Edexcel A Level Further Maths: Further Mechanics 1

Your notes

Momentum & Impulse

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Your notes

Momentum & Impulse in 1D

Impulse-Momentum Principle in 1D

What is momentum?

- Any object that has mass and is moving has **momentum**
- **Momentum** measures the quantity of motion that an object has
- The **momentum** of a particle is defined as the **product** of its **mass ($m$kg)** and its **velocity (v m s^{-1})**
  - Momentum = ***mv***
- The SI unit for momentum is kg m s^{-1}
- Momentum is a **vector** quantity - so it has a **magnitude** and **direction**
  - The **direction of the momentum** of a particle is the **same** as the **direction of motion** of the particle
  - The **momentum is negative** if the **velocity is negative**

What is impulse?

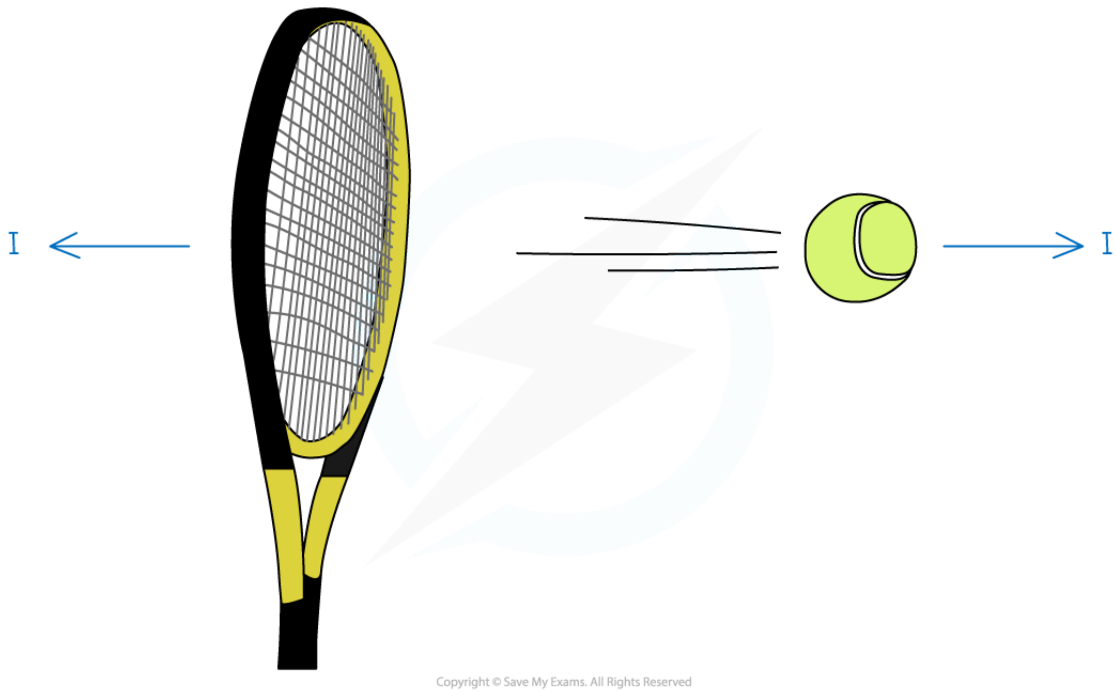
- **Impulse** measures the effect of a force acting on a particle over time, it could be thought of as a "push"
- If a **constant force (F N)** acts on a particle for t **seconds** then the **impulse (I)** of the force is defined to be the **product** of the **force** and **time**
  - $I = Ft$
- The SI unit for impulse is **N s** (newton seconds) which is equivalent to kg m s^{-1}
  - This is the same as the units for momentum
- Impulse is a **vector** quantity – so it has magnitude and direction
  - The **direction of the impulse** of a force is the same as the **direction of the force**

What is the impulse-momentum principle?

- The **Impulse-Momentum Principle** states that impulse is equal to the **change in momentum**
  - $I = mv - mu$
  - or equivalently $I = m(v - u)$
  - where m is the mass, u is the initial velocity and v is the final velocity

What happens when two objects are in contact?

- If two objects are in contact with each other then by **Newton's Third Law** there will be **equal** and **opposite** reaction forces
- This means there will be **equal and opposite impulses**
- For example, consider hitting a tennis ball with a racket, there will be
  - an impulse exerted **by the racket on the ball** which propels the ball forward
  - an impulse exerted **by the ball on the racket** which reduces the velocity of the racket
  - The **magnitudes** of these impulses are **equal** but they are in **opposite** directions



Examiner Tip

- Always define a positive direction and be careful with negatives. Use common sense to see if your answer makes sense - would you expect the velocity to have increased or decreased?
- If two objects collide, remember that the impulse on both objects is the same, just in opposite directions

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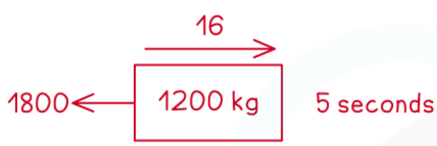
Worked example

A car with mass 1200 kg is driving to the right along a smooth horizontal road with speed 16 m s^{-1}. The driver applied a constant braking force of magnitude 1800 N for 5 seconds.

a) Find the magnitude of the impulse of the braking force.

a)



Use formula "I = Ft"

Impulse = 1800 × 5

We want the magnitude so use the magnitude of the force

9000 N s

Include units N s or kg m s^{-1}

b) State the direction of the impulse.

b) Direction of impulse is the same as the direction of the force

Impulse is to the left – opposite to the motion of the car

c) Find the speed of the car 5 seconds after the braking force was applied.

c) A before/after diagram can help

Use "$I = mv - mu$"

$-9000 = 1200v - 1200(16)$

Negative as impulse is in the opposite direction

$1200v = 1200(16) - 9000$
$1200v = 10200$
$v = 8.5$

Speed = 8.5 m s^{-1}

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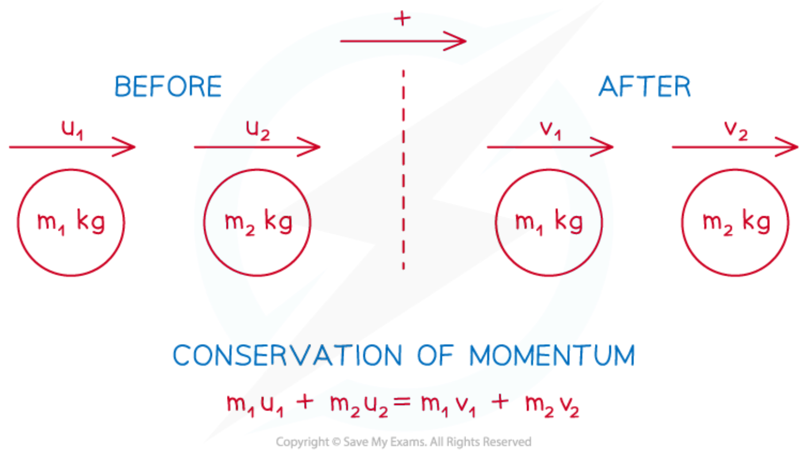
Conservation of Momentum in 1D

What is a direct collision?

- A **direct collision** is when two objects are travelling along the **same straight line** when they collide
- **Before** the collision:
  - **One** of the objects could be stationary
  - The two objects could be travelling in the **same direction** with the faster object behind the slower one
  - The two objects could be travelling in **opposite directions** towards each other
- **After** the collision:
  - **One or both** of the objects could be **stationary**
  - The two objects could be travelling in the **same direction** with the faster object in front of the slower one
  - The two objects could be travelling in **opposite directions** away from each other
  - The two objects could **coalesce** (merge to form one object) and travel in either direction
- **Explosions** work like direct collisions and are when an object **separates** into two objects travelling along the same straight line
  - An example of this is a **bullet being fired from a gun**, the bullet moves forwards and the gun recoils backwards
  - For an explosion it is possible that the object is **initially stationary** and then splits into two objects **moving in opposite directions**

What is meant by conservation of momentum?

- The **principle of conservation of momentum** states that when two objects collide the **total momentum is unchanged**
  - **Total** momentum **before** collision = **Total** momentum **after** collision
  - This only works if there are **no external forces** acting on the objects
- If an object changes direction after a collision then its velocity changes between positive and negative
  - It is important to be clear about which direction is positive
- It can be written as: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
  - One object has mass m_1 kg, velocity $u_1\,\mathrm{m\,s^{-1}}$ before the collision and $v_1\,\mathrm{m\,s^{-1}}$ after the collision
  - The other object has mass m_2 kg, velocity $u_2\,\mathrm{m\,s^{-1}}$ before the collision and $v_2\,\mathrm{m\,s^{-1}}$ after the collision



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How do I use conservation of momentum to solve collision problems?

- **STEP 1**: Choose the **positive direction**
- **STEP 2**: Draw a **before/after diagram**
  - Clearly show the mass, speeds and directions
  - If a direction is unknown, then choose any direction and if you get a negative value for its velocity it means it is travelling in the opposite direction
  - If the two objects coalesce then you can either consider them as two particles moving in the same direction with the same speed or consider them as one particle and add together their masses
- **STEP 3**: Form an **equation** using the **conservation of momentum**
  - Be careful with negatives
  - If an arrow is in the opposite direction to the positive direction, then its velocity is negative
- **STEP 4**: **Solve** and give answer in **context**
  - You might need to find the speed and/or direction after a collision

Examiner Tip

- Always draw clear diagrams with arrows!
- When considering directions, use common sense - if two particles are travelling in the same direction then they will not collide if the faster one is in front. Two particles can't go through each other (at least not at this level of mathematics…) so if two objects are travelling towards each other, then at least one of them must change direction after the collision.

Your notes

Worked example

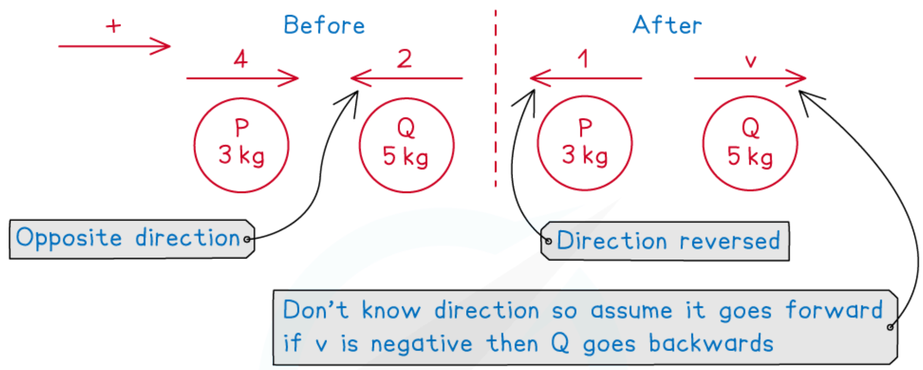
Two particles *P* and Q, with masses 3 kg and 5 kg respectively, are travelling in opposite directions towards each other along the same straight line on a smooth horizontal table when they collide directly. Immediately before the collision the speeds of *P* and Q are 4 m s^{-1} and 2 m s^{-1} respectively. Immediately after the collision the direction of motion of *P* is reversed and its speed is 1 m s^{-1}.

Find the speed of Q immediately after the collision, and state whether the direction of motion of Q is changed by the collision.

Step 1: Choose positive direction

Let the initial direction of P be positive

Step 2: Draw a before/after diagram



Step 3: Use conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3(4) + 5(-2) = 3(-1) + 5(v)$ Negative as arrows go backwards

$12 - 10 = -3 + 5v$

$2 = 5v - 3$

Step 4: Solve

$5v - 3 = 2$

$v = 1$

Speed = 1 ms^{-1}

v = 1 is positive so Q moves forwards after collision
Before collision Q moved backwards

Direction of motion of Q is reversed

Your notes

Momentum & Impulse with Vectors

Impulse-Momentum Principle with Vectors

How do impulse and momentum work in 2D?

- Impulse and momentum can be used in 2D as they are vector quantities
  - Impulse in 2D essentially works the **same** way as impulse in 1D
- For a **constant force** given by the vector $\mathbf{F}\ N = (F_x\mathbf{i} + F_y\mathbf{j})$ N acting on a particle for t seconds the **impulse** is given by the **vector**:
  - $\mathbf{I} = \mathbf{F}t = (F_x\mathbf{i} + F_y\mathbf{j})t$
  - The units are still N s (equivalent to kg m s^{-1})
- For a particle of mass m the impulse is still equal to the **change in momentum**
  - $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$ or $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$
  - where **u** m s^{-1} is the initial velocity vector and **v** m s^{-1} is the final velocity vector
  - $\mathbf{I},\ \mathbf{v},\ \mathbf{u}$ are all vector quantities, and m is a scalar
  - If using column vectors, this equation would look as follows:
    - $\begin{pmatrix} \mathrm{I}_x \\ \mathrm{I}_y \end{pmatrix} = m\left(\begin{pmatrix} \mathrm{v}_x \\ \mathrm{v}_y \end{pmatrix} - \begin{pmatrix} \mathrm{u}_x \\ \mathrm{u}_y \end{pmatrix}\right)$

How are questions different in 2D?

- You could be asked to work out the **magnitude** of the impulse
  - You would need to find the two components of the impulse vector and then use **Pythagoras**
- You could be asked to work out the **direction** of the impulse
  - You would need to find the two components of the impulse vector and then use **SOHCAHTOA (right-angled trigonometry)**
  - You might need to find the angle between the impulse and the vector $\mathbf{i}$ or $\mathbf{j}$ so always draw a sketch
- If you know the magnitude and direction of the impulse or a velocity, then you might have to **resolve** it into horizontal and vertical components

Examiner Tip

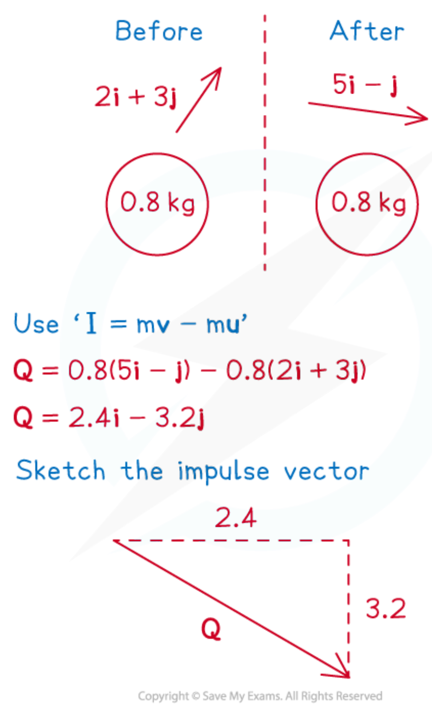
- Be careful with negatives, especially when adding and subtracting vectors
- When finding angles and directions always sketch a diagram. Read the question carefully to help you decide where the angle should be measured from

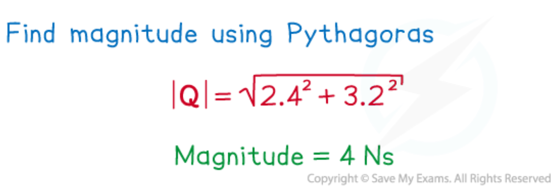
Your notes

Worked example

A ball of mass 0.8 kg is moving with velocity $(2\mathrm{i}+3\mathrm{j})\ \mathrm{m\ s^{-1}}$ when it receives an impulse Q N s. Immediately after receiving the impulse, the velocity of the ball is $(5\mathrm{i}-\mathrm{j})\ \mathrm{m\ s^{-1}}$.

Find the magnitude of the impulse **Q** N, and its angle from the vector **j**.





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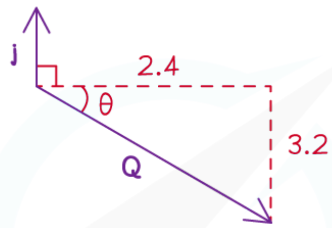
Find the angle by using trigonometry



$$\tan\theta = \frac{3.2}{2.4}$$

$$\theta = \tan^{-1}\left(\frac{3.2}{2.4}\right) = 53.130...$$

Add 90° to get angle between **Q** and **j**

$$90 + 53.130... = 143.130...$$

Angle between **Q** and **j** is 143° (3sf)