A Level · Edexcel · Maths

4 hours 32 questions

5.9 Modelling with Trigonometric Functions (A Level only)

Easy (8 questions)	/50
Medium (8 questions)	/56
Hard (8 questions)	/55
Very Hard (8 questions)	/55
Total Marks	**/216**

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Easy Questions

1 (a) The length of a spring, l cm, at time t seconds, is modelled by the function

$$l = 8 + 2\sin t, t \geq 0.$$

Write down

(i) the natural length of the spring,
(ii) the maximum length of the spring,
(iii) the minimum length of the spring.

(3 marks)

(b) (i) Find the length of the spring after 5 seconds.
(ii) Find the time at which the length of the spring first reaches 9.5 cm.

(2 marks)

(c) Give one criticism of this model for large values of t.

(1 mark)

2 (a) A dolphin is swimming such that it is diving in and out of the sea at a constant speed.

The height, h cm, of the dolphin, relative to sea level $(h = 0)$, at time t sconds, is to be modelled using the formula $h = A\sin(Bt)$ where A are B constants.

On each jump and dive the dolphin reaches a height of 70 cm above sea level and a depth of 70 cm below sea level.

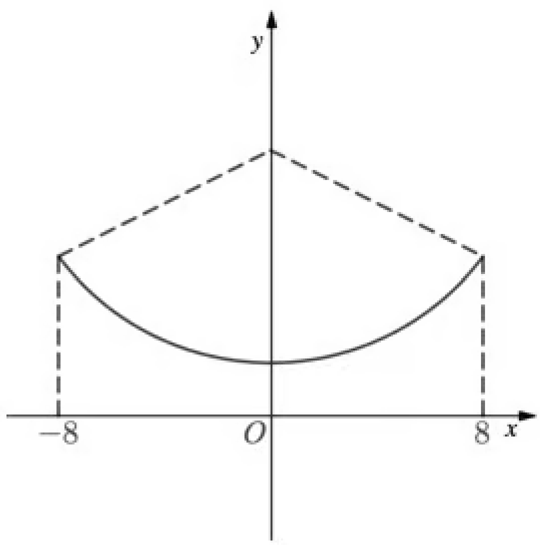
Write down the value of a.

(1 mark)

(b) Starting at sea level, the dolphin takes π seconds to jump out of the water, dive back under and return to sea level.
Given that $0 \leq B \leq 2$, determine the value of B.

(2 marks)

3 The path of a swing boat fairground ride that swings forwards and backwards is modelled as the arc of a circle, radius $8\ m$, as shown in the diagram below.



Ground level is represented by the x-axis.

The value of x represents the horizontal displacement, in metres, of the swing boat relative to the origin.

The value of y represents the height, in metres, of the swing boat above ground level.

The height of the swing boat is modelled using

$$y = 12 - \sqrt{100 - x^2}, \qquad -8 \leq x \leq 8$$

(i) Find the height of the boat when it's horizontal displacement is 6 m.

(ii) Find the horizontal distance from the origin when the boat is 5 m above the ground, giving your answer to three significant figures.

(iii) Find the maximum height the swing boat reaches.

(iv) When at its maximum height, find the angle of elevation of the swing boat from the origin.

(5 marks)

4 (a) The height, h m, of water in a reservoir is modelled by the function

$$h(t) = 6 + A\sin(t),\ t \geq 0,$$

where t is the time in hours after midday.

A is a positive constant.

Write down the height of the water in the reservoir at midday.

(1 mark)

(b) The minimum height the water is 3 m.

(i) Write down the value of A.

(ii) Find the maximum height of the water.

(2 marks)

(c) Find the height of the water at

(i) 2pm,

(ii) midnight,

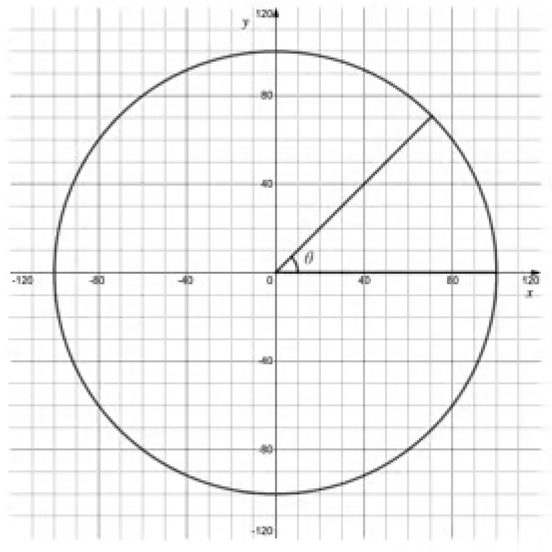
giving your answers to two decimal places.

(3 marks)

5 (a) A Ferris wheel is modelled as a circle with centre $(0,0)$ and radius 100 m.

There are 32 passenger “pods” which are evenly spaced around the Ferris wheel.

A pod’s position can be determined by the angle, θ radians, which is measured anticlockwise from the positive x-direction, as shown in the diagram below.



The coordinates of a pod, (x, y), are given by $(100\cos\theta, 100\sin\theta)$.

(i) Find the angle, in radians, between each pod.

(ii) Find the coordinates, to one decimal place, of the first pod located **anticlockwise above** the positive x-axis.

(3 marks)

(b) (i) Write down the angle θ for the passenger pod located at the point $(-100,0)$.

(ii) Determine the angle θ for the pod located at the point $(50, 50\sqrt{3})$.

(3 marks)

6 (a) As part of a quality control test, a lifejacket is thrown into the sea.

The height, h m, of the lifejacket above or below sea level $(h=0)$, at time t seconds after first hitting the water, is modelled by the equation $h=-e^{-0.6t}\sin 2t$

(i) Find the height of the lifejacket after 1.5 seconds.
Is the lifejacket above or below sea level at this point in time?

(ii) Excluding the case when $t=0$ find the value of t the first time the lifejacket is at sea level.

(3 marks)

(b) The lifejacket reaches its furthest point below sea level after 0.64 seconds.
Find the distance below sea level this is, giving your answer to three significant figures.

(2 marks)

(c) According to the model, what should happen to the lifejacket as time progresses?

(1 mark)

7 (a) The number of daylight hours, h, is modelled using the function

$$h = 12 + 5\sin(d - 1)^\circ, \qquad d \geq 1$$

where d is the day number on which the model applies.

(i) Write down the number of daylight hours on day 1.

(ii) Work out the number of daylight hours on day 136.

(3 marks)

(b) (i) Find the days on which there are 9.5 daylight hours.

(ii) Hence find the number of days in a year for which there are less than 9.5 daylight hours.

(5 marks)

(c) Explain why the model does not quite cover a whole year before repeating itself.

(1 mark)

8 (a) The alternating voltage, V, in an electrical circuit, t seconds after it is switched on, is modelled by the function

$$V = 20\cos \pi t + 20\sqrt{3}\sin \pi t$$

where t is measured in seconds.

Use the identity $R\cos(\pi t - \alpha) = R\cos\alpha\cos\pi t + R\sin\alpha\sin\pi t$ to show that

$$20\cos \pi t + 20\sqrt{3}\sin \pi t$$

can be written as

$$40\cos(\pi t - \frac{\pi}{3}).$$

(3 marks)

(b) (i) Write down the maximum voltage in the electrical circuit.

(ii) Find the voltage at time $t = 0$.

(iii) Find the voltage after two seconds.

(3 marks)

(c) After how many seconds does the voltage first equal -20 volts?

(3 marks)

Medium Questions

1 (a) A small spring is extended to its maximum length and released from rest.

The length of the spring, l cm , at time t seconds, where the angle is given in radians, is then modelled by the function

$$l = 5 + 3\cos 2t, \qquad t \geq 0$$

(i) Write down the natural length of the spring.

(ii) Write down the maximum extension of the spring.

(2 marks)

(b) (i) Find the length of the spring after 6 seconds.

(ii) Find the time at which the length of the spring first reaches 4 cm.

(3 marks)

(c) State one criticism of this model as time passes.

(1 mark)

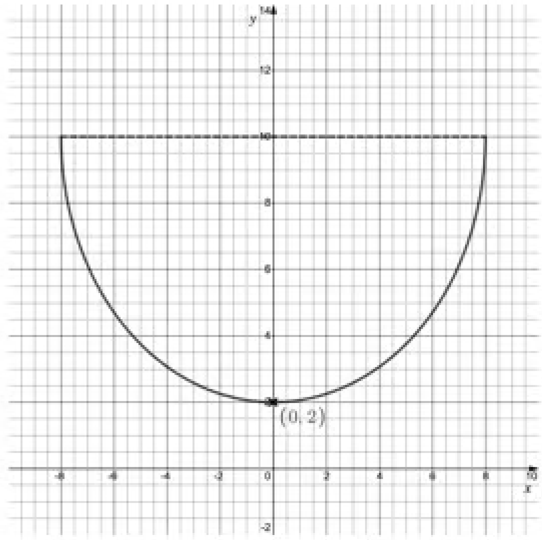
2 A dolphin is swimming such that it is diving in and out of the water at a constant speed. On each jump and dive the dolphin reaches a height of 2 m above sea level and a depth of 2 m below sea level.

Starting at sea level, the dolphin takes $\frac{2\pi}{3}$ seconds to jump out of the water, dive back in and return to sea level.

Write down a model for the height, h m, of the dolphin, relative to sea level, at time t seconds, in the form $h = A\ \sin(Bt)$ where A are B constants to be found.

(3 marks)

3 (a) The path of a swing boat fairground ride that swings forwards and backwards is modelled as a semi-circle, radius 8 cm, as shown in the diagram below.



Ground level is represented by the x-axis and represents the height of the boat above ground level. The path of the boat is given by the formula

$$y = 10 - \sqrt{64 - x^2} \qquad -8 \leq x \leq 8$$

The boat's initial position is at the point $(0,2)$.

(i) Find the height of the boat when it is 2 m horizontally from its initial position.

(ii) When the boat is at a height of 6 m, find its exact horizontal distance from the origin.

(4 marks)

(b) Given that the x-coordinate of the boat is also given by

$$x = 8\sin\left(\frac{\pi}{6}t\right)$$

where t seconds is the time since the boat was released from its initial position, find the time it takes the boat to swing from one end of the ride to the other.

(3 marks)

4 (a) The height, h m, of water in a reservoir is modelled by the function

$$h(t) = A + B\sin\left(\frac{\pi}{6}t\right), \quad t \geq 0$$

where t is the time in hours after midnight. A and B are positive constants.

In terms of A and B, write down the natural height of the water in the reservoir, as well as its maximum and minimum heights.

(3 marks)

(b) The maximum level of water is 3m higher than its natural level.

The level of water is three times higher at its maximum than at its minimum.

Find the maximum, minimum and natural water levels.

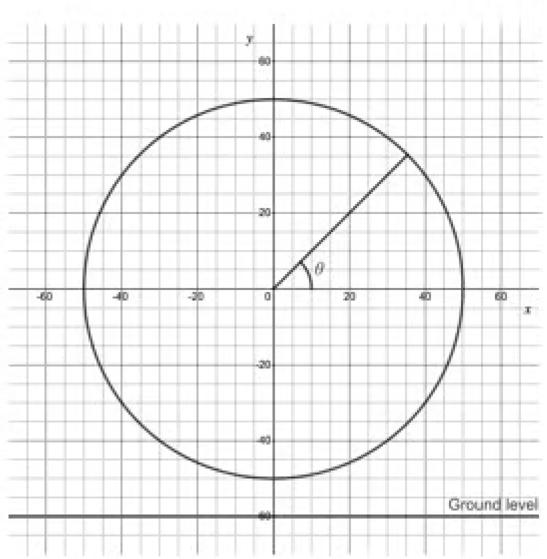
(3 marks)

(c) (i) How many times per day does the water reach its maximum level?

(ii) Find the times of day when the water level is at its minimum?

(3 marks)

5 (a) A Ferris wheel with p passenger "pods" is modelled as a circle with centre $(0,0)$ and radius 50 m. A pod's position can be determined by the angle, θ radians, which is measured anticlockwise from the positive x-direction, as shown in the diagram below.



The coordinates of a pod, (x, y), are given by $(A\cos(\theta), A\sin(\theta))$, where A is a positive constant. Ground level is represented by the line with equation y=-60.

(i) Write down the value of the constant A.

(ii) The angle between each pod is $\frac{\pi}{12}$ radians. Find the value of p.

(iii) Find the maximum height above the ground of a passenger pod during one complete rotation of the Ferris wheel.

(4 marks)

(b) Find, to three significant figures, the angle θ for a passenger pod located at the point $(30,40)$.

(2 marks)

6 (a) A lifejacket falls over the side of a boat from a height of 3 m.
The height, h m, of the lifejacket above or below sea level $(h=0)$, at time t seconds after falling, is modelled by the equation $h=3e^{-0.7t}\cos 4t$.

The lifejacket reaches its furthest point below sea level after 0.742 seconds.
Find the total distance it has fallen, giving your answer to three significant figures.

(2 marks)

(b) Write down the value of t for the first three times the lifejacket is at sea level.

(2 marks)

(c) (i) Find the value of $3e^{-0.7t}$ when $t=6.2$.

(ii) Hence justify why, from 6.2 seconds on, the lifejacket will always be within 4 centimetres of sea level.

(3 marks)

7 (a) The number of daylight hours, h, in the UK, during a day d days after the spring equinox (the day in spring when the number of daylight hours is 12), is modelled using the function

$$h = 12 + \frac{9}{2}\sin\left(\frac{2\pi}{365}d\right)$$

(i) Find the number of daylight hours during the day that is 100 days after the spring equinox.

(ii) Find the number of days after the spring equinox that the two days occur during which the number of daylight hours is closest to 9.

(5 marks)

(b) For how many days of the year does the model suggest that the number of daylight hours exceeds 15 hours? Give your answer as a whole number of days.

(3 marks)

8 (a) The alternating voltage, V, in an electrical circuit, t seconds after it is switched on, is modelled by the function

$$V = 55\sqrt{3}\sin\frac{\pi t}{30} + 55\cos\frac{\pi t}{30}$$

Show that

$$55\sqrt{3}\sin\frac{\pi t}{30} + 55\cos\frac{\pi t}{30}$$

can be written as

$$R\sin\left(\frac{\pi t}{30} + \alpha\right)$$

where $R = 110$ and $\alpha = \frac{\pi}{6}$.

(3 marks)

(b) (i) Find the voltage at time $t = 0$.

(ii) Find the voltage after one minute.

(4 marks)

(c) After how many seconds does the voltage first equal -55 volts?

(3 marks)

Hard Questions

1 (a) The length of a spring, l cm, at time t seconds, after being released from rest, is modelled by the function

$$l = a + b\cos 4t, \qquad t \geq 0$$

Describe what the constants a and b represent in terms of the length of the spring.

(2 marks)

(b) Given that the minimum length the spring can attain is 12 cm and its maximum length is 30 cm, find the values of a and b.

(2 marks)

(c) (i) A similar spring, with the same values of a and b, has length modelled by

$$l = a + b\cos 2t, \qquad t \geq 0$$

Compare the motion of the two springs.

(ii) Suggest one way in which the model (for both springs) could be improved.

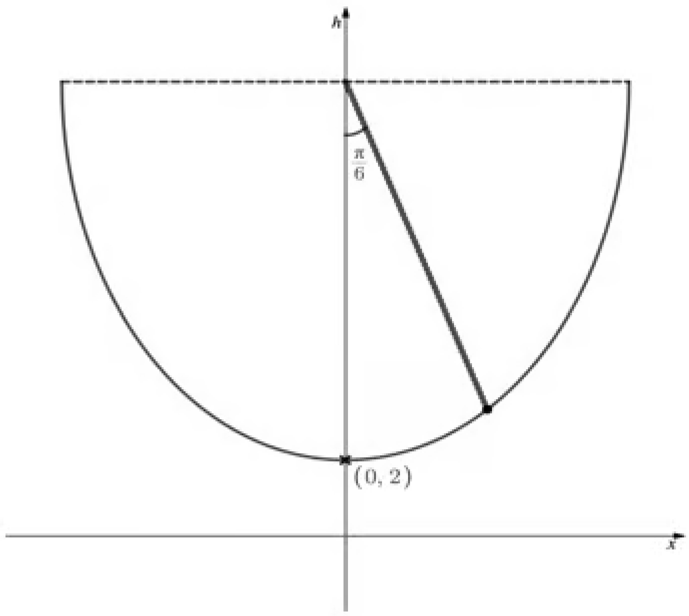
(3 marks)

2 A hovering helicopter moves up and down at a constant rate between the heights of 200 m and 220 m. It takes the helicopter $\frac{\pi}{5}$ seconds to move between these two heights.

Write down a model in the form $h = A + B\cos(Ct)$ for the height, h m, of the helicopter at time t seconds, where A, B are C constants to be found.
State the initial height of the helicopter suggested by your model.

(3 marks)

3 (a) The path of a swing boat fairground ride that swings forwards and backwards is modelled as a semi-circle, radius 10 m, as shown in the diagram below.



At time t seconds, the x-coordinate of the boat is modelled by the function

$$x(t) = 10\sin\left(\frac{\pi}{5}t\right), \qquad t \geq 0$$

and the height, h m , of the boat above the ground, at time t seconds, is modelled by

$$h(t) = 12 - 10\left|\cos\left(\frac{\pi}{5}t\right)\right|, \qquad t \geq 0.$$

Verify that the initial position of the boat is $(0,2)$.

(2 marks)

(b) (i) Write down the coordinates of the boat when it is at its maximum height.

(ii) Find the time it takes the boat to swing between these two points.

(3 marks)

(c) Find the position of the boat when it has swung through an angle of $\frac{\pi}{6}$ anticlockwise from the y-axis, as shown in the diagram above.
Find the time at which the boat first reaches this position.

(2 marks)

4 (a) The height, h m, of water in a reservoir is modelled by the function

$$h(t) = A + B\sin(Ct), \qquad t \geq 0$$

where t is the time, in hours, after midnight. A, B and C are positive constants.

(i) Given that the water level rises and falls through one and a half cycles in a 24 hour period, find the value of C.

(ii) The height of water reaches its minimum of 1 m just once per day.
Find the time of day when this occurs.

(iii) The maximum height of water is 11 m. Find the values of A and B.

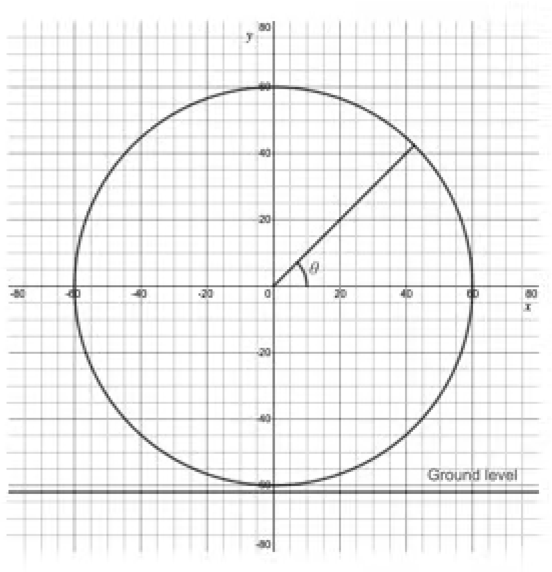
(5 marks)

(b) The reservoir is only capable of holding water to a maximum height of 10 m
Should the water level exceed this, an overflow reservoir is available.

During which times of day will the overflow reservoir be in use?
Give your answers to the nearest minute.

(3 marks)

5 (a) A Ferris wheel with 30 passenger "pods" is modelled as a circle with centre $(0,0)$ and radius 60 m. A pod's position can be determined by the angle θ radians, which is measured anticlockwise from the positive x-direction, as shown in the diagram below.



The coordinates of a pod, (x,y) are given by $(A\cos(\theta), B\sin(\theta))$ where A and B are positive constants. Ground level is represented by the line with equation y=-62.

(i) Write down the values of A and B.

(ii) The pods are evenly distributed around the wheel.
Find the angle between each pod.

(3 marks)

(b) Find the height above the ground of a passenger pod when $\theta = \frac{7\pi}{6}$ radians.

(3 marks)

(c) Find the angle θ, to three significant figures, for a passenger pod located at the point $(48, -36)$.

(2 marks)

(d) What would you be able to say about the Ferris wheel in the case where $A \neq B$?

(1 mark)

6 (a) A lifejacket falls over the side of a boat from a height of 4 m above sea level.
The height, h m , of the lifejacket above or below sea level $(h = 0)$ at time t seconds after falling, is modelled by the equation $h = Ae^{-kt}\cos 2t$, where A are k positive constants.

(i) Write down the value of A.

(ii) Briefly explain how the constant k affects the height of the lifejacket over time.

(2 marks)

(b) After 2.054 seconds the lifejacket is 1 m below sea level. Find the value of k and determine whether the lifejacket is rising or sinking.

(4 marks)

7 (a) The number of daylight hours, h, in the UK, d days after the spring equinox (the day in spring when the number of daylight hours is 12) is modelled using the function

$$h = A + B\sin\left(\frac{2\pi}{365}d\right)$$

where A are B constants.

(i) Write down the value of A.

(ii) Given that the maximum number of daylight hours is 16.5, write down the value of B.

(2 marks)

(b) For how many days of the year does the number of daylight hours remain below 10? Give your answer as a whole number of days.

(2 marks)

(c) If the spring equinox falls on the 21st March, find the dates throughout the year when there are 16 hours of daylight.

(3 marks)

(d) The model needs to be adjusted every four years. Suggest a reason why.

(1 mark)

8 (a) The alternating voltage, V , in an electrical circuit t seconds after it is switched on is modelled by the function

$$V = 55\sqrt{2}\left(\sin\frac{\pi t}{60} + \cos\frac{\pi t}{60}\right)$$

Express

$$55\sqrt{2}\left(\sin\frac{\pi t}{60} + \cos\frac{\pi t}{60}\right)$$

in the form

$$R\sin\left(\frac{\pi t}{60} + \alpha\right)$$

where R are α constants to be found. $R > 0$ and α is acute.

(3 marks)

(b) Find the voltage when the circuit is switched on.

(2 marks)

(c) (i) Write down the maximum voltage and the time at which this first occurs.

(ii) Find the time it takes the voltage to complete one period (cycle).

(2 marks)

Very Hard Questions

1 (a) The length of a spring, l cm, at time t seconds, after being released from rest, is modelled by the function

$$l = a + b\cos ct, \qquad t \geq 0$$

where a, b and c are constants.

(i) Describe the effect the constant c has on the model.

(ii) Explain how you know the spring is stretched to its maximum length before being released.

(2 marks)

(b) (i) It takes $\frac{\pi}{10}$ seconds from release until the spring first returns to its starting length .Find the value of c.

(ii) Given that the maximum length of the spring is twice its minimum length, find a relationship between a and b.

(3 marks)

(c) Explain why the function would not be appropriate for modelling the length of a spring if $b \geq a$.

(1 mark)

2 (a) The height above ground, h m , of a drone used as part of an air display is modelled by the function $h = A + B\sin(Ct + D)$, where t is the time in seconds after launch A, B, C and D are constants.

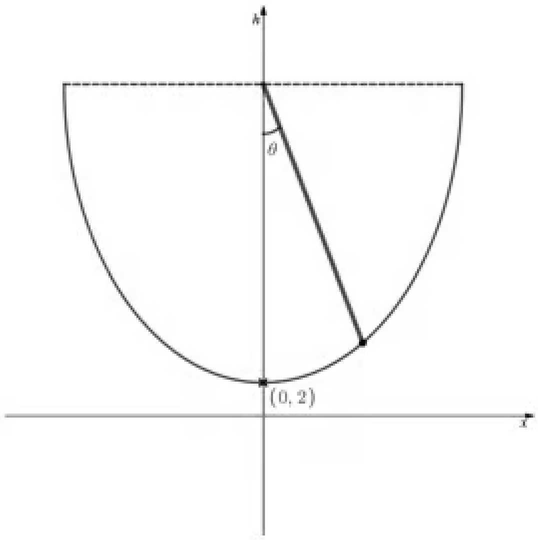
The drone is launched upwards from a height of 23 m and $\frac{\pi}{6}$ seconds later it reaches its maximum height of 26 m. The minimum height the drone reaches is 14 m.
Find the value of the constants A, B, C and D given that D is acute.

(4 marks)

(b) The drone's lights switch off when its height drops below 17 m.
Show that the drone's lights are on for two-thirds of its flight.

(3 marks)

3 (a) A swing boat fairground ride is modelled as moving forwards and backwards along the path of a semi-circle, radius 18 m, as shown in the diagram below.



Show that, for $0 \leq \theta \leq \frac{\pi}{2}$,

(i) the x-coordinate of the boat is given by $x = 18 \sin \theta$,

(ii) the y-coordinate is given by $y = 20 - 18 \cos \theta$.

(3 marks)

(b) The model is refined so that the coordinates of the boat can be calculated from the time, t seconds, after the boat is set in motion. The x and y coordinates are now given by

$$x = 18 \sin Bt \qquad\qquad y = 20 - 18|\cos Bt|$$

where B is a constant.

(i) Briefly explain why the modulus of cos θ is required for the y- coordinate.

(ii)

Given that the time between the boat reaching its maximum height at either end of the ride is 8 seconds, find the value of B.

(3 marks)

(c) For $0 \leq t \leq 4$, find the times when the boat is equidistant from the ground and horizontally from the origin.

(3 marks)

4 (a) The height of water, h m , in a reservoir is modelled by the function

$$h(t) = A + B \sin Ct, \qquad t \geq 0$$

where t is the time in hours after midnight. A, B and C are positive constants.

Briefly explain how each of the constants A, B and C affect the height of the water in the reservoir.

(2 marks)

(b) Show that the height of water will first be at its minimum level at time

$$t = \frac{3\pi}{2C}$$

hours after midnight.

(2 marks)

(c) Show that the rate of change of the height of water in the reservoir is at its greatest every

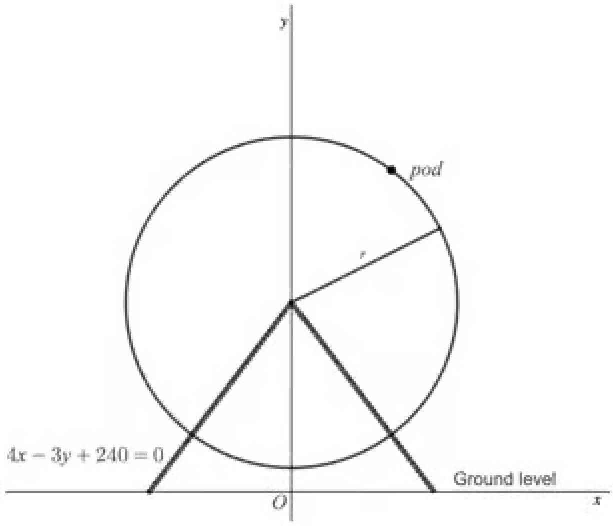
$$\frac{k\pi}{C}, \qquad k \in \mathbb{Z}_0^+$$

hours after midnight.

(3 marks)

5 (a) Engineers are designing a Ferris wheel with passenger "pods".
The wheel is modelled as a circle with centre $(0, a)$ and radius r meters.

One of the pods is to be located at the point with coordinates $(42, 136)$.



The thick lines on the diagram represent two symmetrical ground supports for the Ferris wheel each going from its centre to ground level.

The left-hand support is represented by the equation $4x - 3y + 240 = 0$.
The x-axis represents ground level.

(i) Find the equation of the circle.

(ii) How far from the ground is the lowest point of the Ferris wheel?

(2 marks)

(b) The p pods are to be evenly distributed around the wheel.
Ideally the engineers would like no more than three pods to be within the intersection of the supports at any one time. Find the maximum value of p this design approach allows.

(3 marks)

(c) For both strength and aesthetic reasons, both the ground supports will be made in two sections. Thinner materials will be used within the wheel so as not to obstruct the view of, and from, the Ferris wheel and thicker material will be used for the lower base supports outside the wheel.

Find the percentage of the thicker material required.

(2 marks)

6 (a) The height, h m, of a helicopter, t seconds after take-off, is modelled by the function

$$h = 12 + 2\tan\left(\frac{1}{2}t - \frac{\pi}{2}\right) \qquad 0 < t \leq 6$$

The time lag between the pilot firing up the helicopter and leaving the ground is accounted for in the model by negative values of h for the period $0 < t \leq 6$.

Find the value of α to two significant figures.

(2 marks)

(b) Show that the helicopter rises just 4 m between the times of $\frac{\pi}{2}$ seconds and $\frac{3\pi}{2}$ seconds

(2 marks)

(c) Find the height of the helicopter at the point at which the model ceases to be valid.

(2 marks)

7 (a) The number of daylight hours, h , in the UK, d days after the spring equinox (the day in spring when the number of daylight hours is 12) is modelled using the function

$$h = 12 + B\sin\left(\frac{2\pi}{C}d\right)$$

where B and C are constants.

Explain the meaning of the constants B and C in the context of this model.

(2 marks)

(b) During a normal year (not a leap year), the maximum number of daylight hours is 16 hours and 38 minutes.

Find the total number of daylight hours in the first half of the year.
(Assume a year in this sense starts on the spring equinox, when $d = 0$.
Give your answer to the nearest 10 hours.

(3 marks)

8 (a) The alternating voltage, V, in a domestic electrical circuit, t seconds after it is switched on is modelled by the function

$$V = 115 \sin \omega t + 115\sqrt{3} \cos \omega t$$

Express

$$115 \sin \omega t + 115\sqrt{3} \cos \omega t$$

in the form

$$R \sin(\omega t + \alpha)$$

where R and α are constants to be found. $R > 0$ and is α acute.

(2 marks)

(b) In the UK, domestic electricity runs at a frequency, f, of 50 Hertz (Hz).

The constant ω, is given by $\omega = 2\pi f$.

(i) Find the initial voltage when a domestic appliance (such as a kettle or TV) is switched on.

(ii) Find the time at which the voltage first turns negative.

(4 marks)

(c) (i) Find the period of one cycle of voltage in the UK.

(ii) In the US, the period of one cycle is $\frac{1}{60}$ seconds

Write down the frequency of US domestic electricity.

(2 marks)