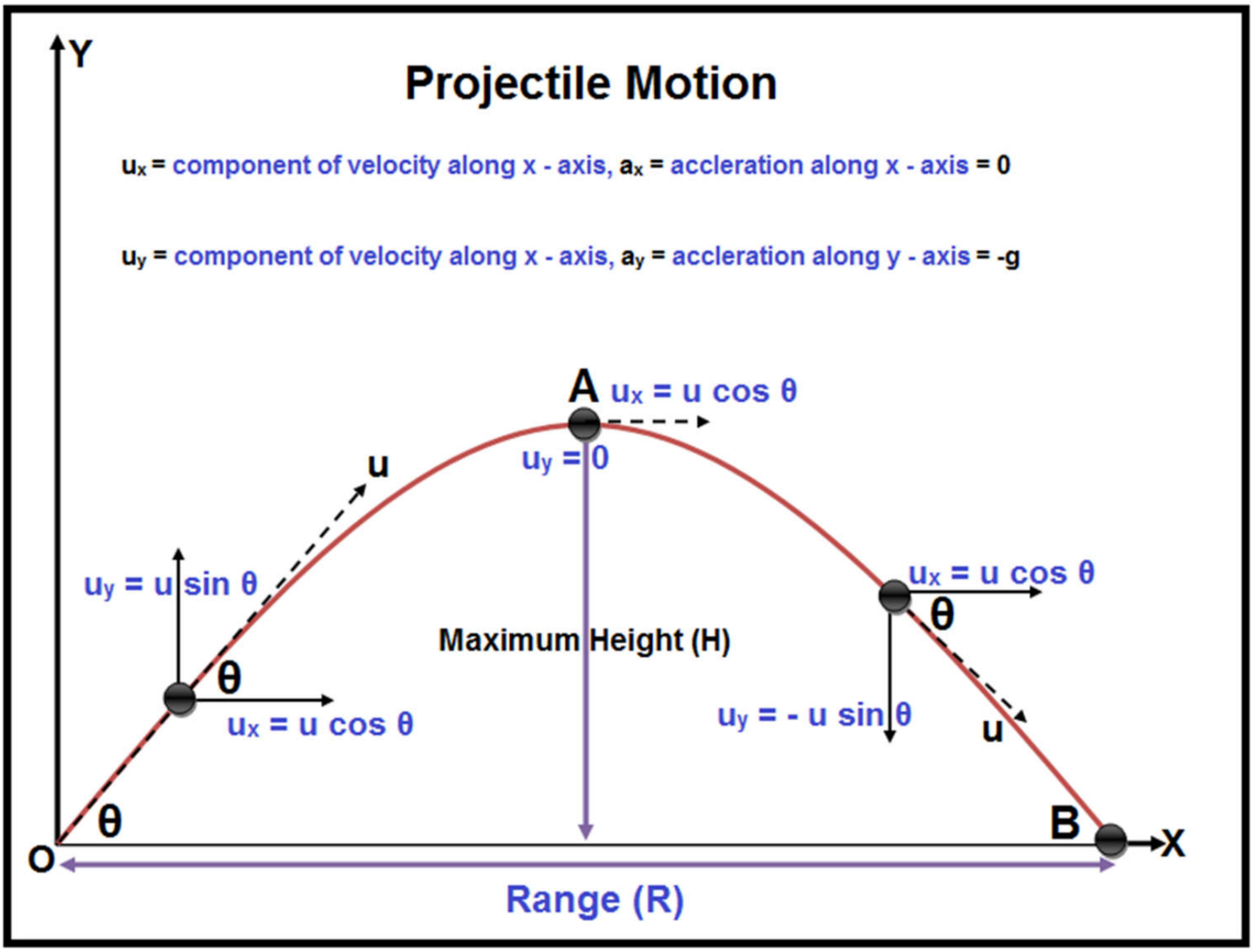
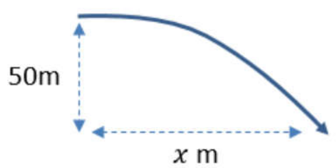


Projectile Motion

In Mechanics Year 1 we already encountered problems of vertical motion of objects when projected vertically. We used "SUVAT" equations where the acceleration was g ms^{-2}.
In this chapter we allow the object to be **projected sideways**!

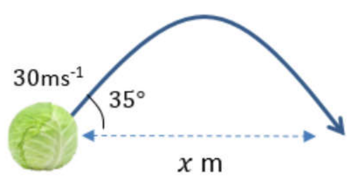
1:: Horizontally projected

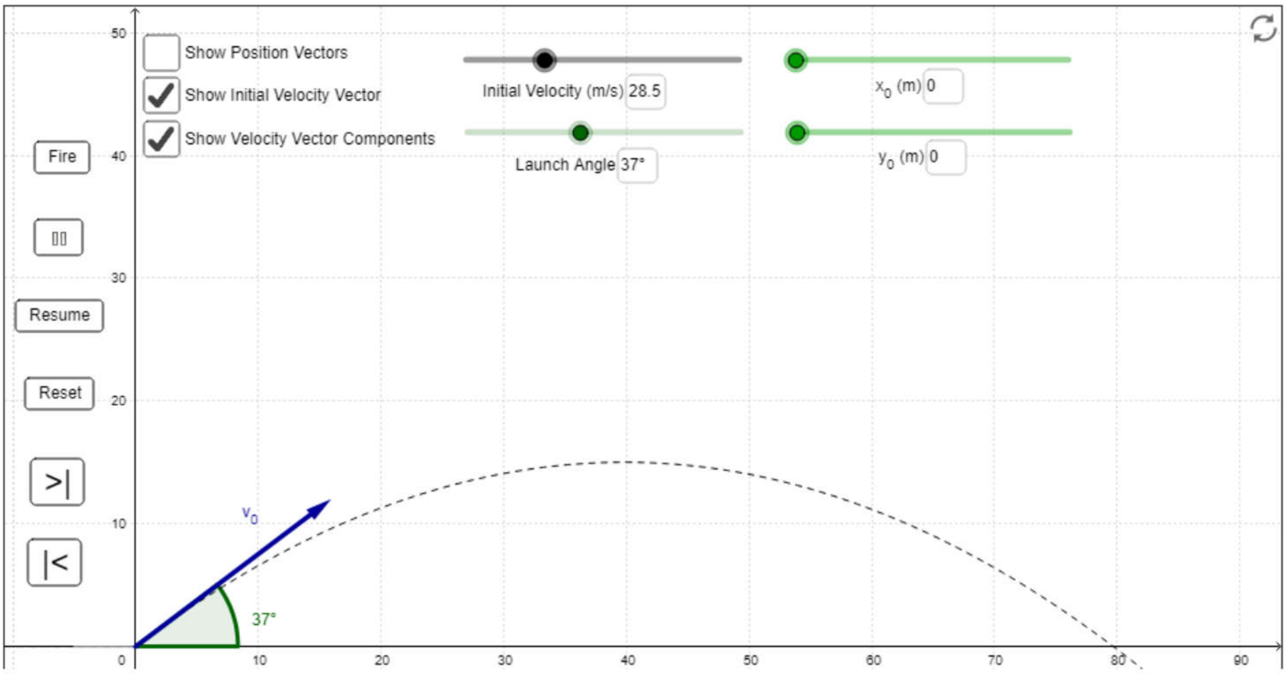
"A particle is projected horizontally at 20 ms^{-1}, at a distance 50m above the ground. How far along the ground does it travel?"



2:: Projection at any angle

"A cabbage is projected from ground level at 30 ms^{-1} at an angle of 35°. How far away is the cabbage when it hits the ground?"





Acceleration in each direction

Consider vertical and horizontal motion separately

In **vertical** direction, acceleration downwards is g ms^{-2}. (it is constant)

Use SUVAT equations as before.

In **horizontal** direction, acceleration is 0 ms^{-2}.

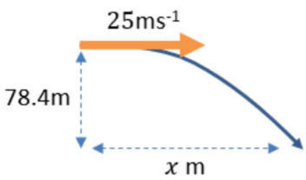
Constant velocity, so can use standard $speed = \frac{distance}{time}$

A particle is projected horizontally at 25 ms^{-1} from a point 78.4 metres above a horizontal surface. Find:

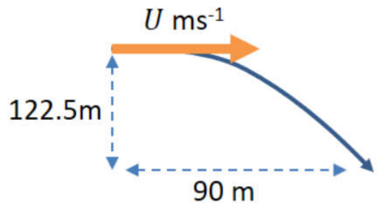
(a) the time taken by the particle to reach the surface

(b) the horizontal distance travelled in that time.

(c) the distance of the impact point from the original point.

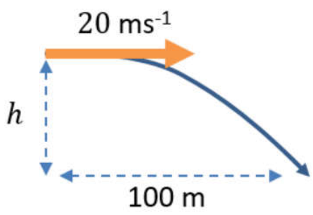


A particle is projected horizontally with a speed of U ms^{-1} from a point 122.5m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 90m away from the starting point.
Find the initial speed of the particle.



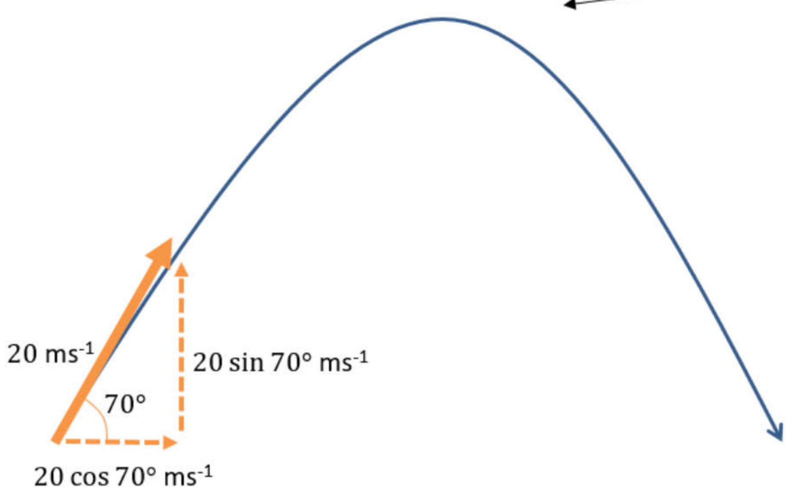
Your Turn

A particle is projected horizontally with a speed of 20 ms^{-1} from a point h m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 100m away from the starting point. Determine the value of h.



Components of velocity

Just as **we split forces into its horizontal and vertical components**, in order to consider forces in the horizontal and vertical directions respectively, we can do **exactly the same with velocity**!



When the object is at its highest point:

We know that the scalar form of velocity is **speed**, and thus we just find the **magnitude** of the velocity vector:

$$\begin{pmatrix}12\\5\end{pmatrix} ms^{-1}$$
$$\Rightarrow \sqrt{\mathbf{12^2+5^2}}$$
$$= \mathbf{13\ ms^{-1}}$$

A particle P is projected from a point O on a horizontal plane with speed 28 ms^{-1} and with angle of elevation 30°. After projection, the particle moves freely under gravity until it strikes the plane at a point A. Find:

(a) the greatest height above the plane reached by P

(b) the time of flight of P

(c) the distance OA

Projected from above ground

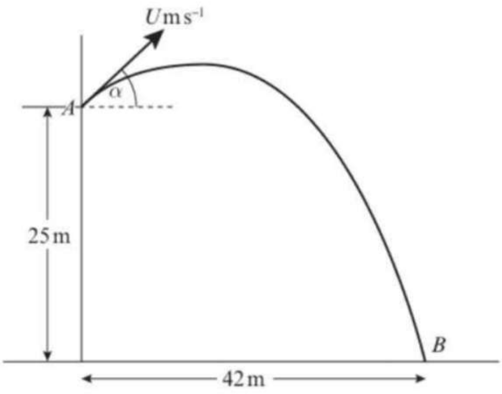
A particle is projected from a point O with speed V ms^{-1} and at an angle of elevation of θ, where $\tan\theta = \frac{4}{3}$. The point O is 42.5m above a horizontal plane. The particle strikes the plane at a point A, 5 s after it is projected.
(a) Show that $V = 20$. (b) Find the distance between O and A.

Time above a given point

A particle is projected from a point O with speed 35 ms^{-1} and at an angle of elevation of 30°. The particle moves freely under gravity. Find the length of time for which the particle is 15 m or more above O.

Finding speed at a given point

In this question use $g = 10\,\text{m}\,\text{s}^{-2}$.
An object is projected with speed $U\,\text{m}\,\text{s}^{-1}$ from a point A at the top of a vertical building. The point A is 25 m above the ground. The object is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$. The object hits the ground at the point B, which is at a horizontal distance of 42 m from the foot of the building, as shown in the diagram. The object is modelled as a particle moving freely under gravity.



Find:

a the value of U **(6 marks)**

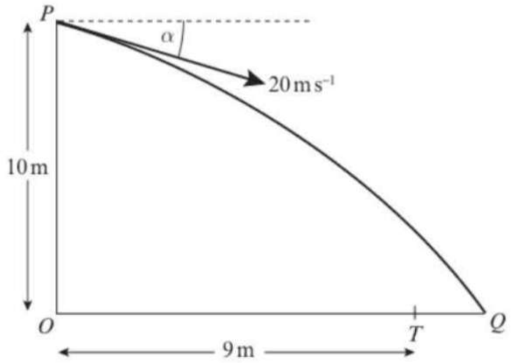
b the time taken by the object to travel from A to B **(2 marks)**

c the speed of the object when it is 12.4 m above the ground, giving your answer to 2 significant figures. **(5 marks)**

8 In this question use $g = 10\,\text{m}\,\text{s}^{-2}$.

A stone is thrown from a point P at a target, which is on horizontal ground. The point P is 10 m above the point O on the ground. The stone is thrown from P with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of α below the horizontal, where $\tan\alpha = \frac{3}{4}$.

The stone is modelled as a particle and the target as a point T. The distance OT is 9 m. The stone misses the target and hits the ground at the point Q, where OTQ is a straight line, as shown in the diagram. Find:



a the time taken by the ball to travel from P to Q **(5 marks)**

b the distance TQ. **(4 marks)**

The point A is on the path of the ball vertically above T.

c Find the speed of the ball at A. **(5 marks)**

Edexcel M2(Old) May 2012 Q7

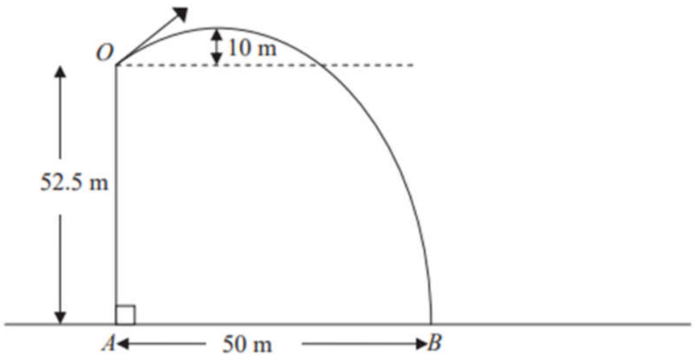


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.
(3)

(b) Find the speed of projection.
(9)

(c) Find the time after projection when the stone is moving parallel to OB.
(5)

Ex 6C

Projectile Formulae

There's nothing new here, but you may be asked to prove more general results regarding projectile motion.

A particle is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal and moves freely under gravity until it hits the plane at point B.
Given that that acceleration due to gravity is g, find expressions for:

(a) the time of flight, T

(b) the range, R, on the horizontal plane.

A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

(a) Show that $y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha)$

A particle is projected from a point O on a horizontal plane, with speed 28 ms^{-1} at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32m from O and at a height of 8m above the plane.

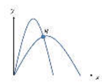
(b) Find the two possible values of α, giving your answers to the nearest degree.

A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

(a) Show that $y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha)$

A particle is projected from a point O on a horizontal plane, with speed 28 ms^{-1} at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32m from O and at a height of 8m above the plane.

(b) Find the two possible values of α, giving your answers to the nearest degree.



General Results

✏ For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \frac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \frac{U \sin \alpha}{g}$
- Range on horizontal plane $= \frac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory: $y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$
  where y is vertical height of particle and x horizontal distance.

Ex 6D