A Level · Edexcel · Maths

4 hours · 35 questions

7.4 Further Applications of Differentiation (A Level only)

Easy (8 questions)	/48
Medium (9 questions)	/54
Hard (9 questions)	/49
Very Hard (9 questions)	/59
Total Marks	**/210**

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Easy Questions

1 (a) A curve has equation $y = x^3 + 2x + 1$.

Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(3 marks)

(b) (i) Write down the value of x for which $\frac{d^2y}{dx^2} = 0$.

(ii) By considering the sign of $\frac{d^2y}{dx^2}$ just either side of the value of x found in part (i), show that this is a point of inflection.

(3 marks)

(c) Hence, or otherwise, find the coordinates of the point of inflection on the curve with equation $y = x^3 + 2x + 1$.

(1 mark)

2 (a) Given $y = x^3 - 6x^2$ show that $\dfrac{d^2y}{dx^2} = 6x - 12$.

(2 marks)

(b) Solve the inequality $\dfrac{d^2y}{dx^2} < 0$ and hence determine the set of values for which the graph of $y = x^3 - 6x^2$ is concave.

(2 marks)

3 (a) In a computer animation, the side length, s mm, of a square is increasing at a constant rate of 2 millimetres per second.

(i) Write down the value of $\frac{\mathrm{d}s}{\mathrm{d}t}$, where t is time and measured in seconds.

(ii) Write down a formula for the area, A mm^2, of the square and hence find $\frac{\mathrm{d}A}{\mathrm{d}s}$.

(3 marks)

(b) Use the chain rule to find an expression for $\frac{\mathrm{d}A}{\mathrm{d}t}$ in terms of s and hence find the rate at which the area is increasing when $s=10$.

(3 marks)

4 (a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the point where $x = 2$ for the curve with equation $y = x^3 - 6x^2 + 9x + 4$.

(4 marks)

(b) Show that $x = 2$ is a point of inflection.

(2 marks)

(c) Explain why $x = 2$ is not a stationary point.

(1 mark)

5 The graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a continuous function has the following properties:

- The graph intercepts the x-axis at $(-a, 0)$ and the y-axis at $(0, b)$
- It is concave in the interval $(-\infty, c)$
- It is convex in the interval (c, ∞)
- a, b and c are such that $0 < a < b < c$

Sketch a graph of the function.

Label the points where the graph intercepts the coordinate axes and label the point of inflection.

(3 marks)

6 (a) The side length, x cm, of a cube increases at a constant rate of 0.1 $\text{cm } s^{-1}$.

(i) Write down the value of $\frac{\mathrm{d}x}{\mathrm{d}t}$, where t is time and measured in seconds.

(ii) Write down a formula for the volume, $V \text{ cm}^3$, of the cube and hence find $\frac{\mathrm{d}V}{\mathrm{d}x}$.

(3 marks)

(b) Use the chain rule to find an expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of x and hence find the rate at which the volume is increasing when the side length of the cube is 4 cm.

(3 marks)

7 (a) The rate at which the radius, r cm, of a sphere increases over time (t seconds) is directly proportional to the temperature (T °C) of its immediate surroundings.

Write down an equation linking $\frac{\mathrm{d}r}{\mathrm{d}t}$, T and the constant of proportionality, k.

(2 marks)

(b) When the surrounding temperature is 20 °C, the radius of the sphere is increasing at a rate of 0.4 $\mathrm{cm}\ s^{-1}$.

Find the value of k.

(3 marks)

8 (a) (i) Write down a formula for the volume of a cube, $V\ \text{cm}^3$, and the surface area, $S\ \text{cm}^2$, of a cube, in terms of the side length of a cube, x cm.

(ii) Show that $\frac{dx}{dV} = \frac{1}{3x^2}$ and find an expression for $\frac{dS}{dx}$.

(5 marks)

(b) The volume of a cube is decreasing at a constant rate of 0.6 $\text{cm}^3\ s^{-1}$.

(i) Explain why $\frac{dV}{dt}$, where t is time in seconds, has the value of -0.6.

(ii) Use the chain rule to find an expression for $\frac{dS}{dt}$ in terms of $\frac{dS}{dx}$, $\frac{dx}{dV}$ and $\frac{dV}{dt}$.

(iii) Hence write $\frac{dS}{dt}$ in terms of x and find the rate at which the area of the cube is decreasing at the instant when its side length is 5 cm.

(5 marks)

Medium Questions

1 Find the coordinates of the point of inflection on the curve with equation

$$y = x^3 + 3x^2 - 2$$

(5 marks)

2 (a) Given $y = x^2 + \ln x - 2x$, where $x > 0$, show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2 - \frac{1}{x^2}$$

(3 marks)

(b) Solve the inequality

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} > 0$$

and hence determine the set of values for which the graph of $y = x^2 + \ln x - 2x$ is convex.

(3 marks)

3 In a computer animation, the radius of a circle increases at a constant rate of 1 millimetre per second. Find the rate, per second, at which the area of the circle is increasing at the time when the radius is 8 millimetres. Give your answer as a multiple of π.

(4 marks)

4 (a) Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

(4 marks)

(b) Find the nature of the stationary points found in part (a).

(3 marks)

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

(3 marks)

(d) Explain why, in this case, the point of inflection is not a stationary point.

(1 mark)

5 The graph of a continuous function has the following properties:

The function is concave in the interval $(-\infty, a)$.

The function is convex in the interval (a, ∞).

The graph of the function intercepts the x-axis at the points $(b, 0)$, $(c, 0)$ and $(d, 0)$, where b, c and d are such that $d > c > b > 0$.

The x-coordinates of the turning points of the function are e and f, which are such that $f > e$.

The graph of the function intercepts the y-axis at $(0, g)$

Given that the value of the function is positive when $x = a$, sketch a graph of the function. Be sure to label the x-axis with the x-coordinates of the stationary points and the point of inflection, and also to label the points where the graph crosses the coordinate axes.

(4 marks)

6 The side length of a cube increases at a rate of $0.1\ \text{cm s}^{-1}$.
Find the rate of change of the volume of the cube at the instant the side length is 5 cm.
You may assume that the cube remains cubical at all times.

(5 marks)

7 (a) In the production process of a glass sphere, hot glass is blown such that the radius, r cm, increases over time (t seconds) in direct proportion to the temperature (T °C) of the glass. Find an expression, in terms of r and T, for the rate of change of the volume (V cm^3) of a glass sphere.

(5 marks)

(b) When the temperature of the glass is 1200 °C, a glass sphere has a radius of 2 cm and its volume is increasing at a rate of 5 cm^3s^{-1}.
Find the rate of increase of the radius at this time.

(3 marks)

8 An ice cube, of side length x cm, is melting at a constant rate of 0.8 cm^3 s^{-1}.

Assuming that the ice cube remains in the shape of a cube whilst it melts, find the rate at which its surface area is melting at the point when its side length is 2 cm^2.

(6 marks)

9 A bowl is in the shape of a hemisphere of radius 8 cm.

The volume of liquid in the bowl is given by the formula

$$V = 8\pi h^2 - \frac{1}{3}\pi h^3$$

where h cm is the depth of the liquid (ie the height between the bottom of the bowl and the level of the liquid).

Liquid is leaking through a small hole in the bottom of the bowl at a constant rate of 5 cm^3s^{-1}. Find the rate of change of the depth of liquid in the bowl at the instant the height of liquid is 3 cm.

(5 marks)

Hard Questions

1 Determine the number of points of inflection on the curve with equation

$$y = x^4 + 3x^3 + 2$$

and determine their coordinates.

(5 marks)

2 Find the values for which the curve defined by the function

$$fx = x^2 - e^{2x} + 1$$

is concave.

(4 marks)

3 A spherical air bubble's surface area is increasing at a constant rate of $4\pi \text{ cm}^2 s^{-1}$.
Find an expression for the rate at which the radius is increasing per second.
(The surface area of a sphere is $4\pi r^2$.)

(4 marks)

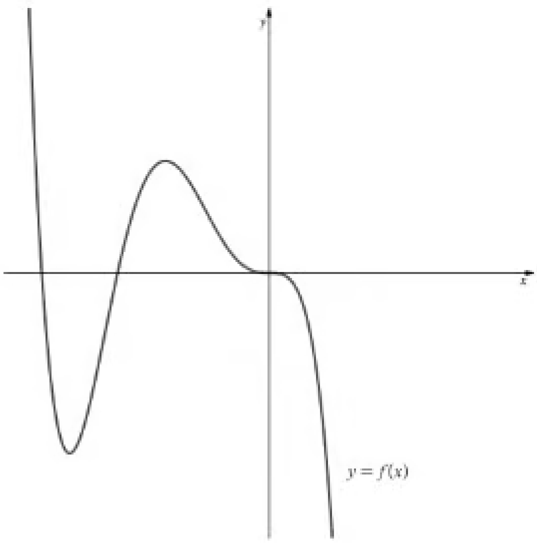
4 (a) Find the coordinates of the stationary points, and their nature, on the graph with equation $y = 4x - x^2 - 2x^3$.

(4 marks)

(b) Determine the coordinates of any points of inflection on the graph and hence state the intervals in which the graph is convex and concave.

(4 marks)

5 The diagram below shows a sketch of the graph with equation $y = \mathrm{f}(x)$.



On the sketch, mark the approximate locations of the following ...

(i) ... intercepts with the coordinate axes using the letter C

(ii) ... stationary points using the letter S

(iii) ... points of inflection using the letter I

Also highlight sections of the curve where the graph is convex.

(4 marks)

6 A cone, stood on its vertex, of radius 3 cm and height 9 cm, is being filled with sand at a constant rate of $0.2\ \mathrm{cm}^3 s^{-1}$.

Find the rate of change of the depth of sand in the cone at the instant the radius of the sand is 1.2 cm.

(6 marks)

7 The material used to make a spherical balloon is designed so that it can be inflated at a maximum rate of $16\ \mathrm{cm}^3 s^{-1}$ without bursting.

Given that the radius of the balloon is determined by the function $r(t) = \frac{t}{\pi} + \frac{1}{2}$, $t \geq 0$

show that the maximum time the balloon can be inflated for, without bursting, is $\frac{3}{2}\pi$

seconds.

(6 marks)

8 An ice lolly which is in the shape of a cylinder of radius r cm and length 8r cm is melting at a constant rate of $0.4\ \mathrm{cm^3 s^{-1}}$.

Assuming that the ice lolly remains in the shape of a cylinder (mathematically similar to the original cylinder) whilst it melts, find the rate at which its surface area is melting at the point when it’s radius is 0.3 cm.

(6 marks)

9 The volume of liquid in a hemispherical bowl is given by the formula

$$V = \frac{1}{3}\pi h^2(3R - h)$$

where R is the radius of the bowl and h is the depth of liquid

(ie the height between the bottom of the bowl and the level of the liquid).

In a particular case, a bowl is leaking liquid through a small hole in the bottom at a rate directly proportional to the depth of liquid.

Show that the depth of liquid in the bowl is decreasing by

$$\frac{k}{\pi(2R-h)}$$

where k is a constant.

(6 marks)

Very Hard Questions

1 (a) Show that there are two points of inflection on the curve with equation

$$y = x^2 e^{0.3x}$$

(5 marks)

(b) Find the coordinates of the two points of inflection on the curve with equation

$$y = x^2 e^{0.3x}$$

giving your answers to three significant figures.

(2 marks)

2 Show that the shape of the curve with equation

$$y = 2x + 4\sin 3x \qquad\qquad 0 < x < \pi$$

is convex in the interval $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$.

(5 marks)

3 A plant pot in the shape of square-based pyramid (stood on its vertex) is being filled with soil at a rate of 72 cm^3s^{-1}.
The plant pot has a height of 1 m and a base length of 40 cm.
Find the rate at which the depth of soil is increasing at the moment when the depth is 60 cm.

(The volume of a pyramid is a third of the area of the base times the height.)

(6 marks)

4 Find the coordinates of the stationary points, and their nature, on the graph with equation $y = e^{0.4x}(x^2 + 3x - 4)$.

Find the coordinates of any points of inflection, determine if these are also stationary points and find the intervals in which the graph is convex and concave.

Give values to three significant figures.

(7 marks)

5 (a) Explain why the graph of $y = 3x^3$ has a point of inflection which is also a stationary point but the the graph of $y = 3x^3 + 2x$ has a point of inflection that is not a stationary point.

(4 marks)

(b) On the same diagram, sketch the graphs of $y = 3x^3$ and $y = 3x^3 + 2x$.

(2 marks)

6 An expanding spherical air bubble has radius, r cm, at a time, t seconds, determined by the function $r(t) = 0.3 + 0.1t^2$.

The bubble will burst if the rate of expansion of its volume exceeds $4t \text{ cm}^3 \text{ s}^{-1}$.

Find, to one decimal place, the length of time the bubble expands for.

(6 marks)

7 A small conical pot, stood on its base, is being filled with salt via a small hole at its vertex. The cone has a height of 6 cm and a radius of 2 cm.

Salt is being poured into the pot at a constant rate of 0.3 $\mathrm{cm^3s^{-1}}$.
Find, to three significant figures, the rate of change in depth of the salt at the instant when the pot is half full by volume.

(7 marks)

8 (a) A large block of ice used by sculptors is in the shape of a cuboid with dimensions x m by $2x$ m by $5x$ m. The block melts uniformly with its surface area decreasing at a constant rate of k m^2 s^{-1}. You may assume that as the block melts, the shape remains mathematically similar to the original cuboid.

Show that the rate of melting, by volume, is given by

$$\frac{15kx}{34} \text{ m}^3 \text{ s}^{-1}.$$

(5 marks)

(b) In the case when $k = 0.2$, the block of ice remains solid enough to be sculpted as long as the rate of melting, by volume, does not exceed $0.05 \text{ m}^3 \text{ s}^{-1}$.

Find the value of x for the largest block of ice that can be used for ice sculpting under such conditions, giving your answer as a fraction in its lowest terms.

(3 marks)

9 The volume of liquid in a hemispherical bowl is given by the formula

$$V = \frac{1}{3}\pi h^2(3R - h)$$

where R is the radius of the bowl and h is the depth of liquid.

(ie the height between the bottom of the bowl and the level of the liquid).

In a particular case, liquid is leaking through a small hole in the bottom of a bowl at a rate directly proportional to the depth of liquid.

When the bowl is full, the rate of volume loss is equal to π.

Show that the rate of change of the depth of the liquid is inversely proportional to

$$R(h-2R)$$

(7 marks)