Chapter 5: Probability

1 :: Basic Probability

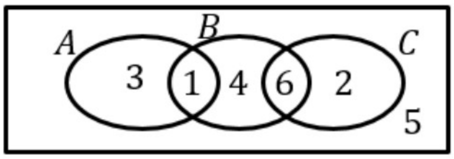
"I throw two fair die. Calculate the probability the sum of the two dice is more than 6."

2 :: Venn Diagrams

"Out of 50 students, 12 play both piano and drums, 30 play piano and 25 play drums. Find the probability a randomly chosen student plays neither instrument."

3 :: Mutually Exclusive/Independent Events

Determine whether A and B are independent.



4 :: Tree Diagrams

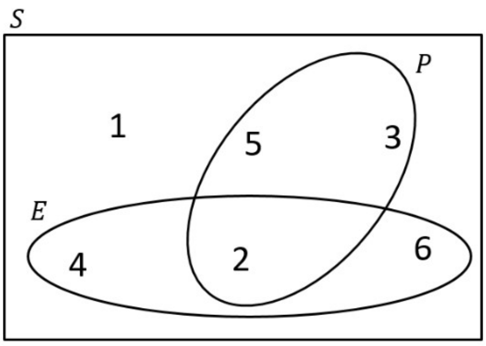
"The probability I hit a target is 0.3. If I hit it, the probability I hit again on the next shot is 0.4. If I miss, the probability I hit on the next shot is 0.1. If I shoot 3 times, what's the probability I hit on the first and third shot?"

Probability concepts

✎ An **experiment** is a repeatable process that gives rise a number a number of **outcomes**.

✎ An **event** is a set of <u>one or more</u> of these outcomes.

(We often use capital letters to represent them)



$E =$ "rolling an even number"
$P =$ "rolling a prime number"

✎ A **sample space** is the set of all possible outcomes.

Because we are dealing with sets, we can use a **Venn diagram**, where

- the numbers are the individual outcomes,
- the sample space is a rectangle and
- the events are sets, each a subset of the sample space.

(You do not need to use set notation like ∩ and ∪ until Year 2!)

Two fair spinners each have four sectors numbered 1 to 4. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded.
Find the probability of the spinners indicating a sum of
(a) exactly 5 (b) more than 5

Spinner 1

Spinner 2	+	1	2	3	4
	1				
	2				
	3				
	4				

If the sample space is the amalgamation of two underlying experiments, a table is a helpful way to list the outcomes.

$$P(5) =$$

$$P(>5) =$$

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 7$	$7 \leq t < 9$	$9 \leq t < 11$	$11 \leq t < 13$	$13 \leq t < 15$
Frequency	6	13	12	5	4

A student is chosen at random. Find the probability for a group of students to complete a number puzzle
(a) In under 9 minutes (b) in over 10.5 minutes.

$$P(\leq 9) =$$

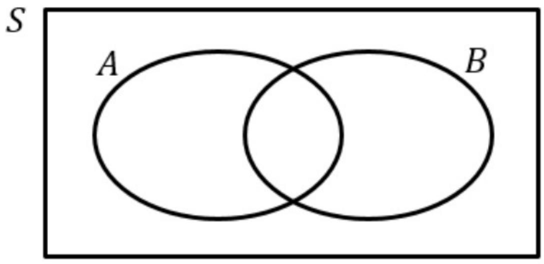
$$P(\geq 10.5) =$$

This is an estimate because we're assuming the people are equally distributed across the interval.

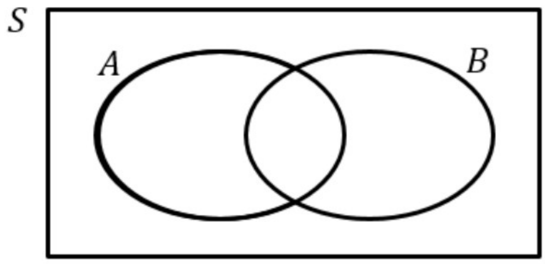
Ex 5A

Venn Diagrams

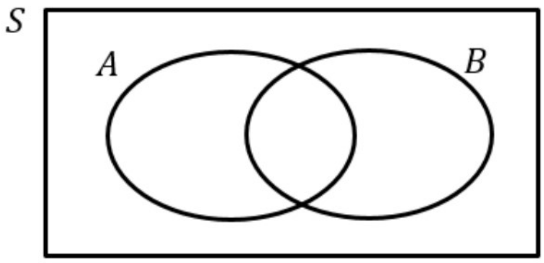
Venn Diagrams allow us to combine events, e.g. "A happened **and** B happened".



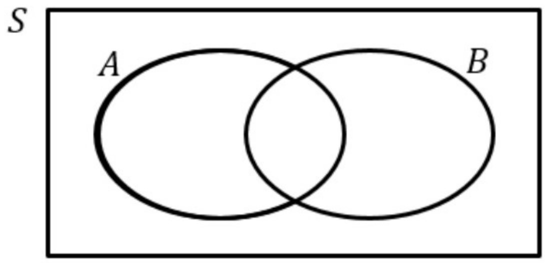
The event "A **and** B"
Known as the **intersection** of A and B.



The event "A **or** B"
Known as the **union** of A and B.



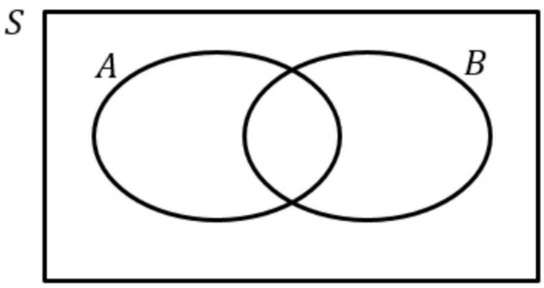
The event "not A"
Known as the **complement** of A.



These can be combined,
e.g. "A and not B".

We can either put frequencies or probabilities into the Venn Diagram.

Given that $P(A) = 0.6$ and $P(A \text{ or } B) = 0.85$, find the probability of:
a) $P(\text{not } A \text{ and } B)$
b) $P(\text{neither } A \text{ nor } B)$



In a class of 30 students, 7 are in the choir, 5 are in the school band, and 2 are in the choir and the school band. A student is chosen at random from the class.

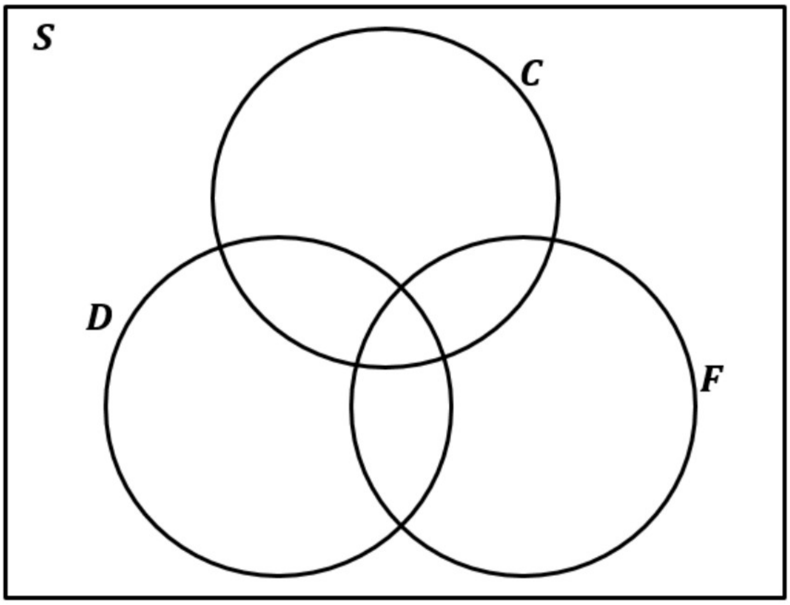
a) Draw a Venn diagram to represent this information
b) Find the probability that the student is not in the band
c) Find the probability that the student is not in the choir nor in the band

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Fill in this Venn Diagram, and hence answer the following questions:

a) $P(owns\ dog\ only)$
b) $P(does\ not\ own\ tropical\ fish)$
c) $P(does\ not\ own\ dogs,\ cats, or\ tropical\ fish)$

Tip: Start from the centre frequency and work your way outwards using subtraction.



Your Turn

Tip: You'll lose a mark if you don't have an outside box!

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The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run	48 swim
60 cycle	40 run and swim
30 swim and cycle	35 run and cycle
25 do all three	

(*a*) Draw a Venn Diagram to represent these data. **(4)**

Find the probability that a randomly selected person from the survey

(*b*) takes none of these types of exercise, **(2)**
(*c*) swims but does not run, **(2)**
(d) takes at least two of these types of exercise. **(2)**

Ex 5B

Mutually Exclusive Events

- If two events are mutually exclusive **they can't happen at the same time**.
- The Venn Diagram would look like:

- If A and B are mutually exclusive then:
  - $\boldsymbol{P(A\ and\ B)} =$
  - $\boldsymbol{P(A\ or\ B)} =$

e.g. the events picking a heart from a standard deck of cards and picking a diamond from a standard deck of cards are mutually exclusive

e.g. if no one in our maths class studies drama, then the events 'studying maths' and 'studying drama' are mutually exclusive

Independent Events

- If two events are independent
  then whether one event happens does not affect the probability of the other happening.
- If A and B are independent then:
  - $P(A \text{ and } B) = P(A) \times P(B)$

Note: Independence does not affect how the circles interact in a Venn Diagram.

Are events A and B ***Independent*** or ***Dependent***?

A = Being in a car accident
B = Riding in a car to work

A = Winning a scratch card
B = Winning the lottery

A = Running out of petrol on a journey
B = Forgetting your umbrella on a rainy day

A = Parking your car without checking the restrictions
B = Receiving a parking ticket

A = Going skiing in France
B = Breaking a bone in your arm

A = Having a maths degree
B = Getting a job in the Civil Service

A = 'tr' recorded for rainfall in Cambourne
B = Mean temperature above 20°C in Perth

A = Daily total rainfall above 6mm in Leeming in 1987
B = Daily mean total cloud cover above 6 oktas in Leeming in 1987

Example

1 2 3 4

1 I pick one of the four numbers 1, 2, 3, 4 at random. What's the probability that:

a) I pick a multiple of 2: $\frac{1}{2}$

b) I pick a multiple of 4: $\frac{1}{4}$

2 Explain (conceptually) why these two events are not independent.

3 Show that the events are not independent.

This is a VERY common exam question.
Either show that $P(A \text{ and } B) = P(A) \times P(B)$
or that $P(A \text{ and } B) \neq P(A) \times P(B)$

Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$.

a) Find $P(A\ or\ B)$

b) Find $P(A\ but\ not\ B)$

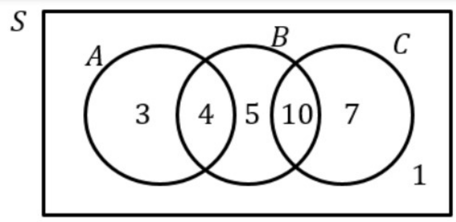
c) Find $P(neither\ A\ nor\ B)$

Events A and B are independent and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$.
Find $P(A\ and\ B)$.

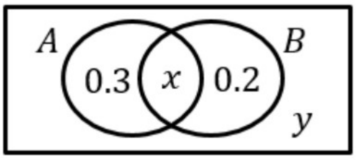
The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes.

a) Find the probability that a student chosen at random watches B or C or both.

b) Determine whether watching A and watching B are statistically independent.



The Venn diagram shows the probability of each event. Given that A and B are independent, determine the possible values of x.



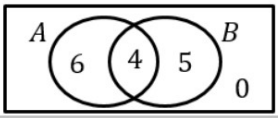
Your Turn

There are three events A, B, C. The events A and B are mutually exclusive.

a) Draw a Venn diagram which represents this information.

b) If $P(A) = 0.1$ and $P(B) = 0.6$, determine $P(neither\ A\ nor\ B)$

The Venn diagram shows the number of people who like each of two different colours. Determine if A and B are independent.



Tree Diagrams

At GCSE we saw that tree diagrams were an effective way of showing the outcome of two events which happen **in succession**.

There are 3 yellow and 2 green counters in a bag. I take two counters at random. Determine the probability that:

a) They are of the same colour.
b) They are of different colours.

Repeated Events

The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5^{th} shot.

The probability I am on time to school on any day is 0.95. Determine the probability that I am late on Monday and Wednesday, but not on Tuesday, Thursday and Friday.

Ex 5D

Exam Questions

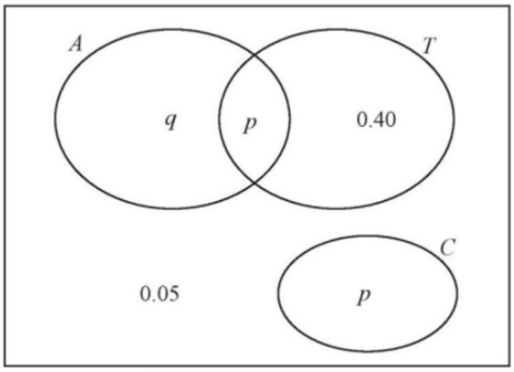
3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.

T represents the event that a student takes part in Tennis.

C represents the event that a student takes part in Cricket.

p and q are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of p.

(1)

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.

(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

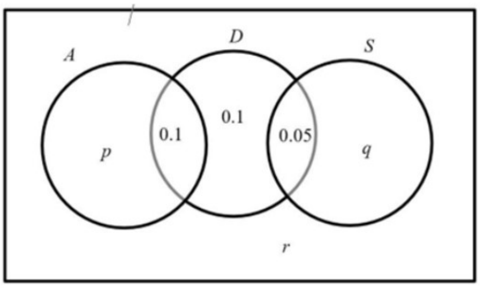
(1)

4. Alyona, Dawn and Sergei are sometimes late for school.
The events A, D and S are as follows

A Alyona is late for school

D Dawn is late for school

S Sergei is late for school



The Venn diagram below shows the three events A, D and S and the probabilities associated with each region of D. The constants p, q and r each represent probabilities associated with the three separate regions outside D.

(a) Write down 2 of the events A, D and S that are mutually exclusive. Give a reason for your answer.
(1)

The probability that Sergei is late for school is 0.2
The events A and D are independent.

(b) Find the value of r
(4)

Dawn and Sergei's teacher believes that when Sergei is late for school, Dawn tends to be late for school.

(c) State whether or not D and S are independent, giving a reason for your answer.
(1)

(d) Comment on the teacher's belief in the light of your answer to part (c).
(1)

2. A factory buys 10% of its components from supplier A, 30% from supplier B and the rest from supplier C. It is known that 6% of the components it buys are faulty.

Of the components bought from supplier A, 9% are faulty and of the components bought from supplier B, 3% are faulty.

(a) Find the percentage of components bought from supplier C that are faulty.

(3)

A component is selected at random.

(b) Explain why the event "the component was bought from supplier B" is not statistically independent from the event "the component is faulty".

(1)