OCR A Level Physics

Your notes

Electric Potential & Energy

Contents

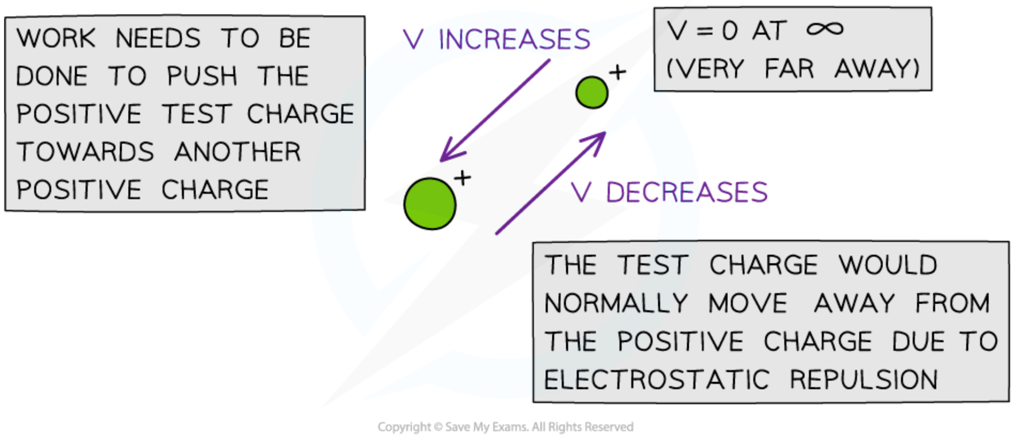
- Electric Potential
- Calculating Electric Potential
- Capacitance of an Isolated Sphere
- Force-Distance Graph
- Electric Potential Energy

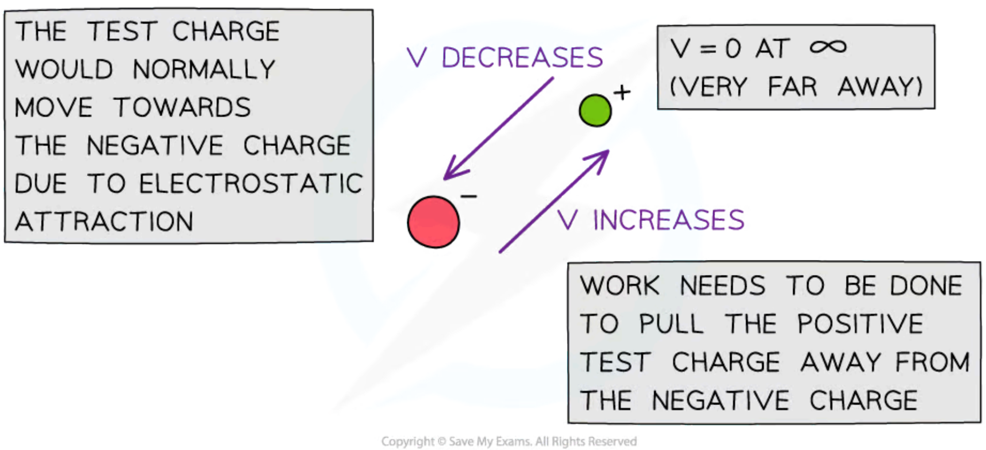
Your notes

Electric Potential

- In order to move a positive charge closer to another positive charge, work must be done to overcome the force of repulsion between them
  - Similarly, to move a positive charge away from a negative charge, work must be done to overcome the force of attraction between them
- Energy is therefore transferred to the charge that is being pushed upon
  - This means its **potential energy** increases
- If the positive charge is free to move, it will start to move away from the repelling charge
  - As a result, its potential energy decreases back to 0
- This is analogous to the gravitational potential energy of a mass increasing as it is being lifted upwards and decreasing as it falls
- The electric potential at a point is defined as:

  The work done per unit positive charge in bringing a point test charge from infinity to a defined point
- Electric potential is a **scalar** quantity
  - This means it doesn't have a direction
- Even though electric potential is a scalar quantity, it can take a positive or negative sign, this is because the electric potential is:
  - **Positive** around an isolated positive charge
  - **Negative** around an isolated negative charge
  - **Zero** at infinity
- Positive work is done by the mass from infinity to a point around a positive charge and negative work is done around a negative charge. This means:
  - When a positive test charge moves closer to a **negative** charge, its electric potential **decreases**
  - When a positive test charge moves closer to a **positive** charge, its electric potential **increases**
- To find the potential at a point caused by multiple charges, the total potential is the sum of the potential from each charge





The electric potential V decreases in the direction the test charge would naturally move in due to repulsion or attraction

Calculating Electric Potential

Your notes

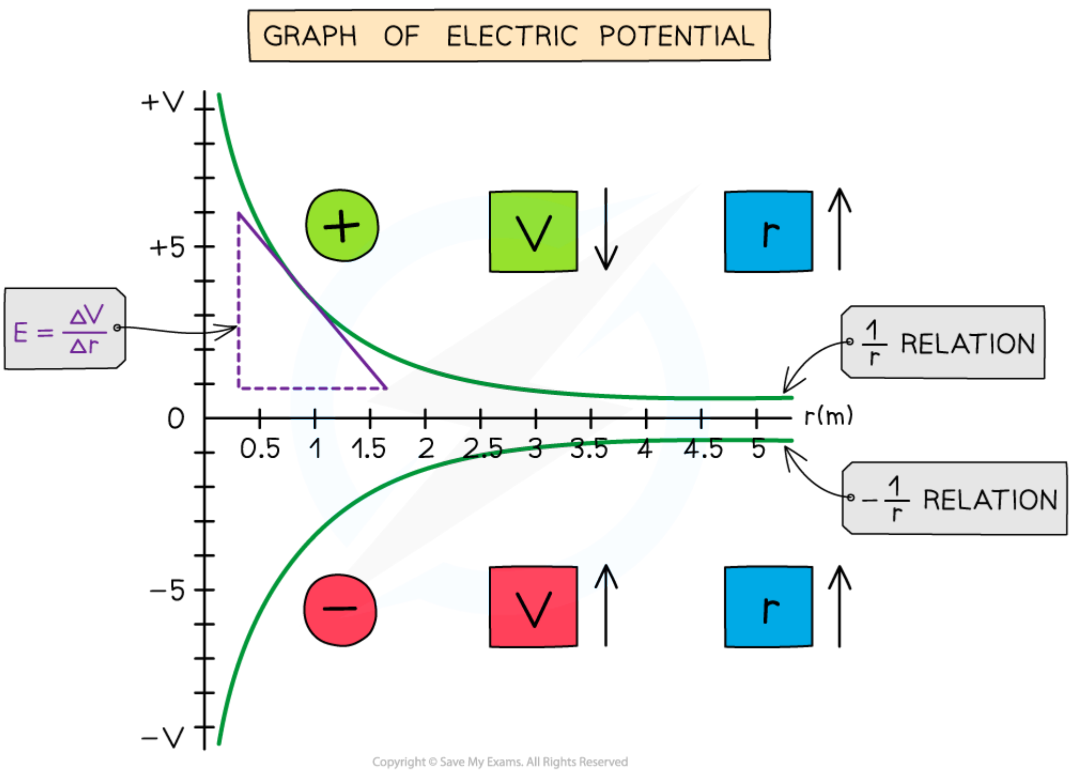
Calculating Electric Potential

- The **electric potential** in the field due to a **point charge** is defined as:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Where:
  - V = the electric potential (V)
  - Q = the point charge producing the potential (C)
  - ε_0 = permittivity of free space (F m^{-1})
  - r = distance from the centre of the point charge (m)
- This equation shows that for a positive (+) charge:
  - As the distance from the charge r **decreases**, the potential V **increases**
  - This is because more work has to be done on a positive test charge to overcome the repulsive force
- For a negative (–) charge:
  - As the distance from the charge r **decreases**, the potential V **decreases**
  - This is because less work has to be done on a positive test charge since the attractive force will make it easier
- The graph of potential V against distance r for a negative or positive charge is:

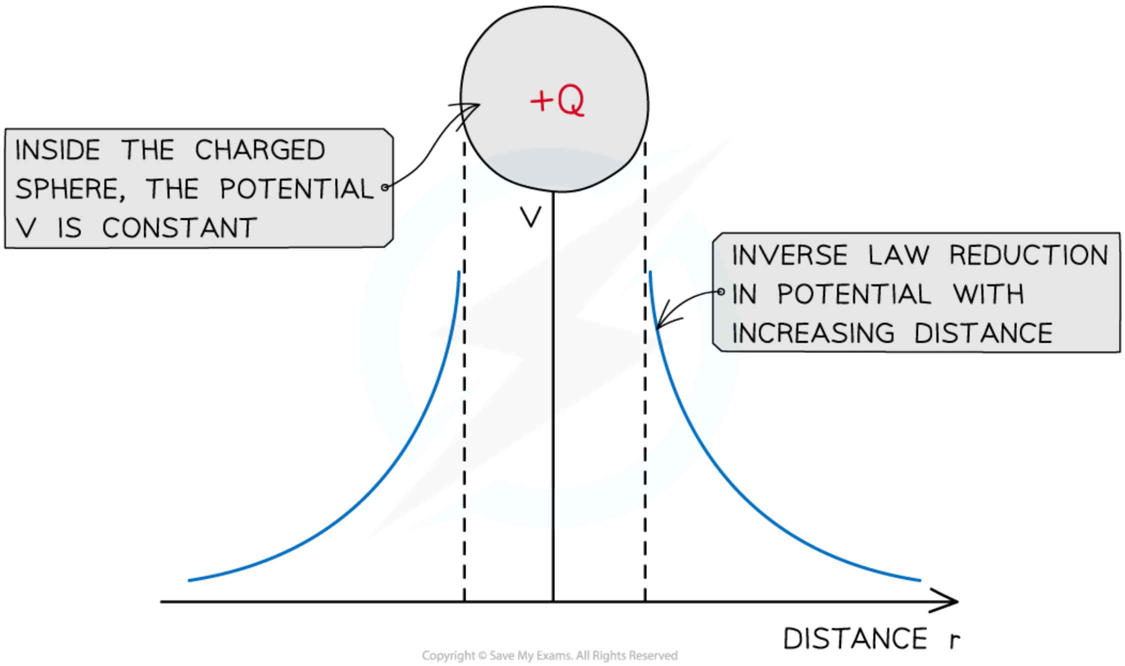
Your notes



The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- Unlike the **gravitational potential** equation, the minus sign in the electric potential equation will be included in the charge
- The electric potential varies according to 1 / r
  - Note, this is different to electric field strength, which varies according to 1 / r^2

Your notes



The potential changes as an inverse law with distance near a charged sphere

- **Note:** this equation still applies to a conducting sphere. The charge on the sphere is treated as if it concentrated at a point in the sphere from the point charge approximation

Worked Example

A Van de Graaf generator has a spherical dome of radius 15 cm. It is charged up to a potential of 240 kV.

Calculate:

a) The charge stored on the dome

b) The potential at a distance of 30 cm from the dome

Answer:

Part (a)

Step 1: Write down the known quantities

- Radius of the dome, r = 15 cm = 15×10^{-2} m

- Potential difference, V = 240 kV = 240 × 10^3 V

Step 2: Write down the equation for the electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Rearrange for charge Q

$$Q = V4\pi\varepsilon_0 r$$

Step 4: Substitute in values

$$Q = (240 \times 10^3) \times (4\pi \times 8.85 \times 10^{-12}) \times (15 \times 10^{-2})$$

$$Q = 4.0 \times 10^{-6}\ \text{C} = 4.0\ \mu\text{C}$$

Part (b)

Step 1: Write down the known quantities

- Q = charge stored in the dome = 4.0 μC = 4.0 × 10^{-6} C
- r = radius of the dome + distance from the dome = 15 + 30 = 45 cm = 45 × 10^{-2} m

Step 2: Write down the equation for electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Substitute in values and calculate final answer

$$V = \frac{(4.0 \times 10^{-6})}{(4\pi \times 8.85 \times 10^{-12}) \times (45 \times 10^{-2})} = 79.93 \times 10^3 = \mathbf{80\ kV}\ (2\text{ s.f.})$$

Your notes

Your notes

Capacitance of an Isolated Sphere

Capacitance of an Isolated Sphere

- The capacitance, C, of a charged sphere, is defined as the **charge per unit potential at the surface of the sphere**

$$C = \frac{Q}{V}$$

- Where:
  - C = capacitance (F)
  - Q = charge (C)
  - V = potential difference (V)
- The charge on the surface of a spherical conductor can be considered as a point charge at its centre
- The potential V of an isolated point charge is given by:

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

- Where:
  - R = radius of sphere (m)
  - ε_0 = permittivity of free space
- The charge, Q, is **not** the charge of the capacitor itself, it is the charge stored **on** the surface of the spherical conductor
- Combining these equations gives an expression for the capacitance of an isolated sphere:

$$V = \frac{Q}{4\pi\varepsilon_0 R} = \frac{Q}{C}$$

$$\mathbf{C = 4\pi\varepsilon_0 R}$$

Worked Example

Your notes

Lightning can be simulated in a laboratory using an isolated metal sphere to investigate electrical discharge.

A sphere of radius 75 cm is charged to a potential of 1.5 MV.

Following the electrical discharge, the sphere loses 95% of its energy.

Calculate:

a) The capacitance of the sphere.

b) The potential of the sphere after discharging.

Answer:

Part (a)

Step 1: List the known quantities

- Radius of sphere, R = 75 cm = 75×10^{-2} m
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

Step 2: Write out the equation for the capacitance of a charged sphere

$$C = 4\pi\varepsilon_0 R$$

Step 3: Calculate the capacitance

$$C = 4\pi \times (8.85 \times 10^{-12}) \times (75 \times 10^{-2})$$

$$C = 8.34 \times 10^{-11} \text{ F}$$

Part (b)

Step 1: List the known quantities

- Original potential, V_1 = 1.5 MV = 1.5×10^6 V
- Final potential = V_2
- Original energy = E_1
- Final energy, $E_2 = 0.05\,E_1$

Step 2: Write out the equation for the energy stored by a capacitor

$$E = \frac{1}{2}CV^2$$

Step 3: Write out equations for energy before and after discharge

$$E_1 = \frac{1}{2}CV_1^{\,2}$$

$$E_2 = \frac{1}{2} C V_2^{\ 2}$$

Step 4: Equate the two expressions and simplify

- Since $E_2 = 0.05\, E_1$

$$\frac{1}{2} C V_2^{\ 2} = 0.05\left(\frac{1}{2} C V_1^{\ 2}\right)$$

$$V_2^{\ 2} = 0.05\left(V_1^{\ 2}\right)$$

$$V_2 = \sqrt{0.05}\ V_1$$

Step 5: Calculate the final potential, V_2

$$V_2 = \sqrt{0.05} \times (1.5 \times 10^6) = 3.35 \times 10^5 \text{ V}$$

Your notes

Force-Distance Graph

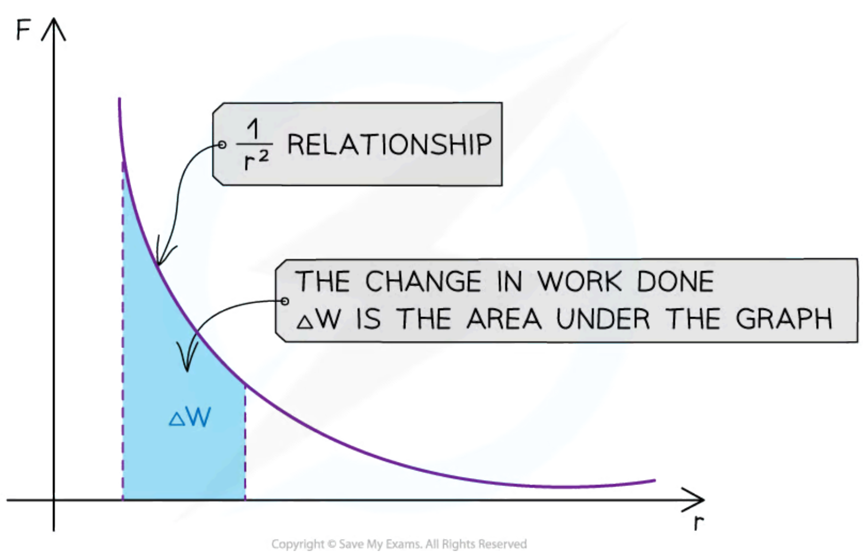
Your notes

Force-Distance Graph for a Point Charge

- **The key features of the force-distance graph are**:
  - The values for F are all positive
  - As r increases, F against r follows a $1/r^2$ relation (inverse square law)
  - The **area** under this graph is the work done ΔW
  - The graph has a steep decline as r increases
- The area under the graph can be estimated by counting squares, if it is plotted on squared paper, or by splitting it into trapeziums and summing the area of each trapezium
  - The **inverse square law** relation means that if the distance r increases by a factor of 2, F would decrease by a factor of 4
- This is a graphical representation of the equation:

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

- Where Q, q and $4\pi\varepsilon_0$ are constants



Your notes

Examiner Tips and Tricks

Drawing, interpreting or calculations using graphs are common exam questions. The graph of F against r should start off **steeper** and decrease **rapidly** compared to that of V against r, to distinguish it as an inverse square law ($1/r^2$) relation instead of just $1/r$.

Your notes

Electric Potential Energy

Electric Potential Energy

- When a mass with charge moves through an electric field, work is done
- The work done in moving a charge q is given by:

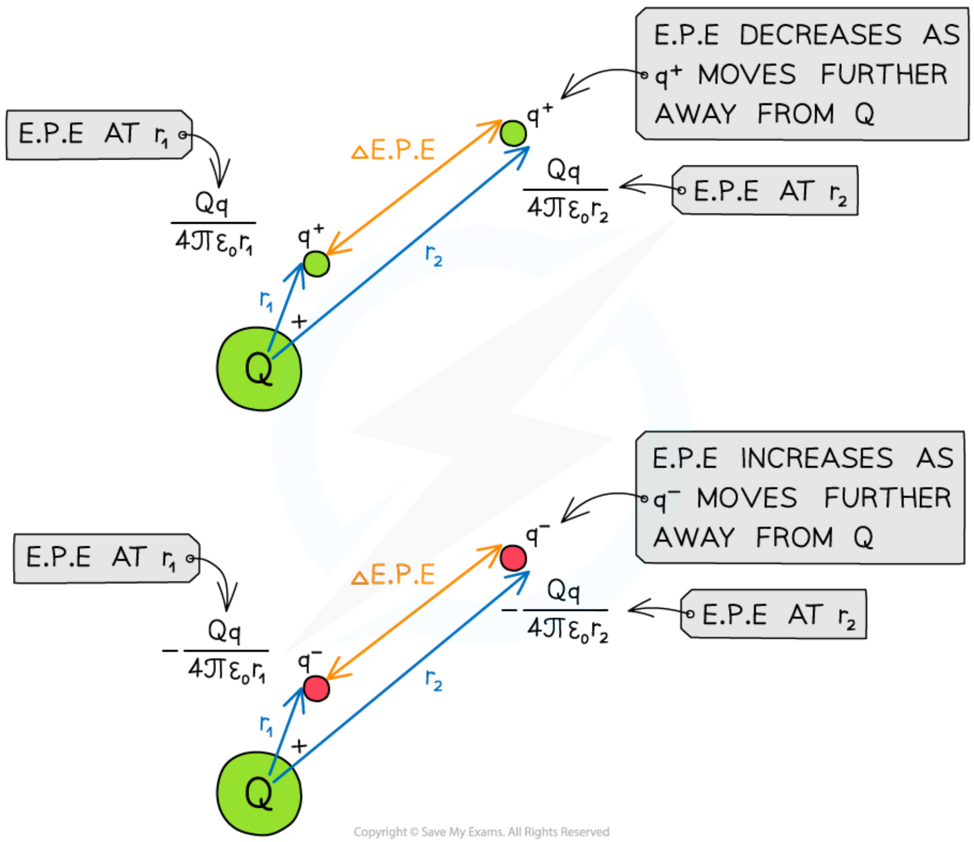
$$\Delta W = q\Delta V$$

- Where:
  - ΔW = change in work done (J)
  - q = charge (C)
  - ΔV = change in electric potential (J C^{-1})
- This change in work done is equal to the change in **electric potential energy** (E.P.E)
  - When $V = 0$, then the E.P.E = 0
- The change in E.P.E, or work done, for a point charge q at a distance r_1 from the centre of a larger charge Q, to a distance of r_2 further away can be written as:

$$\Delta E.P.E = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

- Where:
  - Q = charge that is producing the electric field (C)
  - q = charge that is moving in the electric field (C)
  - r_1 = first distance of q from the centre of Q (m)
  - r_2 = second distance of q from the centre of Q (m)

Your notes



Work is done when moving a point charge away from another charge

- Work is done when a positive charge in an electric field moves **against** the electric field lines or when a negative charge moves **with** the electric field lines

Worked Example

The potentials at points **R** and **S** due to the +7.0 nC charge are 675 V and 850 V respectively.

Calculate how much work is done when a +3.0 nC charge is moved from **R** to **S**.

Answer:

Step 1: Write down the known quantities

- p.d. at **R**, $V_1 = 675$ V
- p.d. at **S**, $V_2 = 850$ V

- Charge, $q = +3.0 \text{ nC} = +3.0 \times 10^{-9}$ C

Step 2: Write down the work done equation

$$W = q\Delta V$$

Step 3: Substitute in the values into the equation

$$W = (3.0 \times 10^{-9}) \times (850 - 675) = 5.3 \times 10^{-7} \text{ J}$$

Examiner Tips and Tricks

Remember that *q* in the work done equation is the charge that is being moved, whilst Q is the charge which is producing the potential. Make sure not to get these two mixed up, as both could be given in the question (like the worked example) and you will be expected to choose the correct one.

Your notes