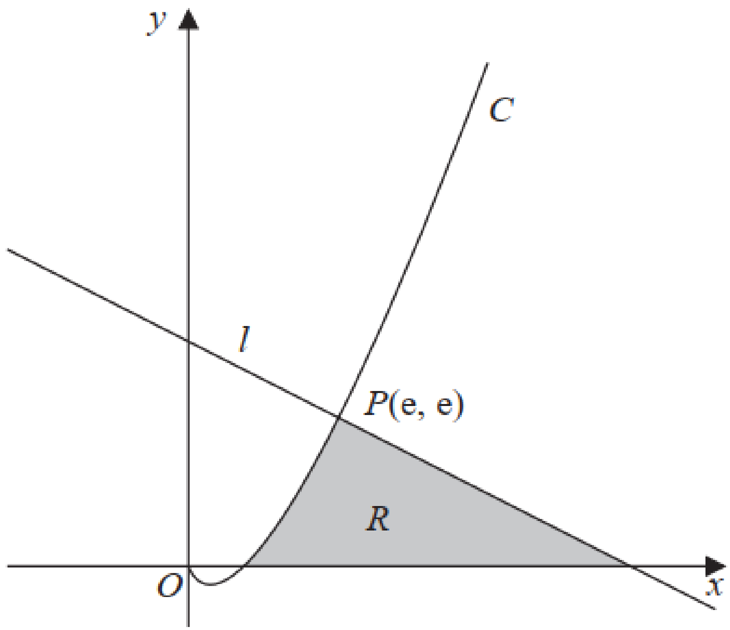

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x\ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

A2 2018 Paper 2

Integration

Question	Scheme	Marks	AOs
13	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

HOME

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

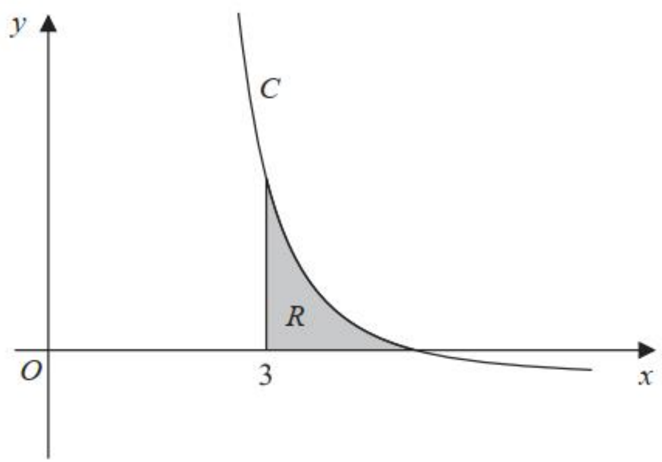

Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a\ln 2 + b\ln 3$, where a and b are rational constants to be found.

(8)

A2 2019 Paper 1

Integration

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The asymptote is found where $2x - q = 0$ Hence $q = 4$	B1	This mark is given for explaining that the asymptote at $x = 2$ is a solution of $2x - q = 0$
	$y = \dfrac{p - 3x}{(2x-4)(x+3)}$ $\dfrac{1}{2} = \dfrac{p-9}{(6-4)(3+3)}$	M1	This mark is given for substituting $x = 3$, $y = \dfrac{1}{2}$ (and $q = 4$)
	$p - 9 = 6$ $p = 15$	A1	This mark is given for solving for p and showing that $p = 15$, as required
(b)	$\dfrac{15-3x}{(2x-4)(x+3)} = \dfrac{A}{(2x-4)} + \dfrac{B}{(x+3)}$	M1	This mark is given for a method to use partial fractions
	$= \dfrac{1.8}{(2x-4)} - \dfrac{2.4}{(x+3)}$	M1	This mark is given for finding values for A and B
	$= \dfrac{0.9}{(x-2)} - \dfrac{2.4}{(x+3)}$	A1	This mark is given for a fully simplified expression
	$I = \displaystyle\int \frac{15-3x}{(2x-4)(x+3)}\,dx$ $= m \ln (2x - 4) + n \ln (x + 3)$	M1	This mark is given for a method to integrate to find the area of R
	$= 0.9 \ln (2x - 4) + 2.4 \ln (x + 3)$	A1	This mark is given for a correct expression for the area of R
	Area $R = \Big[0.9\ln(2x-4) - 2.4\ln(x+3)\Big]_3^5$	M1	This mark is given for deducing an expression for the area of R ($y = 0$ when $x = 5$)
	$= [0.9 \ln 6 - 2.4 \ln 8] - [0.9 \ln 2 - 2.4 \ln 6]$ $= [0.9 \ln 6 + 2.4 \ln 6] - [7.2 \ln 2 + 0.9 \ln 2]$ $= 3.3 \ln 6 - 8.1 \ln 2$ $= 3.3 \ln 3 + 3.3 \ln 2 - 8.1 \ln 2$	M1	This mark is given for a method to find the exact area of R
	$= 3.3 \ln 3 - 4.8 \ln 2$	A1	This mark is given for a correct value of the area of R with $a = 3.3$ and $b = 4.8$

HOME

14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4 - \sqrt{h}} = -8\ln\left|4 - \sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}\left(4 - \sqrt{h}\right)}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

HOME

A2 2019 Paper 2

Integration

(a)	$\mathrm{d}h = -2(4-u)\,\mathrm{d}u$	B1	This mark is given for finding an expression for dh
	$\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \int \frac{-2(4-u)\,\mathrm{d}u}{4-\sqrt{h}}$	M1	This mark is given for substituting $u = 4-\sqrt{h}$ into the integral
	$= \int -\frac{8}{u} + 2\,\mathrm{d}u$	M1	This mark is given for a method to find a simplified version of the integral
	$-8\ln u + 2u + c$ $= -8\ln(4-\sqrt{h}) + 2(4-\sqrt{h}) + c$	M1	This mark is given for integrating with respect to u to produce an expression in terms of h
		A1	This mark is given for a correct expression for the integral
	$= -8\ln(4-\sqrt{h}) - 2\sqrt{h} + k$	A1	This mark is given for a full proof to arrive at the answer as shown (appreciating that $k = c + 8$)
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$	M1	This mark is given for a setting $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$
	$0 < h < 16$	A1	This mark is given for deducing the range of the heights of the trees for this model
(c)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20} \Rightarrow \frac{\mathrm{d}h}{(4-\sqrt{h})} = \frac{t^{0.25}\mathrm{d}t}{20}$	B1	This mark is given for separating the variables
	$-8\ln(4-\sqrt{h}) - 2\sqrt{h} + k = \frac{t^{1.25}}{25}$	M1	This mark is given for a method to integrate both sides of the equation
		A1	This mark is given for integrating both sides of the equation correctly
	When $t = 0$ and $h = 1$, $-8\ln 3 - 2 + k = 0$ $k = 2 + 8\ln 3$	M1	This mark is given for substituting values of $t = 0$ and $h = 1$ to find a value for k
	When $h = 12$, $-8\ln(4-\sqrt{12}) - 2\sqrt{12} + 2 + 8\ln 3 = \frac{t^{1.25}}{25}$	M1	This mark is given for a method to find a value for t by substituting $h = 12$ into the equation
	$t^{1.25} = 221.2795 \Rightarrow t = \sqrt[1.25]{221.2795}$	M1	This mark is given for simplifying to find an expression for t
	$t = 75.2$ years	A1	This mark is given for correctly finding the time the tree would take to reach a height of 12 metres

HOME

A2 2020 Paper 1

Integration

This doesn't have to use integration, but you could set it up as a simple differential equation:

8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation.*)

(2)

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)
Notes:			

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = e^{\pm t}$, $n = Ae^{\pm t}$, $n = Ae^{\pm kt}$ condoning $n = A + Be^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = Ae^{kt}$, $n = e^{kt+c}$, $n = e^{kt}e^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = e^{kt+c}$ followed by $n = e^{kt} + e^{c}$ o.e. $n = e^{kt} + A$ such as is M1 A0

....

You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^{t}$ using the same marking criteria as above

FYI $\frac{dn}{dt} = Ak^{t} \times \ln k = \ln k \times n$ so $\frac{dn}{dt} \propto n$

....

10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

Question	Scheme	Marks	AOs
10 (a)	$x = u^2 + 1 \Rightarrow \mathrm{d}x = 2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\int \frac{3\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int \frac{3\times 2u\,\mathrm{d}u}{\left(u^2+1-1\right)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p = 2, q = 3$	B1	1.1b
	$= \int \frac{3\times 2\cancel{u}\,\mathrm{d}u}{u^{\cancel{2}}(3+2u)} = \int \frac{6\,\mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u} \Rightarrow A = ..., B = ...$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6\,\mathrm{d}u}{u(3+2u)} = 2\ln u - 2\ln(3+2u)$ $(+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u = "3", u = "2"$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3 - 2\ln 9) - (2\ln 2 - 2\ln 7) = 2\ln\frac{7}{6}$	M1	1.1b
	$\ln\frac{49}{36}$	A1	2.1
		(6)	
	(10 marks)		

14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

HOME

A2 2020 Paper 1

Integration

Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c)	B1	3.1b
	Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt} = -c$ and $\frac{dV}{dr} = 4\pi r^2$	M1	2.1
	$-c = 4\pi r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{dr}{dt} = -\frac{k}{r^2} \Rightarrow \int r^2 dr = \int -k dt$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3} = -kt(+\alpha)$	A1	1.1b
	Uses $t = 0, r = 40 \Rightarrow \alpha = ...$ $\quad \alpha = \frac{64000}{3}$	M1	1.1b
	Uses $t = 5, r = 20$ & $\alpha = ... \Rightarrow k = ...$	M1	3.4
	$r^3 = 64000 - 11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. "$64000 - 11200t$" ... $0 \Rightarrow t...$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
			(10 marks)

1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\, \mathrm{d}x$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, \mathrm{d}x$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, \mathrm{d}x = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

A2 2020 Paper 2

Integration

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	$3 \times$ their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3×1.5
	
	If (a) is incorrect allow $3 \times$ their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\|\frac{4.535-4.50}{4.535}\right\| \times 100 = 0.77\%$ or $\left\|\frac{4.535-4.51}{4.535}\right\| \times 100 = 0.55\%$ or $\left\|\frac{4.535-4.51425}{4.535}\right\| \times 100 = 0.46\%$ or $\left\|\frac{4.50}{4.535}\right\| \times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
			(5 marks)

HOME

6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} \, \mathrm{d}x$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=\ldots, B=\ldots, C=\ldots$ **or** $x+2\,\overline{)\,x^2+8x-3}$ quotient $x+6$; $\underline{x^2+2x}$; $6x-3$; $\underline{6x+12}$; -15	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2+8x-3}{x+2}\,dx = \int x+6-\frac{15}{x+2}\,dx = \ldots -15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2) \quad (+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2+8x-3}{x+2}\,dx = \left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ $= (18 + 36 - 15\ln8) - (0 + 0 - 15\ln2)$ $= 18 + 36 - (15 - 45)\ln2$ or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	$= 54 - 30\ln2$	A1	1.1b
		(4)	
			(7 marks)

HOME

A2 2020 Paper 2

Integration

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,\mathrm{d}t$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

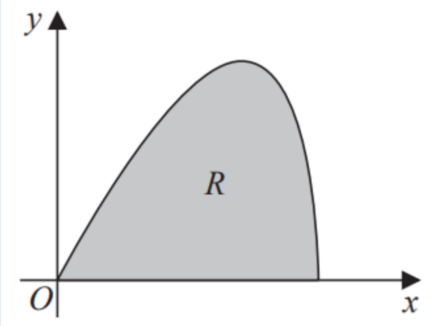

Figure 3

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

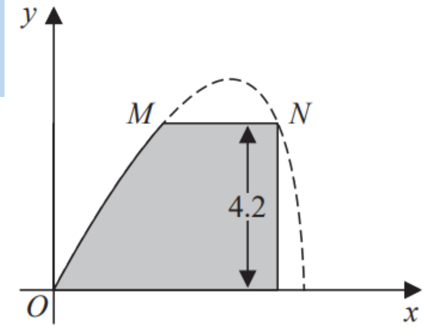

Figure 4

HOME

A2 2020 Paper 2

Integration

Question	Scheme	Marks	AOs
12(a)(i)	$y\times\frac{dx}{dt}=5\sin 2t\times 6\cos t$ or $5\times 2\sin t\cos t\times 6\cos t$	M1	1.2
	(Area =) $\int 5\sin 2t\times 6\cos t\,dt=\int 5\times 2\sin t\cos t\times 6\cos t\,dt$ or $\int 5\sin 2t\times 6\cos t\,dt=\int 60\sin t\cos^2 t\,dt$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\ dt$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 60\sin t\cos^2 t\,dt=-20\cos^3 t$	M1 A1	1.1b 1.1b
	Area = $\left[-20\cos^3 t\right]_0^{\frac{\pi}{2}}=0-(-20)=20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t=4.2\Rightarrow\sin 2t=\frac{4.2}{5}$	M1	3.4
	$t=0.4986...,1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t=0.4986...\Rightarrow x=2.869...$ $t=1.072\Rightarrow x=5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

HOME

A2 2021 Paper 1

Integration

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

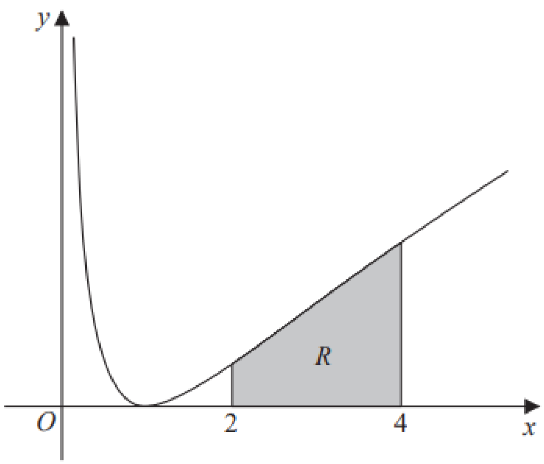

Figure 2

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.

(5)

HOME

A2 2021 Paper 1

Integration

Question	Scheme	Marks	AOs
11(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{2}\{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int (\ln x)^2 \, dx = x(\ln x)^2 - \int x \times \frac{2\ln x}{x} \, dx$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x \, dx = x(\ln x)^2 - 2\left(x\ln x - \int dx\right)$ $= x(\ln x)^2 - 2\int \ln x \, dx = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2 \, dx = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2\times 4\ln 4 + 2\times 4 - \left(2(\ln 2)^2 - 2\times 2\ln 2 + 2\times 2\right)$ $= 4(2\ln 2)^2 - 16\ln 2 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
			(8 marks)

BICEN MATHS

HOME

12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = \int_p^q \frac{2(u-1)^3}{u}\,du$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = A - B\ln 5$$

where A and B are constants to be found.

(4)

A2 2021 Paper 2

Integration

Question	Scheme	Marks	AOs
12(a)	$u=1+\sqrt{x} \Rightarrow x=(u-1)^2 \Rightarrow \dfrac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x} \Rightarrow \dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\displaystyle\int \frac{x}{1+\sqrt{x}}dx=\int \frac{(u-1)^2}{u}2(u-1)du$ or $\displaystyle\int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times 2x^{\frac{1}{2}}du=\int \frac{2x^{\frac{3}{2}}}{u}du=\int \frac{2(u-1)^3}{u}du$	M1	2.1
	$\displaystyle\int_0^{16} \frac{x}{1+\sqrt{x}}dx=\int_1^5 \frac{2(u-1)^3}{u}du$	A1	1.1b
		(3)	
(b)	$\displaystyle 2\int \frac{u^3-3u^2+3u-1}{u}du=2\int \left(u^2-3u+3-\frac{1}{u}\right)du=\ldots$	M1	3.1a
	$=(2)\left[\dfrac{u^3}{3}-\dfrac{3u^2}{2}+3u-\ln u\right]$	A1	1.1b
	$=2\left[\dfrac{5^3}{3}-\dfrac{3(5)^2}{2}+3(5)-\ln 5-\left(\dfrac{1}{3}-\dfrac{3}{2}+3-\ln 1\right)\right]$	dM1	2.1
	$=\dfrac{104}{3}-2\ln 5$	A1	1.1b
		(4)	
		(7 marks)	

HOME

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

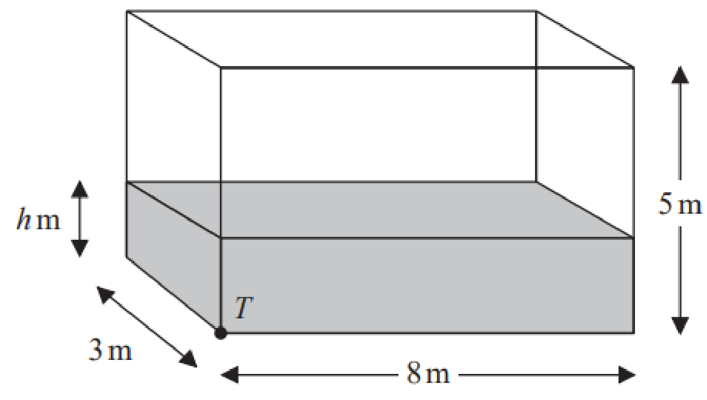

Figure 5

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of $0.48\,\text{m}^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h\,\text{m}^3$ per minute

(a) Show that, according to the model,

$$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$$

(4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + B\mathrm{e}^{-kt}$$

where A, B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

A2 2021 Paper 2

Integration

Question	Scheme	Marks	AOs
14(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24$ or $\frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\frac{dh}{dt}$	M1	2
	$1200\frac{dh}{dt} = 24 - 5h$ *	A1*	1
		(4)	
(b)	$1200\frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24-5h}\, dh = \int dt$ $\Rightarrow$ e.g. $\alpha \ln(24-5h) = t(+c)$ oe or $1200\frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24-5h}$ $\Rightarrow$ e.g. $t(+c) = \alpha \ln(24-5h)$ oe	M1	3
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = \ldots(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24-5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24-5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = \ldots$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	

(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8, \frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \to h = 4.8$ • As $e^{-\frac{t}{240}} > 0,\ h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a
		(2)	

HOME

4. (a) Express $\lim\limits_{\delta x \to 0} \sum\limits_{x=2.1}^{6.3} \frac{2}{x}\delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \ln k$$

where k is a constant to be found.

(2)

A2 2022 Paper 1

Integration

4 (a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\,dx$	B1
		(1)
(b)	$= [2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1$	M1
	$= \ln 9$ CSO	A1
		(2)

HOME

12. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

12	$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{1}{x} \, dx$	M1
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1
	$\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1
	$= \frac{7}{16}e^8 + \frac{1}{16}$	A1
		(5)

16.

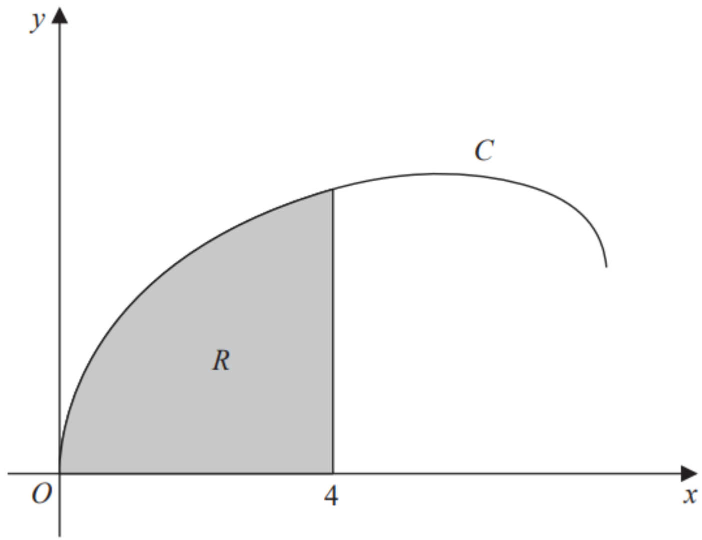

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t\cos t\right)\mathrm{d}t$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

A2 2022 Paper 1

Integration

16 (a)	Attempts $y.\frac{dx}{dt} = (2\sin 2t + 3\sin t)\times 16\sin t\cos t$ and uses $\sin 2t = 2\sin t\cos t$	M1
	Correct expanded integrand. **Usually** for one of $(R) = \int \underline{\underline{48\sin^2 t\cos t + 16\sin^2 2t}}\,dt$ $(R) = \int \underline{\underline{48\sin^2 t\cos t + 64\sin^2 t\cos^2 t}}\,dt$ $(R) = \int \underline{\underline{24\sin 2t\sin t + 16\sin^2 2t}}\,dt$	A1
	Attempts to use $\cos 4t = 1 - 2\sin^2 2t = \left(1 - 8\sin^2 t\cos^2 t\right)$	M1
	$R = \int_0^a 8 - 8\cos 4t + 48\sin^2 t\cos t\,dt$ *	A1*
	Deduces $a = \frac{\pi}{4}$	B1
		(5)
(b)	$\int 8 - 8\cos 4t + 48\sin^2 t\cos t\,dt = 8t - 2\sin 4t + 16\sin^3 t$	M1 A1
	$\left[8t - 2\sin 4t + 16\sin^3 t\right]_0^{\frac{\pi}{4}} = 2\pi + 4\sqrt{2}$	M1 A1
		(4)

HOME

14. (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

(3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,\text{m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,\text{m}^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

(5)

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m^3

(2)

HOME

A2 2022 Paper 2

14(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = \ldots,\ B = \ldots$	M1
	Either $A = 2$ or $B = -1$	A1
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1
		(3)
(b)	$\int \frac{1}{V}\,\mathrm{d}V = \int \frac{3}{(2t-1)(t+1)}\,\mathrm{d}t$	B1
	$\int \frac{2}{2t-1} - \frac{1}{t+1}\,\mathrm{d}t = \ldots\ln(2t-1) - \ldots\ln(t+1)(+c)$	M1
	$\ln V = \ln(2t-1) - \ln(t+1)(+c)$	A1ft
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*
		(5)
	(b) Alternative separation of variables:	
	$\int \frac{1}{3V}\,\mathrm{d}V = \int \frac{1}{(2t-1)(t+1)}\,\mathrm{d}t$	B1
	$\frac{1}{3}\int \frac{2}{2t-1} - \frac{1}{t+1}\,\mathrm{d}t = \ldots\ln(2t-1) - \ldots\ln(t+1)(+c)$	M1
	$\frac{1}{3}\ln 3V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1)(+c)$	A1ft
	Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3}\ln 3\right)$	M1
	$\frac{1}{3}\ln V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1) + \frac{1}{3}\ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*
		(5)
(c)	(i) 30 (minutes)	B1
	(ii) 6 (m^3)	B1
		(2)

Vectors

7.

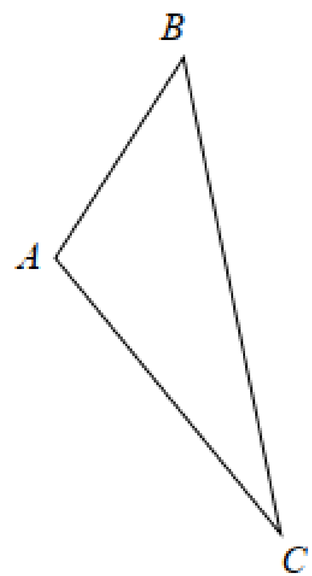

Figure 2

Figure 2 shows a sketch of a triangle ABC.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

Question	Scheme	Marks	AOs
7	Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k}=3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert=\sqrt{14},\ \lvert AC\rvert=\sqrt{61},\ \lvert BC\rvert=\sqrt{91}$	A1ft	1.1b
	$\cos BAC=\dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC=105.9°$ *	A1*	1.1b
		(5)	
		(5 marks)	

2. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

A2 2018 Paper 2

Vectors

Question	Scheme	Marks	AOs
2	$\overrightarrow{OA}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k},\ \overrightarrow{OB}=4\mathbf{i}-2\mathbf{j}+3\mathbf{k},\ \overrightarrow{OC}=a\mathbf{i}+5\mathbf{j}-2\mathbf{k},\ a<0$ $\overrightarrow{AB}=\overrightarrow{BD},\ \lvert\overrightarrow{AB}\rvert=4$		
(a)	E.g. $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}$ **or** $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}=\overrightarrow{OB}+\overrightarrow{OB}-\overrightarrow{OA}=2\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AB}$		
	$=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\ \left\{=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$ **or** $=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\right)\ \left\{=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$	M1	3.1a
	$=\begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert=4\Rightarrow\right\}(a-2)^2+(5-3)^2+(-2--4)^2=(4)^2$ $\Rightarrow(a-2)^2=8\Rightarrow a=\ldots$ **or** $\Rightarrow a^2-4a-4=0\Rightarrow a=\ldots$	dM1	2.1
	(as $a<0\Rightarrow$) $a=2-2\sqrt{2}$ (or $a=2-\sqrt{8}$)	A1	1.1b
		(3)	
			(5 marks)

HOME

3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as OC is parallel to AB, so $OABC$ is a trapezium.	A1	2.4
		(2)	
			(4 marks)

6.

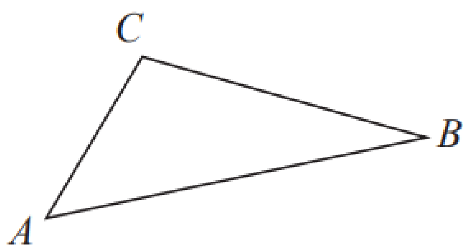

Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3)

6(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=\ldots$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)=\text{"}2^2+3^2+1^2\text{"},\left(AB^2\right)=3^2+4^2+5^2,\ \left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\dfrac{50+18-14}{2\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$ *	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\cdot(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow \cos ABC=\dfrac{27}{\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$ *	A1*	2.1
			(5 marks)

9.

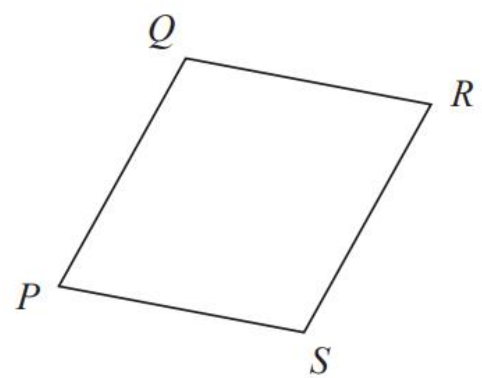

Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram *PQRS* is a rhombus.

(2)

(b) Find the exact area of the rhombus *PQRS*.

(4)

9(a)	Attempts both $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2+3^2+(-4)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{5^2+(-2)^2}$	M1
	States that $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{29}$ so $PQRS$ is a rhombus	A1
		(2)
(b)	Attempts BOTH $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ AND $\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	M1
	Correct $\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	A1
	Correct method for area $PQRS$. E.g. $\frac{1}{2}\times\left\|\overrightarrow{PR}\right\|\times\left\|\overrightarrow{QS}\right\|$	dM1
	$=\sqrt{517}$	A1
		(4)

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13. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

(3)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $\left|\overrightarrow{OD}\right|$, writing your answer as a fully simplified surd.

(3)

13(a)	Attempts two of the relevant vectors $\pm\overrightarrow{AB} = \pm(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$ $\pm\overrightarrow{AC} = \pm(-20\mathbf{i}+(p+3)\mathbf{j}+5\mathbf{k})$ $\pm\overrightarrow{BC} = \pm(-16\mathbf{i}+(p-4)\mathbf{j}+4\mathbf{k})$	M1
	Uses two of the three vectors in such a way as to find the value of p. E.g. $p+3=5\times 7$	dM1
	$p=32$	A1
		(3)
	(a) Alternative:	
	$r_{AB} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$	M1
	$4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow \lambda=5$ $4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow p=35-3$	dM1
	$p=32$	A1
(b)	Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$	M1
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$ $\overrightarrow{OA}=4\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ $4\lambda-32=-12\Rightarrow\lambda=\ldots$ OR $6\lambda-10=20\Rightarrow\lambda=\ldots$	dM1
	$\left\lvert\overrightarrow{OD}\right\rvert = 5\times\sqrt{4^2+6^2}=10\sqrt{13}$	A1
		(3)

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