A Level · Edexcel · Maths

4 hours 32 questions

2.13 Further Modelling with Functions (A Level only)

Easy (8 questions)	/40
Medium (8 questions)	/58
Hard (8 questions)	/58
Very Hard (8 questions)	/59
Total Marks	**/215**

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Easy Questions

1 ***Note: For this question ensure you are working in degrees.***

The height, h cm, of water in a wave tank, at time t seconds after the tank is switch on, is measured according to the function

$$h(t) = 20\sin(15t)° \qquad t \geq 0$$

The model is designed so that a height of 0 cm represents calm, still water.

(i) What is the maximum height the water will reach?

(ii) How long does it take the water to first reach its maximum height?

(iii) Find the height of the water after 8 seconds.

(3 marks)

2 A skydiver jumps from a moving aircraft when it is directly above a fixed point, *O*, on the ground. The trajectory of the skydiver is then modelled by the function

$$h(x) = 4900 - x^2$$

where h m is the height of the skydiver above the ground and x m is the horizontal distance along the ground from point *O*.

(i) At what height does the skydiver jump from?

(ii) How far above the ground is the skydiver when their horizontal distance from O is 50 m?

(iii) Explain why the model is not suitable for values of x larger than 70 m.

(4 marks)

3 (a) An Aerobatic Display Team use smoke trails as part of their flying display.
At the start of the display, each aeroplane holds a tank containing 100 litres of smoke.
The amount of smoke remaining in a tank is inversely proportional to 10 more than the total time that the smoke has been used for.

Show that

$$W = \frac{1000}{t + 10}$$

where W litres is the amount of smoke left in the tank t seconds after the smoke first started being used.

(3 marks)

(b) Find the amount of smoke left in the tank after 40 seconds of use.

(3 marks)

(c) Explain why the model suggests the tank never empties.

(1 mark)

4 (a) The linear function $T(h) = 38 - 1.2h$ is used to model the temperature of a human body, $T\,°C$, h hours after death.

(i) According to this model, what would the expected temperature be of the body of a human who died 6 hours ago?

(ii) If the temperature of a body is 27.2 °C, how long ago did the person die?

(3 marks)

(b) Police suspect a murder victim was killed at around 11 am.
The body was discovered at 3.30 pm the same day.
If the police's suspicions are correct, what should the temperature of the body have been when it was discovered?

(2 marks)

5 A gardener is modelling the number of hours of daylight his allotment receives at different times of the year using the function

$$h(t) = 11 + 7\cos\left(\frac{2\pi t}{365}\right) \qquad t \geq 0$$

where h is the number of hours of daylight on a given day, and t is the time measured in whole days. Note that $t = 0$ corresponds to the first day of the model.

(i) Find the number of hours of daylight on the 10th

(ii) Write down the maximum and minimum number of daylight hours the model predicts.

(iii) Suggest, with a reason, the season when this model would start.

(5 marks)

6 (a) The revenue (money received before expenses, tax, etc.) Mod*x* makes from selling maths t-shirts is calculated by multiplying the selling price by the number of items sold.
Last year a company sold 8000 t-shirts, at a price of £12 each, thus giving them a revenue of $£96\,000$.

This year, in order to maximise their revenue, Mod*x* intend to increase the t-shirt price by $£x$. It is expected, however, that each price increase of $£1$ will reduce the number of items sold by 400.

The expected revenue for this year is thus modelled by the equation

$$R = 96\,000 + 3200x - 400x^2$$

where $£x$ is the increase in price per item.

By factorising $3200x - 400x^2$ and completing the square, show that the expected revenue can be rewritten as $R = 102\,400 - 400(x-4)^2$.

(3 marks)

(b) (i) Write down the maximum revenue the company should expect this year.

(ii) What price should the company sell the item at this year order to maximise revenue?

(3 marks)

7 (a) A business owner is investing £30 000 at an interest rate of 1.14% and models the value of their investment using the formula

$$V(t) = 30\,000\left(1 + \frac{1.14}{100}\right)^t$$

where *V* is the value of the investment t years after the initial £30 000 investment.

Find the value of the investment after 7 years.

(1 mark)

(b) Find the number of whole years it will take for the value of the investment to double.

(3 marks)

(c) Another business owner uses a similar model to invest £25 000 at an interest rate of 1.3%. Write down the formula for this model, in the form $V = f(t)$.

(1 mark)

8 (a) The flight of a hot air balloon ascending from the ground to its cruising altitude is modelled according to the function

$$a(t) = 5t^3 + 5t^2 \qquad 0 \leq t \leq 10$$

where t is the time of ascent in minutes and a is the altitude in feet.

Find the altitude of the hot air balloon after:

(i) 4 minutes,

(ii) 6 minutes.

(2 marks)

(b) Find the change in altitude of the hot air balloon between 5 minutes and 9 minutes.

(2 marks)

(c) Explain why the model should not be used for larger values of t.

(1 mark)

Medium Questions

1 ***Note: For this question ensure you are working in degrees.***

A wave tank is used to simulate the sea at high tide.
At a certain point along the tank the height of water is measured relative to calm sea level which has a height of 0 cm.
The height of water in the tank is modelled by the function

$$h(t) = 15\sin(18t)° \qquad t \geq 0$$

where h cm is the height of water and t seconds is the time after the simulation begins.

(i) According to the model what is the maximum height the water will reach?

(ii) How frequent are the waves generated by the tank?

(iii) How many waves are generated per minute?

(iv) How often is the water at calm sea level?

(v) Give one criticism of using this model to simulate actual sea waves.

(5 marks)

2 A skydiver jumps from a moving aircraft at a point directly above a fixed point, *O*, on the ground. The trajectory of the skydiver is then modelled by the function

$$h(x) = 3200 - 0.5x^2$$

where h m is the height of the skydiver above the ground and x m is the horizontal distance along the ground from point *O*.

(i) Explain the significance of the value 3200 in the model.

(ii) How much ground has the skydiver covered when they land?

(iii) Sketch a graph of h against x.

(iv) Explain why the model is not suitable for values of x larger than 80 m.

(7 marks)

3 (a) The White Blades Aerobatic Display Team use white smoke trails as part of their flying display.

At the start of the display, each aeroplane holds a tank containing 180 litres of white smoke fluid.

The amount of white smoke fluid remaining in an aeroplane's tank is inversely proportional to the total time that white smoke has been produced plus 25 seconds.

Find an equation to model the relationship between the amount of white smoke fluid left in the tank, W , and the total time, t , that white smoke has been produced.

(3 marks)

(b) The smoke-producing mechanism becomes unreliable after the amount of white smoke fluid remaining in the tank drops below 7% of the initial amount.
Given this restriction, find the maximum whole number of minutes that each aeroplane can reliably produce white smoke.

(3 marks)

(c) State a problem with the model for large values of t.

(1 mark)

4 (a) The linear function

$$T(t) = 37.7 - 1.5t$$

is used to model the temperature of a human body, $T\,°C$, t hours after death.

(i) According to this model, what would the expected temperature be of the body of a human who died 12 hours ago?

(ii) If the temperature of a body is 18.2 °C, how long ago did the person die?

(3 marks)

(b) (i) What does the value 37.7 in the model represent?

(ii) Criticise the model for large values of t.

(2 marks)

(c) This model only gives the approximate temperature, and historical data suggests that actual temperatures can be within 20% of those predicted by the model.

Police suspect a murder victim was killed around 6 hours prior to the body being discovered. The temperature of the body when discovered was 23.5 °C.
Does the temperature of the body support the suspicions of the police regarding the time of the murder? Fully justify your answer.

(3 marks)

5 A gardener is modelling the number of hours of daylight his allotment receives at different times of the year using the function

$$h(t) = 12 + 5\sin\left(\frac{2\pi t}{365}\right) \qquad t \geq 0$$

where h is the number of hours of daylight on a given day, and t is the time measured in whole days. Note that $t = 0$ corresponds to the first day of the model.

(i) Find the number of hours of daylight on the 100th day.

(ii) Write down the maximum and minimum number of daylight hours the model predicts.

(iii) Assuming the allotment is located in the UK, give a reason why the first day of the model most likely does ***not*** correspond to 1st january.

(5 marks)

6 (a) The revenue (money received before expenses, tax, etc.) a company makes from selling a particular item is calculated by multiplying the selling price by the number of items sold.

Last year BuoysToys sold 6000 remote-controlled boats at a price of £40 each, thus giving them a revenue of $£40 \times 6000 = £240\ 000$.

This year they wish to increase the price of the remote-controlled boat in order to maximise the revenue but realise that a price increase will also reduce the number of the toys sold.

BuoysToys expect that 200 fewer remote-controlled boats will be sold this year compared to last year for every £2 increase they make in the price.

Show that the expected revenue for this year, $£R$, would be

$$R = 240\ 000 + 4000x - 400x^2$$

where x is the number of £2 increases BuoysToys make to the price of the remote-controlled boat.

(3 marks)

(b) By completing the square show that

$$R = 250\ 000 - 400(x - 5)^2$$

(3 marks)

(c) Hence find the price at which BuoysToys should sell the remote-controlled boat this year in order to maximise revenue. State the revenue expected.

(2 marks)

7 (a) In a simple model of an investment account, the function

$$V(t) = I(1 + \frac{r}{100})^t$$

is used where I is the initial amount invested, $r\,\%$ is the interest rate, and V is the value of the investment t years later.

For an initial investment of £1000, find the value after 12 years at an interest rate of 0.8%.

(2 marks)

(b) After investing £400 for 8 years the value of the investment is £448.82.
Find the interest rate to three significant figures.

(3 marks)

(c) Find the least number of years £20 000 would need to be invested at an interest rate of 3.4% in order for its value to have doubled.

(4 marks)

(d) Describe a refinement to the model that would more realistically reflect the way savings and investment accounts work.

(1 mark)

8 (a) A hot air balloon is modelled ascending from the ground to its cruising height according to the function

$$a(t) = 8t^3 - 132t^2 + 726t \qquad 0 \leq t \leq 11$$

where t is the time of ascent in minutes and a is the altitude in feet.

(i) It takes 11 minutes for the hot air balloon to reach its cruising altitude. Find the cruising altitude.

(ii) Show the hot air balloon rises by just 250 feet between 3 and 8 minutes. How does this suggest the pilot flew the hot air balloon during its ascent.

(4 marks)

(b) Show that there is only one real solution to the equation $a(t) = 0$ and hence explain why the model cannot be used indefinitely for the altitude of the hot air balloon.

(4 marks)

Hard Questions

1 (a) ***Note: For this question ensure you are working in degrees.***

A wave tank is used to simulate the sea at high tide.
At a certain point along the tank the height of water is measured relative to the calm water level which has a height of $0\ cm$.
The height of water in the tank is modelled by the function

$$h(t) = 12\cos(20t)° \qquad t \geq 0$$

where h cm is the height of water and t seconds is the time after the peak of the first wave passes the measuring point.

Sketch a graph of h against t for $0 \leq t \leq 54$.

(3 marks)

(b) (i) What is the maximum height the water reaches according to the model?

(ii) How frequent are the waves generated by the tank?

(iii) How often is the water at its calm level?

(iv) When will the peak of the 12^{th} wave pass the measuring point?

(4 marks)

(c) Comment on the suitability of using this model to simulate actual sea waves.

(1 mark)

2 (a) A skydiver jumps from a moving aircraft at an altitude of 10 000 feet directly above a fixed point, *O*, on the ground. The trajectory of the skydiver is then modelled by the function

$$h(x) = 3048 - 0.5x^2$$

where h m is the height of the skydiver above the ground and x m is the horizontal distance along the ground from point *O*.

(i) Explain why the value 10 000 does not appear in the model.

(ii) Find the height of the skydiver at the point when they have covered a ground distance of 60 m.

(iii) How much ground has the skydiver covered when they land on the ground?

(4 marks)

(b) (i) Using a straight line between the start and end points of the skydive, find an approximation for the distance travelled by the skydiver.

(ii) Given that the skydive took 84 seconds, find an approximation for the average speed of the skydiver. State whether this is an underestimate or an overestimate.

(3 marks)

3 (a) The Blue Blades Aerobatic Display Team use white and blue smoke trails as part of their flying display.

At the start of the display, each aeroplane holds a tank of 360 litres of white smoke fluid and a separate tank of blue smoke fluid.

A function of the form

$$w = \frac{k}{t+p} - q$$

is used to model the amount of white smoke fluid remaining in its tank, w litres, after time t seconds of use. k, p and q are positive constants.

(i) Given that $q = 90$ and it takes 50 seconds for the amount of white smoke fluid in its tank to halve, find the values of k and p.

(ii) Hence find the number of minutes of use that a tank of 360 litres of white smoke fluid will last for.

(6 marks)

(b) A similar model used for the blue smoke is as follows

$$b = \frac{8400}{t+40} - 30$$

Find the initial amount of blue smoke fluid in the tank, and determine how many minutes of use the blue smoke fluid will last for.

(3 marks)

4 (a) The function

$$t(T) = 25 \ln\left(\frac{98.6 - R}{T - R}\right) \qquad R < T \leq 98.6$$

is used as a model to estimate the time, t hours, since the death of a human body. $T\,°F$ is the temperature of the body at a given time.

$R\,°F$ is the ambient (surrounding) temperature, assumed to be constant.

(i) If a body records a temperature of $81\,°F$ at an ambient temperature of $70\,°F$, estimate how long ago the person died.

(ii) A suspected murder victim's body was discovered 15 hours after the victim died. Assuming the victim was found in a room of fixed temperature $70\,°F$, what temperature should the body have registered when it was discovered?

(5 marks)

(b) (i) What does the value 98.6 in the model represent?

(ii) Describe a problem with using the model for a body temperature reading that is close to the ambient temperature.

(2 marks)

5 A gardener is modelling the number of hours of daylight his allotment receives at different times of the year using the function

$$h(t) = 12 - a\sin\left(\frac{2\pi t}{365}\right) \qquad t \geq 0$$

where h is the number of hours of daylight on a given day, t is the time measured in whole days, and a is a positive constant.

(i) Given that the maximum amount of daylight predicted by the model is 16 hours write down the value of a.

(ii) The gardener is also a keen golfer.
In order to have enough daylight to play golf after working in the garden there needs to be at least 9 hours daylight in the day.
On approximately how many days of the year can the gardener ***not*** play golf.

(5 marks)

6 The revenue (money received before expenses, tax, etc.) a company makes from selling a particular item is calculated by multiplying the selling price by the number of items sold.

Last year Toys2Go sold 120 000 doll's houses at a price of £80 each.

Toys2Go wish to maximise their revenue from sales of the doll's houses this year by increasing the price. However a price increase will also lead to a reduction in the number of doll's houses sold.

Toys2Go expect that 5000 fewer doll's houses will be sold this year compared to last year for every £5 increase they make to the price.

(i) Show that the expected revenue for this year, £ R , would be

$$R = 10000000 - 25000(x-4)^2$$

where x is the number of £5 increases Toys2Go make to the price of a doll's house.

(ii) Hence find the price Toys2Go should sell the doll's house for this year in order to maximise revenue. State the revenue expected.

(6 marks)

7 (a) The extremely rare Bouncing Unicorn has the ability to jump repeatedly with precision, such that both the height of the jump and the length of the jump remain constant.

The way in which Bouncing Unicorns jump can be modelled by the function

$$h(x) = |a \sin bx| \quad x \geq 0$$

where x is the horizontal distance covered and h is the height, both measured in metres. a and b are positive constants.

(i) Explain the meaning of the constant b in the context of the model.

(ii) A fully-grown Bouncing Unicorn jumps to a maximum height of 2.5 metres covering a horizontal distance of $\frac{2\pi}{3}$ metres in the process.

Write down the values of a and b for a fully-grown Bouncing Unicorn.

(3 marks)

(b) A fully-grown Bouncing Unicorn takes 3 seconds to complete one jump.
Estimate the amount of time during a single jump that a Bouncing Unicorn spends 1.5 metres or more above the ground.
State any assumptions you make for this question.

(5 marks)

8 (a) In a simple model of an investment account, the function

$$V(t) = I\left(1 + \frac{r}{100}\right)^t$$

is used where I is the initial amount invested, $r\,\%$ is the interest rate, and V is the value of the investment at the end of t years.

(i) For an initial investment of £7500, find the interest rate if, after 5 years, the investment value is £8250.

(ii) Find the least number of years an initial amount of money would need to be invested at an interest rate of 5.6% in order for its value to triple.

(5 marks)

(b) In 1998 the interest rate was 6.33%.
In 2019 the interest rate was 1.39%.
Assuming the interest rate remains constant from the initial time of investment, find roughly how many times greater the initial investment would have to be in 2019 compared to 1998 if the aim in both cases was to have an investment worth a million pounds 25 years later.

(3 marks)

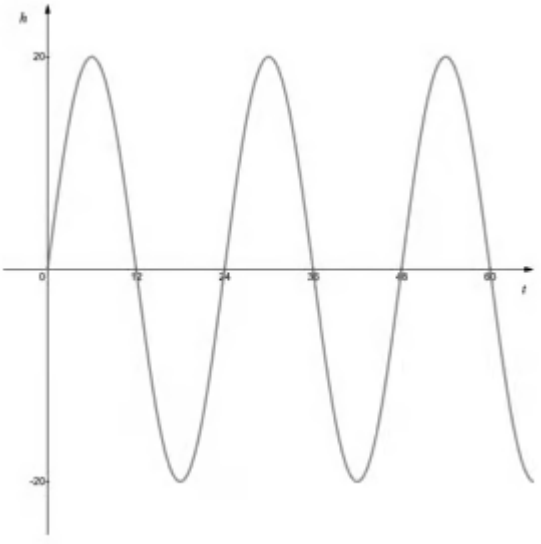
Very Hard Questions

1 (a) ***Note: For this question ensure you are working in degrees.***

A wave tank is used to simulate the sea at high tide.

At a certain point along the tank the height of water is measured relative to the calm water level which has a height of 0 cm.

The graph of the height of water, h cm against the time after the simulation is started, t seconds is shown below.



(i) According to the graph what is the maximum height the water will reach

(ii) How frequent are the waves generated by the tank?

(iii) Use the graph to write down a function for the model in the form
$h(t) = A\sin(Bt)°$ $t \geq 0$

(4 marks)

(b) Write down a model of the form $h(t) = A\sin(Bt)°$ that could be used to generate waves of double the amplitude and at a frequency of 15 waves per minute.

(2 marks)

(c) Suggest a way the model can be improved for simulating actual sea waves.

(1 mark)

2 A skydiver jumps from a moving aircraft directly above a fixed point, *O*, on the ground. The trajectory of the skydiver is then modelled by the function

$$h(x) = 4000 - 0.1x^2$$

where h m is the height of the skydiver above the ground and x m is the horizontal distance along the ground from point *O*.

(i) From what altitude does the skydiver jump from the aircraft?

(ii) Find the height of the skydiver at the point when they have covered a ground distance of 60 m.

(iii) How much ground has the skydiver covered when they land on the ground?

(iv) The landing zone is in the shape of a circle with an area of $90000m^2$.
Assuming the skydiver lands exactly in the centre of this circle determine whether the point O lies within the landing zone or not.

(6 marks)

3 (a) The Red Blades Aerobatic Display Team use white and red smoke trails as part of their flying display.

At the start of the display, each aeroplane holds a tank containing 270 litres of white smoke fluid, and another tank containing 90 litres of red smoke fluid.

A function of the form

$$w = \frac{30\,375}{t+p} - q$$

is proposed to model the amount of white smoke fluid remaining in its tank, w litres, after time t seconds of its use. p and q are positive constants.

The white smoke fluid runs out after 6 minutes of use.
Find the values of p and q.

(5 marks)

(b) A similar model for the red smoke fluid is also used

$$r = \frac{36\,000}{t+240} - 60$$

Find the time at which the white smoke fluid tank and the red smoke fluid tank have the same amount of fluid in them. Comment on your answer.

(4 marks)

4 A gardener wants to model the number of hours of daylight his allotment receives at different times of the year using a function of the form

$$h(t) = a + b\sin\left(\frac{2\pi}{365}t\right) \qquad t \geq 0$$

where h is the number of hours of daylight on a given day, t is the time measured in whole days, and a and b are positive constants. Note that $t = 0$ corresponds to the first day of the model.

(i) Given that the model needs to predict a maximum of 17 hours daylight and a minimum of 7 hours daylight find the values of a and b.

(ii) Explain the significance of the value $\frac{2\pi}{365}$ in the model.

(iii) Suggest, with a reason, the date of the year that the model starts on.

(4 marks)

5 The revenue (money received before expenses, tax, etc.) a company makes from selling a particular item is calculated by multiplying the selling price by the number of items sold.

Last year Toys-were-Us sold 3 million computer games at a price of £50 each.

Toys-were-Us wish to maximise their revenue from sales of computer games this year. However a change in price could lead to a change in the number of computer games sold.

Toys-were-Us expect there will be a change of 500 000 computer games sold for every £2 change they make in the price.

So for every £2 increase to the price, sales drop by 500 000.

For every £2 decrease in price, sales increase by 500 000.

Find the price Toys-were-Us should sell computer games at this year in order to maximise revenue. State the expected number of games they will sell and the expected revenue.

(6 marks)

6 (a) The rare Leaping Unicorn jumps in such a way that the length of a jump is always the same distance. However, the maximum height a Leaping Unicorn reaches during a jump reduces gradually over time as the unicorn tires.

The way in which Leaping Unicorns jump can be modelled by the function

$$h(x) = |A(e - kx)\sin x| \qquad x \geq 0$$

where x is the horizontal distance covered and h is the height, both measured in metres.
A and k are both positive constants.

(i) Write down the length of a Leaping Unicorn jump.

(ii) Briefly describe how changing the value of the constant k would affect the model.

(2 marks)

(b) During its first jump, a Leaping Unicorn reaches a maximum height of 1.288 metres after covering 1.471 metres over the ground.
Find the values of A and k.

(5 marks)

(c) What is the total distance of ground covered by a Leaping Unicorn when it is at the maximum height of its third jump?

(2 marks)

7 (a) In a simple model of an investment account, the function

$$V(t) = I\left(1 + \frac{r}{100}\right)^t$$

is used where I is the initial amount invested, $r\,\%$ is the interest rate, and

V is the value of the investment at the end of t years.

Find the interest rate required for an amount of money invested for 8 years to double in value.

(3 marks)

(b) An investor is comparing two options offered by a local bank.

Option 1	**Option 2**
10 year investment	10 year investment
Interest rate is 3.4%	Interest rate is 3.85%
At the end of the 10 years your account will be credited with a £500 bonus.	

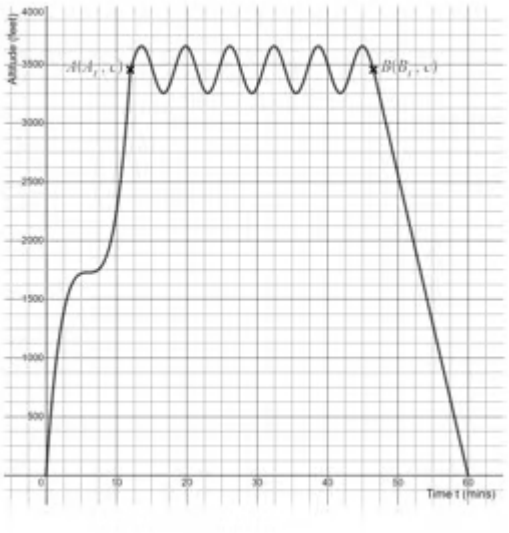
(i) Find the least amount of money an investor would need in order for Option 2 to give a greater return than Option 1.

(ii) What advice would you give to a customer with £8100 to invest?
Justify your answer.

(5 marks)

8 (a) The flight path of a hot air balloon is planned according to the graph below.

The path is made up of three segments - ascent, cruising and descent.



Point $A(A_t, c)$ is the point where the flight path changes from ascent to cruising.

Point $B(B_t, c)$ is the point where the flight path changes from cruising to descent.

The functions for the ascent and cruising segments of the flight are given below.

Ascent:	$\mathrm{f}(t) = 8(t-6)^3 + 1728$	$0 \le t \le A_t$
Cruise:	$\mathrm{g}(t) = 3456 + 200\sin(t-12)$	$A_t \le t \le B_t$

where $\mathrm{f}(t)$ and $\mathrm{g}(t)$ give the altitude in feet at a time t minutes after the commencement of the balloon's flight.

The balloon begins and ends the cruising segment of its flight at an altitude midway between its minimum and maximum cruising altitudes.

Use the information given to deduce

(i) The values of , A_t, B_t and c

(ii) The difference between the maximum and minimum cruising altitudes.

(5 marks)

(b) The total flight time is planned to be 60 minutes. The descent part of the journey is modelled by a linear function, $\mathrm{h}(t)$, where $B_t \leq t \leq 60$. Find an equation for $\mathrm{h}(t)$.

(4 marks)

(c) Describe a problem with attempting to model hot air balloon flights in this manner.

(1 mark)