Please check the examination details below before entering your candidate information

Candidate surname	Other names

MME Edexcel
Level 3 GCE

Centre Number	Candidate Number

MME Edexcel Practice Papers

Morning (Time: 2 hours)	Paper Reference 1MME

Mathematics (A-A*)

Advanced

Paper 1: Pure Mathematics 1

You must have: Mathematical Formulae and Statistical Tables, Calculator	Total Marks

Candidates may use any approved calculator.
Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. (a) Prove by **contradiction** that $\log 7$ is an irrational number. **(5)**

Given that x, a and b are positive real numbers, with $a > b$ and $x^2 > ab$

(b) use proof by **contradiction** to show that,

$$\frac{x+a}{\sqrt{x^2+a^2}} - \frac{x+b}{\sqrt{x^2+b^2}} > 0$$

(8)

(c) Given that,

$$P = T^2 - 1 \text{ and } T = 2^q - 1, q \in \mathbb{N}$$

prove that 2^{q+1} is a factor of P.

(2)

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Question 1 continued

(Total for Question 1 is 15 marks)

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2. The function f is defined by

$$f(x) = \begin{cases} x + 2 & x \leq 2 \\ (x-2)^2 + 4 & x > 2 \end{cases}$$

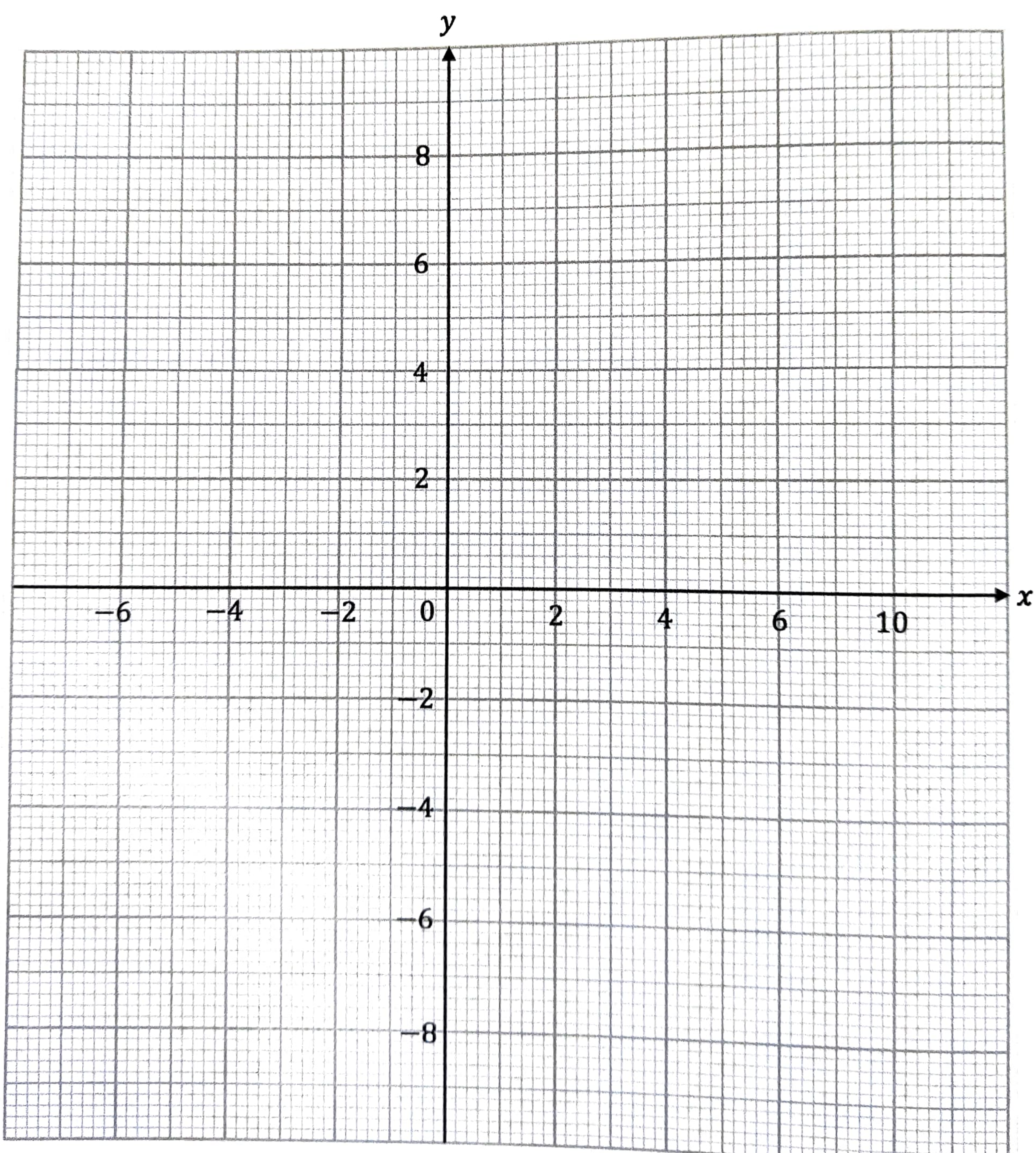
(a) Sketch the graph of $f(x)$ on the axes below.

(3)

(b) Find an expression for $f^{-1}(x)$.

You must specify its domain.

(6)



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Question 2 continued

(Total for Question 2 is 9 marks)

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3.

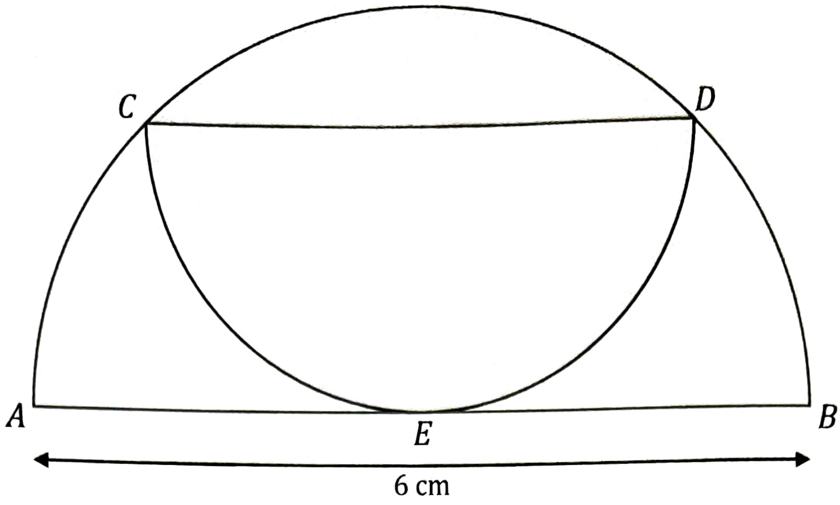

Figure 1

Figure 1 is constructed as follows.

A semicircle with diameter AB of 6 cm is first drawn.

Then another semicircle is drawn, with its diameter CD parallel to AB.

The semicircle with CD as its diameter is circumscribed by the semicircle with AB as its diameter, as shown in the figure.

Show that the area of the shaded region can be written as $\left(\frac{9}{2}(\pi - 1)\right)$ cm^2.

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

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4. A geometric progression's 1^{st}, 2^{nd} and 3^{rd} terms are also the respective 7^{th}, 4^{th} and 2^{nd} terms of an arithmetic progression.

Find the common ratio of the geometric progression.

(7)

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Question 4 continued

(Total for Question 4 is 7 marks)

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5. The radius of a circle is changing, subject to time, such that,

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{3r^2}$$

Show that the rate at which the area of the circle changes can be written as

$$\sqrt{\frac{a\pi^3}{bA}}$$

where a and b are integer constants to be determined.

You **must** show your working.

(6)

(Total for Question 5 is 6 marks)

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6. The following recurrence relation describes the Fibonacci sequence,

$$u_{n+2} = u_{n+1} + u_n \qquad u_1 = 1, u_2 = 1$$

The ratio of consecutive terms converges to a limit ϕ, known as the Golden Ratio.

Show, by using the recurrence relation, that $\phi = \frac{1}{2}(1 + \sqrt{5})$.

(7)

Turn over ▶

Question 6 continued

(Total for Question 6 is 7 marks)

7. The point $P(x, y)$ lies on a circle with centre at $(2,0)$ and radius 2.

Find exactly, the greatest value of $x + y$, for all possible positions of the point P.

(12)

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8. A curve C is defined in the largest real domain by the equation,

$$y = \log_x 3$$

The point P, where $x = 3$, lies on C.

The normal to C at P meets C again at the point Q.

(a) Show that the x-coordinate of Q is a solution to the equation,

$$\ln 3 = \ln x\,(1 + x \ln 27 - 3 \ln 27)$$

(9)

(b) Use an iterative formula of the form $x_{n+1} = e^{f(x_n)}$, with a suitable starting value, to find the coordinates of Q, correct to 3 decimal places.

(4)

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Question 8 continued

(Total for Question 8 is 13 marks)

9.

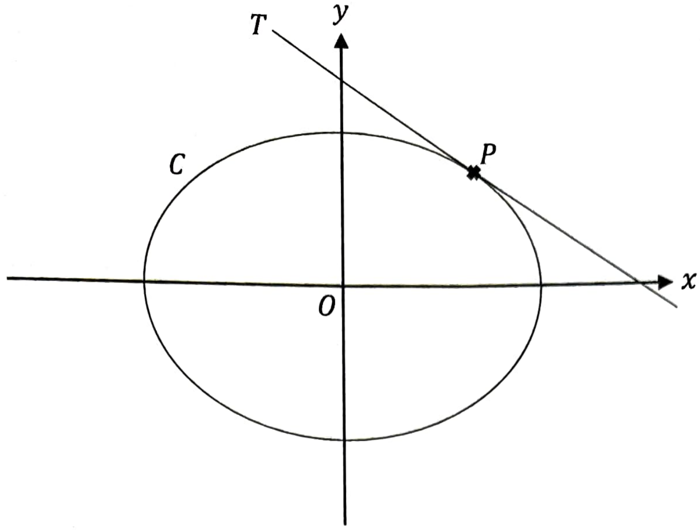

Figure 2

Figure 2 shows the curve C with parametric equations,

$$x = 2\cos\theta\,, y = 4\sin\theta\,, \qquad 0 \le \theta \le 2\pi$$

The point P lies on C where $\theta = \alpha$, where $0 < \theta < \frac{\pi}{2}$.

The line T is a tangent to C at P.

(a) Find the equation of the tangent at P in the form $\mathrm{a}x + \mathrm{b}y = \mathrm{c}$, where a and b are functions in α, and c is an integer constant.

(6)

The tangent meets the coordinate axes at the points A and B.

The area of the triangle OAB, where O is the origin, is less than 16 square units.

(b) Find the range of the possible values of α.

(7)

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Question 9 continued

(Total for Question 9 is 13 marks)

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10. With respect to a fixed origin, the points A and B have position vectors $20\boldsymbol{i} + 18\boldsymbol{j} - 12\boldsymbol{k}$ and $12\boldsymbol{i} - 6\boldsymbol{j} + 20\boldsymbol{k}$, respectively.

The position vector of the point C has $\boldsymbol{i}$-component equal to 4.

The distance of C from both A and B is 24 units.

Show that one of the two possible position vectors of C is $4\boldsymbol{i} + 10\boldsymbol{j} + 4\boldsymbol{k}$ and determine the other.

(12)

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Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

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