



OCR A Level Physics



Your notes

Measurements & Uncertainties

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Your notes

Sources of Uncertainty

Random & Systematic Errors

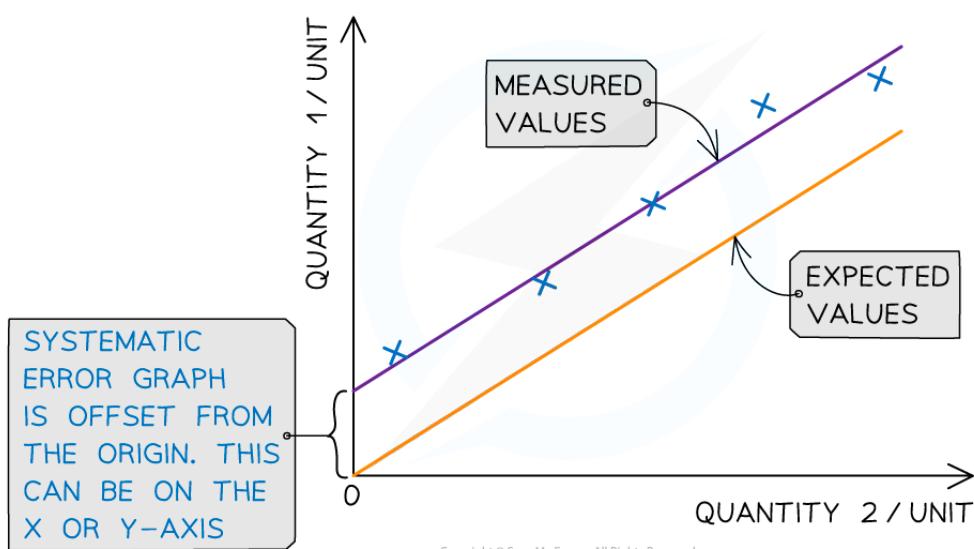
- Measurements of quantities are made with the aim of finding the true value of that quantity
- In reality, it is impossible to obtain the true value of any quantity as there will always be a degree of uncertainty
- The uncertainty is an estimate of the difference between a measurement reading and the true value
- Random and systematic errors are two types of measurement errors that lead to uncertainty

Random error

- Random errors cause unpredictable fluctuations in an instrument's readings as a result of uncontrollable factors, such as environmental conditions
- This affects the **precision** of the measurements taken, causing a wider spread of results about the mean value
- To **reduce** random error:
 - **Repeat** measurements several times and calculate an average from them

Systematic error

- Systematic errors arise from the use of faulty instruments used or from flaws in the experimental method
- This type of error is repeated consistently every time the instrument is used or the method is followed, which affects the **accuracy** of all readings obtained
- To **reduce** systematic errors:
 - Instruments should be **recalibrated**, or different instruments should be used
 - Corrections or adjustments should be made to the technique



Systematic errors on graphs are shown by the offset of the line from the origin

Zero error

- This is a type of systematic error which occurs when an instrument gives a reading when the **true reading is zero**
 - For example, a top-ban balance that starts at 2 g instead of 0 g
- Zero errors can be removed by taking the difference of the offset from each value
 - Eg. If a scale starts at 2 g instead of 0 g, a measurement of 50 g would actually be $50 - 2 = 48$ g
 - The offset could be positive or negative

Precision & Accuracy

Precision

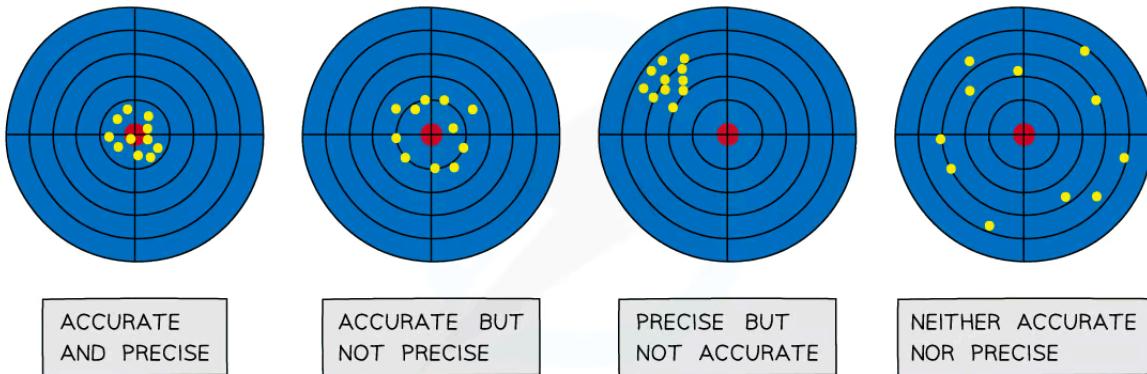
- Precise measurements are ones in which there is very little spread about the mean value, in other words, how close the measured values are to each other
- If a measurement is repeated several times, it can be described as precise when the values are very similar to, or the same as, each other
- The precision of a measurement is reflected in the values recorded - measurements to a greater number of decimal places are said to be more **precise** than those to a whole number

Accuracy



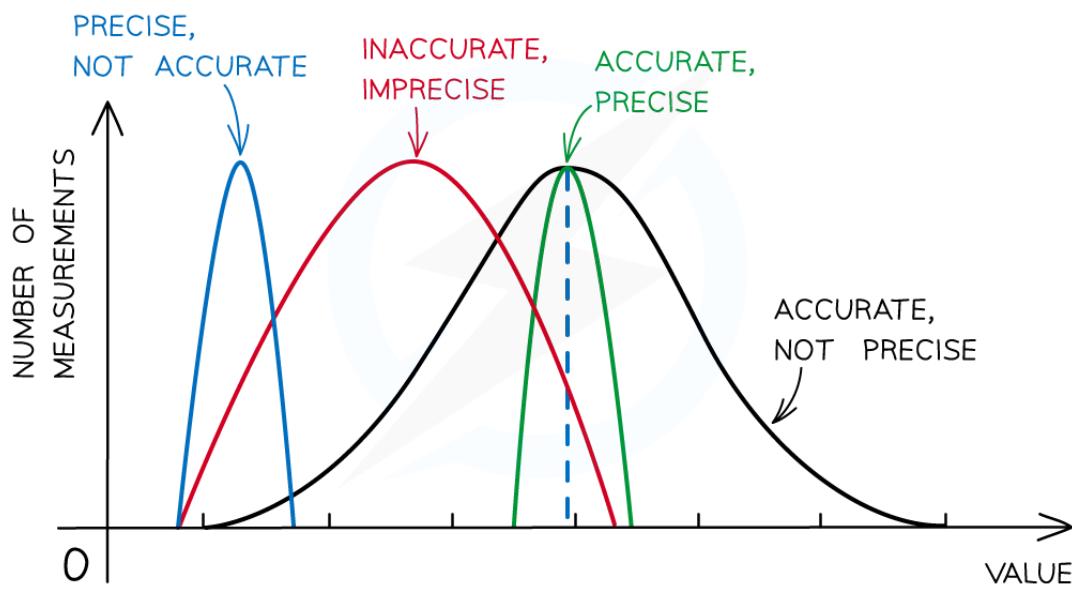
Your notes

- A measurement is considered accurate if it is close to the true value
- The accuracy can be increased by repeating measurements and finding a mean of the results
- Repeating measurements also helps to identify anomalies that can be omitted from the final results



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The difference between precise and accurate results



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Representing precision and accuracy on a graph



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Examiner Tips and Tricks

It is a very common mistake to confuse precision with accuracy - measurements can be precise but **not** accurate if each measurement reading has the same error. Make sure you learn that **precision** refers to the ability to take multiple readings with an instrument that are close to each other, whereas **accuracy** is the closeness of those measurements to the true value.



Your notes

Calculating Uncertainties

Combining Uncertainties

Absolute & Percentage Uncertainties

- There is always a degree of uncertainty when measurements are taken; the uncertainty can be thought of as the difference between the **actual** reading taken (caused by the equipment or techniques used) and the **true value**
- Uncertainties are **not** the same as errors
 - Errors can be thought of as issues with equipment or methodology that cause a reading to be different from the true value
 - The uncertainty is a range of values around a measurement within which the true value is expected to lie, and is an **estimate**
- For example, if the true value of the mass of a box is 950 g, but a systematic error with a balance gives an actual reading of 952 g, the uncertainty is ± 2 g
- These uncertainties can be represented in a number of ways:
 - **Absolute Uncertainty:** where uncertainty is given as a fixed quantity
 - **Fractional Uncertainty:** where uncertainty is given as a fraction of the measurement
 - **Percentage Uncertainty:** where uncertainty is given as a percentage of the measurement

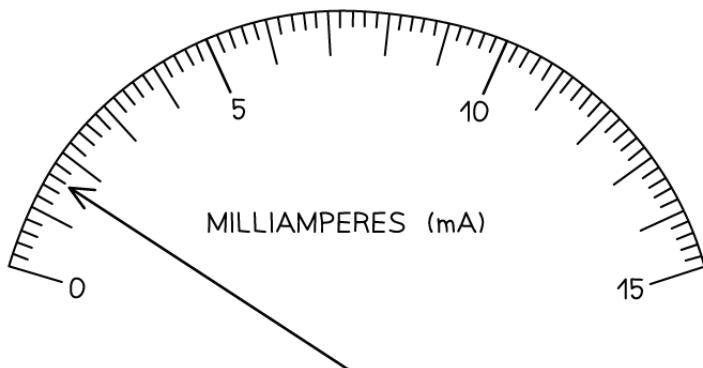
$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

- To find uncertainties in different situations:
 - **The uncertainty in a reading:** \pm half the smallest division
 - **The uncertainty in a measurement:** at least ± 1 smallest division
 - **The uncertainty in repeated data:** half the range i.e. $\pm \frac{1}{2}$ (largest - smallest value)
 - **The uncertainty in digital readings:** \pm the last significant digit unless otherwise quoted

$$\text{▪ The uncertainty in the natural log of a value: absolute uncertainty in } \ln(x) = \frac{\text{uncertainty in } x}{x}$$



Your notes



SMALLEST DIVISION = 0.2 mA

READING (I) = 1.6 mA

$$\text{ABSOLUTE UNCERTAINTY } (\Delta I) = \frac{1}{2} \times 0.2 \text{ mA} = 0.1 \text{ mA}$$

$$I = 1.6 \pm 0.1 \text{ mA}$$

$$\text{FRACTIONAL UNCERTAINTY} = \frac{\text{UNCERTAINTY}}{\text{VALUE}} = \frac{0.1}{1.6} = \frac{1}{16}$$

$$I = 1.6 \pm \frac{1}{16} \text{ mA}$$

$$\text{PERCENTAGE UNCERTAINTY (\%)} = \frac{\text{UNCERTAINTY}}{\text{VALUE}} \times 100 = \frac{0.1}{1.6} \times 100 = 6.2\%$$

$$I = 1.6 \pm 6.2\% \text{ mA}$$

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How to calculate absolute, fractional and percentage uncertainty

- Always make sure your absolute or percentage uncertainty is to the same number of **significant figures** as the reading

Combining Uncertainties

- When combining uncertainties, the rules are as follows:

Adding / Subtracting Data

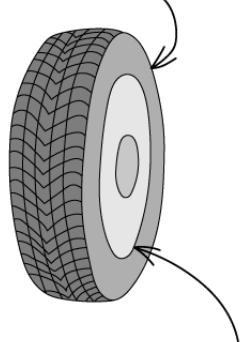
- Add together the absolute uncertainties



Your notes

ADDING / SUBTRACTING DATA

DIAMETER OF TYRE (d_1) = 55.0 ± 0.5 cm



DIAMETER OF INNER TYRE (d_2) = 21.0 ± 0.7 cm

DIFFERENCE IN DIAMETERS ($d_1 - d_2$) = $55.0 - 21.0 = 34.0$ cm

UNCERTAINTY IN DIFFERENCE = $\pm(0.5 + 0.7) = \pm 1.2$ cm

$$d_1 - d_2 = 34.0 \pm 1.2 \text{ cm}$$

Multiplying / Dividing Data

- Add the percentage or fractional uncertainties

MULTIPLYING / DIVIDING DATA



Your notes



$$\text{DISTANCE} = 50.0 \pm 0.1 \text{ m}$$

$$\text{TIME} = 5.00 \pm 0.05 \text{ s}$$

$$\text{SPEED } (v) = \frac{\text{DISTANCE } (s)}{\text{TIME } (t)}$$

$$v = \frac{50.0}{5.0} = 10.0 \text{ ms}^{-1}$$

$$\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t} = \frac{0.1}{50.0} + \frac{0.05}{5.00} = 0.002 + 0.01 = 0.012$$

$$\text{ABSOLUTE UNCERTAINTY } (\Delta v) = 10.0 \times 0.012 = \pm 0.12 \text{ ms}^{-1}$$

$$v = 10.0 \pm 0.12 \text{ ms}^{-1}$$

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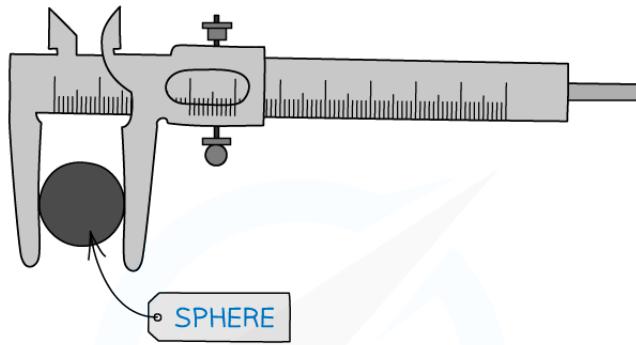
Raising to a Power

- **Multiply** the percentage uncertainty by the power



Your notes

RAISING TO A POWER



$$V = \frac{4}{3} \pi r^3$$

$$r = 2.50 \pm 0.02 \text{ cm}$$

$$V = \frac{4}{3} \pi (2.50)^3 = 65.5 \text{ cm}^3$$

$$\frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r} = 3 \times \frac{0.02}{2.50} = 0.024$$

$$\text{ABSOLUTELY UNCERTAINTY } (\Delta V) = 65.5 \times 0.024 = 1.57 \text{ cm}^3$$

$$\text{PERCENTAGE UNCERTAINTY } (\% \Delta V) = 100 \times 0.024 = 2.4\%$$

Copyright © Save My Exams. All Rights Reserved**Examiner Tips and Tricks**

Remember:

- Absolute uncertainties (denoted by Δ) have the same units as the quantity
- Percentage uncertainties have no units
- The uncertainty in constants, such as π , is taken to be zero

Uncertainties in trigonometric and logarithmic functions will not be tested in the exam, so just remember these rules and you'll be fine!



Your notes

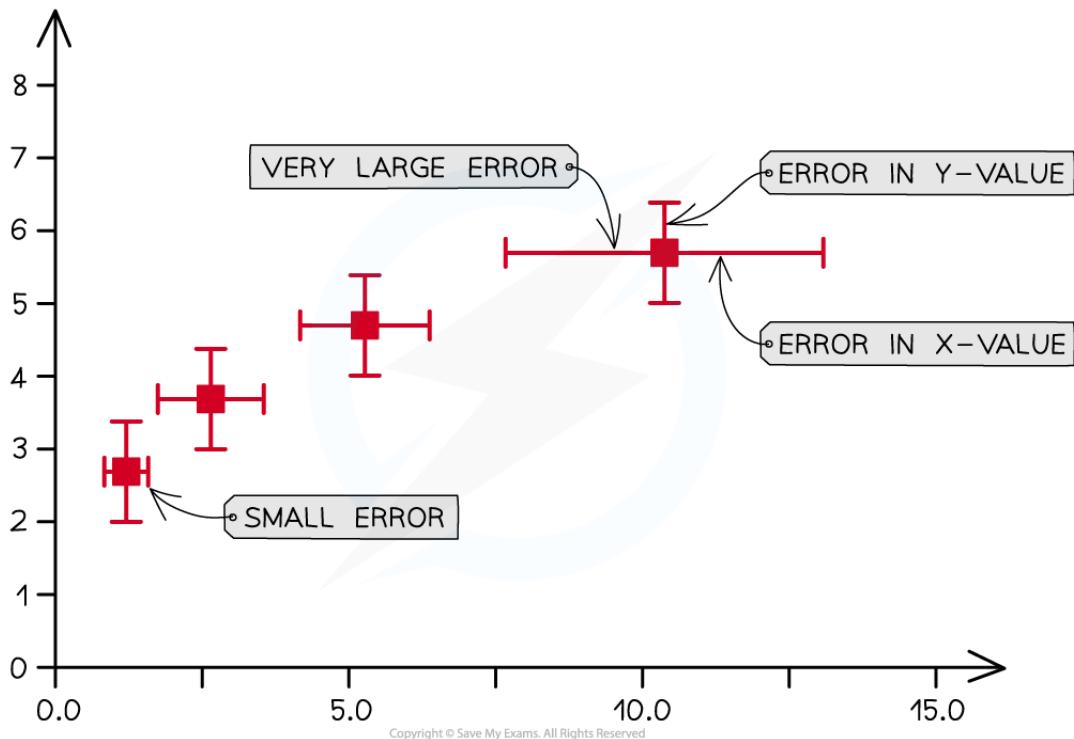


Your notes

Determining Uncertainties from Graphs

Error Bars

- The uncertainty in a measurement can be shown on a graph as an **error bar**
- This bar is drawn above and below the point (or from side to side) and shows the **uncertainty** in that measurement
- Error bars are plotted on graphs to show the **absolute uncertainty** of values plotted
- Usually, error bars will be in the vertical direction, for y-values, but can also be plotted horizontally, for x-values



Representing error bars on a graph

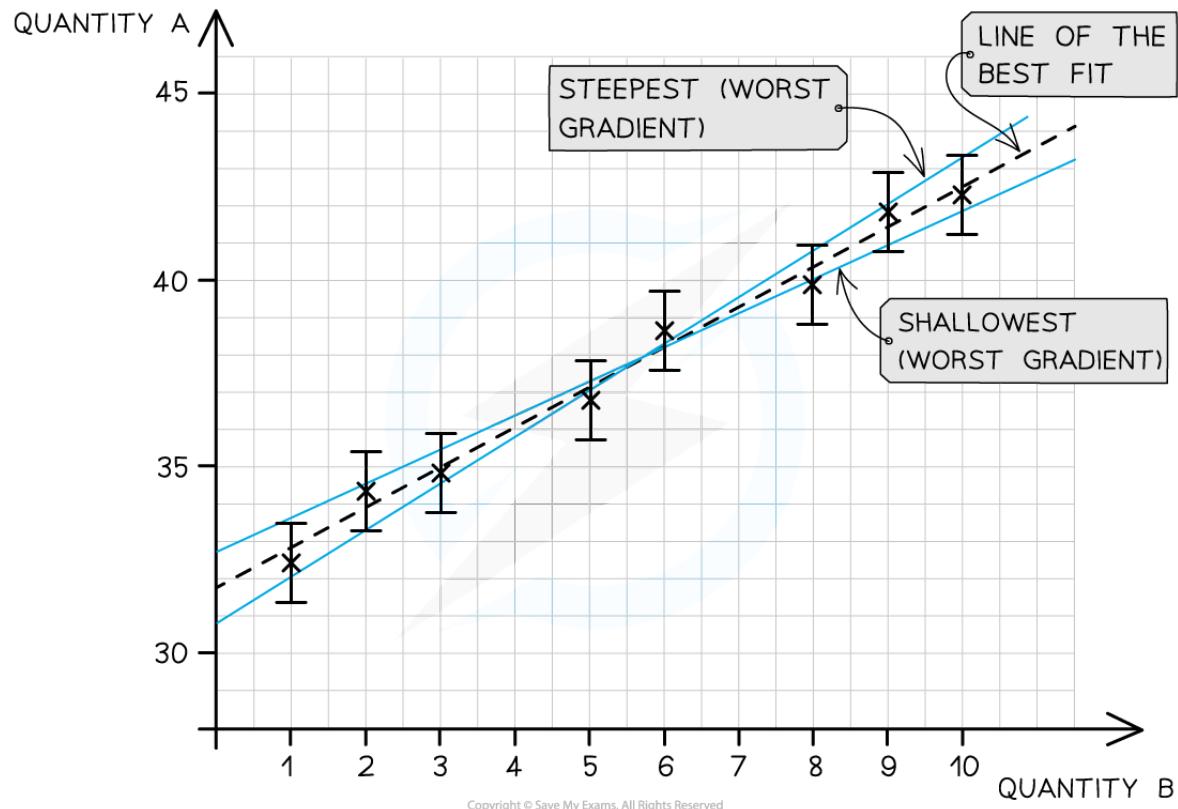
Determining Uncertainties from Graphs

- To calculate the uncertainty in a gradient, two lines of best fit should be drawn on the graph:



Your notes

- The 'best' line of best fit, which passes as close to the points as possible
- The 'worst' line of best fit, either the steepest possible or the shallowest possible line which fits within all the error bars



The line of best fit passes as close as possible to all the points. The steepest and shallowest lines are known as the worst fit

- The percentage uncertainty in the **gradient** can be found using:

$$\text{Percentage uncertainty} = \frac{\text{best gradient} - \text{worst gradient}}{\text{best gradient}} \times 100\%$$

- The percentage uncertainty in the **y-intercept** can be found using:

$$\text{Percentage uncertainty} = \frac{\text{best y intercept} - \text{worst y intercept}}{\text{best y intercept}} \times 100\%$$

Percentage Difference



Your notes

- The percentage difference gives an indication of how close the experimental value achieved from an experiment is to the accepted value
 - It is **not** a percentage uncertainty
- The percentage difference is defined by the equation:

$$\text{Percentage Difference} = \frac{\text{Experimental Value} - \text{Accepted Value}}{\text{Accepted Value}} \times 100 \%$$

- The experimental value is sometimes referred to as the 'measured' value
- The accepted value is sometimes referred to as the 'true' value
 - This may be labelled on a component such as the capacitance of a capacitor or the resistance of a resistor
 - Or, from a databook
- For example, the acceleration due to gravity g is known to be 9.81 m s^{-2} . This is its accepted value
 - From an experiment, the value of g may be found to be 10.35 m s^{-2}
 - Its percentage difference would therefore be 5.5%
- The smaller the percentage difference, the more **accurate** the results of the experiment



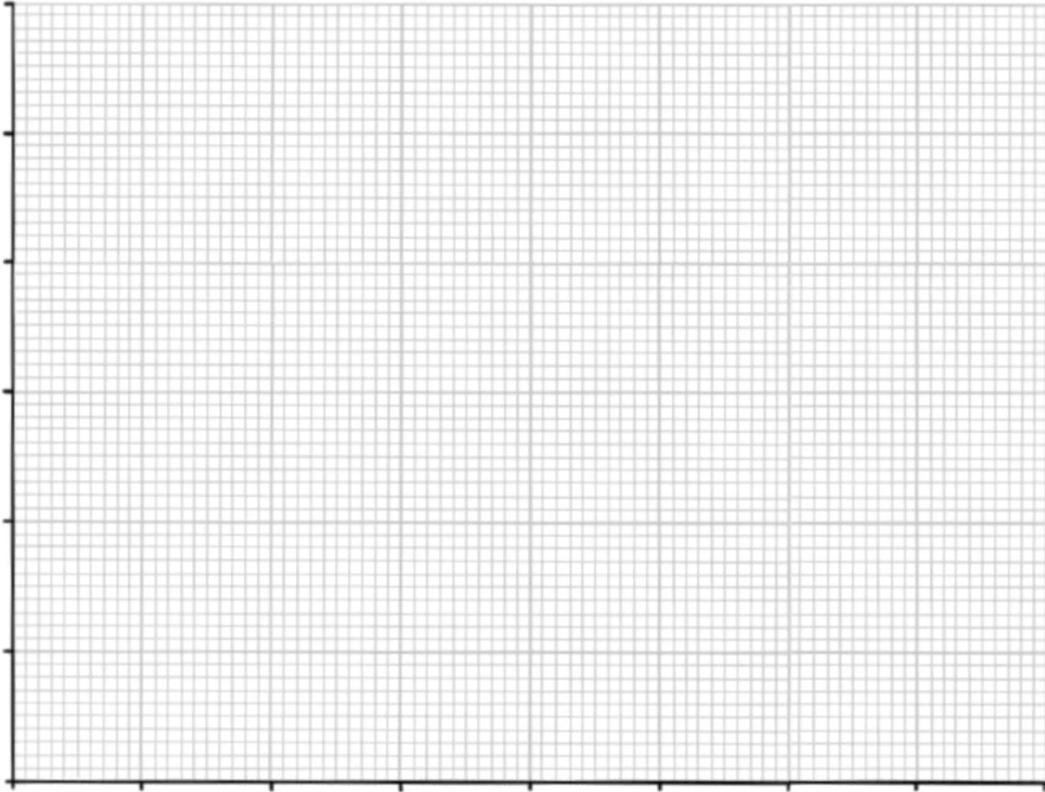
Worked Example

On the axes provided, plot the graph for the following data and draw error bars and lines of best and worst fit.

Force / N	10	20	30	40	50	60	70	80
Extension / mm	8.5 ± 1	11 ± 0.5	15 ± 1	15 ± 2	20 ± 1.5	19.5 ± 2	22 ± 0.5	26 ± 1

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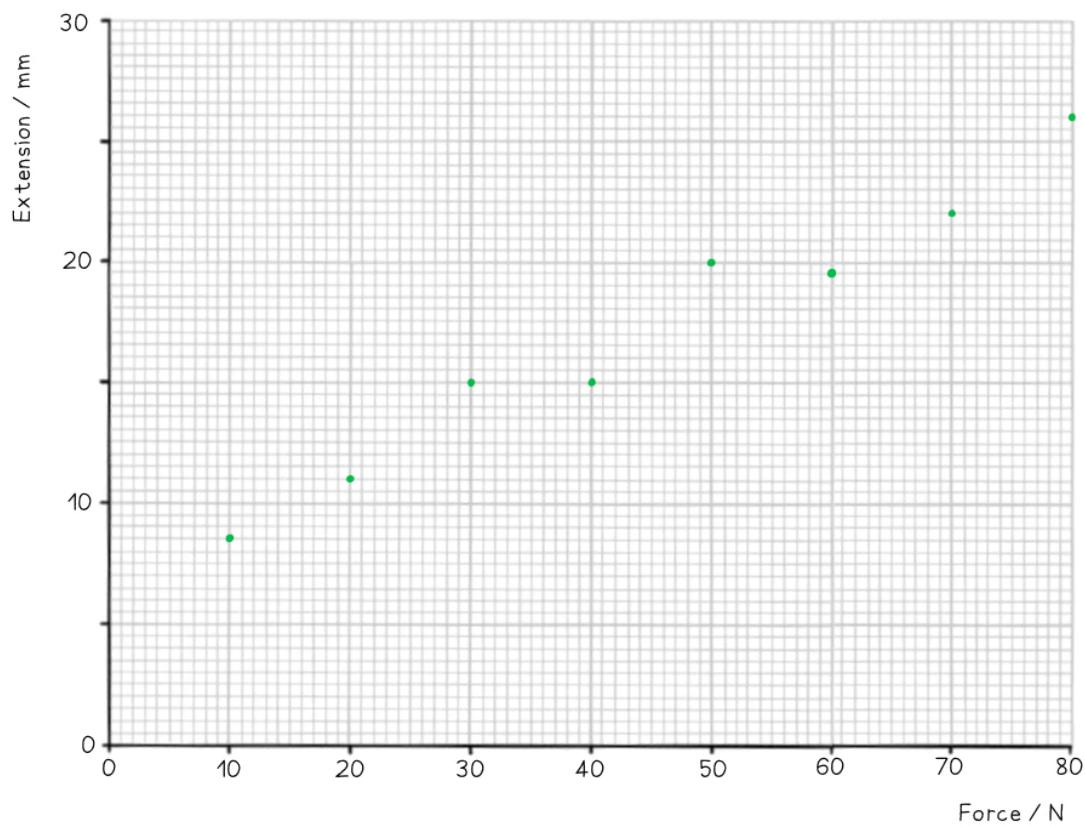
Find the percentage uncertainty in the gradient from your graph.



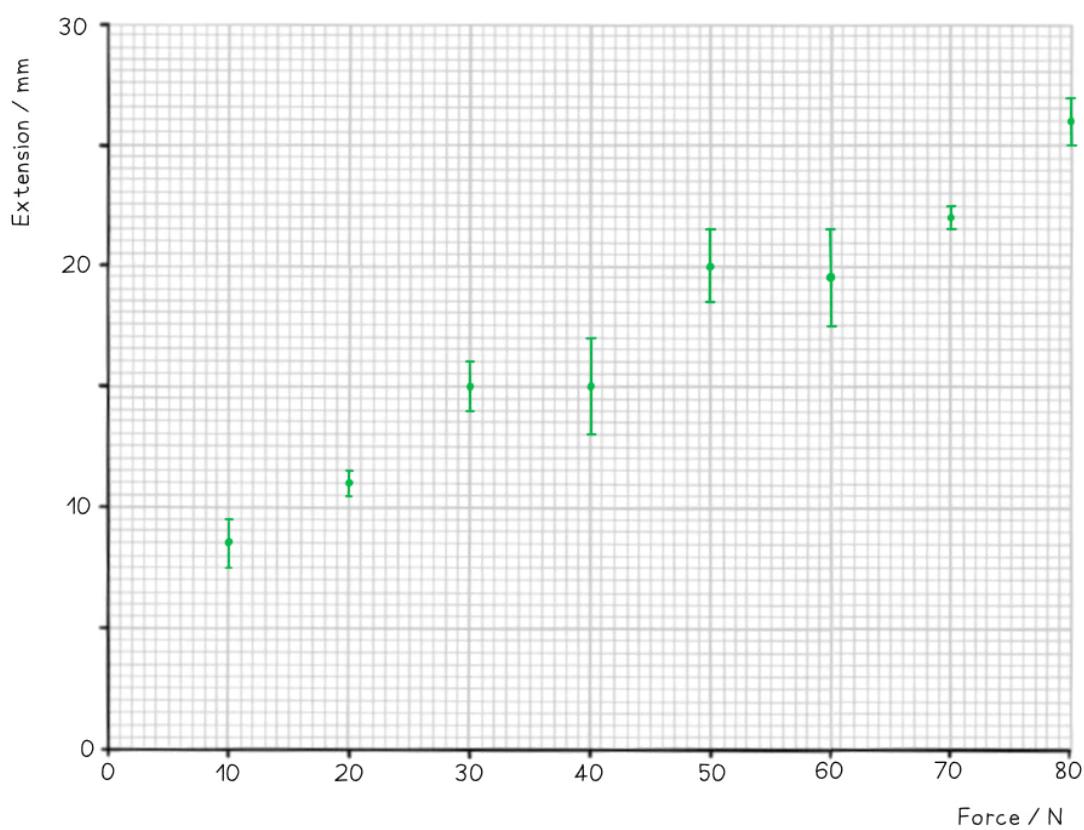
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Answer:

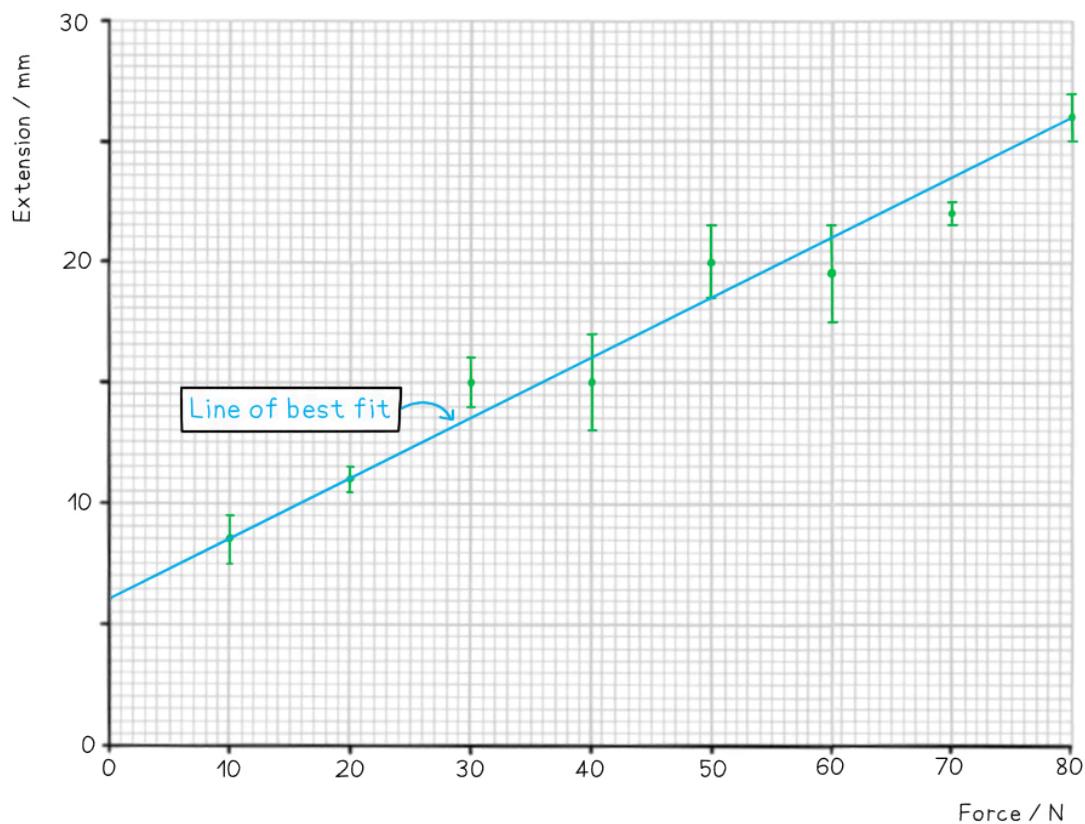
Step 1: Draw sensible scales on the axes and plot the data


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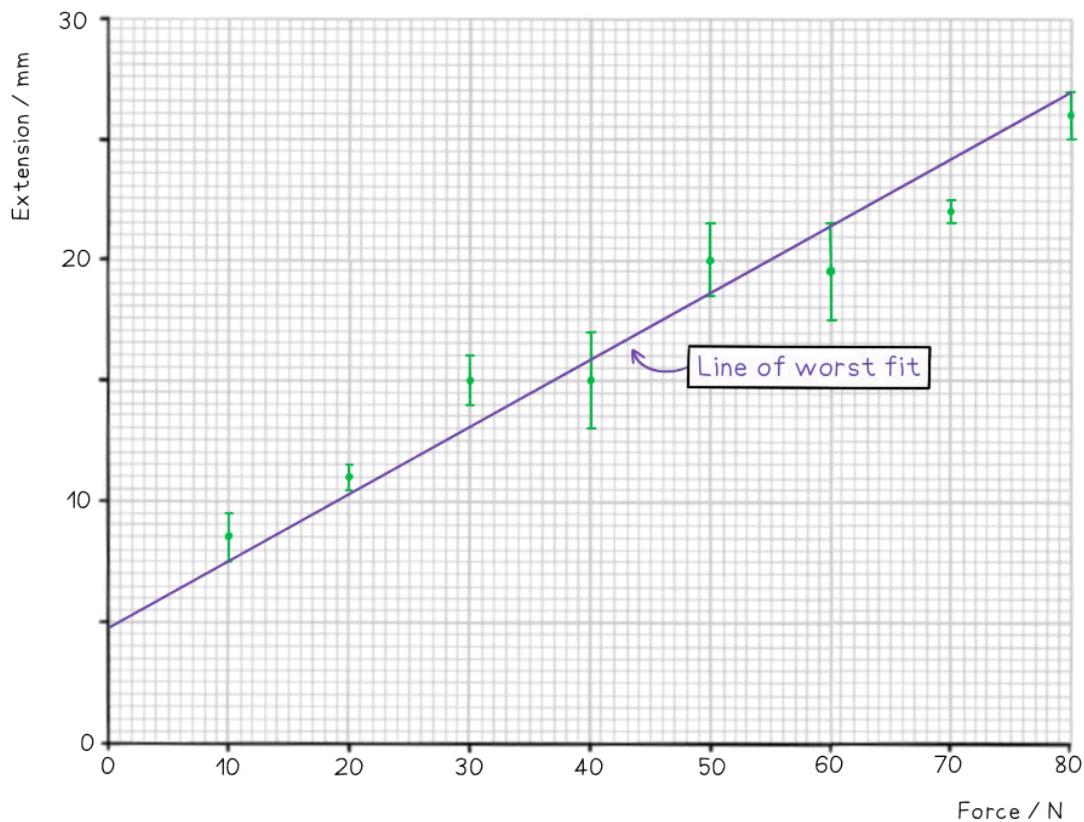
Step 2: Draw the errors bars for each point

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Step 3: Draw the line of best fit

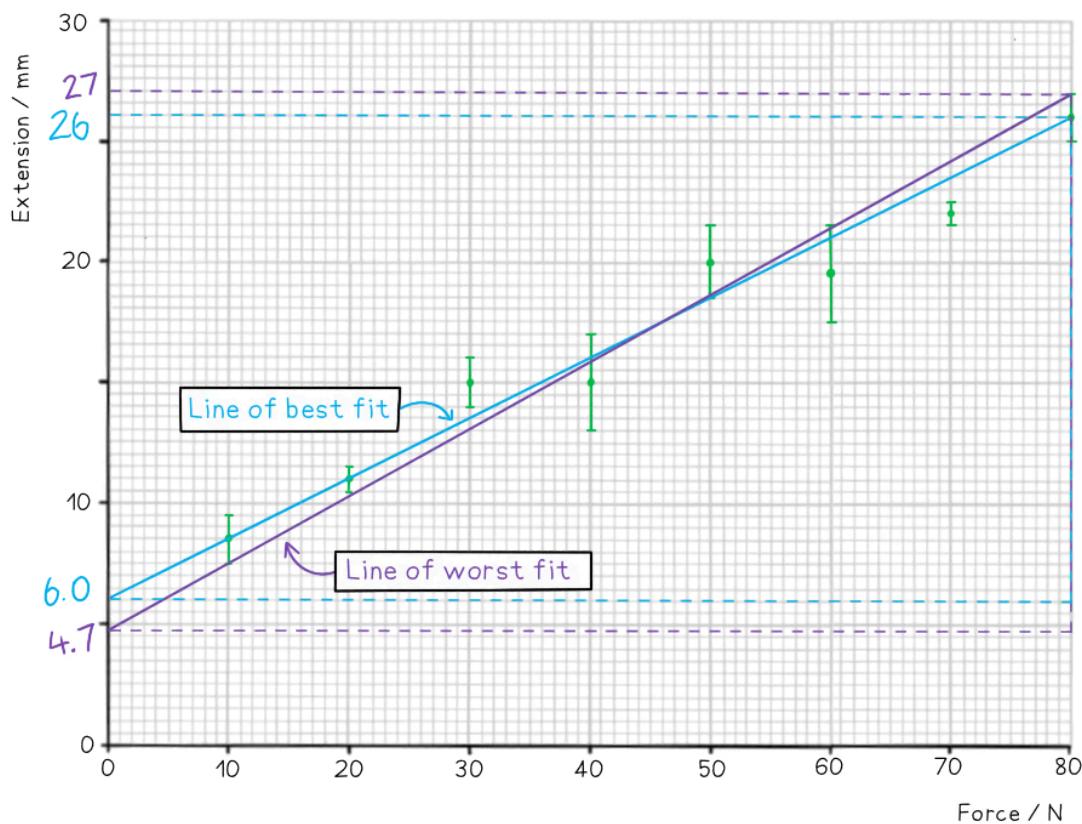

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Step 4: Draw the line of worst fit



Your notes

Step 5: Work out the gradient of each line and calculate the percentage uncertainty



$$\text{Best gradient} = \frac{\Delta y}{\Delta x} = \frac{26 - 6}{80 - 0} = 0.25$$

$$\text{Worst gradient} = \frac{\Delta y}{\Delta x} = \frac{27 - 4.7}{80 - 0} = 0.28$$

$$\text{Percentage uncertainty} = \frac{0.28 - 0.25}{0.25} \times 100\% = 12\%$$



Examiner Tips and Tricks

When drawing graphs make sure to follow these rules to gain full marks:

- Ensure the scale is sensible and takes up as much paper as possible
- Label the axes with a quantity and a unit
- Precisely plot the points to within 0.5 squares
- Leave a roughly equal number of points above and below the best fit line
- Draw the error bars accurately



Your notes