A Level · Edexcel · Maths

4 hours 37 questions

3.3 Further Forces & Newton's Laws (A Level only)

Easy (8 questions)	/54
Medium (10 questions)	/65
Hard (10 questions)	/66
Very Hard (9 questions)	/69
Total Marks	**/254**

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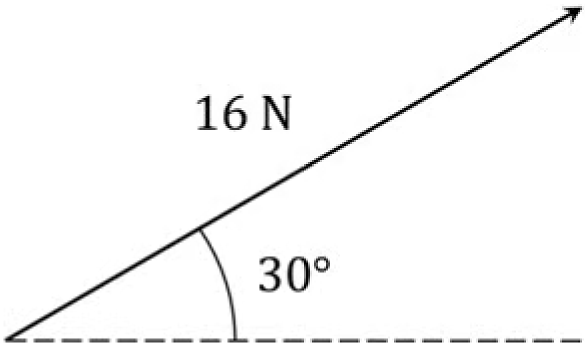
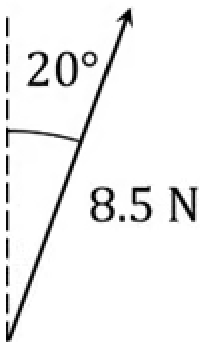
Easy Questions

1 (a) Use trigonometry (SOHCAHTOA) to resolve each of the following forces into its horizontal and vertical components.

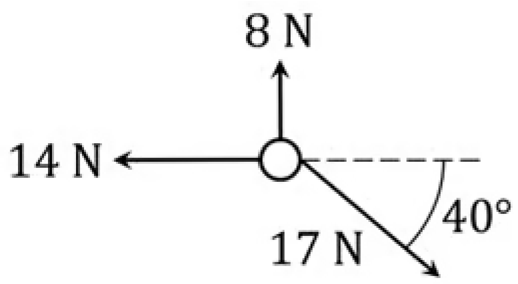


(2 marks)



(b) **(2 marks)**

2 (a) The following force diagram shows three forces acting on a particle:



Use trigonometry to resolve the 17 N force into its horizontal and vertical components.

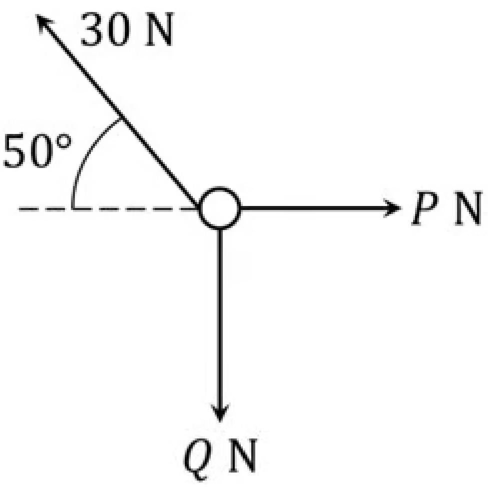
(2 marks)

(b) Hence find the magnitude and direction of the resultant forces acting on the particle in the horizontal and vertical directions.

(2 marks)

3 (a) A particle is said to be *in equilibrium* when the total force acting on it in any direction is zero.

The following force diagram shows three forces acting on a particle:



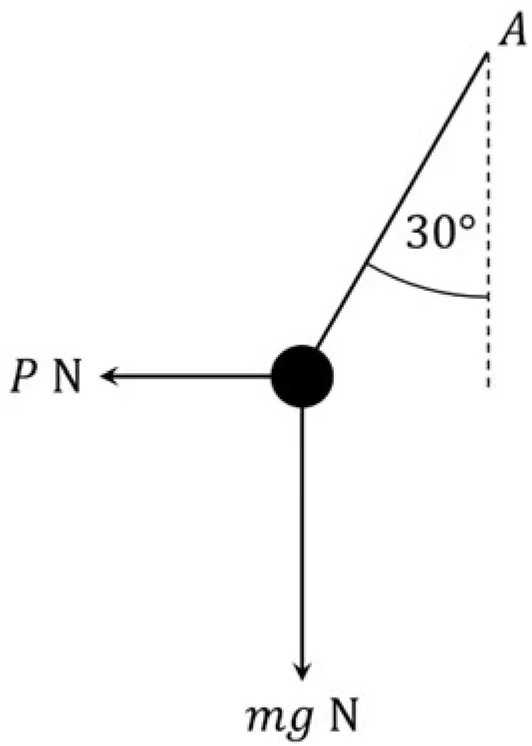
Use trigonometry to resolve the 30 N force into its horizontal and vertical components. Give your answers correct to 3 significant figures.

(2 marks)

(b) Given that the particle is in equilibrium, find the values of P and Q.

(2 marks)

4 (a) A particle of mass m kg is suspended by a light inextensible string, with the other end of the string attached to a fixed point A. With the string at an angle of $30°$ to the vertical, equilibrium is maintained by a horizontal force of P N which acts on the particle as shown in the diagram below:



Given that g is the constant of acceleration due to gravity, explain what the downward force of mg N represents in the diagram.

(1 mark)

(b) The tension in the string, T, is a force acting on the particle along the line of the string, in the direction from the particle towards A. The tension in the string is found to be 16 N.

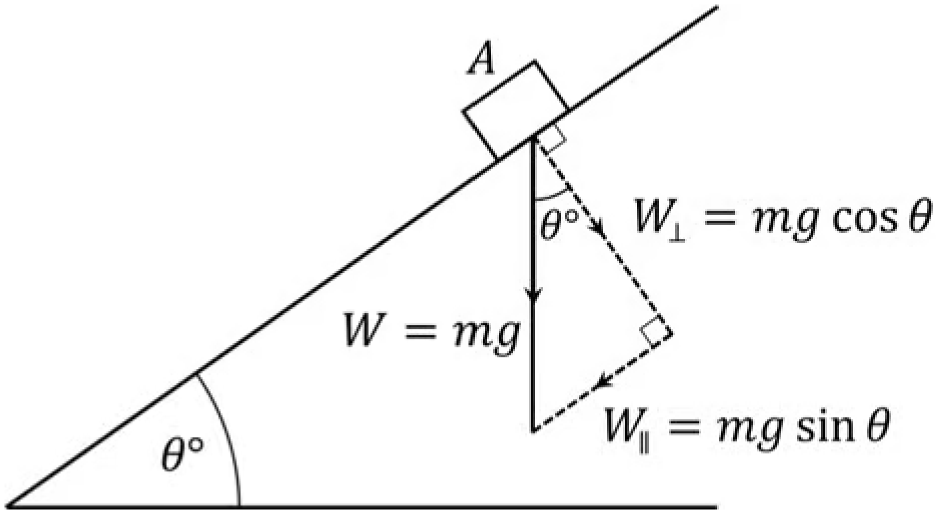
Use trigonometry to resolve T into its horizontal and vertical components.

(2 marks)

(c) Hence find the values of P and m.

(2 marks)

5 (a) The following diagram depicts a particle A of mass m kg on a slope that is angled at $\theta°$ to the horizontal:



The weight of the particle is indicated by the force $W = mg$. As shown, the weight may be decomposed into its components $W_\perp$ and $W_\parallel$, where $W_\perp = mg\cos\theta$ is the component of the weight perpendicular to the surface of the slope and $W_\parallel = mg\sin\theta$ is the component of the weight parallel to the surface of the slope.

A particle of mass 5 kg is sliding down a smooth slope that is angled at $30°$ to the horizontal.

By considering the forces acting on the particle in directions parallel to the surface of the slope, and by using the Newton's Second Law of Motion equation $F = ma$, calculate the acceleration of the particle down the slope.

(3 marks)

(b) By considering the forces acting on the particle in directions perpendicular to the surface of the slope, and by using the fact that the resultant force perpendicular to the slope is zero, calculate the normal reaction force of the slope on the particle.

(3 marks)

6 For an object on a rough surface, the force of friction will always act to oppose the motion of the object along the surface. The maximum frictional force, F_{MAX}, may be found by the formula

$$F_{MAX} = \mu R$$

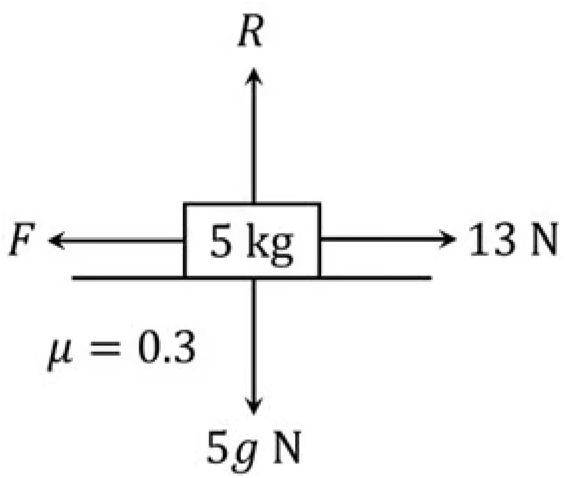
where μ is a constant called the *coefficient of friction*, and R is the normal reaction force exerted by the surface on the object.

If the object is at rest and the sum of other forces acting on the object parallel to the surface is less than or equal to F_{MAX}, then the object will remain at rest. In that case the actual frictional force will be equal in magnitude and opposite in direction to the sum of the other forces. If the sum of the other forces in the parallel direction is greater than F_{MAX}, then the object will begin to accelerate in the direction of those other forces. In that case the force of friction will be equal to F_{MAX}, and it will act in the direction opposite to the direction of motion of the object.

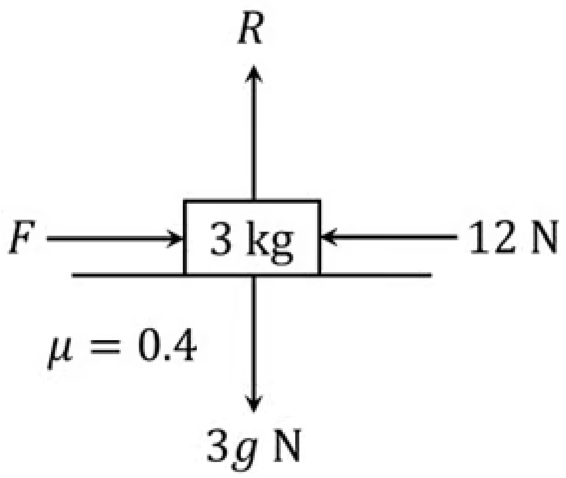
For an object in motion, the force of friction will always be equal to F_{MAX}, and will act in the direction opposite to the direction of motion of the object.

In each of the situations depicted below, the object is initially at rest. Find the magnitude of the frictional force F that will act upon the object in each case, and determine whether the object will remain at rest or begin to move.

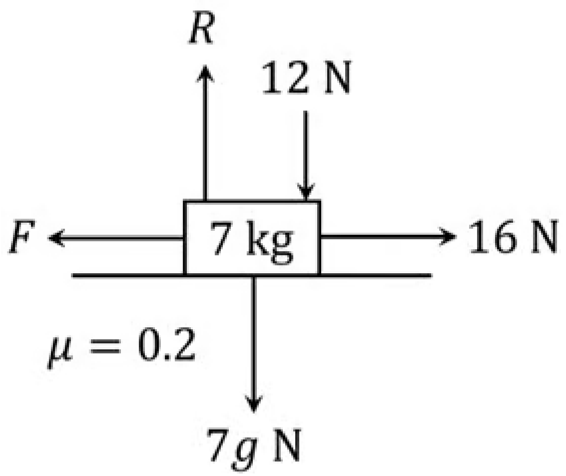
(i)



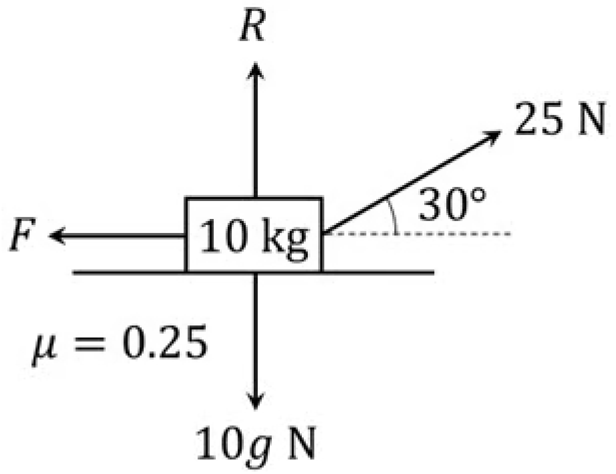
(ii)


R
F
3 kg
12 N
$\mu = 0.4$
$3g$ N

(iii)


R
12 N
F
7 kg
16 N
$\mu = 0.2$
$7g$ N

(iv)

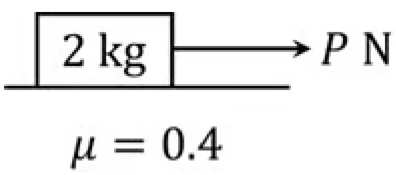

R
25 N
30°
F
10 kg
$\mu = 0.25$
$10g$ N

(13 marks)

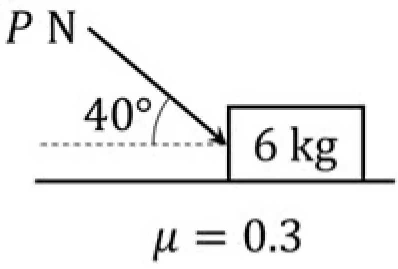
7 The maximum frictional force between an object and a rough surface, F_{MAX}, may be calculated using the formula $F_{MAX} = \mu R$, where μ is the coefficient of friction and R is the normal reaction force exerted by the surface on the object. For a stationary object, when the magnitude of the sum of other forces parallel to the surface is exactly equal to F_{MAX} the object is said to be *in limiting equilibrium.* An object in limiting equilibrium is on the point of moving under the influence of the non-frictional forces.

In each of the following examples, the object shown is at rest on a rough horizontal surface, and is on the point of moving under the influence of the force P. By first decomposing forces into components parallel and perpendicular to the surface where necessary, calculate the value of P in each instance.

(i)



(ii)



(7 marks)

8 (a) This question involves a particle of mass 5 kg in motion on a rough plane which is inclined at $30°$ to the horizontal. The coefficient of friction between the particle and the plane is 0.3.

Initially the particle is projected up a line of greatest slope of the plane, and moves upwards until the combined forces of friction and gravity cause it to come to rest.

By first decomposing the weight of the particle into components parallel and perpendicular to the plane, determine the frictional force acting on the particle as it moves up the plane. Be sure to give both the magnitude and the direction of the force.

(3 marks)

(b) Using your results from part (a), along with the Newton's Second Law of Motion equation $F = ma$, determine the acceleration of the particle while it is moving up the plane. Be sure to give both the magnitude and the direction of the acceleration.

(3 marks)

(c) After coming momentarily to rest, the particle begins to slide back down the plane.

Determine the frictional force acting on the particle as it slides down the plane, being sure to give both the magnitude and the direction of the force.

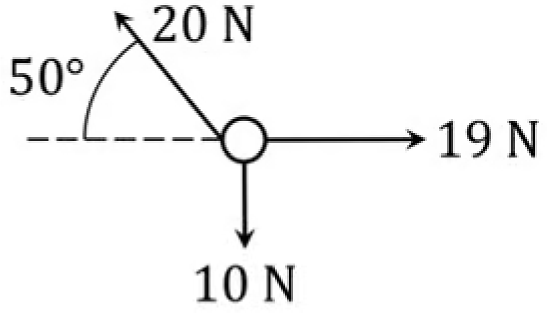
(2 marks)

(d) Determine the acceleration of the particle while it is sliding down the plane, being sure to give both the magnitude and the direction of the acceleration.

(3 marks)

Medium Questions

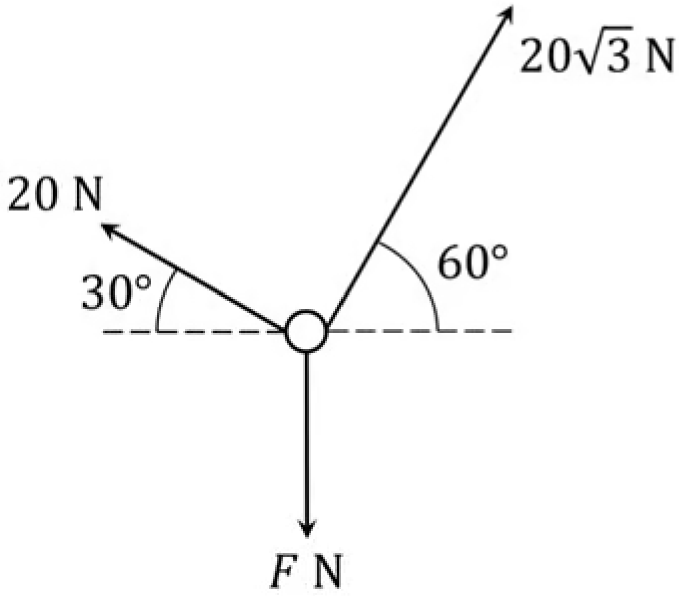
1 The following force diagram shows three forces acting on a particle:



Find the magnitude and direction of the resultant forces acting on the particle in the horizontal and vertical directions.

(4 marks)

2 (a) The following force diagram shows three forces acting on a particle:



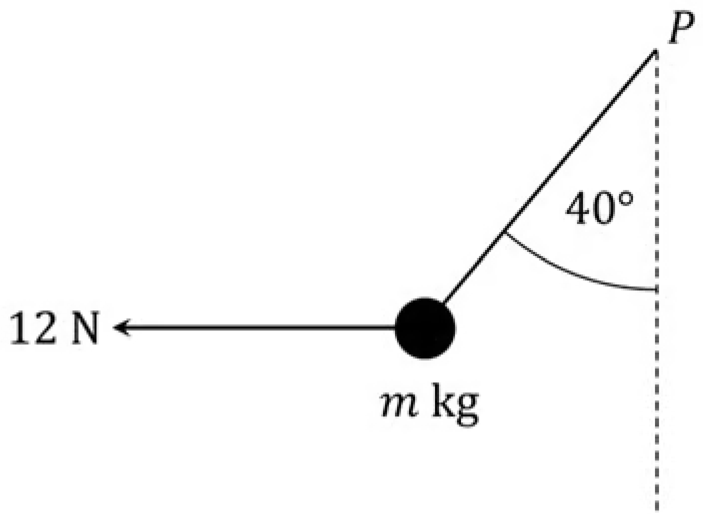
Show that the resultant force on the particle in the horizontal direction is zero.

(3 marks)

(b) Given that the particle is in equilibrium, find the exact value of F.

(3 marks)

3 A particle of mass m kg is suspended by a light inextensible string, with the other end of the string attached to a fixed point P. With the string at an angle of $40°$ to the vertical, equilibrium is maintained by a horizontal force of 12 N which acts on the particle as shown in the diagram below:



Find

(i) the tension in the string

(ii) the value of m.

(5 marks)

4 **(a)** A particle of mass 10 kg is sliding down a smooth slope that is angled at $15°$ to the horizontal.

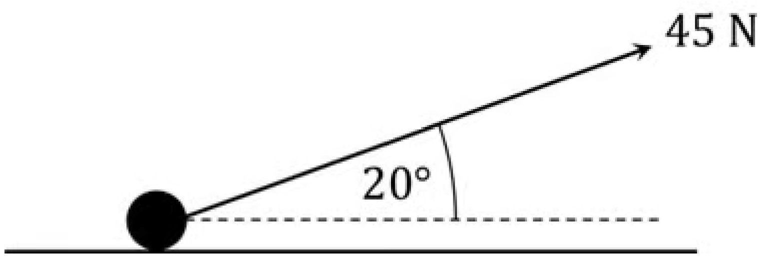
Calculate the acceleration of the particle down the slope.

(3 marks)

(b) Calculate the normal reaction force of the slope on the particle.

(3 marks)

5 (a) A particle of mass 7 kg is being pulled along a rough horizontal plane by a force of 45 N acting at an angle of $20°$ to the horizontal.



The coefficient of friction between the particle and the plane is 0.4, and the particle is moving to the right under the influence of the force.

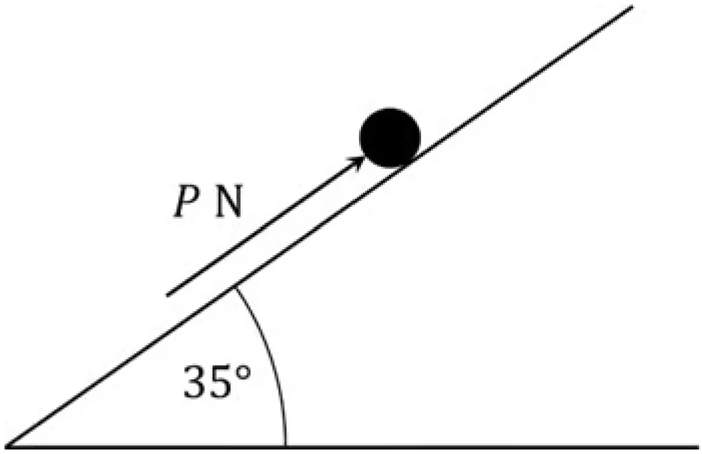
By first finding the normal reaction force, calculate the frictional force between the particle and the plane and state its direction.

(4 marks)

(b) Hence calculate the acceleration of the particle.

(3 marks)

6 A particle of mass 0.7 kg is on a rough plane which is angled at $35°$ to the horizontal. The particle is being held at rest by a force of P N acting up the plane along the line of greatest slope.



Given that the coefficient of friction between the particle and the plane is 0.35, and that the particle is on the point of slipping up the plane, find the value of P.

(6 marks)

7 A particle of mass 3 kg is released from rest on a rough plane which is inclined at $15°$ to the horizontal. The coefficient of friction between the particle and the plane is 0.2.

Determine the distance the particle will slide down the plane in the first 3 seconds after it is released.

(5 marks)

8 (a) A particle of mass 3 kg is projected up a rough plane which is inclined at an angle of $25°$ to the horizontal. It is projected up the line of greatest slope with an initial velocity of $u \text{ m s}^{-1}$, and it comes to instantaneous rest after moving a distance of 1.91 m up the slope. The coefficient of friction between the particle and the slope is 0.27.

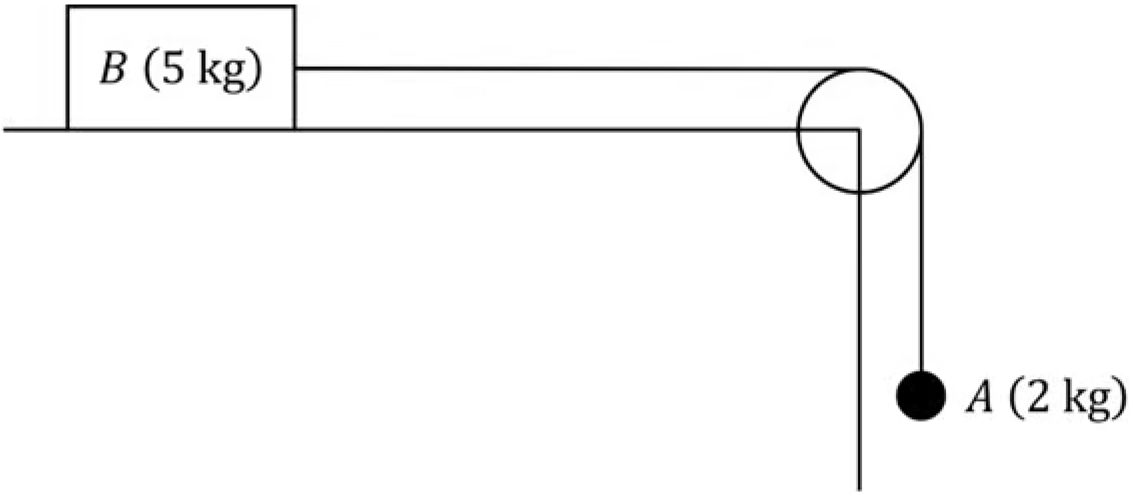
Show that while the particle is moving up the plane it experiences a deceleration of $(\sin 25 + 0.27 \cos 25)\, g \text{ m s}^{-2}$, where g is the constant of acceleration due to gravity.

(4 marks)

(b) Hence find the value of u.

(3 marks)

9 A box B of mass 5 kg rests on a rough horizontal table. It is connected by a light inextensible string to a metal sphere A of mass 2 kg. The string passes over a smooth light fixed pulley at the edge of the table so that A is hanging vertically downwards as shown in the diagram below:



The string between B and the pulley is horizontal, and the coefficient of friction between B and the table is 0.35.

The system is released from rest with the string taut.

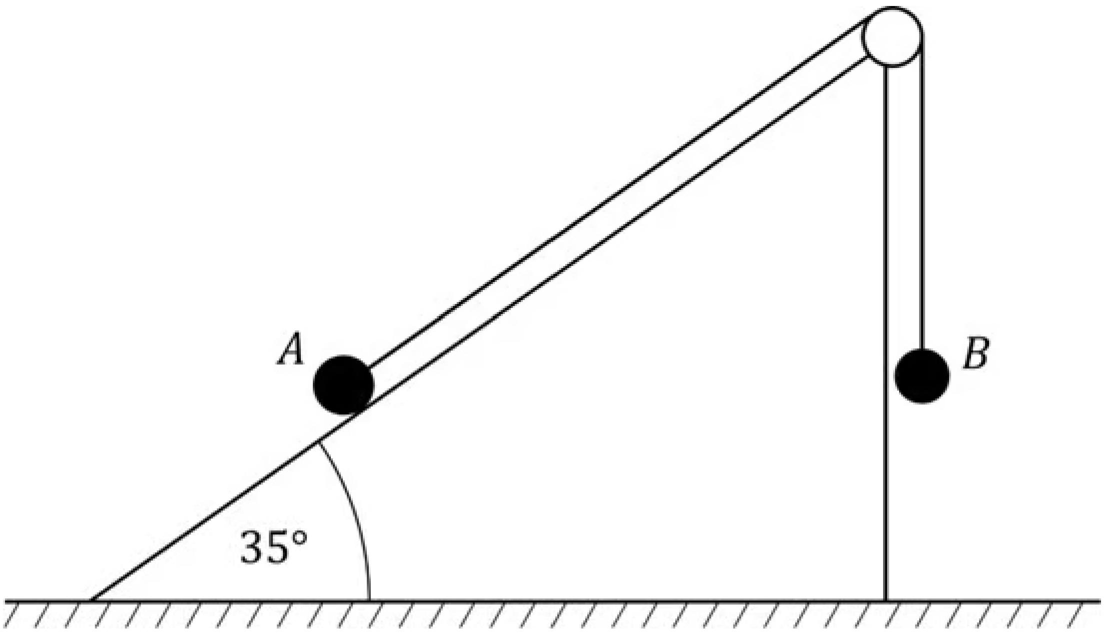
Calculate

(i) the acceleration of the two objects
(ii) the tension in the string

as A descends.

(6 marks)

10 (a) Two particles A and B, of masses 4 kg and 3 kg respectively, are connected by means of a light inextensible string. Particle A is held motionless on a rough fixed plane inclined at $35°$ to the horizontal. The string passes over a smooth light pulley fixed at the top of the plane so that B is hanging vertically downwards as shown in the diagram below:



The string between A and the pulley lies along a line of greatest slope of the plane, and B hangs freely from the pulley. The coefficient of friction between particle A and the plane is 0.15.

The system is released from rest with the string taut.

Calculate the acceleration of the two objects and the tension in the string as B descends.

(7 marks)

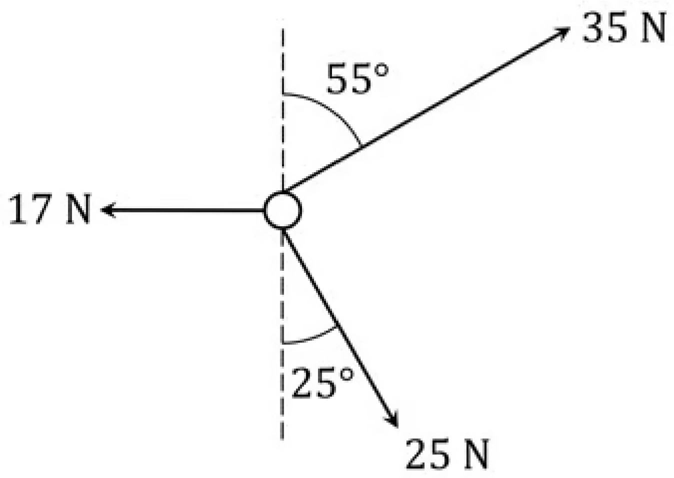
(b) After descending for 3.2 seconds, particle B strikes the ground and immediately comes to rest. Particle A continues to move up the slope until the forces of gravity and friction cause it to come momentarily to rest.

Find the total distance travelled by particle A between the time that the system is first released from rest and the time that particle A comes momentarily to rest again after B has struck the ground.

(6 marks)

Hard Questions

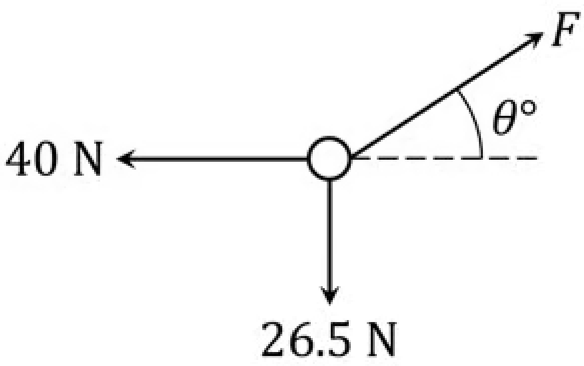
1 The following force diagram shows three forces acting on a particle:



Find the magnitude and direction of the resultant forces acting on the particle in the horizontal and vertical directions.

(5 marks)

2 The following force diagram shows three forces acting on a particle:



Given that the particle is in equilibrium, find:

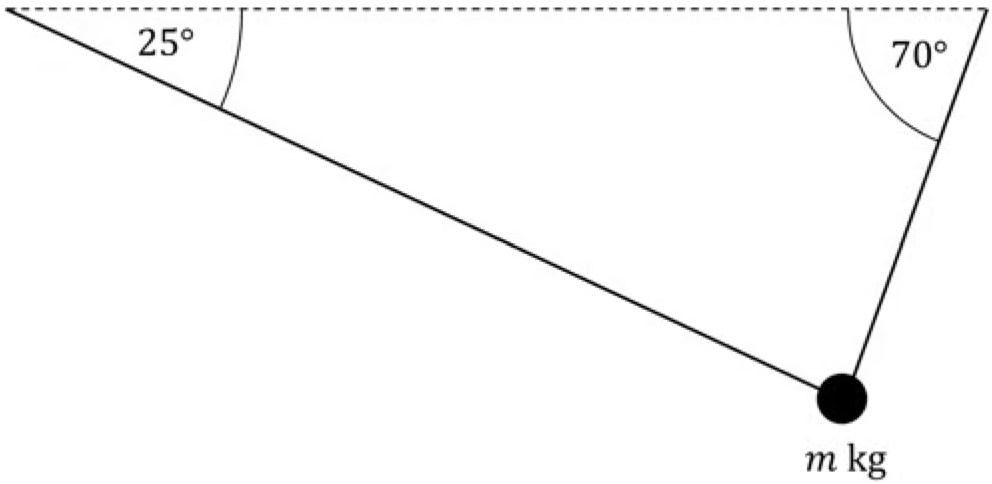
(i) the magnitude of force F

(ii) the angle, θ, that F makes with the horizontal.

Give both your answers correct to 3 significant figures.

(4 marks)

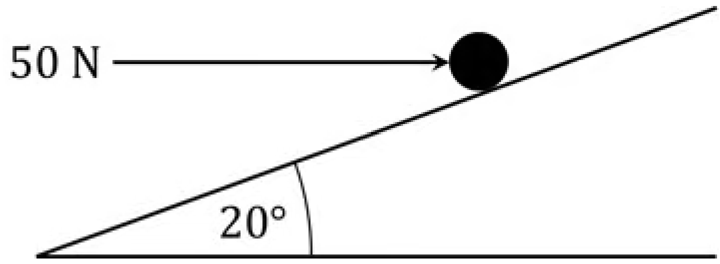
3 A particle of mass m kg hangs in equilibrium, suspended by two light inextensible strings. The strings are inclined at 25° and 70° to the horizontal, as shown in the diagram below:



Given that the tension in the string angled at 70° to the horizontal is 56 N, find the value of m.

(6 marks)

4 (a) A particle of mass 12 kg is being pushed up a smooth slope by a force of 50 N that acts horizontally. The slope is inclined at $20°$ to the horizontal, as shown in the diagram below:



Calculate the acceleration of the particle up the slope.

(3 marks)

(b) Calculate the normal reaction force of the slope on the particle.

(3 marks)

5 (a) A particle of mass 15 kg is sliding down a rough slope that is angled 25° to the horizontal. The coefficient of friction between the particle and the slope is 0.3.

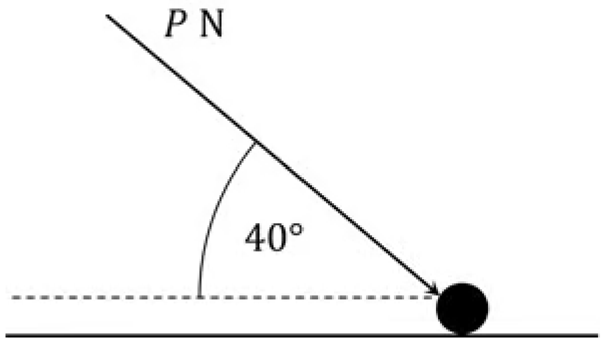
Calculate the frictional force between the particle and the slope.

(3 marks)

(b) Hence calculate the acceleration of the particle down the slope.

(3 marks)

6 A particle of mass 0.9 kg is at rest on a rough horizontal plane. A force of magnitude P N is acting on the particle at an angle of 40° to the horizontal.



Given that the coefficient of friction between the plane and the particle is 0.3, and that the particle is on the point of moving to the right under the influence of the force, find the value of P.

(6 marks)

7 A particle of mass m kg is released from rest on a rough plane inclined at $\theta°$ to the horizontal, where $45° < \theta < 90°$. The coefficient of friction between the particle and the plane is μ.

Given that the particle remains motionless after it is released, show that $\mu > 1$.

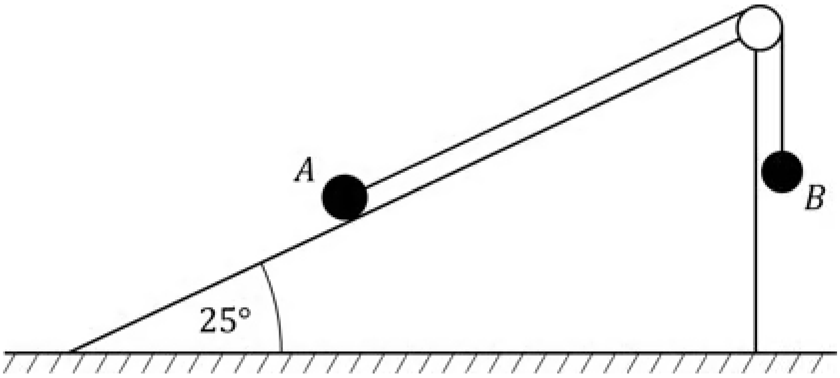
(5 marks)

8 A particle of mass 2 kg is projected up a rough plane which is inclined at an angle of 20° to the horizontal. It is projected up the line of greatest slope with an initial velocity of $u \text{ m s}^{-1}$, and it comes to instantaneous rest after moving a distance of 4.85 m up the slope. The coefficient of friction between the particle and the slope is 0.2.

Find the value of u.

(7 marks)

9 Two particles A and B, of masses 2.7 kg and 2.2 kg respectively, are connected by means of a light inextensible string. Particle A is held motionless on a rough fixed plane inclined at 25° to the horizontal. The string passes over a smooth light pulley fixed at the top of the plane so that B is hanging vertically downwards as shown in the diagram below:

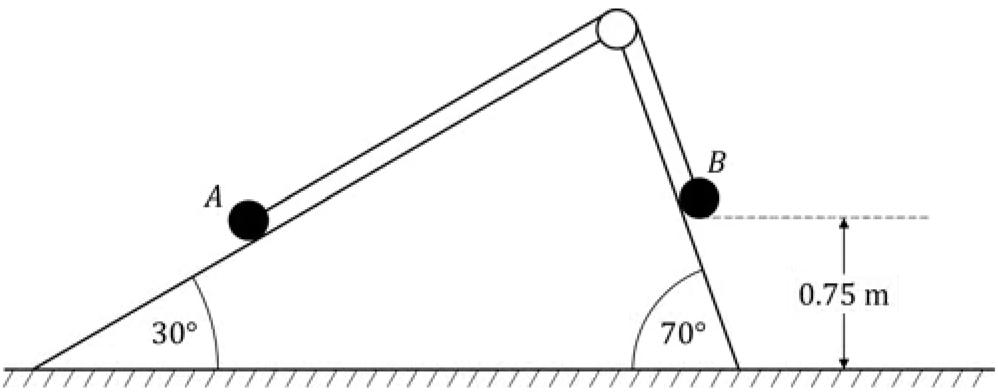


The string between A and the pulley lies along a line of greatest slope of the plane, and B hangs freely from the pulley. The coefficient of friction between particle A and the plane is μ.

The system is released from rest with the string taut. Given that particle B descends 1.82 m in the first 3 seconds after it is released, find the value of μ.

(8 marks)

10 Two particles A and B, of identical mass, are connected by means of a light inextensible string. Particle A is held motionless on a rough fixed plane inclined at $30°$ to the horizontal, and that plane is connected at its top to another rough fixed plane inclined at $70°$ to the horizontal. The string passes over a smooth light pulley fixed at the top of the two planes so that B is hanging downwards in contact with the second plane. This situation is shown in the diagram below:



The parts of the string between A and the pulley and between B and the pulley each lie along a line of greatest slope of the respective planes. The coefficient of friction between the particles and the planes is 0.15 in both cases.

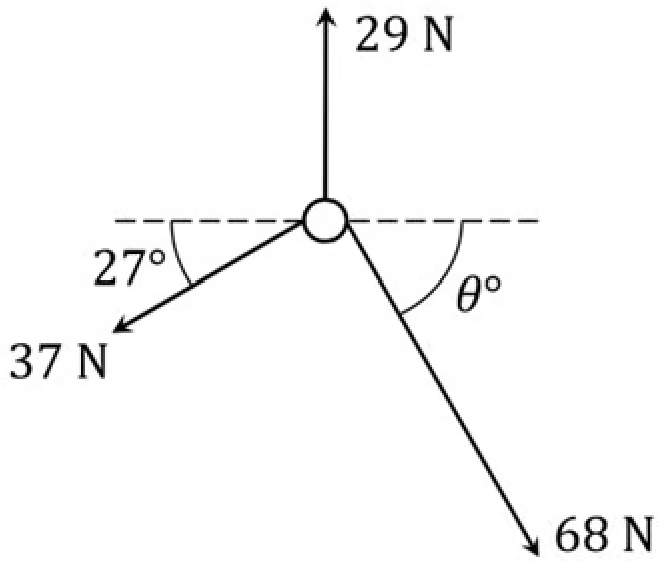
The system is released from rest with the string taut, and with particle B a vertical distance of 0.75 m from the ground. Particle B descends along the slope until it reaches the ground, at which point it immediately comes to rest. Particle A continues to move up the slope until the forces of gravity and friction cause it to come momentarily to rest.

Find the total distance travelled by particle A between the time that the system is first released from rest and the time that particle A comes momentarily to rest again after B has reached the ground.

(13 marks)

Very Hard Questions

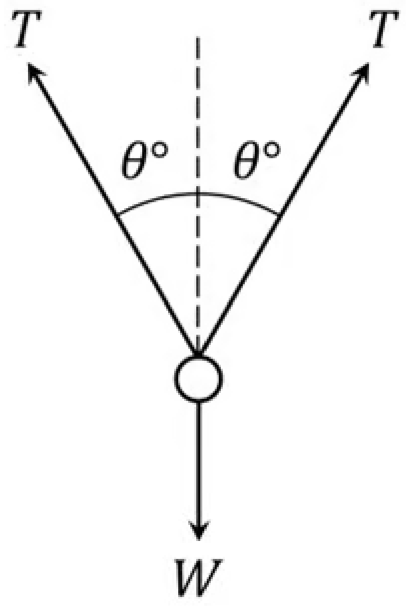
1 The following force diagram shows three forces acting on a particle:



Given that the resultant force on the particle in the vertical direction is 44.8 N downwards, find the size of the angle θ. Give your answer in degrees correct to 3 significant figures.

(5 marks)

2 (a) In a warehouse, a lifting device is used to lift a load of weight W by means of two lifting cables as shown in the diagram below:



The tension, T, is the same in each of the lifting cables, and the angle that each of the cables makes with the vertical, θ, is also the same. For purposes of this question any influences on the motion of the load other than the weight of the load and the tension in the two cables may be ignored.

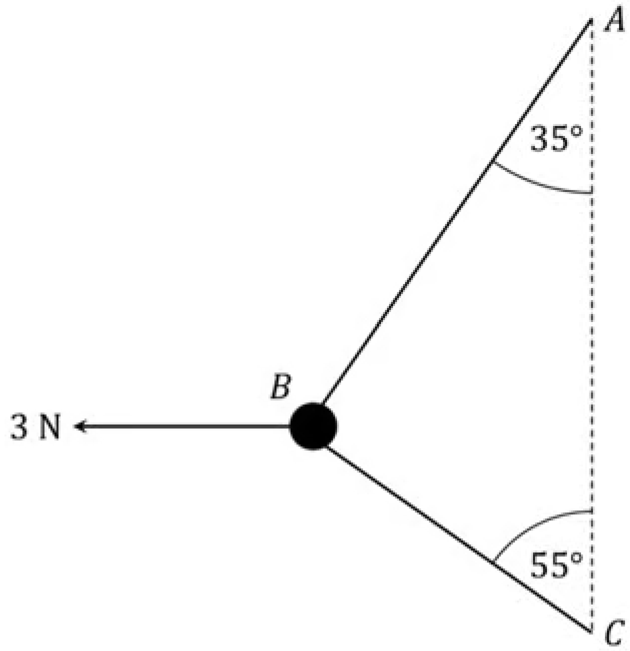
Given that the load is initially motionless, find an inequality in terms of T and θ that the weight W must satisfy if it is to be lifted by the cables.

(3 marks)

(b) Given that the load must still be accelerating in the upwards direction when $\theta = 45°$, find an inequality in terms of W that the tension T must satisfy.

(3 marks)

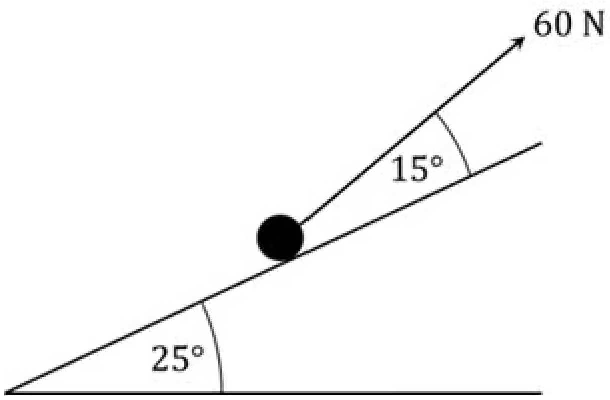
3 A smooth bead B with a mass of m grams has been threaded onto a light inextensible string. The ends of the string are fixed to two points A and C, where A is vertically above C. The bead is held in equilibrium by a horizontal force of 3 N, with sections AB and BC of the string making angles of $35°$ and $55°$ respectively with the vertical as shown in the diagram below:



Find the value of m.

(6 marks)

4 (a) A particle is being pulled up a smooth slope by a force of 60 N that acts at an angle of $15°$ to the slope. The slope is inclined at $25°$ to the horizontal, as shown in the diagram below:



The particle experiences an acceleration of $0.3\ \mathrm{m\,s^{-2}}$ parallel to the slope in the upwards direction.

Use the information provided above to calculate the mass of the particle.

(3 marks)

(b) Calculate the normal reaction force of the slope on the particle.

(3 marks)

5 A laboratory apparatus contains a rough inclined plane that may be inclined at different angles to the horizontal. When the plane is at an angle of θ_1 to the horizontal, where $0 < \theta_1 < 90°$, a small mass is released from rest on the plane and remains motionless. After the angle is increased to θ_2, where $\theta_2 < 90°$, the same small mass is released from rest on the plane and begins to slide down the slope of the plane.

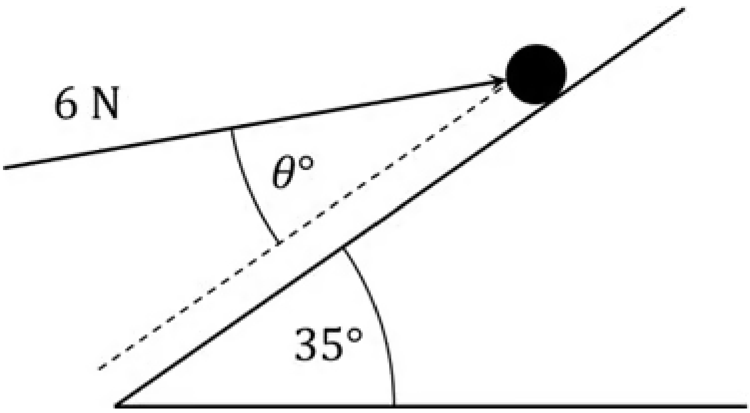
Show that

$$\tan \theta_1 \le \mu < \tan \theta_2$$

where μ is the coefficient of friction between the small mass and the plane.

(6 marks)

6 A particle of mass 0.5 kg is at rest on a rough plane which is inclined at $35°$ to the horizontal. The particle is being acted upon by a force of 6 N, directed at an angle of $\theta°$ to the plane. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane.



Given that the coefficient of friction between the particle and the plane is 0.4, and that the particle is on the point of slipping up the plane, find the value of θ.

(9 marks)

7 A particle of mass 0.8 kg is released from rest on a rough slope that is angled at $\theta°$ to the horizontal, where $\tan\theta = \frac{2}{7}$. After 4 seconds the speed of the particle is 1.35 m s^{-1}.

Find the value of the coefficient of friction μ between the particle and the slope.

(7 marks)

8 (a) A particle of mass m kg is projected up a rough plane which is inclined at an angle of $\theta°$ to the horizontal. It is projected up the line of greatest slope with an initial velocity of u metres per second, and it comes to instantaneous rest in t_1 seconds after moving a distance of s metres up the slope. The coefficient of friction between the particle and the slope is μ.

Show that:

(i) $$t_1 = \frac{u}{g(\sin\theta + \mu\cos\theta)}$$

(ii) $$s = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$

(7 marks)

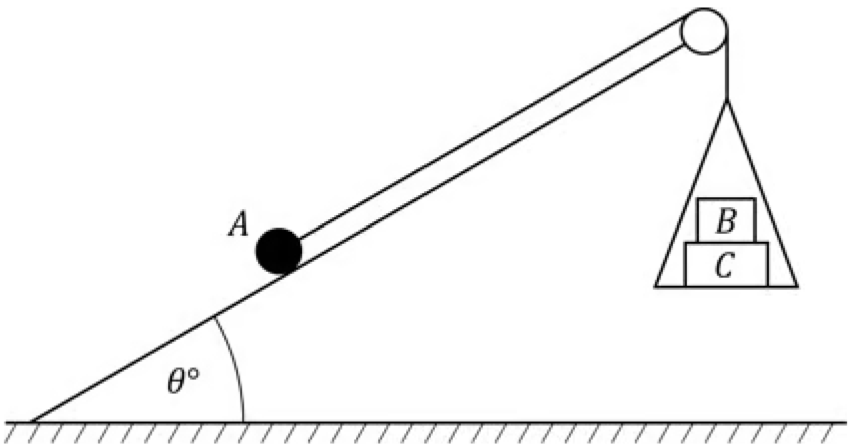
(b) After coming to instantaneous rest, the particle begins to slide back down the slope, and after t_2 seconds it has returned to its starting point.

Find an expression for t_2 in terms of u, g, μ and θ.

(4 marks)

9 (a) You have landed on the planet Hephaestia, where the gravitational constant of acceleration g has a different value than it does on Earth.

Your spaceship contains a device which may be used to find the value of g on any planet. In this device a particle A with mass m_A kg is connected by a light inextensible string to a light scale-pan. A force meter C with mass m_C kg is placed in the scale-pan, and a small block B with mass m_B kg is placed on top of C. A is held in place on a rough plane angled at $\theta°$ to the horizontal, where the coefficient of friction between A and the plane is μ. The string passes over a smooth light pulley fixed at the top of the plane so that the scale-pan is hanging vertically downwards as shown in the diagram below:



With the string between A and the pulley lying along a line of greatest slope of the plane, A is projected down the plane with a velocity of v m s^{-1} parallel to the string. After a time of t seconds the system comes momentarily to rest, and then the scale-pan begins to descend under the force of gravity, pulling mass A up the slope behind it.

When the scale-pan is initially moving upwards, the force exerted by B on C is denoted by $F_{BC\uparrow}$. When the scale-pan begins to descend, the force exerted by B on C is denoted by $F_{BC\downarrow}$. The force meter is only able to record the *difference* ΔF between these two values, where $\Delta F = F_{BC}\downarrow - F_{BC}\uparrow$.

Show that

$$\Delta F = \left(\frac{2m_A m_B}{m_A + m_B + m_C}\right) g\,\mu\,\cos\theta$$

(11 marks)

(b) For your ship's device, $m_A = 3$, $m_B = 1$, $m_C = 2$, $\theta = 30$ and $\mu = 0.4$.

Find the value of g on Hephaestia, given that the value recorded for ΔF is 1.286 N.

(2 marks)