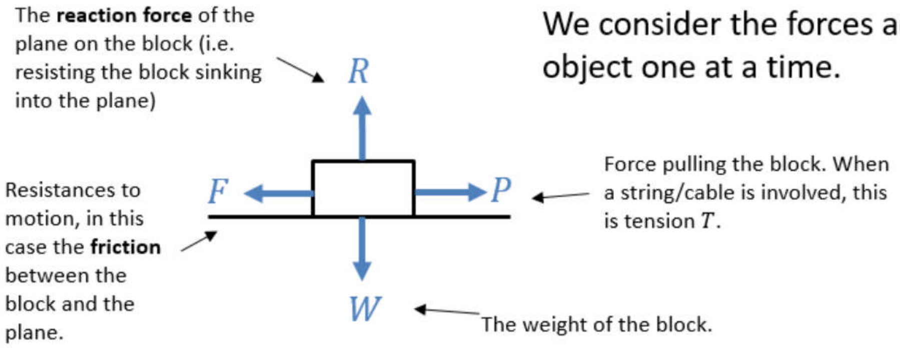
Forces (Year 1)

Force Diagrams and Common Forces

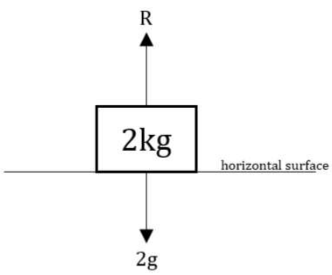
We consider the forces acting on each object one at a time.

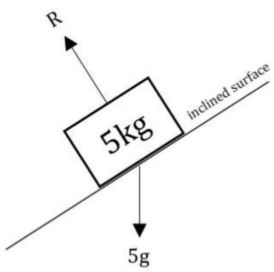


Recall that we often model an object as a **particle**, i.e. a point with negligible dimensions.

weight = mass $\times$ g (where g is the acceleration due to gravity, $g = 9.8ms^{-2}$
$W = mg$
Weight acts vertically downwards (obviously)

The normal reaction (sometimes called the contact force) is the force which acts on a box/particle from the surface that it is on.
It is called a **normal** reaction because it acts normal (perpendicular) to the surface.
It is called a normal **reaction** because it has reacted to the forces in the opposing direction.
For example, when you are sat on a chair, your weight acts down, but the chair (surface) has a reaction force upwards which stops you falling to the floor. This is the normal reaction.
We use the letter R for the normal reaction.





Note that the weight acts vertically downwards, but the normal reaction is perpendicular to the slope

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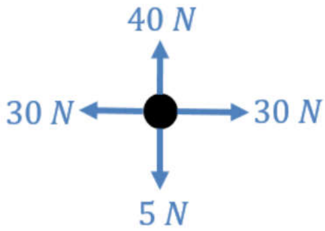
Let's consider the forces acting on a body suspended from a light inextensible string

Newton's 1st Law of Motion states than an object at rest will stay at rest and that an object moving with constant velocity will remain at that velocity unless an unbalanced force acts on the object.

In other words, if the object is not accelerating, the **forces are balanced in every direction**, e.g. forces up = forces down and forces left = forces right.

We use $R(\ )$ to 'resolve' the forces in a particular direction. **This is standard notation expected in exams.**

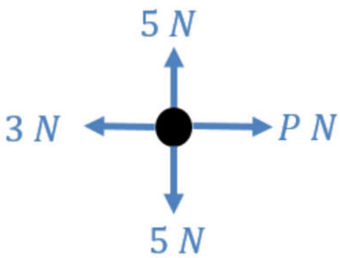
The '**resultant force**' is the overall force acting on the object. An object will accelerate in the direction of the resultant force.



$R(\uparrow)$:

$R(\rightarrow)$:

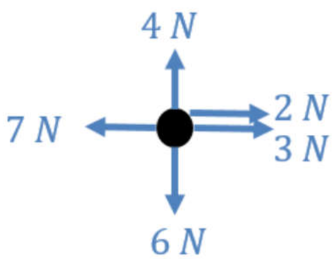
Finding Resultant Forces



$R(\uparrow)$:

$R(\rightarrow)$:

I do not usually write the 'N' on diagrams



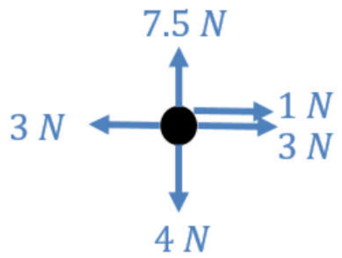
$R(\downarrow)$:

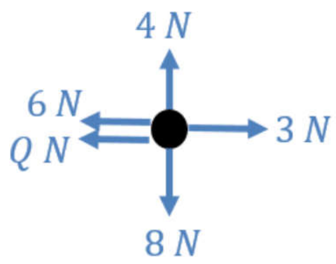
$R(\leftarrow)$:

or

$R(\uparrow)$:

$R(\rightarrow)$:



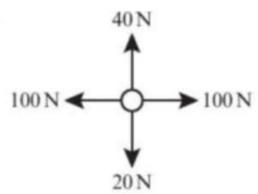


12 Each diagram shows the forces acting on a particle.

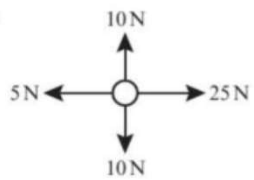
i Work out the size and direction of the resultant force.

ii Describe the motion of the particle.

a



b



13 A truck is moving along a horizontal level road. The truck's engine provides a forward thrust of 10 000 N. The total resistance is modelled as a constant force of magnitude 1600 N.

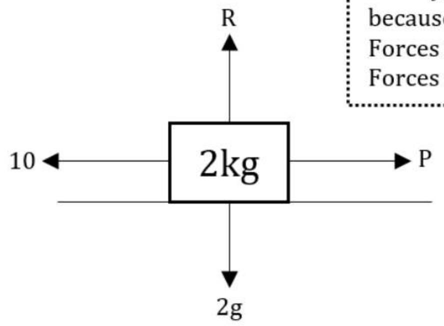
a Modelling the truck as a particle, draw a force diagram to show the forces acting on the truck.

b Calculate the resultant force acting on the truck.

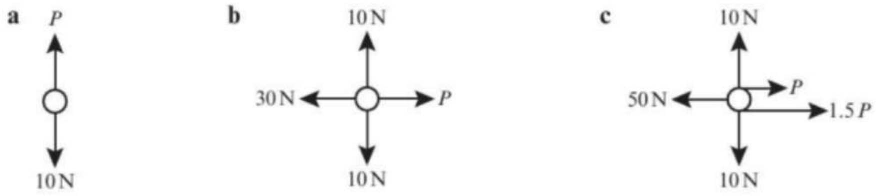
Particles in equilibrium - at rest/constant velocity

If the particle is at rest or moving at constant velocity, there is no resultant force.
Left = right and up = down

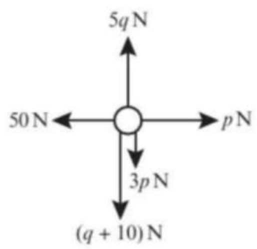
Given that this particle is rest, work out the value of P and R.
Clearly, R = 2g and P = 10, because it is in equilibrium.
Forces left = forces right.
Forces up = forces down



8 Given that each of the particles is stationary, work out the value of P:



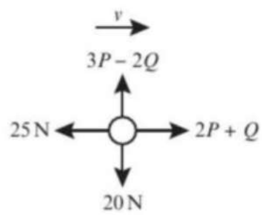
10 The diagram shows a particle acted on by a set of forces.
Given that the particle is at rest, find the value of p and the value of q.



11 Given that the particle in this diagram is moving with constant velocity, v, find the values of P and Q.

Problem-solving

Set up two simultaneous equations.



Forces as Vectors

Forces have direction, and therefore we can naturally write them as vectors, either in $\boldsymbol{i}$-$\boldsymbol{j}$ notation or as column vectors.

✏ You can find the resultant of two or more forces given as vectors by adding the vectors.

The forces $2\boldsymbol{i} + 3\boldsymbol{j}$, $4\boldsymbol{i} - \boldsymbol{j}$, $-3\boldsymbol{i} + 2\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

(b) Work out the magnitude and bearing of the resultant force.

Edexcel M1 May 2009 Q2

A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$. **(2)**

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(b) find the value of p. **(4)**

Tip: If a vector is parallel to say $\begin{pmatrix}1\\2\end{pmatrix}$, then it could be any multiple of it, i.e. $\begin{pmatrix}k\\2k\end{pmatrix}$

Your Turn

Edexcel M1 Jan 2012 Q3

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\ \text{N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\ \text{N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\ \text{N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q. **(3)**

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Find

(b) the magnitude of $\mathbf{R}$, **(2)**

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$. **(3)**

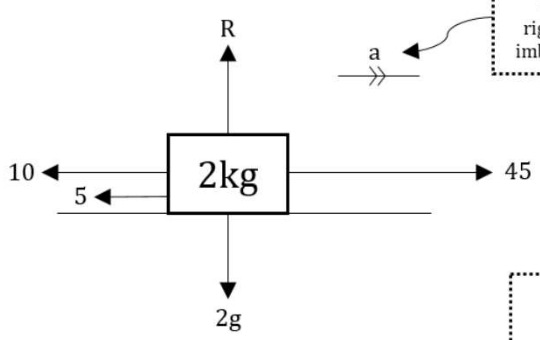
Ex 10B

Forces and Acceleration

✏ Newton's 2nd Law of Motion: $F = ma$
(where the **resultant** force F and acceleration a are in the <u>same direction</u>)

This 'feels' right: if we doubled the force, we double the rate at which it accelerates. Similarly, if we have an object of twice the mass, we'd require twice the force to make it accelerate at the same rate.

It is clearly going to accelerate to the right as there is an imbalance of forces



Notice how we find the resultant force by doing the forces to the right minus the forces to the left

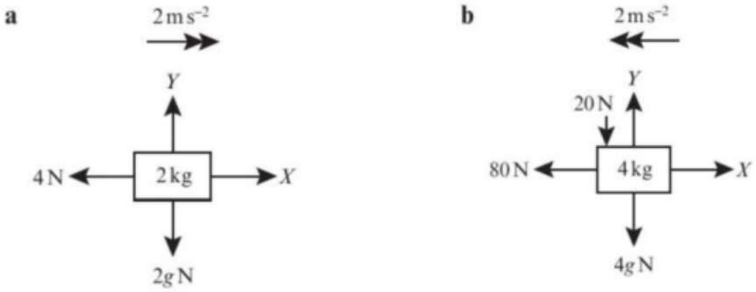
Work out the acceleration for the particle.

Using F = ma

$45 - 10 - 5 = 2a$
$30 = 2a$
$15 = a$

So the particle will accelerate at 15ms^{-2} to the right

In each of these diagrams the body is accelerating as shown. Find the magnitudes of the unknown forces X and Y.



A car of 2000kg has a driving force of 800N and forces of 200N resisting its motion. Determine its acceleration.

A child has a mass of 50kg. What is the gravitational force acting on the child? (i.e. its weight)

A falling sheep of mass 70kg experiences air resistance of 300 N. Determine the sheep's acceleration as it plummets towards the ground.

Combining F=ma with SUVAT equations

Since $F = ma$ involves both force and acceleration, it allows us to connect calculations involving forces with a calculations involving $suvat$ values.

A body of mass 5kg is pulled along a rough horizontal table by a horizontal force of magnitude 20N against a constant friction force of magnitude 4N. Given that the body is initially at rest, find:
(a) the acceleration of the body
(b) the distance travelled by the body in the first 4 seconds
(c) the magnitude of the normal reaction between the body and the table

Edexcel M1 May 2012 Q5 (abridged)

A particle P is projected vertically upwards from a point A with speed u m s^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1}.

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertical distance that P sinks into the ground before coming to rest.
(4)

Motion in 2 dimensions - using vectors again!

In Chapter 8 we saw that many physical quantities could have both direction and magnitude, and therefore could be represented as a vector:

Can be a vector:	Force, acceleration, velocity, displacement
Scalar only:	Mass, area, volume

This naturally means that $\boldsymbol{F} = m\boldsymbol{a}$ works with vectors too.

Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(3\boldsymbol{i} + 8\boldsymbol{j})$ N acts upon a particle of mass 0.5 kg.
(a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})$ ms^{-2}.
(b) Find the magnitude and bearing of the acceleration of the particle.

Three forces act on a particle of mass 3kg. Find the acceleration of the particle.

$$F_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} N, \qquad F_2 = \begin{pmatrix} -5 \\ 4 \end{pmatrix} N, \qquad F_3 = \begin{pmatrix} 6 \\ -5 \end{pmatrix} N$$

A boat is modelled as a particle of mass 60 kg being acted on by three forces.

$$F_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N, \qquad F_2 = \begin{pmatrix} 10q \\ 20q \end{pmatrix} N, \qquad F_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of p and q.

Ex 10D

Connected Particles - horizontal surface

Up to now we've only considered one particle at a time.
When we have multiple connected objects moving in the same direction, **they can be considered as a single particle**:

Two particles, P and Q, of masses 5kg and 3kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40N along a rough horizontal plane. Particle P experiences a frictional force of 10N and particle Q experiences a frictional force of 6N.

(a) Find the acceleration of the particles.
(b) Find the tension in the string.
(c) Explain how the modelling assumptions that the string is light and inextensible have been used.

Tension vs Thrust

tension
rigid bar/rod

thrust
rigid bar/rod

a) A car of mass 900 kg tows a trailer of mass 600 kg by means of a rigid tow bar. The car experiences a resistance of 200 N and the trailer a resistance of 300 N. If the car engine exerts a driving force of 3000 N, find the acceleration of the system and the tension in the tow bar.

b) The driver sees a set of traffic lights ahead that have turned red. She reduces the driving force produced by the engine to zero, and applies an additional resistant braking force of 700N. Find the deceleration of the system and the thrust in the tow bar.

Ex 10E
Not Q4, 6, 7

Connected Particles - hanging strings

Two particles *A* and *B* of masses 3kg and 2kg respectively are connected by a light inextensible string. Particle *B* hangs directly below particle *A*.
A force of 65N is applied vertically upwards causing the particles to accelerate.
Find:

i) The magnitude of the acceleration
ii) The tension in the string.

Connected Particles - lifts/boxes on boxes

Edexcel M1 May 2013 Q2

A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s^{-2}. By modelling the cable as being light and inextensible, find

(a) the tension in the cable, **(3)**

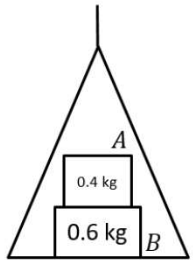
(b) the magnitude of the force exerted on the woman by the floor of the lift. **(3)**

✏ Newton's 3rd Law: For every action there is an equal and opposite reaction.

Therefore when two bodies A and B are in contact, if body A exerts a force on body B, then body B exerts a force on body A that is equal in magnitude and acts in the opposite direction.

A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400g and the mass of B is 600g. A rests on top of B, as shown in the diagram.
The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.



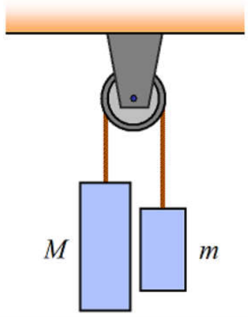
Tip: We can use Newton's 3rd Law to reverse "force of A on B" to "force of B on A" and vice versa.

Ex 10E
Q4, 6, 7

Connected Particles - pulleys

Why can't we just model both particles as a single particle as before?

Under what conditions is the tension in each part of the string the same?



Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

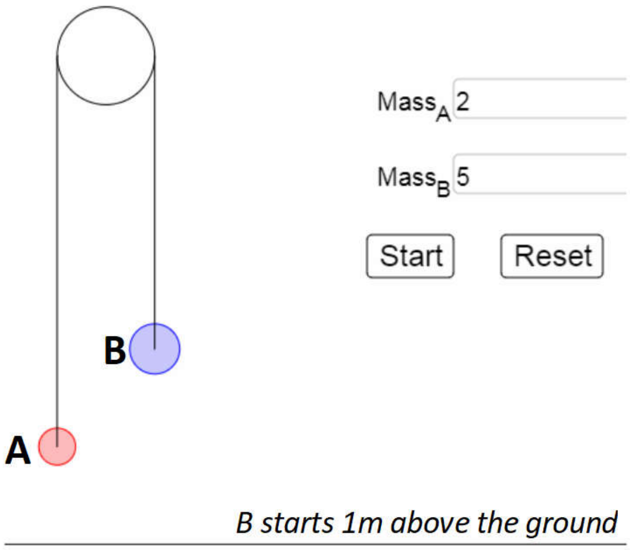
(a) Write down an equation of motion for P and for Q.

(b) Find the acceleration of each mass.

(c) Find the tension in the string.

(d) Find the force exerted on the pulley by the string.

(e) Find the distance moved by Q in the first 4 s, assuming that P does not reach the pulley.



After B hits the floor, what happens next?

Can you describe what happens to A?

(describe its acceleration, speed, etc. Use as much mechanics language as possible!)

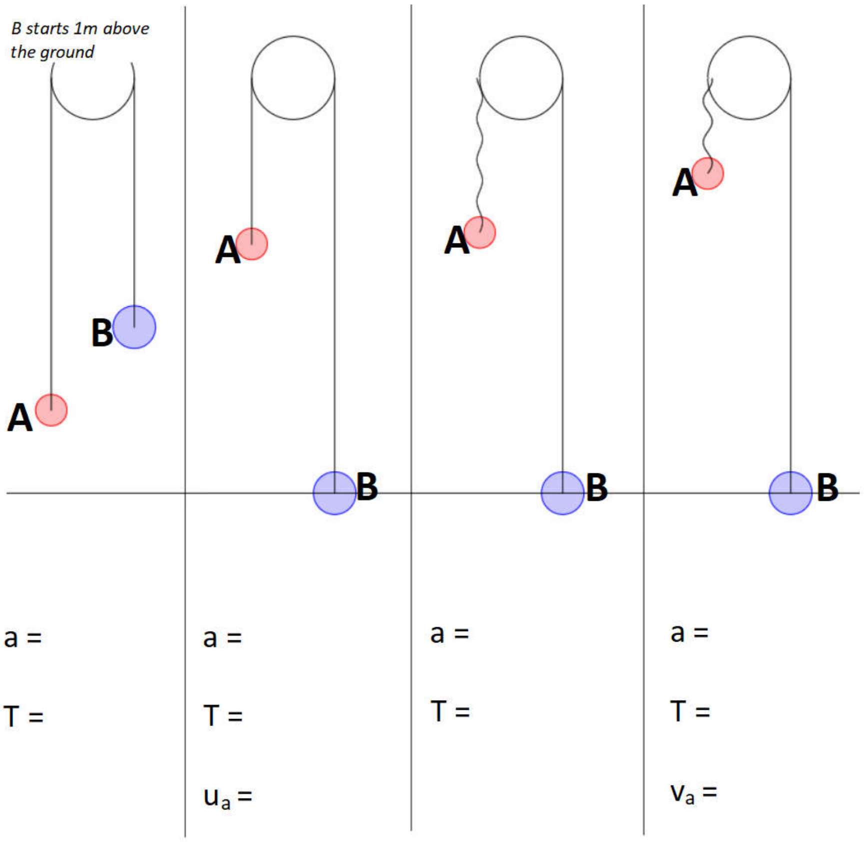
What is the acceleration of A after they are released? What is the tension in the string?

https://www.geogebra.org/m/fyysxw3s

Checklist

F=ma
F=ma
Solve

SUVAT: v ⇨ u
New acceleration: T = 0
SUVAT
Solve problem



How far does A travel before reaching its greatest height?

Checklist

F=ma
F=ma
Solve

SUVAT: v ⇨ u
New acceleration: T = 0
SUVAT
Solve problem

Ex 10F Q1, 2, 3

Checklist

F=ma
F=ma
Solve

SUVAT: v ⇨ u
New acceleration: T = 0
SUVAT
Solve problem

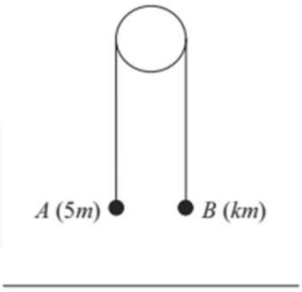


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.
(3)

(b) Find the value of k.
(3)

(c) State how you have used the information that the pulley is smooth.
(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane.
(7)

Connected Particles - pulleys on tables

Two particles A and B of masses 0.4kg and 0.8kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5m above horizontal ground. A frictional force of magnitude 0.08g opposes the motion of particle A. The system is released from rest. Find:

(a) The acceleration of the system
(b) The tension in the string
(c) The time taken for B to reach the ground
(d) The total distance travelled by A before it first comes to rest.

Checklist

F=ma
F=ma
Solve

SUVAT: v ⇨ u
New acceleration: T = 0
SUVAT
Solve problem

Ex 10F Q4, 5