A Level · Edexcel · Maths

3 hours · 36 questions

5.5 Reciprocal & Inverse Trigonometric Functions (A Level only)

Easy (12 questions)	/42
Medium (8 questions)	/47
Hard (8 questions)	/48
Very Hard (8 questions)	/51
Total Marks	**/188**

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Easy Questions

1 Sketch the graph of $y = \text{cosec } x$, for $-180° \leq x \leq 180°$.

(2 marks)

2 (a) Write down the domain and range for the function $\arccos\,\theta$.

(2 marks)

(b) Hence sketch the graph of $y = \arccos\,\theta$.

(2 marks)

3 Solve the equation $\cot x = 3$, for $-\pi \leq x \leq \pi$, giving your answers to three significant figures.

(3 marks)

4 Sketch the graph of $y = \sec\theta$, for $-\pi \leq \theta \leq \pi$.

Label any points of intersection with the coordinate axes and state the equations of any asymptotes.

(4 marks)

5 Starting with the identity

$$\sin^2 x + \cos^2 x \equiv 1$$

show that

(i) $1 + \cot^2 x \equiv \text{cosec}^2 x$

(ii) $\tan^2 x + 1 \equiv \sec^2 x$

(4 marks)

6 Show that

$\sec^2 \theta \sin \theta \equiv \tan \theta \sec \theta$

(3 marks)

7 (a) Write down the domain and range for the function $\arcsin\ \theta$.

(2 marks)

(b) Hence sketch the graph of $y = \arcsin\ \theta$.

(2 marks)

8 Solve the equation $\text{cosec}^2\ x - 2\ \text{cosec}\ x - 8 = 0$ for $0° \leq x \leq 360°$, giving your answers to one decimal place where appropriate.

(3 marks)

9 Show that

$$\cot\ x\ \text{cosec}\ x\ \sec\ x \equiv 1 + \cot^2\ x$$

(3 marks)

10 Solve the equation $\sec\ \theta\ \tan\ \theta - \sec\ \theta = 0$, for $0 \leq x \leq 2\pi$, giving your answers in exact form.

(4 marks)

11 (a) Write down the domain and range for the function $\arctan\ \theta$.

(2 marks)

(b) Hence sketch the graph of $y = \arctan\ \theta$.

(2 marks)

12 (a) Sketch the graph of $y = 2\sec 2x$, for $-\pi \leq x \leq \pi$.

(2 marks)

(b) Draw a suitable line on your graph to show that the equation $2\sec 2x = 4$ has four solutions in the range $-\pi \leq x \leq \pi$.

(2 marks)

Medium Questions

1 (a) Use the definitions of the secant, cosecant and cotangent functions to show that

$$\sec\theta \cot\theta \equiv \operatorname{cosec}\theta.$$

(2 marks)

(b) Hence solve, in the range $0 \le \theta \le 2\pi$, the equation

$$\sec\theta \cot\theta = -2$$

(3 marks)

2 (a) Show that the equation

$$3 - \sec\theta = \frac{2}{\sec\theta}$$

can be rewritten in the form

$$(\sec\theta - 2)(\sec\theta - 1) = 0$$

(2 marks)

(b) Hence solve, in the range $0 \leq \theta \leq 2\pi$, the equation

$$3 - \sec\theta = \frac{2}{\sec\theta}$$

(4 marks)

3 (a) Using the double angle formula $\sin 2A \equiv 2 \sin A \cos A$, show that the equation

$$\sec x \operatorname{cosec} x - 5 = \operatorname{cosec} 2x$$

can be rewritten in the form

$$\text{coesec } 2x = 5.$$

(3 marks)

(b) Hence solve, in the range $0 \leq x \leq 2\pi$, the equation

$$\sec x \operatorname{cosec} x - 5 = \operatorname{cosec} 2x$$

giving your answers correct to 3 significant figures.

(3 marks)

4 (a) Show that the equation

$$\tan^2 x = 6 \sec x - 10$$

can be rewritten in the form

$$(\sec x - 3)^2 = 0$$

(3 marks)

(b) Hence solve, in the range $0 \leq x \leq 2\pi$, the equation

$$\tan^2 x = 6 \sec x - 10$$

giving your answers correct to 3 significant figures.

(3 marks)

5 Given that x satisfies the equation $\arccos x = k$, where $0 < k < \frac{\pi}{2}$

(i) state the range of possible values of x,

(ii) express both sin k and tan k in terms of x.

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(5 marks)

6 (a) Prove that for $0 \leq x \leq 1$, $\arcsin x = \arccos\sqrt{1-x^2}$.

(4 marks)

(b) Explain why this is not true for $-1 \leq x < 0$.

(2 marks)

7 (i) Sketch, in the interval $-2\pi \leq \theta \leq 2\pi$, the graph of $y = 3 + 2 \text{ cosec } \theta$, include asymptotes and label the coordinates of all maximum and minimum points.

(ii) Hence, deduce the number of solutions to the equation $3 + 2 \text{ cosec } \theta = \frac{1}{2}$ in the interval $-2\pi \leq \theta \leq 2\pi$.

(5 marks)

8 (a) The function f is defined as $\mathrm{f}(x) = \arccos x$, $-1 \leq x \leq 1$, and the function g is such that $\mathrm{g}(x) = \mathrm{f}(3x)$.

Sketch the graph of $y = \mathrm{f}(x)$ and state the range of f.

(3 marks)

(b) Sketch the graph of $y = \mathrm{g}(x)$ and state the domain of g.

(3 marks)

(c) Find the inverse function $\mathrm{g}^{-1}(x)$ and state its domain.

(2 marks)

Hard Questions

1 (a) Rewrite tan θ cosec θ as a single trigonometric function.

(2 marks)

(b) Hence solve, in the range $-\pi < \theta \leq \pi$, the equation

$$\tan\theta \operatorname{cosec}\theta = -\frac{2\sqrt{3}}{3}$$

(3 marks)

2 Solve, in the range $0 \leq \theta \leq 2\pi$, the equation

$$\frac{2}{\operatorname{cosec}\theta} - \operatorname{cosec}\theta = 1$$

(6 marks)

3 Using the double angle formula $\sin 2A \equiv 2 \sin A \cos A$, find the solutions to the equation

$$\sec x \operatorname{cosec} x - 75 = 5 \operatorname{cosec} 2x$$

in the range $-\pi < x \leq \pi$. Give your answers correct to 3 significant figures.

(6 marks)

4 (a) Show that the equation

$$2\ \cot^2 x = 1 - 5\operatorname{cosec} x$$

can be rewritten in the form

$$(2\ \operatorname{cosec} x - 1)(\operatorname{cosec} x + 3) = 0$$

(3 marks)

(b) Hence solve, in the range $0 \le x \le 2\pi$, the equation

$$2\ \cot^2 x = 1 - 5\operatorname{cosec} x$$

giving your answers correct to 3 significant figures.

(3 marks)

5 Given that x satisfies the equation $\arcsin x = k$, where $-\dfrac{\pi}{2} < k < 0$,

(i) state the range of possible values of x,

(ii) express both cos k and tan k in terms of x.

(5 marks)

6 Prove that for $-1 \leq x \leq 0, \arccos x = \pi - \arcsin\sqrt{1 - x^2}$.

(7 marks)

7 (i) Sketch, in the interval $-2\pi \leq \theta \leq 2\pi$, the graph of $y = -5 + \frac{1}{2}\sec\theta$, include asymptotes and label the coordinates of all maximum and minimum points.

(ii) Hence deduce the range of values for k for which the equation $-5 + \frac{1}{2}\sec\theta = \mathrm{k}$ has no solutions.

(5 marks)

8 (a) The function f is defined as $\mathrm{f}(x) = \arctan x$, $x \in \mathbb{R}$, and the function g is such that $\mathrm{g}(x) = \frac{2}{\pi}\mathrm{f}(x) - 1$.

Sketch the graph of $y = \mathrm{f}(x)$ and state the range of f.

(3 marks)

(b) Sketch the graph of $y = \mathrm{g}(x)$ and state the range of g.

(3 marks)

(c) Find the inverse function $\mathrm{g}^{-1}(x)$ and state its domain.

(2 marks)

Very Hard Questions

1 Solve, in the range $-\pi < \theta \leq \pi$, the equation

$$\frac{\sec\theta \cot\theta}{\operatorname{cosec}\theta \tan\theta} = -\sqrt{3}$$

(5 marks)

2 Solve, in the range $0 \leq \theta \leq 2\pi$, the equation

$$6\sec\theta + \frac{2\sqrt{3}}{\sec\theta} = -3 - 4\sqrt{3}$$

Leaving your answers as exact values.

(6 marks)

3 Using the double angle formulae $\sin 2A \equiv 2\sin A \cos A$ and $\cos 2A \equiv \cos^2 A - \sin^2 A$, find the solutions to the equation

$$(\operatorname{cosec} x - \sec x)\left(\frac{1}{\sec x} + \frac{1}{\operatorname{cosec} x}\right) = \cot 2x + 3$$

in the range $-\pi < x \leq \pi$. Give your answers correct to 3 significant figures.

(6 marks)

4 Solve, in the range $0 \leq x \leq 2\pi$, the equation

$$3\cot^2 x - 4\sqrt{3} = \left(6 - 2\sqrt{3}\right)\operatorname{cosec} x - 3$$

Leaving your answers as exact values.

(6 marks)

5 Given that x satisfies the equation $\arctan x = k$, where $-\frac{\pi}{2} \leq k \leq 0$

(i) state the range of possible values of x,

(ii) express both sin k and cos k in terms of x.

(5 marks)

6 Prove that for $x \leq -1$,

$$\arcsin\frac{1}{x} = -\arccos\left(-\frac{\sqrt{x^2-1}}{x}\right)$$

(7 marks)

7 (a) Sketch, in the interval $-2\pi \leq \theta \leq \pi$, the graph of $y = 2 + 3\sec\left(\theta + \frac{\pi}{2}\right)$, include asymptotes and label the coordinates of all maximum and minimum points.

(3 marks)

(b) Deduce the maximum and minimum values of $\dfrac{1}{2 + 3\sec\left(\theta + \frac{\pi}{2}\right)}$.

(4 marks)

8 (a) The function f is defined as $\mathrm{f}: x \mapsto \arcsin x, \ -1 \leq x \leq 1$, and the function g is such that

$$\mathrm{g}(x) = \frac{4\mathrm{f}\left(\frac{x}{3}\right)}{\pi} + 2$$

Sketch the graph of $y = \mathrm{g}(x)$ and state the domain and range of g.

(4 marks)

(b) Define the inverse function g^{-1} in the form $\mathrm{g}^{-1}: x \mapsto \ldots$

(2 marks)

(c) Over the same domain as g, the function h is defined as $\mathrm{h}: x \mapsto p \arccos(\mathrm{q}x)$.

Given that $\mathrm{h}(x) = -\mathrm{g}(x)$ for all x in the two functions' common domain, determine the values of p and q.

(3 marks)