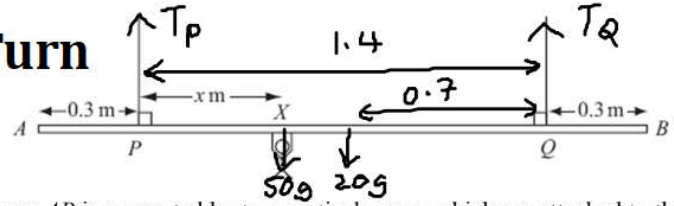
May 2009

Your Turn

Hanging bars/rods



A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(a) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(b) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(d) find the value of x.

(3)

M(Q) $20g \times 0.7 + 50g(1.4 - x) = T_P \times 1.4$

$137.2 + 686 - 50gx = 1.4T_P$

$588 - 350x = T_P$

$0 < x < 1.4$

$x = 0 \quad T_P = 588$

$x = 1.4 \quad T_P = 98$

$98 < T_P < 588$

R↕ $T_Q + T_P = 70g$

$T_Q = 70g - (588 - 350x) = 98 + 350x$

$98 < T_Q < 588$

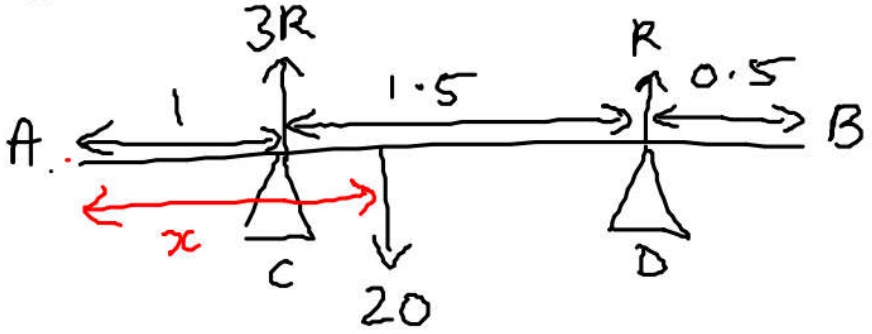
d) $T_Q = 3T_P$

$x = 1.19$

Non-uniform Rods

What if the rod is non-uniform? We cannot assume that its centre of mass lies at the centre! We must have its weight acting at a different point...

e.g. A rod AB is 3m long and has weight 20N. It is in a horiztonal position resting on supports at points C and D, where AC = 1m and AD = 2.5m. The magnitude of the reaction at C is three times the magnitude of the reaction at D. Find the distance of the centre of mass of the rod from A.



(R↕) $3R + R = 20$

$4R = 20$

$R = 5$.

M(A) $3R \times 1 + R \times 2.5 = 20x$

$15 + 12.5 = 20x$

$\frac{27.5}{20} = x$

$x = 1.375$

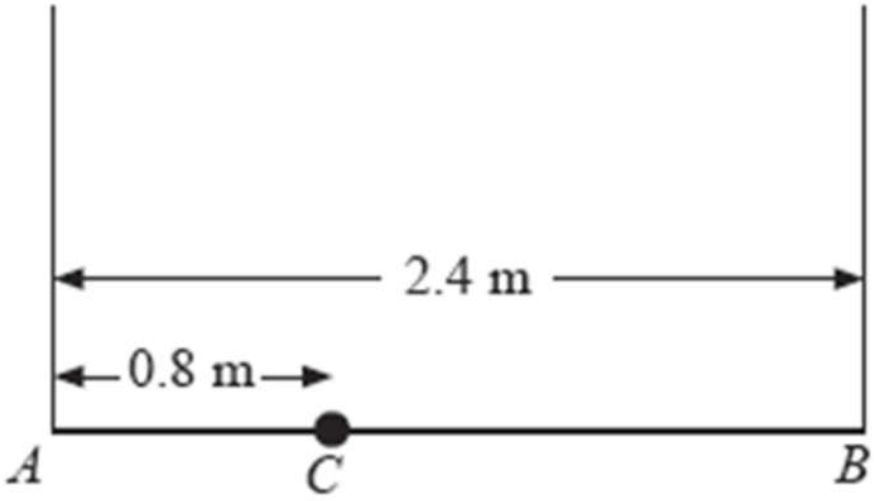
$= 1.4\text{m}$ or 1.38m (3sf).

Hint: take moments about the point where you have an unknown

May 2008

Your Turn

Non-uniform



A plank AB has mass 12 kg and length 2.4 m. A load of mass 8 kg is attached to the plank at the point C, where $AC = 0.8$ m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in Figure 2. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.

(a) Find the tension in the rope attached at B.

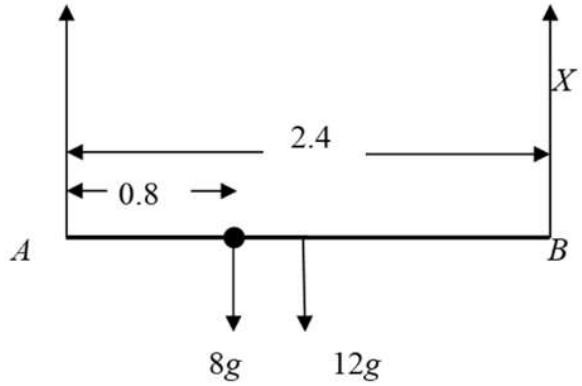
(4)

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 10 N greater than the tension in the rope attached at B.

(b) Find the distance of the centre of mass of the plank from A.

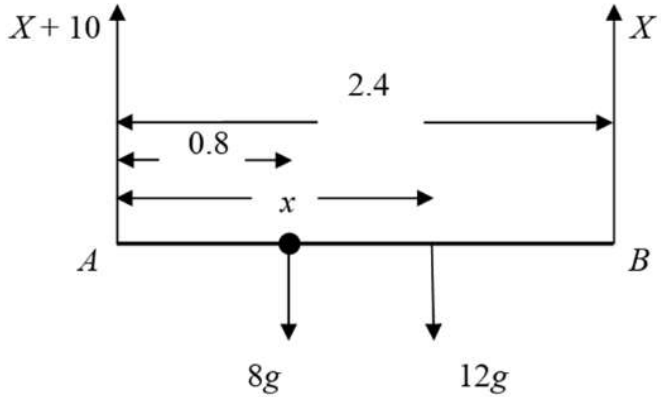
(6)

6. (a)



$M(A)$	$8g \times 0.8 + 12g \times 1.2 = X \times 2.4$			M1 A1
	$X \approx 85$ (N)	accept 84.9, $\frac{26g}{3}$		DM1 A1 (4)

(*b*)



$R(\uparrow)$	$(X+10) + X = 8g + 12g$		M1 B1 A1
	$(X = 93)$		
$M(A)$	$8g \times 0.8 + 12g \times x = X \times 2.4$		M1 A1
	$x = 1.4$ (m)	accept 1.36	A1 (6)

Jan 2012 **Your Turn**

Non-uniform

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D. $T_C = 0$

(a) Show that $GD = \frac{5}{2}d$.

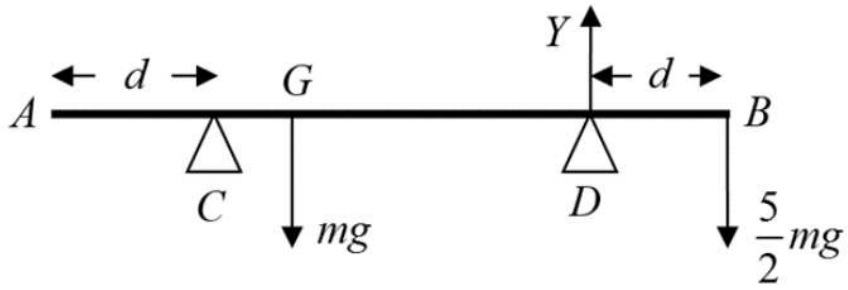
(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

4. (a)

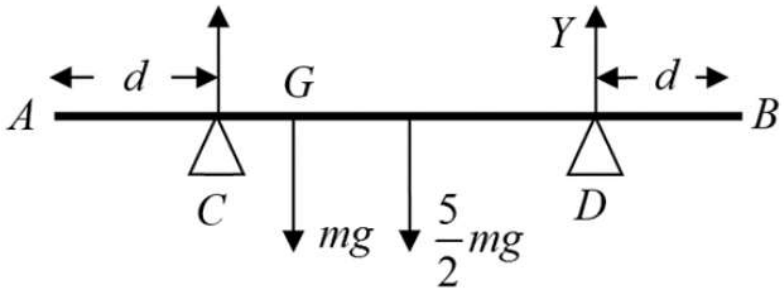


M(D) $\quad mg \times GD = \frac{5}{2} mg \times d$ — M1 A1

$GD = \frac{5}{2} d$ * — DM1 A1

(4)

(b)



M(C) $\quad mg \times \frac{d}{2} + \frac{5}{2} mg \times \frac{3}{2} d = Y \times 3d$ — M1 A2(1, 0)

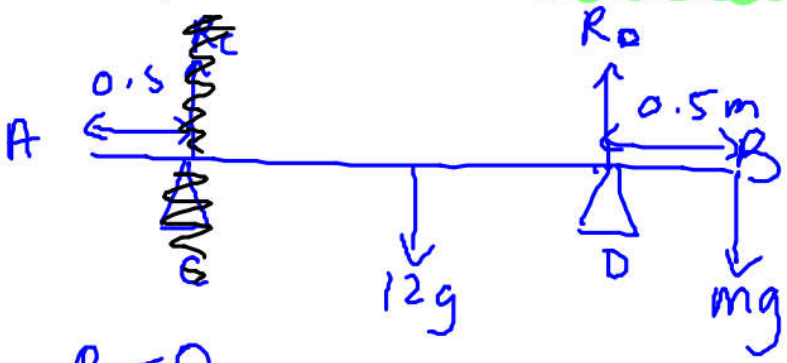
Leading to $\quad Y = \frac{17}{12} mg$ — DM1 A1

(5)

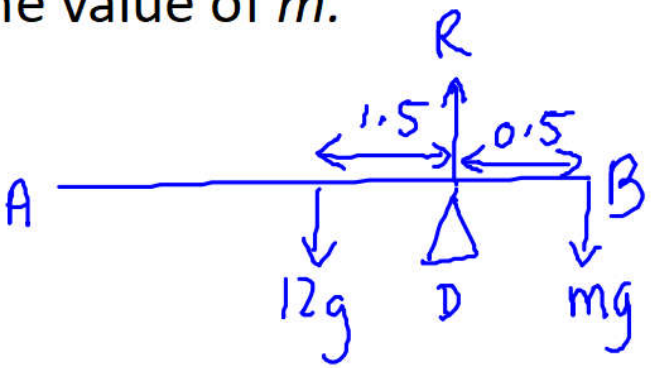
(9 marks)

On the Point of Tilting/Tipping

A uniform rod AB of length 4m and mass 12kg is resting in a horizontal position on supports at C and D, with AC=DB=0.5m.
When a particle of mass *m*kg is placed on the rod at point B the rod is on the point of turning about D. Find the value of *m*.



$R_C = 0$



M(D) $12g \times 1.5 = 0.5mg$

$18 = 0.5m$

$36 = m$

✎ When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or tension in any other wire/string) is zero.

8 A uniform rod AB has length 4 m and mass 8 kg. It is resting in a horizontal position on supports at points C and D were $AC = 1$ m and $AD = 5$ m. A particle of mass m kg is placed at point E where $AE = 3.3$ m. Given that the rod is about to tilt about D, calculate the value of m.

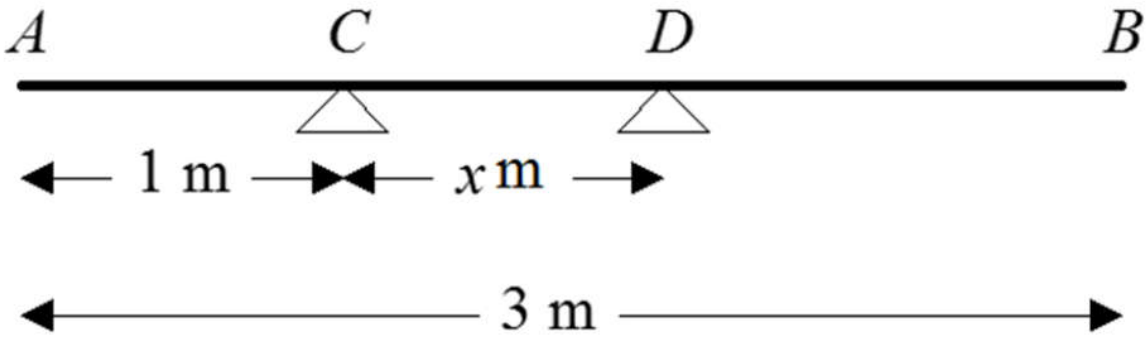
9 A uniform bar AB of length 6 m and weight 40 N is resting in a horizontal position on supports at points C and D where $AC = 2$ m and $AD = 2.5$ m. When a particle of weight 30 N is attached to the bar at point E the bar is on the point of tilting about C. Calculate the distance AE.

10 A plank AB of mass 12 kg and length 3 m is in equilibrium in a horizontal position resting on supports at C and D where $AC = 0.7$ m and $DB = 1.1$ m. A boy of mass 32 kg stands on the plank at point E. The plank is about to tilt about D. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance AE.

5, 2/3, 2.05

Jan 2007

Your Turn



On the point of tipping

A uniform plank AB has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $CD = x$ m, as shown in Figure 2. The reaction of the support on the plank at D has magnitude 80 N. Modelling the plank as a rod,

(a) show that $x = 0.75$.

(3)

A rock is now placed at B and the plank is on the point of tilting about D. Modelling the rock as a particle, find

(b) the weight of the rock,

(4)

(c) the magnitude of the reaction of the support on the plank at D.

(2)

(d) State how you have used the model of the rock as a particle.

(1)

2.	(a)	$A(C)$ $80 \times x = 120 \times 0.5$ $x = 0.75$ *		M1 A1 A1	3
	(b)	Using reaction at $C = 0$ $A(D)$ $120 \times 0.25 = W \times 1.25$ $W = 24$ (N)		B1 M1 A1 A1	4
	(c)	$R = 24 + 120 = 144$ (N)		M1 A1ft	2
	(d)	The weight of the rock acts precisely at B		B1	1 10

Your Turn

6. A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S.

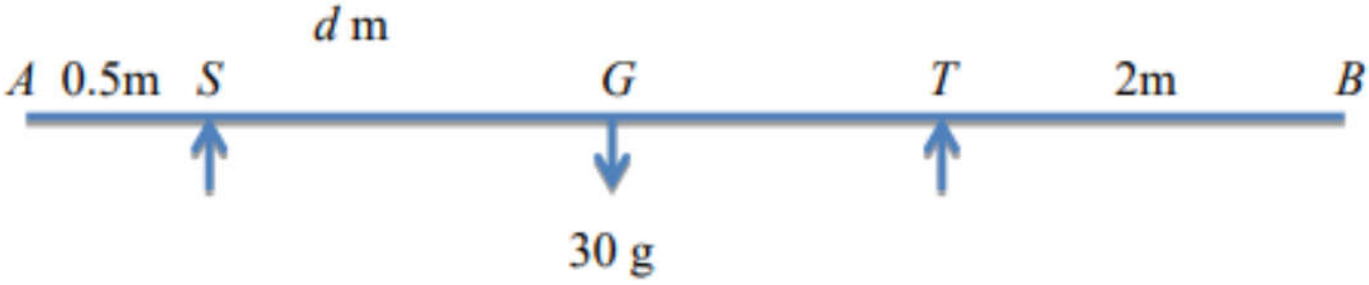
When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T.

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i) the value of d,

(ii) the value of M.

(7)



$M(S)$: $Mg \times 0.5 = 30g(d - 0.5)$ M1 A1

$M(T)$: $Mg \times 2 = 30g(4 - d)$ M1 A1

dividing: $4 = \dfrac{(4 - d)}{(d - 0.5)}$ => (i) $d = 1.2$ **DM1** A1

=> (ii) $M = 42$ A1

Your Turn

4.

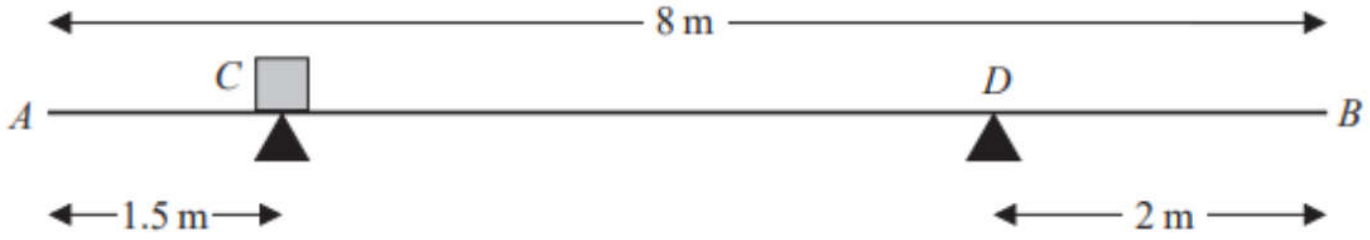


Figure 2

A plank AB of mass 20 kg and length 8 m is resting in a horizontal position on two supports at C and D, where $AC = 1.5$ m and $DB = 2$ m. A package of mass 8 kg is placed on the plank at C, as shown in Figure 2. The plank remains horizontal and in equilibrium. The plank is modelled as a uniform rod and the package is modelled as a particle.

(a) Find the magnitude of the normal reaction

(i) between the plank and the support at C,

(ii) between the plank and the support at D.

(6)

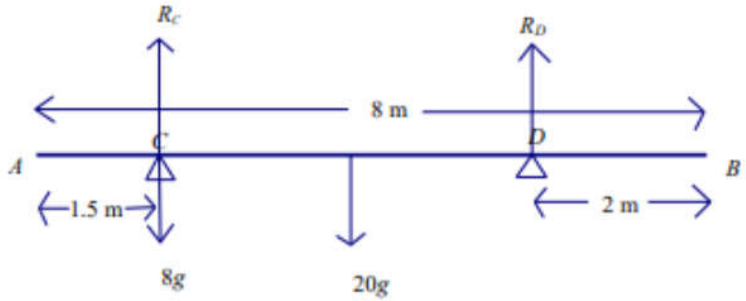
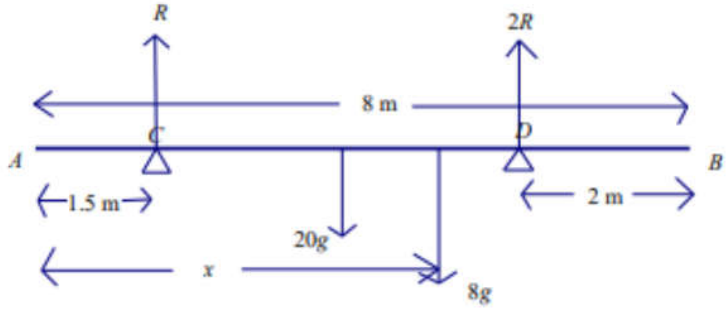
The package is now moved along the plank to the point E. When the package is at E, the magnitude of the normal reaction between the plank and the support at C is R newtons and the magnitude of the normal reaction between the plank and the support at D is $2R$ newtons.

(b) Find the distance AE.

(6)

(c) State how you have used the fact that the package is modelled as a particle.

(1)

4a		
	Moments about D: $20g\times 2+8g\times 4.5=R_C\times 4.5$ **OR** Resolve: $R_C+R_D=28g$	M1A1
(i)	$R_C=\frac{152}{9}g\,(=166 \text{ or } 170)$	A1
	Moments about C: $20g\times 2.5=R_D\times 4.5$ **OR** Resolve: $R_C+R_D=28g$	M1A1
(ii)	$R_D=\frac{100}{9}g\,(=109 \text{ or } 110)$	A1
		(6)
4b		
	Moments about A: $R\times 1.5+2R\times 6=20g\times 4+8g\times x$	M1A1
	Resolve: $3R=28g$, $\left(R=\frac{28}{3}g\,(=91.5)\right)$	M1A1
	Substitute for R and solve for x: $\frac{27}{2}\times\frac{28}{3}g=80g+8g\times x$	M1
	$126=80+8x$, $8x=46$, $x=5.75\,(\text{m})$	A1
		(6)
4c	The weight of the package acts at point C (or E)	B1 (1)
		[13]