OCR A Level Physics

Your notes

Linear & Projectile Motion

Contents

SUVAT Equations

Your notes

SUVAT Equations

- The SUVAT equations are the equations of motion used for objects in constant acceleration
- They contain the following variables:
  - ***s*** = displacement (m)
  - ***u*** = initial velocity (m s^{-1})
  - ***v*** = final velocity (m s^{-1})
  - ***a*** = acceleration (m s^{-2})
  - ***t*** = time (s)
- The 4 SUVAT equations are:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{(v + u)}{2}t \qquad v^2 = u^2 + 2as$$

- These are all given on the data sheet
- All the variables, apart from **time *t***, are vector quantities
  - This means they can either be positive or negative depending on their direction
- The key terms to look out for are:
  - 'Starts from rest', or if the initial velocity is not mentioned, this means ***u*** = 0
  - 'Starts from rest' means an object starts from rest at ***x*** = 0 when ***t*** = 0
  - If an object is only 'falling due to gravity' then ***a*** = *g* = 9.81 m s^{-2}
  - It doesn't matter which way is positive or negative for the scenario, as long as it is consistent for all the vector quantities

Your notes

- For example, if downwards is negative then for a ball travelling upwards, ***s*** must be positive and ***a*** must be negative
- SUVAT equations are used for motion with **constant acceleration** in a straight line
  - For example, an object falling in a uniform gravitational field without air resistance

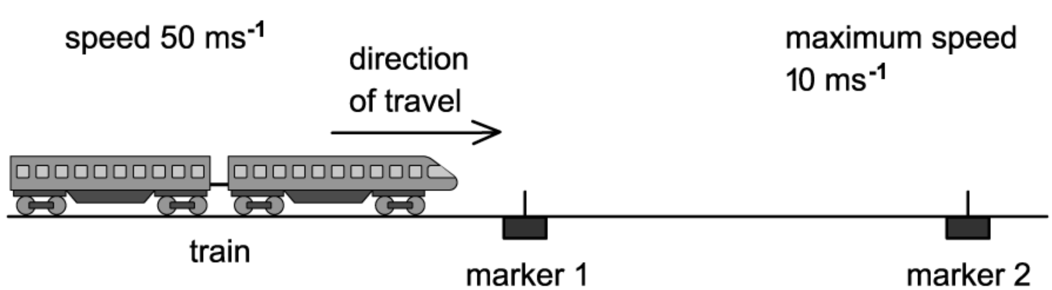
How to Use the SUVAT Equations

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion $a = \pm 9.81$ m s^{-2}, an object which starts or finishes at rest will have $u = 0$ or $v = 0$
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links *s, u, a* and *t* is $s = ut + ½at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

- Sometimes the question may have to be split into two, where the SUVAT equations will have to be used twice
  - For example, if there are two masses connected over a pulley and one mass continues moving after the other has stopped

Worked Example

The diagram shows an arrangement to stop trains that are travelling too fast.



At marker 1, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker 2 at no more than 10 m s^{-1}.

The train carries a detector that notes the times when the train passes each marker and will apply an emergency brake if the time between passing marker 1 and marker 2 is less than 20 s.

Trains coming from the left travel at a speed of 50 m s^{-1}.

Determine how far marker 1 should be placed from marker 2.

Answer:

STEP 1

OUR KNOWN VARIABLES ARE

- $u = 50\ ms^{-1}$
- $v = 10\ ms^{-1}$
- $t = 20\ s$

AND WE ARE ASKED TO FIND DISTANCE, s.

STEP 2

SO THE EQUATION THAT LINKS u, v, t AND s IS

$$s = \frac{(u+v)}{2}t$$

STEP 3

NO REARRANGING IS REQUIRED SO WE SIMPLY PLUG IN THE VARIABLES:

$$s = \frac{(50+10)}{2} \times 20 = 30 \times 20 = 600\ m$$

Examiner Tips and Tricks

This is arguably the most important section of this topic, you can always be sure there will be one, or more, questions in the exam about solving problems with SUVAT equations

The best way to master this section is to practice as many questions as possible!

Your notes

Your notes

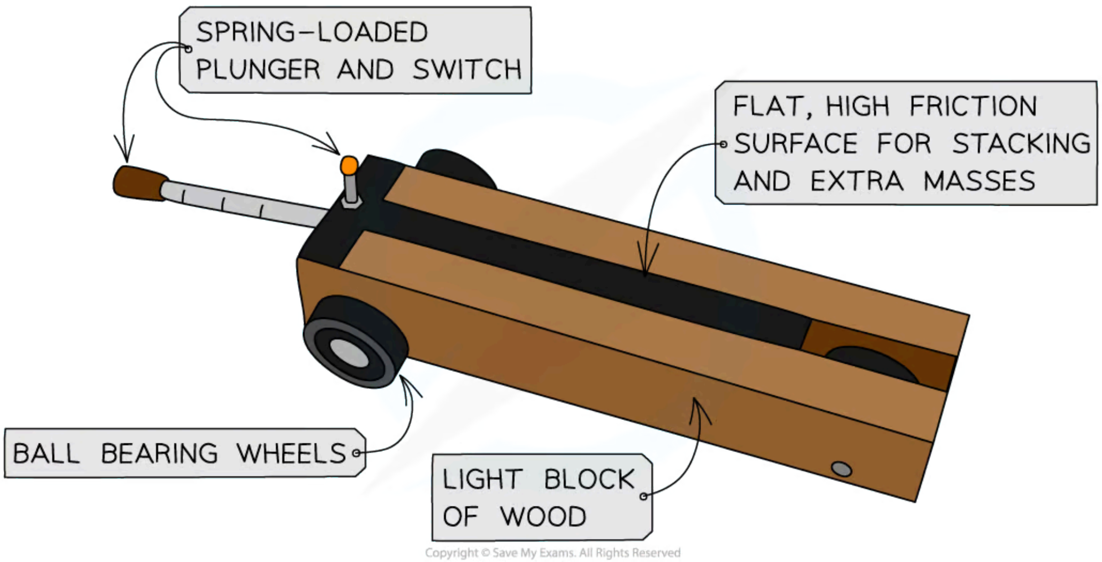
Investigating Motion & Collisions

- There are a number of techniques and procedures that can be used to investigate the motion of objects
- A few examples of experiments that could be carried out are:
  - Measurement of speed and acceleration of trolleys down a ramp
  - Collisions between trolleys on an air track to investigate momentum and energy changes
  - Determination of acceleration due to gravity, *g*
  - The terminal velocity of a falling object
- Typically, these experiments require the use of one or more of the following:
  - Trolleys
  - Air-Track Gliders
  - Ticker Timers
  - Light Gates
  - Data Loggers
  - Video Techniques

Trolleys

- Trolleys are essential when studying motion and collisions
- They can be used to investigate speed, acceleration, and momentum
- They are composed of
  - A light block of wood or plastic
  - Ball-bearing wheels (to reduce friction)
  - A spring-loaded plunger (for collisions)
  - A flat top surface to allow stacking or additional masses to be added

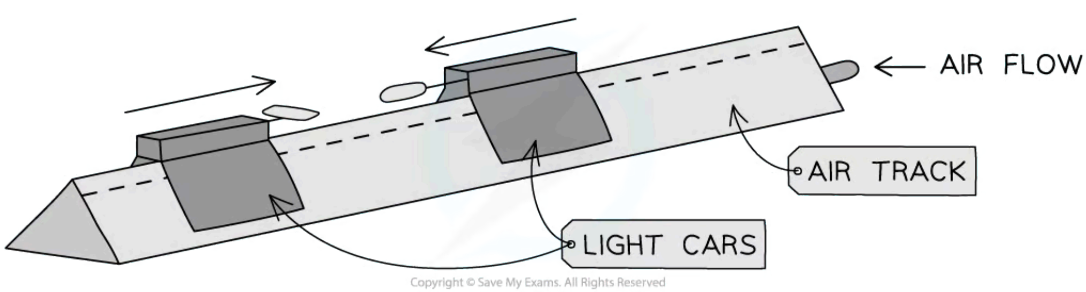
Your notes



Trolleys are essential in physics experiments for speed, acceleration, and momentum

Air-Track Gliders

- This technique can also be used to investigate the conservation of momentum resulting from a collision
- Air tracks are the optimum equipment to use for this as they reduce the friction
- Hence the energy lost to overcoming friction is minimised and the collision is kept as elastic as possible



Air tracks are used to investigate collisions and the principle of conservation of momentum

Ticker Timers

- Another way of analysing motion in a physics lab is to use ticker tape

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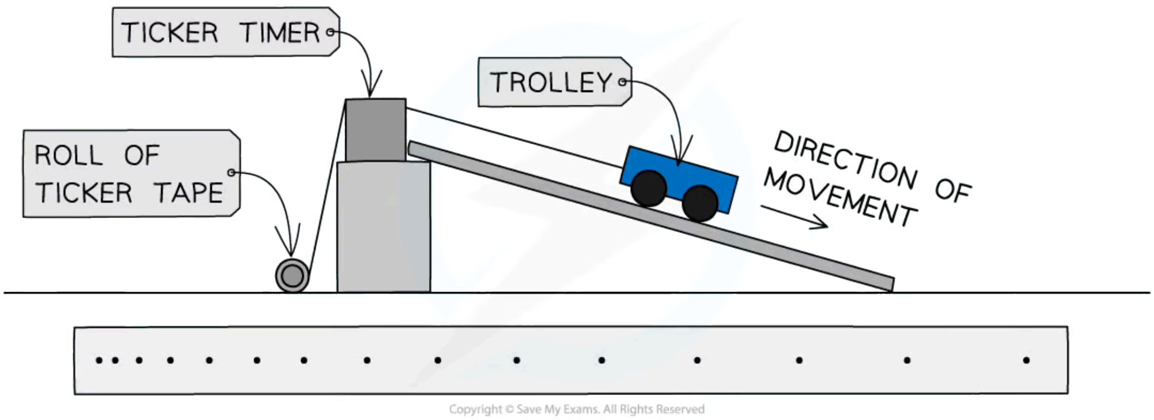
- A long tape is attached to a moving trolley and threaded through a device that places a tick upon the tape at regular intervals of time
- The ticker timer will produce a certain number of dots per second on the tape, which will travel at the same speed as the trolley
- The distance between dots and the time can then be used to determine the velocity



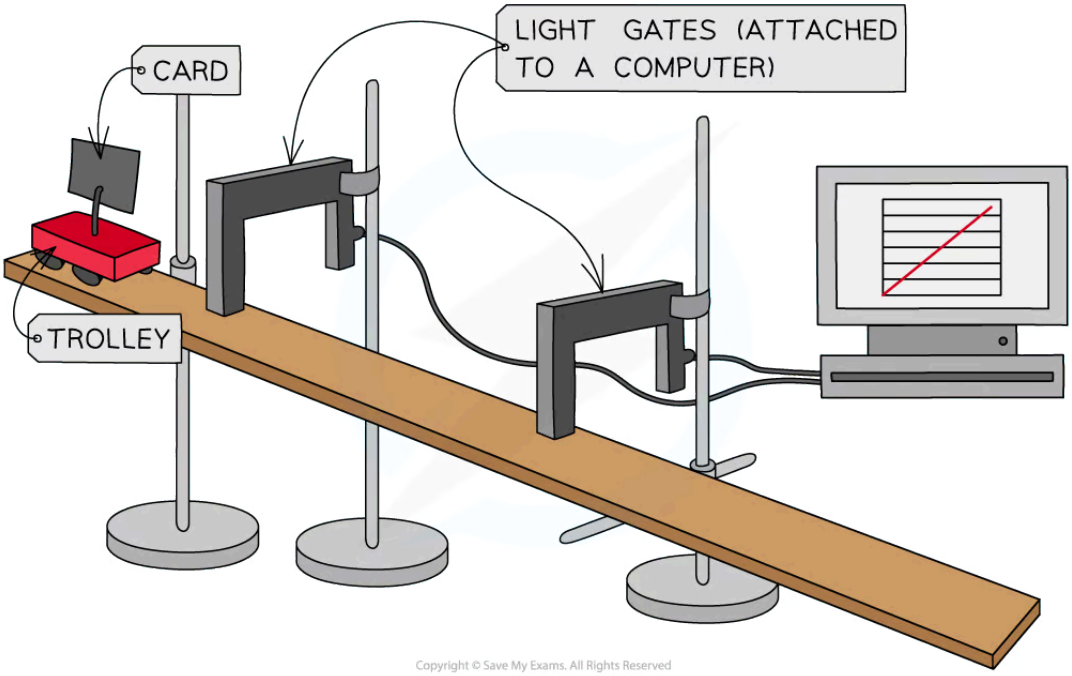
The distance between successive dots increases down the ramp shows that the acceleration of the trolley is constant

Light Gates

- Light gates provide the most accurate way of measuring the time taken for a trolley to move through a set distance
- A card is attached to the top of the trolley as this is will interrupt the light beams on the gates
- The trolley is released from the top of the ramp, with one light gate just in front of the release point and the other at the bottom of the ramp
- The time taken to travel between the light gates, *t*, can be used to work out the initial speed, *u*, and final speed, *v*
- The acceleration, *a*, can then be calculated using the equation:

$$v = u + at$$

Your notes



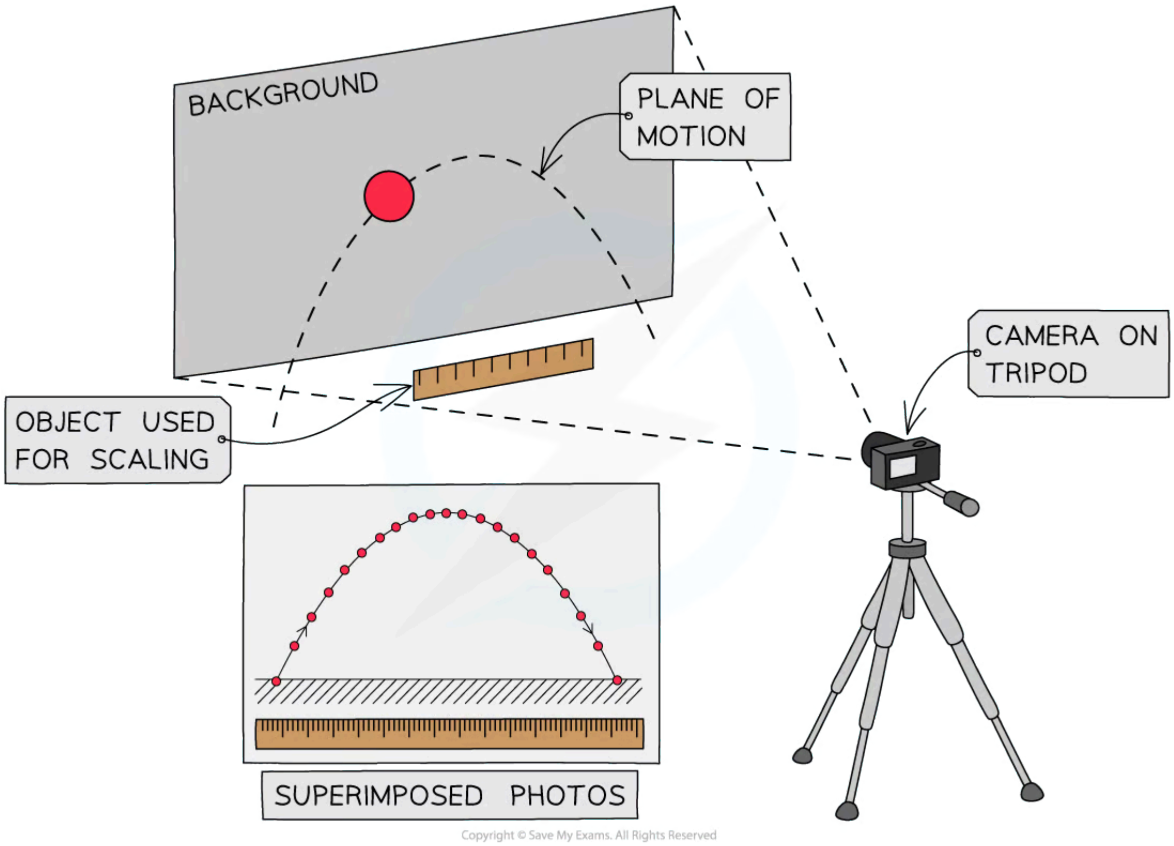
Set up for investigating acceleration down a ramp using light gates

Data Loggers

- A data logger is an electronic device that records data and stores it for further analysis
- It can be set to record at regular time intervals or when triggered by a sensor, such as a light gate
- Data loggers are often used as a more accurate way of measuring time and eliminate the error from the human reflex speed needed to stop and start a stopwatch

Video Techniques

- Taking videos or successive photographs of objects in motion is a useful method for determining
  - The acceleration of freefall
  - Projectile motion
  - Terminal velocity
- This technique has two requirements:
  - The frames per second must be known, as this can be used to determine the time taken
  - The distance must be known, usually from placing a ruler in the shot with the object



Set up for investigating projectile motion using a camera

Your notes

Acceleration & Free Fall

Acceleration g of Free Fall

- The acceleration of free fall, g, is defined as:

  The acceleration of any object in response to the gravitational attraction between the Earth and the object

- Any object released on the Earth will accelerate **downwards** to the centre of the Earth as long as there are no external forces acting on it
- On Earth, the acceleration of free fall is equal to $\boldsymbol{g = 9.81\ m\ s^{-2}}$

Determining g in the Laboratory

Aims of the Experiment

- The overall aim of the experiment is to calculate the value of the acceleration due to gravity, *g*
- This is done by measuring the time it takes for a ball-bearing to fall a certain distance. The acceleration is then calculated using an equation of motion

Variables

- Independent variable = height, *h*
- Dependent variable = time, *t*
- Control variables:
  - Same steel ball-bearing
  - Same electromagnet
  - Distance between ball-bearing and top of the glass tube

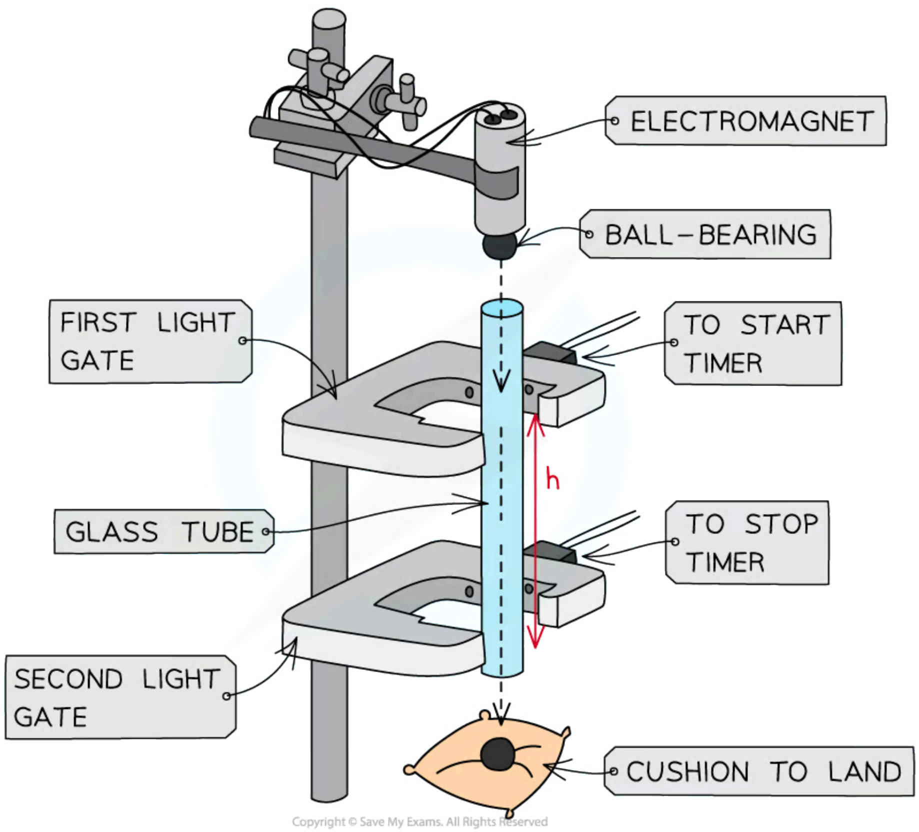
Equipment List

Your notes

Equipment	Purpose
Metre ruler	To measure the distance between the light gates
Steel ball–bearing	To measure the distance and time taken to drop. The ball must be made from a magnetic material (iron, steel etc.)
Electromagnet	To drop the ball–bearing through the glass tube at specific height every time
Two light gates	To determine the time taken to drop a certain distance
Timer	To measure the time taken for the ball to drop between the light–gates. The timer must be activated to start when the ball passes the first light gate and stop when it passes the second
Tall clamp stand (retort stand)	To hold the glass tube and electromagnet in line with each other
Glass tube	To guide the ball–bearing vertically downwards
Cushion	To stop the ball–bearing from being damaged or damaging the surface when landing

- Resolution of measuring equipment:
    - Metre ruler = 1 mm
    - Timer = 0.01 s

Method



Apparatus setup to measure the distance and time for the ball bearing to drop

This method is an example of the procedure for varying the height the ball-bearing falls and determining the time taken – this is just one possible relationship that can be tested

1. Set up the apparatus by attaching the electromagnet to the top of a tall clamp stand. Do not switch on the current till everything is set up
2. Place the glass tube directly underneath the electromagnet, leaving space for the ball-bearing. Make sure it faces directly downwards and not at an angle
3. Attach both light gates around the glass tube at a starting distance of around 10 cm
4. Measure this distance between the two light gates as the height, *h* with a metre ruler
5. Place the cushion directly underneath the end of the glass tube to catch the ball-bearing when it falls through
6. Switch the current on the electromagnet and place the ball-bearing directly underneath so it is attracted to it

7. Turn the current to the electromagnet off. The ball should drop
8. When the ball drops through the first light gate, the timer starts
9. When the ball drops through the second light gate, the timer stops
10. Read the time on the timer and record this as time, *t*
11. Increase *h* (eg. by 5 cm) and repeat the experiment. At least 5 – 10 values for *h* should be used
12. Repeat this method at least 3 times for each value of *h* and calculate an average *t* for each

- An example of a table with some possible heights would look like this:

Example Table of Results

HEIGHT h/m	TIME t_1/s	TIME t_2/s	TIME t_3/s	AVERAGE TIME t/s
0.10				
0.15				
0.20				
0.25				
0.30				
0.35				

Analysis of Results

- The acceleration is found by using one of the SUVAT equations
- The known quantities are
    - Displacement $s = h$
    - Time taken = t
    - Initial velocity $u = u$
    - Acceleration $a = g$

Your notes

- The following SUVAT equation can be rearranged:

$$s = ut + \frac{1}{2}at^2$$

$$2s = 2ut + at^2$$

$$\frac{2s}{t} = 2u + at$$

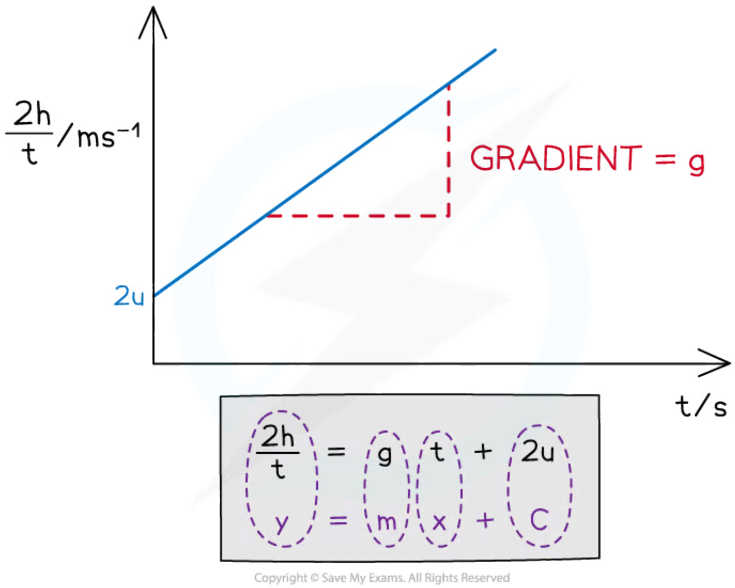
- Substituting in the values and rearranging it as a straight line equation gives:

$$\frac{2h}{t} = gt + 2u$$

- Comparing this to the equation of a straight line: y = mx + c
  - y = 2h/*t* (m s^{-1})
  - x = *t*
  - Gradient, m = a = *g* (m s^{-2})
  - y-intercept = 2*u*

1. Plot a graph of the 2*h*/*t* against *t*
2. Draw a line of best fit
3. Calculate the gradient - this is the acceleration due to gravity *g*
4. Assess the uncertainties in the measurements of *h* and *t*. Carry out any calculations needed to determine the uncertainty in *g* due to these

Your notes



The graph of 2h/t against t produces a straight-line graph where the acceleration is the gradient

Evaluating the Experiment

Systematic Errors:

- Residue magnetism after the electromagnet is switched off may cause *t* to be recorded as longer than it should be

Random Errors:

- Large uncertainty in *h* from using a metre rule with a precision of 1 mm
- Parallax error from reading *h*
- The ball may not fall accurately down the centre of each light gate
- Random errors are reduced through repeating the experiment for each value of *h* at least 3–5 times and finding an average time, *t*

Safety Considerations

- The electromagnetic requires current
  - Care must be taken to not have any water near it
  - To reduce the risk of electrocution, only switch on the current to the electromagnet once everything is set up

- A cushion or a soft surface must be used to catch the ball-bearing so it doesn't roll off / damage the surface
- The tall clamp stand needs to be attached to a surface with a G clamp so it stays rigid

Your notes

Worked Example

A student investigates the relationship between the height that a ball-bearing is dropped between two light gates and the time taken for it to drop.

Height h/m	Time t_1/s	Time t_2/s	Time t_3/s	Average time t/s
0.10	0.14	0.15	0.14	
0.20	0.18	0.20	0.21	
0.30	0.25	0.25	0.26	
0.40	0.29	0.28	0.30	
0.50	0.32	0.31	0.32	
0.60	0.34	0.35	0.34	

Calculate the value of g from the table.

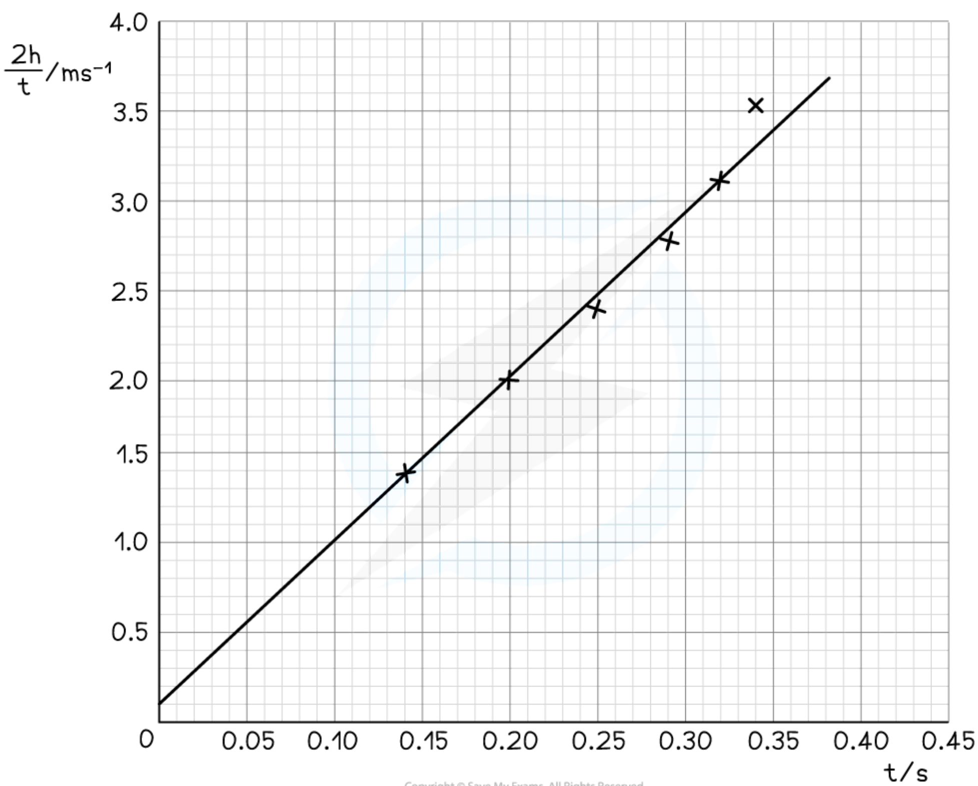
Answer:

Step 1: Complete the table

- Calculate the average time for each height
- Add an extra column 2h / t

Height h/m	Time t_1/s	Time t_2/s	Time t_3/s	Average time t/s	$\frac{2h}{t}$/ms^{-1}
0.10	0.14	0.15	0.14	0.14	1.43
0.20	0.18	0.20	0.21	0.20	2.00
0.30	0.25	0.25	0.26	0.25	2.40
0.40	0.29	0.28	0.30	0.29	2.76
0.50	0.32	0.31	0.32	0.32	3.13
0.60	0.34	0.35	0.34	0.34	3.53

Step 2: Draw graph of 2h/t against time t

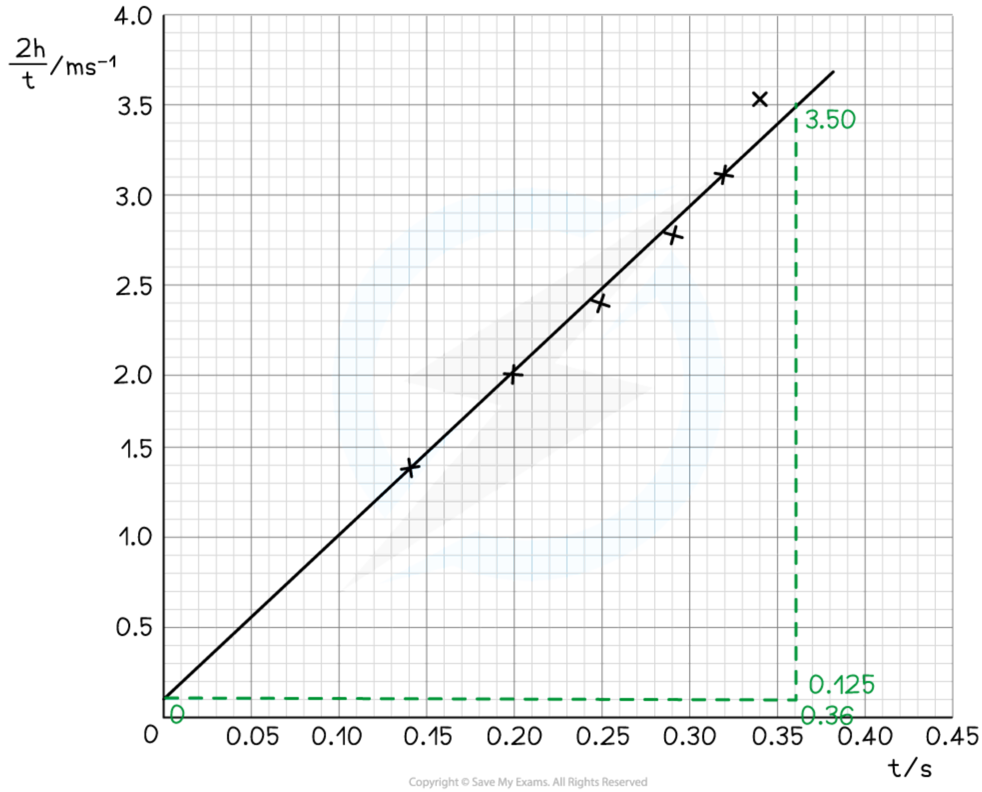


Your notes

Your notes

- Make sure the axes are properly labelled and the line of best fit is drawn with a ruler

Step 3: Calculate the gradient of the graph



- The gradient is calculated by:

$$g = \frac{3.50 - 0.125}{0.36 - 0} = 9.375 = \mathbf{9.38\ m\ s^{-2}}$$

Your notes

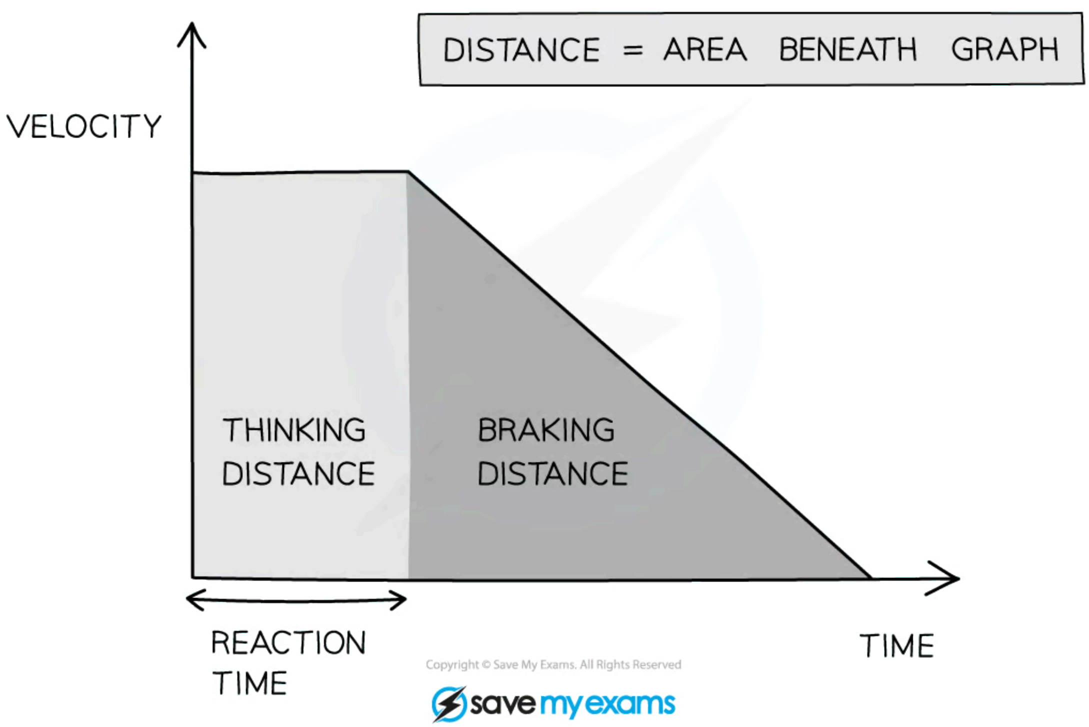
Braking & Reaction Times

Braking & Reaction Times

- The **stopping** distance of a car is the distance it travels in the time it takes to stop in response to an emergency
- The stopping distance consists of two parts:
  - The **thinking** distance
  - The **braking** distance
- Therefore, the stopping distance is equal to:

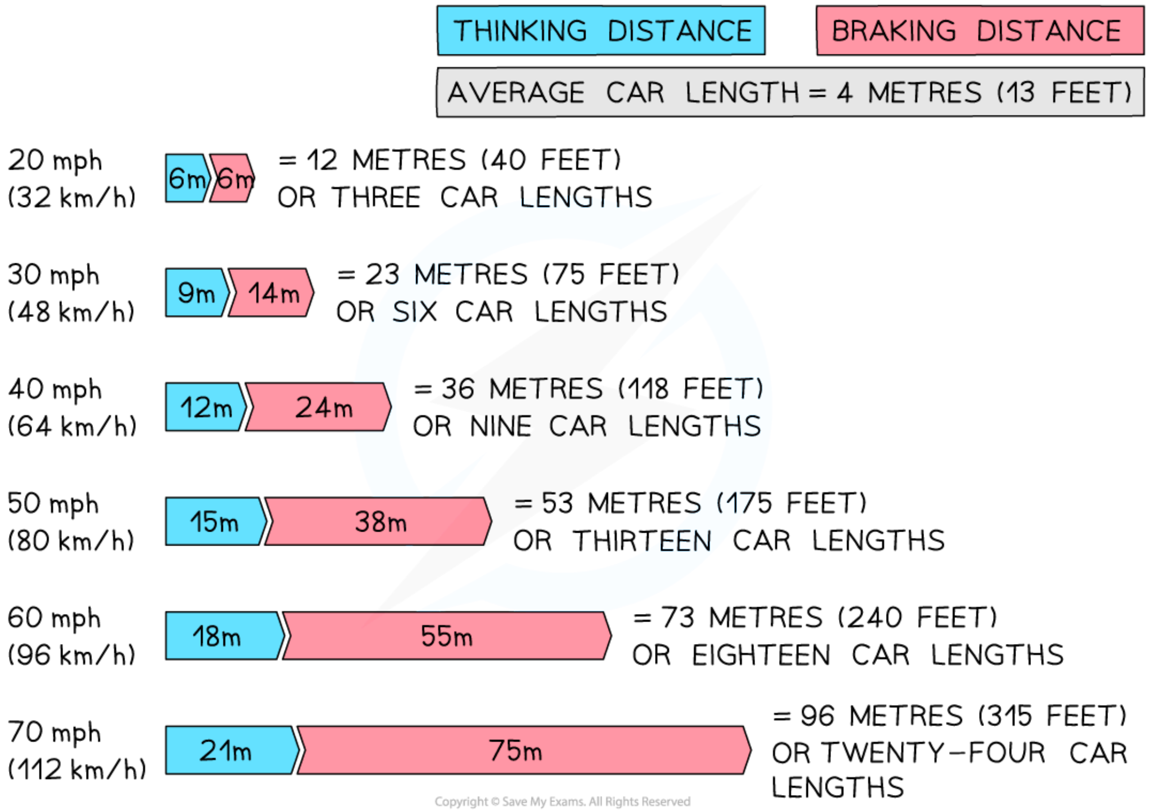
Stopping distance = Thinking distance + Braking distance

- The graph below shows how the velocity of a car will typically change during an emergency stop



A velocity-time graph for a stopping car showing thinking and braking distance

- The stopping distance increases considerably with the speed of a car
    - This is mostly because a car will travel further whilst braking before coming to rest



Estimate of stopping distances for various speeds

- The image illustrates the following important principles:
    - The thinking distance **increases proportionally** with speed (i.e. if speed **doubles**, the thinking distance **doubles**)
    - The braking distance increases at an **even faster rate** with speed
- For a typical family car, these speeds and stopping distances are summarised in the table below:

Table of Stopping Distances for a Family Car

Your notes

Speed (mph)	Speed (m/s)	Stopping Distance (m)
20	9	12
30	14	23
40	18	36
50	22	53
60	27	73
70	31	96

Your notes

Thinking Distance

- Thinking distance is defined as:

  The distance travelled by the vehicle from when the driver sees a problem and the brakes are applied

- The thinking distance is proportional to the initial speed, *u*, of the vehicle
- The thinking distance can be calculated from:

Thinking distance = Initial speed × Reaction time

- Where:
  - Reaction time = the time taken by the driver to respond to the problem
- Factors that affect thinking distance are:
  - The initial speed of the vehicle
  - Intoxication ie. consumption of alcohol or drugs
  - Distractions ie. using a mobile phone
  - Tiredness (since reaction times increase when tired)

Braking Distance

- Braking distance is defined as:

  The distance travelled by the vehicle after the driver has applied the brake

- The braking distance of the vehicle is proportional to the square of the initial speed of the car, u^2
- This is because all of the vehicle's kinetic energy ($\frac{1}{2}mv^2$) must be dissipated by the brakes in order to come to a stop
- The work done by the brakes is given by:

$$\textbf{Work Done = Braking Force × Braking Distance} = \tfrac{1}{2}mv^2$$

- Factors that affect braking distance are:
  - The initial speed of the vehicle
  - Mass of the vehicle
  - Poor road conditions eg. icy, wet
  - Car conditions eg. worn brakes

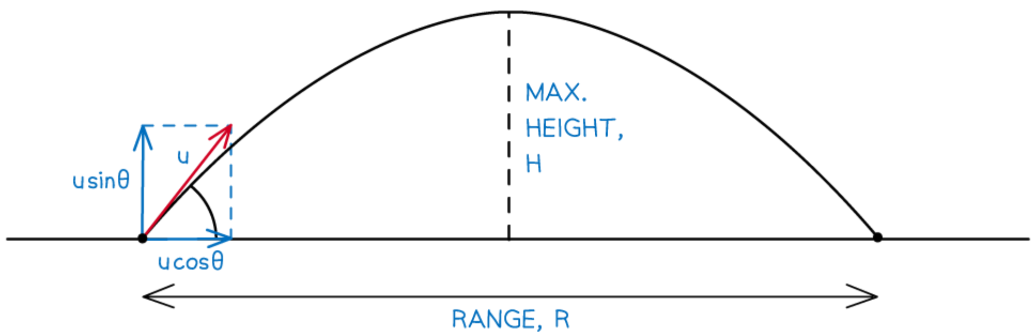
Your notes

Projectile Motion

Your notes

Vertical & Horizontal Motion of a Projectile

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance travelled by the projectile



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ \text{ms}^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \dfrac{u\sin\theta}{g}$

$2t = \dfrac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

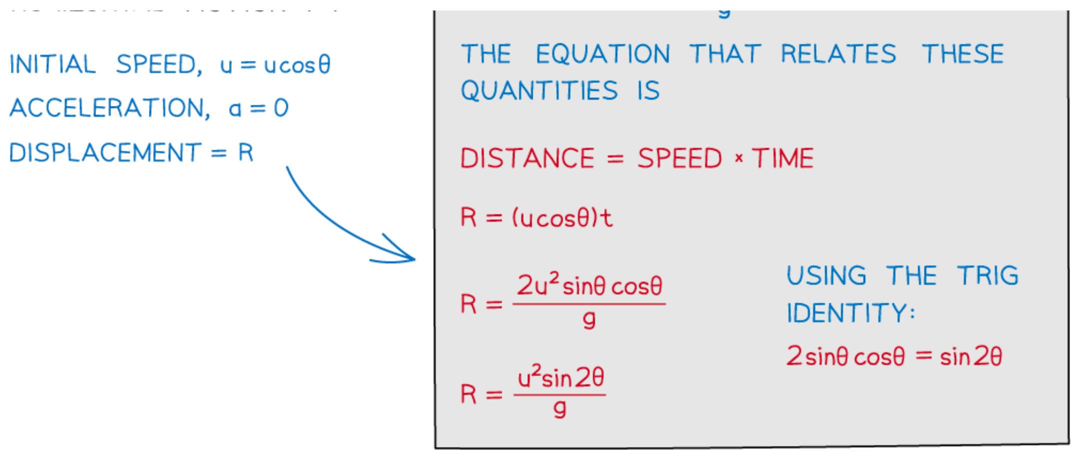
$2gH = (u\sin\theta)^2$

$H = \dfrac{(u\sin\theta)^2}{2g}$

HORIZONTAL MOTION (→)

RANGE (R)

$u = u\cos\theta \quad t = \dfrac{2u\sin\theta}{g} \quad a = 0 \quad R = ?$



How to find the time of flight, maximum height and range

Your notes

Problems Involving Projectile Motion

- There are two main considerations for solving problems involving two-dimensional motion of a projectile
  - Constant velocity in the horizontal direction
  - Constant acceleration in a perpendicular direction
- The only force acting on the projectile, after it has been released, is **gravity**

- There are three possible scenarios for projectile motion:
  - **Vertical** projection
  - **Horizontal** projection
  - **Projection** at an **angle**

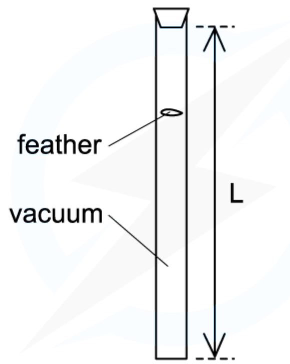
Worked Example

To calculate vertical projection (free fall)

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.

Your notes



What is the length of the tube, L?

Answer:

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81 \text{ ms}^{-2}$ $\quad u = 0$ $\quad t = 0.5 \text{ s}$ $\quad L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2 \text{ m}$$

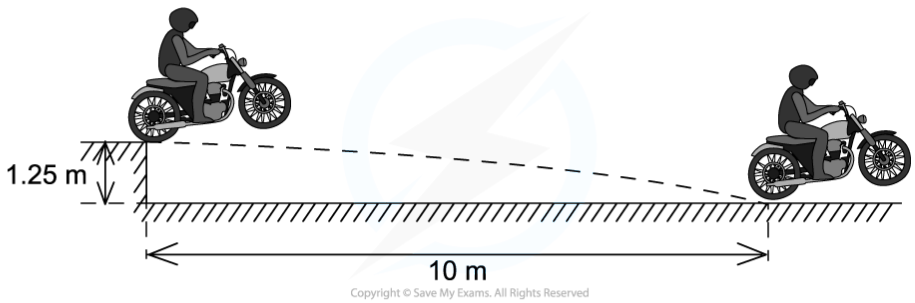
Worked Example

To calculate horizontal projection

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off?

Your notes



Answer:

IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $a = 9.81$ ms^{-2} $u = 0$ $t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $a = 0$ $t = 0.5$ s $u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20 \text{ ms}^{-1}$$

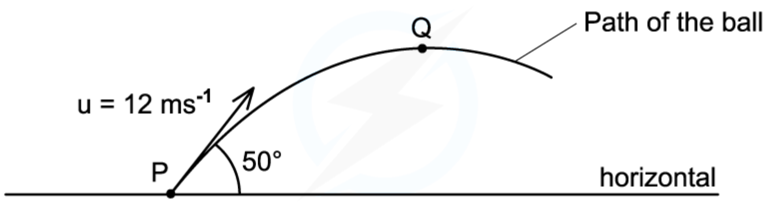
Your notes

Worked Example

To calculate projection at an angle

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q?



Answer:

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $\quad a = -9.81\ \text{ms}^{-2}$ $\quad v = 0$ $\quad H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ \text{m}$$

Examiner Tips and Tricks

Make sure you don't make these common mistakes:

- Forgetting that deceleration is negative as the object rises
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres