100 Days of A Level Maths

Week 1

Days 1 – 7

Day 1

Express y in terms of x:

$$2^y \times 8^x = \frac{1}{2\sqrt{2}}$$

Day 2

Simplify:

$$\sqrt{20} + 2\sqrt{45} - \sqrt{80}$$

Day 3

Solve:

$$x^6 + 9x^3 + 8 = 0$$

Day 4

Solve:

$$3^{2x} - 28(3^x) + 27 = 0$$

Day 5

Find the values of k for which the equation $5x^2 - 2x + k = 0$ has exactly one solution

Day 6

Find the range of values of p for which $x^2 + 4x + p = 0$ has no real solutions

Day 7

Solve $x^2 - 3x - 10 > 0$ giving your answer in set notation

100 Days of A Level Maths

Week 2

Days 8 – 14

Day 8

Sketch $y = (x - 2)(x - 3)(4 - x)$

Day 9

Find the equation of the straight line with gradient 3 that passes through $(2, -4)$

Day 10

Given that the distance between $(2, y)$ and $(5,7)$ is $3\sqrt{10}$, show that $y^2 - 14y - 32 = 0$

Day 11

Write down the equation of a circle with centre $(5, -6)$ and radius $2\sqrt{3}$

Day 12

Find the centre and radius of the circle with equation $x^2 + y^2 + 12x - 4y = 9$

Day 13

Divide $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$

Day 14

Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$, find the value of a

100 Days of A Level Maths

Week 3

Days 15 – 21

Day 15

Given that $n \in \mathbb{N}$, prove that $n^2 + 2$ is not divisible by 4

Day 16

Use Pascal's Triangle to find the expansion of $(x + 2y)^4$

Day 17

The coefficient of x^2 in the expansion of $(1 + qx)^5$ is 490. Find the values of q

Day 18

Find the value of x in the triangle

Day 19

Simple rule for trig equations

Day 20

Solve for $0 < \theta < 180°$

$\cos 2\theta = \frac{1}{2}$

Day 21

Solve, for $0 < \theta < 720°$

$\sin \theta = 0.3$

100 Days of A Level Maths

Week 4

Days 22 – 28

Day 22

Solve $4\,cos^2\theta - 7\sin\theta - 2 = 0$ for $0° < \theta \leq 360°$

Day 23

Given that $\sin x \cos y = 3\cos x \sin y$, express $\tan x$ in terms of $\tan y$

Day 24

Given that $\boldsymbol{c} = 3\boldsymbol{i} + 4\boldsymbol{j}$ and $\boldsymbol{d} = \boldsymbol{i} - 2\boldsymbol{j}$

find t if $\boldsymbol{d} - t\boldsymbol{c}$ is parallel to $-2\boldsymbol{i} + \boldsymbol{j}$

Day 25

Given that $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\boldsymbol{a} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, find the magnitude of $\boldsymbol{b}$

Day 26

Differentiate the following with respect to x

(i) $y = 3x^5 + 2x^7$

(ii) $y = 4\sqrt{x} + \frac{2}{x^2}$

Day 27

$f(x) = 3x^2 + \frac{24}{x} + 2$

Find the exact range of values for x for which $f(x)$ is an increasing function

Day 28

Find the x coordinates for the stationary points of the curve $y = 2x^3 - 15x^2 + 24x + 6$

100 Days of A Level Maths

Week 5

Days 29 – 35

Day 29

Given that $\theta = 3r^2 - 4r^{-1}$ find $\frac{d^2\theta}{dr^2}$

Day 30

The curve with equation $y = ax^2 + bx + 5$ has gradient 0 at $(2,1)$. Find a and b.

Day 31

Work out $\int \left(5x^4 - x^{-\frac{3}{2}} - 12x^{-5}\right) dx$

Day 32

Given that $f'(x) = 2x^3 - \frac{1}{x^2}$ and that $f(1) = 2$, find $f(x)$.

Day 33

The graph shows $y = x^2(2 - x)$. Find the value of R.

Day 34

Rewrite each statement using logarithms or powers

(i) $3^{-2} = \frac{1}{9}$

(ii) $a^4 = b$

(iii) $\log_4 16 = 2$

(iv) $\log_x y = z$

Day 35

Solve the equation

$\log_{10} 4 + 2\log_{10} x = 2$

100 Days of A Level Maths

Week 6

Days 36 – 42

Day 36

Solve $2^x e^{x+1} = 3$ giving your answer as an exact value

Day 37

The temperature T°C of a cup of tea is given by $T = 55e^{-\frac{t}{8}} + 20, \quad t \geq 0$ where t is time in minutes

(a) State the starting temperature of the tea

(b) Find the time it takes for the tea to cool to 50°C, to 2 decimal places

Day 38

Prove that there exist no integers a and b for which $21a + 14b = 1$

Day 39

Prove that if ab is irrational then at least one of a or b must be irrational

Day 40

Express $\frac{16x-1}{(3x+2)(x-1)}$ as partial fractions

Day 41

The equation $|6-x| = \frac{1}{2}x + k$ has no solutions. Find the range of values of k

Day 42

Given that $f(x) = e^x + 3$, find $f^{-1}(x)$, and draw $f(x)$ and $f^{-1}(x)$ on the same axes, indicating any intersections and equations of asymptotes.

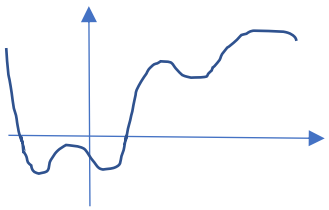
100 Days of A Level Maths

Week 7

Days 43 – 49

Day 43

Given that $y = f(x)$ is drawn below, sketch the graphs of $|f(x)|$ and $f(|x|)$



Day 44

The first 3 terms of an arithmetic sequence are:

$$4,\ \ 2k,\ \ 5k - 10$$

Show that the sum of the first n terms is a square number

Day 45

In a geometric series $S_\infty = \frac{8}{7}S_6$

Find the possible values of r

Day 46

Given that $a_1 = 3$ and $a_{n+1} = \frac{a_n - 3}{a_n - 2}$ find $\sum_{r=1}^{100} a_r = a_1 + a_2 + \cdots + a_{100}$

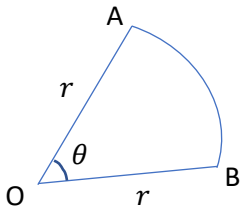
Day 47

Find the binomial expansion of $(4 + 5x)^{\frac{1}{2}}$ up to and including the x^2 term

Day 48

Given that θ is small and measured in radians, find an approximate value for $\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$

Day 49



The area of the sector is 11cm^2

Given that the perimeter of the sector is 4 times the length of the arc AB, find the value of r.

100 Days of A Level Maths

Week 8

Days 50 – 56

Day 50

Prove that $\frac{cosec\ x - \cot x}{1 - \cos x} \equiv cosec\ x$

Day 51

Show that $\tan\theta = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found if $3\ tan^2\theta + 4\ sec^2\theta = 5$

Day 52

Given that $\tan\frac{\pi}{4} = 1$ and $\tan\frac{\pi}{8} = \sqrt{2} - 1$ use the addition formula for tan to find an exact value for $\tan\frac{3\pi}{8}$

Day 53

Solve for $0 \leq x \leq \frac{\pi}{2}$

$4 \sin x = \sec x$

Day 54

Prove that $cosec\ 2x + \cot 2x \equiv \cot x$

Day 55

Given that $7 \cos\theta + 24 \sin\theta = R\cos(\theta - \alpha)$ find the value of R and the value of α to 2 dp in degrees.

Day 56

Find the cartesian equation of the curve with parametric equations $x = \sin t$ and $y = \sin 2t$

100 Days of A Level Maths

Week 9

Days 57 – 63

Day 57

Find where the curve with parametric equations $x = t^2 + t$ and $y = 4t + 4$, $t > 0$, crosses the line with equation $y = x - 6$

Day 58

1. $\frac{d}{dx}(e^x)$
2. $\frac{d}{dx}(2^x)$
3. $\frac{d}{dx}(\ln x)$
4. $\frac{d}{dx}(\sin x)$
5. $\frac{d}{dx}(\cos x)$
6. $\frac{d}{dx}(\tan x)$
7. $\frac{d}{dx}(\sec x)$
8. $\frac{d}{dx}(\cot x)$
9. $\frac{d}{dx}(cosec\ x)$

Day 59

The Chain Rule

$$f(blah) \rightarrow f'(blah) \times blah'$$

e.g. find $f'(x)$:

1. $f(x) = (x^4 + 7x)^5$

2. $f(x) = e^{\sin x + x^2}$

3. $f(x) = \ln(3x^2 + 4)$

Day 60

The Product Rule

If $y = uv$, then $\frac{dy}{dx} = uv' + vu'$

e.g differentiate $y = \sin(x^2)\ln(\cos x)$

Day 61

The Quotient Rule

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{vu\prime - uv\prime}{v^2}$

e.g find $\frac{dy}{dx}$ when $y = \frac{\sin x}{e^{2x}}$

Day 62

Given that $x = 2\sin t$ and $y = \sqrt{2}\cos 2t$, show that $\frac{dy}{dx} = k\sin t$ where k is a constant to be found.

Day 63

Find an expression for $\frac{dy}{dx}$ in terms of x and y:

$$3y^2 - 2y + 2xy = x^3$$

100 Days of A Level Maths

Week 10

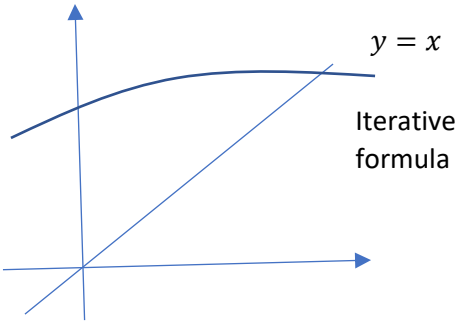
Days 64 – 70

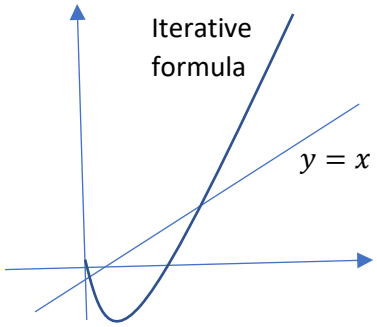
Day 64

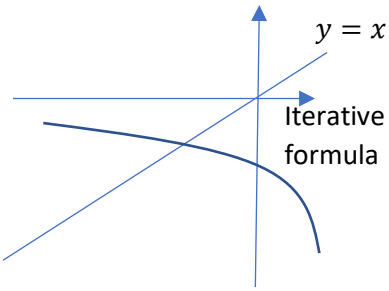
Given that $V = \frac{1}{3}\pi r^3$ and that $\frac{dV}{dt} = 8$, find $\frac{dr}{dt}$ when $r = 3$

Day 65

Use the graphs of the iterative functions to decide whether the root will converge or diverge







Day 66

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Using $x_1 = 1$ as a first approximation as a root of $2x^3 + x^2 - 1 = 0$, find a 2nd approximation, using the Newton-Raphson method.

Day 67

Integrate the following:

(a) $\int \cos x \ dx$

(b) $\int sec^2 x \ dx$

(c) $\int e^x \ dx$

(d) $\int \frac{1}{x}\, dx$

(e) $\int 3\sin x\ dx$

(f) $\int 4\sec x \tan x\ dx$

Day 68 *TRY $y = \cdots$*

Integrate the following:

(a) $\int \sin(2x+1)\ dx$

(b) $\int 4e^{3x}\, dx$

(c) $\int 3\sec 4x \tan 4x\ dx$

(d) $\int (3x+2)^5\, dx$

Day 69

$\int sin^2 x\, dx$

Day 70 *General patterns*

$\int \frac{2x+1}{x^2+x} dx$

$\int \cos x\, e^{\sin x} dx$

100 Days of A Level Maths

Week 11

Days 71 – 77

Day 71

$$\int 2\cos 3x \ sin^4 3x \ dx$$

Day 72

$$\int \frac{x+1}{x} dx$$

$$\int \frac{x+1}{x-1} dx$$

Day 73

$$\int 2x \sin(3x-1) dx$$

Day 74

$$\int \ln x \, dx$$

Day 75

Use the substitution $u = x + 1$ to find $\int x\sqrt{x+1}\,dx$

Day 76

Solve $(x + x^2)\frac{dy}{dx} = y(1 + 2x)$

Day 77

Solve $(1 + x^2)\frac{dy}{dx} = x\,sec\,y$

100 Days of A Level Maths

Week 12

Days 78 – 84

Day 78

Given that A has coordinates $(5, 0, 3)$ and B has coordinates $(4, 2, k)$, and the distance $AB = 3$, find the values of k.

Day 79

Find the angle that $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$ makes with the positive y axis.

Day 80

Given that $3\boldsymbol{i} + (p + 2)\boldsymbol{j} + 120\boldsymbol{k} = p\boldsymbol{i} - qj + 4pqr\boldsymbol{k}$

find the values of p, q and r

Exam Questions

Day 81

(a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

Day 82

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of k,

(4)

Day 83 and 84

A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

100 Days of A Level Maths

Week 13

Days 85 – 91

Day 85

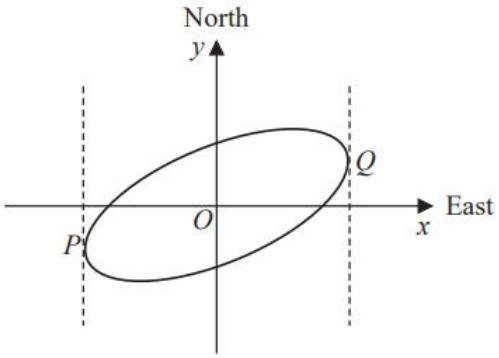


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\frac{dy}{dx} = \frac{y - x}{3y - x}$

(4)

Day 86

The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

Day 87

$$g(x) = \frac{2x + 5}{x - 3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

Day 88

Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Day 89

(a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

Day 90

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

Day 91

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

100 Days of A Level Maths

Week 14

Days 92 – 98

Day 92

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

Day 93

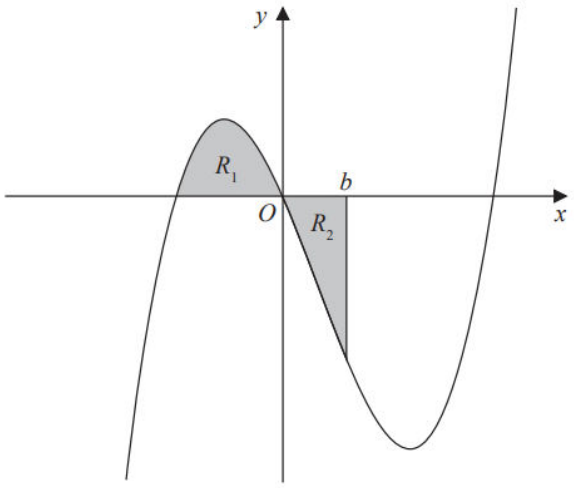


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

Day 94

Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

Day 95

The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

Day 96

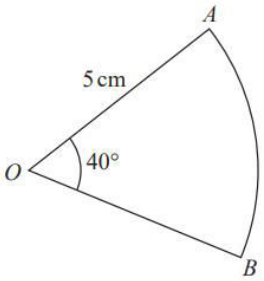


Figure 1

Figure 1 shows a sector AOB of a circle with centre O, radius 5 cm and angle $AOB = 40°$

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2} r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500\,\text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

Day 97

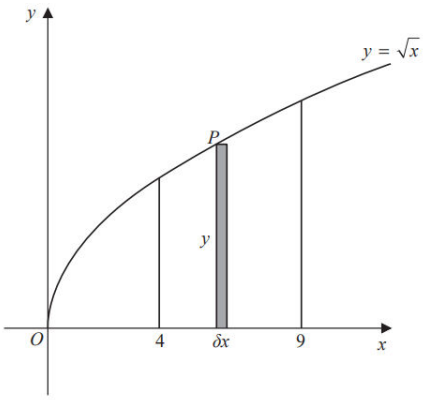


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x$$

(3)

Day 98

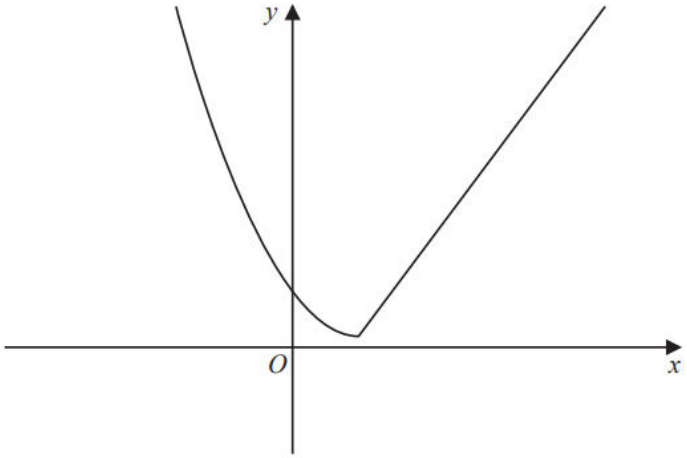


Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $gg(0)$.

(2)

100 Days of A Level Maths

Week 15

Days 99 – 100

Day 99

(i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

Day 100

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)