SUVAT equations (Part 2)

The other SUVAT equations can be derived using $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$.

Eliminating t:

$v = u + at$

$\frac{v-u}{a} = t$

$s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$

$s = \frac{v^2-u^2}{2a}$

$2as = v^2 - u^2$

$v^2 = u^2 + 2as$

$v^2 = u^2 + 2as$

Eliminating v:

$$v = u + at$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$s = \left(\frac{2u+at}{2}\right)t$$

$$s = \frac{2ut+at^2}{2}$$

$$s = ut + \frac{1}{2}at^2$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

Note: Because this is quadratic in t, we typically end up with two different possible times.

Eliminating u:

$$v = u + at$$

$$v - at = u$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$s = \left(\frac{v-at+v}{2}\right)t$$

$$s = \left(\frac{2v-at}{2}\right)t$$

$$s = \frac{2vt-at^2}{2}$$

$$s = vt - \frac{1}{2}at^2$$

$$\boxed{s = vt - \frac{1}{2}at^2}$$

A particle is moving along a straight line from A to B with constant acceleration 5 ms^{-2}. The velocity of the particle is 3 ms^{-1} in the direction $\overrightarrow{AB}$. The velocity of the particle at B is 18 ms^{-1} in the same direction. Find the distance from A to B.

- $v = u + at$
- $s = \left(\frac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

$a = 5$
$u = 3$
$v = 18$
$s = ?$

$v^2 = u^2 + 2as$
$18^2 = 3^2 + 2 \times 5 \times s$
$315 = 10s$
$s = 31.5\text{m}$

A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2}. At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1} travelling towards a point A, where $OA = 20$ m. Find:
(a) the times when the particle passes through A
(b) the value of t when the particle returns to O.

↳ origin

$a = -4$
$u = 13$
$s = 20$
$t = ?$

a) $s = ut + \frac{1}{2}at^2$
$20 = 13t - 2t^2$
$0 = -2t^2 + 13t - 20$
$t = 4 \quad t = 2.5$

$s = 13t - 2t^2$

b) $a = -4$
$u = 13$
$s = 0$
$t = ?$

$0 = 13t - 2t^2$
$0 = t(13 - 2t)$
$t = 0 \text{ or } t = 6.5$

Further Example - exam style

7. A car is moving along a straight horizontal road with constant acceleration.

There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m.

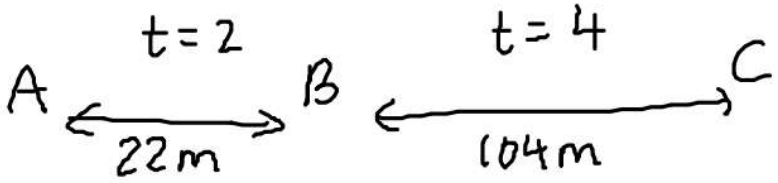
The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)



A to C

$s = 126$

$a = ?$

$t = 6$

$u = u$

A to B

$s = 22$

$a = ?$

$t = 2$

$u = u$

$s = ut + \frac{1}{2}at^2$

A to C $\quad 126 = 6u + \frac{1}{2}a \times 36$

$126 = 6u + 18a$ (x, y)

A to B $\quad 22 = 2u + \frac{1}{2}a \times 4$

$22 = 2u + 2a$ (x, y)

$u = 6\,ms^{-1}$ $a = 5\,ms^{-2}$.

Your Turn

Edexcel M1 May 2013 Q4

A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed u m s^{-1}, ($u < 34$), and 10 seconds later passes a point B with speed 34 m s^{-1}. Given that $AB = 240$ m, find

(a) the value of u,

(3)

(b) the time taken for the lorry to move from A to the mid-point of AB.

(6)

a)

$u = ?$
$s = 240$
$t = 10$
$v = 34$

$s = \left(\frac{u+v}{2}\right)t$

$240 = \left(\frac{u+34}{2}\right)10$

$48 = u + 34$

$u = 14$

b)

$s = 120$
$t = ?$
$u = 14$
$a = 2$

We need a

$v = u + at$
$34 = 14 + 10a$
$20 = 10a$
$a = 2$

$s = ut + \frac{1}{2}at^2$

$120 = 14t + \frac{1}{2} \times 2t^2$

$120 = 14t + t^2$

$0 = t^2 + 14t - 120$

$t = 6, \ t = -20$

$t > 0$

Ex 9D
Evens