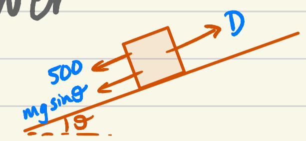
Power



Form 2 equations, then solve simultaneously

I) $P = Fv$ where F is the driving force, D

II) a) Constant speed
- Equilibrium, forces balanced

or b) Acceleration,
- $F = ma$, where F is the resultant

Elastics

Form 2 equations, then solve simultaneously

I) $T = \dfrac{\lambda x}{l}$

II) a) At rest
- Equilibrium, forces balanced

or b) Accelerating
- $F = ma$, where F is the resultant

Tips: Max velocity when $a = 0$, i.e. at its equilibrium position

Max acceleration when $v = 0$, i.e. just after it is released.

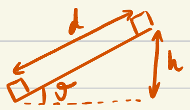
Energy

Types of Energy

KE $\frac{1}{2}mv^2$

GPE mgh or $mgd\sin\theta$

EPE $\dfrac{\lambda x^2}{2l}$



Work Done...

... against friction $F_r \times d$

... by engine/person $F \times d$

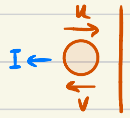
Work-Energy Principle

initial KE + initial GPE + initial EPE + work done by engine/person = final KE + final GPE + final EPE + work done against friction

initial KE	initial GPE	initial EPE	work done by engine/person	final KE	final GPE	final EPE	work done against friction
$v=$	$h=$	$x=$	$F=$ $d=$	$v=$	$h=$	$x=$	$F_r=$ $d=$

1D Collisions

Impulse-Momentum Principle



$I = m(v-u)$ $\quad I = Ft$

Take direction of impulse as positive direction

Principle of Conservation of Linear Momentum, PCLM

Initial momentum = final momentum Direction matters!

Newton's Law of Restitution, NLR

e is the coefficient of restitution

$0 \le e \le 1$ If $e = 0$, inelastic (not bouncy)

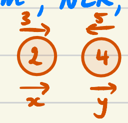
If $e = 1$, perfectly elastic (bouncy)

$e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

Strategy:

PCLM, NLR, simultaneous equations

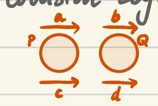
eg.



PCLM $3\times 2 - 5\times 4 = 2x + 4y$

NLR $e = \dfrac{y-x}{3+5}$

Collision Logic



$a > b$

$d > c$

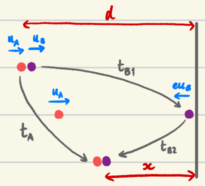
If P reverses $c < 0$

If Q stays same $d > 0$

Distance Problems

Constant velocity, use dist = speed × time

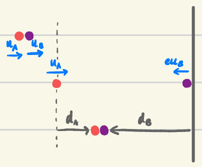
Collision, wall bounce, 2nd collision – "Find x"



Strategy 1 (Times)

- Compare the time it takes between collisions for A and B

$t_A = t_{B1} + t_{B2}$ Use $t = \dfrac{\text{dist}}{\text{speed}}$



Strategy 2 (Distance Ratios)

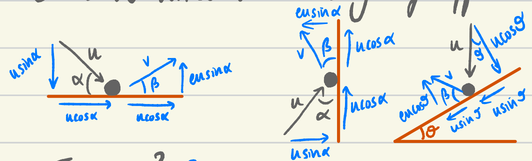
- If we know how far apart A and B are when B hits the wall, then speeds will be in same ratio as distances travelled i.e. $u_A : eu_B$ is equivalent to $d_A : d_B$

2D Collisions – one sphere

Impulse-Momentum Principle

$\underline{I} = m(\underline{v} - \underline{u})$

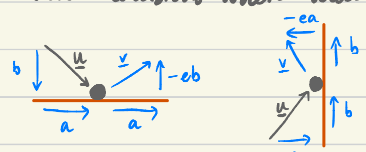
Collisions with known angle of approach



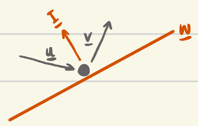
Find v? Pythagoras

β? $\tan^{-1}$

Vector collisions where wall is in $\underline{i}$ or $\underline{j}$ direction



Vector collisions where wall is not $\underline{i}$ or $\underline{j}$ direction



Conservation of velocity parallel to wall

$\underline{u}.\underline{w} = \underline{v}.\underline{w}$

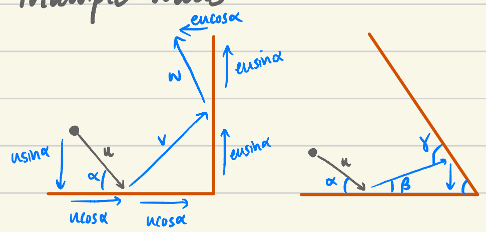
Impact law for perpendicular to wall

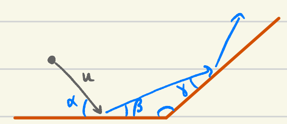
$-e\,\underline{u}.\underline{I} = \underline{v}.\underline{I}$

$\underline{I}$? Use $\underline{I} = m(\underline{v} - \underline{u})$

$\underline{I} \leftrightarrow \underline{w}$? Switch $\underline{i}$ and $\underline{j}$ comp and negate one.

Multiple Walls

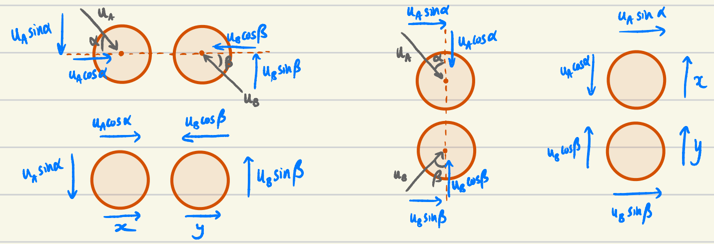




Use sum of angles in a triangle

2D Collisions – two spheres

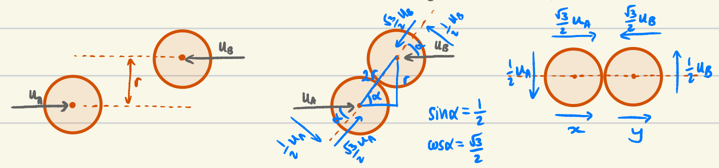
Collisions with known angles to line of centres



Strategy: PCLM, NLR, Sim equations

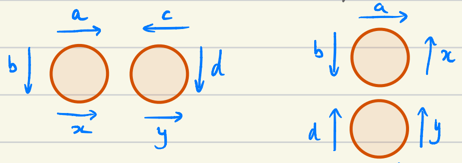
Put "back together" using Pythag. and Trig.

Collisions with unknown angles: geometric problems



... then same strategy as before

Vector collisions where line of centres is along $\underline{i}$ or $\underline{j}$



Vector collisions with unknown line of centres

3 of 4 known velocities? Vector version of PCLM

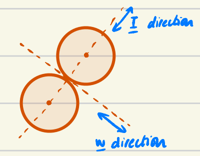
Direction of line of centres is same as direction of impulse, $\underline{I}$

Conservation perp. to line of centres

$(\underline{u}_A - \underline{u}_B).\underline{w} = (\underline{v}_A - \underline{v}_B).\underline{w}$

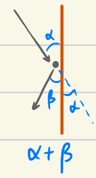
Impact law parallel to line of centres

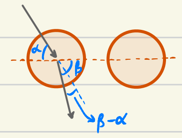
$-e(\underline{u}_A - \underline{u}_B).\underline{I} = (\underline{v}_A - \underline{v}_B).\underline{I}$



Angles of Deflection

How much the original path is rotated/turned





If $\underline{u}$ and $\underline{v}$ are known...

$\cos\theta = \dfrac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|}$