Partial Fractions

If the **denominator is a product of a linear terms**, it can be split into the sum of 'partial fractions', where **each denominator is a single linear term**.

$$\frac{6x-2}{(x-3)(x+1)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$$

Notation reminder: $\equiv$ means 'equivalent/identical to', and indicates that both sides are equal for **all** values of x.

Method 1: Substitution   Find A and B

$$\frac{6x-2}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{6x-2}{(x-3)(x+1)} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

$$6x-2 = A(x+1)+B(x-3)$$

$x=-1$

$-8 = -4B$

$B=2$

$x=3$

$16 = 4A$

$A=4$

$$\frac{6x-2}{(x-3)(x+1)} = \frac{4}{x-3} + \frac{2}{x+1}$$

Method 2: Comparing Coefficients

LHS   RHS

$6x-2 = A(x+1)+B(x-3)$

x coefficients

$6 = A+B$

$6-B=A$

$A=4$

constants

$-2 = A-3B$

$-2 = 6-B-3B$

$-8 = -4B$

$B=2$

Given that $\frac{6x^2+5x-2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$, find the values of the constants A, B, C.

$$6x^2 + 5x - 2 = A(x-1)(2x+1) + Bx(2x+1) + Cx(x-1)$$

Method 1 - substitution

$x = 1$

$$9 = 3B$$
$$\underline{3 = B}$$

$2x+1=0$

$x = -\frac{1}{2}$

$$6\times\left(-\tfrac{1}{2}\right)^2 + 5\left(-\tfrac{1}{2}\right) - 2 = C\left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{2}-1\right)$$
$$\frac{3}{2} - \frac{5}{2} - 2 = \frac{3}{4}C$$
$$-3 = \frac{3}{4}C$$
$$\underline{\underline{C = -4}}$$

$x = 0$

$$-2 = A(-1)(1)$$
$$-2 = -A$$
$$\underline{\underline{A = 2}}$$

$$\frac{6x^2+5x-2}{x(x-1)(2x+1)} = \frac{2}{x} + \frac{3}{x-1} - \frac{4}{2x+1}$$

Your Turn

C4 June 2005 Q3a

Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

$$\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$$

$$5x+3 = A(x+2) + B(2x-3)$$

compare x $\quad 5 = A+2B \longrightarrow A = 5-2B$

const. $\quad 3 = 2A-3B$

$$3 = 2(5-2B)-3B$$

$$3 = 10-4B-3B$$

$$-7 = -7B$$

$$\underline{\underline{B=1}}$$

$$A = 5-2 = \underline{\underline{3}}$$

subst.

$x=-2$

$-7 = -7B$

$\underline{\underline{B=1}}$.

$x = \frac{3}{2}$

$\frac{21}{2} = \frac{7}{2}A$

$\underline{\underline{A=3}}$

Ex 1D

1adf

3

5

7

Partial Fractions - repeated linear factors

Suppose we wished to express $\frac{2x+1}{(x+1)^2}$ as $\cancel{\frac{A}{x+1}+\frac{B}{x+1}}$. What's the problem?

$$\frac{2x+1}{(x+1)^2} \neq \frac{A+B}{x+1}$$

correct method

$$\frac{2x+1}{(x+1)^2} = \boxed{\frac{A}{x+1} + \frac{B}{(x+1)^2}}$$

$$\frac{2x+1}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2}$$

Q Split $\frac{11x^2+14x+5}{(x+1)^2(2x+1)}$ into partial fractions.

$$\frac{11x^2+14x+5}{(x+1)^2(2x+1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(2x+1)}$$

$$11x^2+14x+5 = A(x+1)(2x+1) + B(2x+1) + C(x+1)^2$$

Subst. $x=-1$

$11-14+5 = -B$

$2 = -B$

$\underline{B=-2}$

$x = -\frac{1}{2}$

$\frac{3}{4} = \frac{1}{4}C$

$\underline{C=3}$

Compare constant

$5 = A+B+C$

$5 = A-2+3$

$\underline{4=A}$

compare x^2 coeff.

$11 = 2A + C$

$11 = 2A + 3$

$8 = 2A$

$\underline{A=4}$

The problem is resolved by having the factor **both squared and non-squared**.

Your Turn

C4 June 2011 Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants A, B and C. **(4)**

$$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$$

$x = 1$

$9 = 3B$

$\underline{B = 3}$

$x = -\frac{1}{2}$

$\frac{9}{4} = \frac{9}{4}C$

$\underline{C = 1}$

compare x^2 coefficient

$9 = 2A + C$

$9 = 2A + 1$

$8 = 2A$

$\underline{A = 4}$

A mixture of substitution and comparing coefficients can be very effective.

Ex 1E

Q1

3

5

7

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

$$\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{11-2P}$$

$$1 = A(11-2P) + BP$$

$P = 0$ $\qquad\qquad$ $P = \frac{11}{2}$

$1 = 11A$ $\qquad\qquad$ $1 = \frac{11}{2}B$

$A = \frac{1}{11}$ $\qquad\qquad$ $B = \frac{2}{11}$

$$\frac{1}{P(11-2P)} = \frac{1/11 \ {\times 11}}{P \ {\times 11}} + \frac{2/11 \ {\times 11}}{11-2P \ {\times 11}}$$

$$= \frac{1}{11P} + \frac{2}{11(11-2P)}$$