A Level · Edexcel · Maths

3 hours 35 questions

7.5 Implicit Differentiation (A Level only)

Easy (8 questions)	/40
Medium (9 questions)	/60
Hard (9 questions)	/49
Very Hard (9 questions)	/49
Total Marks	**/198**

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Easy Questions

1 Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ when

(i) $x^2 + y = 3,$

(ii) $5x^4 + y^2 - 4 = 0,$

(iii) $\sin 3x - 3y = 0$

(iv) $e^x + e^y = 2x.$

(8 marks)

2 Find the gradient of the curve with equation $3y^2 - 2x^3 = 10$ at the point (1 , 2).

(3 marks)

3 Show that if $x - \sin y = 0$ then $\frac{dy}{dx} = \sec y$.

(3 marks)

4 (a) The curve C has equation

$$y^2 - 4x + 2 = 0.$$

Show that C intersects the x-axis at the point $\left(\frac{1}{2}, 0\right)$.

(2 marks)

(b) (i) Find an expression for $\frac{dy}{dx}$.

(ii) Explain why the curve C does not have any stationary points.

(3 marks)

5 (a) Show that the point $\left(-\frac{4}{\pi}, \frac{\pi}{2}\right)$ lies on the curve with equation $2\cos 2y = xy$.

(2 marks)

(b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3 marks)

6 (a) Find the gradient of the curve with equation $12x^2 - 4y^2 + 24 = 0$ at the point (1 , 3).

(2 marks)

(b) Hence, find an equation of the tangent to the curve at the point (1 , 3).

(2 marks)

7 (a) Given $3x^2 - 2y = xy$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3 marks)

(b) Hence show that the stationary points of $3x^2 - 2y = xy$ lie on the line $y = 6x$.

(2 marks)

8 (a) The curve C has equation $x^3 + 9xy^2 = 54.$

Find the gradient of the tangent to C at the point (3 , 1).

(4 marks)

(b) Hence find the equation of the normal to C at (3 , 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3 marks)

Medium Questions

1 Find an expression for $\frac{dy}{dx}$ in terms of x and y for the following

(i) $2xy + y^2 = 4$

(ii) $3\sin y - y = 2x - 1$

(4 marks)

2 Find the gradient of the curve with equation

$$3x^2y + 4x - y = 39$$

at the point with coordinates (2 , 3).

(4 marks)

3 (a) The curve C has equation

$$\frac{1}{5}x^2e^y = 5$$

Show that C intersects the x-axis at the points (-5 , 0) and (5 , 0).

(2 marks)

(b) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

(2 marks)

(c) Hence find the gradients of C at the two points where C intercepts the x-axis.

(2 marks)

4 Show that if $y = \arcsin x$, then

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

(3 marks)

5 (a) Show that the point $(0, \pi)$ lies on the curve with equation

$$3\tan y = 2xy$$

(2 marks)

(b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3 marks)

(c) Find the gradient at the point $(0, \pi)$.

(2 marks)

(d) Hence find an equation of the tangent to the curve at the point $(0, \pi)$.

(3 marks)

6 (a) The curve C is defined by the equation $\ln y = 1 - xy$.

The point $P(1, 1)$ lies on C.

Show that

$$\frac{dy}{dx} = -\frac{y^2}{1+xy}$$

(4 marks)

(b) Find the gradient of the tangent to C at point P, and hence find the gradient of the normal to C at point P.

(3 marks)

(c) Find the equation of the normal to C at point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(2 marks)

7 (a) Find an expression for $\frac{dy}{dx}$ in terms of x and y given that

$$2x^2 - y = xy^2$$

(3 marks)

(b) Show that $\frac{dy}{dx} = 0$ when $4x = y^2$.

(2 marks)

(c) Hence, or otherwise, find the coordinates of the stationary points.

(3 marks)

8 (a) The curve C is given by the equation $e^{xy} = y - x$.

Find the coordinates of the points where C intersects the coordinate axes.

(2 marks)

(b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3 marks)

(c) Show that the tangents to C, at the points where it meets the coordinate axes, have equations

$y = 2x + 1$ and $2y = x + 1$

(4 marks)

(d) The tangents meet at point Q.
Find the distance OQ, where O is the origin.

(4 marks)

9 Use implicit differentiation to show that

$$\frac{\mathrm{d}}{\mathrm{dx}}\left[a^x\right] = a^x \ln a$$

where a is a constant.

(3 marks)

Hard Questions

1 Find an expression for $\frac{dy}{dx}$ in terms of x and y for the following

(i) $2ye^x + 5x^2y^2 = 8,$

(ii) $3x\tan y = 2x^2.$

(4 marks)

2 (a) Given that

$$y^2 + 4x^2 - e^y = 0,$$

find the positive value of x when $y = 0$.

(1 mark)

(b) Hence, or otherwise, find the value of the gradient of

$$y^2 + 4x^2 - e^y = 0$$

at the point where $y = 0$ and x is positive.

(4 marks)

3 Show that if $y = \arccos 2x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{\sqrt{1-4x^2}}.$$

(3 marks)

4 (a) The curve C has equation $2xy^2 - x^2 = 16$, Line L has equation $x = 4$.

Show that the two points where C intersects L have equal gradients.

(5 marks)

(b) What else can you deduce about the two points where C and L intercept?

(1 mark)

5 Verify that the point (-1 , 0) lies on the curve with equation

$$3xe^y + 2x + 5 = 4y$$

and find the equation of the tangent to the curve at the point (-1 , 0).
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5 marks)

6 (a) Show that the derivative function of the curve given by

$$\ln y - 2xy^3 = 8$$

is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^4}{1 - 6xy^3}.$$

(5 marks)

(b) Find the equation of the normal to the curve given in part (a) at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3 marks)

7 Show that the stationary points on the curve with equation

$$xy^2 - 4x^2 = 64$$

occur when $x = 4$, and find the **exact** y-coordinates of the stationary points.

(6 marks)

8 (a) Verify that the point $A(1\ ,\ 1)$ lies on the curve with equation

$$\ln(xy) + xy^2 = 1.$$

(1 mark)

(b) The tangent at point A intercepts the x-axis at point B and the y-axis at point C.
Find the area of the triangle OBC.

(8 marks)

9 Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}[a^{kx}] = ka^{kx}\ln a$$

where a and k are constants.

(3 marks)

Very Hard Questions

1 Find an expression for $\frac{dy}{dx}$ in terms of x and y for the following

(i) $e^{xy} + \ln(xy) = \text{cosec}(x) + 4$

(ii) $4\cos(x^2y) - 3e^{x^2y} = 4e^y$

(5 marks)

2 Find the gradient at the point where $x = -2$ and y is an integer on the curve with equation $x^2y^2 - 5x = 22y$.

(5 marks)

3 Show that if $2y = \arctan x^2$, then

$$\frac{dy}{dx} = \frac{x}{1 + x^4}$$

(3 marks)

4 An ellipse has equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Find an expression for $\frac{dy}{dx}$ and hence show that the gradient of the ellipse at any point where it meets a line of the form $y = kx \ (k \neq 0)$ is independent of x and y.

(4 marks)

5 The curve C is described by the equation

$$\ln y + x^2y^2 = 9.$$

Show that the tangents of the two points on C where $y = 1$ meet at the point $\left(0, \frac{37}{19}\right)$.

(5 marks)

6 (a) The curve C is described by the equation

$$3x^2 + 2xy^3 + 16 = 0.$$

Show that the normal to C at the point where $x = -4$ is parallel to the normal to C at the point where $x = 4$.

(4 marks)

(b) Find the distance between the y-axis intercepts of these two normals.

(4 marks)

7 Find the stationary points and determine their nature for the curve with equation $y^2 = 3x^2 - 2xy + 3$.

(8 marks)

8 The curve C is defined by

$$e^{\sin xy} = 1 \qquad \{y > 0\}$$

Points A and B have coordinates $\left(\frac{\pi}{2}, 2\right)$ and $\left(-\frac{\pi}{2}, 2\right)$ respectively.

The tangents to C at points A and B intersect at the point P.
The tangent to C at point A intersects the x-axis at point Q.
The tangent to C at point B intersects the x-axis at point R.

Find the area of triangle PQR.

(8 marks)

9 Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[a^{x^k}\right] = ka^{x^k}x^{k-1}\ln a$$

where a and k are constants.

(3 marks)