A Level · Edexcel · Maths

4 hours 34 questions

4.3 Normal Distribution (A Level only)

4.3.1 The Normal Distribution / 4.3.2 Normal Distribution - Calculations / 4.3.3 Standard Normal Distribution

Easy (9 questions)	/57
Medium (9 questions)	/53
Hard (8 questions)	/52
Very Hard (8 questions)	/59
Total Marks	**/221**

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Easy Questions

1 (a) A continuous random variable can take any value within a given range. Many naturally occurring continuous quantities can be modelled using the Normal Distribution, for example the height of human beings; the mass of new born puppies or the distribution of all A Level maths exam results.

Give a different example of a quantity that could be modelled using the normal distribution.

(1 mark)

(b) The graph of the normal distribution has a characteristic bell shape that is symmetrical about the mean, μ. If X has a normal distribution with a mean, μ , and variance, σ^2, then it can be written as $X \sim N(\mu, \sigma^2)$.

For $X \sim N(\mu, \sigma^2)$, state:

(i) $\mathrm{P}(X < \mu)$

(ii) $\mathrm{P}(X \geq \mu)$

(iii) $\mathrm{P}(X = \mu)$

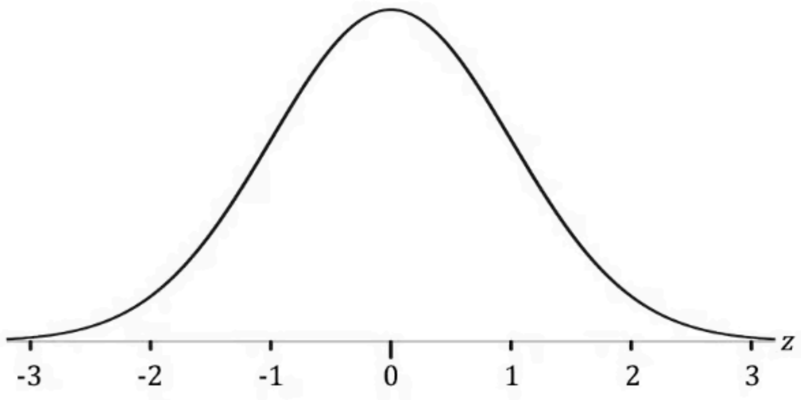
(3 marks)

(c) Using your answers to part (b), or otherwise, explain why there is no difference between $\geq$ and $>$ or $\leq$ and $<$ when calculating normal probabilities.

(1 mark)

2 (a) The graph of the normal distribution has a maximum point at the mean and points of inflection at $\mu \pm \sigma$, where σ is the standard deviation.

The following diagram shows the distribution of the standard normal variable, $Z \sim N(0,1)$ with mean $\mu = 0$, and variance $\sigma^2 = 1$..



Write down the standard deviation of the standard normal variable, Z .

(1 mark)

(b) Write down the values that correspond to:

(i) the maximum point on the curve.

(ii) the points of inflection on the curve.

(3 marks)

(c) For a normal distribution, probabilities can be calculated by finding the area under the graph. Approximately 68% of the data lies within one standard deviation of the mean, 95% within two and 99.7% of the data lies within three standard deviations of the mean

Using the given properties, find the values of a, b, and c in the following statements:

(i) $\mathrm{P}(-1 \leq Z \leq 1) = a$

(ii) $\mathrm{P}(-b \leq Z \leq b) = 0.997$

(iii) $\mathrm{P}(-c \leq Z \leq 0) = 0.475$

(3 marks)

3 (a) For the random variable $X \sim N(20, 4^2)$ find, using the properties of the normal distribution:

(i) $P(16 \leq X \leq 24)$

(ii) $P(20 \leq X \leq 24)$

(iii) $P(X \leq 24)$

(3 marks)

(b) By first writing the standard deviation of the random variable $Y \sim N(10,4)$, find:

(i) $P(0 \leq Y \leq 20)$

(ii) $P(0 \leq Y \leq 16)$

(iii) $P(8 \leq Y \leq 14)$

(4 marks)

4 (a) One method to find probabilities for the normal distribution is to use the normal cumulative distribution function on your calculator.

For the random variable, $X \sim N(32,9)$,

(i) Write down the mean and standard deviation.

(ii) Draw a sketch of the graph, labelling the mean and the points of inflection clearly.

(3 marks)

(b) With the help of your diagram, explain how you should know, without carrying out any calculations, that $\mathrm{P}(X \leq 34) > 0.5$.

Define a suitable lower bound you could use on your calculator to calculate $\mathrm{P}(X \leq 34)$..

(2 marks)

(c) Use the normal cumulative distribution function on your calculator to find the following probabilities, giving all answers to four decimal places.

(i) $\mathrm{P}(X \leq 34)$

(ii) $\mathrm{P}(X > 30)$

(iii) $\mathrm{P}(31 \leq X \leq 35)$

(3 marks)

5 (a) The heights H cm, of young fig trees on a farm in Australia are normally distributed with mean, 90 cm, and standard deviation, 7 cm. They are modelled as $H \sim N(90, 7^2)$..

Find, giving all answers to four decimal places.:

(i) $\mathrm{P}(H \leq 91)$

(ii) $\mathrm{P}(H > 91)$

(iii) $\mathrm{P}(H \geq 89)$

(3 marks)

(b) Explain why it was only necessary to use the normal distribution function on your calculator for part (i) in question (a).

(2 marks)

(c) The fig trees need to be moved to a more spacious area once they reach a height of one metre. The heights of the fig trees are measured at the start of each day.

(i) Find the probability a fig tree chosen at random is more than one metre tall.

(ii) If, on a particular day, the farmer has 100 fig trees, how many would they expect to have to move that day?

(2 marks)

6 (a) The slithering speeds, S kmph, of a population of garden snails are modelled as a normal distribution with $S \sim N(0.04, \sigma^2)$.

Use the properties of the normal distribution to find , given that 68% of snails chosen at random from the population slither with speeds in the range 0.03 kmph and 0.05 kmph.

(1 mark)

(b) Given that 60% of snails slither with a speed of no more than s kmph, draw a diagram to show that $s > 0.04$ kmph.

Use the inverse normal distribution function on your calculator to find the value of s, giving your answer in kmph to four decimal places.

(2 marks)

(c) (i) With the help of a diagram, show that if $\mathrm{P}(S \leq a) = 0.7$, and $\mathrm{P}(S \geq b) = 0.3$, then $a = b$.

(ii) Hence, or otherwise, find the value of b, such that 30% of the garden snails slither with a speed of greater than b kmph.

(2 marks)

7 (a) For the standard normal distribution, $Z \sim N(0,1^2)$,the probability $\mathrm{P}(Z < z)$ can be written as $\Phi(z)$

Using the normal cumulative distribution function on your calculator with the parameters for the standard normal distribution, write the following in the form $\mathrm{P}(Z < z)$, and find each solution to four decimal places.

(i) $\Phi(1.2)$

(ii) $\Phi(-0.6)$

(iii) $1 - \Phi(0.8)$

(3 marks)

(b) A random variable $X \sim N(\mu, \sigma^2)$ can be coded to model the standard normal variable $Z \sim N(0,1)$, using the formula:

$$Z = \frac{X-\mu}{\sigma}$$

For the random variable $X \sim N(54, 5^2)$, write in terms of $\Phi(z)$ and hence find:

(i) $\mathrm{P}(X \leq 60)$

(ii) $\mathrm{P}(X < 51)$

(iii) $\mathrm{P}(X \geq 58)$

(3 marks)

8 (a) The percentage points of the normal distribution table below provides z-values that correspond to given probabilities. It gives values of p such that $\mathrm{P}(Z > z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0365	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

From the table, $\mathrm{P}(Z > 0.2533) = 0.4$. On a sketch of the normal distribution curve, show that $\mathrm{P}(Z < -0.2533) = 0.4$ and $\mathrm{P}(Z > -0.2533) = 0.6$.

(2 marks)

(b) Use the percentage points table for the standard normal distribution to find the value of z for which $\mathrm{P}(Z > z) = 0.05$.

(1 mark)

(c) The weights, W kg, of watermelons arriving for packing at Walter's Wacky Watermelon Warehouse are modelled as $W \sim N(10, 2.5^2)$. Walter keeps the heaviest 5% of watermelons to enter into a weekly competition and sends the rest to the farmers market to be sold.

(i) By substituting the values of the mean and standard deviation into the formula connecting W and Z, show that $W = 2.5Z + 10$.

(ii) Use your answer to part (b) to find w such that $\mathrm{P}(W > w) = 0.05$, and thus find the lightest weight of a watermelon that Walter would enter into the competition.

(3 marks)

9 (a) A soft serve ice cream machine is set to produce serving portion sizes, X, that are normally distributed with a mean of 100 ml and a standard deviation of σ ml. It is given that 10% of the servings produced by the machine are less than 98 ml.

(i) Find, to four decimal places, the value of z such that $\mathrm{P}(Z < z)$ = 0.1.

(ii) By substituting your answer to part (i) into the formula connecting X and Z,show that $\sigma = 1.56\ ml$,,to two decimals places.

(3 marks)

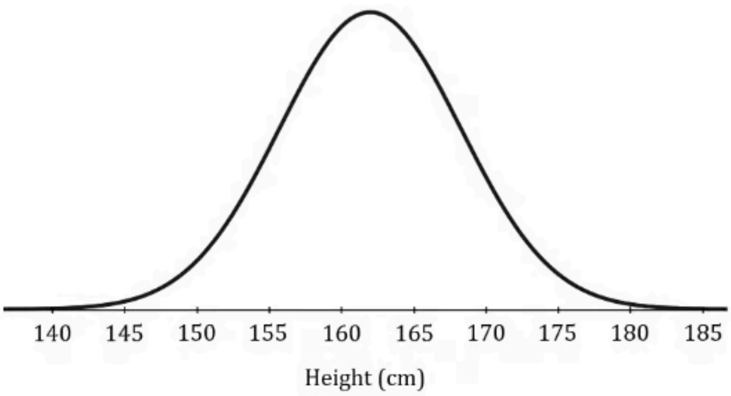
(b) The containers for the soft serve ice cream are designed to hold a volume, V ml with distribution $V \sim N(\mu, 15^2)$..

Given that 20% of the containers can hold a volume of more than 150 ml, find the value of μ to one decimal place.

(3 marks)

Medium Questions

1 (a) The following diagram shows the distribution of heights, in cm, of adult women in the UK:



The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

Write down the values of the height that correspond to:

(i) the maximum point on the curve

(ii) the points of inflection on the curve

(3 marks)

(b) Use the properties of the normal distribution to suggest a range of heights within which the heights of

(i) 68%

(ii) 95%

(iii) nearly all

of adult women in the UK will fall.

(3 marks)

2 (a) For the random variable $X \sim N(23, 4^2)$ find the following probabilities:

(i) $P(X < 20)$

(ii) $P(X \geq 29)$

(iii) $P(20 \leq X < 29)$

(3 marks)

(b) For the random variable $Y \sim N(100, 225)$ find the following probabilities:

(i) $P(Y \leq 90)$

(ii) $P(Y > 140)$

(iii) $P(85 \leq Y \leq 115)$

(3 marks)

3 (a) The weight, W g, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim N(200,\ 1.75^2)$.

Find:

(i) $\mathrm{P}(W < 195)$

(ii) $\mathrm{P}(W > 203)$

(2 marks)

(b) Heledd buys a pack containing 12 of the chocolate bars. It may be assumed that the 12 bars in the pack represent a random sample.

Find the probability that all of the bars in the pack have a weight of at least 195 g.

(2 marks)

4 (a) The random variable $X \sim N(330,\ 10^2)$.

Find the value of a, to 2 decimal places, such that:

(i) $\mathrm{P}(X < a) = 0.25$

(ii) $\mathrm{P}(X > a) = 0.25$

(iii) $\mathrm{P}(315 \leq X \leq a) = 0.5$

(4 marks)

(b) The random variable $Y \sim N(10,10)$.

Find the value of b and the value of c, each to 2 decimal places, such that:

(i) $\mathrm{P}(Y < b) = 0.4$

(ii) $\mathrm{P}(Y > c) = 0.25$

(2 marks)

(c) Use a sketch of the distribution of Y to explain why $\mathrm{P}(b \leq Y \leq c) = 0.35$.

(2 marks)

5 (a) The test scores, X, of a group of RAF recruits in an aptitude test are modelled as a normal distribution with $X \sim N(210,\ 27.8^2)$.

(i) Find the values of a and b such that $\mathrm{P}(X < a) = 0.25$ and $\mathrm{P}(X > b) = 0.25$.

(ii) Hence find the interquartile range of the scores.

(3 marks)

(b) Those who score in the top 30% on the test move on to the next stage of training.

One of the recruits, Amelia, achieves a score of 231. Determine whether Amelia will move on to the next stage of training.

(2 marks)

6 (a) For the standard normal distribution $Z \sim N(0, 1^2)$, find:

(i) $\mathrm{P}(Z < 1.5)$

(ii) $\mathrm{P}(Z > -0.8)$

(iii) $\mathrm{P}(-2.1 < Z < -0.3)$

(4 marks)

(b) The random variable $X \sim N(2,\ 0.1^2)$.

By using the coding relationship between X and Z, re-express the probabilities from parts (a) (i), (ii) and (iii) in the forms $\mathrm{P}(X < a)$, $\mathrm{P}(X > b)$ and $\mathrm{P}(c < X < d)$ respectively, where a, b, c and d are constants to be found.

(3 marks)

7 (a) The table below shows the percentage points of the normal distribution. The values z in the table are those which a random variable $Z \sim N(0,1)$ exceeds with probability p.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0365	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

(i) Use the percentage points table for the standard normal distribution to find the value of z for which $\mathrm{P}(Z > z) = 0.2$.

(ii) Use your answer to part (a)(i) along with the properties of the normal distribution to work out the values of a and b for which $\mathrm{P}(Z < a) = 0.2$ and $\mathrm{P}(Z < b) = 0.8$.

(3 marks)

(b) The weights, W kg, of coconuts grown on the Coconutty As They Come coconut plantation are modelled as a normal distribution with mean 1.25 kg and standard deviation 0.38 kg. The plantation only considers coconuts to be exportable if their weight falls into the 20% to 80% interpercentile range.

Use your answer to part (a)(ii) to find the range of possible weights, to the nearest 0.01 kg, for an exportable coconut.

(2 marks)

8 (a) A machine is used to fill cans of a particular brand of soft drink. The volume, V ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation σ ml. Given that 15% of the cans contain more than 333.4 ml of soft drink, find:

the value of σ

(2 marks)

(b) $\mathrm{P}(320 \le V \le 340)$.

(1 mark)

(c) Six cans of the soft drink are chosen at random.

Find the probability that all of the cans contain less than 329 ml of soft drink.

(3 marks)

9 (a) The random variable $X \sim N(\mu, \sigma^2)$. It is known that $P(X > 36.88) = 0.025$ and $P(X < 27.16) = 0.1$

Find the values of a and b for which $P(Z > a) = 0.025$ and $P(Z < b) = 0.1$, where Z is the standard normal variable. Give your answers correct to 4 decimal places.

(2 marks)

(b) Use your answers to part (a), along with the relationship between Z and X, to show that the following simultaneous equations must be true:

$$\mu + 1.96\sigma = 36.88$$

$$\mu - 1.2816\sigma = 27.16$$

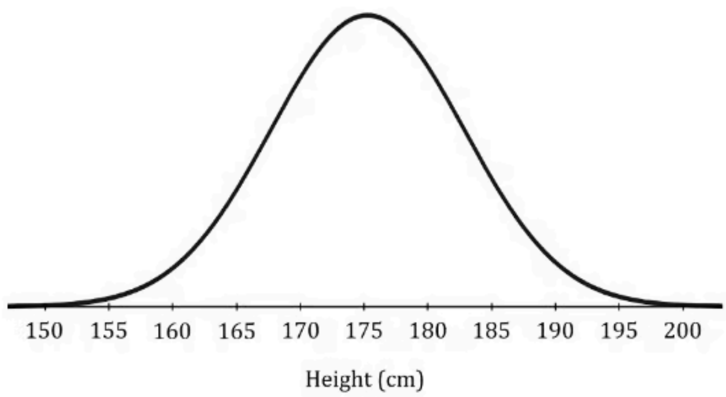
(2 marks)

(c) By solving the simultaneous equations in (b), determine the values of μ and σ. Give your answers correct to 2 decimal places.

(2 marks)

Hard Questions

1 (a) The following diagram shows the distribution of heights, in cm, of adult men in the UK:



The distribution of heights follows a normal distribution, with a mean of 175.3 cm and a standard deviation of 7.6 cm.

Write down the values of the height that correspond to:

(i) the line of symmetry of the curve.

(ii) the points of inflection on the curve.

(3 marks)

(b) Use the properties of the normal distribution to determine whether each of the following statements is likely to be true. In each case give a reason for your answer.

(i) 95% of adult men in the UK will have a height less than 190.5 cm.

(ii) 68% of adult men in the UK will have a height between 167.7 cm and 182.9 cm.

(iii) 81.5% of adult men in the UK will have a height between 160.1 cm and 182.9 cm.

(4 marks)

(c) Paul, a renowned mathematics educationalist, has a height of 196 cm.

Find the percentage of adult men in the UK who have a height that is less than Paul's.

(2 marks)

2 (a) The weight, W kg, of the feed in a sack of partridge feed produced by a certain manufacturer is modelled as $W \sim N(20, 0.01)$.

Find:

(i) $\mathrm{P}(W < 19.75)$

(ii) $\mathrm{P}(W > 20.15)$

(2 marks)

(b) Roger buys ten sacks of the manufacturer's partridge feed to feed to the partridges who have begun showing up at his backyard bird feeding station.

Using one of your answers to part (i), along with the properties of the normal distribution, find the probability that all ten sacks contain feed with a weight that is within 250 g of 20 kg.

(3 marks)

3 (a) The random variable $X \sim N(13, 4^2)$.

Find the value of a, to 3 decimal places, such that:

(i) $P(X > a) = 0.3$

(ii) $P(a \leq X \leq 14) = 0.5$

(3 marks)

(b) The random variable $Y \sim N(20, 44)$..

Find the value of b and the value of c, each to 3 decimal places, such that:

(i) $P(Y < b) = 0.13$

(ii) $P(Y > c) = 0.27$

(2 marks)

(c) Use a sketch of the distribution of Y, along with the properties of the normal distribution, to explain why $P(b < Y < c) = 0.6$..

(3 marks)

4 (a) The test scores, X, of a group of Royal Navy recruits in an aptitude test are modelled as a normal distribution with $X \sim N(520,\ 89.9^2)$.

Find the interquartile range of the scores.

(3 marks)

(b) Those who score in the top 1% on the test are eligible to join the submarine service.

One of the recruits, Mervyn, is a keen would-be submariner. He achieves a score of 750 on the test. Determine whether Mervyn will be eligible to join the submarine service.

(2 marks)

5 (a) For the standard normal variable $Z \sim N(0, 1^2)$, the function Φ is defined by

$$\Phi(a) = P(Z < a)$$

Find:

(i) $\Phi(1.2)$

(ii) $\Phi(0.3)$

(iii) $\Phi(-2.1)$

(3 marks)

(b) The random variable $X \sim N(30, 2^2)$.

Use your answers from part (a), along with the relationship between X and Z, to work out the following probabilities:

(i) $\mathrm{P}(X < 30.6)$

(ii) $\mathrm{P}(X > 32.4)$

(iii) $\mathrm{P}(25.8 \leq X < 27.6)$

(4 marks)

6 (a) The table below shows the percentage points of the normal distribution. The values z in the table are those which a random variable $Z \sim N(0,1)$ exceeds with probability p.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0365	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Use the percentage points table for the standard normal distribution, along with the general properties of the normal distribution, to work out the values of a and b for which $\mathrm{P}(Z < a) = 0.15$ and $\mathrm{P}(Z < b) = 0.85$..

(3 marks)

(b) A new agricultural standardisation scheme has been proposed that will measure the bendiness of bananas in terms of a unit called the 'bent'. It is found that the bendiness measurements, B, of bananas grown on the Yes We Have Some Bananas banana plantation can be modelled as a normal distribution with mean 55.3 bents and standard deviation 6.1 bents. Because the owners of the plantation are committed to making sure that the bananas they export are neither too bendy nor not bendy enough, the plantation only considers bananas to be exportable if their bendiness falls into the 15% to 85% interpercentile range of the plantation's banana bendiness measurements.

Use your answer to part (a) to find the range of possible bendiness measurements, to the nearest hundredth of a bent, for an exportable banana.

(2 marks)

7 (a) A machine is used to fill bags of potatoes for a supermarket chain. The weight, W kg, of potatoes in the bags is normally distributed with mean 3 kg and standard deviation σ kg.

Given that 7% of the bags contain a weight of potatoes that is at least 50 g more than the mean, find:

$\mathrm{P}(2.9 \leq W \leq 3.1)$.

(3 marks)

(b) Twelve of the bags of potatoes are chosen at random.

Find the probability that not more than one of the bags will contain less than 2.96 kg of potatoes.

(4 marks)

8 (a) The random variable $X \sim N(\mu, \sigma^2)$. It is known that $P(X > 34.451) = 0.001$ and $P(X < 14.792) = 0.2$

Use the relationship between X and the standard normal variable Z to show that the following simultaneous equations must be true:

$$\mu + 3.0902\sigma = 34.451$$

$$\mu - 0.8416\sigma = 14.792$$

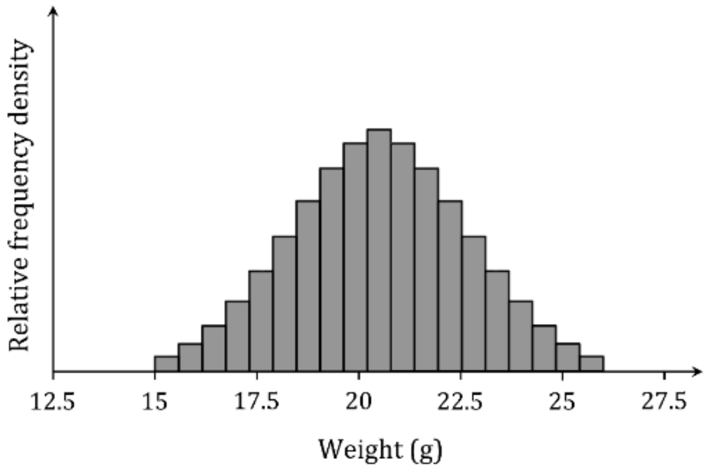
(4 marks)

(b) By solving the simultaneous equations in (a), determine the values of μ and σ.

(2 marks)

Very Hard Questions

1 (a) The following histogram shows the distribution of weights, in grams, of a population of dormice in the UK:



(i) Explain why it would be appropriate to use a normal distribution to model the distribution of weights of the dormouse population.

(ii) Explain how the histogram could be altered so as to better approximate the smooth curve of the normal distribution.

(3 marks)

(b) The mean and standard deviation for the weights of the dormouse population are calculated to be 20.5 g and 2.6 g respectively. A normal curve is drawn corresponding to these values.

Write down the values of the weight that correspond to the line of symmetry and the points of inflection of the normal curve.

(3 marks)

(c) Use the properties of the normal distribution to determine whether each of the following statements is likely to be true. In each case give a reason for your answer.

(i) 84% of the dormice have a weight that is less than 23.1 g.

(ii) More than 99% of the dormice have a weight that is between 15.3 g and 28.3 g.

(iii) 18.5% of the dormice have a weight that is either less than 17.9 g or greater than 25.7 g.

(4 marks)

2 The weight, W kg, of the feed in a sack of pheasant feed produced by a certain manufacturer is modelled as $W \sim N\left(20, \dfrac{1}{3600}\right)$.

Roger buys twelve sacks of the manufacturer's pheasant feed to feed to the pheasants who have begun showing up at his backyard bird feeding station.

Find the probability that all twelve sacks contain feed with a weight that is within 35 g of 20 kg.

(5 marks)

3 (a) The random variable $X \sim N(2.35,\ 0.3^2)$.

Find the value of a, to 3 decimal places, such that:

(i) $\mathrm{P}(X > a) = 0.005$

(ii) $\mathrm{P}(a \leq X \leq 3) = 0.47$

(3 marks)

(b) The random variable $Y \sim N(15,101)..$

Find the value of b and the value of c, each to 3 decimal places, such that:

(i) $\mathrm{P}(Y > b) = 0.97$

(ii) $\mathrm{P}(Y < c) = 0.23$

(2 marks)

(c) Use a sketch of the distribution of Y, along with the properties of the normal distribution, to explain why $\mathrm{P}(b \leq Y \leq c) = 0.2..$

(3 marks)

4 (a) The distribution of the test scores, X, of a group of British Army officer cadets on an aviation aptitude test is modelled as a normal distribution with $X \sim N(120, 26.5^2)$.

Only cadets who score in the top 10% on the test are eligible to proceed directly to helicopter pilot training. Cadets whose scores are between the 40th and 90th percentiles, however, are eligible to resit the test in an attempt to improve their scores.

Given that it is only possible to receive an integer number of marks as a score on the test, determine the range of test scores for which cadets would be eligible to resit the test.

(4 marks)

(b) (i) Find $\text{P}(X > 200)$

(ii) The maximum score it is possible to receive on the test is 200. Use this fact, and your answer to part (b)(i), to criticise the model being used for the score distribution.

(3 marks)

5 (a) For the standard normal variable $Z \sim N(0,1^2)$, the function Φ is defined by

$$\Phi(a) = P(Z < a), \quad a \in R$$

The constants q, r and s are positive real numbers.

Find an expression for each of the following probabilities, giving your answers as simply as possible in terms of $\Phi(q), \Phi(r)$, and $\Phi(s)$.

(i) $\mathrm{P}(Z > q)$

(ii) $\mathrm{P}(-s < Z < r)$

(3 marks)

(b) The random variable $X \sim N(\mu, \sigma)$.

Find an expression for each of the following probabilities. You should give your answers as simply as possible in terms of the function Φ, where the argument of the function should in each case be given in terms of μ and σ.

(i) $\mathrm{P}(X < 103)$

(ii) $\mathrm{P}(X > -7)$

(iii) $\mathrm{P}(-80 < X \leq 2500)$

(4 marks)

6 The table below shows the percentage points of the normal distribution. The values z in the table are those which a random variable $Z \sim N(0,1)$ exceeds with probability p.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0365	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

The Monkey Puzzle Tree Marketing Board has proposed a new scheme that will measure the puzzlingness of monkey puzzle trees in terms of a unit called the 'fuddle'. It is found that the puzzlingness measurements, X, of monkey puzzle trees grown on the We ♥ Puzzling Monkeys monkey puzzle tree plantation can be modelled as a normal distribution with mean μ fuddles and standard deviation σ fuddles. Because the owners of the plantation are committed to making sure that the monkey puzzle trees they sell to gardeners are neither too puzzling nor not puzzling enough, the plantation only considers monkey puzzle trees to be saleable if their puzzlingness falls into the 10% to 97.5% interpercentile range of the plantation's monkey puzzle tree puzzlingness measurements.

Using the percentage points table for the standard normal distribution, or otherwise, find the range of possible puzzlingness measurements for a saleable monkey puzzle tree. Your answer should be given in terms of μ and σ.

(5 marks)

7 A machine is used to produce the 3-metre barge poles sold by the You Would Touch It With One Of Ours barge pole company. The actual length, L m, of the barge poles is normally distributed with mean 3 m, standard deviation σ m, and interquartile range 0.01712 m.

Twenty of the barge poles are chosen at random.

Find the probability that at most two of the barge poles will be shorter than 3 m by 1 cm or more.

(7 marks)

8 (a) An archaeologist has devoted his life to studying ancient Greek vases produced by a particular Boeotian pottery workshop. The vases were made to a standard pattern, and after measuring a very large number of them the archaeologist has found that 5% of the vases have a mass greater than 2.237 kg, while only 1% of them have a mass less than 1.906 kg.

Given that the masses of the vases may be assumed to be distributed normally, find the mean and standard deviation of the distribution.

(6 marks)

(b) The archaeologist has found that vases made by the workshop with a mass less than 1.93 kg are particularly fragile and require special care.

A museum has just purchased a collection of k vases produced by the workshop. The k vases may be assumed to be a random sample.

Given that there is a less than 15% chance that the collection contains vases that are particularly fragile and require special care, find the greatest possible value of k. Your answer should be supported by clear algebraic working.

(4 marks)