OCR A Level Physics

Your notes

EM Radiation From Stars

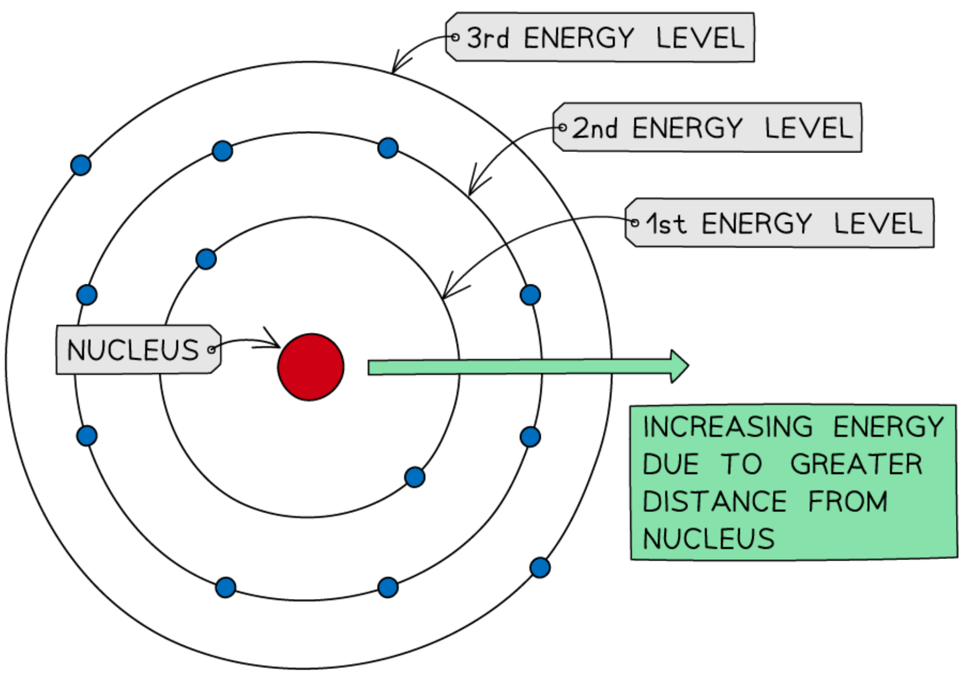
Contents

Electron Energy Levels

Your notes

Electron Energy Levels

- A **continuous spectrum** is a spectrum that appears to contain all frequencies over a comparatively wide range
  - The Sun's spectrum is not continuous as it shows dark lines where some frequencies are missing
- Bohr explained the existence of **spectral lines** using the new photon model of electromagnetic radiation
- In addition, Bohr proposed a model of the atom which includes:
  - Electrons orbiting the nucleus
  - Electrons only occupying specific orbits associated with a **discrete** set of energies
  - The orbits are also called shells or **energy levels**
  - Electrons can gain or lose energy by moving from one allowed energy level to another



Electrons in an atom can have only certain specific (discrete) energies within an atom

- Energy is required for an electron to move from a lower to a higher energy level
  - This transition is called an **excitation**
- Energy is released if the electron moves from a higher to a lower energy level
  - This transition is called a **de-excitation**
- Electrons can move to a higher or lower energy level by absorbing or emitting electromagnetic radiation with a frequency, *f*
- The energy required for **excitation** of a gas can be provided by:
  - A photon of a specific frequency
  - Energy absorbed from the surroundings (through heating)
  - Energy supplied by an electric field
- The energy released during **de-excitation** is emitted as electromagnetic radiation of a specific frequency
  - This frequency depends on the difference of energy between the specific energy levels involved in the transition

Your notes

Examiner Tips and Tricks

The values of energy level are all given using the unit electron volt (eV), because we are dealing with very small energy values. 1 eV is equal to 1.6×10^{-19} Joules, so to convert between J and eV, remember:

$$J \xrightarrow{\div 1.6 \times 10^{-19}} eV$$

$$eV \xrightarrow{\times 1.6 \times 10^{-19}} J$$

Negative Values of Energy Levels

- The energy of an electron is taken to be zero when it is **infinitely far from the nucleus**
  - The forces of interaction between the electron and the nucleus are practically zero
  - Energy is regarded to be 0 eV and the electron is said to be free from the atom
- Energy levels have **negative values**

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- Therefore, any energy value for an electron inside the atom will be negative
  - This is because external energy is required to remove an electron from its energy level
- The negative values also mean that electrons are **confined** within the atom
- The value of the energy level is equal to

  The amount of energy required to remove an electron from that energy level

- The energy level with the most negative value is the lowest energy level $n = 1$
  - This is called the **ground state**
- The value for the energy level of the ground state is equal to

  The energy required to remove an electron from the atom

- For hydrogen, the energy value related to the ground state is equal to –13.6 eV
  - Hence, 13.6 eV is the energy required to remove an electron from the ground state of an atom of hydrogen
- The complete removal of an electron from an atom is known as **ionisation**
  - If a photon of energy more than 13.6 eV is absorbed, then the excess energy after the electron has been ionised will be transferred to it as kinetic energy

Examiner Tips and Tricks

Electron energy levels are defined in a similar way to gravitational potential – when at infinity these quantities can be considered to be zero – this is to emphasise that when a mass is far away from another mass, the gravitational force no longer has an effect.

So, similarly, when the electron is far away from the nucleus, the electric force by the nucleus no longer has an effect on the electron.

Your notes

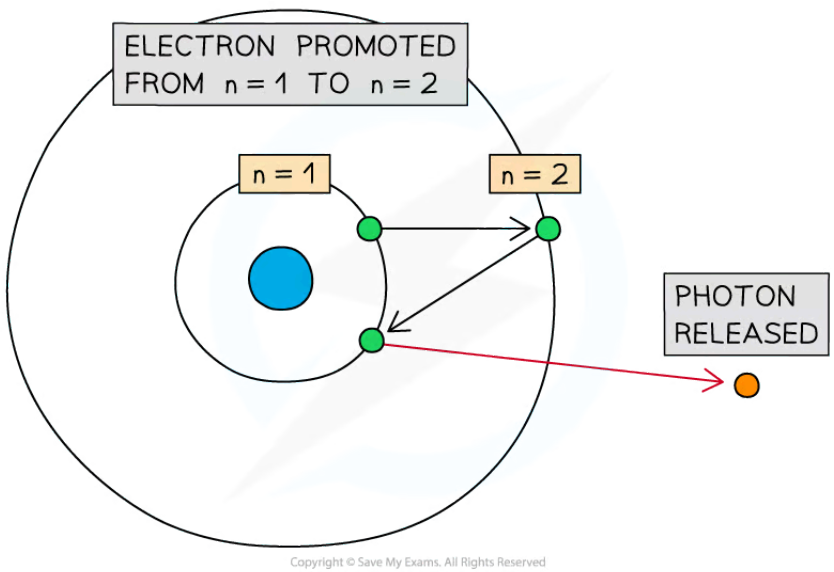
Emission Spectra & Energy Levels

Emission Spectra

- An emission line spectrum is produced when:

  An excited electron in an atom moves from a higher to a lower energy level and emits a photon with an energy corresponding to the difference between these energy levels

- Each element produces a unique **emission line spectrum** due to its unique set of energy levels
  - Hot gases produce emission line spectra, such as stars
- When the atoms of a gas are **excited**, electrons gain energy and move to higher energy levels



When an electron moves from a higher energy level to a lower energy level, a photon is released

- Electrons cannot stay in a continuous state of excitation, so they will move back to lower energy levels through **de-excitation**
  - During de-excitation, energy must be conserved, so transitions result in an **emission of photons** with discrete frequencies (or wavelengths) specific to that element
  - Since there are many possible electron transitions for each atom, there are many different radiated wavelengths
  - This creates a **line spectrum** consisting of a series of bright lines against a dark background

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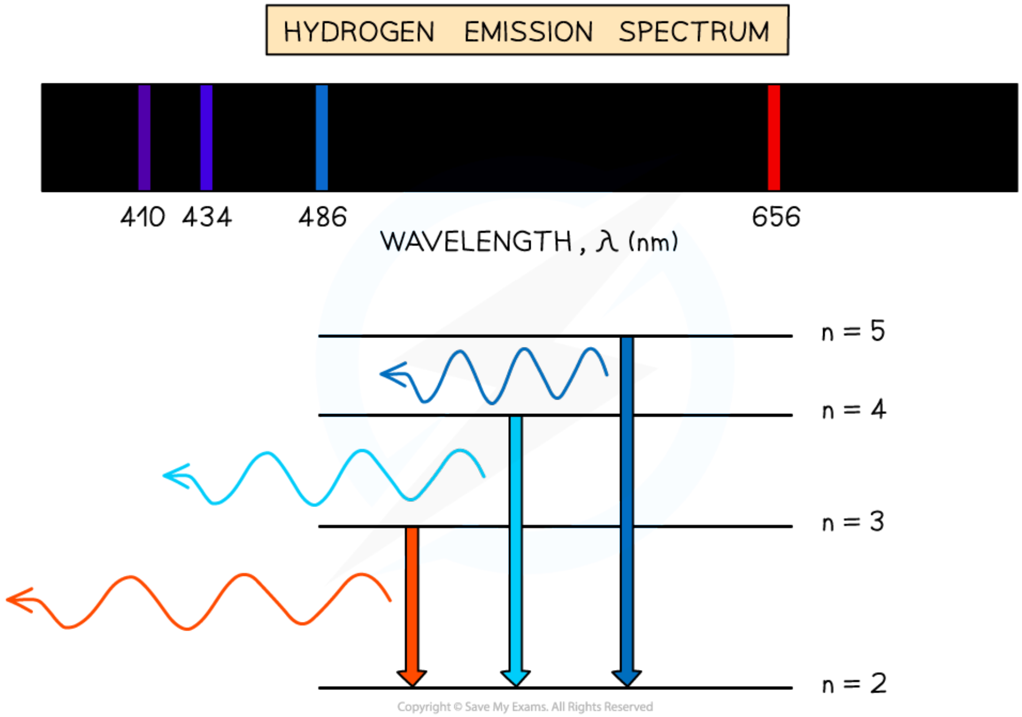
- An emission line spectrum acts as a **fingerprint of the element**



An example of the emission line spectrum of hydrogen

Examiner Tips and Tricks

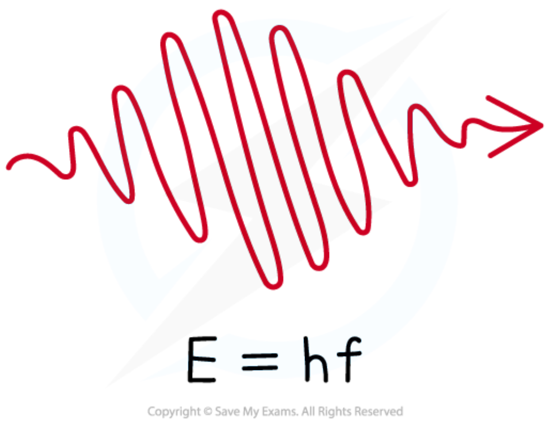
You need to be able to explain how an emission line is produced in an emission spectrum. You need to state that an electron makes a transition to a lower energy level **and** emits a photon. Both points need to be made to gain the mark.

Difference in Discrete Energy Levels

- Each line of the emission spectrum corresponds to a different **energy level** transition within the atom
  - Electrons can transition between energy levels absorbing or emitting a **discrete amount of energy**
  - An excited electron can transition down to the next energy level or move to a further level closer to the ground state
- For example, if an atom has six energy levels:

Your notes

- At low temperatures, most electrons will occupy the ground state n = 1
- At high temperatures, electrons may be excited to the most excited state n = 6



Energy and frequency of a photon are directly proportional

- The energy of a photon is given by the equation:

$$E = hf$$

- Using energy can be written as:

$$E = \frac{hc}{\lambda}$$

- Where:
  - E = energy of the photon (J)
  - h = Planck's constant(J s)
  - c = speed of light (m s^{-1})
  - f = frequency (Hz)
  - λ = wavelength (m)
- The energy required to move from one energy level to another is given by the difference of energy between the two energy levels:

$$\Delta E = E_1 - E_2$$

- Where:
  - E_1 = energy associated with the level that the electron has left (eV)

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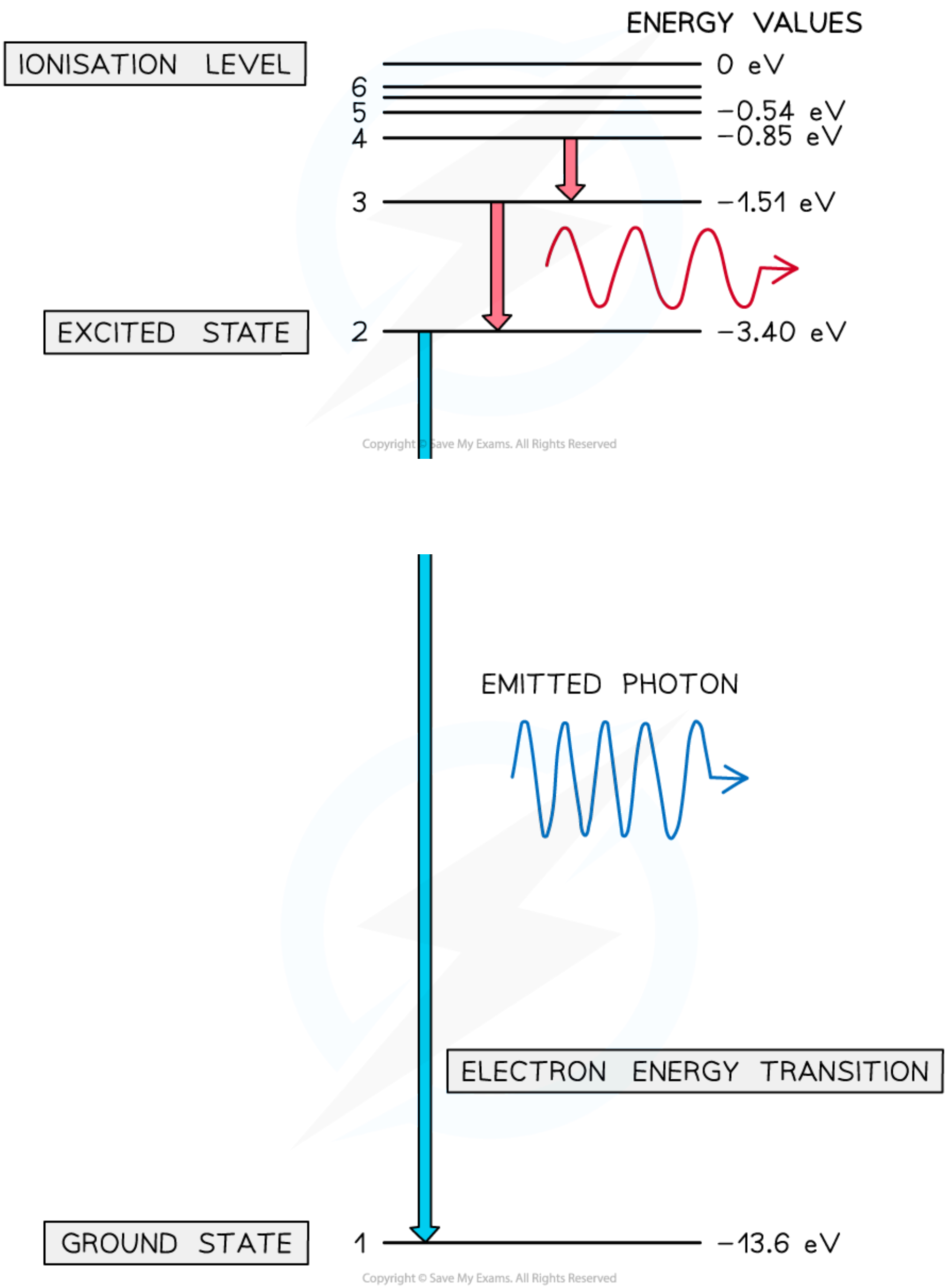
- E_2 = energy associated with the level that the electron moves to (eV)
- The difference of energy corresponds to the energy of the absorbed (or emitted) photon:

$$\Delta E = E_1 - E_2 = hf = \frac{hc}{\lambda}$$

- For each transition, a photon will be emitted with a specific wavelength
- In the case of hydrogen, all wavelengths are in the visible range:
  - From n = 6 to n = 2 - violet
  - From n = 5 to n = 2 - blue
  - From n = 4 to n = 2 - light blue
  - From n = 3 to n = 2 - red

If the emitted photons are in the visible range, wavelengths can be represented as lines of the respective colour against a black background

- Emitted photons can have a range of wavelengths spanning the whole electromagnetic spectrum
  - The wavelength is inversely proportional to the energy level transition associated with the emitted photon
- For example, the transitions for hydrogen will be as follows:
  - To n = 1 (ground level) – ultraviolet, highest energy, high frequency, short wavelength
  - To n = 2 – visible light, violet is the highest energy, red is the lowest energy
  - To n = 3 – infrared light, lowest energy, low frequency, longest wavelength



The larger the energy transition, the longer the wavelength of the emitted photon

Worked Example

Your notes

Your notes

The value of the energy level in eV for the hydrogen atom is given by the **Bohr formula**:

$$E_n = \frac{-13.6}{n^2}$$

Where n is an integer 1, 2, 3 etc.

Determine the frequency of the emitted photon when an electron makes a transition between levels n = 4 and n = 2.

Answer:

Step 1: List the known quantities

- Transition between n = 4 and n = 2
- Planck's constant, $h = 6.63 \times 10^{-34}$ J s

Step 2: Determine an equation for the change in energy Δ*E*

$$\Delta E = E_4 - E_2$$

$$\Delta E = \left(\frac{-13.6}{4^2}\right) - \left(\frac{-13.6}{2^2}\right)$$

Step 3: Calculate the change in energy, in eV, for the photon using the given equation

$$\Delta E = -13.6 \times \left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 2.55 \text{ eV}$$

Step 4: Convert the calculated energy from eV to Joules

$$\Delta E = 2.55 \times (1.60 \times 10^{-19}) = 4.08 \times 10^{-19} \text{ J}$$

Step 5: Identify the appropriate equation to determine the frequency, knowing Δ*E*

$$\Delta E = hf$$

Step 6: Rearrange the equation for frequency, *f*

$$f = \frac{\Delta E}{h}$$

Step 7: Substitute the known values and calculate the frequency for the photons

$$f = \frac{4.08 \times 10^{-19}}{6.63 \times 10^{-34}} = 6.15 \times 10^{14} \text{ Hz}$$

Your notes

Examiner Tips and Tricks

Although the difference in energy $\Delta E = E_1 - E_2$ can be expressed in eV, you need to convert this value in Joules when you are asked to calculate either the frequency or wavelength of the emitted photon.

You are expected to be able to calculate the frequency or the wavelength of a photon, given a specific transition on an energy levels diagram or to identify a specific transition on a given diagram when provided with the value of frequency or wavelength.

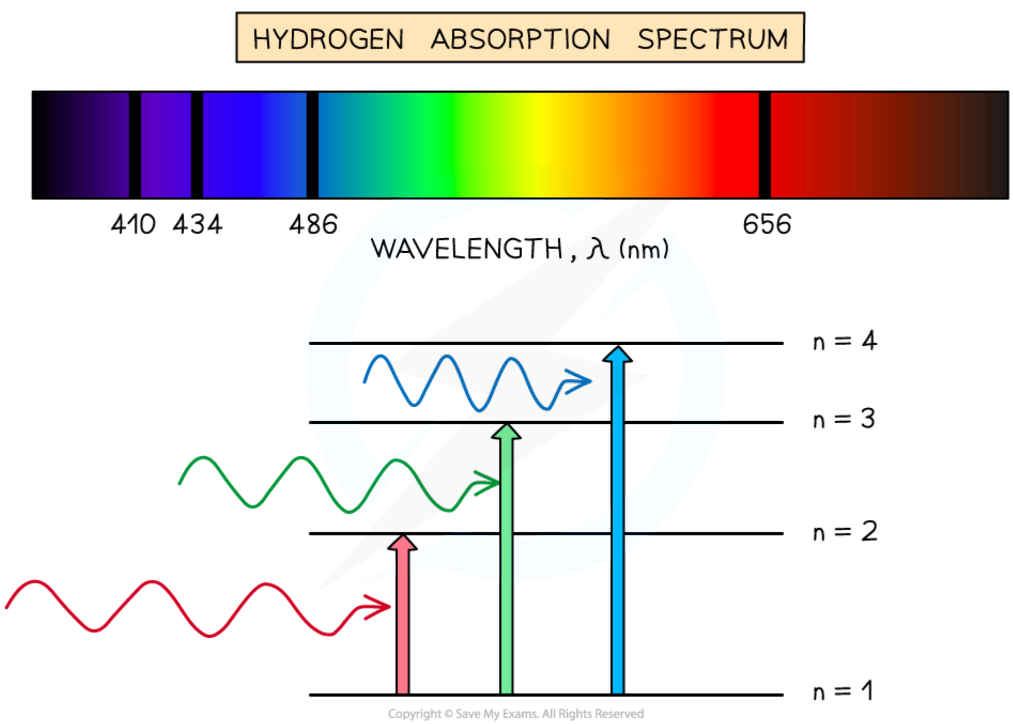
You are not expected to know the Bohr formula as given in the worked example - this is just an example of an unfamiliar context you could be given that you have to apply your knowledge to.

Your notes

Identifying Elements Within Stars Using Spectral Lines

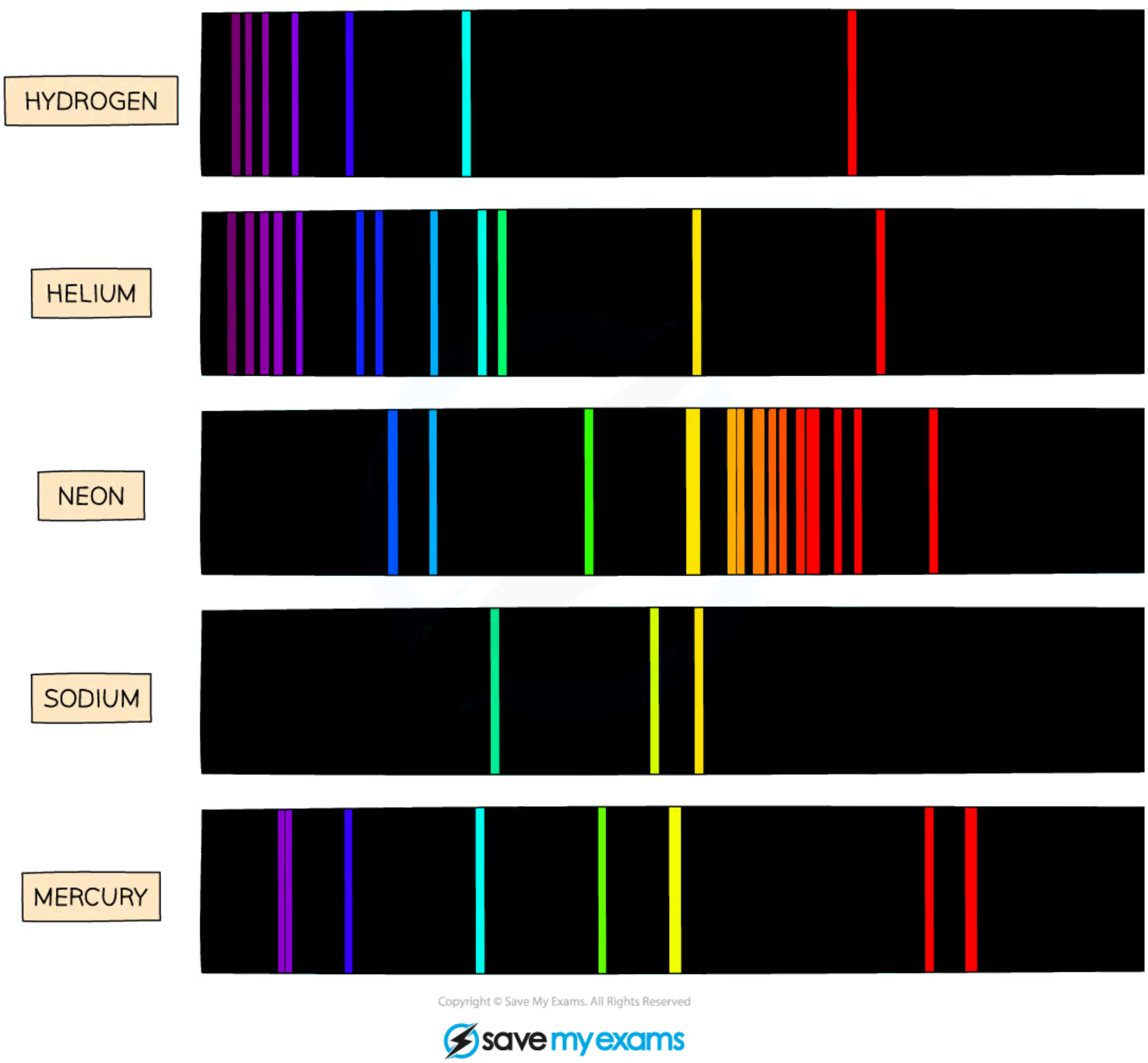
Identifying Elements Within Stars Using Spectral Lines

- Photons produced by fusion reactions in a star's core move towards the layers of gas in the outer atmosphere of the star
  - The photons have all frequencies of the electromagnetic spectrum - this is known as a **continuous spectrum**
  - Photons are absorbed by the gas atoms, which excite and re-emit other photons of various frequencies in random directions
- The light from a star can be analysed using **spectroscopy**
  - The atmospheres of stars are not hot enough to produce an emission line spectrum
  - Therefore, stars are found to emit an **absorption line spectrum**



An absorption line will appear in a spectrum if an absorbing material is placed between a source and the observer

- An absorption line spectrum is the equivalent of an emission line spectrum but it is made of **dark lines** on top of a **continuous spectrum**
    - The dark lines represent the frequencies or wavelengths that are absorbed by a medium, such as a gas, when light passes through it
- Each gas produces a unique pattern of spectral lines due to the specific transition between the element's energy levels
    - The presence of spectral lines in a star's absorption spectrum act as fingerprints
    - They can be used to determine the presence of a **certain element** within the star



Your notes

Emission line spectra are unique to each element

Your notes

- The **chemical composition** of a star can be investigated even when extremely distant
  - If the element is present in the star, its characteristic pattern of spectral lines will appear as dark lines in the absorption line spectrum of the star
- The Sun is predominantly made up of **hydrogen** and **helium** gas
  - The chemical composition of the Sun can be verified using the **emission line spectra** of the two gases compared with the **absorption line spectrum** of the Sun
- For example, the hydrogen emission line spectrum includes lines at:
  - 2 nm, 486.3 nm and 656.5 nm
- While helium spectrum includes lines at:
  - 7 nm and 587.7 nm
- The same wavelengths can be seen as dark lines on top of the Sun's continuous spectrum

Examiner Tips and Tricks

Given an absorption line spectrum for a specific star, you can be asked to identify a star of similar chemical composition. It is important to pay attention to the spacing between the lines to be able to correctly identify the most similar star to the given one.

Continuous, Emission Line & Absorption Line Spectrum

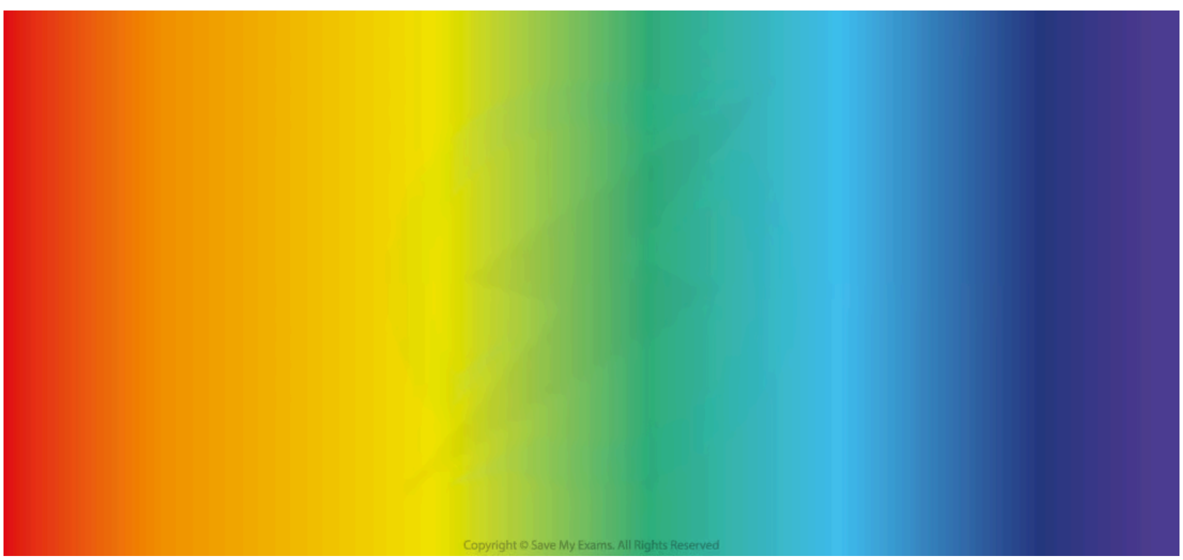
Your notes

Continuous, Emission Line & Absorption Line Spectrum

- There are three kinds of light spectra:
  - **Continuous emission** spectra
  - **Emission line** spectra
  - **Absorption line** spectra

Continuous Line Spectra

- Photons emitted from the core of a star contain all the wavelengths and frequencies of the electromagnetic spectrum
  - This is called a **continuous spectrum**
- Continuous spectra are produced from **hot, dense sources**, such as the cores of stars

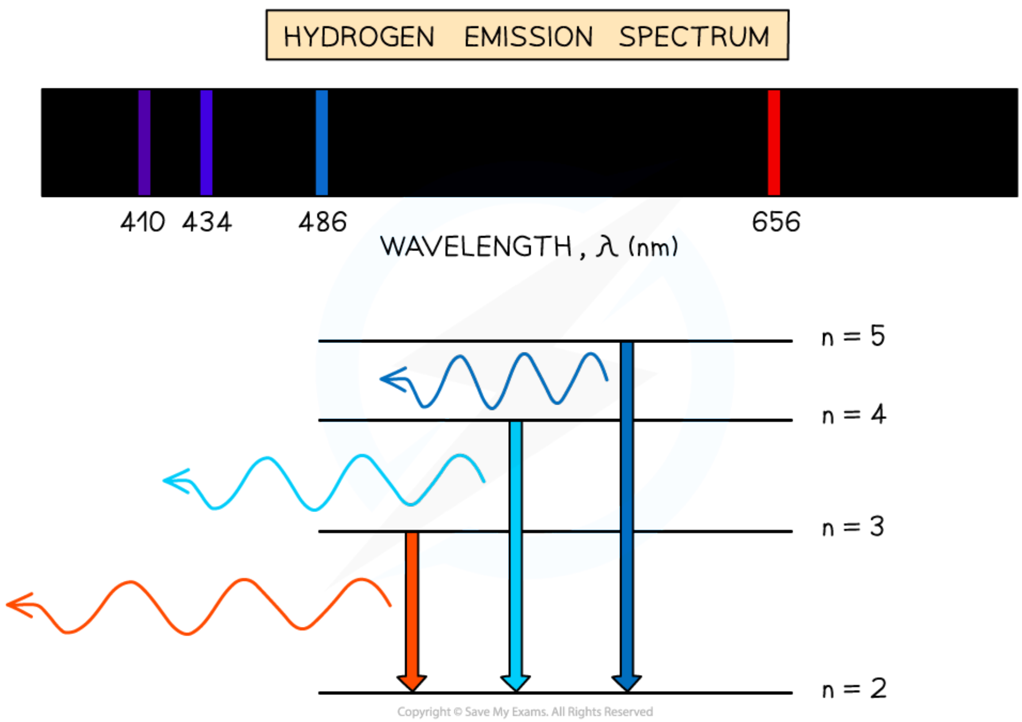


A continuous spectrum is one which emits all wavelengths of light

Emission Line Spectra

Your notes

- When an electron transitions from a higher energy level to a lower energy level, this results in the **emission** of a photon
    - Each transition corresponds to a different wavelength of light and this corresponds to a line in the spectrum
- The resulting emission spectrum contains a set of discrete wavelengths
    - It is represented by **coloured lines** on a **black** background
- Emission line spectra are produced by **hot, low pressure gases**



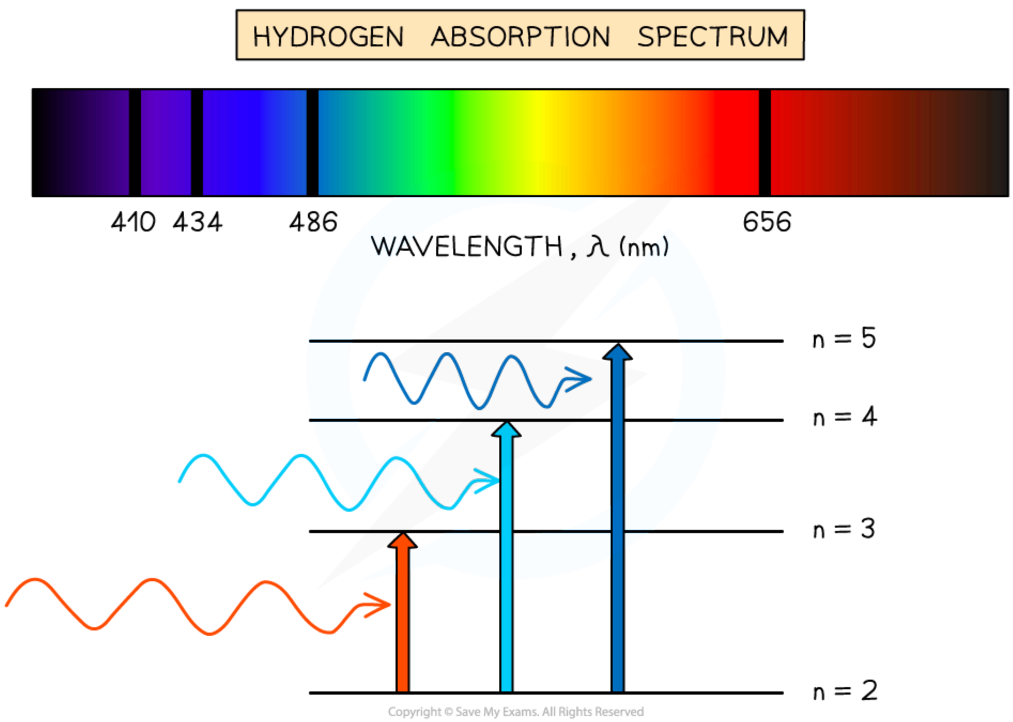
The emission spectrum of hydrogen

Absorption Spectra

- An atom can be raised to an excited state by the absorption of a photon
- When white light passes through a **cool, low pressure gas** it is found that light of certain wavelengths are missing
    - This type of spectrum is called an absorption spectrum
- An absorption spectrum consists of a continuous spectrum

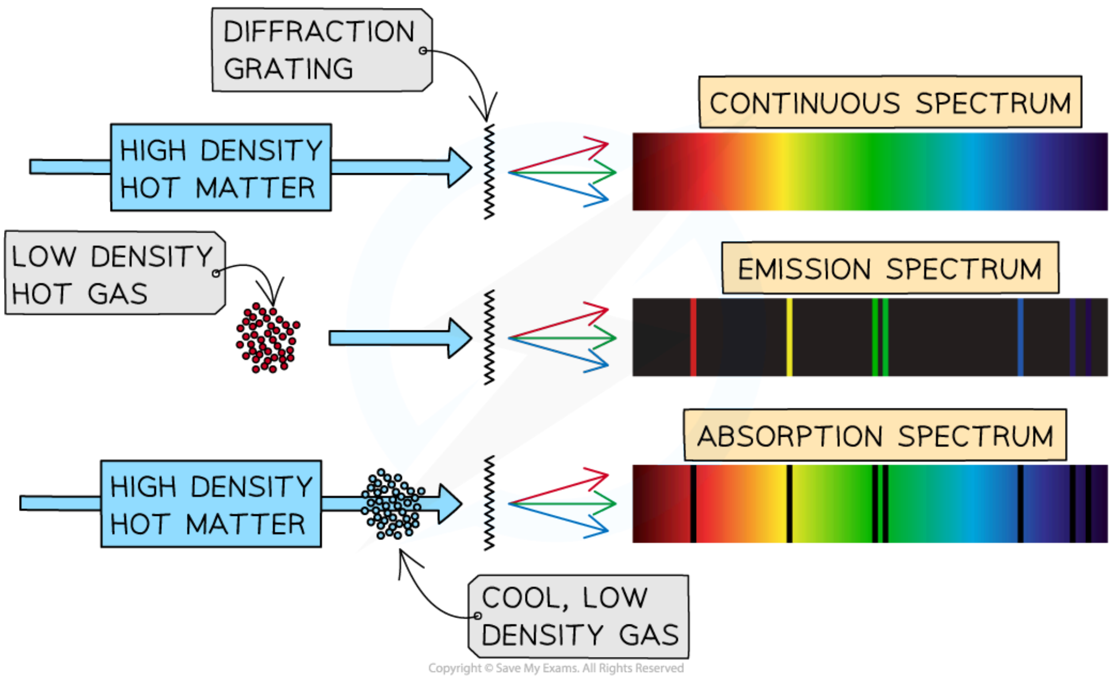
Your notes

- This is represented by a background of all the colours with dark lines at certain wavelengths
- These dark lines correspond exactly to the **differences in energy levels** in an atom
- When these electrons return to lower levels, the photons are emitted in **all directions**, rather than in the original direction of the white light
    - Therefore, some wavelengths appear to be missing
- The wavelengths missing from an absorption spectrum are the same as their corresponding emission spectra of the same element



The absorption spectrum of hydrogen

Your notes


DIFFRACTION GRATING
HIGH DENSITY HOT MATTER
CONTINUOUS SPECTRUM
LOW DENSITY HOT GAS
EMISSION SPECTRUM
ABSORPTION SPECTRUM
HIGH DENSITY HOT MATTER
COOL, LOW DENSITY GAS
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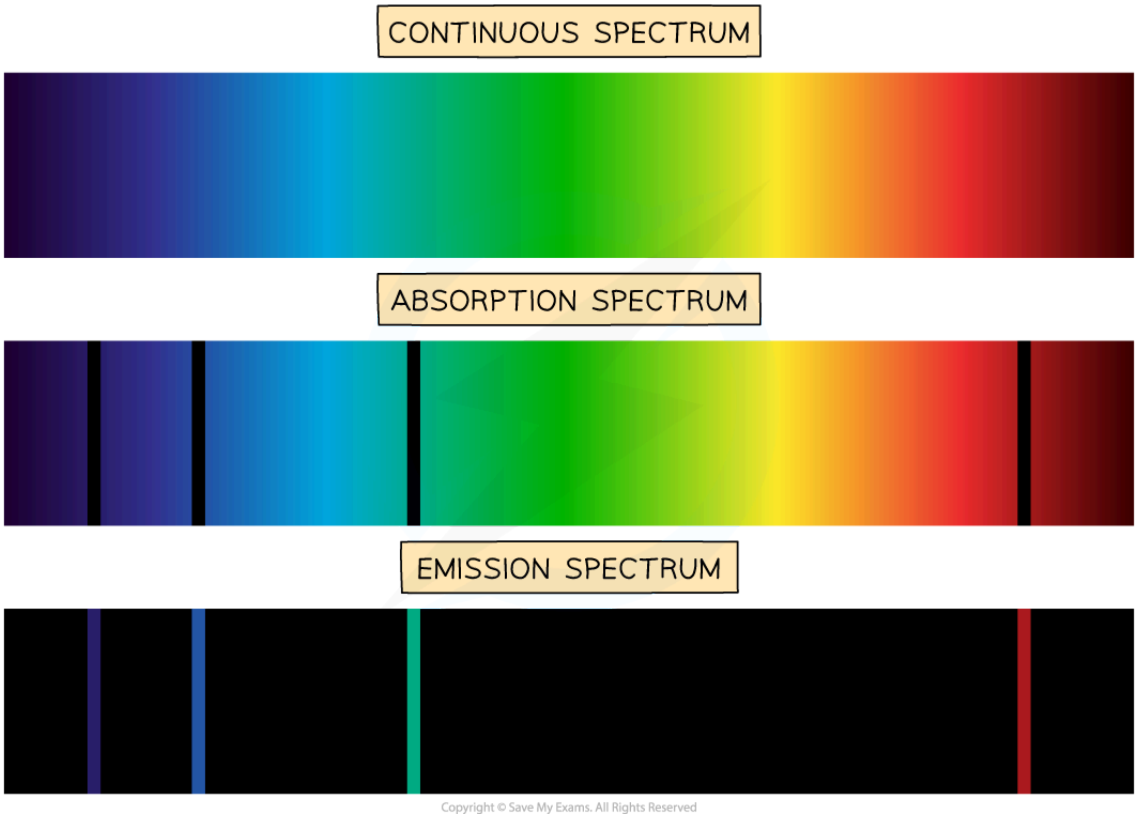


Your notes

Worked Example

An absorption line at a wavelength of 588 nm is observed in the spectrum from a star.

Determine the difference between the energy levels for the atoms in the gas responsible for this absorption line. Give your answer in electron volts.

Answer:

Step 1: List the known quantities

- Wavelength, λ = 588 nm = 5.88×10^{-7} m
- Planck's constant, h = 6.63×10^{-34} J s
- Speed of light, c = 3.00×10^{8} m s^{-1}

Step 2: Identify the appropriate equation

$$\Delta E = \frac{hc}{\lambda}$$

Step 3: Substitute the known values and calculate Δ*E*

$$\Delta E = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})}{5.88 \times 10^{-7}} = 3.38 \times 10^{-19} \text{ J}$$

Step 4: Convert from J to eV

$$\Delta E = \frac{3.38 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.11 \text{ eV}$$

Worked Example

Explain why:

a) Hot, dense sources produce continuous spectra

b) Hot, low pressure gases produce emission spectra

c) Hot, dense sources observed through cool, low pressure gases produce absorption spectra

Answer:

Part (a)

Hot, dense sources, such as the cores of stars, produce continuous spectra because:

- In a hot, dense material, the atoms or molecules are so close together that they interact with one another
- This leads to a spread of energy states that are not clearly defined
- Therefore, photons of all frequencies are emitted leading to an uninterrupted band of colour

Part (b)

Hot, low pressure gases produce emission line spectra, because:

- Hot gases produce emission line spectra when photons are emitted due to the transition of electrons between discrete energy levels in atoms of the gas
- The line spectrum has certain, fixed frequencies related to the differences in energy between the various energy levels of the atoms of the gas
- In a low pressure gas, the atoms or molecules are not close together
- This means the energy levels of the gas atoms or molecules are clearly quantised and well-defined

Your notes

- Therefore, only photons which correspond to the differences in energy between the energy levels of a bound electron are seen

Part (c)

Hot, dense sources observed through cold gases produce absorption spectra because:

- Atoms of different elements in the cold gas absorb energy emitted from the hot source but only at particular energy values
- These particular energy values correspond to the differences in energy between the energy levels of a bound electron
- This means that particular frequencies of light are absorbed, creating black lines in the continuous emission spectrum

Examiner Tips and Tricks

You will be required to identify the part of the electromagnetic spectrum to which an absorption or an emission line belongs. To help with this, make sure you are familiar with the wavelength values.

You should also be able to determine the colour by knowing the wavelength when the absorption or emission lines are in the visible range.

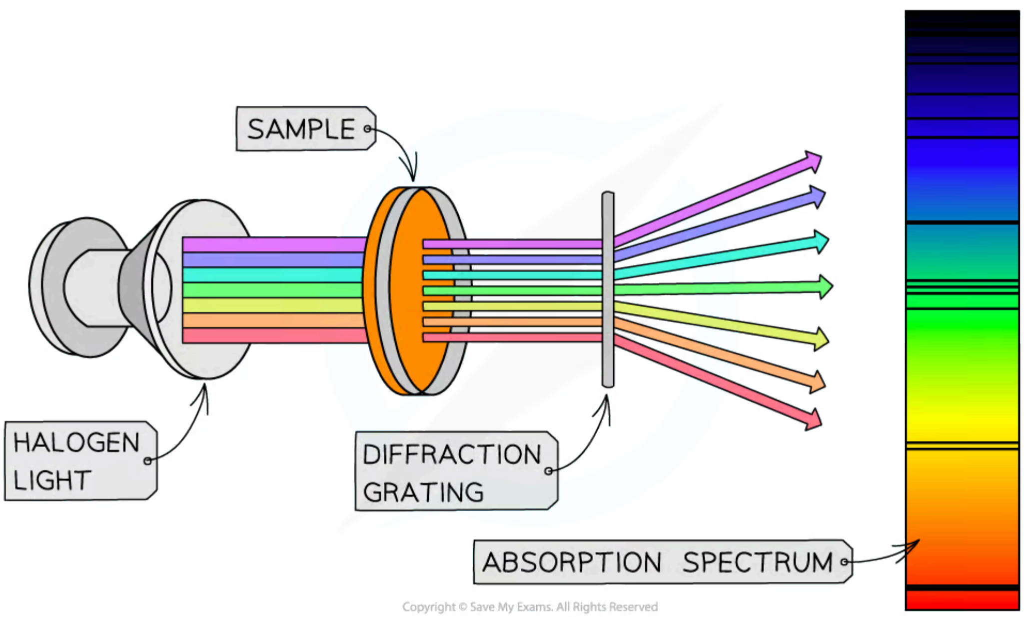
Your notes

Transmission Diffraction Grating

Your notes

Transmission Diffraction Grating

- **Dispersion** is the separation of visible white light into a spectrum of its colours
  - This can be done using a glass prism or a diffraction grating
- Transmission diffraction gratings are useful for separating light of different wavelengths with high resolution in order to:
  - Analyse light from stars
  - Analyse the composition of a star



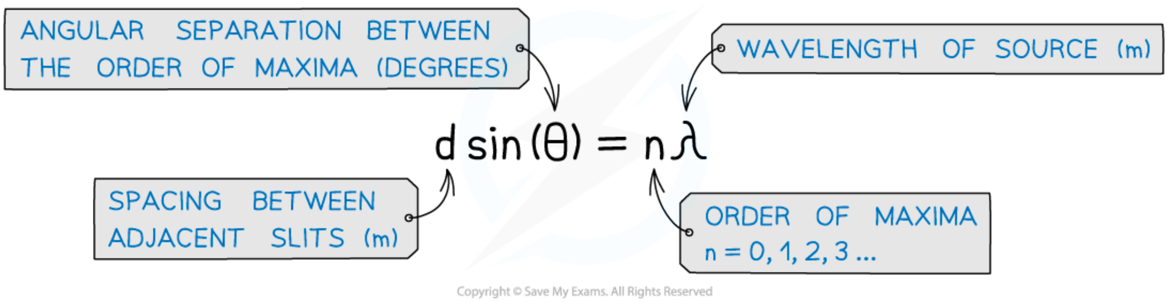
Diffraction gratings are most commonly used in spectrometers to analyse light from stars

- A **transmission diffraction grating** is a glass or plastic slide containing a large number of regularly spaced, parallel **slits** or lines
- It is used to analyse spectral line wavelengths from the light emitted by stars
  - The angular dispersion (separation) of the colours is much greater using a transmission diffraction grating than an optical prism
  - Using diffraction gratings results in sharper fringes compared to using a double slit

Condition for Maxima for a Diffraction Grating

Your notes

- The angles at which the maxima of intensity (constructive interference) are produced can be deduced by the diffraction grating equation:



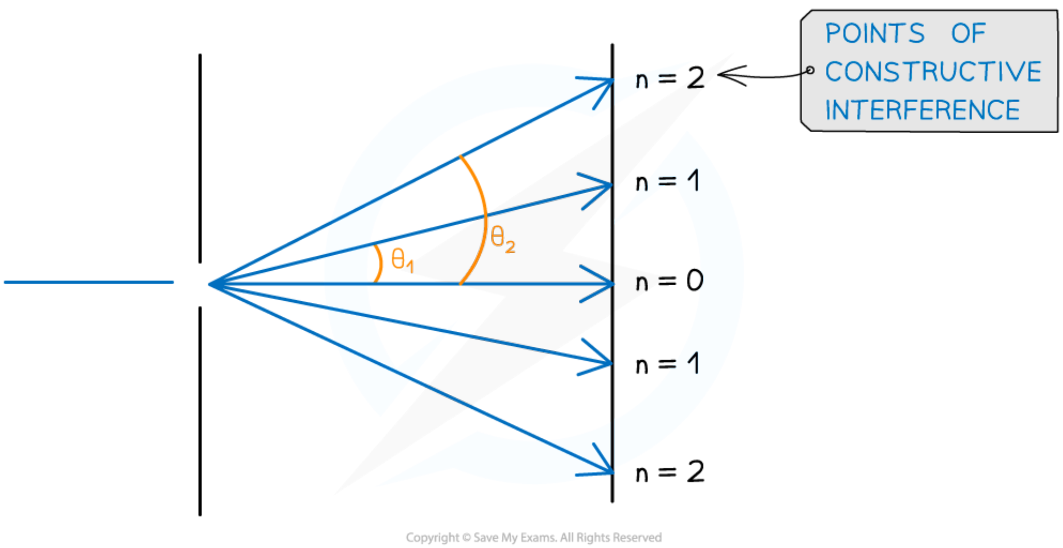
- Exam questions sometime state the **lines per m** (or per mm, per nm etc.) on the grating which is represented by the symbol *N*
- *d* can be calculated from *N* using the equation

$$d = \frac{1}{N}$$

Angular Separation

- The angular separation of each maxima is calculated by rearranging the grating equation to make θ the subject
- The angle θ is taken from the centre meaning the higher orders are at greater angles

Your notes



Angular separation

- The angular separation between two angles is found by subtracting the smaller angle from the larger one
- The angular separation between the first and second maxima n_1 and n_2 is $\boldsymbol{\theta_2 - \theta_1}$

Orders of Maxima

- The maximum angle to see orders of maxima is when the beam is at right angles to the diffraction grating
  - This means $\theta = 90^\circ$ and $\sin\theta = 1$
- The highest order of maxima visible is therefore calculated by the equation:

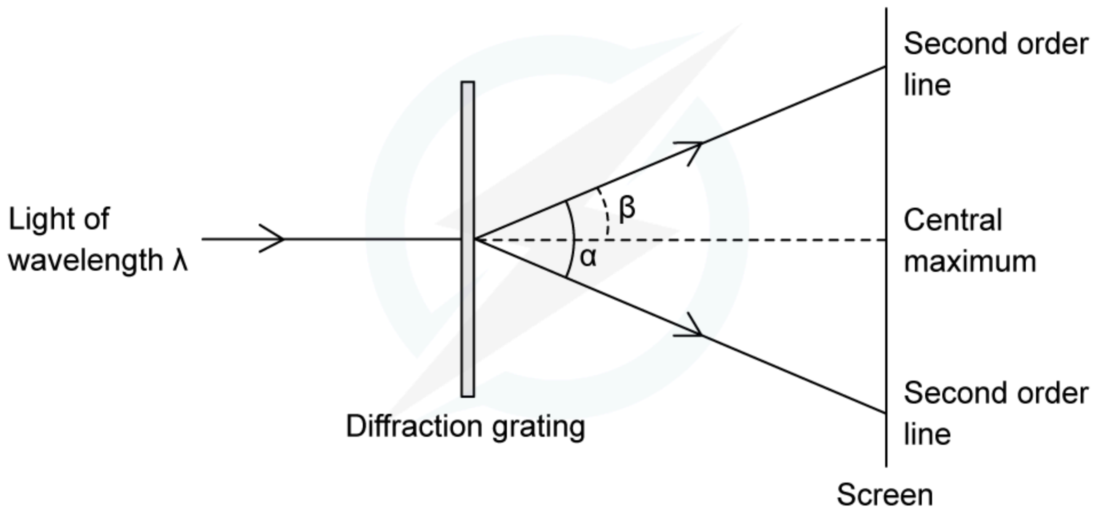
$$n = \frac{d}{\lambda}$$

- Note that since n must be an integer, if the value is a decimal it must be rounded **down**
  - E.g If n is calculated as 2.7 then $n = 2$ is the highest order visible

Your notes

Worked Example

An experiment was set up to investigate light passing through a diffraction grating with a slit spacing of 1.7 μm. The fringe pattern was observed on a screen. The wavelength of the light is 550 nm.



Calculate the angle α between the two second-order lines.

Answer:

Your notes

STEP 1

DIFFRACTION GRATING EQUATION

$d \sin(\theta) = n\lambda$

$n = 2$ FOR THE SECOND ORDER LINE

$D = 1.7\ \mu m$

$\lambda = 550\ nm$

STEP 2

REARRANGE FOR $\sin(\theta)$

$$\sin(\theta) = \frac{n\lambda}{d}$$

STEP 3

SUBSTITUTE IN VALUES

$$\sin(\theta) = \frac{2 \times 550 \times 10^{-9}}{1.7 \times 10^{-6}} = 0.64705... = 0.65 \ (2\ s.f.)$$

STEP 4

FIND θ THROUGH THE INVERSE SINE

$$\sin^{-1}(0.65) = 40.54^\circ$$

STEP 5

θ IS ANGLE FROM THE CENTRE TO THE SECOND ORDER LINE (β ON THE DIAGRAM)

$$\alpha = \theta \times 2 = 81^\circ \ (2\ s.f.)$$

Examiner Tips and Tricks

Take care that the angle θ is the correct angle taken from the centre and **not** the angle taken between two orders of maxima.

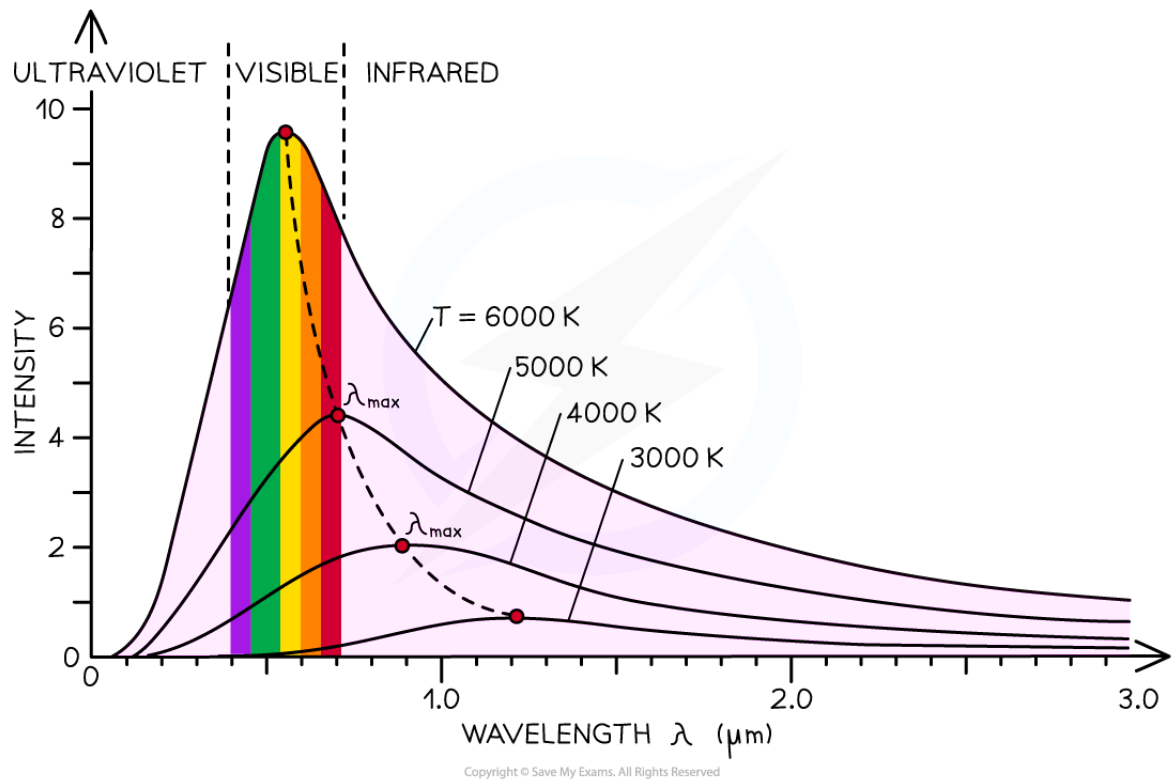
Your notes

Wein's Displacement Law

Wien's Law

Black Body Radiator

- An ideal black body radiator is one that absorbs and emits all wavelengths.
  - A black body is a theoretical object, however, stars are the best approximation there is
- The radiation emitted from a black body has a **characteristic spectrum** that is determined by the temperature alone



The intensity-wavelength graph shows how thermodynamic temperature links to the peak wavelength for four different bodies

Wien's Displacement Law

- Wien's displacement law relates the observed wavelength of light from an object to its surface temperature, it states:

The black body radiation curve for different temperatures peaks at a wavelength that is inversely proportional to the temperature

- This relation can be written as:

$$\lambda_{max} \propto \frac{1}{T}$$

- Where:
  - λ_{max} = the maximum wavelength emitted by an object at the peak intensity (m)
  - T = the surface temperature of an object (K)
- Wien's Law equation is given by:

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

- This equation shows that the higher the temperature of a body:
  - The shorter the wavelength at the peak intensity, so **hotter** objects tend to be **white or blue,** and **cooler** objects tend to be **red or yellow**
  - The greater the intensity of the radiation at each wavelength

Table to compare surface temperature and star colour

Colour of star	Temperature / K
blue	> 33 000
blue–white	10 000 – 30 000
white	7 500 – 10 000
yellow–white	6 000 – 7 500
yellow	5 000 – 6 000
orange	3 500 – 5 000
red	< 3 500

Your notes

Your notes

Worked Example

The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm.

Determine which of these two stars, Betelgeuse or Rigel, is cooler.

Answer:

Step 1: List the known quantities

- Maximum emission wavelength of Rigel = 263 nm = 263×10^{-9} m
- Maximum emission wavelength of Betelgeuse = 828×10^{-9} m

Step 2: Write down Wien's displacement law

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

Step 3: Rearrange for temperature T

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}}$$

Step 3: Calculate the surface temperature of each star

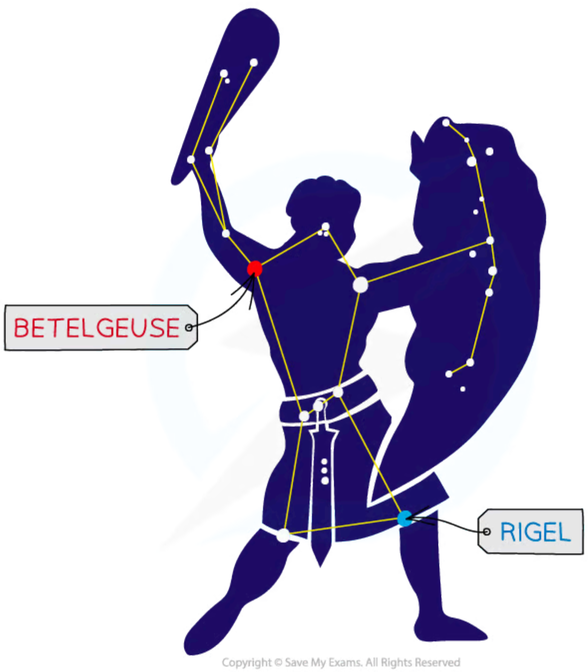
$$\text{Rigel:} \quad T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{263 \times 10^{-9}} = 11026 = \mathbf{11\ 000\ K}$$

$$\text{Betelgeuse:} \quad T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{828 \times 10^{-9}} = 3502 = \mathbf{3\ 500\ K}$$

Step 4: Write a concluding sentence

- Betelgeuse has a surface temperature of 3500 K, therefore, it is much cooler than Rigel

Your notes



The Orion Constellation; cooler stars, such as Betelguese, appear red or yellow, while hotter stars, such as Rigel, appear white or blue

Examiner Tips and Tricks

Note that the temperature used in Wien's Law is in **Kelvin** (K). Remember to convert from $^{\circ}$C if the temperature is given in degrees in the question before using the Wien's Law equation.

Your notes

Stefan's Law

Stefan's Law

- An objects luminosity depends on two factors:
  - Its surface temperature
  - Its surface area
- The relationship between these is known as **Stefan's Law** or the **Stefan–Boltzmann Law**, which states:

  The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body

- So Stefan's Law shows that the luminosity of a star is directly proportional:
  - To its radius $L \propto r^2$
  - To its surface area $L \propto 4\pi r^2$
  - To its surface absolute temperature $L \propto T^4$
- Stefan's Law equation is given by:

$$L = 4\pi r^2 \sigma T^4$$

- Where:
  - L = luminosity of the star (W)
  - r = radius of the star
  - σ = the Stefan-Boltzmann constant
  - T = surface temperature of the star (K)
- The surface area of a star (or other spherical object) can be calculated using: $A = 4\pi r^2$
  - Where r = radius of the star

Worked Example

The surface temperature of Proxima Centuri, the nearest star to Earth, is 3000 K and its luminosity is 6.506×10^{23} W.

Calculate the radius of Proxima Centuri in kilometres and show your working clearly.

Answer:

Step 1: List the known quantities:

- Surface temperature, $T = 3000$ K
- Luminosity, $L = 6.506 \times 10^{23}$ W
- Stefan's constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}

Step 2: Write down Stefan's Law

$$L = 4\pi r^2 \sigma T^4$$

Step 3: Rearrange the equation for r

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Step 4: Substitute into the equation

$$r = \sqrt{\frac{6.506 \times 10^{23}}{4\pi \times (5.67 \times 10^{-8}) \times 3000^4}} = 106\,173\,971 \text{ m}$$

Step 5: Write the final answer to the correct amount of significant figures

- The radius of Proximal Centuri is 106 200 km (4 s.f.)

Examiner Tips and Tricks

Remember to convert temperatures into Kelvin.

Check the values you obtain in your calculations. Do they make sense? Do they fit in with the magnitudes of other stars/objects that you know about?

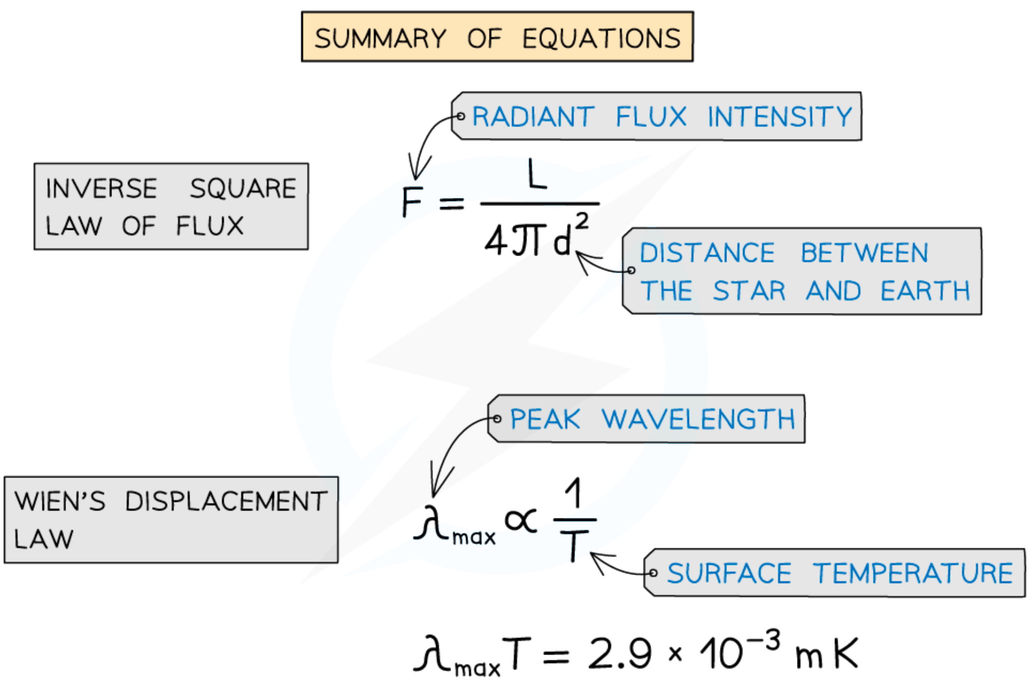
Your notes

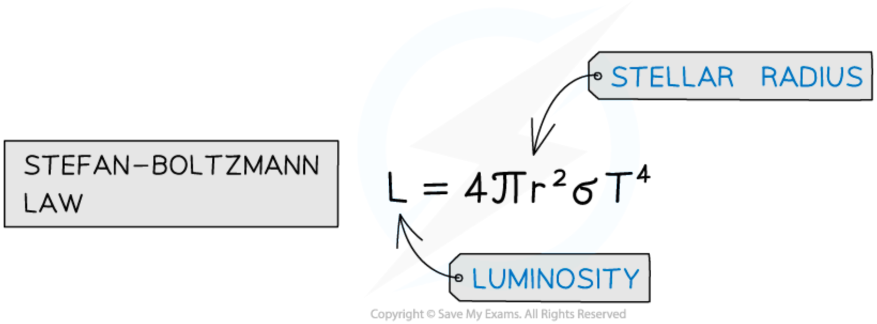
Your notes

Estimating the Radius of Stars

Estimating the Radius of Stars

- The radius of a star can be estimated by combining Wien's displacement law and the Stefan–Boltzmann law
- The procedure for this is as follows:
  - Using Wien's displacement law to find the surface temperature of the star
  - Using the inverse square law of flux equation to find the luminosity of the star (if given the radiant flux and stellar distance)
  - Then, using the Stefan-Boltzmann law, the stellar radius can be obtained





Worked Example

Betelguese is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm.

Calculate the ratio of the radius of Betelgeuse r_B to the radius of the Sun r_S.

Radius of the Sun $r_s = 6.95 \times 10^8$ m

Answer:

Step 1: List the known quantities

- Luminosity of Betelgeuse, $L = 4.49 \times 10^{31}$ W
- Peak wavelength of Betelgeuse, $\lambda_{max} = 850 \text{ nm} = 850 \times 10^{-9}$ m
- Radius of the Sun, $r_s = 6.95 \times 10^8$ m

Step 2: Write down Wien's displacement law

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

Step 3: Rearrange Wien's displacement law to find the surface temperature of Betelguese

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{850 \times 10^{-9}} = 3410 \text{ K}$$

Step 4: Write down the Stefan-Boltzmann law

$$L = 4\pi r^2 \sigma T^4$$

Step 5: Rearrange for r and calculate the stellar radius of Betelguese

Your notes

$$r_B = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{(4.49 \times 10^{31})}{4\pi \times (5.67 \times 10^{-8}) \times (3410)^4}} = 6.83 \times 10^{11} \text{ m}$$

Step 6: Calculate the ratio r_B / r_s

$$\frac{r_B}{r_s} = \frac{6.83 \times 10^{11}}{6.95 \times 10^{8}} = \mathbf{983}$$

- Therefore, the radius of Betelguese is about 1000 times larger than the Sun's radius