A Level · Edexcel · Maths

3 hours 32 questions

9.1 Parametric Equations (A Level only)

Easy (8 questions)	/44
Medium (8 questions)	/44
Hard (8 questions)	/46
Very Hard (8 questions)	/60
Total Marks	**/194**

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Easy Questions

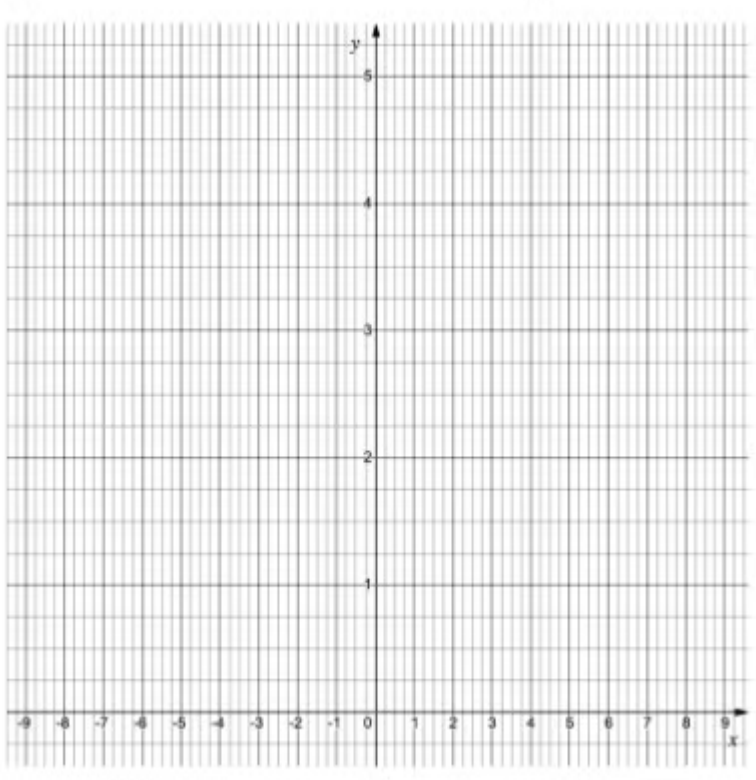
1 (a) The curve C has parametric equations $x = 4t$ and $y = t^2$.

Complete the table below.

t	-2	-1	0	1	2
x					
y					

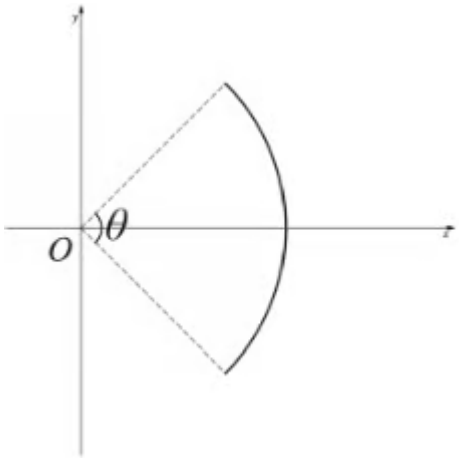
(2 marks)

(b) Plot the graph of C on the axes below.



(2 marks)

2 (a) A company's logo is created using an arc of a circle as shown in the diagram below.



When the end points of the arc are joined to the origin, they form the minor sector of a circle with angle θ radians at the centre.

The arc is formed using the parametric equations $x = \sin t$ and $y = \cos t$, with domain $\frac{\pi}{4} \leq t \leq \frac{3\pi}{4}$.

Write down the size of the angle θ.

(1 mark)

(b) i) By finding both x^2 and y^2 show that $x^2 + y^2 = 1$.

ii) Hence write down the radius of the sector.

(3 marks)

(c) Use the formula "$l = r\theta$" to find the length of the arc.

(1 mark)

3 A curve C has parametric equations $x = t - 3$ and $y = t^2 - 4$.

Find a Cartesian equation for the curve C in the form $y = \mathrm{f}(x)$.

(3 marks)

4 (a) A curve is defined parametrically by the equations $x = 2\sin\theta$ and $y = 2\cos\theta$.

(i) Find expressions, in terms of θ, for x^2 and y^2.

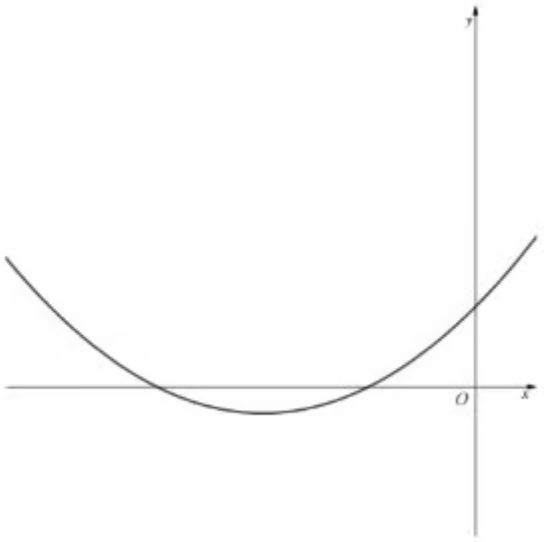
(ii) Hence find the Cartesian equation for the curve.

(4 marks)

(b) Describe the shape of the curve.

(2 marks)

5 (a) A sketch of the graph with parametric equations $x = 4t - 8$ and $y = t^2 - 1$ is shown below.



Find the value of t when $x = 0$ and hence write down the coordinates of the y-axis intercept.

(2 marks)

(b) Find the values of t when $y = 0$ and hence write down the coordinates of the x-axis intercepts.

(3 marks)

6 (a) A curve C is defined by the parametric equations $x = t + 1$ and $y = t^2 - 1$.

Show that the equation of the curve can be given in the Cartesian form $y = \mathrm{f}(x)$, with $\mathrm{f}(x) = x^2 - 2x$.

(2 marks)

(b) Sketch the graph of $y = \mathrm{f}(x)$, labelling any points where the graph intercepts the coordinate axes.

(3 marks)

(c) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

(ii) Hence show the coordinates of the minimum point on the graph of $y = \mathrm{f}(x)$ are (1 , -1).

(4 marks)

(d) Given $t \in \mathbb{R}$, write down the domain and range of $\mathrm{f}(x)$.

(2 marks)

7 The curve defined parametrically by $x = pt^3$ and $y = 2t + 1$, where p is a non-zero constant, passes through the point Q (16 , 5).

(i) Show that at point Q, $t = 2$.

(ii) Hence show that $p = 2$.

(4 marks)

8 (a) A curve C is defined by the parametric equations $x = e^t$ and $y = |t|$, where $t \in \mathbb{R}$.

Show that $y = f(x)$, where $f(x) = |\ln x|$.

(2 marks)

(b) Sketch the graph of C, labelling any points where the graph intercepts the coordinate axes.

(2 marks)

(c) Determine the domain and range for the function $f(x)$.

(2 marks)

Medium Questions

1 (a) The curve C has parametric equations

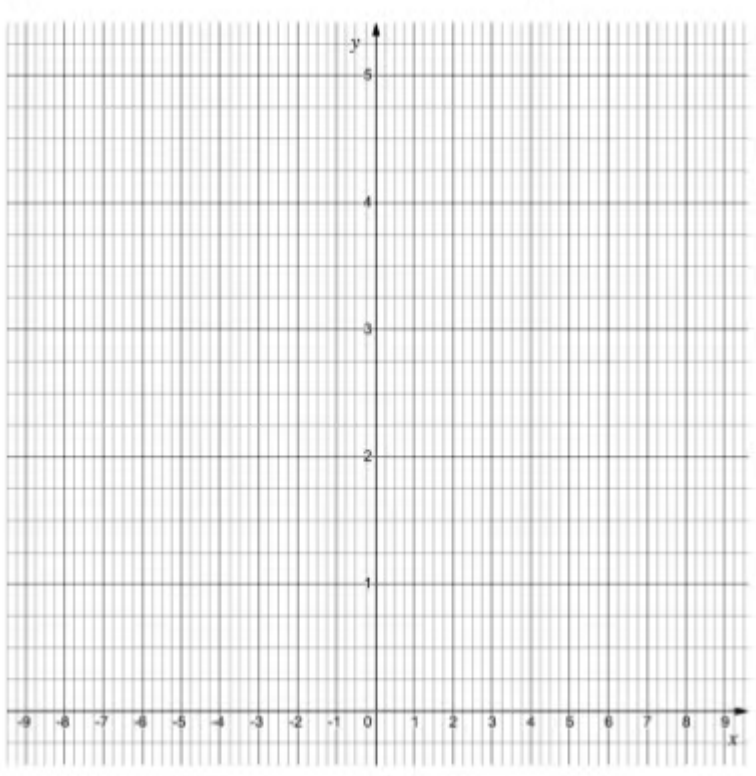
$$x = t^3 \quad \text{and} \quad y = t^2$$

Complete the table below.

t	-2	-1	0	1	2
x					
y					

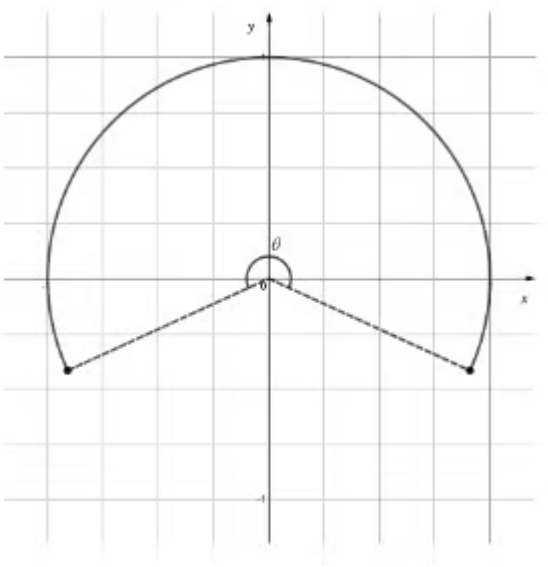
(2 marks)

(b) Plot the graph of C on the axes below.



(2 marks)

2 A company's logo is created using an arc of a circle as shown in the diagram below.



When the end points of the arc are joined to the origin, they form the major sector of a circle with angle θ radians at the centre.

The arc is formed using the parametric equations

$$x = \sin t \qquad y = \cos t$$

with domain $-2 \leq t \leq 2$.

Find the size of the angle θ, as well as the length of the arc used in the logo.

(6 marks)

3 A curve C has parametric equations

$$x = 2t - 1 \quad y = 4t^2 + 3$$

Find a Cartesian equation for the curve C in the form $y = f(x)$.

(3 marks)

4 (i) Find the Cartesian equation for the curve defined by the parametric equations

$$x = 4\cos\theta \quad y = 4\sin\theta$$

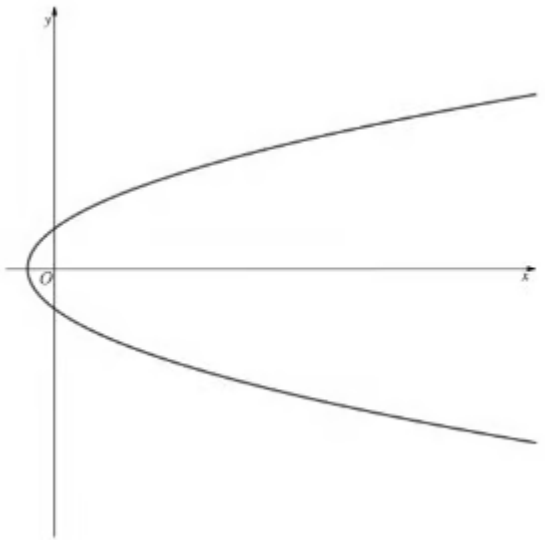
(ii) Describe the shape of the curve.

(4 marks)

5 (a) A sketch of the graph with parametric equations

$$x = t^2 - 4 \quad y = 3t$$

is shown below.



Find the value of t when $y = 0$ and hence write down the coordinates of the x-axis intercept.

(2 marks)

(b) Find the values of t when $x = 0$ and hence write down the coordinates of the y-axis intercepts.

(3 marks)

(c) Find the coordinates of the points where the graph intersects the line with equation $x = 12$.

(3 marks)

6 (a) A curve C is defined by the parametric equations

$$x = t^2 + 1 \quad y = t^2 - 1$$

Show that the equation of the curve can be given in the Cartesian form $y = \mathrm{f}(x)$, with $\mathrm{f}(x) = x - 2$.

(2 marks)

(b) Assuming there are no restrictions on the values of t, find the domain and range of $\mathrm{f}(x)$.

(2 marks)

(c) Sketch the graph of $y = \mathrm{f}(x)$.

(3 marks)

7 (a) The curve defined parametrically by

$$x = pt^3 \quad y = pt^2$$

where p is a non-zero constant, passes through the point Q (3 , -3).

Show that at point Q, $t = -1$.

(3 marks)

(b) Hence, or otherwise, find the value of p.

(2 marks)

8 (a) A curve C is defined by the parametric equations

$$x = \cos^2 t \quad y = \sin^2 t \quad -\pi \leq t \leq \pi$$

Show that $y = 1 - x$.

(2 marks)

(b) Determine the ranges of x and y in the given domain of t.

(2 marks)

(c) Sketch the graph of C.

(3 marks)

Hard Questions

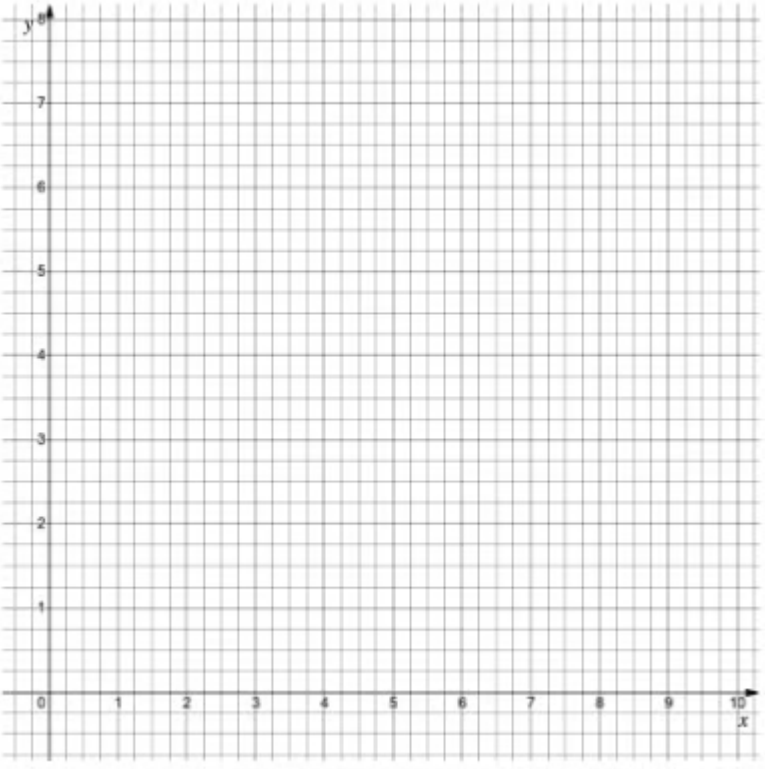
1 The curve C has parametric equations

$$x = t^2 \quad \text{and} \quad y = e^t$$

(i) Complete the table below.
Give values to three significant figures where appropriate.

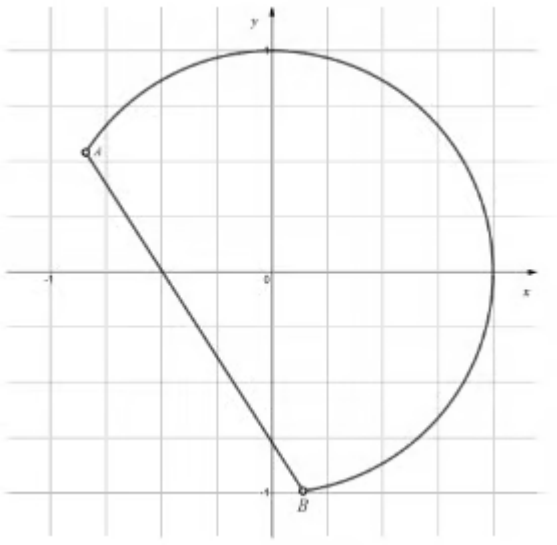
t	-3	-2	-1	0	1	2
x						
y						

(ii) Plot the graph of C on the axes below.



(4 marks)

2 A company's logo is created using an arc of a circle and a straight line as shown in the diagram below.



The arc part of the logo is formed using the parametric equations

$$x = \sin\phi \quad y = \cos\phi$$

with domain $-1 \leq \phi \leq 3$.

Find the equation of the line passing through points A and B, stating values to three significant figures where appropriate.

(3 marks)

3 Find a Cartesian equation for the curve that has parametric equations

$$x = 4t^2 + 1 \quad y = \ln 2t \quad t > 0$$

giving your answer in the form $y = f(x)$.

(3 marks)

4 (a) A curve is defined by the parametric equations

$$x = 2\cos 3\theta + 3 \quad y = 2\sin 3\theta - 2 .$$

By changing these equations into Cartesian form, show that this is the equation of a circle, and determine its centre and radius.

(5 marks)

(b) (i) Given that θ is non-negative, write down a minimum possible domain for θ that will produce a complete circle.

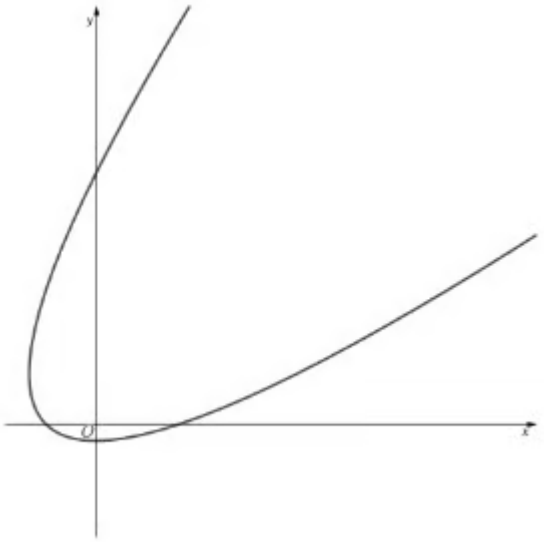
(ii) Give a restricted domain for θ that would create a semicircle.

(3 marks)

5 (a) A sketch of the graph with parametric equations

$$x = t^2 + 4t \quad y = t^2 - 1$$

is shown below.



Find the coordinates of all points where the graph intersects the coordinate axes.

(5 marks)

(b) Find the coordinates of the points where the graph intersects the line with equation $x - 5y + 19 = 0$.

(4 marks)

6 (a) A curve C is defined by the parametric equations

$$x = t^4 - 4t^2 \quad y = 4t \quad t \in \mathbb{R}$$

Show that the Cartesian equation of C can be written in the form $x = \frac{1}{256}\,\mathrm{f}(y)$, where $\mathrm{f}(y)$ is a function of y.

(3 marks)

(b) Sketch the curve C.

(4 marks)

7 The curve defined parametrically by

$$x = pt^3 + 4 \quad y = pt^2 - 2$$

where p is a constant, passes through the point (139 , 43).

Find the value of p.

(4 marks)

8 (a) A curve C is defined by the parametric equations

$$x = \arccos t \quad y = \sqrt{1 - t^2} \quad -1 \leq t \leq 1$$

(i) Determine the ranges of x and y in the given domain of t.

(ii) Show that $y = \sin x$.

(5 marks)

(b) Sketch the graph of C.

(3 marks)

Very Hard Questions

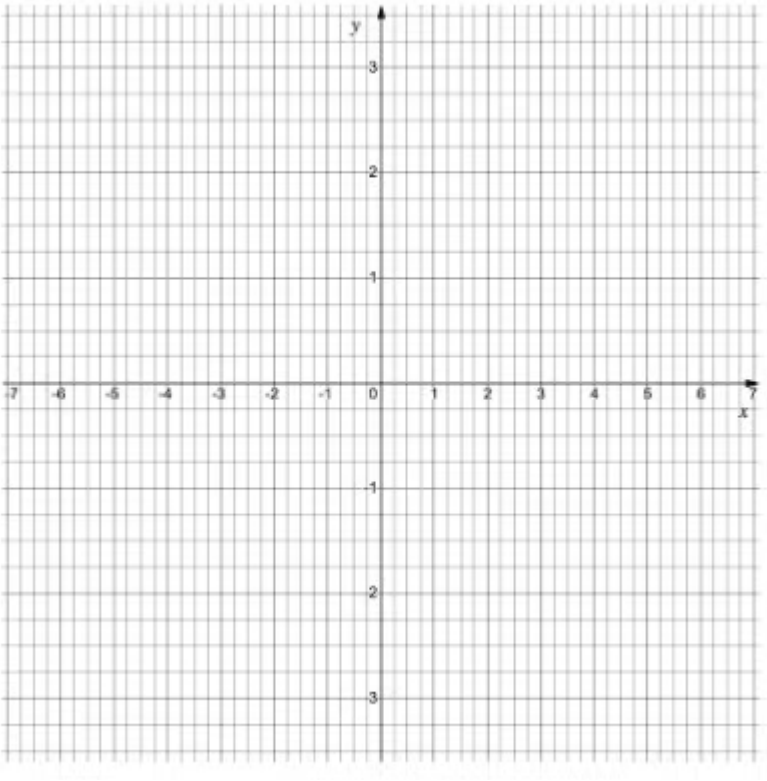
1 The curve C has parametric equations

$$x = t\ln(t^2 + 0.75) \quad \text{and} \quad y = t$$

(i) Complete the table below.
Give values to three significant figures where appropriate.

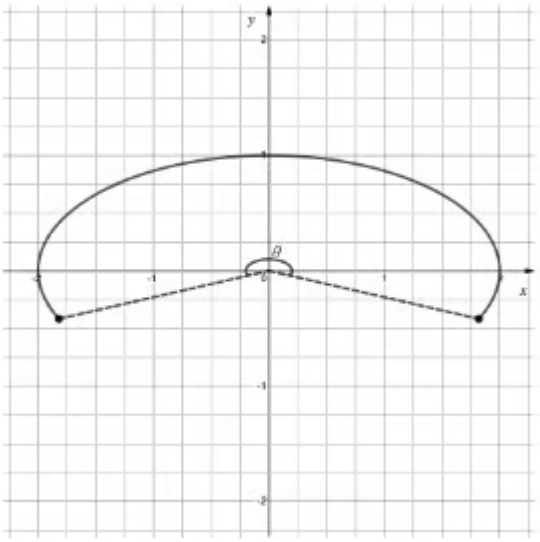
t	-3	-2	-1	-0.5	0	0.5	1	2	3
x									
y									

(ii) Plot the graph of C on the axes below.



(5 marks)

2 A company's logo is a major arc of an ellipse as shown in the diagram below.



When the end points of the arc are connected to the origin, they form an elliptic sector with angle θ radians at the centre.

The logo is formed using the parametric equations

$$x = 2\sin\phi \quad y = \cos\phi$$

with domain $-2 \leq \phi \leq 2$.

Find the angle θ, giving your answer to three significant figures.

(4 marks)

3 (a) Find a Cartesian equation for the curve that has parametric equations

$$x = (\ln 2t)^2 \quad y = t\sin 2t \quad t \geq \frac{1}{2}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(4 marks)

(b) (i) For the curve in part (a), explain why the domain must be restricted to $t > 0$.

(ii) How would your answer to part (a) change if the domain were changed to

$0 < t < \frac{1}{2}$?

(3 marks)

4 (a) A curve is defined by the parametric equations

$$x = a\cos 2t + p \quad y = b\sin 2t + q$$

where a and b are non-zero constants, and p and q are constants.

(i) Show that if $|a| = |b|$ then this is the equation of a circle, giving the equation of the circle in Cartesian form.

(ii) State the centre and radius of the circle.

(6 marks)

(b) In the case where $a = -6$ and $b = 6$, the curve is used to represent the position of a robot at time t.

i) Find the total distance travelled by the robot between times $t = 0$ and $t = 2\pi$.

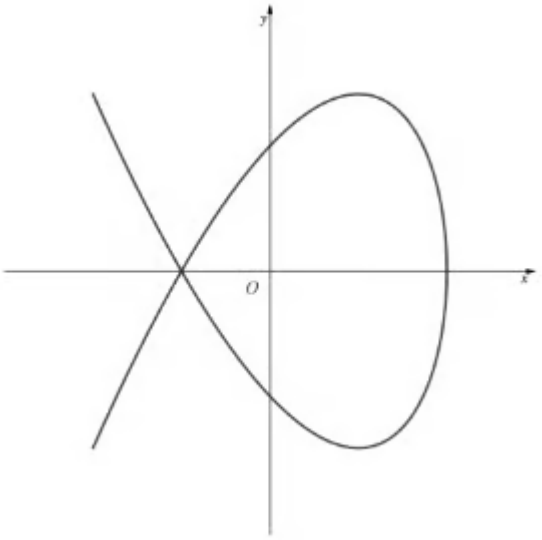
ii) In which direction (clockwise or anticlockwise) does the robot traverse the curve?

(4 marks)

5 (a) A sketch of the graph with parametric equations

$$x = \cos 2\phi \qquad y = \sin 3\phi \qquad \frac{\pi}{2} \leq \phi \leq \frac{3\pi}{2}$$

is shown below.



Find the coordinates of the points where the graph crosses the coordinate axes.

(6 marks)

(b) Find the coordinates of the points where the graph intersects the square with vertices at (-1 , 1), (1 , 1), (1 , -1), (-1 , -1).

(6 marks)

6 (a) A curve C has parametric equations

$$x = t^2 - 4 \quad y = t^3 - 4t \quad t \in \mathbb{R}$$

Determine the ranges of x and y in the given domain of t.

(2 marks)

(b) Show that the Cartesian equation of C can be written in the form $y^2 = x^2(x + 4)$.

(3 marks)

(c) Sketch the graph of C.

(4 marks)

7 The curve defined parametrically by

$$x = mt^3 - 6 \quad y = mt + t^2$$

where m is a constant, passes through the point (-22 , 0).

Find the possible values of m.

(4 marks)

8 (a) Show that $\cos\theta - \sqrt{3}\sin\theta$ can be written as $2\cos\left(\theta + \frac{\pi}{3}\right)$.

(3 marks)

(b) A curve C is defined by the parametric equations

$$x = \theta + \frac{\pi}{3} \qquad y = \cos\theta - \sqrt{3}\sin\theta \quad 0 \leq \theta \leq 2\pi$$

Sketch the graph of C, showing clearly the beginning and end points of the curve, as well as the points of intersection with the coordinate axes and any minimum and maximum points.

(6 marks)