Edexcel A Level Further Maths: Core Pure

7.1 Polar Coordinates

Your notes

Contents

7.1.1 Polar Coordinates

Your notes

Intro to Polar Coordinates

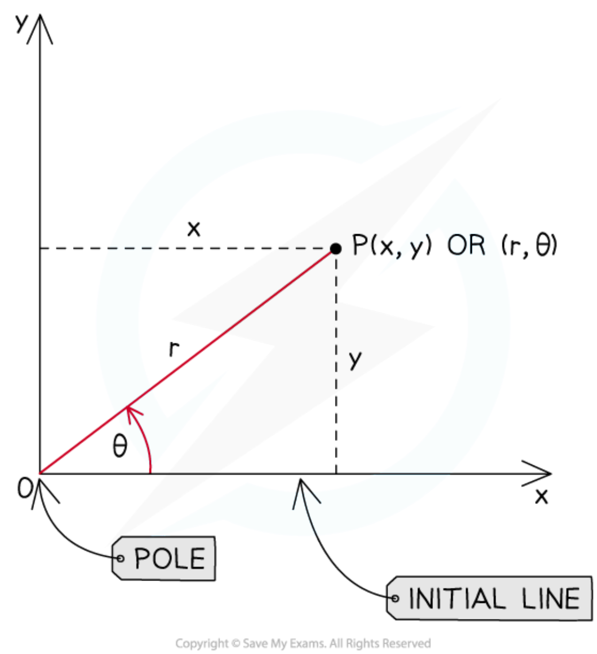
What are polar coordinates?

- **Polar coordinates** are an alternative way (to Cartesian coordinates) to describe the position of a point in 2D (or 3D) space
- In 2D, the position of a point is described using an **angle,** θ and a **distance,** r
  - This is akin to "*aiming in the right direction*", then "*travelling so far in that direction*"
- Polar coordinates generally make working with **circles, spirals** and similar shapes easier
  - (3D) polar coordinates are beyond the A level syllabus but they are used with objects based on spheres such as the planets in the solar system

How do I describe the position of a point using polar coordinates?

- Point ***P*** would be described by the coordinates (r, θ)
- θ is measured in radians, anti-clockwise from the **initial line** (equivalent to the positive x-axis)
  - Negative values of θ can be used (*clockwise* from the initial line)
- r is the (straight line) **distance** between the **pole** (origin) and point P
  - r is usually given as a function of θ, $r = \mathrm{f}(\theta)$
  - equations can be given implicitly too, e.g. $r^2 = \mathrm{f}(\theta)$
- A **half-line** starts at the **pole** and extends outwards in the **direction** of θ
  - The equation of a haf-line will be of the form $\theta = \alpha$, where α is a constant
  - The line represents **positive** values of r
  - Negative values of r are possible but are not included in Edexcel A level Further Mathematics

Your notes



What is the connection between polar coordinates and Cartesian coordinates?

- $r^2 = x^2 + y^2$
  $r\cos\theta = x$
  $r\sin\theta = y$
  $\tan\theta = \dfrac{y}{x}$
- These results are **not** provided in the formulae booklet
  - they are easily derived from a **sketch** and **basic trigonometry**
- Be careful solving $\tan\theta = \dfrac{y}{x}$ so that θ locates point P in the correct **quadrant**
  - Always use a sketch to ensure θ is measured from the **initial line**
- Check the **domain** of θ to see if negative values are used
  - e.g. $0 \le \theta < 2\pi$ as opposed to $-\pi \le \theta < \pi$
- This is very similar to the **modulus-argument** form of a **complex number**
  - $z = x + iy = r(\cos\theta + i\sin\theta)$ where $x = r\cos\theta$ and $y = r\sin\theta$

How do I convert from polar coordinates to Cartesian coordinates?

To convert the point $P(r, \theta)$ to $P(x, y)$

- Find the x-coordinate using $x = r\cos\theta$
- Find the y-coordinate using $y = r\sin\theta$

Your notes

- In both cases take care with which **quadrant** P lies in
  - A **sketch** is the easiest way to double check

How do I convert from Cartesian coordinates to Polar coordinates?

To convert the point $P(x, y)$ to $P(r, \theta)$

- Find r using **Pythagoras'** theorem
- r will (generally) take the **positive** square root since it is a **distance** (from the pole)
  - (It is possible for r to be negative, depending on the nature of $f(\theta)$)
- Find θ by using a **sketch** in association with $\tan\theta = \frac{y}{x}$
  - Use the sketch to ensure θ locates point P in the correct quadrant
  - There may be the need to add or subtract π to get θ in the correct quadrant

Examiner Tip

- Ensure your calculator is in radians mode when working with polar coordinates
- Note how polar coordinates (r, θ) are given in the order r then θ, even though $r = f(\theta)$

Your notes

Worked example

(a) Convert the polar coordinates $\left(3, \frac{\pi}{4}\right)$ to Cartesian coordinates.

$r = 3, \quad \theta = \frac{\pi}{4}$

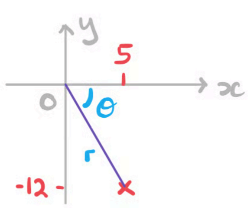
$\therefore\ x = r\cos\theta = 3\cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$y = r\sin\theta = 3\sin\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$\therefore$ Cartesian coordinates are $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

(b) Convert the Cartesian coordinates $(5, -12)$ to polar coordinates.

Sketch a diagram to ensure the correct quadrant is considered



$r^2 = 5^2 + 12^2 = 169, \quad r = 13$

$\theta = -\tan^{-1}\left(\frac{12}{5}\right) = -1.176\,005\ldots$

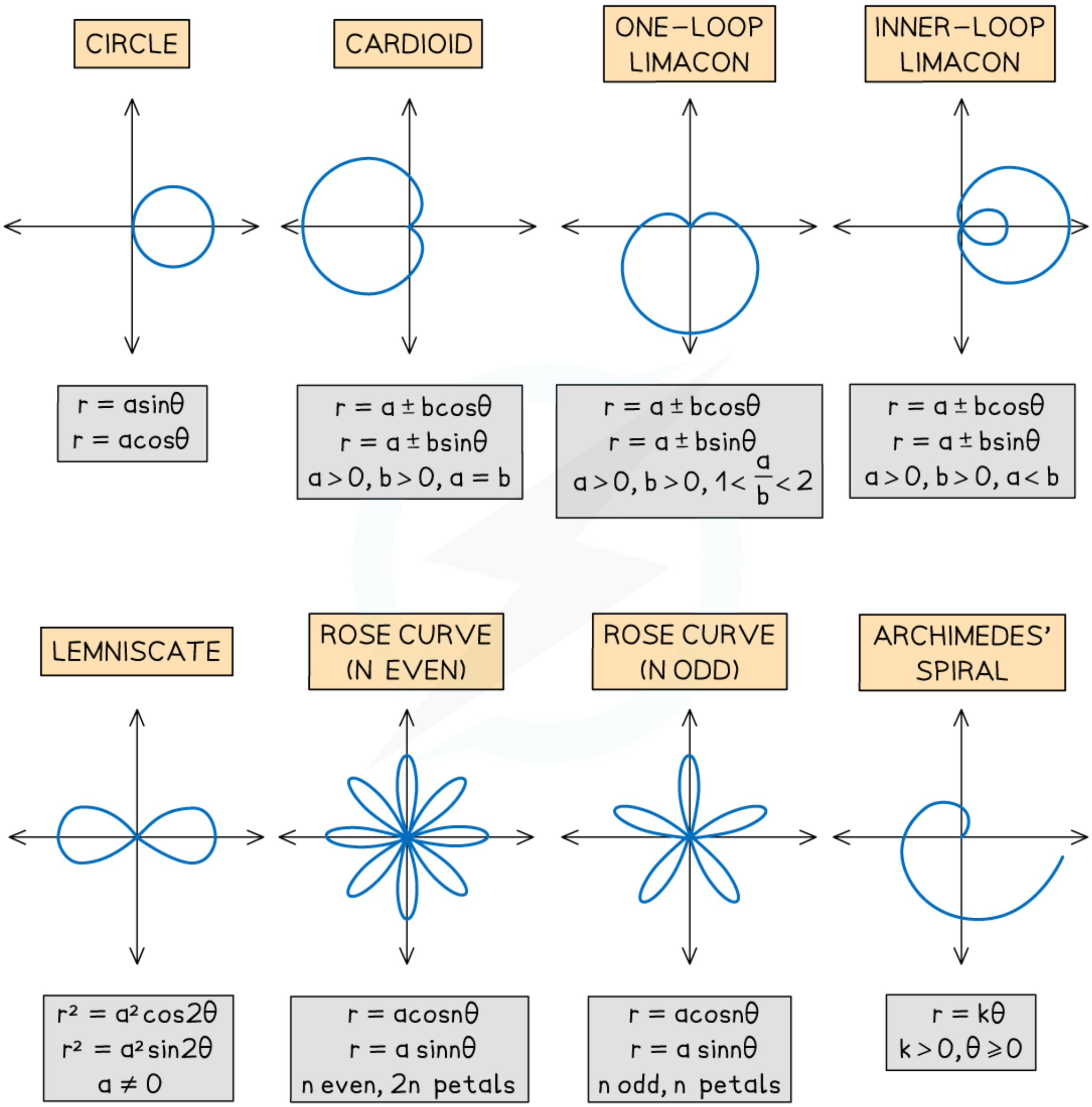
$\therefore$ Polar coordinates are $(13, -1.18)$ (3 s.f.)

Your notes

Sketching Curves in Polar Form

How do I sketch curves given in polar coordinates/polar form?

- Recognising common graphs and the style of their equations is important
- There are three basic equations to be familiar with
  - $\theta = \alpha$ is the equation of a **half-line** from the **pole** in the **direction** α radians **anti-clockwise** from the **initial line**
  - $r = a$ is a **circle**, **centre** at the **pole** with radius a
  - $r = k\theta$ is a **spiral**, starting at the **pole** where k is a **positive** constant
- Other common types of polar curve encountered are summarised in the diagram below



- The cardioid and one-loop limacon have a **cusp** at the pole
- For **Rose Curves** when n is **even, half** of the **petals** are where $r > 0$ and **half** of them are where $r < 0$
- For **Rose Curves** when n is **odd**, the **petals** are drawn **twice** – once when $r > 0$ and once when $r < 0$
  - (The positive and negative petals sit on top of each other)
  - Some graphing software will plot negative values of r with a dotted curve

How are horizontal and vertical lines described in polar coordinates?

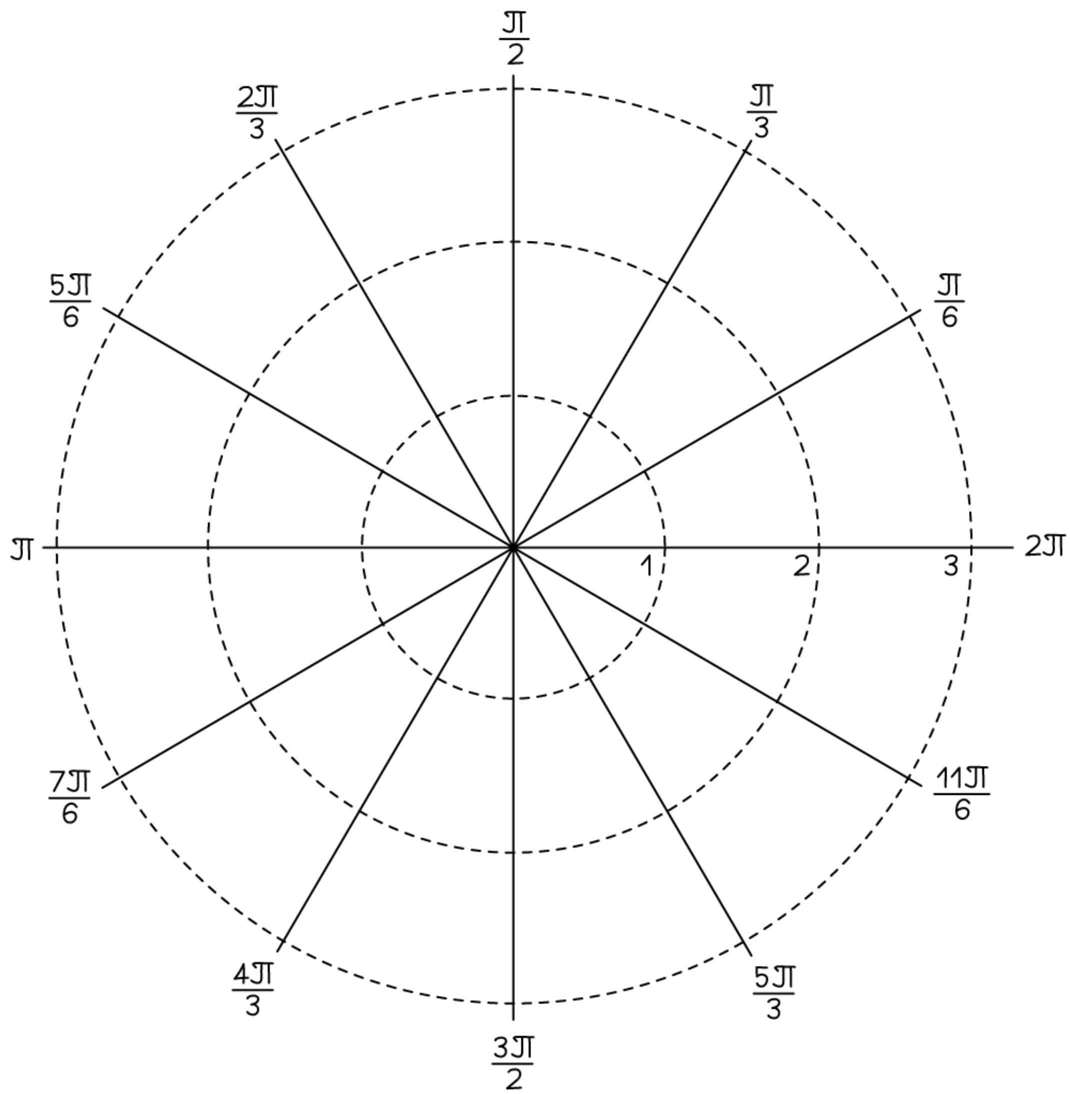
Your notes

- **Straight lines** have **polar equations** of the form $r = a\sec(\alpha - \theta)$
  - For the **horizontal** line corresponding to $y = a$, $\alpha = \frac{k\pi}{2}$ where *k* is **odd**
  - For the **vertical** line corresponding to $x = a$, $\alpha = k\pi$ where *k* is an **integer**
- **Diagonal** lines are formed using other values of α

How do I plot curves given in polar coordinates/polar form?

For more unusual polar equations a table of *r* and θ values can be generated

- Using the table points, can be plotted and joined on polar graph paper
- Values of θ may be given, e.g. every $\frac{\pi}{12}$ radians



- Where they are not given, think about common multiples of π that suit the question
  - e.g. if 3θ is involved in the question, $\frac{\pi}{3}$, $\frac{\pi}{6}$ or $\frac{\pi}{12}$ may be suitable
- Use a calculator to find the corresponding values of r
  - Be accurate but using decimals here is fine
- It is usual for questions to only require the plotting of part of a polar curve
  e.g. plotting within a **domain** of θ that completes a 'loop'
  e.g. a restricted **domain** of θ that produces only **positive** values of r
- When practising problems and revising have some graphing software running so you can quickly check your sketches against an accurate diagram

Your notes

Worked example

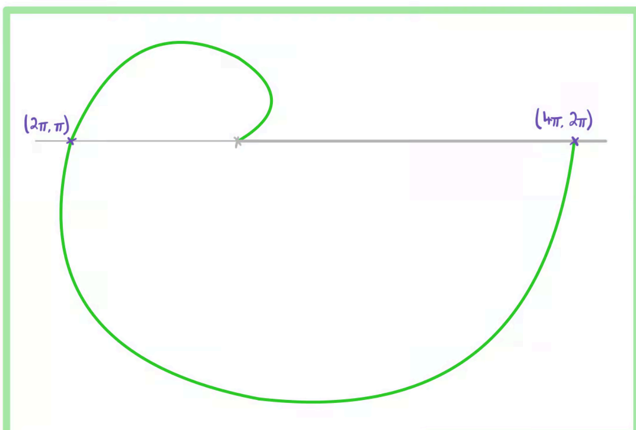
On separate diagrams, sketch the graph of the following polar curves

(i) $r = 2\theta$ for $0 \leq \theta < 2\pi$

(ii) $r = 3\cos 2\theta$

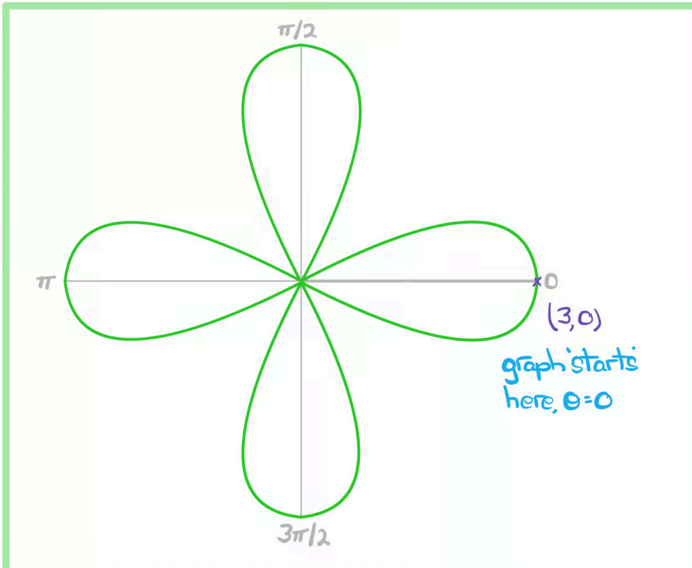
(iii) $r^2 = \sin 2\theta$ for $0 \leq \theta \leq \dfrac{\pi}{2}$

i. $r = 2\theta$ is a spiral



Your notes

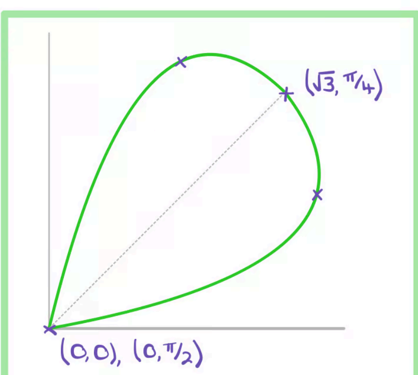
ii. $r = 3\cos 2\theta$ is a rose curve with 4 petals



iii. Construct a table of values, every $\pi/8$ radians

θ	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
r	0	$1.456...$	$1.732...$	$1.456...$	0

$(\sqrt{3})$



Your notes

Polar Curve to Cartesian Equation

How do I convert a polar equation to a Cartesian equation?

- For equations of the form $r = \mathrm{f}(\theta)$ **square both** sides
  - Some questions may define r^2 rather than r
    - r^2 can then be replaced by $x^2 + y^2$
- To eliminate θ, some manipulation and use of trigonometric identities may be needed
  - Aim to convert terms involving θ into either the form $r\cos\theta$ or $r\sin\theta$ then convert to x and y
- e.g. If $r = 2\sin\theta$ then $r^2 = 4\sin^2\theta, \quad x^2 + y^2 = 4\left(\frac{y^2}{r^2}\right), \quad (x^2 + y^2)^2 = 4y^2$
- Awkward powers of r may be involved but these can be manipulated into terms of r^2 too
  e.g. $r^3 = (r^2)^{\frac{3}{2}}$

How do I convert a Cartesian equation to a polar equation?

- In general substitute $x = r\cos\theta$ and $y = r\sin\theta$ into the Cartesian equation and simplify/rearrange
- Trigonometric identities may be involved
- If you spot them, there may be some shortcuts
  e.g. 'hidden' sums of x^2 and y^2 such as in $(x + y)^2$

Examiner Tip

- When converting a polar equation to a Cartesian equation, unless required by the question, do not worry about rearranging into the form $y = \mathrm{f}(x)$
- Make any obvious simplifications but otherwise an implicit Cartesian form is fine

Your notes

Worked example

(a) Find a Cartesian equation of the polar curve $r = 3 + \sin 2\theta$.

Since $\sin 2\theta = 2\sin\theta\cos\theta$, multiply through by r^2 to generate $r\sin\theta$ (y) and $r\cos\theta$ (x)...

$$r^3 = 3r^2 + 2r^2\sin\theta\cos\theta$$

$$(x^2+y^2)^{3/2} = 3(x^2+y^2) + 2(r\cos\theta)(r\sin\theta)$$

awkward powers of r!

$$\boxed{\therefore \text{ A Cartesian equation is } (x^2+y^2)^{3/2} = 3(x^2+y^2) + 2xy}$$

(b) Find a polar equation in the form $r^2 = \mathrm{f}(\theta)$ for the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$.

Substitute $x = r\cos\theta$ and $y = r\sin\theta$ in and rearrange

$$\frac{(r\cos\theta)^2}{9} + \frac{(r\sin\theta)^2}{25} = 1$$

$$r^2(25\cos^2\theta + 9\sin^2\theta) = 225$$

$$r^2(25\cos^2\theta + 9(1-\cos^2\theta)) = 225$$

$$r^2(16\cos^2\theta + 9) = 225$$

$$\boxed{\therefore r^2 = \frac{225}{16\cos^2\theta + 9}}$$

$$\left(r^2 = \frac{225}{25 - 16\sin^2\theta}\right)$$

Your notes

Your notes

Intersections of Polar Curves

How do I find the intersections of two curves given in polar form?

- This is essentially the same as solving **simultaneous equations**
  - The aim is to eliminate one of the variables (usually r) and solve for the other
  - Any previous skills used to eliminate variables may still be useful here
- The general approach is to write the two equations in the forms $r = \mathrm{f}(\theta)$ and $r = \mathrm{g}(\theta)$
  - Then solve $\mathrm{f}(\theta) = \mathrm{g}(\theta)$
  - If required, substitute θ into $\mathrm{f}(\theta)$ or $\mathrm{g}(\theta)$ to find r
- Be aware that polar curves are often given in the form $r^2 = \mathrm{f}(\theta)$
  - Working with r^2 rather than r may be easier
- Skills beyond basic simultaneous equations include
  - using trigonometric identities and solving trigonometric equations
  - "squaring and adding" (this is a common technique)
    - this can produce very useful $\sin^2\theta$ and/or $\cos^2\theta$ terms!

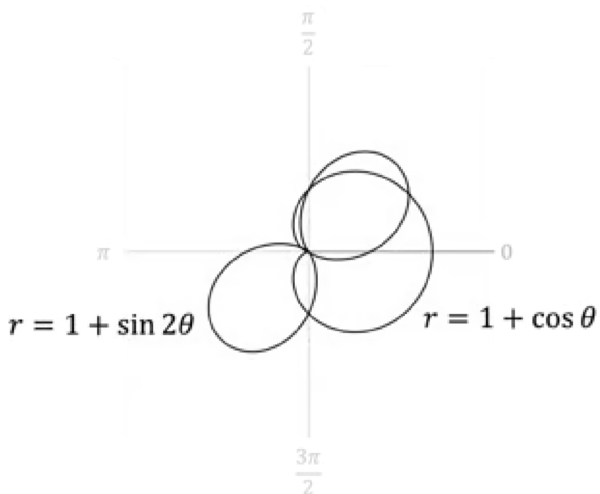
Examiner Tip

- Calculators are unlikely to be able to solve these types of simultaneous equations directly
- They may have a 'solve' mode you can use once the equation has been reduced to a single variable
- However look out for when questions require ***exact*** answers

Your notes

Worked example

The diagram below shows a sketch of the polar graphs of $r = 1 + \cos\,\theta$ and $r = 1 + \sin\,2\theta$ for $0 \le \theta < 2\pi$.



a) Find the smallest positive values of θ for which each curve crosses the pole.

Your notes

$1+\cos\theta=0$

$\cos\theta=-1$

$\theta=\pi$

$1+\sin 2\theta=0$

$\sin 2\theta=-1$

$2\theta=\frac{3\pi}{2}$

$\theta=\frac{3\pi}{4}$

$\therefore$ $r=1+\cos\theta$ crosses the pole when $\theta=\pi$ $(0,\pi)$

$r=1+\sin 2\theta$ crosses the pole when $\theta=\frac{3\pi}{2}$ $\left(0,\frac{3\pi}{2}\right)$

b) For $r>0$, find the points of intersection between the two curves for $0\le\theta<2\pi$.

Your notes

Curves intersect when

$$1 + \sin 2\theta = 1 + \cos\theta$$

$$2\sin\theta\cos\theta = \cos\theta$$

$$\cos\theta(2\sin\theta - 1) = 0$$

Don't cancel trig. terms!

$\therefore \cos\theta = 0, \quad \sin\theta = 1/2$

For $0 \leq \theta < 2\pi$

$\theta = \pi/2,\ 3\pi/2 \qquad \theta = \pi/6,\ 5\pi/6$

$r = 1,\ 1 \qquad r = \frac{1}{2}(2+\sqrt{3}),\ \frac{1}{2}(2-\sqrt{3})$ ($r > 0$ in all cases)

$\therefore$ Intersections have polar coordinates

$$\left(\frac{2+\sqrt{3}}{2}, \frac{\pi}{6}\right), \left(1, \frac{\pi}{2}\right), \left(\frac{2-\sqrt{3}}{2}, \frac{5\pi}{6}\right), \left(1, \frac{3\pi}{2}\right)$$

Your notes

7.1.2 Calculus with Polar Coordinates

Finding Tangents to Polar Curves

What is the gradient/tangent of a polar curve?

- Gradients (and tangents) are the same as using Cartesian coordinates
  - i.e. a gradient of 1 in Cartesian coordinates is still a gradient of 1 in polar coordinates
  - a 45° line from "bottom left" to "top right" is a gradient of 1 in both systems
  - the equation of a tangent to a polar curve should be written in polar form

How do I find the tangents to a polar curve?

- Finding the gradient - and so the equation of a tangent - to a polar curve is based on parametric differentiation in Cartesian form
- Since $x = r\cos\theta$, $y = r\sin\theta$ and $r = f(\theta)$, it follows that
  - $x = f(\theta)\cos\theta$
  - $y = f(\theta)\sin\theta$
- Then, using **parametirc** differentiation the gradient is given by
  - $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \div \dfrac{\mathrm{d}x}{\mathrm{d}\theta}$
  - From which the Cartesian equation can be found
  - Which can then be converted into polar form using $x = r\cos\,\theta$ and $y = r\sin\,\theta$

How do I find horizontal and vertical tangents to a polar curve?

- Many questions only concern tangents that are **horizontal** and/or **vertical** to the curve
- **Horizontal** tangents are described as being **"parallel** to the **initial line"**
  - Horizontal tangents occur where $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 0$
- **Vertical** tangents are described as being **"perpendicular** to the **initial line"**
  - Vertical tangents occur where $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 0$
- Questions require finding the coordinates of points that have horizontal or vertical tangents (rather than finding the equations of the tangents)
  - Coordinates should be in polar form, i.e. (r, θ)
- In some cases, **both** $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ and $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 0$ at a particular point
  - Under these cases the polar curve has a **cusp**
  - But vice versa is **not** necessarily true

- A polar curve with a cusp does **not** necessarily mean $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$

Your notes

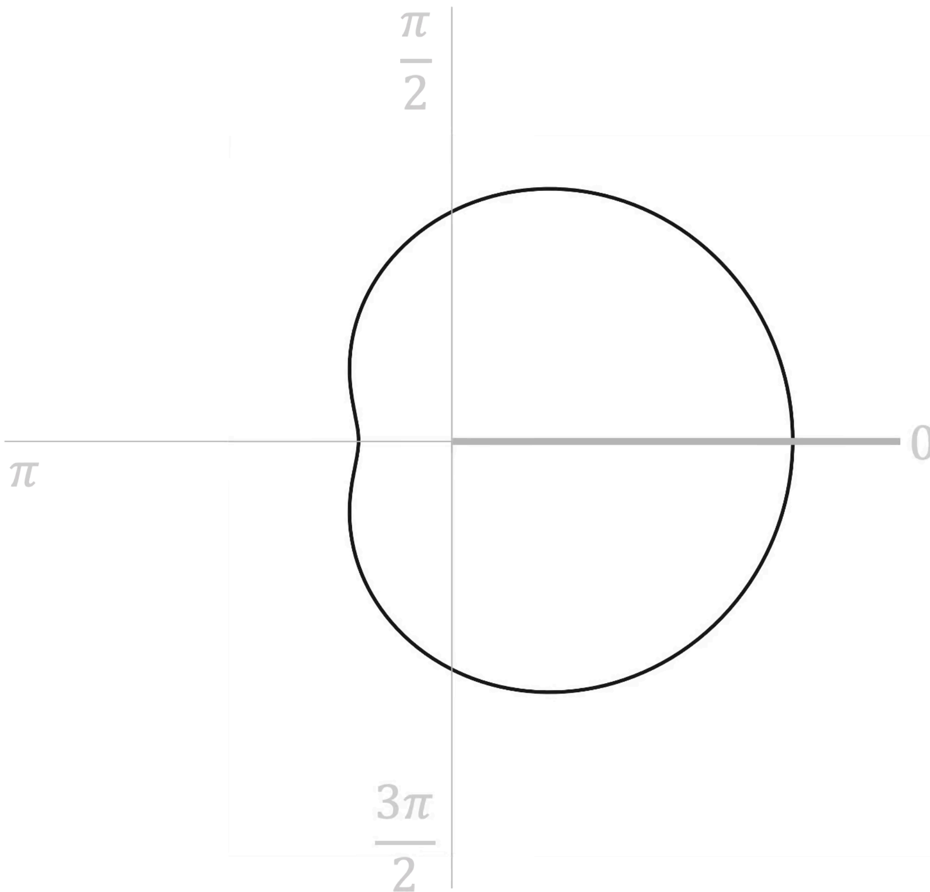
Examiner Tip

- If not provided, sketch the graph of the polar curve
  - This will help you to spot how many horizontal/vertical tangents there are
  - You could use a graphical calculator to help you do this

Your notes

Worked example

A sketch of the polar curve C, with equation $r = 3 + 2\cos\theta$, where $0 \leq \theta < 2\pi$ is shown below.



Find the coordinates of the points on C where the tangents are (i) parallel, (ii) perpendicular to the initial line, giving values to 2 significant figures where appropriate.

Your notes

(i) Parallel tangents occur when $\frac{dy}{d\theta} = 0$

$y = r\sin\theta = 3\sin\theta + 2\cos\theta\sin\theta = 3\sin\theta + \sin 2\theta$

$\therefore \frac{dy}{d\theta} = 3\cos\theta + 2\cos 2\theta = 0$

$3\cos\theta + 2(2\cos^2\theta - 1) = 0$

$4\cos^2\theta + 3\cos\theta - 2 = 0$

Use calculator to solve

$\therefore \cos\theta = \frac{-3 \pm \sqrt{41}}{8}$

$\therefore \theta = 1.131\ 402\ \ldots,$

$2\pi - 1.131\ 402\ldots = 5.151\ 782\ldots$

$\cos\theta = \frac{-3 \pm \sqrt{41}}{8}$ has no solutions

$r = 3.850\ 781\ldots,\ 3.850\ 781\ldots$

Your notes

(ii) Perpendicular tangents occur when $\frac{dx}{d\theta} = 0$

$x = r\cos\theta = 3\cos\theta + 2\cos^2\theta$

$\therefore \frac{dx}{d\theta} = -3\sin\theta - 4\cos\theta\sin\theta = -\sin\theta(3 + 4\cos\theta) = 0$

$\therefore \sin\theta = 0$ $\qquad$ $\cos\theta = -0.75$

$\theta = 0, \pi$ $\qquad$ $\theta = 2.418\ 858\ldots,\ 3.864\ 326\ldots$

$r = 5, 1$ $\qquad$ $r = 1.5,\ 1.5$

$\therefore$ (i) Parallel tangents occur at $(3.9, 1.1)$ and $(3.9, 5.2)$ (2 s.f.)

(ii) Perpendicular tangents occur at $(5, 0), (1, \pi), (1.5, 2.4)$ and $(1.5, 2.9)$ (2 s.f.)

Your notes

Finding Areas enclosed by Polar Curves

To find the area enclosed by a polar curve (or part of) it is first crucial to know how to find the area of a sector in polar coordinates

How do I find the area of a sector given by a polar curve?

- In polar coordinates, the area of a **sector,** A, is given by

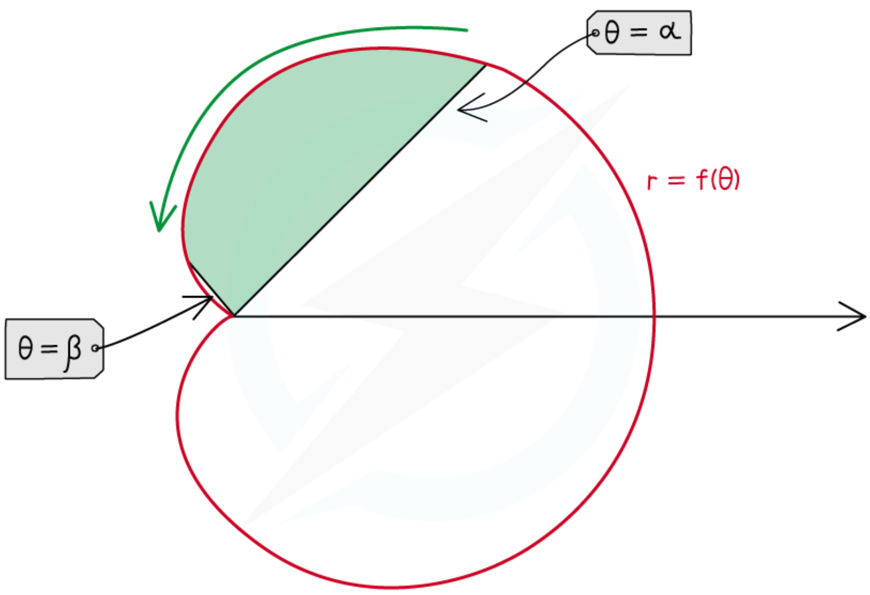
$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

where $r = f(\theta)$ and $\beta > \alpha$

- The sector is bounded by the curve $r = f(\theta)$ and the two half-lines $\theta = \alpha$ and $\theta = \beta$
  - This **is** given in the formula booklet
- If $f(\theta)$ is **constant** then the formula gives the area of the sector of a circle with centre angle $\beta - \alpha$

What is meant by the area enclosed by a polar curve?

- The area enclosed by a polar curve refers to an area bounded by a curve $r = f(\theta)$ between the half-lines $\theta = \alpha$ and $\theta = \beta$



- This can be considered as the area created by a 'sweeping' hand of a clock (but going anticlockwise!) moving between α and β

Your notes

- The integral calculates the sum of an infinite number of sectors which start at $\theta = \alpha$ and end at $\theta = \beta$
- This is the polar equivalent of the sum of an infinite number of rectangles under a curve in Cartesian coordinates

How do I find the area enclosed by a polar curve?

- STEP 1
  If not given, a sketch of the curve is helpful
  Identify the half-lines $\theta = \alpha$ and $\theta = \beta$ between which the area lies
  This may involve solving equations
  Always look for **symmetry** – many problems can be found by finding "half the area" and "doubling" – for example only finding an area above the initial line
- STEP 2
  Find r^2 and manipulate it into an integrable form
  This may involve using trigonometric identities and/or common integration techniques such as reverse chain rule, 'adjust and compensate'
  Set up the integral using the formula $\text{Area} = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, \mathrm{d}\theta$
- STEP 3
  Evaluate the integral and interpret the answer
  Remember to double/scale-up the integral value to find the area if symmetry has been used

Examiner Tip

- The use of symmetry in these problems can make them a lot easier so do always look to use it
- Calculators may be able to evaluate integrals but remember they usually expect *x* to be the 'input' variable
  - Calculators may not always produce *exact* values so check what is required by the question

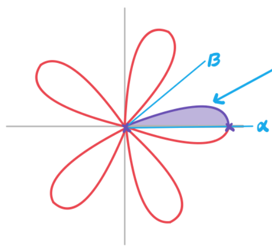
Your notes

Worked example

Find the exact area of one loop of the curve with polar equation $r = 3\cos 5\theta$.

STEP 1 Sketch the graph

$r = 3\cos 5\theta$ is a rose curve with 5 petals



Using symmetry consider the top half of the first loop

$\alpha = 0 \quad (r = 3)$

$\beta = \pi/10 \quad (r = 0)$

STEP 2: $r^2 = 9\cos^2 5\theta = 9\left[\frac{1}{2}(1+\cos 10\theta)\right]$

$\therefore$ Area of one loop $= 2\left[\frac{1}{2}\int_0^{\pi/10} \frac{9}{2}(1+\cos 10\theta)\, d\theta\right]$

$= \frac{9}{2}\int_0^{\pi/10}(1+\cos 10\theta)\, d\theta$

STEP 3: Area $= \frac{9}{2}\left[\theta + \frac{1}{10}\sin 10\theta\right]_0^{\pi/10}$

$= \frac{9}{2}\left(\frac{\pi}{10} - 0\right)$

$\boxed{\therefore \text{Area of one loop is } \frac{9\pi}{20} \text{ units}^2}$

Your notes

Finding Areas enclosed by Multiple Polar Curves

What is meant by the area enclosed by multiple polar curves?

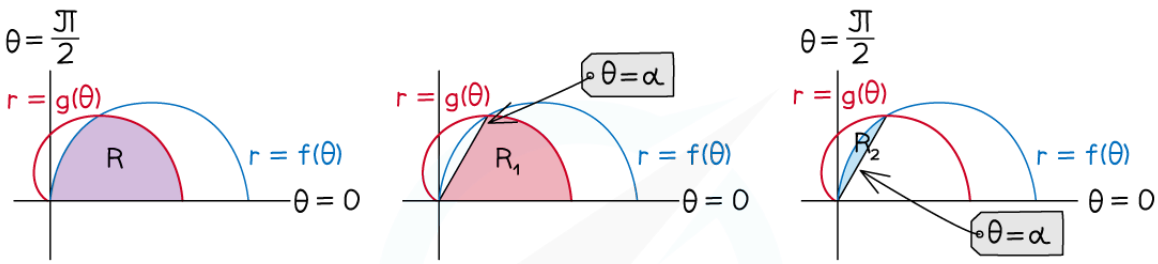
- An area enclosed by **multiple** polar curves could be
  - an **area** between **two** polar curves
  - an **area partially** enclosed by one polar curve and **partially** enclosed by another

How do I find the area enclosed by multiple polar curves?

- STEP 1
  If not given, a sketch, on the same diagram, of the curves is helpful
  Identify any half-lines that are needed by looking for intersections between the curves
  Identify any relevant values of θ such that $r=0$ (i.e. intersections with the pole)
  This may involve solving equations in relevant ranges of θ
  Look for **symmetry** to simplify the problem
- STEP 2
  Find r^2 for both curves, manipulating them into integrable forms
  This may involve using trigonometric identities and or common integration techniques such as reverse chain rule, 'adjust and compensate'
  Set up an integral for each partial area using the formula $\text{Area} = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, \mathrm{d}\theta$
- STEP 3
  Evaluate the integrals
  Double/scale-up each integral as necessary if symmetry has been used
  Total the partial integrals to find the entire area required

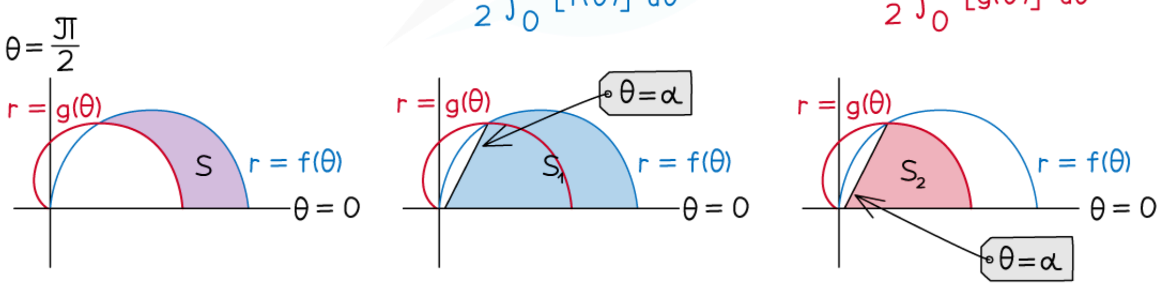
AREA OF R = AREA OF R_1 + AREA OF R_2

$$\frac{1}{2}\int_0^{\alpha}[g(\theta)]^2\,d\theta + \frac{1}{2}\int_{\alpha}^{\frac{\pi}{2}}[f(\theta)]^2\,d\theta$$



AREA OF S = AREA OF S_1 − AREA OF S_2

$$\frac{1}{2}\int_0^{\alpha}[f(\theta)]^2\,d\theta - \frac{1}{2}\int_0^{\alpha}[g(\theta)]^2\,d\theta$$



Your notes

Examiner Tip

- Graph sketches do not have to be accurate, but should enable you to visualise the problem and get an idea of where intersections and half-lines are
- Look out for when *exact* areas are required and whether your calculator can produce these using its integration function

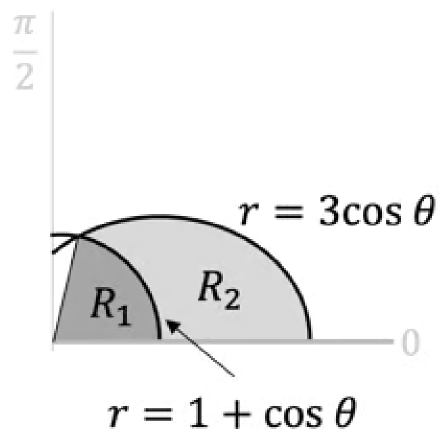
Your notes

Worked example

A sketch of the polar curves defined by the following equations is shown below

$$r = 1 + \cos\,\theta \quad 0 \le \theta \le \frac{\pi}{2}$$

$$r = 3\cos\,\theta \qquad 0 \le \theta \le \frac{\pi}{2}$$



a) Find the area labelled R_1.

First find the intersection of the two curves

$1 + \cos\theta = 3\cos\theta$

$\cos\theta = \frac{1}{2}$, $\theta = \frac{\pi}{3}$ $(0 \le \theta \le \pi/2)$

$\therefore$ Area of $R_1 = \int_0^{\pi/3} \frac{1}{2}(1+\cos\theta)^2 \, d\theta$

Using a calculator (exact answer not required)

Area of $R_1 = 1.759\,676\ldots$

$\boxed{\therefore \text{Area of } R_1 = 1.76 \text{ units}^2 \ (3 \text{ s.f.})}$

Your notes

a) Find the area labelled R_2.

STEP 1: R_2 will be the difference between two areas

"R_2 =  "

$\alpha = 0$, $\beta = \pi/3$

STEP 2: $r^2 = (3\cos\theta)^2 = 9\cos^2\theta$

$$\therefore \text{Area of } R_2 = \int_0^{\pi/3} \frac{9}{2}\cos^2\theta \, d\theta - 1.759\,676\ldots$$

from part (a)

STEP 3: Using a calculator

Area of R_2 = $3.330\,473\ldots - 1.759\,676\ldots$

$$\boxed{\therefore \text{Area of } R_2 = \frac{\pi}{2} \text{ units}^2 \quad (1.57 \text{ to } 3 \text{ s.f.})}$$