OCR A Level Physics

Your notes

Nuclear Fission & Fusion

Contents

Energy & Mass Equation

Your notes

Energy & Mass Equation

- Einstein showed in his **theory of relativity** that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$E = mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})
- Some examples of mass-energy equivalence are:
  - The **fusion** of hydrogen into helium in the centre of the sun
  - The **fission** of uranium in nuclear power plants
  - Nuclear **weapons**
  - High-energy **particle collisions** in particle accelerators

Energy Released in Nuclear Reactions

- The binding energy is equal to the amount of energy released in forming the nucleus, and can be calculated using:

$$E = (\Delta m)c^2$$

- Where:
  - E = Binding energy released (J)
  - Δm = mass defect (kg)
  - c = speed of light (m s^{-1})

- The daughter nuclei produced as a result of both fission and fusion have a higher binding energy per nucleon than the parent nuclei
- Therefore, energy is released as a result of the mass difference between the parent nuclei and the daughter nuclei

Your notes

Worked Example

When Uranium−235 nuclei are fissioned by slow-moving neutrons, two possible reactions are:

Reaction 1: $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{139}_{54}Xe + ^{95}_{38}Sr + 2^{1}_{0}n$ + energy

Reaction 2: $^{235}_{92}U + ^{1}_{0}n \rightarrow 2\,^{116}_{46}Pd + xc$ + energy

a) For reaction 2, identify the particle c and state the number x of such particles generated in the reaction.

b) The binding energy per nucleon *E* for a number of nuclides is given by the table below. Use the table to show that the energy produced in reaction 1 is about 210 MeV.

c) The energy produced in reaction 2 is 163 MeV. Suggest, with supporting reason, which one of the two reactions is more likely to happen.

nuclide	E / MeV
$^{95}_{38}Sr$	8.74
$^{139}_{54}Xe$	8.39
$^{235}_{92}U$	7.60

Answer:

Part (a)

Step 1: Balance the number of protons on each side (bottom number)

$$92 = (2 \times 46) + xn_p \text{ (where } n_p \text{ is the number of protons in c)}$$

$$xn_p = 92 - 92 = 0$$

Therefore, c must be a neutron

Step 2: Balance the number of nucleons on each side

$$235 + 1 = (2 \times 116) + x$$

$$x = 235 + 1 - 232 = 4$$

Therefore, 4 neutrons are generated in the reaction

Part (b)

Step 1: Find the binding energy of each nucleus

Total binding energy of each nucleus = Binding energy per nucleon × Mass number

Binding energy of $^{95}Sr = 8.74 \times 95 = 830.3$ MeV

Binding energy of $^{139}Xe = 8.39 \times 139 = 1166.21$ MeV

Binding energy of $^{235}U = 7.60 \times 235 = 1786$ MeV

Step 2: Calculate the difference in energy between the products and reactants

$$\text{Energy released in reaction 1} = E_{Sr} + E_{Xe} - E_U$$

$$\text{Energy released in reaction 1} = 830.3 + 1166.21 - 1786$$

$$\text{Energy released in reaction 1} = 210.5 \text{ MeV}$$

Part (c)

- Since reaction 1 releases more energy than reaction 2, its end products will have a higher binding energy per nucleon
  - Hence, they will be more stable
- This is because the more energy is released, the further it moves up the graph of binding energy per nucleon against nucleon number (A)
  - Since at high values of A, binding energy per nucleon gradually decreases with A
- Nuclear reactions will tend to favour the more stable route, therefore, reaction 1 is more likely to happen

Your notes

Your notes

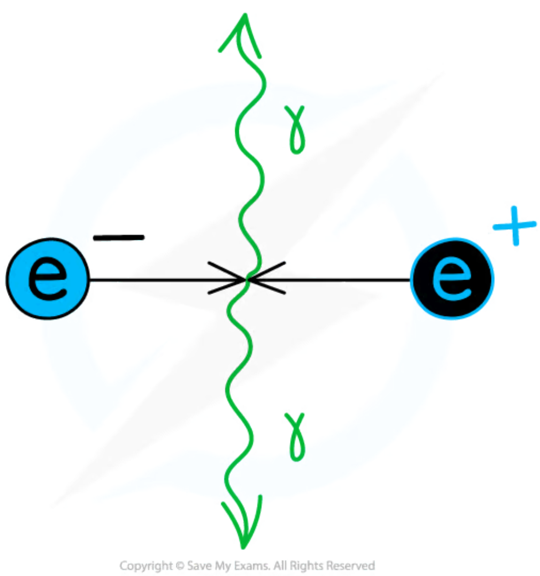
Particle-Antiparticle Pairs

Annihilation of Matter & Antimatter

Annihilation

- When a particle meets its antiparticle partner, the two will **annihilate**
- Annihilation is:

  When a particle meets its equivalent anti-particle they both are destroyed and their mass is converted into energy in the form of two gamma ray photons



When an electron and positron collide, their mass is converted into energy in the form of two photons emitted in opposite directions

Pair Production

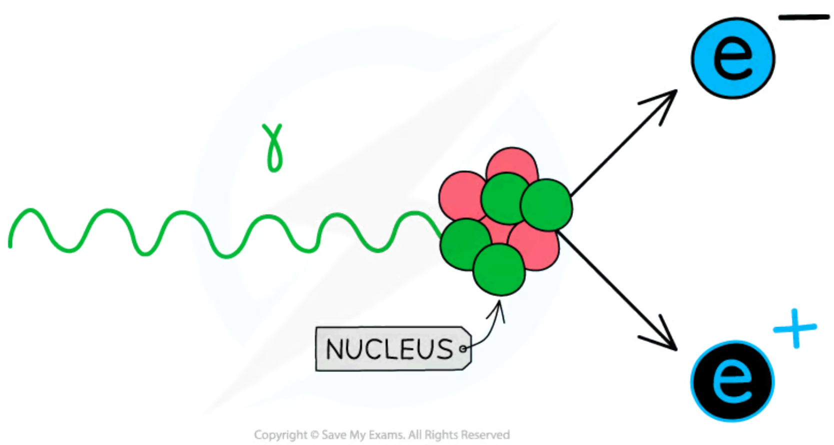
- Pair production is the opposite of annihilation
- Pair production is:

  When a photon interacts with a nucleus or atom and the energy of the photon is used to create a particle-antiparticle pair

- The presence of a nearby neutron is essential in pair production so that the process conserves both energy and momentum

Your notes

- A single photon alone cannot produce a particle-anti-particle pair or the conservation laws would be broken
- Pair creation is a case of energy being converted into matter



When a photon with enough energy interacts with a nucleus it can produce an electron-positron pair

- This means the energy of the photon must be above a certain value to provide the **total rest mass energy** of the particle-antiparticle pair
- Einstein's famous mass-energy relation showed that **energy** can be converted into **mass**, and vice versa
- It is given by:

$$\Delta E = c^2 \Delta m$$

- Where:
    - Δm = rest mass of the particle (kg)
    - c = speed of light (m s^{-1})
    - ΔE = rest mass energy of the particle (J)
- Therefore, in order to create a particle & anti-particle pair, the energy carried by a single photon must be **at least** twice the rest-mass energy required, i.e.

$$\mathbf{2\Delta E = 2(c^2 \Delta m)}$$

- This also means if a particle meets its anti-particle and annihilates, the energy carried away by **each** of the two photons E_{photon} is given by:

$$E_{photon} = hf = \frac{hc}{\lambda} = c^2 \Delta m$$

Your notes

Worked Example

Calculate the maximum wavelength of one of the photons produced when a proton and antiproton annihilate each other.

Answer:

Step 1: Write down the known quantities

Rest mass energy of a proton (and antiproton) = 938.257 MeV

$$1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$$

Step 2: Write the equation for the minimum photon energy

$$E_{min} = hf_{min} = E$$

Step 3: Write energy in terms of wavelength

$$f_{min} = \frac{c}{\lambda_{max}}$$

$$E_{min} = \frac{hc}{\lambda_{max}} = E$$

Step 4: Rearrange for wavelength

$$\lambda_{max} = \frac{hc}{E}$$

Step 5: Substitute in values

$$\lambda_{max} = \frac{\left(6.63 \times 10^{-34}\right) \times \left(3.0 \times 10^{8}\right)}{938.257 \times \left(1.60 \times 10^{-13}\right)} = 1.32 \times 10^{-15} \text{ m}$$

Examiner Tips and Tricks

Since the Planck constant is in Joules (J) remember to always convert the rest mass-energy from MeV to J.

Your notes

Mass Defect & Binding Energy

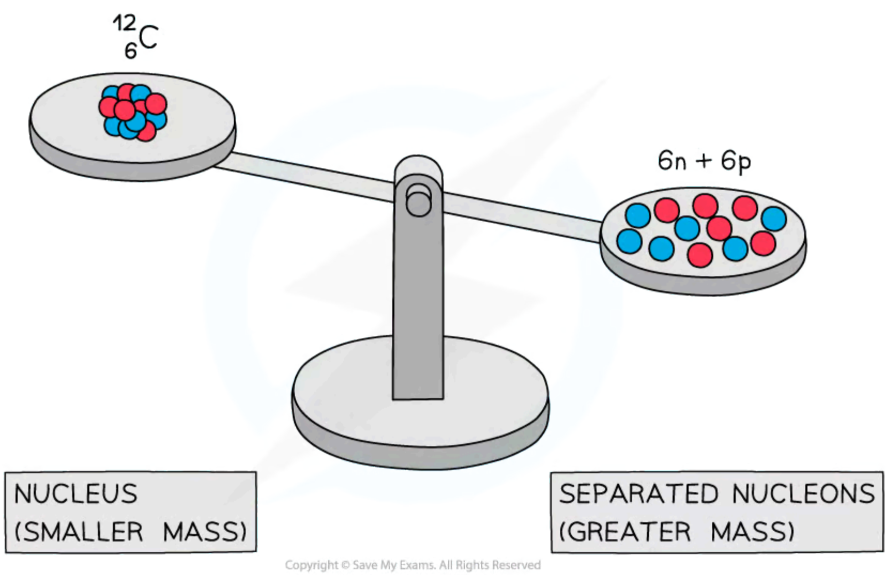
Your notes

Mass Defect & Binding Energy

Mass Defect & Binding Energy

- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
- This difference in mass is known as the **mass defect** or **mass deficit**
- Mass defect is defined as:

  The difference between the measured mass of a nucleus and the sum total of the masses of its constituents



A system of separated nucleons has a greater mass than a system of bound nucleons

- Due to the equivalence of mass and energy, this decrease in mass implies that energy is released in the process
- Since nuclei are made up of neutrons and protons, there are forces of repulsion between the positive protons
  - Therefore, it takes energy, ie. the binding energy, to hold nucleons together as a nucleus
- Binding energy is defined as:

Your notes

The energy required to break a nucleus into its constituent protons and neutrons

- Energy and mass are proportional, so, the total energy of a nucleus is less than the sum of the energies of its constituent nucleons
- The formation of a nucleus from a system of isolated protons and neutrons is therefore an exothermic reaction - meaning that it releases energy

Examiner Tips and Tricks

Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put into the nucleus to pull it apart.

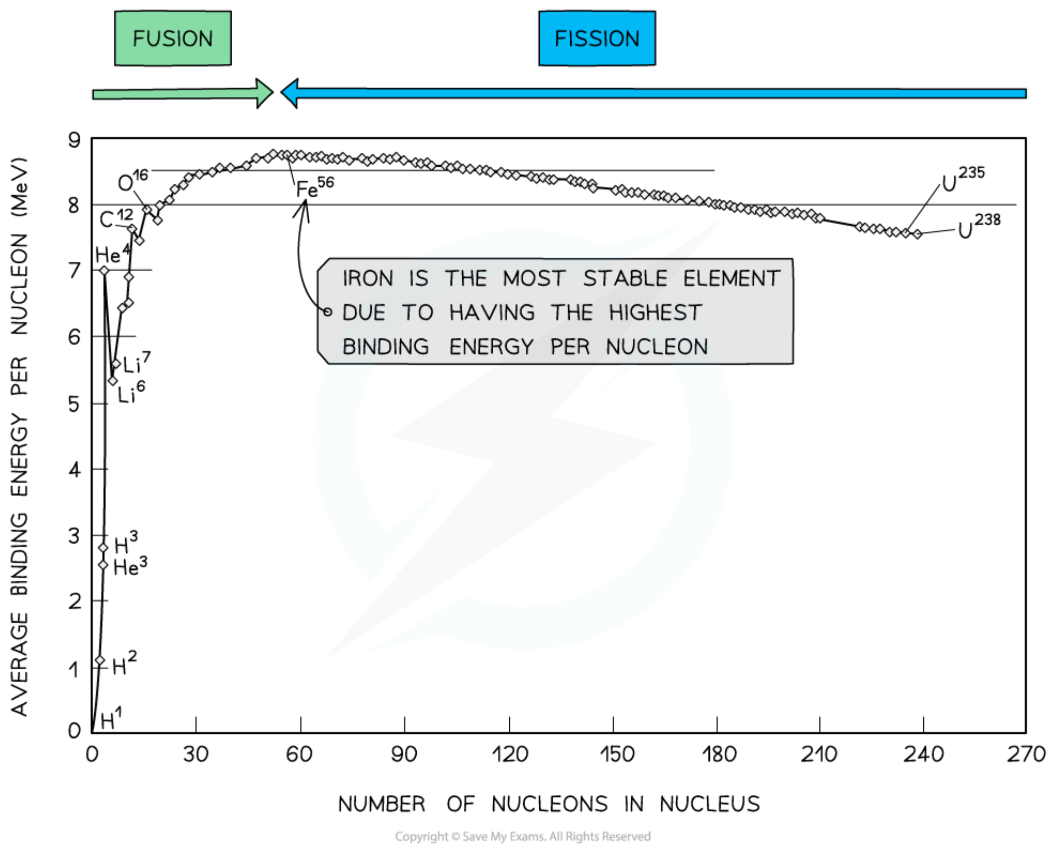
Binding Energy per Nucleon Graph

- In order to compare nuclear stability, it is more useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability
  - In other words, it requires more energy to pull the nucleus apart
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the most stable of all the elements

Your notes



By plotting a graph of binding energy per nucleon against nucleon number, the stability of elements can be inferred

Key Features of the Graph

- At low values of A:
  - Nuclei tend to have a lower binding energy per nucleon, hence, they are generally less stable
  - This means the lightest elements have weaker electrostatic forces and are the most likely to undergo **fusion**
- Helium (^{4}He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend
  - Helium-4 is a particularly stable nucleus hence it has a high binding energy per nucleon
  - Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together

- At high values of A:
  - The general binding energy per nucleon is high and gradually decreases with A
  - This means the heaviest elements are the most unstable and likely to undergo **fission**

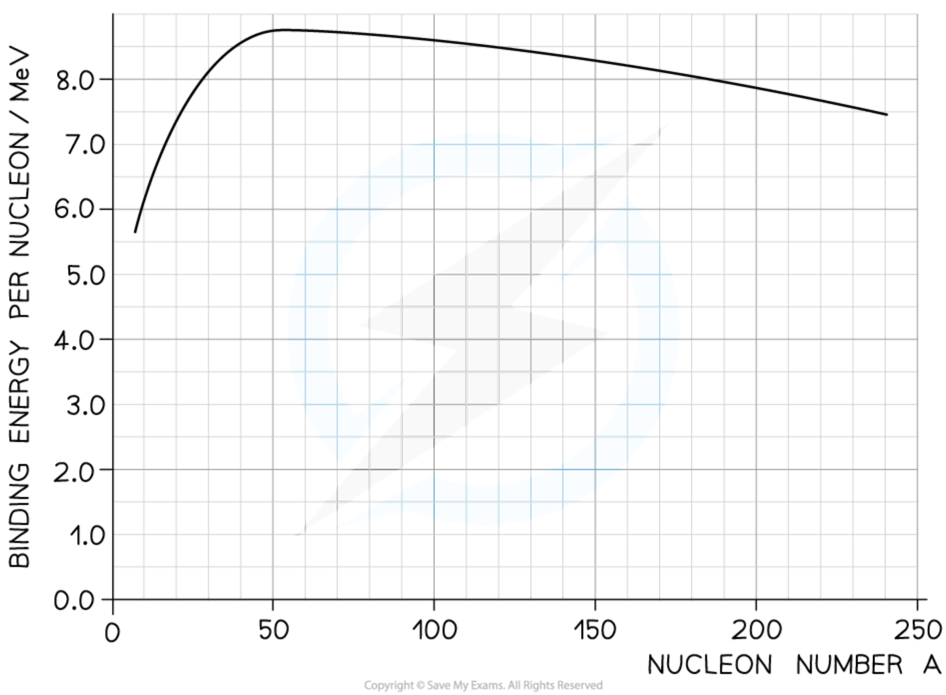
Your notes

Worked Example

The following equation represents one possible decay of the induced fission of a nucleus of uranium-235.

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{91}_{38}\text{Sr} + ^{142}_{54}\text{Xe} + 3^{1}_{0}\text{n}$$

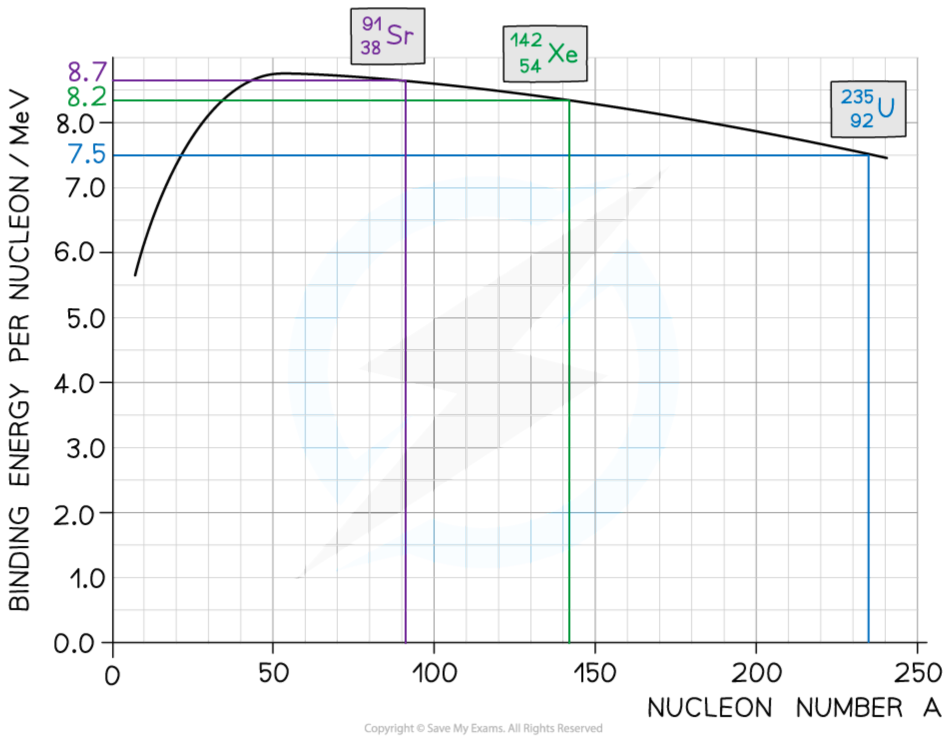
The graph shows the binding energy per nucleon plotted against nucleon number A.



Calculate the energy released by the fission of uranium-235.

Answer:

Step 1: Use the graph to identify each isotope's binding energy per nucleon



- Binding energy per nucleon (U-235) = 7.5 MeV
- Binding energy per nucleon (Sr-91) = 8.2 MeV
- Binding energy per nucleon (Xe-142) = 8.7 MeV

Step 2: Determine the binding energy of each isotope

$$\text{Binding energy} = \text{Binding Energy per Nucleon} \times \text{Mass Number}$$

- Binding energy of U-235 nucleus = (235 × 7.5) = 1763 MeV
- Binding energy of Sr-91 = (91 × 8.2) = 746 MeV
- Binding energy of Xe-142 = (142 × 8.7) = 1235 MeV

Step 3: Calculate the energy released

$$\text{Energy released} = \text{Binding energy after (Sr + Xe)} - \text{Binding energy before (U)}$$

$$\text{Energy released} = (1235 + 746) - 1763 = 218 \text{ MeV}$$

Examiner Tips and Tricks

Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

- You will be expected to draw the best fit curve AND a cross to show the anomaly that is helium
- Do not begin your curve at A = 0, this is not a nucleus!
- Make sure to correctly label both axes AND units for binding energy per nucleon
- You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

Your notes

Calculating Binding Energy

Your notes

Calculating Binding Energy

- **Binding energy** can be calculated using the equation:

$$\Delta E = c^2 \Delta m$$

- Where Δm is the mass defect of a nucleus
- This can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

  - Where:
    - Z = proton number
    - A = nucleon number
    - m_p = mass of a proton (kg)
    - m_n = mass of a neutron (kg)
    - m_{total} = measured mass of the nucleus (kg)

Worked Example

What is the binding energy per nucleon of iron-56 ($^{56}_{26}Fe$) in MeV?

Mass of a neutron = 1.675×10^{-27} kg
Mass of a proton = 1.673×10^{-27} kg
Mass of $^{56}_{26}Fe$ nucleus = 9.288×10^{-26} kg

Answer:

Step 1: Calculate the mass defect

Number of protons, Z = 26

Your notes

Number of neutrons, $A - Z = 56 - 26 = 30$

Mass defect, $\Delta m = Zm_p + (A - Z)m_n - m_{total}$

$$\Delta m = (26 \times 1.673 \times 10^{-27}) + (30 \times 1.675 \times 10^{-27}) - (9.288 \times 10^{-26})$$

$$\Delta m = 8.680 \times 10^{-28} \text{ kg}$$

Step 2: Calculate the binding energy of the nucleus

Binding energy, $\Delta E = c^2 \Delta m$

$$E = (8.680 \times 10^{-28}) \times (3.00 \times 10^8)^2 = 7.812 \times 10^{-11} \text{ J}$$

Step 3: Calculate the binding energy per nucleon

$$\text{Binding energy per nucleon} = \frac{E}{A}$$

$$\frac{E}{A} = \frac{7.812 \times 10^{-11}}{56} = 1.395 \times 10^{-12} \text{ J}$$

Step 4: Convert to MeV

$J \rightarrow eV$: divide by 1.6×10^{-19}

$eV \rightarrow MeV$: divide by 10^6

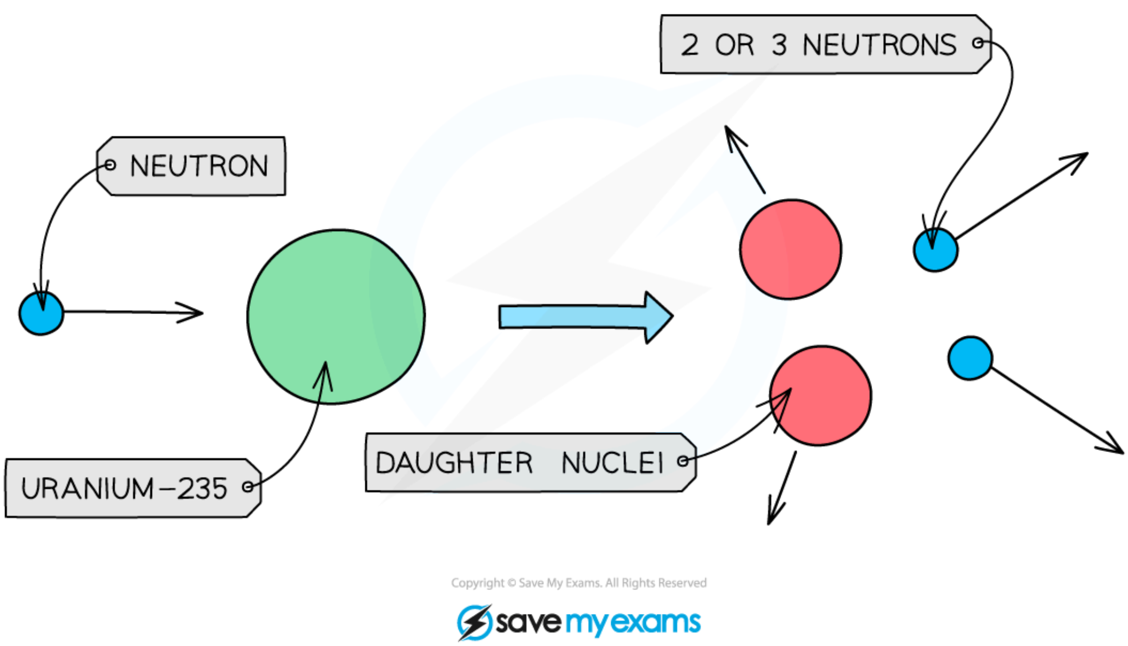
$$\text{Binding energy per nucleon} = \frac{1.395 \times 10^{-12}}{1.6 \times 10^{-19}} = 8\,718\,750 \text{ eV} = \mathbf{8.7\ MeV} \text{ (2 s.f.)}$$

Nuclear Fission

Your notes

Nuclear Fission

- There is a lot of energy stored within the nucleus of an atom
  - This energy can be released in a nuclear reaction such as **fission** or **fusion**
- Nuclear fission is defined as:
- **The splitting of a large, unstable nucleus into two smaller nuclei**
- Isotopes of **uranium** and **plutonium** both undergo fission and are used as fuels in nuclear power stations
- During fission, when a neutron collides with an unstable nucleus, the nucleus splits into **two smaller nuclei** (called daughter nuclei) as well as **two or three neutrons**
  - Gamma rays are also emitted



Large nuclei can decay by fission to produce smaller nuclei and neutrons with a lot of kinetic energy

- The products of fission move away very **quickly**
  - Energy transferred is from **nuclear potential energy** to kinetic energy

Your notes

Spontaneous Fission

- It is **rare** for nuclei to undergo fission without additional energy being put into the nucleus
- When nuclear fission occurs in this way it is called **spontaneous fission**

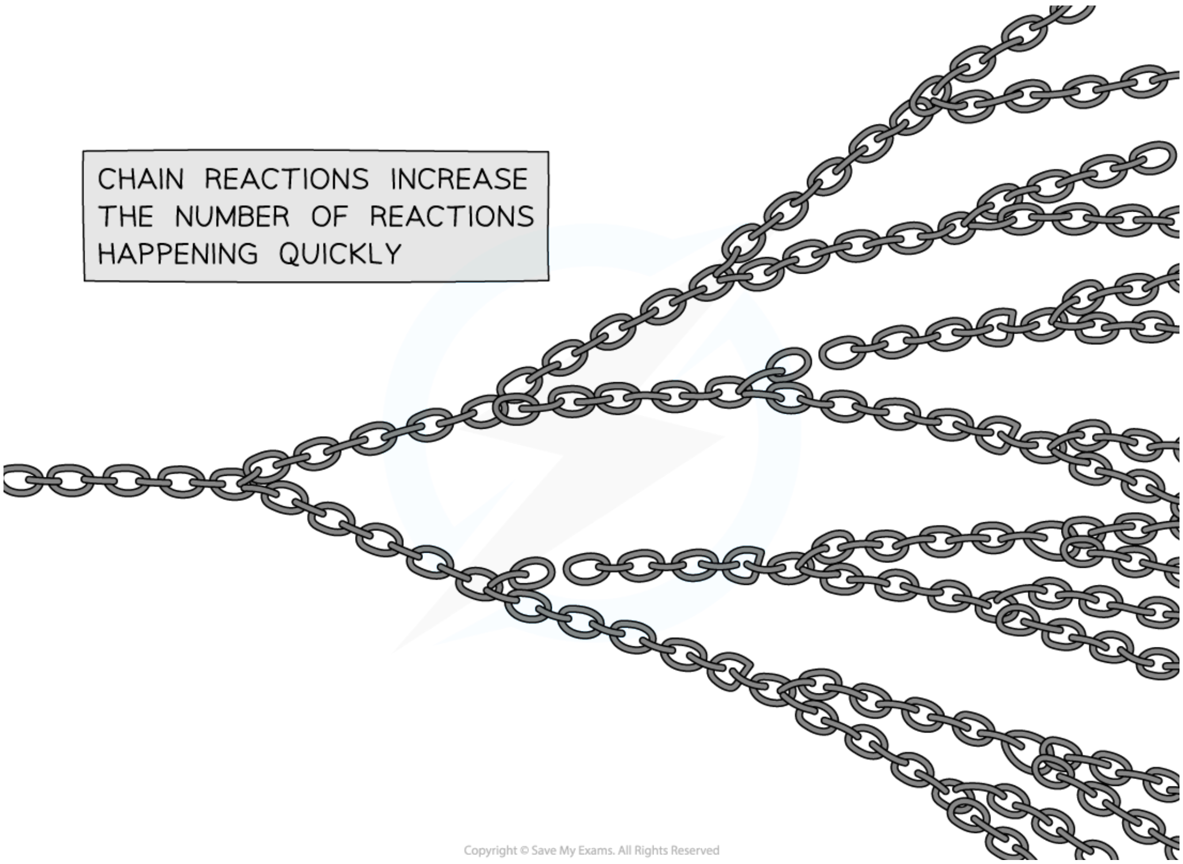
Induced Fission

- Usually, for fission to occur the unstable nucleus must first **absorb** a **neutron**
- Take, for example, uranium-235, which is commonly used as a fuel in nuclear reactors
- It has a very long half-life of 700 million years
- This means that it would have low activity and energy would be released very slowly
  - This is unsuitable for producing energy in a nuclear power station
- During induced fission, a **neutron** is absorbed by the uranium-235 nucleus to make uranium-236
- This is very unstable and splits by nuclear fission almost immediately

Chain Reactions

- Only one extra neutron is required to induce a Uranium-235 nucleus to split by fission
- During the fission, it produces two or three neutrons which move away at high speed
- Each of these new neutrons can start another fission reaction, which again creates further **excess neutrons**
- This process is called a **chain reaction**

Your notes



The neutrons released by each fission reaction can go on to create further fissions, like a chain that is linked several times – from each chain comes two more

Worked Example

During a particular spontaneous fission reaction, plutonium-239 splits as shown in the equation below:

$$^{239}_{94}Pu \longrightarrow {}^{112}_{46}Pd + {}^{124}_{48}Cd + \ldots$$

Which answer shows the section missing from this equation?

Your notes

A	^3_0n
B	$^0_0\gamma$
C	$^4_2\alpha$
D	$3\,^1_0n$

Answer: D

Step 1: Identify the different mass and atomic numbers

- Pu (Plutonium) has mass number 239 and atomic number 94
- Pd (Palladium) has mass number 112 and atomic number 46
- Cd (Cadmium) has mass number 124 and atomic number 48

Step 2: Calculate the mass and atomic number of the missing section

- Mass number is equal to the difference between the mass numbers of the reactants and the products

$$239 - (112 + 124) = 3$$

- Atomic number is equal to the difference between the atomic numbers of the reactants and the products

$$94 - (46 + 48) = 0$$

- The answer is therefore not **B** or **C**

Step 3: Determine the correct notation

- Neutrons have a mass number of 1
- The answer is therefore not **A**
- Therefore, this must be three neutrons, which corresponds to **D**

Examiner Tips and Tricks

Fission and fusion are very different processes. Fusion comes from the word "fuse" as in bind/stick together. This brings small nuclei together. Fission is the opposite. It is the breaking down of large nuclei like Uranium.

Your notes

Nuclear Fission Reactor & Waste

Structure of a Fission Reactor

Moderator

The purpose of a moderator: To slow down neutrons

- The moderator is a material that surrounds the fuel rods and control rods inside the reactor core
- The fast-moving neutrons produced by the fission reactions slow down by colliding with the molecules of the moderator, causing them to lose some momentum
- The neutrons are slowed down so that they are in **thermal equilibrium** with the moderator, hence the term 'thermal neutron'
  - This ensures neutrons can react efficiently with the uranium fuel

Control Rods

Purpose of a control rod: To absorb neutrons

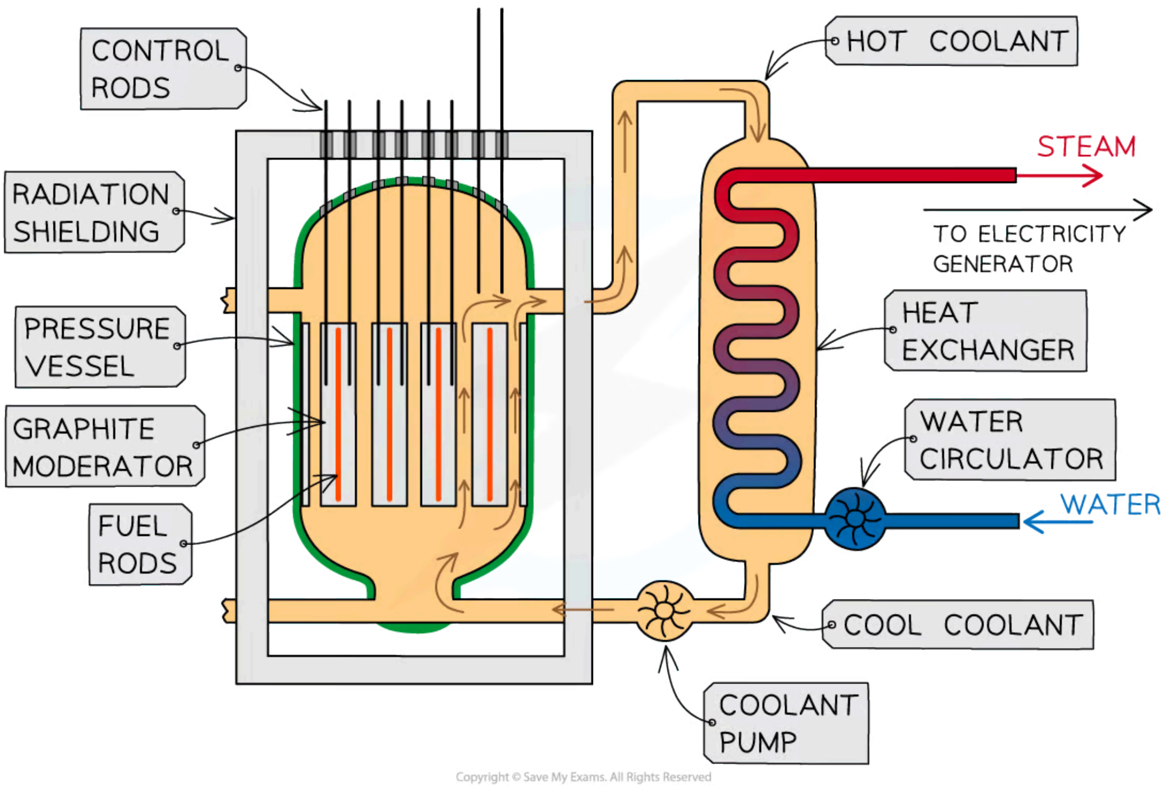
- The number of neutrons absorbed is controlled by varying the depth of the control rods in the fuel rods
  - Lowering the rods further **decreases** the rate of fission, as more neutrons are absorbed
  - Raising the rods **increases** the rate of fission, as fewer neutrons are absorbed
- This is adjusted automatically so that exactly one fission neutron produced by each fission event goes on to cause another fission
- In the event the nuclear reactor needs to shut down, the control rods can be lowered all the way so no reaction can take place

Coolant

The purpose of coolant: To remove the heat released by the fission reactions

- The coolant carries the heat to an external boiler to produce steam
- This steam then goes on to power electricity-generating turbines

Your notes



Components of a nuclear reactor

Environmental Impact of Nuclear Waste

The End of the Reactor Process

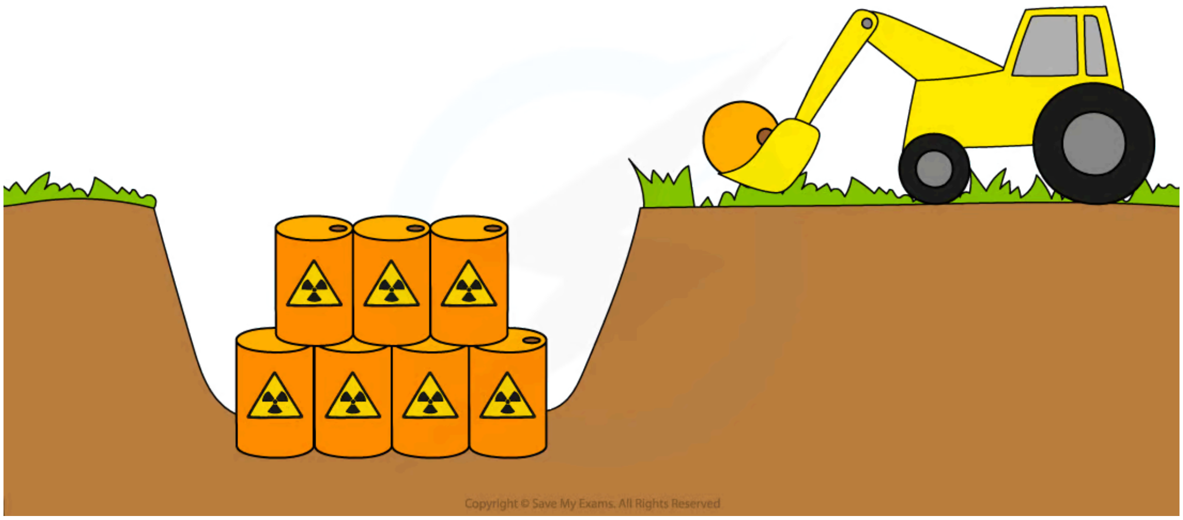
- Within the fuel rods, nuclei of **uranium-238** quickly decay into nuclei of **plutonium-239**
  - These nuclei are extremely radioactive
  - They have a long half-life of 24 000 years
- So, **plutonium-239** decays slowly
  - It will **remain radioactive for a very long time**
  - So, it presents a risk of **contamination** for a long time
  - It is classified as **high-level radioactive waste**

Types of Radioactive Waste

- There are three main types of **nuclear waste**:

- Low level
- Intermediate level
- High level
- **Low-level waste**
  - This is waste such as clothing, gloves and tools which may be lightly contaminated
  - This type of waste will be radioactive for a few years, so must be encased in concrete and stored a few metres underground until it can be disposed of with regular waste
- **Intermediate-level waste**
  - This is everything between daily used items and the fuel rods themselves
  - Usually, this is the waste produced when a nuclear power station is decommissioned and taken apart
  - This waste will have a longer half-life than the low-level waste, so must be encased in cement in steel drums and stored securely underground
- **High-level waste**
  - This waste comprises of the unusable fission products from the fission of uranium-235 or from spent fuel rods
  - This is by far the **most dangerous** type of waste as it will remain radioactive for thousands of years
  - As well as being highly radioactive, the spent fuel roads are **extremely hot** and must be handled and stored much more carefully than the other types of waste
- How high-level waste is treated:
  - The waste is initially placed in cooling ponds of water close to the reactor for a number of years
  - Isotopes of plutonium and uranium are harvested to be used again
  - Waste is mixed with molten glass and made solid (this is known as **vitrification**)
  - Then it is encased in containers made from steel, lead, or concrete
  - This type of waste must be stored very **deep** underground

Your notes



Depending on the activity of radioactive waste, it is buried in different ways

Environmental Considerations

- Isotopes with long half-lives must not enter our water and food supplies
- Burial locations must be geologically stable, secure from attack, and designed for safety
- Space for such locations is limited

Nuclear Fusion

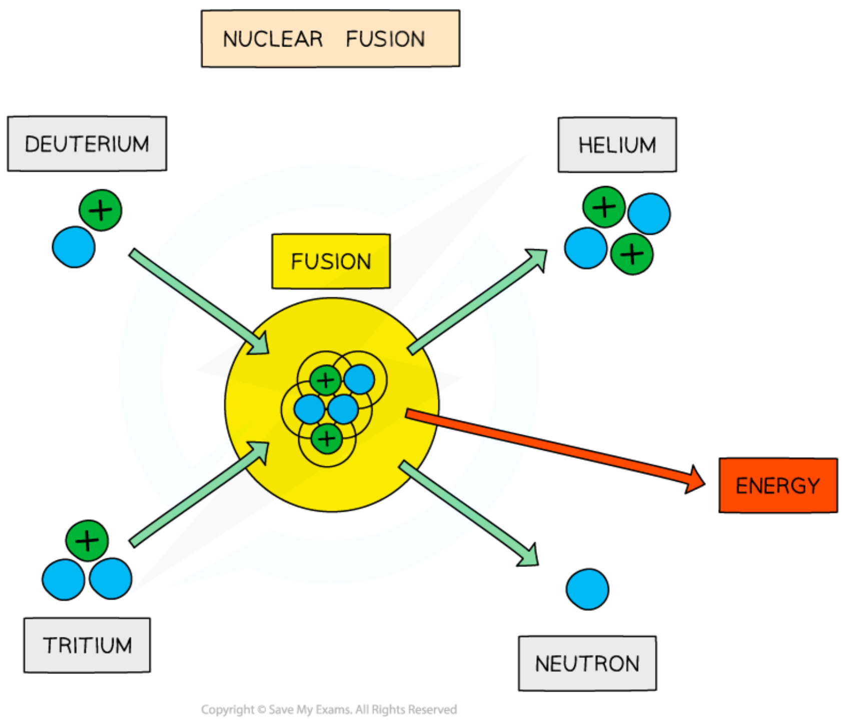
Your notes

Nuclear Fusion

- Fusion is defined as:

  Small nuclides combine together to make larger nuclei, releasing energy

- Low mass nuclei (such as hydrogen and helium) can undergo fusion and release energy



The fusion of deuterium and tritium to form helium with the release of energy

- For two nuclei to fuse, both nuclei must have high kinetic energy
  - This is because the protons inside the nuclei are positively charged, which means that they repel one another
- It takes a great deal of energy to overcome the electrostatic force, so this is why it can only be achieved in an extremely high-energy environment, such as a star's core
- When two protons fuse, the element deuterium is produced

- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning

Your notes

Examiner Tips and Tricks

In the fusion process, the mass of the new heavier nucleus is less than the mass of the constituent parts of the nuclei fused together, as some mass is converted into energy.

Not all of this energy is used as binding energy for the new larger nucleus, so energy will be released from this reaction. The binding energy per nucleon afterwards is higher than at the start.

Your notes

Balancing Nuclear Equations

Balancing Nuclear Equations

- Nuclear reactions, such as fission and fusion, can be represented using nuclear equations (which are similar to chemical equations in Chemistry)

  For example:

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{90}_{38}Sr + ^{144}_{54}Xe + 2\,^{1}_{0}n$$

- The above equation represents a fission reaction in which a Uranium nucleus is hit with a neutron and splits into two smaller nuclei – a Strontium nucleus and a Xenon nucleus, releasing two neutrons in the process
- In the above reaction:

  The sum of top (nucleon) numbers on the left-hand side equals the sum of top number on the right-hand side:

$$\mathbf{235 + 1 = 236 = 90 + 144 + 2 \times 1}$$

The same is true for the lower (proton) numbers:

$$\mathbf{92 + 0 = 92 = 38 + 54 + 2 \times 0}$$

- By balancing equations in this way, you can determine, for example, the number of neutrons emitted by a process like this

  Example:

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{96}_{37}Rb + ^{137}_{55}Cs + N\,^{1}_{0}n$$

- In the above example, balancing the numbers on the top shows that 3 neutrons must be released in the reaction (i.e. $N = 3$)

Your notes