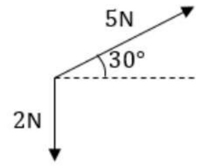
Forces (Year 2)

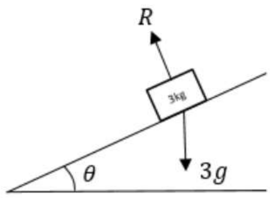
1:: Resolving components

"Determine the magnitude and direction of the resultant force."



2:: Inclined Planes

"A block of mass 3kg is placed on a smooth slope with angle of inclination θ where $\tan\theta = \frac{3}{4}$. Determine the acceleration of the block down the slope."

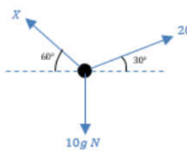


3:: $F \leq \mu R$

Understand that the maximum friction is μR, where μ is the coefficient of friction of the surface, and R is the normal reaction force of the surface on the object. Use to solve inclined plane problems when the surface is rough.

4:: Unknown forces for bodies in equilibrium.

"If the particle is in equilibrium, determine the magnitude of the force X."



5:: Static problem involving weight, tension and pulleys

A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.

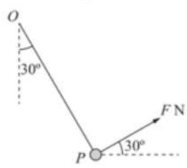


Figure 3

Find

(i) the value of F,

(ii) the tension in the string.

(8)

6:: Objects in motion on inclined planes

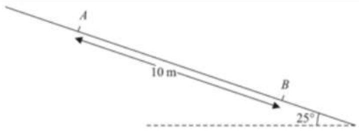


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B, **(3)**

(b) the acceleration of P, **(2)**

(c) the coefficient of friction between P and the plane. **(5)**

7:: Connected particles requiring resolution of forces.

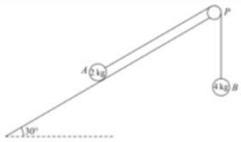


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

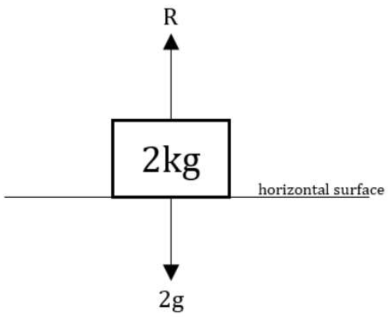
Find the tension in the string immediately after the particles are released.

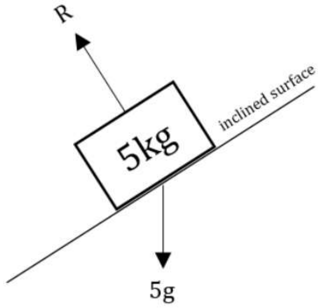
(9)

Mechanics essentials

weight = mass × g (where g is the acceleration due to gravity, $g = 9.8ms^{-2}$
$W = mg$
Weight acts vertically downwards (obviously)

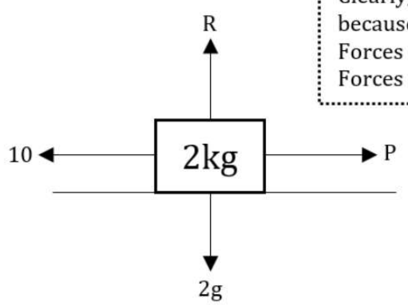
The normal reaction (sometimes called the contact force) is the force which acts on a box/particle from the surface that it is on.
It is called a **normal** reaction because it acts normal (perpendicular) to the surface.
It is called a normal **reaction** because it has reacted to the forces in the opposing direction.
For example, when you are sat on a chair, your weight acts down, but the chair (surface) has a reaction force upwards which stops you falling to the floor. This is the normal reaction.
We use the letter R for the normal reaction.





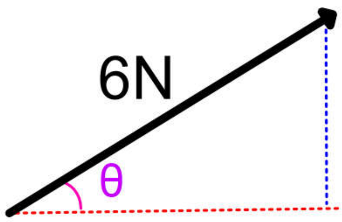
Note that the weight acts vertically downwards, but the normal reaction is perpendicular to the slope

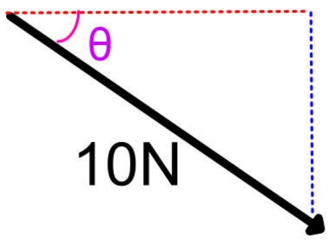
Newton's First Law
"An object will remain at rest or will continue to move with constant velocity unless acted upon by an external force"
Essentially, this means that something will not move, or move with no acceleration if there is no overall resultant force. It means that all the forces are balanced.
We call this **equilibrium** (think of the word 'equal')



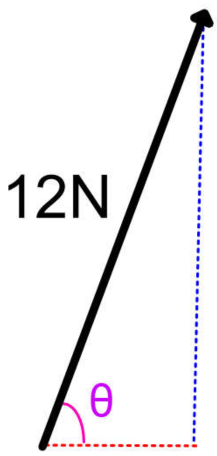
Given that this particle is rest, work out the value of P and R.
Clearly, R = 2g and P = 10, because it is in equilibrium.
Forces left = forces right.
Forces up = forces down

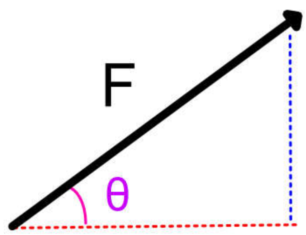
Resolving Forces into x and y components





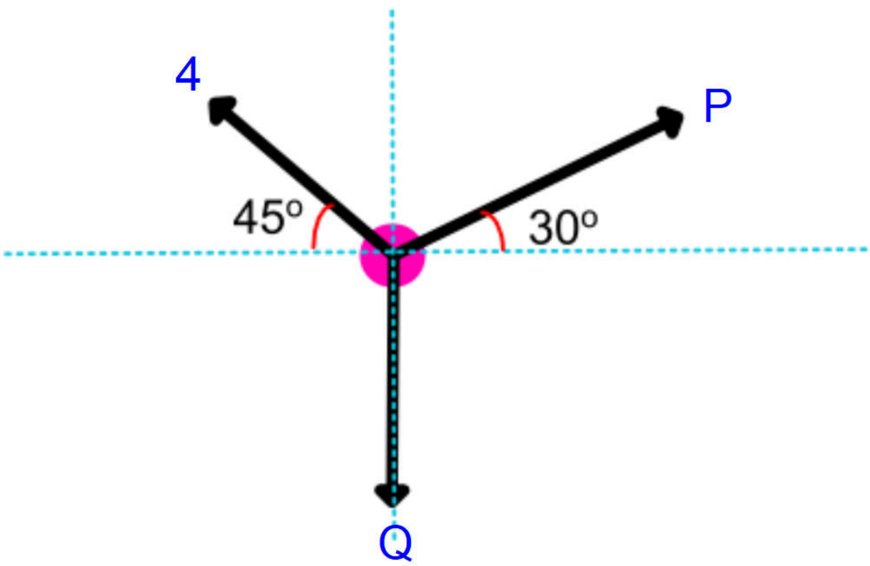
Forces can be considered by resolving them (splitting them) into 2 perpendicular forces.
Perpendicular forces do not interact with each other.
These triangles obey the rules of SOH CAH TOA and Pythagoras





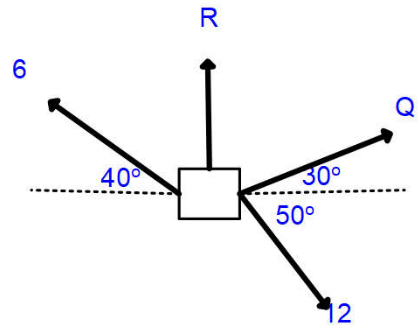
Tip: If F is the magnitude/the hypotenuse, use $F\cos\theta$ for the side adjacent to the angle and $F\sin\theta$ for the side opposite it.

Statics - particles not moving

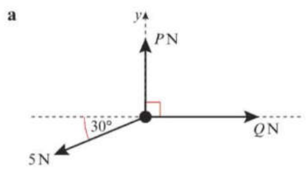


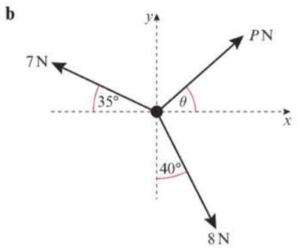
'Static' means there is no movement. This means there is no acceleration, so the particle is in *equilibrium*.
All the forces are balanced in *any direction*.
Forces left = forces right
Forces up = forces down

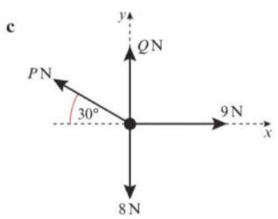
Further Example



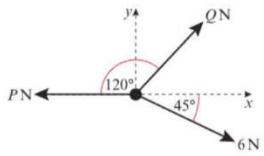
Draw a second diagram



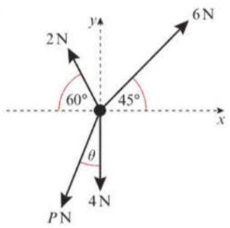




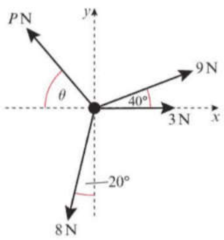
d



e

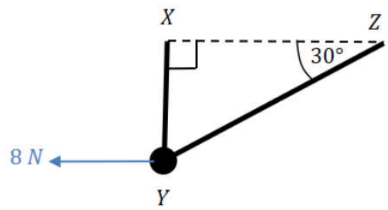


f



a **i** $Q - 5\cos 30° = 0$ **ii** $P - 5\sin 30° = 0$
iii $Q = 4.33\text{N}$ $P = 2.5\text{N}$
b **i** $P\cos\theta + 8\sin 40° - 7\cos 35° = 0$
ii $P\sin\theta + 7\sin 35° - 8\cos 40° = 0$
iii $\theta = 74.4°$ (allow $74.3°$) $P = 2.20\text{N}$ (allow 2.19)
c **i** $9 - P\cos 30° = 0$
ii $Q + P\sin 30° - 8 = 0$
iii $Q = 2.80\text{N}$ $P = 10.4\text{N}$
d **i** $Q\cos 60° + 6\cos 45° - P = 0$
ii $Q\sin 60° - 6\sin 45° = 0$
iii $Q = 4.90\text{N}$ $P = 6.69\text{ N}$
e **i** $6\cos 45° - 2\cos 60° - P\sin\theta = 0$
ii $6\sin 45° + 2\sin 60° - P\cos\theta - 4 = 0$
iii $\theta = 58.7°$ $P = 3.80\text{N}$
f **i** $9\cos 40° + 3 - P\cos\theta - 8\sin 20° = 0$
ii $P\sin\theta + 9\sin 40° - 8\cos 20° = 0$
iii $\theta = 13.6°$ $P = 7.36\text{N}$

A smooth bead Y is threaded on a light inextensible string. The ends of the string are attached to two fixed points, X and Y, on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 8 N acting parallel to ZX. The bead Y is vertically below X and $\angle XZY = 30°$ as shown in the diagram.
Find the tension in the string and the weight of the bead.



As the bead is smooth, the two parts of the string can be considered as a single piece of string, and therefore the tension is the same throughout.

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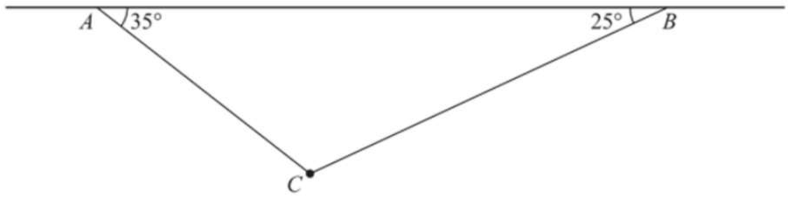


Figure 1

A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

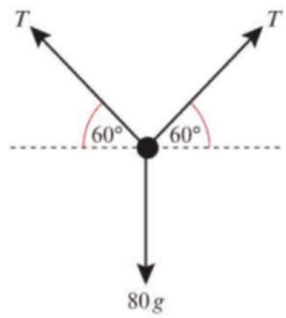
(i) the tension in the string AC,

(ii) the tension in the string BC.

(8)

The particle can't move along the string, so we have two separate strings with separate tensions. Introduce suitable variables for the tensions of each, e.g. T_1 and T_2.

8 A parachutist of mass 80 kg is attached to a canopy by two lines, each with tension T. The parachutist is falling with constant velocity, and experiences a resistance to motion due to air resistance equal to one quarter of her weight. Show that the tension in each line, T, is $20\sqrt{3}\,g$ N.



(3 marks)

<u>Newton's Second Law</u>

"An object will accelerate if there is an overall resultant force on the object. The acceleration is proportional to this force, and inversely proportional to its mass."

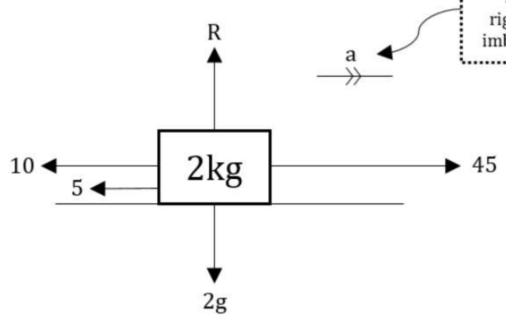
In other words

$$F = ma$$

Where **F** = resultant force, **m** = mass, **a** = acceleration

The resultant force is found by finding the difference between the forces in one direction, and the forces in the opposing direction. This tells you the overall force in one direction.

It is clearly going to accelerate to the right as there is an imbalance of forces



Notice how we find the resultant force by doing the forces to the right minus the forces to the left

Work out the acceleration for the particle.

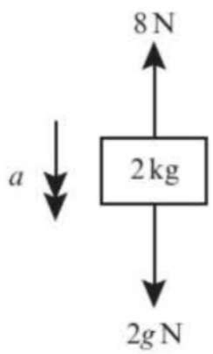
Using F = ma

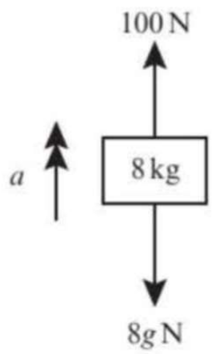
$$45 - 10 - 5 = 2a$$
$$30 = 2a$$
$$15 = a$$

So the particle will accelerate at $15ms^{-2}$ to the right

Dynamics - forces causing motion (Year 1 recap)

In each situation, the forces acting on the body cause it to accelerate as shown in the diagram. Find the value of *a*.





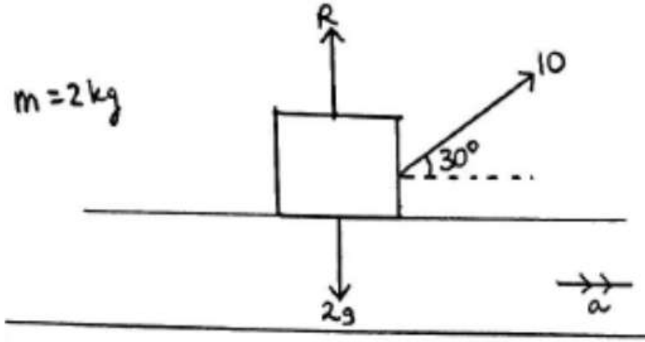
Keywords:

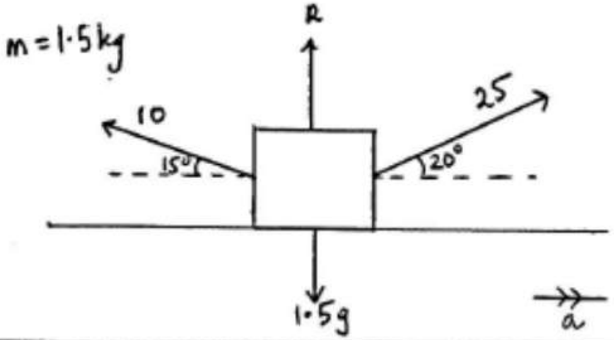
- Resultant force
- Resolve

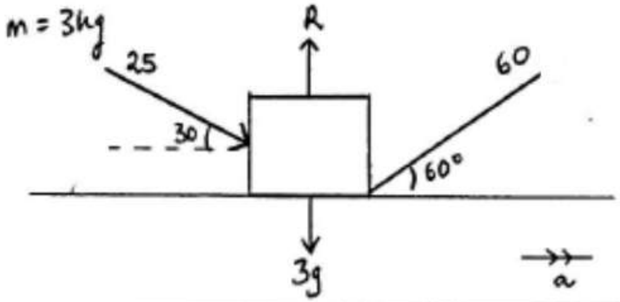
Key ideas:

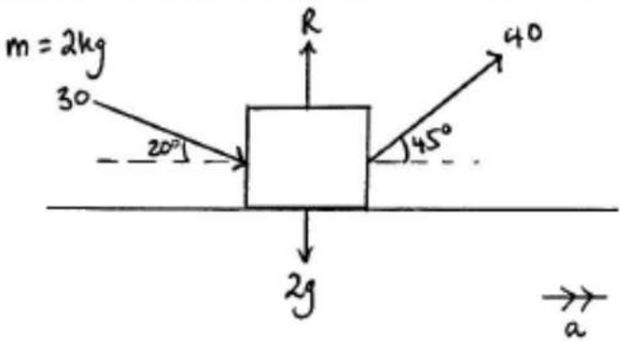
- F = ma
- F is the *resultant force*
- Direction of force corresponds to positive or negative value

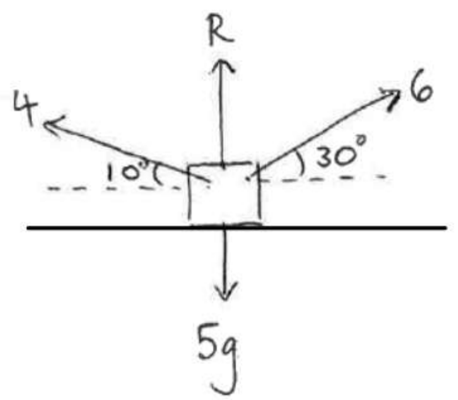
	New Diagram	Find the value of R and a



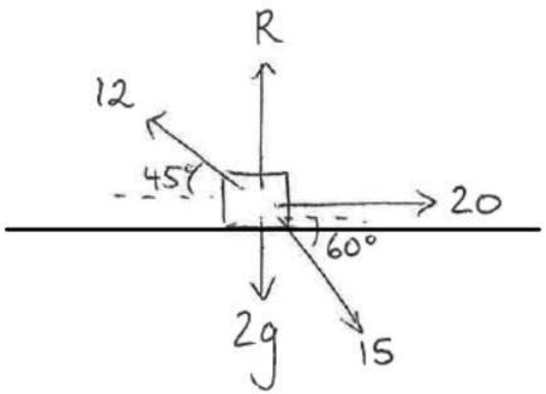




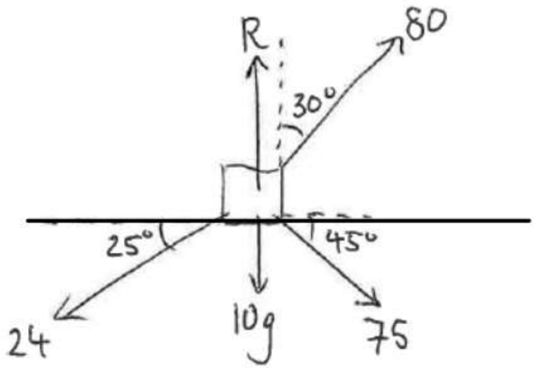




R
4
6
10°
30°
5g



R
12
45°
20
60°
2g
15



R
80
30°
25°
45°
24
10g
75

A box of mass 8kg lies on a smooth horizontal floor. A force of 10N is applied at an angle of 30° causing the box to accelerate horizontally along the floor.

(a) Work out the acceleration of the box.

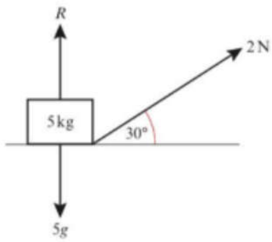
(b) Calculate the normal reaction between the box and the floor.

Your Turn

5 A box of mass 5 kg lies on a smooth horizontal floor. The box is pulled by a force of 2 N applied at an angle of 30° to the horizontal, causing the box to accelerate horizontally along the floor.

a Work out the acceleration of the box.

b Work out the normal reaction of the box with the floor.



E **6** A force P is applied to a box of mass 10 kg causing the box to accelerate at $2\,\mathrm{m\,s^{-2}}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 45° to the plane, work out the value of P. **(3 marks)**

E **7** A force of 20 N is applied to a box of mass m kg causing the box to accelerate at $0.5\,\mathrm{m\,s^{-2}}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 25° to the plane, work out the value of m. **(3 marks)**

5 **a** $\frac{\sqrt{3}}{5}\,\mathrm{m\,s^{-2}}$ **b** 48 N

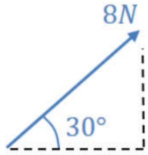
6 $20\sqrt{2}$ N

7 36.3 kg (3 s.f.)

Writing forces in vector form

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively. Also write your answer in column vector form.

1



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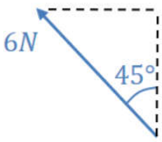

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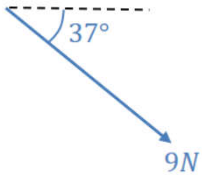
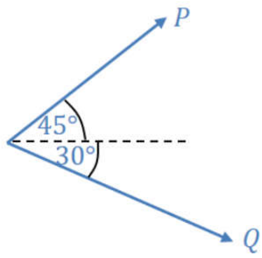


Combining Forces

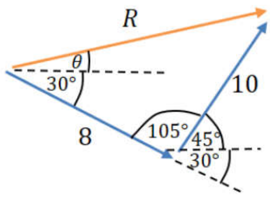
Two forces P and Q act on a particle as shown.
P has a magnitude of 10N and Q has a magnitude of 8N.
Work out the magnitude and direction of the resultant force.



Method 1: Finding total x and y components of force.

Method 2: Using Triangle Law for vector addition.

Yuk



We can avoid resolving components by drawing the force vectors in a chain, then finding the vector from the start to end point. The resultant vector (orange) geometrically represents the same of the vectors.

Use cosine rule to get magnitude of R:

$R^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos(105°)$

$R = 14.3\ N$

Use sine rule to get θ:

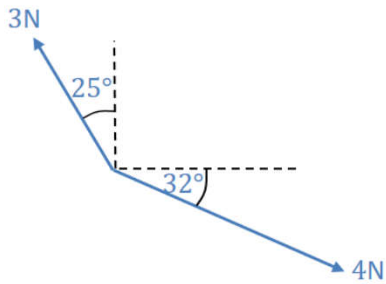
$$\frac{\sin(\theta + 30°)}{10} = \frac{\sin(105°)}{14.332}$$

$$\sin(\theta + 30°) = \frac{10\sin(105°)}{14.332} \quad \ldots$$

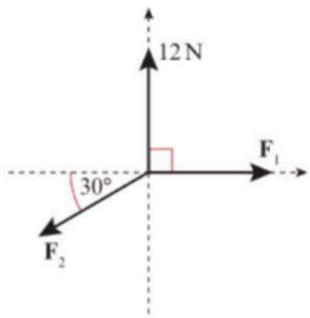
$\theta = 12.4°$

Your Turn

A particle has forces acting on it as indicated in the diagram. Determine the magnitude and direction (anticlockwise from the positive x direction) of the resultant force.



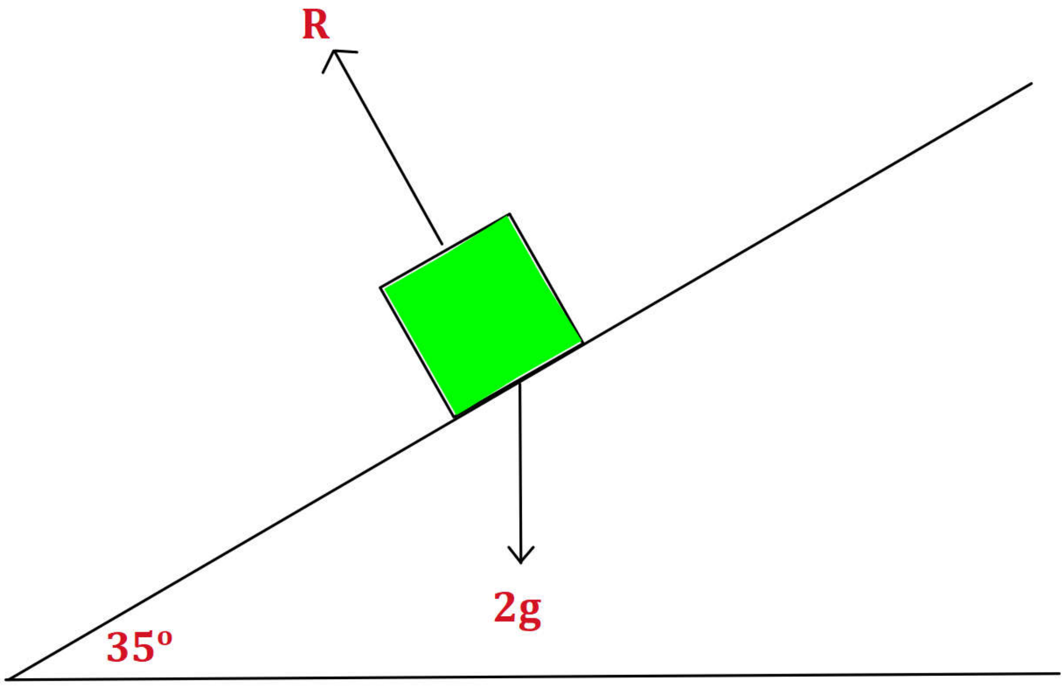
9 A system of forces act upon a particle as shown in the diagram.
The resultant force on the particle is $(2\sqrt{3}\mathbf{i} + 2\mathbf{j})$ N.
Calculate the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.



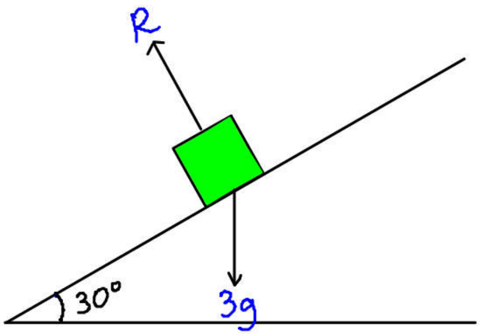
(3 marks)

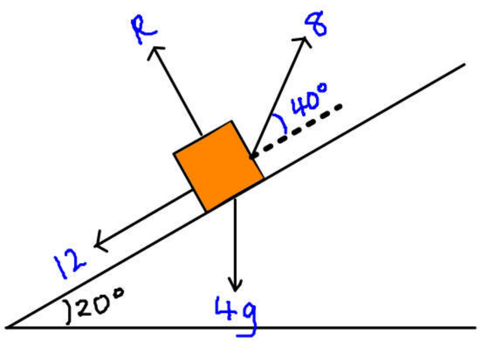
9 $\mathbf{F}_1 = 12\sqrt{3}$N, $\mathbf{F}_2 = 20$N

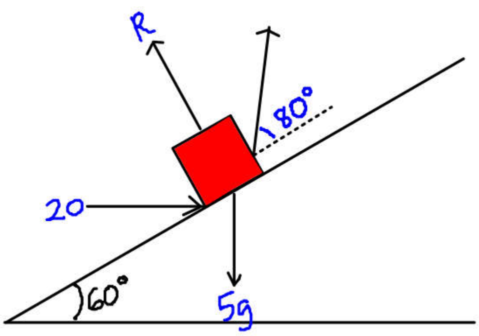
Inclined Planes

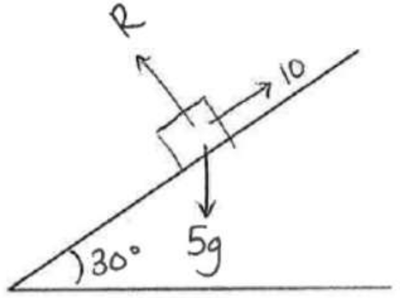


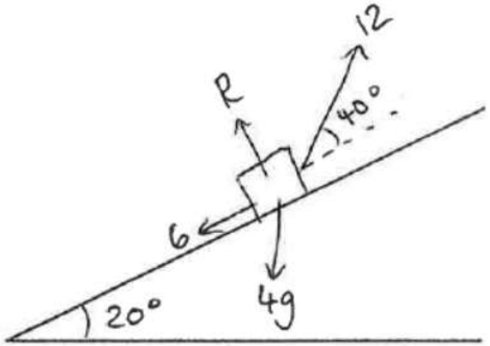
For problems involving inclined planes, resolve forces parallel and perpendicular to the plane.

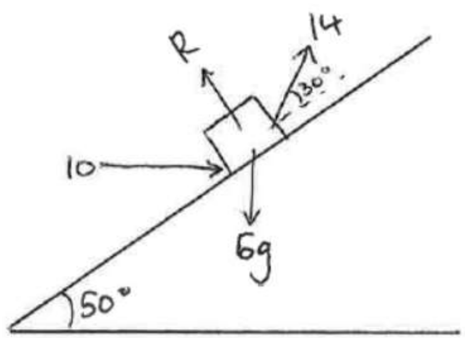




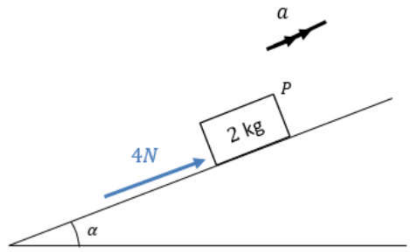



R
10
5g
30°

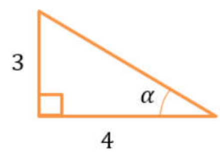

12
R
40°
6
4g
20°


14
R
30°
10
6g
50°

A particle P of mass 2kg is moving on a smooth slope and is being acted on by a force of 4N that acts parallel to the slope, as shown.
The slope is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. Work out the acceleration of the particle.



Hint: Don't find α explicitly. We can find $\cos \alpha$ and $\sin \alpha$ by forming a suitable triangle such that $\tan \alpha$ would be $\frac{3}{4}$:



$\cos \alpha =$

$\sin \alpha =$

A particle of mass m is pushed up a smooth slope, inclined at $30°$ by a force of magnitude $5g$ N acting at angle of $60°$ to the slope, causing the particle to accelerate up the slope at 0.5 ms^{-2}.
Show that the mass of the particle is $\left(\frac{5g}{1+g}\right)$ kg

Exercise 5B

1 A particle of mass 3 kg slides down a smooth slope that is inclined at 20° to the horizontal.

a Draw a force diagram to represent all the forces acting on the particle.

b Work out the normal reaction between the particle and the plane.

c Find the acceleration of the particle.

2 A force of 50 N is pulling a particle of mass 5 kg up a smooth plane that is inclined at 30° to the horizontal. Given that the force acts parallel to the plane,

a draw a force diagram to represent all the forces acting on the particle

b work out the normal reaction between the particle and the plane

c find the acceleration of the particle.

3 A particle of mass 0.5 kg is held at rest on a smooth slope that is inclined at an angle α to the horizontal. The particle is released. Given that $\tan\alpha = \frac{3}{4}$, calculate:

a the normal reaction between the particle and the plane

b the acceleration of the particle.

(E) 4 A force of 30 N is pulling a particle of mass 6 kg up a rough slope that is inclined at 15° to the horizontal. The force acts in the direction of motion of the particle and the particle experiences a constant resistance due to friction.

a Draw a force diagram to represent all the forces acting on the particle. **(4 marks)**

Given that the particle is moving with constant speed,

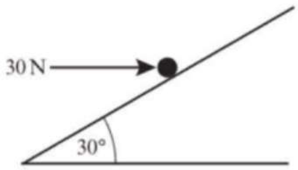
b calculate the magnitude of the resistance due to friction. **(5 marks)**

(E) 5 A particle of mass m kg is sliding down a smooth slope that is angled at 30° to the horizontal. The normal reaction between the plane and the particle is 5 N.

a Calculate the mass m of the particle. **(3 marks)**

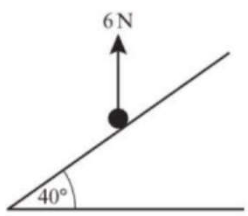
b Calculate the acceleration of the particle. **(3 marks)**

(E/P) 6 A force of 30 N acts horizontally on a particle of mass 5 kg that rests on a smooth slope that is inclined at 30° to the horizontal as shown in the diagram.
Find the acceleration of the particle.



(4 marks)

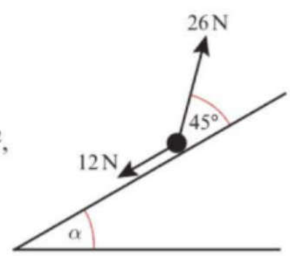
(E/P) 7 A particle of mass 3 kg is moving on a rough slope that is inclined at 40° to the horizontal. A force of 6 N acts vertically upon the particle. Given that the particle is moving at a constant velocity, calculate the value of F, the constant resistance due to friction.



(4 marks)

(E/P) 8 A particle of mass m kg is pulled up a rough slope by a force of 26 N that acts at an angle of 45° to the slope. The particle experiences a constant frictional force of magnitude 12 N.

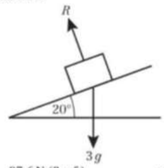
Given that $\tan\alpha = \frac{1}{\sqrt{3}}$ and that the acceleration of the particle is $1\,\text{m s}^{-2}$, show that $m = 1.08$ kg (3 s.f.).



(5 marks)

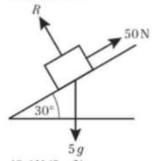
Exercise 5B

1 a



b 27.6 N (3 s.f.) c 3.35 ms⁻¹

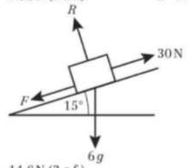
2 a



b 42.4 N (3 s.f.) c $5.1\,\text{ms}^{-2}$

3 a 3.92 N (3 s.f.) b $5.88\,\text{ms}^{-2}$ (3 s.f.)

4 a



b 14.8 N (3 s.f.)

5 a 0.589 kg (3 s.f.) b $4.9\,\text{ms}^{-2}$

6 $0.296\,\text{ms}^{-2}$ (3 s.f.)

7 15.0 N (3 s.f.)

8 R(↗): $26\cos 45 - mg\sin\alpha - 12 = m \times 1$

$13\sqrt{2} - 12 = m + \frac{1}{2}mg$

$m = \frac{13\sqrt{2} - 12}{1 + \frac{g}{2}}$

$m = 1.08$ kg (3 s.f.)