A Level · Edexcel · Further Maths

42 mins 4 questions

Graphical Solution of LP problems

Solving a Linear Programming Problem Graphically

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Total Marks /42

1 (a) The manager of a factory is planning the production schedule for the next three weeks for a range of cabinets. The following constraints apply to the production schedule.

- The total number of cabinets produced in week 3 cannot be fewer than the total number produced in weeks 1 and 2
- At most twice as many cabinets must be produced in week 3 as in week 2
- The number of cabinets produced in weeks 2 and 3 must, in total, be at most 125

The production cost for each cabinet produced in weeks 1, 2 and 3 is £250, £275 and £200 respectively.

The factory manager decides to formulate a linear programming problem to find a production schedule that minimises the total cost of production.

The objective is to minimise $250x + 275y + 200z$

Explain what the variables x, y and z represent.

(1 mark)

(b) Write down the constraints of the linear programming problem in terms of x, y and z.

(2 marks)

(c) Due to demand, exactly 150 cabinets must be produced during these three weeks. This reduces the constraints to

$$x + y \leqslant 75$$
$$x + 3y \geqslant 150$$
$$x \geqslant 25$$
$$y \geqslant 0$$

which are shown in Diagram 1 in the answer book.

Given that the manager does not want any cabinets left unfinished at the end of a week,

(i)

use a graphical approach to solve the linear programming problem and hence determine the production schedule which minimises the cost of production. You should make your method and working clear.

(ii) Find the minimum total cost of the production schedule.

(8 marks)

2 (a) A shop sells two types of watch, analogue watches and digital watches.

The shop manager knows that, each month, she should order at least 60 watches in total.
In addition, at most 80% of the watches she orders must be digital.

Let x be the number of analogue watches ordered and let y be the number of digital watches ordered.

Write down inequalities, in terms of x and y, to model these constraints.

(2 marks)

(b) Two further constraints are

$$y + 3x \geq 140$$

$$4y + x \geq 80$$

Represent all these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4 marks)

(c) The cost to the shop of ordering an analogue watch is five times the cost of ordering a digital watch. The shop manager wishes to minimise the total cost.

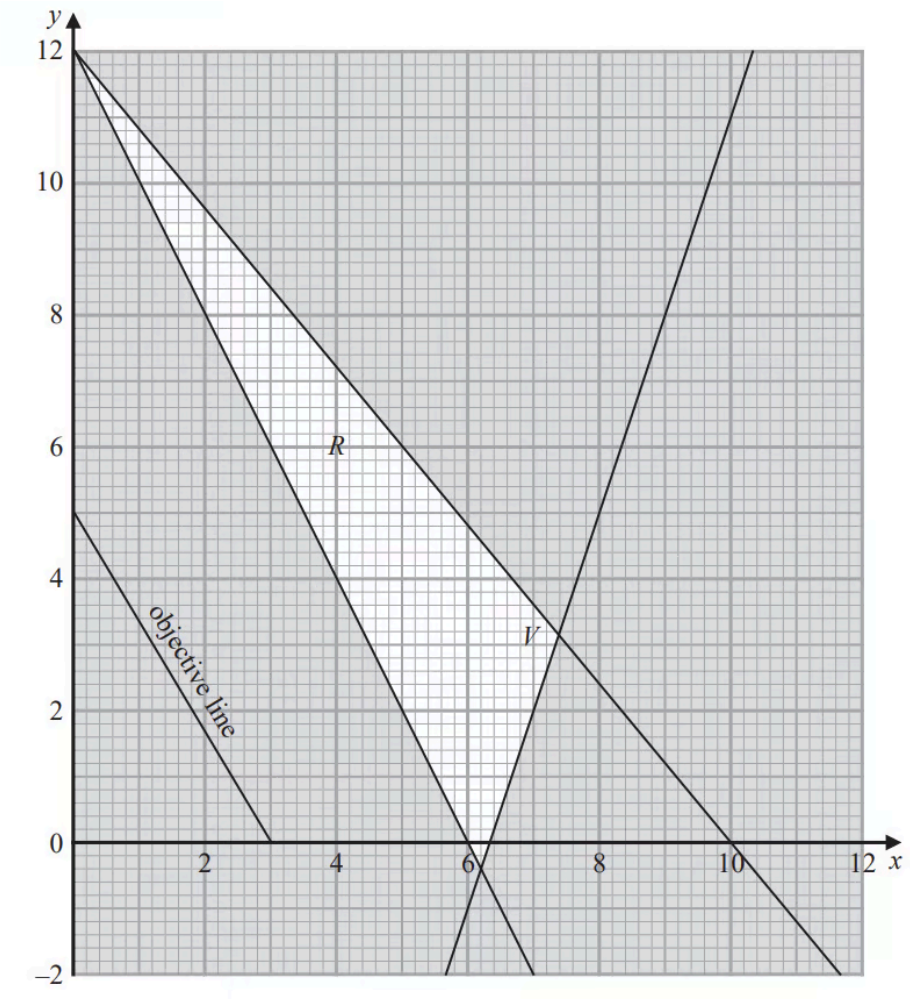
Determine the number of each type of watch the shop manager should order.
You must make your method clear.

(3 marks)

(d) Given that the minimum total cost of ordering the watches is £4455

Determine the cost of ordering one analogue watch and the cost of ordering one digital watch. You must make your method clear.

(3 marks)



3 **Figure 3**

Figure 3 shows the constraints of a linear programming problem in x and y, where R is the feasible region. Figure 3 also shows an objective line for the problem and the optimal vertex, which is labelled as V.

The value of the objective at V is 556

Express the linear programming problem in algebraic form. List the constraints as simplified inequalities with integer coefficients and determine the objective.

(9 marks)

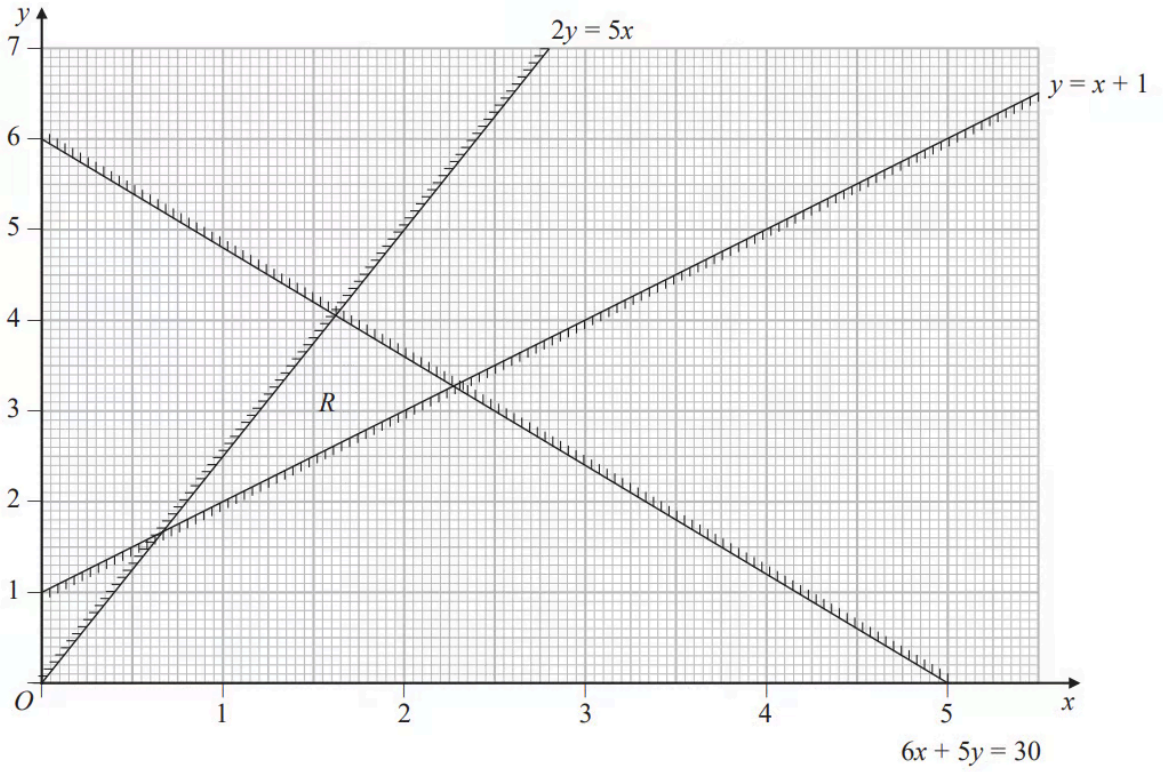


Figure 3

4 (a) Figure 3 shows the constraints of a linear programming problem in x and y, where R is the feasible region.

Write down the inequalities that define R.

(2 marks)

(b) The objective is to maximise P, where $P = 3x + y$

Obtain the exact value of P at each of the three vertices of R and hence find the optimal vertex, V.

(4 marks)

(c) The objective is changed to maximise Q, where $Q = 3x + ay$. Given that a is a constant and the optimal vertex is still V, find the range of possible values of a.

(4 marks)