Set A

Please check the examination details below before entering your candidate information

Candidate surname	Other names
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MME Edexcel
Level 3 GCE

Centre Number					Candidate Number			
1	7	7	2	1	–	–	–	–

MME Edexcel Practice Papers

Morning (Time: 2 hours)	Paper Reference **1MME**

Mathematics
Advanced
Paper 1: Pure Mathematics 1

Worked Solutions

All content is provided as is, without any warranty. I do not take liability for incorrect answers/working.

You must have: Mathematical Formulae and Statistical Tables, Calculator	Total Marks

Candidates may use any approved calculator.
Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1. Find the coordinates and the nature of the stationary points of the function,

$$f(x) = 2x^3 - 3x^2 - 36x + 2, \qquad x \in \mathbb{R}$$

(7)

$\frac{dy}{dx} = 6x^2 - 6x - 36$

$\frac{d^2y}{dx^2} = 12x - 6$

$\frac{dy}{dx} = 0 \longrightarrow 6x^2 - 6x - 36 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad y = -79 \quad \rightarrow (3, -79)$ (min)

$x = -2 \quad y = 46 \quad (-2, 46)$ (max)

$\left.\frac{d^2y}{dx^2}\right|_{x=3} \longrightarrow 12(3) - 6 = 30 > 0 \quad \therefore$ minimum

$\left.\frac{d^2y}{dx^2}\right|_{x=-2} \longrightarrow 12(-2) - 6 = -30 < 0 \quad \therefore$ maximum

(Total for Question 1 is 7 marks)

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2. Given that angle x is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{\sin x \cos 2x - \sin x}{x^2 \tan 4x}$$

$\sin x \simeq x$

$\cos x \simeq 1 - \frac{x^2}{2}$

$\tan x \simeq x$

Express your answer in its simplest form.

(4)

$$\frac{x\left(1 - \frac{(2x)^2}{2} - 1\right)}{x^2(4x)} = \frac{-\frac{4x^3}{2}}{4x^3} = \boxed{-\frac{1}{2}}$$

(Total for Question 2 is 4 marks)

Turn over

3. **Figure 1** shows a circle with a centre O and radius r cm

The angle in the shaded sector AOB shown is θ.

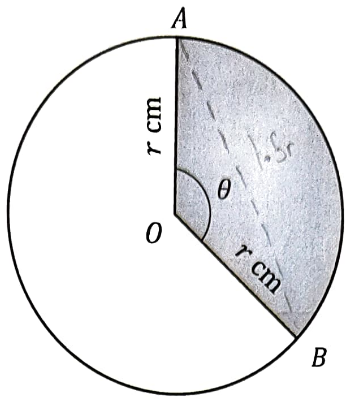

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Figure 1

(a) The length of the **chord** $AB = 1.5r$

Find angle θ in radians.

Give your answer to 3 decimal places.

(3)

(b) The area of the sector is 16π cm^2

Find the perimeter of the shaded sector.
Give your answer to 2 decimal places.

(4)

a) $\cos\theta = \dfrac{r^2+r^2-\left(\frac{3}{2}r\right)^2}{2r^2}$

$\theta = \cos^{-1}\left(\dfrac{2r^2-\frac{9}{4}r^2}{2r^2}\right) = \cos^{-1}\left(\dfrac{-\frac{1}{4}r^2}{2r^2}\right) = \cos^{-1}\left(-\frac{1}{8}\right)$

$= 1.696$ rad

b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\cos^{-1}\left(-\frac{1}{8}\right)\right)$

$16\pi = \frac{1}{2}r^2\theta$

$\dfrac{16\pi}{\frac{1}{2}\theta} = r^2 \longrightarrow r = \sqrt{\dfrac{16\pi}{\frac{1}{2}\theta}} = 7.6987\ldots$

Arc length $= r\theta \approx 13.058$cm $\rightarrow P = l + r + r = 28.4555.. \rightarrow 28.46$cm

4. A curve C has the parametric equations,

$$x = 2\sin t - 1, \qquad y = 4 + 2\cos 2t, \qquad 0 \le t \le 2\pi$$

(a) Show that the cartesian equation of C is:

$$y = 6 - (x+1)^2$$

(3)

(b) The line $y = \frac{1}{3}x + k$, where k is a constant, crosses the curve C at two distinct points.

Find the range of possible values of k.

(5)

a) $\frac{x+1}{2} = \sin t \qquad \frac{y-4}{2} = \cos 2t = 1 - 2\sin^2 t$

$\frac{y-4}{2} = 1 - 2\left(\frac{x+1}{2}\right)^2$

$\frac{y-4}{2} = 1 - \frac{1}{2}(x+1)^2$

$y - 4 = 2 - (x+1)^2$

$y = 6 - (x+1)^2$

b) $\frac{1}{3}x + k = 6 - (x+1)^2$

$\frac{1}{3}x + k = 6 - x^2 - 2x - 1$

$x^2 + \frac{7}{3}x + k - 5 = 0 \longrightarrow b^2 - 4ac > 0$

$\frac{49}{9} - 4(k-5) > 0$

$\frac{229}{9} - 4k > 0$

$k < \frac{229}{36}$

5. **(a)** Using binomial expansion, show that:

$$\sqrt{\frac{4-x}{1+3x}} \approx 2 - \frac{13}{4}x + \frac{455}{64}x^2$$

(6)

(b) Hence, or otherwise show that an approximation for $\sqrt{3}$ can be written as $\frac{k}{256}$, where k is an integer to be found.

(3)

(c) Explain why $x = \frac{11}{6}$ and the binomial expansion above, should not be used to find an approximation for $\sqrt{\frac{1}{3}}$

(2)

$\sqrt{\frac{4-x}{1+3x}} = (4-x)^{\frac{1}{2}}(1+3x)^{-\frac{1}{2}}$

$(4-x)^{\frac{1}{2}} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} \to 2\left(1+\frac{1}{2}\left(-\frac{x}{4}\right) + \left(\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\right)\left(-\frac{x}{4}\right)^2\right)$

$= 2\left(1-\frac{1}{8}x - \frac{1}{128}x^2\right)$

$(1+3x)^{-\frac{1}{2}} = 1 + -\frac{1}{2}(3x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(3x)^2 = 1 - \frac{3}{2}x + \frac{27}{8}x^2$

$\therefore$

$(4-x)^{\frac{1}{2}} \times (1+3x)^{-\frac{1}{2}} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2\right)\left(1-\frac{3}{2}x+\frac{27}{8}x^2\right)$

$= 2\left(1-\frac{3}{2}x+\frac{27}{8}x^2 - \frac{1}{8}x + \frac{3}{16}x^2 - \frac{1}{128}x^2\right)$

$= 2 - \frac{13}{4}x + \frac{455}{64}$

Question 5 continued

b) $\sqrt{3} = \sqrt{\frac{4-x}{1+3x}}$

$3 = \frac{4-x}{1+3x} \rightarrow 9x+3 = 4-x$

$x = \frac{1}{10}$

$\rightarrow 2 - \frac{13}{4}x + \frac{455}{64}(x^2) = \frac{447}{256}$

$k = 447$

c) The expansion is only valid for $|a| \leq 1$

$\sqrt{\frac{1}{3}} \rightarrow |x| \leq 1$

$|3x| \leq 1 \rightarrow \frac{11}{6} > 1, \therefore$ not valid.

(Total for Question 5 is 11 marks)

6. (a) Find the values of θ for which,

$$\tan^2\theta = 9, \qquad -\pi \leq \theta \leq \pi$$

Give your solution to 2 decimal places.

(2)

(b) Solve the equation,

$$4\cos^2 t = 3 - \sin t, \qquad 0 \leq t \leq 2\pi$$

Give your solution to 2 decimal places.

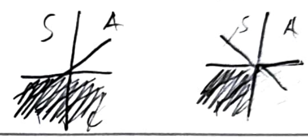
(4)

a) $\tan^2\theta = 9$

$\tan\theta = \pm 3$

$\theta = \tan^{-1} \pm 3$

$\theta = \boxed{\begin{array}{l} \pm\ 1.25 \text{ rad} \\ 1.89 \text{ rad} \end{array}}$



interval $= -\pi \leq \theta \leq \pi$

$\therefore$ ignore bottom left

b) $4\cos^2 t = 3 - \sin t$

$4 - 4\sin^2 t = 3 - \sin t$

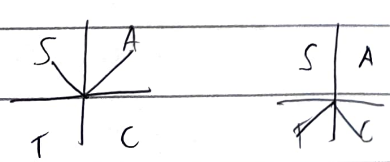
$4\sin^2 t - \sin t - 1 = 0$

$x = \sin t \rightarrow 4x^2 - x - 1 = 0 \rightarrow a = 4,\ b = -1,\ c = -1$

$x = \dfrac{1 \pm \sqrt{17}}{8}$

$\sin t = \dfrac{1 \pm \sqrt{17}}{8}$

$\boxed{t = 0.70,\ 3.54,\ 2.45,\ 5.88}$



7. Lewis is training for a half-marathon, which takes place in just over 3 weeks time. He is going to follow a training plan where he increases the distance by 4% each day.

The first day he runs 10 miles.

(a) How far will Lewis run on the 7th day?

(3)

(b) Over the 21 days of the plan how far will Lewis have run altogether?

Give your answer to the nearest mile.

(4)

$10(1.04)^{d-1}$ $d =$ day no.

$\therefore$ a) $10(1.04)^{7-1} \approx 12.7$ miles

b)

$S_{21} = \frac{10(1-1.04^{21})}{1-1.04} = 319.692...$

$\hookrightarrow$ 320 miles

(Total for Question 7 is 7 marks)

8. **Figure 2** shows a sketch of the curve C with equation $y = xe^x$,

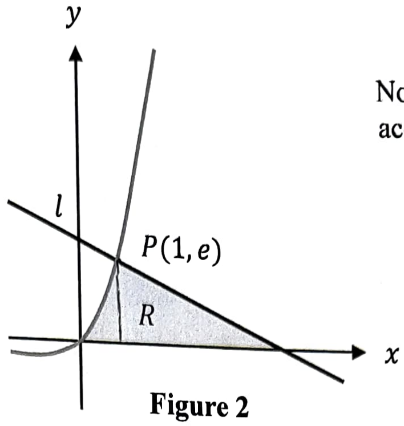


Figure 2

The line l is the normal to C at the point $(1, e)$.

(a) Find the equation of the normal, l, in the form $y = mx + c$.

(5)

(b) Find the area of the region, R, enclosed by the curve C, the line l and the x-axis.

(5)

a) $\frac{dy}{dx} = xe^x + e^x$ $\qquad \left.\frac{dy}{dx}\right|_{x=1} = 2e$

$\therefore \left.\frac{dy}{dx}\right|_{Norm} = -\frac{1}{2e}$

$y - e = -\frac{1}{2e}(x-1)$

$\boxed{y = -\frac{1}{2e}x + \frac{1}{2e} + e}$

b) $R = \int_0^1 xe^x \, dx + \frac{1}{2} \times 2e^2 \times e$

$\int_0^1 xe^x = \left[xe^x - e^x\right]_0^1 = 1$

$R = 1 + e^3$

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9. The number of bacteria, B, in a petri dish after, t minutes, can be modelled by the following exponential equation,

$$B = ae^{kt}$$

where a and k are constants.

At the start of the experiment, there are 20 bacteria in the petri dish.

After one hour there are 54 bacteria in the petri dish.

(a) Calculate the value of k to 2 significant figures.

(2)

(b) The petri dish is left on the side in the lab at $15:00$, what time, to the nearest minute, does the number of bacteria in the dish exceed 3000?

(3)

a) $54 = 20e^{60k}$

$\ln\left(\frac{54}{20}\right) = 60k \rightarrow k = \frac{\ln\frac{54}{20}}{60} \approx \underline{\underline{0.017}}$

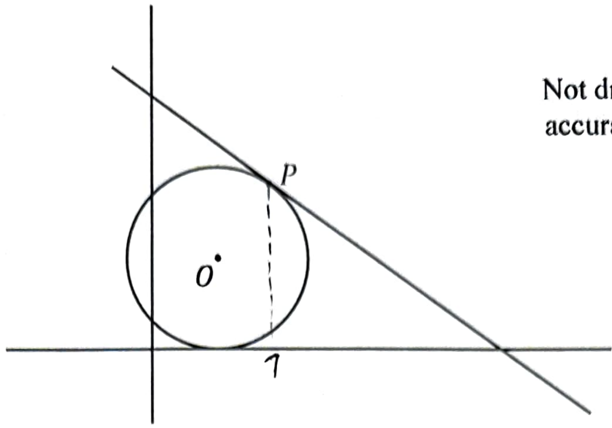
$= 0.0165541\ldots$

b) $3000 = 20e^{kt}$

$\frac{\ln\frac{3000}{20}}{k} = t = 303\text{mins} \longrightarrow \text{Time} = 20:03$

Turn over

10. The circle, C, has centre at O, (3,5)



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The line between O and the point P on the circle has the equation,

$$y = \frac{3}{4}x + \frac{11}{4}$$

At P, $x = 7$

(a) Show that the tangent to C at the point P has equation,

$$y = -\frac{4}{3}x + \frac{52}{3}$$

(3)

(b) Find the equation of the circle, C.

(3)

(c) Find the equation of the second tangent of the circle with the same gradient in the form:

$$y = -\frac{4}{3}x + k \text{ where } k \neq \frac{52}{3}$$

(3)

a) $m_T = -\frac{4}{3} \rightarrow x = 7, \; y = \frac{3}{4}(7) + \frac{11}{4} = 8$

$y - 8 = -\frac{4}{3}(x - 7)$

$y = -\frac{4}{3}x + \frac{28}{3} + 8 = \underline{\underline{-\frac{4}{3}x + \frac{52}{3}}}$

Turn over ▶

Question 10 continued

b) $r = \sqrt{(7-3)^2 + (8-5)^2} = \sqrt{3^2+4^2} = 5$

$(x-3)^2 + (y-5)^2 = 25$

c) $(a,b) \longrightarrow (3,5) \longrightarrow (7,8)$

$\xleftarrow[y-3]{x-4}$ $\xrightarrow[y+3]{x+4}$

$(-1, 2)$

$y-2 = -\frac{4}{3}x - \frac{4}{3}$

$y = -\frac{4}{3}x + \frac{2}{3}$

(Total for Question 10 is 9 marks)

11. Prime integers, p_i , are labelled in ascending order such that,

$$p_1 = 2 , p_2 = 3 , \ldots \ p_5 = 11 \ldots$$

Show that there is no largest prime number.

(6)

Contradiction

Assume that there is a largest prime number P_N

$\boxed{a} = (P_1 \times P_2 \ldots P_N) + 1$

a is larger than P_N, ∴ cannot be prime.

However, a has no prime factors as they all leave remainders of 1

There are no larger primes, ∴ a must be prime, contradicting the assumption

∴ statement is true.

(Total for Question 11 is 6 marks)

12. Show that,

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \equiv 2\operatorname{cosec} 2\theta$$

(4)

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{2}\sin 2\theta} = \frac{1}{\frac{1}{2}}\operatorname{cosec} 2\theta$$

$$= 2\operatorname{cosec} 2\theta$$

(as required)

(Total for Question 12 is 4 marks)

13. **(a)** Show that $(2x+1)$ is a factor of $f(x) = 2x^3 - 3x^2 + 4x + 3$

(3)

(b) Express $(x^3 - 3x^2 - 7x + 2)$ in the form $(x+2)(Ax^2 + Bx + C) + R$

Where A, B, C and R are integer constants to be determined

(4)

a) $2x+1 \overline{) 2x^3 - 3x^2 + 4x + 3}$ quotient $x^2 - 2x + 3$

$2x^3 + x^2$

$0 \quad -4x^2 - 4x$

$-4x^2 - 2x$

$0 \quad -6x + 3$

$6x + 3$

0

remainder $= 0$, ∴ a factor

$2x+1 = 0$

$x = -\frac{1}{2}$

$f(-\frac{1}{2}) = 0$, ∴ a factor

b) $x+2 \overline{) x^3 - 3x^2 - 7x + 2}$ quotient $x^2 - 5x + 3$

$x^3 + 2x^2$

$-5x^2 - 7x$

$-5x^2 - 10x$

$3x + 2$

$3x + 6$

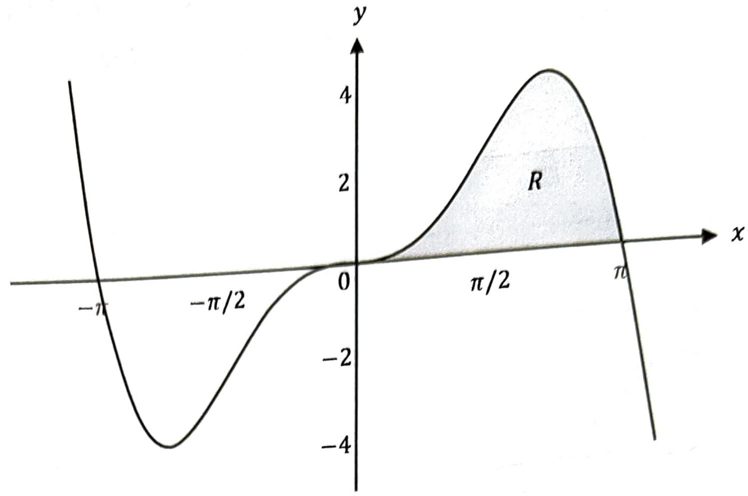
$-4 \leftarrow R$

$$\boxed{(x+2)(x^2 - 5x + 3) - 4}$$

(Total for Question 13 is 7 marks)

14. Consider the graph of

$$f(x) = x^2 \sin x$$



Find the area of the region, R, enclosed between $f(x)$ and the x-axis for $0 \leq x \leq \pi$

(9)

$A_R = \int_0^\pi x^2 \sin x \, dx = -x^2 \cos x - \int 2x \cos x \, dx$

$u = x^2 \rightarrow \frac{du}{dx} = 2x$

$\frac{dv}{dx} = \sin x \rightarrow v = -\cos x$

$2 \times \int x \cos x \, dx \longrightarrow x \sin x - \int \sin x \, dx$

$u = x \rightarrow \frac{du}{dx} = 1 \qquad = x \sin x + \cos x$

$\frac{dv}{dx} = \cos x \rightarrow v = \sin x$

$\left[-x^2 \cos x - 2(x \sin x + \cos x) \right]_0^\pi$

$= (-\pi)^2 \times \cos \pi - 2\left(\pi \times 0 + 2(-1)\right) - -2$

$= \underline{\pi^2 - 4}$

Question	Paper 1 Mark Scheme	Marks
1	$f'(x) = 6x^2 - 6x - 36 = 0$	M1
	$(x-3)(x+2) = 0$	M1
	Turning points at $x = 3$ and $x = -2$	M1
	Hence $f'(x) = 0$ when $x = 3$ or $x = -2$, $(3, -79), (-2, 46)$	M1
	$f''(x) = 12x - 6$	M1
	$x = 3$: $f''(3) = 30 > 0 \Rightarrow$ minimum	A1
	$x = -2$: $f''(-2) = -30 < 0 \Rightarrow$ maximum	A1
		(7)
2	In the limit of the small angle approximation when $x \to 0$,	M1
	$\sin x \approx x \qquad \tan x \approx x \qquad \cos x \approx 1 - \frac{x^2}{2}$	
	$\frac{\sin x \cos 2x - \sin x}{x^2 \tan 4x} \approx \frac{x\left(1 - \frac{(2x)^2}{2} - 1\right)}{x^2(4x)}$	M1
	Begins process to simplify e.g. $\frac{x\left(1 - \frac{(2x)^2}{2} - 1\right)}{x^2(4x)} = \frac{\left(-\frac{4x^3}{2}\right)}{4x^3}$	M1
	$-\frac{1}{2}$	B1
		(4)
3(a)	Cosine rule states, $\cos\theta = \frac{(b^2+c^2-a^2)}{2bc}$ hence,	M1
	$\theta = \arccos\left(\frac{(r^2+r^2-(1.5r)^2)}{2(r)(r)}\right)$	
	$\theta = \arccos\left(-\frac{1}{8}\right)$	M1
	$\theta = 1.696$ rad	B1
	Allow alternative methods	**(3)**
3(b)	Area of a sector, $A = \frac{1}{2}r^2\theta$	M1
	$A = \frac{1}{2}r^2 \times \arccos\left(-\frac{1}{8}\right) = 0.8481 \ldots r^2$	
	Thus, $r^2 = \frac{16\pi}{0.8481 \ldots}$	
	$r = \pm\sqrt{\frac{16\pi}{0.8481 \ldots}} = +7.6987 \ldots$ cm	M1
	(reject negative solution as r represents a real physical length)	

Question	Paper 1 Mark Scheme	Marks
3(b) cont.	Length of arc , $s = r\theta = 7.6987 \ldots \times 1.696 \ldots$ $s = 13.058$ cm Perimeter of sector AOB , $P = r + r + s$ $P = 28.45559408$ $P = 28.46$ cm	M1 B1 **(4)**
4(a)	Rearranging $x = 2\sin t - 1, \quad \sin t = \frac{x+1}{2}$ and $y = 4 + 2\cos 2t, \quad \cos 2t = \frac{y-4}{2}$ By use of the double angle formula : $\cos 2x = 1 - 2\sin^2 x$ $\frac{y-4}{2} = 1 - 2\left(\frac{x+1}{2}\right)^2$ Process to simplify and rearrange resulting in given form $\frac{y-4}{2} = 1 - \frac{1}{2}(x+1)^2,$ $y - 4 = 2 - (x+1)^2,$ $y = 6 - (x+1)^2$	M1 M1 A1 **(3)**
4(b)	Equating the cartesian equation of the curve and line, $\frac{1}{3}x + k = 6 - (x+1)^2$ Process to rearrange to quadratic $\frac{1}{3}x + k = 6 - x^2 - 2x - 1$ $x^2 + \frac{7}{3}x + (k-5) = 0$ Hence to have two real solutions the discriminant of the quadratic must be greater than zero i.e. $b^2 - 4ac > 0$ $\frac{49}{9} - 4(k-5) > 0$ Begins process of rearranging for k $\frac{229}{9} - 4k > 0,$ $k < \frac{229}{36}$	M1 M1 M1 M1 B1 **(5)**

Question	Paper 1 Mark Scheme	Marks
5(a)	$\sqrt{\frac{4-x}{1+3x}} = (4-x)^{\frac{1}{2}}(1+3x)^{-\frac{1}{2}}$ $(4-x)^{\frac{1}{2}} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} \approx 2\left(1+\left(\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)^2}{2!}\right)$ $\approx 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2\right)$ $(1+3x)^{-\frac{1}{2}} \approx 1+\left(-\frac{1}{2}\right)(3x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(3x)^2}{2!} \approx 1-\frac{3}{2}x+\frac{27}{8}x^2$ $2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}(1+3x)^{-\frac{1}{2}} \approx 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2\right)\left(1-\frac{3}{2}x+\frac{27}{8}x^2\right)$ $= 2\left(1-\frac{3}{2}x+\frac{27}{8}x^2-\frac{1}{8}x+\frac{3}{16}x^2-\frac{1}{128}x^2+O(x^3)\right) = 2-\frac{13}{4}x+\frac{455}{64}x^2$	M1 M1 M1 M1 M1 B1 **(6)**
5(b)	$\frac{4-x}{1+3x} = 3$ $4-x = 3+9x$ $1 = 10x$ Hence selecting $x = \frac{1}{10}$ will approximate the value of $\sqrt{3}$ $\sqrt{3} \approx 2-\frac{13}{4}\left(\frac{1}{10}\right)+\frac{455}{64}\left(\frac{1}{10}\right)^2 \approx \frac{447}{256}$	M1 M1 A1 **(3)**
5(c)	As each of the binomial expansions of the form $(1+a)^n$ have powers that are not positive integers. The resultant series expansion is infinite and only valid under the condition $-1 \le a \le 1$ Hence in this instance, the two expansions are valid when, $-1 \le -\frac{x}{4} \le 1, \quad -1 \le 3x \le 1$ Hence the combined expansion is only valid when $-\frac{1}{3} \le x \le \frac{1}{3}$ $(\lvert 3x \rvert \le 1)$ Thus $x = \frac{11}{6}$ can not be used.	B1 B1 **(2)**
6(a)	$\tan^2\theta = 9$ $\tan\theta = \pm 3$ $\theta = -1.25, 1.25, 1.89, -1.89$	M1 B1 **(2)**

Question	Paper 1 Mark Scheme	Marks
6(b)	$4\cos^2 t = 3 - \sin t$ Using $\sin^2 x + \cos^2 x = 1$ $4(1 - \sin^2 t) = 3 - \sin t$ $4 - 4\sin^2 t = 3 - \sin t$ $4\sin^2 t - \sin t - 1 = 0$ Attempting to solve quadratic e.g. $\sin t = \frac{1 \pm \sqrt{(-1)^2 - (4\times4\times-1)}}{8} = \frac{1\pm\sqrt{17}}{8}$ $t = 0.70, 3.54, 2.45, 5.88$	M1 M1 M1 B1 **(4)**
7(a)	The plan follows a geometric sequence of the form $a_n = ar^{n-1}$, where a is the value the first term, r is the common ratio and where n is the number of days. Hence using $a = 10$ and $r = 1.04$ $a_n = 10(1.04)^{7-1}$ Hence by day 7 Lewis will run $10(1.04)^6 = 12.65 \ldots$ miles	M1 M1 B1 **(3)**
7(b)	States or attempts to use, $S_n = \frac{a(1-r^n)}{1-r}$ Correct substitution $s_{21} = \frac{10(1-1.04^{21})}{(1-1.04)}$ $s_{21} = 319.6920172$ Hence the total distance run after 3 weeks = 320 miles (to the nearest mile)	M1 M1 M1 B1 **(4)**
8(a)	$\frac{dy}{dx} = xe^x + e^x$ Gradient of tangent at $(1, e) = 2e$ Gradient of normal at $(1, e) = -\frac{1}{2e}$ Process to find equation $y - e = -\frac{1}{2e}(x-1)$ $y = -\frac{1}{2e}x + \frac{1}{2e} + e$	M1 M1 M1 M1 B1 **(5)**

Question	Paper 1 Mark Scheme	Marks
8(b)	At $y = 0,\ x = 1 + 2e^2$ Area of the triangle formed between point P and the x-axis $= \frac{1}{2} \times 2e^2 \times e\ = e^3$ $\int_0^1 xe^x\,dx\ = [xe^x] - \int_0^1 e^x dx\ = [xe^x - e^x]_0^1$ $[xe^x - e^x]_0^1 = e - e - (0 - 1) = 1$ So total area of $R = 1 + e^3$	M1 M1 M1 M1 B1 **(5)**
9(a)	$54 = 20e^{k\times 60}$ $k = \frac{1}{60} \times \ln\left(\frac{54}{20}\right) = 0.01655419622$ $k = 0.017$ (2 sf)	M1 B1 **(2)**
9(b)	$t = \frac{\ln\left(\frac{B}{a}\right)}{k}$ $\frac{\ln\left(\frac{3000}{20}\right)}{0.01655419622} = 303$ minutes $20{:}03$ (Also give (3) marks for answer $19{:}55$ using $k = 0.017$)	M1 M1 B1 **(3)**
10(a)	The line OP is perpendicular to the tangent, so the tangent at C must have gradient $-\frac{4}{3}$ Beginning of process to find equation When $x = 7, y = \frac{3}{4}(7) + \frac{11}{4} = 8$ Valid workings resulting in, $y = -\frac{4}{3}x + \frac{52}{3}$	M1 M1 A1 **(3)**
10(b)	The distance between $(3,5)$ and $(7,8)$ is the radius, which is $\sqrt{(7-3)^2 + (8-5)^2} = \sqrt{4^2 + 3^2} = 5$ $(x-3)^2 + (y-5)^2 = 25$ Award 1 mark for one side of eqn incorrect	M1 B2 **(3)**

Question	Paper 1 Mark Scheme	Marks
10(c)	Identifies the tangent would be at $(-1,2)$ $y-2=-\frac{4}{3}(x+1)$ $y=-\frac{4}{3}x+\frac{2}{3}$	M1 M1 B1 **(3)**
11	Assuming the contrary to the statement, let the largest prime number be p_N. Let the number q be formed the product of all prime numbers up to p_N and the addition of 1 to that product, i.e. $q=(p_1 \times p_2 \times p_5 \dots \times p_N)+1$ The assumption states that p_N is the largest prime number, thus no number greater than the value of p_N can have any prime factors. The quotient of q with any of the prime numbers, p_i takes the form $$\frac{q}{p_i}=\frac{(p_1 \times p_2 \times p_5 \dots \times p_N)+1}{p_i}=X_i+\frac{1}{p_i}$$ Hence the quotient $\frac{q}{p_i}$ can not be an integer since X_i is an integer and $\frac{1}{p_i}$ is not. Hence p_i is not a devisor of q. As q is not divisible by any of the set of primes p_i, q must itself be prime. Hence there is a contradiction as $q>p_N$ and prime hence it follows there is no largest prime number.	M1 M1 M1 M1 M1 A1 **(6)**
12	Valid start to process e.g. $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}$ $=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ As $\sin^2\theta+\cos^2\theta=1$ $=\frac{1}{\cos\theta\sin\theta}$ $\sin 2\theta=2\cos\theta\sin\theta$ $=\frac{1}{\frac{1}{2}\sin 2\theta}=2\operatorname{cosec} 2\theta$	M1 M1 M1 A1 **(4)**
13(a)	States or attempts to use the factor theorem $f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{8}\right)-3\left(\frac{1}{4}\right)+4\left(-\frac{1}{2}\right)+3=0$ Since $f\left(-\frac{1}{2}\right)=0$, $(2x+1)$ is a factor of $f(x)$	M1 M1 A1 **(3)**

Question	Paper 1 Mark Scheme	Marks
13(b)	Valid start to process e.g. $(x+2)(Ax^2+Bx+C)+R = Ax^3+Bx^2+Cx+2Ax^2+2Bx+2C+R$	M1
	Valid process to find one constant e.g. $Ax^3 = x^3 \Rightarrow A = 1$	M1
	Continuation of process to find another constant e.g. $Bx^2+2Ax^2 = -3x^2 \Rightarrow B+2(1) = -3 \Rightarrow B = -5$	M1
	$(x^3-3x^2-7x+2) = (x+2)(x^2-5x+3)-4$	B1
		(4)
14	States or attempt to use integration by parts	M1
	Let $u = x^2$, $u' = 2x$ $v = -\cos x$, $v' = \sin x$	M1
	$\int_0^{\pi} x^2 \sin x \, dx = [-x^2\cos x]_0^{\pi} - \int_0^{\pi} -2x\cos x \, dx$	M1
	$= [-x^2\cos x]_0^{\pi} + 2\int_0^{\pi} x\cos x \, dx$	B1
	Let $u = x$, $u' = 1$ $v = \sin x$, $v' = \cos x$	M1
	$= [-x^2\cos x]_0^{\pi} + [2x\sin x]_0^{\pi} + 2\int_0^{\pi} \sin x \, dx$	M1
	$= [-x^2\cos x + 2x\sin x + 2\cos x]_0^{\pi}$	B1
	$= [(-\pi^2 \times (-1) + 2\pi \times 0 + 2(-1)) - (-0 \times 1 + 0 + 2)]$	M1
	$= \pi^2 - 4$	B1
		(9)