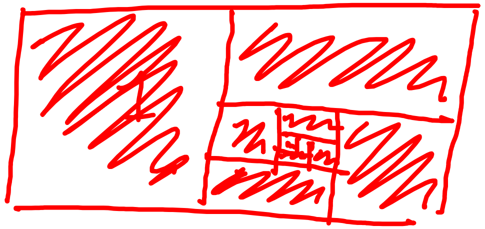
Sum to Infinity - Divergent vs Convergent

What can you say about the sum of each series up to infinity?



✗ $1 + 2 + 4 + 8 + 16 + \ldots$ Divergent

✗ $1 - 2 + 3 - 4 + 5 - 6 + \ldots$ Divergent

✓ $1 + 0.5 + 0.25 + 0.125 + \ldots$ Convergent

$$S_\infty = \frac{1}{1-\frac{1}{2}} = \underline{\underline{2}}$$

- The infinite series will converge provided that $-1 < r < 1$ (which can be written as $|r| < 1$), because the terms will get smaller.

- Provided that $|r| < 1$, what happens to r^n as $n \to \infty$?

$r^n \to 0$

✎ A geometric series is convergent if $|r| < 1$.

- How therefore can we use the $S_n = \frac{a(1-r^n)}{1-r}$ formula to find the sum to infinity, i.e. S_∞?

$$\frac{a(1-0)}{1-r}$$

✎ For a <u>convergent</u> geometric series,

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

$\boldsymbol{a} =$ 1 $\quad \boldsymbol{r} =$ $\frac{1}{2}$ $\quad \boldsymbol{S_\infty} =$ $\frac{1}{1-\frac{1}{2}}$

$= 2$

$27, -9, 3, -1, \ldots$

$a = 27 \quad r = -\frac{1}{3} \quad S_\infty = \frac{27}{1 - -\frac{1}{3}} = \frac{81}{4} = 20.25$

$p, p^2, p^3, p^4, \ldots$

where $-1 < p < 1$

$a = p \quad r = p \quad S_\infty = \underline{\underline{\frac{p}{1-p}}}$

$p, 1, \frac{1}{p}, \frac{1}{p^2}, \ldots$

$a = p \quad r = \frac{1}{p} \quad S_\infty = \frac{p}{1-\frac{1}{p}} \begin{matrix} \times p \\ \times p \end{matrix} = \underline{\underline{\frac{p^2}{p-1}}}$

The fourth term of a geometric series is 1.08 and the seventh term is 0.23328.

a) Show that this series is convergent.

b) Find the sum to infinity of this series.

$|r|<1$ if convergent

find r

find a then use $S_\infty = \frac{a}{1-r}$

a)

$U_4 = 1.08$ $\quad$ $U_7 = 0.23328$

$1.08 = ar^3$ $\quad$ $0.23328 = ar^6$

$$\frac{0.23328}{1.08} = \frac{ar^6}{ar^3}$$

$0.216 = r^3$

$r = 0.6$

Because $|r|<1$,

the series is convergent.

b) $1.08 = a \times 0.6^3$

$a = 5$

$5 + 3 + 1.8 + 1.08 + 0.648 ...$

$$S_\infty = \frac{5}{1-0.6}$$

$= \underline{\underline{12.5}}$

For a geometric series with first term a and common ratio r, $S_4 = 15$ and $S_\infty = 16$.
a) Find the possible values of r.
b) Given that all the terms in the series are positive, find the value of a.

a) $S_n = \frac{a(1-r^n)}{1-r}$ $\quad \cancel{u_n = ar^{n-1}}$

$S_4 = 15$, $n = 4$

$15 = \frac{a(1-r^4)}{1-r}$

$S_\infty = 16$

$16 = \frac{a}{1-r}$

$15 = 16(1-r^4)$

$\frac{15}{16} = 1 - r^4$

$r^4 = \frac{1}{16}$

$r = \pm\frac{1}{2}$

b) $r = \frac{1}{2}$

$16 = \frac{a}{1-\frac{1}{2}}$

$a = 16 \times \frac{1}{2}$

$a = 8$

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

$$S_\infty = \frac{8}{7} \times S_6$$

$$\frac{\cancel{a}}{\cancel{1-r}} = \frac{8}{7} \times \frac{\cancel{a}(1-r^6)}{\cancel{1-r}}$$

$$\frac{7}{8} = 1 - r^6$$

$$r^6 = 1 - \frac{7}{8}$$

$$()^{1/3} \quad \sqrt[3]{\ }$$

$$r^6 = \frac{1}{8} \quad \left(\frac{1}{8}\right)^{1/3}$$

$$r^2 = \frac{1}{2}$$

$$r = \pm\frac{1}{\sqrt{2}}$$

$$\underline{\underline{k = 2}}$$

Your Turn

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6. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(*a*) the common ratio, **(2)**

(*b*) the first term, **(2)**

(*c*) the sum to infinity, **(2)**

(*d*) the smallest value of n for which the sum of the first n terms of the series exceeds 1000. **(4)**

Ex 3E