A-Level Mathematics

Predicted Paper 2023

Edexcel

Paper 1 – Pure Mathematics

Name ....

Date ....

2 hours allowed

Calculator Paper

Maximum Mark: 100

Grade boundaries

These are VERY rough guesses! Getting an A on this paper does not guarantee you the same mark in the exam.

- A* 75%
- A 55%
- B 45%
- C 35%
- D 25%
- E 15%

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01 Prove by contradiction that there is no greatest even integer.

[3 marks]

02 The daily world production of oil can be modelled using

$$V = 9 + 98\left(\frac{T}{28}\right)^3 - 49\left(\frac{T}{28}\right)^4$$

where V is the volume of oil in millions of barrels, and T is time in years since 1st January 2000.

a) The model is used to predict the time, T, when oil production will fall to zero.

Show that V satisfies the equation

$$T = \sqrt[3]{56T^2 + \frac{112896}{T}}$$

[3 marks]

b) Use the iterative formula $T_{n+1} = \sqrt[3]{56{T_n}^2 + \frac{112869}{T_n}}$, with $T_0 = 21$, to find the values of T_1, T_2 and T_3, giving your answers to three decimal places.

[2 marks]

c) Explain the relevance of using $T_0 = 21$.

[1 mark]

d) From 1st January 2000 the daily use of oil by one technological developing country can be modelled as

$$V = 6 \times 1.05^T$$

Use the models to show that the country's use of oil and the world production of oil will be equal during the year 2050.

[4 marks]

03 A circle with centre *C* has equation $x^2 + y^2 + 6x - 8y = 24$

a) Find the coordinates of *C* and the radius of the circle.

[3 marks]

b) The points *P* and *Q* lie on the circle, the origin is the midpoint of the chord *PQ*.

Show that *PQ* has length $n\sqrt{6}$, where n is an integer.

[5 marks]

04

a) Solve, for $-360° \leq \theta \leq 360°$, the equation

$$4sin2\theta = 6tan\theta$$

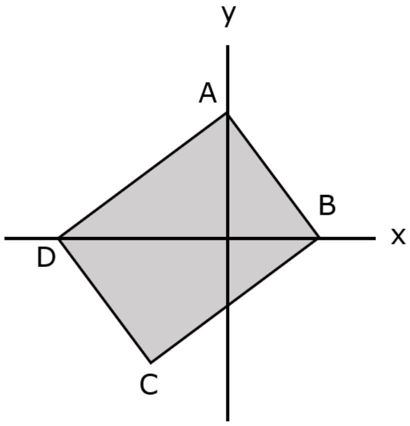
[6 marks]

b) Deduce the smallest positive solution to the equation

$$4sin(2x - 40^o) = 6tan(x - 20^o)$$

[2 marks]

05 The diagram shows a rectangle *ABCD*. The point A lies on the y-axis and the points *B* and *D* lie on the x-axis.



Given that the straight line through the points *A* and *B* has equation $2y + 3x = 24$

a) Show that the straight line through the points *A* and *D* has equation

$$3y - 2x = 36$$

[4 marks]

b) Find the area of the rectangle *ABCD*.

[3 marks]

06 Solve the equation

$$sec^2\theta = 3 - tan\ \theta$$

for values of θ between $0° \leq \theta \leq 360°$

Give solutions to 2 decimal places where necessary.

[5 marks]

07 The depth of water, D meters, in a harbour on a particular day is modelled by the formula

$$D = 6 + 2sin(20t)° \qquad 0 \leq t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:00 am and it takes 2 hours to load its cargo.

The boat requires the depth of water to be at least 5 metres before it can leave the harbour.

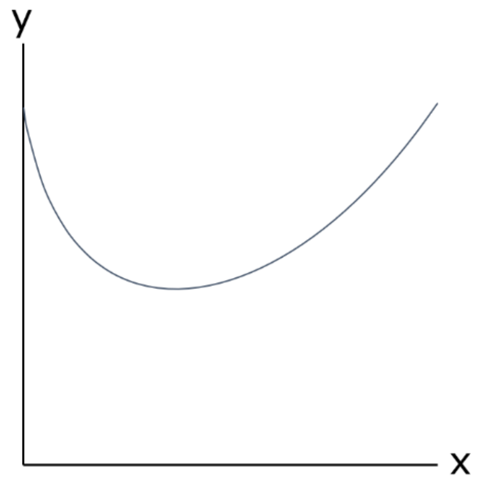
a) Find the depth of water in the harbour when the boat enters the harbour.
Give your answer to 1 decimal place.

[1 mark]

b) Find, to the nearest minute, the latest time that day that the boat can leave the harbour.

[4 marks]

08 The diagram shows a sketch of the curve C with equation $y = x^x$, $x > 0$



a) Find by taking logarithms, the x-coordinate of the turning point of C.

[5 marks]

b) The point $P(a, 3)$ lies on C.

Show that $1.8 < a < 1.9$

[2 marks]

c) A possible iteration formula that could be used in an attempt to find a is

$$x_{n+1} = 3{x_n}^{1-x_n}$$

Using this formula with $x_1 = 1.8$, find x_4 to 3 decimal places.

[2 marks]

d) Describe the long-term behaviour of the iterative formula for x_n and comment on its suitability as an estimate.

[2 marks]

09

a) Use the trapezium rule, with two strips each of equal width, to show that $\int_0^4 \frac{1}{4+\sqrt{x}}\, dx \approx \frac{5}{12} + \frac{4-\sqrt{2}}{7}$

[5 marks]

b) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{4+\sqrt{x}}\, dx$$

[6 marks]

c) Find the percentage difference between the exact value found in (b) and the estimate found from the trapezium rule in (a) for:

$$\int_0^4 \frac{1}{4+\sqrt{x}}\,dx$$

[2 marks]

10

a) Three consecutive terms in an arithmetic sequence are $4e^{-p}, 4,\ 4e^{p}$.

Find the value of p. Give your answers in an exact form.

[6 marks]

b) Prove that there are possible values of p for which $4e^{-p}$, $4,\ 4e^{p}$ are consecutive terms of a geometric sequence.

[4 marks]

11 A curve is given by the equation

$$x^3 + 3y^2 = 11$$

By using implicit differentiation find $\frac{dy}{dx}$ in terms of x and y.

[4 marks]

12

a) Show that $\frac{2x}{(x-1)(x-3)^2} = \frac{A}{(x-1)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$

where A, B and C are constants to be found.

[3 marks]

b) Evaluate

$$\int_4^6 \frac{1}{2(x-1)} - \frac{1}{2(x-3)} + \frac{3}{(x-3)^2}\, dx$$

Giving your answer in an exact simplified form.

[5 marks]

13 The height x metres, of the fireworks in a fireworks display satisfies the differential equation $\frac{dx}{dt} = \frac{6\sin 2t}{3\sqrt{x}}$, where t is the time in seconds after the display begins.

a) Solve the differential equation, given that initially the fireworks have zero height.
Express your answer in the form $x = f(t)$.

[7 marks]

b) Find the maximum height of the fireworks, giving your answer to the nearest cm.

[1 mark]

END OF QUESTIONS

MARKING GUIDANCE

1	A1M for assuming that there is a greatest positive even integer $$n = 2m$$ A1M for looking for a larger even integer $$n + 2 = 2m + 2$$ $$n + 2 = 2(m + 1)$$ A1M for stating that $n + 2$ is even because 2 is a factor and also that $n + 2 > n$ which shouldn't be possible.
2 (a)	A1M for $0 = 9 + 98\left(\frac{T}{28}\right)^3 - 49\left(\frac{T}{28}\right)^4$ A1M for $$49\left(\frac{T}{28}\right)^4 = 9 + 98\left(\frac{T}{28}\right)^3$$ $$\frac{T^4}{12544} = 9 + \frac{T^3}{224}$$ $$\frac{T^4}{12544} = \frac{9}{T} + \frac{T^3}{224}$$ A1M for $$T = \sqrt[3]{\frac{112896}{T} + 56T^2}$$
2 (b)	A1M for $T_1 = 31.097$ A1M for $T_2 = 38.660$ and $T_3 = 44.245$
2 (c)	A1M for stating that $T_0 = 21$ represents the year 2021

2 (d)	A2M for using $9+98\left(\frac{T}{28}\right)^3-49\left(\frac{T}{28}\right)^4=6\times1.05^T$ with $T=50$ A1M for $T=49.9985$ A1M for stating that this is close to 50 years which would give 2050
3 (a)	A1M for finding circle equation $(x+3)^2+(y-4)^2=49$ A1M for Centre (–3, 4) A1M for Radius 7
3 (b)	A1M for OC^2 = $(-3)^2 + (4)^2 = 25$ A2M for $OP^2 = r^2 - OC^2 = 49 - 25 = 24$ A1M for $PQ = 2OP = 2\sqrt{24}$ A1M for $PQ = 4\sqrt{6}$
4 (a)	A1M for use of trig identities $8sin\theta cos\theta=6\frac{sin\theta}{cos\theta}$ A1M for rearrangement $8cos^2\theta-6=0$ A1M for $\cos\theta=\pm\sqrt{\frac{3}{4}}$ A1M for $\sin\theta=0$, $\theta=0°,180°,360°,-180°,-360°$ A1M for $\cos\theta=\sqrt{\frac{3}{4}},\ \theta=30°,330°,-30°,-330°$ A1M for $\cos\theta=-\sqrt{\frac{3}{4}},\ \theta=150°,210°,-150°,-210°$
4 (b)	A1M for $x-20°=0°$ A1M for $x=20°$
5 (a)	A1M for gradient of *AB* $=-\frac{3}{2}$ A1M for y intercept of *AB* at (0, 12) A1M for equation of *AD* $y=\frac{2x}{3}+12$ A1M for rearrangement $3y=2x+36$

5 (b)	A1M for x intercepts $(8, 0)$ and $(-18, 0)$ A1M for lengths $\sqrt{12^2 + 8^2} = 4\sqrt{13}$ and $\sqrt{12^2 + -18^2} = 6\sqrt{13}$ A1M for $4\sqrt{13}\ \times\ 6\sqrt{13}\ = 312$
6	A1M for $(tan^2\theta + 1) = 3 - tan\,\theta$ A1M for $tan^2\theta + tan\,\theta - 2 = 0$ A1M for $(tan\,\theta - 1)(tan\,\theta + 2) = 0$ and $tan\,\theta = 1,\ tan\,\theta = -2$ A1M for $\theta =\ 45°,\ 225°$ A1M for $\theta =\ 116.57°,\ 296.57°$
7 (a)	A1M for $D = 6 + 2sin(20 \times 6)° = 7.7m$
7 (b)	A1M for $6 + 2sin(20t)° = 5m$ A1M for $sin(20t)° = -0.5$ A1M for $t = 10.5\ and\ 16.5$ A1M for 16.30 (or 4:30pm)
8 (a)	A1M for converting $y = x^x$ to $ln\,y\ =\ x\,ln\,x$ A2M for $\frac{1}{y}\frac{dy}{dx}\ =\ 1 +\ ln\,x$ A1M for using $\frac{dy}{dx} = 0$ to find $1 +\ ln\,x = 0$ and $ln\,x = -1$ A1M for $x = e^{-1}$
8 (b)	A1M for $1.8^{1.8} = 2.88065\ldots$ and $1.9^{1.9} = 3.38557\ldots$ A1M for stating that these results are on either side of 3

8 (c)	A1M for $$x_1 = 1.8$$ $$x_2 = 3(1.8)^{1-(1.8)} = 1.87457...$$ $$x_3 = 3(1.87458)^{1-(1.87458)} = 1.73159...$$ $$x_4 = 3(1.73159)^{1-(1.73159)} = 2.00758...$$ A1M for $x_4 = 2.008$
8 (d)	A1M for the answers are oscillating up and down A1M for the equation seems unsuitable as the answers are not converging between 1.8 and 1.9
9 (a)	A1M for correct form of the trapezium rule with h=2 A1M for $$\frac{2}{2}(\frac{1}{4+\sqrt{0}} + \frac{1}{4+\sqrt{4}} + 2(\frac{1}{4+\sqrt{2}}))$$ A1M for $$\frac{1}{4} + \frac{1}{6} + \frac{2}{4+\sqrt{2}}$$ A1M for $$\frac{1}{4} + \frac{1}{6} + \frac{4-\sqrt{2}}{7}$$ A1M for $$\frac{5}{12} + \frac{4-\sqrt{2}}{7}$$

9 (b)	A1M for $x = u^2$ differentiated to $dx = 2u\,du$ A1M for $\int_0^4 \frac{dx}{4+\sqrt{x}} = \int_0^2 \frac{2u}{4+u}$ A1M for $2\int_0^2 \frac{4+u-4}{4+u} du = 2\int_0^2 1 - \frac{4}{4+u}$ A1M for $2[u - 4ln(u+4)]_0^2$ A1M for $2([2 - 4ln(6)] - [0 - 4ln(4)])$ A1M for $-8\ln(6) + 8ln(4) + 4$
9 (c)	A1M for $\frac{(-8\ln(6)+8\ln(4)+4)-\left(\frac{5}{12}+\frac{4-\sqrt{2}}{7}\right)}{(-8\ln(6)+8\ln(4)+4)} \times 100$ A1M for -3.94%
10 (a)	A2M for the difference between two terms $4e^p - 4 = 4 - 4e^{-p}$ A1M for rearrangement $4e^{2p} - 8e^p + 4 = 0$ A2M for solving quadratic $e^p = 1$ A1M for $p = ln\,1 = 0$

10 (b)	A1M for assuming it is possible $a = 4e^{-p},\ ar = 4,\ ar^2 = 4e^p$ A1M for finding the ratio between two consecutive terms $\frac{ar}{a} = \frac{4}{4e^{-p}} = \frac{1}{e^{-p}}$ $\frac{ar^2}{ar} = \frac{4e^p}{4} = e^p$ A1M for making the ratios equal $\frac{1}{e^{-p}} = e^p$ A1M for rearranging to find 1=1, therefore there are possible values
11	A3M for $3x^2 + 6y\frac{dy}{dx} = 0$ A1M for $\frac{dy}{dx} = \frac{-3x^2}{6y} = -\frac{x^2}{2y}$
12 (a)	$2x = A(x-3)^2 + B(x-1)(x-3) + C(x-1)$ A1M for when $x = 3$ then $6 = 2C$ and $C = 3$ A1M for when $x = 1$ then $2 = A(-2)^2$ and $A = \frac{1}{2}$ A1M for $B = -\frac{1}{2}$
12 (b)	A3M for integration $\left[\frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x-3) - \frac{3}{x-3}\right]_4^6$ A1M for correct substitution of limits: $\left(\frac{1}{2}\ln 5 - \frac{1}{2}\ln 3 - 1\right) - \left(\frac{1}{2}\ln 3 - \frac{1}{2}\ln 1 - 3\right)$ A1M for exact value $\frac{1}{2}\ln 5 - \ln 3 + 2$

13 (a)	A1M for $3\sqrt{x}\frac{dx}{dt} = 6\sin 2t$ A1M for $\int 3\sqrt{x}\, dx = \int 6\sin 2t\ dt$ A1M for $\int 3x^{\frac{1}{2}}\, dx = \int 6\sin 2t\ dt$ A1M for integration $2x^{\frac{3}{2}} = -3\cos 2t\ +\ c$ A1M for substitution of initial conditions $2(0)^{\frac{3}{2}} = -3\cos(2(0))\ +\ c$ $c = 3$ A1M for $2x^{\frac{3}{2}} = -3\cos 2t\ +\ 3$ A1M for $= (\frac{3}{2} - \frac{3}{2}\cos 2t)^{\frac{2}{3}}$
13 (b)	$A1M\ for\ \cos 2t = -1 \quad x = \left(\frac{3}{2} + \frac{3}{2}\right)^{\frac{2}{3}} = 208cm$