A Level · Edexcel · Further Maths

34 mins   3 questions

8.1 First Order Differential Equations

8.1.1 Intro to Differential Equations / 8.1.2 Solving First Order Differential Equations / 8.1.3 Modelling using First Order Differential Equations

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Total Marks **/34**

1 (a) A pond initially contains 1000 litres of unpolluted water.

The pond is leaking at a constant rate of 20 litres per day.

It is suspected that contaminated water flows into the pond at a constant rate of 25 litres per day and that the contaminated water contains 2 grams of pollutant in every litre of water.

It is assumed that the pollutant instantly dissolves throughout the pond upon entry.

Given that there are x grams of the pollutant in the pond after t days,

(a) show that the situation can be modelled by the differential equation,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 50 - \frac{4x}{200 + t}$$

(4 marks)

(b) (b) Hence find the number of grams of pollutant in the pond after 8 days.

(5 marks)

(c) (c) Explain how the model could be refined.

(1 mark)

2 (a) A sample of bacteria in a sealed container is being studied.

The number of bacteria, P, in thousands, is modelled by the differential equation

$$(1 + t)\frac{\mathrm{d}P}{\mathrm{d}t} + P = t^{\frac{1}{2}}(1 + t)$$

where t is the time in hours after the start of the study.

Initially, there are exactly 5000 bacteria in the container.

(a) Determine, according to the model, the number of bacteria in the container 8 hours after the start of the study.

(6 marks)

(b) (b) Find, according to the model, the rate of change of the number of bacteria in the container 4 hours after the start of the study.

(4 marks)

(c) (c) State a limitation of the model.

(1 mark)

3 (a) A tank at a chemical plant has a capacity of 250 litres. The tank initially contains 100 litres of pure water.

Salt water enters the tank at a rate of 3 litres every minute. Each litre of salt water entering the tank contains 1 gram of salt.

It is assumed that the salt water mixes instantly with the contents of the tank upon entry.

At the instant when the salt water begins to enter the tank, a valve is opened at the bottom of the tank and the solution in the tank flows out at a rate of 2 litres per minute.

Given that there are S grams of salt in the tank after t minutes,

a) show that the situation can be modelled by the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = 3 - \frac{2S}{100 + t}$$

(4 marks)

(b) (b) Hence find the number of grams of salt in the tank after 10 minutes.

When the concentration of salt in the tank reaches 0.9 grams per litre, the valve at the bottom of the tank must be closed.

(5 marks)

(c) (c) Find, to the nearest minute, when the valve would need to be closed.

(3 marks)

(d) (d) Evaluate the model.

(1 mark)