



# OCR A Level Physics



Your notes

## Simple Harmonic Oscillations

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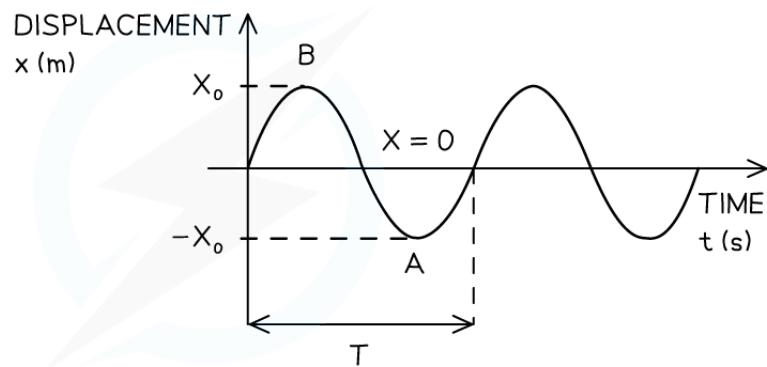
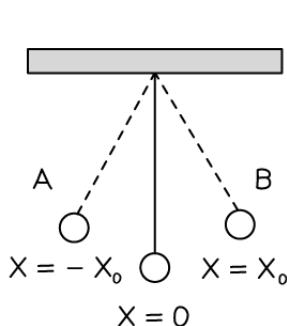


Your notes

## Describing Oscillations

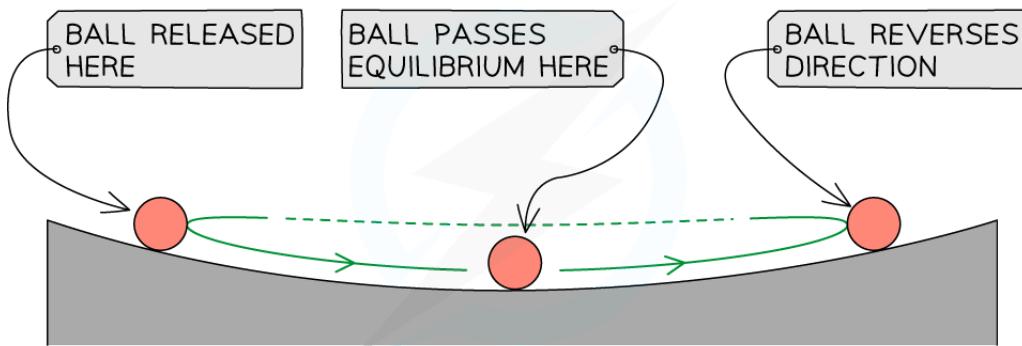
- An **oscillation** is defined as follows:

The repetitive variation with time  $t$  of the displacement  $x$  of an object about the equilibrium position ( $x = 0$ )

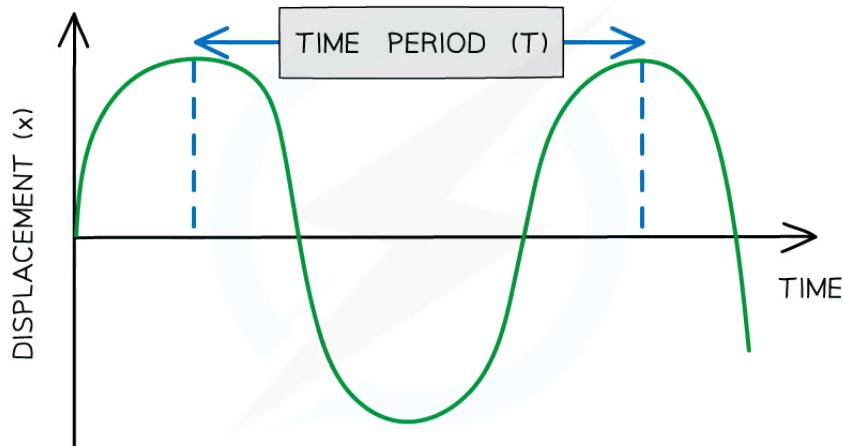

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A pendulum oscillates between A and B. On a displacement-time graph, the oscillating motion of the pendulum is represented by a wave, with an amplitude equal to  $x_0$

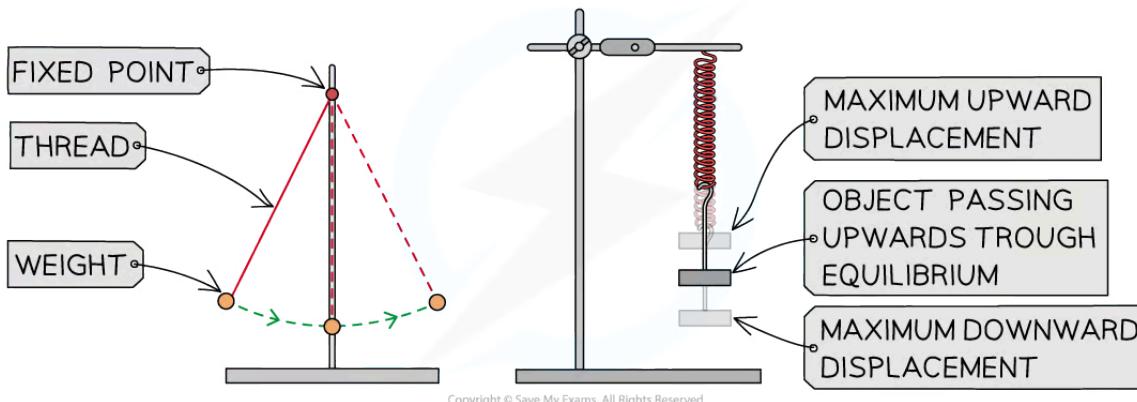
- Displacement (x)** of a wave is the distance of a point on the wave from its equilibrium position
  - It is a vector quantity; it can be positive or negative


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- **Period ( $T$ )** or time period, is the **time interval** for one complete oscillation and it is measured in seconds (s)
  - If the oscillations have a **constant period**, they are said to be **isochronous**

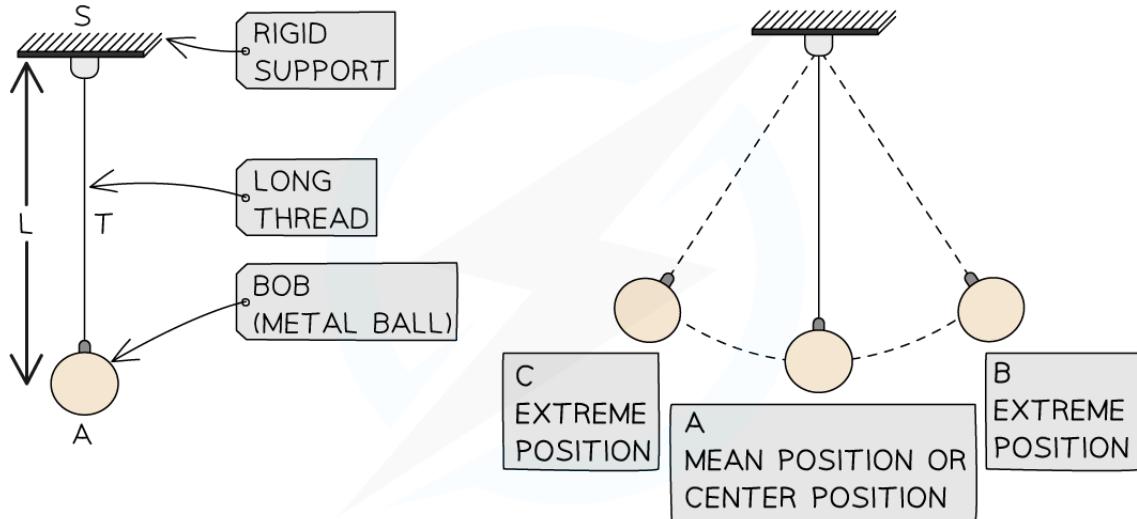

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**Diagram showing the time period of a wave oscillation**



**A diagram showing one oscillation for both types of SHM**

- **Amplitude ( $x_0$ )** is the **maximum value of the displacement** on either side of the equilibrium position is known as the **amplitude** of the oscillation



The diagram shows the amplitude of an oscillation is measured from the central mean position to an extreme position.

- Frequency ( $f$ ) is the **number of oscillations per second** and it is measured in hertz (Hz)
- The frequency and the period of the oscillations are related by the following equation:

$$f = \frac{1}{T}$$

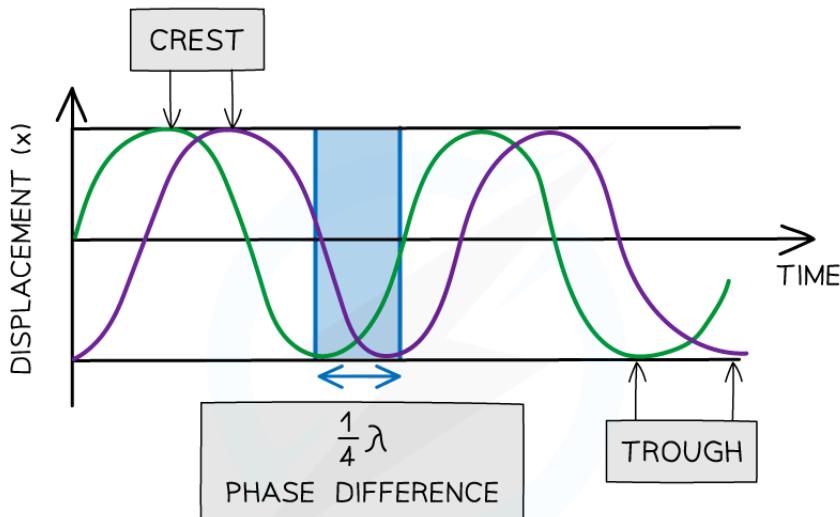
**FREQUENCY (Hz)**

**PERIOD (S)**

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- The **phase difference** between two waves is a measure of **how much a point or a wave is in front or behind another**
- This can be found from the relative positive of the crests or troughs of two different waves of the same frequency
  - When the crests or troughs are aligned, the waves are **in phase**
  - When the crest of one wave aligns with the trough of another, they are in **antiphase**

- The diagram below shows the green wave **leads** the purple wave by  $\frac{1}{4}\lambda$



$$\text{FRACTION OF } \lambda = \frac{1}{4} \lambda \quad \text{FRACTION } \times 360^\circ = \frac{1}{4} \times 360^\circ = 90^\circ \quad \text{FRACTION } \times 2\pi = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$$

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### Two waves $\frac{1}{4}\lambda$ out of phase

- In contrast, the purple wave is said to **lag** behind the green wave by  $\frac{1}{4}\lambda$
- Phase difference is measured in **fractions of a wavelength, degrees or radians**
- The phase difference can be calculated from two different points on the same wave or the same point on two different waves
- The phase difference between two points can be described as:
  - In phase** is  $360^\circ$  or  $2\pi$  radians
  - In anti-phase** is  $180^\circ$  or  $\pi$  radians



### Worked Example

A child on a swing performs 0.2 oscillations per second. Calculate the period of the child's oscillations.

**Answer:**

**Step 1: Write down the frequency of the child's oscillations**

$$f = 0.2 \text{ Hz}$$



**Step 2: Write down the relationship between the period  $T$  and the frequency  $f$**

$$T = \frac{1}{f}$$

**Step 3: Substitute the value of the frequency into the above equation and calculate the period**

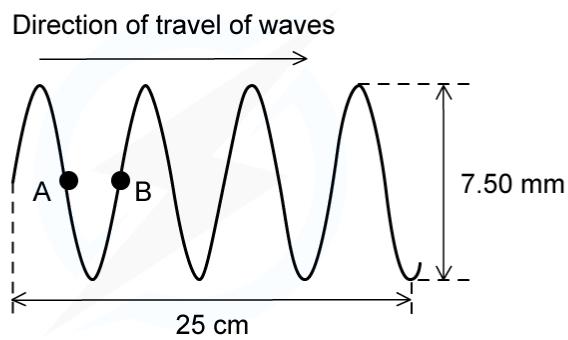
$$T = \frac{1}{0.2}$$

$$T = 5 \text{ s}$$



### Worked Example

Plane waves on the surface of water at a particular instant are represented by the diagram below.



The waves have a frequency of 2.5 Hz. Determine:

- The amplitude
- The wavelength
- The phase difference between points A and B

Answer:



Your notes

A. THE AMPLITUDE

MAXIMUM DISPLACEMENT FROM THE EQUILIBRIUM POSITION

$$7.50 \text{ mm} \div 2 = 3.75 \text{ mm}$$

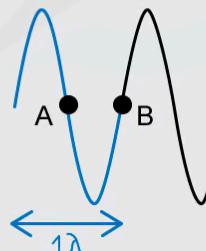
B. THE WAVELENGTH

DISTANCE BETWEEN POINTS ON SUCCESSIVE OSCILLATIONS  
OF THE WAVE THAT ARE IN PHASE

FROM DIAGRAM:  $25\text{cm} = 3\frac{3}{4}$  WAVELENGTHS

$$1\lambda = 25 \text{ cm} \div 3\frac{3}{4} = 6.67 \text{ cm}$$

C. THE PHASE DIFFERENCE BETWEEN POINTS A AND B



POINTS A AND B HAVE  $\frac{1}{2}\lambda$  DIFFERENCE =  $\frac{1}{2} \times 360^\circ = 180^\circ$

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### Examiner Tips and Tricks

When labelling the wavelength and time period on a diagram:

- Make sure that your arrows go from the **very top** of a wave to the very top of the next one
- If your arrow is too short, you will lose marks

- The same goes for labelling amplitude, don't draw an arrow from the bottom to the top of the wave, this will lose you marks too.



Your notes



Your notes

## Angular Frequency

- ## Angular Frequency
- Angular frequency,  $\omega$ , in oscillations is equivalent to angular velocity in circular motion
  - It can be defined as:

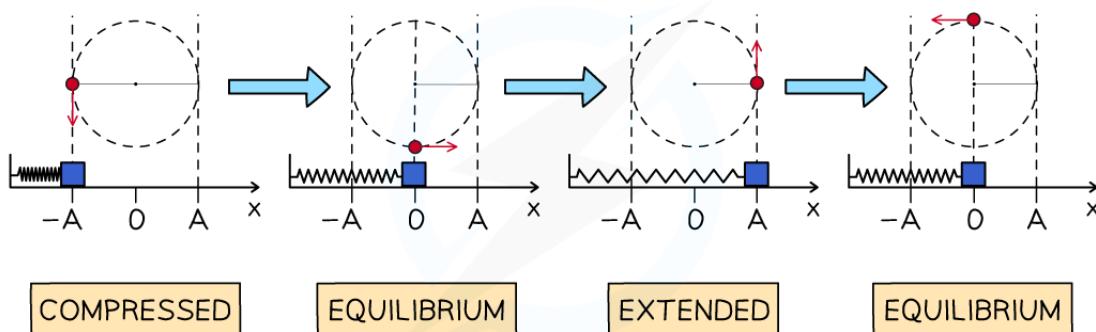
The rate of change of angular displacement with respect to time

- This can be written as an equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Where:

- $\omega$  = angular frequency ( $\text{rad s}^{-1}$ )
- $2\pi$  = circumference of a circle
- $T$  = time period (s)
- $f$  = frequency of oscillation (Hz)



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### Worked Example

A cuckoo in a cuckoo clock emerges from a fully compressed position to a fully extended position in 1.5 seconds.

Calculate the angular frequency of the cuckoo as it emerges from the clock.

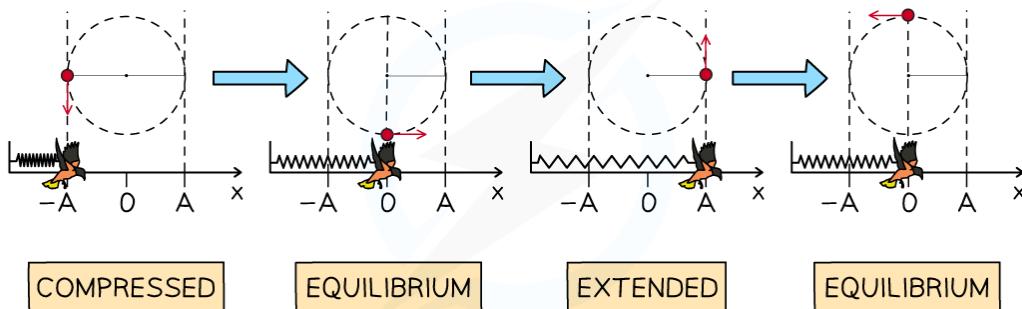


Your notes

**Answer:**

### Step 1: Consider the motion of the cuckoo

- The cuckoo goes from being fully compressed to fully extended which means that it travels for an **angular displacement** of half a circle and not a full circle
- So, instead of  $2\pi$  the distance will be  $\pi$



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### Step 2: Substitute into the equation for angular velocity and time period

$$\omega = \frac{2\pi}{T} = \frac{\pi}{1.5} = 2.09 \text{ rad s}^{-1}$$

### Step 3: State the final answer

- The angular frequency of the cuckoo as it emerges from the clock is  $2.1 \text{ rad s}^{-1}$  (2 s.f.)



Your notes

## Conditions for Simple Harmonic Motion

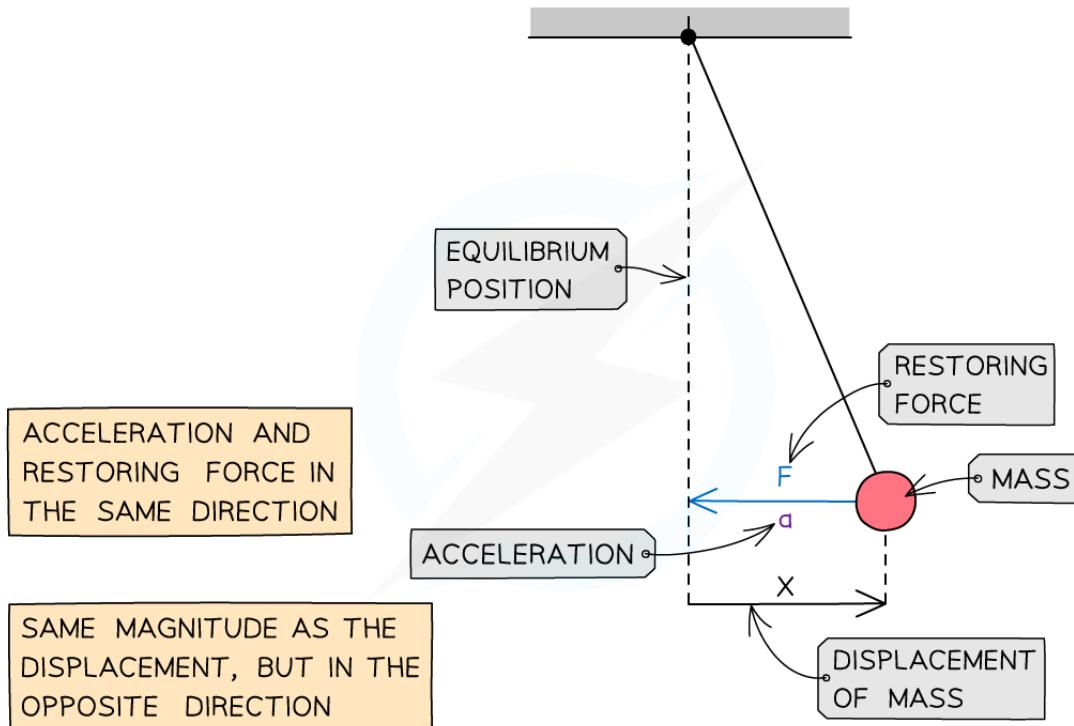
### Simple Harmonic Motion

- Simple harmonic motion (SHM) is a specific type of oscillation
- An oscillation is said to be SHM when:
 

**The acceleration of a body is proportional to its displacement but acts in the opposite direction**
- Acceleration  $a$  and displacement  $x$  can be represented by the defining equation of SHM:

$$a \propto -x$$

- The two **conditions** required for an object to be simple harmonic motion are therefore:
  - The acceleration is **proportional** to the displacement
  - The acceleration is in the **opposite direction** to the displacement


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**Force, acceleration and displacement of a pendulum in SHM**



## Worked Example

Explain why a person jumping on a trampoline is not an example of simple harmonic motion.

**Answer:**

### Step 1: Recall the conditions for simple harmonic motion

- The conditions required for SHM:
  - The restoring force/acceleration is **proportional** to the displacement
  - The restoring force/acceleration is in the **opposite direction** to the displacement

### Step 2: Consider the forces in the scenario given

- When the person is not in contact with the trampoline, the restoring force is equal to their weight, which is constant
- The value of their weight does not change, even if they jump higher (increase displacement)

### Step 3: Write a concluding sentence

- The restoring force on the person is not proportional to their distance from the equilibrium position, therefore, this scenario does not fulfil the conditions for SHM



Your notes



Your notes

## Time Period & Frequency

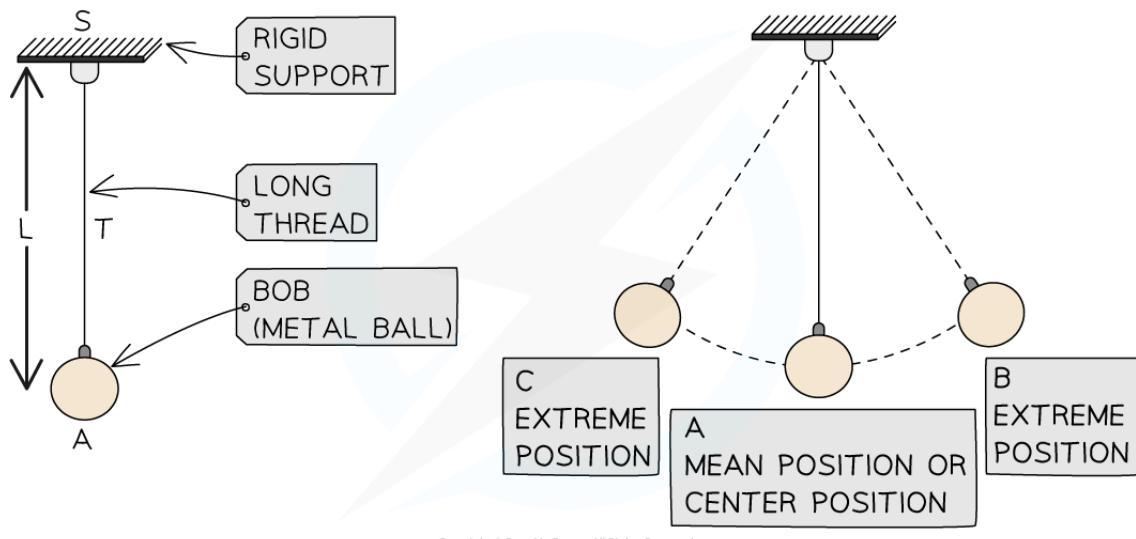
# Period & Amplitude of a Simple Harmonic Oscillator

- An oscillator in simple harmonic motion is an **isochronous** oscillation, which means:

**The period of oscillation is independent of the amplitude**

- Therefore, an object is said to perform **simple harmonic oscillations** when all of the following apply:

- The oscillations are isochronous
- There is a central equilibrium point
- The object's displacement, velocity and acceleration change continuously
- There is a **restoring force** always directed towards the equilibrium point
- The magnitude of the restoring force is proportional to the displacement

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## Determining the Period & Frequency of Simple Harmonic Oscillations

### Equipment List



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Apparatus	Purpose
Clamp stand	Used to hold the spring, masses, and pendulum string vertically
Spring	Used to calculate the spring constant and provide the oscillations
Fiducial marker (e.g. Needle)	To mark the equilibrium position
Mass hanger + 50 g masses	To hang from the spring and vary the mass
Stopwatch	To measure the time for a certain number of oscillations
Metre ruler	To check all oscillations have the same amplitude (mass on spring) To measure the length of the pendulum (simple pendulum)
Pendulum bob on 2 m long string	To provide the oscillations for simple pendulum
Wooden Block	To clamp the string in place to create the pivot for the pendulum

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- Resolution of measuring equipment:
  - Stopwatch = 0.01s
  - Metre Ruler = 1mm

## SHM in a Mass-Spring System

- The overall aim of this experiment is to calculate the spring constant of a mass-spring system
- This is done by investigating how the time period of the oscillations varies with the mass
  - This is just one example of how this required practical might be carried out

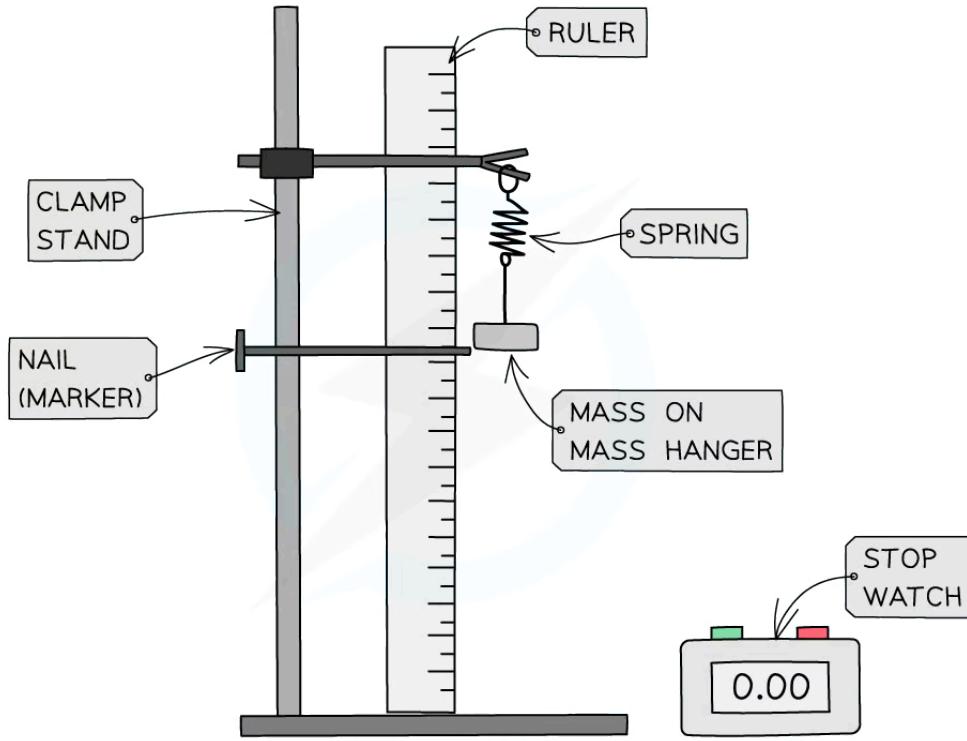
### Variables

- Independent variable = mass,  $m$
- Dependent variable = time period,  $T$
- Control variables:

- Spring constant,  $k$
- Number of oscillations



## Method



### The setup of apparatus to detect oscillations of a mass–spring system

1. Set up the apparatus, with no masses hanging on the holder to begin with (just the 100 g mass attached to it)
  2. Pull the mass hanger vertically downwards between 2–5 cm as measured from the ruler and let go. The mass hanger will begin to oscillate
  3. Start the stopwatch when it passes the nail marker
  4. Stop the stopwatch after 10 complete oscillations and record this time. Divide the time by 10 for the time period (which is the mean)
  5. Add a 50 g mass to the holder and repeat the above between 8–10 readings. Make sure the mass is pulled down by the same length before letting go
- An example table might look like this:

TIME TAKEN FOR 10 OSCILLATIONS	$T_{10}/\text{s}$	$T/\text{s}$	$T^2/\text{s}^2$
0.05			
0.10			
0.15			
0.20			
0.25			
0.30			
0.35			
0.40			

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## Analysing the Results

- The time period of a mass-spring system is given by:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

- Where:

- $T$  = time period (s)
- $m$  = mass (kg)
- $k$  = spring constant ( $\text{N m}^{-1}$ )

- Squaring both sides of the equation gives:

$$T^2 = 4\pi^2 \frac{m}{k}$$



Your notes

- Comparing this to the equation of a straight line:  $y = mx$

- $y = T^2$
- $x = m$
- Gradient =  $4\pi^2/k$

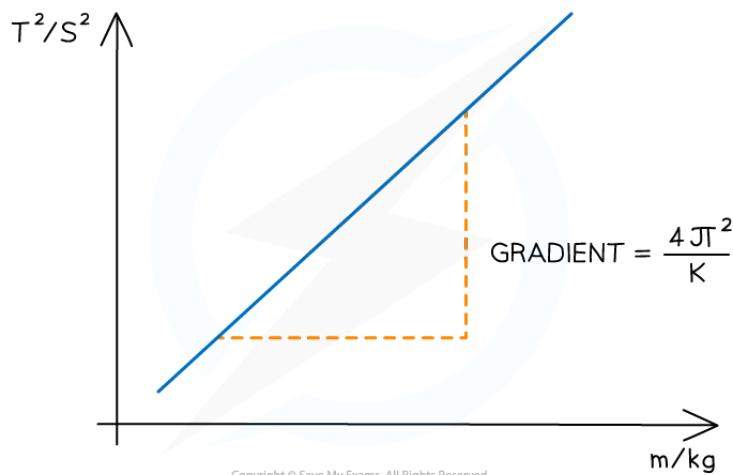
1. Plot a graph of  $T^2$  against  $m$  and draw a line of best fit

2. Calculate the gradient

3. The spring constant,  $k$ , is therefore equal to:

$$k = \frac{4\pi^2}{\text{gradient}}$$

- The spring constant can also be found using the Hooke's Law equation ( $F = -kx$ )
- An experiment can be carried out and  $k$  found from the gradient of a plot of the force  $F$  against the extension  $x$ 
  - The two values could then be compared



## SHM in a Simple Pendulum

- The overall aim of this experiment is to calculate the acceleration due to gravity from a simple pendulum

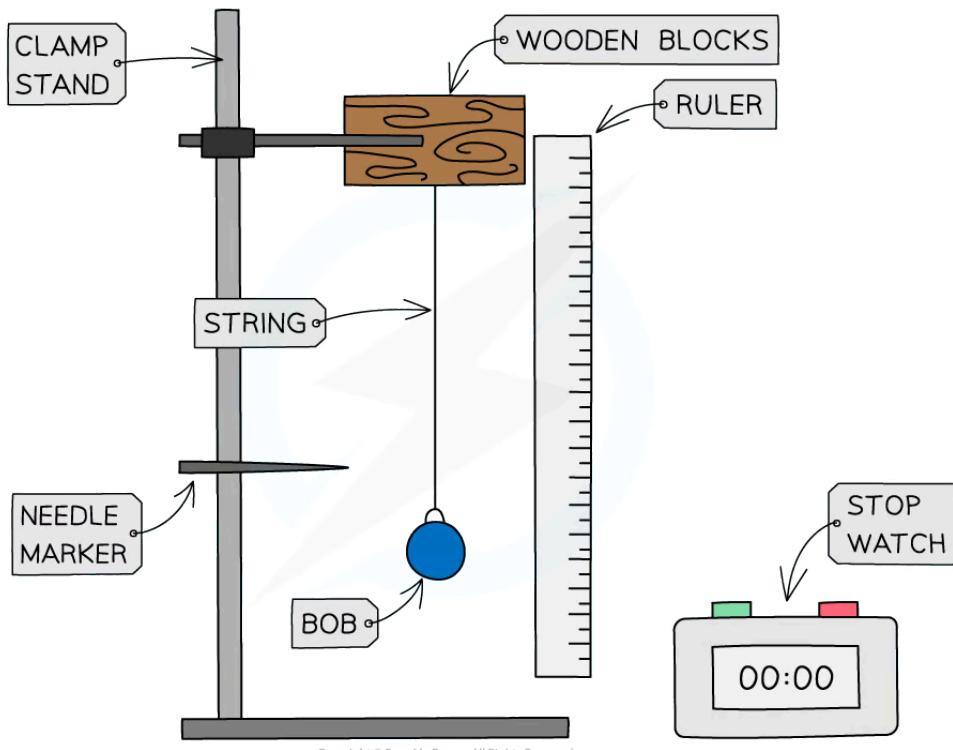
- This is done by investigating how the time period of its oscillations varies with its length
  - This is just one example of how this required practical might be carried out



### Variables

- Independent variable = length,  $L$
- Dependent variable = time period,  $T$
- Control variables:
  - Mass of pendulum bob,  $m$
  - Number of oscillations

### Method

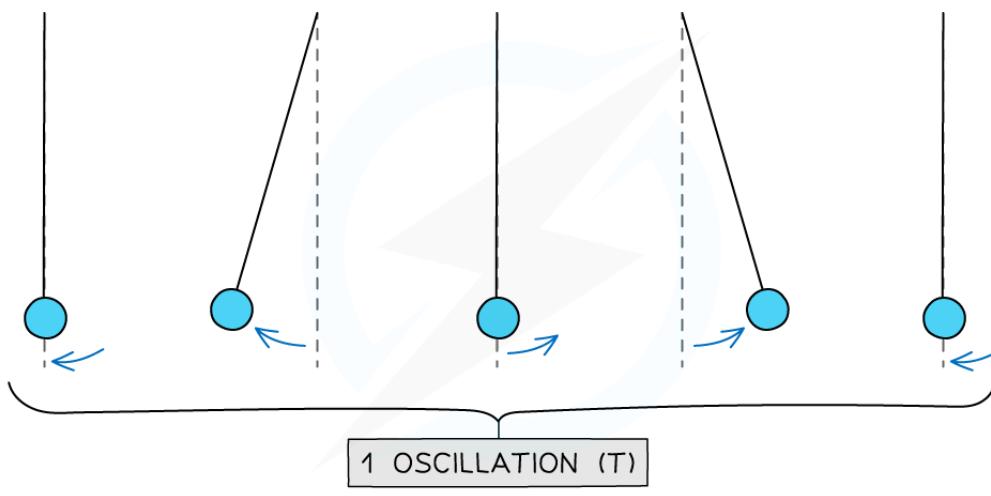


- Set up the apparatus, with the length of the pendulum at 0.2 m
- Make sure the pendulum hangs vertically downwards at equilibrium and inline directly in front of the needle marker
- Pull the pendulum to the side at a very small angle then let go. The pendulum will begin to oscillate



Your notes

4. Start the stopwatch when the pendulum passes the needle marker. One complete oscillation is when the pendulum passes through the equilibrium, then to one amplitude and the other and then back to the equilibrium again (not just from side to side)
5. Stop the stopwatch after 10 complete oscillations and record this time. Divide the time by 10 for the time period (which is the mean)
6. Increase the length of the pendulum by adjusting the string and the wooden block and repeat the above for 8–10 readings. The ruler is used to measure the string and ensure it is measured from the wooden blocks to the centre of mass of the bob. Also, make sure the mass is pulled to the side by the same angle before letting go for the oscillations



- An example table might look like this:



Your notes

LENGTH / m	$T_{10}/s$	$T/s$	$T^2/s^2$
0.2			
0.4			
0.6			
0.8			
1.0			
1.2			
1.4			
1.6			
1.8			
2.0			

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## Analysing the Results

- The time period of a simple pendulum is given by:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

- Where:
  - $T$  = time period (s)
  - $L$  = length of the pendulum (m)
  - $g$  = acceleration due to gravity ( $\text{m s}^{-2}$ )
- Squaring both sides of the equation gives

$$T^2 = 4\pi^2 \frac{L}{g}$$



Your notes

- Comparing this to the equation of a straight line:  $y = mx + c$

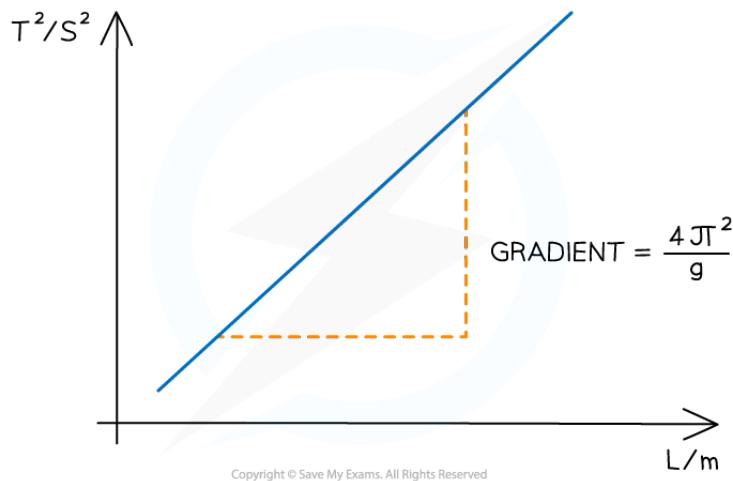
- $y = T^2$
- $x = L$
- gradient  $m = 4\pi^2/g$
- $c = 0$

1. Plot a graph of  $T^2$  against  $L$  and draw a line of best fit

2. Calculate the gradient

3. The acceleration due to gravity is equal to:

$$g = \frac{4\pi^2}{\text{gradient}}$$



## Evaluating the Experiments

### Systematic Errors:

- Reduce parallax error by being at eye level with the marker

**Random Errors:**

- Record the time taken for 10 full oscillations, then divide by 10 for one period, to reduce random errors
- The oscillations may not completely go from side to side, and end up in a circle. Therefore, keep the amplitudes relatively small (only a few cm) and repeat the readings if they do take a different trajectory
- A motion tracker and data logger could provide a more accurate value for the time period and reduce the random errors in starting and stopping the stopwatch (due to reflex times)
- The equation for the time period of a pendulum bob only works for small angles, so make sure the pendulum is not pulled too far out to the side for the oscillations
- For the mass-spring system, the oscillations may not stay completely vertical. Therefore, keep the amplitudes relatively small (only a few cm) and repeat the reading making sure they are vertical



Your notes

**Safety Considerations**

- The suspended masses or pendulum bob could damage the surface if they were to fall. Make sure to keep a soft surface directly below the equipment
- Only pull down the mass and spring system a few centimetres for the oscillations, as larger oscillations could cause the masses to fall off and damage the equipment
- The wooden blocks must be tightly clamped together to hold the string for the pendulum in place, otherwise the pendulum may dislodge during oscillations and fall off

**Worked Example**

A student investigates the relationship between the time period and the mass on a mass-spring system that oscillates with simple harmonic motion. They obtain the following results.



Your notes

Mass/kg	$T_{10}$ /s	T/s
0.05	6.3	0.63
0.10	6.9	0.69
0.15	7.4	0.74
0.20	7.9	0.79
0.25	8.2	0.82
0.30	8.5	0.85
0.35	8.8	0.88
0.40	9.1	0.91

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Calculate the value of the spring constant of the spring used in this experiment

**Answer:**

**Step 1: Complete the table**

Add the extra column  $T^2$  and calculate the values



Your notes

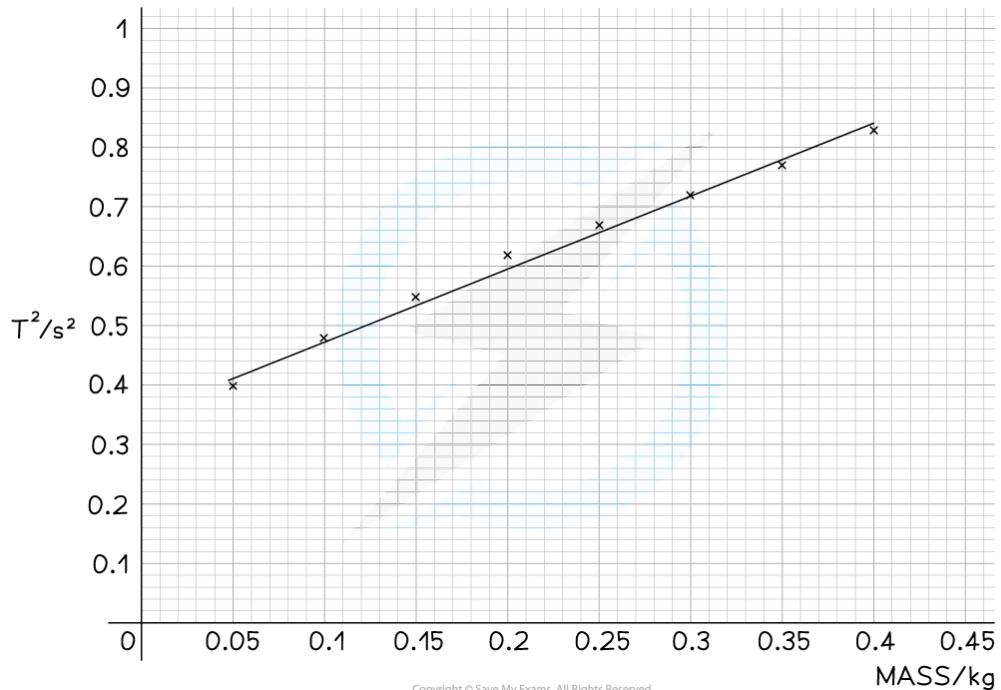
Mass/kg	$T_{10} / \text{s}$	$T/\text{s}$	$T^2/\text{s}^2$
0.05	6.3	0.63	0.40
0.10	6.9	0.69	0.48
0.15	7.4	0.74	0.55
0.20	7.9	0.79	0.62
0.25	8.2	0.82	0.67
0.30	8.5	0.85	0.72
0.35	8.8	0.88	0.77
0.40	9.1	0.91	0.83

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**Step 2: Plot the graph of  $T^2$  against the mass  $m$**



Your notes

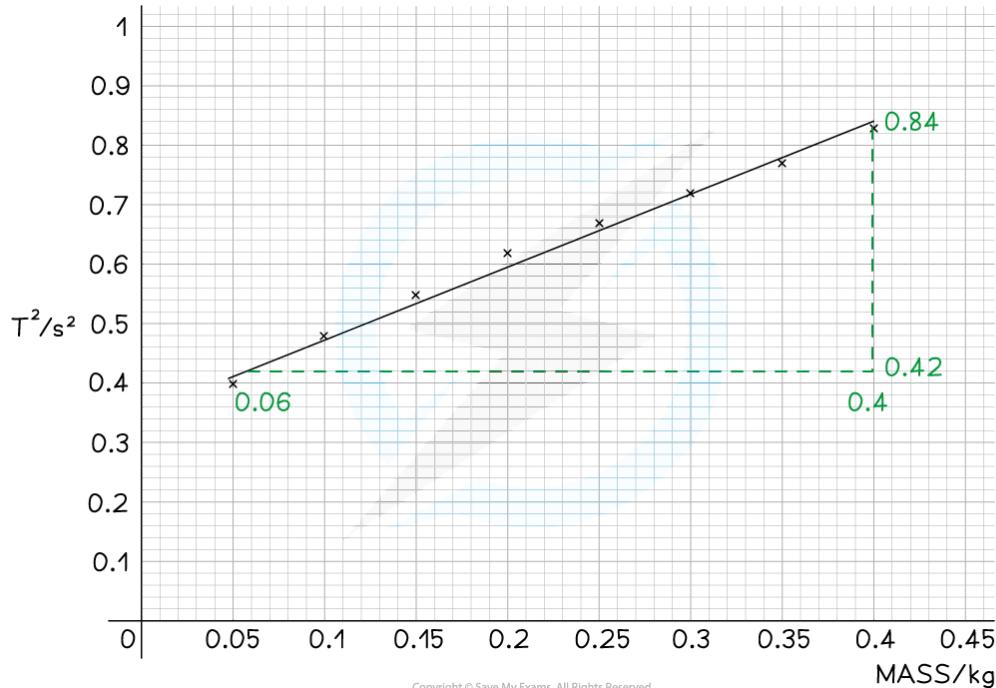


Make sure the axes are properly labelled and the line of best fit is drawn with a ruler

### Step 3: Calculate the gradient of the graph



Your notes



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The gradient is calculated by:

$$\text{gradient} = \frac{0.84 - 0.42}{0.4 - 0.06} = 1.23529$$

#### Step 4: Calculate the spring constant, $k$

$$k = \frac{4\pi^2}{\text{gradient}} = \frac{4\pi^2}{1.23529} = 32.0 \text{ N m}^{-1}$$

## Acceleration & Displacement



Your notes

# Acceleration & Displacement of an Oscillator

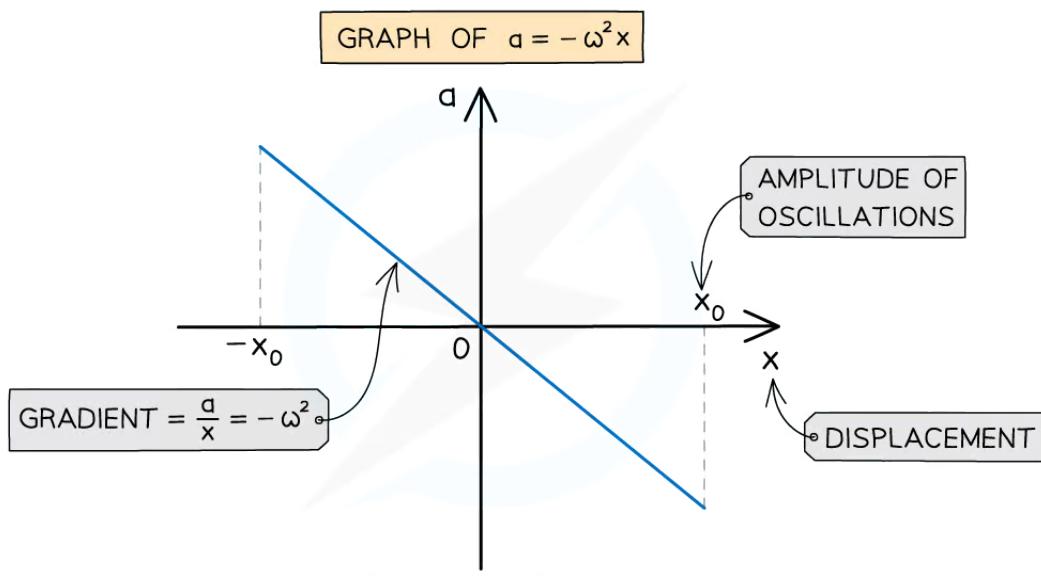
- The acceleration of an object oscillating in simple harmonic motion is:

$$a = -\omega^2 x$$

- Where:

- $a$  = acceleration ( $\text{m s}^{-2}$ )
- $\omega$  = angular frequency ( $\text{rad s}^{-1}$ )
- $x$  = displacement (m)

- This is used to find the acceleration of an object in SHM with a particular angular frequency  $\omega$  at a specific displacement  $x$
- The equation demonstrates:
  - The acceleration reaches its maximum value when the displacement is at a maximum ie.  $x = x_0$  (amplitude)
  - The minus sign shows that when the object is displaced to the **right**, the direction of the acceleration is to the **left**





Your notes

**The acceleration of an object in SHM is directly proportional to the negative displacement**

- The graph of acceleration against displacement is a straight line through the origin sloping downwards (similar to  $y = -x$ )
- Key features of the graph:
  - The gradient is equal to  $-\omega^2$
  - The maximum and minimum displacement  $x$  values are the amplitudes  $-x_0$  and  $+x_0$
- A solution to the SHM acceleration equation is the displacement equation:

$$x = x_0 \sin(\omega t)$$

- Where:
  - $x$  = displacement (m)
  - $x_0$  = amplitude (m)
  - $t$  = time (s)
- This equation can be used to find the position of an object in SHM with a particular angular frequency and amplitude at a moment in time
  - **Note:** This version of the equation is only relevant when an object begins oscillating from the equilibrium position ( $x = 0$  at  $t = 0$ )
- The displacement will be at its maximum when  $\sin(\omega t)$  equals 1 or  $-1$ , when  $x = x_0$
- If an object is oscillating from its amplitude position ( $x = x_0$  or  $x = -x_0$  at  $t = 0$ ) then the displacement equation will be:

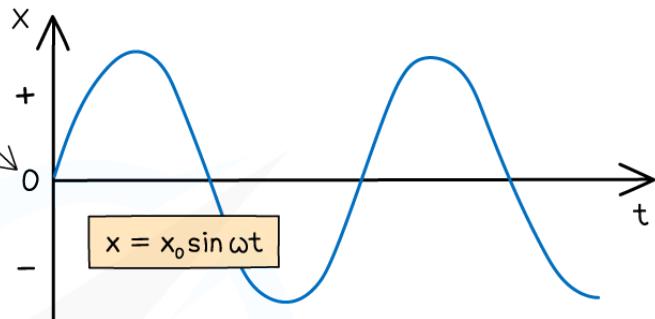
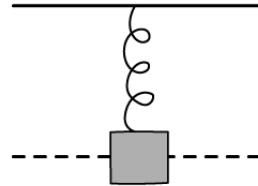
$$x = x_0 \cos(\omega t)$$

- This is because the cosine graph starts at a maximum, whilst the sine graph starts at 0

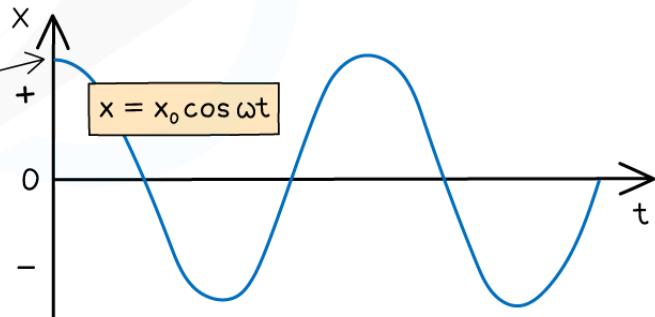
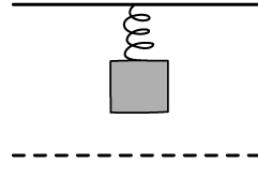


Your notes

MASS ON SPRING STARTS OSCILLATING AT  $t=0$  AT THE EQUILIBRIUM



MASS ON SPRING STARTS OSCILLATING AT  $t=0$  AT MAXIMUM DISPLACEMENT



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**These two graphs represent the same SHM. The difference is the starting position**



## Worked Example

A mass of 55 g is suspended from a fixed point by means of a spring. The stationary mass is pulled vertically downwards through a distance of 4.3 cm and then released at  $t = 0$ .

The mass is observed to perform simple harmonic motion with a period of 0.8 s.

Calculate the displacement  $x$  in cm of the mass at time  $t = 0.3$  s.

**Answer:**

**Step 1: Write down the SHM displacement equation**

- Since the mass is released at  $t = 0$  at its maximum displacement, the displacement equation will be with the cosine function:

$$x = x_0 \cos(\omega t)$$

**Step 2: Calculate angular frequency**

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85 \text{ rad s}^{-1}$$



Your notes

- Remember to use the value of the time period given, not the time where you are calculating the displacement from

**Step 3: Substitute values into the displacement equation**

$$x = 4.3 \cos(7.85 \times 0.3) = -3.0369\dots = -3.0 \text{ cm (2 s.f.)}$$

- Make sure the calculator is in **radians mode**
  - The negative value means the mass is 3.0 cm on the opposite side of the equilibrium position to where it started (3.0 cm above it)

**Examiner Tips and Tricks**

Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative, you could lose a mark if not!

Also, remember that your calculator must be in **radians** mode when using the cosine and sine functions. This is because the angular frequency  $\omega$  is calculated in  $\text{rad s}^{-1}$ , **not** degrees.

You often have to convert between time period  $T$ , frequency  $f$  and angular frequency  $\omega$  for many exam questions – so make sure you revise the equations relating to these.

## Velocity



Your notes

# Velocity of an Oscillator

- The velocity of an object in simple harmonic motion varies as it oscillates back and forth
  - Since velocity is a **vector**, the velocity of the oscillator is its speed in a certain direction
- The **maximum velocity** of an oscillator is at the **equilibrium position** i.e. when its displacement is 0 ( $x = 0$ )
- The velocity of an oscillator in SHM is defined by:

$$v = v_0 \cos(\omega t)$$

- Where:

- $v$  = velocity ( $\text{m s}^{-1}$ )
- $v_0$  = maximum velocity ( $\text{m s}^{-1}$ )
- $\omega$  = angular frequency ( $\text{rad s}^{-1}$ )
- $t$  = time (s)
- This is a cosine function if the object starts oscillating from the equilibrium position ( $x = 0$  when  $t = 0$ )
- How the velocity  $v$  changes with the oscillator's displacement  $x$  is defined by:

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

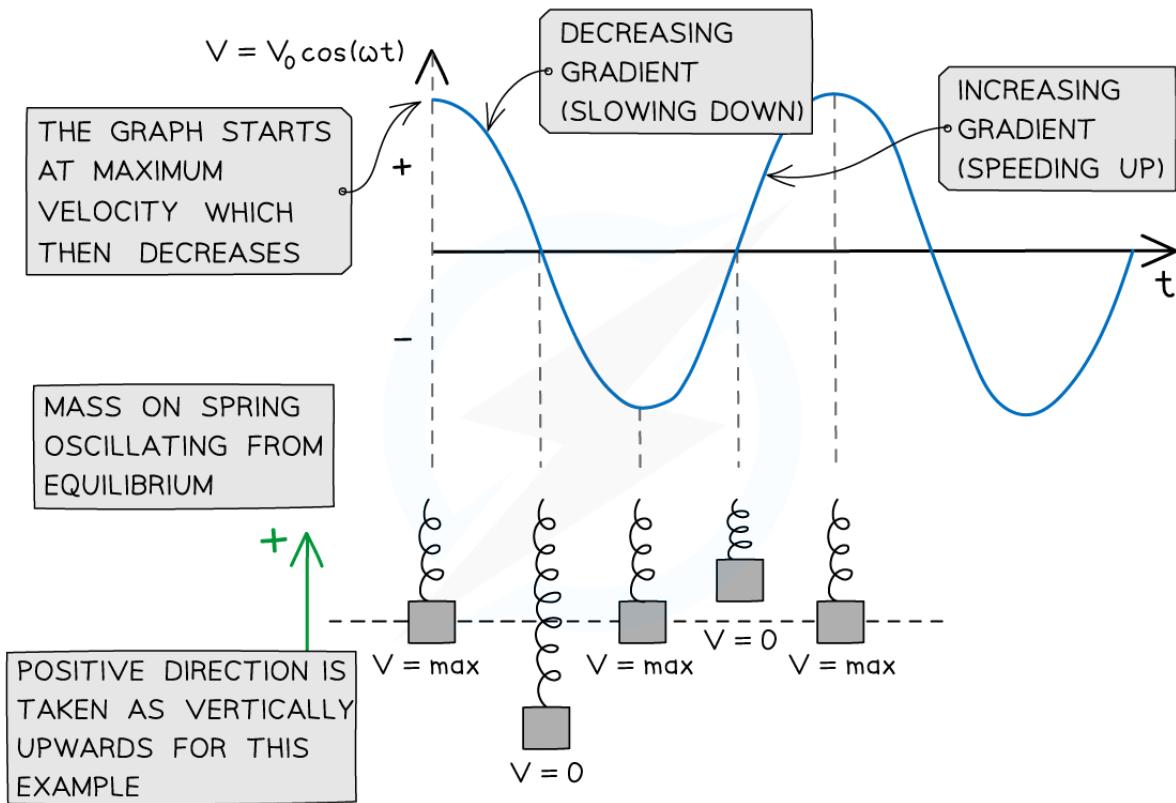
- Where:

- $x$  = displacement (m)
- $x_0$  = amplitude (m)
- $\pm$  = 'plus or minus'. The value can be negative or positive
- This equation shows that when an oscillator has a greater amplitude  $x_0$ , it has to travel a greater distance in the same time and hence has greater velocity,  $v$
- When the velocity is at its maximum (at  $x = 0$ ), the equation becomes:

$$v_0 = \omega x_0$$



Your notes


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*The variation of the speed of a mass on a spring in SHM over one complete cycle*



## Worked Example

A simple pendulum oscillates with simple harmonic motion with an amplitude of 15 cm. The frequency of the oscillations is 6.7 Hz.

Calculate the speed of the pendulum at a position of 12 cm from the equilibrium position.

**Answer:**

**Step 1: Write out the known quantities**

- Amplitude of oscillations,  $x_0 = 15 \text{ cm} = 0.15 \text{ m}$
- Displacement at which the speed is to be found,  $x = 12 \text{ cm} = 0.12 \text{ m}$
- Frequency,  $f = 6.7 \text{ Hz}$

**Step 2: Oscillator speed with displacement equation**

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$



Your notes

- Since the speed is being calculated, the  $\pm$  sign can be removed as direction does not matter in this case

**Step 3: Write an expression for the angular frequency**

- Equation relating angular frequency and normal frequency:

$$\omega = 2\pi f = 2\pi \times 6.7 = 42.097\dots$$

**Step 4: Substitute in values and calculate**

$$v = (2\pi \times 6.7) \times \sqrt{(0.15)^2 - (0.12)^2}$$

$$v = 3.789 = 3.8 \text{ m s}^{-1} \text{ (2.s.f)}$$

**Examiner Tips and Tricks**

You often have to convert between time period  $T$ , frequency  $f$  and angular frequency  $\omega$  for many exam questions – so make sure you revise the equations relating to these:



Your notes

**GENERAL****ACCELERATION**

IN OPPOSITE DIRECTION

DISPLACEMENT

SQUARE OF ANGULAR FREQUENCY

$$\text{TIME PERIOD} \rightarrow T = \frac{2\pi}{\omega}$$

$$\star \omega = 2\pi f \leftarrow \text{FREQUENCY}$$

**MAXIMUM VELOCITY**

$$\star V_{\max} = \omega x_0 \leftarrow \text{AMPLITUDE (MAXIMUM DISPLACEMENT)}$$

**MAXIMUM ACCELERATION**

$$\star a_{\max} = \omega^2 x_0$$

$$V = \pm \omega \sqrt{(x_0^2 - x^2)} \leftarrow \text{DISPLACEMENT}$$

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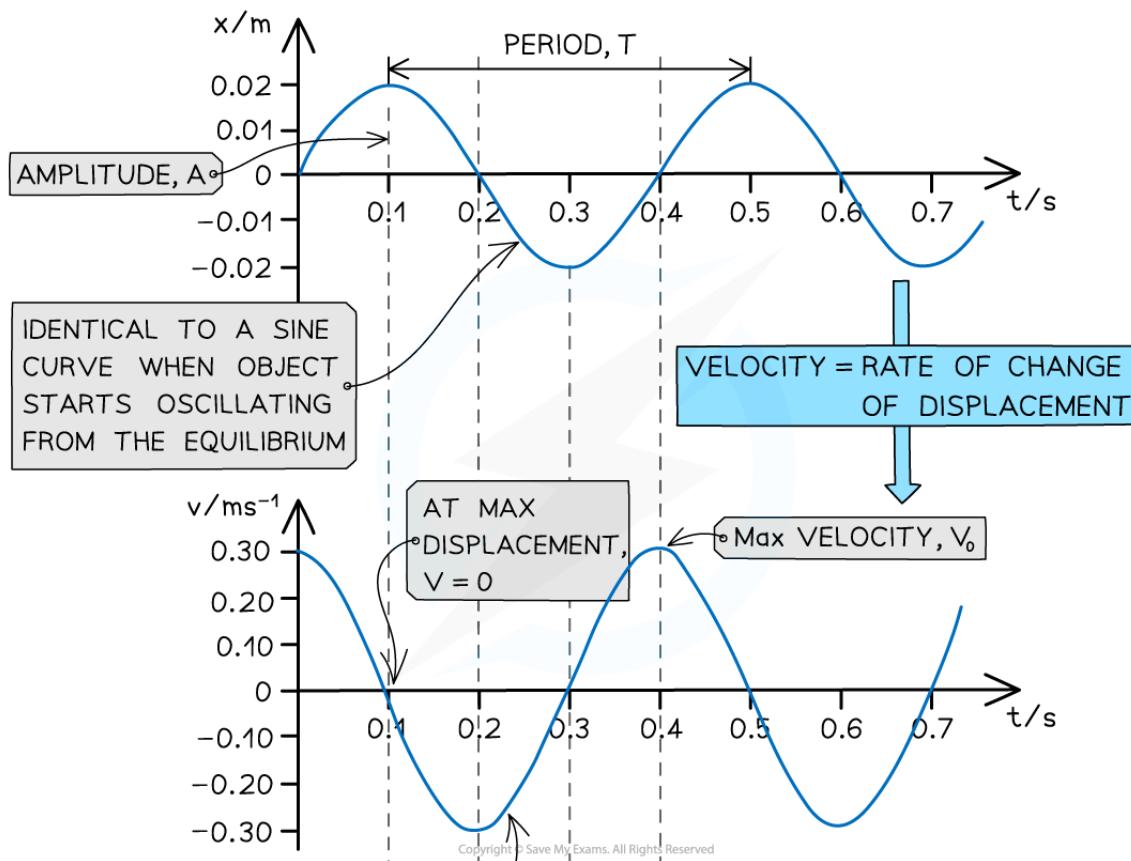
## SHM Graphs



Your notes

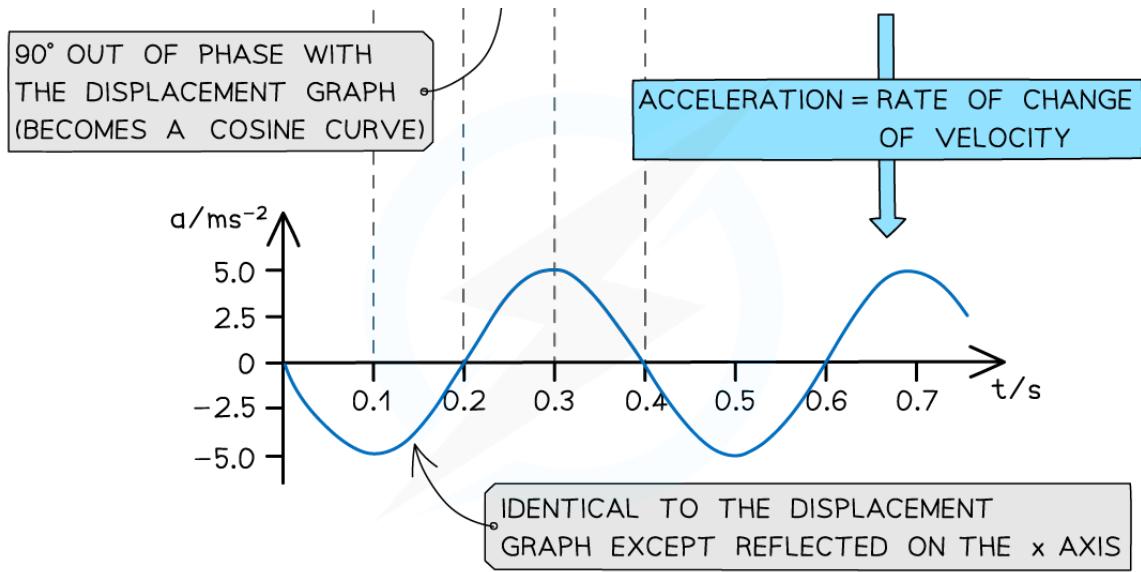
## SHM Graphs

- The displacement, velocity and acceleration of an object in simple harmonic motion can be represented by graphs against time
- All undamped SHM graphs are represented by **periodic functions**
  - This means they can all be described by sine and cosine curves





Your notes



**The displacement, velocity and acceleration graphs in SHM are all 90° out of phase with each other**

▪ **Key features of the displacement-time graph:**

- The amplitude of oscillations A can be found from the maximum value of x
- The time period of oscillations T can be found from reading the time taken for one full cycle
- The graph might not always start at 0
- If the oscillations starts at the positive or negative amplitude, the displacement will be at its maximum

▪ **Key features of the velocity-time graph:**

- It is 90° out of phase with the displacement-time graph
- Velocity is equal to the rate of change of displacement
- So, the velocity of an oscillator at any time can be determined from the **gradient of the displacement-time graph**:

$$v = \frac{\Delta x}{\Delta t}$$

- An oscillator moves the fastest at its equilibrium position



Your notes

- Therefore, the velocity is at its **maximum** when the **displacement is zero**
- **Key features of the acceleration–time graph:**
  - The acceleration graph is a reflection of the displacement graph on the x axis
  - This means when a mass has positive displacement (to the right) the acceleration is in the opposite direction (to the left) and vice versa
  - It is  $90^\circ$  out of phase with the velocity–time graph
  - Acceleration is equal to the rate of change of velocity
  - So, the acceleration of an oscillator at any time can be determined from the **gradient of the velocity–time graph:**

$$a = \frac{\Delta v}{\Delta t}$$

- The maximum value of the acceleration is when the oscillator is at its **maximum displacement**

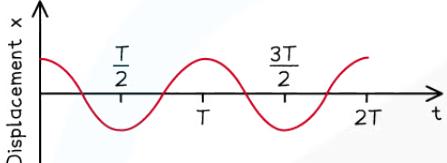
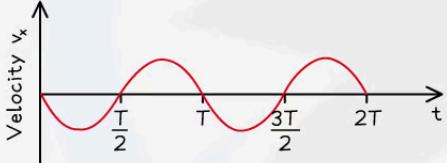
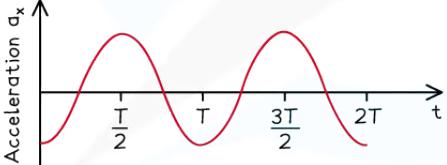
## Summary of Equations & Graphs for SHM

- A summary of the equations and graphs of simple harmonic motion are shown in the table
- Note that the equations differ depending on the starting point of the oscillator

**Summary table of equations and graphs for displacement, velocity and acceleration**



Your notes

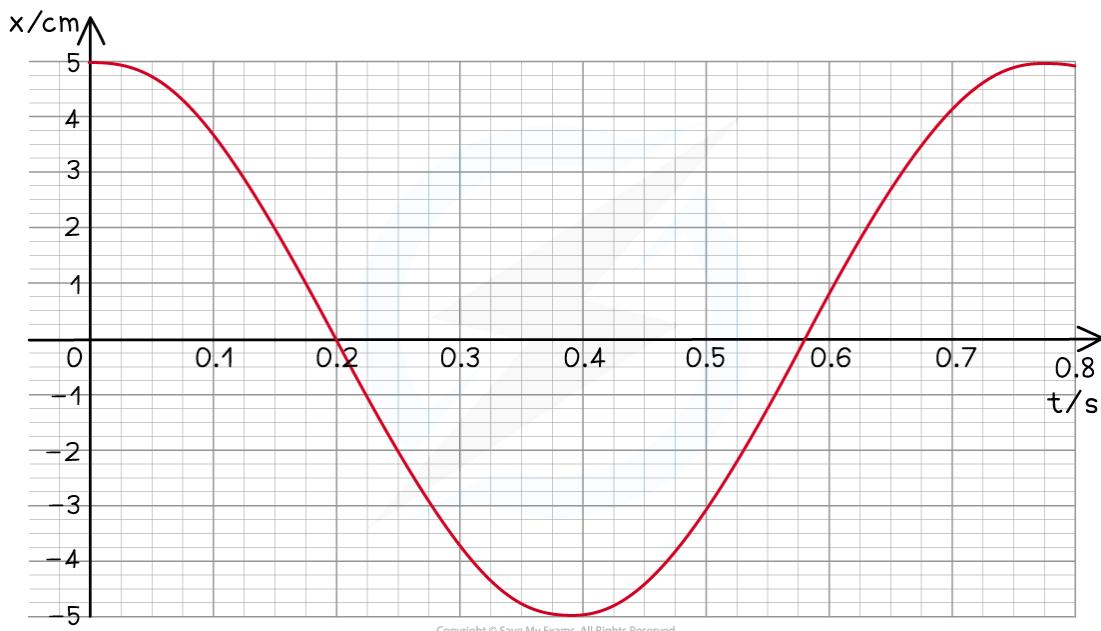
		Object starts in centre of motion	Object starts at extremes of motion
<b>Displacement</b>		$x = x_0 \sin \omega t$	$x = x_0 \cos \omega t$
<b>Velocity</b>		$v = \omega x_0 \cos \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$	$v = -\omega x_0 \sin \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$
<b>Acceleration</b>		$a = -x_0 \omega^2 \sin \omega t$ $a = -\omega^2 x$	$a = -x_0 \omega^2 \cos \omega t$ $a = -\omega^2 x$

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## Worked Example

A swing is pulled 5 cm and then released. The variation of the horizontal displacement  $x$  of the swing with time  $t$  is shown on the graph below.



The swing exhibits simple harmonic motion.

Use data from the graph to determine at what time the velocity of the swing is first at its maximum.

**Answer:**

- The velocity is at its **maximum** when the displacement  $x = 0$
- Reading the value of time when  $x = 0$ , from the graph, this is equal to  $0.2$  s
- Therefore, at **0.2 s**, the swing will reach its maximum velocity for the first time



### Examiner Tips and Tricks

These graphs might not look identical to what is in your textbook, depending on where the object starts oscillating from at  $t = 0$  (on either side of the equilibrium, or at the equilibrium).

However, if there is no damping, they will all always be general sine or cosine curves – make sure to pay particular attention to the difference between the graph shapes produced when the oscillator starts at the **equilibrium position** or **maximum displacement**