

2.7 Graphs of Functions

2.7.1 Sketching Polynomials / 2.7.2 Reciprocal Graphs - Sketching / 2.7.3 Solving Equations Graphically / 2.7.4 Proportional Relationships

Easy (10 questions)	/38
Medium (10 questions)	/50
Hard (10 questions)	/49
Very Hard (10 questions)	/54
Total Marks	/191

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Easy Questions

- 1 Sketch the graph of $y = 6x - 12$, labelling any points where the graph intersects the coordinate axes.

(3 marks)

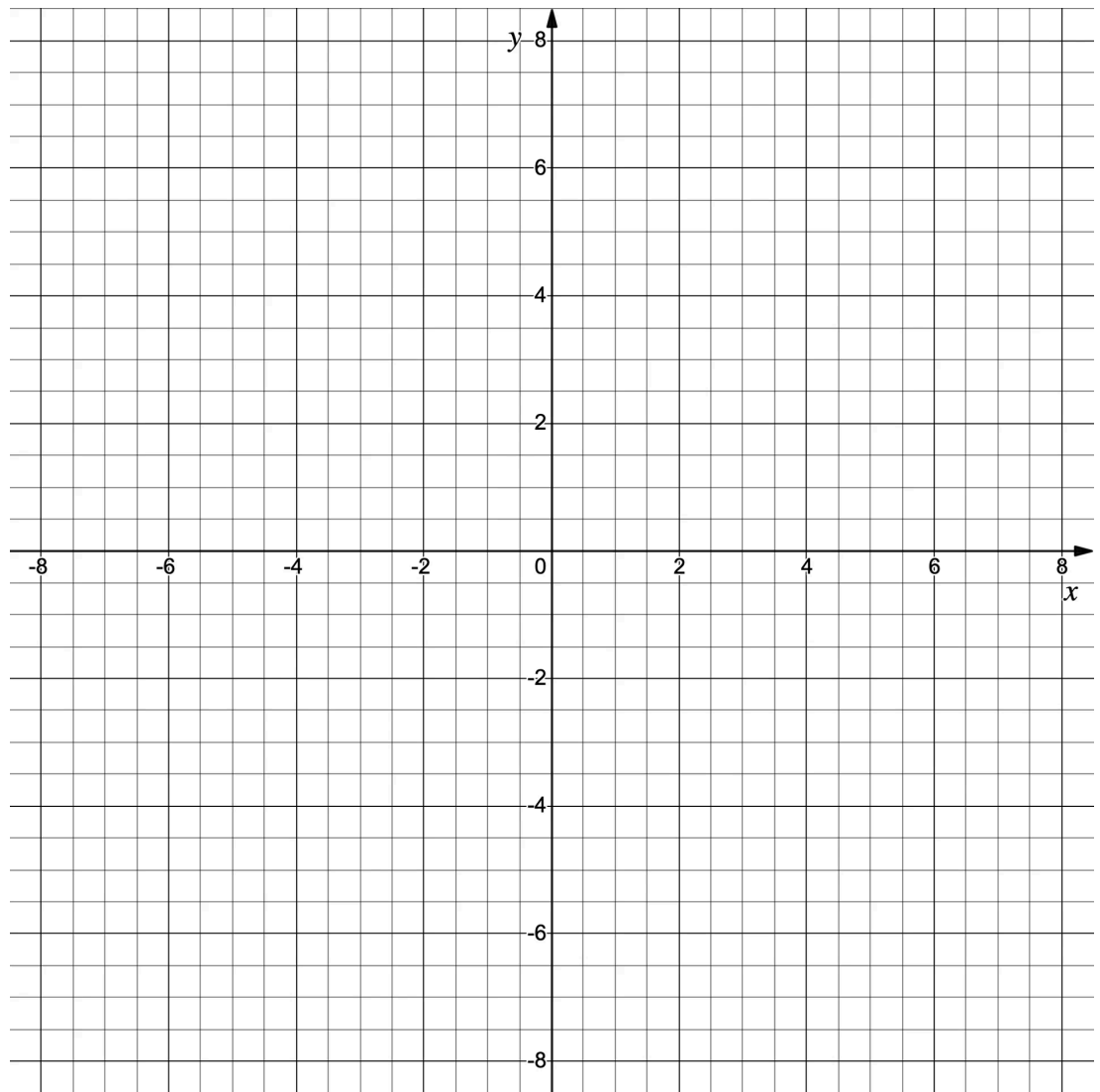
- 2 Sketch the graph of $y = x^2 - 1$, labelling any points where the graph intersects the coordinate axes.

(3 marks)

- 3 Sketch the graph of $y = \frac{1}{x}$, labelling any points where the graph intersects the coordinate axes and stating the equations of any asymptotes.

(3 marks)

- 4 (i) On the axes below, sketch the graphs of both $y = x$ and $y = -x + 2$.



(ii) Using your graph, or otherwise, find the solution to the simultaneous equations

$$y = x \text{ and } y = -x + 2.$$

(3 marks)

5 (a) (i) Write down the value of $x^2 + 3x - 4$ when $x = 0$.

(ii) Factorise $x^2 + 3x - 4$

(2 marks)

(b) Sketch the graph of $y = x^2 + 3x - 4$, labelling any points where the graph intersects the coordinate axes.

(3 marks)

- 6 (a)** Express $2x^3 + 2x^2 - 12x$ in the form $ax(x + b)(x + c)$, where a , b and c are integers to be found.

(2 marks)

- (b)** Hence sketch the graph of $y = 2x^3 + 2x^2 - 12x$ labelling any points where the graph intersects the coordinate axes.

(3 marks)

- 7** y is proportional to x .

When $x = 2$, $y = 10$.

- (i) Find the constant of proportionality
- (ii) Sketch the graph of y against x .

(3 marks)

- 8** By sketching the graphs of $y = x^3$ and $y = \frac{1}{x}$ on the same diagram show that there are two real solutions to the equation $x^3 = \frac{1}{x}$.

(3 marks)

9 (a) Use the factor theorem to show that $(x - 2)$ is a factor of the function

$$f(x) = x^3 - 2x^2 - 4x + 8.$$

(2 marks)

(b) Hence, or otherwise, express $f(x)$ as a product of three linear factors.

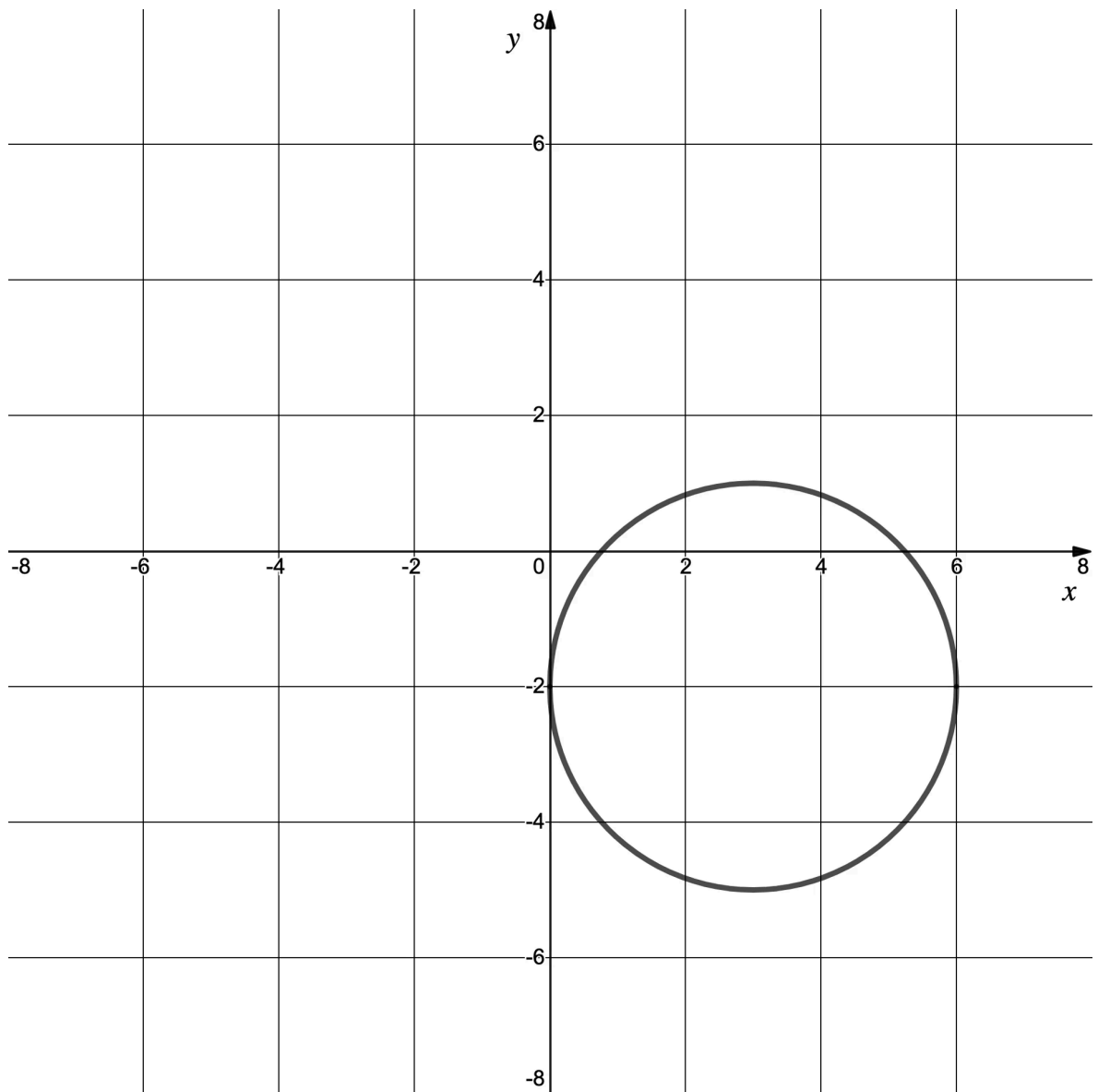
(2 marks)

(c) Sketch the graph of $y = f(x)$ labelling any points where the graph intersects the coordinate axes.

(3 marks)

10 The diagram below shows the graph of a circle with equation $(x - 3)^2 + (y + 2)^2 = 9$.

Add straight lines passing through the point to the diagram to show how the circle can have either no, one or two intercepts. All lines must pass through the point $(6, -4)$.



(3 marks)

Medium Questions

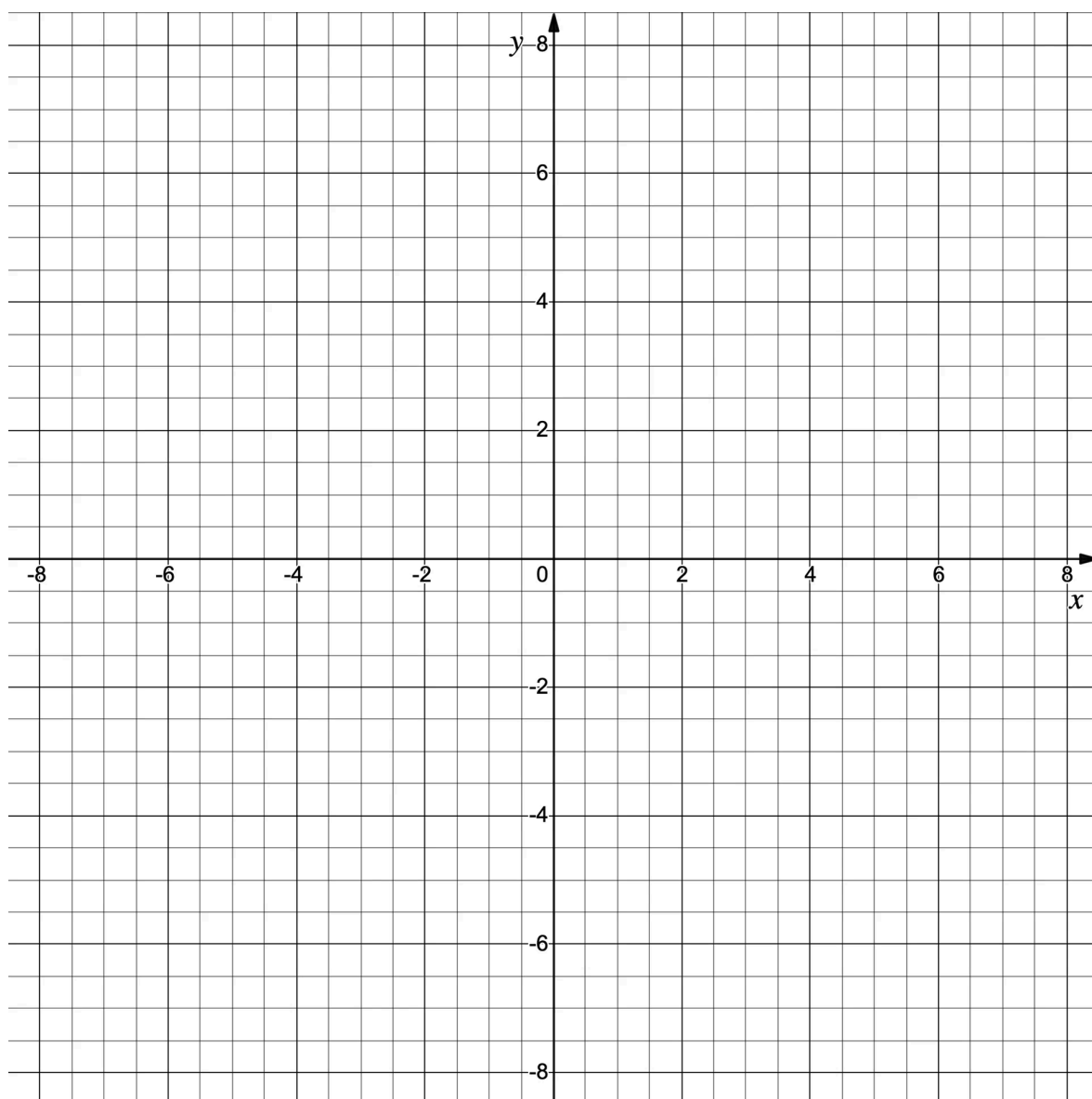
- 1 Sketch the graph of $y = (x + 3)^3$ labelling any points where the graph intersects the coordinate axes.

(3 marks)

- 2 Sketch the graph of $y = \frac{1}{x^2}$ and write down the equations of any asymptotes.

(2 marks)

3 (a) On the axes below sketch the graphs of both $y = 3x - 2$ and $y = x + 4$.



(2 marks)

(b) Using your graph, or otherwise, find the solution to the simultaneous equations

$$\begin{aligned} 3x - y &= 2 \text{ and} \\ x - y &= -4. \end{aligned}$$

(1 mark)

- 4 y is inversely proportional to x . When $x = 3$, $y = 12$. Find the constant of proportionality and sketch the graph of y against x .

(3 marks)

- 5 Sketch the graph of $y = 2x^2(x + 3)$ labelling any points where the graph intersects the coordinate axes.

(3 marks)

6 (a) On the same diagram, sketch the graphs of $y = x(x + 2)(x - 1)$ and $y = \frac{1}{x}$.

(3 marks)

(b) Use your graph to determine the number of solutions to the equation

$$x(x + 2)(x - 1) = \frac{1}{x}.$$

(1 mark)

- 7 (a)** A machine computes a calculation in time, t seconds, that is proportional to the number of processes, p , involved. For a calculation involving 10 processes the machine takes 0.01 seconds.

Show that the constant of proportionality is 0.001.

(2 marks)

- (b)** Hence write down an equation linking the number of processes (p) to the time taken (t).

(2 marks)

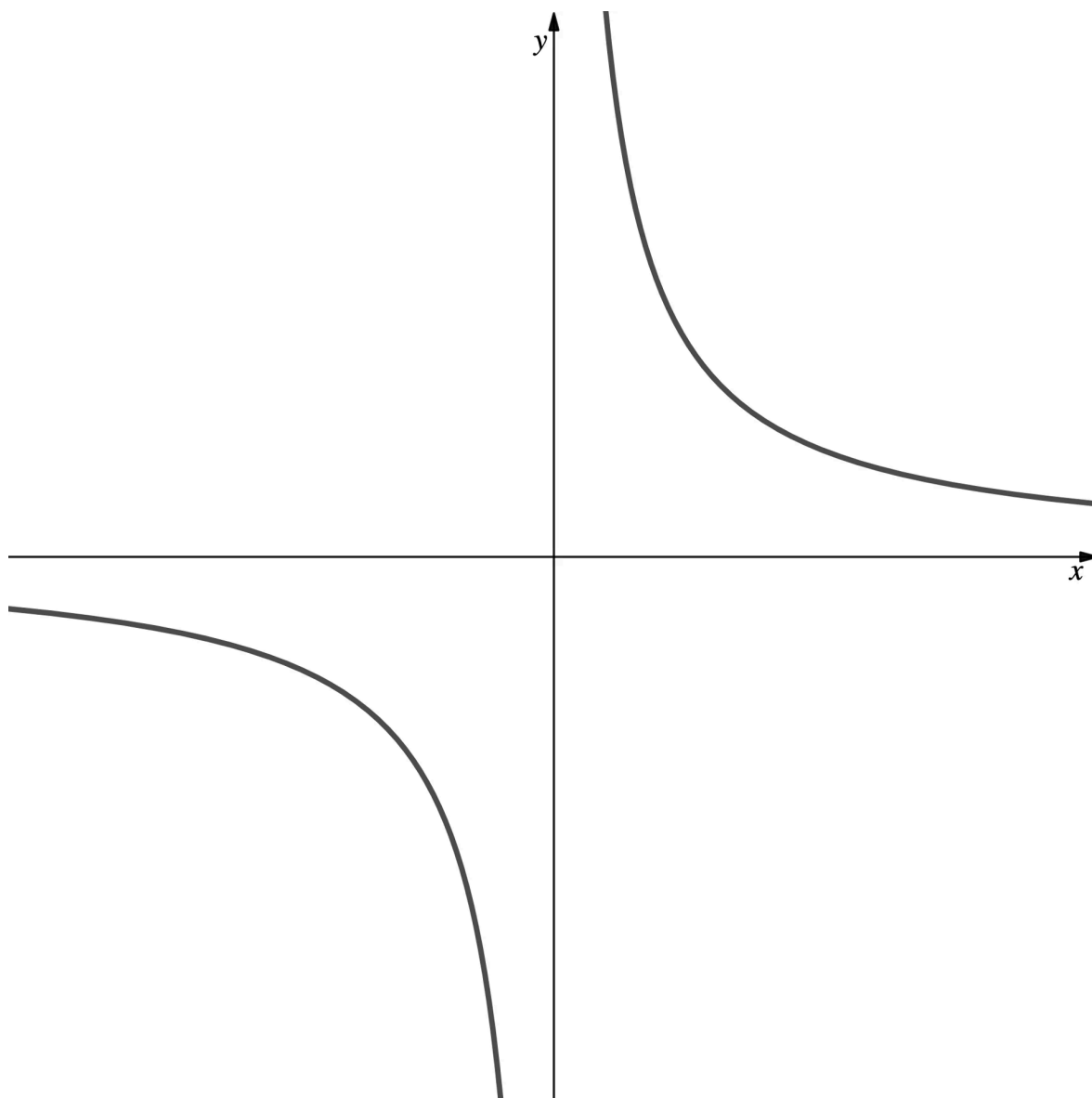
- (c)** Find the time it takes for the machine to compute a calculation involving 200 processes.

(2 marks)

- (d)** How many processes are involved for a calculation taking 2.3 seconds?

(2 marks)

- 8 (a)** The diagram below shows the graph of $y = \frac{a}{x}$, where $a > 0$.



Sketch the graph of $y = \frac{a}{x}$, where $a < 0$.

(2 marks)

- (b)** State which graph the following points must lie on and find the value of a in each case.

- (i) $(-4, -5)$
- (ii) $(-0.02, 250)$

(4 marks)

9 (a) Show that $(x + 2)$ is a factor of $2x^3 - 3x^2 - 11x + 6$.

(1 mark)

(b) Fully factorise $2x^3 - 3x^2 - 11x + 6$.

(2 marks)

(c) Sketch the graph of $y = 2x^3 - 3x^2 - 11x + 6$.

Label any points where the graph crosses the coordinate axes.

(4 marks)

10 (a) Solve the equation $x^3 - x^2 - 2x + 4 = 4x + 4$.

(3 marks)

(b) Write down the x -coordinates of the points of intersection between the graphs of $y = x^3 - x^2 - 2x + 4$ and $y = 4x + 4$.

(2 marks)

(c) Find the y -coordinates of the points of intersection between the graphs of $y = x^3 - x^2 - 2x + 4$ and $y = 4x + 4$.

(2 marks)

(d) On the same diagram, sketch the graphs of $y = x^3 - x^2 - 2x + 4$ and $y = 4x + 4$.

(4 marks)

Hard Questions

- 1 Given that $(x + 1)$ is a factor of $x^3 - 4x^2 + x + 6$, sketch the graph of $y = x^3 - 4x^2 + x + 6$. Label any points where the graph intersects the coordinate axes.

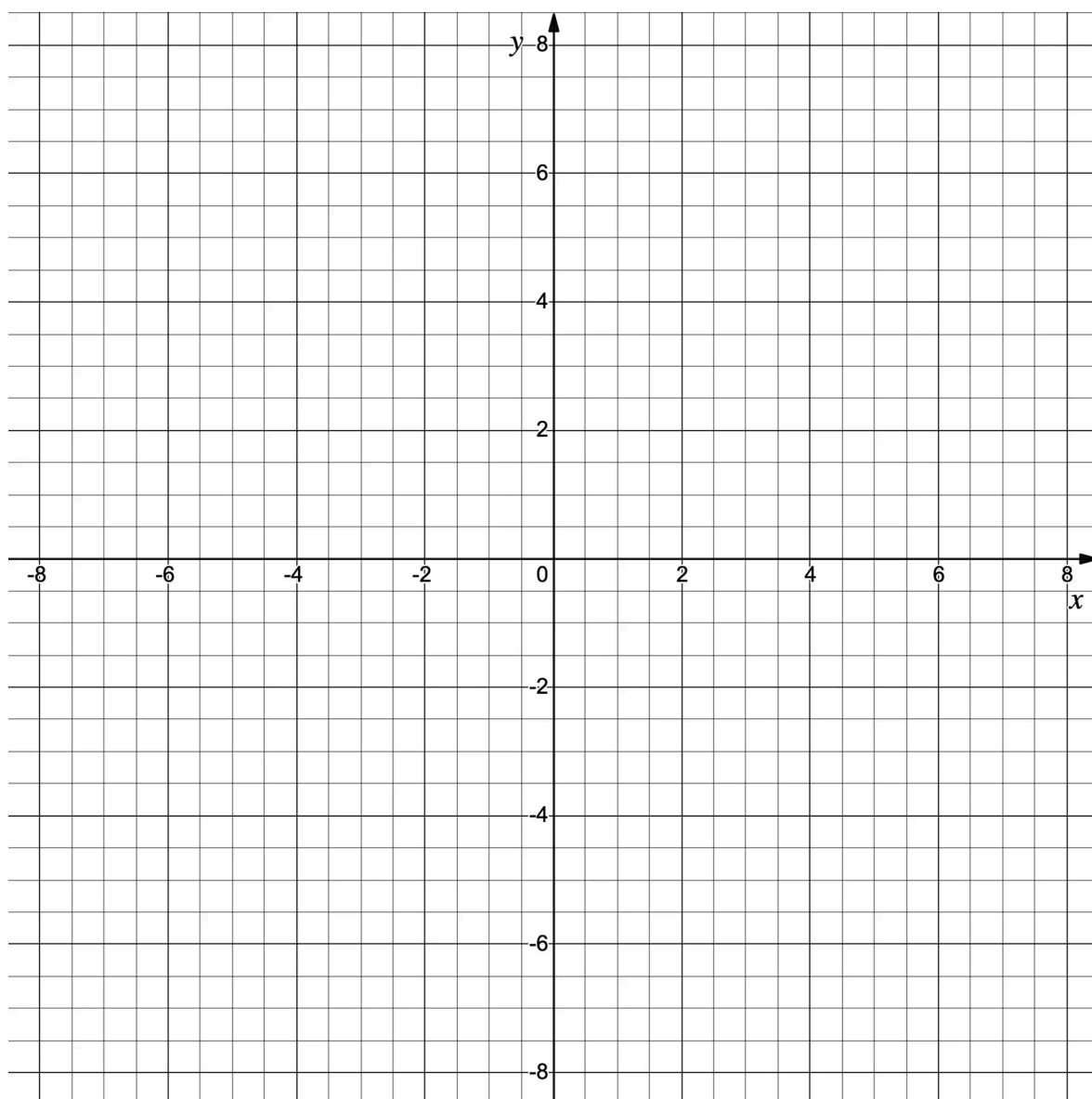
(There is no need to label any stationary points.)

(4 marks)

- 2 Sketch the graph of $y = \frac{-2}{x^2}$ and write down the equations of any asymptotes.

(2 marks)

3 (a) On the axes below sketch the graphs of both $y = x^2 + 2x - 3$ and $y = x - 1$.



(3 marks)

(b) Using your graph, or otherwise, find the solutions to the simultaneous equations

$$\begin{aligned}y &= (x + 3)(x - 1) \\x - y &= 1\end{aligned}$$

(2 marks)

- 4 y is inversely proportional to x . When $x = 2$, $y = 10$. Find the constant of proportionality and sketch the graph of y against x .

(4 marks)

- 5 Sketch the graph of $y = 3x^3 - 2x^2 - x$ labelling any points where the graph intersects the coordinate axes.

(3 marks)

6 (a) On the same diagram, sketch the graphs of $y = x^3 - 2x^2 - 8x$ and $y = \frac{1}{x}$.

(3 marks)

(b) Use your graph to determine the number of solutions to the equation

$$x^3 - 2x^2 - 8x = \frac{1}{x}.$$

(1 mark)

- 7 (a)** A machine computes a calculation in time, t seconds, that is proportional to the square of the number of processes, p , involved. For a calculation involving 8 processes the machine takes 0.032 seconds.

Find an equation linking the number of processes, (p) to the time taken, (t) seconds.

(3 marks)

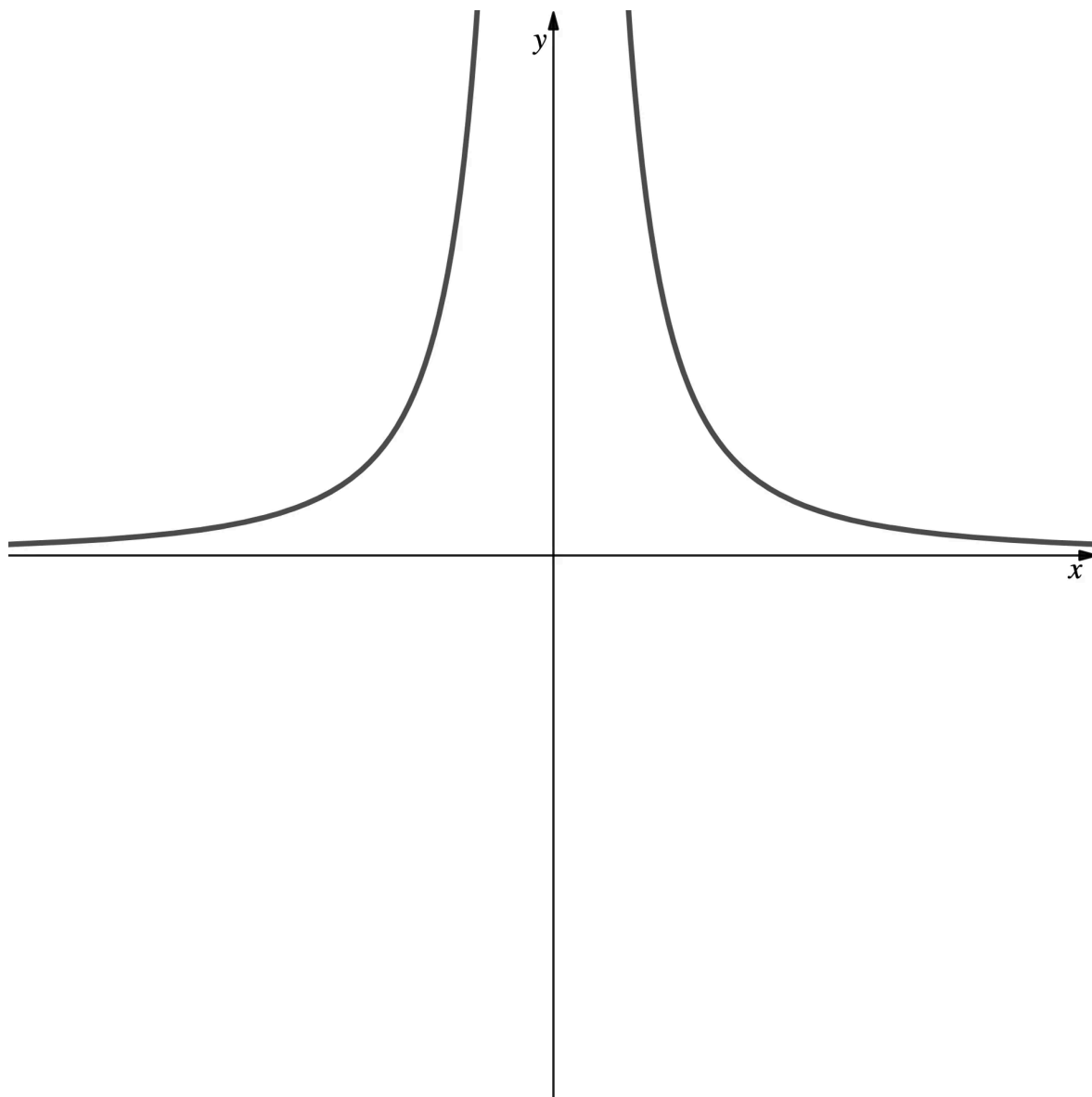
- (b)** How many processes are involved for a calculation taking 0.2 seconds?

(2 marks)

- (c)** Find the time it takes for the machine to compute a calculation involving 30 processes.

(2 marks)

- 8 (a)** The diagram below shows the graph of the equation $y = \frac{a}{x^2}$, where $a > 0$.



Sketch the graph of $y = \frac{a}{x^2}$, where $a < 0$.

(2 marks)

- (b)** Given that m is a negative real number, state with a reason, which graph passes through the point (m, m^4) .

(2 marks)

- 9 (a)** Show that $(x + 3)$ is a factor of the function $f(x) = 6x^3 + 23x^2 + 11x - 12$ and hence, or otherwise, fully factorise $f(x)$.

(3 marks)

- (b)** Sketch the graph of $y = f(x)$.
Label any points where the graph crosses the coordinate axes.

(4 marks)

- 10 (a)** Find the coordinates of the points of intersection between the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

(4 marks)

- (b)** On the same diagram, sketch the graphs of $y = x^3 - x^2 - 4x + 4$ and $y = 2x + 4$.
Label the coordinates of any points of intersection between the two graphs.
Also label any points where the graphs intersect the coordinate axes.

(5 marks)

Very Hard Questions

- 1 $(x - 2)$ and $(x + 3)$ are factors of $f(x)$, where $f(x) = x^4 - 9x^3 + 9x^2 + 85x - 150$.

Sketch the graph of $y = f(x)$ labelling any points where the graph intersects the coordinate axes. (There is no need to label any stationary points).

(5 marks)

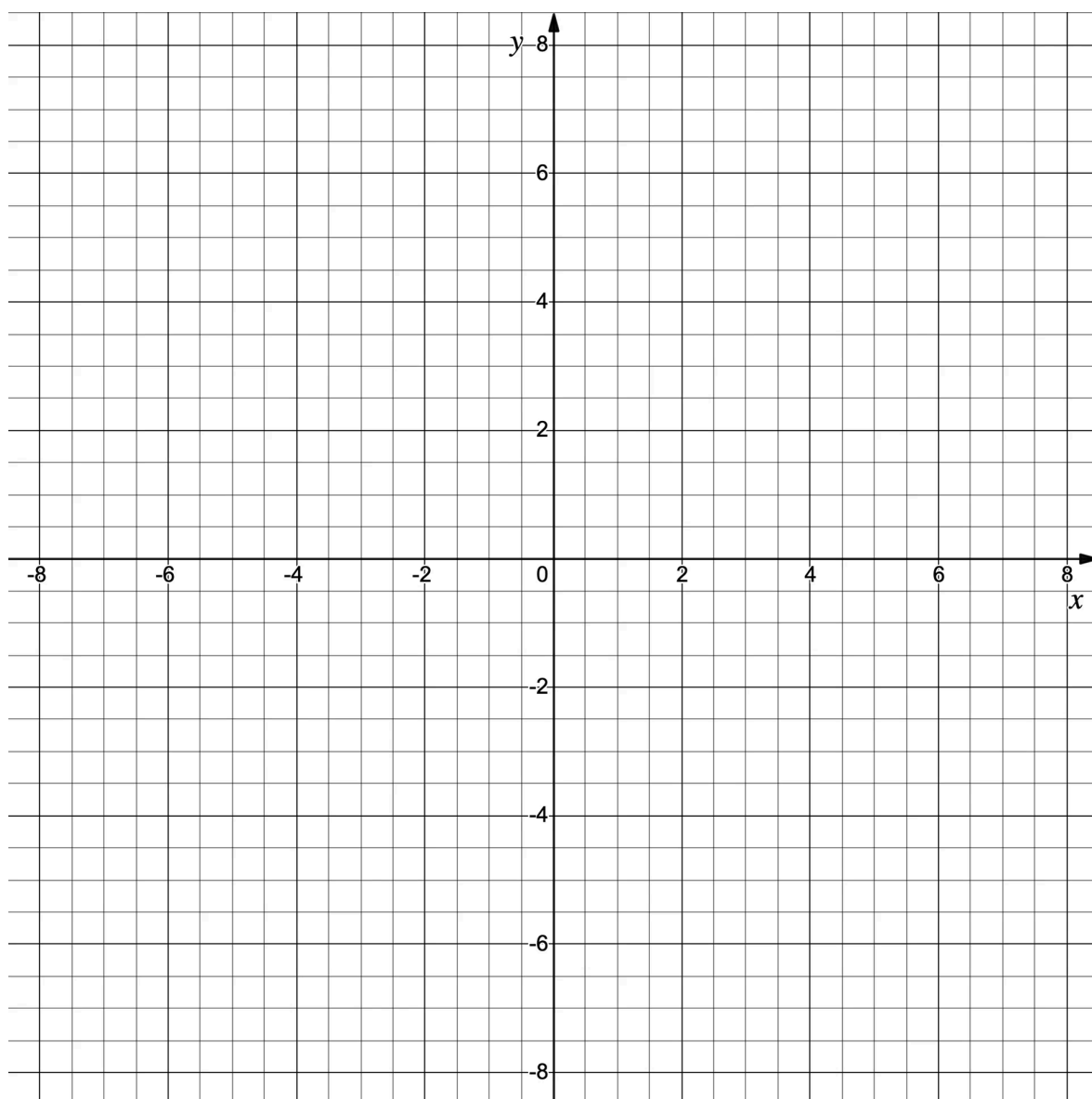
2 (a) On the same diagram, sketch the graphs of $y = \frac{1}{x^2}$ and $y = \frac{-3}{x^2}$.

(3 marks)

(b) Write down the equation(s) of any lines of symmetry and asymptotes for the two graphs in part (a).

(2 marks)

3 (a) On the axes below sketch the graphs of both $y = (x - 1)^2$ and $y = 2 - x^2 - x$.



(3 marks)

(b) Using your graph, or otherwise, find the solutions to the equation

$$x^2 - 2x + 1 = 2 - x^2 - x.$$

(2 marks)

- 4 y is inversely proportional to the square of x . When $x = 4$, $y = 8$. Find the constant of proportionality and sketch the graph of y against x .

(4 marks)

- 5 Sketch the graph of $y = 3x^3 + 2x^2 - 3x + 10$ labelling any points where the graph intersects the coordinate axes.

(4 marks)

6 (a) On the same diagram, sketch the graphs of $y = x^3 - 3x^2 - 6x + 8$ and $y = \frac{3}{x^2}$.

(4 marks)

(b) Write down the number of solutions to the equation

$$x^5 - 3x^4 - 6x^3 + 8x^2 = 3.$$

(1 mark)

- 7 (a)** A machine computes a calculation in time, t seconds, that is proportional to the cube root of the number of processes, p , involved. For a calculation involving 8 processes the computer takes 6.4×10^{-4} seconds.

How many processes are involved for a calculation taking 1.28×10^{-3} seconds?

(4 marks)

- (b)** Find the time it takes for the machine to compute a calculation involving 250 processes.

(2 marks)

- 8 (a)** On separate diagrams, sketch the graphs of $y = \frac{a}{x^2}$, where $a > 0$ and $y = \frac{a}{x^2}$, where $a < 0$.

(3 marks)

- (b)** One of the graphs passes through the point with coordinates (m, m^6) .

Write a in terms of m and, justifying your answer, state which graph this point must lie on.

(3 marks)

9 (a) Fully factorise $4x^3 + 17x^2 + 20x + 4$.

(3 marks)

(b) Sketch the graph of $y = 4x^3 + 17x^2 + 20x + 4$.
Label any points where the graph crosses the coordinate axes.

(2 marks)

10 On the same diagram, sketch the graphs of $4y = x^3 - 5x^2 - 12x + 36$ and $x + y - 6 = 0$.

Label the coordinates of any points of intersection between the two graphs.
Also label any points where the graphs intersect the coordinate axes.

(9 marks)