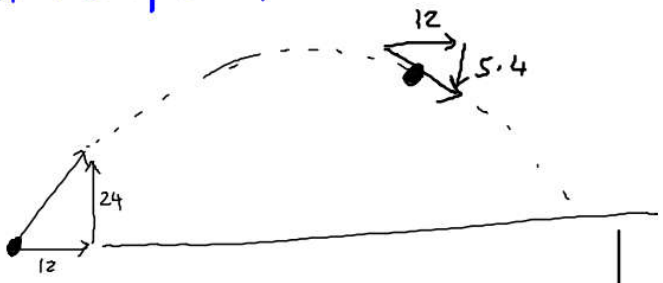
Ex 6C)

Q8

velocity $(12\underline{i} + 24\underline{j})$ ms^{-1}.

a) The position vector of P after 3s - where is it?

b) the speed of P after 3 seconds.



a) horizontal.

speed = 12

time = 3

distance = $12 \times 3 = 36$

vertical ↑+

$t = 3$

$u = 24$

$a = -9.8$

$s = ?$

$s = 24 \times 3 - 4.9 \times 3^2$

$s = 27.9$

$\begin{pmatrix} 36 \\ 27.9 \end{pmatrix}$ m

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{s} = \begin{pmatrix} 12 \\ 24 \end{pmatrix} 3 + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \times 3^2$

$\underline{s} = \begin{pmatrix} 36 \\ 72 \end{pmatrix} + \begin{pmatrix} 0 \\ -44.1 \end{pmatrix} = \begin{pmatrix} 36 \\ 27.9 \end{pmatrix}$

b) horizontal speed = 12 (constant

vertically ↑+

$u = 24$

$a = -9.8$

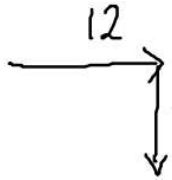
$t = 3$

$v =$

$v = u + at$

$= 24 - 9.8 \times 3$

$= -5.4$



$\underline{v} = \begin{pmatrix} 12 \\ -5.4 \end{pmatrix}$

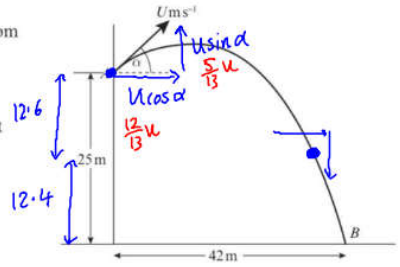
$v = \sqrt{12^2 + 5.4^2}$

$= 13.2$ (3sf) ms^{-1}

Finding speed at a given point

$\sin\alpha = \frac{5}{13}$ $\cos\alpha = \frac{12}{13}$

In this question use $g = 10\,\text{m s}^{-2}$.
An object is projected with speed $U\,\text{m s}^{-1}$ from a point A at the top of a vertical building.
The point A is 25 m above the ground.
The object is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$. The object hits the ground at the point B, which is at a horizontal distance of 42 m from the foot of the building, as shown in the diagram.
The object is modelled as a particle moving freely under gravity.



Find:

a the value of U **(6 marks)**

b the time taken by the object to travel from A to B **(2 marks)**

c the speed of the object when it is 12.4 m above the ground, giving your answer to 2 significant figures. **(5 marks)**

vert. mot. ↓+

$s = 25$
$a = 10$
$u = -\frac{5}{13}U$
$t = ?$

$s = ut + \frac{1}{2}at^2$

$25 = -\frac{5}{13}Ut + 5t^2$

horizontal motion

speed $= \frac{12}{13}U$ dist $= 42$ time $= t$

$t = \frac{42}{\frac{12}{13}U} = \frac{91}{2U}$

$25 = -\frac{5}{13}U \times \frac{91}{2U} + 5 \times \left(\frac{91}{2U}\right)^2$

$25 = -\frac{35}{2} + \frac{41405}{4U^2}$

$42.5 = \frac{41405}{4U^2}$

$170U^2 = 41405$

$U^2 = 243.55\ldots$

$U = 15.6\,\text{ms}^{-1}$ (3sf)

b) $t = \frac{91}{2 \times 15.606\ldots}$

$= 2.92$ secs (3sf)

speed when 12.4m above the ground

horizontal speed $= \frac{12}{13} \times 15.606\ldots = 14.40588\ldots$ → constant.

vertical speed ↓+

$a = 10$
$u = -\frac{5}{13} \times 15.606 = -6.002$
$s = 12.6$
$v = ?$

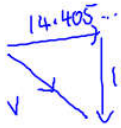
$v^2 = u^2 + 2as$

$= (-6.002)^2 + 2 \times 10 \times 12.6$

$v^2 = 288.08294\ldots$

$v = 16.97\ldots$

speed



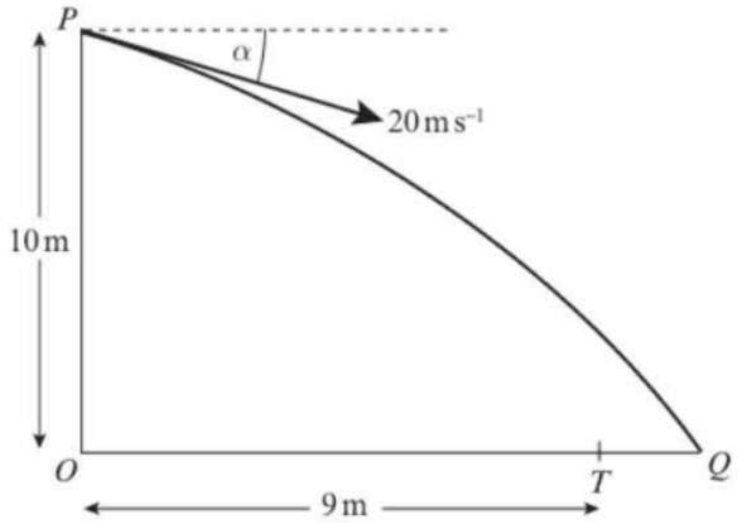
$v = \sqrt{14.405^2 + 16.97\ldots^2}$

$= 22.260\ldots$

$= 22\,\text{ms}^{-1}$ (2sf)

8 In this question use $g = 10\,\text{m s}^{-2}$.

A stone is thrown from a point P at a target, which is on horizontal ground. The point P is 10 m above the point O on the ground. The stone is thrown from P with speed $20\,\text{m s}^{-1}$ at an angle of α below the horizontal, where $\tan\alpha = \frac{3}{4}$.



The stone is modelled as a particle and the target as a point T. The distance OT is 9 m. The stone misses the target and hits the ground at the point Q, where OTQ is a straight line, as shown in the diagram. Find:

a the time taken by the ball to travel from P to Q **(5 marks)**

b the distance TQ. **(4 marks)**

The point A is on the path of the ball vertically above T.

c Find the speed of the ball at A. **(5 marks)**

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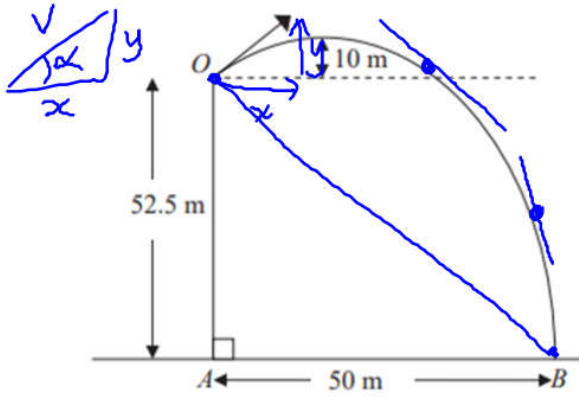


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.
(3)

(b) Find the speed of projection.
(9)

(c) Find the time after projection when the stone is moving parallel to OB.
(5)

a) vert. ↑+

$u = y$
$a = -9.8$
$s = 10$
$v = 0$
$v^2 = u^2 + 2as$
$0 = y^2 - 196$
$196 = y^2$
$y = 14$.

b) vert. ↓+

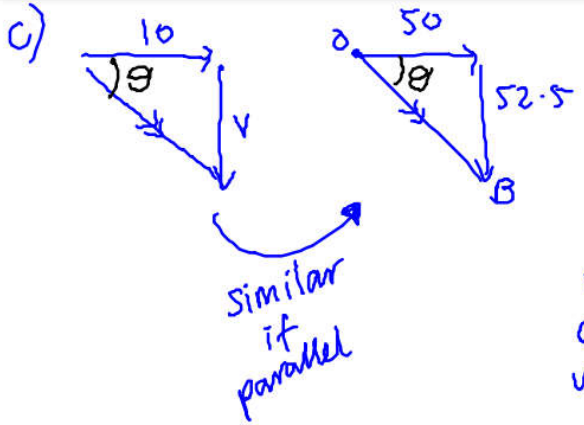
$u = -14$
$a = 9.8$
$s = 52.5$
$t = ?$
$52.5 = -14t + 4.9t^2$
$0 = 4.9t^2 - 14t - 52.5$
$t = 5 \quad t = -2.142$

horiz.

$t = 5$
speed = x dist = 50
$x = \frac{50}{5} = 10$

speed $= \sqrt{10^2 + 14^2} = 17.2 \text{ ms}^{-1}$

c)



$v = \frac{52.5}{5}$

$v = 10.5$

$v = 10.5$
$a = 9.8$
$u = -14$
$t = ?$

$v = u + at$
$10.5 = -14 + 9.8t$
$t = 2.5$

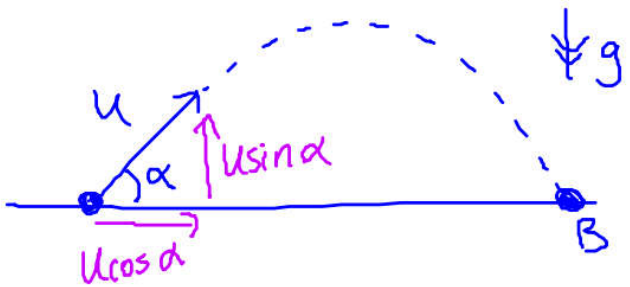
$\frac{52.5}{50} = \frac{v}{10}$
$v = 10.5$

Ex 6C

Projectile Formulae

There's nothing new here, but you may be asked to prove more general results regarding projectile motion.

A particle is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal and moves freely under gravity until it hits the plane at point B.
Given that that acceleration due to gravity is g, find expressions for:
(a) the time of flight, T
(b) the range, R, on the horizontal plane.



vertical motion ↑+

$u = U\sin\alpha$ $\quad s = ut + \frac{1}{2}at^2$

$a = -g$ $\quad 0 = U\sin\alpha \times T - \frac{1}{2}gT^2$

$s = 0$ $\quad 0 = UT\sin\alpha - \frac{1}{2}gT^2$

$t = T$ $\quad 0 = T(U\sin\alpha - \frac{1}{2}gT)$

$T = 0$

$U\sin\alpha - \frac{1}{2}gT = 0$

$U\sin\alpha = \frac{1}{2}gT$

$\frac{2U\sin\alpha}{g} = T$

b) horizontal motion

speed $= U\cos\alpha$

dist $= R$

time $= \frac{2U}{g}\sin\alpha$

$R = U\cos\alpha \times \frac{2U}{g}\sin\alpha$

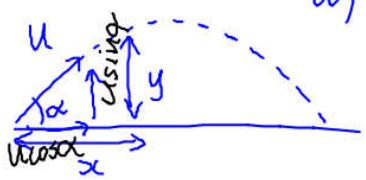
$R = \frac{2U^2}{g}\sin\alpha\cos\alpha = \frac{U^2}{g}\sin 2\alpha$

A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

(a) Show that $y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$

A particle is projected from a point O on a horizontal plane, with speed 28 ms^{-1} at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32m from O and at a height of 8m above the plane.

(b) Find the two possible values of α, giving your answers to the nearest degree.



a) horizontally.

dist = x

speed = $u\cos\alpha$

time = t

time = $\frac{x}{u\cos\alpha}$

vertically ↑+

$u = u\sin\alpha$

$t = \frac{x}{u\cos\alpha}$

$a = -g$

$s = y$

$s = ut + \frac{1}{2}at^2$

$y = u\sin\alpha \times \frac{x}{u\cos\alpha} + \frac{1}{2}(-g)\times\frac{x^2}{u^2\cos^2\alpha}$

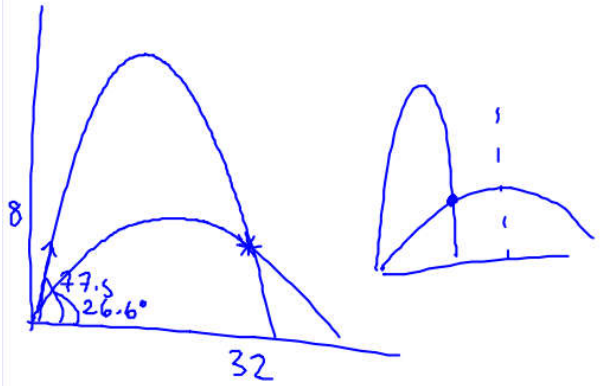
$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$

b) $u = 28$ $x = 32$ $y = 8$

$8 = 32\tan\alpha - \frac{32^2\times 9.8}{2\times 28^2}(1+\tan^2\alpha)$

$8 = 32\tan\alpha - \frac{32}{5} - \frac{32}{5}\tan^2\alpha$

$0 = -\frac{32}{5}\tan^2\alpha + 32\tan\alpha - \frac{72}{5}$



$\tan\alpha = \frac{9}{2}$ $\quad \tan\alpha = \frac{1}{2}$

$\alpha = \tan^{-1}\left(\frac{9}{2}\right)$ $\quad \alpha = 26.6°$

$= 77.5°$

General Results

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \frac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \frac{U \sin \alpha}{g}$
- Range on horizontal plane $= \frac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory: $y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$
  where y is vertical height of particle and x horizontal distance.

Ex 6D