A Level · Edexcel · Further Maths

3 hours 14 questions

Elastic Collisions in 1D

Newton's Law of Restitution / Energy in 1D Collisions / Successive Collisions in 1D

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Total Marks **/156**

1 (a) A particle of mass $m$kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point O between two fixed parallel vertical walls.

The point O is equidistant from the two walls and the walls are 4 m apart.

At time t = 0 the particle is projected from O with speed u ms^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{3}{4}$.

The magnitude of the impulse on the particle due to the first impact with a wall is λmu Ns.

Find the value of λ.

(3 marks)

(b) The particle returns to O, having bounced off each wall once, at time $t = 7$ seconds.

Find the value of u.

(5 marks)

2 (a) A particle P of mass $2m$ and a particle Q of mass $5m$ are moving along the same straight line on a smooth horizontal plane.

They are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is $2u$ and the speed of Q is u.

The direction of motion of Q is reversed by the collision.

The coefficient of restitution between P and Q is e.

Find the range of possible values of e.

(8 marks)

(b) Given that $e = \frac{1}{3}$

show that the kinetic energy lost in the collision is $\frac{40mu^2}{7}$.

(5 marks)

(c) Without doing any further calculation, state how the amount of kinetic energy lost in the collision would change if $e > \frac{1}{3}$.

(1 mark)

3 (a) A particle P of mass $3m$ is moving in a straight line on a smooth horizontal floor. A particle Q of mass $5m$ is moving in the opposite direction to P along the same straight line.

The particles collide directly.

Immediately before the collision, the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

Show that the speed of Q immediately after the collision is $\frac{u}{8}(9e + 1)$.

(6 marks)

(b) Find the range of values of e for which the direction of motion of P is not changed as a result of the collision.

(2 marks)

(c) When P and Q collide they are at a distance d from a smooth fixed vertical wall, which is perpendicular to their direction of motion. After the collision with P, particle Q collides directly with the wall and rebounds so that there is a second collision between P and Q. This second collision takes place at a distance x from the wall.

Given that $e = \frac{1}{18}$ and the coefficient of restitution between Q and the wall is $\frac{1}{3}$,

find x in terms of d.

(6 marks)

4 (a) Two particles, A and B, of masses $2m$ and $3m$ respectively, are moving on a smooth horizontal plane. The particles are moving in opposite directions towards each other along the same straight line when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u. In the collision the impulse of A on B has magnitude $5mu$.

Find the coefficient of restitution between A and B.

(9 marks)

(b) Find the total loss in kinetic energy due to the collision.

(4 marks)

5 (a) Three particles, P, Q and R, are at rest on a smooth horizontal plane. The particles lie along a straight line with Q between P and R. The particles Q and R have masses m and km respectively, where k is a constant.

Particle Q is projected towards R with speed u and the particles collide directly.

The coefficient of restitution between each pair of particles is e.

Find, in terms of e, the range of values of k for which there is a second collision.

(9 marks)

(b) Given that the mass of P is km and that there is a second collision,

write down, in terms of u, k and e, the speed of Q after this second collision.

(1 mark)

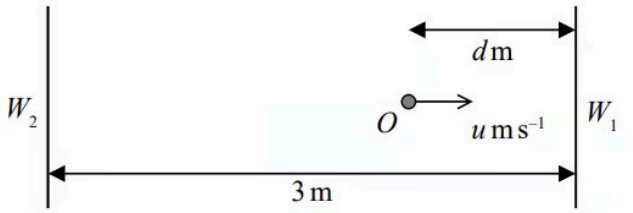

Figure 1

6 (a)

Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart.

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1.

At time $t = 0$, the particle is projected from O towards W_1 with speed u ms^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$.

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

Show that $T = \frac{45 - 5d}{4u}$.

(6 marks)

(b) The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

Find the least possible value of T, giving your answer in terms of u. You must give a reason for your answer.

(2 marks)

7 (a) A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e.

Find the range of possible values of e, justifying your answer.

(8 marks)

(b) Given that Q loses 75% of its kinetic energy as a result of the collision,

find the value of e.

(3 marks)

8 (a) Two particles P and Q have masses m and $4m$ respectively. The particles are at rest on a smooth horizontal plane. Particle P is given a horizontal impulse, of magnitude I, in the direction PQ. Particle P then collides directly with Q. Immediately after this collision, P is at rest and Q has speed w. The coefficient of restitution between the particles is e.

Find I in terms of m and w.

(2 marks)

(b) Show that $e = \frac{1}{4}$.

(1 mark)

(c) Find, in terms of m and w, the total kinetic energy lost in the collision between P and Q.

(2 marks)

9 (a) Three particles A, B and C are at rest on a smooth horizontal plane. The particles lie along a straight line with B between A and C.

Particle B has mass $4m$ and particle C has mass km, where k is a positive constant.

Particle B is projected with speed u along the plane towards C and they collide directly.

The coefficient of restitution between B and C is $\frac{1}{4}$.

Find the range of values of k for which there would be no further collisions.

(8 marks)

(b) The magnitude of the impulse on B in the collision between B and C is $3mu$.

Find the value of k.

(4 marks)

10 (a) A small ball, of mass m, is thrown vertically upwards with speed $\sqrt{8gH}$ from a point O on a smooth horizontal floor. The ball moves towards a smooth horizontal ceiling that is a vertical distance H above O. The coefficient of restitution between the ball and the ceiling is $\frac{1}{2}$.

In a model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to air resistance of constant magnitude $\frac{1}{2}mg$.

Using this model,

use the work-energy principle to find, in terms of g and H, the speed of the ball immediately before it strikes the ceiling.

(5 marks)

(b) Find, in terms of g and H, the speed of the ball immediately before it strikes the floor at O for the first time.

(5 marks)

(c) In a simplified model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to no air resistance.

Using this simplified model,

explain, without any detailed calculation, why the speed of the ball, immediately before it strikes the floor at O for the first time, would still be less than $\sqrt{8gH}$.

(1 mark)

11 (a) Two particles, A and B, have masses $3m$ and $4m$ respectively. The particles are moving in the same direction along the same straight line on a smooth horizontal surface when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u.

The coefficient of restitution between A and B is e.

Show that the direction of motion of each of the particles is unchanged by the collision.

(8 marks)

(b) After the collision with A, particle B collides directly with a third particle, C, of mass $2m$, which is at rest on the surface.

The coefficient of restitution between B and C is also e.

Show that there will be a second collision between A and B.

(6 marks)

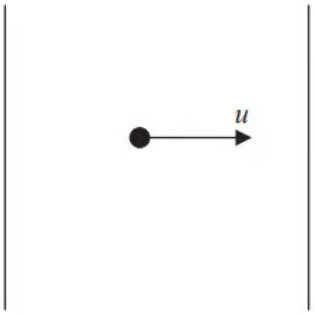

Figure 2

12 (a) A particle of mass em is at rest on a smooth horizontal plane between two smooth fixed parallel vertical walls, as shown in the plan view in Figure 2. The particle is projected along the plane with speed u towards one of the walls and strikes the wall at right angles. The coefficient of restitution between the particle and each wall is e and air resistance is modelled as being negligible.

Using the model,

find, in terms of m, u and e, an expression for the total loss in the kinetic energy of the particle as a result of the first two impacts.

(3 marks)

(b) Given that e can vary such that $0 < e < 1$ and using the model,

find the value of e for which the total loss in the kinetic energy of the particle as a result of the first two impacts is a maximum.

(4 marks)

(c) Describe the subsequent motion of the particle.

(2 marks)

13 (a) Two particles, P and Q, have masses m and em respectively. The particles are moving on a smooth horizontal plane in the same direction along the same straight line when they collide directly. The coefficient of restitution between P and Q is e, where $0 < e < 1$.

Immediately before the collision the speed of P is u and the speed of Q is eu.

Show that the speed of Q immediately after the collision is u.

(6 marks)

(b) Show that the direction of motion of P is unchanged by the collision.

(3 marks)

(c) The magnitude of the impulse on Q in the collision is $\frac{2}{9}mu$.

Find the possible values of e.

(4 marks)

14 (a) Two particles, A and B, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle A has mass $5m$ and particle B has mass $3m$.

The coefficient of restitution between A and B is e, where $e > 0$

Immediately **after** the collision the speed of A is v and the speed of B is $2v$.

Given that A and B are moving in the same direction after the collision,

find the set of possible values of e.

(8 marks)

(b) Given also that the kinetic energy of A immediately after the collision is 16% of the kinetic energy of A immediately before the collision,

find

i) the value of e,

ii) the magnitude of the impulse received by A in the collision, giving your answer in terms of m and v.

(6 marks)