# Coordinate Geometry - Straight Line Graphs

There is little new theory since GCSE, but the algebraic manipulation is harder.

**1**:: $y = mx + c$ , Gradient & Determining Equations

Find the equation of the line passing through $(2,3)$ and $(7,5)$, giving your equation in the form $ax + by + c = 0$, where $a, b, c$ are integers.

**NEW! since GCSE**

The equation $y - y_1 = m(x - x_1)$ for a line with given gradient and going through a given point.

**2**:: Parallel/Perpendicular Lines

A line is perpendicular to $3x + 8y - 11 = 0$ and passes through $(0, -8)$. Find the equation of the line.

**3**:: Lengths and Areas

The line $2x + 3y = 6$ crosses the $x$-axis and $y$-axis at the points $A$ and $B$ respectively. Determine:

(a) The length $AB$ and

(b) The area $OAB$.

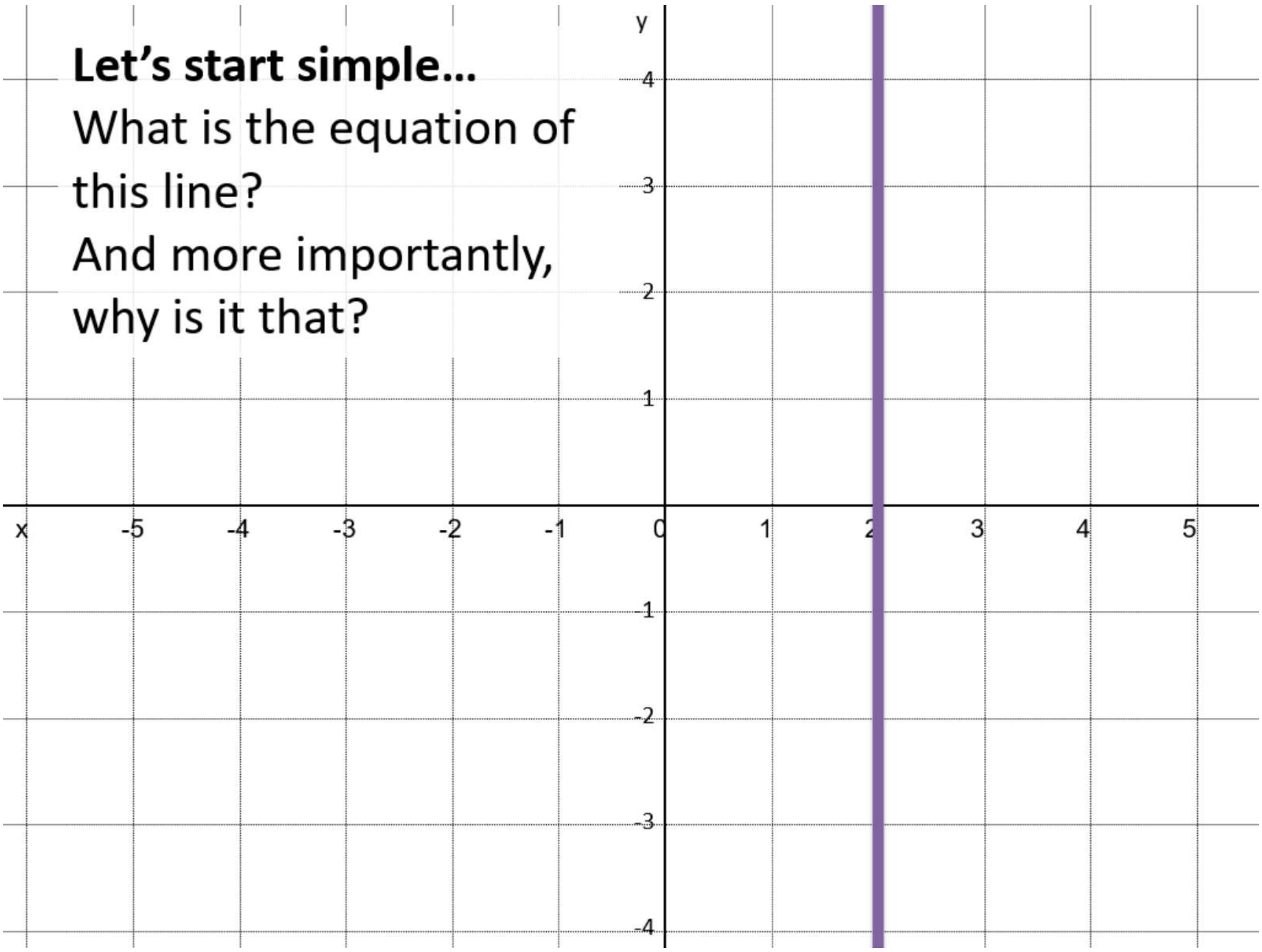
**4**:: Modelling

A plumber charges a fixed cost plus a unit cost per day. If he charges £840 for 2 days work and ...

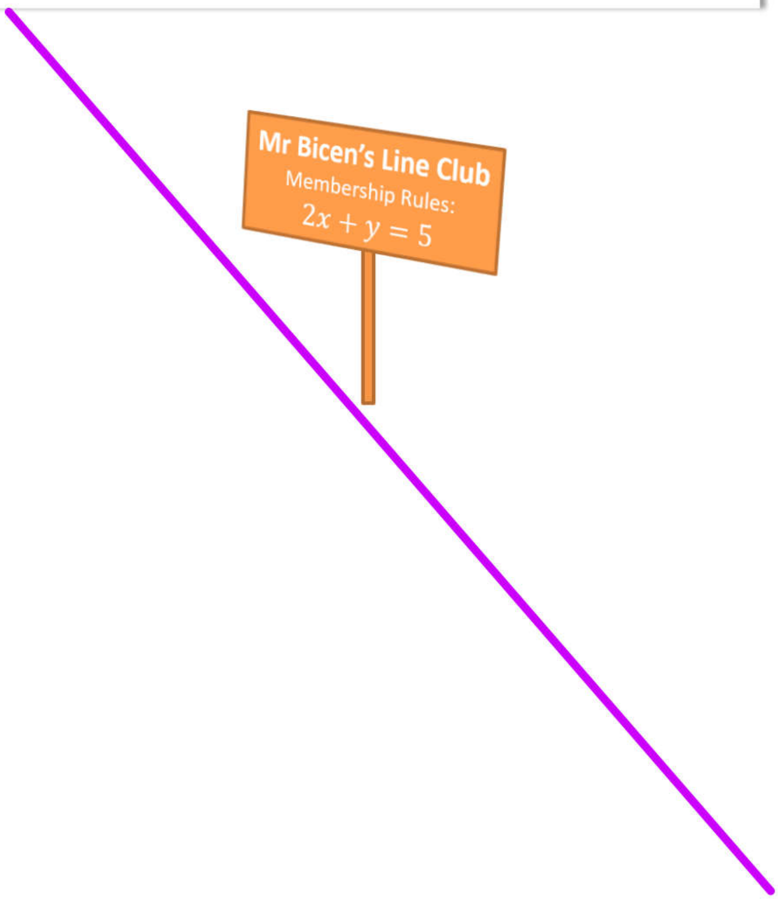
**Let's start simple...**

What is the equation of this line?

And more importantly, why is it that?



✏ A line consists of all points which satisfy some equation in terms of $x$ and/or $y$.




$(3,2)$

$(3,-1)$

$(0,5)$

This means we can **substitute** the values of a coordinate into our equation whenever we know the point lies on the line.

The point $(5, a)$ lies on the line with equation $y = 3x + 2$. Determine the value of $a$.

Find the coordinate of the point where the line $2x + y = 5$ cuts the $x$-axis.

# Your Turn

Determine where the line $x + 2y = 3$ crosses the:

a) $y$-axis:

b) $x$-axis:

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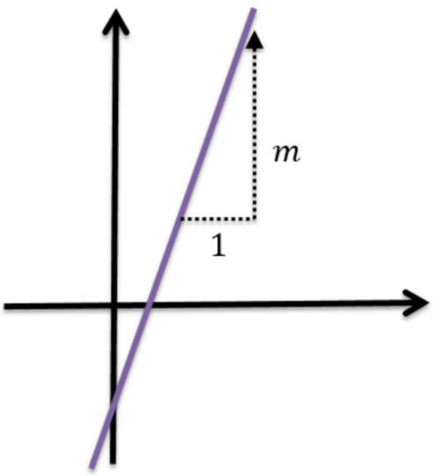
What mistakes do you think it's easy to make?

- **Mixing up $x/y$: Putting answer as $(0, 3)$ rather than $(3, 0)$.**
- **Setting $y = 0$ to find the $y$-intercept, or $x = 0$ to find the $x$-intercept.**

# Recap of Gradient

The steepness of a line is known as the **gradient**.
It tells us what $y$ changes by as $x$ increases by 1.




So if the $y$ value increased by 6 as the $x$ value increased by 2, what is $y$ increasing by for each unit increase of $x$?
How would that give us a suitable formula for the gradient $m$?

$$\boxed{m = \frac{\Delta y}{\Delta x}}$$

$\Delta$ is the (capital) Greek letter "delta" and means **"change in"**.

**Textbook Note:**
You can also use $m = \frac{y_2 - y_1}{x_2 - x_1}$ for two points $(x_1, y_1)$ and $(x_2, y_2)$ which is the same thing

Find the gradient of the line that goes through the points:

1 $(1, 4) \quad (3, 10)$

2 $(5, 7) \quad (8, 1)$

3 $(2, 2) \quad (-1, 10)$

4 Show that the points $\mathrm{A}(3,4), B(5,5), C(11,8)$ all lie on a straight line.

The line joining $(2, -5)$ to $(4, a)$ has gradient -1. Work out the value of $a$.

One form we can put a straight line equation in is:

$$y = mx + c$$

Determine the gradient and $y$-intercept of the line with equation $4x - 3y + 5 = 0$

Make $y$ the subject so we have the form
$$y = mx + c$$
Put $y$ on the side it's positive.

Divide **each** term by 3; don't write $y = \frac{4x+5}{3}$ otherwise it's not in the form $y = mx + c$

This is algebra, so use improper fractions, and not mixed numbers or recurring decimals.

At GCSE, $\boldsymbol{y = mx + c}$ was the main form you would express a straight line equation, sometimes known as the '**slope-intercept form**'.
But another common form is $\boldsymbol{ax + by + c = 0}$, where $a, b, c$ are integers.
This is known as the '**standard**' form.

Express $y = \frac{1}{3}x - \frac{2}{3}$ in the form $ax + by + c = 0$, where $a, b, c$ are integers.

**Your Turn**

Express $y = \frac{2}{5}x + \frac{3}{5}$ in the form $ax + by + c = 0$, where $a, b, c$ are integers.

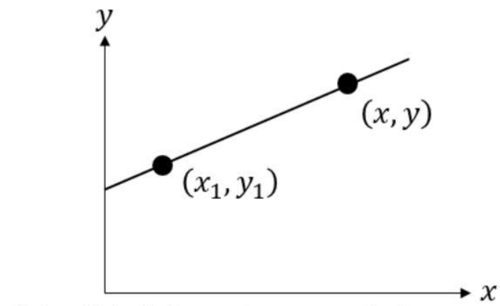
Ex 5A/5B

# Equations using two points/point + gradient

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

How would you have done this at **<u>GCSE</u>**?

# A New Way!




**Notes**: Note that $x_1$ and $y_1$ are constants while $x$ and $y$ are variables. The latter are variables because as these 'vary', we get different points on the line.

Suppose that $(x_1, y_1)$ is some fixed point on the line that we specify (e.g. $(3,5)$). Suppose that $(x, y)$ represents a generic point on the line, which is allowed to change as we consider different points on this line.
Then:

$m =$

Thus:

✏ The equation of a line that has gradient $m$ and passes through a point $(x_1, y_1)$ is:

$$y - y_1 = m(x - x_1)$$

Let's revisit:

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

In a nutshell: You can use this formula whenever you have (a) a gradient and (b) any point on the line.

| Gradient | Point | (Unsimplified) Equation |
|---|---|---|
| $3$ | $(1,2)$ | |
| $5$ | $(3,0)$ | |
| $2$ | $(-3,4)$ | |
| $\frac{1}{2}$ | $(1,-5)$ | |
| $9$ | $(-4,-4)$ | |

**Important Side Note**: I've found that many students shun this formula and just use the GCSE method. Please persist with it – it'll be much easier when fractions are involved. Further Mathematicians, don't even think about using the GCSE method, because you'll encounter massive headaches when you consider algebraic points. Trust me on this one!

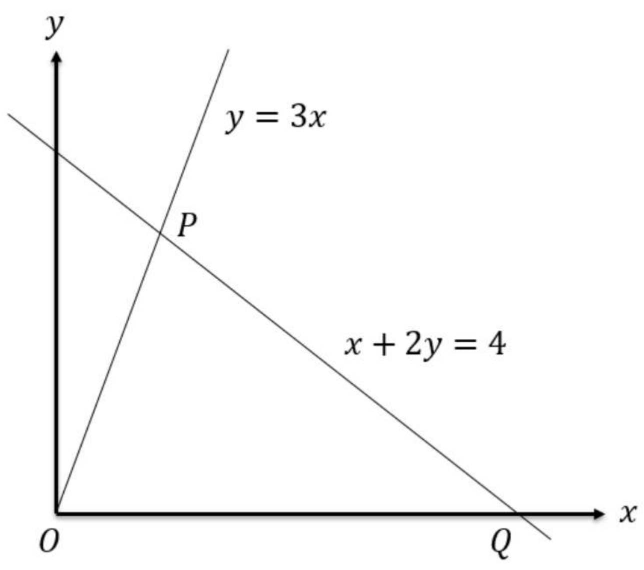
# Using 2 points

Find the equation of the line that goes through $(4,5)$ and $(6,2)$, giving your equation in the form $ax + by + c = 0$.

**Your Turn:**
Find the equation of the line that goes through $(-1,9)$ and $(4,5)$, giving your equation in the form $ax + by + c = 0$.

Ex 5C

# Intersection of 2 lines




The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point $P$.

a) Determine the coordinates of $P$.

b) The line $x + 2y = 4$ intersects the $x$-axis at the point $Q$. Determine the coordinate of $Q$.

# C1 Edexcel May 2013 Q6

The straight line $L_1$ passes through the points $(-1, 3)$ and $(11, 12)$.

(*a*) Find an equation for $L_1$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(4)**

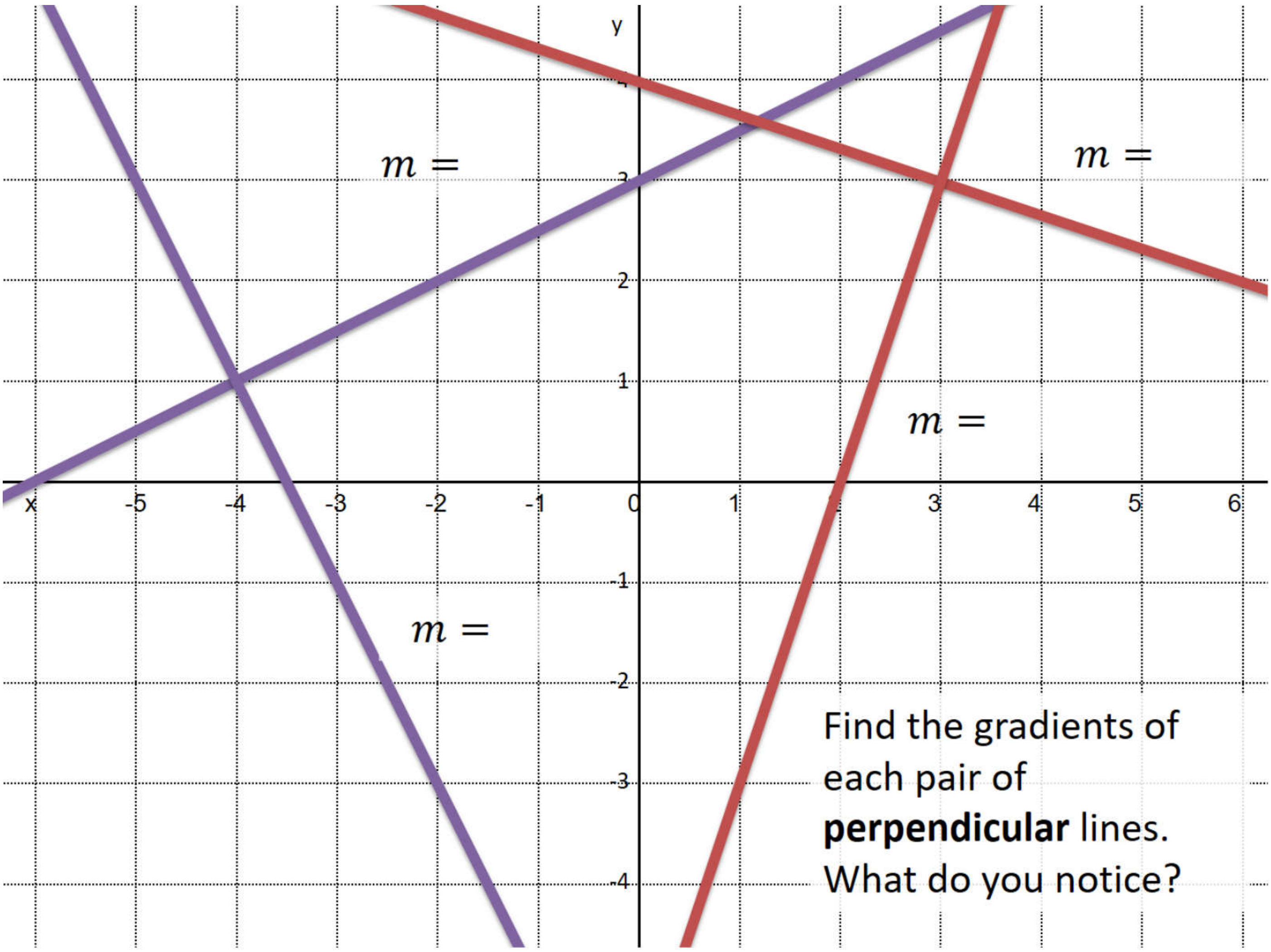
The line $L_2$ has equation $3y + 4x - 30 = 0$.

(*b*) Find the coordinates of the point of intersection of $L_1$ and $L_2$ .

**(3)**

| | | | |
|---|---|---|---|
| | $(-1, 3)$, $(11, 12)$ | | |
| (a) | $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-3}{11-(-1)} = \frac{3}{4}$ | M1: Correct method for the gradient. A1: Any correct fraction or decimal | M1 A1 |
| | $y - 3 = \frac{3}{4}(x+1)$ or $y - 12 = \frac{3}{4}(x - 11)$ or $y = \frac{3}{4}x + c$ with attempt at substitution to find $c$ | Correct straight line method using either of the given points and a numerical gradient | M1 |
| | $4y - 3x - 15 = 0$ | Or equivalent with integer coefficients (= 0 is required) | A1 |
| | This A1 should only be awarded in [illegible] | | |
| | | | (4) |
| (b) | Solves their equation from part (a) and $L_2$ simultaneously to eliminate one variable | Must reach as far as an equation in x only or in y only. (Allow slips in the algebra) | M1 |
| | $x = 3$ or $y = 6$ | One of $x = 3$ or $y = 6$ | A1 |
| | **Both $x = 3$ and $y = 6$** | Values can be un-simplified fractions. | A1 |
| | **Fully correct answers with no working can score 3/3 in (b)** | | |
| | | | (3) |

Ex 5D



# Perpendicular Lines

The gradients of parallel lines are equal.
If two lines are perpendicular, then the gradient of one is the **negative reciprocal** of the other.

$$m_1 = -\frac{1}{m_2}$$

To **show** that two lines are perpendicular:

$$m_1 m_2 = -1$$

| Gradient | Gradient of Perpendicular Line |
|---|---|
| $2$ | |
| $-3$ | |
| $\frac{1}{4}$ | |
| $5$ | |
| $-\frac{2}{7}$ | |
| $\frac{7}{5}$ | |

1 A line is goes through the point (9,10) and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

2 A line $L_1$ goes through the points $A(1,3)$ and $B(3,-1)$. A second line $L_2$ is perpendicular to $L_1$ and passes through point B. Where does $L_2$ cross the x-axis?

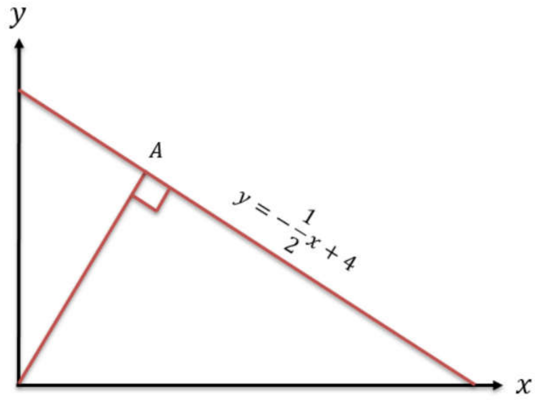
3 Are the following lines parallel, perpendicular, or neither?

$$y = \frac{1}{2}x$$
$$2x - y + 4 = 0$$

# Your Turn

1 A line goes through the point (4,7) and is perpendicular to another line with equation $y = 2x + 2$.
What is the equation of the line?
Put your answer in the form $ax + by + c = 0$, where $a, b, c$ are integers.

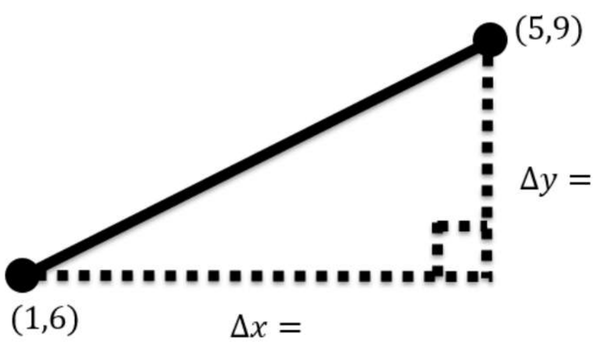
2 Determine the point $A$.
(note that A passes through origin)




Ex 5E/5F

# Distance between 2 points

Recall: Δ (said 'delta') means "change in".




How could we find the **distance** between these two points?

Distance between two points:
$$\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Examples:

**Distance between:**

$(3,4)$ and $(5,7)$

$(5,1)$ and $(6,-3)$

$(0,-2)$ and $(-1,3)$

**Note**: Unlike with gradient, we don't care if the difference is positive or negative (it's being squared to make it positive anyway!)

Your Turn:

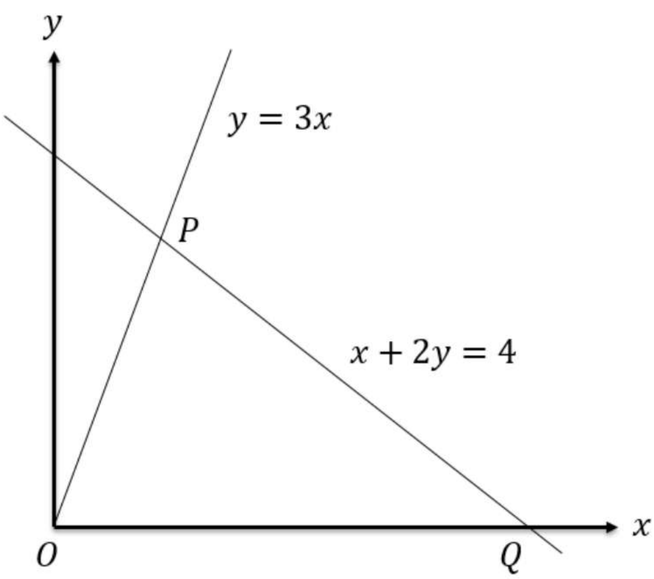
**Distance between:**

$(1,10)$ and $(4,14)$

$(3,-1)$ and $(0,1)$

$(-4,-2)$ and $(-12,4)$

# Area of Shapes

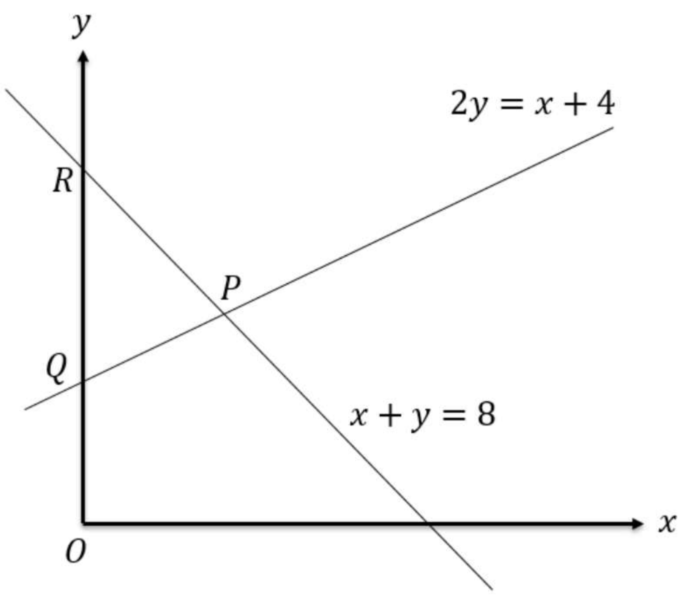



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point $P$.

a) Determine the coordinates of $P$.

*(We did this in a previous lesson)*

b) The line $x + 2y = 4$ intersects the $x$-axis at the point $Q$. Determine the area of the triangle $OPQ$.

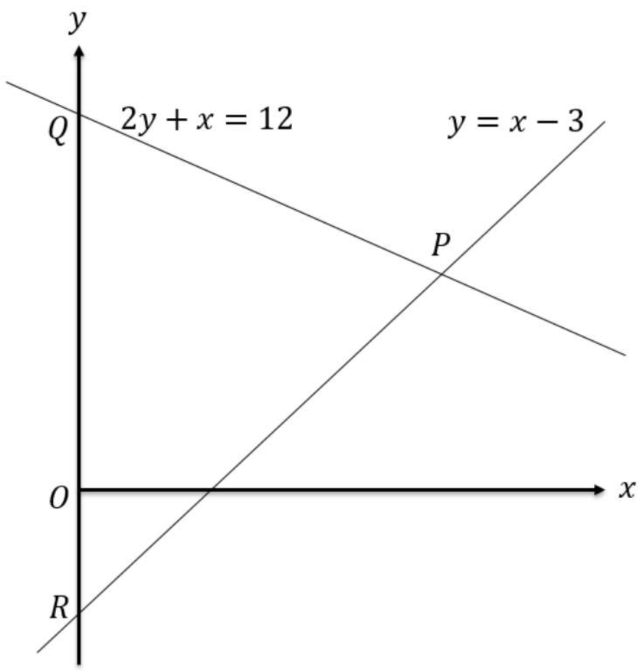



a Determine the length of $PQ$.

**Tip**: When finding areas of triangles in exam questions, one line is often vertical or horizontal. You should generally choose this to be the 'base' of your triangle.

b Determine the area $PQR$.

**Your Turn**




a) Determine the coordinate of $P$.

b) Determine the area of $PQR$.

c) Determine the length $PQ$.

**Extension Problem**

*[MAT 2001 1C]*

What is the shortest distance from the origin to the line $3x + 4y = 25$?

# Modelling with Linear Equations

We saw in Chapter 2 that lots of things in real life have a 'quadratic' relationship, e.g. vertical height with time. Lots of real life variables have a 'linear' relationship, i.e. **there is a fixed increase/decrease in one variable each time the other variable goes up by 1 unit.**

Examples

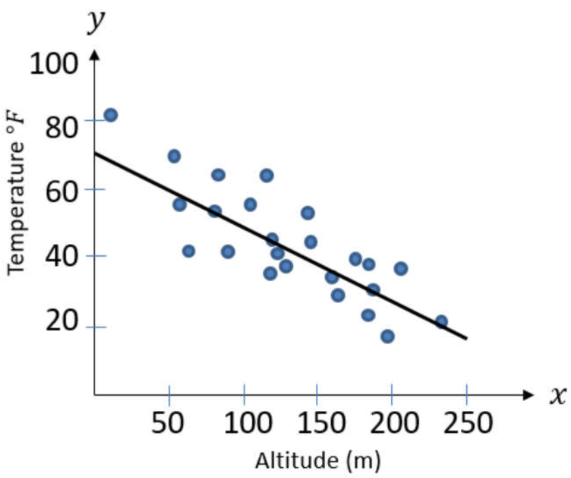
Car sales made and take home pay.

The relationship between Celsius and Fahrenheit.


Temperature and altitude (in a particular location)

*(And a pure maths one:)*
The $n$th term of an arithmetic series.

3, 5, 8, 11, 14, …




The temperature $y$ at different points on a mountain is recorded at different altitudes $x$. Suppose we were to use a linear model $y = mx + c$.

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a Determine $m$ and $c$ (you can assume the line goes through $(0,70)$ and $(250,20)$.

b **Interpret** the meaning of $m$ and $c$ **in this context**.

c Predict at what altitude the temperature reaches $0°F$.

**3.** A tank, which contained water, started to leak from a hole in its base.

The volume of water in the tank 24 minutes after the leak started was $4 \text{ m}^3$.

The volume of water in the tank 60 minutes after the leak started was $2.8 \text{ m}^3$.

The volume of water, $V \text{ m}^3$, in the tank $t$ minutes after the leak started, can be described by a linear model between $V$ and $t$.

(*a*) Find an equation linking $V$ with $t$.

**(4)**

Use this model to find

(*b*) (i) the initial volume of water in the tank,

(ii) the time taken for the tank to empty.

**(3)**

(*c*) Suggest a reason why this linear model may not be suitable.

**(1)**

**(Total for Question 3 is 8 marks)**

# Your Turn

The height, $H$ metres, of a plant was measured $t$ years after planting.

Exactly 2 years after planting, the height of the plant was 1.43 metres.
Exactly 5 years after planting, the height of the plant was 3.23 metres.

Using a linear model,

(a) find an equation linking $H$ with $t$.

The height of the plant was approximately 55cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a)

Ex 5H Q5-9