# Differentiate with respect to x

$f(x) = \ln(\sec x)$

$f'(x) = \frac{1}{\sec x} \times \sec x \tan x = \tan x$

$\int \tan x \, dx = \ln|\sec x| + c$

$g(x) = \sin^4 x$

$= (\sin x)^4$

$g'(x) = 4\sin^3 x \cos x$

$h(x) = \tan x \sin x$

$u = \tan x \quad v = \sin x$

$u' = \sec^2 x \quad v' = \cos x$

$h'(x) = \sin x \sec^2 x + \tan x \cos x$

$= \tan x \sec x + \sin x$

$m(x) = 3x^2 \cot x$

$u = 3x^2 \quad v = \cot x$

$u' = 6x \quad v' = -\operatorname{cosec}^2 x$

$m'(x) = 6x \cot x - 3x^2 \operatorname{cosec}^2 x$

$n(x) = e^{\operatorname{cosec} 2x}$

$n'(x) = e^{\operatorname{cosec} 2x} \times -2\operatorname{cosec} x \cot x$

Often in exam questions, you will be given $x$ in terms of $y$, but want to find $\frac{dy}{dx}$ in terms of $x$.

**The key is to make use of an appropriate trig identity, e.g:**

$$\sin^2 x + \cos^2 x \equiv 1 \qquad 1 + \tan^2 x \equiv \sec^2 x$$

$\tan^2 y = \sec^2 y - 1$

Given that $x = \tan y$, express $\frac{dy}{dx}$ in terms of $x$.

$x = \tan y$ } diff. w.r.t. $y$

$\frac{dx}{dy} = \sec^2 y$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$ ☆ becomes in terms of $x$

$\frac{dy}{dx} = \frac{1}{x^2+1}$

$x = \tan y$

$x^2 = \tan^2 y$

$x^2 = \sec^2 y - 1$

$x^2 + 1 = \sec^2 y$

$\frac{1}{x^2+1} = \frac{1}{\sec^2 y}$

**Your turn**: given that $x = 2\sin y$, express $dy/dx$ in terms of $x$

$x = 2\sin y$

$\frac{dx}{dy} = 2\cos y$

$\frac{dy}{dx} = \frac{1}{2\cos y}$

$x = 2\sin y$

$\frac{x}{2} = \sin y$

$\frac{x^2}{4} = \sin^2 y$

$\frac{x^2}{4} = 1 - \cos^2 y$

$\cos^2 y = 1 - \frac{x^2}{4}$

$\cos y = \sqrt{1 - \frac{x^2}{4}}$

$\sin^2 y + \cos^2 y = 1$

$2\cos y = 2\sqrt{1 - \frac{x^2}{4}}$

$\frac{1}{2\cos y} = \frac{1}{2\sqrt{1 - \frac{x^2}{4}}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{1 - \frac{x^2}{4}}}$ ✓✓

$= \frac{1}{2\sqrt{\frac{4-x^2}{4}}} = \frac{\cancel{2}\sqrt{4-x^2}}{\cancel{2}}$

$= \frac{1}{\sqrt{4-x^2}}$

more simplified

# Parametric Differentiation

✎ If $x$ and $y$ are given as functions of a parameter $t$, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{dy}{\cancel{dt}} \times \frac{\cancel{dt}}{dx}$$

$$= \frac{dy}{dx}$$

Find the gradient at the point $P$ where $t = 2$, on the curve given parametrically by

$x = t^3 + t, \qquad y = t^2 + 1, \qquad t \in \mathbb{R}$

$$x = t^3 + t \qquad y = t^2 + 1$$

$$\frac{dx}{dt} = 3t^2 + 1 \qquad \frac{dy}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{2t}{3t^2+1} \qquad t = 2 \qquad \frac{dy}{dx} = \frac{2 \times 2}{3 \times 2^2 + 1} = \frac{4}{13}$$

Find the equation of the normal at the point $P$ where $\theta = \frac{\pi}{6}$, to the curve with parametric equations $x = 3\sin\theta$, $y = 5\cos\theta$

$x = 3\sin\theta \qquad y = 5\cos\theta$

$\frac{dx}{d\theta} = 3\cos\theta \qquad \frac{dy}{d\theta} = -5\sin\theta$

$\frac{dy}{dx} = -\frac{5\sin\theta}{3\cos\theta}$

$\frac{dy}{dx} = -\frac{5}{3}\tan\theta$

Find m and a coordinate.

P $x = 3\sin\frac{\pi}{6} = \frac{3}{2} \qquad y = 5\cos\frac{\pi}{6} = \frac{5\sqrt{3}}{2}$

$\frac{dy}{dx} = -\frac{5}{3}\tan\frac{\pi}{6}$

$= -\frac{5}{3} \times \frac{\sqrt{3}}{3} = -\frac{5\sqrt{3}}{9}$

normal gradient $\frac{9}{5\sqrt{3}}$

$y - y_1 = m(x - x_1)$

$y - \frac{5\sqrt{3}}{2} = \frac{9}{5\sqrt{3}}\left(x - \frac{3}{2}\right)$

This is the normal line's equation.

Ex 9G

Q2, 4, 6, 8, 10

$x = 3 - 2\sin t \qquad y = t\cos t$

PRODUCT RULE.