A Level · Edexcel · Maths

4 hours 43 questions

4.2 General Binomial Expansion (A Level only)

Easy (10 questions)	/44
Medium (11 questions)	/55
Hard (11 questions)	/58
Very Hard (11 questions)	/55
Total Marks	**/212**

Scan here to return to the course
or visit savemyexams.com

Easy Questions

1 Find, in ascending powers of x, the binomial expansion of

$$(1-x)^{-1}$$

up to and including the term in x^2.

(2 marks)

2 (a) Find the first three terms, in ascending powers of x, in the binomial expansion of

$$(1+x)^{-2}$$

(2 marks)

(b) State the values of x for which your expansion in part (a) is valid.

(1 mark)

3 (a) Show that

$$\sqrt{4-4x} = 2(1-x)^{\frac{1}{2}}$$

(2 marks)

(b) Hence find, in ascending powers of x, the first three terms in the binomial expansion of

$$\sqrt{4-4x}$$

(2 marks)

(c) Using $x = 0.02$, use your expansion from part (b) to find an approximation to $2\sqrt{0.98}$.

(2 marks)

4 Find, in ascending powers of x, the binomial expansion of

$$(1+2x)^{-\frac{1}{2}}$$

up to and including the term in x^3.

(4 marks)

5 (a) Find the first three terms, in ascending powers of x, in the binomial expansion of

$$(1-\frac{1}{2}x)^{\frac{1}{3}}$$

(4 marks)

(b) State the values of x for which your expansion in part (a) is valid.

(1 mark)

6 Find the coefficient of the term in x^2 in the binomial expansion of

$$(1-3x)^{-3}$$

(2 marks)

7 The function $\mathrm{f}(x)$ is given by

$$\mathrm{f}(x)=(1-px)^{-4}$$

where p is an integer.

Find the coefficient of the term in x^3 in the binomial expansion of $\mathrm{f}(x)$, in terms of p.

(2 marks)

8 (a) Given that x is small such that x^3 and higher powers of x can be ignored show that

$$(1-\frac{1}{3}x)^{-2} \approx 1+\frac{2}{3}x+\frac{1}{3}x^2$$

(3 marks)

(b) Using a suitable value of x in the result from part (a), find an approximation for the value of $(0.94)^{-2}$.

(2 marks)

9 It is given that

$$f(x)=\sqrt{1+ax} \quad and \quad g(x)=\sqrt[3]{1-ax}$$

where a is a non-zero constant.

In their binomial expansions, the coefficient of the x^2 term for $f(x)$ is equal to the coefficient of the x term for $g(x)$.

Find the value of a.

(5 marks)

10 (a) Show, as partial fractions, that

$$\frac{5-x}{(1+x)(1-x)} \equiv \frac{3}{1+x} + \frac{2}{1-x}$$

(3 marks)

(b) Find the first three terms, in ascending powers of x, of the binomial expansion of

(i) $3(1+x)^{-1}$,

(ii) $2(1-x)^{-1}$

(4 marks)

(c) Hence show that the first three terms, in ascending powers of x, in the binomial expansion of

$$\frac{5-x}{(1+x)(1-x)}$$

are

$$5 - x + 5x^2$$

(2 marks)

(d) Write down the values of x for which this expansion converges.

(1 mark)

Medium Questions

1 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{(1-x)^2}$$

up to and including the term in x^3.

(2 marks)

2 (a) Find the first three terms, in ascending powers of x, in the binomial expansion of

$$\sqrt{1+2x}$$

(2 marks)

(b) State the values of x for which your expansion in part (a) is valid.

(1 mark)

(c) Using a suitable value of x, use your expansion from part (a) to estimate $\sqrt{1.06}$, giving your answer to 3 significant figures.

(1 mark)

3 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{(4+8x)^2}$$

up to and including the term in x^3.

(3 marks)

4 (a) Use the binomial expansion to show that the first three terms in the expansion of $(1+2x)^{-3}$ are $1-6x+24x^2$.

(3 marks)

(b) Hence, or otherwise, find the expansion of $(1+x)(1+2x)^{-3}$ up to and including the term in x^2.

(2 marks)

5 (a) The function $\mathrm{f}(x)$ is given by

$$f(x) = \sqrt{4 - sx}$$

where s is an integer.

(i) Find the coefficient of the term in x in the binomial expansion of $\mathrm{f}(x)$, in terms of s.

(ii) Find the coefficient of the term in x^2 in the binomial expansion of $\mathrm{f}(x)$, in terms of s.

(2 marks)

(b) In the binomial expansion of $\mathrm{f}(x)$, the coefficient of the term in x is equal to the coefficient of the term in x^2.
Find the value of s.

(2 marks)

6 (a) The functions $f(x)$ and $g(x)$ are given as follows

$$f(x) = \left(1 - \frac{1}{2}x\right)^{\frac{1}{2}} \qquad g(x) = (2 + x)^{-2}$$

(i) Expand $f(x)$, in ascending powers of x up to and including the term in x^2.

(ii) Find the values for x for which the expansion is valid.

(3 marks)

(b) (i) Expand $g(x)$, in ascending powers of x up to and including the term in x^2.

(ii) Find the values for x for which the expansion is valid.

(3 marks)

(c) (i) Find the expansion of $\frac{\sqrt{1 - \frac{1}{2}x}}{(2 + x)^2}$ in ascending powers of x, up to and including the term in x^2.

(ii) Find the values for x for which the expansion is valid.

(2 marks)

7 In the expansion of $\left(1 - \frac{1}{4}x\right)^n$, where n is a negative integer, the coefficient of the term in x^2 is $\frac{3}{8}$.

Find the value of n.

(3 marks)

8 (a) Express $\frac{2}{(1-x)(1+x)}$ in partial fractions.

(3 marks)

(b) Use the binomial expansion to find the first three terms, in ascending powers of x, in each of $(1-x)^{-1}$ and $(1+x)^{-1}$

(2 marks)

(c) Hence show that $\frac{2}{(1-x)(1+x)} \approx 2+2x^2$

(2 marks)

(d) Write down the values of x for which your expansion in part (c) converges.

(1 mark)

9 (a) Given that x is small such that x^3 and higher powers of x can be ignored show that

$$\left(1-\frac{1}{3}x\right)^{-1}(2-x)^{-2} \approx \frac{1}{4}+\frac{1}{3}x+\frac{43}{144}x^2$$

(3 marks)

(b) For which values of x is the approximation in part (a) valid?

(2 marks)

(c) (i) Use your calculator to find the exact fraction of $\left(1-\frac{1}{3}x\right)^{-1}(2-x)^{-2}$ when $x=0.5$

(ii) Use your calculator to find the fraction from the approximation $\frac{1}{4}+\frac{1}{3}x+\frac{43}{144}x^2$ when $x=0.5$

(iii) Find the percentage error in the approximation, giving your answer to two decimal places.

(3 marks)

10 It is given that

$$f(x)=\sqrt{9+px} \quad \text{and} \quad g(x)=\sqrt[4]{16+px}$$

In their binomial expansions, the coefficient of the x^2 term for $\mathrm{f}(x)$ is equal to the coefficient of the x term for $g(x)$.

Find the value of p.

(3 marks)

11 (a) Express $\frac{12-x}{(x+2)(3-x)}$ in partial fractions.

(2 marks)

(b) Using binomial expansions, up to and including terms in x^2 show that

$$\frac{12-x}{(x+2)(3-x)} \approx 2-\frac{1}{2}x+\frac{5}{12}x^2$$

(3 marks)

(c) Explain why the approximation in part (b) is only valid for $|x|<2$.

(2 marks)

Hard Questions

1 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{(1-2x)^3}$$

up to and including the term in x^3.

(2 marks)

2 (a) Use the first three terms, in ascending powers of x in the binomial expansion of

$$(1+4x)^{\frac{1}{3}}$$

to estimate the value of $\sqrt[3]{1.2}$, giving your answer to three significant figures.

(3 marks)

(b) Explain why your approximation in part (a) is valid.

(1 mark)

3 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{(4+x)^3}$$

up to and including the term in x^3.

(4 marks)

4 (a) Use the binomial expansion to expand $\left(1-\frac{1}{2}x\right)^{\frac{1}{3}}$ up to and including the term in x^2.

(2 marks)

(b) Hence, or otherwise, expand $(1-x)\left(1-\frac{1}{2}x\right)^{\frac{1}{3}}$ up to and including the term in x^2.

(2 marks)

5 In the expansion of $\frac{1}{(3+px)^3}$ the coefficient of the term in x^2 is double the coefficient of the term in x^3. Find the value of p.

(3 marks)

6 (a) The functions $f(x)$ and $g(x)$ are given as follows

$$f(x) = (4+3x)^{\frac{1}{2}} \qquad g(x) = (9-2x)^{-\frac{1}{2}}$$

Expand $f(x)$, in ascending powers of x up to and including the term in x^2.

(2 marks)

(b) Expand $g(x)$, in ascending powers of x up to and including the term in x^2.

(2 marks)

(c) Find the expansion of $\sqrt{\frac{4+3x}{9-2x}}$ in ascending powers of x, up to and including the term in x^2.

(2 marks)

(d) Find the values of x for which your expansion in part (c) is valid.

(2 marks)

7 In the expansion of $\left(1-\frac{4}{3}x\right)^n$, where n is a real number, the coefficient of the term in x^2 is $-\frac{16}{81}$.

Find the possible values of n.

(3 marks)

8 (a) Express $\frac{4+5x-x^2}{(1-x)(1+x)^2}$ in partial fractions.

(3 marks)

(b) Use the binomial expansion to find the first three terms, in ascending powers of x, in each of $(1-x)^{-1}$, $(1+x)^{-1}$, and $(1+x)^{-2}$.

(3 marks)

(c) Hence express $\frac{4+5x-x^2}{(1-x)(1+x)^2}$ as the first three terms of a binomial expansion in ascending powers of x.

(2 marks)

(d) Write down the values of x for which your expansion in part (c) converges.

(1 mark)

9 (a) Given that x is small such that x^3 and higher powers of x can be ignored show that

$$(2+3x)^{-1}(3-2x)^{-2} \approx \frac{1}{18} - \frac{1}{108}x + \frac{19}{216}x^2$$

(3 marks)

(b) Find the percentage error between your calculator answer and the approximation in part (a) when $x = 0.1$, giving your answer to one decimal place.

(3 marks)

(c) For which values of x is the approximation in part (a) valid?

(2 marks)

10 (a) In the binomial expansion of $\sqrt{4+\frac{p}{q}x}$ where $p<0<q$, the coefficient of the x^2 term is equal to the coefficient of the x^3 term.

Show that $p=-8q$.

(3 marks)

(b) Given further that $pq=-8$ find the values of p and q.

(2 marks)

11 (a) Express $\frac{1-7x}{(x+2)(3-x)}$ in the form $\frac{A}{x+2}+\frac{B}{3-x}$, where A and B are integers to be found.

(3 marks)

(b) Hence, or otherwise, find the binomial expansion of $\frac{1-7x}{(x+2)(3-x)}$, in ascending powers of x, up to and including the term in x^2.

(3 marks)

(c) The expansion in part (b) is to be used to approximate the value of a fraction.

(i) If $x=0.1$, which fraction is being approximated?

(ii) Which fraction does the approximation give?

(2 marks)

Very Hard Questions

1 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{\left(1-\frac{1}{3}x\right)^4}$$

up to and including the term in x^3.

(2 marks)

2 (a) Use the first three terms, in ascending powers of x, in the binomial expansion of

$$\frac{1}{\sqrt{1-\frac{1}{2}x}}$$

to estimate the value of $\frac{1}{\sqrt{0.95}}$, giving your answer to two decimal places.

(3 marks)

(b) Explain why you would not be able to use your expansion to approximate $\frac{1}{\sqrt{3}}$.

(1 mark)

3 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{(3-2x)^4}$$

up to and including the term in x^3.

(3 marks)

4 Expand $\left(1-\frac{1}{2}x\right)(9+3x)^{-\frac{1}{2}}$ up to and including the term in x^2.

(4 marks)

5 In the expansion of $\frac{1}{(8+2qx)^{\frac{1}{3}}}$, the coefficient of the term in x^2 is one-seventh of the coefficient of the term in x^3. Find the value of q.

(4 marks)

6 The functions $f(x)$ and $g(x)$ are given as follows

$$f(x) = 8 - x \qquad g(x) = 8 + 2x$$

Find the binomial expansion of $\sqrt[3]{\frac{f(x)}{g(x)}}$, in ascending powers of x, up to and including the term in x^2. Also find the values of x for which your expansion is valid.

(5 marks)

7 In the expansion of $(16-2x)^n$, where n is a real number, the coefficient of the term in x^2 is $16^n \times \frac{5}{2048}$.

Given that $|n| < 1$ find the value of n.

(4 marks)

8 (a) Express $\frac{2(2-5x+x^2)}{(x+2)(2-x)^2}$ in partial fractions.

(3 marks)

(b) Express $\frac{2(2-5x+x^2)}{(x+2)(2-x)^2}$ as the first three terms of a binomial expansion in ascending powers of x.

(5 marks)

(c) Write down the values of x for which your expansion in part (b) converges.

(1 mark)

9 (a) Given that x is small such that x^3 and higher powers of x can be ignored show that

$$(4-3x)^{-2}(2-x)^{-3} \approx \frac{1}{128} + \frac{3}{128}x + \frac{87}{2048}x^2$$

(4 marks)

(b) Find the percentage error between your calculator answer and the approximation in part (a) when $x = 0.2$, giving your answer to one decimal place.

(3 marks)

(c) For which values of x is the approximation in part (a) valid?

(2 marks)

10 It is given that

$$f(x) = \sqrt{4+ax} \quad \text{and} \quad g(x) = \sqrt[4]{16+bx}$$

The binomial expansions of $f(x)$ and $g(x)$ have the following properties:

(i) The coefficient of the x^3 term in the expansion of $f(x)$ is 72 times larger than the coefficient of the x^2 term in the expansion of $g(x)$.

(ii) The coefficient of the x term in the expansion of $f(x)$ is 24 times larger than the

coefficient of the x term in the expansion of $g(x)$.

Find the values of a and b.

(4 marks)

11 (a) Find the binomial expansion of $\frac{15}{(x-4)(5x-2)'}$, in ascending powers of x, up to and including the term in x^2.

(5 marks)

(b) Explain why the expansion found in part (a) cannot be used when $x = 0.6$.

(2 marks)