# Using Matrices For Simultaneous Equations

✎ If $\mathbf{A}\begin{pmatrix}x\\y\\z\end{pmatrix} = \underset{\sim}{\boldsymbol{v}}$ then $\begin{pmatrix}x\\y\\z\end{pmatrix} = \mathbf{A}^{-1}\boldsymbol{v}$

$$\begin{pmatrix}-1 & 6 & -2\\6 & -2 & -1\\-2 & 3 & 5\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}21\\-16\\24\end{pmatrix}$$

$A\underset{\sim}{x} = \underset{\sim}{v}$

$A^{-1}A\underset{\sim}{x} = A^{-1}\underset{\sim}{v}$

$\underset{\sim}{x} = A^{-1}\underset{\sim}{v}$

$\underset{\sim}{x} = \begin{pmatrix}x\\y\\z\end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix}a\\b\\c\end{pmatrix}$

[Textbook] Use an inverse matrix to solve the simultaneous equations:

$$-x + 6y - 2z = 21$$
$$6x - 2y - z = -16$$
$$-2x + 3y + 5z = 24$$

We can write using a matrix multiplication:

$$\begin{pmatrix}-1 & 6 & -2\\6 & -2 & -1\\-2 & 3 & 5\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}21\\-16\\24\end{pmatrix}$$

If we multiplied out the LHS it's easy to see this gives us the equations in the original question.

Find inverse of LHS matrix:

$$\frac{1}{189}\begin{pmatrix}7 & 36 & 10\\28 & 9 & 13\\-14 & 9 & 34\end{pmatrix}$$

Use your calculator to find this directly.

$$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{1}{189}\begin{pmatrix}7 & 36 & 10\\28 & 9 & 13\\-14 & 9 & 34\end{pmatrix}\begin{pmatrix}21\\-16\\24\end{pmatrix}$$

$$= \begin{pmatrix}-1\\4\\2\end{pmatrix}$$

$$\therefore x = -1, y = 4, z = 2$$

On your calc.

$$\begin{pmatrix}-1 & 6 & -2\\6 & -2 & -1\\-2 & 3 & 5\end{pmatrix}^{-1}\begin{pmatrix}21\\-16\\24\end{pmatrix} = \begin{pmatrix}-1\\4\\2\end{pmatrix}$$

**Calculator Tip**: You could check your answer using the simultaneous equation solver.