Algebraic Laws

Index Laws

$a^n \times a^m = a^{n+m}$

$a^n \div a^m = a^{n-m}$

$(a^n)^m = a^{nm}$

$a^{-n} = \frac{1}{a^n}$

$a^{1/n} = \sqrt[n]{a}$

$a^0 = 1$

Log Laws

$\log_a n = x \Leftrightarrow a^x = n$

$\ln xy = \ln x + \ln y$

$\ln \frac{x}{y} = \ln x - \ln y$

$\ln x^k = k \ln x$

$\ln \frac{1}{x} = -\ln x$

$\ln e = 1$

$\ln 1 = 0$

Factor Theorem

If $f(a)=0$, then $(x-a)$ is a factor and vice versa.

Inequalities

negating flips $<$ to $>$

Partial Fractions

$\frac{x}{(x+a)^2(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+b)}$

Geometry

Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Area of a triangle

$\frac{1}{2} ab \sin C$

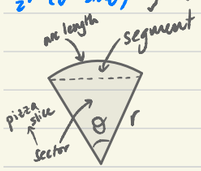
GCSE Circle Theorems

Arc Length $r\theta$

Sector Area $\frac{1}{2} r^2 \theta$

Segment Area $\frac{1}{2} r^2 (\theta - \sin\theta)$

} must be in radians



Functions

Definition

"1-to-many" not functions

Terminology

domain: input/x, range: output/y

Composite Functions

$fg(x) \rightarrow g$ then f

Inverse Functions

- domain $\leftrightarrow$ range
- reflect in $x=y$
- $x \leftrightarrow y$, rearrange

Graphs + Coordinates

Midpoint

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Gradient

$m = \frac{\Delta y}{\Delta x}$

Perpendicular Gradients

If m_1 and m_2 are perp $m_1 = -\frac{1}{m_2}$

Equation of a Line

$y - y_1 = m(x - x_1)$

The Discriminant

$b^2 - 4ac > 0$ 2 roots

$b^2 - 4ac = 0$ repeated root

$b^2 - 4ac < 0$ No real roots

Reciprocal Graphs

$y = \frac{k}{x}$ $y = -\frac{k}{x}$

$y = \frac{k}{x^2}$ $y = -\frac{k}{x^2}$

Circles

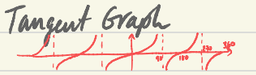
$(x-a)^2 + (y-b)^2 = r^2$

Centre (a, b), radius $= r$

Sine Graph

Cosine Graph

Tangent Graph



Transformations, $f(x)$

$f(x)+a$ $\begin{pmatrix}0\\a\end{pmatrix}$	$f(x+a)$ $\begin{pmatrix}-a\\0\end{pmatrix}$
$af(x)$ ↕ $\times a$	$f(ax)$ ↔ $\times \frac{1}{a}$
$-f(x)$ flip/reflection below x-axis reflects up	$f(-x)$ flip/reflection right of y-axis reflects left
$\lvert f(x)\rvert$	

Vectors

Position Vectors

$\overrightarrow{AB} = \underline{b} - \underline{a}$

Parallel Condition

$\underline{a}$ and $\underline{b}$ are parallel

if $\underline{a} = \lambda \underline{b}$, λ is a constant

Magnitude

If $\underline{a} = x\underline{i} + y\underline{j} + z\underline{k}$

then $|\underline{a}| = \sqrt{x^2+y^2+z^2}$

Unit Vector

$\hat{\underline{a}} = \frac{1}{|\underline{a}|}\underline{a}$

Angles with Axes

$\cos\theta_x = \frac{x}{|\underline{a}|}$

$\cos\theta_y = \frac{y}{|\underline{a}|}$

$\cos\theta_z = \frac{z}{|\underline{a}|}$

Binomial Expansion

Binomial Coefficient

F $^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Formula

F $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$ for $|x|<1$

Other form

$(a+bx)^n = a^n\left(1+\frac{bx}{a}\right)^n$ for $\left|\frac{bx}{a}\right|<1$

Series

Arithmetic n^{th} term

$u_n = a + (n-1)d$

Arithmetic Sum

♥F $S_n = \frac{n}{2}(2a + (n-1)d)$

♥F $S_n = \frac{n}{2}(a + L)$

Geometric n^{th} term

$u_n = ar^{n-1}$

Geometric Sums

♥F $S_n = \frac{a(1-r^n)}{1-r}$, $S_\infty = \frac{a}{1-r}$

Increasing Sequence

... if $u_{n+1} > u_n$ for all n

Decreasing Sequence

... if $u_{n+1} < u_n$ for all n

Periodic Sequence

... if $u_{n+k} = u_n$ for all n

... and its period/order is k

Trigonometry

Radians

$2\pi = 360°$ $\pi = 180°$ $\frac{\pi}{2} = 90°$

$\frac{\pi}{3} = 60°$ $\frac{\pi}{4} = 45°$ $\frac{\pi}{6} = 30°$

Small Angle Approximations

F $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$

Exact Trig Values

$\sin 30 = \frac{1}{2}$	$\cos 30 = \frac{\sqrt{3}}{2}$	$\tan 30 = \frac{1}{\sqrt{3}}$
$\sin 60 = \frac{\sqrt{3}}{2}$	$\cos 60 = \frac{1}{2}$	$\tan 60 = \sqrt{3}$
$\sin 45 = \frac{1}{\sqrt{2}}$	$\cos 45 = \frac{1}{\sqrt{2}}$	$\tan 45 = 1$

Tangent Definition

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Solving Equations

$\sin\theta = \sin(180 - \theta) \pm 360$

$\cos\theta = \cos(360 - \theta) \pm 360$

$\tan\theta \quad \pm 180$

Reciprocal Trig Functions

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\frac{1}{\operatorname{cosec}\theta} = \sin\theta$

$\sec\theta = \frac{1}{\cos\theta}$, $\frac{1}{\sec\theta} = \cos\theta$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Co-function

$\sin\theta = \cos(90-\theta)$, $\cos\theta = \sin(90-\theta)$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Addition Formulae

♥F $\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$

♥F $\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$

♥F $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Formulae

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Rearranged Double Angle

$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

} used in integration

Harmonic Identity ($R\cos(\theta \pm \alpha)$)

If $R\cos\alpha = a$, $R\sin\alpha = b$

$R = \sqrt{a^2+b^2}$ $\tan\alpha = \frac{b}{a}$

Differentiation

First Principles

F $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

First Derivatives

$f'(x) < 0$ decreasing $f'(x) = 0$ stationary $f'(x) > 0$ increasing

Second Derivatives

$f''(x) < 0$ concave/max $f''(x) = 0$ point of inflection $f''(x) > 0$ convex/min

Functions → derivatives

$ax^n \rightarrow anx^{n-1}$

$a^x \rightarrow \ln a \cdot a^x$

$e^x \rightarrow e^x$

$\ln x \rightarrow \frac{1}{x}$

$\sin x \rightarrow \cos x$

$\cos x \rightarrow -\sin x$

♥F $\tan x \rightarrow \sec^2 x$

♥F $\sec x \rightarrow \sec x \tan x$

♥F $\cot x \rightarrow -\operatorname{cosec}^2 x$

♥F $\operatorname{cosec} x \rightarrow -\operatorname{cosec} x \cot x$

$f(blah) \xrightarrow{\text{CHAIN}} f'(blah) \times blah'$

$uv \xrightarrow{\text{PRODUCT}} uv' + u'v$ $u \times v$

♥F $\frac{u}{v} \xrightarrow{\text{QUOTIENT}} \frac{vu' - uv'}{v^2}$ $u' \propto v'$

$f(y) \xrightarrow{\text{IMPLICIT}} f'(y) \times \frac{dy}{dx}$

Reciprocals

$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

Parametric

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Connected Rates of Change

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

"rate" means $\frac{}{dt}$

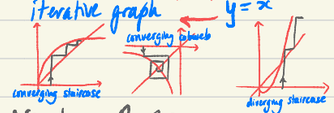
Numerical Methods

Root

Change in sign and function continuous over the interval

Staircase/Cobwebs

iterative graph $y = x$



Newton-Raphson

F $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ ← fails if $=0$

Integration

Functions → integrals (+c)

$ax^n \rightarrow \frac{a}{n+1}x^{n+1}$

$e^x \rightarrow e^x$

$\frac{1}{x} \rightarrow \ln|x|$

$\cos x \rightarrow \sin x$

$\sin x \rightarrow -\cos x$

♥F $\sec^2 x \rightarrow \tan x$

$\sec x \tan x \rightarrow \sec x$

$\operatorname{cosec}^2 x \rightarrow -\cot x$

$\operatorname{cosec} x \cot x \rightarrow -\operatorname{cosec} x$

$f'(ax+b) \rightarrow \frac{1}{a}f(ax+b)$

Plus more in formula book

Reverse Chain Rule

Consider then Scale

Substitution

Substitution/Limits/Derivative of u

"Sir loves derivatives"

Integration by Parts

♥F $\int uv'\,dx = uv - \int u'v\,dx$ $u \times v'$

$\ln x$ takes priority for u

Parametric Integration

$\int y \frac{dx}{dt}\,dt$, change limits

Tips: "Sir really adores partial fractions"

a) Split the numerator

$\frac{x-1}{x} = \frac{x}{x} - \frac{1}{x}$

b) Reverse Chain Rule

$\int \frac{x}{x^2-3}\,dx$, $\int \cos x \sin^n x\,dx$

c) Algebraic Division

$\frac{x+1}{x-1}$ $\frac{x^2}{x+1}$

d) Partial Fractions

$\frac{2}{(x+3)(x+2)}$ $\frac{1}{x^2+x}$

Trapezium Rule

F $\int y\,dx \approx \frac{1}{2}h(\text{first} + \text{last} + 2(\text{middle}))$

Key: F in the formula booklet

♥F in the formula booklet but recommend learning by heart