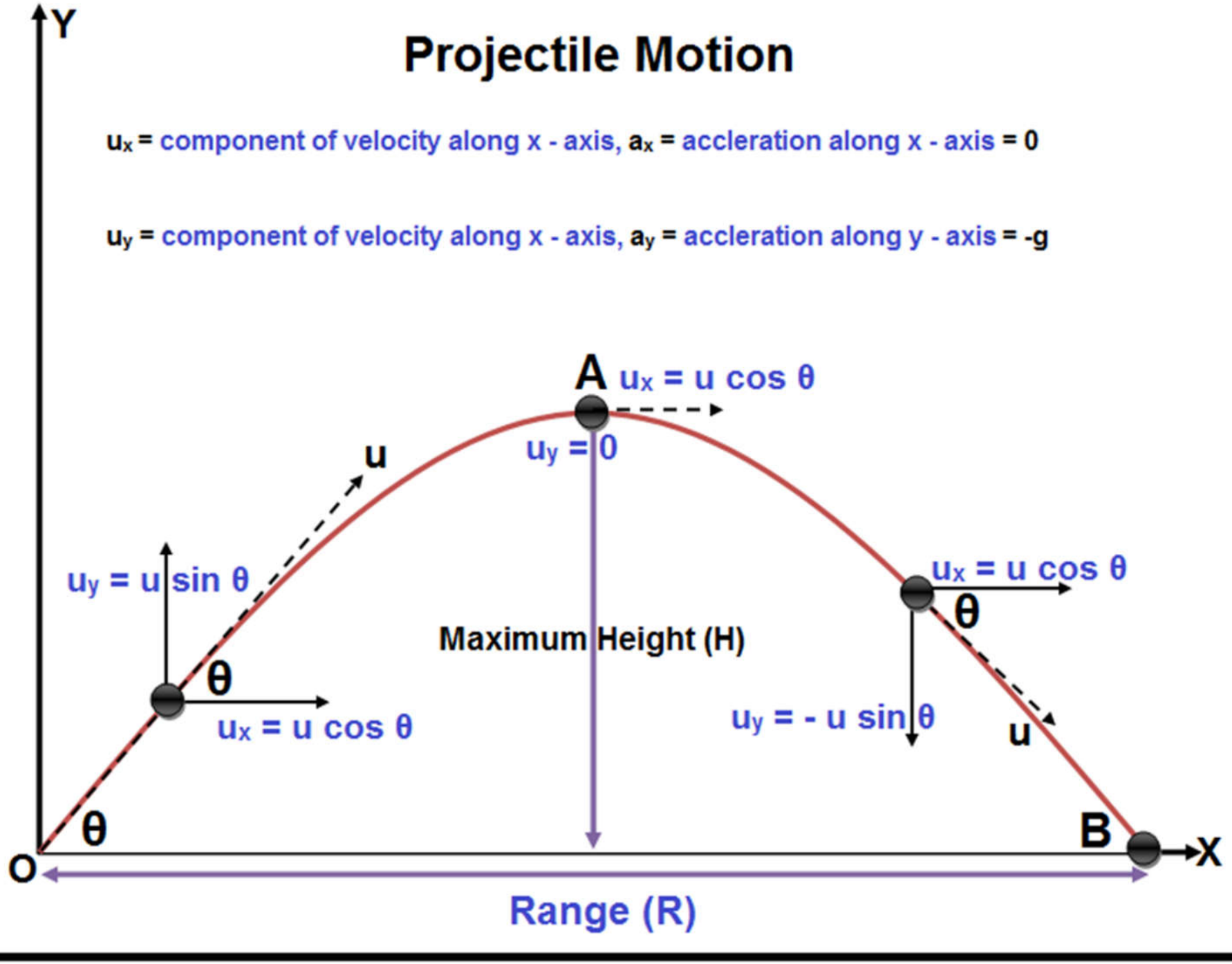
Projectile Motion

u_x = component of velocity along x - axis, a_x = accleration along x - axis = 0

u_y = component of velocity along x - axis, a_y = accleration along y - axis = -g

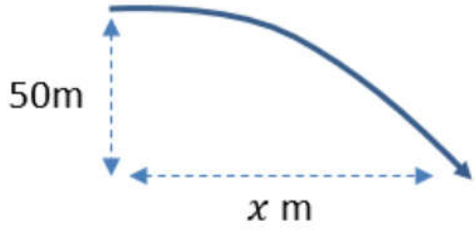


Projectile Motion

In Mechanics Year 1 we already encountered problems of vertical motion of objects when projected vertically. We used "SUVAT" equations where the acceleration was g ms^{-2}.
In this chapter we allow the object to be **projected sideways**!

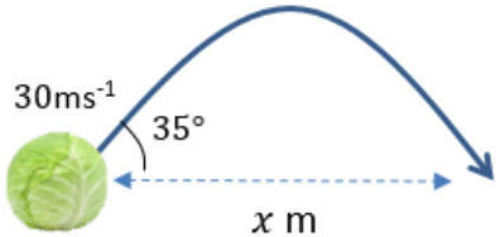
1:: Horizontally projected

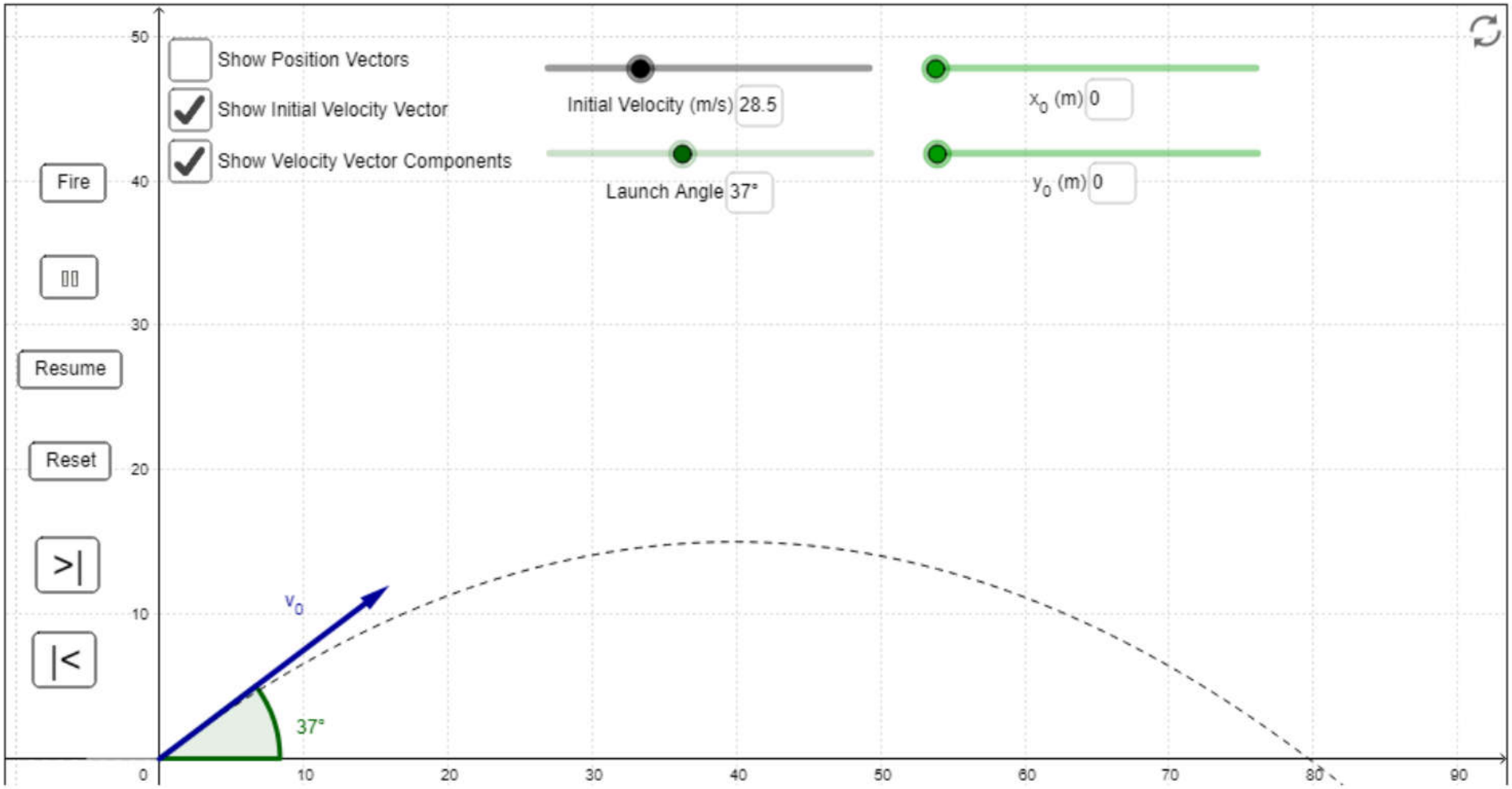
"A particle is projected horizontally at 20 ms^{-1}, at a distance 50m above the ground. How far along the ground does it travel?"



2:: Projection at any angle

"A cabbage is projected from ground level at 30 ms^{-1} at an angle of 35°. How far away is the cabbage when it hits the ground?"




Show Position Vectors
Show Initial Velocity Vector
Show Velocity Vector Components
Initial Velocity (m/s) 28.5
Launch Angle 37°
x_0 (m) 0
y_0 (m) 0
Fire
Resume
Reset
v_0
37°
0
10
20
30
40
50
60
70
80
90

Acceleration in each direction

Consider vertical and horizontal motion separately

In **vertical** direction, acceleration downwards is g ms^{-2}. (it is constant)

Use SUVAT equations as before.

In **horizontal** direction, acceleration is 0 ms^{-2}.

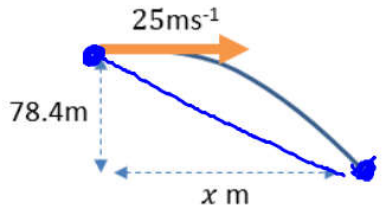
Constant velocity, so can use standard $speed = \frac{distance}{time}$

A particle is projected horizontally at 25 ms^{-1} from a point 78.4 metres above a horizontal surface. Find:

(a) the time taken by the particle to reach the surface

(b) the horizontal distance travelled in that time.

(c) the distance of the impact point from the original point.



vertical motion ↓+

a) $u = 0$

$a = 9.8$

$s = 78.4$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$78.4 = 4.9t^2$

$t = 4$

b) horizontal motion

speed = 25

dist = x

time = 4

$x = 25 \times 4$

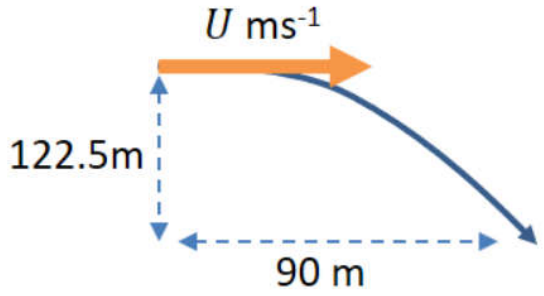
$x = 100$m

c) Pyth.

$dist = \sqrt{78.4^2 + 100^2}$

$= 127$m

A particle is projected horizontally with a speed of U ms^{-1} from a point 122.5m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 90m away from the starting point.
Find the initial speed of the particle.



↓9.8

Vert. ↓+

$u = 0$
$s = 122.5$
$a = 9.8$
$t = ?$

$s = ut + \frac{1}{2}at^2$

$122.5 = 4.9t^2$

$t = 5$.

horizontal

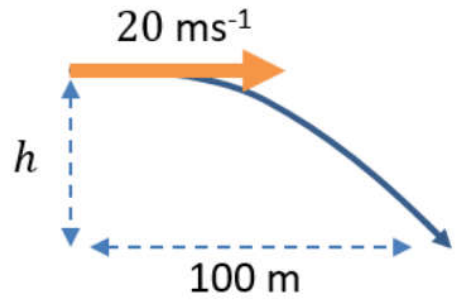
dist = 90
time = 5
speed = u

$u = \frac{90}{5}$

$u = 18$ ms^{-1}

Your Turn

A particle is projected horizontally with a speed of 20 ms^{-1} from a point h m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 100m away from the starting point. Determine the value of h.



Ex 6A

Q 4, 5, 6, 7

horiz.

speed = 20

dist = 100

time = $\frac{100}{20}$ = 5

vert. ↓+

a = 9·8 s = ?

u = 0

t = 5

$s = ut + \frac{1}{2}at^2$

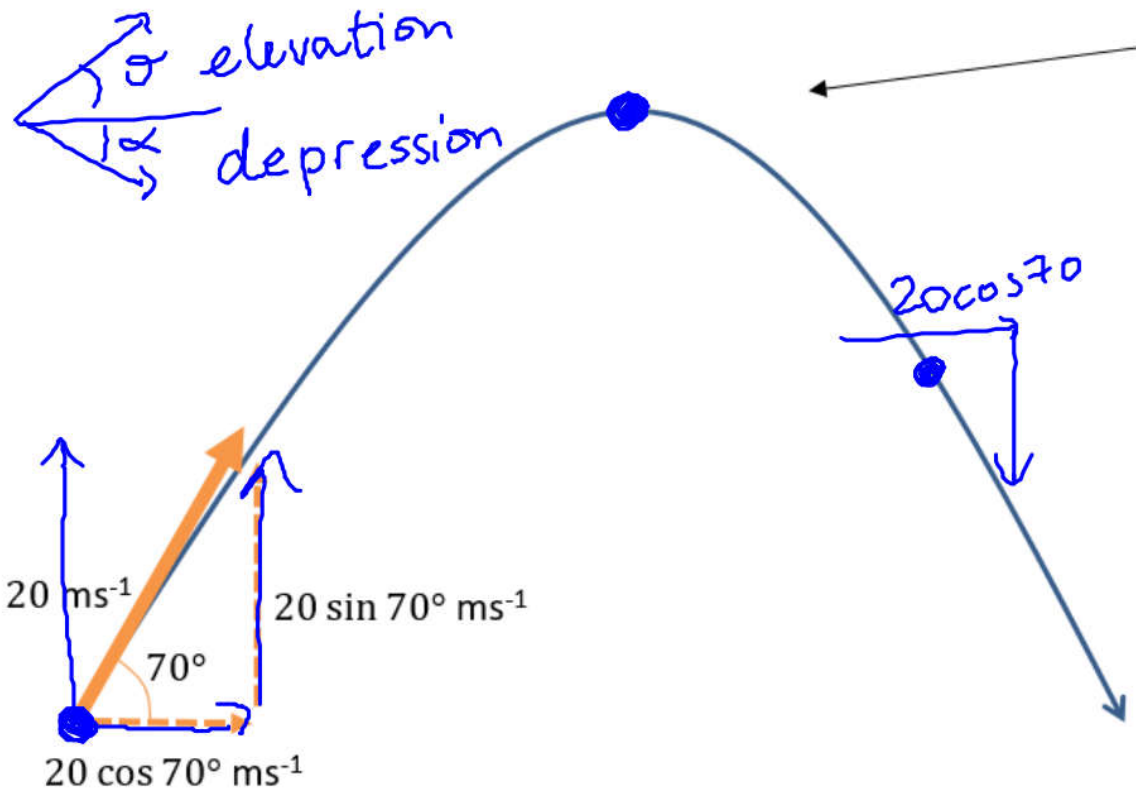
$= 4.9 \times 5^2$

$= 122.5m$

$= 123m$ (3sf)

Components of velocity

Just as **we split forces into its horizontal and vertical components**, in order to consider forces in the horizontal and vertical directions respectively, we can do **exactly the same with velocity**!

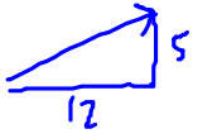


When the object is at its highest point:

vert. velocity = 0

20cos70

We know that the scalar form of velocity is **speed**, and thus we just find the **magnitude** of the velocity vector:



$$\begin{pmatrix} 12 \\ 5 \end{pmatrix} ms^{-1}$$

$$\Rightarrow \sqrt{12^2 + 5^2}$$

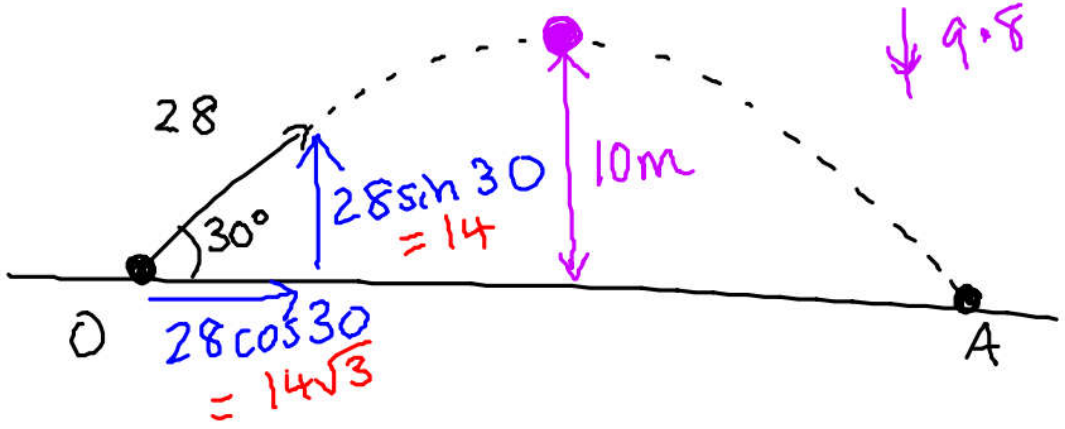
$$= \mathbf{13\ ms^{-1}}$$

A particle P is projected from a point O on a horizontal plane with speed 28 ms^{-1} and with angle of elevation 30°. After projection, the particle moves freely under gravity until it strikes the plane at a point A. Find:

(a) the greatest height above the plane reached by P

(b) the time of flight of P

(c) the distance OA



a) vert mot ↑+

↓9.8

$u = 14$

$a = -9.8$

$s = ?$

$v = 0$

$v^2 = u^2 + 2as$

$0 = 14^2 - 19.6s$

$s = 10\text{m}$

b) vert. mot ↑+

$s = 0$

$a = -9.8$

$u = 14$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 14t - 4.9t^2$

$0 = t(14 - 4.9t)$

$t = 0$ $t = 2.86$ (3sf)

c) horiz.

dist = speed x time

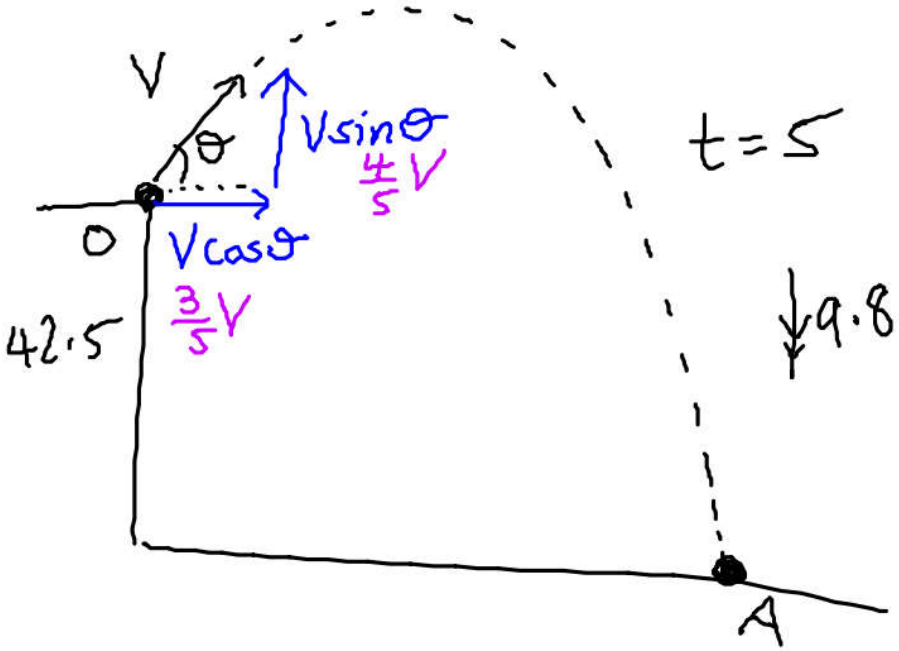
$= 14\sqrt{3} \times 2.86$

$= 69.3\text{ m}$ (3sf)

Projected from above ground

$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$.

A particle is projected from a point O with speed V ms^{-1} and at an angle of elevation of θ, where $\tan\theta = \frac{4}{3}$. The point O is 42.5m above a horizontal plane. The particle strikes the plane at a point A, 5 s after it is projected.
(a) Show that $V = 20$. (b) Find the distance between O and A.



vert. mot. ↓+

$s = 42.5$ $a = 9.8$ $u = -\frac{4}{5}V$ $t = 5$

$42.5 = -\frac{4}{5}V \times 5 + 4.9 \times 5^2$

$42.5 = -4V + 122.5$

$4V = 80$

$V = 20$

b) horizontal

dist = speed x time

$= \frac{3}{5}V \times 5$

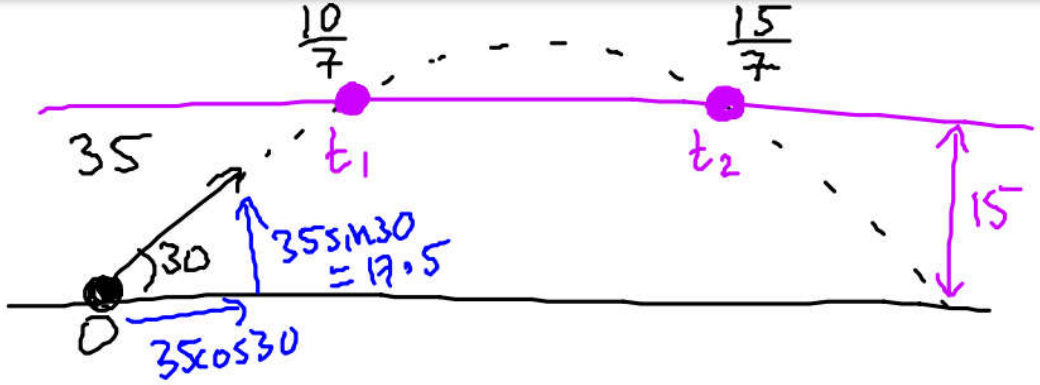
$= \frac{3}{5} \times 20 \times 5 = 60$m

$OA = \sqrt{60^2 + 42.5^2}$

$= 73.5$m (3sf)

Time above a given point

A particle is projected from a point O with speed 35 ms^{-1} and at an angle of elevation of 30°. The particle moves freely under gravity. Find the length of time for which the particle is 15 m or more above O.



vert. mot ↑+

$s = 15$
$a = -9.8$
$u = 17.5$
$t = ?$

$s = ut + \frac{1}{2}at^2$

$15 = 17.5t - 4.9t^2$

$0 = -4.9t^2 + 17.5t - 15$

$t_1 = \frac{10}{7}$ $t_2 = \frac{15}{7}$

Time above $= \frac{15}{7} - \frac{10}{7} = \frac{5}{7} = \underline{0.714}$ (3sf)