OCR A Level Physics

Your notes

Damping

Contents

Energy in SHM

Your notes

Kinetic & Potential Energies

- During simple harmonic motion, energy is constantly exchanged between two forms:
  - **Kinetic** energy
  - **Potential** energy
- The potential energy could be in the form of:
  - Gravitational potential energy (for a **pendulum**)
  - Elastic potential energy (for a horizontal mass on a **spring**)
  - Or **both** (for a vertical mass on a spring)
- The speed of an oscillator is at a maximum when displacement $x = 0$, so:

  The kinetic energy of an oscillator is at a maximum when the displacement is zero

- This is because kinetic energy is equal to $\frac{1}{2}mv^2$ so when the oscillator moves at maximum velocity (at the equilibrium position) it reaches its maximum value of kinetic energy
- Therefore, the kinetic energy is zero at maximum displacement $x = x_0$, so:

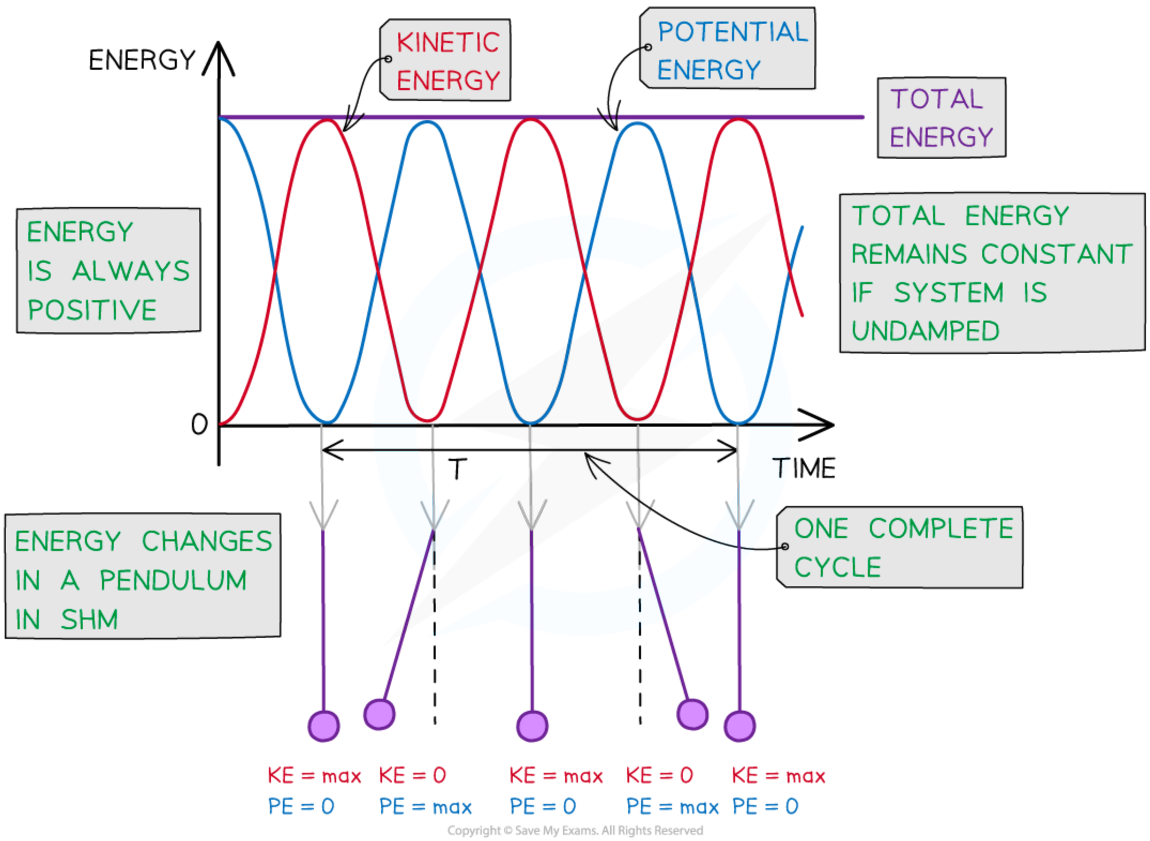
  The potential energy of an oscillator is at a maximum when the displacement (both positive and negative) is at a maximum, $x = \pm x_0$

- This is because the kinetic energy is transferred to potential energy as the height above the equilibrium position increases, since potential energy is equal to *mgh*
- A simple harmonic system is therefore constantly converting between kinetic and potential energy
- When one increases, the other decreases and vice versa, therefore:

  The total energy of a simple harmonic system always remains constant

- The total energy is, therefore, equal to the sum of the kinetic and potential energies

Your notes



The kinetic and potential energy of an oscillator in SHM vary periodically

- **The key features of the energy-time graph are**:
    - Both the kinetic and potential energies are represented by periodic functions (sine or cosine) which are varying in opposite directions to one another
    - When the potential energy is 0, the kinetic energy is at its maximum point and vice versa
    - The **total energy** is represented by a **horizontal straight line** directly above the curves at the maximum value of both the kinetic or potential energy
    - Energy is **always positive** so there are no negative values on the y axis
- **Note:** kinetic and potential energy go through **two** complete cycles during one **period** of oscillation
    - This is because one complete oscillation reaches the maximum displacement **twice** (positive and negative)

Energy-Displacement Graphs

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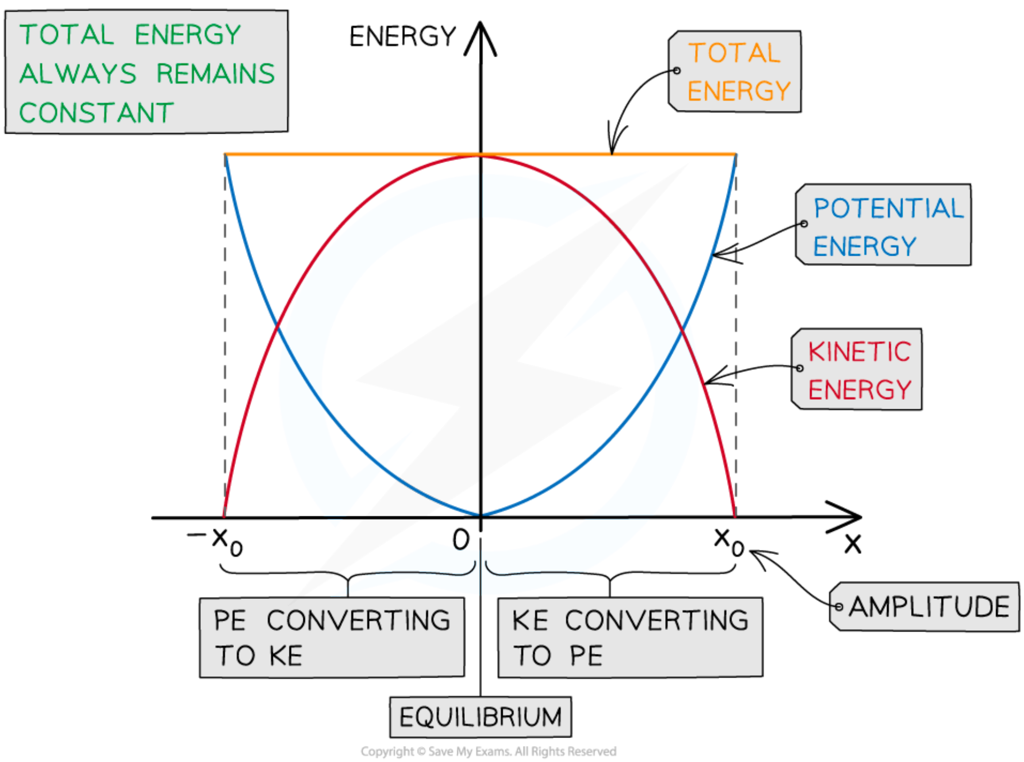
The total energy of system undergoing simple harmonic motion is defined by:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

- Where:
  - E = total energy of a simple harmonic system (J)
  - m = mass of the oscillator (kg)
  - ω = angular frequency (rad s^{-1})
  - x_0 = amplitude (m)
- The energy-displacement graph for **half** a cycle looks like:



Potential and kinetic energy v displacement in half a period of an SHM oscillation

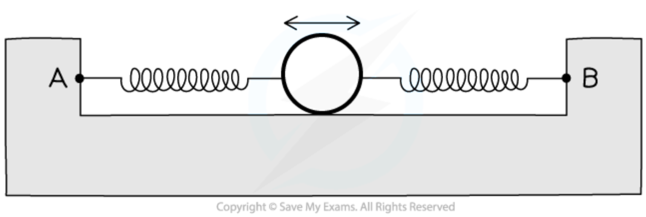
- **The key features of the energy-displacement graph:**
  - Displacement is a vector, so, the graph has both **positive** and **negative** x values

- The potential energy is always at a maximum at the amplitude positions x_0 and 0 at the equilibrium position (x = 0)
- This is represented by a **'U' shaped curve**
- The kinetic energy is the opposite: it is 0 at the amplitude positions x_0 and maximum at the equilibrium position x = 0
- This is represented by a **'n' shaped curve**
- The total energy is represented by a **horizontal straight line** above the curves

Your notes

Worked Example

A ball of mass 23 g is held between two fixed points A and B by two stretch helical springs, as shown in the figure below



The ball oscillates along the line AB with simple harmonic motion of frequency 4.8 Hz and amplitude 1.5 cm. Calculate the total energy of the oscillations.

Answer:

Step 1: Write down all known quantities

- Mass, m = 23 g = 23 × 10^{-3} kg
- Amplitude, x_0 = 1.5 cm = 0.015 m
- Frequency, f = 4.8 Hz

Step 2: Write down the equation for the total energy of SHM oscillations:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

Step 3: Write an expression for the angular frequency

$$\omega = 2\pi f = 2\pi \times 4.8$$

Step 4: Substitute values into energy equation

$$E = \frac{1}{2} \times (23 \times 10^{-3}) \times (2\pi \times 4.8)^2 \times (0.015)^2$$

$$E = 2.354 \times 10^{-3} = 2.4 \text{ mJ}$$

Your notes

Examiner Tips and Tricks

You may be expected to draw as well as interpret energy graphs against time or displacement in exam questions. Make sure the sketches of the curves are as even as possible and **use a ruler** to draw straight lines, for example, to represent the total energy.

Free & Forced Oscillations

Your notes

Free & Forced Oscillations

Free Oscillations

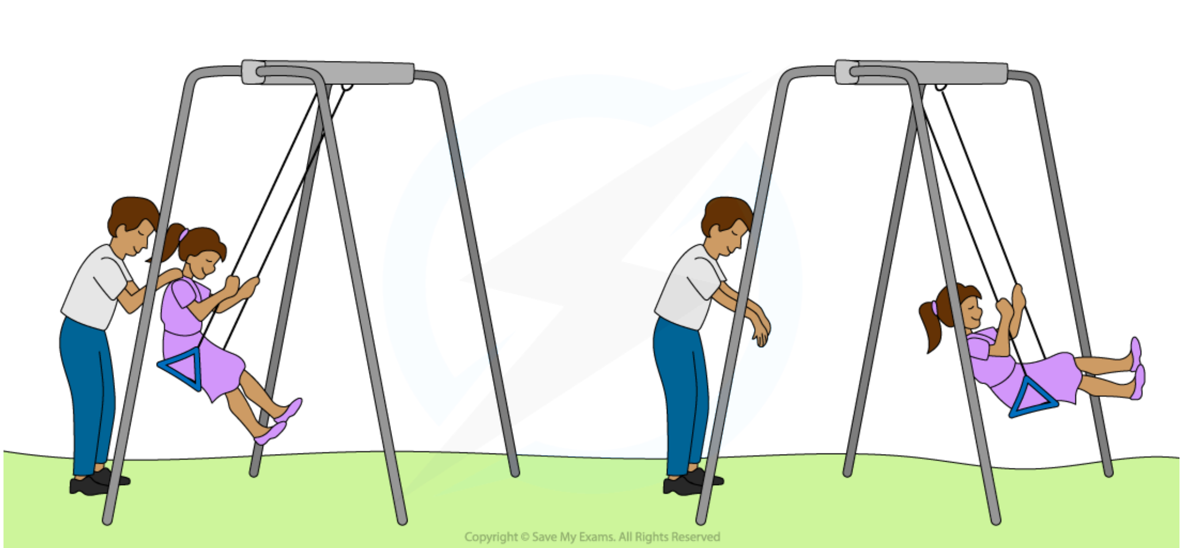
- Free oscillations occur when there is no transfer of energy to or from the surroundings
  - This happens when an oscillating system is displaced and then left to oscillate
- In practice, this only happens in a vacuum. However, anything vibrating in air is still considered a free vibration as long as there are **no external forces** acting upon it
- Therefore, a **free oscillation** is defined as:

  An oscillation where there are only internal forces (and no external forces) acting and there is no energy input
- A free vibration always oscillates at its **resonant frequency**

Forced Oscillations

- In order to sustain oscillations in a simple harmonic system, a periodic force must be applied to replace the energy lost in damping
  - This periodic force **does work** against the resistive force responsible for decreasing the oscillations
  - It is sometimes known as an external **driving** force
- These are known as **forced oscillations** (or vibrations), and are defined as:

  Oscillations acted on by a periodic external force where energy is given in order to sustain oscillations
- Forced oscillations are made to oscillate at the same frequency as the oscillator creating the external, periodic driving force
- For example, when a child is on a swing, they will be pushed at one end after each cycle in order to keep swinging and prevent air resistance from damping the oscillations
  - These extra pushes are the forced oscillations, without them, the child will eventually come to a stop



Worked Example

State whether the following are free or forced oscillations:

(i) Striking a tuning fork

(ii) Breaking a glass from a high pitched sound

(iii) The interior of a car vibrating when travelling at a high speed

(iv) Playing the clarinet

Answer:

Part (i)

Striking a tuning fork

This is a free vibration. When a tuning fork is struck, it will vibrate at its natural frequency and there are no other external forces

Part (ii)

Breaking a glass from a high pitched sound

This is a forced vibration. The glass is forced to vibrate at the same frequency as the sound until it breaks. The frequency of the high-pitched sound is the external driving frequency

Part (iii)

The interior of a car vibrating when travelling at a particular speed

This is a forced vibration. The interior of the car vibrates at the same frequency as the wheels travelling over a rough surface at a high speed

Part (iv)

Playing the clarinet

This is a forced vibration. The air from the player's lungs is used to sustain the vibration in the air column in a clarinet to create and hold a sound. The air column inside the clarinet mimics the vibrations at the same frequency as the air forced into the mouthpiece of the clarinet (the reed).

Examiner Tips and Tricks

Avoid writing 'a free oscillation is not forced to oscillate'. Mark schemes are mainly looking for a reference to internal and external forces and energy transfers

Your notes

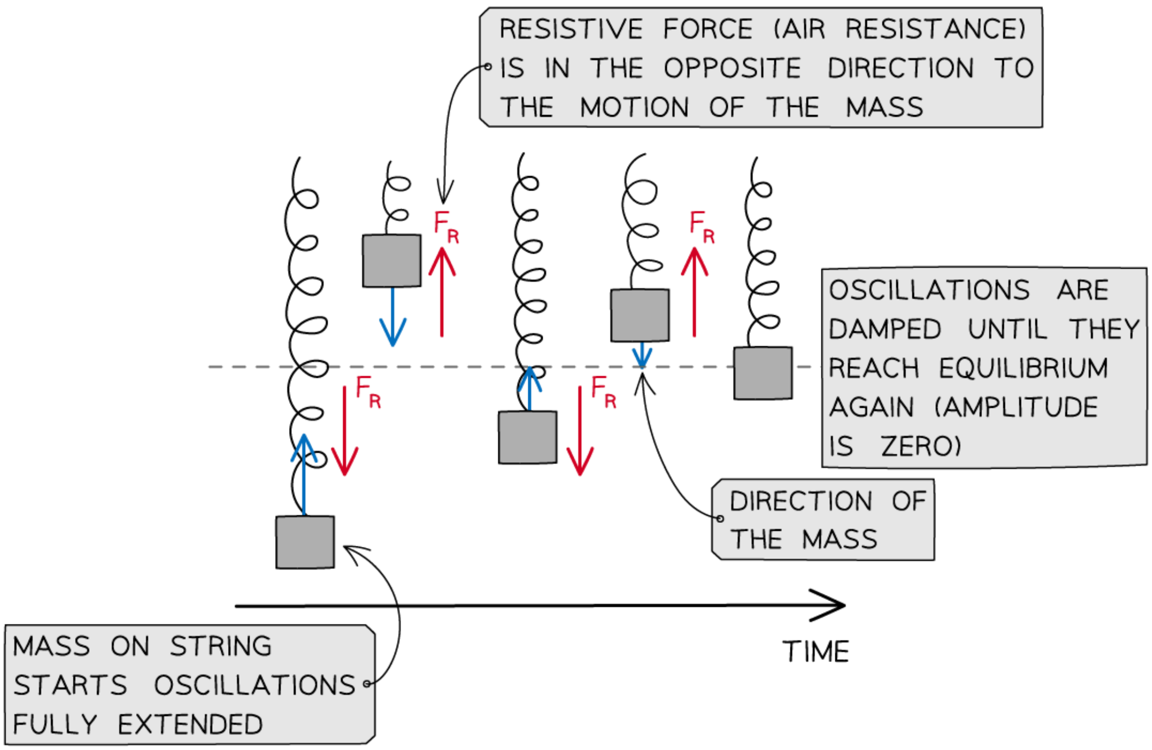
Damping

Your notes

The Effects of Damping

- In practice, all oscillators eventually stop oscillating
  - Their amplitudes decrease rapidly, or gradually
- This happens due to **resistive forces**, such as friction or air resistance, which act in the opposite direction to the motion, or **velocity**, of an oscillator
- Resistive forces acting on an oscillating simple harmonic system cause **damping**
  - These are known as **damped** oscillations
- Damping is defined as:

  The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system
- Damping continues to have an effect until the oscillator comes to rest at the equilibrium position
- A key feature of simple harmonic motion is that the **frequency** of damped oscillations **does not change** as the amplitude decreases
  - For example, a child on a swing can oscillate back and forth once every second, but this time remains the same regardless of the amplitude



Damping on a mass on a spring is caused by a resistive force acting in the opposite direction to the motion, or velocity. This continues until the amplitude of the oscillations reaches zero

Forced & Damped Oscillations

Types of Damping

- There are three degrees of damping depending on how quickly the amplitude of the oscillations decrease:
  - Light damping
  - Critical damping
  - Heavy damping

Light Damping

- When oscillations are lightly damped, the amplitude does not decrease linearly
  - It decays exponentially with time

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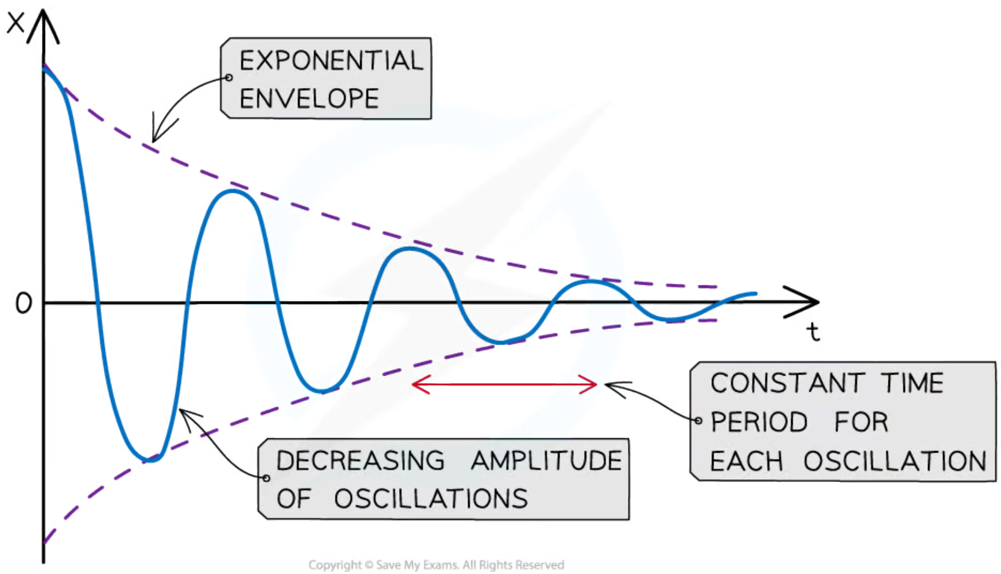
- When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with gradually decreasing amplitude
  - For example, a swinging pendulum decreasing in amplitude until it comes to a stop



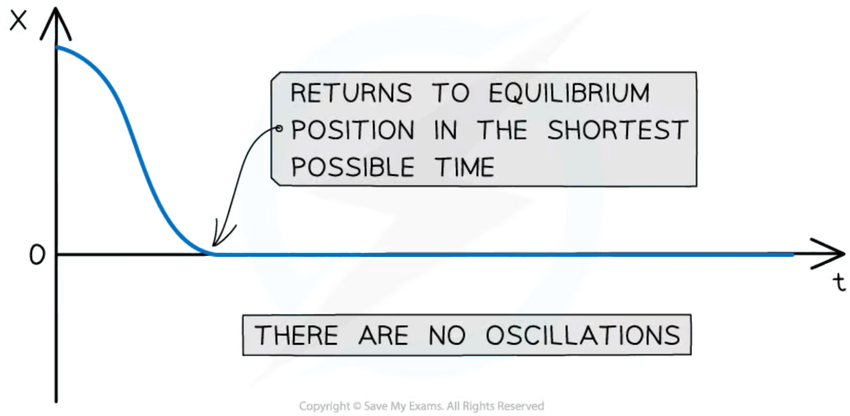
A graph for a lightly damped system consists of oscillations decreasing exponentially

- **Key features of a displacement-time graph for a lightly damped system:**
  - There are many oscillations represented by a sine or cosine curve with gradually decreasing amplitude over time
  - This is shown by the height of the curve decreasing in both the positive and negative displacement values
  - The amplitude decreases exponentially
  - The frequency of the oscillations remain constant, this means the time period of oscillations must stay the same and each peak and trough is equally spaced

Critical Damping

- When a critically damped oscillator is displaced from the equilibrium, it will return to rest at its equilibrium position in the shortest possible time **without** oscillating
  - For example, car suspension systems prevent the car from oscillating after travelling over a bump in the road

Your notes



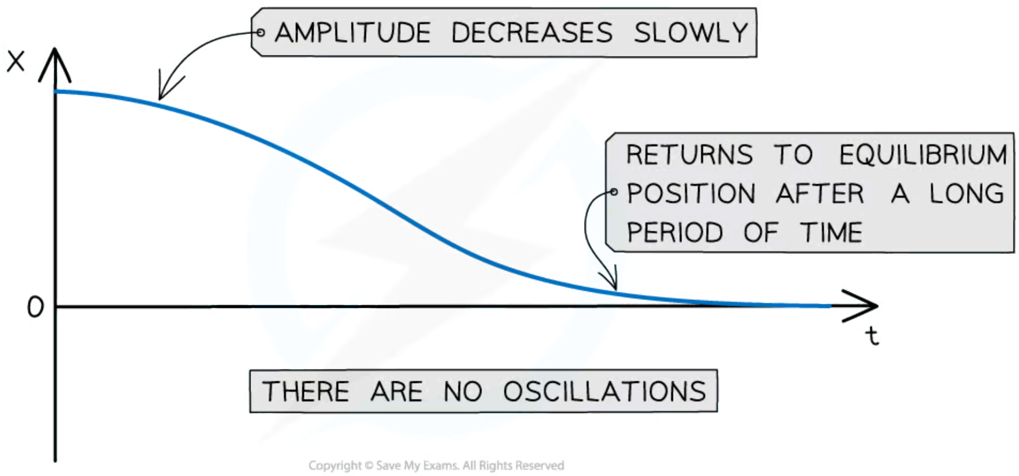
The graph for a critically damped system shows no oscillations and the displacement returns to zero in the quickest possible time

- **Key features of a displacement–time graph for a critically damped system:**
  - This system does **not** oscillate, meaning the displacement falls to 0 straight away
  - The graph has a fast decreasing gradient when the oscillator is first displaced until it reaches the x axis
  - When the oscillator reaches the equilibrium position ($x = 0$), the graph is a horizontal line at $x = 0$ for the remaining time

Heavy Damping

- When a heavily damped oscillator is displaced from the equilibrium, it will take a long time to return to its equilibrium position **without** oscillating
- The system returns to equilibrium more slowly than the critical damping case
  - For example, door dampers are used on doors to prevent them slamming shut

Your notes



A heavy damping curve has no oscillations and the displacement returns to zero after a long period of time

- **Key features of a displacement-time graph for a heavily damped system:**
  - There are no oscillations. This means the displacement does not pass zero
  - The graph has a slow decreasing gradient from when the oscillator is first displaced until it reaches the x axis
  - The oscillator reaches the equilibrium position (x = 0) after a long period of time, after which the graph remains a horizontal line for the remaining time

Worked Example

A mechanical weighing scale consists of a needle that moves to a position on a numerical scale depending on the weight applied. Sometimes, the needle moves to the equilibrium position after oscillating slightly, making it difficult to read the number on the scale at which it is pointing.

Suggest, with a reason, whether light, critical or heavy damping should be applied to the mechanical weighing scale to read the scale more easily.

Answer:

Step 1: Consider light damping

- Ideally, the needle should not oscillate before settling
- This means the scale should have either **critical** or **heavy damping**

Step 2: Consider heavy damping

- Since the scale is read straight away after a weight is applied, ideally, the needle should settle as quickly as possible
- Heavy damping would mean the needle will take some time to settle on the scale

Step 3: Consider critical damping

- Therefore, **critical damping** should be applied to the weighing scale so the **needle can settle as quickly as possible** to read from the scale

Your notes

Examiner Tips and Tricks

Make sure not to confuse **resistive** force and **restoring** force:

- Resistive force is what **opposes the motion / velocity** of the oscillator and causes damping
- Restoring force is what brings the oscillator **back to the equilibrium position**

Resonance

Your notes

Resonance

- The frequency of forced oscillations is referred to as the **driving frequency, *f*,** or the frequency of the applied force
- All oscillating systems have a **natural frequency, f_0**, this is defined as this is the frequency of an oscillation when the oscillating system is allowed to oscillate freely
- Oscillating systems can exhibit a property known as **resonance**
- When the driving frequency approaches the natural frequency of an oscillator, the system gains more energy from the driving force
  - Eventually, when they are equal, the oscillator vibrates with its maximum amplitude, this is **resonance**
- Resonance is defined as:

  When the frequency of the applied force to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations increases significantly

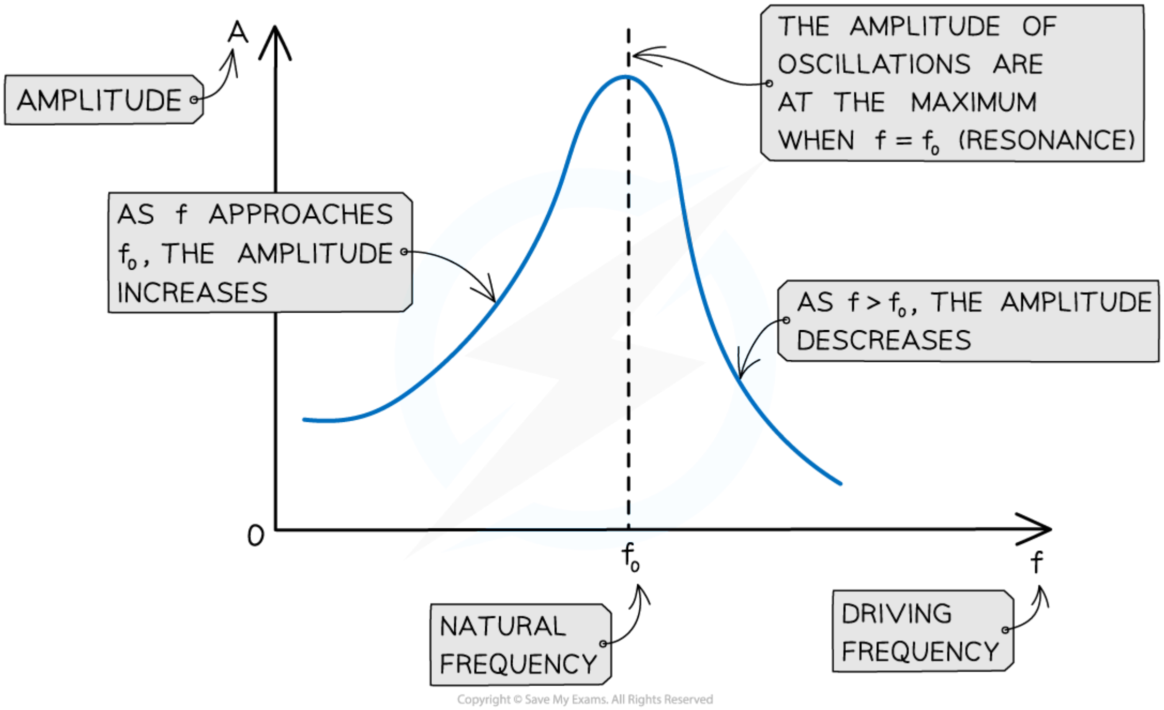
- For example, when a child is pushed on a swing:
  - The swing plus the child has a fixed natural frequency
  - A small push after each cycle increases the amplitude of the oscillations to swing the child higher. This frequency at which this push happens is the driving frequency
  - When the driving frequency is exactly equal to the natural frequency of the swing oscillations, resonance occurs
  - If the driving frequency does not quite match the natural frequency, the amplitude will increase but not to the same extent as when resonance is achieved
- This is because, at resonance, energy is transferred from the driver to the oscillating system **most efficiently**
  - Therefore, at resonance, the system will be transferring the maximum kinetic energy possible

Amplitude-Frequency Graphs

- A graph of driving frequency *f* against amplitude *A* of oscillations is called a **resonance curve**. It has the following key features:
  - When $f < f_0$, the amplitude of oscillations increases

Your notes

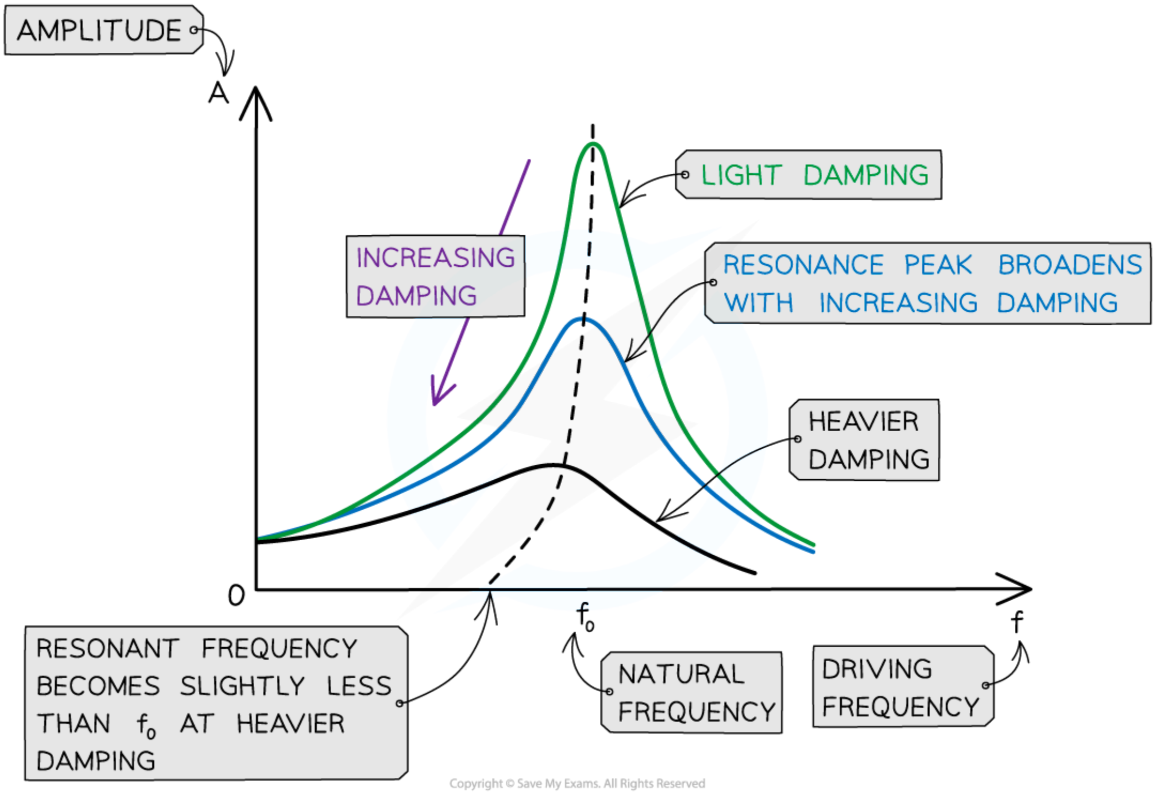
- At the peak where $f = f_0$, the amplitude is at its maximum. This is **resonance**
- When $f > f_0$, the amplitude of oscillations starts to decrease



The maximum amplitude of the oscillations occurs when the driving frequency is equal to the natural frequency of the oscillator

The Effects of Damping on Resonance

- Damping **reduces** the amplitude of resonance vibrations
- The height and shape of the resonance curve will therefore change slightly depending on the degree of damping
  - **Note:** the natural frequency f_0 of the oscillator will remain the same
- As the degree of damping is increased, the resonance graph is altered in the following ways:
  - The amplitude of resonance vibrations **decrease**, meaning the peak of the curve lowers
  - The resonance peak **broadens**
  - The resonance peak moves slightly to the **left** of the natural frequency when heavily damped
- Therefore, damping reduced the sharpness of resonance and reduces the amplitude at resonant frequency



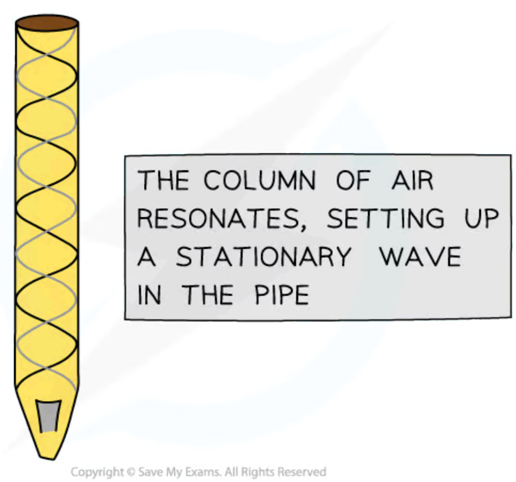
As damping is increased, resonance peak lowers, the curve broadens and moves slightly to the left

Your notes

Examples of Forced Oscillations & Resonance

Examples of Forced Oscillations & Resonance

- Resonance occurs for any forced oscillation where the frequency of the driving force is equal to the natural frequency of the oscillator
    - For example, a glass smashing from a high pitched sound wave at the right frequency
- Some other practical examples of forced oscillations and resonance include:
    - An organ pipe
    - Radio receivers
    - Microwave oven
    - Magnetic resonance imaging (MRI)
- **In an organ pipe**
    - Air molecules vibrate in an air column setting up a stationary wave in the pipe
    - This causes the air molecules to resonate leading to an increase in amplitude of sound

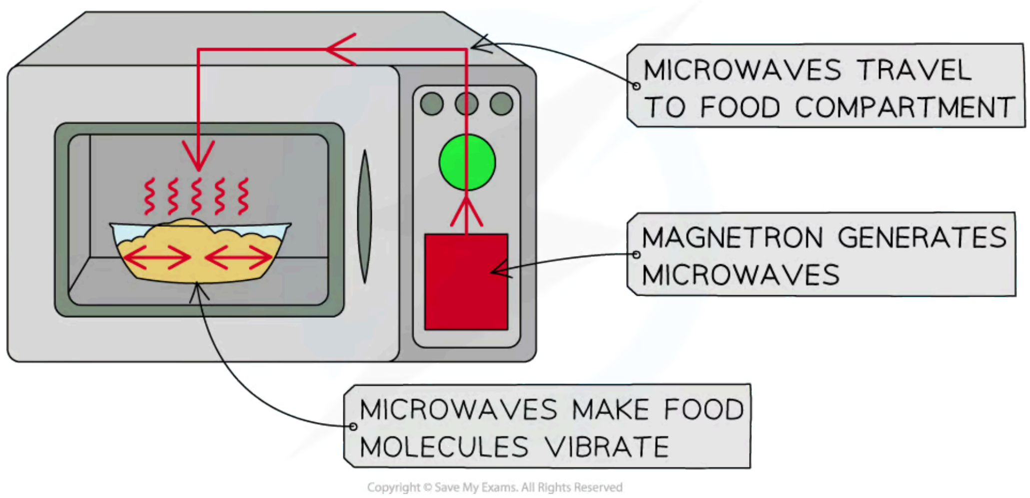


Standing waves forming inside an organ pipe from resonance

- **Radio receivers**
    - The radio is "tuned" by setting its natural frequency equal to that of a radio station

- The radio tuned so that the electric circuit resonates at the same frequency as the specific broadcast
- The resonance of the radio waves allows the signal to be amplified by the receiver to listen

- **Microwave oven**
  - Conventional cooking methods involve transferring heat energy by conduction or convection
  - A microwave transfers heat energy by radiation i.e. microwaves of a particular frequency that resonate with the water molecules in food



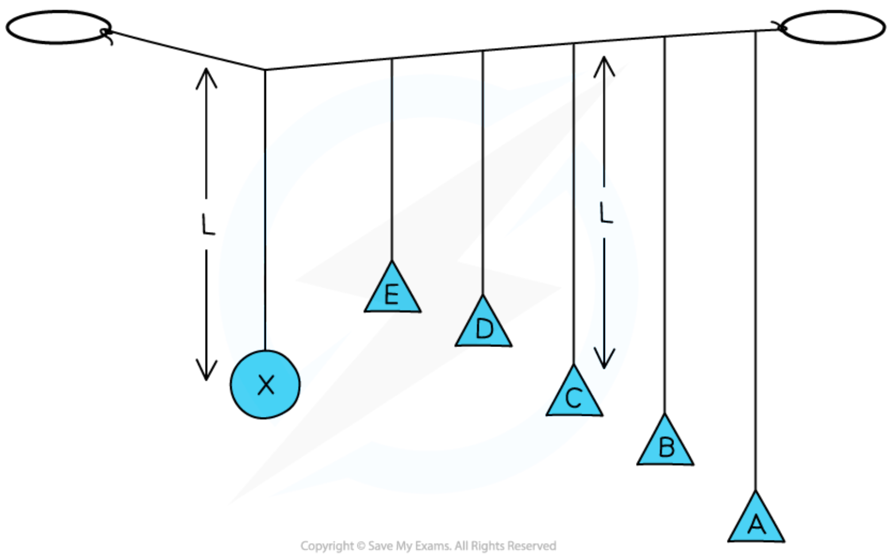
- **Magnetic resonance imaging (MRI)**
  - This type of scanner is a widely used medical diagnostic tool used to look at organs and structures inside the body
  - The atomic nuclei in the body are made to resonate with incoming radio waves (of the order of 100 MHz)
  - The signals are then sent to a computer to create digital scans and provide a detailed image of the scanned area

Barton's Pendulums

- A mechanical system commonly used to show resonance is **Barton's pendulums**
- A set of light pendulums labelled **A–E** are suspended from a string
  - A heavy pendulum **X**, with a length *L*, is attached to the string at one end and will act as the driving pendulum
- When pendulum **X** is released, it pushes the string and begins to drive the other pendulums

Your notes

- Most of the pendulums swing with a low amplitude but pendulum **C** with the same length *L* has the **largest** amplitude
  - This is because its natural frequency is **equal** to the frequency of pendulum X (the driving frequency)



Barton's pendulums helps display resonance

- The phase of the oscillations relative to the driver are:
  - Pendulums **E** and **D** with lengths < *L* are in phase
  - Pendulum **C** with length = *L* is 0.5π out of phase
  - Pendulums **B** and **A** with lengths > *L* are π out of phase