

# Algebraic Fractions and Partial Fractions

[Edexcel C3 June 2013(R) Q1]

Express

$$\frac{3x + 5}{x^2 + x - 12} - \frac{2}{x - 3}$$

as a single fraction in its simplest form.

Express the following as a single fraction, giving your answer in its simplest form.

$$\frac{10x + 4}{3x^2 + 4x + 1} - \frac{3}{x + 1}$$

Extra practice, Ex 1B and 1C

## Partial Fractions

If the **denominator is a product of a linear terms**, it can be split into the sum of 'partial fractions', where **each denominator is a single linear term**.

$$\frac{6x - 2}{(x - 3)(x + 1)} \equiv \frac{A}{x - 3} + \frac{B}{x + 1}$$

**Notation reminder:**  $\equiv$  means 'equivalent/identical to', and indicates that both sides are equal for all values of  $x$ .

**Method 1:** Substitution

**Method 2:** Comparing Coefficients

Given that  $\frac{6x^2+5x-2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$ , find the values of the constants  $A, B, C$ .

## Your Turn

C4 June 2005 Q3a

Express  $\frac{5x+3}{(2x-3)(x+2)}$  in partial fractions.

(3)

## Partial Fractions - repeated linear factors

Suppose we wished to express  $\frac{2x+1}{(x+1)^2}$  as  $\frac{A}{x+1} + \frac{B}{x+1}$ . What's the problem?

Q

Split  $\frac{11x^2+14x+5}{(x+1)^2(2x+1)}$  into partial fractions.

The problem is resolved by having the factor **both squared and non-squared**.

### Your Turn

C4 June 2011 Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants  $A$ ,  $B$  and  $C$ .

(4)

# Partial Fractions - dealing with improper fractions

In Pure Year 1, we saw that the '**degree**' of a polynomial is the highest power, e.g. a quadratic has degree 2.

An algebraic fraction is **improper** if the degree of the numerator is **at least** the degree of the denominator.

$$\frac{x^2 - 3}{x + 2}$$

$$\frac{x + 1}{x - 1}$$

$$\frac{x^3 - x^2 + 3}{x^2 - x}$$

A partial fraction is still improper if the degree is the same top and bottom.

Questions might take one of two forms:

- Do the division to express as a quotient and a remainder, e.g.  $\frac{x+1}{x-1} \rightarrow 1 + \frac{2}{x-1}$
- Express as partial fractions, e.g.  $\frac{x^2+x}{(x+1)(x-2)} = A + \frac{B}{x+1} + \frac{C}{x-2}$

You know for example that as  $7 \div 3 = 2 \text{ rem } 1$ , we could write:

$$\frac{7}{3} = 2 + \frac{1}{3}$$

Similarly in general:

$$\frac{F(x)}{\text{divisor}} = Q(x) + \frac{\text{remainder}}{\text{divisor}}$$

Quotient

If  $\frac{x^2+5x-9}{x+2} = Ax + B + \frac{C}{x+2}$ , determine the values of  $A$ ,  $B$  and  $C$ .

## Your Turn

Edexcel C4 June 2013 Q1

Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$ .

(4)

**Tip:** There's a missing  $x$  term in the numerator and missing  $x$  term in the denominator. Use  $+0x$  to avoid gaps.

Ex 1F

## Dealing with Improper Fractions

Q

Split  $\frac{3x^2 - 3x - 2}{(x-1)(x-2)}$  into partial fractions.

**Method 1:** Algebraic Division

**Q**Split  $\frac{3x^2-3x-2}{(x-1)(x-2)}$  into partial fractions.**Method 2:** Using one identity

## Your Turn

C4 Jan 2013 Q3

Express  $\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$  in partial fractions.**(4)**