9. An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish F swims from a point A to a point B.

The octopus is modelled as a fixed particle at the origin O.

Fish F is modelled as a particle moving in a straight line from A to B.

Relative to O, the coordinates of A are $(-3, 1, -7)$ and the coordinates of B are $(9, 4, 11)$, where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish F.

shortest dist between point $(0,0,0)$, line AB.

(7)

(b) Criticise the model in relation to fish F.

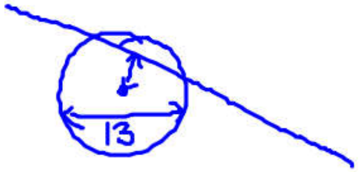
(1)

(c) Criticise the model in relation to the octopus.

(1)

Question	Scheme	Marks	AOs
9(a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{ \overrightarrow{OF} \cdot \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$ $\Rightarrow -36+144\lambda+3+9\lambda-126+324\lambda=0 \Rightarrow 477\lambda-159=0$	dM1	1.1b
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{ \overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2+(2)^2+(-1)^2}$	dM1	3.1a
	$= \sqrt{6}$ **or** $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

9. A small comet C is passing near to a planet. The planet can be modelled as a sphere with centre O taken as a fixed point in space, so that the motion of the comet is relative to the origin O.



The diameter of the planet is 13 000 km.

The comet is monitored by satellites orbiting the planet.

When the monitoring begins the comet is at position $146\mathbf{i} + 234\mathbf{j} - 85\mathbf{k}$ and is moving with vector $-21\mathbf{i} - 33\mathbf{j} + 13\mathbf{k}$ every hour, where the units are in thousands of kilometres.

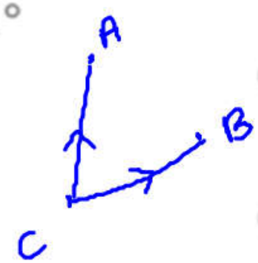
Assuming the comet maintains a straight line course throughout its motion,

(a) determine whether or not the comet will collide with the planet.

Shortest dist between point (0,0,0) and line of comet **(6)**

Two of the satellites, A and B, have position vectors $\overrightarrow{OA} = 5\mathbf{i} + 12\mathbf{k}$ and $\overrightarrow{OB} = 4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}$ at the beginning of monitoring. They return to these positions every 4 hours.

(b) Find the expected angle ACB between the comet and the satellites A and B when they first return to their initial positions. Give your answer to the nearest 0.1°



$t=4$ **(4)**

(c) Give a reason why the answer to (b) may differ from the true value. **(1)**

Question	Scheme	Marks	AOs
9(a)	Forms a correct strategy to find the minimum distance between the comet and O. Using $\overrightarrow{OC} = 146\mathbf{i} + 234\mathbf{j} - 85\mathbf{k} + \lambda(-21\mathbf{i} - 33\mathbf{j} + 13\mathbf{k})$ **Way 1** Attempts dot product of $\overrightarrow{OC}$ and the direction $\mathbf{d}$ to form an equation in λ, then uses λ to find min distance or its square. **Way 2** Attempts distance formula for $\overrightarrow{OC}$ in terms of λ, then completes the square in λ to find min distance or its square. **Way 3** Attempts dot product to find angle, θ between the line and $\overrightarrow{OX} = 146\mathbf{i} + 234\mathbf{j} - 85\mathbf{k}$ and use a trigonometric approach to find the minimum distance within a right angle triangle.	M1	
	Way 1 $\overrightarrow{OC} \cdot \mathbf{d} = 0 \Rightarrow \begin{pmatrix} 146-21\lambda \\ 234-33\lambda \\ -85+13\lambda \end{pmatrix} \cdot \begin{pmatrix} -21 \\ -33 \\ 13 \end{pmatrix} = 0 \Rightarrow -3066 + 441\lambda -$ $7722 + 1089\lambda - 1105 + 169\lambda = 0 \Rightarrow 1699\lambda = 11893 \Rightarrow \lambda = \ldots$ **Way 2** Min distance, d, given by $d^2 = (146-21\lambda)^2 + (234-33\lambda)^2 + (-85+113\lambda)^2 = \ldots$ **Way 3** $\cos\theta = \left(\overrightarrow{OX} \cdot \mathbf{d}\right) / \left(\left\|\overrightarrow{OX}\right\| \mathbf{d} \right\|\right)$ $= \dfrac{\pm(146\times(-21) + 234\times(-33) + (-85)\times 13)}{\sqrt{146^2 + 234^2 + (-85)^2}\sqrt{(-21)^2 + (-33)^2 + 13^2}} = -0.9997\ldots$	M1	
	Way 1 $\lambda = 7$ **Way 2** So $d^2 = 1699\lambda^2 - 23786\lambda + 83297$ **Way 3** $\theta = 178.653\ldots°$ or $1.34653\ldots°$ (oe) or $\sin\theta = 0.023499\ldots$	A1	1.1b
	Way 1 So distance is $d = \sqrt{(146-7\times 21)^2 + (234-7\times 33)^2 + (-85+7\times 13)^2} = \ldots\left(=\sqrt{46} = 6.782\ldots\right)$ **Way 2** $= 1699\left[(\lambda-7)^2 - 49\right] + 83297 = 1699(\lambda-7)^2 + 46$, so $d_{\min} = \sqrt{\text{their } 46}$ or $d_{\min}^2 = \text{their } 46$ **Way 3** So $d_{\min} = \sqrt{146^2 + 234^2 + (-85)^2}\sin\theta = \ldots(= 6.782\ldots)$	M1	1.1b
	Interprets situation correctly and compares their minimum distance with the radius of the planet with correct units, e.g. 6500km compared with 6782km or 6.5^2 with 46.	M1	3.1b
	The closest distance of the comet to the planet is more than a radius away from the centre, so comet (just) misses planet.	A1	3.2a
		(6)	
(b)	$C_{\lambda=4} = \begin{pmatrix} 62 \\ 102 \\ -33 \end{pmatrix}$, so need $\mathbf{d}_1 = \begin{pmatrix} 5 \\ 0 \\ 12 \end{pmatrix} - \begin{pmatrix} 62 \\ 102 \\ -33 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} 4 \\ 12 \\ -3 \end{pmatrix} - \begin{pmatrix} 62 \\ 102 \\ -33 \end{pmatrix}$	M1	3.1a
	$\mathbf{d}_1 = \begin{pmatrix} -57 \\ -102 \\ 45 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -58 \\ -90 \\ 30 \end{pmatrix}$	A1	1.1b
	$\cos\angle ACB = \dfrac{(-57)(-58) + (-102)(-90) + (45)(30)}{\sqrt{(-57)^2 + (-102)^2 + 45^2}\sqrt{(-58)^2 + (-90)^2 + 30^2}}$ $\left(= \dfrac{13836}{\sqrt{15678}\sqrt{12364}} = 0.9937\ldots\right)$	M1	1.1b
	$\angle ACB = 6.4°$ (awrt) $(6.399\ldots°)$	A1	1.1b
		(4)	
(c)	The comet may not follow a straight line course, (as e.g. gravity when nearing the planet will affect it).	B1	3.2b
		(1)	
		(11 marks)	

4. Part of the mains water system for a housing estate consists of water pipes buried beneath the ground surface. The water pipes are modelled as straight line segments. One water pipe, W, is buried beneath a particular road. With respect to a fixed origin O, the road surface is modelled as a plane with equation $3x - 5y - 18z = 7$, and W passes through the points $A(-1, -1, -3)$ and $B(1, 2, -3)$. The units are in metres.

(a) Use the model to calculate the acute angle between W and the road surface.

angle between line AB and a plane $\sin\theta = \left|\frac{\underline{n}\cdot\underline{d}}{|\underline{n}||\underline{d}|}\right|$ **(5)**

A point $C(-1, -2, 0)$ lies on the road. A section of water pipe needs to be connected to W from C.

(b) Using the model, find, to the nearest cm, the shortest length of pipe needed to connect C to W.

C → point, W → line

shortest distance between point C and a line AB **(6)**

Question	Scheme	Marks	AOs
4(a)	Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle.	M1	3.1a
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$	M1 A1	1.1b 1.1b
	$\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha=\text{"}-9\text{"}$ $\theta=90-\arccos\left(\dfrac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\dfrac{9}{\sqrt{13}\sqrt{358}}\right)$ Angle between pipe and road $=7.58°$ (3sf) or 0.132 radians (3sf) (Allow $-7.58°$ or -0.132 radians)	M1 A1	1.1b 3.2a
		(5)	
(b)	$W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$C\text{ to }W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$	M1	3.4
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=\ldots$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=\ldots$ or $(2t)^2+(3t+1)^2+(-3)^2=\ldots$ or $(2+2t)^2+(4+3t)^2+(-3)^2=\ldots$	M1	3.1b
	$t=-\dfrac{3}{13}$ or $\lambda=-\dfrac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\dfrac{6}{13}\mathbf{i}+\dfrac{4}{13}\mathbf{j}-3\mathbf{k}$ or $(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\dfrac{3}{13}\right)^2+\dfrac{121}{13}$ or $(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\dfrac{16}{13}\right)^2+\dfrac{121}{13}$ or $\dfrac{\mathrm{d}\left((2t)^2+(3t+1)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\dfrac{3}{13}\Rightarrow C\text{ to }W\text{ is }-\dfrac{6}{13}\mathbf{i}+\dfrac{4}{13}\mathbf{j}-3\mathbf{k}$ Or $\dfrac{\mathrm{d}\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\dfrac{16}{13}\Rightarrow(C\text{ to }W)_{\min}\text{ is }-\dfrac{6}{13}\mathbf{i}+\dfrac{4}{13}\mathbf{j}-3\mathbf{k}$	A1	1.1b
	$d=\sqrt{\left(-\dfrac{6}{13}\right)^2+\left(\dfrac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\dfrac{121}{13}}$	ddM1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
			(11 marks)

Figure 2 shows a sketch of a shelter against a wall. The shelter consists of two rectangular wooden boards, $OABC$ and $BCDG$, which can be modelled as parts of planes. Board $OABC$ is vertical and parallel to the wall and the ground may be assumed to be horizontal.
The points E and F are at the foot of the wall directly below D and G respectively.

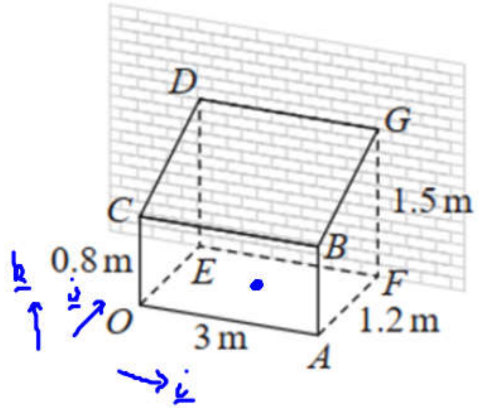


Figure 2

The length OC is 0.8 m, the length OA is 3 m and the board $OABC$ is 1.2 m away from the wall. The points D and G are 1.5 m above the ground.

To model the shelter, take O as the origin, the vector $\mathbf{i}$ to be 1 m in the direction of $\overrightarrow{OA}$, the vector $\mathbf{j}$ to be 1 m in the direction of $\overrightarrow{OE}$ and the vector $\mathbf{k}$ to be 1 m in the direction of $\overrightarrow{OC}$.

(a) Find an equation of the plane $BCDG$, giving your answer in the form $\mathbf{r} \bullet \mathbf{n} = d$

(5)

In order to support the roof of the shelter, one end of a pole is attached to the ground at the centre of the rectangle $OAFE$ and the other end to a point on the roof. Modelling the pole as a rod,

(b) find, to the nearest mm, the shortest possible length for the pole.

shortest dist between point and plane → formula

(3)

(c) State a limitation of the assumption that the boards can be modelled as planes.

(1)

Question	Scheme	Marks	AOs
5(a)	$\overrightarrow{OC} = 0.8\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 0.8\mathbf{k}$ and $\overrightarrow{OD} = 1.2\mathbf{j} + 1.5\mathbf{k}$, or $\overrightarrow{CB} = 3\mathbf{i}$, and $\overrightarrow{CD} = 1.2\mathbf{j} + 0.7\mathbf{k}$	B1	3.3
	So plane has equation $\mathbf{r} =$ their $\overrightarrow{OC}$ + their $\lambda\overrightarrow{CB}$ + their $\mu\overrightarrow{CD}$ (oe) OR $(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}).(3\mathbf{i}) = 0$ and $(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}).(1.2\mathbf{j} + 0.7\mathbf{k}) = 0$ leading to a =..., b =... and c =... (may use vector product)	M1	1.1b
	Equation is $\mathbf{r} = 0.8\mathbf{k} + \lambda(3\mathbf{i}) + \mu(1.2\mathbf{j} + 0.7\mathbf{k})$ OR normal is $\mathbf{n} = p(7\mathbf{j} - 12\mathbf{k})$	A1	1.1b
	$x = 3\lambda$, $y = 1.2\mu$ and $z = 0.8 + 0.7\mu \Rightarrow 70y - 120z = -96$ OR $(0.8\mathbf{k}).(7\mathbf{j} - 12\mathbf{k}) = -9.6 \Rightarrow d = -9.6$	M1	1.1b
	Equation is $\mathbf{r}.(7\mathbf{j} - 12\mathbf{k}) = -9.6$ (or a multiple e.g. $\mathbf{r}.(70\mathbf{j} - 120\mathbf{k}) = -96$)	A1	2.5
		(5)	
(b)	Full attempt to find the minimum distance from the centre of the base rectangle to the plane – e.g. using the distance formula for closest point, or first finding the intersection point then finding the distance. Must have correct starting point (1.5, 0.6, 0).	M1	3.1b
	E.g. Minimum distance $= \dfrac{\lvert 0\times1.5 + 7\times0.6 + (-12)\times0 + 9.6\rvert}{\sqrt{0^2 + 7^2 + (-12)^2}} = \ldots$	M1	3.4
	$= 0.993$ m or 99.3 cm or 993 mm (to 3 s.f.) Accept awrt.	A1	1.1b
		(3)	
(c)	E.g. the boards will not have negligible thickness, which should be taken into account in the model, or wooden boards will bow and so not form planes.	B1	3.5b
		(1)	

(9 marks)

2. The plane Π passes through the point A and is perpendicular to the vector $\mathbf{n}$

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

where O is the origin,

(a) find a Cartesian equation of Π.

(2)

With respect to the fixed origin O, the line l is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$$

The line l intersects the plane Π at the point X.

(b) Show that the acute angle between the plane Π and the line l is 21.2° correct to one decimal place.

(4)

(c) Find the coordinates of the point X.

(4)

Question	Scheme	Marks	AOs
2(a)	$\mathbf{r}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}5\\-3\\-4\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}$	M1	1.1b
	$3x-y+2z=10$	A1	2.5
		(2)	
(b)	$\begin{pmatrix}3\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\-5\\3\end{pmatrix}=8$	B1	1.1b
	$\sqrt{(3)^2+(-1)^2+(2)^2}\,.\sqrt{(-1)^2+(-5)^2+(3)^2}\ \cos\alpha = \text{"}-3+5+6\text{"}$	M1	1.1b
	$\theta=90^\circ-\arccos\left(\frac{8}{\sqrt{14}.\sqrt{35}}\right)$ or $\sin\theta=\frac{8}{\sqrt{14}.\sqrt{35}}$	M1	2.1
	$\theta=21.2^\circ$ (1 dp) * cso	A1*	1.1b
		(4)	
(c)	$3(7-\lambda)-(3-5\lambda)+2(-2+3\lambda)=10\Rightarrow\lambda=\ldots$	M1	3.1a
	$\lambda=-\frac{1}{2}$	A1	1.1b
	$\overrightarrow{OX}=\begin{pmatrix}7\\3\\-2\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix}=\begin{pmatrix}\ldots\\\ldots\\\ldots\end{pmatrix}$	M1	1.1b
	$X(7.5, 5.5, -3.5)$	A1ft	1.1b
		(4)	
		(10 marks)	

3. (a) Find, in terms of the real constant k, the determinant of the matrix

$$\mathbf{M} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{pmatrix}$$

(2)

Three distinct planes, Π_1, Π_2 and Π_3, are defined by the equations

$$\Pi_1 : \mathbf{r} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 4$$

$$\Pi_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\Pi_3 : x + ky + 2z = -1$$

where λ and μ are scalar parameters.

(b) Find an equation in Cartesian form for

(i) Π_1

(ii) Π_2

(4)

Given that the three planes Π_1, Π_2 and Π_3 form a sheaf,

(c) use the answer to part (a) to explain why $k = -1$

(2)

Question	Scheme	Marks	AOs
3(a)	$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{vmatrix} = 3(3\times 2 - k\times -1) - 2(2\times 2 - 1\times -1) + 1(2\times k - 1\times 3)$	M1	1.1b
	$= 5k + 5$	A1	1.1b
		(2)	
(b)	(i) $3x + 2y + z = 4$	B1	1.1b
	(ii) EITHER $y = 2 - \lambda \Rightarrow \lambda = 2 - y$ OR $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = 0 = 0 \Rightarrow \mathbf{n} = A\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$	M1	1.1b
	EITHER $x = 1 + (2 - y) + \mu \Rightarrow z = 3 - (2 - y) + 2(x - (2 - y) - 1)$ OR $d = A\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 5A$	M1	1.1b
	$\Rightarrow 2x + 3y - z = 5$	A1	1.1b
		(4)	
(c)	The planes meet when all three equations are satisfied, so we can find where they meet by solving $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$	B1	3.1a
	If the planes form a sheaf, then they must share a common line. But if $k \neq -1$ the determinant of the matrix is non-zero, so the equation has unique solution and hence the planes would meet in a single point. Therefore, we must have $k = -1$.	B1	2.3
		(4)	
			(8 marks)

8. The line l_1 has equation $\dfrac{x-2}{4} = \dfrac{y-4}{-2} = \dfrac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(7)

Question	Scheme	Marks	AOs
8	$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=\ldots$	M1	1.1b
	$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10,\ 0,\ -4)$	A1	1.1b
	$2+t-2(4-2t)-6+t=6 \Rightarrow t=\ldots$	M1	3.1a
	$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1),4+6(-2),-6+6(1))$	M1	3.1a
	$(8,\ -8,\ 0)$	A1	1.1b
	$\begin{pmatrix}10\\0\\-4\end{pmatrix}-\begin{pmatrix}8\\-8\\0\end{pmatrix}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
	$\mathbf{r}=\begin{pmatrix}10\\0\\-4\end{pmatrix}+k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r}-\begin{pmatrix}10\\0\\-4\end{pmatrix}\right)\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\mathbf{0}$	A1	2.5
		(7)	
			(7 marks)

7. The plane Π_1 has equation

$$\mathbf{r}.(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = -8$$

(a) Find the perpendicular distance from the point (8, 2, 10) to Π_1

(3)

The plane Π_2 has equation

$$\mathbf{r} = \lambda(\mathbf{i} + 3\mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $4\mathbf{i} + \mathbf{j} - 7\mathbf{k}$ is perpendicular to Π_2

(2)

(c) Find, to the nearest degree, the acute angle between Π_1 and Π_2

(3)

(d) Find a vector equation of the line of intersection of the planes Π_1 and Π_2

(4)

7(a)	$\mathbf{r} = 8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k} + k(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$ or $(8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 16 - 6 + 40$	M1	1.1b
	$(8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k} + k(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = -8 \Rightarrow k = -2$ $\Rightarrow d = \lvert 2(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) \rvert = \sqrt{116}$ or $2\sqrt{29}$ Or $d = \dfrac{(8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) + 8}{\sqrt{2^2 + 3^2 + 4^2}} = \dfrac{58}{\sqrt{29}}$	M1 A1	3.1a 1.1b
		(3)	
(b)	$(4\mathbf{i} + \mathbf{j} - 7\mathbf{k}).(\mathbf{i} + 3\mathbf{j} + \mathbf{k}) = 4 + 3 - 7 = 0$ $(4\mathbf{i} + \mathbf{j} - 7\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 8 - 1 - 7 = 0$	M1	1.1b
	As $4\mathbf{i} + \mathbf{j} - 7\mathbf{k}$ is perpendicular to both direction vectors of Π_2 then it must be perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$(4\mathbf{i} + \mathbf{j} - 7\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 8 - 3 - 28 = -23$	M1	1.1b
	$\sqrt{4^2 + 1^2 + 7^2}\sqrt{2^2 + 3^2 + 4^2}\cos\theta = -23$ $\Rightarrow \cos\theta = \dfrac{-23}{\sqrt{66}\sqrt{29}}$	M1	2.1
	$\theta = 58°$	A1	1.1b
		(3)	
(d)	$4x + y - 7z = 0$ and $2x - 3y + 4z = -8$		
	$x = 0 \to \left(0, \frac{56}{17}, \frac{8}{17}\right)$, $y = 0 \to \left(-\frac{28}{15}, 0, -\frac{16}{15}\right)$, $z = 0 \to \left(-\frac{4}{7}, \frac{16}{7}, 0\right)$ $\Rightarrow dir = 17\mathbf{i} + 30\mathbf{j} + 14\mathbf{k}$	M1 A1	3.1a 1.1b
	$\mathbf{r} = \frac{56}{17}\mathbf{j} + \frac{8}{17}\mathbf{k} + \lambda(17\mathbf{i} + 30\mathbf{j} + 14\mathbf{k})$	M1 A1	1.1b 2.5
		(4)	