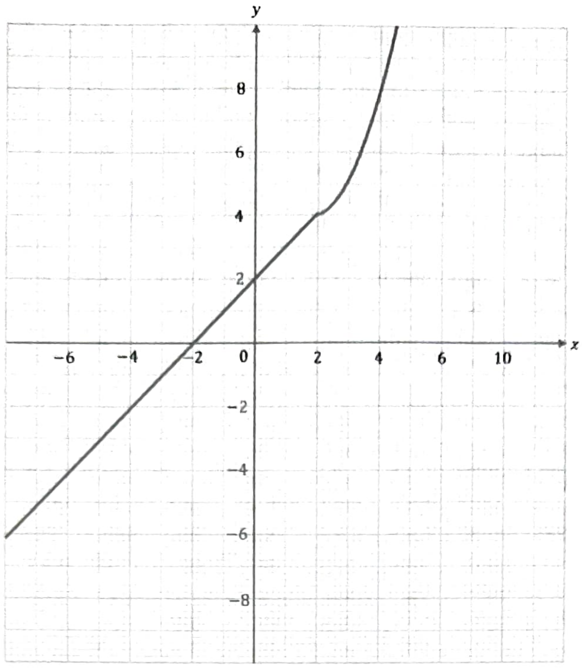
MME Revise
A-A* Edexcel
A Level Mathematics
Papers

Paper 1: Pure Mathematics 1
Paper 2: Pure Mathematics 2
Paper 3: Statistics and Mechanics

<u>**Grade Boundaries**</u>

For grade boundaries visit:

1(a)	Suppose that $\log 7$ is rational $\log 7 = \frac{a}{b}$, where a and b are positive irrational numbers Begin process to rearrange e.g. $7 = 10^{\frac{a}{b}}$ $7^b = 10^a$ The integer powers of 7 are all odd, while the integer powers of 10 are even, hence $\log 7$ is irrational by contradiction	M1 M1 M1 M1 B1 **[5]**
1(b)	Suppose that $\frac{x+a}{\sqrt{x^2+a^2}} - \frac{x+b}{\sqrt{x^2+b^2}} \leq 0$ $\frac{x+a}{\sqrt{x^2+a^2}} \leq \frac{x+b}{\sqrt{x^2+b^2}}$ Since both sides are positive, $\frac{(x+a)^2}{x^2+a^2} \leq \frac{(x+b)^2}{x^2+b^2}$ $\frac{x^2+2ax+a^2}{x^2+a^2} \leq \frac{x^2+2bx+b^2}{x^2+b^2}$ $1 + \frac{2ax}{x^2+a^2} \leq 1 + \frac{2bx}{x^2+b^2}$ $\frac{2ax}{x^2+a^2} \leq \frac{2bx}{x^2+b^2}$ Since $2x > 0$ $\frac{a}{x^2+a^2} \leq \frac{b}{x^2+b^2}$ Since both denominators are positive, $a(x^2+b^2) \leq b(x^2+a^2)$ $ax^2 + ab^2 \leq bx^2 + ba^2$ $x^2(a-b) - ab(a-b) \leq 0$ $(a-b)(x^2-ab) \leq 0$ But $a > b \Rightarrow a - b > 0$ and $x^2 > ab \Rightarrow x^2 - ab > 0$ Thus $(a-b)(x^2-ab) > 0$ Hence, proof by contradiction	M1 M1 M1 M1 M1 M1 M1 A1 **[8]**
1(c)	$P = (2^q - 1)^2 - 1 = 2^{2q} - 2^{q+1} + 1 - 1 = 2^{2q} - 2^{q+1}$ $P = 2^{q+1}(2^{q-1} - 1)$ Hence 2^{q+1} is a factor of P.	M1 A1 **[2]** **[15 marks]**

Question	Paper 1 – Pure Mathematics 1	Marks
2(a)		B1 $x + 2$ plotted correctly B1 General shape and positioning of $(x-2)^2+4$ correct B1 Intersection at $(2,4)$, must clearly be the minimum point of $(x-2)^2+4$ [3]
2(b)	Process to find inverse of $f_1(x) = x + 2$ $f_1^{-1}(x) = x - 2$ Begin process to find inverse of $f_2(x) = (x-2)^2 + 4$ Must show evidence of considering both roots e.g. $x - 2 = \pm\sqrt{x-4}$ As domain is $x > 2$ RHS is + ve $x - 2 = \sqrt{x-4}$ $f_2^{-1}(x) = 2 + \sqrt{x-4}$ Fully correct inverse including domain $f^{-1}(x) = \begin{cases} x-2 & x \le 4 \\ 2+\sqrt{x-4} & x > 4 \end{cases}$	M1 A1 M1 M1 A1 A1 [6] [9 marks]
3	Indicates that CDE is isosceles and $\angle CED = 90°$ May be implied in working or indicated on diagram $\lvert CE\rvert^2 + \lvert DE\rvert^2 = \lvert CD\rvert^2$ $\left(\frac{6}{2}\right)^2 + \left(\frac{6}{2}\right)^2 = \lvert CD\rvert^2 = 18$ $\lvert CD\rvert = 3\sqrt{2}$ Finds area of shaded semicircle $\frac{1}{2}\pi\left(\frac{3\sqrt{2}}{2}\right)^2 = \frac{9}{4}\pi\ \text{cm}^2$	M1 M1 M1 A1

Question	Paper 1 – Pure Mathematics 1	Marks
3 cont.	Process to find area of segment $\frac{90}{360}\times\pi\left(\frac{6}{2}\right)^2-\frac{1}{2}\times 3^2\times\sin 90=\left(\frac{9}{4}\pi-\frac{9}{2}\right)\text{cm}^2$ Process to find total area resulting in desired form $\frac{9}{4}\pi+\left(\frac{9}{4}\pi-\frac{9}{2}\right)=\frac{9}{2}(\pi-1)\text{ cm}^2$	M1 A1 [6] [6 marks]
4	Let, $a=1^{\text{st}}$ term of arithmetic progression $d=$ common difference of arithmetic progression $A=1^{\text{st}}$ term of geometric progression $r=$ common ratio of arithmetic progression Hence, $A=a+6d$ $Ar=a+3d$ $Ar^2=a+d$ Finds two equations linking 3 variables e.g. $A=a+6Ar^2-6a$ **and** $Ar=a+3Ar^2-3a$ Eliminates second variable, r must remain e.g. $5a=6Ar^2-A$ **and** $2a=3Ar^2-Ar$ $\frac{5a}{2a}=\frac{6Ar^2-A}{3Ar^2-Ar}$ $\frac{5}{2}=\frac{6Ar^2-A}{3Ar^2-Ar}$ Finds equation in r alone e.g. $\frac{5}{2}=\frac{6r^2-1}{3r^2-r}$ Begins process to solve e.g. rearranges to quadratic $3r^2-5r+2=0$ $r=\frac{2}{3}, r\neq 1$ $r=\frac{2}{3}$ Accept sensible alternative methods	 M1 M1 M1 M1 M1 M1 Both must be considered to be awarded A1 Do not award if $r=1$ is not ruled out [7] [7 marks]
5	Finds equation for dr in terms of r and A Let $A=$ area of the circle $A=\pi r^2$ $\frac{dA}{dr}=2\pi r$ $dr=\frac{dA}{2\pi r}$ Eliminates dr $\frac{\frac{dA}{2\pi r}}{dt}=\frac{1}{3r^2}$	M1 M1

Question	Paper 1 – Pure Mathematics 1	Marks
5 cont.	Rearranges for $\frac{dA}{dt}$ and simplifies $\frac{dA}{dt} = \frac{2\pi r}{3r^2} = \frac{2\pi}{3r}$ Start of process to rearrange to given form e.g. $\frac{dA}{dt} = \frac{2\pi^2}{3\pi r}$ $\left(\frac{dA}{dt}\right)^2 = \frac{4\pi^4}{9\pi^2 r^2}$ $\left(\frac{dA}{dt}\right)^2 = \frac{4\pi^3}{9\pi r^2}$ Substitution of $A = \pi r^2$ e.g. $\left(\frac{\mathrm{dA}}{\mathrm{dt}}\right)^2 = \frac{4\pi^3}{9A}$ Continuation of process of rearranging resulting in, $\frac{dA}{dt} = +\sqrt{\frac{4\pi^3}{9A}}$, Do not accept $\pm\sqrt{\frac{4\pi^3}{9A}}$ since $r > 0$ $a = 4, b = 9$ Accept sensible alternative methods	M1 M1 M1 M1 *a* and *b* may be stated or implied **[6]** **[6 marks]**
6	$u_{n+2} = u_{n+1} + u_n$ So, $\frac{u_{n+2}}{u_{n+1}} = \frac{u_{n+1}}{u_{n+1}} + \frac{u_n}{u_{n+1}}$ $\frac{u_{n+2}}{u_{n+1}} \longrightarrow \phi$ as $n \longrightarrow \infty$ $\frac{u_n}{u_{n+1}} \longrightarrow \frac{1}{\phi}$ as $n \longrightarrow \infty$ $\phi = 1 + \frac{1}{\phi}$ $\phi^2 = \phi + 1$ $\phi^2 - \phi - 1 = 0$ $\phi = \frac{(1 \pm \sqrt{5})}{2}$ $\phi \neq \frac{1}{2}(1 - \sqrt{5})$ Since the ratio of consecutive positive terms cannot be negative. So only positive result valid, as given.	M1 M1 M1 M1 M1 M1 A1 Must include valid reasoning for discounting the negative results **[7]** **[7 marks]**

Question	Paper 1 – Pure Mathematics 1	Marks
7	$(x-2)^2+y^2=4$	M1
	Begins process to rearrange for y, e.g. $y^2=4-(x-2)^2$	M1
	$y=\pm\sqrt{4x-x^2}$ Since negative y's will yield a smaller $x+y$. $y=+(4x-x^2)^{\frac{1}{2}}$	M1
	Expression to be maximised is, $f(x,y)=x+y$	M1
	$f(x)=x+(4x-x^2)^{\frac{1}{2}}$	M1
	$f'(x)=1+\frac{1}{2}(4-2x)(4x-x^2)^{-\frac{1}{2}}$	M1
	$1+\frac{1}{2}(4-2x)(4x-x^2)^{-\frac{1}{2}}=0$	M1
	Begins process to solve, e.g. $0=(4x-x^2)^{\frac{1}{2}}+2-x$ $x-2=(4x-x^2)^{\frac{1}{2}}$	M1
	$x^2-4x+4=4x-x^2$ $2x^2-8x+4$	M1
	$x=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(2\times 4)}}{2\times 2}=2\pm\sqrt{2}$	M1
	Only $2+\sqrt{2}$ is valid, since the negative result does not satisfy $x-2=(4x-x^2)^{\frac{1}{2}}$	M1 Must include reasoning for ruling out negative result
	Max $x+y=$ $(2+\sqrt{2})+\left(4(2+\sqrt{2})-(2+\sqrt{2})^2\right)^{\frac{1}{2}}=2+2\sqrt{2}$	B1
		[12] **[12 marks]**
8(a)	Attempts to use, $\log_a b=\dfrac{\log_c b}{\log_c a}$	M1
	$y=\log_x 3=\dfrac{\log_e 3}{\log_e x}=\dfrac{\ln 3}{\ln x}$	M1
	$\dfrac{dy}{dx}=-1\times(\ln 3)\times(\ln x)^{-2}\times\dfrac{1}{x}=-\dfrac{\ln 3}{x(\ln x)^2}$	M1
	Substitute $x=3$ $-\dfrac{\ln 3}{3(\ln 3)^2}=-\dfrac{1}{3(\ln 3)}=-\dfrac{1}{\ln 27}$	M1
	The normal has gradient $\ln 27$ and $P=(3,1)$	M1

Question	Paper 1 – Pure Mathematics 1	Marks
8(a) cont.	Equation of normal is given by, $y-1=\ln 27\,(x-3)$	M1
	Begins process to solve simultaneously with curve, e.g. $\frac{\ln 3}{\ln x}-1=x\ln 27-3\ln 27$	M1
	Begins process to rearrange for $\ln 3$ e.g. $\ln 3-\ln x=x\ln x\ln 27-3\ln x\ln 27$	M1
	Full process resulting in given form $\ln 3=\ln x\,(1+x\ln 27-3\ln 27)$	A1
		[9]
8(b)	Finds iterative formula, $\ln x=\dfrac{\ln 3}{1+x\ln 27-3\ln 27}$ $x=e^{\frac{\ln 3}{1+x\ln 27-3\ln 27}}$ $x_{n+1}=e^{\left(\frac{\ln 3}{1+x_n\ln 27-3\ln 27}\right)}$	M1
	Identifies valid starting value, e.g. $x_0=0.5$	M1
	Uses iterative formula to find x-coordinate e.g. $x_1=0.8592\ldots$ $x_2=0.8341\ldots$ …	A1
	Critical values $x_3=0.8361$ and $x_4=0.8360$ must be seen	
	Thus $x=0.836$	
	$y=\dfrac{\ln 3}{\ln(0.836)}=-6.133$	A1
		[4]
		[13 marks]
9(a)	$\dfrac{dy}{dx}=\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1
	At least one of, $\frac{dy}{d\theta}=4\cos\theta$ **or** $\frac{dx}{d\theta}=-2\sin\theta$	M1
	$\dfrac{dy}{dx}=\dfrac{4\cos\theta}{-2\sin\theta}=-\dfrac{2}{\tan\theta}$	M1
	Begins process to find equation, e.g. $y-4\sin\alpha=-\frac{2}{\tan\alpha}(x-2\cos\alpha)$	M1
	Begins process to rearrange to desired form, $y\sin\alpha-4\sin^2\alpha=-2x\cos\alpha+4\cos^2\alpha$	M1
	Full process resulting in desired form, e.g. $y\sin\alpha+2x\cos\alpha=4(\sin^2\alpha+\cos^2\alpha)$ $y\sin\alpha+2x\cos\alpha=4$	B1
		[6]

Question	Paper 1 – Pure Mathematics 1	Marks
9(b)	$A = \left(0, \dfrac{4}{\sin\alpha}\right)$ **and** $B = \left(\dfrac{2}{\cos\alpha}, 0\right)$	M1 Or vice versa
	Area of triangle $OAB = \dfrac{1}{2} \times \dfrac{4}{\sin\alpha} \times \dfrac{2}{\cos\alpha} = \dfrac{8}{\sin 2\alpha}$	M1
	$\dfrac{8}{\sin 2\alpha} < 16$	M1
	As α is acute, $\sin 2\alpha > 0$ $\dfrac{1}{2} < \sin 2\alpha$	M1
	$2\alpha = \dfrac{\pi}{6}$ $2\alpha = \dfrac{5\pi}{6}$	B1 Must have both for mark
	$\alpha = \dfrac{\pi}{12}$ $\alpha = \dfrac{5\pi}{12}$	B1 Must have both for mark
	$\dfrac{\pi}{12} < \alpha < \dfrac{5\pi}{12}$	B1
		[7]
		[13 marks]
10	$A(20,18,-12)\ B(12,-6,20)\ C(4,y,z)$	
	$\overrightarrow{AC} = (4,y,z) - (20,18,-12) = (-16, y-18, z+12)$	M1
	$\overrightarrow{BC} = (4,y,z) - (12,-6,20) = (-8, y+6, z-20)$	M1
	$\lvert\overrightarrow{AC}\rvert = \sqrt{(-16)^2 + (y-18)^2 + (z+12)^2} = 24$	M1
	$y^2 - 36y + z^2 + 24z = -148$ (*)	M1
	$\lvert\overrightarrow{BC}\rvert = \sqrt{(-8)^2 + (y+6)^2 + (z-20)^2} = 24$	M1
	$y^2 + 12y + z^2 - 40z = 76$ (**)	M1
	(*) subtract (**): $-48y + 64z = -224$ $4z + 14 = 3y$	M1
	$9(y^2 + 12y + z^2 - 40z) = 9(76)$ $9y^2 + 108y + 9z^2 - 360z = 684$	M1
	$(4z+14)^2 + 36(4z+14) + 9z^2 - 360z = 684$	M1 Students may also eliminate z
	$25z^2 - 104z + 16 = 0$ $(25z-4)(z-4) = 0$ $z = 4$ or $\dfrac{4}{25}$	A1

Question	Paper 1 – Pure Mathematics 1	Marks
10 cont.	$z = 4$: $4(4) + 14 = 3y \Rightarrow y = 10$ $(4,10,4)$ $z = \frac{4}{25}$: $4\left(\frac{4}{25}\right) + 14 = 3y \Rightarrow y = \frac{122}{25}$ $\left(4, \frac{122}{25}, \frac{4}{25}\right)$	A1 A1 [12] [12 marks]

END

Question	Paper 2 – Pure Mathematics 2	Marks
1	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ Where h is the height of the cone in mm, and V is the volume of the cone in mm^3 $\frac{dh}{dt}=\frac{dh}{dV}\times k$	M1
	$\frac{r}{h}=\frac{18}{72}$ $r=\frac{1}{4}h$	M1
	$V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{1}{4}h\right)^2h=\frac{1}{48}\pi h^3$	M1
	$\frac{dV}{dh}=\frac{1}{16}\pi h^2$ $\frac{dh}{dV}=\frac{16}{\pi h^2}$	M1
	$\frac{dh}{dt}=\frac{16}{\pi h^2}\times k=\frac{16k}{\pi h^2}$	M1
	12.5 minutes = 750 s $V=k\ \text{mm}^3\text{s}^{-1}\times 750\ \text{s}=750k\ \text{mm}^3$	M1
	$\frac{1}{48}\pi h^3=750k$	M1
	$\pi h^3=36000k$	M1
	$\frac{2}{75}=\frac{16k}{\pi h^2}$	M1
	$\pi h^2=600k$	M1
	$\pi h^3=36000k$ $\pi h^2=600k$ $\Rightarrow h=60$	M1
	$\pi 60^2=600k$ $k=6\pi$	B1
		[12]
		[12 marks]

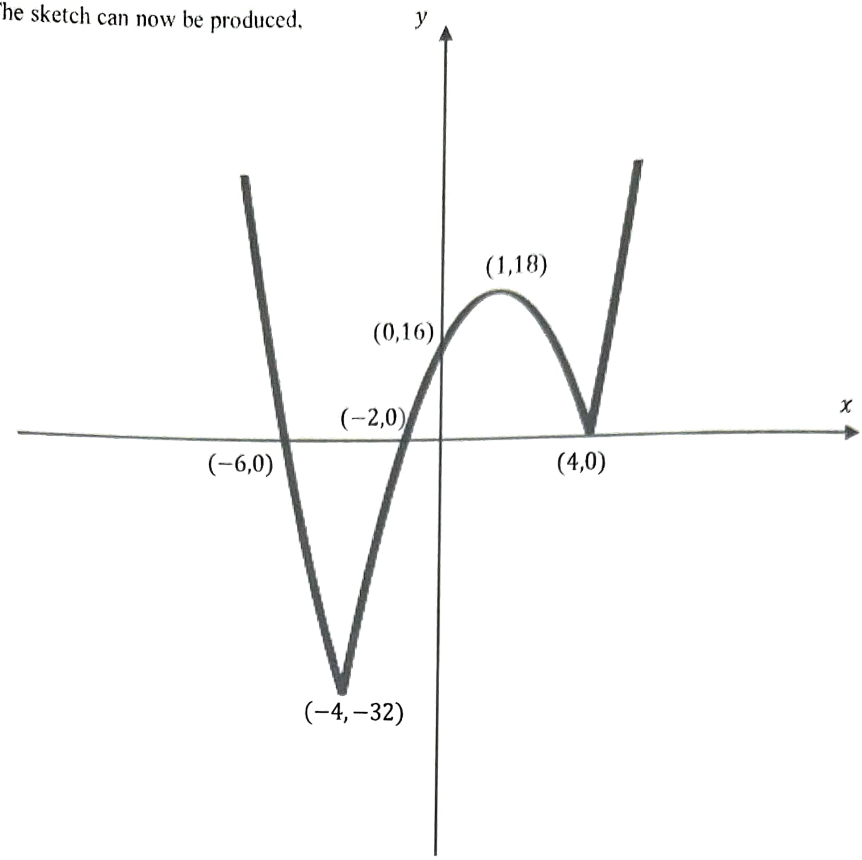
Question	Paper 2 – Pure Mathematics 2	Marks
2	$u_1 = \frac{1}{5}$ $\frac{1}{k+c} = \frac{1}{5}$ $c = 5 - k$	M1
	$u_2 = \frac{10 \times \frac{1}{5}}{1 + 16 \times \frac{1}{5}} = \frac{10}{21}$ $u_3 = \frac{10 \times \frac{10}{21}}{1 + 16 \times \frac{10}{21}} = \frac{100}{181}$	M1
	$u_2 = \frac{a}{ka + c} = \frac{10}{21}$ $ka + c = \frac{21a}{10}$	M1
	$u_3 = \frac{a^2}{ka^2 + c} = \frac{100}{181}$ $ka^2 + c = \frac{181a^2}{100}$	M1
	$ka + (5 - k) = \frac{21a}{10}$ $k(a - 1) = \frac{21a}{10} - 5$	M1
	$ka^2 + (5 - k) = \frac{181a^2}{100}$ $k(a^2 - 1) = \frac{181a^2}{100} - 5$	M1
	$\frac{k(a^2 - 1)}{k(a - 1)} = \frac{\frac{181a^2}{100} - 5}{\frac{21a}{10} - 5}$ $(a + 1) = \frac{181a^2 - 500}{210a - 500}$	M1
	$210a^2 - 290a - 500 = 181a^2 - 500$ $29a^2 - 290a = 0$	M1
	$a(a - 10) = 0$ $a = 10, a \neq 0$	M1
	$k(10 - 1) = \frac{21(10)}{10} - 5$ $k = \frac{16}{9}$	M1
	$c = 5 - \frac{16}{9}$ $c = \frac{29}{9}$	M1
	$u_n = \frac{10^{n-1}}{\frac{29}{9} + \frac{16}{9} \times 10^{n-1}}$	B1
		[12]
		[12 marks]

Question	Paper 2 – Pure Mathematics 2	Marks
3	$\int \ln\left(\frac{x^2+1}{x^2}\right) \times \frac{\sqrt{x^2+1}}{x^4} dx$	M1
	$\int \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \ln\left(1+\frac{1}{x^2}\right) \times \frac{1}{x^3} dx$	M1
	$\int \sqrt{1+\frac{1}{x^2}} \ln\left(1+\frac{1}{x^2}\right) \times \frac{1}{x^3} dx$	
	$u = \sqrt{1+\frac{1}{x^2}}$	M1
	$u^2 = 1+\frac{1}{x^2}$ $2u\,du = -\frac{2}{x^3} dx$	M1
	$\int u \ln(u^2) \times \frac{1}{x^3} \times (-x^3) du$	M1
	$\int -u^2 \ln u^2\, du$	M1
	$-2\int u^2 \ln u\, du$	
	Attempts to use integration by parts, $t = \ln u \Rightarrow \frac{dt}{du} = \frac{1}{u}$ $\frac{dv}{du} = u^2 \Rightarrow v = \frac{u^3}{3}$	M1
	$-2\left(\frac{u^3}{3}\ln u - \int \frac{u^3}{3} \times \frac{1}{u} du\right)$	M1
	$-\frac{2u^3}{3}\ln u + \frac{2u^3}{9} + C$	M1
	$\frac{2}{9}u^3(1-3\ln u) + C$	M1
	$\frac{2}{9}\left(\sqrt{1+\frac{1}{x^2}}\right)^3 \left(1-3\ln\left(\sqrt{1+\frac{1}{x^2}}\right)\right) + C$	M1
	$\frac{2}{9x^3}(x^2+1)^{\frac{3}{2}}\left(1-3\ln\left(\sqrt{\frac{x^2+1}{x^2}}\right)\right) + C$	B1
		[12]
		[12 marks]

Question	Paper 2 – Pure Mathematics 2	Marks
4(a)	Since $x > 0$, $y = \dfrac{x}{(x-1)^{\frac{1}{2}}}$	M1
	$\dfrac{dy}{dx} = \dfrac{(x-1)^{\frac{1}{2}} \times 1 - x \times \frac{1}{2}(x-1)^{-\frac{1}{2}}}{\left((x-1)^{\frac{1}{2}}\right)^2}$	M1
	$\dfrac{dy}{dx} = \dfrac{(x-2)}{2(x-1)^{\frac{3}{2}}}$	M1
	Let P be a generic point on the curve for which $y > 0$ Then, $\dfrac{dy}{dx}\Big\vert_{x=P} = \dfrac{P-2}{2(P-1)^{\frac{3}{2}}}$	M1
	Equation of tangent at P takes the form, $y - \dfrac{P}{(P-1)^{\frac{1}{2}}} = \dfrac{P-2}{2(P-1)^{\frac{3}{2}}}(x-P)$	M1
	Substitute $x = 1$ $2 - \dfrac{P}{(P-1)^{\frac{1}{2}}} = \dfrac{P-2}{2(P-1)^{\frac{3}{2}}}(1-P)$	M1
	$2 - \dfrac{P}{(P-1)^{\frac{1}{2}}} = \dfrac{2-P}{2(P-1)^{\frac{1}{2}}}$	M1
	$4(P-1)^{\frac{1}{2}} - 2P = 2 - P$	M1
	$16(P-1) = P^2 + 4P + 4$	M1
	$P^2 - 12P + 20 = 0$ $(P-2)(P-10) = 0$ $P = 2$ or 10	M1
	Hence, two tangents in the form $y - \dfrac{P}{(P-1)^{\frac{1}{2}}} = \dfrac{P-2}{2(P-1)^{\frac{3}{2}}}(x-P)$ where $P = 2$ or 10	A1 **[11]**
4(b)	Substitutes $P = 2$, $y - \dfrac{2}{(2-1)^{\frac{1}{2}}} = \dfrac{2-2}{2(2-1)^{\frac{3}{2}}}(x-2)$	M1
	$y - 2 = 0$	A1
	Substitutes $P = 10$, $y - \dfrac{10}{(10-1)^{\frac{1}{2}}} = \dfrac{10-2}{2(10-1)^{\frac{3}{2}}}(x-10)$	M1
	$y - \dfrac{10}{3} = \dfrac{4}{27}(x-10)$	M1
	$4x - 27y + 50 = 0$	A1
		[5]
		[16 marks]

Question	Paper 2 – Pure Mathematics 2	Marks
5	States or uses, $\sin(-x) \equiv -\sin x$ $\cos(-x) \equiv \cos x$	M1
	$f(-x) \equiv \dfrac{e^{-\sin x \cos x} + 1}{e^{-\sin x \cos x} - 1}$	M1
	$f(-x) \equiv \dfrac{e^{-\sin x \cos x} \times e^{\sin x \cos x} + 1 \times e^{\sin x \cos x}}{e^{-\sin x \cos x} \times e^{\sin x \cos x} - 1 \times e^{\sin x \cos x}}$	M1
	$f(-x) \equiv \dfrac{e^{0} + e^{\sin x \cos x}}{e^{0} - e^{\sin x \cos x}} = \dfrac{1 + e^{\sin x \cos x}}{1 - e^{\sin x \cos x}}$	M1
	$f(-x) \equiv \dfrac{e^{\sin x \cos x} + 1}{-1(e^{\sin x \cos x} - 1)}$	M1
	$f(-x) \equiv \dfrac{e^{\sin x \cos x} + 1}{-1(e^{\sin x \cos x} - 1)} \equiv -\dfrac{e^{\sin x \cos x} + 1}{e^{\sin x \cos x} - 1}$	M1
	$-\dfrac{e^{\sin x \cos x} + 1}{e^{\sin x \cos x} - 1} \equiv -f(x)$ Hence, $f(-x) = -f(x)$	A1
		[7] **[7 marks]**
6	$M = \frac{1}{2}B = (7, -1, 3)$	M1
	$N = \frac{1}{3}A = (4, -12, -4)$	M1
	$\overrightarrow{NP} = k\overrightarrow{NB} = k(\boldsymbol{b} - \boldsymbol{n})$	M1
	$\overrightarrow{NP} = k\big((14, -2, 6) - (4, -12, -4)\big) = (10k, 10k, 10k)$	M1
	$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{ON} + \overrightarrow{NP}$ $= -(7, -1, 3) + (4, -12, -4) + (10k, 10k, 10k)$ $= (10k - 3, 10k - 11, 10k - 7)$	M1
	$\overrightarrow{PA} = \overrightarrow{PN} + \overrightarrow{NA}$ $= -(10k, 10k, 10k) + 2(4, -12, -4)$ $= (8 - 10k, -10k - 24, -10k - 8)$	M1
	$\overrightarrow{MP} = \lambda\overrightarrow{PA}$ $(10k - 3, 10k - 11, 10k - 7) = \lambda(8 - 10k, -10k - 24, -10k - 8)$	M1
	Equate any two components, e.g. $10k - 3 = 8\lambda - 10k\lambda$ **and** $10k - 11 = -10k\lambda - 24\lambda$	M1
	Subtract the two equations, $8\lambda + 3 = 11 - 24\lambda$ $\lambda = \frac{1}{4}$	M1

Question	Paper 2 – Pure Mathematics 2	Marks
6 cont.	Substitute to find k, e.g. $10k - 3 = 8\left(\frac{1}{4}\right) - 10k\left(\frac{1}{4}\right)$ $k = \frac{2}{5}$	M1
	$\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP}$ $= (4,-12,-4) + \left(10\left(\frac{2}{5}\right), 10\left(\frac{2}{5}\right), 10\left(\frac{2}{5}\right)\right)$ $= (8,-8,0) = P$	M1
		[11]
		[11 marks]
7	$S_\infty = 1 + \frac{2}{4(1)} + \frac{2\times 3}{4^2(1\times 2)} + \frac{2\times 3\times 4}{4^3(1\times 2\times 3)} + \frac{2\times 3\times 4\times 5}{4^4(1\times 2\times 3\times 4)} + \ldots$	M1
	$S_\infty = 1 + \frac{2}{1!}\left(\frac{1}{4}\right)^1 + \frac{2\times 3}{2!}\left(\frac{1}{4}\right)^2 + \frac{2\times 3\times 4}{3!}\left(\frac{1}{4}\right)^3 + \frac{2\times 3\times 4\times 5}{4!}\left(\frac{1}{4}\right)^4 + \ldots$	M1
	S_∞ $= 1 + \frac{-2}{1!}\left(-\frac{1}{4}\right)^1 + \frac{(-2)(-3)}{2!}\left(-\frac{1}{4}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(-\frac{1}{4}\right)^3 + \frac{(-2)(-3)(-4)(-5)}{4!}\left(-\frac{1}{4}\right)^4$ $+ \ldots$	M1
	$S_\infty = \left(1 - \frac{1}{4}\right)^{-2}$	M1
	$S_\infty = \frac{16}{9}$	B1
		[5]
		[5 marks]
8(a)	$x = 0$ $2\lvert 0^2 - 16\rvert + 4(0-4) = 16$	M1
	Two cases to consider, $x^2 - 16 \geq 0$ and $x^2 - 16 \leq 0$ May be stated or implied by working	M1
	For $x^2 - 16 \geq 0$ $y = +2(x^2 - 16) + 4(x-4)$ $y = 2(x-4)(x+4) + 4(x-4)$	M1
	$y = (x-4)(2(x+4)+4)$ $y = (x-4)(2x+12)$ $x = 4$ or $x = -6$	M1
	For $x^2 - 16 \leq 0$ $y = -2(x^2 - 16) + 4(x-4)$ $y = -2(x-4)(x+4) + 4(x-4)$	M1
	$y = (x-4)(-2(x+4)+4)$ $y = (x-4)(-2x-4)$ $x = 4$ or $x = -2$	M1

Question	Paper 2 – Pure Mathematics 2	Marks
8(a) cont.	The sketch can now be produced. 	B1 For general shape B1 For x −intercepts correct B1 For turning points correct **[9]**
8(b)	$\int_{-6}^{-2} 2\lvert x^2-16\rvert + 4(x-4)\,dx = \int_{-6}^{-4} (x-4)(2x+12)dx + \int_{-4}^{-2} (x-4)(-2x-4)dx$ $= \int_{-6}^{-4} 2x^2+4x-48\,dx + \int_{-4}^{-2} -2x^2+4x+16\,dx$ $= \left[\frac{2}{3}x^3+2x^2-48x\right]_{-6}^{-4} + \left[-\frac{2}{3}x^3+2x^2+16x\right]_{-4}^{-2}$ $= \left(\left(\frac{2}{3}(-4)^3+2(-4)^2-48(-4)\right) - \left(\frac{2}{3}(-6)^3+2(-6)^2-48(-6)\right)\right)$ $+ \left(\left(-\frac{2}{3}(-2)^3+2(-2)^2+16(-2)\right) - \left(-\frac{2}{3}(-4)^3+2(-4)^2+16(-4)\right)\right)$ $= \frac{544}{3} - 216 - \frac{56}{3} - \frac{32}{3} = -64$ Hence the area is 64 square units	M1 M1 M1 M1 B1 **[5]** **[14 marks]**
9(a)	$fg(x) = f(g(x))$ $f(2\sqrt{x-1}) = 8\sqrt{x-1} + 2$ Domain: Must satisfy both $x \geq 12$ and $2\sqrt{x-1} \leq 16$	M1 May be stated or implied B1 M1

Question	Paper 2 – Pure Mathematics 2	Marks
9(a) cont.	$2\sqrt{x-1} \leq 16$ $\sqrt{x-1} \leq 8$ $x - 1 \leq 64$ $x \leq 65$ Hence, $12 \leq x \leq 65$	B1
	Since $y = 8\sqrt{x-1} + 2$ is $y = \sqrt{x}$ after a stretch and a translation, the extrema of the range occur at the ends of the domain's interval may be stated or implied A graph is also sufficient to convey this	M1
	At least one of, $8\sqrt{12-1} = 8\sqrt{11}$ or $8\sqrt{65-1} = 64$	M1
	Range: $8\sqrt{11} \leq fg(x) \leq 64$	B1
		[7]
9(b)	$f(x)$'s output must not satisfy $g(x)$'s domain to prevent $gf(x)$ being formed	M1 May be stated or implied
	Hence, $4x + 2 < 12$	M1
	$4x + 2 < 12$ $x < \frac{5}{2}$	B1
	$\alpha = \frac{5}{2}$	B1
		[4]
		[11 marks]

END