The Complete A Level Pure Maths Revision Sheet

Mr Astbury Maths

ALEGBRA

Indices

$a^m \times a^n \rightarrow a^{m+n}$ $\qquad a^0 \rightarrow 1$

$a^m \div a^n \rightarrow a^{m-n}$ $\qquad a^1 \rightarrow a$

$(a^m)^n \rightarrow a^{mn}$ $\qquad a^{\frac{1}{n}} \rightarrow \sqrt[n]{a}$

$a^{-m} \rightarrow \frac{1}{a^m}$ $\qquad a^{\frac{m}{n}} \rightarrow \sqrt[n]{a^m}$

Difference of Two Squares

$a^2 - b^2 \rightarrow (a-b)(a+b)$

Surds

$\sqrt{ab} \rightarrow \sqrt{a} \times \sqrt{b}$ $\qquad \sqrt{\frac{a}{b}} \rightarrow \frac{\sqrt{a}}{\sqrt{b}}$

$\frac{1}{\sqrt{a}} \rightarrow \frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$ $\qquad \frac{1}{a \pm \sqrt{b}} \rightarrow \frac{1}{a \pm \sqrt{b}} \times \frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$

Discriminant

$b^2 - 4ac < 0 \rightarrow$ No real roots

$b^2 - 4ac = 0 \rightarrow$ One real root

$b^2 - 4ac > 0 \rightarrow$ Two real roots

Factor Theorem

$f(a) = 0 \leftrightarrow (x-a)$ is a factor of $f(x)$

Numbers

$\mathbb{R} \rightarrow$ Reals $\qquad \mathbb{N} \rightarrow$ Naturals e.g. 1, 2, 3, ...

$\mathbb{Z} \rightarrow$ Integers $\qquad \mathbb{Q} \rightarrow$ Rationals

FUNCTIONS

Transformations

$f(x)+a \rightarrow$ Translation Up $\qquad f(x)-a \rightarrow$ Translation Down

$f(x-a) \rightarrow$ Translation Right $\qquad f(x+a) \rightarrow$ Translation Left

$af(x) \rightarrow$ Stretch in y axis by scale factor a

$f(ax) \rightarrow$ Stretch in x axis by scale factor $\frac{1}{a}$

$-f(x) \rightarrow$ Reflection in x axis

$f(-x) \rightarrow$ Reflection in y axis

Domain & Range

Domain $\rightarrow$ Inputs, x $\qquad$ Range $\rightarrow$ Outputs, y

Domain of $f^{-1}(x) \rightarrow$ Range of $f(x)$

Range of $f^{-1}(x) \rightarrow$ Domain of $f(x)$

SEQUENCES

Arithmetic

n^{th} term $\rightarrow a_n = a + (n-1)d$ $\qquad a =$ first term

Sum $\rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$ $\qquad d =$ difference

Geometric

n^{th} term $\rightarrow a_n = ar^{n-1}$ $\qquad a =$ first term

Sum $\rightarrow S_n = \frac{a(1-r^n)}{1-r}$ $\qquad r =$ ratio

Sum to infinity $\rightarrow S_\infty = \frac{a}{1-r}$

Convergent $\rightarrow |r| < 1$

Recursive

Period, order $k \rightarrow a_{n+k} = a_n$

COORDINATE GEOMETRY

Straight Lines

Gradient $\rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation $\rightarrow y = mx + c$ $\qquad$ Alternative $\rightarrow y - y_1 = m(x - x_1)$

Parallel $\rightarrow$ Same gradient

Perpendicular $\rightarrow m_1 \times m_2 = -1$

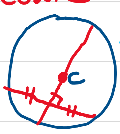
Distance $\rightarrow \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Midpoint $\rightarrow \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

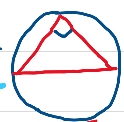
Circles

Equation $\rightarrow (x-a)^2 + (y-b)^2 = r^2$

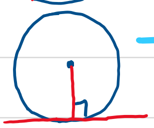
Centre $\rightarrow (a, b)$ $\qquad$ Radius $\rightarrow r$



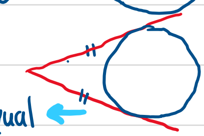
$\rightarrow$ Perpendicular bisector of chord passes Centre



Angle in a semi-circle is 90° $\leftarrow$



$\rightarrow$ Tangent and radius 90°



Tangents that meet are equal $\leftarrow$

BINOMIAL

Year 1

$(a+b)^n \; n \in \mathbb{N} \rightarrow a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + b^n$

Year 2

$(1+x)^n \rightarrow 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

$(a+bx)^n \rightarrow a^n\left(1 + \frac{b}{a}x\right)^n$ $\qquad$ Valid $\rightarrow \left|\frac{bx}{a}\right| < 1$

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EXPONENTIALS

Log Rules

$\log_a x = n \rightarrow a^n = x$

$\log x + \log y \rightarrow \log xy$

$\log x - \log y \rightarrow \log \frac{x}{y}$

$\log x^k \rightarrow k \log x$

$\log_a a \rightarrow 1$

$\log_a 1 \rightarrow 0$

$e^{\ln x} \rightarrow \ln e^x \rightarrow x$

$y = ax^n \rightarrow \log y = n \log x + \log a$

$y = ab^x \rightarrow \log y = (\log b)x + \log a$

NUMERICAL METHODS

Root

Change of sign $\rightarrow$ $f(x)$ is continuous

Newton Raphson

$x_{n+1} \rightarrow x_n - \frac{f(x_n)}{f'(x_n)}$

VECTORS

Vectors

$\vec{AB} \rightarrow \vec{OB} - \vec{OA}$

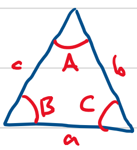
Distance $\rightarrow |a| = \sqrt{x^2 + y^2 + z^2}$

Parallel $\rightarrow \vec{AB} = \lambda \vec{CD}$

Collinear $\rightarrow$ Lie on the same straight line

TRIGONOMETRY

Sine and Cosine Rules



Sine $\rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$

Cosine $\rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

Area of Triangle $\rightarrow \frac{1}{2}ab\sin C$

Radians

$2\pi \rightarrow 360$, $\pi \rightarrow 180$, $\frac{\pi}{2} \rightarrow 90$

$\frac{\pi}{3} \rightarrow 60$, $\frac{\pi}{4} \rightarrow 45$, $\frac{\pi}{6} \rightarrow 30$

1 Radian $\rightarrow \frac{180}{\pi}$

Sectors

Area $\rightarrow \frac{1}{2}r^2\theta$

Arc $\rightarrow r\theta$

Segment $\rightarrow \frac{1}{2}r^2(\theta - \sin\theta)$

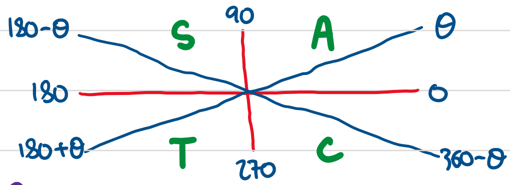
Exact Values

$\sin 30 \rightarrow \frac{1}{2}$ $\cos 30 \rightarrow \frac{\sqrt{3}}{2}$ $\tan 30 \rightarrow \frac{1}{\sqrt{3}}$

$\sin 45 \rightarrow \frac{1}{\sqrt{2}}$ $\cos 45 \rightarrow \frac{1}{\sqrt{2}}$ $\tan 45 \rightarrow 1$

$\sin 60 \rightarrow \frac{\sqrt{3}}{2}$ $\cos 60 \rightarrow \frac{1}{2}$ $\tan 60 \rightarrow \sqrt{3}$

CAST



Small Angle

$\sin\theta \approx \theta$ $\tan\theta \approx \theta$ $\cos\theta \approx 1 - \frac{\theta^2}{2}$

Reciprocals

$\sec x \rightarrow \frac{1}{\cos x}$

$\operatorname{cosec} x \rightarrow \frac{1}{\sin x}$

$\cot x \rightarrow \frac{1}{\tan x}$

Indentities

$\sin^2 x + \cos^2 x \equiv 1$

$1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

$\tan^2 x + 1 \equiv \sec^2 x$

Compound Angle

$\sin(A \pm B) \rightarrow \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) \rightarrow \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) \rightarrow \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Double Angle

$\sin 2\theta \rightarrow 2\sin\theta\cos\theta$

$\cos 2\theta \rightarrow \cos^2\theta - \sin^2\theta$

$\rightarrow 2\cos^2\theta - 1$

$\rightarrow 1 - 2\sin^2\theta$

$\tan 2\theta \rightarrow \frac{2\tan\theta}{1 - \tan^2\theta}$

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Rsin(x+α)

$R\sin(x\pm\alpha) \rightarrow a\sin x \pm b\cos x$

$R\cos(x\pm\alpha) \rightarrow a\cos x \mp b\sin x$

$R \rightarrow \sqrt{a^2+b^2}$ $\quad \alpha \rightarrow \tan^{-1}\left(\frac{b}{a}\right)$

DIFFERENTIATION

First Principles

$f'(x) \rightarrow \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Gradients

$\frac{dy}{dx} < 0 \rightarrow$ Decreasing $\quad \frac{dy}{dx} > 0 \rightarrow$ Increasing

$\frac{dy}{dx} = 0 \rightarrow$ Stationary Points

$\frac{d^2y}{dx^2} < 0 \rightarrow$ Concave, Max $\quad \frac{d^2y}{dx^2} > 0 \rightarrow$ Convex, Min

$\frac{d^2y}{dx^2} = 0 \rightarrow$ Point of Inflection IFF $\frac{d^2y}{dx^2}$ Changes Sign.

Chain Rule

$y=f(u)$ and $u=f(x)$ then $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$

Product Rule

$y=uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient Rule

$y=\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Parametric

$y=f(t)$ and $x=f(t)$ then $\rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Standard Results

$ax^n \rightarrow nax^{n-1}$

$\sin kx \rightarrow k\cos kx$

$\cos kx \rightarrow -k\sin kx$

$e^{kx} \rightarrow ke^{kx}$

$\ln x \rightarrow \frac{1}{x}$

$a^x \rightarrow a^x \ln a$

$\tan kx \rightarrow k\sec^2 kx$

$\operatorname{cosec} kx \rightarrow -k\operatorname{cosec} kx\cot kx$

$\sec kx \rightarrow k\sec kx\tan kx$

$\cot kx \rightarrow -k\operatorname{cosec}^2 kx$

$(\text{anything})^n \rightarrow n(\text{anything})^{n-1} \times \frac{d(\text{anything})}{dx}$

INTEGRATION

Standard Results

$ax^n \rightarrow \frac{ax^{n+1}}{n+1}+c$

$e^{kx} \rightarrow \frac{1}{k}e^{kx}+c$

$\frac{1}{x} \rightarrow \ln|x|+c$

$\sin kx \rightarrow -\frac{1}{k}\cos kx + c$

$\cos kx \rightarrow \frac{1}{k}\sin kx + c$

$\sec^2 kx \rightarrow \frac{1}{k}\tan kx + c$

$\tan kx \rightarrow \frac{1}{k}\ln|\sec kx| + c$

$\cot kx \rightarrow \frac{1}{k}\ln|\sin kx| + c$

$\operatorname{cosec} kx \rightarrow -\frac{1}{k}\ln|\operatorname{cosec} kx + \cot kx| + c$

$\sec kx \rightarrow \frac{1}{k}\ln|\sec kx + \tan kx| + c$

$f'(ax+b) \rightarrow \frac{1}{a}f(ax+b)+c$

Reverse Chain Rule

$\int \frac{f'(x)}{f(x)} \rightarrow \ln|f(x)|+c$

$\int f'(x)[f(x)]^n \rightarrow \frac{1}{n+1}[f(x)]^{n+1}+c$

Substitution

Steps $\rightarrow$ Differentiate u to find dx, Convert limits

Parts

Formula $\rightarrow \int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$ Remember $\rightarrow$ LATE

Fractions

$\frac{x+3}{x} \rightarrow$ Split $\quad 1+\frac{3}{x}$

$\frac{x+1}{x-1} \rightarrow$ Divide $\quad 1+\frac{2}{x-1}$

$\frac{x+1}{x(x-1)} \rightarrow$ Partial $\quad \frac{-1}{x}+\frac{2}{x-1}$

Trapezium Rule

Formula $\rightarrow \int_a^b y\,dx = \frac{h}{2}[\text{First}+\text{Last}+2(\text{Middles})]$

$h \rightarrow \frac{b-a}{n}$

Parametric

Formula $\rightarrow \int y\frac{dx}{dt}\,dt$

Limit Sum

$\lim_{\delta x\to 0}\sum_{x=a}^{b} f(x)\delta x \rightarrow \int_a^b f(x)\,dx$