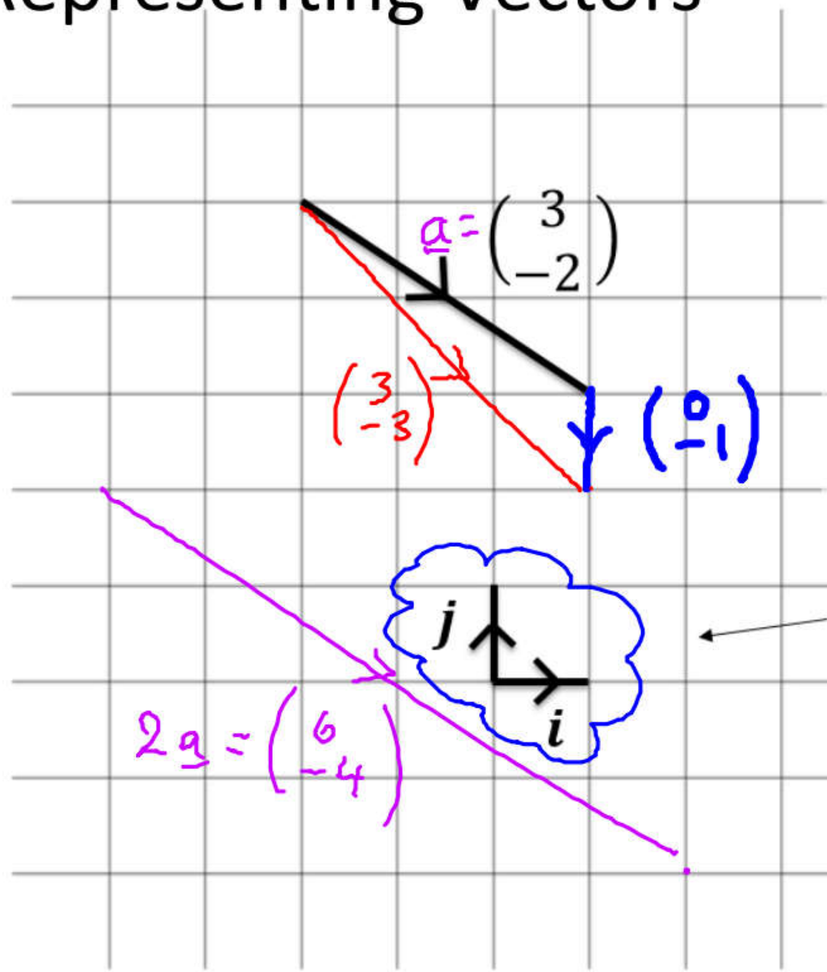
Representing Vectors



You should already be familiar that the value of a vector is the **displacement** in the x and y direction (if in 2D).

$$\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 3 \\ -3 \end{pmatrix}}$$

$$2\boldsymbol{a} = 2\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

✏ A **unit vector** is a vector of magnitude 1.
$\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit vectors in the x-axis and y-axis respectively.

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

e.g. $\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 4\boldsymbol{i} + 3\boldsymbol{j}$

If $\boldsymbol{a} = 3\boldsymbol{i}, \quad \boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j}, \quad \boldsymbol{c} = \boldsymbol{i} - 2\boldsymbol{j}$ then:

$\underline{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \underline{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

1) Write $\boldsymbol{a}$ in vector form.
2) Find $\boldsymbol{b} + 2\boldsymbol{c}$ in $\boldsymbol{i}, \boldsymbol{j}$ form.

$$\underline{b} + 2\underline{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} = 3\underline{i} - 3\underline{j}$$

Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

Your Turn

a λ if $\boxed{\mathbf{c} + \lambda\mathbf{d}}$ is parallel to $\mathbf{i} + \mathbf{j}$

b μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$

d t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

λ lambda

parallel means "is a multiple of"

$$\underline{c} + \lambda \underline{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} \lambda \\ -2\lambda \end{pmatrix}$$

$$\underline{c} + \lambda \underline{d} = \begin{pmatrix} 3 + \lambda \\ 4 - 2\lambda \end{pmatrix}$$

$$\begin{pmatrix} 3 + \lambda \\ 4 - 2\lambda \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 + \lambda \\ 4 - 2\lambda \end{pmatrix} = \begin{pmatrix} x \\ x \end{pmatrix}$$

i // $3 + \lambda = x$

j // $4 - 2\lambda = x$

$3 + \lambda = 4 - 2\lambda$

$3 + 3\lambda = 4$

$3\lambda = 1 \qquad \lambda = \frac{1}{3}$

The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$. Find:

a the value of p **(4 marks)**

b the resultant of vectors **a** and **b**. **(1 mark)**

Skip Ex 11B

Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

Your Turn

a λ if $\mathbf{c} + \lambda\mathbf{d}$ is parallel to $\mathbf{i} + \mathbf{j}$

b μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$

d t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

$$\underset{\sim}{c} - s\underset{\sim}{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - s\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3-s \\ 4+2s \end{pmatrix}$$

$$\begin{pmatrix} 3-s \\ 4+2s \end{pmatrix} = x\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3-s \\ 4+2s \end{pmatrix} = \begin{pmatrix} 2x \\ x \end{pmatrix}$$

$3 - s = 2x$

$4 + 2s = x$

$3 - s = 2(4 + 2s)$

$3 - s = 8 + 4s$

$-5 = 5s$

$\underline{s = -1}$

$\mu = mu$

μ

The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$.

Find:

→ the vector which is the result of 2 other vectors added.

a the value of p **(4 marks)**

b the resultant of vectors **a** and **b**. **(1 mark)**

Skip Ex 11B

Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

Your Turn

a λ if $\mathbf{c} + \lambda\mathbf{d}$ is parallel to $\mathbf{i} + \mathbf{j}$

b μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$

d t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

$$\mu\underline{c} + \underline{d} = \mu\begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}1\\-2\end{pmatrix}$$

$$= \begin{pmatrix}3\mu\\4\mu\end{pmatrix} + \begin{pmatrix}1\\-2\end{pmatrix}$$

$$= \begin{pmatrix}3\mu+1\\4\mu-2\end{pmatrix}$$

$$\begin{pmatrix}3\mu+1\\4\mu-2\end{pmatrix} = x\begin{pmatrix}1\\3\end{pmatrix}$$

$$\begin{pmatrix}3\mu+1\\4\mu-2\end{pmatrix} = \begin{pmatrix}x\\3x\end{pmatrix}$$

$$3\mu+1 = x$$
$$4\mu-2 = 3x$$

$$4\mu-2 = 3(3\mu+1)$$
$$4\mu-2 = 9\mu+3$$
$$-5 = 5\mu \qquad \mu = -1$$

The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$.
Find:

a the value of p **(4 marks)**

b the resultant of vectors **a** and **b**. **(1 mark)**

Skip Ex 11B

Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

Your Turn

a λ if $\mathbf{c} + \lambda\mathbf{d}$ is parallel to $\mathbf{i} + \mathbf{j}$

b μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$

d t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

$$\underline{d} - t\underline{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} - t\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} -3t \\ -4t \end{pmatrix}$$

$$\underline{d} - t\underline{c} = \begin{pmatrix} 1-3t \\ -2-4t \end{pmatrix}$$

$$\begin{pmatrix} 1-3t \\ -2-4t \end{pmatrix} = x\begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$1 - 3t = -2x$ $\quad \times -3 \quad$ $-3 + 9t = 6x$

$-2 - 4t = 3x$ $\quad \times 2 \quad$ $-4 - 8t = 6x$

$$-3 + 9t = -4 - 8t$$

$$17t = -1$$

$$t = -\frac{1}{17}$$

The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$. Find:

a the value of p **(4 marks)**

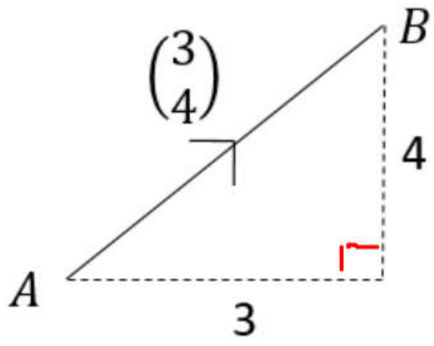
b the resultant of vectors **a** and **b**. **(1 mark)**

Skip Ex 11B

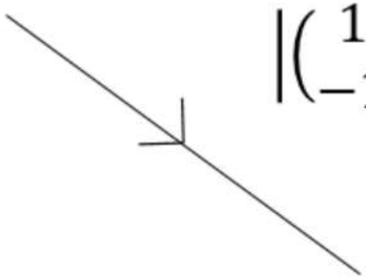
Magnitude of a Vector

The magnitude $|a|$ of a vector a is its length.

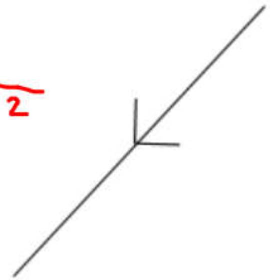
If $\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad |\boldsymbol{a}| = \sqrt{x^2 + y^2}$



$$\left|\overrightarrow{AB}\right| = \sqrt{3^2 + 4^2} = 5$$



$$\left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| = \sqrt{1^2 + 1^2}$$
$$= \sqrt{2}$$



$$\left|\begin{pmatrix} -5 \\ -12 \end{pmatrix}\right| = \sqrt{5^2 + 12^2}$$
$$= 13$$

$\boldsymbol{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $\quad |\boldsymbol{a}| = \sqrt{4^2 + 1^2} = \sqrt{17}$

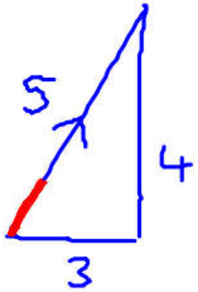
$\boldsymbol{b} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\quad |\boldsymbol{b}| = \sqrt{2^2 + 0^2} = 2$

Unit Vectors

length

A unit vector is a vector whose magnitude is 1

There's certain operations on vectors that require the vectors to be 'unit' vectors. We just scale the vector so that its magnitude is now 1.



$$\boldsymbol{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$|\underline{a}| = \sqrt{3^2 + 4^2} = 5$

$\hat{\underline{a}} = \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$

If $\boldsymbol{a}$ is a vector, then the unit vector $\hat{\boldsymbol{a}}$ in the same direction is

$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

$\frac{1}{|\underline{a}|}\ \underline{a}$

Test Your Understanding: Convert the following vectors to unit vectors.

$$\boldsymbol{a} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

$|\underline{a}| = \sqrt{12^2 + 5^2} = 13$

$\hat{\underline{a}} = \frac{1}{13}\begin{pmatrix} 12 \\ -5 \end{pmatrix} = \begin{pmatrix} 12/13 \\ -5/13 \end{pmatrix}$

$$\boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$|\underline{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

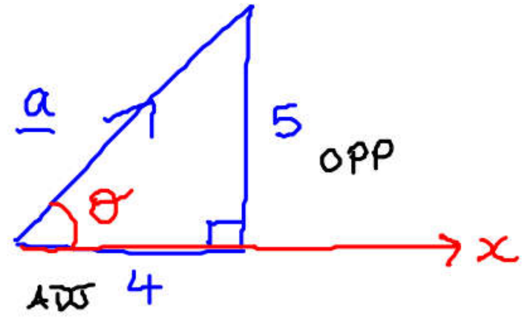
$\hat{\underline{b}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

Direction of Vectors

Find the angle between $\boldsymbol{a}$ and the positive x − axis

$$\boldsymbol{a} = \begin{pmatrix}4\\5\end{pmatrix}$$

Sketch of the vector



SOH CAH TOA

theta

$\tan\theta = \frac{5}{4}$

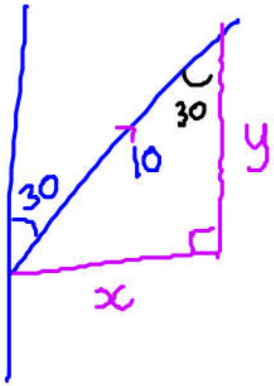
$\theta = \tan^{-1}\left(\frac{5}{4}\right)$

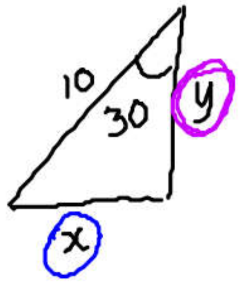
$\theta = 51\cdot3^\circ$

Vector $\boldsymbol{a}$ has magnitude 10 and makes an angle of 30° with $\boldsymbol{j}$
Find $\boldsymbol{a}$ in $\boldsymbol{i}, \boldsymbol{j}$ and column vector format.

up $\uparrow y$

Sketch





SOH CAH TOA

$\sin 30 = \frac{x}{10}$

$10\sin 30 = x$

$x = 5$

SOH CAH TOA

$\cos 30 = \frac{y}{10}$

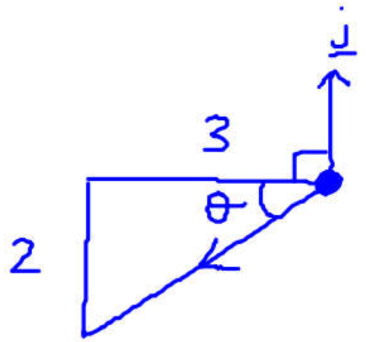
$10\cos 30 = y$

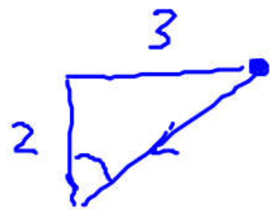
$y = 5\sqrt{3}$

$\underline{a} = 5\underline{i} + 5\sqrt{3}\underline{j} = \begin{pmatrix}5\\5\sqrt{3}\end{pmatrix}$

Ex 11C
2ab
3ab
4, 5
6ab
10, 11

Find the angle $-3\underline{i} - 2\underline{j}$ makes with $\underline{j}$.





$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.7°$

angle is $90 + 33.7 = \underline{\underline{123.7°}}$

on the left of $\underline{j}$.