

# Simplex Algorithm

Simplex Algorithm - Slack Variables & Initial Tableau / Two-stage Simplex Method / Big-M Method

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Total Marks

/74

**1 (a)** A linear programming problem in  $x$ ,  $y$  and  $z$  is described as follows.

Maximise  $P = 2x + 2y - z$

subject to  $3x + y + 2z \leq 30$

$$x - y + z \geq 8$$

$$4y + 2z \geq 15$$

$$x, y, z \geq 0$$

Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

**(1 mark)**

**(b)** Write your answer in the [Answer Booklet](#).

Set up the initial tableau for solving this linear programming problem using the big-M method.

**(7 marks)**

**(c)** After a first iteration of the big-M method, the tableau is

b.v.	$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$a_1$	$a_2$	Value
$s_1$	3	0	1.5	1	0	0.25	0	-0.25	26.25
$a_1$	1	0	1.5	0	-1	-0.25	1	0.25	11.75
$y$	0	1	0.5	0	0	-0.25	0	0.25	3.75
$P$	$-(2 + M)$	0	$2 - 1.5M$	0	$M$	$-0.5 + 0.25M$	0	$0.5 + 0.75M$	$7.5 - 11.75M$

State the value of each variable after the first iteration.

**(1 mark)**

**(d)** Explain why the solution given by the first iteration is not feasible.

**(1 mark)**

**(e)** Taking the most negative entry in the profit row to indicate the pivot column,

Obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

**(2 marks)**

- 2 (a)** A maximisation linear programming problem in  $x$ ,  $y$  and  $z$  is to be solved using the two-stage simplex method.

The partially completed initial tableau is shown below.

Basic variable	$x$	$y$	$z$	$S_1$	$S_2$	$S_3$	$a_1$	$a_2$	Value
$S_1$	1	2	3	1	0	0	0	0	45
$a_1$	3	2	0	0	-1	0	1	0	9
$a_2$	-1	0	4	0	0	-1	0	1	4
$P$	-2	-1	-3	0	0	0	0	0	0
$A$									

Using the information in the above tableau, formulate the linear programming problem. State the objective and list the constraints as inequalities.

**(4 marks)**

- (b)** Complete the bottom row of Table 1 in the [answer book](#). You should make your method and working clear.

**(2 marks)**

- (c)** The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

Basic variable	$x$	$y$	$z$	$S_1$	$S_2$	$S_3$	$a_1$	$a_2$	Value
$S_1$	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$-\frac{7}{12}$	$-\frac{3}{4}$	$\frac{147}{4}$
$a_1$	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	3
$a_2$	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{7}{4}$
$P$	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{45}{4}$
$A$	0	0	0	0	0	0	1	1	0

- (i) Explain how the above tableau shows that a basic feasible solution has been found for the original linear programming problem.
- (ii) Write down the basic feasible solution for the second stage.

**(3 marks)**

- (d) Write your answer in the [Answer Booklet](#).

Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method, to obtain a new tableau,  $T$ . Make your method clear by stating the row operations you use.

**(5 marks)**

- (e) (i) Explain, using  $T$ , whether or not an optimal solution to the original linear programming problem has been found.

- (ii) Write down the value of the objective function.
- (iii) State the values of the basic variables.

**(3 marks)**

- 3 (a)** Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- $x$  represent the number of hours swimming
- $y$  represent the number of hours cycling
- $z$  represent the number of hours running

Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

**(5 marks)**

- (b)** Write your answer in the [Answer Booklet](#).

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

Set up an initial tableau for solving this problem using the two-stage Simplex method. As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6 marks)

- (c) The following tableau  $T$  is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	Value
$y$	0	1	0	1	0	1	11
$s_2$	0	0	5	-2	1	-5	62
$x$	1	0	1	0	0	-1	28
$P$	0	0	-1	1	0	1	11

Obtain a suitable pivot for a second iteration. You must give reasons for your answer.

(2 marks)

- (d) Write your answer in the [Answer Booklet](#).

Starting from tableau  $T$ , solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use.



(5 marks)

- 4 (a)** A garden centre makes hanging baskets to sell to its customers. Three types of hanging basket are made, *Sunshine*, *Drama* and *Peaceful*. The plants used are categorised as *Impact*, *Flowering* or *Trailing*.

Each *Sunshine* basket contains 2 *Impact* plants, 4 *Flowering* plants and 3 *Trailing* plants.

Each *Drama* basket contains 3 *Impact* plants, 2 *Flowering* plants and 4 *Trailing* plants.

Each *Peaceful* basket contains 1 *Impact* plant, 3 *Flowering* plants and 2 *Trailing* plants.

The garden centre can use at most 80 *Impact* plants, at most 140 *Flowering* plants and at most 96 *Trailing* plants each day.

The profit on *Sunshine*, *Drama* and *Peaceful* baskets are £12, £20 and £16 respectively.

The garden centre wishes to maximise its profit.

Let  $x$ ,  $y$  and  $z$  be the number of *Sunshine*, *Drama* and *Peaceful* baskets respectively, produced each day.

Formulate this situation as a linear programming problem, giving your constraints as inequalities.

**(5 marks)**

- (b)** State the further restriction that applies to the values of  $x$ ,  $y$  and  $z$  in this context.

**(1 mark)**

- (c)** The Simplex algorithm is used to solve this problem. After one iteration, the tableau is

b.v.	$x$	$y$	$z$	$r$	$s$	$t$	Value
$r$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	1	0	$-\frac{3}{4}$	8
$s$	$\frac{5}{2}$	0	2	0	1	$-\frac{1}{2}$	92
$y$	$\frac{3}{4}$	1	$\frac{1}{2}$	0	0	$\frac{1}{4}$	24
$P$	3	0	-6	0	0	5	480

State the variable that was increased in the first iteration. Justify your answer.

**(2 marks)**

- (d)** Determine how many plants in total are being used after only one iteration of the Simplex algorithm.

**(1 mark)**

- (e)** Explain why for a second iteration of the Simplex algorithm the 2 in the  $z$  column is the pivot value.

**(2 marks)**

- (f)** After a second iteration, the tableau is

b.v.	$x$	$y$	$z$	$r$	$s$	$t$	Value
$r$	$\frac{3}{8}$	0	0	1	$\frac{1}{4}$	$-\frac{7}{8}$	31
$s$	$\frac{5}{4}$	0	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	46
$y$	$\frac{1}{8}$	1	0	0	$-\frac{1}{4}$	$\frac{3}{8}$	1
$P$	$\frac{21}{2}$	0	0	0	3	$\frac{7}{2}$	756

Use algebra to explain why this tableau is optimal.

**(1 mark)**

**(g)** State the optimal number of each type of basket that should be made.

**(1 mark)**

**(h)** The manager of the garden centre is able to increase the number of *Impact* plants available each day from 80 to 100. She wants to know if this would increase her profit.

Use your final tableau to determine the effect of this increase. (You should not carry out any further calculations.)

**(2 marks)**

**5 (a)** A linear programming problem in  $x$ ,  $y$  and  $z$  is described as follows.

$$\text{Maximise } P = 3x + 2y + 2z$$

$$\text{subject to } 2x + 2y + z \leq 25$$

$$x + 4y \leq 15$$

$$x \geq 3$$

Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

**(1 mark)**

**(b)** The big-M method is to be used to solve this linear programming problem.

Define, in this context, what  $M$  represents. You must use correct mathematical language in your answer.

**(1 mark)**

**(c)** The initial tableau for a big-M solution to the problem is shown below.

b.v.	$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$t_1$	Value
$s_1$	2	2	1	1	0	0	0	25
$s_2$	1	4	0	0	1	0	0	15
$t_1$	1	0	0	0	0	-1	1	3
$P$	$-(3+M)$	-2	-2	0	0	$M$	0	$-3M$

Explain clearly how the equation represented in the b.v.  $t_1$  row was derived.

**(1 mark)**

**(d)** Show how the equation represented in the b.v.  $P$  row was derived.

(2 marks)

(e) Write your answer in the [Answer Booklet](#).

The tableau obtained from the first iteration of the big-M method is shown below.

b.v.	$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$t_1$	Value
$s_1$	0	2	1	1	0	2	-2	19
$s_2$	0	4	0	0	1	1	-1	12
$t_1$	1	0	0	0	0	-1	1	3
$P$	0	-2	-2	0	0	-3	$3+M$	9

Solve the linear programming problem, starting from this second tableau. You must

- give a detailed explanation of your method by clearly stating the row operations you use and
- state the solution by deducing the final values of  $P$ ,  $x$ ,  $y$  and  $z$ .

(7 marks)