Save My Exams

A Level · Edexcel · Maths

⏱ 3 hours   ❓ 26 questions

5.1 Hypothesis Testing

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Easy (7 questions)	/43
Medium (8 questions)	/41
Hard (6 questions)	/40
Very Hard (5 questions)	/41
Total Marks	**/165**

Easy Questions

1 A hypothesis test uses a sample of data in an experiment to test a statement made about the value of a population parameter (p).

Explain, in the context of hypothesis testing, what is meant by:

(i)   'sample of data',

(ii)   'population parameter'

(iii)   'null hypothesis',

(iv)   'alternative hypothesis',

(v)   'test statistic'.

(5 marks)

2 (a) From previous research, Marta has found that in general there is a 15% chance that any given customer ordering food at her restaurant will choose a salad. She wants to test whether people are more inclined to eat salads when it is sunny out.

   (i)   Clearly defining the value of the population parameter (p), state a suitable null hypothesis that Marta could use for this test.

   (ii)   State a suitable alternative hypothesis that Marta could use for this test.

   (iii) Give an example of a test statistic that Marta could use to carry out this test.

(4 marks)

(b) After carrying out the test, Marta had evidence to conclude that people are more likely to eat salads when the sun is out. State whether she accepted or rejected the null hypothesis you have written in part (a)(i).

(1 mark)

3 For the following null and alternative hypotheses, state whether the test is a one-tailed or a two-tailed test and give a suitable example context for each problem.

   (i) $H_0 : p = 0.5, H_1 : p > 0.5.$

   (ii) $H_0 : p = \dfrac{1}{6}, H_1 : p \neq \dfrac{1}{6}.$

   (iii) $H_0 : p = 0.3, H_1 : p < 0.3.$

4 In a quiz, students have to choose the correct answer to each question from three possible options. There is only one correct answer for each question. Ethan got k answers correct, and he claims that he merely guessed the answer to every question but his teacher believes he used some knowledge in the quiz. She uses the null hypothesis $H_0 : p = \dfrac{1}{3}$ to test her belief at the 10% significance level.

    (i)   If the teacher wishes to test to see if Ethan was trying to get the answers correct, rather than guessing them at random, write down the alternative hypothesis she should use and explain the conditions under which the null hypothesis would be rejected.

    (ii)   If the teacher wishes to test to see if Ethan was trying to get the answers incorrect, rather than guessing them at random, write down the alternative hypothesis she should use and explain the conditions under which the null hypothesis would be rejected.

    (iii)   If the teacher wishes to test to see whether Ethan was not guessing the answers at random, but she is uncertain whether he was using his knowledge to get them right or to get them wrong, write down the alternative hypothesis she should use and explain the conditions under which the null hypothesis would be rejected.

5 (a) A hypothesis test at the 4% significance level is carried out on a spinner with four sectors using the following hypotheses:

$$H_0 : p = \frac{1}{4}, H_1 : p \neq \frac{1}{4},$$

(i) Describe what the parameter, p, could be defined as.

(ii) In the context of this question, explain how the significance level of 4% should be used.

(iii) If the significance level were instead given as 10%, would the probability of incorrectly rejecting the null hypothesis be likely to increase or decrease? Give a reason for your answer.

(4 marks)

(b) The spinner is spun 50 times and it is decided to reject the null hypothesis if there are less than 7 or more than 18 successes.

(i) The critical regions for this test are given as $X \leq a$ and $X \geq b$. Write down the values of a and b.

(ii) State the set of values for which the null hypothesis would be accepted.

(2 marks)

6 (a) Two volunteers at a national park, Owen and Cathy, have begun a campaign to stop people leaving their litter behind after visiting the park. To see whether their campaign has had an effect, Owen conducts a hypothesis test at the 10% significance level, using the following hypotheses:

$$H_0 : p = 0.2, \quad H_1 : p \neq 0.2$$

(i) State the percentage of people who left litter behind in the national park before the start of the campaign.

(ii) State whether this is a one-tailed or two-tailed test.

(2 marks)

(b) Owen observes a random sample of 100 people at the national park and finds that 14 of them left litter behind. He calculates that if H_0 were true, then the probability of 14 or less people leaving litter would be 0.08044.

With reference to the hypotheses above, state with a reason whether Owen should accept or reject his null hypothesis.

(2 marks)

(c) Cathy conducted her own hypothesis test at the 10% significance level, using the same sample data as Owen, but instead she used the following hypotheses:

$$H_0 : p = 0.2, \quad H_1 : p < 0.2$$

(i) Explain how Cathy's hypothesis test is different to Owen's.

(ii) Using these hypotheses, state whether the sample results given in part (b) should lead Cathy to accept or reject her null hypothesis. Give a reason for your answer.

7 (a) A drinks manufacturer, BestBubbles, claims that in taste tests more than 50% of people can distinguish between its drinks and those of a rival brand. The company decides to test its claim by having 20 people each taste two drinks and then attempt to determine which was made by BestBubbles and which was made by the rival company. The random variable X represents the number of people who correctly identify the drink that was made by BestBubbles.

(i) State, giving a reason, whether this is a one-tailed or a two-tailed test.

(ii) Write down the null and alternative hypotheses for this test.

(2 marks)

(b) Under the null hypothesis, it is given that:

$$P(X = 13) = 0.07393$$

$$P(X = 14) = 0.03696$$

$$P(X > 14) = 0.02069$$

(i) Calculate $P(X \geq 14)$ and $P(X \geq 13)$.

(ii) Given that a 10% level of significance was used, write down the critical value and the critical region for this test.

(iii) State the actual level of significance for this test.

(5 marks)

(c) In fact, 15 of the 20 people correctly identify the drink made by BestBubbles.

    (i)   State whether there is sufficient evidence to reject the null hypothesis at the 10% significance level.

    (ii)   Write a conclusion for this hypothesis test in the context of the question.

(2 marks)

Medium Questions

1 (a) Explain what you understand by a critical region of a test statistic.

(2 marks)

(b) Nationally 44% of A Level mathematics students identify as female. The headteacher of a particular school claims that the proportion of A Level mathematics students in the school who identify as female is higher than the national average.

   (i)   State a suitable null hypothesis to test the headteacher's claim.

   (ii) State a suitable alternative hypothesis to test the headteacher's claim.

(2 marks)

(c) The headteacher takes a random sample of 60 A Level mathematics students and records the number of them who identify as female, x. For a test at the 10% significance level the critical region is $X \geq 32$.

Given that $x = 36$, comment on the headteacher's claim.

(2 marks)

2 (a) The probability of a chicken laying an egg on any given day is 65%. Two farmers, Amina and Bert, have 30 chickens each. They believe that the probability of their chickens laying an egg on any given day is different to 65%.

    (i)   State a suitable null hypothesis to test the farmers' belief.

    (ii)   State a suitable alternative hypothesis for a two-tailed test.

(2 marks)

(b) During a specific day, Amina and Bert each record the number of their 30 chickens that lay an egg. At the 5% significance level the critical regions for this test are $X \leq 13$ and $X \geq 25.$.

Write down the critical values for the hypothesis test.

(1 mark)

(c) (i)   Given that for Amina $x = 12$, comment on her belief.

    (ii)   Given that for Bert $x = 24$, comment on his belief.

(4 marks)

3 (a) A memory experiment involves having participants read a list of 20 words for two minutes and then recording how many of the words they can recall. Peter, a psychologist, claims that more than 60% of teenagers can recall all the words. Peter takes a random sample of 40 teenagers and records how many of them recall all the words.

     (i)   State a suitable null hypothesis to test the psychologist's claim.

     (ii)   State a suitable alternative hypothesis to test the psychologist's claim.

(2 marks)

(b) Given that the critical value for the test is $x = 19$, state the outcome of the test if

     (i)   18 out of the 40 teenagers recall all the words

     (ii)   19 out of the 40 teenagers recall all the words

     (iii)   20 out of the 40 teenagers recall all the words.

(3 marks)

4 (a) A machine produces toys for a company. It was found that 8% of the toys it was producing were faulty. After an engineer works on the machine, she claims that the proportion of faulty toys should now have decreased.

State suitable null and alternative hypotheses to test this claim.

(2 marks)

(b) After the engineer is finished, the manager of the company takes a random sample of 100 toys and finds that 2 of them are faulty.

Given that $P(X \leq 2) = 0.01127$ when $X \sim B(100, 0.08)$, determine the outcome of the hypothesis test using a 1% level of significance. Give your conclusion in context.

(2 marks)

5 (a) After it was estimated that only 72% of patients were turning up for their appointments at Pearly Teeth dental surgery, the owner began sending text message reminders to the patients on the day before their appointments. In order to test whether the reminders have increased the proportion of patients turning up to their appointments, the owner decides to conduct a hypothesis test at the 5% level of significance using the next 160 patients scheduled for appointments as a sample.

State suitable null and alternative hypotheses to test this claim.

(2 marks)

(b) Describe, in context, the test statistic for this test.

(1 mark)

(c) Out of the 160 patients used for the sample, 127 turned up for their appointments. The p-value for $x = 127$ is 0.02094.

Determine the outcome of the hypothesis test, giving your conclusion in context.

(2 marks)

6 (a) Chase buys a board game which contains a six-sided dice. He rolls the dice 150 times and obtains the number six on 15 occasions. Chase wishes to test his belief that the dice is not fair.

   (i)   State a suitable null hypothesis to test Chase's belief.

   (ii)   State a suitable alternative hypothesis for a two-tailed test.

(2 marks)

(b) Given that $P(X \leq 15) = 0.01452$ when $X \sim B(150, \frac{1}{6})$, test Chase's belief that the dice is not fair, using a 2% level of significance.

(3 marks)

7 (a) A test of the null hypothesis $H_0 : p = 0.3$ is carried out for the random variable $X \sim B(25, p)$. The observed value of the test statistic is $x = 3$. You are given the following probabilities:

$$P(X < 3) = 0.00896$$

$$P(X = 3) = 0.02428$$

$$P(X \leq 3) = 0.03324$$

Determine the outcome of the test, with reasons, when the alternative hypothesis is:

$H_1 : p < 0.3$ with a 1% level of significance.

(2 marks)

(b) $H_1 : p \neq 0.3$ with a 5% level of significance.

(2 marks)

8 (a) A two-tailed test of the null hypothesis $H_0 : p = 0.23$ is carried out for the random variable $X \sim B(60, p)$.

Write down the alternative hypothesis.

(1 mark)

(b) One of the critical regions is $X \geq 20$. You are given the following probabilities:

$$P(X \leq 8) = 0.04603$$

$$P(X \leq 9) = 0.08932$$

$$P(X \leq 10) = 0.15526$$

Given that a 10% level of significance is used, determine the other critical region. Give a reason for your answer by using a relevant probability.

(2 marks)

(c) You are also given that $P(X \geq 20) = 0.04427..$

Find the actual level of significance of this test.

(2 marks)

Hard Questions

1 (a) Joel is a manager at a swimming pool and claims that less than half of customers wear goggles in the water. Joel forms a sample using the next 100 swimmers and he notes that 42 of them wear goggles.

If $X \sim B(100,0.5)$ then:

$$P(X < 42) = 0.0443$$

$$P(X \leq 42) = 0.0666$$

$$P(X = 42) = 0.0223$$

$$P(X \geq 42) = 0.9557$$

$$P(X > 42) = 0.9334$$

Stating your hypotheses clearly, test Joel's claim using a 5% level of significance.

(4 marks)

(b) Joel discovers that there was a family of 12 people included in the sample, all of whom wore goggles.

Explain how this information affects the conclusion to the hypothesis test.

(2 marks)

2 (a) At *Hilbert's Hotel* three quarters of customers leave feedback upon departure by writing a comment in a book on the reception desk. Karla, the manager, decides to get rid of the feedback book and instead leaves a feedback form in each room. To test whether this new system has made a difference to the proportion of guests who leave feedback, Karla forms a sample using the next 80 room bookings. Once the 80 sets of guests leave *Hilbert's Hotel*, Karla counts that 65 feedback forms have been completed.

When $X \sim B(80, \frac{3}{4})$ the following probabilities are given:

$$P(X < 65) = 0.8792$$

$$P(X \leq 65) = 0.9260$$

$$P(X = 65) = 0.0468$$

$$P(X \geq 65) = 0.1208$$

$$P(X > 65) = 0.0740$$

Test, using a 10% level of significance, whether there is evidence to suggest that the feedback forms have changed the proportion of guests who leave feedback. State your hypotheses clearly.

(4 marks)

(b) Karla repeats the same test, with the same hypotheses, the following week and finds that 53 out of the 80 sets of guests fill in their feedback forms. This leads to the null hypothesis being rejected. Karla claims that this shows that there is evidence that the proportion of guests leaving feedback has decreased.

Explain whether Karla's claim is valid.

3 (a) Explain one advantage of using critical regions instead of p-values for a hypothesis test.

(1 mark)

(b) A test of the null hypothesis $H_0 : p = 0.2$ against the alternative hypothesis $H_1 : p < 0.2$ is carried out for the random variable $X \sim B(n, p)$.

x	$P(X = x)$
0	0.000406
1	0.003549
2	0.015085
3	0.041484
4	0.082968

The table below shows the probabilities for different values that $X \sim B(n, 0.2)$ can take:

Calculate the p-value for the test statistic $x = 3$.

(2 marks)

(c) (i) Using a 5% level of significance, find the critical region for the test.

(ii) State the actual level of significance for the test.

(3 marks)

4 (a) A group of high school statistics students are investigating the probability of winning a game called *Chi Squares.* Their teacher claims that they have more than a 60% chance of winning the game. To test the claim, they play 30 games of *Chi Squares* and win 80% of them. They perform a hypothesis test using a 5% level of significance. Below are shown the solutions of two students, Gertrude and Nate:

Gertrude's solution	Nate's solution
$H_0 : p = 0.6$ $H_1 : p \geq 0.6$	$H_0 : p = 0.6$ $H_1 : p > \dfrac{24}{30} = 0.8$
Let X be the number of games won, $X \sim B(30, 0.6)$ $P(X = 24) = 0.0115$ $0.0115 < 0.05$ do not reject H_0	Let be the number of games won, $X \sim B(30, 0.6)$ $P(X > 24) = 0.0057$ $0.0057 < 0.05$ so reject H_0

You are given that the students have correctly calculated their probabilities.

Identify and explain the **three** mistakes made by Gertrude.

(3 marks)

(b) Identify and explain the **two** mistakes made by Nate.

(2 marks)

(c) Use the information above to find the p-value for the test statistic $x = 24$, showing your calculation clearly.

(1 mark)

5 (a) Explain what you understand by the significance level of a hypothesis test.

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(1 mark)

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(b) (i) Give an advantage of using a lower significance level for a hypothesis test.

(ii) Give a disadvantage of using a lower significance level for a hypothesis test.

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(2 marks)

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(c) For each of the following scenarios, explain whether a 1%, 5% or 10% level of significance would be most appropriate.

(i) A shopkeeper takes a sample of 10 cartons of milk to test whether the amount of milk in a carton has decreased.

(ii) A doctor takes a sample of 100 patients to test whether there is an improvement to the recovery rate of an illness when a new drug is used, compared with the current best treatment regime.

(iii) A manager takes a sample of 100 employees to test whether their level of job satisfaction has changed after new working hours have been introduced.

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(6 marks)

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6 (a) The table below shows the cumulative probabilities for different values that $X \sim B(10,0.5)$ can take:

x	$P(X \leq x)$
0	0.000977
1	0.010742
2	0.054688
3	0.171875
4	0.376953
5	0.623047

Kieran collects coins and suspects that one of them is biased. To test his suspicion Kieran flips the coin 10 times and records the number of times, T, that it lands on tails.

Stating your hypotheses clearly, find the critical regions for the test using a 10% level of significance.

(3 marks)

(b) Calculate the probability of incorrectly rejecting the null hypothesis.

(2 marks)

(c) The coin lands on heads on each of the 10 flips. Kieran claims that the coin is definitely biased.

Comment on the validity of Kieran's claim.

(1 mark)

(d) Describe one adjustment Kieran could make to his test to give a more reliable conclusion.

(1 mark)

Very Hard Questions

1 (a) In the context of hypothesis testing, explain the term:

(i)   critical region

(ii)   critical value

(3 marks)

(b) The table below shows the probabilities for different values that $X \sim B(40, 0.8)$ can take:

x	$P(X = x)$
40	0.000133
39	0.001329
38	0.006480
37	0.020520
36	0.047452

A test of the null hypothesis $H_0 : p = 0.8$ against the alternative hypothesis $H_1 : p > 0.8$ is carried out for the random variable $X \sim B(40, p)$.

Using a 5% level of significance, find the values of X which would lead to the rejection of the null hypothesis.

(3 marks)

(c) A second test is carried out with the same null hypothesis against the alternative hypothesis $H_1 : p \neq 0.8.$

Given that $x = 38$ is a critical value, find the minimum level of significance for the test.

(3 marks)

2 (a) Meditest is a company manufacturing medical tests which are used to determine whether a patient has a certain illness. Meditest claims that the tests are 95% accurate, however a particular hospital will only purchase the tests if they are more than 95% accurate. Meditest test the accuracy of their product using a sample of 250 patients with the illness and agree on a 1% level of significance. They discover that the tests are accurate for 245 out of the 250 patients.

If $X \sim B(250, 0.95)$ then $P(X = 245) = 0.008515$ and $P(X > 245) = 0.004571$.

Stating your hypotheses clearly, test whether Meditest's product is more than 95% accurate using a 1% level of significance.

(5 marks)

(b) Meditest notice that they would have had sufficient evidence to reject the null hypothesis using a 5% level of significance. They change the level of significance from 1% to 5% and report to the hospital that their product is more than 95% accurate.

Comment on the validity of Meditest's report to the hospital.

(1 mark)

3 (a) Frank is the owner of a factory which has recently opened near a school where Hilda is the headteacher. Before the factory opened, the attendance rate at the school was good 90% of the time. Hilda claims that the proportion of days when the attendance rate is good has decreased and she suspects this is due to the fumes from the factory making the children sick. Frank disagrees and claims that the factory has made no difference to the attendance rate. To test their claims a sample of 40 days is taken and on 32 days the attendance rate is good.

If $X \sim B(40, 0.9)$ then:

$$P(X < 32) = 0.015495$$

$$P(X = 32) = 0.026407$$

$$P(X > 32) = 0.958098$$

Stating your hypotheses clearly, test Hilda's claim using a 5% level of significance. Give your answer in context.

(4 marks)

(b) Explain whether the outcome of the test supports Hilda's suspicion.

(1 mark)

(c) Stating your hypotheses clearly, test Frank's claim using a 5% level of significance.

(3 marks)

(d) Suggest a reason why Frank might want to use a two-tailed test.

(1 mark)

(e) State, with a reason, whether a one-tailed test or a two-tailed test would have been more appropriate for this scenario.

(2 marks)

4 (a) Given that $X \sim B(40, 0.3)$ then:

$$P(X \leq 5) = 0.008618$$

$$P(X = 6) = k$$

$$P(X = 7) = 0.031522$$

When a sample of size 40 is used to test $H_0 : p = 0.3$ against $H_1 : p < 0.3$, it is known that $x = 6$ is the critical value using a 5% level of significance. Use the probabilities above to find upper and lower bounds for the value of k.

(4 marks)

(b) When a sample of size 40 is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$, it is known that $x = 6$ is one of the two critical values using a 5% level of significance. Use the probabilities above to find an improvement for one of the bounds for the value of k.

(2 marks)

5 (a) If $X \sim B(n,p)$ then $P(X=0) = (1-p)^n$ and $P(X=n) = p^n$.

A sample of size 30 is used to test the null hypothesis $H_0 : p = 0.9$ against $H_1 : p > 0.9$ the alternative hypothesis using a k% level of significance.

Given that there is at least one value that leads to the rejection of the null hypothesis, find the range of values for k.

(2 marks)

(b) A sample of size 100 is used to test the null hypothesis $H_0 : p = q$ against the alternative hypothesis $H_1 : p < q$ using a 5% level of significance.

Given that there are no critical values for this test, find the range of values for q.

(3 marks)

(c) A sample of size m is used to test the null hypothesis $H_0 : p = 0.2$ against the alternative hypothesis $H_1 : p \neq 0.2$ using a 1% level of significance.

Given that there is exactly one critical region for this test, find the range of values for m.

(4 marks)