# Inverse matrices for inverse transformations

$BAx = y$

$x = A^{-1}B^{-1}y$

✎ Suppose $\boldsymbol{x}$ and $\boldsymbol{y}$ are column vectors. Then if $\boldsymbol{Ax} = \boldsymbol{y}$, then $\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{y}$.

The inverse matrix therefore allows us to retrieve the original point/position vector before a transformation.

The triangle $T$ has vertices at $A$, $B$ and $C$. The matrix $M = \begin{pmatrix} 4 & -1 \\ 3 & 1 \end{pmatrix}$ transforms $T$ to the triangle $T'$ with vertices at $A'(4,3)$, $B'(4,10)$ and $C'(-4,-3)$. Determine the coordinates of $A$, $B$ and $C$.

$$M = \begin{pmatrix} 4 & -1 \\ 3 & 1 \end{pmatrix} \qquad M^{-1} = \frac{1}{7}\begin{pmatrix} 1 & 1 \\ -3 & 4 \end{pmatrix}$$

$$\frac{1}{7}\begin{pmatrix} 1 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 4 & -4 \\ 3 & 10 & -3 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 & 14 & -7 \\ 0 & 28 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & -1 \\ 0 & 4 & 0 \end{pmatrix}$$

$A(1,0)$, $B(2,4)$, $C(-1,0)$

## Edexcel June 2012 Q9

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}.$$

(*a*) Find det **M**. $= -23$ **(1)**

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$\underline{j} \leftarrow \underline{i}$

(*e*) describe fully the single geometrical transformation represented by **A**. **(2)**

Rotation 90° anticlockwise, centre (0, 0).

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(*f*) Find **B**. **(4)**

$\underline{i} \rightarrow \underline{j}$

$$M = BA$$
$$MA^{-1} = BAA^{-1}$$
$$MA^{-1} = B$$

$$A^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = B$$

$$B = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$$

**Tip:** If $\mathbf{M} = \mathbf{BA}$, make sure you multiply the <u>end</u> of each by $\mathbf{A}^{-1}$ :

$$\mathbf{MA}^{-1} = \mathbf{BAA}^{-1}$$
$$\mathbf{MA}^{-1} = \mathbf{BI} = \mathbf{B}$$

Ex 7F

Q 3, 5, 10, 12

**5.**

$$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that $\mathbf{M}$ is non-singular.

(2)

The hexagon $R$ is transformed to the hexagon $S$ by the transformation represented by the matrix $\mathbf{M}$.

Given that the area of hexagon $R$ is 5 square units,

(b) find the area of hexagon $S$.

(1)

The matrix $\mathbf{M}$ represents an enlargement, with centre (0, 0) and scale factor $k$, where $k > 0$, followed by a rotation anti-clockwise through an angle $\theta$ about (0, 0).

(c) Find the value of $k$.

(2)

(d) Find the value of $\theta$.

(2)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** | $\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$ | M1 | 1.1a |
| | $\mathbf{M}$ is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$ | A1 | 2.4 |
| | | **(2)** | |
| **(b)** | $\text{Area}(S) = 4(5) = 20$ | B1ft | 1.2 |
| | | **(1)** | |
| **(c)** | $k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$ | M1 | 1.1b |
| | $= 2$ | A1ft | 1.1b |
| | | **(2)** | |
| **(d)** | $\cos\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$ | M1 | 1.1b |
| | $\theta = 60°$ or $\frac{\pi}{3}$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(7 marks)** |

**1.**

$$\mathbf{P} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The matrices **P** and **Q** represent linear transformations, $P$ and $Q$ respectively, of the plane.

The linear transformation $M$ is formed by first applying $P$ and then applying $Q$.

(a) Find the matrix **M** that represents the linear transformation $M$.

**(2)**

(b) Show that the invariant points of the linear transformation $M$ form a line in the plane, stating the equation of this line.

**(3)**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | $\mathbf{M} = \mathbf{QP} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \times \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$ | M1 | 1.1a |
| | $= \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | $\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow -\frac{1}{2}x - \frac{\sqrt{3}}{2}y = x$ and $-\frac{\sqrt{3}}{2}x + \frac{1}{2}y = y$ | M1 | 3.1a |
| | $\Rightarrow -y\sqrt{3} = 3x$ and $y = -x\sqrt{3}$ | M1 | 1.1b |
| | First equation gives $y = -\frac{3x}{\sqrt{3}} = -x\sqrt{3}$, so equations are the same, hence $M$ fixes all points on the line $y = -x\sqrt{3}$. | A1ft | 2.1 |
| | | **(3)** | |
| | | | **(5 marks)** |