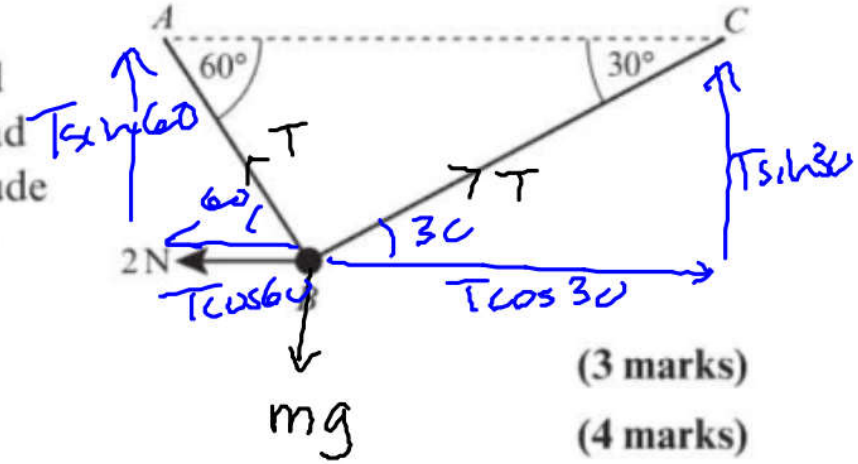
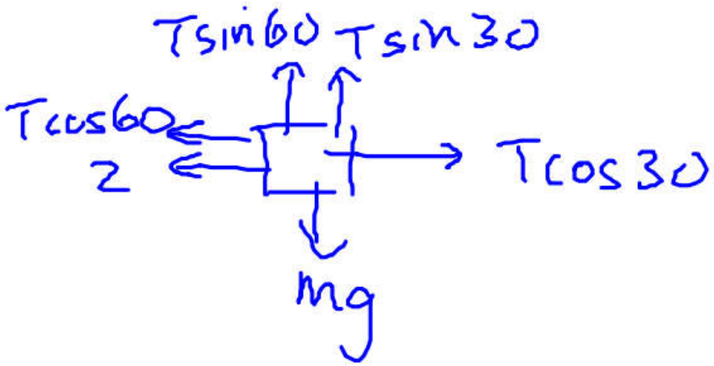
5 A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points, A and C, on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 2 N acting parallel to CA. The sections of string make angles of 60° and 30° with the horizontal. Find:



a the tension in the string **(3 marks)**

b the mass of the bead. **(4 marks)**

c State how you have used the modelling assumption that the bead is smooth in your calculations. **(1 mark)**



Newton's Second Law

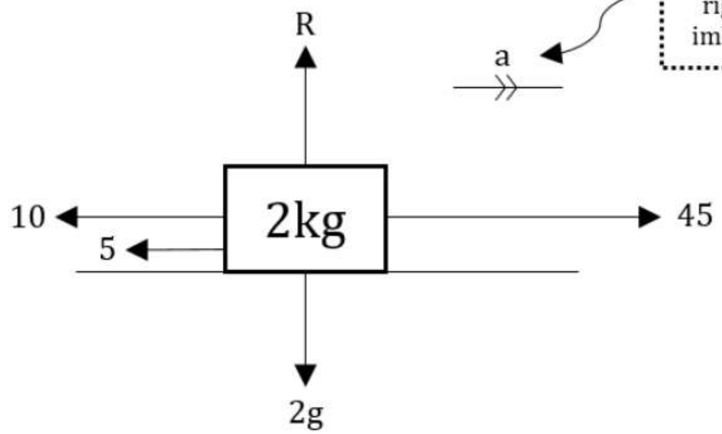
"An object will accelerate if there is an overall resultant force on the object. The acceleration is proportional to this force, and inversely proportional to its mass."

In other words

$$F = ma$$

Where **F** = resultant force, **m** = mass, **a** = acceleration

The resultant force is found by finding the difference between the forces in one direction, and the forces in the opposing direction. This tells you the overall force in one direction.



It is clearly going to accelerate to the right as there is an imbalance of forces

Notice how we find the resultant force by doing the forces to the right minus the forces to the left

Work out the acceleration for the particle.

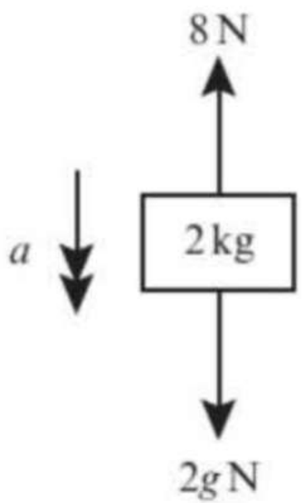
Using $F = ma$

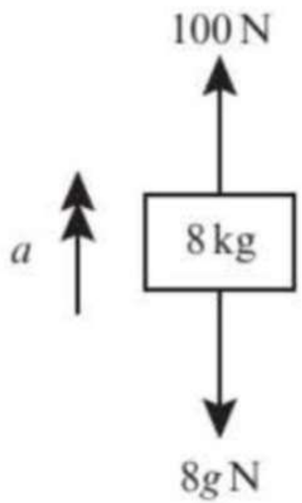
$$45 - 10 - 5 = 2a$$
$$30 = 2a$$
$$15 = a$$

So the particle will accelerate at 15ms^{-2} to the right

Dynamics - forces causing motion (Year 1 recap)

In each situation, the forces acting on the body cause it to accelerate as shown in the diagram. Find the value of *a*.





$\downarrow$ $F = ma$

$2g - 8 = 2a$

$\underline{5.8 \text{ ms}^{-2}} = a$

$\uparrow$ $F = ma$

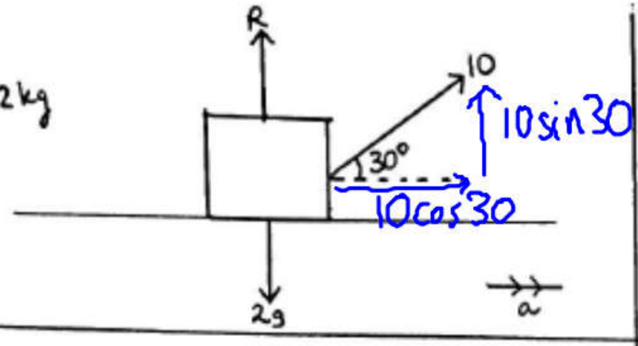
$100 - 8g = 8a$

$a = \underline{2.7 \text{ ms}^{-2}}$

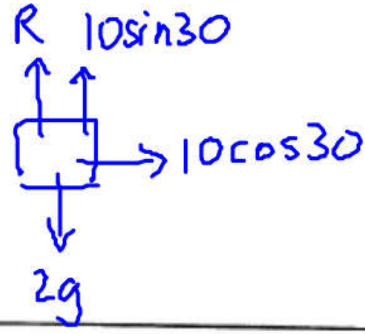
Keywords:
- Resultant force
- Resolve

Key ideas:
- F = ma
- F is the *resultant force*
- Direction of force corresponds to positive or negative value

$m = 2kg$



New Diagram



Find the value of R and a

R ↕ $R + 10\sin 30 = 2g$

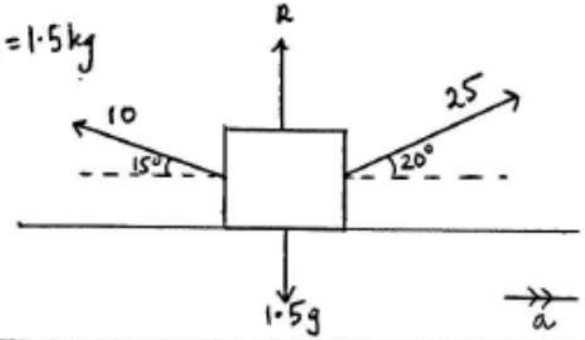
$R = \underline{14.6}\text{ N}$

$F = ma$ →

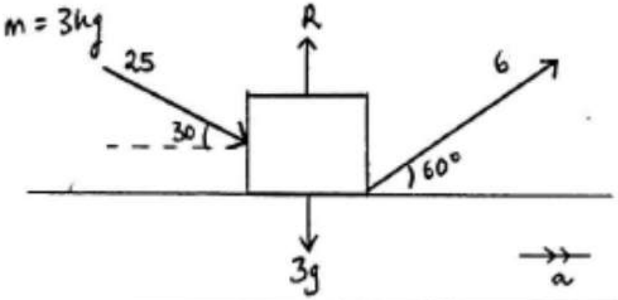
$10\cos 30 = 2a$

$a = \underline{4.33\ ms^{-2}}$

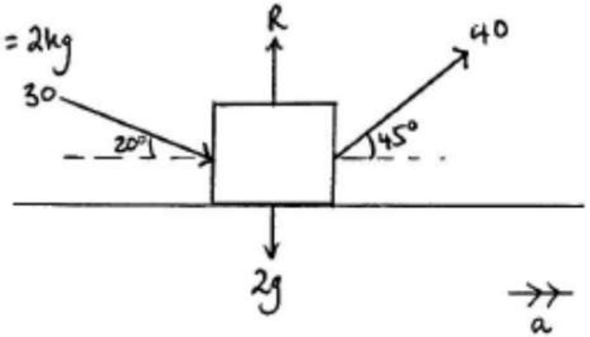
$m = 1.5kg$

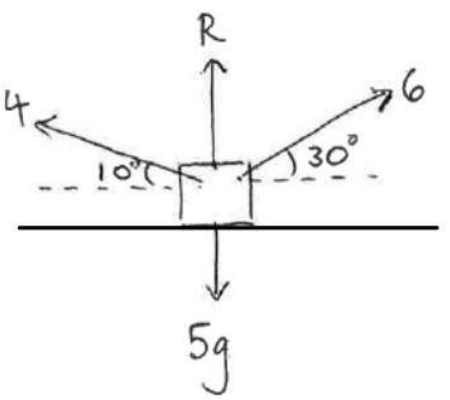


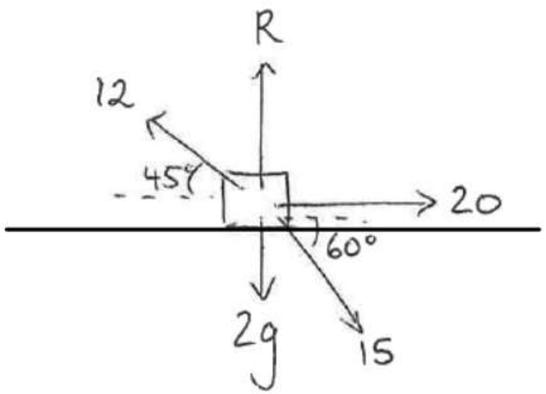
$m = 3kg$

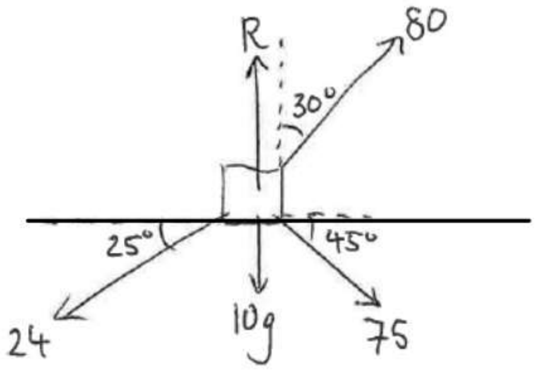


$m = 2kg$




R
4
6
10°
30°
5g

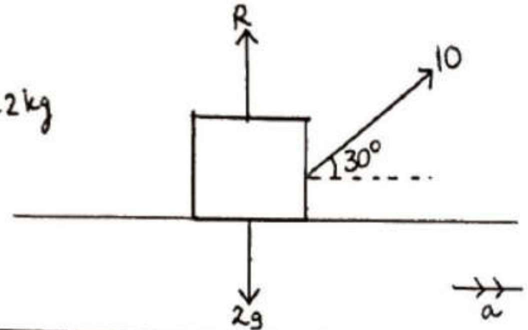

R
12
45°
20
60°
2g
15

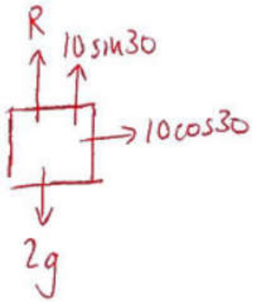

R
80
30°
25°
45°
24
10g
75

New Diagram

Find the value of R and a

m = 2kg





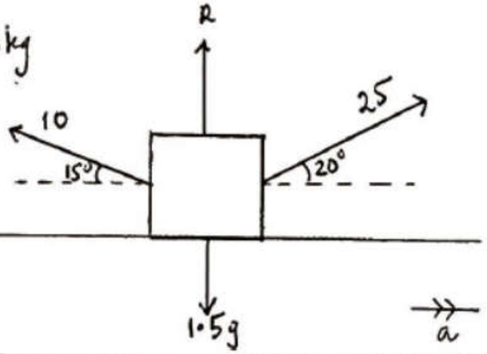
$R + 10\sin 30 = 2g$

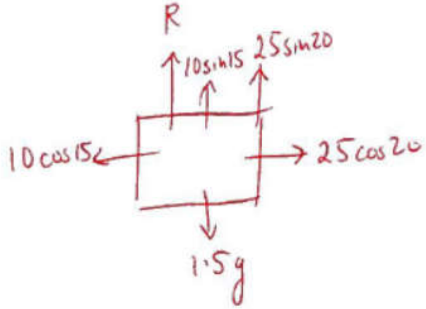
$R = 14.6N$

$10\cos 30 = 2a$

$a = 4.33ms^{-}$

m = 1.5kg





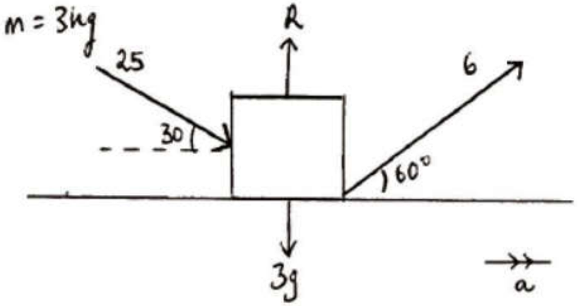
$R + 10\sin 15 + 25\sin 20 = 1.5g$

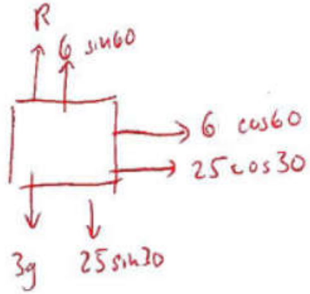
$R = 3.56N$

$25\cos 20 - 10\cos 15 = 1.5a$

$a = 9.22ms^{-2}$

m = 3kg





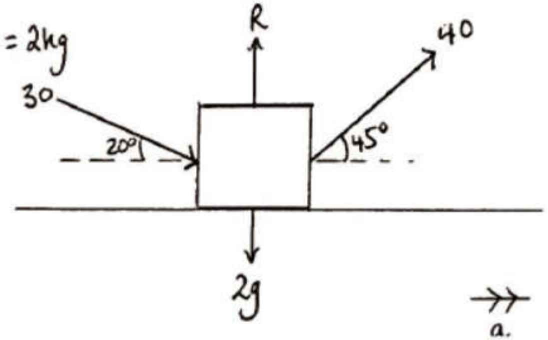
$R + 6\sin 60 = 3g + 25\sin 30$

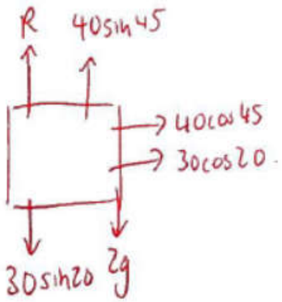
$R =$ **36.7N**

$6\cos 60 + 25\cos 30 = 3a$

$a =$ **$8.22ms^{-2}$**

m = 2kg



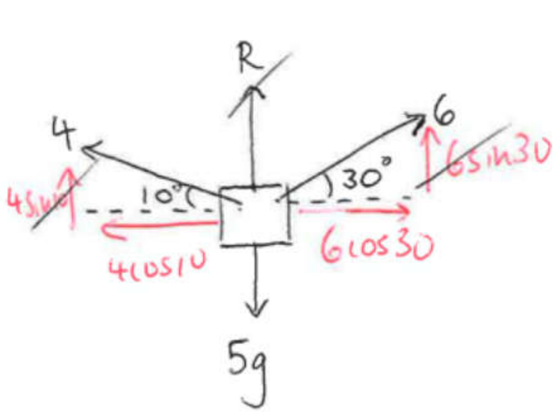


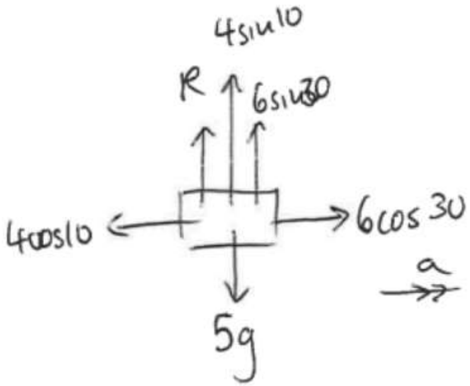
$R + 40\sin 45 = 30\sin 20 + 2g$

$R = 1.58N$

$40\cos 45 + 30\cos 20 = 2a$

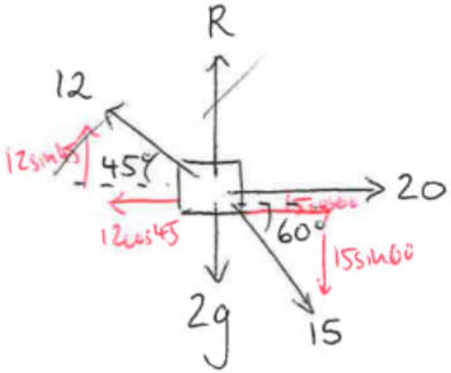
$a = 28.2ms^{-2}$

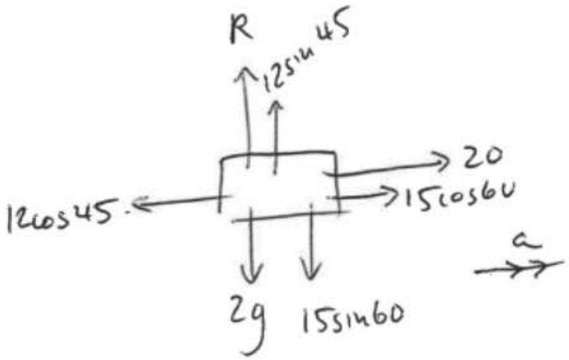




$$R + 4\sin 10 + 6\sin 30 = 5g$$
$$R = \underline{\underline{45.3}}\,N$$

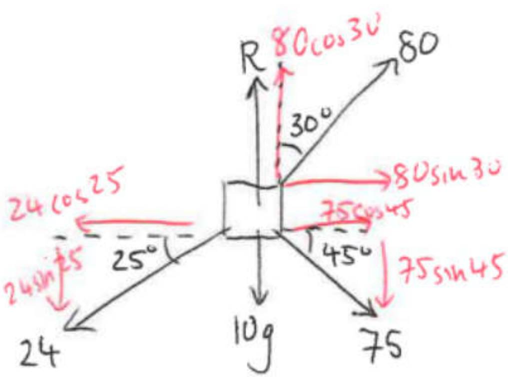
$$6\cos 30 - 4\cos 10 = 5a$$
$$a = \underline{\underline{0.251}}\,ms^{-2}$$

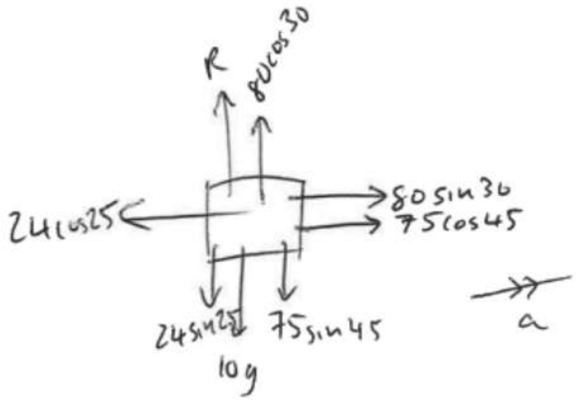




$$R + 12\sin 45 = 2g + 15\sin 60$$
$$R = \underline{\underline{24.1}}\ N$$

$$20 + 15\cos 60 - 12\cos 45 = 2a$$
$$a = \underline{\underline{9.51}}\,ms^{-2}$$





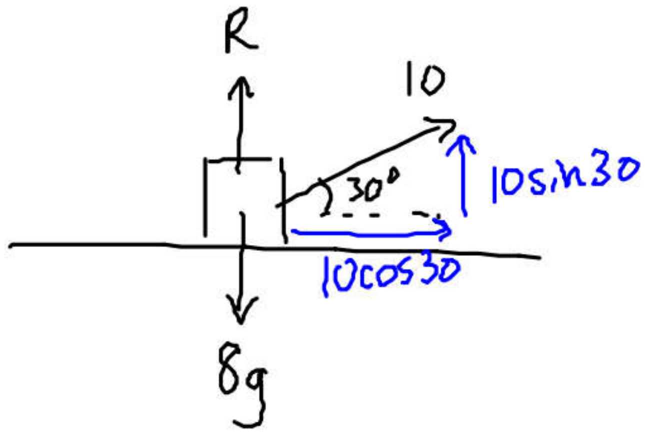
$$R + 80\cos 30 = 24\sin 25 + 10g + 75\sin 45.$$
$$R = \underline{\underline{91.9}}\ N$$

$$80\sin 30 + 75\cos 45 - 24\cos 25 = 10a$$
$$a = \underline{\underline{7.13}}\,ms^{-2}$$

A box of mass 8kg lies on a smooth horizontal floor. A force of 10N is applied at an angle of 30° causing the box to accelerate horizontally along the floor.

(a) Work out the acceleration of the box.

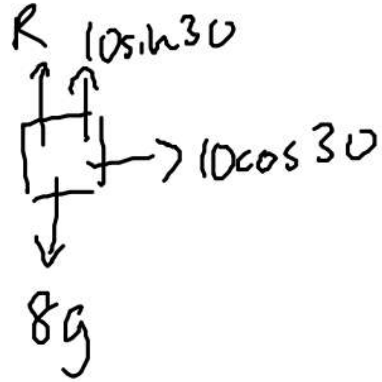
(b) Calculate the normal reaction between the box and the floor.



a) $F = ma \rightarrow$

$10\cos 30 = 8a$

$a = \underline{\underline{1.08\ ms^{-2}}}$ (3sf).



b) $R \updownarrow$

$R + 10\sin 30 = 8g$

$R + 5 = 8g$

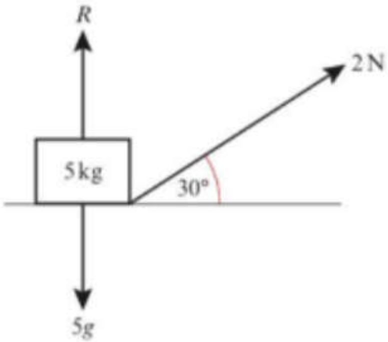
$R = 8g - 5$

$R = \underline{\underline{73.4N}}$

Note, $R < 8g$, not $= 8g$ $\underline{\underline{78.4}}$

Your Turn

5 A box of mass 5 kg lies on a smooth horizontal floor. The box is pulled by a force of 2 N applied at an angle of 30° to the horizontal, causing the box to accelerate horizontally along the floor.



a Work out the acceleration of the box.

b Work out the normal reaction of the box with the floor.

(E) 6 A force P is applied to a box of mass 10 kg causing the box to accelerate at $2\,\text{m s}^{-2}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 45° to the plane, work out the value of P. **(3 marks)**

(E) 7 A force of 20 N is applied to a box of mass m kg causing the box to accelerate at $0.5\,\text{m s}^{-2}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 25° to the plane, work out the value of m. **(3 marks)**

5 **a** $\frac{\sqrt{3}}{5}\,\text{m s}^{-2}$ **b** 48 N

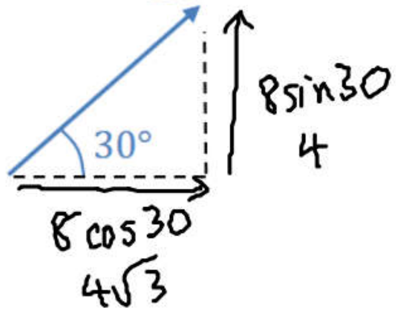
6 $20\sqrt{2}$ N

7 36.3 kg (3 s.f.)

Writing forces in vector form

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively. Also write your answer in column vector form.

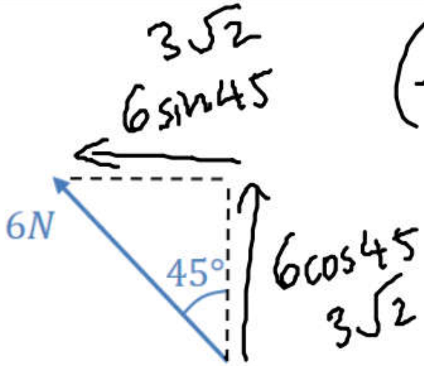
1



$(4\sqrt{3}\underline{i} + 4\underline{j})\text{N}$

$\begin{pmatrix} 4\sqrt{3} \\ 4 \end{pmatrix} \text{N}$

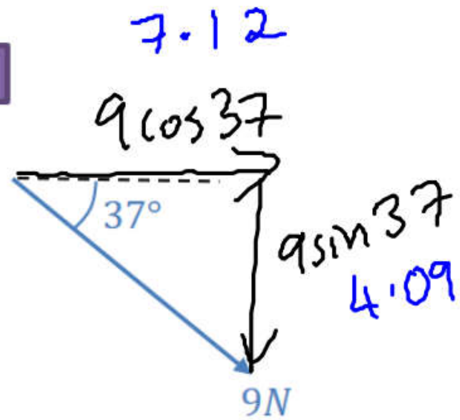
2



$(-3\sqrt{2}\underline{i} + 3\sqrt{2}\underline{j})\text{N}$

$\begin{pmatrix} -3\sqrt{2} \\ 3\sqrt{2} \end{pmatrix} \text{N}$

3

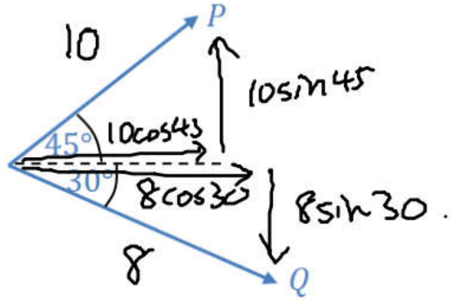


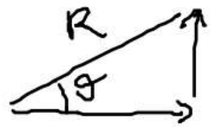
$(7.12\underline{i} - 4.09\underline{j})\text{N}$

$\begin{pmatrix} 7.12 \\ -4.09 \end{pmatrix}$

Combining Forces

Two forces P and Q act on a particle as shown.
P has a magnitude of 10N and Q has a magnitude of 8N.
Work out the magnitude and direction of the resultant force.





Method 1: Finding total x and y components of force.

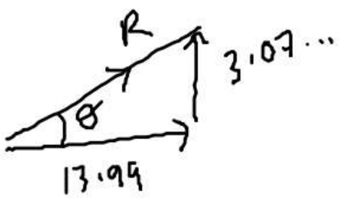
$$\underline{P} = \begin{pmatrix} 10\cos 45 \\ 10\sin 45 \end{pmatrix}$$

$$\underline{Q} = \begin{pmatrix} 8\cos 30 \\ -8\sin 30 \end{pmatrix}$$

$$\underline{R} = \underline{P} + \underline{Q} = \begin{pmatrix} 10\cos 45 + 8\cos 30 \\ 10\sin 45 - 8\sin 30 \end{pmatrix}$$

$$\underline{R} = \begin{pmatrix} 13.9992\ldots \\ 3.0710\ldots \end{pmatrix}$$

$$|\underline{R}| = \sqrt{13.9..^2 + 3.07...^2} = \underline{\underline{14.3N}} \text{ (3sf)}$$

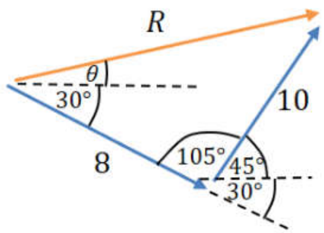


$$\theta = \tan^{-1}\left(\frac{3.0710\ldots}{13.9992\ldots}\right)$$

$= 12.4°$ above the horizontal

Method 2: Using Triangle Law for vector addition.

Yuk



We can avoid resolving components by drawing the force vectors in a chain, then finding the vector from the start to end point. The resultant vector (orange) geometrically represents the same of the vectors.

Use cosine rule to get magnitude of R:
$R^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos(105°)$
$R = 14.3\ N$

Use sine rule to get θ:

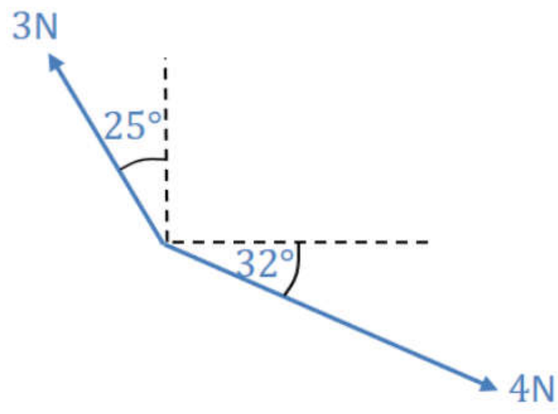
$$\frac{\sin(\theta + 30°)}{10} = \frac{\sin(105°)}{14.332}$$

$$\sin(\theta + 30°) = \frac{10\sin(105°)}{14.332} \quad \ldots$$

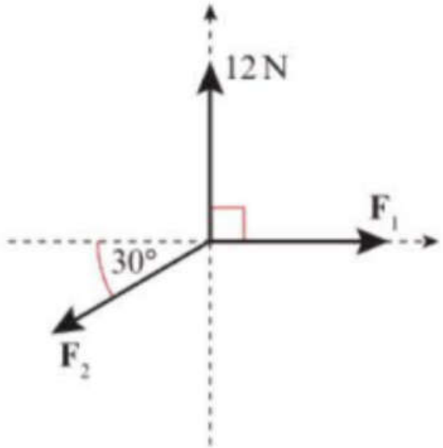
$\theta = 12.4°$

Your Turn

A particle has forces acting on it as indicated in the diagram. Determine the magnitude and direction (anticlockwise from the positive x direction) of the resultant force.



9 A system of forces act upon a particle as shown in the diagram.
The resultant force on the particle is $(2\sqrt{3}\mathbf{i} + 2\mathbf{j})$ N.
Calculate the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.



(3 marks)

9 $\mathbf{F}_1 = 12\sqrt{3}$ N, $\mathbf{F}_2 = 20$ N