A Level · Edexcel · Maths

3 hours 41 questions

7.1 Differentiation

7.1.1 Definition of Gradient / 7.1.2 First Principles Differentiation / 7.1.3 Differentiating Powers of x

Easy (11 questions)	/36
Medium (10 questions)	/40
Hard (10 questions)	/42
Very Hard (10 questions)	/48
Total Marks	**/166**

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Easy Questions

1 Differentiate

(i) $5x$,

(ii) $2x^3$

(iii) $x^{\frac{1}{2}}$

(3 marks)

2 Write down the formula that should be used as a starting point when explaining differentiation from first principles.

(2 marks)

3 (a) Write down the gradient of the line with equation $y = k$, where k is a constant.

(1 mark)

(b) Find the gradient at the point where $x = 8$ for the following functions

(i) $\mathrm{f}(x) = 3x^2$,

(ii) $\mathrm{f}(x) = 4x^3 - 2x$,

(iii) $\mathrm{f}(x) = 3x^{\frac{1}{3}}$.

(3 marks)

4 A student is trying to show that the derivative of $7x^2$ is $14x$ using first principles.
Their working is shown below.
Find and explain their error.

STEP 1

$$\mathrm{f}(x) = 7x^2$$

STEP 2

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

STEP 3

$$f'(x) = \lim_{h \to 0} \frac{7(x+h)^2 - 7x^2}{h}$$

STEP 4

$$f'(x) = \lim_{h \to 0} \frac{7x^2 + 14hx + 7h^2 - 7x^2}{h}$$

STEP 5

$$f'(x) = \lim_{h \to 0} \frac{h(14x + 7h)}{h}$$

STEP 6

$$f'(x) = 14x + 7h$$

When h=0, $14x + 7h = 14x$

$$\therefore f'(x) = 14x$$

(3 marks)

5 (i) Expand $(x+3)(x-2)$.

(ii) Hence differentiate $(x+3)(x-2)$.

(3 marks)

6 Given that $y = 2x^{\frac{1}{2}} + 3x^{-1}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2 marks)

7 Find the x-coordinate of the point on the curve $y = 5x^2 - 16x$ where the gradient is 4.

(3 marks)

8 Find the coordinates of the points on the curve $y = 2x^3 - 9x^2 + 12x$ where the gradient is 0.

(4 marks)

9 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $y = \left(\sqrt{x}\right)^3 + \frac{2}{\sqrt{x}}$.

(3 marks)

10 (a) The function $f(x)$ is given by

$$f(x) = \frac{2x^{\frac{1}{3}} + 3x^{\frac{2}{3}}}{x}.$$

Show that $f(x)$ can be written in the form $f(x) = ax^b + cx^d$, where a, b, c and d are constants to be found.

(3 marks)

(b) Find $f'(x)$.

(3 marks)

11 Prove, from first principles, that the derivative of $4x$ is 4.

(3 marks)

Medium Questions

1 Prove, from first principles, that the derivative of $-3x$ is -3.

(3 marks)

2 Prove, from first principles, that the derivative of $2x^2$ is $4x$.

(4 marks)

3 **(a)** For each of the following, find $\frac{dy}{dx}$ in terms of x:

$$y = 4x^2 - 3x + 19$$

(1 mark)

(b) $y = x^3 - 5x^2 + 14x - 1$

(2 marks)

(c) $y = 4x^{\frac{3}{2}} - 3x^{-1}$

(2 marks)

4 Given that $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, $x > 0$, find $\frac{dy}{dx}$.

(3 marks)

5 (a) For each of the following, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x:

$$y = (2x + 3)(3x - 1)$$

(2 marks)

(b) $y = x^3 \left(\frac{1}{x^3} - \frac{2}{x^2} + \frac{3}{x}\right)$

(2 marks)

6 (a) The function f is defined by $f(x) = 2x^3 - x^2 - 4x + 3$.

Find $f'(x)$.

(2 marks)

(b) Solve the equation $f'(x) = 0$.

(2 marks)

7 (a) A curve has the equation $y = 3x - 4x^{-2}$, $x \neq 0$.

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(2 marks)

(b) Find the coordinates of the point on the curve where the gradient is 2.

(2 marks)

8 (a) The function f is defined by $f(x) = x^3 - 6x^2 - cx + 12$.

Find $f'(x)$.

(2 marks)

(b) Given that the equation $f'(x) = 0$ has exactly one real solution, find the value of c.

(2 marks)

9 (a) A curve is described by the equation $\frac{y}{x-3} = x^2 + 1$.

Make y the subject of the equation.

(1 mark)

(b) Hence find $\frac{dy}{dx}$.

(1 mark)

(c) Find the coordinates of the point on the curve where the gradient is -2.

(2 marks)

10 (a) The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1,\ 13)$, and a gradient of -3 at the point (1, 3).

By considering $\frac{\mathrm{d}y}{\mathrm{d}x}$ show that $2a + b = -3$ and $-2a + b = -7$.

(2 marks)

(b) Hence find the values of a and b.

(1 mark)

(c) By considering a point that you know to be on the curve, find the value of c.

(2 marks)

Hard Questions

1 Prove, from first principles, that the derivative of ax^2 is $2ax$, where a is a constant.

(4 marks)

2 Prove, from first principles, that the derivative of $2x^3$ is $6x^2$.

(5 marks)

3 (a) For each of the following, find $\frac{dy}{dx}$ in terms of x:

$$y = -3x^3 + 5x^2 - 3x + \sqrt{13}$$

(2 marks)

(b) $y = 9x^{\frac{1}{3}} - 6x^{-\frac{1}{3}}$

(2 marks)

4 Given that $y = \frac{1}{\sqrt{x}}\left(1 + \frac{1}{x}\right)$, $x > 0$, find $\frac{dy}{dx}$.

(3 marks)

5 (a) For each of the following, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x:

$y = (2x - 1)^2 (x + 1)$

(3 marks)

(b) $y = \frac{1}{x^5} (x^2 + \sqrt{x} - 1)$

(3 marks)

6 The function f is defined by $\mathrm{f}(x) = x^3 - 4x^2 + 6x - 9$. Show that there are no solutions to the equation $\mathrm{f}'(x) = 0$.

(4 marks)

7 (a) A curve has the equation $y = \frac{3}{8}x^{\frac{4}{3}} - 12x^{\frac{1}{3}}$.

Show that $\frac{\mathrm{d}x}{\mathrm{d}y} = ax^{-\frac{2}{3}}(x+b)$, where a and b are rational numbers to be found.

(3 marks)

(b) Hence find the coordinates of the point on the curve where the gradient is 0.

(2 marks)

8 A curve has the equation $y = 4x^3 + bx^2 + 3x - 17$, where b is a constant. Given that there is only one point on the curve where the gradient is zero, determine the possible values of b.

(4 marks)

9 A curve is described by the equation $4y^2 - 3x^5 = 0, \quad y > 0$.

By rearranging the equation to make y the subject, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2 marks)

10 The curve with equation $y = ax^2 + bx + c$ has a gradient of 8 at the point $(-2, 0)$, and a gradient of -10 at the point $(1, -3)$. Find the values of a, b and c.

(5 marks)

Very Hard Questions

1 Prove, from first principles, that the derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$.

(5 marks)

2 (a) Show that $\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)=h$.

(2 marks)

(b) Prove, from first principles, that the derivative of $\sqrt{x}$ is $\dfrac{1}{2\sqrt{x}}$.

(4 marks)

3 (a) For each of the following, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x:

$$y = -\frac{5}{4}x^3 + \frac{3}{5}x^2 - x\sqrt{2} + \pi$$

(2 marks)

(b) $y = \frac{3}{2}x^{\frac{4}{5}} - \frac{10}{3}x^{-\frac{4}{5}}$

(2 marks)

4 Given that $y = \left(\frac{1}{x} - \frac{1}{x\sqrt{x}}\right)^2$, $x > 0$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4 marks)

5 (a) For each of the following, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x:

$$y = \frac{2x^3 - 5x^2 - 3x}{2x + 1}$$

(3 marks)

(b) $y = \left(\sqrt{x} + 3 - \frac{4}{\sqrt{x}} \right)^2$

(4 marks)

6 The function f is defined by $\mathrm{f}(x) = 2x^3 + px^2 + 3x - 16$. Determine the range of values for p for which the equation $\mathrm{f}'(x) = 0$ has at least one real solution.

(5 marks)

7 A curve has the equation $y = x\sqrt{x} + \dfrac{48}{\sqrt{x}}$, $x > 0$. Find the coordinates of the point on the curve where the gradient is 0.

(5 marks)

8 The function f is defined by $\mathrm{f}(x) = x^n - x$, $n \in \mathbb{N}$, $n \geq 2$. Determine the relationship between the value of n and the number of real solutions to the equation $\mathrm{f}'(x) = 0$.

(4 marks)

9 A curve is described by the equation $\dfrac{\sqrt{y}}{-1 + \sqrt{x}} = \dfrac{1}{x}$, $x > 1$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3 marks)

10 The curve with equation $y = ax^2 + bx + c$ passes through the point (-1, 4). At the point (2, 7) the gradient of the curve is 7. Find the values of a, b and c.

(5 marks)