Set A

Please check the examination details below before entering your candidate information

Candidate surname	Other names
KHAMBHAITA	PRAVEET

MME Edexcel
Level 3 GCE

Centre Number: 1 7 7 2 1

Candidate Number: – – – –

MME Edexcel Practice Papers

Morning (Time: 2 hours)	Paper Reference **2MME**

Mathematics
Advanced
Paper 2: Pure Mathematics 2

Worked Solutions

All content is provided as is, without any warranty. I do not take liability for incorrect answers/working.

You must have:
Mathematical Formulae and Statistical Tables, Calculator

Total Marks: 100

Candidates may use any approved calculator.
Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. **Figure 1** shows a sketch of the curve C with equation $y = h(x)$,

$$h(x) = 1 + \sqrt{x+3}, \qquad x \geq -3$$

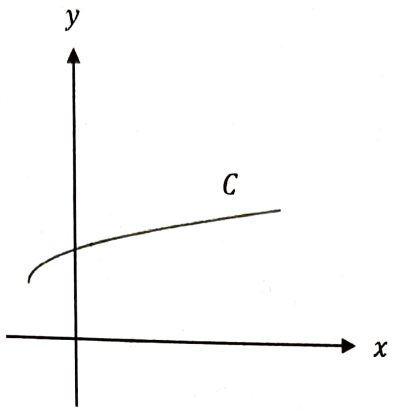

Figure 1

(a) State the range of $h(x)$.

(1)

(b) Find $h^{-1}(x)$ and state its domain.

(3)

(c) Find the exact value of x that satisfies the equation,

$$h(x) = x$$

(4)

(d) State the value of m such that,

$$h(m) = h^{-1}(m)$$

(1)

a) $x = -3 \rightarrow y = 1$ Range: $y \geq 1$

b) $x = 1 + \sqrt{y+3}$

$x - 1 = \sqrt{y+3}$

$(x-1)^2 - 3 = y$ $\rightarrow$ domain of $h^{-1}(x)$ = range of $h(x) \rightarrow x \geq 1$

$h^{-1}(x) = (x-1)^2 - 3$

d) $m = \frac{3 \pm \sqrt{17}}{2}$

c) $1 + \sqrt{x+3} = x$

$x + 3 = (x-1)^2 \rightarrow x + 3 = x^2 - 2x + 1$

$x^3 - 3x - 2 = 0 \longrightarrow x = \frac{3 \pm \sqrt{17}}{2}$

2. Relative to a fixed origin, O, the position vectors A, B and C are,

$$A: \left(ai + \sqrt{2}j + 5k\right) \qquad B: (2i - 3j - 4k) \qquad C: (bi + 3j - 2k)$$

Where a and b are constants and $a > 0$ and $b < 0$

Given that, $|\overrightarrow{OA}| = 6$

(a) Find the value of a.

(3)

(b) D is the position vector such that $\overrightarrow{AB} = \overrightarrow{BD}$

Find the position vector D.

(2)

(c) Given $|\overrightarrow{BC}| = 7$, find the value of b.

(2)

a) $|\overrightarrow{OA}| \Rightarrow a^2 + 2 + 25 = 36$

$a^2 = 9$

$a = \pm 3 \rightarrow a = 3$ as $a > 0$

b) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -i + \left(-3-\sqrt{2}\right)j - 9k$

$\overrightarrow{AB} = \overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB}$

$\overrightarrow{BD} + \overrightarrow{OB} = \overrightarrow{OD} = i + \left(-6-\sqrt{2}\right)j - 13k$

c) $|\overrightarrow{BC}| = (b-2)^2 + 6^2 + 2^2 = 7^2$

$(b-2)^2 = 9$

$b^2 - 4b + 4 = 9$

$b^2 - 4b - 5 = 0$

$(b+1)(b-5) = 0$

$b = -1 \;/\; \cancel{b=5}$

$b < 0$

$b = -1$

(Total for Question 2 is 7 marks)

3. The equation $3x^2 - 4x - 3e^{-x} = 0$ has exactly one real root.

(a) Show that the Newton-Raphson formula can be written in the form,

$$x_{n+1} = \frac{3x_n^2 + 3(x_n + 1)e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$$

(4)

(b) Using $x_1 = 2$ and the formula given in part (a), find the values of x_2, x_3 and x_4

(3)

(c) State an approximation for the root to 3 decimal places.

(1)

(d) Explain why not all values for x_1 will give a suitable approximation.

(2)

a)

$f(x) = 3x^2 - 4x - 3e^{-x}$

$f'(x) = 6x - 4 + 3e^{-x}$

$$x_{n+1} = x_n - \frac{3x_n^2 - 4x_n - 3e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$$

$$= \frac{x_n(6x_n - 4 + 3e^{-x_n}) - (3x_n^2 - 4x_n - 3e^{-x_n})}{6x_n - 4 + 3e^{-x_n}}$$

$$= \frac{6x_n^2 - 4x_n + 3x_n e^{-x_n} - 3x_n^2 + 4x_n + 3e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$$

$$= \frac{3x_n^2 + 3(x_n + 1)e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$$

b) $x_2 = 1.572449245$

$x_3 = 1.489015406$

$x_4 = 1.48569061$

c) $x \approx 1.486$

d) Values close to points of inflection will not cross x-axis in most cases

(Total for Question 3 is 10 marks)

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4. Find algebraically the exact solutions to the equations

(a) $$\ln(2 - x) + \ln(10 - 5x) = 2\ln(x + 2) \quad -2 < x < 2$$

(5)

(b) $$2^x e^{2x+1} = 5$$

Give your answer in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

a) $\cancel{\ln}(2-x)(10-5x) = \cancel{\ln}(x+2)^2$

$20 - 20x + 5x^2 = x^2 + 4x + 4$

$4x^2 - 24x + 16 = 0$

$x = \frac{24 \pm \sqrt{320}}{8} = 3 \pm \sqrt{5} \rightarrow x = 3 - \sqrt{5}$ as $x < 2$

b) $2^x e^{2x+1} = 5$

$\ln 2^x e^{2x+1} = \ln 5$

$x\ln 2 + 2x + 1 = \ln 5$

$x(2 + \ln 2) = \ln 5 - 1$

$\boxed{x = \frac{\ln 5 - 1}{2 + \ln 2}}$

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5. **(a)** Express $5\sin\theta + 12\cos\theta$ in the form $R\sin(\theta+\alpha)$ where, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give your answer to 3 decimal places.

(4)

(b) Hence, or otherwise, find the solution to the equation,

$$5\sin\theta + 12\cos\theta = 6 \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

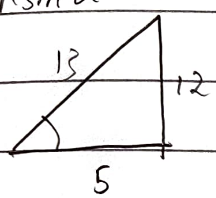
Give your answer to 3 decimal places.

(4)

a) $R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = 5$ $\qquad R = \sqrt{5^2+12^2} = 13$

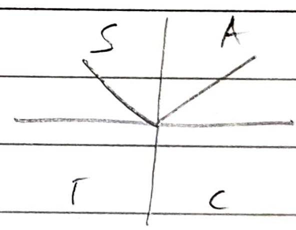
$R\sin\alpha = 12$



$\alpha = \tan^{-1}\left(\frac{12}{5}\right)$

$5\sin\theta + 12\cos\theta = 13\sin\left(\theta + \tan^{-1}\left(\frac{12}{5}\right)\right)$

b) $13\sin\left(\theta+\tan^{-1}\left(\frac{12}{5}\right)\right) = 6$



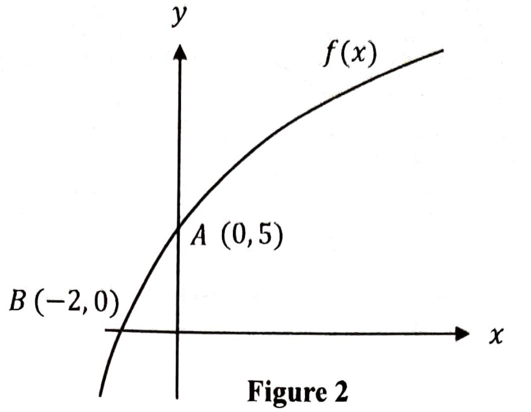
$\theta + \tan^{-1}\left(\frac{12}{5}\right) = \sin^{-1}\left(\frac{6}{13}\right)$

$= 0.4797\ldots / 2.66186\ldots$

$\boxed{\theta = -0.696 \;/\; \theta = 1.486}$

6. **Figure 2** shows part of the curve with equation $y = f(x), \quad x \in \mathbb{R}$

The curve passes through the points $A\ (0,5)$ and $B(2,0)$ as shown.



Not drawn accurately

Figure 2

On separate diagrams, sketch the curves with following equations making sure to clearly label any intersections with the axes.

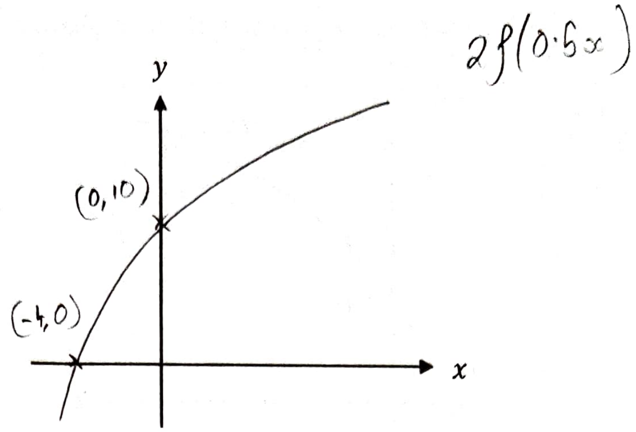
(a) $y = 2f(0.5x)$ **(2)**

(b) $y = f(|x|) - 5$ **(2)**

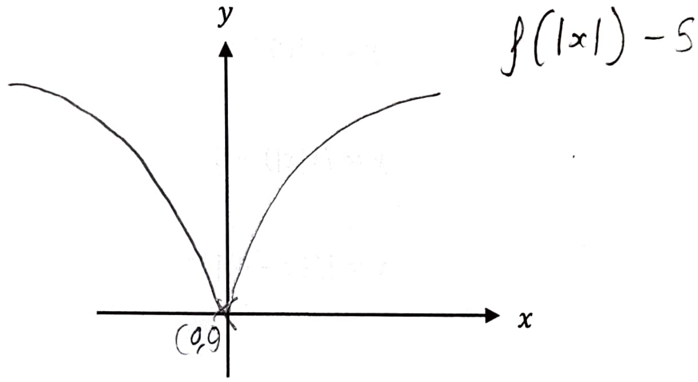
(c) $y = |f(x-4)|$ **(2)**

Turn over

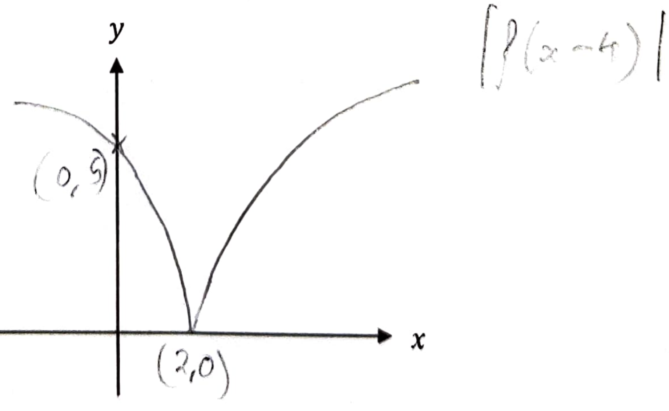
(a)



(b)



(c)



(Total for Question 6 is 6 marks)

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7.

$$y = \frac{2(1+\sin x)^2}{3\cos^2 x}$$

$\frac{4\pi}{24}$ $\frac{\pi}{24}$

x	$\frac{\pi}{6}$	$\frac{5\pi}{24}$	$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{\pi}{3}$
y	2	2.7413	3.8856	5.7856	9.2855

(a) Calculate the missing values in the table.

(2)

(b) Use all values from the completed table in part **(a)** and the trapezium rule to find an estimate for

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2(1+\sin x)^2}{3\cos^2 x}\,dx$$

Give your answer to 4 decimal places.

(3)

b) $\frac{1}{2} \times \frac{\pi}{24}\left(2 + 2(2.7413 + 3.8856 + 5.7856) + 9.2855\right)$

$= 2.3634$

Turn over

8. Show that when $y = \tan^{-1} x$,

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

(6)

$y = \tan^{-1}(x)$

$x = \tan y \rightarrow x = \frac{\sin y}{\cos y}$

$\frac{dx}{dy} = \frac{\cos^2 y + \sin^2 y}{\cos^2 y}$

$= \frac{1}{\cos^2 y}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y}$ $\boxed{\frac{1}{1+x^2}}$

$x = \tan y$

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9. Given that,

(a)
$$\frac{2x^2 + 11x - 3}{(x+4)(x-1)} = A + \frac{B}{x+4} + \frac{C}{x-1}$$

find the values of the constants, A, B and C.

(5)

(b) Given,

$$f(x) = \frac{2x^2 + 11x - 3}{(x+4)(x-1)}, \qquad x > 1$$

Show that $f(x)$ is strictly decreasing

(5)

a) $2x^2 + 11x - 3 = A(x+4)(x-1) + B(x-1) + C(x+4)$

$x = 1 \rightarrow 5C = 10 \rightarrow \underline{\underline{C = 2}}$

$x = -4 \rightarrow -5B = -15$

$\underline{\underline{B = 3}}$

$Ax^2 = 2x^2, \therefore \underline{A = 2}$

$$\boxed{\frac{2x^2 + 11x - 3}{(x+4)(x-1)} = 2 + \frac{3}{x+4} + \frac{2}{x-1}}$$

b) $\frac{d}{dx}\left(2 + \frac{3}{x+4} + \frac{2}{x-1}\right) = -\frac{3}{(x+4)^2} - \frac{2}{(x-1)^2} = f'(x)$

both > 0, $\therefore f'(x) < 0$, strictly decreasing

Turn over ▶

10. For any arithmetic sequence, prove the sum of the first n positive integers, S_n is given by,

$$S_n = \sum_{i=1}^{n} a_i = \frac{1}{2}n(a + l)$$

where a and l are the are the first and last terms in the sequence respectively.

(7)

$S_n = a + (a+d) + (a+2d) + \ldots a + (n-1)d$

$S_n = a+(n-1)d + a(n-2)d \ldots a+d + a$

$2S_n = (2a + (n-1)d) \times n$

$S_n = \frac{1}{2}n(2a+(n-1)d)$

Since $a + (n-1)d$ is the last term, replace with l

$S_n = \frac{1}{2}n(a+l)$ $\boxed{\text{as required}}$

(Total for Question 10 is 7 marks)

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11. Evaluate the following integrals,

(a)

$$I = \int \sin^3 x \cos x \; dx$$

(5)

(b)

$$I = \int \cos^4 x \; dx$$

(7)

a) $\int \sin^3 x \cos x \, dx = \int u^3 \, du = \frac{u^4}{4}$

$u = \sin x \rightarrow \frac{du}{dx} = \cos x$ $\quad = \frac{\sin^4 x}{4}$

$dx = \frac{du}{\cos x}$

b) $\int \cos^4 x \, dx$

$\cos^4 x = \left(\frac{1}{2}(1+\cos 2x)\right)^2$

$= \frac{1}{4}\left(1 + 2\cos 2x + \cos^2 2x\right)$

$= \frac{1}{4}\left[\left(1 + 2\cos 2x\right) + \left(\frac{1}{2}\left(1+\cos 4x\right)\right)\right]$

$= \frac{1}{4} + \frac{1}{8} + \frac{2\cos 2x}{4} + \frac{\cos 4x}{8}$

$= \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

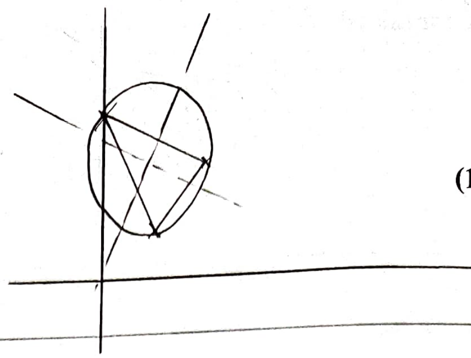
$\int \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x = \boxed{\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x}$

12. A triangle has vertices (1,1), (2,3), (0,5).

Find the equation of its circumcircle.

(10)

To find equation
- Find equations of 2 perpendicular bisectors
- Make equal and find centre C
- Find radius w/ C and 1 other point



Bisector 1: $(1,1)+(2,3)$

$m = \frac{3-1}{2-1} = 2 \longrightarrow m_{PB1} = -\frac{1}{2}$

$MP = \left(\frac{3}{2}, 2\right) \longrightarrow y-2 = -\frac{1}{2}\left(x-\frac{3}{2}\right)$

$\boxed{y = -\frac{1}{2}x + \frac{11}{4}}$

Bisector 2: $(0,5)+(2,3)$

$m = -1 \longrightarrow m_{PB2} = 1$

$MP = (1,4) \longrightarrow y-4 = x-1$

$\boxed{y = x+3}$

Make equal:

$x+3 = -\frac{1}{2}x + \frac{11}{4}$

$\frac{3}{2}x = -\frac{1}{4}$

$x = -\frac{1}{6} \longrightarrow y = -\frac{1}{6}+3 = \frac{17}{6}$

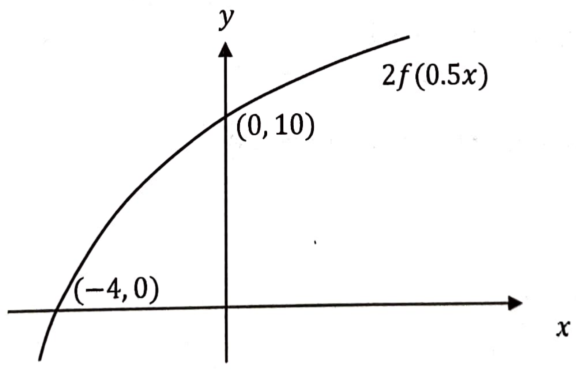
Find radius w/ dist. formula

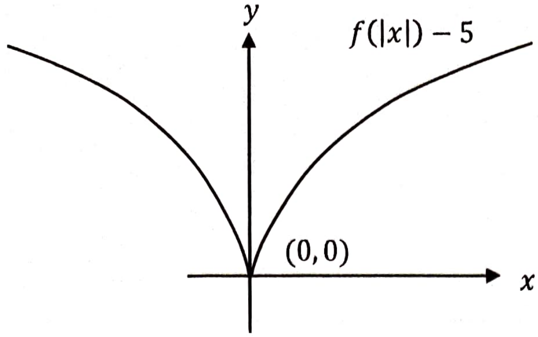
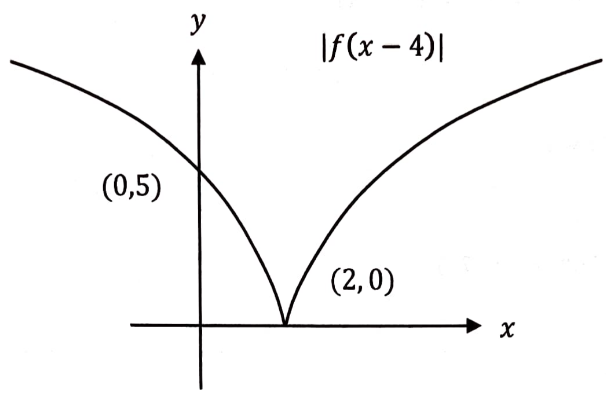
$r = \sqrt{\left(-\frac{1}{6}-1\right)^2 + \left(\frac{17}{6}-1\right)^2} = \frac{\sqrt{170}}{6}$

Equation of circumcircle: $\boxed{\left(x+\frac{1}{6}\right)^2 + \left(y-\frac{17}{6}\right)^2 = \frac{170}{36}}$

Question	Paper 2 Mark Scheme	Marks
1(a)	$y \geq 1$	B1 **(1)**
1(b)	Start to process to find inverse e.g. $x = 1 + \sqrt{y+3}$ $(x-1)^2 = y + 3$ $h^{-1}(x) = (x-1)^2 - 3$ $x \geq 1$	M1 B1 A1 **(3)**
1(c)	Valid start to process $h(x) = 1 + \sqrt{x+3} = x$ $(x+3)^{1/2} = (x-1)$ $(x+3) = (x-1)^2$ $x + 3 = x^2 - 2x + 1$ $x^2 - 3x - 2 = 0$ $x = \frac{3-\sqrt{17}}{2}$ $x = \frac{3+\sqrt{17}}{2}$	M1 M1 B1 B1 **(4)**
1(d)	$m = \frac{3+\sqrt{17}}{2}$ or $m = \frac{3-\sqrt{17}}{2}$	B1 **(1)**
2(a)	Correct start to process $\lvert\overrightarrow{OA}\rvert = a^2 + \sqrt{2}^2 + 5^2 = 6^2$ $a^2 + 2 + 25 = 36$ $a^2 = 9$ $a = \pm 3$ $a = 3$, as > 0	M1 M1 B1 **(3)**
2(b)	Correct start to process $\overrightarrow{AB} = (2-3)i + (-3-\sqrt{2})j + (-4-5)k = -i + (-3-\sqrt{2})j - 9k$ So $D = (2-1)i + \left(-3 + \left(-3-\sqrt{2}\right)\right)j + (-9-4)k$ $D = i + (-6-\sqrt{2})j - 13k$	M1 B1 **(2)**

Question	Paper 2 Mark Scheme	Marks
2(c)	$\lvert\overrightarrow{BC}\rvert = (b-2)^2 + 6^2 + 2^2 = 7^2$ $b = -1$ since $b < 0$	M1 B1 **(2)**
3(a)	$f(x) = 3x^2 - 4x - 3e^{-x}$ $f'(x) = 6x - 4 + 3e^{-x}$ $x_{n+1} = x_n - \dfrac{3x_n^2 - 4x_n - 3e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$ Start of process to rearrange to given form e.g. $x_{n+1} = \frac{x_n(6x_n-4+3e^{-x_n})-(3x_n^2-4x_n-3e^{-x_n})}{6x_n-4+3e^{-x_n}}$ $x_{n+1} = \dfrac{6x_n^2 - 4x_n + 3x_ne^{-x_n} - 3x_n^2 + 4x_n + 3e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$ $x_{n+1} = \dfrac{3x_n^2 + 3(x_n+1)e^{-x_n}}{6x_n - 4 + 3e^{-x_n}}$	M1 M1 M1 M1 **(4)**
3(b)	$x_2 = 1.572449245$ $x_3 = 1.489015406$ $x_4 = 1.48569061$	B1 B1 B1 **(3)**
3(c)	$x \approx 1.486$ (3 dp)	B1 **(1)**
3(d)	Values of x_1 that are close to points of inflection, either local maxima or minima, will not approximate a root as the tangent at those initial values will likely not cross the x-axis as its gradient will be (close to) zero.	B2 **(2)**
4(a)	Valid start to process e.g. $\ln(2-x)(10-5x) = \ln(x+2)^2$ $(2-x)(10-5x) = (x+2)^2$ $20 - 20x + 5x^2 = x^2 + 4x + 4$ $4x^2 - 24x + 16 = 0$ $x = \dfrac{24 \pm \sqrt{320}}{8} = 3 \pm \sqrt{5}$ $x = 3 - \sqrt{5}$ only as $x < 2$	M1 M1 M1 B1 B1 **(5)**

Question	Paper 2 Mark Scheme	Marks
4(b)	$\ln(2^x e^{2x+1}) = \ln 5$	M1
	$\ln 2^x + \ln e^{2x+1} = \ln 5$	M1
	$x \ln 2 + 2x + 1 = \ln 5$	M1
	$x(2 + \ln 2) = -1 + \ln 5$	M1
	$x = \dfrac{-1 + \ln 5}{2 + \ln 2}$	A1
		(5)
5(a)	$5\sin\theta + 12\cos\theta = R\sin(\theta + \alpha)$ $5\sin\theta + 12\cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	M1
	$R^2 = 5^2 + 12^2 = 169$ $R = 13$	B1
	So, $5\sin\theta = 13\sin\theta\cos\alpha$ $\alpha = \arccos\dfrac{5}{13} = 1.176$	B1
	$5\sin\theta + 12\cos\theta = 13\sin(\theta + 1.176)$	B1
		(4)
5(b)	$13\sin(\theta + 1.176) = 6$	M1
	$\theta + 1.176 = \arcsin\dfrac{6}{13} = 0.4797\ldots, 2.6619\ldots$	M1
	$\theta = 1.486$	A1
	$\theta = -0.696$	A1
		(4)
6(a)		B2
		(2)

Question	Paper 2 Mark Scheme	Marks
6(b)		B2 **(2)**
6(c)		B2 Allow small section below x-axis on original graph reflected **(2)**
7(a)	x: $\frac{\pi}{6}$, $\frac{5\pi}{24}$, $\frac{\pi}{4}$, $\frac{7\pi}{24}$, $\frac{\pi}{3}$ y: 2, 2.7413, 3.8856, 5.7856, 9.2855	B2 1 mark for each correct **(2)**
7(b)	States or attempts to use $\frac{1}{2}h\left((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\right)$ Correct substitution $\frac{1}{2} \times \frac{\pi}{24}\left((2 + 9.2855) + 2(2.7413 + 3.8856 + 5.7856)\right)$ 2.3634	M1 M1 A1 **(3)**
8	Let $x = \tan y$ hence $x = \frac{\sin y}{\cos y}$ $\frac{dx}{dy} = \frac{\cos y\,(\cos y) - \sin y\,(-\sin y)}{\cos^2 y}$ $\frac{dx}{dy} = \frac{1}{\cos^2 y}$ $\frac{dy}{dx} = \frac{1}{\sec^2 y}$	M1 M1 M1 M1

Question	Paper 2 Mark Scheme	Marks
8 cont.	$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y}$	M1
	$\frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$	A1
		(6)
9(a)	$A + \frac{B}{x+4} + \frac{C}{x-1} \to A(x+4)(x-1) + B(x-1) + C(x+4) = 2x^2 + 11x - 3$	M1
	Correct process to find a variable e.g. $x = 1 \Rightarrow 5C = 10$	M1
	Finds two variables e.g. $C = 2$ and B $= 3$	A1
	Process to find A $Ax^2 = 2x^2$	M1
	$\frac{2x^2+11x-3}{(x+4)(x-1)} = 2 + \frac{3}{x+4} + \frac{2}{x-1}$	B1
		(5)
9(b)	$\frac{d}{dx}\frac{3}{x+4} = -\frac{3}{(x+4)^2}$	M1
	$\frac{d}{dx}\frac{2}{x-1} = -\frac{2}{(x-1)^2}$	M1
	$f'(x) = -\frac{3}{(x+4)^2} - \frac{2}{(x-1)^2}$	M1
	$(x+4)^2 > 0$ and $(x-1)^2 > 0$	M1
	Hence $f'(x) < 0$ so $f(x)$ is strictly decreasing	A1
		(5)
10	$S_{\text{forward}} = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d)$.	M1
	$S_{\text{reverse}} = (a+(n-1)d) + (a+(n-2)d) + \cdots + (a+d) + a$	M1
	Summing the two series $2S_n = (2a+(n-1)d) + (2a+(n-1)d) + \cdots + (2a+(n-1)d)$,	M1
	Thus we simply have n lots of $(2a+(n-1)d)$ on the RHS, so	M1
	$2S_n = n(2a+(n-1)d)$	
	$S_n = \frac{1}{2}n(2a+(n-1)d)$,	M1
	$S_n = \frac{1}{2}n(a+a+(n-1)d)$,	M1
	where $(a+(n-1)d)$, is the last term in the series so can be replaced with $l = (a+(n-1)d)$, $S_n = \frac{1}{2}n(a+l)$	A1
		(7)

Question	Paper 2 Mark Scheme	Marks
11(a)	$u = \sin x$	M1
	$du = \cos x \, dx$	M1
	Substitutes u and du $\int u^3 \, du$	M1
	$\int u^3 du = \frac{1}{4}u^4 + c$	M1
	$\int \sin^3 x \cos x \, dx = \frac{1}{4}\sin^4 x + c$	B1
		(5)
11(b)	$I = \int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx$	M1
	$\int (\cos^2 x)^2 \, dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$	M1
	$= \int \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x) \, dx$	M1
	$= \int \frac{1}{4}\left(1 + 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\right) dx$	M1
	$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + c$	B2
	$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$	B1
		(7)

Question	Paper 2 Mark Scheme	Marks
12	Find gradient of perpendicular bisector through $(1,1)$ and $(2,3)$ Gradient $= \frac{3-1}{2-1} = 2$ Perpendicular bisector gradient $= -\frac{1}{2}$	M1
	Find midpoint of $(1,1)$ and $(2,3)$ $\left(\frac{1+2}{2}, \frac{1+3}{2}\right) = \left(\frac{3}{2}, 2\right)$	M1
	Process to find equation of perpendicular bisector $y - 2 = -\frac{1}{2}\left(x - \frac{3}{2}\right)$ $y = -\frac{1}{2}x + \frac{11}{4}$	M1
	Find gradient of perpendicular bisector through $(0,5)$ and $(2,3)$ Gradient $= \frac{5-3}{0-2} = -1$ Perpendicular bisector gradient $= 1$	M1
	Find midpoint of $(0,5)$ and $(2,3)$ $\left(\frac{2+0}{2}, \frac{3+5}{2}\right) = (1,4)$	M1
	Process to find equation of perpendicular bisector $y - 4 = x - 1$ $y = x + 3$	M1
	Find centre of circle via perpendicular bisector intersection $x + 3 = -\frac{1}{2}x + \frac{11}{4}$ $\frac{3}{2}x = -\frac{1}{4}$ $x = -\frac{1}{6}$	A1
	$y = -\frac{1}{6} + 3 = \frac{17}{6}$	A1
	Process to find radius $r = \sqrt{\left(-\frac{1}{6} - 1\right)^2 + \left(\frac{17}{6} - 1\right)^2}$ $r = \frac{\sqrt{170}}{6}$	A1
	Centre $= \left(-\frac{1}{6}, \frac{17}{6}\right)$ Form equation $\left(x - \frac{-1}{6}\right)^2 + \left(y - \frac{17}{6}\right)^2 = \left(\frac{\sqrt{170}}{6}\right)^2$ $\left(x + \frac{1}{6}\right)^2 + \left(y - \frac{17}{6}\right)^2 = \frac{170}{36}$	B1 (10)