Coordinate Geometry - Straight Line Graphs

There is little new theory since GCSE, but the algebraic manipulation is harder.

1:: $y = mx + c$, Gradient & Determining Equations

Find the equation of the line passing through $(2,3)$ and $(7,5)$, giving your equation in the form $ax + by + c = 0$, where a, b, c are integers.

NEW! since GCSE
The equation $y - y_1 = m(x - x_1)$ for a line with given gradient and going through a given point.

2:: Parallel/Perpendicular Lines

A line is perpendicular to $3x + 8y - 11 = 0$ and passes through $(0, -8)$. Find the equation of the line.

3:: Lengths and Areas

The line $2x + 3y = 6$ crosses the x-axis and y-axis at the points A and B respectively. Determine:
(a) The length AB and
(b) The area OAB.

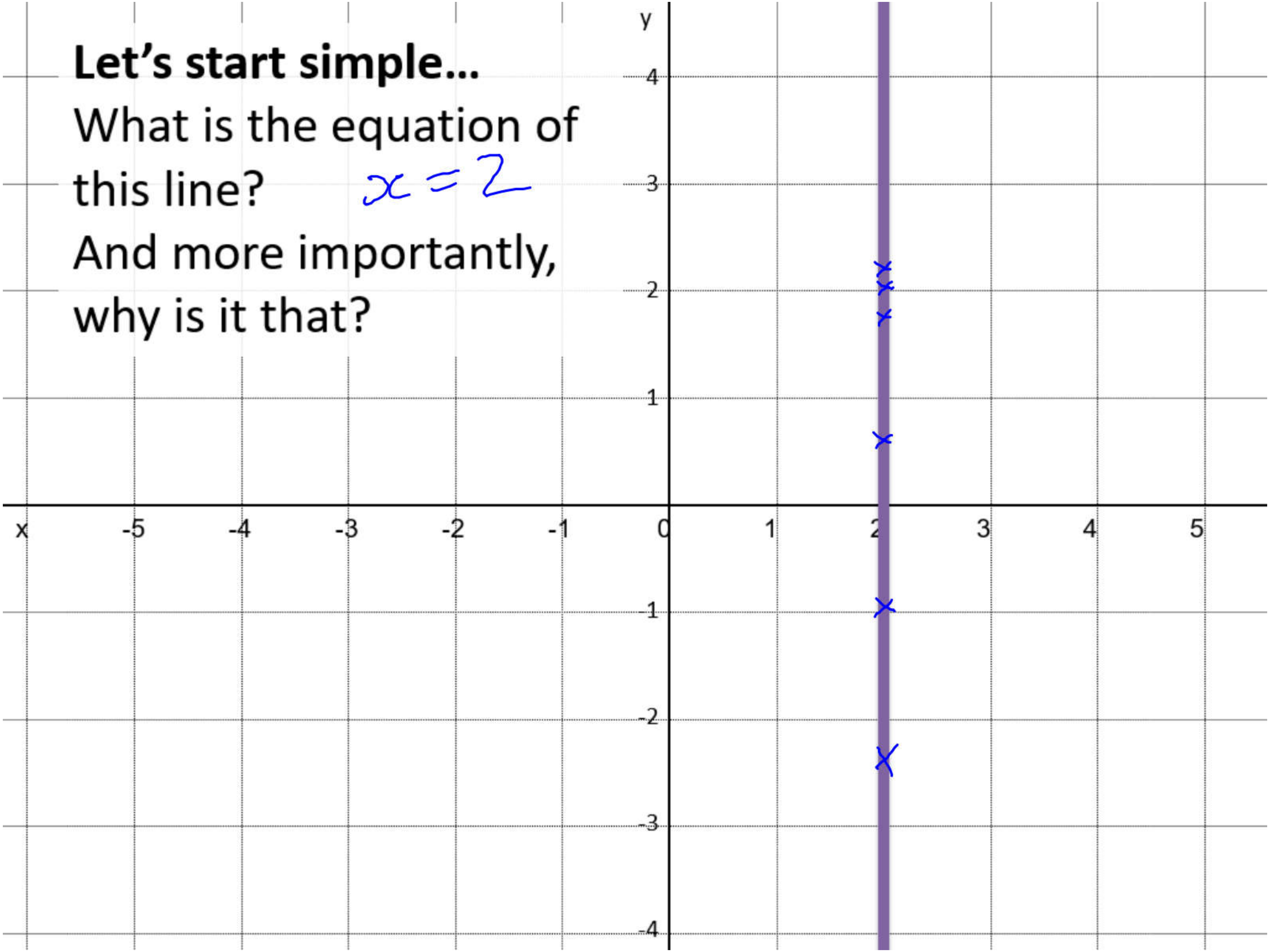
4:: Modelling

A plumber charges a fixed cost plus a unit cost per day. If he charges £840 for 2 days work and ...

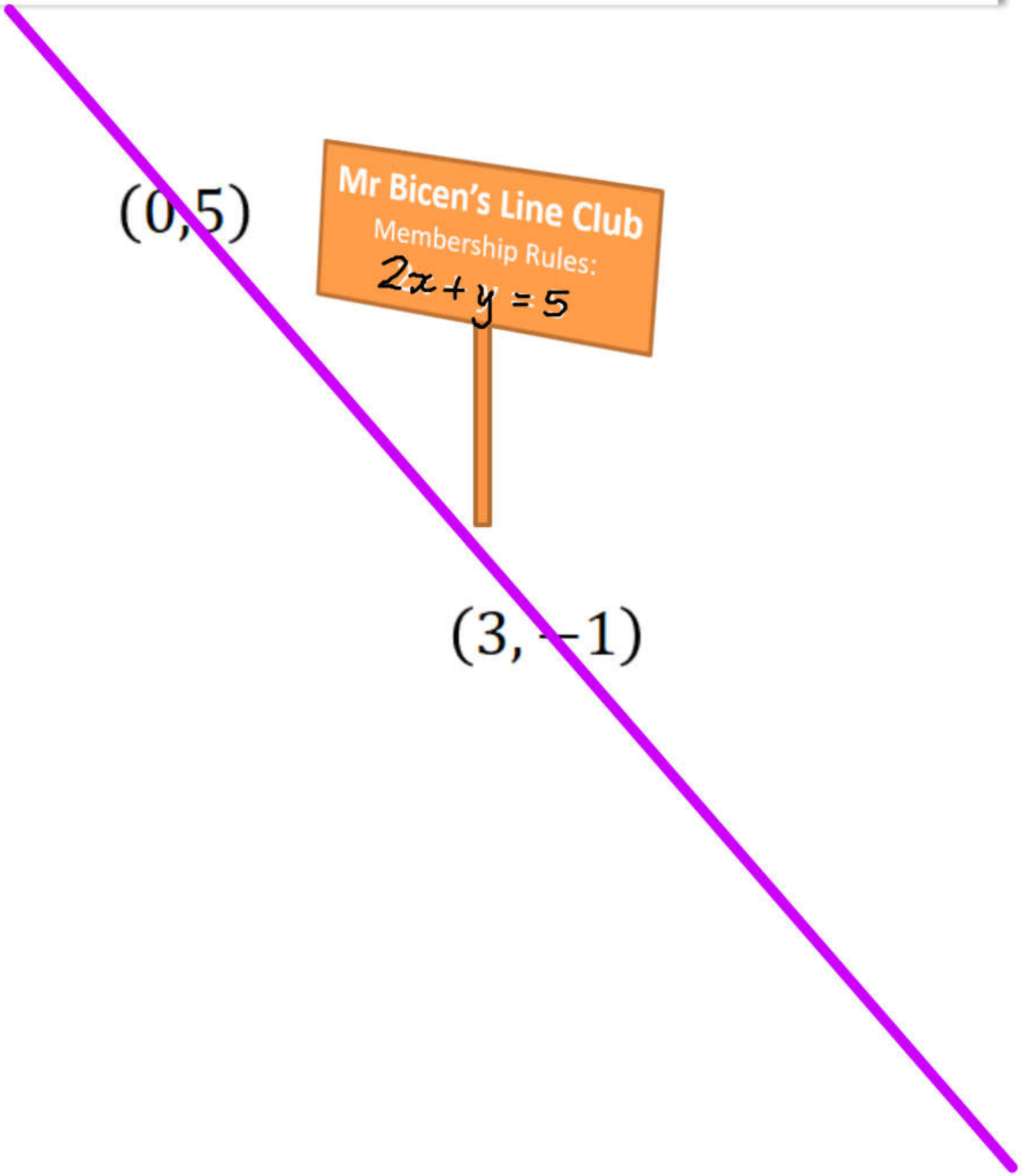
Let's start simple…

What is the equation of this line? $x = 2$

And more importantly, why is it that?



A line consists of all points which satisfy some equation in terms of x and/or y.



This means we can **substitute** the values of a coordinate into our equation whenever we know the point lies on the line.

The point $(5, a)$ lies on the line with equation $y = 3x + 2$. Determine the value of a.

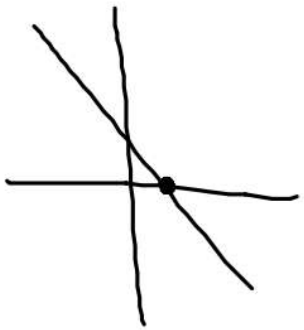
$x = 5 \quad y = a$

$y = 3x + 2$

$a = 3 \times 5 + 2$

$\underline{\underline{a = 17}}$.

Find the coordinate of the point where the line $2x + y = 5$ cuts the x-axis



$y = 0$

$2x = 5$

$x = \frac{5}{2}$

$\left(\frac{5}{2}, 0\right)$

x-coordinate

Your Turn

Determine where the line $x + 2y = 3$ crosses the:

a) y-axis:

$x = 0$ $\quad 2y = 3$

$y = \frac{3}{2}$ $\quad \left(0, \frac{3}{2}\right)$

b) x-axis:

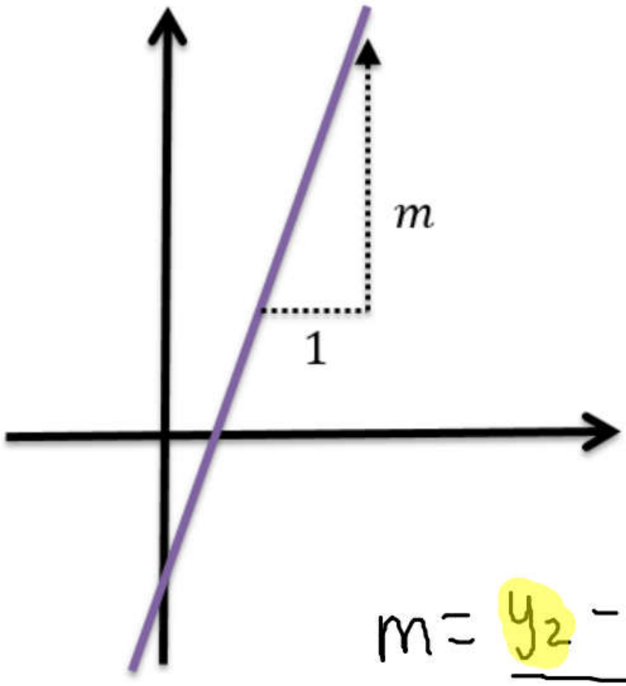
$y = 0$ $\quad x = 3$ $\quad (3, 0)$

What mistakes do you think it's easy to make?

- **Mixing up x/y: Putting answer as $(0, 3)$ rather than $(3, 0)$.**
- **Setting $y = 0$ to find the y-intercept, or $x = 0$ to find the x-intercept.**

Recap of Gradient

The steepness of a line is known as the **gradient**.
It tells us what y changes by as x increases by 1.



So if the y value increased by 6 as the x value increased by 2, what is y increasing by for each unit increase of x? 3
How would that give us a suitable formula for the gradient m?

δ

$$\boxed{m = \frac{\Delta y}{\Delta x}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Δ is the (capital) Greek letter "delta" and means **"change in"**.

Textbook Note:
You can also use $m = \frac{y_2 - y_1}{x_2 - x_1}$ for two points (x_1, y_1) and (x_2, y_2) which is the same thing

Find the gradient of the line that goes through the points:

1 $(1,4) \quad (3,10)$

$$m = \frac{10-4}{3-1} = \frac{6}{2} = 3$$

$$m = \frac{4-10}{1-3} = \frac{-6}{-2} = 3$$

2 $(5,7) \quad (8,1)$

$$m = \frac{1-7}{8-5} = -\frac{6}{3} = -2$$

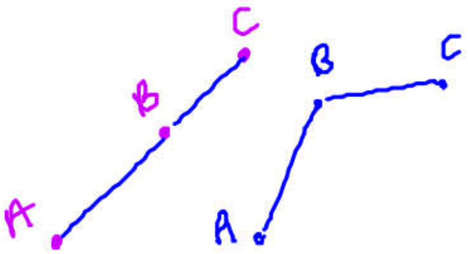
3 $(2,2) \quad (-1,10)$

$$m = \frac{10-2}{-1-2} = \frac{8}{-3} = -\frac{8}{3} \text{ or } -\frac{8}{3}$$

4 Show that the points $A(3,4), B(5,5), C(11,8)$ all lie on a straight line.

BC

AB, and AC should have equal gradient



$$m_{AC} = \frac{8-4}{11-3} = \frac{4}{8} = \frac{1}{2}$$

$$m_{AB} = \frac{5-4}{5-3} = \frac{1}{2}$$

As gradients $m_{AC} = m_{AB} = \frac{1}{2}$,

A, B and C are collinear → lies on same line

The line joining $(2, -5)$ to $(4, a)$ has gradient -1. Work out the value of a.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-1 = \frac{a - -5}{4 - 2}$$

$$-1 = \frac{a + 5}{2}$$

$$-2 = a + 5$$

$$\underline{-7 = a}.$$

One form we can put a straight line equation in is:

$$\boxed{y =} mx + c$$

gradient

y-intercept.

LHS must be $y =$

$3y + 2x = 4$

$m \neq 2$

$c \neq 4$

$\boxed{y =}$

$m =$

$c =$

m c

Determine the gradient and y-intercept of the line with equation $4x - 3y + 5 = 0$

$$4x - 3y + 5 = 0$$

$$4x + 5 = 3y$$

$$\frac{4x}{3}$$

same

$$\frac{4}{3}x + \frac{5}{3} = y$$

$$m = \frac{4}{3} \qquad c = \frac{5}{3}$$

Make y the subject so we have the form

$$y = mx + c$$

Put y on the side it's positive.

Divide **each** term by 3; don't write $y = \frac{4x+5}{3}$ otherwise it's not in the form $y = mx + c$

This is algebra, so use improper fractions, and not mixed numbers or recurring decimals.

At GCSE, $\boldsymbol{y = mx + c}$ was the main form you would express a straight line equation, sometimes known as the '**slope-intercept form**'.
But another common form is $\boldsymbol{ax + by + c = 0}$, where a, b, c are integers.
This is known as the '**standard**' form.

Express $y = \frac{1}{3}x - \frac{2}{3}$ in the form $ax + by + c = 0$, where a, b, c are integers.

$y = \frac{1}{3}x - \frac{2}{3}$

$3y = x - 2$

$0 = x - 3y - 2$ or $-x + 3y + 2 = 0$

Your Turn

Express $y = \frac{2}{5}x + \frac{3}{5}$ in the form $ax + by + c = 0$, where a, b, c are integers.

$5y = 2x + 3$

$0 = 2x - 5y + 3$

Ex 5A/5B

Equations using two points/point + gradient

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

How would you have done this at **GCSE**?

$y = mx + c$

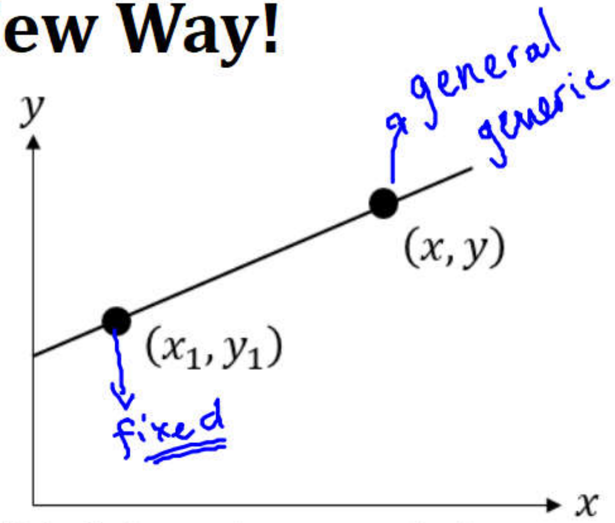
$5 = 2 \times 3 + c$

$5 = 6 + c$

$c = -1$

$y = 2x - 1$

A New Way!



Notes: Note that x_1 and y_1 are <u>constants</u> while x and y are <u>variables</u>. The latter are variables because as these 'vary', we get different points on the line.

Suppose that (x_1, y_1) is some fixed point on the line that we specify (e.g. (3,5)). Suppose that (x, y) represents a generic point on the line, which is allowed to change as we consider different points on this line.

Then:

$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = y - y_1$$

Thus:

✏ The equation of a line that has gradient m and passes through a point (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

Let's revisit:

Find the equation of the line that goes through $\underset{x_1,\ y_1}{(3,5)}$ and has gradient $\underset{m}{2}$.

$$y - 5 = 2(x - 3)$$

$$y - 5 = 2x - 6$$

$$\underline{\underline{y = 2x - 1}}$$

In a nutshell: You can use this formula whenever you have (a) a gradient and (b) any point on the line.

Gradient	Point	(Unsimplified) Equation
3	$(1,2)$	$\boldsymbol{y-2=3(x-1)}$
5	$(3,0)$	$\boldsymbol{y=5(x-3)}$
2	$(-3,4)$	$\boldsymbol{y-4=2(x+3)}$
$\frac{1}{2}$	$(1,-5)$	$\boldsymbol{y+5=\frac{1}{2}(x-1)}$
9	$(-4,-4)$	$\boldsymbol{y+4=9(x+4)}$

Important Side Note: I've found that many students shun this formula and just use the GCSE method. Please persist with it – it'll be much easier when fractions are involved. Further Mathematicians, <u>don't even think</u> about using the GCSE method, because you'll encounter massive headaches when you consider algebraic points. Trust me on this one!

Using 2 points

Find the equation of the line that goes through $(4,5)$ and $(6,2)$, giving your equation in the form $ax + by + c = 0$.

gradient and a point.

$m = \frac{2-5}{6-4} = -\frac{3}{2}$

$(\underset{x_1}{4}, \underset{y_1}{5}) \quad (6,2)$

$y - 5 = -\frac{3}{2}(x-4)$

$2y - 10 = -3(x-4)$

$2y - 10 = -3x + 12$

$2y - 10$

$3x + 2y - 22 = 0$ ✓✓

$-3x - 2y + 22 = 0$ ✓✓

Your Turn:

Find the equation of the line that goes through $(-1,9)$ and $(4,5)$, giving your equation in the form $ax + by + c = 0$.

$m = \frac{5-9}{4--1} = -\frac{4}{5}$

$x_1 = -1 \quad y_1 = 9$

$y - 9 = -\frac{4}{5}(x+1)$

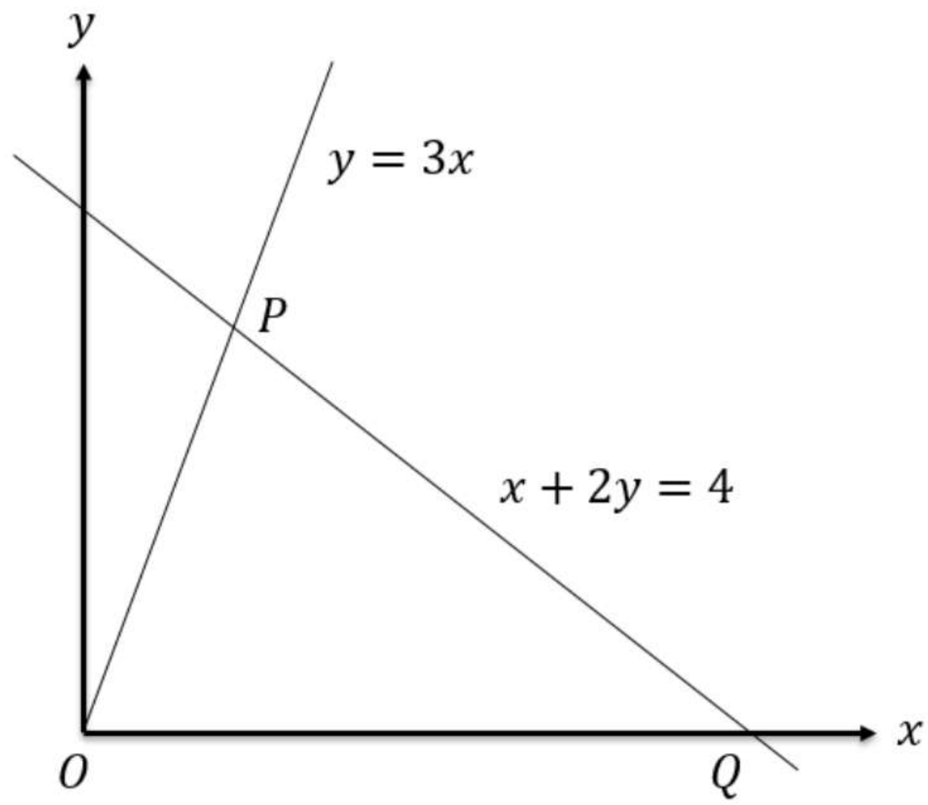
$5y - 45 = -4(x+1)$

$5y - 45 = -4x - 4$

$4x + 5y - 41 = 0$

Ex 5C

Intersection of 2 lines



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P.

a) Determine the coordinates of P.

$y = 3x$

$x + 2y = 4$

$x + 2(3x) = 4$

$x + 6x = 4$

$7x = 4$

$x = \frac{4}{7}$

$y = 3 \times \frac{4}{7}$

$= 12/7$

$P\left(\frac{4}{7}, \frac{12}{7}\right)$

b) The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the coordinate of Q.

$y = 0$

$x + 2y = 4$

$\underline{\underline{x = 4}}$

C1 Edexcel May 2013 Q6

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(*a*) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(*b*) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

-	$(-1, 3)$, $(11, 12)$		
(a)	$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{12-3}{11-(-1)}, = \dfrac{3}{4}$	M1:Correct method for the gradient A1: Any correct fraction or decimal	M1,A1
	$y - 3 = \frac{3}{4}(x+1)$ or $y - 12 = \frac{3}{4}(x - 11)$ or $y = \frac{3}{4}x + c$ with attempt at substitution to find c	Correct straight line method using either of the given points and a numerical gradient.	M1
	$4y - 3x - 15 = 0$	Or equivalent with integer coefficients (= 0 is required)	A1
	This A1 should only be awarded in (a)		
			(4)
(b)	Solves their equation from part (a) and L_2 simultaneously to eliminate one variable	Must reach as far as an equation in x only or in y only. (Allow slips in the algebra)	M1
	$x = 3$ or $y = 6$	One of $x = 3$ or $y = 6$	A1
	Both $x = 3$ and $y = 6$	Values can be un-simplified fractions.	A1
	Fully correct answers with no working can score 3/3 in (b)		
			(3)

Ex 5D