Integrating $\ln x$ and definite integration

Find $\int \ln x \; dx$, ~~leaving your answer in terms of natural logarithms.~~

$u = \ln x \quad v = x$

$u' = \frac{1}{x} \quad v' = 1$

$$\int \ln x \, dx = x \ln x - \int 1 \, dx$$

$$= x \ln x - x + c$$

Check $\frac{d}{dx}(x \ln x - x) = 1 + \ln x - 1 = \ln x$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

not decimals

Find $\int_1^2 \ln x \, dx$, leaving your answer in terms of natural logarithms.

$$\int_1^2 \ln x \, dx = \left[x \ln x\right]_1^2 - \int_1^2 1 \, dx$$

$$= \left[x \ln x\right]_1^2 - \left[x\right]_1^2$$

$$= \left[x \ln x - x\right]_1^2$$

$$= 2\ln 2 - 2 - (1 \ln 1 - 1)$$

$$= 2\ln 2 - 2 + 1 = \underline{\underline{2\ln 2 - 1}}$$

In general:

$$\int_a^b u \frac{dv}{dx} dx = [uv]_a^b - \int_a^b v \frac{du}{dx} dx$$

Your Turn

Find $\int_0^{\frac{\pi}{2}} x \sin x \; dx$

$u = x \qquad v = -\cos x$

$u' = 1 \qquad v' = \sin x$

$$= \left[-x\cos x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos x \; dx$$

$$= \left[-x\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\pi/2} \cos x \; dx$$

$$= \left[-x\cos x\right]_0^{\pi/2} + \left[\sin x\right]_0^{\pi/2}$$

$$= 0 + 1 - 0$$

$$= \underline{\underline{1}}.$$

Evaluate the following:

a $\int_0^{\ln 2} x e^{2x} \, dx$ b $\int_0^{\frac{\pi}{2}} x \sin x \, dx$ c $\int_0^{\frac{\pi}{2}} x \cos x \, dx$ d $\int_1^2 \frac{\ln x}{x^2} \, dx$

e $\int_0^1 4x(1+x)^3 \, dx$ f $\int_0^{\pi} x \cos \frac{1}{4} x \, dx$ g $\int_0^{\frac{\pi}{3}} \sin x \ln(\sec x) \, dx$

Ex 11F Q4 onwards

4 Evaluate the following:

a $\cancel{\int_0^{\ln 2} xe^{2x}\,dx}$ **b** $\cancel{\int_0^{\frac{\pi}{2}} x\sin x\,dx}$ **c** $\cancel{\int_0^{\frac{\pi}{2}} x\cos x\,dx}$ **d** $\int_1^2 \frac{\ln x}{x^2}\,dx$

e $\int_0^1 4x(1+x)^3\,dx$ **f** $\int_0^{\pi} x\cos\frac{1}{4}x\,dx$ **g** $\int_0^{\frac{\pi}{3}} \sin x\ln(\sec x)\,dx$

g) $\int_0^{\frac{\pi}{3}} \sin x \ln(\sec x)\,dx = \left[-(\ln\sec x)\cos x\right]_0^{\pi/3} - \int_0^{\pi/3} -\cos x\tan x$

$u = \ln(\sec x)$ $\quad v = -\cos x$

$u' = \frac{\cancel{\sec x}\tan x}{\cancel{\sec x}}$ $\quad v' = \sin x$

$= \tan x$

$= \left[-(\ln\sec x)\cos x\right]_0^{\frac{\pi}{3}} + \int_0^{\frac{\pi}{3}} \sin x\,dx$

$= \left[\cos x\ln\cos x\right]_0^{\frac{\pi}{3}} + \left[-\cos x\right]_0^{\pi/3}$

$-\ln\sec x = \ln(\sec x)^{-1}$

$= \ln\frac{1}{\sec x}$

$= \ln\cos x$

$= \left[\cos x\ln\cos x - \cos x\right]_0^{\frac{\pi}{3}}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\cos 0 = 1$

$= \left(\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\right) - \left(0 - 1\right)$

$= \frac{1}{2}\ln\frac{1}{2} - \frac{1}{2} + 1$

$= \frac{1}{2}\ln\frac{1}{2} + \frac{1}{2} = \frac{1}{2}\left(\ln\frac{1}{2} + 1\right)$

$= \frac{1}{2}\left(\ln 2^{-1} + 1\right)$

$= \frac{1}{2}\left(-\ln 2 + 1\right)$

$\ln 2$

$= \frac{1}{2}\left(1 - \ln 2\right)$

One final unusual one…

$u = \sin x \quad v = e^x$
$u' = \cos x \quad v' = e^x$

$u = \cos x \quad v = e^x$
$u' = -\sin x \quad v' = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - \int \underline{e^x \cos x} \, dx$$

$$= e^x \sin x - \left(e^x \cos x - \int -e^x \sin x \, dx \right)$$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) + c$$

SKILL #7: Using Partial Fractions

Find $\int \frac{2}{x^2-1}\,dx$

$$\frac{2}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$$

$$2 = A(x+1) + B(x-1)$$

$x = 1$

$2 = 2A$

$A = 1$

$x = -1$

$2 = -2B$

$B = -1$

$$\int \frac{2}{x^2-1}\,dx = \int \left(\frac{1}{x-1} - \frac{1}{x+1}\right) dx$$

$$= \ln|x-1| - \ln|x+1| + c$$

$$= \ln\left|\frac{x-1}{x+1}\right| + c$$

$$= \ln\left|\frac{x-1}{x+1}\right| + \ln k$$

$$= \ln\left|\frac{k(x-1)}{x+1}\right|$$

Find $\int \frac{x-5}{(x+1)(x-2)}\,dx$

$$\frac{x-5}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

$$x-5 = A(x-2) + B(x+1)$$

$x=2$: $-3 = 3B$, $\underline{\underline{B=-1}}$

$x=-1$: $-6 = -3A$, $\underline{\underline{A=2}}$

$$\int \frac{x-5}{(x+1)(x-2)}\,dx = \int \left(\frac{2}{x+1} - \frac{1}{x-2} \right) dx \quad (+c)$$

$$= 2\ln|x+1| - \ln|x-2| + \ln k$$

$$= \ln(x+1)^2 - \ln|x-2| + \ln k.$$

$$= \ln\left|\frac{(x+1)^2}{x-2}\right| + \ln k$$

$$= \ln\left|\frac{k(x+1)^2}{x-2}\right|$$

$$\ln\left|\frac{(x+1)^2}{x-2}\right| + c$$

Find $\int \frac{8x^2-19x+1}{(2x+1)(x-2)^2}\,dx$

$$\frac{8x^2-19x+1}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$8x^2-19x+1 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$$

$x=2$

$-5 = 5C$

$\underline{\underline{C=-1}}$

$x = -\frac{1}{2}$

$\frac{25}{2} = \frac{25}{4}A$

$\underline{\underline{A=2}}$

compare x^2 coefficient.

$8 = A + 2B$

$8 = 2 + 2B$

$\underline{\underline{B=3}}$.

$-(x-2)^{-2}$

$$\int \frac{8x^2-19x+1}{(2x+1)(x-2)^2} = \int \left(\frac{2}{2x+1} + \frac{3}{x-2} - \frac{1}{(x-2)^2}\right) dx$$

$$= \ln|2x+1| + 3\ln|x-2| + (x-2)^{-1} + c$$

$$= \ln|(2x+1)(x-2)^3| + \frac{1}{x-2} + c$$

Ex 11G Q1, 3, 4, 6