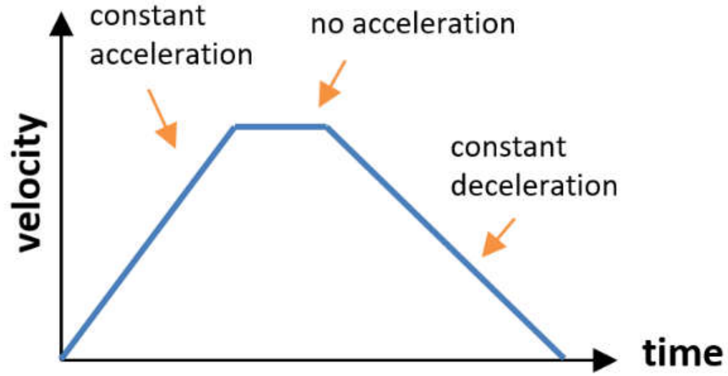
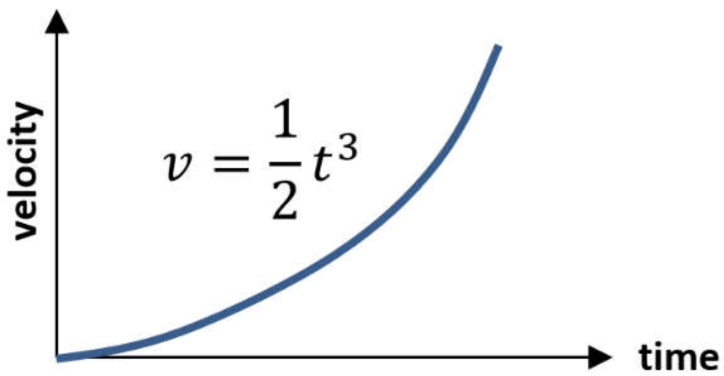
Variable Acceleration (Year 1)

Functions of time

Up to now, the acceleration has always been constant in any particular period of time... but this isn't realistic, as there should be a **smooth change** from acceleration to constant velocities.



It is possible to specify either the displacement, velocity or acceleration as any function of time (i.e. an expression in terms of t). This allows the acceleration to constantly change.



The velocity-time graph of a body is shown above, where $v = \frac{1}{2}t^3$.

(a) What is the velocity after 4 seconds have elapsed? $t=4$

(b) How many seconds have elapsed when the velocity of the body is 108 ms^{-1}?

$v=108$

a) $t=4$

$v = \frac{1}{2} \times 4^3$

$v = \underline{\underline{32ms^{-1}}}$.

b) $v=108$

$v = \frac{1}{2}t^3$

$108 = \frac{1}{2}t^3$

$216 = t^3$

$\underline{\underline{t = 6}}$.

A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 2t^2 - 16t + 24$. Find

(a) The initial velocity $t=0$

(b) The values of t when the body is instantaneously at rest. $v=0$

(c) The value of t when the velocity is 64 ms^{-1}. $v=64$

(d) The greatest speed of the body in the interval $0 \le t \le 5$.

a) $t=0$

$v = 2\times0^2 - 16\times0 + 24$

$\underline{v=24}$

b) $0 = 2t^2 - 16t + 24$

$\underline{t=6}$ and $\underline{t=2}$

c) $v=64$

$64 = 2t^2 - 16t + 24$

$0 = 2t^2 - 16t - 40$

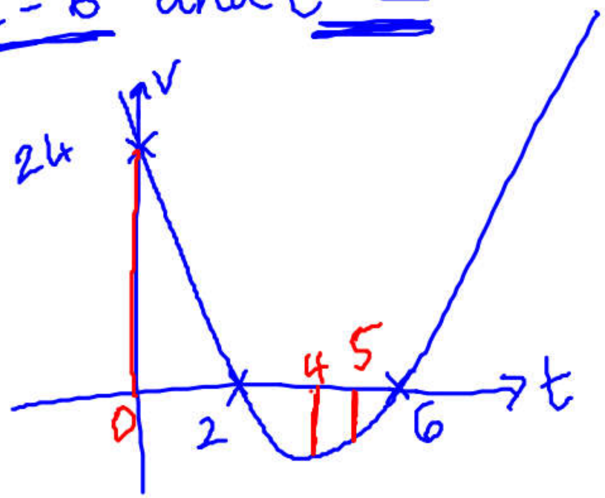
$0 = t^2 - 8t - 20$

$0 = (t-10)(t+2)$

$\underline{t=10}$, $\cancel{t=-2}$

$t>0$

d)



$t=4$

$v = 2\times4^2 - 16\times4 + 24$

$v = 32 - 64 + 24$

$= \underline{-8}$

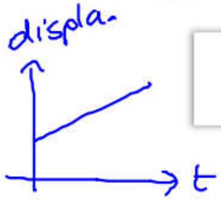
So max speed is $\underline{24\text{ ms}^{-1}}$

Ex 11A

Using Differentiation

Earlier on, we saw that velocity v is the rate of change of displacement s (i.e. the gradient). But in Pure, we know that we can use differentiation to find the gradient function:

the rate of change of displacement. the rate of change of velocity.

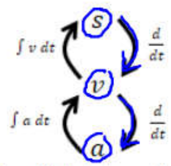


$$v = \frac{ds}{dt}$$ metres / secs and similarly... $$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

velocity is the rate of change of displacement

acceleration is the rate of change of velocity

Memory Tip: I picture interchanging between s, v, a as differentiating to go downwards and integrating to go upwards:



(We will do integration a bit later)

A particle P is moving on the x-axis. At time t seconds, the displacement x metres from O is given by $x = t^4 - 32t + 14$. Find:

(a) the velocity of P when $t = 3$

(b) The value of t when P is instantaneously at rest $v=0$

(c) The acceleration of P when $t = 1.5$

a) $x = t^4 - 32t + 14$

$v = \frac{dx}{dt} = 4t^3 - 32$

$\boxed{v = 4t^3 - 32}$

$t = 3$

$v = 4\times3^3 - 32 = \underline{76\text{ms}^{-1}}$

b) $v = 0$

$0 = 4t^3 - 32$

$8 = t^3$

$\underline{t = 2}$.

c) $v = 4t^3 - 32$

$a = \frac{dv}{dt} = 12t^2$

$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

$t = 1.5$ $a = 12\times1.5^2$

$= \underline{27\text{ms}^{-2}}$

disp. $x = t^4 - 32t + 14$

vel. $\frac{dx}{dt} = 4t^3 - 32$

acc. $\frac{d^2x}{dt^2} = 12t^2$

$a = \_\ \_\ \_\ \_$.

A cat's displacement from a house, in metres, is $t^3 - \frac{3}{2}t^2 - 36t$ where t is in seconds.

(a) Determine the velocity of the cat when $t = 2$.

(b) At what time will the cat be instantaneously at rest? $v = 0$

(c) What is the cat's acceleration after 5 seconds?

a)

$$s = t^3 - \frac{3}{2}t^2 - 36t$$

$$\frac{ds}{dt} = v = 3t^2 - 3t - 36$$

$t = 2$, $v = 3 \times 2^2 - 3 \times 2 - 36$

$= -30\,ms^{-1}$

b) $0 = 3t^2 - 3t - 36$

$t = 4$, $\cancel{t = -3}$

$$s = t^3 - \frac{3}{2}t^2 - 36t$$

$$v = 3t^2 - 3t - 36$$

$$a = 6t - 3$$

c) $\frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 3$

$a = 6t - 3$

$t = 5$

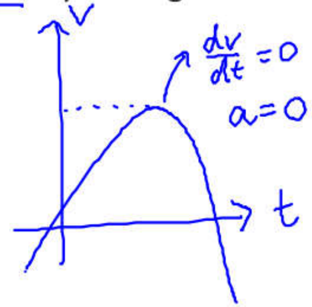
$a = 6 \times 5 - 3$

$= 27\,ms^{-2}$

Ex 11B

Maxima and Minima Problems

Recall from Pure that at minimum/maximum points, the gradient is 0. We could therefore for example find where the velocity is minimum/maximum by finding when $\frac{dv}{dt} = 0$ (i.e. when the acceleration is 0).



A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight ~~lien~~ Line before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 0.6t + 0.4t^2 - 0.2t^3, \qquad 0 \le t \le 3$$

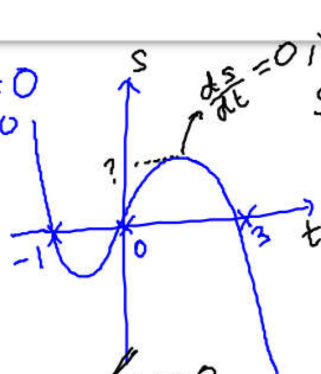
(a) Justify the restriction $0 \le t \le 3$ Draw a graph

(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

$0.6t + 0.4t^2 - 0.2t^3 = 0$

$-0.2t^3 + 0.4t^2 + 0.6t = 0$

Roots are $t = 0, 3, -1$



$s \geq 0$ for the yo-yo, so when $0 \le t \le 3$, $s \geq 0$. s cannot be negative

b) max. s is when $\frac{ds}{dt} = 0$ / $v = 0$

$s = 0.6t + 0.4t^2 - 0.2t^3$

$\frac{ds}{dt} = 0.6 + 0.8t - 0.6t^2$

$0.6 + 0.8t - 0.6t^2 = 0$

$t = 1.86851$

$t = -0.535$ X

sub into $s = 0.6t + 0.4t^2 - 0.2t^3$

$s = \underline{\underline{1.21\text{ m}}}$ (3sf)

A dolphin escapes from Seaworld and its velocity as it speeds away from the park, is $t^3 - 9t^2 - 48t + 500$ (in ms^{-1}), until it reaches its maximum velocity, and then subsequently remains at this velocity.

(a) When does the dolphin reach its maximum velocity? $a = 0$ or $\frac{dv}{dt} = 0$

(b) What is this maximum velocity?

a)

$$v = t^3 - 9t^2 - 48t + 500$$

$$\frac{dv}{dt} = a = 3t^2 - 18t - 48$$

$$0 = 3t^2 - 18t - 48$$

$$0 = t^2 - 6t - 16$$

$$0 = (t-8)(t+2)$$

$$\underline{\underline{t = 8}}, \ \cancel{t = -2}$$

$$t = 8$$

$$v = 8^3 - 9(8)^2 - 48(8) + 500$$

$$= \underline{\underline{52\,ms^{-1}}}$$

Your Turn

Edexcel M2 June 2013 Q3a,b

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(a) the times when P is instantaneously at rest, **(3)**

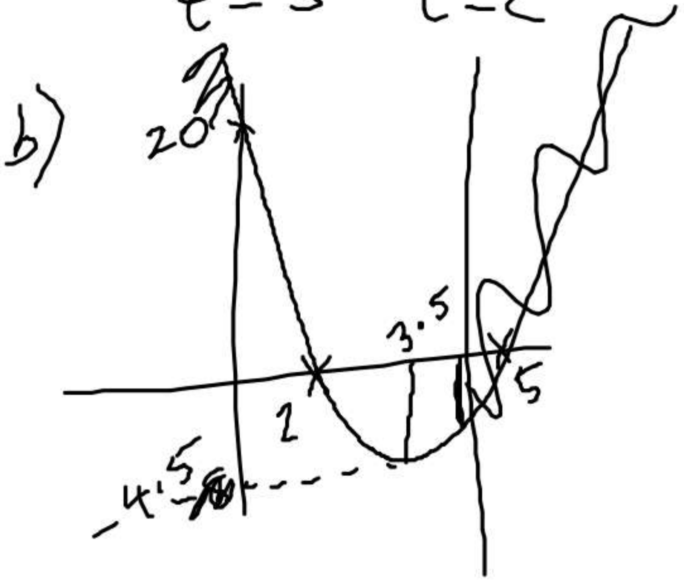
(b) the greatest speed of P in the interval $0 \leq t \leq 4$, **(5)**

a	$v = 0 = 2t^2 - 14t + 20$	M1
	$= 2(t-2)(t-5)$	M1
	$t = 2$ or $t = 5$	A1
b	$(t = 0)$, $v = 20$ (m s^{-1})	B1
	$a = 4t - 14 = 0$	M1
	$t = \frac{7}{2}$, $\quad v = 2 \times \frac{3}{2} \times \frac{-3}{2} = \frac{-9}{2}$	M1A
		A1
	Max speed = 20 ms^{-1}	

a) $0 = 2t^2 - 14t + 20$

$t = 5 \quad t = 2$

b)



$v = 2t^2 - 14t + 20 \qquad \frac{dv}{dt} = 0$

$\frac{dv}{dt} = 4t - 14$

$4t = 14$

$t = 3.5$

$t = 3.5 \quad v = 2 \times 3.5^2 - 14 \times 3.5 + 20$

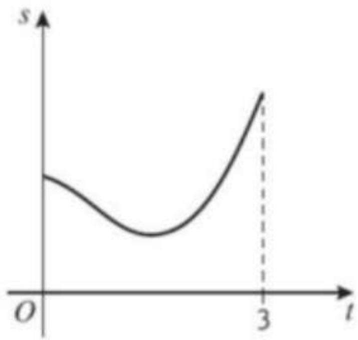
$= -4.5$

max speed is $20 ms^{-1}$

Ex 11C

1 A particle P moves in a straight line such that its distance, s m, from a fixed point O at time t is given by:

$$s = 0.4t^3 - 0.3t^2 - 1.8t + 5,\ 0 \leqslant t \leqslant 3$$



The diagram shows the displacement–time graph of the motion of P.

a Determine the time at which P is moving with minimum velocity.

$a = 0$

b Find the displacement of P from O at this time.

c Find the velocity of P at this time.

Diff. twice for (a)

$\frac{dv}{dt} = 0 \quad a = 0$

4 A particle P moves along the x-axis. Its velocity, v m s^{-1} in the positive x-direction, at time t seconds is given by:

$$v = 2t^2 - 3t + 5,\ t \geqslant 0$$

a Show that P never comes to rest.

b Find the minimum velocity of P.

$v = 2t^2 - 3t + 5 \quad t \geqslant 0$

$a = 4t - 3$