Algebraic Fractions and Partial Fractions

[Edexcel C3 June 2013(R) Q1]

Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

Express the following as a single fraction, giving your answer in its simplest form.

$$\frac{10x+4}{3x^2+4x+1} - \frac{3}{x+1}$$

Extra practice, Ex 1B and 1C

Partial Fractions

If the **denominator is a product of a linear terms**, it can be split into the sum of 'partial fractions', where **each denominator is a single linear term**.

$$\frac{6x-2}{(x-3)(x+1)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$$

Notation reminder: \equiv means 'equivalent/identical to', and indicates that both sides are equal for **all** values of x.

Method 1: Substitution

Method 2: Comparing Coefficients

Given that $\frac{6x^2+5x-2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$, find the values of the constants A, B, C.

Your Turn

C4 June 2005 Q3a

Express
$$\frac{5x+3}{(2x-3)(x+2)}$$
 in partial fractions.

(3)

Partial Fractions - repeated linear factors

Suppose we wished to express $\frac{\frac{a}{2x+1}}{(x+1)^2}$ as $\frac{A}{x+1} + \frac{B}{x+1}$. What's the problem?

Q Split $\frac{11x^2+14x+5}{(x+1)^2(2x+1)}$ into partial fractions.

The problem is resolved by having the factor both squared and non-squared.

Your Turn

C4 June 2011 Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants A, B and C.

Partial Fractions - dealing with improper fractions

In Pure Year 1, we saw that the 'degree' of a polynomial is the highest power, e.g. a quadratic has degree 2.

An algebraic fraction is **improper** if the degree of the numerator is **at least** the degree of the denominator.

$$\frac{x^2-3}{x+2}$$

$$\frac{x+1}{x-1}$$

$$\frac{x^3 - x^2 + 3}{x^2 - x}$$

A partial fraction is still improper if the degree is the same top and bottom.

Questions might take one of two forms:

- Do the division to express as a quotient and a remainder, e.g. $\frac{x+1}{x-1} \rightarrow 1 + \frac{2}{x-1}$
- Express as partial fractions, e.g. $\frac{x^2+x}{(x+1)(x-2)} = A + \frac{B}{x+1} + \frac{C}{x-2}$

You know for example that as $7 \div 3 = 2 \ rem \ 1$, we could write:

$$\frac{7}{3} = 2 + \frac{1}{3}$$

Similarly in general:

$$\frac{F(x)}{divisor} = Q(x) + \frac{remainder}{divisor}$$

If $\frac{x^2+5x-9}{x+2} = Ax + B + \frac{C}{x+2}$, determine the values of A, B and C.

Your Turn

Edexcel C4 June 2013 Q1

Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

Tip: There's a missing x term in the numerator and missing x term in the denominator. Use +0x to avoid gaps.

find the values of the constants a, b, c, d and e.

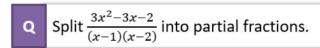
(4)

Ex 1F

Dealing with Improper Fractions

Q Split $\frac{3x^2-3x-2}{(x-1)(x-2)}$ into partial fractions.

Method 1: Algebraic Division



Method 2: Using one identity

Your Turn

C4 Jan 2013 Q3

Express
$$\frac{9x^2 + 20x - 10}{(x+2)(3x-1)}$$
 in partial fractions.

(4)