**A Level · Edexcel · Maths**

3 hours 43 questions

# 2.4 Inequalities

2.4.1 Linear Inequalities / 2.4.2 Quadratic Inequalities / 2.4.3 Inequalities on Graphs

| | |
|---|---|
| Easy (12 questions) | /48 |
| Medium (10 questions) | /42 |
| Hard (10 questions) | /50 |
| Very Hard (11 questions) | /57 |
| **Total Marks** | **/197** |

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# Easy Questions

**1** Solve the inequalities:

(i) $2x \geq 8$
(ii) $3 + 2x < 11$
(iii) $5 + x > 4x - 1$

**(3 marks)**

**2** Solve the inequalities:

(i) $2x - 9 \geq 5(x - 3)$
(ii) $3(5 - x) < 2(9 - 2x)$

**(4 marks)**

**3 (a)** Write down the solutions to $(x-3)(x-8)=0$.

**(2 marks)**

**(b)** Sketch the graph of $y=(x-3)(x-8)$, clearly showing the coordinates of the points where the graph intercepts the $x$-axis.

**(2 marks)**

**(c)** Hence, or otherwise, solve the inequality $(x-3)(x-8)<0$.

**(2 marks)**

**4 (a)** Find the discriminant for the quadratic function $x^2 + 8x + 15$.

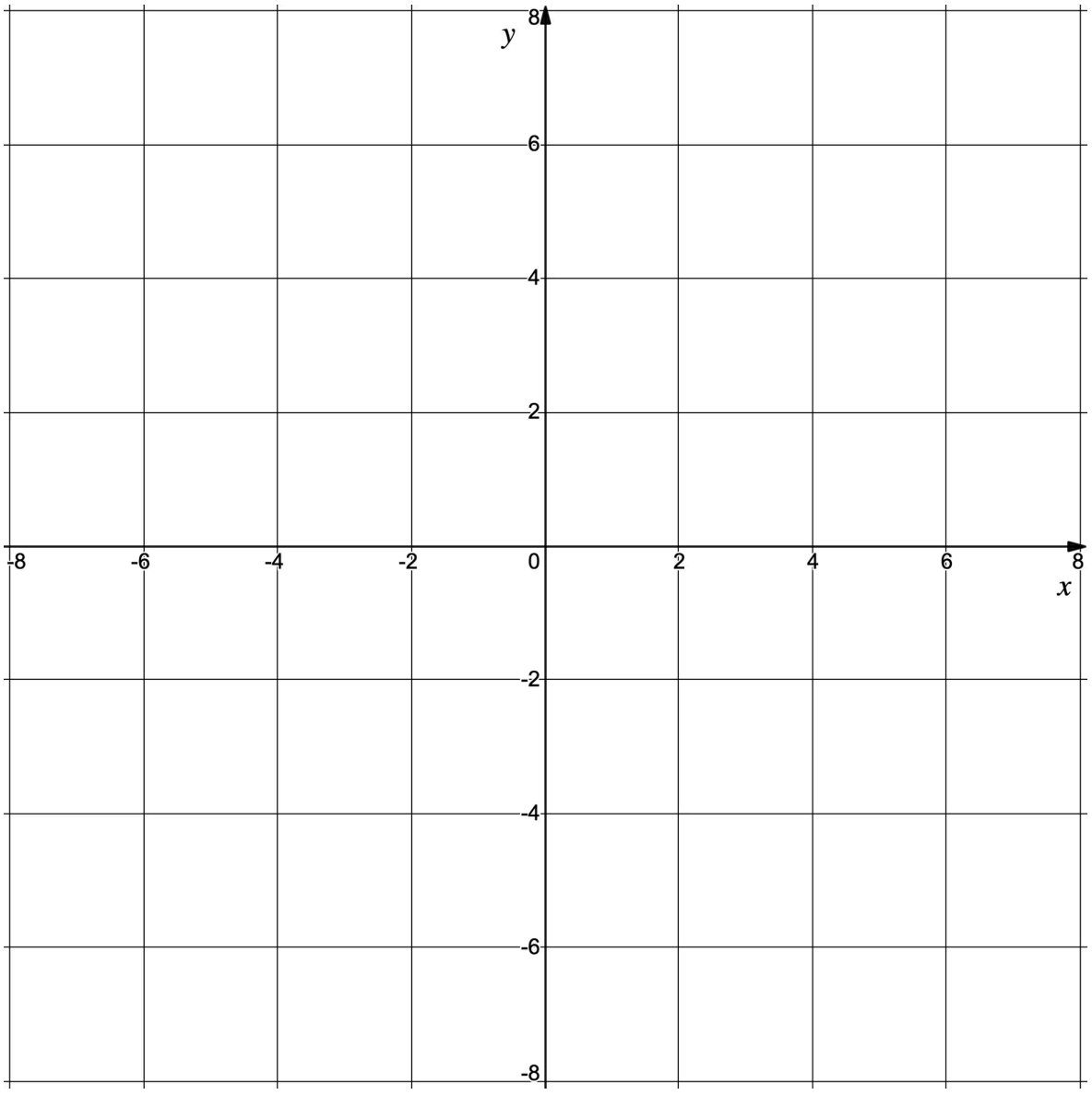
**(2 marks)**

**(b)** Write down the number of real solutions to the equation $x^2 + 8x + 15 = 0$.

**(2 marks)**

**5** On the axes below, show the region bounded by the inequalities

$$x \geq 0$$
$$y \leq 4$$
$$x \leq 5$$
$$y \geq 1$$




**(4 marks)**

**6 (a)** (i) Solve the equation $9 - x^2 = 0$.

(ii) Use symmetry to write down the coordinates of the turning point on the graph of $y = 9 - x^2$.

**(3 marks)**

**(b)** Sketch the graph of $y = 9 - x^2$ and hence solve the inequality $9 - x^2 \geq 0$.

**(3 marks)**

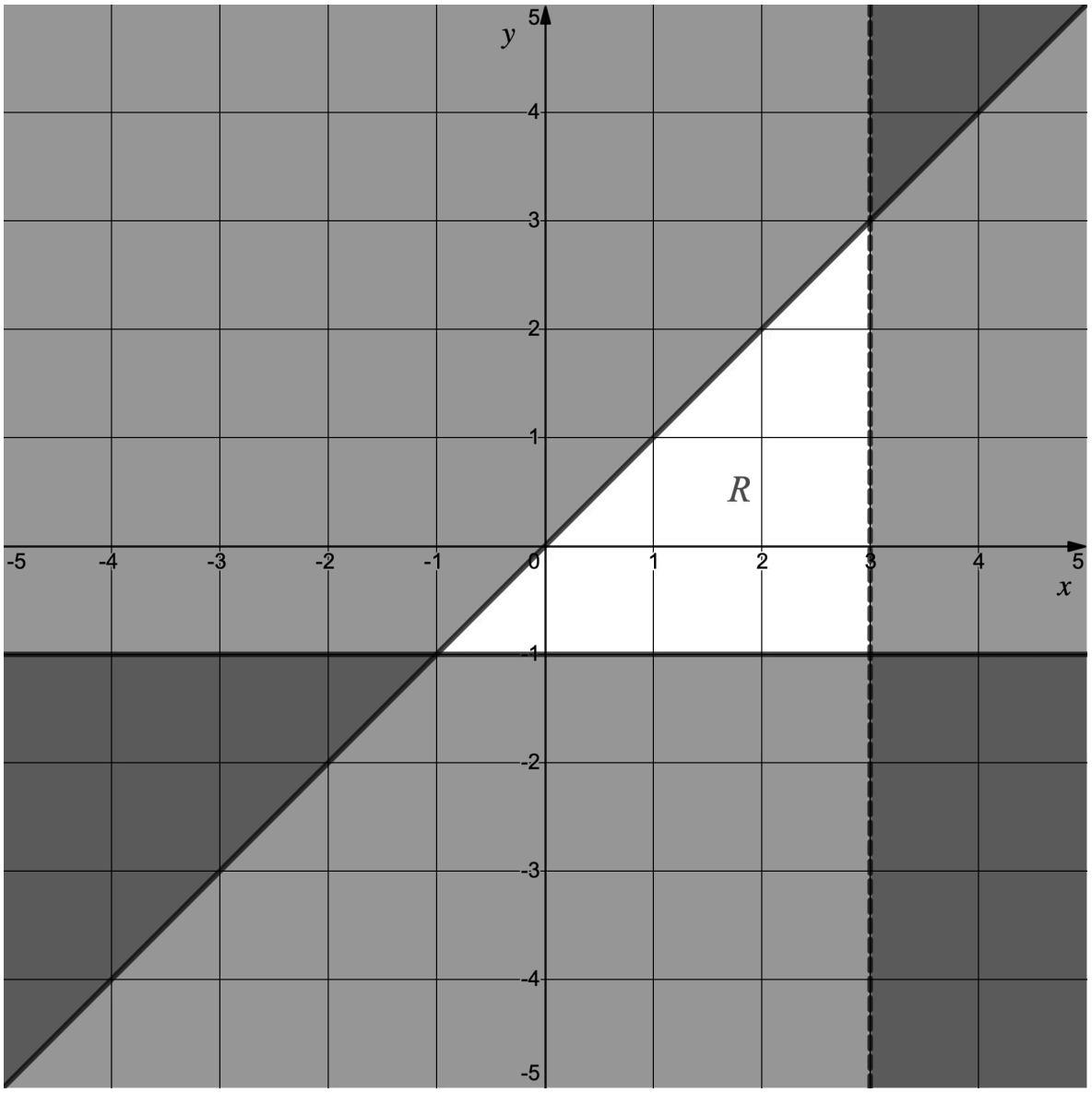
**7 (a)** Write down, in terms of $k$, the discriminant of $x^2 + 8x + 4k$.

**(1 mark)**

**(b)** Hence find the values of for $k$ which the equation $x^2 + 8x + 4k = 0$ has two real and distinct solutions.

**(2 marks)**

**8** Write down the three inequalities that define the region $R$ shown in the diagram below.




**(3 marks)**

**9** The total cost to a company manufacturing $c$ cables is $(500 + 3c)$ pence.

The total income from selling all $c$ cables is $(5c - 3500)$ pence.

What is the minimum number of cables the company needs to sell in order to recover their costs?

**(4 marks)**

**10** The equation $x^2 + kx + 4 = 0$, where $k$ is a constant, has no real roots.

Find the possible value(s) of k.

**(4 marks)**

**11** Solve the inequality $6x - 7 \leq 35$, giving your answer in set notation.

**(4 marks)**

**12** Solve the inequality $6 \leq 8x - 2 \leq 22$.

**(3 marks)**

# Medium Questions

**1** Solve the inequality $3x + 4 \leq 5(x - 1)$.

**(3 marks)**

**2** Solve the inequality $x^2 - 5x > 6$.

**(4 marks)**

**3** The equation $kx^2 + 2kx + 4 = 0$, where $k$ is a constant, has two distinct real roots.

Find the possible value(s) of k.

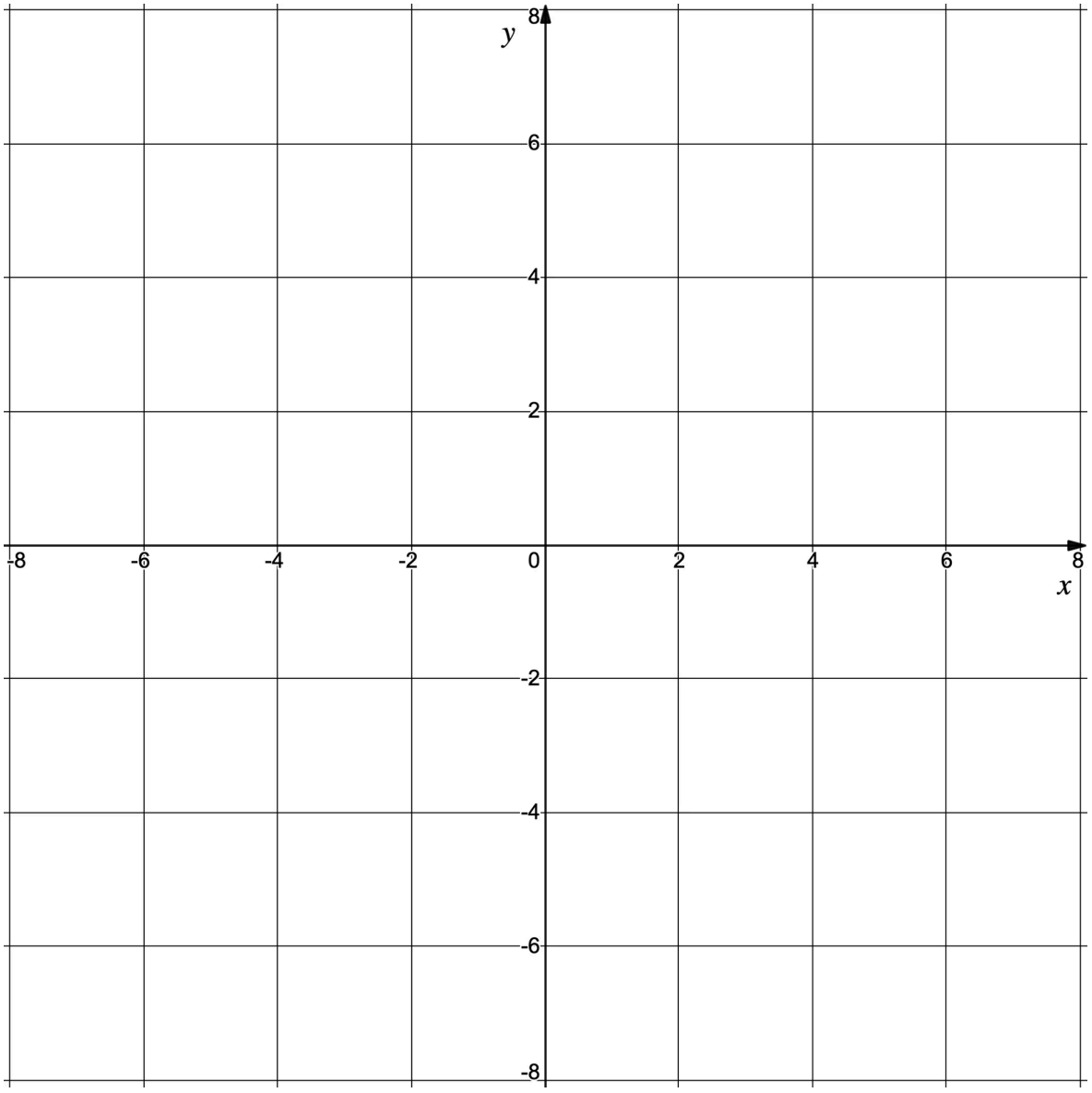
**(4 marks)**

**4** On the axes below show the region satisfied by the inequalities

$$x + 2y > 3$$
$$y \leq x + 4$$

$$y + 3x < 8$$

Label this region R.




**(5 marks)**

**5** Find the values of $x$ that satisfy the inequalities

$$x^2 + 3x > 4$$
$$4x + 1 > 4$$

**(5 marks)**

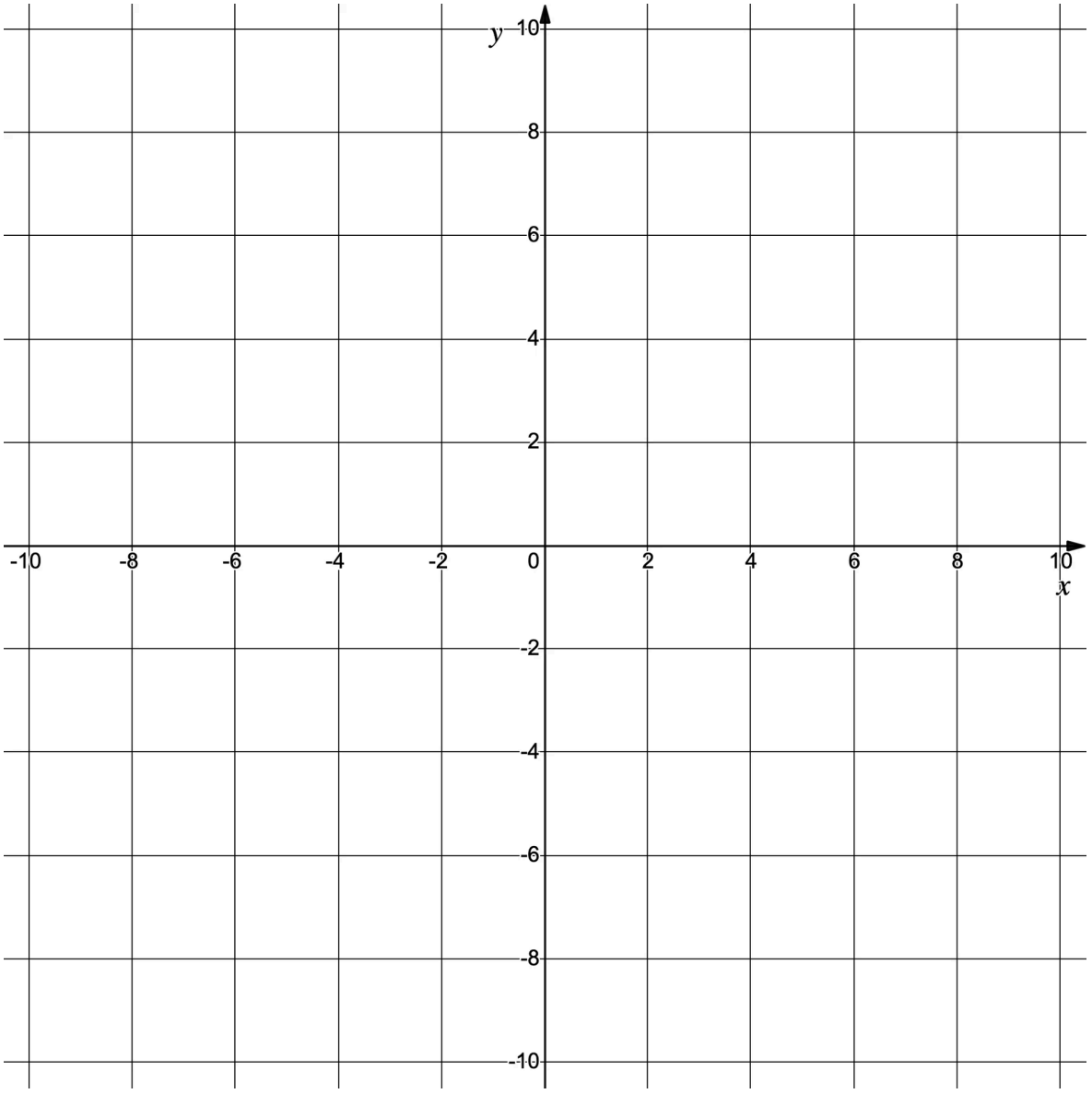
**6** Solve the inequality $-2 \leq 3x - 4 \leq 5$, giving your answer in set notation.

**(4 marks)**

**7 (a)** The cross section of a tunnel is in the shape of the region defined by the inequalities

$$y \leq 5 - \frac{x^2}{5} \qquad\qquad y \geq 0$$

On the axes below show the region satisfying the inequalities

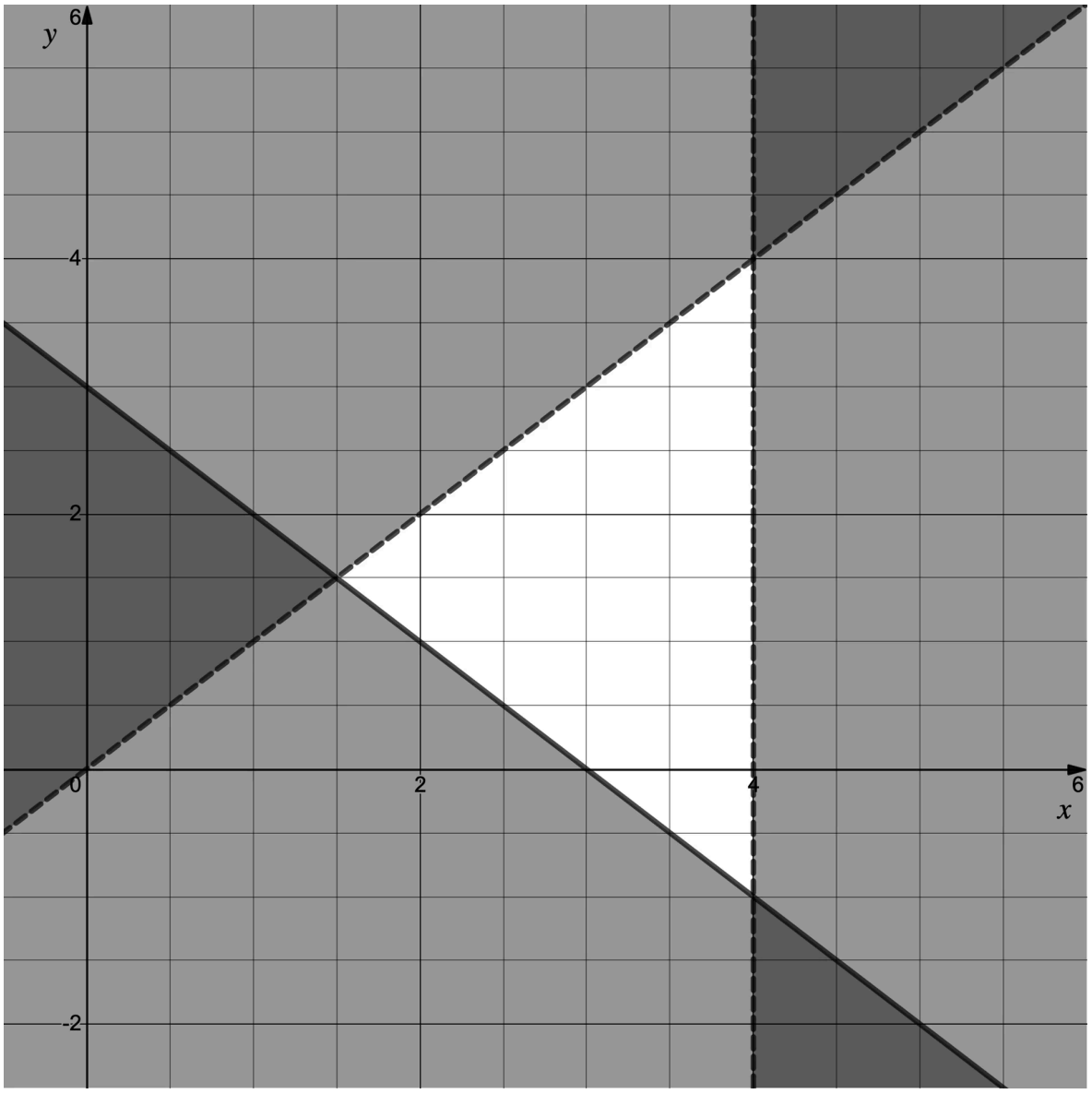



**(3 marks)**

**(b)** Given that $x$ and $y$ are in metres write down the height and the maximum width of the tunnel.

**(2 marks)**

**8** Write down the inequalities that define the region R shown in the diagram below.



**(4 marks)**

**9** The total cost to a company manufacturing $c$ cables is $(100 + 5c)$ pence.

The total income from selling all $c$ cables is $(30c - c^2)$ pence.

What is the minimum number of cables the company needs to sell in order to recover their costs?

**(4 marks)**

**10** A stone is projected vertically upwards from ground level.

The distance above the ground, $d$ m at $t$ seconds after launch, is given by

$$d(t) = 12t - 4.9t^2$$

How long does the stone remain $2$ m above the ground?

**(4 marks)**

# Hard Questions

**1** Solve the inequality $(x+2)^2 > 5$.

**(3 marks)**

**2** Solve the inequality $\frac{5}{3x^2+2} \leq 2$.

**(4 marks)**

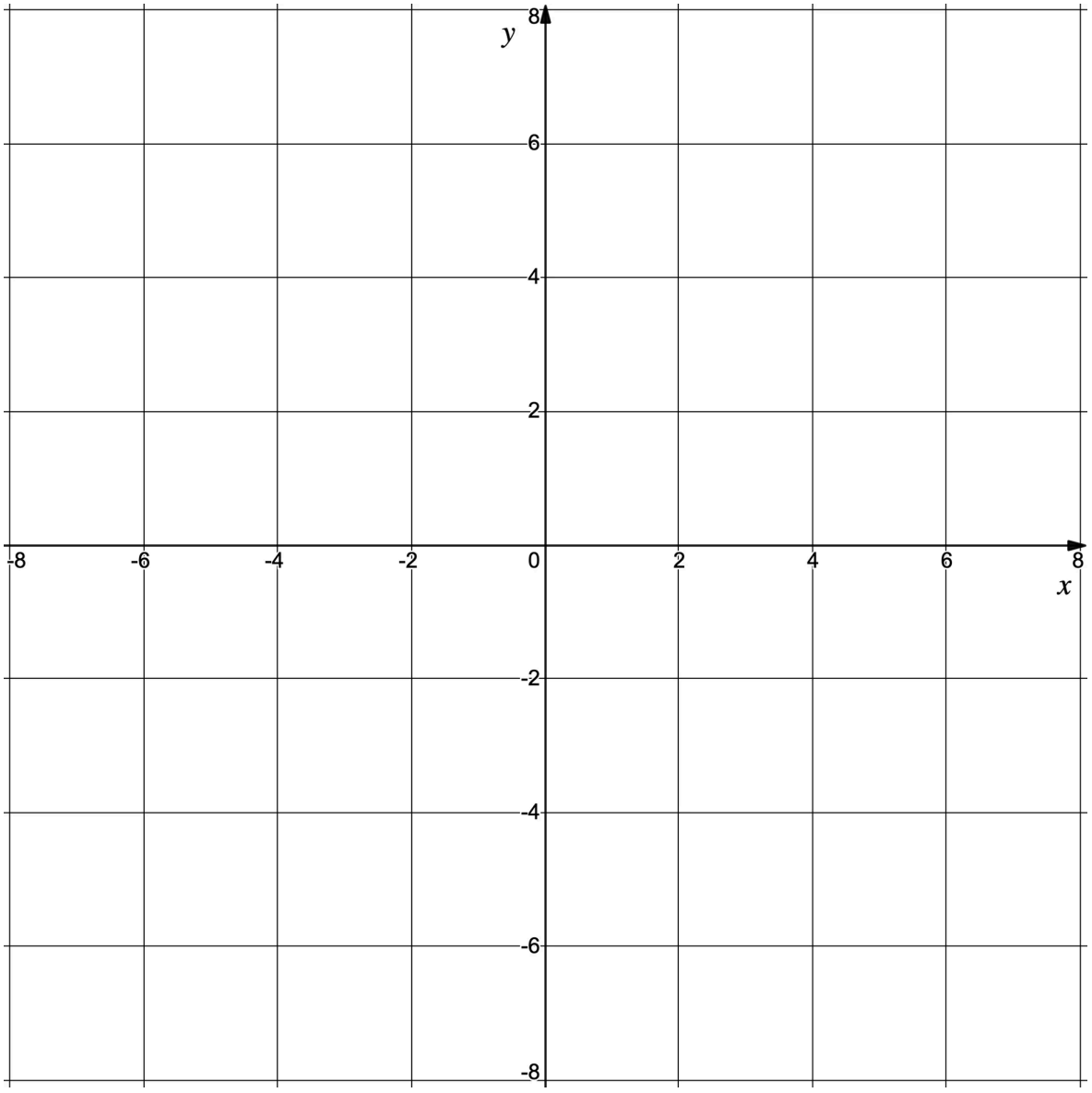
**3** The equation $(kx)^2 + (k-2)x + 1 = 0$, where $k$ is a constant, has two distinct real roots. Find the possible values of k.

**(4 marks)**

**4** On the axes below show the region satisfied by the inequalities

$$y + x > x^2$$
$$5y < 20 - 4x$$
$$y - 1 \geq 0$$

Label this region R.



**(5 marks)**

**5** Find the values of $x$ that satisfy the inequalities

$$x^2 + x < 2$$
$$x^2 < 4$$

**(5 marks)**

**6 (a)** Solve the inequality $-2 \leq x^2 - 4 \leq 5$.

**(4 marks)**

**(b)** Find the values of $x$ that satisfy the inequalities

$$x^2 + 4x - 3 \leq 2 - x^2 - 5x$$
$$8 - 2x^2 \leq 2x(2x + 1)$$

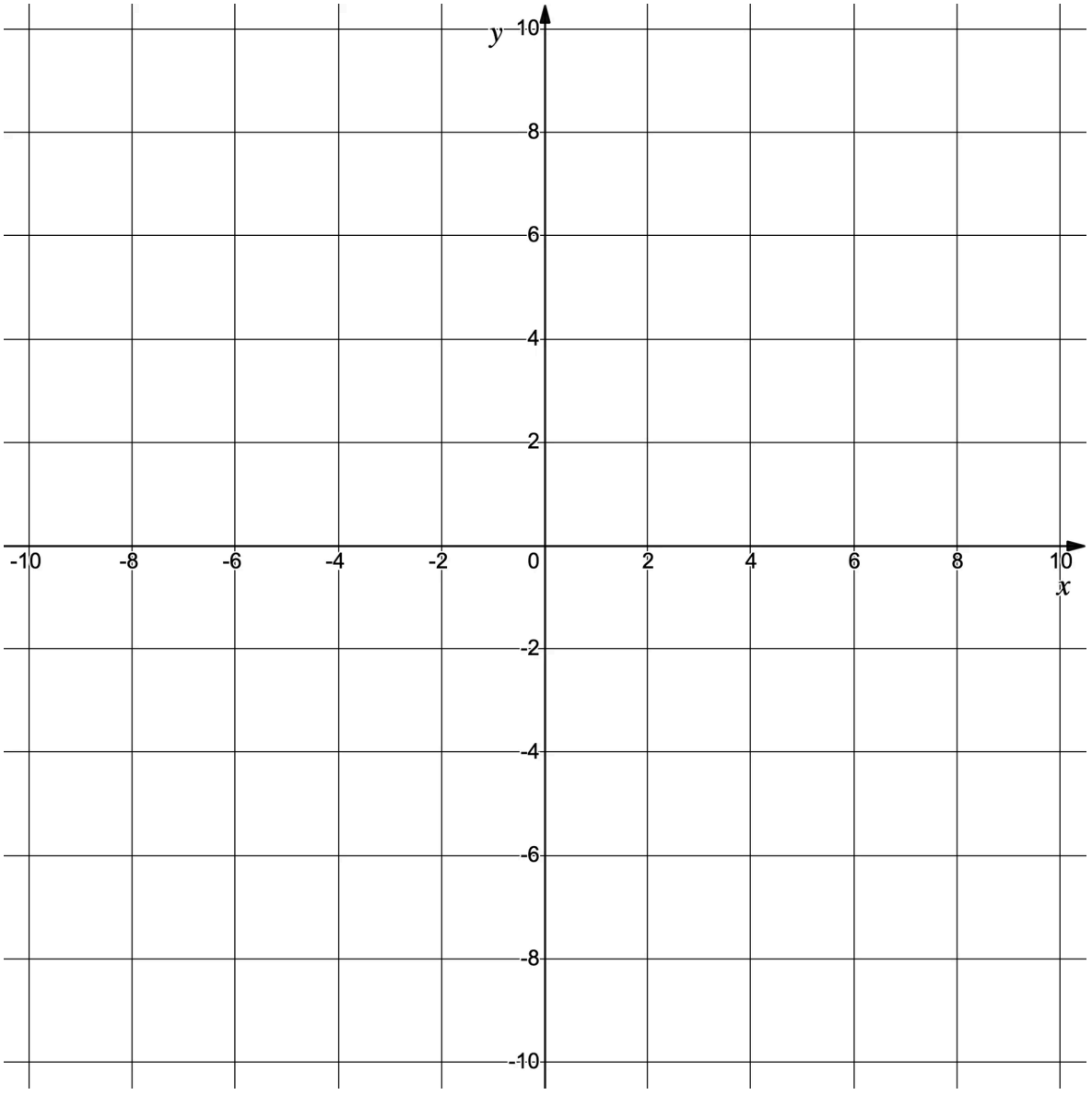
Give your answer in set notation.

**(4 marks)**

**7 (a)** The cross section of a tunnel is in the shape of the region defined by the inequalities

$$x^2 + y^2 \leq 25 \qquad\qquad y \geq 0$$

On the axes below show the region satisfying the inequalities




**(3 marks)**

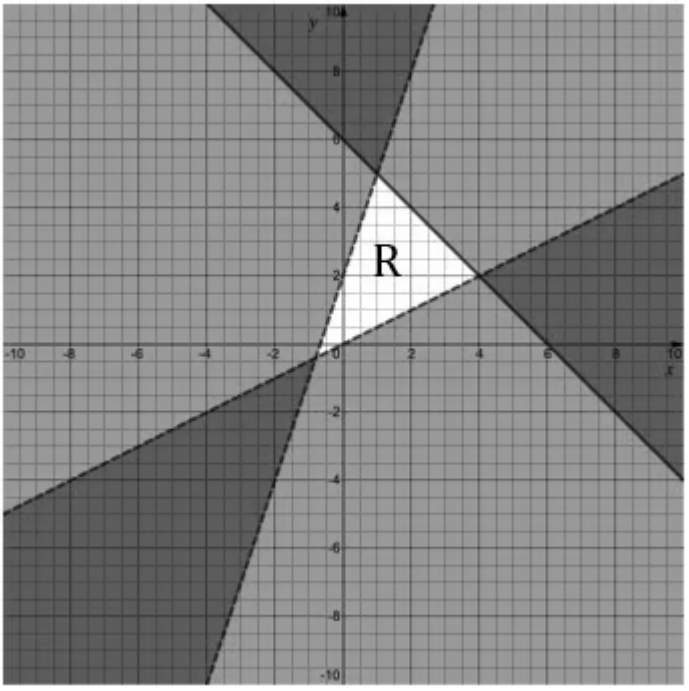
**(b)** Given that $x$ and $y$ are in metres, write down the height and the maximum width of the tunnel.

**(2 marks)**

**(c)** Find the area of the cross-section of the tunnel.

**(2 marks)**

**8** Write down the inequalities that define the region R shown in the diagram below.



**(4 marks)**

**9 (a)** An electronics company can produce $c$ cables at a total cost of $(200 + 10c)$ pence.
The cables can be sold for $(40 - c)$ pence each.

Show that the total income from selling $c$ cables is $(40c - c^2)$ pence

**(2 marks)**

**(b)** What is the minimum number of cables the company needs to sell in order to make a profit?

**(4 marks)**

**10** A stone is projected vertically upwards from a height of $1.5$ m.
It's height, above its starting position, $d$ m at time $t$ seconds after launch, is given by

$$d(t) = 16t - 4.9t^2$$

How long does the stone remain $3$ m above the ground?

**(4 marks)**

# Very Hard Questions

**1** Solve the simultaneous inequalities

$$t^2 - 2t - 15 < 0 \text{ and}$$
$$t^2 + 14 \leq 9t.$$

**(4 marks)**

**2** Solve the inequality $\frac{4x^2 - 11}{(x+1)^2} \geq 4$.

**(4 marks)**

**3** The equation $(k+1)t^2 + 2(k+2)t = 3(k+3)$ has real roots.

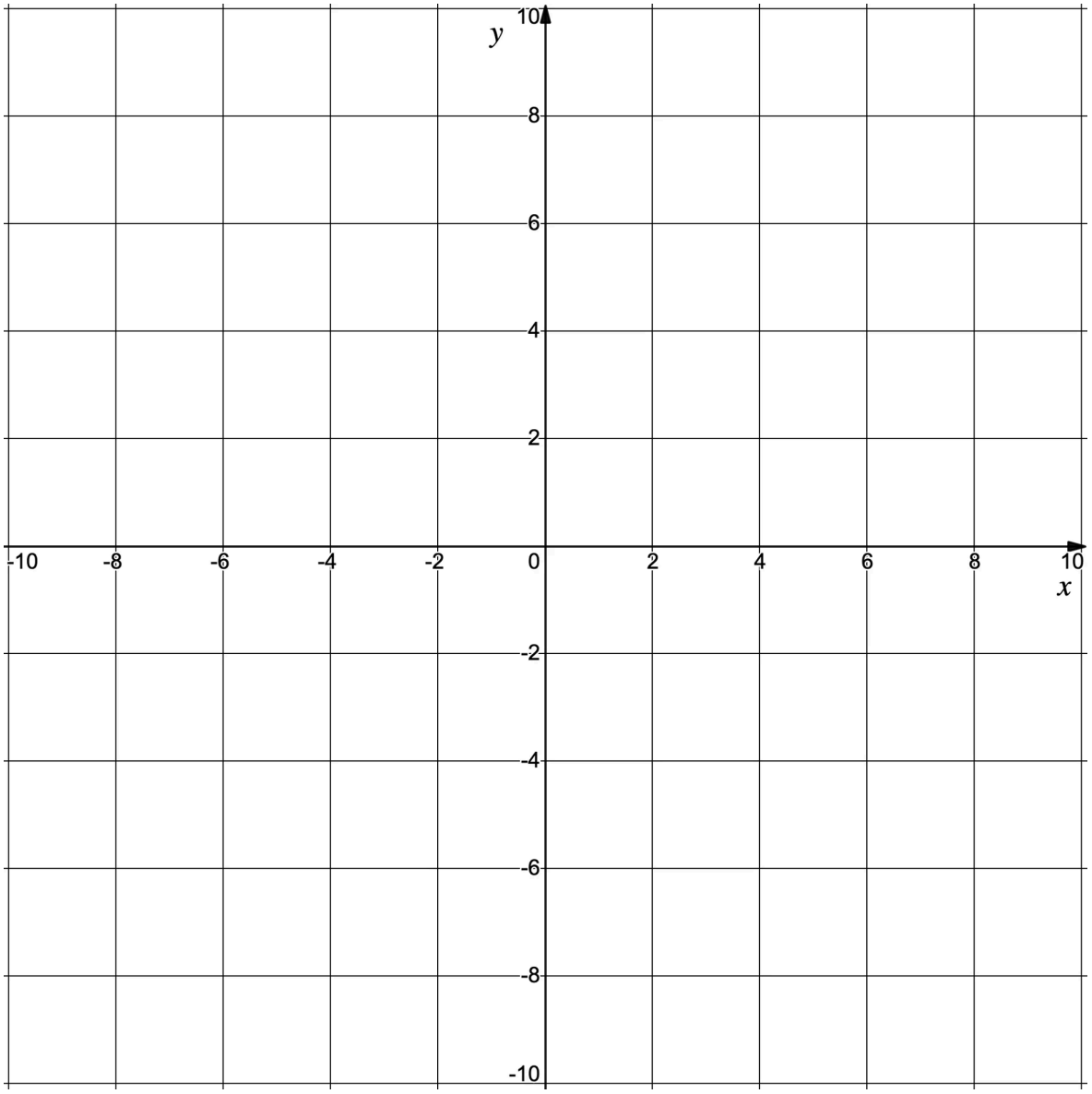
Find the possible values of $k$.

**(3 marks)**

**4 (a)** On the axes below show the region satisfied by the inequalities

$$x^2 - 9 \leq y$$
$$y \leq (2 + x)(2 - x)$$

Label this region R.




**(3 marks)**

**(b)** Write down the equation(s) of any line(s) of symmetry of the region R.

**(1 mark)**

**5** Solve the inequality $-6 \leq x^2 + 3x - 4 \leq 6$, giving your answer in set notation.

**(6 marks)**

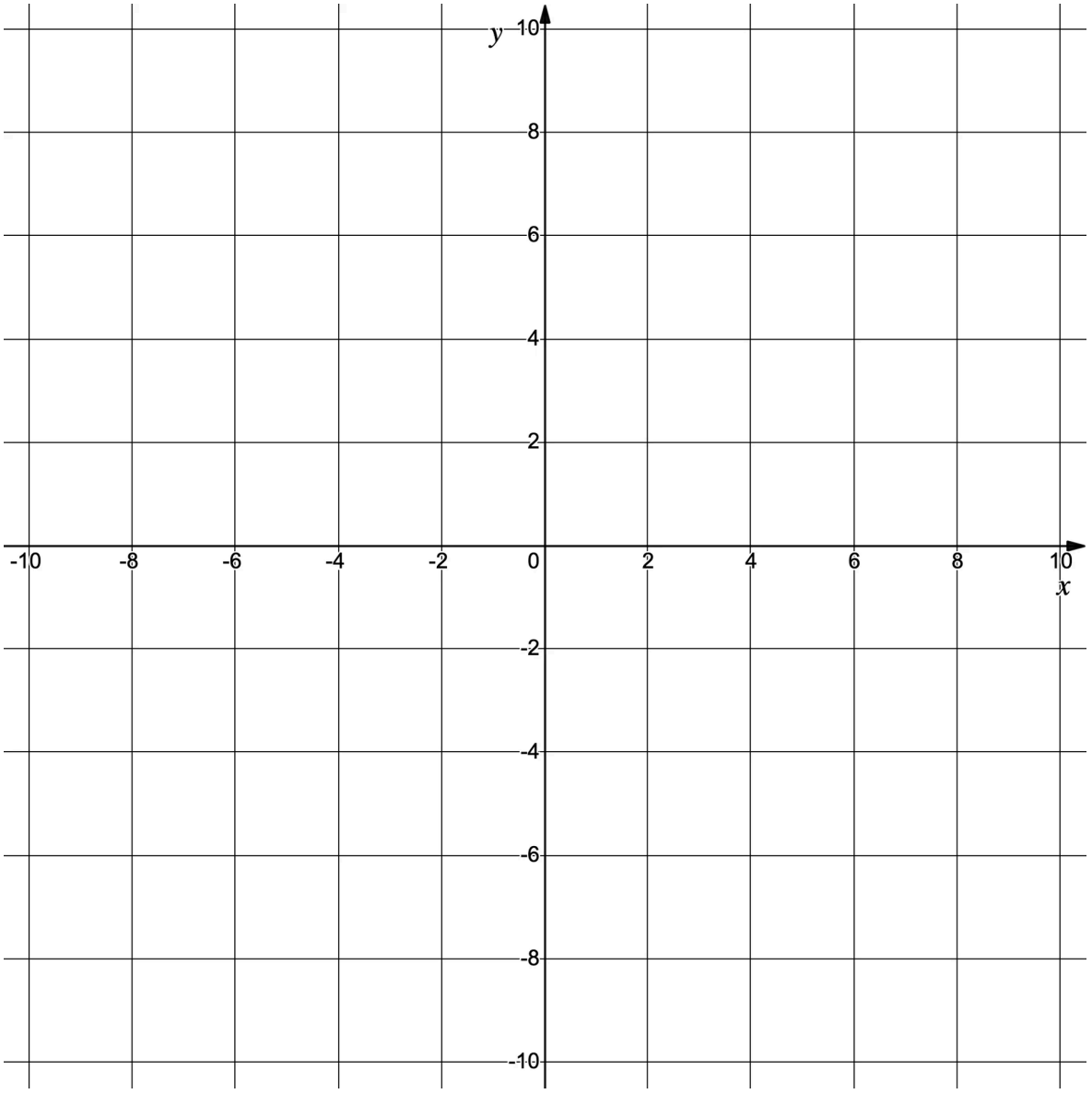
**6** Solve the inequality $2x^2 + 1 \leq x^2 + 10x - 8 < 2x^2 - 7x + 52$, giving your answer in interval notation.

**(5 marks)**

**7 (a)** The cross section of a tunnel is in the shape of the region defined by the inequalities

$$y \leq 6 - \frac{x^2}{6} \qquad\qquad y \geq 0$$

On the axes below show the region satisfying the inequalities




**(2 marks)**

**(b)** Given that $x$ and $y$ are in metres, write down the height and the maximum width of the tunnel.

**(2 marks)**

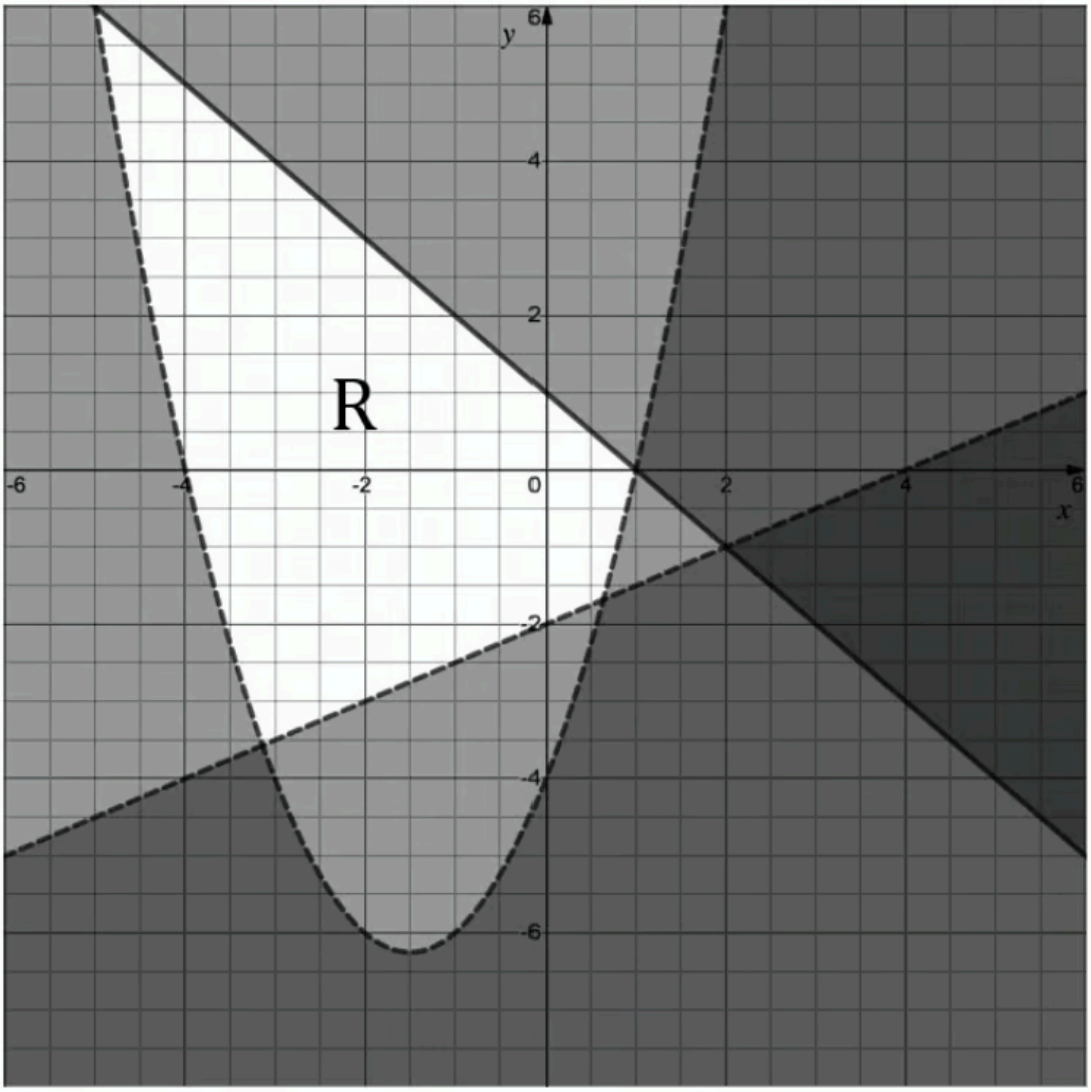
**(c)** Using a semi-circle of radius 6, estimate the area of the cross-section of the tunnel.

**(3 marks)**

**(d)** Given that the tunnel is to be 20 m in length estimate the volume of earth that will need to be removed in order to build the tunnel.

**(2 marks)**

**8** Write down the inequalities that define the region R shown in the diagram below.




**(3 marks)**

**9 (a)** An electronics company can produce $c$ cables at a total cost of $(160 + 12c)$ pence.
The cables can then be sold for $(38 - c)$ pence each.

Find the minimum and maximum number of cables the company needs to sell in order to make a profit?

**(5 marks)**

**(b)** How many cables does the company need to sell to make the maximum profit?

**(1 mark)**

**10** A stone is projected vertically upwards from a height of $2$ m.
It's height, above it's starting position, $d_1$ m, at time $t$ seconds after launch, is given by

$$d_1(t) = 13.2t - 4.9t^2$$

At the same time a second stone is projected upwards from a height of $2.3$ m.
It's height, above its starting position, is given by

$$d_2(t) = 13t - 4.9t^2$$

For how long are both stones simultaneously at least $4$ m above the ground?

**(5 marks)**

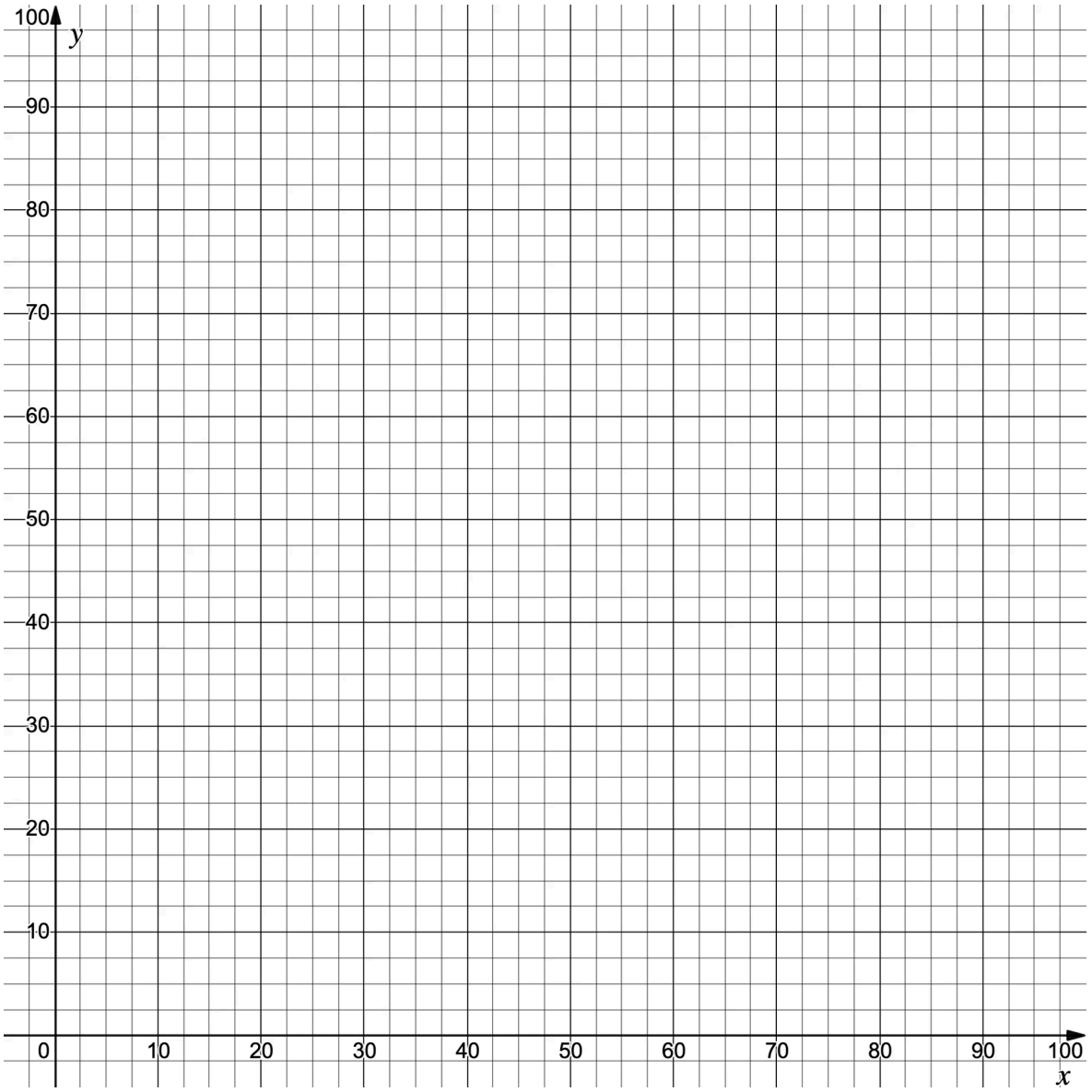
**11 (a)** A company produces $x$ chairs and $y$ tables in a day. They sell every chair and every table they produce. Due to the manufacturing processes involved the number of chairs and tables they can make in a day are limited by the following inequalities:

$$y \leq x + 20 \qquad y \leq -2x + 80$$
$$y \geq 3x - 45 \qquad x \geq 0, y \geq 0$$

Briefly explain why the inequalities $x \geq 0$ and $y \geq 0$ are appropriate.

**(1 mark)**

**(b)** On the axes below show the region within which the company can produce $x$ chairs and $y$ tables per day.




**(4 marks)**

**(c)** The company's profit, £$P$, per day, is given by the formula $P = 3x + 2y$.
Given that the maximum profit lies on a vertex of the region found in part (b), find the number of chairs and tables the company should make in order to maximise its daily profit.

**(3 marks)**