

OCR A Level Physics



Newton's Laws of Motion & Momentum

Contents

- * Newton's Three Laws of Motion
- * Linear Momentum
- * Impulse
- * Impulse on a Force-Time Graph
- * Conservation of Momentum
- * Collisions



Newton's Three Laws of Motion

Your notes

Newton's Three Laws of Motion

Newton's First Law

Newton's First Law states:

A body will remain at rest or move with constant velocity unless acted on by a resultant force

- If the forces on a body are balanced (the resultant force is 0), the body must be either:
 - At rest
 - Moving at a constant velocity
- Since force is a vector, it is easier to split the forces into **horizontal** and **vertical** forces
- If the forces are balanced:
 - The forces to the left = the forces to the right
 - The forces up = the forces down
- The resultant force is the single force obtained by combining all the forces on the body



Worked Example

If there are no external forces acting on the car, other than friction, and it is moving at a constant velocity, what is the value of the frictional force F?



Answer:

SINCE THE CAR IS MOVING AT CONSTANT VELOCITY, THERE IS NO RESULTANT FORCE.

Your notes

THIS MEANS THE DRIVING AND FRICTIONAL FORCES ARE BALANCED.

F IS ALSO EQUAL TO 6 kN

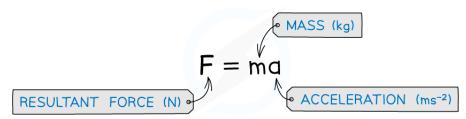
Copyright © Save My Exams. All Rights Reserve

Newton's Second Law

Newton's Second Law states:

The resultant force is equal to the rate of change in momentum. The change in momentum is in the same direction as the resultant force

■ This can also be written as:



Copyright © Save My Exams. All Rights Reserved

- This relationship means that objects will accelerate if there is a resultant force acting upon them
- This is derived from the definition of momentum as follows:

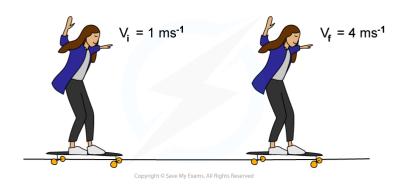
- An unbalanced force on a body means it experiences a resultant force
 - If the resultant force is along the direction of motion, it will speed up (accelerate) or slow down (decelerate) the body
 - If the resultant force is at an angle, it will change the direction of the body





Worked Example

A girl is riding her skateboard down the road and increases her speed from $1\,\mathrm{m\,s^{-1}}$ to $4\,\mathrm{m\,s^{-1}}$ in 2.5 s. If the force driving her forward is 72 N, calculate the combined mass of the girl and the skateboard.



Answer:



STEP 1 NEWTON'S SECOND LAW STATES THE RESULTANT FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM $F = \frac{\Delta P}{\Delta t}$

STEP 2 FIND CHANGE IN MOMENTUM
$$\Delta p$$

$$\Delta p = FINAL MOMENTUM - INITIAL MOMENTUM$$

$$\Delta p = mv_f - mv_i$$

STEP 3 SUBSTITUTE ALL VALUES INTO NEWTON'S SECOND LAW
$$72\,\text{N} = \frac{\text{m}(4-1)}{2.5}$$
 MASS m IS CONSTANT SO CAN BE TAKEN OUT AS FACTOR

STEP 4 REARRANGE FOR MASS m $m = \frac{72 \times 2.5}{3} = 60 \text{ kg}$

opyright © Save My Exams. All Rights Reserve



Examiner Tips and Tricks

The direction you consider positive is your choice, as long as the signs of the numbers (positive or negative) are consistent throughout the question. It is a general rule to consider the direction the object is initially travelling in as positive. Therefore all vectors in the direction of motion will be positive and opposing vectors, such as drag forces, will be negative.

Newton's Third Law

Newton's Third Law states:

If body A exerts a force on body B, then body B will exert a force on body A of equal magnitude but in the opposite direction

- This means that every force has a paired equal and opposite force
 - Newton's Third Law force pairs must act on two **different** objects
 - Newton's Third Law force pairs must also be of the **same type** e.g. gravitational or frictional

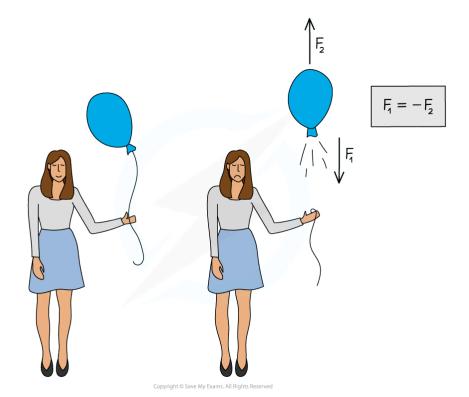




Worked Example

Using Newton's third law, describe why when a balloon is untied, it travels in the opposite direction.

Answer:



THE AIR INSIDE THE BALLOON WILL RUSH OUT WITH THE FORCE $\mathbf{F_4}$.

THIS WILL PRODUCE AN EQUAL AND OPPOSITE FORCE ON THE BALLOON ${\sf F_2}$ FORCING THE BALLOON TO MOVE THROUGH THE AIR IN THE OPPOSITE DIRECTION.

Copyright © Save My Exams. All Rights Reserved







Examiner Tips and Tricks

You may have heard Newton's Third Law as: 'For every action there is an equal and opposite reaction'. However, try and avoid using this definition since it is unclear on **what** the forces are acting on and can be misleading.

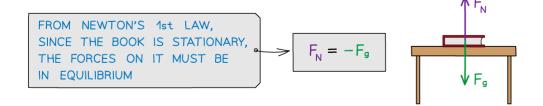






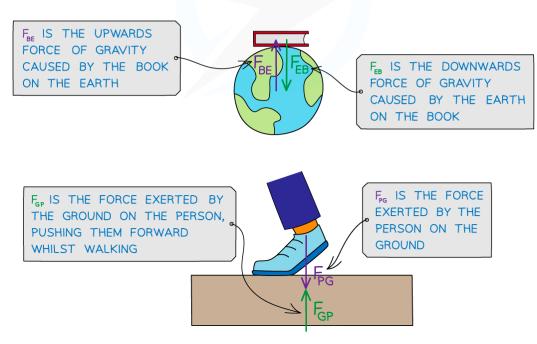
SCENARIO 1:

NOT A NEWTON'S THIRD LAW PAIR SINCE BOTH FORCES ARE ACTING ON THE **SAME** OBJECT - THE BOOK



SCENARIO 2:

THESE ARE NEWTON'S THIRD LAW PAIRS SINCE BOTH FORCES ARE ACTING ON DIFFERENT OBJECTS



Copyright © Save My Exams. All Rights Reserved



Newton's Third Law force pairs are only those that act on different objects





Linear Momentum

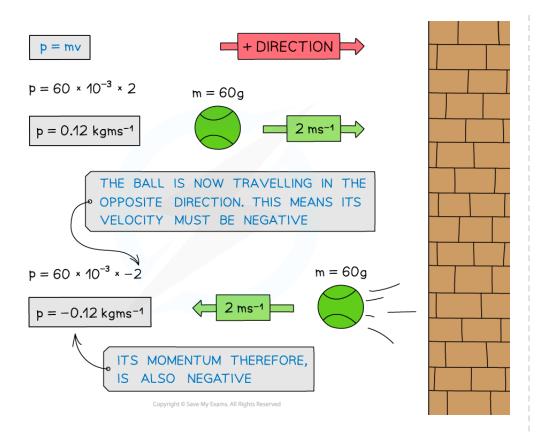
Your notes

Linear Momentum

- When an object with **mass** is in motion and therefore has a **velocity**, the object also has **momentum**
- Linear momentum is the momentum of an object that is moving in only one dimension
- The linear momentum of an object remains **constant** unless the system is acted upon by an external resultant force
- Momentum is defined as the product of mass and velocity

$$p = mv$$

- Where:
 - $p = momentum, measured in kg m s^{-1}$
 - \blacksquare m mass, measured in kg
 - V = velocity, measured in m s⁻¹
- Momentum is a **vector** quantity with both **magnitude** and **direction**
 - The initial direction of motion is usually assigned the positive direction





When the ball is travelling in the opposite direction, its velocity is negative. Since momentum = mass \times velocity, its momentum is also negative



Worked Example

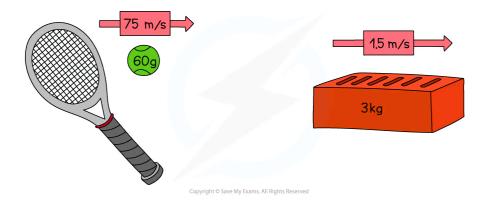
A tennis ball of mass 60 g travels to the right with a speed of 75 m s^{-1} .

A brick of mass 3 kg is thrown to the right at a speed of $1.5 \,\mathrm{m \, s^{-1}}$.

Determine which object has the greatest momentum.



Head to www.savemyexams.com for more awesome resources





Answer:

Copyright © Save My Exams. All Rights Reserve

- Both the tennis ball and the brick have the same momentum
- Even though the brick is much heavier than the ball, the ball is travelling much faster than the brick
- This means that on impact, they would both exert a similar force (depending on the time it takes for each to come to rest)



Examiner Tips and Tricks

Since the SI units for momentum are kg m s⁻¹:

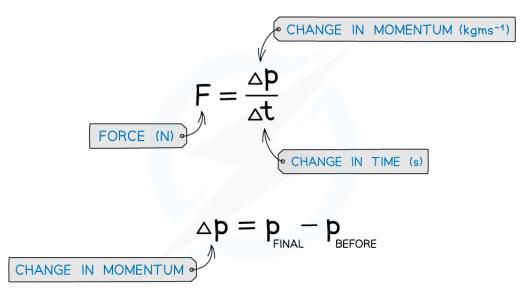
- If the mass is given in grams, you need to convert to kg by dividing the value by 1000
- If the velocity is given in km s⁻¹, you need to convert to m s⁻¹ by multiplying the value by 1000
- The direction you consider positive is your choice, as long the signs of the numbers (positive or negative) are consistent with this throughout the question

Impulse

Your notes

Force & Momentum

- Force is defined as the rate of change of momentum on a body
- The change in momentum is defined as the final momentum minus the initial momentum
- These can be expressed as follows:



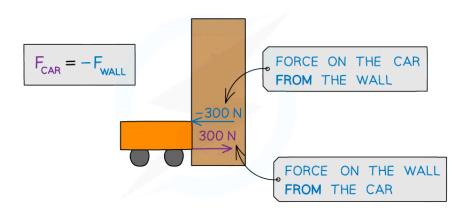
Copyright © Save My Exams. All Rights Reserved

Direction of Forces

- Force and momentum are **vectors** so they can take either positive or negative values
- The force that is equal to the rate of change of momentum is still the **resultant force**
- A force on an object will be negative if it is directed in the opposite motion to its initial velocity
 - This means that the force is **produced by** the object it has collided with



Head to www.savemyexams.com for more awesome resources





The wall produces a force of -300N on the car and (due to Newton's Third Law) the car also produces a force of 300 N back onto the wall



Worked Example

A car of mass 1500 kg hits a wall at an initial velocity of $15\,\mathrm{m\,s^{-1}}$.

It then rebounds off the wall at $5 \,\mathrm{m\,s^{-1}}$ and comes to rest after 3.0 s.

Calculate the average force experienced by the car.

Answer:

STEP 1 FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM

$$F = \frac{\Delta p}{\Delta t}$$

STEP 2 CHANGE IN MOMENTUM

Δp = FINAL MOMENTUM - INITIAL MOMENTUM

STEP 3 INITIAL MOMENTUM

INITIAL MOMENTUM = MASS x INITIAL VELOCITY

$$P_i = m \times v_i$$

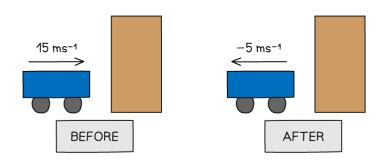
= 1500 kg × 15 ms⁻¹

 $P_i = 22500 \, \text{kgms}^{-1}$

Copyright © Save My Exams. All Rights Reserved







STEP 4 FINAL MOMENTUM

FINAL MOMENTUM = MASS × FINAL VELOCITY

$$P_f = m \times v_f$$

$$= 1500 \text{ kg} \times -5 \text{ ms}^{-1}$$

$$P_f = -7500 \text{ kgms}^{-1}$$

STEP 5 CALCULATE CHANGE IN MOMENTUM
$$\Delta p$$

$$\Delta p = -7500 - 22500 = -30000 \text{ kgms}^{-1}$$

STEP 6 SUBSTITUTE THIS VALUE BACK INTO THE FORCE EQUATION
$$F = \frac{\Delta p}{\Delta t} = \frac{-30000}{3} = -10000N$$

Copyright © Save My Exams. All Rights Reserve



Examiner Tips and Tricks

In an exam question, carefully consider what produces the force(s) acting. Look out for words such as '**from'** or '**acting on'** to determine this and don't be afraid to draw a force diagram to figure out what is going on.

Impulse

- The force and momentum equation can be rearranged to find the impulse
- Impulse, *I*, is equal to the **change in momentum**:



$I = F\Delta t = \Delta p = mv - mu$



- *l* = impulse (N s)
- F = force(N)
- *t* = time (s)
- $p = momentum (kg m s^{-1})$
- *m* = mass (kg)
- $v = \text{final velocity (m s}^{-1})$
- u = initial velocity (m s⁻¹)
- This equation is only used when the force is **constant**
 - Since the impulse is proportional to the force, it is also a vector
 - The impulse is in the same direction as the force
- The unit of impulse is **N s**
- The impulse quantifies the effect of a force acting over a time interval
 - This means a small force acting over a long time has the same effect as a large force acting over a short time

Examples of Impulse

- An example in everyday life of impulse is when standing under an umbrella when it is raining, compared to hail (frozen water droplets)
 - When rain hits an umbrella, the water droplets tend to splatter and fall off it and there is only a very small change in momentum
 - However, hailstones have a larger mass and tend to bounce back off the umbrella, creating a greater change in momentum
 - Therefore, the impulse on an umbrella is **greater** in hail than in rain
 - This means that **more force** is required to hold an umbrella upright in hail compared to rain









Since hailstones bounce back off an umbrella, compared to water droplets from rain, there is a greater impulse on an umbrella in hail than in rain



Worked Example

A 58 g tennis ball moving horizontally to the left at a speed of 30 m s $^{-1}$ is struck by a tennis racket which returns the ball back to the right at 20 m s $^{-1}$.

- (i) Calculate the impulse delivered to the ball by the racket
- (ii) State which direction the impulse is in

Answer:

Part (i)

Step 1: Write the known quantities



 $Head to \underline{www.savemyexams.com} for more awe some resources$

Taking the initial direction of the ball as positive (the left)



■ Final velocity, $v = -20 \text{ m s}^{-1}$

• Mass, $m = 58 \text{ g} = 58 \times 10^{-3} \text{ kg}$

Step 2: Write down the impulse equation

Impulse
$$I = \Delta p = m(v - u)$$

Step 3: Substitute in the values

$$I = (58 \times 10^{-3}) \times (-20 - 30) = -2.9 \text{ N s}$$

Part (ii)

Direction of the impulse

- Since the impulse is negative, it must be in the opposite direction to which the tennis ball was initial travelling (since the left is taken as positive)
 - Therefore, the direction of the impulse is to the right



Examiner Tips and Tricks

Remember that if an object changes direction, then this must be reflected by the change in sign of the velocity. As long as the magnitude is correct, the final sign for the impulse doesn't matter as long as it is consistent with which way you have considered positive (and negative). For example, if the left is taken as positive and therefore the right as negative, an impulse of $20\,\mathrm{N}\,\mathrm{s}$ to the right is equal to $-20\,\mathrm{N}\,\mathrm{s}$

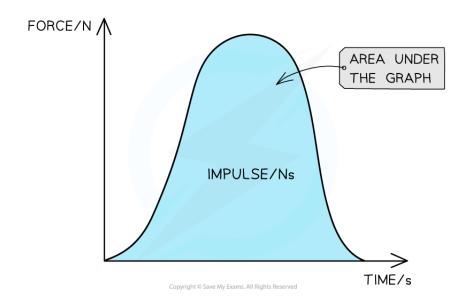


Impulse on a Force-Time Graph

Your notes

Impulse on a Force-Time Graph

- In real life, forces are often not constant and will vary over time
- If the force is plotted against time, the impulse is equal to the area under the force-time graph



When the force is not constant, the impulse is the area under a force-time graph

This is because

Impulse = $F\Delta t$

- Where:
 - F = force(N)
 - $\Delta t = \text{change in time (s)}$
- The impulse is therefore equal whether there is
 - A small force over a long period of time
 - A large force over a small period of time
- The force-time graph may be a **curve** or a **straight line**
 - If the graph is a curve, the area can be found by counting the squares underneath

If the graph is made up of straight lines, split the graph into sections. The total area is the sum of the areas of each section



$$\mathsf{F} \! \downarrow = \frac{\mathsf{\Delta} \mathsf{p}}{\mathsf{\Delta} \mathsf{t} \! \uparrow}$$

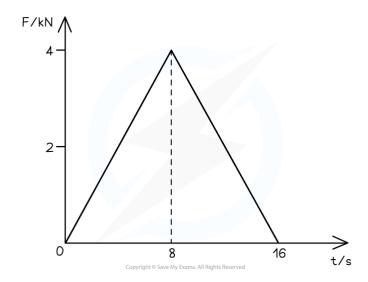
THE SAME CHANGE IN MOMENTUM OVER A LONGER PERIOD OF TIME WILL PRODUCE LESS FORCE (AND VICE VERSA)

Copyright © Save My Exams. All Rights Reserved



Worked Example

A ball of mass 3.0 kg, initially at rest, is acted on by a force F which varies with t as shown by the graph.



Calculate the velocity of the ball after 16 s.

Answer:

Step 1: List the known quantities

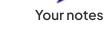
- Mass, m = 3.0 kg
- Initial velocity, $u = 0 \text{ m s}^{-1}$ (since it is initially at rest)

Step 2: Calculate the impulse



- The impulse is the area under the graph
- The graph can be split up into two right-angled triangles with a base of 8 s and a height of 4 kN

Area =
$$(\frac{1}{2} \times 8 \times (4 \times 10^3)) + (\frac{1}{2} \times (16 - 8) \times (4 \times 10^3))$$



Area = Impulse = 32×10^3 N s

Step 3: Write the equation for impulse

Impulse,
$$I = \Delta p = m(v - u)$$

Step 4: Substitute in the values

$$I = mv$$

$$32 \times 10^{3} = 3.0 \times v$$

$$v = (32 \times 10^{3}) \div 3.0$$

$$v = 10666 \text{ m s}^{-1} = 11 \text{ km s}^{-1}$$



Examiner Tips and Tricks

Some maths tips for this section: Rate of Change

- 'Rate of change' describes how one variable changes with respect to another
- In maths, how fast something changes with **time** is represented as dividing by **\Delta t** (e.g. acceleration is the rate of change in velocity)
- More specifically, **\Delta t** is used for finite and quantifiable changes such as the difference in time between two events

Areas

• The area under a graph may be split up into different shapes, so make sure you're comfortable with calculating the area of squares, rectangles, right-angled triangles and trapeziums!

Conservation of Momentum

Your notes

The Principle of Conservation of Momentum

• The principle of conservation of linear momentum states:

The total momentum before a collision is equal to the total momentum after a collision, provided no external force acts

Therefore:

momentum before = momentum after

- Momentum is a **vector** quantity, therefore:
 - opposing vectors can cancel each other out, resulting in a net momentum of zero
 - an object that collides with another object and rebounds, has a positive velocity before the collision and a negative velocity after
- Momentum, just like energy, is always conserved
- If objects A and B collide, their momenta before and after are related by the following equation:

$$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$$

- Where:
 - p_{Ai} = initial momentum of A, measured in kg m s⁻¹
 - p_{Bi} = initial momentum of B, measured in kg m s⁻¹
 - p_{Af} = final momentum of A, measured in kg m s⁻¹
 - p_{Bf} = final momentum of B, measured in kg m s⁻¹

Conservation of momentum example: collision

- $\,\blacksquare\,$ Ball A moves with an initial velocity of u_A
- Ball A collides with Ball B which is stationary
- After the collision, both balls travel in opposite directions
 - Taking the direction of the initial motion of Ball A as the positive direction (to the right)

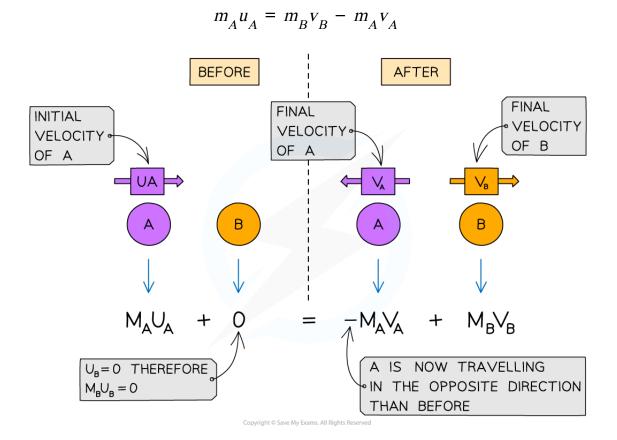
■ The total momentum before the collision is

$$p_{before} = m_A u_A + 0$$



$$p_{after} = -m_A^{} V_A^{} + m_B^{} V_B^{}$$

- The minus sign shows that Ball A travels in the **opposite** direction to the initial travel
- If an object is stationary like Ball B is before the collision, then it has a momentum of **0**
- From the conservation of momentum, one can equate these expressions



The conservation of momentum for two objects A and B colliding then moving apart

External and Internal Forces

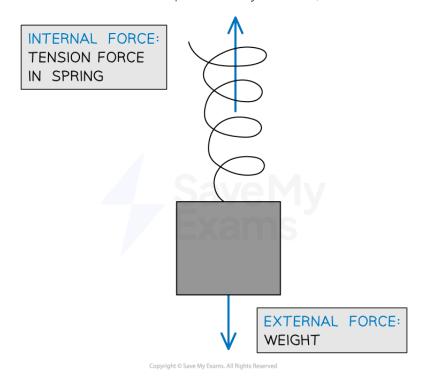
Note that the definition of the law of conservation of momentum states that it only applies when no external forces act





- External forces are forces that act on a structure or system from outside e.g. friction and weight
- Internal forces are forces exchanged by the particles in the system e.g. tension in a string
- Forces which are internal or external will depend on the system itself, as shown in the diagram below:





Internal and external forces on a mass on a spring

- Systems with no external forces may be described as 'closed' or 'isolated'
 - These are keywords that refer to a system that is not affected by external forces
 - In these systems, total momentum is conserved
- For example, a swimmer diving from a boat:
 - The diver will move **forwards**, and, to conserve momentum, the boat will move **backwards**
 - This is because the momentum beforehand was zero and no external forces were present to affect the horizontal motion of the diver or the boat



Worked Example



Head to www.savemyexams.com for more awesome resources

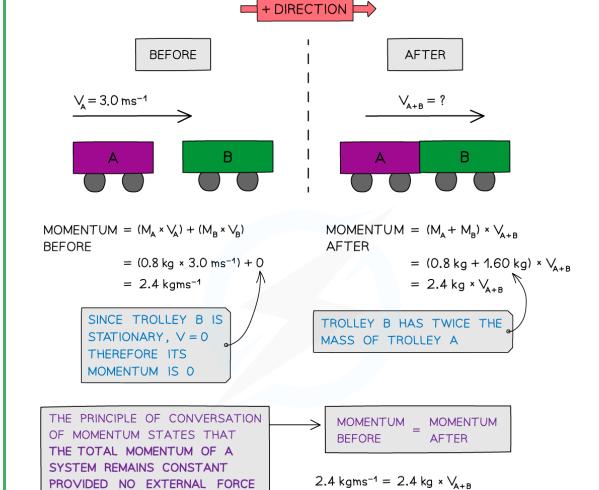
Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** whilst travelling at 3.0 m s⁻¹.

Trolley **B** has twice the mass of trolley **A**. On impact, the trolleys stick together.

Using the conservation of momentum, calculate the common velocity of both trolleys after the collision.

Answer:

ACTS ON IT





 $V_{A+B} = 1.0 \text{ ms}^{-1}$

 $V_{A+B} = \frac{2.4 \text{ kgms}^{-1}}{2.4 \text{ kg}}$ REARRANGING FOR V_{A+B}



Collisions



Collisions in One & Two Dimensions

One-dimensional Problems

- Momentum (p) is equal to: $p = m \times v$
- One-dimensional momentum problems are when collisions are taken place in just the x (horizontal) or just the y (vertical) direction
- Using the conservation of linear momentum, it is possible to calculate missing velocities and masses of components in the system
- This is shown in the example below:

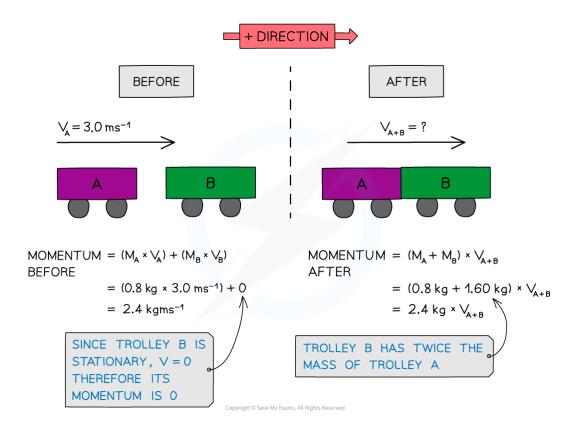


Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** at a velocity of $3.0 \,\mathrm{ms}^{-1}$. Trolley **B** has twice the mass of trolley **A**. The trolleys stick together. Using the conservation of momentum, calculate the common velocity of both trolleys after the collision. Determine whether this is an elastic or inelastic collision.

Answer:







THE PRINCIPLE OF CONVERSATION
OF MOMENTUM STATES THAT
THE TOTAL MOMENTUM OF A
SYSTEM REMAINS CONSTANT
PROVIDED NO EXTERNAL FORCE
ACTS ON IT

MOMENTUM = MOMENTUM AFTER

 $2.4 \text{ kgms}^{-1} = 2.4 \text{ kg} \times V_{A+B}$

$$V_{A+B} = \frac{2.4 \text{ kgms}^{-1}}{2.4 \text{ kg}}$$
 REARRANGING FOR V_{A+B}

 $V_{A+B} = 1.0 \text{ ms}^{-1}$

b) IS THIS AN ELASTIC OR INELASTIC COLLISION?

KINETIC ENERGY =
$$1/2 \text{ mv}^2$$

ENERGY
$$= \frac{1}{2} \times M_A \times (V_A)^2 + \frac{1}{2} \times M_B \times (V_B^2)$$

$$= \frac{1}{2} \times 0.8 \times (3.0)^2 + 0$$

$$= 3.6 \text{ J}$$

KINETIC ENERGY
$$= \frac{1}{2} \times M_{A+B} \times (\bigvee_{A+B})^{2}$$
$$= \frac{1}{2} \times 2.4 \times (1.0)^{2}$$

 $= \frac{1}{2} \times M_{A+B} \times (V_{A+B})^{2}$ $= \frac{1}{2} \times 2.4 \times (1.0)^{2}$ THIS IS AN INELASTIC
COLLISION SINCE KINETIC
ENERGY IS **NOT** CONSERVED

Converight © Savo My Evame All Digi

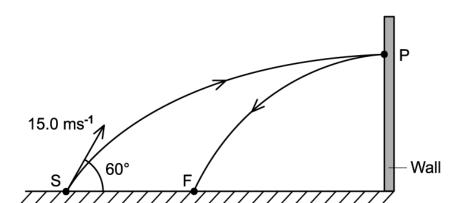
Two-dimensional Momentum Problems

- Since momentum is a vector, in 2D it can be split up into its x and y components
- Two-dimensional momentum problems are when collisions are taken place in both the x (horizontal) and the y (vertical) direction
- Using the conversation of linear momentum as well as resolving vectors, it is possible to calculate changes in momentum
- This is shown in the example below:



Worked Example

A ball is thrown at a vertical wall. The path of the ball is shown below





The ball is thrown from **S** with an initial velocity of 15.0 m s⁻¹ at 60.0° to the horizontal. The mass of the ball is 60×10^{-3} kg and rebounds at a velocity of 4.6 m s⁻¹. Calculate the change in momentum of the ball if it rebounds off the wall at **P**.

Answer:

STEP 1 CHANGE IN MOMENTUM EQUATION $\Delta P = m(\bigvee_f - \bigvee_i)$

CALCULATE INITIAL VELOCITY

CHANGE IN MOMENTUM IS ONLY DUE TO THE HORIZONTAL VELOCITIES

V_i = 15.0 cos(60.0) = 7.5 ms⁻¹

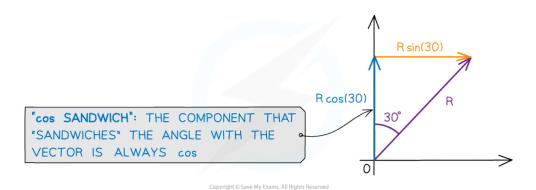
STEP 3 SUBSTITUTE VALUES INTO $\triangle P$ EQUATION $\triangle P = 60 \times 10^{-3} (-4.6 - 7.5) = -0.73 \text{ Ns}$ NEGATIVE BECAUSE THE BALL IS NOW TRAVELLING IN THE OPPOSITE DIRECTION TO ITS INITIAL VELOCITY

Copyright © Save My Exams. All Rights Reserve



Examiner Tips and Tricks

If an object is stationary or at rest, it's velocity equals **0**, therefore, the momentum and kinetic energy are also equal to 0. When a collision occurs in which two objects are stuck together, treat the final object as a single object with a mass equal to the **sum** of the two individual objects. In 2D problems, make sure you're confident resolving vectors. Here is a small trick to remember which component is cosine or sine of the angle for a vector **R**:



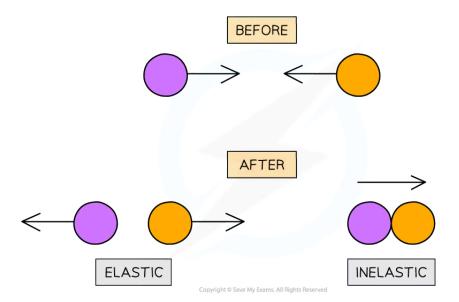
Resolving vectors with sine and cosine

Elastic & Inelastic Collisions

- In both collisions and explosions, momentum is always conserved
- However, **kinetic energy** might not always be
- A collision (or explosion) is either:
 - Elastic if the kinetic energy is conserved
 - Inelastic if the kinetic energy is not conserved
- Collisions are when objects striking against each other
 - Elastic collisions are commonly those where objects colliding do not stick together and then move in opposite directions
 - Inelastic collision are commonly those where objects collide and stick together after the collision



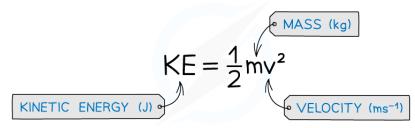






Elastic collisions are where two objects move in opposite directions. Inelastic collisions are where two objects stick together

- An explosion is commonly to do with **recoil**
 - For example, a gun recoiling after shooting a bullet or an unstable nucleus emitting an alpha particle and a daughter nucleus
- To find out whether a collision is elastic or inelastic, compare the kinetic energy before and after the collision
- The equation for kinetic energy is:



Copyright © Save My Exams. All Rights Reserved

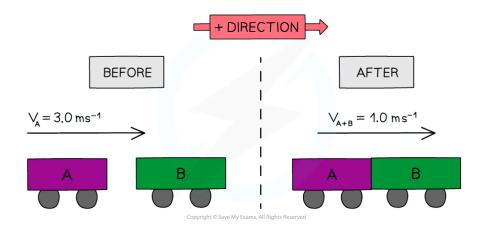


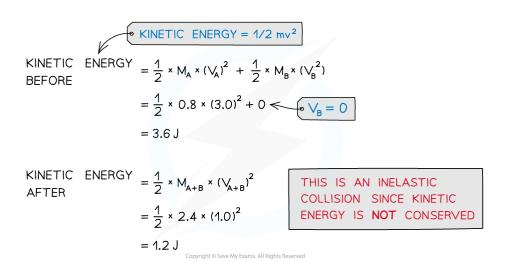
Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** at speed 3.0 m s⁻¹. Trolley **B** has twice the mass of trolley **A**. The trolleys stick together and travel at a velocity of 1.0 m s⁻¹. Determine whether this is an elastic or inelastic collision.

Your notes

Answer:







Examiner Tips and Tricks



If an object is stationary or at rest, it's velocity equals **0**, therefore, the momentum and kinetic energy are also equal to 0. When a collision occurs in which two objects are stuck together, treat the final object as a single object with a mass equal to the **sum** of the two individual objects.

