A Level · Edexcel · Maths

3 hours 33 questions

4.4 Geometric Sequences & Series (A Level only)

Easy (9 questions)	/37
Medium (8 questions)	/45
Hard (8 questions)	/46
Very Hard (8 questions)	/56
Total Marks	**/184**

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Easy Questions

1 Identify which of the following are geometric sequences.
For those that are, write down the first term and the common ratio.

(i) $3,8,13,18,...$
(ii) $5,15,45,135,...$
(iii) $5,-10,20,-40,...$
(iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, ...$

(4 marks)

2 (a) Evaluate

$$\sum_{r=1}^{5} 3(2^r)$$

(2 marks)

(b) Evaluate.

$$\sum_{r=4}^{8} (-1)^r(2^r)$$

(2 marks)

3 Write down a formula for the n^{th} term of each of the following geometric sequences

(i) $3, 12, 48, 192,...$

(ii) First term: $a = 5$

Common ratio: $r = -2$

(iii) $a = 16, \ r = \frac{1}{2}$

(3 marks)

4 Find the 5^{th} and 10^{th} terms in each of the following geometric sequences

(i) $u_n = 2(3)^n$

(ii) $u_n = 10\,000(1.02)^n$

(iii)

$u_n = 3^{-n}$

(3 marks)

5 (a) The 3^{rd} and 6^{th} terms of a geometric sequence are 10 and 270 respectively, Find the first term and the common ratio.

(3 marks)

(b) The 12^{th} term of a geometric sequence is 16 times greater than the 8^{th} term. Find the possible values of the common ratio.

(2 marks)

6 (a) Find the sum of the first 12 terms of the geometric series that has first term 5 and common ratio $\frac{3}{2}$, giving your answer to the nearest whole number.

(2 marks)

(b) Find the sum to infinity of the geometric series that has first term 4 and common ratio $\frac{1}{8}$.

(2 marks)

7 (a) The first term of a geometric sequence is 2.

The 6^{th} term of the sequence is 486.

The sum of the first terms is 177 146.

Find the common ratio.

(2 marks)

(b) Show that $3^n = 177\ 147$.

(2 marks)

(c) Hence find the value of n.

(2 marks)

8 (a) The first term of a geometric sequence is 6.

The sum to infinity is 8.

Show that the common ratio is 0.25.

(2 marks)

(b) Briefly explain why the geometric sequence with first term 6 and common ratio 0.25 has a sum to infinity.

(1 mark)

9 (a) A geometric series is given by

$$k(k+1)+k(k+1)^2+k(k+1)^3+k(k+1)^4+\cdots$$

where k is a constant such that $|k+1|<1$.

Write down a formula for the n^{th} term of the series, in terms of k.

(1 mark)

(b) Show that the sum to infinity is $-(k+1)$.

(2 marks)

(c) The sum to infinity is $-\frac{1}{4}$. Find the value of k.

(2 marks)

Medium Questions

1 (a) The first three terms of a geometric sequence are given by x^2, $4x$, and $x + 14$ respectively, where $x > 0$.

Show that $x^3 - 2x^2 = 0$.

(2 marks)

(b) Find the value of the 15th term of the sequence.

(3 marks)

(c) State, with a reason, whether 8192 is a term in the sequence.

(1 mark)

2 (a) A geometric sequence has first term 900 and a common ratio r where $r > 0$. The 18th term of the sequence is 18.

Show that r satisfies the equation $17\log r + \log 50 = 0$.

(3 marks)

(b) Hence or otherwise find the value of r correct to 3 significant figures.

(1 mark)

3 A geometric series has first term 19 and common ratio $\frac{2}{3}$.

Given that the sum of the first k terms of the series is greater than 56

(i) show that $k > \dfrac{\log\left(\frac{1}{57}\right)}{\log\left(\frac{2}{3}\right)}$

(ii) hence find the smallest possible value of k.

(5 marks)

4 (a) The sum of the first two terms in a geometric series is 9.31.

The sum of the first four terms in the same series is 11.02.

The common ratio of the series is r.

Show that $\frac{1-r^4}{1-r^2} = \frac{58}{49}$.

(2 marks)

(b) Hence find the two possible values of r.

(2 marks)

5 The first term of a geometric series is a, and its common ratio is 5. A different geometric series has first term b and common ratio 3. The sum of the first three terms of both series is the same.

Find the value of $\frac{a}{b}$, giving your answer as a fraction in simplest terms.

(4 marks)

6 (a) The first three terms in a geometric series are $(k-3)$, k, $(2k+8)$, where $k>0$ is a constant.

(i) Show that $k^2+2k-24=0$.

(ii) Hence find the value of k.

(5 marks)

(b) Find the common ratio, r, of this series.

(1 mark)

(c) Find the sum of the first 12 terms in this series.

(2 marks)

7 (a) Given that the geometric series $-1+3x-9x^2+27x^3+...$ is convergent

find the range of possible values of x.

(3 marks)

(b) find an expression for the sum to infinity, S_∞, in terms of x.

(1 mark)

8 (a) A convergent geometric series has first term 64, and the sum to infinity of the series is 384.

Show that the common ratio, r, of the series is $\frac{5}{6}$.

(2 marks)

(b) Find the difference between the ninth and tenth terms of the series, giving your answer correct to 3 significant figures.

(2 marks)

(c) Calculate the sum of the first eight terms in the series, giving your answer correct to 3 significant figures.

(2 marks)

(d) Given that the sum of the first k terms of the series is greater than 380, find the smallest possible value of k.

(4 marks)

Hard Questions

1 The first three terms of a geometric sequence are given by $x + 12$, $3x$, and x^2 respectively, where x is a non-zero real number.

Find the value of the 102nd term in the sequence.

(5 marks)

2 (a) The second term of a geometric sequence is 4, and the sixteenth term of the same sequence is 9. The common ratio is r, where $r > 0$.

Show that r satisfies the equation, $14 \ln r + \ln\left(\frac{4}{9}\right) = 0$.

(3 marks)

(b) Hence or otherwise find the value of r correct to 3 significant figures.

(1 mark)

3 A geometric series has first term 14 and common ratio $\frac{99}{100}$.

Given that the sum of the first k terms of the series is less than 1000, find the largest possible value of k.

(5 marks)

4 The sum of the first three terms in a geometric series is 8.75.

The sum of the first six terms in the same series is 13.23.

Find the common ratio, r, of the series.

(4 marks)

5 A geometric series has first term a and common ratio $\sqrt{5}$.

Show that the sum of the first ten terms of the series is equal to $ka(\sqrt{5}+1)$, where k is a positive integer to be determined.

(4 marks)

6 (a) The first three terms in a geometric series are $(2k+3)$, k , $(k-2)$, where $k<0$ is a constant.

Find the value of k.

(5 marks)

(b) Find the sum of the first 12 terms in this series.

(3 marks)

7 (a) The second and fifth terms of a geometric series are 13.44 and 5.67 respectively. The series has first term a and common ratio r.

By first determining the values of a and r, calculate the sum to infinity of the series.

(6 marks)

(b) Calculate the difference between the sum to infinity of the series and the sum of the first 20 terms of the series. Give your answer accurate to 2 decimal places.

(2 marks)

8 (a) A geometric series has first term 9, and the sum of the first three terms of the series is 19. The common ratio of the series is r.

Show that $9r^2 + 9r - 10 = 0$.

(3 marks)

(b) Find the two possible values of r.

(2 marks)

(c) Given that the series converges, find the sum to infinity of the series.

(3 marks)

Very Hard Questions

1 The first three terms of a geometric sequence are given by $x + 11$, $5x$, and $3x^2$ respectively, where is a non-zero real number.

Find the value of the sixth term in the sequence, giving your answer as a fraction.

(5 marks)

2 (a) The third term of a geometric sequence is 12, and the nineteenth term of the same sequence is 47. The common ratio of the sequence is r.

Show that one possible value of r satisfies the equation $16 \ln r + \ln\left(\frac{12}{47}\right) = 0$.

(3 marks)

(b) By solving the equation in (a), find the corresponding value of r correct to 3 significant figures.

(1 mark)

(c) Find the other possible value of r (also correct to 3 significant figures), and explain why it is not a valid solution to the equation in (a).

(2 marks)

3 A geometric series has second term 648 and fifth term 375.

Find the smallest value of k such that the sum of the first k terms of the series is greater than 4660.

(6 marks)

4 The sum of the first four terms in a geometric series is 27.2, and the sum of the first eight terms in the same series is 164.9.

Given that the first term of the series is positive, find the common ratio, r, of the series.

(5 marks)

5 A geometric series has first term a, and its terms are connected by the relationship $u_{n+4} = 9u_n$ for all $n \geq 1$.

Given that all the terms of the series are positive, show that the sum of the first twelve terms of the series may be written in the form

$$S_{12} = ka(\sqrt{n} + 1)$$

where k and n are positive integers and $\sqrt{n}$ is a surd.

(4 marks)

6 (a) The first three terms in a geometric series are $(2k+6)$, k, $(k-4)$, where k is a constant.

Find the possible values of k.

(4 marks)

(b) Given that the sum to infinity exists, find the sum to infinity of this series.

(4 marks)

7 (a) The second and third terms of a convergent geometric series are $(x-1)$ and (x^2-1), where x is a real number not equal to 1 or -1.

Find the range of possible values of x.

(5 marks)

(b) Given that the sum to infinity of the series is -6,

find the two possible values of x.

(4 marks)

8 (a) The geometric series $S = u_1 + u_2 + u_3 + ... + u_n + ...$ is convergent, and the sum to infinity of the series is S_∞. The first term of the series is a, and the common ratio is r.

A different series $T = u_1^2 + u_2^2 + u_3^2 + ... + u_n^2 + ...$ is formed by squaring all the terms of the series S above.

Show that $T = u_1^2 + u_2^2 + u_3^2 + ... + u_n^2 + ...$ is also a convergent geometric series.

(4 marks)

(b) The sum to infinity of the series T is T_∞.

Express the ratio $\frac{T_\infty}{S_\infty}$ in terms of a and r, simplifying your answer as far as possible.

(3 marks)

(c) Show that if $T_\infty = S_\infty$, then $u_k^2 = u_{2k-1} + u_{2k}$ for all $k \geq 1$. Comment on what this shows about the relationship between the terms of the two series.

(6 marks)