

# Elastic Collisions in 2D

Oblique Collisions with a Surface / Oblique Collisions of Two Spheres / Problem Solving with Oblique Collisions

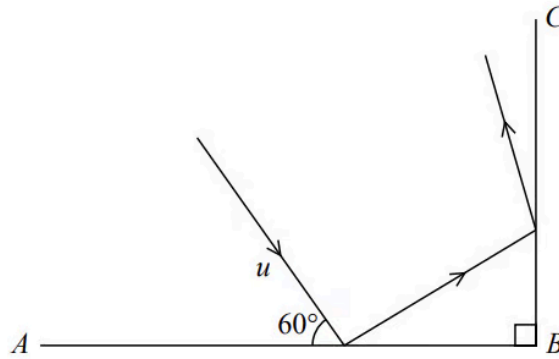
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Total Marks

/108



1 (a)

Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where  $AB$  and  $BC$  are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards  $AB$  with speed  $u \text{ ms}^{-1}$  on a path at an angle of  $60^\circ$  to  $AB$ . The ball hits  $AB$  and then hits  $BC$ .

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall  $AB$  is  $\frac{1}{\sqrt{3}}$ .

The coefficient of restitution between the ball and wall  $BC$  is  $\sqrt{\frac{2}{5}}$ .

Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

**(8 marks)**

- (b)** In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

**(1 mark)**

**2 (a)** [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere  $A$  has mass  $2m$  kg and another smooth uniform sphere  $B$ , with the same radius as  $A$ , has mass  $3m$  kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of  $A$  is  $(3\mathbf{i} + 3\mathbf{j}) \text{ ms}^{-1}$  and the velocity of  $B$  is  $(-5\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ .

At the instant of collision, the line joining the centres of the spheres is parallel to  $\mathbf{i}$ .

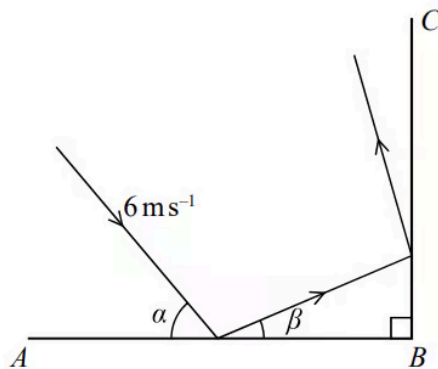
The coefficient of restitution between the spheres is  $\frac{1}{4}$ .

Find the velocity of  $B$  immediately after the collision.

**(7 marks)**

**(b)** Find, to the nearest degree, the size of the angle through which the direction of motion of  $B$  is deflected as a result of the collision.

**(2 marks)**



3 (a)

Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where  $AB$  and  $BC$  are fixed vertical walls with  $AB$  perpendicular to  $BC$ .

A small ball is projected along the floor towards  $AB$  with speed  $6 \text{ ms}^{-1}$  on a path that makes an angle  $\alpha$  with  $AB$ , where  $\tan \alpha = \frac{4}{3}$ . The ball hits  $AB$  and then hits  $BC$ .

Immediately after hitting  $AB$ , the ball is moving at an angle  $\beta$  to  $AB$ , where  $\tan \beta = \frac{1}{3}$ .

The coefficient of restitution between the ball and  $AB$  is  $e$ .

The coefficient of restitution between the ball and  $BC$  is  $\frac{1}{2}$ .

By modelling the ball as a particle and the floor and walls as being smooth,

show that the value  $e = \frac{1}{4}$ .

**(5 marks)**

**(b)** Find the speed of the ball immediately after it hits  $BC$ .

**(4 marks)**

**(c)** Suggest two ways in which the model could be refined to make it more realistic.

**(2 marks)**

**4 (a)** [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere  $A$  has mass  $0.2\text{ kg}$  and another smooth uniform sphere  $B$ , with the same radius as  $A$ , has mass  $0.4\text{ kg}$ .

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of  $A$  is  $(3\mathbf{i} + 2\mathbf{j})\text{ ms}^{-1}$  and the velocity of  $B$  is  $(-4\mathbf{i} - \mathbf{j})\text{ ms}^{-1}$ .

At the instant of collision, the line joining the centres of the spheres is parallel to  $\mathbf{i}$ .

The coefficient of restitution between the spheres is  $\frac{3}{7}$ .

Find the velocity of  $A$  immediately after the collision.

**(7 marks)**

**(b)** Find the magnitude of the impulse received by  $A$  in the collision.

**(2 marks)**

**(c)** Find, to the nearest degree, the size of the angle through which the direction of motion of  $A$  is deflected as a result of the collision.

**(3 marks)**



5 (a) [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular unit vectors in a horizontal plane.]

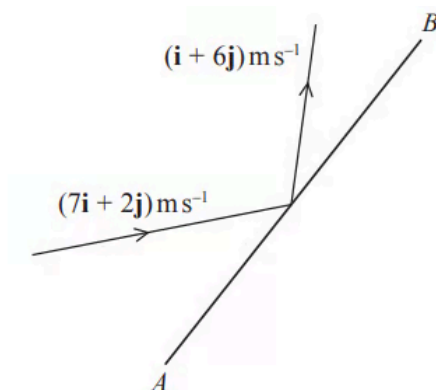


Figure 1

Figure 1 represents the plan view of part of a smooth horizontal floor, where  $AB$  represents a fixed smooth vertical wall.

A small ball of mass 0.5 kg is moving on the floor when it strikes the wall.

Immediately before the impact the velocity of the ball is  $(7\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ .

Immediately after the impact the velocity of the ball is  $(\mathbf{i} + 6\mathbf{j}) \text{ ms}^{-1}$ .

The coefficient of restitution between the ball and the wall is  $e$ .

Show that  $AB$  is parallel to  $(2\mathbf{i} + 3\mathbf{j})$ .

(4 marks)

(b) Find the value of  $e$ .

**(5 marks)**

- 6 (a)** A smooth uniform sphere  $P$  has mass 0.3 kg. Another smooth uniform sphere  $Q$ , with the same radius as  $P$ , has mass 0.2 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of  $P$  is  $(4\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$  and the velocity of  $Q$  is  $(-3\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$ .

At the instant of collision, the line joining the centres of the spheres is parallel to  $\mathbf{i}$ .

The kinetic energy of  $Q$  immediately after the collision is half the kinetic energy of  $Q$  immediately before the collision.

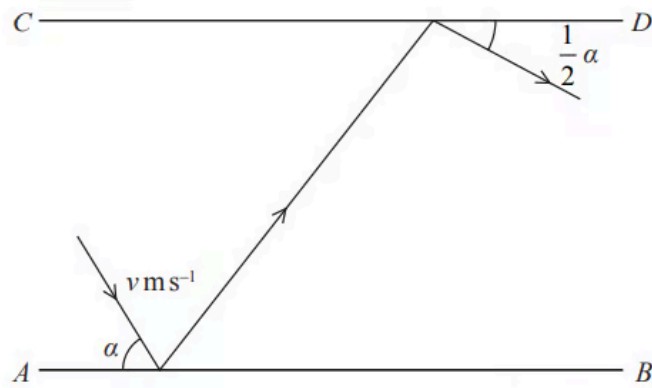
Find

- i) the velocity of  $P$  immediately after the collision,
- ii) the velocity of  $Q$  immediately after the collision,
- iii) the coefficient of restitution between  $P$  and  $Q$ ,  
carefully justifying your answers.

**(11 marks)**

- (b) Find the size of the angle through which the direction of motion of  $P$  is deflected by the collision.

(3 marks)



7 (a)

**Figure 2**

Figure 2 represents the plan view of part of a horizontal floor, where  $AB$  and  $CD$  represent fixed vertical walls, with  $AB$  parallel to  $CD$ .

A small ball is projected along the floor towards wall  $AB$ . Immediately before hitting wall  $AB$ , the ball is moving with speed  $v \text{ ms}^{-1}$  at an angle  $\alpha$  to  $AB$ , where  $0 < \alpha < \frac{\pi}{2}$ .

The ball hits wall  $AB$  and then hits wall  $CD$ .

After the impact with wall  $CD$ , the ball is moving at angle  $\frac{1}{2}\alpha$  to  $CD$ .

The coefficient of restitution between the ball and wall  $AB$  is  $\frac{2}{3}$ .

The coefficient of restitution between the ball and wall  $CD$  is also  $\frac{2}{3}$ .

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

Show that  $\tan\left(\frac{1}{2}\alpha\right) = \frac{1}{3}$ .

**(7 marks)**

- (b)** Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts.

**(4 marks)**

**8 (a)** [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere  $P$  has mass 0.3 kg. Another smooth uniform sphere  $Q$ , with the same radius as  $P$ , has mass 0.5 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of  $P$  is  $(u\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ , where  $u$  is a positive constant, and the velocity of  $Q$  is  $(-4\mathbf{i} + 3\mathbf{j}) \text{ ms}^{-1}$ .

At the instant when the spheres collide, the line joining their centres is parallel to  $\mathbf{i}$ .

The coefficient of restitution between  $P$  and  $Q$  is  $\frac{3}{5}$ .

As a result of the collision, the direction of motion of  $P$  is deflected through an angle of  $90^\circ$  and the direction of motion of  $Q$  is deflected through an angle of  $\alpha^\circ$

Find the value of  $u$ .

**(8 marks)**

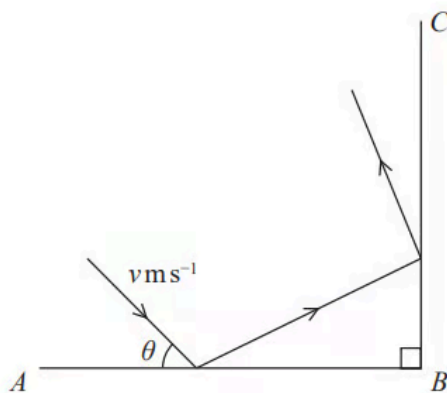
**(b)** Find the value of  $\alpha$ .

**(5 marks)**

**(c)** State how you have used the fact that  $P$  and  $Q$  have equal radii.

**(1 mark)**





9 (a)

Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where  $AB$  and  $BC$  represent fixed vertical walls, with  $AB$  perpendicular to  $BC$ .

A small ball is projected along the floor towards the wall  $AB$ . Immediately before hitting the wall  $AB$  the ball is moving with speed  $v \text{ ms}^{-1}$  at an angle  $\theta$  to  $AB$ .

The ball hits the wall  $AB$  and then hits the wall  $BC$ .

The coefficient of restitution between the ball and the wall  $AB$  is  $\frac{1}{3}$ .

The coefficient of restitution between the ball and the wall  $BC$  is  $e$ .

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall  $AB$ .

Find the exact value of  $\cos \theta$ .

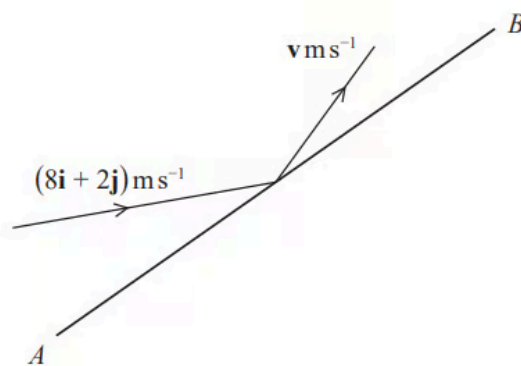
**(5 marks)**

**(b)** The ball loses half of its remaining kinetic energy in the impact with the wall  $BC$ .

Find the exact value of  $e$ .

**(5 marks)**

**10 (a)** [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular unit vectors in a horizontal plane.]



**Figure 3**

Figure 3 represents the plan view of part of a smooth horizontal floor, where  $AB$  is a fixed smooth vertical wall.

The direction of  $\overrightarrow{AB}$  is in the direction of the vector  $(\mathbf{i} + \mathbf{j})$ .

A small ball of mass 0.25 kg is moving on the floor when it strikes the wall  $AB$ .

Immediately before its impact with the wall  $AB$ , the velocity of the ball is  $(8\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ .

Immediately after its impact with the wall  $AB$ , the velocity of the ball is  $\mathbf{v} \text{ ms}^{-1}$ .

The coefficient of restitution between the ball and the wall is  $\frac{1}{3}$ .

By modelling the ball as a particle,

show that  $\mathbf{v} = (4\mathbf{i} + 6\mathbf{j})$ .

**(6 marks)**

**(b)** Find the magnitude of the impulse received by the ball in the impact.

**(3 marks)**