OCR A Level Physics

Your notes

Planetary Motion

Contents

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Kepler's Three Laws of Motion

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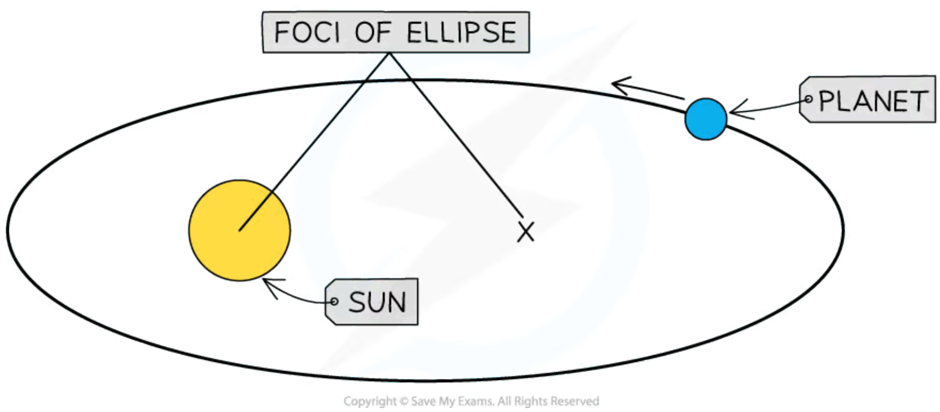
Kepler's First Law

- Kepler's First Law describes the **shape** of planetary orbits
- It states:

The orbit of a planet is an ellipse, with the Sun at one of the two foci



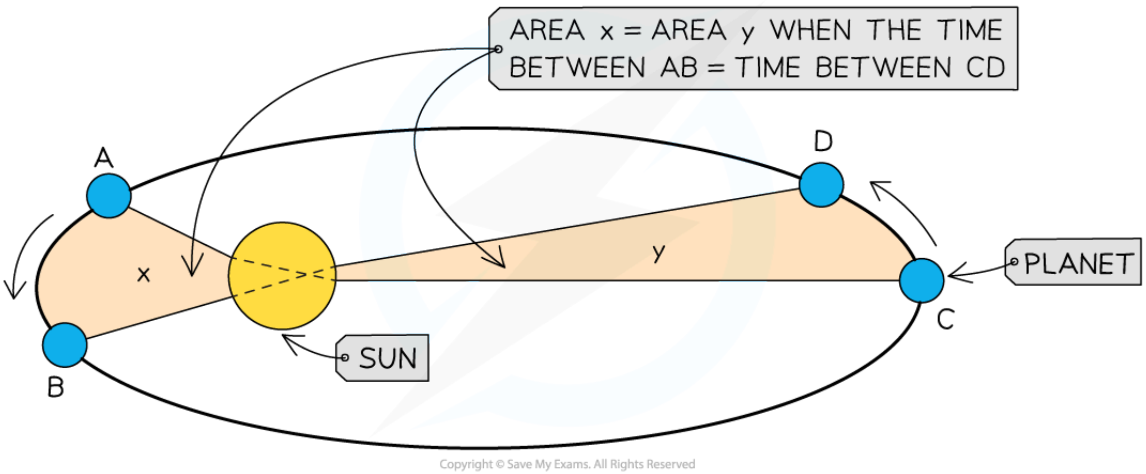
The orbit of all planets are elliptical, and with the Sun at one focus

- An ellipse is just a 'squashed' circle
  - Some planets, like Pluto, have highly elliptical orbits around the Sun
  - Other planets, like Earth, have near circular orbits around the Sun

Kepler's Second Law

- Kepler's Second Law describes the **motion** of all planets around the Sun
- It states:

A line segment joining the Sun to a planet sweeps out equal areas in equal time intervals



- The consequence of Kepler's Second Law is that planets move **faster** nearer the Sun and **slower** further away from it

Kepler's Third Law

- Kepler's Third Law describes the relationship between the **time** of an orbit and its **radius**
- It states;

  The square of the orbital time period T is directly proportional to the cube of the orbital radius r

- Kepler's Third Law can be written mathematically as:

$$T^2 \propto r^3$$

- Which becomes:

$$\frac{T^2}{r^3} = k$$

- Where:
  - T = orbital time period (s)
  - r = mean orbital radius (m)
  - k = constant ($s^2\ m^{-3}$)
- In the case of our solar system, k is constant for all planets orbiting the Sun

Examiner Tips and Tricks

You are expected to be able to describe Kepler's Laws of Motion, so make sure you are familiar with how they are worded.

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Applications of Kepler's Third Law

- Kepler's Third Law, the fact that $T^2 \propto r^3$ applies to **any body in orbit** about some **larger body**
- This means that it can be applied to other systems, not just the planets in our Solar System orbiting the sun, for example:
  - The moons orbiting other planets, like the four moons of Jupiter (Io, Europa, Callisto and Ganymede)
  - **Exoplanets** in orbit about foreign stars
- This is useful because measuring things like **time period** and **orbital radius** are commonplace in modern astrophysics
  - Therefore, **useful** and **interesting** data about the **mass** of orbital systems can be deduced from experimental data

Circular Orbits in Gravitational Fields

Your notes

Centripetal Force on a Planet

- Typically, most planets and satellites have a **near circular** orbit
  - Therefore, the gravitational force F between the Sun and another planet provides the centripetal force needed to stay in an orbit
- Since the gravitational force is centripetal, it is **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass m orbiting the Earth, with mass M, at a distance r from its centre and travelling with linear speed v
- The gravitational force F on the satellite is centripetal, therefore:

$$F = F_{centripetal}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The mass of the satellite m will cancel out on both sides to give:

$$v^2 = \frac{GM}{r}$$

- This can be reduced to give an equation for the **orbital speed *v*,** as:

$$v = \sqrt{\frac{GM}{r}}$$

- Where:
  - v = **orbital speed** of the mass in orbit (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)
  - r = orbital radius (m)

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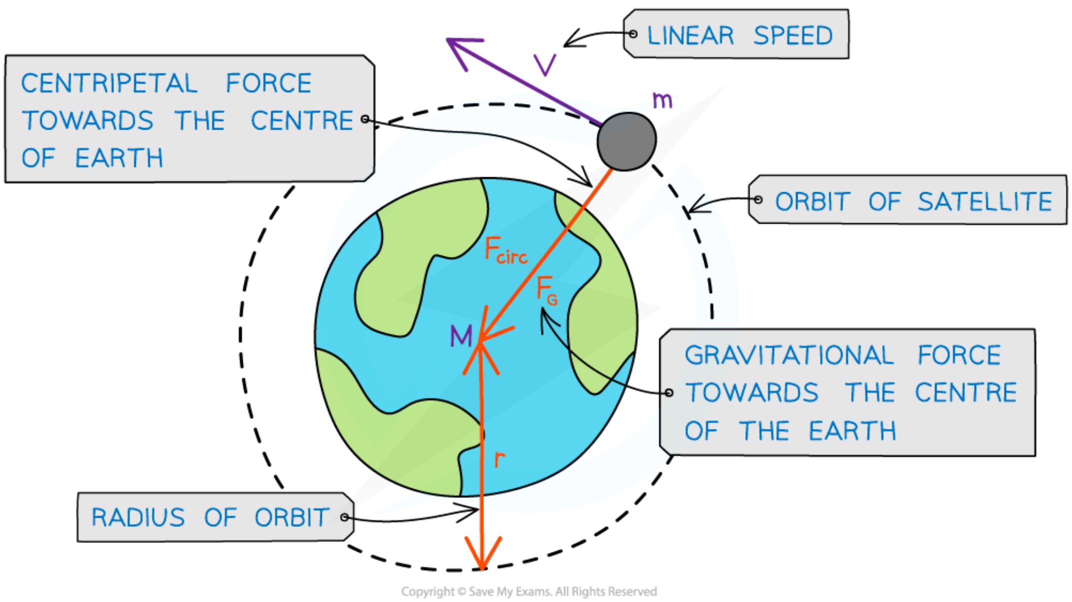
- This means that all satellites, whatever their mass, will travel at the same speed v in a particular orbit radius r
  - Since the direction of a planet orbiting in circular motion is constantly changing, it has centripetal acceleration



A satellite in orbit around the Earth travels in circular motion

Circular Orbits in Gravitational Fields

- **Assuming** a planet or a satellite is travelling in **circular motion** when in orbit, its orbital speed v is given by the distance travelled divided by the orbital period T:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time
  - Where the distance is equal to the circumference of a circle = $2\pi r$
- Recall, orbital speed v is also given by:

$$v^2 = \frac{GM}{r}$$

- Therefore:

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$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Expanding the brackets gives:

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

- Rearranging for T^2 gives the mathematical expression of **Kepler's Third Law** which relates the **time period** T and **orbital radius** r as:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

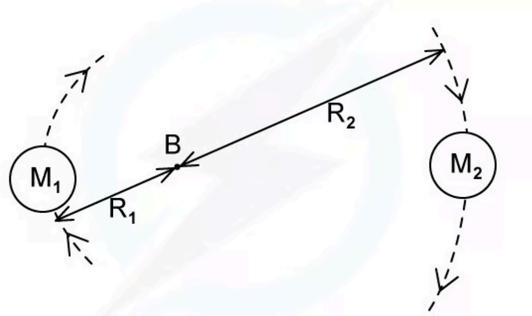
- Where:
  - T = time period of the orbit, or "orbital period" (s)
  - r = orbital radius (m)
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)

Worked Example

A binary star system constant of two stars orbiting about a fixed point **B**.

The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 also has a circular orbit of radius of R_2. Both have linear

speed v and an angular speed ω about **B**.



State the following formula, in terms of G, M_2, R_1 and R_2

(i) The angular speed ω of M_1

(ii) The time period T for each star in terms of angular speed ω

Answer:

Part (i)

(i) The angular speed ω of M_1

Step 1: Equate the centripetal force to the gravitational force

$$M_1 R_1 \omega^2 = \frac{G M_1 M_2}{(R_1 + R_2)^2}$$

Step 2: M_1 cancels on both sides

$$R_1 \omega^2 = \frac{G M_2}{(R_1 + R_2)^2}$$

Step 3: Rearrange for angular velocity ω

$$\omega^2 = \frac{G M_2}{R_1 (R_1 + R_2)^2}$$

Step 4: Square root both sides

$$\omega = \sqrt{\frac{G M_2}{R_1 (R_1 + R_2)^2}}$$

Part (ii)

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(ii) The time period T for each star in terms of angular speed ω

Step 1: Write down the angular speed ω equation with time period T

$$\omega = \frac{2\pi}{T}$$

Step 2: Rearrange for T

$$T = \frac{2\pi}{\omega}$$

Step 3: Substitute in ω from part (i)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{GM_2}}$$

Examiner Tips and Tricks

Many of the calculations in planetary motion questions depend on the equations for circular motion. Be sure to revisit these and understand how to use them! You will be expected to remember the derivation for $T^2 = \frac{4\pi^2}{GM}r^3$, so make sure you understand each step, especially equating the gravitational force and the centripetal force!

Geostationary Orbits

Your notes

Geostationary Orbits & Satellites

- Many communication satellites around Earth follow a **geostationary orbit**
  - This is sometimes referred to as a **geosynchronous** orbit
- This is a specific type of orbit in which the satellite:
  - Remains directly **above the equator**
  - Is in the **plane of the equator**
  - Always orbits at the **same point** above the Earth's surface
  - Moves from **west to east** (same direction as the Earth spins)
  - Has an **orbital time period equal** to Earth's **rotational period** of **24 hours**
- Geostationary satellites are used for **telecommunication** transmissions (e.g. radio) and television broadcast
- A base station on Earth sends the TV signal up to the satellite where it is amplified and broadcast back to the ground to the desired locations
- The satellite receiver dishes on the surface must point towards the same point in the sky
  - Since the geostationary orbits of the satellites are fixed, the receiver dishes can be fixed too

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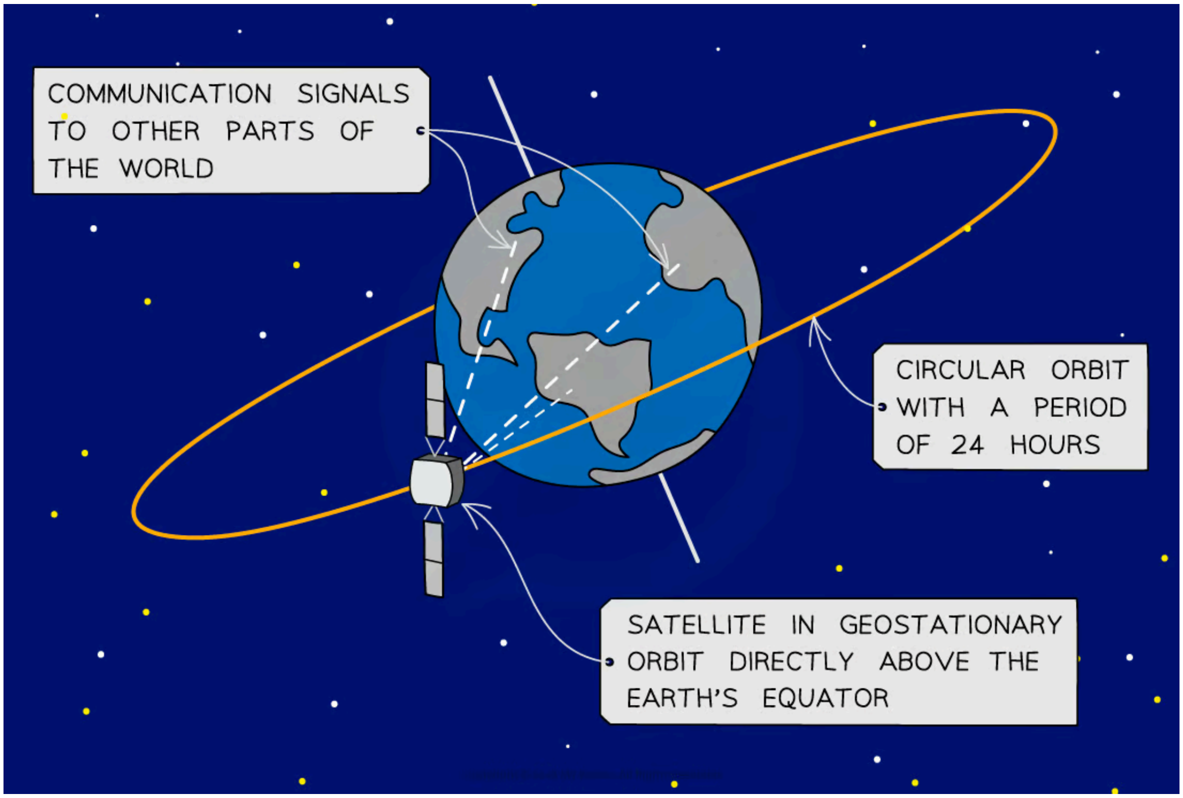

Geostationary satellite in orbit

Worked Example

Calculate the distance above the Earth's surface that a geostationary satellite will orbit.

Mass of the Earth = 6.0×10^{24} kg

Radius of the Earth = 6400 km

Answer:

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STEP 1

KEPLER'S THIRD LAW EQUATION

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

STEP 2

REARRANGE FOR r, THE RADIUS OF THE ORBIT

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

STEP 3

SUBSTITUTE IN VALUES

THE TIME PERIOD T FOR A GEOSTATIONARY ORBIT IS 24 HOURS = 86400s

$$r = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (86400)^2}{4\pi^2}}$$

$$r = 42297523.87\ \text{m} = 4.2 \times 10^7\ \text{m} \quad (2\ \text{s.f.})$$

STEP 4

CALCULATE DISTANCE ABOVE THE EARTH'S SURFACE

r IS THE DISTANCE FROM THE CENTRE OF THE EARTH TO THE SATELLITE

DISTANCE ABOVE SURFACE = RADIUS OF ORBIT − RADIUS OF EARTH

$$= 4.2 \times 10^7 - 6400 \times 10^3$$

$$= 3.6 \times 10^7\ \text{m} \quad (2\ \text{s.f.})$$

Examiner Tips and Tricks

Make sure to memorise the key features of a geostationary orbit, since this is a common exam question. Remember:

- Equatorial orbit
- Moves west to east
- Period of 24 hours

You will also be expected to remember that the time period of the orbit is 24 hours for calculations on a geostationary satellite.