

2.4 Variable Acceleration - 2D (A Level only)

Easy (11 questions)	/51
Medium (10 questions)	/62
Hard (9 questions)	/63
Very Hard (8 questions)	/63
Total Marks	/239

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Easy Questions

1 (a) A particle's position, at time t seconds, is given by the vector

$$\mathbf{r} = \begin{pmatrix} 2t^3 - 1 \\ t^2 + 4 \end{pmatrix} m$$

- (i) Find the coordinates of the starting position of the particle.
- (ii) Find the position vector of the particle after 6 seconds.

(3 marks)

(b) Consider the y -coordinate of the particle to explain why the particle will never pass through the origin.

(1 mark)

2 (a) At time t seconds, a particle moving in a plane has velocity

$$\mathbf{v} = ((2t^3 - 4t)\mathbf{i} + (2t - 3)\mathbf{j}) \text{ m s}^{-1}$$

Use differentiation to find an expression for the acceleration of the particle.

(2 marks)

(b) Use integration to find the displacement of the particle from its initial position.

(3 marks)

3 (a) The acceleration of a particle is modelled using

$$\mathbf{a} = ((2 - 8t)\mathbf{i} + (6t^2)\mathbf{j}) \text{ m s}^{-2}$$

where time t is measured in seconds.

Given that the particle is initially at rest, use integration to find an expression for the velocity of the particle.

(2 marks)

- (b)** (i) Find the velocity of the particle at time $t = 2$ seconds
- (ii) Use Pythagoras' theorem to show that the speed of the particle at time $t = 2$ seconds is 20 m s^{-1} .

(3 marks)

4 (a) The position vector of a particle, at time t seconds, is given by

$$\mathbf{r} = ((\sin t)\mathbf{i} + (\cos 2t)\mathbf{j}) \text{ m} \quad 0 \leq t \leq \pi$$

Differentiate \mathbf{r} with respect to t to find an expression for the velocity of the particle.

(2 marks)

- (b)** (i) Find the time at which the velocity of the particle in the \mathbf{i} -direction is 0.5 m s^{-1} .
- (ii) Hence find the velocity in the \mathbf{j} -direction at this time.

(3 marks)

5 (a) The velocity of a particle at time t seconds is given by

$$\mathbf{v} = \begin{pmatrix} e^t - t \\ 0.5t^4 \end{pmatrix} \text{ m s}^{-1}$$

Given that the particle's motion began at the origin, use integration to find the position vector of the particle at time t seconds.

(2 marks)

(b) Use Pythagoras' theorem to find the distance of the particle from the origin at time $t = 1$ second, giving your answer to three significant figures.

(2 marks)

6 The acceleration of a particle is modelled using the equation

$$\mathbf{a} = \begin{pmatrix} 3t^2 - 1 \\ 5e^{-t} \end{pmatrix} \text{ m s}^{-2}$$

where time t is measured in seconds.

(i) Use integration to find an expression for the velocity of the particle.

- (ii) Given that when $t = 0$, $\mathbf{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ find the value of the constant(s) of integration.

(4 marks)

7 (a) The motion of a particle, starting from the origin, is described by the position vector

$$\mathbf{r} = ((3t^3 - t)\mathbf{i} + (2t^2 - 1)\mathbf{j}) \text{ m}$$

where time t is measured in seconds.

Differentiate \mathbf{r} with respect to t **twice** to find an expression for the **acceleration** of the particle.

(3 marks)

- (b)** (i) Find the acceleration of the particle at time $t = 3$ seconds.
- (ii) Use Pythagoras' theorem to find the magnitude of acceleration of the particle at time $t = 3$ seconds.

(3 marks)

8 (a) The velocity of a particle at time t seconds is given by

$$\mathbf{v} = \begin{pmatrix} 8t^{\frac{1}{2}} + 2t \\ 3t^2 + 5t - 1 \end{pmatrix} \text{ m s}^{-1}$$

Differentiate \mathbf{v} to find the acceleration, $\mathbf{a} \text{ m s}^{-2}$, of the particle at time t seconds.

(2 marks)

(b) Use Pythagoras' theorem to find the magnitude of acceleration of the particle at time $t = 4$ seconds, giving your answer to three significant figures.

(2 marks)

9 (a) A particle, starting from rest at the origin, has acceleration

$$\mathbf{a} = ((6t - 2)\mathbf{i} + (4 - 12t)\mathbf{j}) \text{ m s}^{-2}$$

at time t seconds.

Integrate \mathbf{a} with respect to t *twice* to find an expression for the ***position vector*** of the particle. Remember to account for any constant(s) of integration.

(3 marks)

- (b)** (i) Find the position vector of the particle at time $t = 5$ seconds.
- (ii) Use Pythagoras' theorem to find the distance of the particle from the origin at time $t = 5$ seconds, giving your answer to three significant figures.

(3 marks)

10 (a) The velocity of a particle at time t seconds is given by

$$\mathbf{v} = ((10t - 3t^2)\mathbf{i} + (4t - 5)\mathbf{j}) \text{ m s}^{-1}$$

- (i) Differentiate \mathbf{v} to find the acceleration, $\mathbf{a} \text{ m s}^{-2}$, of the particle at time t seconds.
- (ii) Find the initial acceleration of the particle.

(2 marks)

(b) The particle's initial position is at the point $(4, 5)$.

- (i) Integrate \mathbf{v} to find an expression, in terms of t , for the position vector of the particle.
- (ii) Find the distance of the particle from the origin at time $t = 3$ seconds, giving your answer to three significant figures.

(3 marks)

11 (a) The velocity of a particle at time t seconds is given by

$$\dot{\mathbf{r}} = \begin{pmatrix} 12t^2 - 2t \\ 9t^2 - 1 \end{pmatrix} \text{ m s}^{-1}$$

Differentiate $\dot{\mathbf{r}}$ to find the acceleration, $\ddot{\mathbf{r}}$ m s^{-2} , of the particle at time t seconds.

(1 mark)

(b) Integrate $\dot{\mathbf{r}}$ to find the position vector, \mathbf{r} m , of the particle at time t seconds given that its initial position is the origin.

(2 marks)

Medium Questions

1 (a) The position vector of a boat, sailing on a lake relative to an origin, is

$$\mathbf{r} = (2 \sin t)\mathbf{i} + (2 - 2 \cos t)\mathbf{j} \text{ km}$$

where time t is measured in hours.

- (i) Show that the boat is initially at the origin.
- (ii) Show that the boat takes 2π hours until it first returns to the origin.

(3 marks)

(b) (i) Find the velocity, $\mathbf{v} \text{ km h}^{-1}$, of the boat, at time t hours.

(ii) Find the velocity of the boat at time $t = \frac{2}{3}\pi$ hours.

(3 marks)

2 (a) A particle moving in a plane has velocity, $\mathbf{v} \text{ m s}^{-1}$, at time t seconds, given by

$$\mathbf{v} = \begin{pmatrix} 4t - 3t^2 \\ 6t^2 - 2 \end{pmatrix}$$

- (i) Find an expression for the acceleration of the particle.
- (ii) Find the acceleration of the particle at time $t = 3$ seconds.

(3 marks)

- (b)**
- (i) Find the position vector of the particle given that its initial position is at the origin.
 - (ii) Find the position vector of the particle at time $t = 4$ seconds.

(3 marks)

- 3 (a)** Once an aircraft reaches its cruising height (at time $t = 0$ seconds) its acceleration is modelled by

$$\mathbf{a} = \begin{pmatrix} 30 - 2t \\ 4t - 3 \end{pmatrix} \text{ m s}^{-2} \quad t \geq 0$$

Given that the velocity of the aircraft at $t = 0$ is $\mathbf{v} = \begin{pmatrix} 200 \\ 150 \end{pmatrix} \text{ m s}^{-1}$, find the velocity of the aircraft in terms of t .

(3 marks)

- (b)** Find the speed of the aircraft at time $t = 4$ seconds, giving your answer in kilometres per hour to three significant figures.

(3 marks)

- 4 (a)** An ice skater moves across an ice rink such that their position, at time t seconds relative to an origin, is given by

$$\mathbf{r} = \begin{pmatrix} 0.2t^2 - 0.005t^3 \\ 0.5t + 2 \end{pmatrix} \text{m}$$

- (i) Briefly explain how you can tell the ice skater's motion did not start at the origin.
- (ii) Find the coordinates of the position of the ice skater after 40 seconds.
- (iii) Find the distance between the ice skater and the origin after 40 seconds.

(4 marks)

- (b)** (i) Find an expression for the velocity of the ice skater at time t seconds.
- (ii) Find an expression for the acceleration of the ice skater at time t seconds.

(3 marks)

- 5 (a)** A remote-controlled car is driven around a playground with velocity, $\mathbf{v} \text{ m s}^{-1}$, at time t seconds, given by

$$\mathbf{v} = (0.25)\mathbf{i} + (0.5t - 9)\mathbf{j}$$

Find an expression for the displacement of the remote-controlled car, $\mathbf{s} \text{ m}$, from its initial position.

(2 marks)

- (b)** The remote-controlled car is set in motion from the point $(2, -5)$. Find the position vector \mathbf{r} of the particle at time t seconds.

(1 mark)

- (c)** (i) Find the distance of the remote-controlled car from its initial position after 40 seconds.
- (ii) Find the distance of the remote-controlled car from the origin after 40 seconds.

(4 marks)

6 (a) A spider is crawling across the floor of a house such that it has acceleration

$$\mathbf{a} = (0.1t)\mathbf{i} + (0.6t^2 - 2t)\mathbf{j} \text{ m s}^{-2}$$

at time t seconds after the spider emerged from under the skirting board.

- (i) Given that the spider's initial velocity was $\mathbf{v} = -1.2\mathbf{i} + 1.8\mathbf{j}$, find the velocity of the spider at time t seconds.
- (ii) Find the speed of the spider after 3 seconds.

(4 marks)

- (b)** The point at which the spider emerged from under the skirting board is deemed the origin. Find the position vector of the spider at time t seconds.

(2 marks)

7 (a) The position vector of a particle at time t seconds is given by

$$\mathbf{r} = (12e^{-0.1t})\mathbf{i} + (24e^{-0.2t})\mathbf{j} \text{ m}$$

- (i) Write down the initial position of the particle.
- (ii) Briefly explain why the particle gets closer to the origin but never actually meets it.

(2 marks)

(b) Find an expression for the velocity of the particle at time t seconds.

(2 marks)

- (c)** (i) Find an expression for the acceleration of the particle at time t seconds.
- (ii) Find the magnitude of acceleration of the particle at time $t = 4$ seconds, giving your answer to three significant figures.

(3 marks)

- 8 (a)** A stone is thrown from the edge of a deep cave such that it will fall into the cave with its motion described by the equation

$$\mathbf{v} = \mathbf{i} + (1.5 - 0.3t^2)\mathbf{j} \text{ m s}^{-1}$$

where \mathbf{v} is the velocity of the particle t seconds after the stone is thrown.

Find the speed at which the stone is thrown.

(2 marks)

- (b)** (i) Find the acceleration of the stone at time t seconds.
- (ii) Find the magnitude of the acceleration of the stone at time $t = 2.5$ seconds.

(3 marks)

9 (a) A particle's velocity is modelled by the equation

$$\dot{\mathbf{r}} = \begin{pmatrix} 3t^2 - 6t \\ 4 - 8t^3 \end{pmatrix} \text{ m s}^{-1}$$

where t is the time in seconds.

Given that the particle is initially located at the point $(2, 1)$, find the position vector of the particle, \mathbf{r} m, at time t seconds.

(3 marks)

- (b)** (i) Find the acceleration of the particle, $\ddot{\mathbf{r}}$ m s⁻² at time t seconds.
- (ii) Find the time at which the particle has no acceleration in the horizontal direction.

(3 marks)

10 (a) At time t seconds, a particle P has position vector \mathbf{r} m, where

$$\mathbf{r} = (t^3 - 11t^2 - 16t + 2)\mathbf{i} + (t^3 + 2t - 1)\mathbf{j} \quad t \geq 0$$

Find the velocity of P at time t seconds, where $t \geq 0$.

(2 marks)

- (b)** (i) Find the value of t at the instant when P moves in a direction parallel to \mathbf{j} .
- (ii) Show that P will never move in a direction perpendicular to \mathbf{j} .

(4 marks)

Hard Questions

- 1 (a)** The position of a boat on a small lake, relative to a mooring point located at the origin, is given by the vector

$$\mathbf{r} = \left(-20 \sin\left(\frac{t}{360}\right)\right)\mathbf{i} + \left(20 - 20 \cos\left(\frac{t}{360}\right)\right)\mathbf{j} \text{ m}$$

where time t is measured in seconds.

- (i) Show that the boat is initially at the mooring point.
- (ii) Show that the distance from the mooring point to the boat at time $t = 180\pi$ is $20\sqrt{2}$ m.

(3 marks)

- (b)** (i) Find the velocity, $\mathbf{v} \text{ m s}^{-1}$, of the boat, at time t seconds.
- (ii) Show that the boat has a speed of $\frac{1}{18} \text{ m s}^{-1}$ when it first returns to the mooring point.

(3 marks)

2 (a) A particle moving in the x - y plane has velocity, $\mathbf{v} \text{ m s}^{-1}$, at time t seconds, given by

$$\mathbf{v} = \begin{pmatrix} 0.1t^3 - 3t^2 \\ 2t + 1 \end{pmatrix}$$

- (i) Find the acceleration, $\mathbf{a} \text{ m s}^{-2}$, of the particle at time t and explain how you can tell that the acceleration in the y -direction is constant.
- (ii) Other than $t = 0$, find the time at which the acceleration in the x -direction is zero.

(4 marks)

- (b)** Find the position vector of the particle given that its initial position is at the point $(-3, 5)$.

(3 marks)

- 3 (a)** Once an aircraft reaches its cruising height (at time $t = 0$ hours) its acceleration is modelled by

$$\mathbf{a} = (4t^3 - 6t^2)\mathbf{i} + (0.9t^2 - 1)\mathbf{j} \text{ km h}^{-2}$$

Given that the velocity of the aircraft at $t = 5$ hours is $\mathbf{v} = 400\mathbf{i} + 40\mathbf{j} \text{ km h}^{-1}$, find the velocity of the aircraft in terms of t .

(3 marks)

- (b)** Find the speed of the aircraft when it first reaches its cruising height.

(2 marks)

- 4 (a)** An ice skater moves across a straight section of a frozen river such that their position, at time t seconds relative to an origin is given by

$$\mathbf{r} = \begin{pmatrix} \frac{1}{3}t^2 + \frac{1}{5}t \\ 2t^2 + 7t \end{pmatrix} \text{ m} \quad t \geq 0$$

Find the initial speed of the ice skater giving your answer to three significant figures.

(3 marks)

- (b)** Show that the ice skater's acceleration is constant and find the magnitude of the acceleration, giving your answer to three significant figures.

(3 marks)

- 5 (a)** A remote-controlled car is driven around a large playground with velocity, \mathbf{v}_c m s⁻¹, at time t seconds, given by

$$\mathbf{v}_c = (0.45t^2 + 2t - 16)\mathbf{i} + (0.75t^2 - 1)\mathbf{j}$$

- (i) The remote-controlled car is initially set in motion from position $(-6, 15)$. Find the position vector \mathbf{r}_c of the car at time t seconds.
- (ii) Find the distance of the remote-controlled car from the origin after 15 seconds.

(4 marks)

- (b)** At the same time as the remote-controlled car is started, a remote-controlled truck is also set into motion. The truck has position vector, \mathbf{r}_T m, at time t seconds given by

$$\mathbf{r}_T = (0.15t^3 - 6)\mathbf{i} + (0.25t^3 - 1)\mathbf{j}$$

Determine the time(s) at which the car and the truck will collide, if at all.

(3 marks)

6 (a) A spider is crawling across the floor of a house such that it has acceleration

$$\mathbf{a} = (1.2t)\mathbf{i} + (0.5)\mathbf{j} \text{ m s}^{-2}$$

at time t seconds after the spider emerged from under the skirting board.

After 3 seconds the spider's velocity is $\mathbf{v} = 5.4\mathbf{i} + 1.7\mathbf{j} \text{ m s}^{-1}$.

Find the velocity of the spider at time t seconds.

(4 marks)

(b) After 3 seconds the spider's position, relative to an origin at a corner of the floor, is $(10.4, 5.15)$. Find the distance the spider is from the origin when it emerges from under the skirting board.

(4 marks)

- 7 (a)** A stone is thrown from the edge of a deep cave such that it will fall into the cave with its motion described by the equation

$$\mathbf{v} = (0.2t)\mathbf{i} + (4 - 0.75t^2)\mathbf{j} \text{ m s}^{-1}$$

where $\mathbf{v} \text{ m s}^{-1}$ is the velocity of the stone at time t seconds after it is thrown.

Find the position of the stone – relative to where it is thrown from – after 4 seconds and explain what this means in the context of the question.

(4 marks)

- (b)** The cave has a depth of 384 m. Find the magnitude of the acceleration of the stone as it hits the bottom of the cave.

(3 marks)

8 (a) A particle's velocity is modelled by the equation

$$\dot{\mathbf{r}} = \begin{pmatrix} 0.75e^{1.5t} + 2t \\ 5t - (t+1)^{-1} \end{pmatrix} \text{ m s}^{-1} \quad t \geq 0$$

where t is the time in seconds.

The particle's initial displacement is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Find the position vector of the particle, \mathbf{r} m, at time t seconds.

(3 marks)

(b) Find the magnitude of the acceleration of the particle after 1 second.

(3 marks)

9 (a) At time t seconds, a particle P has acceleration $\mathbf{a} \text{ m s}^{-2}$, where

$$\mathbf{a} = (4t - 3)\mathbf{i} + (4t + 5)\mathbf{j} \quad t \geq 0 \quad .$$

Initially P starts at the origin O and moves with velocity $(-5\mathbf{j}) \text{ m s}^{-1}$.

Find the distance between the origin and the position P of when $t = 6$.

(6 marks)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} + 2\mathbf{j}$.

(5 marks)

Very Hard Questions

- 1 (a)** The displacement of a boat, **s m** on a small lake, relative to a mooring point located at the point $(10, 0)$, is given by the vector

$$\mathbf{s} = \left(-40 \sin\left(\frac{\pi t}{900}\right) \right) \mathbf{i} + \left(30 - 30 \cos\left(\frac{\pi t}{900}\right) \right) \mathbf{j}$$

where time t is measured in seconds.

- (i) Write down the position vector, **r m**, of the boat at time t seconds.
- (ii) Find the difference between the distance the boat is from its mooring point and the distance it is from the origin at the point when $t = 225$ seconds.

(5 marks)

- (b)**
- (i) Find the velocity, **v m s⁻¹**, of the boat, at time t seconds.
 - (ii) Given that one trip around the lake takes half an hour, find the times at which the boat is moving in one direction only during one trip around the lake.

(5 marks)

- 2 Once an aircraft reaches its cruising height (at time $t = 0$ hours) its acceleration is modelled by

$$\mathbf{a} = (3t^2 - 1)\mathbf{i} + (8t + 1)\mathbf{j} \text{ km h}^{-2}$$

Given that at time $t = 2$ hours,

- the \mathbf{i} -component of the aircraft's velocity is positive and double that of the \mathbf{j} -component, and,
- the speed of the aircraft is $22\sqrt{5} \text{ km h}^{-1}$,

find the velocity of the aircraft in terms of t .

(5 marks)

- 3 (a)** An ice skater moves across a straight section of a frozen river such that their position, at time t seconds relative to an origin is given by

$$\mathbf{r} = \begin{pmatrix} 2t^{\frac{1}{2}} + t \\ t^{\frac{1}{2}} \end{pmatrix} \text{ m} \quad 0 \leq t \leq 225$$

- (i) Find the distance the skater is from the origin after 25 seconds.
- (ii) As they skate forwards, the skater slowly crosses the width of the river.
It takes 225 seconds for the skater to cross the river.
How wide is the river?

(3 marks)

- (b)** Show that the magnitude of acceleration of the skater at time t seconds is given by
 $a = 0.25\sqrt{5t^{-3}} \text{ m s}^{-2}$.

(4 marks)

- 4** At time $t = 0$ seconds two remote-controlled cars – one red, one blue - are started and driven around a large playground with velocities, $\mathbf{v}_R \text{ m s}^{-1}$ and $\mathbf{v}_B \text{ m s}^{-1}$ given by

$$\mathbf{v}_R = (0.2t - 1)\mathbf{i} + (t - 5)\mathbf{j}$$

$$\mathbf{v}_B = (0.2t)\mathbf{i} + (t - 7)\mathbf{j}$$

The red car starts from position $(-2.4, 8)$ relative to an origin and the blue car starts from position $(-2.4, -4)$.

Find the time (such that $t \geq 0$) when both remoted-controlled cars are equidistant from the origin. Find the distance of the cars from the origin at this point and show that they are not in the same location.

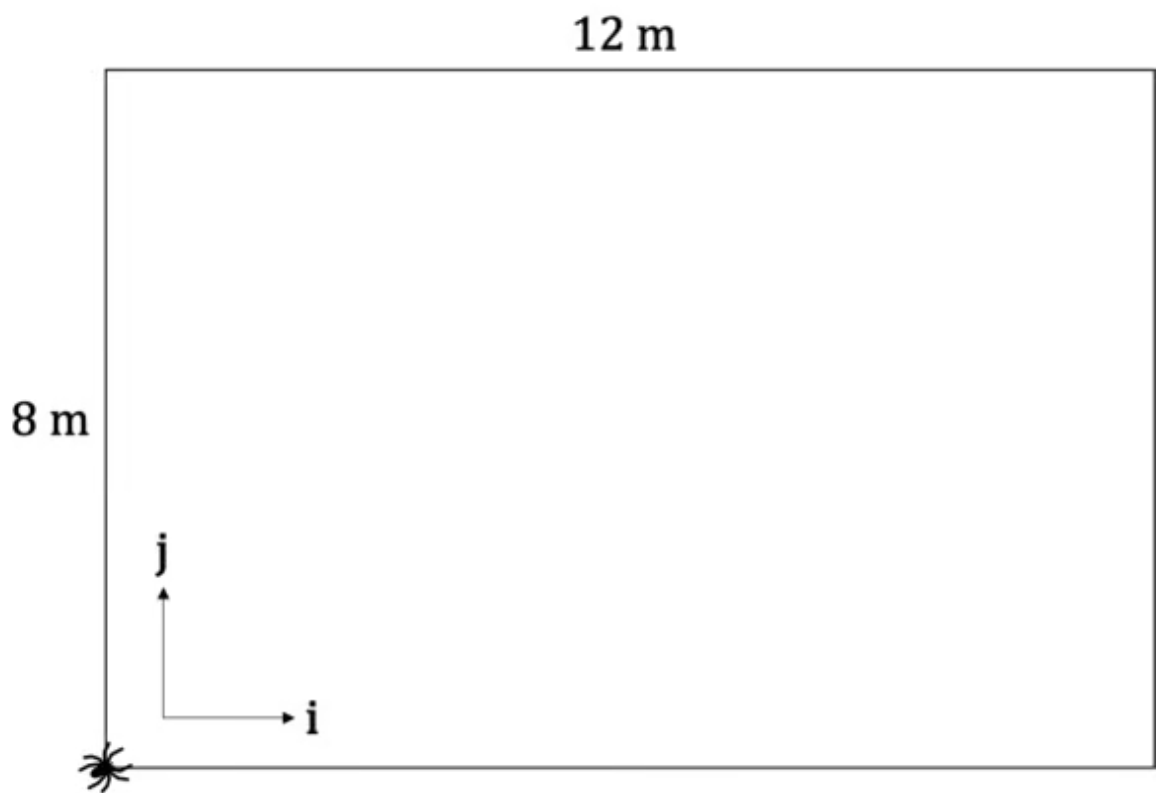
(8 marks)

- 5 A spider is crawling across the floor of a house such that its acceleration is

$$\mathbf{a} = ((0.2t)\mathbf{i} + (0.4t)\mathbf{j}) \text{ m s}^{-2}$$

at time t seconds after the spider emerged from under the skirting board in a corner of the room.

The room measures $12 \text{ m} \times 8 \text{ m}$ and the corner from which the spider emerges is the origin, as shown below.



The spider's velocity is such that

- its speed in the \mathbf{i} -direction after 3 seconds is twice the speed in the \mathbf{i} -direction after 2 seconds,
- its speed in the \mathbf{j} -direction after 3 seconds is three-times the speed in the \mathbf{j} -direction after 1 second,
- neither component of the spider's velocity is ever negative.

Determine whether or not the spider is still in the room (and visible, so not under the skirting board) after 6 seconds.

(8 marks)

- 6 (a)** A stone is thrown from the edge of a deep cave such that it will fall into the cave and has velocity

$$\mathbf{v} = ((0.4t)\mathbf{i} + (2 - 0.3t^2)\mathbf{j}) \text{ m s}^{-1}$$

at time t seconds after it is thrown.

Find the position of the stone at the time it is about to fall into the cave (rather than being in the air above the cave).

(4 marks)

- (b)** Find the maximum height above the cave the stone reaches.

(3 marks)

- (c)** The deepest known cave in the world has a depth of 2212 m. (The Veryovkina Cave in Georgia.) The model above suggests the stone would take around 28 seconds to reach this depth. Consider the average speed and the acceleration of the stone to highlight a problem with this model.

(2 marks)

7 (a) A particle's velocity is modelled by the equation

$$\dot{\mathbf{r}} = \begin{pmatrix} t^{\frac{1}{2}} - t \\ 4(t+1)^{-1} + 5t^{\frac{3}{2}} \end{pmatrix} \text{ m s}^{-1} \quad t \geq 0$$

where t is the time in seconds.

The particle's initial position is $(3, 5)$, find the position vector of the particle, \mathbf{r} m, at time t seconds.

(4 marks)

(b) Find the time at which the particle's acceleration, $\ddot{\mathbf{r}}$ m s⁻² is zero in the horizontal (**i**) direction.

(3 marks)

- 8 (a)** The position vectors in the x - y plane of two particles, A and B , at time t , ($t \geq 0$) are given by

$$\mathbf{r}_A = (3e^{-0.15t})\mathbf{i} + (4e^{0.1t})\mathbf{j}$$

$$\mathbf{r}_B = (-3e^{-0.15t})\mathbf{i} + (4e^{0.1t})\mathbf{j}$$

- (i) Write down the initial position of both particles.
- (ii) Briefly explain what happens to the position of both particles for very high values of t .

(3 marks)

- (b)** (i) Find the velocity of particle A in terms of t .
- (ii) Hence write down the velocity of particle B in terms of t .

(3 marks)

- (c)** (i) On the same diagram sketch the graphs of y against x for both \mathbf{r}_A and \mathbf{r}_B .
- (ii) Without doing any calculations explain why for all values of t ,

$$|\mathbf{r}_A| = |\mathbf{r}_B|, |\mathbf{v}_A| = |\mathbf{v}_B| \text{ and } |\mathbf{a}_A| = |\mathbf{a}_B|$$

(3 marks)