Edexcel A Level Further Maths: Core Pure

Your notes

4.1 Hyperbolic Functions

Contents

4.1.1 Hyperbolic Functions & Graphs

Your notes

Hyperbolic Functions & Graphs

What are the definitions of the hyperbolic functions?

- Hyperbolic sine

$$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

  - This can be pronounced "shine" or "sinch"
- Hyperbolic cosine

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

  - This can be pronounced "cosh"
- Hyperbolic tangent

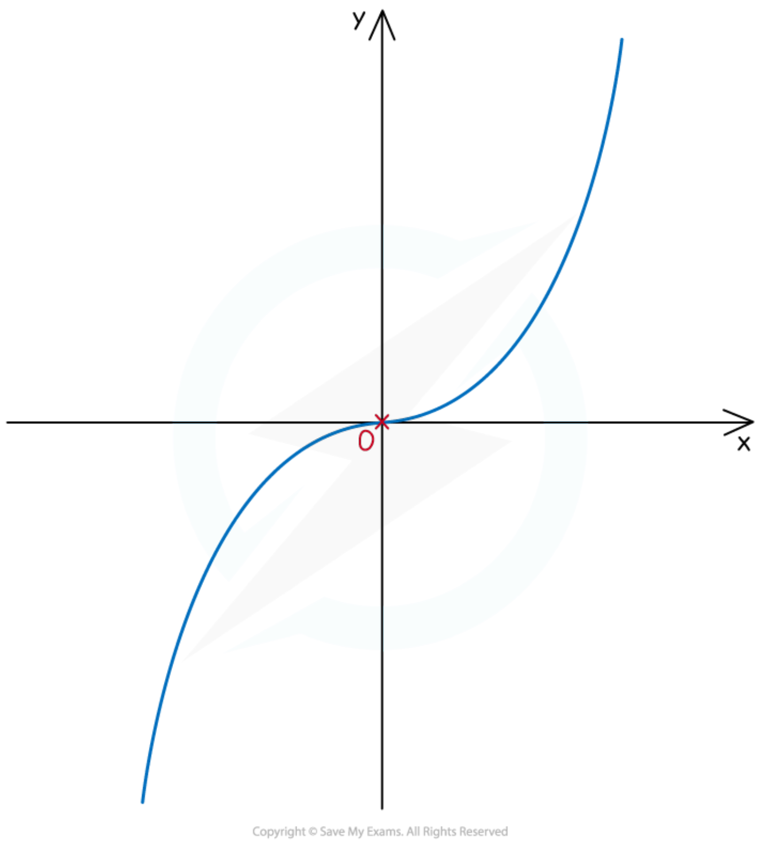
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

  - This can be pronounced "than" or "tanch"

What are the graphs of the hyperbolic functions and their key features?

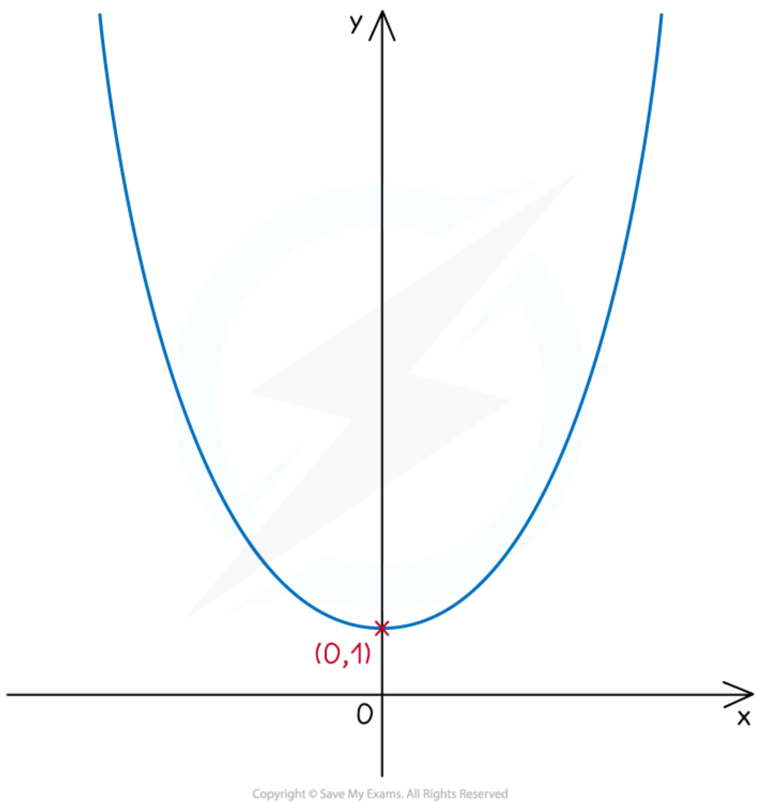
- $y = \sinh x$
  - Domain: $x \in \mathbb{R}$
  - Range: $y \in \mathbb{R}$
  - Non-stationary point of inflection at (0, 0)
  - Its shape is similar to the graph of $y = x^3$

Your notes



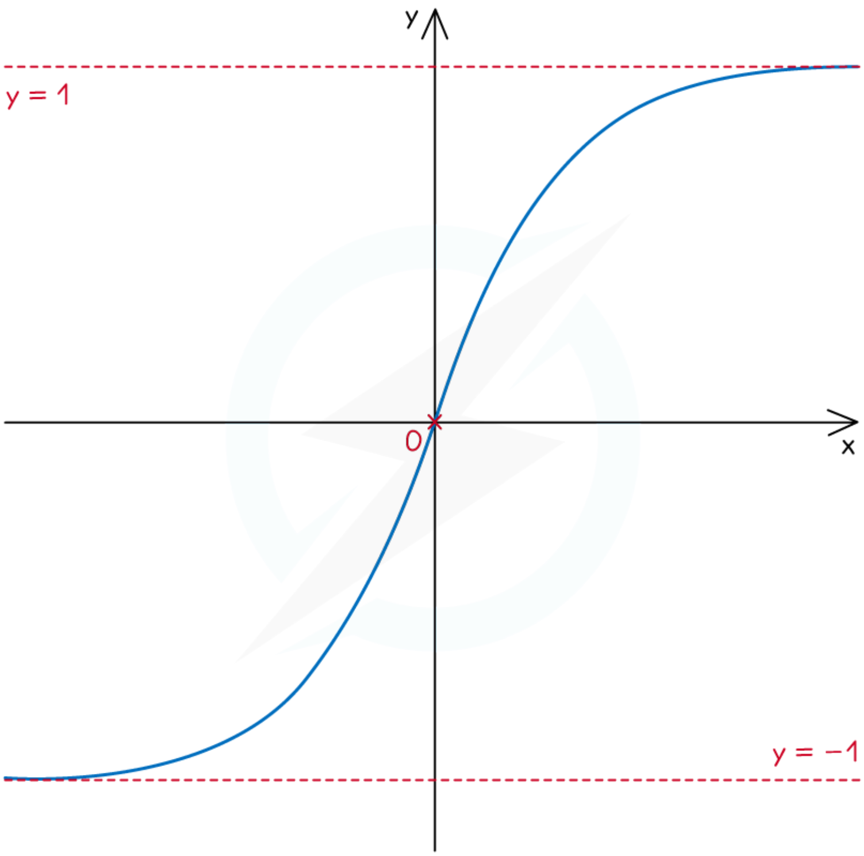
- $y = \cosh x$
  - Domain: $x \in \mathbb{R}$
  - Range: $y \geq 1$
  - Global minimum point at (0, 1)
  - Its shape is similar to the graph of $y = x^2$

Your notes



- $y = \tanh x$
  - Domain: $x \in \mathbb{R}$
  - Range: $-1 < y < 1$
  - Non-stationary point of inflection at (0, 0)
  - Asymptotes at y=1 and y=-1
  - Its shape is similar to the graph of $y = \arctan x$

Your notes



What other features of the hyperbolic functions and graphs do I need to know?

- The graphs of y=sinh$x$ and y=tanh$x$ have rotational symmetry about the origin
  - This means that
    - $\sinh(-x) = -\sinh x$
    - $\tanh(-x) = -\tanh x$
  - $\sinh x$ and are therefore odd functions
- The graph of y=cosh$x$ is symmetrical in the y-axis
  - This means that
    - $\cosh(-x) = \cosh x$
  - $\cosh x$ is therefore an even function

What may I be asked to do with hyperbolic functions and their graphs?

- Sketch graphs and transformations
  - e.g. $y = \sinh(2x) - 4$
    - Write as a transformation of $y = \sinh x$ and apply the transformations in the correct order
  - Where possible label the key features of the transformed graph

Your notes

- Intersections with the coordinate axes
- Equations of any asymptotes
- Coordinates of any turning points
- Find exact values
  - e.g. Find the exact value of $\sinh(\ln(5))$
    - Use the definitions to write in terms of e
    - Use $e^{\ln k} = k$ and $e^{-\ln k} = \frac{1}{k}$

Examiner Tip

- When using a calculator make sure you use sinh, cosh and tanh and **NOT** sin, cos and tan
- Questions asking for values in **exact** form are often easier "to see" without a calculator, using the definitions of sinh and cosh, rather than trying to type in a complicated expression with e and ln

Your notes

Worked example

a) Find the exact values of

(i) $2\cosh(\ln(8)) - 3\sinh(2\ln(2))$

(ii) $3 - \tanh(2\ln(3)) + \tanh(-2\ln(3))$

Use the properties of sinh and cosh to rewrite the expressions

(i) $2\cosh(\ln(8)) - 3\sinh(2\ln(2))$

$-\ln 8 = \ln 8^{-1}$ $2\ln(2) = \ln 2^2 = \ln 4$

$= 2\left(\frac{1}{2}\left(e^{\ln(8)} + e^{-\ln(8)}\right)\right) - 3\left(\frac{1}{2}\left(e^{\ln 4} - e^{-\ln 4}\right)\right)$

$= 8 + 8^{-1} - \frac{3}{2}(4 - 4^{-1})$

$= 8 + \frac{1}{8} - 6 + \frac{3}{8}$ Could also use calculator for this.

$$\boxed{2\cosh(\ln(8)) - 3\sinh(2\ln(2)) = \frac{5}{2}}$$

(ii) $3 - \tanh(2\ln(3)) + \tanh(-2\ln(3)) = 3 - 2\tanh(2\ln(3))$

$\tanh(-x) = -\tanh(x)$

$= 3 - 2\left(\frac{e^{2(2\ln(3))} - 1}{e^{2(2\ln(3))} + 1}\right)$

$= 3 - 2\left(\frac{e^{\ln 3^4} - 1}{e^{\ln 3^4} + 1}\right)$

$= 3 - 2\left(\frac{81 - 1}{81 + 1}\right)$

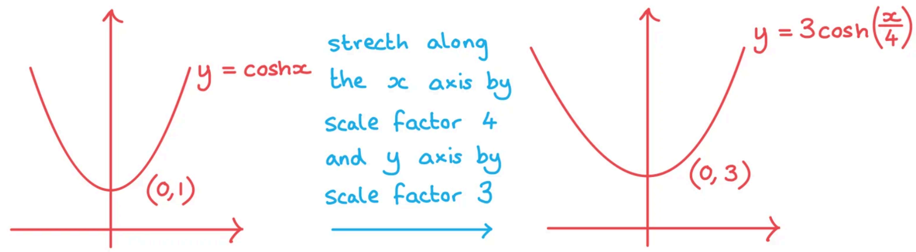
$= 3 - 2\left(\frac{80}{82}\right)$

$$\boxed{3 - \tanh(2\ln(3)) + \tanh(-2\ln(3)) = \frac{43}{41}}$$

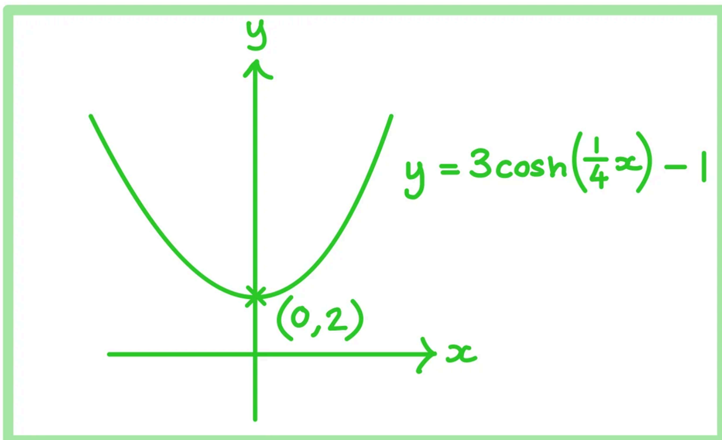
b) Sketch the graph of $y = 3\cosh\left(\frac{1}{4}x\right) - 1$, labelling any points where the graph crosses the coordinate axes and any turning points.

Your notes

Sketch $y = \cosh x$ and apply the transformations in the right order:



Then translate by vector $\begin{pmatrix}0\\-1\end{pmatrix}$



4.1.2 Logarithmic Forms of Inverse Hyperbolic Functions

Your notes

Logarithmic Forms of Inverse Hyperbolic Functions

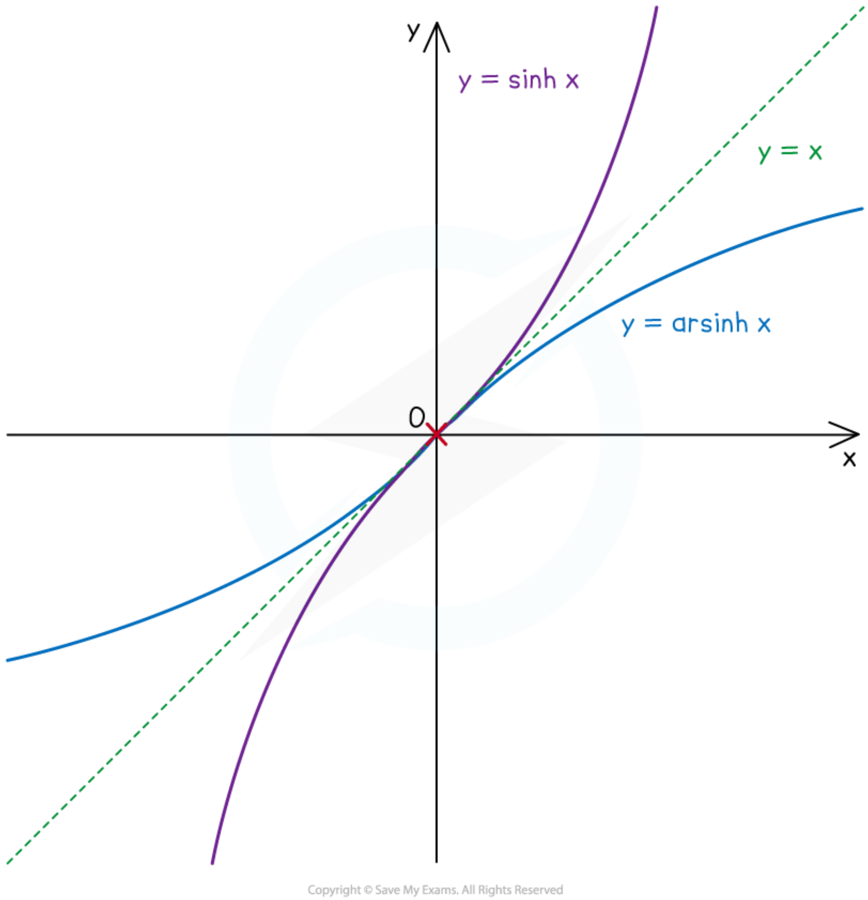
What are the definitions of the inverse hyperbolic functions?

- $\text{arsinh}x = \ln\left(x + \sqrt{x^2 + 1}\right), x \in \mathbb{R}$
- $\text{arcosh}x = \ln\left(x + \sqrt{x^2 - 1}\right), x \geq 1$
  - Since cosh$x$ is a many-to-one function, its domain is restricted to $x \geq 0$ when finding the inverse
  - Therefore, its range is cosh$x \geq 1$
  - So, the domain of the inverse function is $x \geq 1$
- $\text{artanh}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), |x| < 1$
- These three definitions are in the formula booklet

What are the graphs of the inverse hyperbolic functions and their key features?

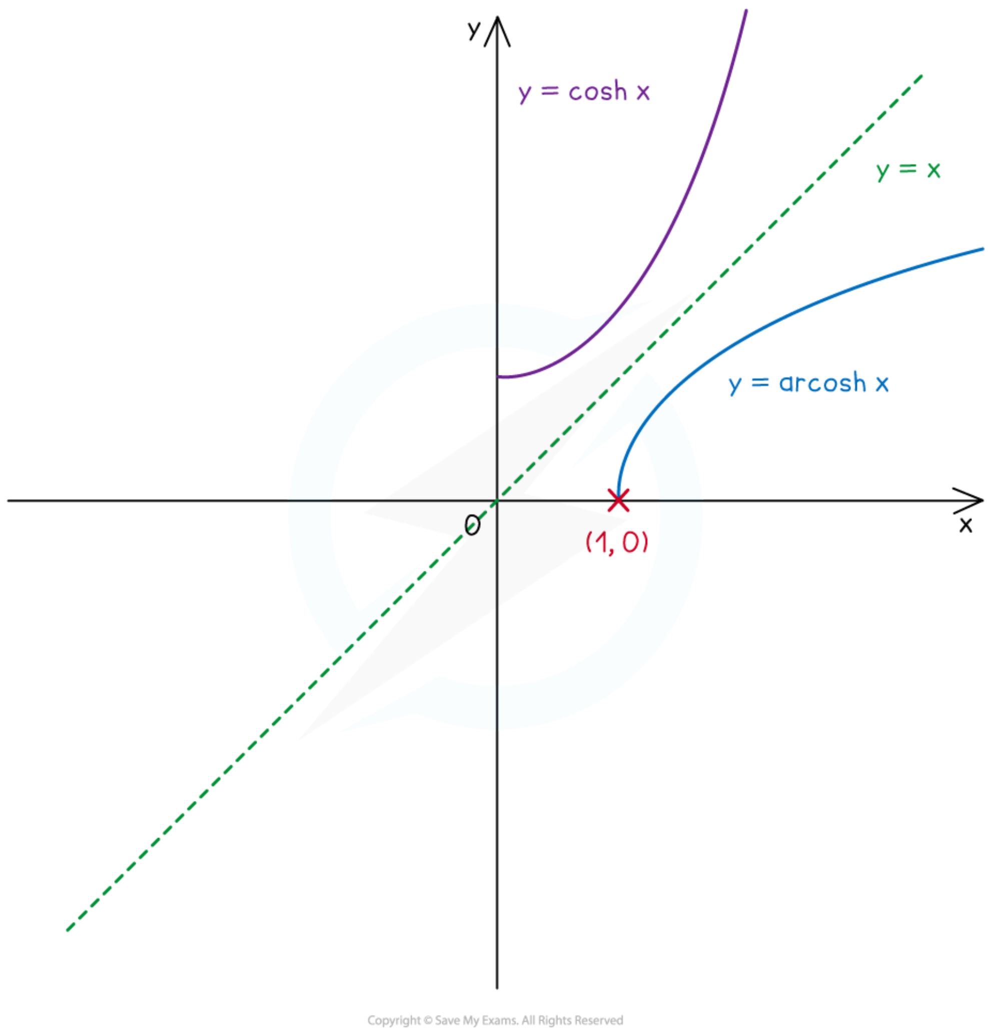
- As they are inverse functions, they are reflections of their original functions in the line y=x
- $y = \text{arsinh}x$
  - Domain: $x \in \mathbb{R}$
  - Range: $y \in \mathbb{R}$

Your notes



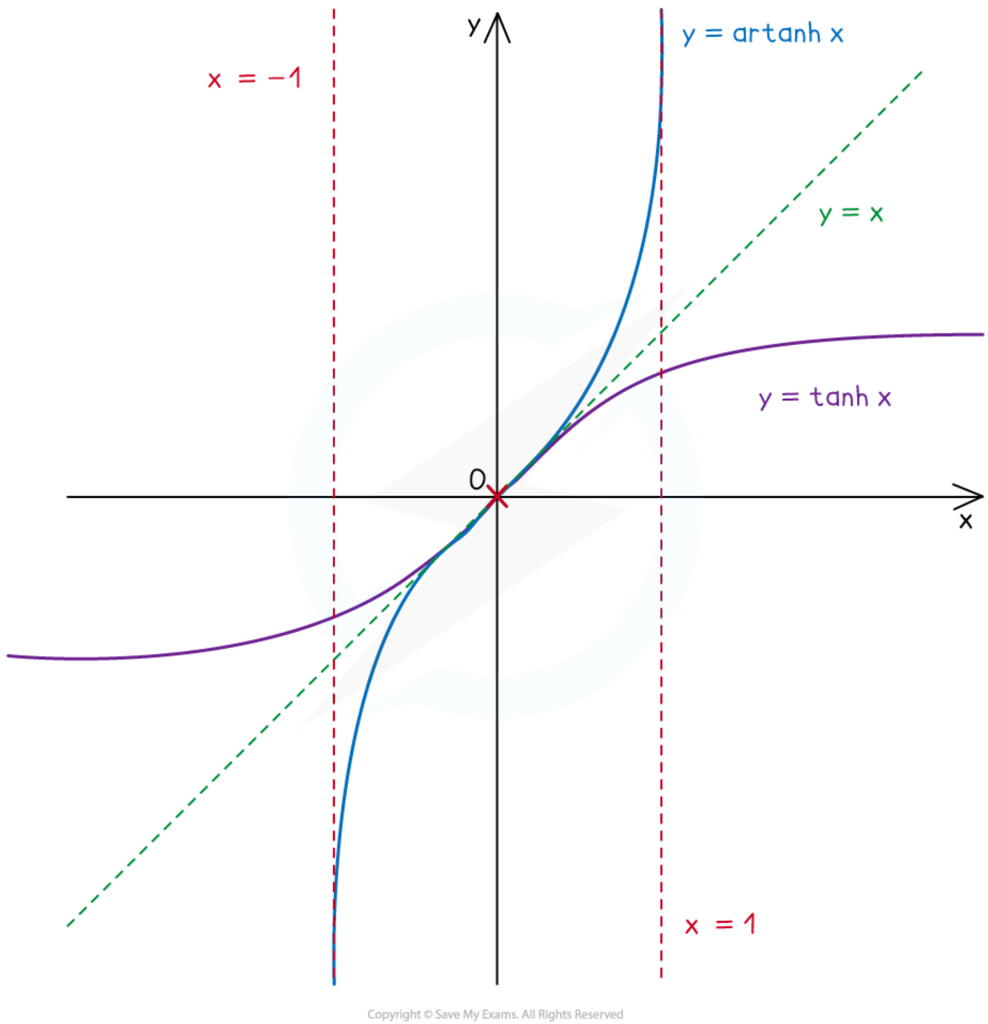
- $y = \text{arcosh}\, x$
  - Domain: $x \geq 1$
  - Range: $y \geq 0$

Your notes



- $y = \text{artanh}\,x$
  - Domain: $-1 < x < 1$
  - Range: $y \in \mathbb{R}$

Your notes



How do I derive the logarithmic formulae for the inverse hyperbolic functions?

- You need to be able to derive each inverse from the definition of the original
- **STEP 1**
  Write in **terms of e**
  - $y = \text{arsinh}x$ and rearrange to $x = \sinh y$
  - $x = \frac{1}{2}(e^y - e^{-y})$
- **STEP 2**
  Form a **quadratic** in terms of e^y
  - Multiply by $2e^y$ and rearrange
  - $(e^y)^2 - 2xe^y - 1 = 0$

Your notes

- **STEP 3**
  Solve the quadratic and find an expression for y
  - $e^y = x \pm \sqrt{x^2+1}$
    - Reject $x - \sqrt{x^2+1}$ as this produces negative values as $\sqrt{x^2+1} > x$ whereas $e^y > 0$
  - $y = \ln\left(x + \sqrt{x^2+1}\right)$
- The derivations of the other two formulae are similar
  - For $\operatorname{arcosh} x$ both $\ln\left(x + \sqrt{x^2-1}\right)$ and $\ln\left(x - \sqrt{x^2-1}\right)$ are well-defined for $x \geq 1$
    - We choose $\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2-1}\right)$ as $\operatorname{arcosh} x \geq 0$ and it can be shown that $\ln\left(x - \sqrt{x^2-1}\right) = \ln\left(\frac{1}{x + \sqrt{x^2-1}}\right) = -\ln\left(x + \sqrt{x^2-1}\right)$

Examiner Tip

- Be careful when working with the circular ("normal") inverse trig functions and the inverse hyperbolic functions
  - Only the "ar" denotes inverse
    - The "c" in arcsin x, arccos x, arctan x indicates the circular functions
    - The hyperbolic functions have "h", but the "h" doesn't come immediately after the "ar": arsinh x, arcosh x, artanh x
  - Be careful not to confuse these, especially when looking them up in the formulae booklet

Your notes

Worked example

Starting from the definition of $\tanh x$, show that

$$\operatorname{artanh}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \text{ when } |x| < 1.$$

$y = \operatorname{artanh}x$ rearrange to $x = \tanh y$

$$x = \tanh y = \frac{\sinh y}{\cosh y} = \frac{\frac{1}{2}(e^{y} - e^{-y})}{\frac{1}{2}(e^{y} + e^{-y})} = \frac{e^{2y}-1}{e^{2y}+1}$$

rearrange $x(e^{2y}+1) = (e^{2y}-1)$

$$x(e^{y})^2 + x = (e^{y})^2 - 1$$

$$x(e^{y})^2 - (e^{y})^2 = -1 - x$$

$$(e^{y})^2 = \frac{-1-x}{x-1} = \frac{1+x}{1-x}$$

$$\therefore e^{y} = \pm\sqrt{\frac{1+x}{1-x}}$$

disregard negative root as $e^{y} > 0$, $|x| < 1$

$$\therefore y = \ln\left(\sqrt{\frac{1+x}{1-x}}\right) = \ln\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$\boxed{\therefore \operatorname{artanh}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ |x| < 1}$$

Your notes

4.1.3 Hyperbolic Identities & Equations

Hyperbolic Identities & Equations

Are there identities linking the hyperbolic functions to the circular trig functions?

- Yes - these can be seen using de Moivre's Theorem to write
  - $\cos x = \frac{1}{2}(\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x})$
  - $\sin x = \frac{1}{2\mathrm{i}}(\mathrm{e}^{\mathrm{i}x} - \mathrm{e}^{-\mathrm{i}x})$
- Compare these with the definitions for the hyperbolic functions
  - $\cosh x = \frac{1}{2}(\mathrm{e}^{x} + \mathrm{e}^{-x})$
  - $\sinh x = \frac{1}{2}(\mathrm{e}^{x} - \mathrm{e}^{-x})$
- Therefore they can be related using the identities
  - $\cos x \equiv \cosh(\mathrm{i}x)$ or $\cosh x \equiv \cos(\mathrm{i}x)$
  - $\sin x \equiv -\mathrm{i}\sinh(\mathrm{i}x)$ or $\sinh x \equiv -\mathrm{i}\sin(\mathrm{i}x)$

What are the **hyperbolic identities?**

- In general, the **hyperbolic identities** are the same as the **circular trigonometric identities** except where there is a product of an even number of sinh terms, in which case the term changes sign
  - e.g. $\cos^2 x + \sin^2 x \equiv 1$ but $\cosh^2 x - \sinh^2 x \equiv 1$
  - This is referred to as Osborn's Rule
    - This occurs because of the connection with the imaginary number i
- All the circular trigonometric identities can be used with hyperbolic functions
- The main hyperbolic identities you are likely to need are
  - $\cosh^2 x - \sinh^2 x \equiv 1$
  - $\sinh 2x \equiv 2\sinh x\cosh x$
  - $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$
    - These are listed in the formulae booklet
- Other identities include
  - $\cosh 2x \equiv 2\cosh^2 x - 1 \equiv 1 + 2\sinh^2 x$
  - $\sinh(A \pm B) = \sinh A\cosh B \pm \sinh B\cosh A$
  - $\cosh(A \pm B) \equiv \cosh A\cosh B \pm \sinh A\sinh B$
- The harmonic identities can also be used with hyperbolic functions
  - $a\cosh x \pm b\sinh x = R\cosh(x \pm \alpha)$

- $a\sinh x \pm b\cosh x = R\sinh(x \pm \alpha)$
- Hyperbolic identities involving tanh*x* exist
  - They are not normally used as it is easier to use sinh*x*, cosh*x* and their definitions
  - If you do use tanh*x* identities, be careful with implied or 'hidden' products of sinh*x* (e.g. $\tanh^2 x$)
- You can prove these identities by using the definitions of the hyperbolic functions in terms of e

Your notes

Do reciprocal hyperbolic functions and identities exist?

- Yes! However, It is usually easier to deal with identities and equations involving these in terms of sinh*x*, cosh*x* and their definitions
- $\operatorname{cosech} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$ (Pronounced "coshec")
- $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$ (Pronounced "shec")
- $\coth x = \dfrac{1}{\tanh x} = \dfrac{\cosh x}{\sinh x} = \dfrac{e^{2x} + 1}{e^{2x} - 1}$ (Pronounced "cough")

How do I use hyperbolic identities to prove other identities?

- Start with the LHS and use the hyperbolic identities to rearrange into the RHS
- This approach can lead to what seems like a dead-end
  - In such cases simplify the LHS as far as possible, ideally so that it is **in terms of sinh*x* and/or cosh*x* only**
  - Then use the sinh*x* and cosh*x* definitions to write the LHS **in terms of e**
  - Repeat this for the RHS so that the LHS and RHS 'meet in the middle'

How do I solve equations involving hyperbolic functions?

- Use identities to create an equation in terms of **sinh*x* or cosh*x* only**
  - This should be a familiar equation to solve – linear, quadratic, etc
  - Find **exact answers** in terms of natural logarithms
    - Using the inverse hyperbolic functions definitions
  - Use your calculator if exact answers are not required
- As with circular trigonometric equations, do not cancel hyperbolic terms, rearrange so the equation **equals zero** and **factorise**
- When solving equations be careful when solving **cosh*x* = k** (for constant k)
  - $\cosh^{-1}x$ is not necessarily the same as arcosh*x*
    - arcosh*x* is, strictly speaking, referring to the inverse function of cosh*x* such that cosh*x* is a one-to-one function
  - Using the graph you can see that the for $k>1$ there are **two solutions** to $\cosh x = k$
    - $x = \pm \operatorname{arcosh} k$
    - This can be written in **logarithmic form** as $\pm\ln\left(k + \sqrt{k^2 - 1}\right)$
    - This can be shown to be **equivalent** to $\ln\left(k \pm \sqrt{k^2 - 1}\right)$

- If $k=1$ then the **only solution** to $\cosh x = k$ is $x=0$
- If $k<1$ then there are **no real solutions** to $\cosh x = k$

Your notes

Examiner Tip

- You can use the A Level Maths section of the formula booklet to remind you of trigonometric identities (such as sin(A±B)) which you can then adapt for the hyperbolic trig functions – don't limit yourself to just the Further Maths section

Your notes

Worked example

a) Using the definitions of $\sinh x$ and $\cosh x$ prove the identity $\cosh 2x = 1 + 2\sinh^2 x$.

$$\text{RHS: } 1 + 2\sinh^2 x \equiv 1 + 2\left(\tfrac{1}{2}\left(e^{x} - e^{-x}\right)\right)^2$$

$$\equiv 1 + 2\left(\tfrac{1}{4}\left(e^{2x} - 2 + e^{-2x}\right)\right)$$

$$\equiv 1 + \tfrac{1}{2}e^{2x} + \tfrac{1}{2}e^{-2x} - 1$$

$$\equiv \tfrac{1}{2}\left(e^{2x} + e^{-2x}\right)$$

$$\equiv \cosh 2x \quad \text{LHS}$$

$$\therefore \cosh 2x \equiv 1 + 2\sinh^2 x$$

b) Find the real solutions, as exact values, to the equation $\cosh 2x = 15 + 3\sinh x$.

Your notes

Use the identity $\cosh 2x \equiv 1 + 2\sinh^2 x$

$$1 + 2\sinh^2 x = 15 + 3\sinh x$$

$$2\sinh^2 x - 3\sinh x - 14 = 0$$

$$(2\sinh x - 7)(\sinh x + 2) = 0$$

$$\sinh x_1 = \frac{7}{2} \quad \text{or} \quad \sinh x_2 = -2$$

$$x_1 = \text{arsinh}\,\frac{7}{2} \qquad x_2 = \text{arsinh}\,\frac{7}{2}$$

$$= \ln\left(\frac{7}{2} + \sqrt{\left(\frac{7}{2}\right)^2 + 1}\right) \qquad = \ln\left(-2 + \sqrt{(-2)^2 + 1}\right)$$

$$= \ln\left(\frac{7 + \sqrt{53}}{2}\right) \qquad = \ln\left(\sqrt{5} - 2\right)$$

$$\boxed{x_1 = \ln\left(\frac{7 + \sqrt{53}}{2}\right) \quad x_2 = \ln\left(\sqrt{5} - 2\right)}$$

Your notes

4.1.4 Differentiating & Integrating Hyperbolic Functions

Differentiating Hyperbolic Functions

What are the **derivatives of the hyperbolic functions?**

- $\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x$
- $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \sinh x$
- $\frac{\mathrm{d}}{\mathrm{d}x}(\tanh x) = \mathrm{sech}^2 x$
- These are given in the **formulae booklet**
  - You can prove them by differentiating the definitions involving e
- Notice that they are **similar to the derivatives of the circular trig functions**
  - Be careful of the difference between the derivatives of cos*x* and cosh*x*
    - One involves a **negative sign** and the other does not

How do I differentiate expressions involving hyperbolic functions?

- The following differentiation skills may be required
  - Chain rule
  - Product rule
  - Quotient rule
  - Implicit differentiation
- Questions may involve showing or proving given results or finding unknown constants
- It is common that derivatives can be written in terms of the **original function**
  - This is due to the derivative of e^x also being e^x giving rise to the **repetition of terms**

What are the **derivatives of the inverse hyperbolic functions?**

- $\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arsinh} x) = \frac{1}{\sqrt{1+x^2}}$
- $\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2-1}}, \; x > 1$
- $\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{artanh} x) = \frac{1}{1-x^2}, \quad |\square| < 1$
- These are given in the **formulae booklet**

How do I prove or show the derivatives of the inverse hyperbolic functions?

- Use the same method for differentiating any inverse function

Your notes

- **STEP 1**
  Write ***x* in terms of *y***
  - $y = \text{arsinh}x$ can be written $x = \sinh y$
- **STEP 2**
  Differentiate with **respect to *y***
  - $\frac{\mathrm{d}x}{\mathrm{d}y} = \cosh y$
- **STEP 3**
  Write the derivative **in terms of *x***
  - $\frac{\mathrm{d}x}{\mathrm{d}y} = \cosh y = \pm\sqrt{\sinh^2 y + 1} = \pm\sqrt{x^2 + 1}$
- **STEP 4**
  Take the **reciprocal**
  - $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\frac{1}{\sqrt{x^2 + 1}}$
- **STEP 5**
  Use the graph to determine whether it is **positive of negative**
  - The graph of $y = \text{arsinh}x$ has a positive gradient everywhere
  - $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{x^2 + 1}}$

Examiner Tip

- It is usually easier to differentiate hyperbolic functions using the **"trig style"** standard results but if you are stuck you can try using their exponential form from the definitions

Your notes

Worked example

a)

Given that $y = 2x\sinh x$, show that $\frac{d^2y}{dx^2} = y + 4\cosh x$.

$y = 2x\sinh x$ Use the product rule

$$\frac{dy}{dx} = 2x\frac{d}{dx}(\sinh x) + \sinh x\frac{d}{dx}(2x)$$

$$= 2x\cosh x + 2\sinh x$$

Use the product rule again:

$$\frac{d^2y}{dx^2} = 2x\frac{d}{dx}(\cosh x) + \cosh x\frac{d}{dx}(2x) + \frac{d}{dx}(2\sinh x)$$

$$= 2x\sinh x + 2\cosh x + 2\cosh x$$

$$= 2x\sinh x + 4\cosh x$$

$$= y + 4\cosh x$$

$$\boxed{\frac{d^2y}{dx^2} = 2x\sinh x + 4\cosh x = y + 4\cosh x}$$

b)

Given that $f(x) = 3\text{arcosh}4x$, show that $f'(5) = \frac{a}{\sqrt{b}}$ where $a \in \mathbb{Q}$ and $b \in \mathbb{N}$ are constants to be found.

Your notes

$$f(x) = 3\operatorname{arcosh}(4x) \Rightarrow f'(x) = 3\frac{d}{dx}\operatorname{arcosh}(4x)$$

use chain rule

$$= 3\left(\frac{1}{\sqrt{(4x)^2-1}}\right)\frac{d}{dx}(4x)$$

$$= \frac{12}{\sqrt{(4x)^2-1}}$$

$$\boxed{f'(5) = \frac{12}{\sqrt{(4\times5)^2-1}} = \frac{12}{\sqrt{399}}}$$

Your notes

Integrating Hyperbolic Functions

What are the **integrals of the hyperbolic functions?**

- These are the reverse results of the derivatives, remembering "+c" of course!
  - $\int \sinh x \, dx = \cosh x + c$
  - $\int \cosh x \, dx = \sinh x + c$
    - These are given in the **integration** section of the **formulae booklet**
  - $\int \operatorname{sech}^2 x \, dx = \tanh x + c$
    - This can be deduced from the **differentiation** section of the **formulae booklet**
- There is also the integral of tanhx
  - $\int \tanh x \, dx = \ln \cosh x + c$
    - This is given in the integration section of the formulae booklet
    - It can be shown using the substitution $u = \cosh x$
  - Since cosh ≥ 1 for all values of x so there is no need for the modulus signs that usually accompany integrals involving ln

How do I integrate expressions involving or resulting in hyperbolic functions?

- The following integration skills may be required
  - Definite integration, area under a curve
  - Reverse chain rule ('adjust' and 'compensate')
  - Substitution
  - Integration by parts
- Hyperbolic identities may be required to rewrite an expression into an integrable form
- For products involving e^x and a hyperbolic function use the definition involving e^x and e^{-x} for the hyperbolic function to write everything in terms of exponentials
  - $\int e^x \cosh x \, dx = \frac{1}{2}\int e^x(e^x + e^{-x}) \, dx = \frac{1}{2}\int e^{2x} + 1 \, dx$

How do I integrate expressions involving inverse hyperbolic functions?

- To integrate inverse hyperbolic functions you would use integration by parts using the same technique as integrating lnx
  - Write the functions as a product with 1
    - e.g. $\operatorname{arsinh} x = 1 \times \operatorname{arsinh} x$
  - Differentiate the inverse function and integrate 1 when integrating by parts
    - $\int \operatorname{arsinh} x \, dx = x \operatorname{arsinh} x - \int \frac{x}{\sqrt{x^2+1}} \, dx = x \operatorname{arsinh} x - \sqrt{x^2+1} + c$

Examiner Tip

- Be aware of what is given in the formula booklet
  - Practise using it to find integrals
  - The results for hyperbolic functions and the inverse circular trig functions are listed together so try not to get confused
- If you can't spot a relevant hyperbolic identity then using exponentials can make the expression easier and quicker to integrate

Your notes

Your notes

Worked example

Find the following integrals:

a) $$\int \sinh 3x \cosh^3 3x \, dx$$

Use integration by substitution.

Let $u = \cosh 3x$, then $\frac{du}{dx} = 3\sinh 3x$

$$\therefore \ dx = \frac{du}{3\sinh 3x}$$

$$\therefore \int \sinh 3x \cosh^3 3x \, dx = \int (\sinh 3x)(u)^3 \frac{du}{3\sinh 3x}$$

$$= \int \frac{1}{3} u^3 \, du$$

$$= \frac{u^4}{12} + c$$

Substitute $u = \cosh 3x$

$$\boxed{\int \sinh 3x \cosh^3 3x \, dx = \frac{\cosh^4 3x}{12} + c}$$

b) $$\int \cosh^4 x - \sinh^4 x \, dx$$

Simplify the expression first:

$$\cosh^4(x) - \sinh^4(x) = (\cosh^2 x + \sinh^2 x)(\cosh^2 x - \sinh^2 x)$$

difference of two squares

$$= \cosh 2x \quad \equiv$$

$\cosh^2 x - \sinh^2 x \equiv 1$

Use reverse chain rule to find $\int \cosh 2x \, dx$

$$\boxed{\int \cosh 2x \, dx = \frac{1}{2}\sinh 2x + c}$$

Your notes