OCR A Level Physics

Your notes

Gravitational Fields

Contents

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Gravitational Fields

Concept of a Gravitational Force Field

- Generally, the idea of a **force field** is any region of space in which a specific type of **object** will experience a **force**
- For example:
  - **Electric** fields are regions in which any object with **charge** experiences an electric force
  - **Magnetic** fields are regions in which any **magnet** experiences a magnetic force
- **Gravitational** fields are a special type of field in which any object with **mass** experiences a **gravitational force**

Defining Gravitational Fields

- Gravitational fields are set up around any object with **mass**
  - These fields affect any other objects with mass in their vicinity
- The Sun, for example, creates a gravitational field around it
  - The Earth, which has mass, experiences the gravitational force due to the Sun
  - This gravitational force keeps the Earth in orbit around the Sun
- Additional effects of the Moon and Sun's gravitational fields can be seen on Earth, such as the cause of tides

Direction of a Gravitational Field

- Gravitational fields represent the **action** of gravitational forces between masses, the direction of these forces can be shown using vectors
  - The direction of the **vector** shows the direction of the **gravitational force** that would be exerted on a **mass** if it was placed at that position in the field
- These vectors are known as **field lines** (or 'lines of force'), which are represented by arrows
  - Therefore, gravitational field lines also show the direction of **acceleration** of a mass placed in the field
- Gravitational field lines are therefore directed toward the centre of mass of a body
  - This is because the gravitational force is **attractive**
  - Therefore, masses always attract each other via the gravitational force

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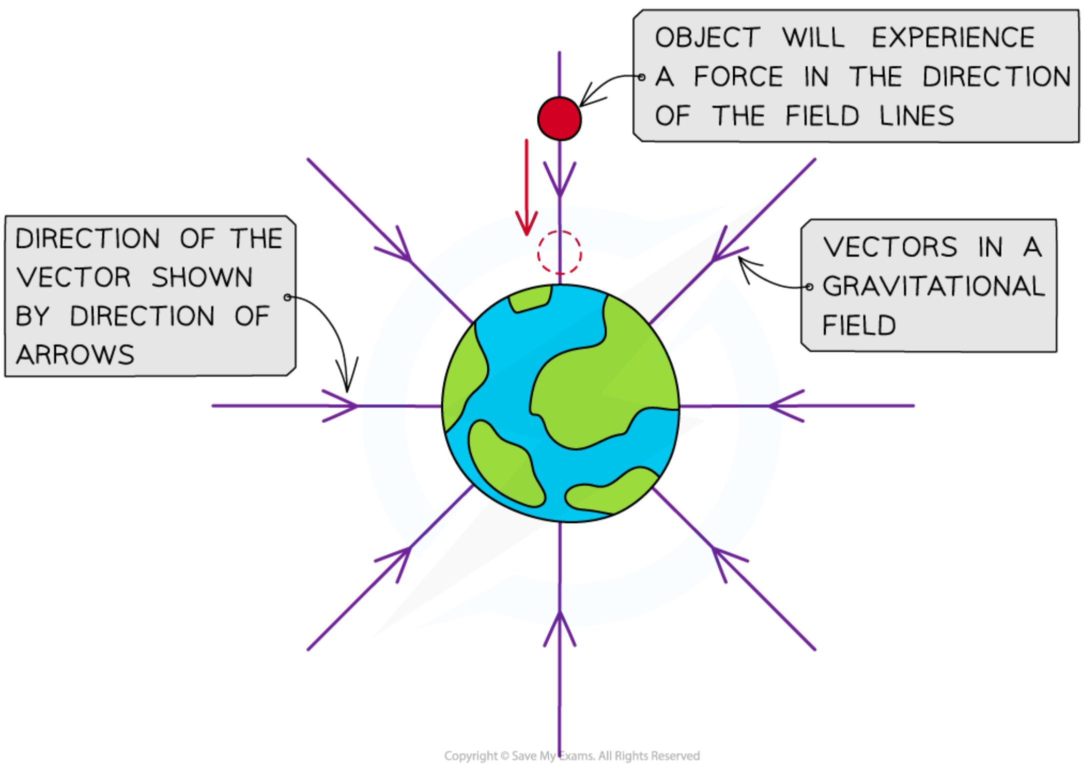
- The gravitational field around a point mass will be **radial** in shape and the field lines will always point towards the centre of mass



The direction of the gravitational field is shown by the vector field lines

Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as point mass when:

  A body covers a very large distance as compared to its size, so, to study its motion, its size or dimensions can be neglected

- An example of this is field lines around planets

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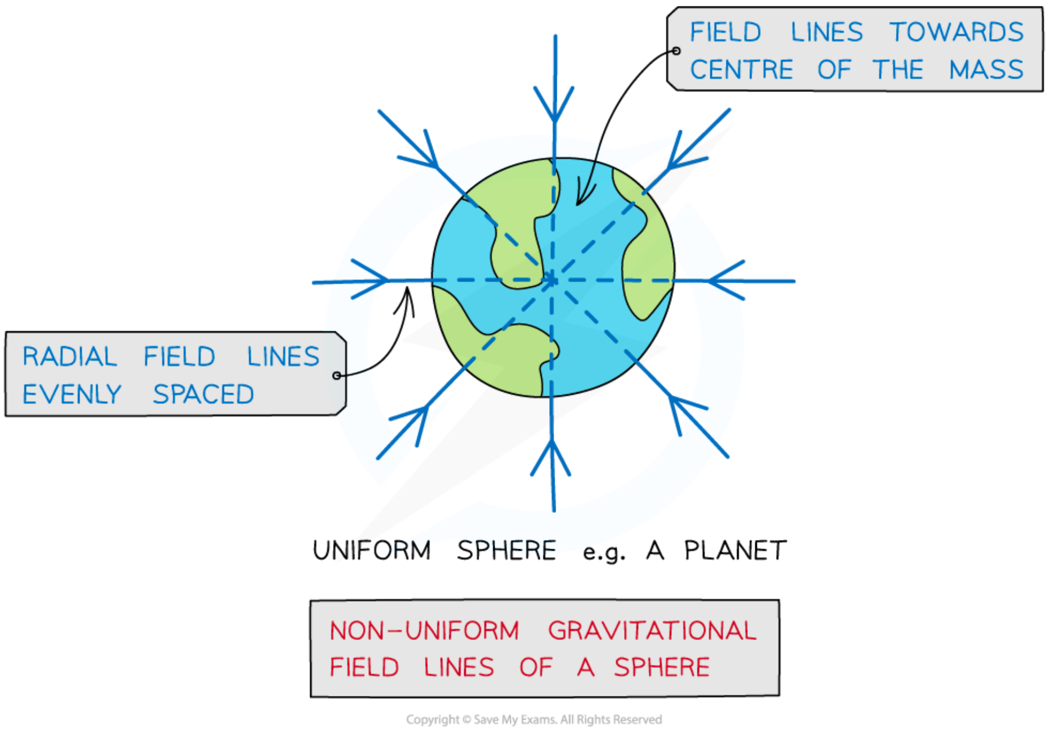

Gravitational field lines around a uniform sphere are identical to those on a point mass

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength g is different depending on how far an object is from the centre of mass of the sphere

Examiner Tips and Tricks

Always label the arrows on the field lines! Gravitational forces are **attractive only**. Remember:

- For a **radial field**: it is towards the centre of the sphere or point charge
- For a **uniform field**: towards the surface of the object e.g. Earth

Gravitational Field Lines

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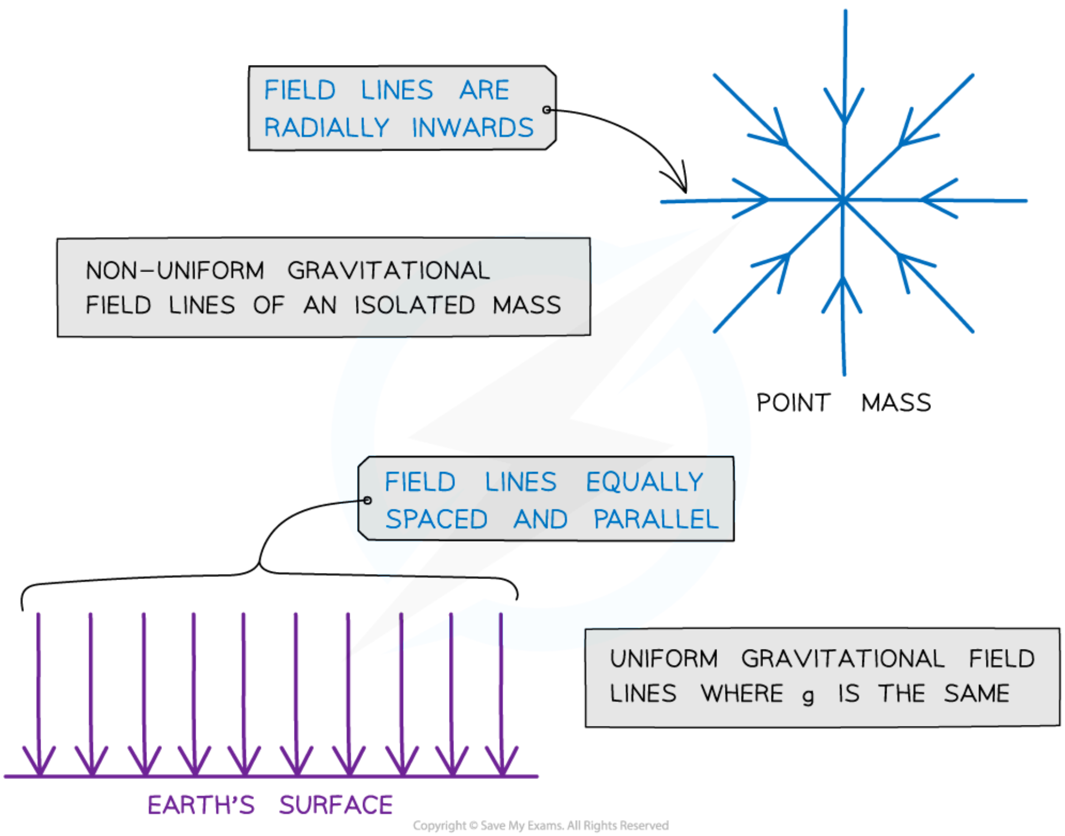
Gravitational Field Lines

- The direction of a gravitational field is represented by gravitational field lines
  - The direction shows the direction of **force**
  - Equivalently, they show the direction of **acceleration** of a **test mass** in the field
- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface



Gravitational field lines for a point mass and a uniform gravitational field

- Radial fields are considered **non-uniform fields**
  - The gravitational field strength *g* is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
  - The gravitational field strength *g* is the same throughout

Examiner Tips and Tricks

You should be able to link gravitational field lines with **vectors**: the density of gravitational field lines show the **magnitude** of the field (i.e., the closer they are, the stronger the field), and they also indicate the field's **direction**.

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Gravitational Field Strength

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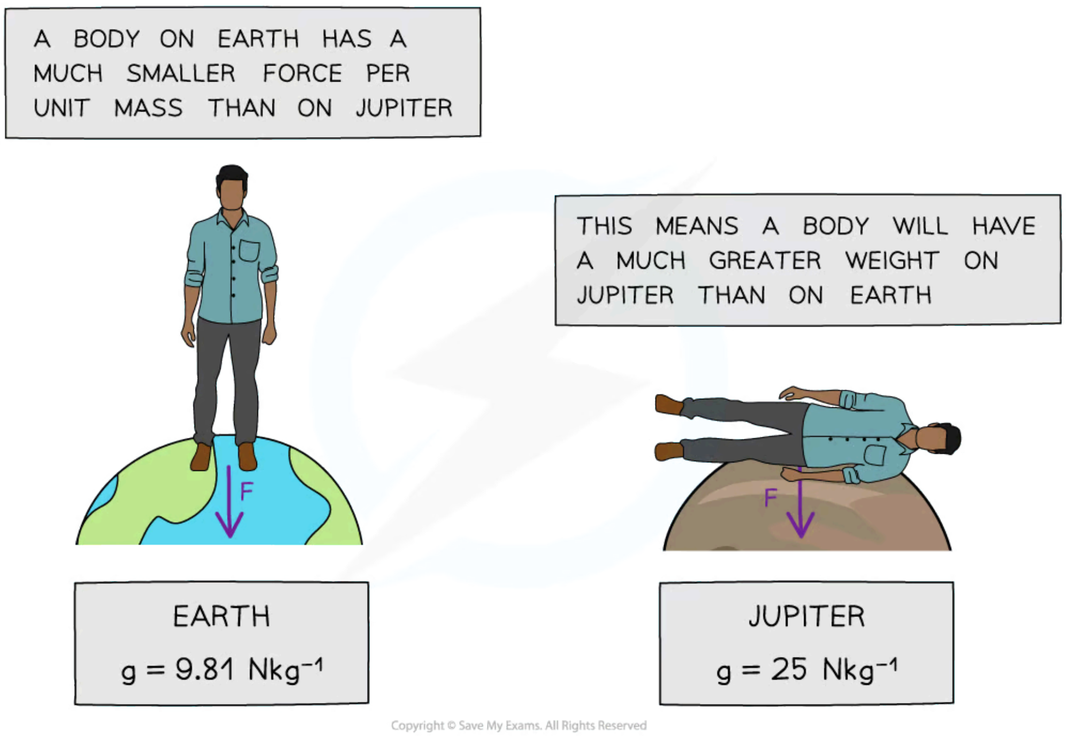
Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
    - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- The **gravitational field strength** g at a point is defined as force F per unit mass m of an object at that point:

$$g = \frac{F}{m}$$

- Where:
    - g = gravitational field strength ($N\ kg^{-1}$)
    - F = force due to gravity, or weight (N)
    - m = mass (kg)
- This equation shows that:
    - The larger the mass of an object, the greater its pull on another mass
    - On planets with a large value of g, the gravitational force per unit mass is **greater** than on planets with a smaller value of g
- An object's mass remains the same at all points in space
    - However, on planets such as Jupiter, the weight of an object will be a lot greater than on a less massive planet, such as Earth
    - This means the gravitational force would be so high that humans, for example, would not be able to fully stand up (or, even worse...)



The weight force on Jupiter would be so large that even standing upright would be difficult

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** (or diameter) of the planet
  - The **mass** (or density) of the planet

Worked Example

Calculate the mass of an object with weight 10 N on Earth.

Answer:

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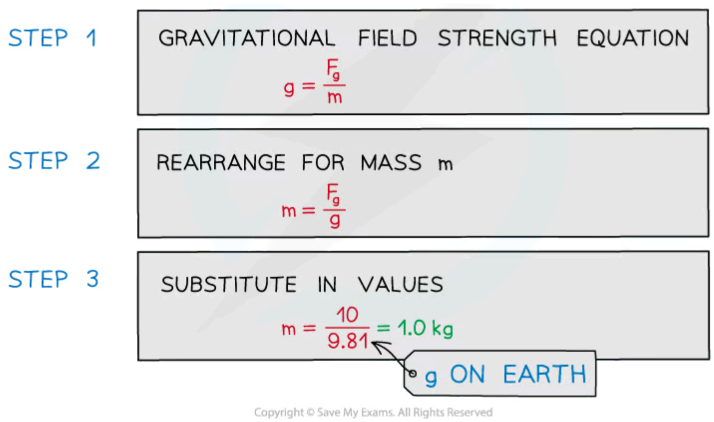


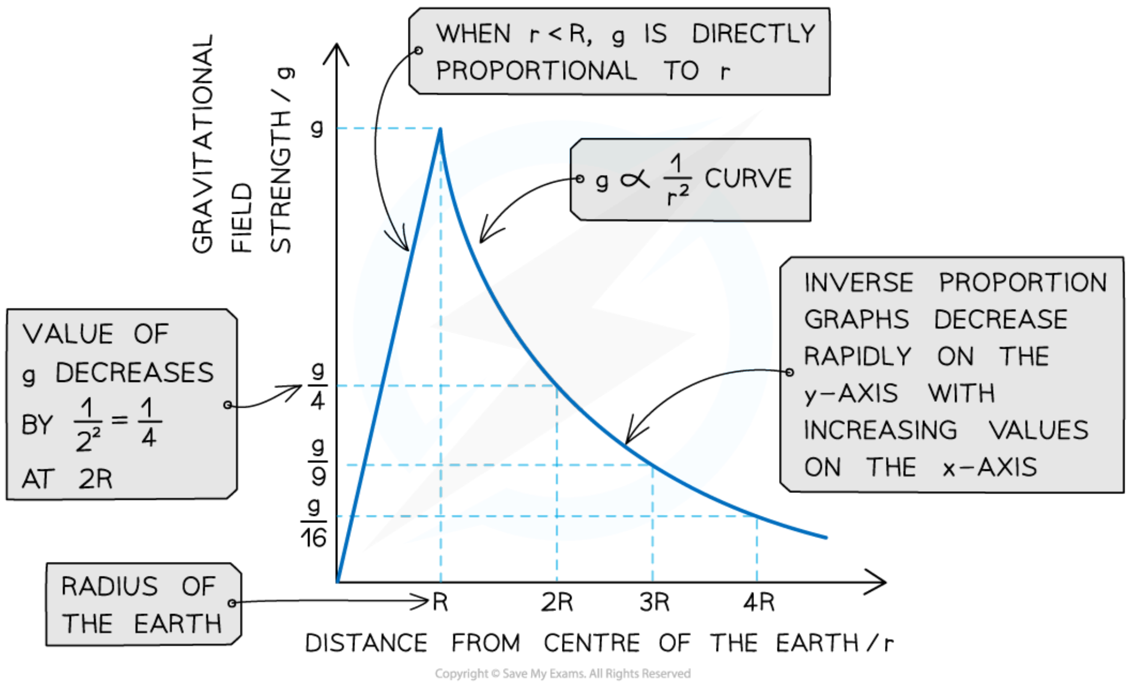
Gravitational Field Strength in a Radial Field

- In a **radial** field (due to a point mass *M*), the gravitational field lines get **further apart** from each other
  - This indicates that the **strength** of the gravitational field **decreases** with distance from the centre of mass of *M*
- The gravitational field strength *g* in a radial field, due to some mass *M*, is given by the equation:

$$g = -\frac{GM}{r^2}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - G = Newton's Gravitational constant (N m^2 kg^{-2})
  - M = mass of the object causing the gravitational field (kg)
  - r = radial distance from the centre of mass of *M* (m)
- **Note:**
  - The negative sign in this equation indicates that the gravitational field is **attractive**
  - In other words, the **direction** of the **gravitational field lines** is **towards** the mass *M*
- On the Earth's surface, *g* has a constant value of 9.81 N kg^{-1}
- However **far outside the Earth's surface, *g* is not constant**
  - *g* decreases as *r* increases by a factor of $1/r^2$
  - This is an **inverse square law relationship** with distance

- When the **magnitude** of *g* is plotted against the distance from the **centre of a planet**, *r* has two parts:
  - When $r < R$ (the radius of the planet), *g* is **directly proportional** to *r*
  - When $r > R$, *g* is **inversely proportional** to r^2



The magnitude of gravitational field strength g against distance r from the Earth's surface follows a 1/r² relationship

Gravitational Field Strength Close to the Earth's Surface

- Near the Earth's surface, the gravitational field is **uniform**
  - Hence, the gravitational field lines are **parallel** and **evenly spaced**
- This means the **gravitational field strength** is **constant** at every point near the Earth's surface
  - Numerically, the gravitational field strength near Earth's surface is equal to the acceleration due to gravity, $\mathbf{g = 9.81\ m\ s^{-2}}$

Worked Example

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Determine the distance from the Earth's surface at which the gravitational field strength decreases by a factor of 0.5.

(The radius of the Earth is 6400 km and its mass is 6.0×10^{24} kg)

Answer:

Step 1: Write the known quantities

- Radius of the Earth R_E = 6400 km = 6400×10^3 m
- Mass of the earth $M_E = 6.0 \times 10^{24}$ kg
- Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Step 2: Recall the value of the gravitational field strength at the Earth's surface

- The gravitational field strength at the Earth's surface g = 9.81 N kg^{-1}

Step 3: Write the equation for gravitational field strength in a radial field

- The Earth creates a **radial** gravitational field (far from its surface) therefore the equation for gravitational field strength g is:

$$g = -\frac{GM}{r^2}$$

Step 4: Determine the distance *r* at which the field strength reduces by a factor of 0.5

- If the field strength decreases by a factor of 0.5, then $g \times 0.5 = 9.81 \times 0.5 = 4.905$ N kg^{-1}
- Therefore, **ignoring** the **negative sign** (as we only want a magnitude):

$$4.905 = \frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{r^2}$$

$$r^2 = \frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{4.905}$$

$$r = \sqrt{\frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{4.905}} = 9.0 \times 10^6 \text{ m}$$

Step 5: Determine the distance from the Earth's surface

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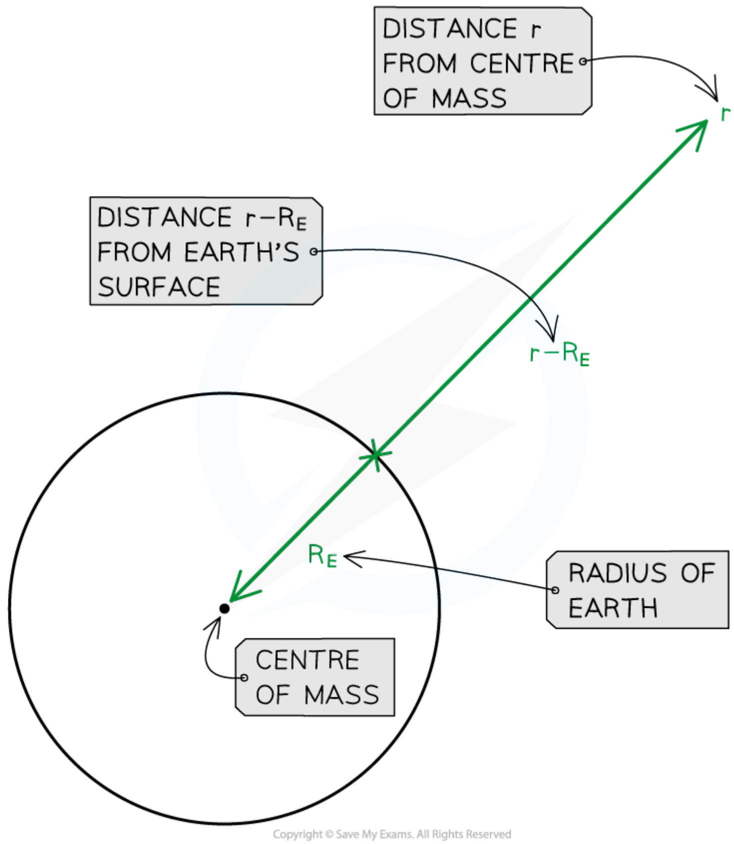

- The value $r = 9.0 \times 10^6$ m is the radial distance from the Earth's **centre of mass**
- Therefore, the gravitational field strength reduces by a factor 0.5 at a distance $r - R_E$

$$r - R_E = (9.0 \times 10^6) - (6400 \times 10^3) = 2.6 \times 10^6 \text{ m}$$

Examiner Tips and Tricks

The equation for the gravitational field strength in a radial field is in terms of the distance *r* from the **centre of mass** of mass *M*. If the exam question is about a planet, remember that you might have to take the planet's radius into account, which is the distance between its centre of mass and its surface! As ever, drawing a labelled diagram of the distances in question really helps.

Newton's Law of Gravitation

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Newton's Law of Gravitation

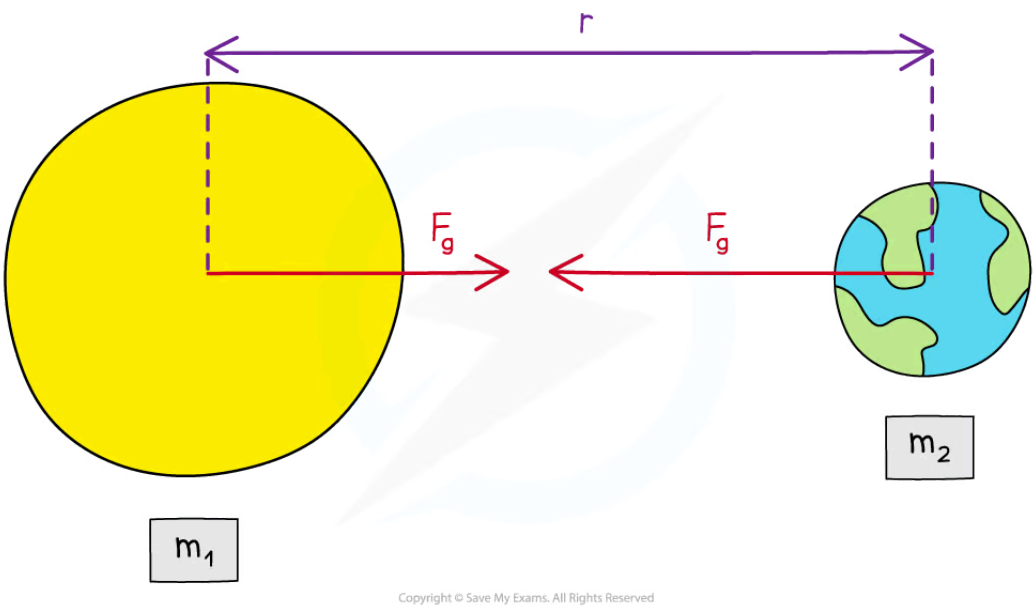
- The gravitational force between two masses, e.g., between the Earth and the Sun, is defined by Newton's Law of Gravitation
- Newton's Law of Gravitation states:

  The gravitational force F between two masses m_1 and m_2 is proportional to the product of their masses and inversely proportional to the square of their separation, r

- In equation form, this is written as:

$$F = -\frac{Gm_1m_2}{r^2}$$

- Where:
  - F = gravitational force between two point masses m_1 and m_2 (N)
  - G = Newton's gravitational constant
  - m_1 and m_2 = mass of body 1 and mass of body 2 (kg)
  - r = distance between the centre of the two masses (m)
- The $1/r^2$ relation is called the 'inverse square law'
  - This means that if the distance between two masses **doubles**, r becomes $2r$
  - Therefore, $1/r^2$ becomes $1/(2r)^2$, which is equal to $1/4r^2$
  - Hence, the gravitational force between the two masses **reduces** by a factor of **four**
- The **negative sign** indicates that the gravitational force F between the two point masses m_1 and m_2 is **attractive**



The gravitational force between two masses is defined by Newton's Law of Gravitation

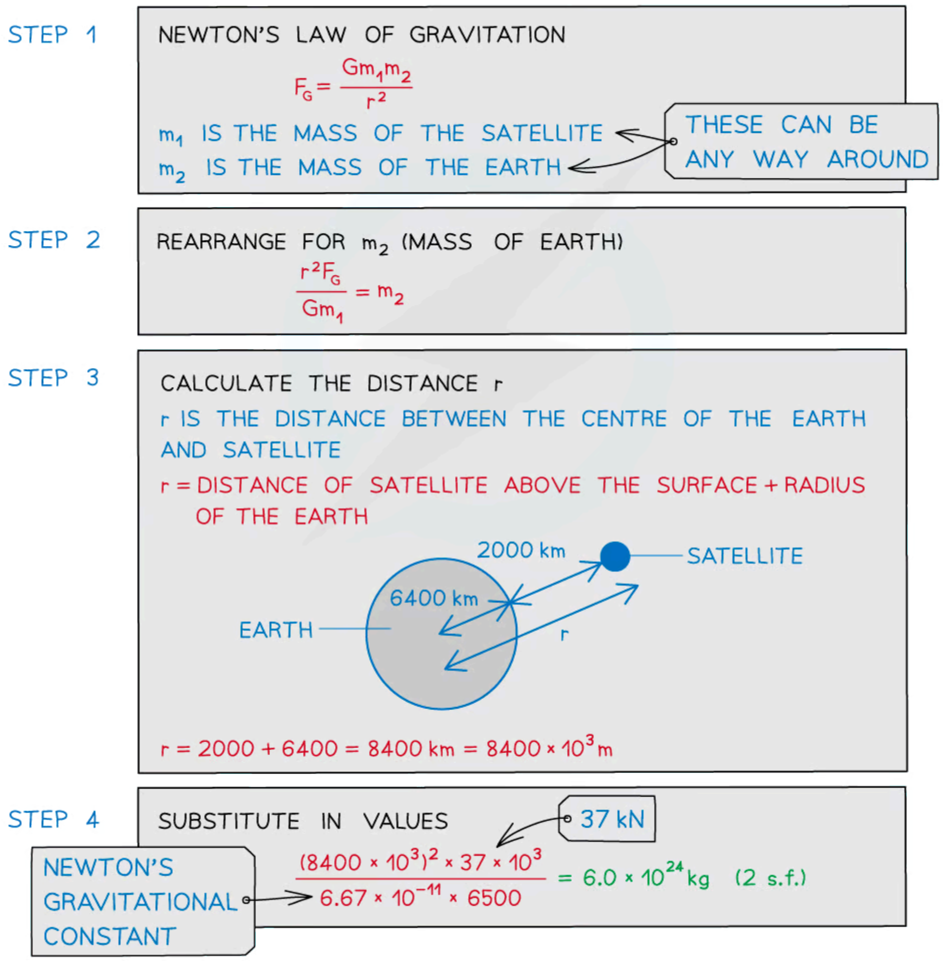
Worked Example

A satellite with mass 6500 kg is orbiting the Earth at 2000 km above the Earth's surface. The magnitude of the gravitational force between them is 37 kN.

Calculate the mass of the Earth.

(Radius of the Earth = 6400 km)

Answer:



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Examiner Tips and Tricks

A few common mistakes to be aware of are:

- forgetting to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured between the **centre** of each mass, which is from the **centre** of the planet to the centre of the satellite!
- forgetting that the **distance** between point masses m_1 and m_2 is **squared**. Remember this whenever you use Newton's Law of Gravitation!
- Note in this worked example, we calculated the **magnitude** of the gravitational force *F*. Therefore, we could ignore the negative sign. Make sure you are aware of this!