A Level · Edexcel · Further Maths

23 mins 3 questions

Graphs

Graph Theory / Eulerian & semi-Eulerian Graphs / Planarity Algorithm

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Total Marks /23

1 (a) A simply connected graph is a connected graph in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

Given that a simply connected graph has exactly four vertices,

(i) write down the minimum number of arcs it can have,

(ii) write down the maximum number of arcs it can have.

(2 marks)

(b) (i) Draw a simply connected graph that has exactly four vertices and exactly five arcs.

(ii) State, with justification, whether your graph is Eulerian, semi-Eulerian or neither.

(3 marks)

(c) By considering the orders of the vertices, explain why there is only one simply connected graph with exactly four vertices and exactly five arcs.

(5 marks)

2 (a) Draw the graph K_5

(1 mark)

(b) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.

(ii) Draw two semi-Eulerian subgraphs of K_5, each having five vertices but with a different number of edges.

(3 marks)

(c) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree.

(2 marks)

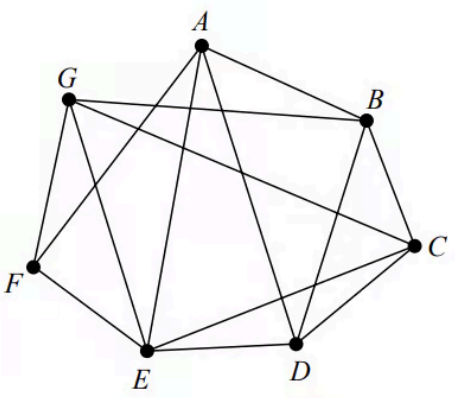

Figure 1

3 (a) Define what is meant by a **planar** graph.

(2 marks)

(b) Starting at A, find a Hamiltonian cycle for the graph in Figure 1.

(1 mark)

(c) Arc AG is added to Figure 1 to create the graph shown in Figure 2.

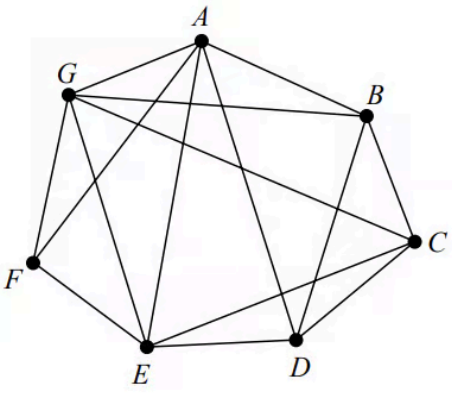

Figure 2

Taking ABCDEFGA as the Hamiltonian cycle,

use the planarity algorithm to determine whether the graph shown in Figure 2 is planar. You must make your working clear and justify your answer.

(4 marks)