Edexcel M1(Old) May 2013(R) Q6

[In this question **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.]*

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours. **(2)**

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n. **(5)**

(c) Find the distance OP. **(4)**

(a)	Use of $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$	M1
	$(-4\mathbf{i}+2\mathbf{j})+(3\mathbf{i}+3\mathbf{j})t = (-4+3t)\mathbf{i}+(2+3t)\mathbf{j}$ $= \begin{pmatrix} -4+3t \\ 2+3t \end{pmatrix}$	A1
(b)	$(6\mathbf{i}+\mathbf{j})+(-2\mathbf{i}+n\mathbf{j})t = (6-2t)\mathbf{i}+(1+nt)\mathbf{j}$	B1
	Position vectors identical $\Rightarrow -4+3t = 6-2t$ **AND** $5t = 10$,	M1
	$\begin{pmatrix} 6-2t \\ 1+nt \end{pmatrix} = \begin{pmatrix} -4+3t \\ 2+3t \end{pmatrix}$ Either equation	A1
	$2+3\times 2 = 1+2n$,	**DM1**
	$n = 3.5$	A1
(c)	Position vector of P is $(-4+6)\mathbf{i}+(2+6)\mathbf{j} = 2\mathbf{i}+8\mathbf{j}$	M1A1
	Distance OP $= \sqrt{2^2+8^2} = \sqrt{68} = 8.25$ (km)	M1A1

Vector methods for projectiles

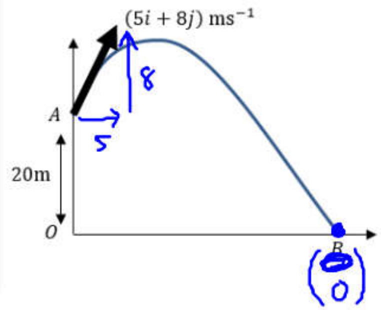
$\underline{r}_0 = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$ $\underline{u} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

A ball is struck by a racket from a point A which has position vector $20j$ m relative to a fixed origin O. Immediately after being struck, the ball has velocity $(5i + 8j)$ ms^{-1}, where i and j are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point B.

(a) Find the speed of the ball 1.5 seconds after being struck.

(b) Find an expression for the position vector, r, of the ball relative to O at time t seconds.

(c) Hence determine the distance OB.



$\downarrow 9.8$

$\underline{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$

a) $\underline{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$

$\underline{u} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$t = 1.5$

$\underline{v} = ?$

$\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 1.5$

$\underline{v} = \begin{pmatrix} 5 \\ -6.7 \end{pmatrix}$

$v = \sqrt{5^2 + 6.7^2} = \underline{8.36 \text{ ms}^{-1}}$ (3sf)

b) $\underline{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ $\underline{r} = ?$

$\underline{u} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$t = t$

$\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = \begin{pmatrix} 0 \\ 20 \end{pmatrix} + \begin{pmatrix} 5 \\ 8 \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2$

$\underline{r} = \begin{pmatrix} 5t \\ 20 + 8t - 4.9t^2 \end{pmatrix}$

Ex 8B evens

c) $\underline{j}$ comp $= 0$

$20 + 8t - 4.9t^2 = 0$

$t = 2.9953\ldots$ or $t = -1.362$

$t > 0$ $t = 2.9953$

$\underline{r} = \begin{pmatrix} 5 \times 2.9953 \\ 0 \end{pmatrix} = \begin{pmatrix} 14.97\ldots \\ 0 \end{pmatrix}$ $OB = \underline{15.0 \text{ m}}$ (3sf)

4. [*In this question the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upward.*]

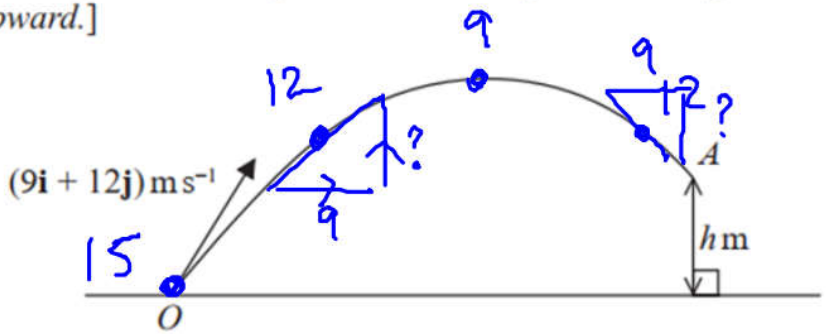


Figure 2

A small ball is projected from the fixed point O on horizontal ground with velocity $(9\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-1}}$

The ball passes through the point A which is h metres vertically above the level of O, as shown in Figure 2.

The velocity of the ball at the instant it passes through the point A is $\lambda(\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$, where λ is a positive constant.

The ball is modelled as a particle moving freely under gravity.

(a) Find the value of h.

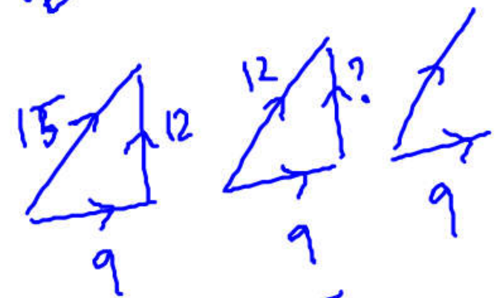
(4)

(b) State the minimum speed of the ball as it moves from O to A.

(1)

(c) Find the length of time for which the speed of the ball is less than $12\,\mathrm{m\,s^{-1}}$

when is speed 12?



(4)

vert. $v = \sqrt{12^2 - 9^2}$

The model could be refined by considering air resistance.

(d) Suggest one other refinement to the model that would make it more realistic.

(1)

Question	Scheme	Marks	AOs
4(a)	$(\lambda\mathbf{i} = 9\mathbf{i})\quad \lambda = 9$	B1	3.3
	Vertical distance:	M1	3.4
	$9^2 = 12^2 - 2gh$	A1ft	1.1b
	$h = 3.2(1)$	A1	1.1b
		(4)	
(b)	Min speed = 9 (m s^{-1})	B1	2.2a
		(1)	
(c)	Vertical component of velocity $= \sqrt{12^2 - 9^2}\left(= \sqrt{63}\right)$	M1	3.1b
	$\Rightarrow -\sqrt{63} = \sqrt{63} - gt$	A1ft	1.1b
	Complete strategy to find the required time	M1	3.1b
	$t = 1.6(2)$ (s)	A1	2.2a
		(4)	
(d)	Consider the dimensions of the ball	B1	3.5c
		(1)	
		(10 marks)	

Edexcel M2(Old) Jan 2012 Q7

[*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

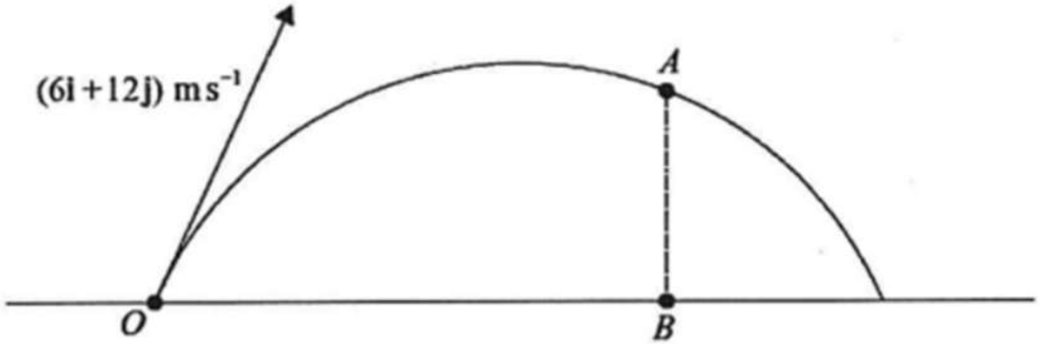


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(*a*) the value of t,

(7)

(*b*) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(*c*) Find the time taken for the ball to travel from O to C.

(3)

(a)	$\mathbf{i} \rightarrow$ distance $= 6t$	B1
	$\mathbf{j} \uparrow$ distance $= 12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	DM1
	$18 = gt$, $t = \frac{18}{g} (= 1.84\text{s})$	A1
(b)	$\mathbf{i} \rightarrow$ speed $= 6$	B1
	$\mathbf{j} \uparrow$ velocity $= 12 - gt = -6$	M1 A1
	$\therefore$ speed at A	
	$= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2} (= 8.49) (\text{ms}^{-1})$	M1 A1
(c)	$\uparrow$ speed $= 12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g} (= 0.61\text{s})$	A1

Variable Acceleration in One Dimension - more complex functions

A particle is moving in a straight line with acceleration at time t seconds given by

$$a = \cos 2\pi t \text{ ms}^{-2}, \qquad t \geq 0$$

The velocity of the particle at time $t = 0$ is $\frac{1}{2\pi}$ ms^{-1}. Find:

(a) an expression for the velocity at time t seconds
(b) the maximum speed
(c) the distance travelled in the first 3 seconds.

$$\begin{tikzcd} s \arrow[d, bend left, "\frac{d}{dt}"] \\ v \arrow[u, bend left, "\int v\,dt"] \arrow[d, bend left, "\frac{d}{dt}"] \\ a \arrow[u, bend left, "\int a\,dt"] \end{tikzcd}$$

a) $v = \int \cos 2\pi t \, dt$

$v = \frac{1}{2\pi}\sin 2\pi t + c$

$t = 0, \quad v = \frac{1}{2\pi}$

$\frac{1}{2\pi} = 0 + c$

$v = \frac{1}{2\pi}\sin 2\pi t + \frac{1}{2\pi}$

if $\sin 2\pi t = -1$

min $v = 0$

So it is always above/on the axis

b) max speed $a = 0$

knowledge of trig functions $-1 \leq \sin\theta \leq 1$

maximum via differentiation

max of $v = \frac{1}{2\pi} \times 1 + \frac{1}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$

c) $\int_0^3 v \, dt = \int_0^3 \left(\frac{1}{2\pi}\sin 2\pi t + \frac{1}{2\pi}\right) dt$

$= \left[-\frac{1}{4\pi^2}\cos 2\pi t + \frac{1}{2\pi}t\right]_0^3$

$= \left(-\frac{1}{4\pi^2} + \frac{3}{2\pi}\right) - \left(-\frac{1}{4\pi^2}\right)$

$= \frac{3}{2\pi} = \underline{0.477 \text{ m}}$ (3sf)

A particle of mass 6kg is moving on the positive x-axis. At time t seconds the displacement, s, of the particle from the origin is given by

$$s = 2t^{\frac{3}{2}} + \frac{e^{-2t}}{3}\ \text{m}, \qquad t \geq 0$$

(a) Find the velocity of the particle when $t = 1.5$.

Given that the particle is acted on by a single force of variable magnitude F N which acts in the direction of the positive x-axis,

(b) Find the value of F when $t = 2$

a)

$s = 2t^{3/2} + \frac{1}{3}e^{-2t}$

$v = 3t^{1/2} - \frac{2}{3}e^{-2t}$

$t = 1.5$

$v = 3(1.5)^{1/2} - \frac{2}{3}e^{-3}$

$= \underline{\underline{3.64\ ms^{-1}}}\ (3sf)$

b)

$F = ma$

$F = 6a$

$a = \frac{3}{2}t^{-1/2} + \frac{4}{3}e^{-2t}$

$t = 2$

$a = \frac{3}{2}(2)^{-1/2} + \frac{4}{3}e^{-4}$

$= 1.0885\ldots$

$F = 6 \times 1.085\ldots$

$= \underline{\underline{6.51N}}\ (3sf)$