Trigonometric Functions (Chapter 6)

This chapter is very similar to the trigonometry chapters in Year 1. The only difference is that new trig functions: sec, $cosec$ and cot, are introduced.

1:: Understanding $sec, cosec, tan$ and draw their graphs.

"Draw a graph of $y = cosec\ x$ for $0 \leq x < 2\pi$."

2:: 'Solve' questions.

"Solve, for $0 \leq x < 2\pi$, the equation
$$2cosec^2 x + \cot x = 5$$
giving your solutions to 3sf."

3:: 'Prove' questions.

"Prove that
$$\sec x - \cos x \equiv \sin x \tan x$$

4:: Inverse trig functions and their domains/ranges.

A new member of the trig family…

$$\cos(x)$$

$$\cos^2(x) = (\cos x)^2$$

The latter form is particularly useful for differentiation in Year 2.

$$\cos^{-1}(x) \ or \ \arccos(x)$$

Be careful: the -1 here doesn't mean a power of -1 UNLIKE $\cos^2 x$ above. This is an unfortunate historical accident. $arccos$ is an notation I prefer because of this reason.

$$\mathbf{sec}(x) = \frac{\mathbf{1}}{\mathbf{cos}(x)}$$

We have a convenient way of representing the reciprocal of the trig functions.

Reciprocal Trigonometric Functions

( ) brackets not *required* here but can be used

$$\sec(x) = \frac{1}{\cos(x)}$$

Short for "**secant**"

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

Short for "**cosecant**"

$$\cot(x) = \frac{1}{\tan(x)} \; or \; \frac{\cos(x)}{\sin(x)}$$

Short for "**cotangent**"

We typically use this version instead of $\frac{1}{\tan x}$ when doing proof questions.

Tip: To remember these, look at the **<u>3rd letter</u>**: *sec*'s 3rd is 'c' so it's 1 over **cos**.

Reciprocals of Reciprocal Trigonometric Functions

$$\frac{1}{\sec x}$$

$$\frac{1}{\operatorname{cosec} x}$$

$$\frac{1}{\cot x}$$

Calculations

You have a calculator in A Level exams, but you should know how to calculate certain values yourself if needed...

$\cot\frac{\pi}{4} =$

$\cot\frac{\pi}{3} =$

$\sec\frac{\pi}{4} =$

$\sec\frac{\pi}{6} =$

$\operatorname{cosec}\frac{\pi}{3} =$

$\operatorname{cosec}\frac{\pi}{2} =$

$\cot\frac{\pi}{6} =$

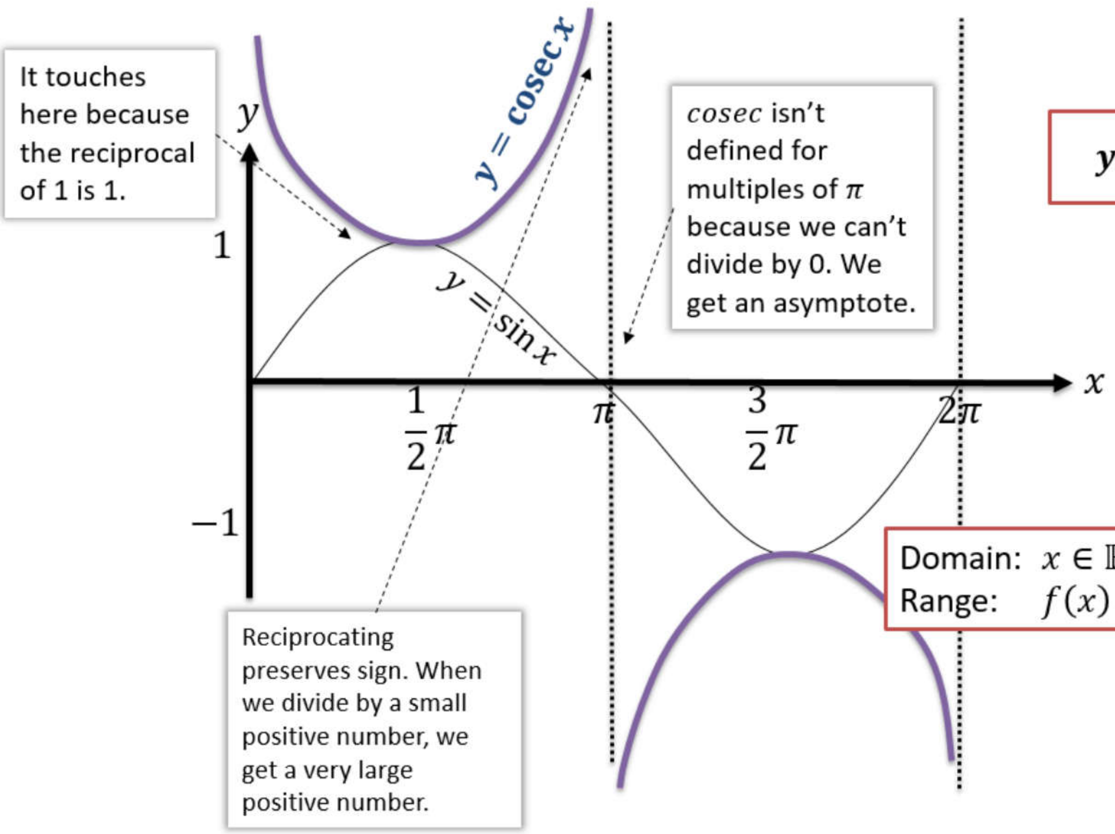
$\sec\frac{5\pi}{3} =$

$\operatorname{cosec}\frac{5\pi}{6} =$

Ex 6A

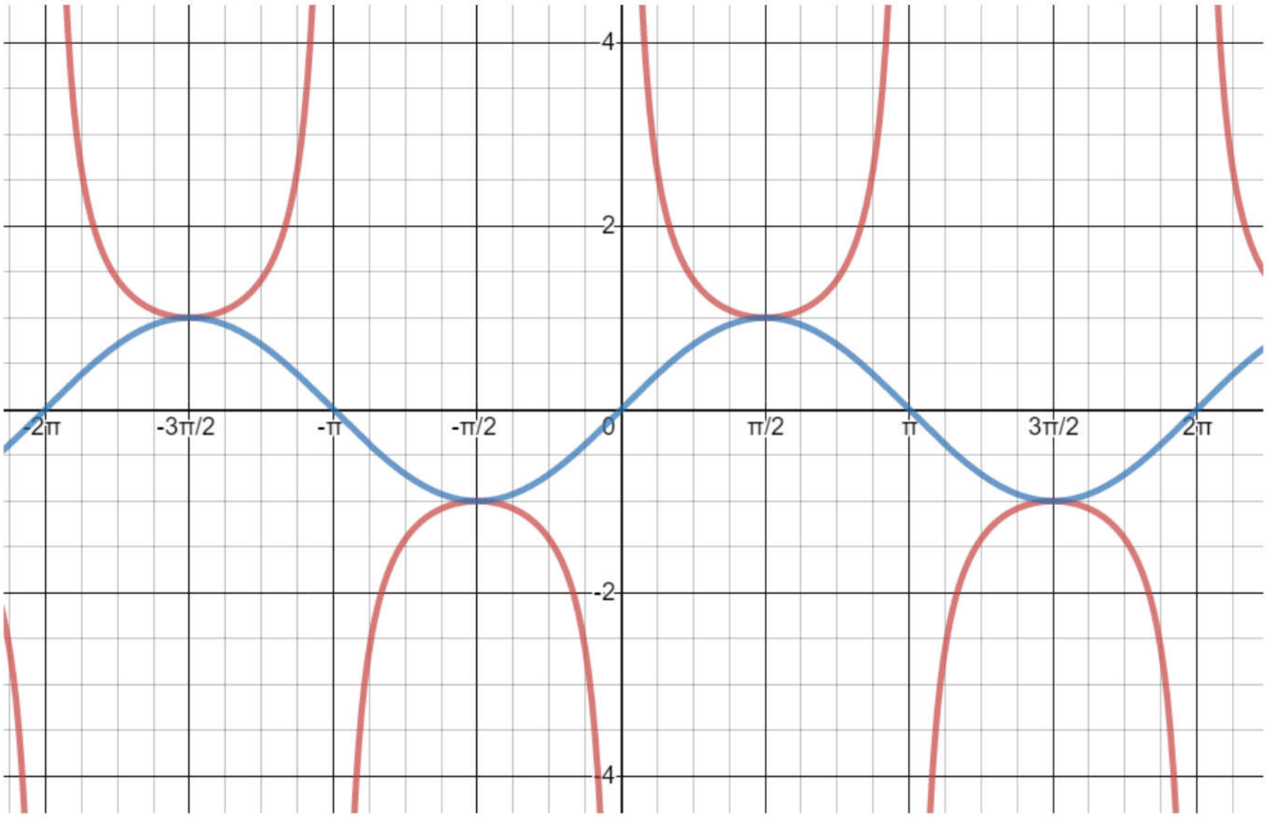
Sketches

To draw a graph of $y = cosec\ x$, start with a graph of $y = \sin x$, then consider what happens when we reciprocate each y value.



$\boldsymbol{y = cosec\ x}$

Domain: $x \in \mathbb{R}, x \neq \pm\pi, \pm 2\pi, \ldots$
Range: $f(x) \leq 1\ or\ f(x) \geq 1$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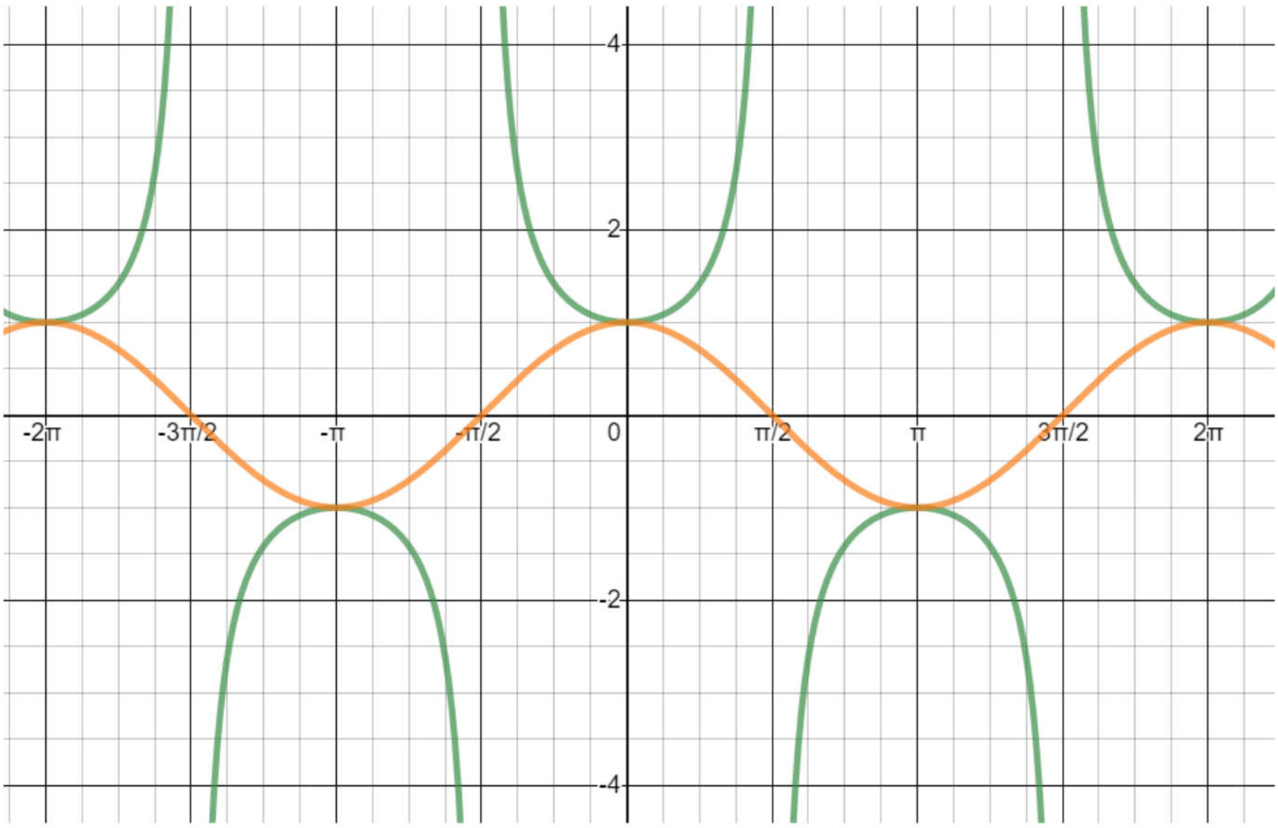
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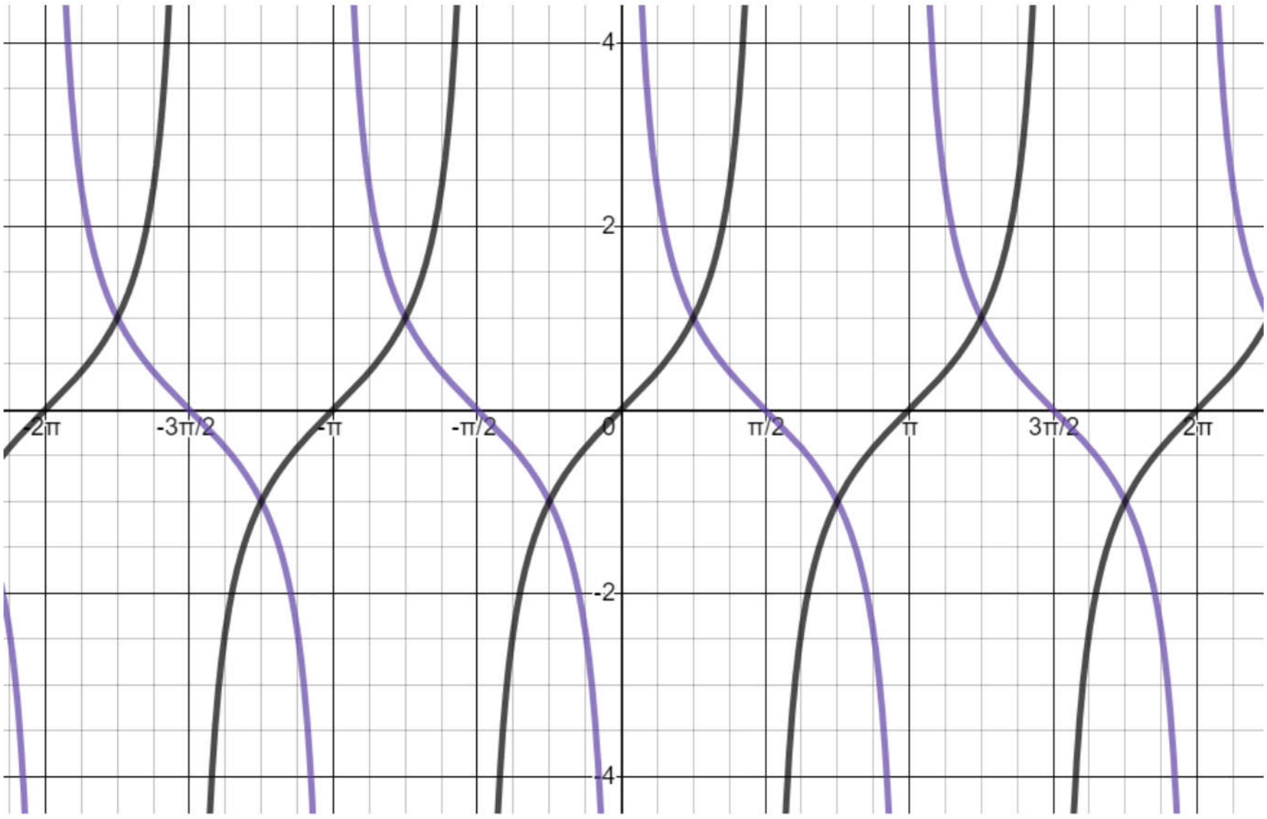

Ex 6B

Using sec, cosec, cot - proving identities

Questions in the exam usually come in two types: (a) 'prove' questions requiring to prove some identity and (b) 'solve' questions.

Tip 1: Get everything in terms of sin and cos first (using $\cot x = \frac{\cos x}{\sin x}$ rather than $\cot x = \frac{1}{\tan x}$)

Tip 2: Whenever you have algebraic fractions being added/subtracted, whether $\frac{a}{b} + \frac{c}{d}$ or $\frac{a}{b} + c$, combine them into one (as we can typically then use $\sin^2 x + \cos^2 x = 1$)

Simplify $\sin\theta \cot\theta \sec\theta$

Simplify $\sin\theta\cos\theta\ (\sec\theta + cosec\ \theta)$

Prove that $\frac{\cot\theta\ cosec\ \theta}{\sec^2\theta + cosec^2\ \theta} \equiv \cos^3\theta$

Your Turn

$\sec x - \cos x \equiv \sin x \tan x$

$(1 + \cos x)(cosec\ x - \cot x) \equiv \sin x$

9. (a) Show that

$$\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}\equiv k\operatorname{cosec} x \qquad x\neq n\pi, \quad n\in\mathbb{Z}$$

where k is a constant to be found.

(4)

(b) Hence explain why the equation

$$\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=1.6$$

has no real solutions.

(1)

Using sec, cosec, cot - solving equations

Solve the following equations in the interval $0 \leq \theta \leq 360°$:

a) $\sec\theta = -2.5$

b) $\cot 2\theta = 0.6$

Solve $\cot\theta = 0$ in the interval $0 \leq \theta \leq 2\pi$.

Your Turn

Solve in the interval $0 \leq \theta < 360°$:

$$cosec\ 3\theta = 2$$

Ex 6C

New Pythagorean Identities

From Year 1 we know:

$$\sin^2 x + \cos^2 x = 1$$

We can create two new identities, which you should memorise:

Dividing by $\cos^2 x$:

Dividing by $\sin^2 x$:

Prove that $cosec^4\,\theta - \cot^4\theta = \frac{1+\cos^2\theta}{1-\cos^2 x}$

Solve the equation $4\,cosec^2\theta - 9 = \cot\theta$ in the interval $0 \leq \theta \leq 360°$

Does this remind you of any equations we've seen in Year 1? How did we deal with them?

Your Turn

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6. (ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

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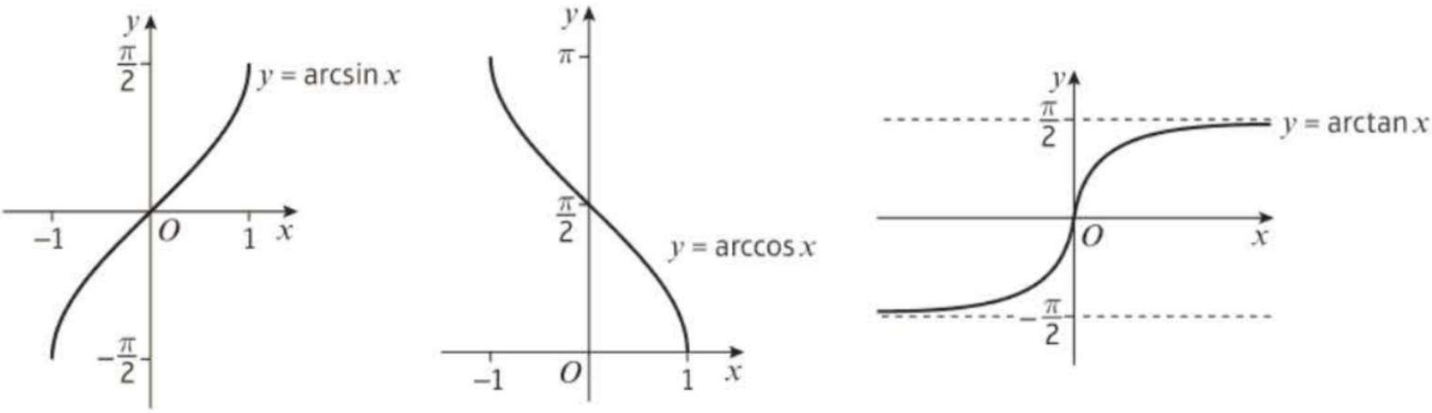
Solve, for $0 \leq x < 2\pi$, the equation

$$2cosec^2 x + \cot x = 5$$

giving your solutions to 3sf.

Graphs of inverse trigonometric functions



You need to know these graphs, but this is a very rarely examined area of content...

They are reflections of sin$x$, cos$x$ and tan$x$ in the line y = x respectively.
This is how inverse functions are sketched - recall Chapter 2.

Ex 6E