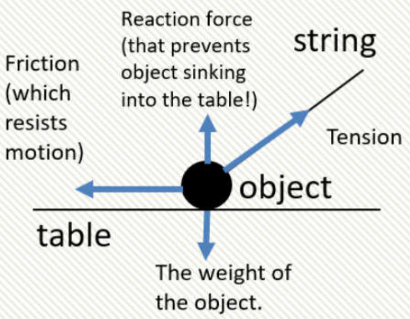
Chapter 8: Modelling in Mechanics

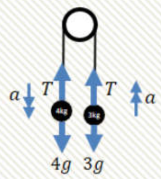
Mechanics, broadly speaking, concerns motion, forces, and how the two interrelate.
This chapter just gives you an overview of what you'll be covering in Year 1 and how it all links together.

Forces

You will later encounter force diagrams. This considers the forces acting at a particular point. Some forces you might consider…



- Forces can be considered as vectors.
- The **magnitude** of the force vector gives the 'size' of the force.
- We often **consider forces in a particular direction**. e.g. If the object above is stationary, the forces left must equal the force right, and forces up equal forces down (Newton's 1st Law).
- Often we need to consider the forces at multiple different points if objects are connected, e.g. with pulleys:



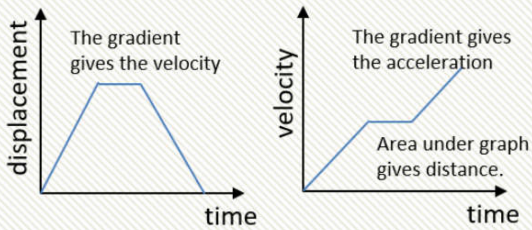
The bridge!

$F = ma$

Newton's 2nd Law allows us to connect the force world (F) with the motion world (acceleration a) if the object is moving.

Motion

At GCSE you may have encountered displacement-time and velocity-time graphs:



Given **constant acceleration** we have 5 quantities of motion ("$suvat$"):

$\boldsymbol{s}$ = displacement
$\boldsymbol{u}$ = initial velocity
$\boldsymbol{v}$ = final velocity
$\boldsymbol{a}$ = acceleration
$\boldsymbol{t}$ = time

which we will see are linked by various equations:

$$s = ut + \frac{1}{2}at^2$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$v^2 = u^2 + 2as$$
$$v = u + at$$

If the **acceleration is not constant**, we can specify displacement/velocity/acceleration as a function of time and differentiate/integrate to change between them.

$s = 2t^3 + 3t \quad \rightarrow \quad v = \frac{ds}{dt} = 6t^2 + 3$

Modelling Assumptions

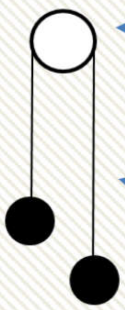
As with many areas of applied maths, we often have to make various modelling assumptions, to make the maths cleaner or to use well-known mathematical approaches.
Here are common modelling assumptions often made in Mechanics:

Particle

Dimensions of object are negligible
Means: Mass of object concentrated at single point. Rotational forces/air resistance can be ignored.

Rough/Smooth surface

Means: Objects in contact with surface does/does not experience friction.



Smooth/light pulley

No friction.
Means: <u>Tension the same in string either side of pulley</u>. Pulley has no mass.

Fro Tip:
Particularly make note of underlined text!

Inextensible string

String does not stretch under load.
Means: <u>Acceleration the same</u> in any connected objects.

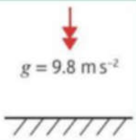
Rod

One dimension is negligible, like a pole or beam.
Means: Mass is concentrated along line. Rigid.

Peg/Support

A support from which a body can be suspended or rested.
Means: Dimensionless and fixed. Can be rough or smooth depending on question.

These are some common models and modelling assumptions that you need to know.

Model	Modelling assumptions
Particle – Dimensions of the object are negligible.	• mass of the object is concentrated at a single point • rotational forces and air resistance can be ignored
Rod – All dimensions but one are negligible, like a pole or a beam.	• mass is concentrated along a line • no thickness • rigid (does not bend or buckle)
Lamina – Object with area but negligible thickness, like a sheet of paper.	• mass is distributed across a flat surface
Uniform body – Mass is distributed evenly.	• mass of the object is concentrated at a single point at the geometrical centre of the body – the **centre of mass**
Light object – Mass of the object is small compared to other masses, like a string or a pulley.	• treat object as having zero mass • tension the same at both ends of a light string
Inextensible string – A string that does not stretch under load.	• acceleration is the same in objects connected by a taut inextensible string
Smooth surface	• assume that there is no friction between the surface and any object on it
Rough surface – If a surface is not smooth, it is rough.	• objects in contact with the surface experience a frictional force if they are moving or are acted on by a force
Wire – Rigid thin length of metal.	• treated as one-dimensional
Smooth and light pulley – all pulleys you consider will be smooth and light.	• pulley has no mass • tension is the same on either side of the pulley
Bead – Particle with a hole in it for threading on a wire or string.	• moves freely along a wire or string • tension is the same on either side of the bead
Peg – A support from which a body can be suspended or rested.	• dimensionless and fixed • can be rough or smooth as specified in question
Air resistance – Resistance experienced as an object moves through the air.	• usually modelled as being negligible
Gravity – Force of attraction between all objects. Acceleration due to gravity is denoted by g. 	• assume that all objects with mass are attracted towards the Earth • Earth's gravity is uniform and acts vertically downwards • g is constant and is taken as 9.8 m s^{-2}, unless otherwise stated in the question

SI units

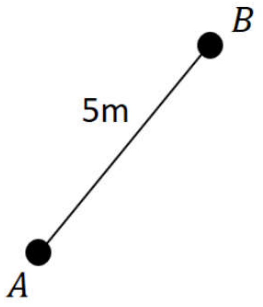
The SI units are a standard system of units, used internationally ("Système International d'unités"). These are the ones you will use:

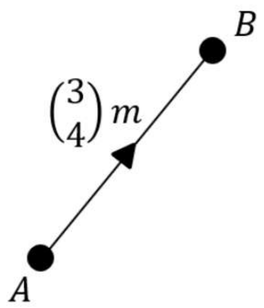
Quantity	**Unit**	**Symbol**
Mass	kilogram	kg
Length/Displacement	metre	m
Time	Seconds	s
Speed/Velocity	metres per second	m s^{-1}
Acceleration	metres per second per second	m s^{-2}
Force/Weight	Newton	N (= kg m s^{-2})

This unit is consistent with force being mass × acceleration

Vectors ↔ Scalars

In Mechanics you will often need to convert to/from the scalar form of a quantity and the vector form.





✎ **A scalar quantity has magnitude (i.e. size) only.**
The 5m is a distance.
The value is always positive.

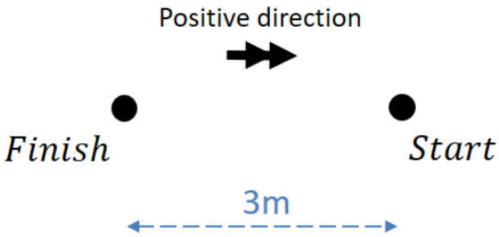
✎ **A vector quantity also has direction.**
The vector equivalent of distance is displacement.

Scalar Form	Vector Form
Distance	
Speed	

Other quantities which can be vectors or scalars:
Force, acceleration

Quantities which can only be scalars:
Time, mass

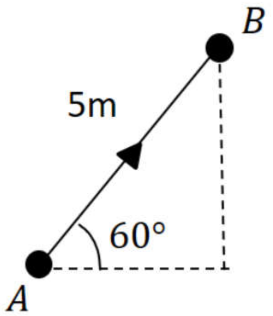
Note: 1-dimensional vectors are still different from scalars. Consider the displacement on a 1-dimensional line in a particular direction. If we'd gone backwards 3 units…



Distance:
Displacement:

Note: we don't write the brackets around 1D vectors. So 1D vectors look like scalars, except they're allowed to be positive or negative.

Scalar Form	Vector Form

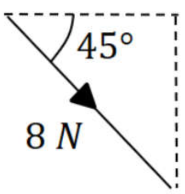


To convert to vector form, just use basic trigonometry to find the x-change and y-change.

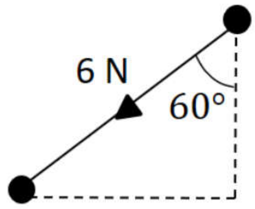
Speed Tip: If x is the magnitude, use $x\cos\theta$ for the side adjacent to the angle and $x\sin\theta$ for the side opposite it.

Velocity:
$$\begin{pmatrix}5\\-12\end{pmatrix} ms^{-1}$$

To convert scalar form, just find the **magnitude** of the vector using Pythagoras.

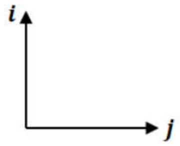


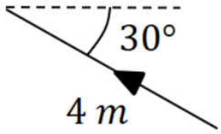
Scalar Form	Vector Form



$$(6\boldsymbol{i} - 8\boldsymbol{j})\ ms^{-2}$$

Recall from Pure Year 1 that $6\boldsymbol{i} - 8\boldsymbol{j}$ is another way of writing $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the positive x and y directions.





A man walks from A to B and then from B to C.
His displacement from A to B is $6\boldsymbol{i} + 4\boldsymbol{j}$ m.
His displacement from B to C is $5\boldsymbol{i} - 12\boldsymbol{j}$ m.
(a) What is the magnitude of the displacement from A to C?
(b) What is the total distance the man has walked in getting from A to C.

A raccoon has a velocity of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}\ ms^{-1}$.
Determine the angle the trajectory of the raccoon makes with the unit vector $\boldsymbol{i}$.

Raccoon

Ex 8D

Exercise 8D

1 a $2.1\ \mathrm{m\,s^{-1}}$ **b** $500\ \mathrm{m}$ **c** $-1.8\ \mathrm{m\,s^{-1}}$
d $-2.7\ \mathrm{m\,s^{-1}}$ **e** $-750\ \mathrm{m}$ **f** $2.5\ \mathrm{m\,s^{-1}}$
2 a $15.6\ \mathrm{m\,s^{-1}}$ **b** $39.8°$
3 a $5\ \mathrm{m\,s^{-2}}$ **b** $143°$
4 a $15.3\ \mathrm{m}$ **b** $24.3\ \mathrm{m}$ **c** $78.7°$