Modelling

$S_n = \frac{n}{2}(a + L)$

Anything involving compound changes (e.g. bank interest) will form a geometric sequence, as there is a constant ratio between terms.
We can therefore use formulae such as S_n to solve problems.

Bruce starts a new company. In year 1 his profits will be £20 000. He predicts his profits to increase by £5000 each year, so that his profits in year 2 are modelled to be £25 000, in year 3, £30 000 and so on. He predicts this will continue until he reaches annual profits of £100 000. He then models his annual profits to remain at £100 000.

a) Calculate the profits for Bruce's business in the first 20 years.
b) State one reason why this may not be a suitable model.
c) Bruce's financial advisor says the yearly profits are likely to increase by 5% per annum. Using this model, calculate the profits for Bruce's business in the first 20 years.

year

20000, 25000, 30000, ..., 100,000 | 100,000, 100,000, 100,000

a) $a = 20000$

$d = 50000$

$u_n = 100000$

$n = ?$

$u_n = a + (n-1)d$

$100000 = 20000 + (n-1)\,5000$

$\frac{80000}{5000} = n - 1$

$16 = n - 1$

$n = 17$

$S_{17} = \frac{17}{2}(20000 + 100000)$

$= £1,020,000$

$S_{20} = S_{17} + 300,000$

$= £1,320,000$

b) It is unlikely for profits to increase by a constant amount.

Ex 3I

b) $a = 20000$

$r = 1.05$

$n = 20$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{20000(1-1.05^{20})}{1-1.05}$$

$$= £661,319.08$$

8. There were 2100 tonnes of wheat harvested on a farm during 2017.

The mass of wheat harvested during each subsequent year is expected to increase by 1.2% per year.

compound

(a) Find the total mass of wheat expected to be harvested from 2017 to 2030 inclusive, giving your answer to 3 significant figures.

(2)

2030 − 2017 + 1 = 14 years

2017 ~~2019~~

Each year it costs

- £5.15 per tonne to harvest the first 2000 tonnes of wheat
- £6.45 per tonne to harvest wheat in excess of 2000 tonnes

(b) Use this information to find the expected cost of harvesting the wheat from 2017 to 2030 inclusive. Give your answer to the nearest £1000

(3)

a) $a = 2100$

$r = 1.012$

$n = 14$

$$S_{14} = \frac{2100(1 - 1.012^{14})}{1 - 1.012} = 31806.9948$$

$= 31800$ tonnes

b) Each year is producing at least 2000 tonnes.

So in 14 years, there will be 2000×14 tonnes @ £5.15

$28000 \times 5.15 = £144{,}200$

Wheat @ £6·45 31807 − 28000 = 3807 tonnes

3807 × 6·45 = £24,555·15

TOTAL = 144200 + 24555·15

= 168,755·15

= £169,000.

Question	Scheme	Marks	AOs
8 (a)	Total amount $= \dfrac{2100(1-(1.012)^{14})}{1-1.012}$ or $\dfrac{2100((1.012)^{14}-1)}{1.012-1}$	M1	3.1b
	$= 31806.9948\ldots = 31800$ (tonnes) (3 sf)	A1	1.1b
		(2)	
	Total Cost $= 5.15(2000(14)) + 6.45(31806.9948\ldots - (2000)(14))$	M1	3.1b
		M1	1.1b
	$= 5.15(28000) + 6.45(3806.9948\ldots) = 144200 + 24555.116\ldots$		
	$= 168755.116\ldots =$ £169000 (nearest £1000)	A1	3.2a
		(3)	
			(5 marks)

12. A company extracted 4500 tonnes of a mineral from a mine during 2018.

The mass of the mineral which the company expects to extract in each subsequent year is modelled to decrease by 2% each year.

(a) Find the total mass of the mineral which the company expects to extract from 2018 to 2040 inclusive, giving your answer to 3 significant figures.

2040 − 2018 + 1 = 23 years of extracting

(2)

(b) Find the mass of the mineral which the company expects to extract during 2040, giving your answer to 3 significant figures.

(2)

The costs of extracting the mineral each year are assumed to be:

- £800 per tonne for the first 1500 tonnes
- £600 per tonne for any amount in excess of 1500 tonnes

The expected cost of extracting the mineral from 2018 to 2040 inclusive is £x million.

$u_n = ar^{n-1}$

(c) Find the value of x, giving your answer to 3 significant figures.

(3)

a) $a = 4500$

$r = 0.98$

$n = 23$

$S_{23} = \dfrac{4500(1-0.98^{23})}{1-0.98}$

$= 83621.8...$

$= 83{,}600$ tonnes

b) $u_{23} = 4500 \times 0.98^{22}$

$= 2885.2...$

$= 2{,}890$ tonnes

c) Each year it does at least 1500 tonnes

$23 \times 1500 \times 800 = 27,600,000 = £27.6$ million

In total the mining company ~~mines~~ mines 83,622 tonnes

$23 \times 1500 = 34500$ @£800

$83,622 - 34500 = 49122$ @£600

$49122 \times 600 = £29.4732$ million

TOTAL in millions = $27.6 + 29.4732 = £57.0732$ million

$= £57.1$ million

$x = 57.1$

Question	Scheme	Marks	AOs
12 (a)	Total mass $= \dfrac{4500(1-(0.98)^{23})}{1-0.98}$ or $\dfrac{4500((0.98)^{23}-1)}{0.98-1}$	M1	3.1b
	$= 83621.86152... = 83600$ (tonnes) (3 sf)	A1	1.1b
		(2)	
(b)	Expected mass in the year 2040 $= 4500(0.98)^{23-1}$	M1	3.4
	$= 2885.268132... = 2890$ (tonnes) (3 sf)	A1	1.1b
		(2)	
(c)	Total cost $= 800(1500(23)) + 600(83621.86152... - 1500(23))$	M1	3.1b
		M1	1.1b
	$= 800(34500) + 600(49121.86152...)$ $= 27600000 + 29473116.91$ $= 57073116.91$		
	$\Rightarrow x =$ (£) 57.1 (million) (3 sf)	A1	3.2a
		(3)	
(c) **Alt 1**	Total cost $= 200(1500(23)) + 600(83621.86152...)$	M1	3.1b
		M1	1.1b
	$= 200(34500) + 600(49121.86152...)$ $= 6900000 + 50173116.91$ $= 57073116.91$		
	$\Rightarrow x =$ (£) 57.1 (million) (3 sf)	A1	3.2a
		(3)	

6. A small company which makes batteries for electric cars has a 10-year plan for growth.

- In year 1 the company will make 2 600 batteries
- In year 10 the company aims to make 12 000 batteries

In order to calculate the number of batteries it will need to make each year, from year 2 to year 9, the company considers two models, Model A and Model B.

$a = 2600$ $d = ?$ $u_{10} = 12000$
$n = 10$

1 2 3 4 5 6 7 8 9 10
2600 12000
9400

In Model A the number of batteries made will increase by the same **number** each year.

(a) Using Model A, determine the number of batteries the company will make in year 2

2600 + 1044 = 3644 batteries

(3)

per year = $9400 \div 9 =$ 1044.44

In Model B the number of batteries made will increase by the same **percentage** each year.

(b) Using Model B, determine the number of batteries the company will make in year 2

Give your answer to the nearest 10 batteries.

$a = 2600$ $n = 10$
$u_{10} = 12000$ $r = ?$

$u_n = ar^{n-1}$
$u_{10} = 2600 \times r^9$
$12000 = 2600 \times r^9$
$\frac{60}{13} = r^9$
$r = \left(\frac{60}{13}\right)^{1/9}$
$r = 1.185225\ldots$

(3)

Sam calculates the total number of batteries made from year 1 to year 10 inclusive using each of the two models.

(c) Calculate the difference between the two totals, giving your answer to the nearest 100 batteries.

(3)

$u_2 = ar$

$= 2600 \times 1.1852\ldots$

$= 3081.5\ldots$

$= 3080$ batteries

c) Arithmetic

$a = 2600$

$d = 1044$

$n = 10$

$S_n = \frac{n}{2}(a + L)$

$S_{10} = \frac{10}{2}(2600 + 12000)$

$= 73,000$

Geometric

$a = 2600$

$r = 1.1852$

$n = 10$

$S_{10} = \frac{2600(1 - 1.1852^{10})}{1 - 1.1852}$

$= 62,741$

difference $= 73000 - 62,741$

$= 10259$

≈ 10300 batteries

Question	Scheme	Marks	AOs
6 (a)	Translates problem into maths $12\,000 = 2600 + 9d \Rightarrow d = (1044.4)$	M1	3.1b
	Uses the AP model to find $2600 + "d"$	M1	3.4
	3 644 or 3 645 (batteries in Year 2)	A1	1.1b
		(3)	
(b)	Translates problem into maths $12\,000 = 2600 \times r^9 \Rightarrow r = (1.185)$	M1	3.1b
	Uses the GP model to find $2600 \times "r"$	M1	3.4
	awrt 3 080 (batteries in Year 2)	A1	1.1b
		(3)	
(c)	Correct attempt at one sum Either $\frac{10}{2}\{2\times 2600 + 9 \times "1044"\}$ or $\frac{2600\left("1.185"^{10} - 1\right)}{"1.185" - 1}$	M1	1.1b
	Attempts both sums and subtracts either way around $\frac{10}{2}\{2\times 2600 + 9 \times "1044"\} - \frac{2600\left("1.185"^{10} - 1\right)}{"1.185" - 1} = \ldots$	dM1	3.1a
	Accept 10 200 or 10 300 batteries	A1	1.1b
		(3)	

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down. 6, 6, 6, 6,

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

a) total time = $6 \times 4 + 6 \times 1.05 + 6 \times 1.05^2$

= 36.915 = 36 mins 55 seconds

b) $r = 5$ $\quad 6 \times 1.05$

$r = 6$ $\quad 6 \times 1.05^2$

$r = 7$ $\quad 6 \times 1.05^3$

$r = n$ $\quad 6 \times 1.05^{n-4}$

Hence, time for rth km is $6 \times 1.05^{r-4}$

c) Race is 20 km

$\underbrace{6, 6, 6, 6}_{24}, \underbrace{6 \times 1.05, 6 \times 1.05^2, \ldots, 6 \times 1.05^{16}}$

$a = 6 \times 1.05$

$r = 1.05$

$n = 16$

$S_{16} = \dfrac{6 \times 1.05 (1 - 1.05^{16})}{1 - 1.05}$

$= 149.04219\ldots$

$S_{20} = 24 + S_{16} = 173.04\ldots$

$= 173$ mins 3 secs.

-173

$\times 60$

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$24 + (6 \times 1.05) + (6 \times 1.05^2)$ minutes	M1	This mark is for a method to find the time taken for the competitor to run 6 km
	= 96.915 minutes = 36 minutes 55 seconds	A1	This mark is given for finding the total time as required
(b)	For example, 5th km $= 6 \times 1.05^1$ 6th km $= 6 \times 1.05^2$ 7th km $= 6 \times 1.05^3$ … rth km $= 6 \times 1.05^{r-4}$	B1	This mark is given for showing the time taken to run the rth km, as required
(c)	$24 + \sum_{r=5}^{20} 6 \times 1.05^{r-4}$	M1	This mark is given for showing the total time to run the race is the time taken for the first 4 km added to the time taken from 5th to 20th km
	$= 24 + 6.3 \times \frac{(1.05^{16} - 1)}{1.05 - 1}$	M1	This mark is given for using $s = a\left(\frac{1 - r^n}{1 - r}\right)$ where $a = 6 \times 1.05 = 6.3$, $r = 1.05$ and $n = 20 - 4 = 16$
	= 24 + 149.04	A1	This mark is given for a correct total time (represented decimally)
	= 173 minutes and 3 seconds	A1	This mark is given for finding a correct total time given in minutes in seconds