OCR A Level Physics

Your notes

Density & Pressure

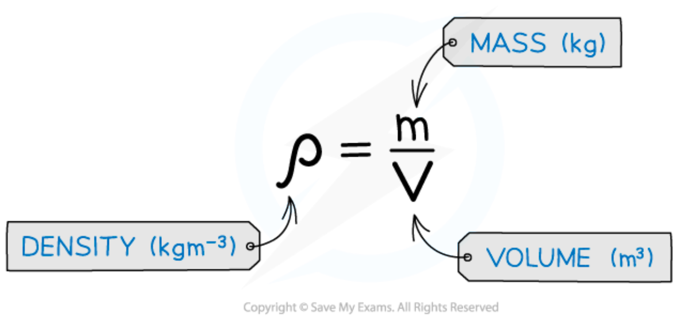
Contents

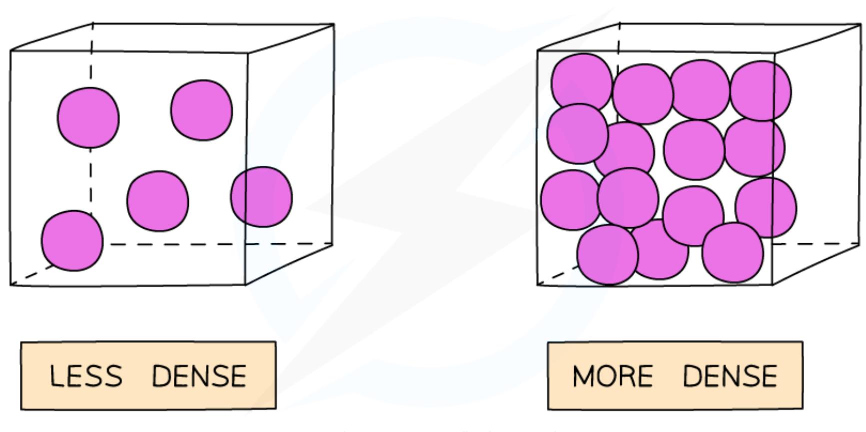
Your notes

Density

Density

- Density is the **mass per unit volume** of an object
  - Objects made from low-density materials typically have a lower mass
  - For example, a balloon is less dense than a small bar of lead despite occupying a larger volume
- The units of density depend on the units used for mass and volume:
  - If the mass is measured in *g* and volume in **cm^3**, then the density will be in **g / cm^3**
  - If the mass is measured in *kg* and volume in **m^3**, then the density will be in **kg / m^3**





Gases are less dense than a solid

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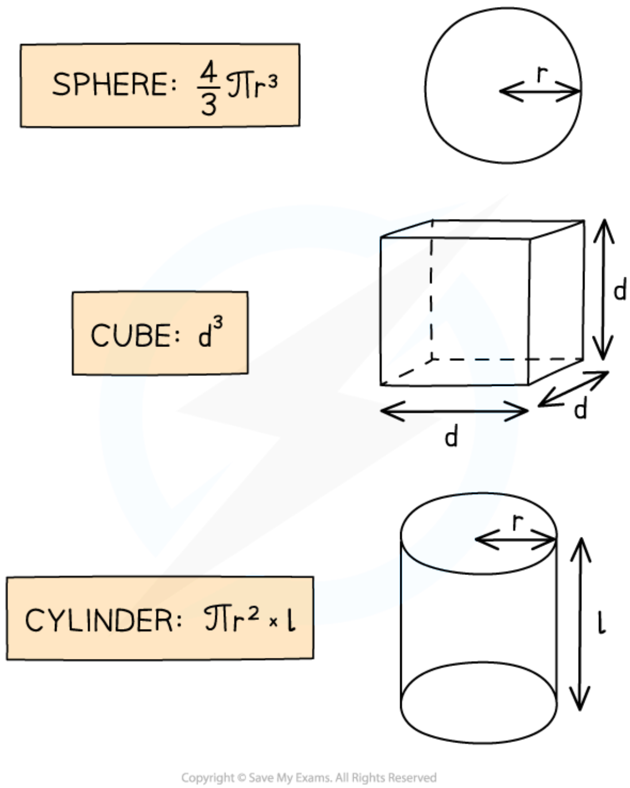
- The volume of an object may not always be given directly, but can be calculated with the appropriate equation depending on the object's shape

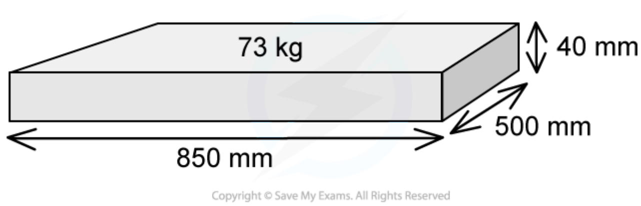


Volumes of common 3D shapes

Worked Example

A paving slab has a mass of 73 kg and dimensions 40 mm × 500 mm × 850 mm.

Calculate the density, in kg m^{-3} of the material from which the paving slab is made.



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Answer:

STEP 1

EQUATION FOR DENSITY

$$\rho = \frac{M}{V}$$

STEP 2

CALCULATE THE VOLUME

$$V = 40\ \text{mm} \times 500\ \text{mm} \times 850\ \text{mm} = 1.7 \times 10^{7}\ \text{mm}^3$$

STEP 3

CONVERT mm^3 TO m^3

$$1\ \text{mm} = 0.001\ \text{m} = 1 \times 10^{-3}\ \text{m}$$

$$1\ \text{mm}^3 = (0.001)^3\ \text{m}^3 = (1 \times 10^{-3})^3\ \text{m}^3 = 1 \times 10^{-9}\ \text{m}^3$$

$$V = 1.7 \times 10^{7} \times 1 \times 10^{-9} = 1.7 \times 10^{-2}\ \text{m}$$

STEP 4

SUBSTITUTE MASS AND VOLUME INTO DENSITY EQUATION

$$\rho = \frac{73\ \text{kg}}{1.7 \times 10^{-2}\ \text{m}^3} = 4300\ \text{kgm}^{-3}\ \ (2\ \text{s.f.})$$

Examiner Tips and Tricks

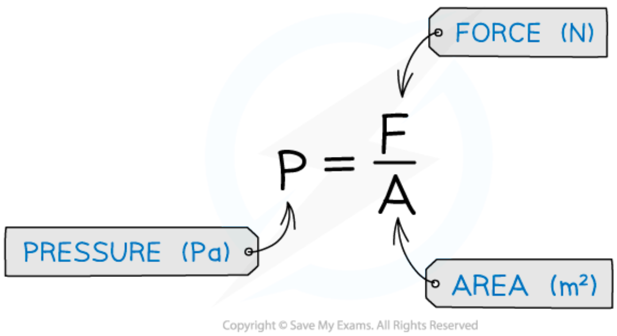
- When converting a **larger** unit to a **smaller** one, you **multiply (×)**
  - E.g. 125 m = 125 × 100 = 12 500 cm
- When you convert a **smaller** unit to a **larger** one, you **divide (÷)**
  - E.g. 5 g = 5 / 1000 = 0.005 or 5 × 10^{-3} kg
- When dealing with squared or cubic conversions, cube or square the conversion factor too
  - E.g. 1 mm^3 = 1 / $(1000)^3$ = 1 × 10^{-9} m^3
  - E.g. 1 cm^3 = 1 / $(100)^3$ = 1 × 10^{-6} m^3

Pressure

Your notes

Pressure

- Pressure tells us how concentrated a force is, it is defined as the **force per unit area**



- This equation shows:
  - If a force is spread over a large area it will result in a small pressure
  - If it is spread over a small area it will result in a large pressure

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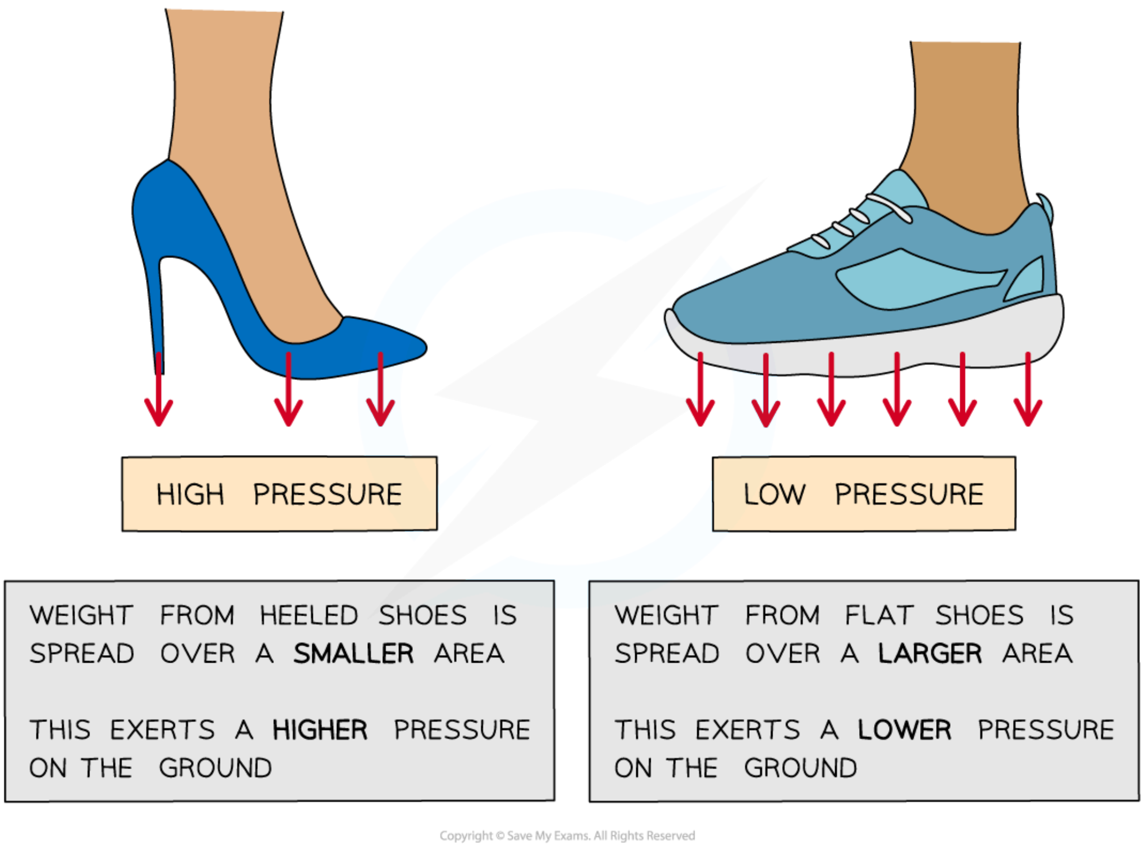


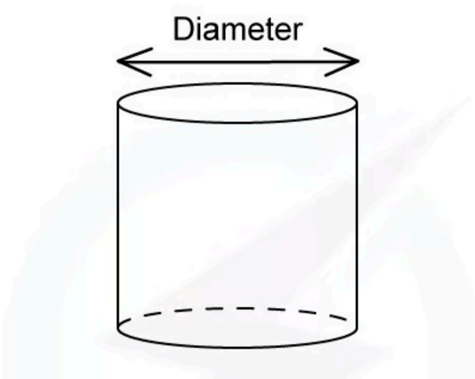
Different pressure is exerted for the same force on different areas

- The units of pressure depend on the units of area:
  - If the area is measured in **cm^2** (and the force in **N**), then the pressure will be in **N/cm^2**
  - If the area is measured in **m^2** (and the force in **N**), then the pressure will be in **N/m^2**
- Pressure can also be measured in pascals, Pa where **1 Pa is the same as 1 N/m^2**
- Pressure, unlike force, is a **scalar** quantity
  - Therefore pressure does not have a specific direction

Worked Example

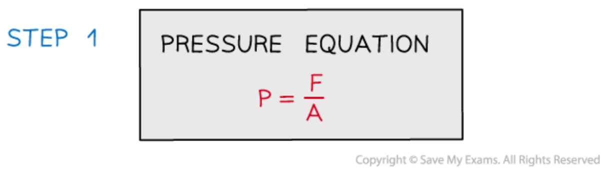
A cylinder is placed on a horizontal surface as shown below

Your notes



The mass of the cylinder is 4.7 kg and the diameter is 8.4 cm. Calculate the pressure produced by the cylinder on the surface in Pa.

Answer:



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STEP 2

CALCULATE THE FORCE PRODUCED BY THE CYLINDER

ONLY FORCE IS THE WEIGHT

$$F = mg = 4.7\,\text{kg} \times 9.81\,\text{ms}^{-2} = 46.1\,\text{N}$$

STEP 3

CALCULATE THE AREA

THE FORCE IS APPLIED ON THE BASE OF THE CYLINFER WHICH IS A CIRCLE

$$\text{AREA OF A CIRCLE} = \pi \times r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{8.4 \times 10^{-2}\,\text{m}}{2}\right)^2 = 5.5 \times 10^{-3}\,\text{m}^2$$

STEP 4

SUBSTITUTE BACK INTO PRESSURE EQUATION

$$P = \frac{46.1\,\text{N}}{5.5 \times 10^{-3}\,\text{m}^2} = 8400\,\text{Pa} \ (2\text{ s.f})$$

Examiner Tips and Tricks

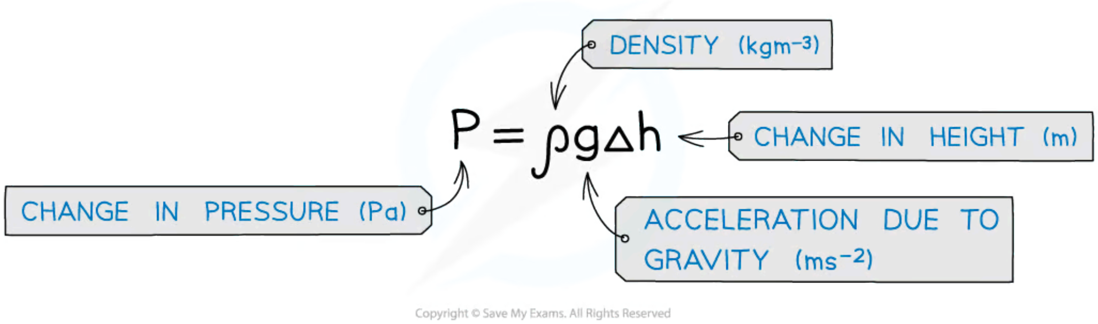
The area referred to is the 'cross-sectional' area of a 3D object. This is the area of the base that the force is applied on. For a cylinder, this will be a circle.

Archimedes' Principle

Archimedes' Principle

Upthrust on an Object in a Fluid

- Pressure increases with depth in a fluid because of the force exerted by the increased weight of the fluid above
- This change in pressure can be calculated using the equation of hydrostatic pressure:



- This equation can be derived in the following way:

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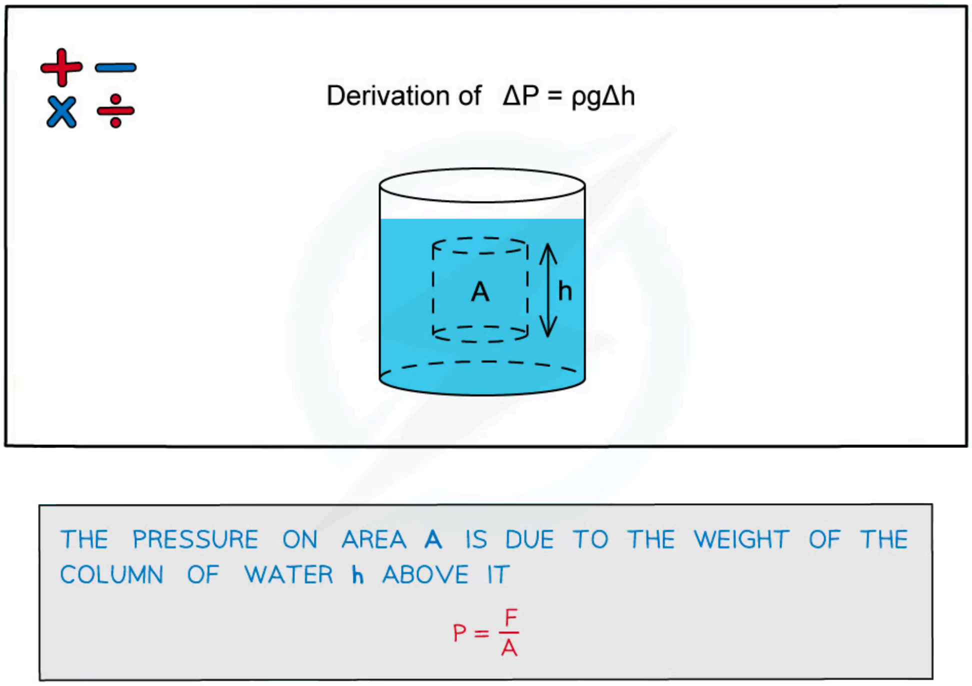

Derivation of ΔP = ρgΔh
A
h
THE PRESSURE ON AREA A IS DUE TO THE WEIGHT OF THE COLUMN OF WATER h ABOVE IT
P = F/A

FORCE IS THE WEIGHT OF THE COLUMN

$$W = mg$$

MASS OF THE COLUMN IS FROM

$$m = \rho \times V$$

WHERE $V = A \times h$

SUBSTITUTE VOLUME, MASS EQUALS

$$m = \rho \times A \times h$$

WEIGHT OF THE COLUMN IS NOW

$$W = \rho \times A \times h \times g$$

SUBSTITUTE BACK INTO PRESSURE EQUATION

$$P = \frac{\rho \times A \times h \times g}{A}$$

$$P = \rho h g$$

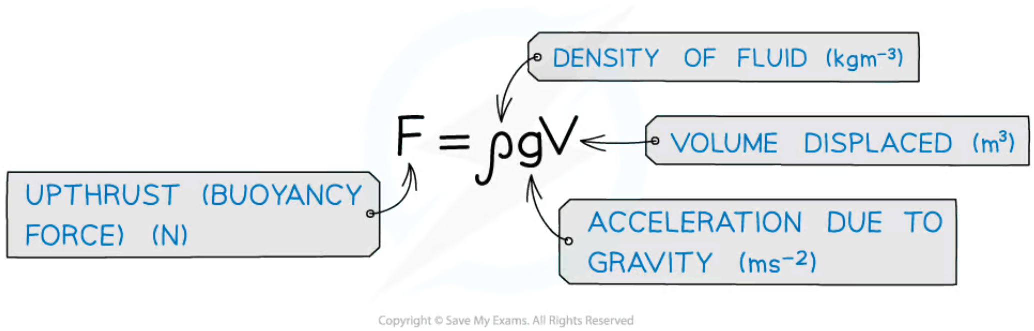
MORE SPECIFICALLY, THE CHANGE IN PRESSURE P IS PROPORTIONAL TO THE CHANGE IN HEIGHT h

EQUATION FOR HYDROSTATIC PRESSURE $\Delta P = \rho g \Delta h$

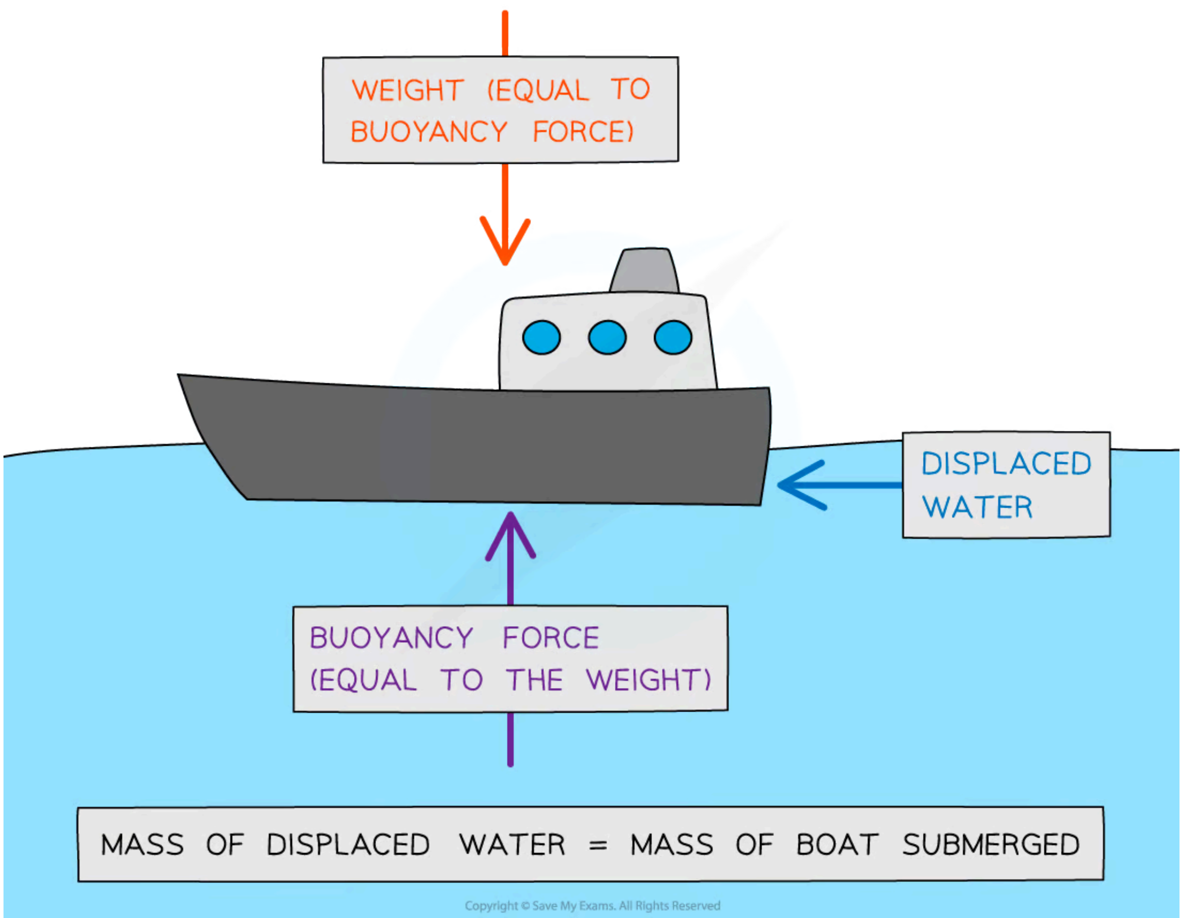
- Archimedes' principle states:

  An object submerged in a fluid at rest has an upward buoyancy force (upthrust) equal to the weight of the fluid displaced by the object

- The object sinks until the weight of the fluid displaced is equal to its own weight
  - Therefore the object floats when the magnitude of the upthrust equals the weight of the object
- The magnitude of upthrust can be calculated by:



- Since $m = \rho V$, upthrust is equal to $F = mg$ which is the weight of the fluid displaced by the object
- Archimedes' Principle explains how ships float:



Your notes

Boats float because they displace an amount of water that is equal to their weight

Worked Example

Atmospheric pressure at sea level has a value of 100 kPa. The density of sea water is 1020 kg m^{-3}.At what depth in the sea would the total pressure be 250 kPa?

A. 20 m

B. 9.5 m

C. 18 m

D. 15 m

ANSWER: D

STEP 1

THE TOTAL PRESSURE = HYDROSTATIC PRESSURE + ATMOSPHERIC PRESSURE

$$250 \times 10^3 = \rho g h + 100 \times 10^3$$

$$250 \times 10^3 = 1020 \times 9.81 \times h + 100 \times 10^3$$

STEP 2

REARRANGING FOR HEIGHT h

$$h = \frac{250 \times 10^3 - 100 \times 10^3}{1020 \times 9.81} = 15 \text{ m}$$

Worked Example

Icebergs typically float with a large volume of ice beneath the water. Ice has a density of 917 kg m^{-3} and a volume of V_i. The density of seawater is 1020 kg m^{-3}.What fraction of the iceberg is above the water?

A. 0.10 V_i **B.** 0.90 V_i **C.** 0.97 V_i

D. 0.20 V_i

Your notes

ANSWER: A

STEP 1

ACCORDING TO ARCHIMEDES' PRINCIPLE, THE UPTHRUST IS EQUAL TO THE WEIGHT OF THE SEAWATER DISPLACED BY THE ICEBERG

ICEBERG WEIGHT $W_i = m_i g$

BUOYANCY FORCE IS THE WEIGHT OF THE DISPLACED WATER $W_w = m_w g$

STEP 2

SINCE THE ICEBERG IS FLOATING, ITS WEIGHT IS EXACTLY EQUAL TO THE BUOYANCY FORCE

$$W_i = W_w$$

$$m_i g = m_w g$$

Your notes

STEP 3

REARRANGE DENSITY EQUATION FOR MASS

$$m = \rho V$$

$$\rho_i V_i g = \rho_w V_w g$$

STEP 4

CANCELLING g SHOWS THE FRACTION OF ICE UNDERWATER IS GIVEN BY RATIO OF DENSITIES

$$\frac{V_i}{V_w} = \frac{\rho_w}{\rho_i}$$

STEP 5

REARRANGE FOR V_w

$$V_w = \frac{\rho_i V_i}{\rho_w}$$

$$V_w = \frac{917}{1020} V_i = 0.9 V_i$$

STEP 6

THEREFORE 90% OF THE ICEBERG'S VOLUME IS SUBMERGED UNDERWATER

THIS MEANS THAT $1 - 0.9 = 0.1\ V_i$ IS ABOVE WATER

Examiner Tips and Tricks

When asked about the **total pressure** remember to also add the atmospheric pressure

Total pressure = Hydrostatic pressure + Atmospheric pressure

Atmospheric pressure (also known as barometric pressure) is equal to **101 325 Pa** Values for pressure can vary widely and depend on metric prefixes such as kPa or MPa. When you're doing calculations make sure all the pressures are in the same units (otherwise you may be out by a factor of 1000!). To be on the safe side, you can convert them all to Pascals.