Techniques of Integration

$\int \ln x \, dx$ $u = \ln x$

$v' = 1$

Substitution

Int. by parts — $\ln x$ takes priority for u
→ may look like a product

Reverse chain rule

Partial fractions

$f(ax+b)$

Standard results eg. $\int \sec^2 x \, dx = \tan x + c$

Use trig identities

Polynomial division

Split the numerator

A Whole Load of Integration

This is it; where all the integration you've seen comes together. You need to find the following integrals without any clue as to how to do them! You could use 'guess and check', partial fractions, parts, substitution or more than one of the above!

1 $\int \cos(3x-1)dx$ standard result.

2 $\int e^{1-x}dx$

3 $\int \frac{2x+1}{\left(x^2+x-1\right)^2}dx$ reverse chain rule

$(2x+1)(x^2+x-1)^{-2}$

4 $\int \cos 2x dx$

5 $\int \ln 2x dx$

6 $\int \frac{x}{\left(x^2-1\right)^3}dx$

7 $\int \sqrt{\underline{2x-3}}dx$ standard result

8 $\int \frac{4x-1}{(x-1)^2(x+2)}dx$ partial fractions

9 $\int x^3 \underline{\ln \overset{u}{x}}dx$ by parts

10 $\int \frac{5}{2x^2-7x+3}dx$ partial fractions

11 $\int (x+1)e^{x^2+2x}dx$ reverse chain rule

12 $\int \frac{\sin x - \cos x}{\sin x + \cos x}dx$ reverse chain rule

13 $\int \underline{x^2} \sin 2x dx$ by parts

14 $\int \sin^3 2x dx$

15) $\int \frac{x-1}{x+1}dx$

1) $\frac{1}{3}\sin(3x-1)+c$

2) $-e^{1-x}+c$

3) $\frac{-1}{x^2+x-1}+c$

4) $\frac{1}{2}\sin(2x)+c$

5) $x\ln(2x)-x+c$

6) $\frac{-1}{4}(x^2-1)^{-2}+c$

7) $\frac{1}{3}(2x-3)^{3/2}+c$

8) $-\ln|x+2|+\ln|x-1|-(x-1)^{-1}+c$

9) $\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+c$

10) $-\ln|2x-1|+\ln|x-3|+c$

11) $\frac{1}{2}e^{x^2+2x}+c$

12) $-\ln|\sin x+\cos x|+c$

13) $\frac{1}{4}\cos 2x-\frac{1}{2}x^2\cos 2x+\frac{1}{2}x\sin 2x+c$

14) $\int \sin^3 2x\,dx$

$=\int \sin 2x(\sin 2x)^2\,dx$

$=\int \sin 2x(1-\cos^2 2x)\,dx$

$=\int(\sin 2x-\sin 2x\cos^2 2x)\,dx$

$=-\frac{1}{2}\cos 2x+\frac{1}{6}\cos^3 2x+c$

$-\sin 2x(\cos 2x)^2$

consider $(\cos 2x)^3$

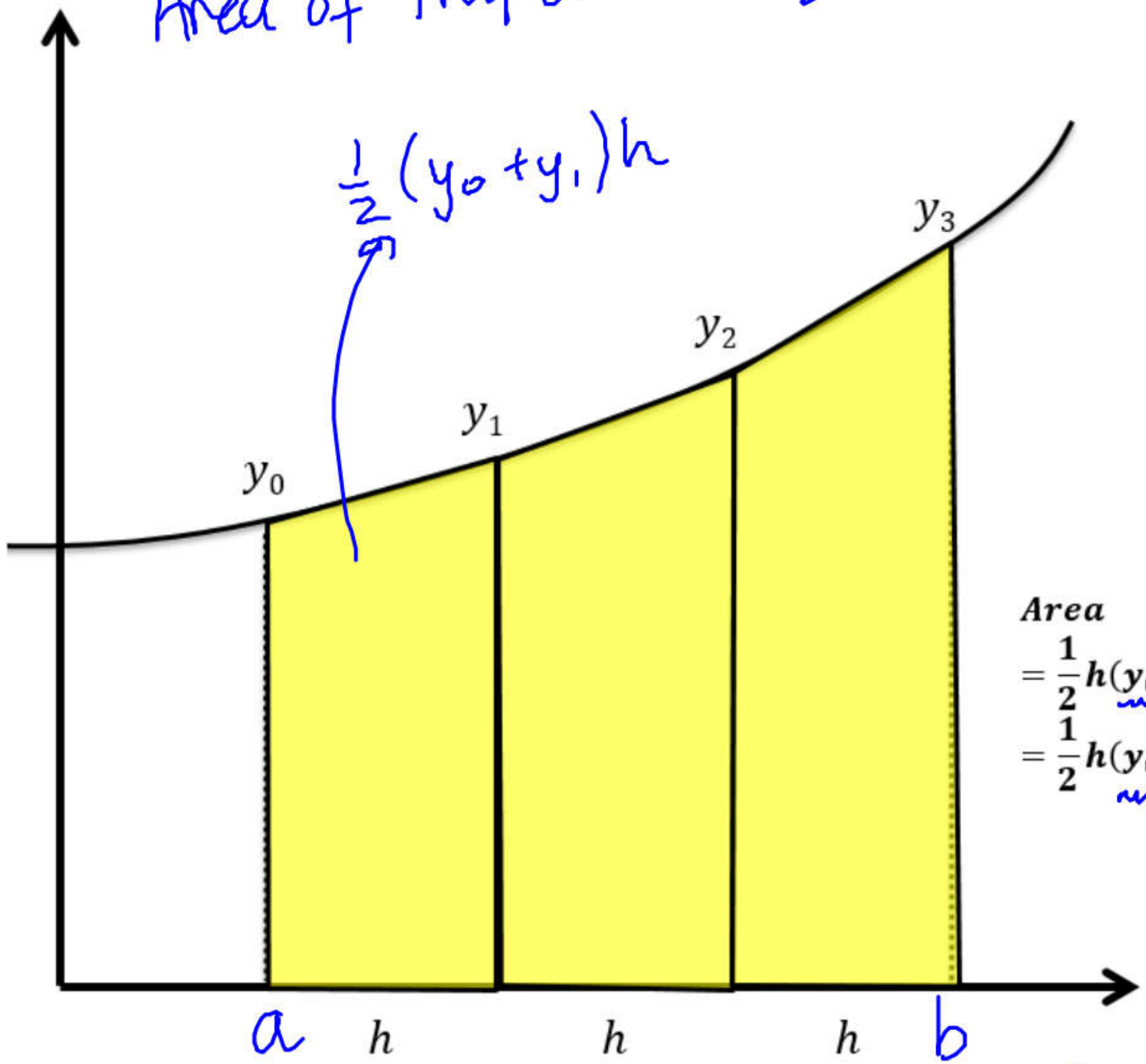
diff. $3(\cos 2x)^2\times(-2\sin 2x)$

$-6\sin 2x\cos^2 2x$

scale $\frac{1}{6}$

Skill #10: Trapezium Rule

Area of trapezium $= \frac{1}{2}(a+b)h$



Sometimes finding the exact area under the graph via integration is difficult. You may be familiar with the idea of **approximating the area under a graph by dividing it into trapeziums of equal width.**

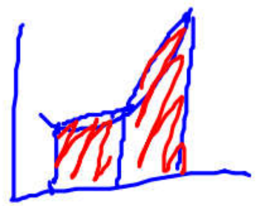
$$\begin{aligned}\textbf{\textit{Area}}\\ &= \frac{1}{2}h(y_0+y_1)+\frac{1}{2}h(y_1+y_2)+\frac{1}{2}h(y_2+y_3)\\ &= \frac{1}{2}h(y_0+2(y_1+y_2)+y_3)\end{aligned}$$

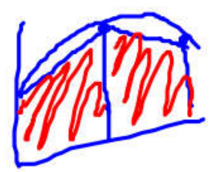
Overestimate? $f''(x) \geq 0$

When $f(x)$ is convex

Underestimate?

When $f(x)$ is concave $f''(x) \leq 0$





In general:

width of each trapezium

$$\int_a^b y\,dx \approx \frac{h}{2}\left(y_1 + 2(y_2 + \cdots + y_{n-1}) + y_n\right)$$

Area under curve

is approximately

Example We're approximating the region bounded between $x = 1$, $x = 3$, the x-axis the curve $y = x^2$, using 4 strips.

h

x	1	1.5	2	2.5	3
y	1	2·25	4	6·25	9

Dividing a gap of 2 into 4 strips means each strip will be width 0.5

$$\int_1^3 x^2\,dx \approx \frac{0.5}{2}\left(1 + 9 + 2(2.25 + 4 + 6.25)\right)$$

$$\approx 8.75$$

$$\int_1^3 x^2\,dx = 8.66 \text{ (2dp)}$$

Edexcel C2 May 2013 (R) Q2

$$y = \frac{x}{\sqrt{(1+x)}}$$

(*a*) Complete the table below with the value of y corresponding to $x = 1.3$, giving your answer to 4 decimal places.

(1)

Tip: You can generate table with Casio calcs . $Mode \rightarrow 3\ (Table)$. Use 'Alpha' button to key in X within the function. Press =

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090	0.8572	0.9037	0.9487

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\, dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

$$Area \approx \frac{0.1}{2}(0.7071 + 2(0.7591 + 0.8090 + 0.8571 + 0.9037) + 0.9487) = 0.416$$

Q

Given $I = \int_0^{\frac{\pi}{3}} \sec x \; dx$

a) Find the exact value of I.

b) Use the trapezium rule with two strips to estimate I.

c) Use the trapezium rule with four strips to find a second estimate of I.

d) Find the percentage error in using each estimate.

Q1, 3, 5, 7

Ex 11I.

a) $I = \int_0^{\frac{\pi}{3}} \sec x \, dx = \left[\ln|\sec x + \tan x|\right]_0^{\frac{\pi}{3}}$

$= \ln\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right) - \ln\left(\sec 0 + \tan 0\right)$

$= \ln(2 + \sqrt{3}) - \ln(1 + 0)$

$= \ln(2+\sqrt{3})$

1.316957897.

b)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$\sec x$	$\sec 0$ 1	$\sec\frac{\pi}{6}$ $\frac{2\sqrt{3}}{3}$	$\sec\frac{\pi}{3}$ 2

$I \approx \frac{\frac{\pi}{6}}{2}\left(1 + 2 + 2\left(\frac{2\sqrt{3}}{3}\right)\right)$

≈ 1.389997951

d) $\frac{1.389997951 - \ln(2+\sqrt{3})}{\ln(2+\sqrt{3})} \times 100$

$= 5.55\%$ (2dp).

$\frac{\text{change}}{\text{original}} \times 100$

original → actual value.