100 Days of A Level Maths - WEEK 1

Day 1

$3^{2y} \times 3^{3x} = \frac{1}{3^2 \times 3^{1/2}}$

$3^{2y+3x} = 3^{-5/2}$ (1) (1)

4 $2y + 3x = -\frac{5}{2}$ (1)

$2y = -\frac{5}{2} - 3x$

$y = -\frac{5}{4} - \frac{3}{2}x$ (1)

Day 2

$\sqrt{4}\sqrt{7} - 3\sqrt{9}\sqrt{7} - \sqrt{25}\sqrt{7}$ (1)

2 $= 2\sqrt{7} - 9\sqrt{7} - 5\sqrt{7}$

$= -12\sqrt{7}$ (1)

Day 3

let $y = x^2$ (1)

$y^2 - y - 6 = 0$ (1)

$(y-3)(y+2) = 0$ (1)

5 $y = 3$ or $y = -2$ (1)

$x^2 = 3$ $\quad x^2 = -2$

$x = \pm\sqrt{3}$ $\quad$ no solutions

(1)

Day 4

Let $y = 4^x$ (1)

$y^2 - 20y + 64 = 0$ (1)

5 $(y-16)(y-4) = 0$ (1)

$y = 16$ or $y = 4$ (1) both

$4^x = 16$ $\quad 4^x = 4$

$x = 2$ or $x = 1$ (1) both

Day 5

$b^2 - 4ac = 0$ (1)

$(-24)^2 - 4 \times 6 \times k = 0$ (1)

3 $576 - 24k = 0$

$24k = 576$

$k = 24$ (1)

Day 6

$b^2 - 4ac < 0$ (1)

$5^2 - 4 \times 1 \times -p < 0$

$25 + 4p < 0$ (1)

3

$4p < -25$

$p < -\frac{25}{4}$ (1)

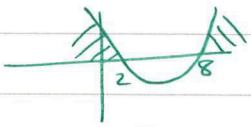
Day 7

$x^2 - 10x + 16 = 0$ (1)

$(x-8)(x-2) = 0$ (1)

$x = 8, \quad x = 2$ (1)

5



(1)

$x < 2$ or $x > 8$

$\{x : x < 2\} \cup \{x : x > 8\}$ (1)

27

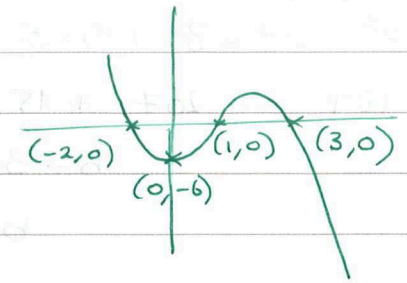
100 Days of A level Maths – week 2

Day 8

$a < 0$ ∴ ∩

[3] roots $x = 1, 3, -2$

$x = 0, y = -1 \times 3 \times 2 = -6$



① roots
① y intercept
① shape

Day 9

[2]

$y - y_1 = m(x - x_1)$

$y - 8 = 5(x + 3)$ ①

$y - 8 = 5x + 15$

$y = 5x + 23$ ① o.e.

Day 10

$\sqrt{(19-9)^2 + (7-x)^2} = 2\sqrt{29}$ ① o.e.

[3]

$10^2 + 49 - 14x + x^2 = 116$

$x^2 - 14x + 149 - 116 = 0$ ①

$x^2 - 14x + 33 = 0$ ①

Day 11

[3]

$(x+3)^2 + (y+7)^2 = 63$

① ① ①

Day 12

[3]

$(x+9)^2 + (y-3)^2 - 81 - 9 = 54$

$(x+9)^2 + (y-3)^2 = 144$ ①

centre $= (-9, 3)$ ①

radius $= 12$ ①

Day 13

[3]

$3x^3 + 23x^2 + 33x - 35 = (x+5)(3x^2 + 8x - 7)$ ② any valid method

∴ $3x^2 + 8x - 7$ ①

Day 14

3

$f(3) = 0$

$7 \times 3^3 - 23 \times 3^2 + 5 \times 3 + b = 0$ (1)

$189 - 207 + 15 + b = 0$ (1)

$b - 3 = 0$

$b = 3$ (1)

/20

100 Days of A level Maths - Week 3 Quiz

Day 15

let $2n$, $2n+2$ $n \in \mathbb{Z}$ be two consecutive even numbers. (1)

5

$(2n+2)^2 - (2n)^2$ (1)

$= 4n^2 + 8n + 4 - 4n^2$

$= 8n + 4$ (1)

$= 4(2n+1)$ (1)

$2n+1 \in \mathbb{Z}$ ∴ the difference of the squares of two consecutive even numbers is always divisible by 4 (1)

Day 16

4

1
1 1
1 2 1
1 3 3 1 (1)

$(2x + 5y)^3$

$= 1 \times (2x)^3 + 3 \times (2x)^2 \times 5y + 3 \times (2x) \times (5y)^2 + 1 \times (5y)^3$ (1) o.e

$= 8x^3 + 3 \times 4x^2 \times 5y + 6x \times 25y^2 + 125y^3$

$= 8x^3 + 60x^2y + 150xy^2 + 125y^3$ (1) (1)

Day 17

$\binom{6}{3} \times 1^3 \times (px)^3 = 14580x^3$ (1)

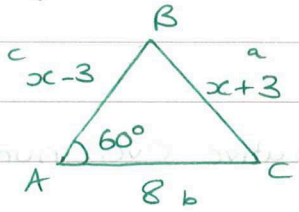
3

$20p^3 = 14580$ (1)

$p^3 = 729$

$p = 9$ (1)

Day 18



$a^2 = b^2 + c^2 - 2bc\cos A$

$(x+3)^2 = 8^2 + (x-3)^2 - 2\times 8\times(x-3)\cos 60$ (1)

$x^2+6x+9 = 64 + x^2 - 6x + 9 - 8(x-3)\times \frac{1}{2}$ (1) (1)

4

$x^2 + 6x + 9 = 64 + x^2 - 6x + 9 - 8x + 24$

$x^2 + 6x + 9 = x^2 - 14x + 97$

$6x + 9 = -14x + 97$ (1)

$20x = 88$

$x = 4.4$ cm. (1)

Day 19

1

$180 - \theta$ where $\theta = \sin^{-1}(k)$ (1)

Day 20

$X = 3\theta$

$\cos X = \frac{\sqrt{3}}{2}$ $\quad 0 < X < 1080$ (1)

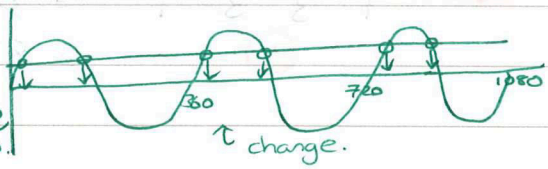
5

$X = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$X = 30$, (1) $360-30, 30+360, 330+360, 390+360, 690+360$ (1)

$X = 30, 330, 390, 690, 750, 1050$ (1)

$\theta = 10°, 110°, 130°, 230°, 250°, 350°$ (1)



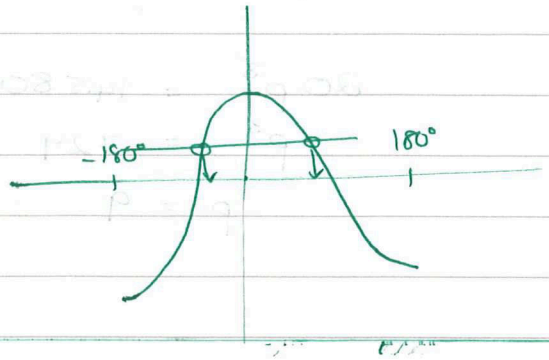
Day 21

$\theta = \cos^{-1}(0.6)$ (1)

$\theta = \pm 53.1°$ (3sf) (1) rounding

(1)

3



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100 Days of A Level Maths – WEEK 4 QUIZ

Day 22

$3(1-\sin^2 x) + 7\sin x - 5 = 0$ (1)

$3 - 3\sin^2 x + 7\sin x - 5 = 0$

$-3\sin^2 x + 7\sin x - 2 = 0$

$3\sin^2 x - 7\sin x + 2 = 0$ (1)

6 $(3\sin x - 1)(\sin x - 2) = 0$ (1)

$\sin x = \frac{1}{3}$ or $\sin x = 2$ (1)

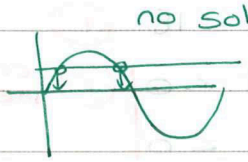
$x = \sin^{-1}\left(\frac{1}{3}\right)$ (1)

$x = 19.4\ldots, 180 - 19.4\ldots$

$x = 19.5°, 161°$ (3sf)

(1)

no soln's



Day 23

$5\,\frac{\cos x \sin y}{\sin x \sin y} = \frac{\sin x \cos y}{\sin x \sin y}$ (1) divide by $\sin x \sin y$

3 $5\,\frac{\cos x}{\sin x} = \frac{\cos y}{\sin y}$

$5\cot x = \cot y$ (1)

$\cot x = \frac{1}{5}\cot y$ (1)

Day 24

$\underline{a} - t\underline{b} = \begin{pmatrix}5\\-1\end{pmatrix} - t\begin{pmatrix}2\\5\end{pmatrix} \parallel \begin{pmatrix}-5\\10\end{pmatrix}$

(1) $\begin{cases} 5 - 2t = -5K & \times -2 \\ -1 - 5t = 10K \end{cases}$ $\quad -10 + 4t = 10K$

3 $\therefore \quad -10 + 4t = -1 - 5t$ (1)

$9t = 9$

$t = 1$ (1)

Day 25

$\begin{pmatrix}-9\\3\end{pmatrix} = \underline{b} - \begin{pmatrix}5\\2\end{pmatrix}$ (1)

4 $\underline{b} = \begin{pmatrix}-9\\3\end{pmatrix} + \begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}-4\\5\end{pmatrix}$ (1)

$|\underline{b}| = \sqrt{(-4)^2 + 5^2}$ (1) ecf their values

$= \sqrt{41}$ (1) cso

Day 26

(i) $\frac{dy}{dx} = 12x^5 - 4x$ (1) (1)

5 (ii) $y = 3x^{1/2} + 8x^{-5}$ (1)

$\frac{dy}{dx} = \frac{3}{2}x^{-1/2} - 40x^{-6}$ (1) (1)

Day 27

$f'(x) = 3x^2 - 3x - 18$ (1)

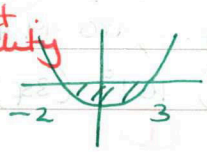
$3x^2 - 3x - 18 < 0$ (1) correct inequality

$x^2 - x - 6 < 0$

5 $x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$ (1) method for solving

$x = -2, \quad x = 3$ (1) correct roots



$-2 < x < 3$ (1) graph & answer

Day 28

$\frac{dy}{dx} = 4x^2 - 12x - 40$ (1)

at stationary points $4x^2 - 12x - 40 = 0$ (1) correct eqtn

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$ (1) method for solving

$x = 5, \quad x = -2$ (1) both correct

30

100 Days of A level Maths - Week 5 Quiz

Day 29

$\frac{ds}{dt} = 15t^2 + 8t^{-3}$ (1)

[3]

$\frac{d^2s}{dt^2} = 30t - 24t^{-4}$ (1) ecf from $\frac{ds}{dt}$ (1) completely correct

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Day 30

$-\frac{1}{4} = a\left(\frac{11}{2}\right)^2 - 11\left(\frac{11}{2}\right) + b \Rightarrow -\frac{1}{4} = \frac{121}{4}a + b - \frac{121}{2}$ (1) method for finding b $\Rightarrow$ $b = \frac{241}{4} - \frac{121}{4}a$

$2ax - 11 = 0$ (1) at $x = 11/2$

[4]

$2a\left(\frac{11}{2}\right) - 11 = 0$ method for finding a

$11a = 11$

$a = 1$ (1)

$b = \frac{241}{4} - \frac{121}{4} = \frac{30}{4}$ (1)

Day 31

$\frac{3x^7}{7} - \frac{x^{-3/2}}{-3/2} - \frac{8x^{-2}}{-2} + c$ (1)

[3]

$= \frac{3}{7}x^7 + \frac{2}{3}x^{-3/2} + 4x^{-2} + c$ (1) two terms correct

(1) all terms correct

Day 32

$f(x) = \frac{12x^4}{4} + \frac{4x^2}{2} - 5x + c$ (1) at least 2 terms correct

[4]

$47 = 3 \times 2^4 + 2 \times 2^2 - 5 \times 2 + c$ (1) ft sub.

$47 = 48 + 8 - 10 + c$

$c = 1$ (1) ft

$f(x) = 3x^4 + 2x^2 - 5x + 1$ (1) c.a.o

Day 33

$y = x(x^2 - 10x + 25)$

$y = x^3 - 10x^2 + 25x$ (1)

Area $= \int_0^5 x^3 - 10x^2 + 25x \, dx$ (1) correct limits

5 $= \left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ (1) at least 2 terms correct

$= \left(\frac{1}{4} \times 5^4 - \frac{10}{3} \times 5^3 + \frac{25}{2} \times 5^2\right) - 0$ (1) correct sub.

$= \frac{625}{12} \left(= 52.083\dot{}\right)$ (1) c.a.o

Day 34

(i) $\log_4 \frac{1}{64} = -3$ (1)

4 (ii) $\log_c d = 3$ (1)

(iii) $7^3 = 343$ (1)

(iv) $y^x = z$ (1)

Day 35

$\log_x 4 + \log_x 5^2 = 2$ (1)

$\log_x (4 \times 5^2) = 2$ (1)

5 $x^2 = 100$ (1)

$x = \pm 10$ (1)

$x > 0 \therefore x = 10$ (1)

28

100 Days of A Level Maths - WEEK 6 QUIZ

Day 36

$$3^{2x} e^{2x-5} = 5$$

$$\ln(3^{2x} e^{2x-5}) = \ln 5 \quad (1)$$

5 $$\ln 3^{2x} + \ln e^{2x-5} = \ln 5 \quad (1)$$

$$2x \ln 3 + 2x - 5 = \ln 5 \quad (1)$$

$$2x(\ln 3 + 1) = \ln 5 + 5 \quad (1)$$

$$x = \frac{\ln 5 + 5}{2(\ln 3 + 1)} \quad (1)$$

Day 37

$$R = 12e^{0.2m}$$

(a) $m = 0$ $\underline{R = 12}$ (1)

(b) $12e^{0.2m} = 200$ (1)

6 $$e^{0.2m} = \frac{50}{3} \quad (1)$$

$$0.2m = \ln\left(\frac{50}{3}\right) \quad (1)$$

$$m = 5\ln\left(\frac{50}{3}\right) \quad (1)$$

$$m = 14.06\ldots$$

$$\therefore m = 15 \text{ months} \quad (1)$$

Day 38

Assume there are integers a and b such that $25a + 15b = 1$ (1)

$$25a + 15b = 1$$

4 $$5a + 3b = \frac{1}{5} \quad (1)$$

But if $a, b \in \mathbb{Z}$, their multiples & sum cannot be a fraction (1)

$\therefore$ there is a contradiction and therefore there does not exist integers a and b for which $25a + 15b = 1$ (1)

Day 39

Given a rational number a and an irrational number b, assume that $a-b$ is rational (1)

let $a = \frac{m}{n}$ $\quad m, n \in \mathbb{Z}$

Given our assumption, let

$$a - b = \frac{p}{q} \quad p, q \in \mathbb{Z}$$

so $\frac{m}{n} - b = \frac{p}{q}$ (1)

4 $\Rightarrow \quad b = \frac{m}{n} - \frac{p}{q}$

$$= \frac{mq - np}{nq}$$

$mq \in \mathbb{Z}$, $np \in \mathbb{Z}$, $nq \in \mathbb{Z}$

so $\frac{mq-np}{nq} \in \mathbb{Q} \therefore b \in \mathbb{Q}$ (1)

this contradicts the assumption that b is irrational

so given a rational number a & an irrational number b, $a-b$ is irrational (1)

Day 40

$$\frac{2x+11}{(x+1)(x+4)} = \frac{A}{x+1} + \frac{B}{x+4}$$

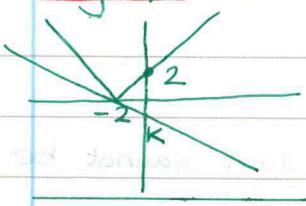
4,3 $\quad 2x+11 = A(x+4) + B(x+1)$

$x = -4 \quad 3 = -3B \Rightarrow B = -1$ (1)

$x = -1 \quad 9 = 3A \Rightarrow A = 3$ (1)

$\frac{3}{x+1} - \frac{1}{x+4}$ (1)

Day 41

3 

one solution when $y=0$, $x=-2$

$y = k - \frac{1}{2}x$ (1)

$0 = k - \frac{1}{2}x - 2 \Rightarrow 0 = k + 1 \Rightarrow k = -1$ (1)

$k \leq -1$ (1)

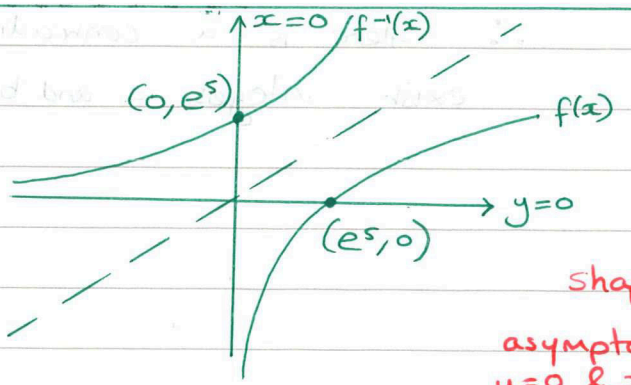
Day 42

$y = \ln x - 5$

$y+5 = \ln x$ (1)

5 $\quad e^{y+5} = x$

$\therefore f^{-1}(x) = e^{x+5}$ (1)



shapes (1)

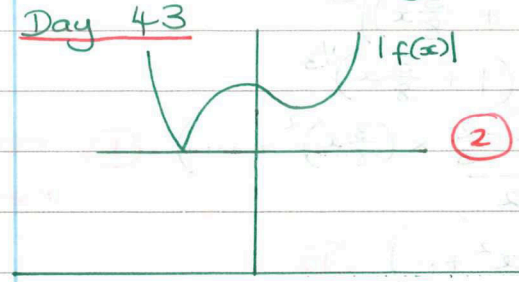
asymptotes (1)
$y=0$ & $x=0$
(not for both branches!)

intercepts (1)

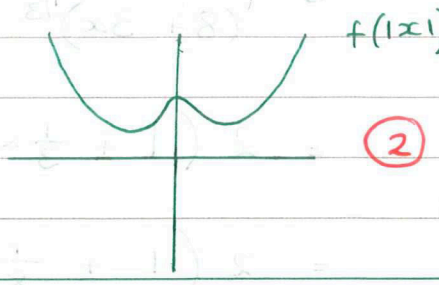
100 Days of A Level Maths – Week 7 Quiz

Day 43

4



(2)



(2)

Day 44

$4k + 7 - 9 = 55 - 2k - (4k + 7)$ (1)

$4k - 2 = 48 - 6k$

$10k = 50$

$k = 5$ (1)

5

$\therefore$ 9, 27, 45 $\Rightarrow$ $a = 9$, $d = 18$ (1)

$S_n = \frac{n}{2}(2a + (n-1)d) \Rightarrow S_n = \frac{n}{2}(2 \times 9 + 18(n-1)) = \frac{n}{2}(18 + 18n - 18)$

$= \frac{n}{2} \times 18n = 9n^2 = (3n)^2$ (1)

$\therefore$ Sum of first n terms is a square number (1)

Day 45

$S_\infty = 16 \Rightarrow \frac{a}{1-r} = 16$

$S_4 = 15 \Rightarrow \frac{a(1-r^4)}{1-r} = 15$ } (1) $\Rightarrow \frac{a}{1-r} = \frac{15}{1-r^4}$

4

$16 = \frac{15}{1-r^4}$ $\quad 1 - r^4 = \frac{15}{16}$ (2) any valid approach to solving

$r^4 = \frac{1}{16}$

$r = \pm \frac{1}{\sqrt[4]{16}} = \pm \frac{1}{2}$ (1)

Day 46

$a_1 = p \quad a_{n+1} = \frac{1}{a_n}$

$a_1 = p \quad a_2 = \frac{1}{p} \quad a_3 = p$ etc. (1)

3

$\sum_{r=1}^{1000} a_r = 500\left(p + \frac{1}{p}\right)$ (1) o.e.

(1) suggestion that 500 is involved

Day 47

$(8+3x)^{1/3} = \left(8\left(1+\frac{3}{8}x\right)\right)^{1/3}$

$= 8^{1/3}\left(1+\frac{3}{8}x\right)^{1/3}$ (1)

$= 2\left(1 + \frac{1}{3}\times\frac{3}{8}x + \frac{\frac{1}{3}\times\frac{-2}{3}}{2}\times\left(\frac{3}{8}x\right)^2 + \ldots\right)$ (1) method, condone 1 error

$= 2\left(1 + \frac{1}{8}x - \frac{1}{64}x^2 + \ldots\right)$ (1)

$= 2 + \frac{1}{4}x - \frac{1}{32}x^2$ (1)

[4]

Day 48

$\frac{\cos 4\theta - 1}{\theta \sin 2\theta} \approx \frac{1 - \frac{(4\theta)^2}{2} - 1}{\theta \times 2\theta}$ (1) (1)

$= \frac{-16\theta^2}{2} \div 2\theta^2$ (1) $= -4$ (1)

[4]

Day 49

$\frac{1}{2}r^2\theta = 25 \Rightarrow r^2\theta = 50$ (1) any correct statement involving area

$r\theta + 2r = 5r\theta$ (1) o.e

$4r\theta = 2r$

$\theta = \frac{2r}{4r} = \frac{1}{2}$ (1) $\theta = \frac{1}{2}$

$r^2 \times \frac{1}{2} = 50$ (1)

$r^2 = 100$ (1)

$r > 0 \quad r = 10$ (1)

[6]

30

100 Days of A Level Maths - WEEK 8 - QUIZ

Day 50

$\operatorname{cosec}\theta - \sin\theta$

$= \frac{1}{\sin\theta} - \sin\theta$ (1)

$= \frac{1 - \sin^2\theta}{\sin\theta}$ (1) common den.

5 $= \frac{\cos^2\theta}{\sin\theta}$ (1)

$= \cos\theta \times \frac{\cos\theta}{\sin\theta}$ (1)

$= \cos\theta \cot\theta$ □ (1)

Day 51

$7\cot^2\theta + 3(1 + \cot^2\theta) = 12$ (1)

$7\cot^2\theta + 3 + 3\cot^2\theta = 12$

4 $10\cot^2\theta = 9$ (1)

$\cot^2\theta = \frac{9}{10}$ (1)

$\cot\theta = \pm\frac{3}{\sqrt{10}}$ (1) $(a = 3 \quad b = 10)$

Day 52

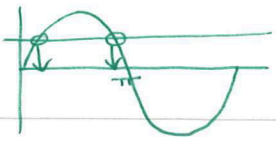
$\tan\frac{7\pi}{12} = \tan\left(\frac{\pi}{6} + \frac{5\pi}{12}\right) = \frac{\tan\frac{\pi}{6} + \tan\frac{5\pi}{12}}{1 - \tan\frac{\pi}{6}\tan\frac{5\pi}{12}}$ (1)

$= \frac{\frac{\sqrt{3}}{3} + 2 + \sqrt{3}}{1 - \frac{\sqrt{3}}{3}(2+\sqrt{3})}$ (1) $= \frac{\frac{4}{3}\sqrt{3} + 2}{1 - \frac{2}{3}\sqrt{3} - 1} = \frac{\frac{4}{3}\sqrt{3} + 2}{-\frac{2}{3}\sqrt{3}}$

4 $= \left(\frac{4}{3}\sqrt{3} + 2\right) \times \frac{-3}{2\sqrt{3}}$ (1) any clear correct steps

$= \frac{4}{3}\sqrt{3} \times \frac{-3}{2\sqrt{3}} + 2 \times \frac{-3}{2\sqrt{3}}$

$= -2 - \sqrt{3}$ (1)



Day 53

$7\cos 2x = \dfrac{1}{\sin 2x}$ (1)

$7\sin 2x \cos 2x = 1$

5 $\frac{7}{2} \times 2\sin 2x \cos 2x = 1$

$\frac{7}{2}\sin 4x = 1$ (1)

$\sin 4x = \frac{2}{7}$

Let $X = 4x$

$\sin X = \frac{2}{7} \qquad 0 \le X \le 2\pi$

$X = \sin^{-1}\left(\frac{2}{7}\right)$ (1)

$X = 0.28..., \pi - 0.28...$

$X = 0.28..., 2.85....$

$x = 0.0724^c \quad 0.713^c \text{ (3sf)}$

(1) (1)

Day 54

$\dfrac{1-\cos 2x}{1+\cos 2x} = \dfrac{1-(1-2\sin^2 x)}{1+(2\cos^2 x - 1)}$ (1) (1)

4 $= \dfrac{1-1+2\sin^2 x}{1+2\cos^2 x - 1} = \dfrac{2\sin^2 x}{2\cos^2 x} = \dfrac{\sin^2 x}{\cos^2 x}$ (1)

$= \tan^2 x$ (1)

Day 55

$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 2$

$R\sin\alpha = 5$ (1)

3 $\tan\alpha = \dfrac{5}{2}$

$\alpha = 68.2^\circ$ (3sf) (1)

$R = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$ (1)

$(2\cos\theta + 5\sin\theta = \sqrt{29}\cos(\theta - 68.2))$

Day 56

$y = \dfrac{\sin 2t}{\cos 2t}$ (1) $= \dfrac{2\sin t\cos t}{1 - 2\sin^2 t}$ (1)

$= \dfrac{2x\cos t}{1-2x^2}$ (1)

5 $\cos t = \sqrt{1-\sin^2 t}$

$= \sqrt{1-x^2}$ (1)

$\therefore \quad y = \dfrac{2x\sqrt{1-x^2}}{1-2x^2}$ (1)

30

100 Days of A Level Maths - WEEK 9

Day 57

$2t^2 - 3t = 2(2t+3) + 9$ (1)

$2t^2 - 3t = 4t + 6 + 9$

$2t^2 - 3t = 4t + 15$

[4/6] $2t^2 - 7t - 15 = 0$ (1)

$(2t+3)(t-5) = 0$ (1)

$t = -3/2$ or $t = 5$

$t > 0 \therefore t = 5$ (1)

$y = 2\times5^2 - 3\times5 = 35$

$x = 2\times5 + 3 = 13$ (1)

$\therefore (13, 35)$ (1)

Day 58

[4/5] 1. e^x (½) 2. $5^x \ln 5$ (1) 3. $\frac{1}{x}$ (½) 4. $\cos x$ (½)

5. $-\sin x$ (½) 6. $\sec^2 x$ (½) 7. $\sec x \tan x$ (½) 8. $-\operatorname{cosec}^2 x$ (½) 9. $-\cot x \operatorname{cosec} x$ (½)

Day 59

[6] 1. $f'(x) = 5(3x^5 - 7)^4 \times (15x^4)$ (1)

$= 75x^4(3x^5 - 7)^4$ (1)

2. $f'(x) = e^{6x - \sin 3x} \times (6 - 3\cos 3x)$ (1)

$= (6 - 3\cos 3x)\, e^{6x - \sin 3x}$ (1)

3. $f'(x) = \frac{1}{7x^2 + 4x} \times (14x + 4)$ (1)

$= \frac{14x + 4}{7x^2 + 4x}$ (1) $\left(= \frac{2(7x+2)}{7x^2 + 4x}\right)$

Day 60

[5] $u = \cos(3x^2)$ $u' = -6x\sin(3x^2)$ (1)

$v = \ln(4x^3)$ $v' = \frac{1}{4x^3} \times 12x^2 = \frac{3}{x}$ (1) | (1)

$\frac{dy}{dx} = \frac{3\cos(3x^2)}{x} - 6x\sin(3x^2)\ln(4x^3)$ (2)

Day 61

[5] $u = e^{2x}$ $u' = 2e^{2x}$ (1)

$v = x^5$ $v' = 5x^4$ (1) | (1)

$\frac{dy}{dx} = \frac{2x^5 e^{2x} - 5x^4 e^{2x}}{x^{10}} = \frac{e^{2x}(2x-5)}{x^6}$ (2)

Day 62

$$\frac{dx}{dt} = 4\sqrt{5}\cos 4t \quad (1)$$

$$\frac{dy}{dt} = -12\sin 4t \quad (1)$$

6

$$\frac{dy}{dx} = \frac{-12\sin 4t}{4\sqrt{5}\cos 4t} \quad (1) = \frac{-3}{\sqrt{5}}\tan 4t \quad (1)$$

$$= -\frac{3\sqrt{5}}{5}\tan 4t \quad (1)$$

$a = 3,\ b = 5$ (1)

Day 63

$$7xy^2 + 2y + 5x + 12 = x^6$$

$$7x\left(2y\frac{dy}{dx}\right) + 7y^2 + 2\frac{dy}{dx} + 5 = 6x^5 \quad (1)$$

5

$$14xy\frac{dy}{dx} + 2\frac{dy}{dx} = 6x^5 - 7y^2 - 5 \quad (1)$$

$$\frac{dy}{dx}(14xy + 2) = 6x^5 - 7y^2 - 5 \quad (1)$$

$$\frac{dy}{dx} = \frac{6x^5 - 7y^2 - 5}{14xy + 2} \quad (1)$$

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100 Days of A level Maths - Week 10 Quiz

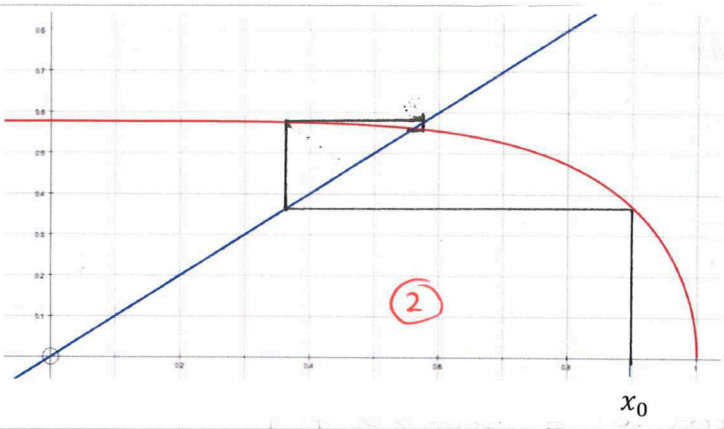
Day 64

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$ (1) $$\frac{dV}{dr} = 27\pi r^2$$ (1)

4 $$= \frac{1}{27\pi r^2} \times 15 = \frac{5}{9\pi r^2}$$ (1)

when $r = 10$ $\frac{dr}{dt} = \frac{15}{27\pi \times 10^2} = \frac{1}{180\pi}$ (1)

Day 65



3 (2)

converges (1)

Day 66

$f(x) = x^2 - 8x + 11$

$f'(x) = 2x - 8$ (1)

3 $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{6^2 - 8\times 6 + 11}{2\times 6 - 8}$$ (1) $= 6.25$ (1)

Day 67

(a) $4\sin x + c$ ($\frac{1}{2}$)

(b) $-\tan x + c$ ($\frac{1}{2}$)

(c) $-5e^x + c$ ($\frac{1}{2}$) ($\frac{1}{2}$) + c

4 (d) $\frac{1}{2}\ln|x| + c$ (1)

(e) $-\frac{1}{9}\cos x + c$ ($\frac{1}{2}$)

(f) $\sec x + c$ ($\frac{1}{2}$)

~~Fro~~ Day 68

(a) $\int \cos 4x \, dx = \frac{1}{4} \sin 4x + c$ (2)

Try $y = \sin 4x$

$\frac{dy}{dx} = 4 \cos 4x$

(b) $\int 7e^{9x} \, dx = \frac{7}{9} e^{9x} + c$ (2)

Try $y = e^{9x}$

$\frac{dy}{dx} = 9e^{9x}$

8 (c) $\int -2 \sec 6x \tan 6x \, dx$

Try $y = \sec 6x$

$\frac{dy}{dx} = 6 \sec 6x \tan 6x$

$= \frac{-2}{6} \sec 6x + c$

$= -\frac{1}{3} \sec 6x + c$ (2)

(d) $\int (6x-1)^{-3} \, dx$

Try $y = (6x-1)^{-2}$

$\frac{dy}{dx} = -2(6x-1)^{-3} \times 6$

$\frac{dy}{dx} = -12(6x-1)^{-3}$

$= \frac{-1}{12} (6x-1)^{-2} + c$

$= \frac{-1}{12(6x-1)^2} + c$ (2)

Day 69 $\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2} \cos 2x + \frac{1}{2}$

3 $\int \underbrace{\frac{1}{2} \cos 2x + \frac{1}{2}}_{(1)} \, dx$

$= \underbrace{\frac{1}{4} \sin 2x}_{(1)} + \underbrace{\frac{1}{2} x}_{(1)} + c$

Try $y = \sin 2x$

$\frac{dy}{dx} = 2 \cos 2x$

Day 70

$\int \frac{5 \sin 2x}{\cos 2x} = -\frac{5}{2} \ln|\cos 2x| + c$ (2)

Try $y = \ln|\cos 2x|$

$\frac{dy}{dx} = \frac{1}{\cos 2x} \times -2 \sin 2x$

$= \frac{-2 \sin 2x}{\cos 2x}$

5 $\int 18x^2 e^{2x^3} \, dx$

Try $y = e^{2x^3}$

$\frac{dy}{dx} = 6x^2 e^{2x^3}$

$= 3e^{2x^3} + c$ (2)

(+1) for +c

/30

100 Days of A Level Maths - WEEK 11

Day 71

Try $y = (\cos(5x-1))^7$ (1)

$\frac{dy}{dx} = 7\cos^6(5x-1) \times -\sin(5x-1) \times 5$

$= -35\sin(5x-1)\cos^6(5x-1)$

3

$\int -\sin(5x-1)\cos^6(5x-1)\,dx$

$= \frac{1}{35}\cos^7(5x-1) + C$ (2)

Day 72

Try $y = \ln|$

$\int \frac{2x+3}{x^2}\,dx = \int \frac{2x}{x^2} + \frac{3}{x^2}\,dx$

3

$= \int \frac{2}{x} + 3x^{-2}\,dx$ (1)

$= 2\ln|x| + \frac{3x^{-1}}{-1} + C$

$= 2\ln|x| - \frac{3}{x} + C$ (1) (1)

3

$\int \frac{5x-3}{x+1}\,dx$

$x+1 \;\overline{)\;5x-3}$ → 5, $-5x-5$, -8

$= \int 5 - \frac{8}{x+1}\,dx$ (1)

$= 5x - 8\ln|x+1| + C$ (1) (1)

Day 73

$\int 2xe^{3x}\,dx$ $\quad v' = e^{3x}$ $\quad v = \frac{1}{3}e^{3x}$

$u = 2x$ $\quad u' = 2$ (1)

4

$= \frac{2}{3}xe^{3x} - \frac{2}{3}\int e^{3x}\,dx$ (1)

$= \frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} + C$ (1) (1)

Day 74

$\int 1 \times \ln 5x\,dx$ $\quad v' = 1$ $\quad v = x$ (1)

$u = \ln 5x$ $\quad u' = 5 \times \frac{1}{5x} = \frac{1}{x}$

3

$\therefore \int \ln 5x\,dx = x\ln 5x - \int 1\,dx$ (1)

$= x\ln 5x - x + C$ (1)

Day 75

$u = 2x+5 \Rightarrow \frac{u-5}{2} = x$

$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$ (1)

$\int (2x-1)\sqrt{2x+5}\,dx$

$= \int \left(2\left(\frac{u-5}{2}\right)-1\right)\sqrt{u} \times \frac{du}{2}$ (1) (1)

6

$= \int \frac{(u-5-1)}{2} u^{1/2}\,du$

$= \frac{1}{2}\int (u-6)u^{1/2}\,du$ (1)

$= \frac{1}{2}\int u^{3/2} - 6u^{1/2}\,du = \frac{1}{2}\left[\frac{u^{5/2}}{5/2} - \frac{6u^{3/2}}{3/2}\right) + c$ (1)

$= \frac{1}{5}u^{5/2} - 2u^{3/2} + c$

$= \frac{1}{5}(2x+5)^{5/2} - 2(2x+5)^{3/2} + c$ (1)

Day 76

$\frac{1}{y}\frac{dy}{dx} = \frac{4x+6x^2}{x^2+x^3}$ (1) separating variables

Try $y = \ln|x^2+x^3|$

$\frac{dy}{dx} = \frac{1}{x^2+x^3} \times 3x^2+2$

$\therefore 2\ldots$

5

$\int \frac{1}{y}\,dy = \int \frac{4x+6x^2}{x^2+x^3}\,dx \Rightarrow \ln|y| = 2\ln|x^2+x^3| + c$ (1) (1)

$\ln|y| = \ln(x^2+x^3)^2 + c$ (1)

$y = e^{\ln(x^2+x^3)^2 + c}$

$y = A(x^2+x^3)^2$ (1)

Day 77

$\sin y \frac{dy}{dx} = \frac{1+2x}{x+x^2}$ (1)

$\int \sin y\,dy = \int \frac{1+2x}{x+x^2}\,dx$ (1)

3

$-\cos y = \ln|x+x^2| + c$

$\cos y = K - \ln|x+x^2|$

$y = \arccos(K - \ln|x+x^2|)$ (1)

30/30

100 Days of A Level Maths – WEEK 12

Day 78

$$\sqrt{(7-k)^2 + (-1-0)^2 + (2-4)^2} = 3 \quad (1)$$

$$49 - 14k + k^2 + 1 + 4 = 9$$

[4]

$$k^2 - 14k + 45 = 0 \quad (1)$$

$$(k-5)(k-9) = 0 \quad (1)$$

$$k = 5, 9 \quad (1)$$

Day 79

$$\cos\theta_x = \frac{x}{|\underline{a}|}$$

[3]

$$|\underline{a}| = \sqrt{4^2 + 2^2 + 1^2} = \sqrt{21} \quad (1)$$

$$\cos\theta_x = \frac{4}{\sqrt{21}} \quad (1)$$

$$\theta_x = 29.205\ldots = 29.2° \text{ (3sf)} \quad (1)$$

Day 80

$$\begin{pmatrix} 3a+b \\ 1 \\ ac \end{pmatrix} = \begin{pmatrix} 7 \\ -b \\ 4 \end{pmatrix} \quad (1)$$

[4]

$b = -1$ (1)

$3a + b = 7 \Rightarrow 3a - 1 = 7 \quad 3a = 8 \quad a = \frac{8}{3}$ (1)

$ac = 4 \qquad 3/2$

$\frac{8}{3}c = 4 \therefore c = \frac{3}{2}$ (1)

Day 81

$$5\sin x = 1 + 2(1 - \sin^2 x) \quad (1)$$

$$5\sin x = 1 + 2 - 2\sin^2 x$$

[2]

$$2\sin^2 x + 5\sin x - 1 - 2 = 0$$

$$2\sin^2 x + 5\sin x - 3 = 0 \quad (1) \text{ s.t } *$$

Day 82

$$\int_k^{4k} \frac{5}{4x-k}\,dx = \left[\frac{5}{4}\ln|4x-k|\right]_k^{4k} \quad (1)$$

[4]

$$= \left(\frac{5}{4}\ln(16k-k)\right) - \left(\frac{5}{4}\ln(4k-k)\right) \quad (1)$$

$$= \frac{5}{4}(\ln 15k - \ln 3k)$$

$$= \frac{5}{4}\ln\left(\frac{15k}{3k}\right) = \frac{5}{4}\ln 5 \quad (1) \text{ independent of } k.$$

(1) with conclusion

Days 83 & 84

17

(a) (i) $\frac{dy}{dx} = 2x^2 + 9x - 5$ (2) 1 for 2 terms correct

(ii) $\frac{d^2y}{dx^2} = 4x + 9$ (1)

(b) $x = -5 \quad \frac{dy}{dx} = 2 \times (-5)^2 + 9(-5) - 5$ (1)

$= 0 \quad \therefore$ stationary point when $x = -5$

(1) $= 0$ with conclusion

(c) at $x = -5$

$\frac{d^2y}{dx^2} = 4(-5) + 9 = -11 < 0 \quad \therefore$ maximum

(1) (1)

/24

100 Days of A Level maths - WEEK 13

Day 85

$\cos 2x + \cos 3y = 1$

$-2\sin 2x - 3\sin 3y \cdot \frac{dy}{dx} = 0$ (1)

(1)

[3]

$\frac{dy}{dx}(3\sin 3y) = -2\sin 2x$

$\frac{dy}{dx} = -\frac{2}{3}\frac{\sin 2x}{\sin 3y}$ (1)

Day 86

$30\,000 \times \left(1 + \frac{r}{100}\right)^3 = 34\,000$ (1)

$\left(1 + \frac{r}{100}\right)^3 = \frac{17}{15}$

[3]

$\frac{r}{100} = \sqrt[3]{\frac{17}{15}} - 1$ (1)

$r = \left(\sqrt[3]{\frac{17}{15}} - 1\right) \times 100 = 4.26$

(1) (2dp)

Day 87

[2]

$f(1) = 2 + \ln(2\times1 - 1)$

$= 2 + \ln 1 = 2$ (1)

$f(2) = 2 + \ln(2\times2 - 1)$

$= 2 + \ln 3$ exact (1)

Day 88

(a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

[3]

$\begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 9 \end{pmatrix}$

$\therefore b_1 = 4 - 3 = 1$

$b_2 = -5 + 4 = -1$

$b_3 = 2 + 9 = 11$

$\therefore B = (1, -1, 11)$ (1)

coordinates

(b) $\overrightarrow{CD} = -\frac{1}{2}\overrightarrow{AB}$ (1) $\therefore$ they are parallel (1)

Day 89

$n = 3$

$n^2 - n + 3 = 3^2 - 3 + 3$

[2]

$= 9$ (1) any correct counterexample.

9 is not prime so the statement is disproved

(1) conclusion

Day 90

(a) $u_2 = pu_1 - 2 = 2p - 2$ (1)

$u_3 = pu_2 - 2$
$= p(2p-2) - 2$
$= 2p^2 - 2p - 2$ (1)

(b) $u_1 = u_3$

$\therefore 2p^2 - 2p - 2 = 2$ (1)

$p^2 - p - 1 = 1$

$p^2 - p - 2 = 0$ (1)

$(p-2)(p+1) = 0 \quad p = 2$ or -1 (1)

But $u_1 \neq u_2$, so $p = -1$ (1)

[8]

(c) $\sum_{n=1}^{1001} u_n = (2 + -4) \times 500 + 2$ (1)

$= -998$ (1)

Day 91

$f(x) = (x+1)(x^2 - 6x + 9)$ (2) → possibly through algebraic long division

$= (x+1)(x-3)^2$ (2) o.e.

[4]

/25

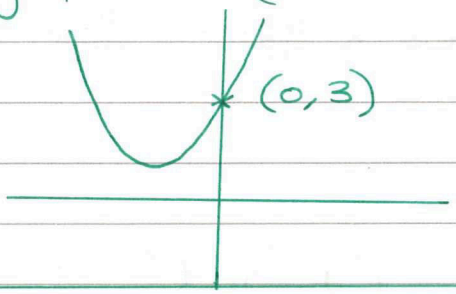
100 Days of A Level Maths - Week 14

Day 92

(a) $x^2 + 2x + 3 = (x+1)^2 + 2$ $\quad a = 1$ (1), $b = 2$ (1)

(b) turning point $= (-1, 2)$

5



(1) Shape, including min in correct quadrant
(1) no x intercepts
(1) $(0, 3)$

Day 93

$x(x^2 - 4) = x^3 - 4x$

$$\int_{-2}^{0} x^3 - 4x \, dx \quad (1)$$

4

$$= \left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} \quad (1)$$

$$= (0) - \left(\frac{(-2)^4}{4} - 2(-2)^2\right) \quad (1)$$

$$= 0 - -4 = 4 \quad (1)$$

Day 94

$\log(ab) = \log(a+b)$

(1) $ab = a + b$ (1)

3

$ab - a = b$

$a(b-1) = b$

$a = \frac{b}{b-1}$ (1)

Day 95

$u = y^2 \qquad u' = 2y$ (1)

$v = \ln y \qquad v' = \frac{1}{y}$ (1)

(a) $\frac{dx}{dy} = \frac{y^2}{y} + 2y \ln y$ (1)

6

$= y + 2y \ln y$ (1)

(b) $\frac{dy}{dx} = \frac{1}{y + 2y \ln y}$ $\qquad y = e$

$\frac{dy}{dx} = \frac{1}{e + 2e \ln e} = \frac{1}{3e}$

(1) (1)

Day 96

3

(a) angle must be in radians, not degrees (1)

(b) $50 \times \frac{\pi}{180} = \frac{5\pi}{18}$ c (1)

Area $= \frac{1}{2} \times 3^2 \times \frac{5\pi}{18} = \frac{5}{4}\pi$ or $3.93\ cm^2$ (3sf)

(1) any rounding or exact value.

Day 97

3

$$\int_2^5 x^{1/3}\,dx = \left[\frac{3}{4}x^{4/3}\right]_2^5$$ (1)

$$= \left(\frac{3}{4} \times 5^{4/3}\right) - \left(\frac{3}{4} \times 2^{4/3}\right)$$ (1) correct integral with attempt to substitute $x = 5$ & 2

$$= 4.523 \quad (4sf)$$ (1)

Day 98

(a) $-2 \leq f(x) \leq 18$ (1) allow y instead of f(x)

(b) $f(-3) = -2$ (1)

3

for eqtn of line from $x = -3$ to $x = 7$:

$$\frac{18 - -2}{7 - -3} = \frac{20}{10} = 2$$

$y = 2x + c$

$18 = 2 \times 7 + c$

$c = 4$

$\therefore y = 2x + 4$

$x = -2$

$y = 2x - 2 + 4$

$= 0$ (1)

27

100 Days of A Level Maths - WEEK 15

Day 99

$r = 8 \quad 15 \times \left(\frac{1}{5}\right)^8 = \frac{3}{78125}$ (1)

[3] $a = \frac{3}{78125}$ $\quad r = \frac{1}{5}$

$S_\infty = \frac{\frac{3}{78125}}{1 - \frac{1}{5}}$ (1) $= \frac{3}{62500}$ $\left(= 4.8 \times 10^{-5}\right)$ (1)

Day 100

$5^x \times 5^{2y} = \frac{1}{5^3 \times 5^{1/2}}$

[3] $5^{x+2y} = 5^{-7/2}$ (1) for either $x+2y$ OR $-7/2$ OR $7/2$

$x + 2y = -7/2$ (1)

$2y = -7/2 - x$

$y = -7/4 - \frac{1}{2}x$ (1)

6