A Level · Edexcel · Maths

3 hours 43 questions

# 6.1 Exponential & Logarithms

6.1.1 Exponential Functions / 6.1.2 Logarithmic Functions / 6.1.3 "e" / 6.1.4 Derivatives of Exponential Functions

| | |
|---|---|
| Easy (12 questions) | /33 |
| Medium (10 questions) | /42 |
| Hard (10 questions) | /40 |
| Very Hard (11 questions) | /50 |
| **Total Marks** | **/165** |

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# Easy Questions

**1** Write down the value of:

(i) $3^3$

(ii) $4^{-2}$

(iii) $9^{0.5}$

**(3 marks)**

**2** Sketch the graph with equation $y = a^x$ , $a > 1$, stating the coordinates of the point where the graph intersects the $y$-axis and the equation of any asymptotes.

Also state whether this equation would represent exponential growth or decay.

**(3 marks)**

**3** The following equations can be used for exponential models.

State whether each one would represent exponential growth or exponential decay.

(i) $y = 3^{-2x}$

(ii) $y = 20(2)^x$

(iii) $y = 30a^{-x}$ where $a > 1$

**(3 marks)**

**4** Write down the value of  in the following statements:

(i) $3^{a} = 27$

(ii) $a^{\frac{1}{3}} = 5$

(iii) $4a^{2} = 64$

**(3 marks)**

**5** Write down the value of  in the following statements:

(i) $\log_{3} a = 4$

(ii) $\log_{a} 216 = 3$

(iii) $\log_{2} 128 = a$

**(3 marks)**

**6** Solve the equation

$$2^{x} = 16$$

**(1 mark)**

**7 (a)** Solve the equation $x^2 - 12x + 27 = 0$.

**(2 marks)**

**(b)** Hence, or otherwise, solve the equation $(3^x)^2 - 12(3^x) + 27 = 0$.

**(3 marks)**

**8** Solve the equation

$$2\log_3 9 = 5x - 6$$

**(2 marks)**

**9** Sketch the graph of $y = e^x$, clearly showing the coordinates of the point where the graph intercepts the $y$-axis and stating the equations of any asymptotes.

**(3 marks)**

**10** Given $y = e^{2x}$:

(i) Write down an expression for $\frac{dy}{dx}$.

(ii) Find the gradient of $y = e^{2x}$ at the point where $x = 0$.

**(2 marks)**

**11** Use a calculator to find the value of

(i) $5 \log_3 7$

(ii) $2 \log_2 3 + 3 \log_3 2$

giving your answers to four significant figures.

**(2 marks)**

**12** Solve the equation $e^{2x} - 16 = 0$, giving your answer in the form $a \ln a$ where $a$ is an integer.

**(3 marks)**

# Medium Questions

**1** On the same axes, sketch the graphs of $y = 2^x$ and $y = 3^x$, labelling any points where the graphs cross the coordinate axes and writing down the equation of any asymptotes.

**(4 marks)**

**2 (a)** Sketch the graph of $y = 2^{-x}$, stating whether this graph indicates exponential growth or exponential decay.

**(3 marks)**

**(b)** Find the exact value of $y$ when $x = 3$.

**(1 mark)**

**3** Write down the value of $a$ in the following statements.

(i) $\log_a 8 = 3$

(ii) $\log a = 2$

(iii) $\ln e^3 = a$

(iv) $\log_5 a = 1$

**(4 marks)**

**4 (a)** Using a calculator, find to 3 significant figures

(i) $\log_2 5 + \log_5 2$

(ii) $\log 25 - \ln 2$

(iii) $\log 200 + \log_5 50 - \log 20 + \ln 10$

**(3 marks)**

**(b)** Solve $3 \log_2 4 + 3x = 5 \log_6 216$.

**(2 marks)**

**5** Solve $2^{2x} - 24(2^x) + 128 = 0$.

**(3 marks)**

**6 (a)** On the same axes, sketch the graphs of $y = e^x$ and $y = e^{-x}$.
Label any points where the graphs intersect the coordinate axes.
Write down the equations of any asymptotes.

**(4 marks)**

**(b)** Write down the gradient of $y = e^x$ at the point $(0,1)$.

**(1 mark)**

**7 (a)** Given $y = e^{4x}$ write down an expression for $\frac{dy}{dx}$.

**(1 mark)**

**(b)** Given $y = 2e^{2x}$ write down an expression for $\frac{dy}{dx}$.

**(1 mark)**

**(c)** Find the gradient $y = 3e^{-2x}$ of at the point where $x = 3$.
Give your answer in the form $pe^q$, where $p$ and $q$ are integers to be found.

**(3 marks)**

**8 (a)** (i) Write down the gradient function of $y = \mathrm{e}^{-3x}$

(ii) Find the gradient of $y = \mathrm{e}^{-3x}$ at the point where $x = 0$.

**(2 marks)**

**(b)** (i) In terms of $e$, write down the gradient of $y = e^{-3x}$ at the point where $x = 2$.

(ii) Find the value for $x$ for which the gradient of $y = e^{-3x}$ is $-3e^{-12}$.

**(2 marks)**

**9 (a)** The function $f(x)$ is defined by $f(x) = 2e^{3x}$ for $x \in \mathbb{R}$

(i) Find $f(-x)$.

(ii) On the same axes, sketch the graphs of $y = f(x)$ and $y = f(-x)$.
Label any points where the graphs intersect the coordinate axes.

**(3 marks)**

**(b)** Describe the transformation from $y = f(x)$ to $y = -f(x)$.

**(2 marks)**

**10** Solve $e^{2x} - 8e^{x} + 15 = 0$, giving your answers to 3 significant figures.

**(3 marks)**

# Hard Questions

**1 (a)** On the same axes, sketch the graphs of $y = 4^x$ and $y = 5^x$.

Label any points of intersection with the coordinate axes.
Write down the equations of any asymptotes.

**(4 marks)**

**(b)** Write down an equation for the graph that is a reflection of $y = 4^x$ in the $y$-axis.

**(1 mark)**

**2 (a)** (i) Sketch the graph of $y = 0.4^x$.

(ii) State whether this graph indicates exponential growth or exponential decay.

**(3 marks)**

**(b)** Find the value of $x$ when $y = 0.064$.

**(1 mark)**

**3 (a)** Find the value of $\log 1000 + \log 10000$

**(1 mark)**

**(b)** Write down the value of $a$ in the statement $6^{\log_6 a} = 36$.

**(1 mark)**

**(c)** Evaluate $\dfrac{2\log_4 64 + 3^{\log_2 8} - \log_5 5}{\log 100}$.

**(2 marks)**

**4 (a)** Solve $2 \log 1000 = x \log_{16} 4$.

**(2 marks)**

**(b)** Solve $3 \log_4 x = \log_4 x + 3 \log_5 25$.

**(2 marks)**

**5** Solve $2(2^{2x}) + 4 = 9(2^x)$.

**(3 marks)**

**6 (a)** Sketch the graph of $y = 12e^{-x}$ for $x \geq 0$.
Label any points of intersection with the coordinate axes.
Write down the equations of any asymptotes.

**(3 marks)**

**(b)** Write down the gradient of $y = 12e^{-x}$ at the point where $x = 0$.

**(1 mark)**

**7 (a)** The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 3\mathrm{e}^{2x}$ for $x \in \mathbb{R}$.

Find $\mathrm{f}(2x)$.

**(2 marks)**

**(b)** Find $\mathrm{f}'(2x)$.

**(2 marks)**

**8** Solve $2e^{2x} = e^{x} + 10$, giving your answer to 3 significant figures.

**(3 marks)**

**9 (a)** Find the gradient of the curve $y = ae^{bx}$ where and $a$ and $b$ are constants.

**(1 mark)**

**(b)** At the point $(0, a)$ the gradient is 12, find $b$ in terms of $a$.

**(2 marks)**

**(c)** Hence write down y in terms of $a$(and $x$) only.

**(1 mark)**

**10 (a)** Show that the equation $e^x - e^{-x} = 0$ has only one real solution.

**(3 marks)**

**(b)** Explain why the equation $e^x + e^{-x} = 0$ has no real solutions.

**(2 marks)**

# Very Hard Questions

**1 (a)** On the same axes, sketch the graphs of $y = 0.3^x$ and $y = 0.5^x$.

Label any points of intersection with the coordinate axes.
Write down the equations of any asymptotes.

**(4 marks)**

**(b)** Write down an equation for the graph that is a reflection of $y = 0.5^x$ in the $y$-axis.

**(1 mark)**

**2 (a)** Sketch the graph of $y = 0.2^{-x}$, stating whether this graph indicates exponential growth or exponential decay.

**(3 marks)**

**(b)** Find the value of $x$ when $y = 625$.

**(1 mark)**

**3 (a)** Without using a calculator, evaluate $\log_4 128$.

Show each stage of your solution carefully.

**(2 marks)**

**(b)** Evaluate $\dfrac{3\log_6 216 - \ln e^5 + 4^{\log_5 625}}{\log 10000}$.

**(2 marks)**

**4** Solve $3^{2(x+1)} + 3 = 28(3^x)$.

**(4 marks)**

**5** Find two values of $x$ for which $\log(x^2) = (\log x)^2$ is true.

**(4 marks)**

**6 (a)** Sketch the graph of $y = 4e^x$ for $x \geq 0$.
Label any points of intersection with the coordinate axes.
Write down the equations of any asymptotes.

**(3 marks)**

**(b)** Find the gradient of $y = 4e^x$ at the point where $x = 3$, giving your answer correct to 3 significant figures.

**(1 mark)**

**(c)** The population growth of population, *P*, at time, *t* years, is modelled by the equation $P = 4e^t$
Write down the initial population.

**(1 mark)**

**7 (a)** The function $f(x)$ is defined by $f(x) = 5e^{3x}$ for $x \in \mathbb{R}$.

Find $f(4x)$.

**(2 marks)**

**(b)** Find $f'(5x)$.

**(2 marks)**

**8 (a)** Find the gradient of the curve $y = \frac{1}{a}e^{-bx}$ where $a$ and $b$ are constants.

**(1 mark)**

**(b)** State a condition on $b$ to ensure $y$ represents exponential decay.

**(1 mark)**

**(c)** At the point $(0, a)$ the gradient is 10. Find $y$ in terms of $a$ (and $x$) only.

**(2 marks)**

**9 (a)** A particle is travelling with velocity, $v$ ms$^{-1}$, at time $t$ seconds.
The velocity of the particle is modelled $v = 0.3e^{kt}$, where $k$ is a constant.

Write down the initial velocity of the particle.

**(1 mark)**

**(b)** Find an expression (in terms of $k$ and $t$) for the acceleration of the particle.

**(2 marks)**

**(c)** After 12 seconds the velocity of the particle is 0.9 ms$^{-1}$.
Find the value of $k$, giving your answer to 3 significant figures.

**(3 marks)**

**(d)** State a problem with the model for large values of $t$

**(1 mark)**

**10** Solve $(e^x - e^{-x})^2 = 0$.

**(4 marks)**

**11** Solve the equation $2e^{3x} - 11e^{2x} + 12e^{x} = 0$, giving answers to 3 significant figures where appropriate.

**(5 marks)**