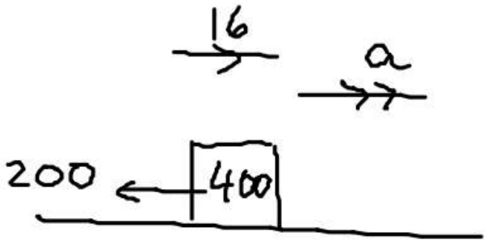
11 The engine of a van of mass 400 kg cuts out when it is moving along a straight horizontal road with speed $16\,\text{m}\,\text{s}^{-1}$. The van comes to rest without the brakes being applied.

In a model of the situation it is assumed that the van is subject to a resistive force which has constant magnitude of 200 N.

a Find how long it takes the van to stop. **(3 marks)**

b Find how far the van travels before it stops. **(2 marks)**

c Comment on the suitability of the modelling assumption. **(1 mark)**



$R \rightarrow,\ F = ma$

$-200 = 400a$

$-0.5 = a$

a)

$a = -0.5$

$u = 16$

$v = 0$

$t = ?$

$v = u + at$

$0 = 16 - 0.5t$

$0.5t = 16$

$\underline{\underline{t = 32}}$.

b)

$a = -0.5$

$u = 16$

$v = 0$

$s = ?$

$v^2 = u^2 + 2as$

$0 = 16^2 + 2 \times (-0.5) \times s$

$s = 16^2 = \underline{\underline{256\text{ m}}}$.

c) The resistance to motion is unlikely to be constant.

Motion in 2 dimensions - using vectors again!

In Chapter 8 we saw that many physical quantities could have both direction and magnitude, and therefore could be represented as a vector:

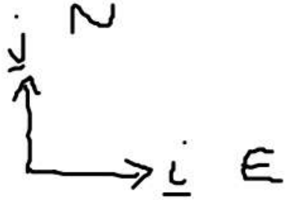
Can be a vector:	Force, acceleration, velocity, displacement
Scalar only:	Mass, area, volume

This naturally means that $\boldsymbol{F} = m\boldsymbol{a}$ works with vectors too.

resultant force

$\underline{F} = m\,\underline{a}$

N



Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(3\boldsymbol{i} + 8\boldsymbol{j})$ N acts upon a particle of mass 0.5 kg.

(a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})$ ms^{-2}.

(b) Find the magnitude and bearing of the acceleration of the particle.

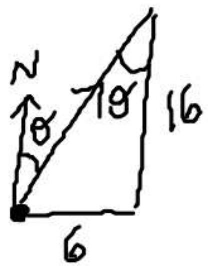
a) $\underline{F} = m\,\underline{a}$

$\begin{pmatrix} 3 \\ 8 \end{pmatrix} = 0.5\,\underline{a}$

$2\begin{pmatrix} 3 \\ 8 \end{pmatrix} = \underline{a}$

$\underline{a} = \begin{pmatrix} 6 \\ 16 \end{pmatrix} = (6\underline{i} + 16\underline{j})\ \text{ms}^{-2}$

b) $|\underline{a}| = \sqrt{6^2 + 16^2} = 2\sqrt{73} = 17.1\ \text{ms}^{-2}$ (3sf)



$\theta = \tan^{-1}\left(\frac{6}{16}\right) = 20.56°$

bearing $\underline{\underline{021°}}$

Three forces act on a particle of mass 3kg. Find the acceleration of the particle.

$m = 3$

$$F_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} N, \qquad F_2 = \begin{pmatrix} -5 \\ 4 \end{pmatrix} N, \qquad F_3 = \begin{pmatrix} 6 \\ -5 \end{pmatrix} N$$

$\underline{F} = m\underline{a}$

$\underline{R} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3\underline{a}$

$\underline{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} ms^{-2}$

A boat is modelled as a particle of mass 60 kg being acted on by three forces.

$m = 60$

$$F_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N, \qquad F_2 = \begin{pmatrix} 10p \\ 20q \end{pmatrix} N, \qquad F_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of $\underline{p}$ and $\underline{q}$.

$\underline{R} = \begin{pmatrix} 80 \\ 50 \end{pmatrix} + \begin{pmatrix} 10p \\ 20q \end{pmatrix} + \begin{pmatrix} -75 \\ 100 \end{pmatrix} = \begin{pmatrix} 5 + 10p \\ 150 + 20q \end{pmatrix}$

$\underline{F} = m\underline{a}$

$\begin{pmatrix} 5 + 10p \\ 150 + 20q \end{pmatrix} = 60 \begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$

$5 + 10p = 60 \times 0.8$

$5 + 10p = 48$

$10p = 43$

$\underline{\underline{p = 4.3}}$

$150 + 20q = 60 \times (-1.5)$

$150 + 20q = -90$

$20q = -240$

$\underline{\underline{q = -12}}$

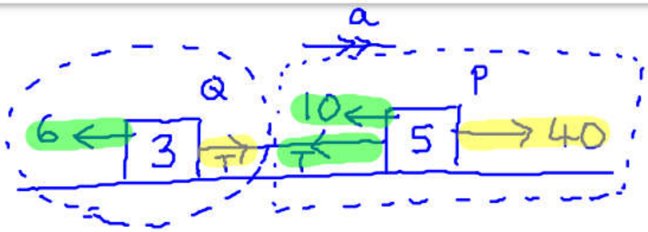
Odd Questions

Ex 10D

Connected Particles - horizontal surface

Up to now we've only considered one particle at a time.
When we have multiple connected objects moving in the same direction, **they can be considered as a single particle**:

> Two particles, P and Q, of masses 5kg and 3kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40N along a rough horizontal plane. Particle P experiences a frictional force of 10N and particle Q experiences a frictional force of 6N.
> (a) Find the acceleration of the particles.
> (b) Find the tension in the string.
> (c) Explain how the modelling assumptions that the string is light and inextensible have been used.



tension $\rightarrow \leftarrow$

a) $R\rightarrow$, $F=ma$

$$40+T-T-10-6=8a$$
$$24=8a$$
$$\underline{\underline{a=3\,ms^{-2}}}$$

b) $R\rightarrow$, $F=ma$, Q

$$T-6=3\times 3$$
$$T-6=9$$
$$\underline{\underline{T=15N}}$$

$R\rightarrow$, $F=ma$, P

$$40-10-T=5\times 3$$
$$30-T=15$$
$$\underline{\underline{T=15N}}$$

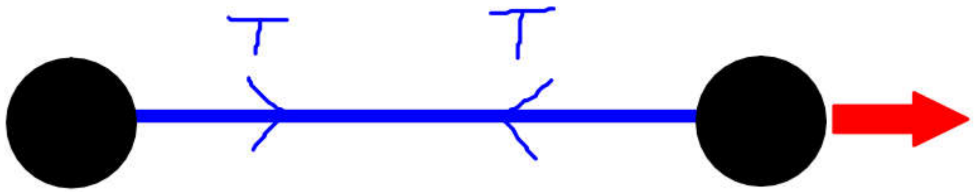
c) Light: the tension in the string is equal throughout.
(a heavy rope would need to support its own weight too, which would be greater at the ends)
Inextensible: the acceleration is equal for both particles.
(the string is not stretching, so both must move at the same rate)

Tension vs Thrust

tension

rigid bar/rod

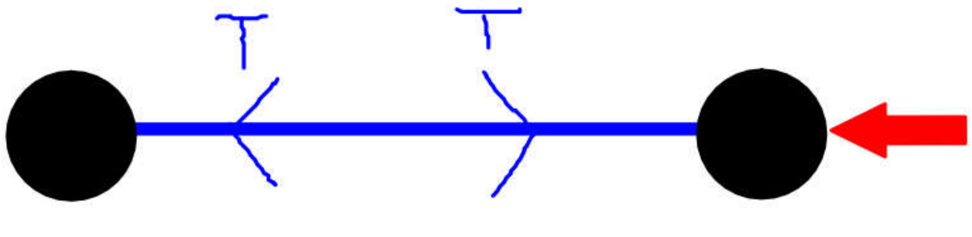
"pulling out"



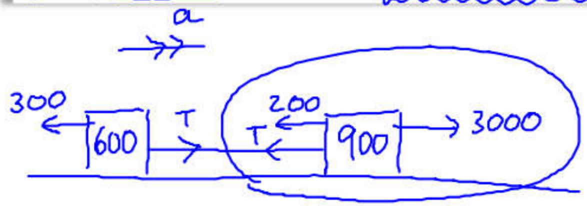
thrust

rigid bar/rod

"pushing in"



a) A car of mass 900 kg tows a trailer of mass 600 kg by means of a rigid tow bar. The car experiences a resistance of 200 N and the trailer a resistance of 300 N. If the car engine exerts a driving force of 3000 N, find the acceleration of the system and the tension in the tow bar.



R→, F=ma, whole system

$3000 + \cancel{T - T} - 200 - 300$

$3000 - 200 - 300 = 1500a$

$2500 = 1500a$

$a = \frac{5}{3}\ ms^{-2}$

$a = \underline{\underline{1.67 ms^{-2}}}$.

F=ma, R→, car

$3000 - 200 - T = 900 \times \frac{5}{3}$

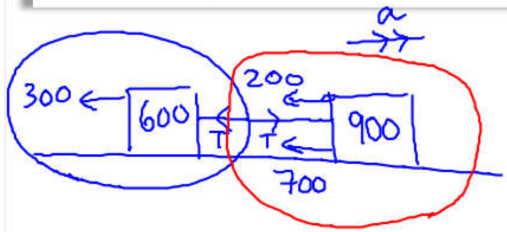
$2800 - T = 1500$

$T = \underline{\underline{1300}}\ N$

$T - 300 = 600 \times \frac{5}{3}$

$T = 1000 + 300.$

b) The driver sees a set of traffic lights ahead that have turned red. She reduces the driving force produced by the engine to zero, and applies an additional resistant braking force of 700N. Find the deceleration of the system and the thrust in the tow bar.



F=ma, R→, whole system

$-200 - 300 - 700 + T - T = 1500a$

$-1200 = 1500a$

$\underline{\underline{-0.8 = a}}$

So deceleration of $\underline{0.8 ms^{-2}}$

F=ma, R→, caravan/trailer

$-T - 300 = 600 \times (-0.8)$

$-T - 300 = -480$

$\underline{\underline{T = 180}}$.

car.

$T - 200 - 700 = 900 \times (-0.8)$

$T - 900 = -720$

$\underline{\underline{T = 180}}$.