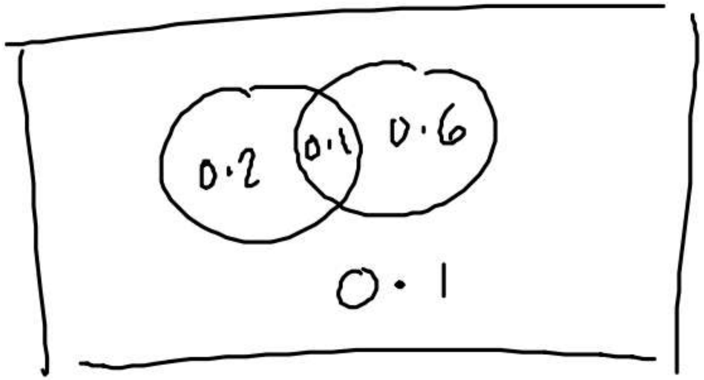
Ex 2D Q11 .

$P(A) = 0.3$

$P(B) = 0.7$

$P(A' \cap B') = 0.1$



a) $P(A \cup B) = 0.9$

$0.9 = 0.3 + 0.7 - P(A \cap B)$

$\underline{P(A \cap B) = 0.1}$

b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.7} = \dfrac{1}{7}$.

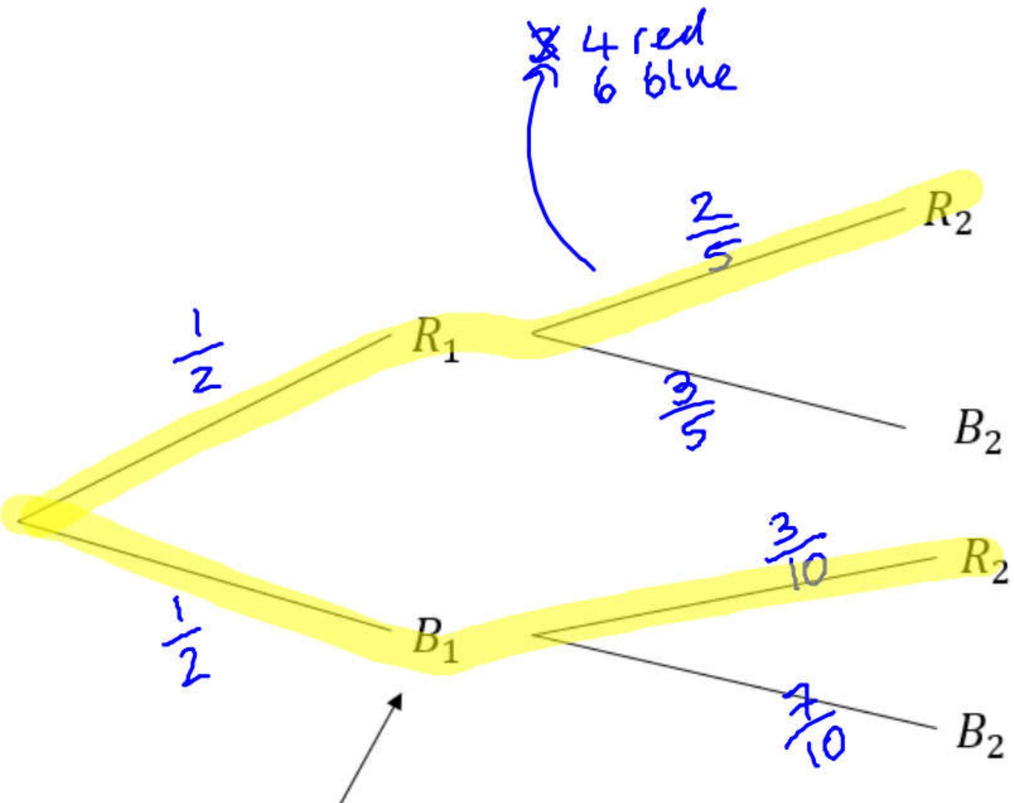
c) $P(A) \times P(B) = 0.7 \times 0.3 = 0.21 \neq 0.1 = P(A \cap B)$

So A and B is not independent.

Probability Trees

We saw probability trees in Year 1. The only difference here is **determining a conditional probability** using your tree.

Example: You have two bags, the first with 5 red balls and 5 blue balls, and the second with 3 red balls and 6 blue balls. You first pick a ball from the first bag, and place it in the second. You then pick a ball from the second bag. Complete the tree diagram.



Tip: Use variable subscripting to indicate what pick you're referring to.

Hence find the probability that:

a) You pick a red ball on your second pick.

$$P(R_2) = P(R_1 \cap R_2) + P(B_1 \cap R_2)$$

$$= \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{10}$$

$$= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$$

b) Given that your second pick was red, the first pick was also red.

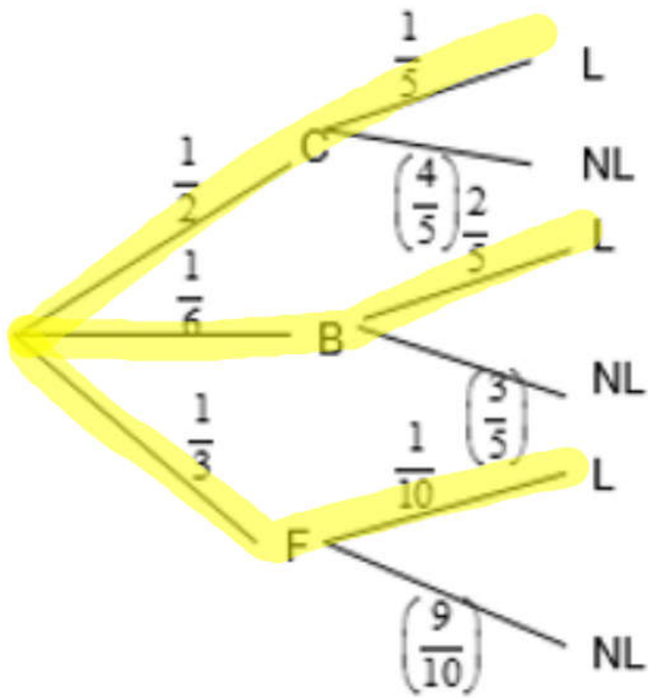
$$P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{7}{20}} = \frac{1/5}{7/20} = \frac{4}{7}$$

Edexcel S1 May 2009 Q2

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}, \frac{2}{5}$ and $\frac{1}{10}$ respectively.

(*c*) Given that Bill is late, find the probability that he did not travel on foot. **(4)**



$$P(F' \mid L) = \frac{P(F' \cap L)}{P(L)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5} + \frac{1}{3} \times \frac{1}{10}} = \frac{\frac{1}{10} + \frac{1}{15}}{\frac{1}{10} + \frac{1}{15} + \frac{1}{30}}$$

$$= \frac{1/6}{1/5} = \frac{5}{6}$$

Edexcel S1

Your Turn

6. **[Jan 2006 Q4]** A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

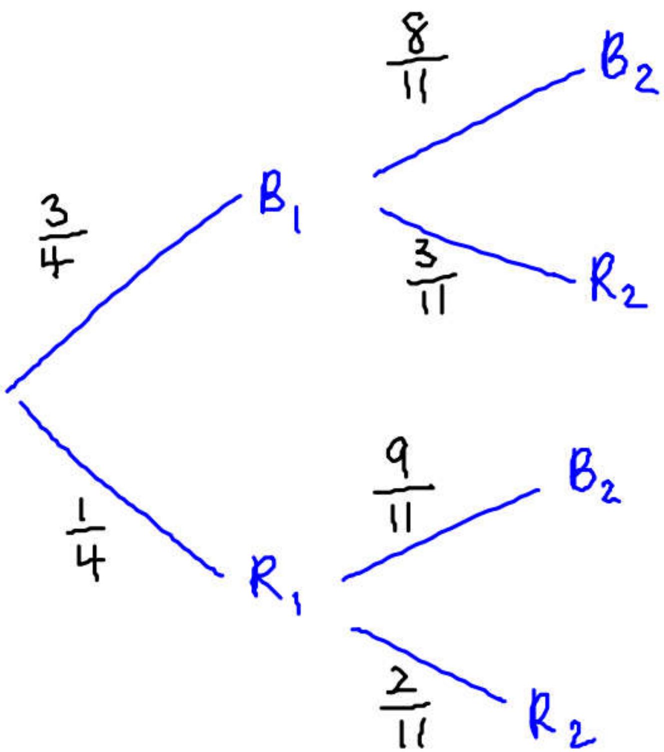
(*a*) Draw a tree diagram to represent the information. **(3)**

Find the probability that

(*a*) the second ball selected is red, **(2)**

(*b*) both balls selected are red, given that the second ball selected is red. **(2)**

a)

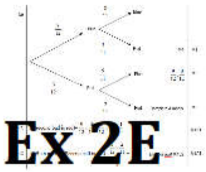


b) $P(R_2) = \frac{3}{4} \times \frac{3}{11} + \frac{1}{4} \times \frac{2}{11}$

$= \frac{9}{44} + \frac{2}{44} = \frac{11}{44} = \underline{\underline{\frac{1}{4}}}.$

c) $P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{\frac{1}{4} \times \frac{2}{11}}{\frac{1}{4}} = \underline{\underline{\frac{2}{11}}}.$

Ex 2E
Q8, 9, 10



Ex 2E

EXAM PRACTICE

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

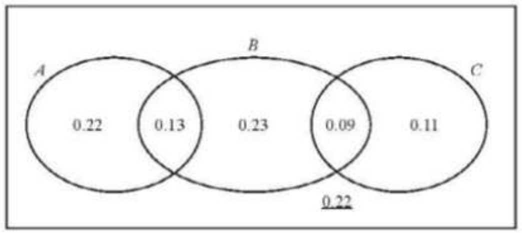
The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

4(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1
	$= \dfrac{3}{5}$ or 0.6	A1
		(2)
(b)	e.g. $\mathrm{P}(A)\times\mathrm{P}(B) = \frac{7}{20}\times\frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A\cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1
		(1)
(c)		B1 M1 A1 M1
		A1
		(5)
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1-[0.56]$ or $1-[0.13+0.23+0.09+0.11]$ o.e.	M1
	$=$ **0.44**	A1
		(2)

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A\cap C$ and $A\cap B\cap C$)No box is B0

M1: for method for finding P($B\cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

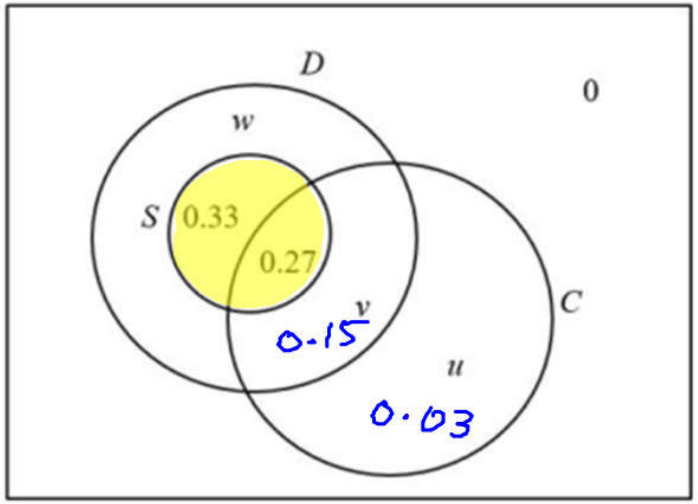
(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

4. The Venn diagram shows the probabilities of students' lunch boxes containing a drink, sandwiches and a chocolate bar.

D is the event that a lunch box contains a drink,
S is the event that a lunch box contains sandwiches,
C is the event that a lunch box contains a chocolate bar,
u, v and w are probabilities.



(a) Write down $P(S \cap D')$ $= 0$

One day, 80 students each bring in a lunch box.
Given that all 80 lunch boxes contain sandwiches and a drink,

(b) estimate how many of these 80 lunch boxes will contain a chocolate bar.

Given that the events S and C are independent and that $P(D \mid C) = \frac{14}{15}$

(c) calculate the value of u, the value of v and the value of w.

b) $P(C \mid S \cap D) = \frac{P(C \cap (S \cap D))}{P(S \cap D)}$

$= \frac{0.27}{0.6} = \frac{9}{20}$

$\frac{9}{20} \times 80 = \underline{\underline{36}}$

c) $P(S) \times P(C) = P(S \cap C)$

$0.6 \times (0.27 + v + u) = 0.27$

$0.27 + v + u = 0.45$

$v + u = 0.18$

$P(D \mid C) = \frac{14}{15}$

(1) $\frac{14}{15} = \frac{P(D \cap C)}{P(C)}$

$\frac{14}{15} = \frac{0.27 + v}{0.27 + v + u}$

(3) $\frac{14}{15} = \frac{0.27 + v}{0.27 + 0.18}$

(7) $0.42 = 0.27 + v$

$v = \underline{\underline{0.15}}$ $\quad u = \underline{\underline{0.03}}$

$w = 0.22$

4(a)	$P(S \cap D') = 0$	B1	1.1b
		(1)	
(b)	$P(C \mid S \cap D) = \frac{0.27}{0.6} = \frac{9}{20} = 0.45$	M1	3.1b
	$\therefore 80 \times "0.45"$	M1	1.1b
	$=36$	A1	1.1b
		(3)	
(c)	$[P(C) \times P(S) = P(C \cap S)]$		
	$P(S) = 0.6,\ P(C) = 0.27 + v + u,\ P(S \cap C) = 0.27$	M1	3.1a
	$0.6 \times (0.27 + u + v) = 0.27$ or $u + v = 0.18$ o.e	A1	1.1b
	$\left[P(D \mid C) = \frac{P(D \cap C)}{P(C)}\right]$ $P(D \cap C) = 0.27 + v$	M1	3.1a
	$\frac{14}{15} = \frac{0.27 + v}{0.27 + v + u}$ or $14u - v = 0.27$ o.e	A1	1.1b
	$15u = 0.45$	M1dd	1.1b
	$u = 0.03 \quad v = 0.15$	A1	1.1b
	$w = 0.22$	A1ft	1.1b
		(7)	

(11 marks)

Notes:

(a) B1: correct answer only

(b) M1: for a correct ratio of probabilities formula with at least one correct value and multiplying by 80

A1: a correct answer

(c) M1: for translating the problem and realising the equation $P(C) \times P(S) = P(C \cap S)$ needs to be used with at least 2 parts correct.

A1: a correct equation

M1: for a correct probability formula with $P(D \cap C) = 0.27 + v$

A1: a second correct equation

M1dd: dependent on the previous 2 method marks being awarded. Solving the two simultaneous equations by eliminating one variable. May be implied by either u or v correct

1. Three bags, A, B and C, each contain 1 red marble and some green marbles.

Bag A contains 1 red marble and 9 green marbles only
Bag B contains 1 red marble and 4 green marbles only
Bag C contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from bag A.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag B.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag C.

(a) Draw a tree diagram to represent this information.

(2)

(b) Find the probability that Sasha selects 3 green marbles.

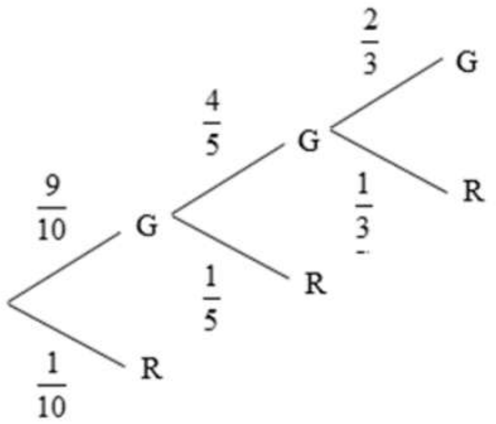
(2)

(c) Find the probability that Sasha selects at least 1 marble of each colour.

(2)

(d) Given that Sasha selects a red marble, find the probability that he selects it from bag B.

(2)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		B1	This mark is given for a correct shape and labels for a tree diagram
		B1	This mark is given for the correct probabilities shown
(b)	$\frac{9}{10} \times \frac{4}{5} \times \frac{2}{3}$	M1	This mark is given for a multiplication of three probabilities
	$= \frac{12}{25}$	A1	This mark is given for the correct probability that Sasha selects three marbles
(c)	$\frac{9}{10} \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{3}$	M1	This mark is given for the addition of two products
	$= \frac{21}{50}$	A1	This mark is given for the correct probability that Sasha selects at least one marble of each colour
(d)	P(red form B | red selected) = $\dfrac{\frac{9}{10} \times \frac{1}{5}}{1 - \frac{12}{25}} = \frac{9}{50} \times \frac{25}{13}$	M1	This mark is given for determining the correct ratio of probabilities
	$= \frac{9}{26}$	A1	This mark is given for the correct probability that Sasha selects a red marble from bag B

3. A company maintains machines.
It has two types of contract, a service contract and a repair contract.
The company classes its customers as new customers or existing customers.
The table gives information about the company's customers.

	Service contract	**Repair contract**
New customer	65	82
Existing customer	231	262

The company is going to survey its customers. It plans to take a sample of 100 of its customers, stratified by customer type and contract type.

(a) Work out how many new customers with repair contracts should be sampled.
(2)

The company has developed a test for a certain fault in the machines it services.
The test sometimes gives incorrect results.

The company collects information from a sample of randomly selected machines.

- 2% of the machines have the fault
- 70% of the machines with the fault test positive for the fault
- 10% of the machines without the fault test positive for the fault.

A machine is selected at random from the sample of the machines, and tests positive for the fault.

(b) (i) Calculate the probability that the machine has the fault.
(4)

(ii) Comment on the usefulness of the company's test.
Give a reason for your answer.
(1)

When the company services the machines, it checks two components, α and β, for wear and tear and replaces these if needed.

Event A is that component α needs to be replaced.

Event B is that component β needs to be replaced.

The probability that component α needs to be replaced is 0.35
The probability that component β needs to be replaced is 0.55
The probability that neither component needs to be replaced is 0.28

(c) Show that events A and B are not independent.
(2)

(d) Find the probability that component α or component β needs to be replaced, but not both.
(2)

Question	Scheme	Marks	AOs
3(a)	$\frac{82}{65+82+231+262} \times 100\ (= 12.8125)$	M1	1.1b
	13	A1	1.1b
		(2)	
(b)(i)	$[F = \text{faulty}, T = \text{tests positive}]\ \ \mathrm{P}(F\|T) = \frac{\mathrm{P}(F \cap T)}{\mathrm{P}(T)}$	M1	3.1b
	$\mathrm{P}(F \cap T) = 0.02 \times 0.7\ [= 0.014]$	M1	1.1b
	$\mathrm{P}(T) = 0.02 \times 0.7 + 0.98 \times 0.1\ [= 0.112]$	M1	1.1b
	$\mathrm{P}(F\|T) = 0.125$	A1	1.1b
		(4)	
b(ii)	Most machines that test positive do not have faults therefore the company's test is not very useful oe	B1	3.2a
		(1)	
(c)	$P(A \cap B) = 0.18$	M1	2.1
	e.g. $P(A) \times P(B) = 0.35 \times 0.55 = 0.1925 \neq P(A \cap B) = 0.18$	A1	1.1b
		(2)	
(d)	$P(A\ or\ B \text{ not both}) = 0.35 + 0.55 - 2 \times 0.18$ oe	M1	3.1b
	=0.54	A1	1.1b
		(2)	
			(10 marks)

Notes:
(a) M1: for a correct calculation for the strata size A1: for 13
(b) M1: for identifying correct calculation M1: for method for finding $P(F \cap T)$ M1: for method for finding $P(T)$ A1: a correct answer
(c) M1: for correctly finding $P(A \cap B)$ oe A1: for a fully correct explanation: correct probabilities and correct comparisons
(d) M1: for a correct expression A1: cao