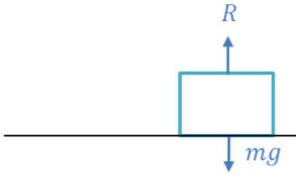
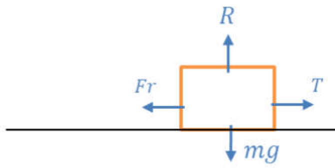
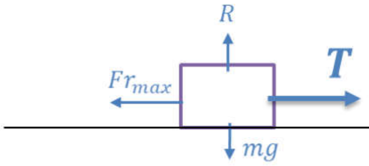
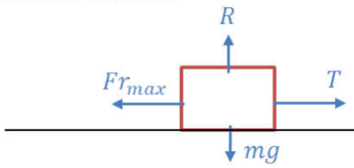
Friction

Scenario 1: A block is on a horizontal rough surface with no forces (other than gravity) acting on it. 	**Comment regarding friction:**	**Equilibrium?**
Scenario 2: A cable is attached to the block and a force applied. The block doesn't move. 	**Comment regarding friction:**	**Equilibrium?**
Scenario 3: The tension is increased until the block starts to move. 	**Comment regarding friction:**	**Equilibrium?**
Scenario 4a/4b: The tension is increased until the block starts to move. 	**Comment regarding friction:**	**a) Equilibrium?** **b) Equilibrium?**

The maximum friction between two surfaces:

$$Fr_{max} = \mu R$$

where μ is the coefficient of friction and R is the normal reaction between two surfaces.

This 'maximum friction' depends on two things:

- How **rough** the surface is (i.e. the rougher the surface, the more force required before the block starts moving).
- How hard the block is pressing against the surface (and more formally, by application of Newton's 3rd Law, how large the **reaction force** $\boldsymbol{R}$ is).

Maximum friction = coefficient of friction (μ) × normal reaction (R)

Maximum $F_r = \mu R$

Note, the coefficient of friction is always greater than 0 and usually less than about 1.5.
The coefficient of friction is specific for the particle and the surface it is on.
A box on sandpaper would have a high coefficient of friction.
A box on ice would have a low coefficient of friction.
A box on a smooth surface would have a coefficient of friction = 0 (i.e. there is no friction)

Friction acts in the **opposite direction to its motion** (obviously)
Friction can be less than this maximum value **if the particle is not moving**.

Materials	Coeff. of Static Friction μ_s
Steel on Steel	0.74
Aluminum on Steel	0.61
Copper on Steel	0.53
Rubber on Concrete	1.0
Wood on Wood	0.25-0.5
Glass on Glass	0.94
Waxed wood on Wet snow	0.14
Waxed wood on Dry snow	-
Metal on Metal (lubricated)	
Ice on Ice	0.1
Teflon on Teflon	0.04
Synovial joints in humans	0.01

R = 15

μ = 0.3

What happens to the box...

... if a force of 10N is applied?

... if a force of 3N is applied?

... if a force of 4.5N is applied?

What is the value of the friction...

... if an opposing force of 2N is applied?

... if an opposing force of 4.5N is applied?

... if an opposing force of 14N is applied?

A block of mass 5kg lies at rest on rough horizontal ground. The coefficient of friction between the block and the ground is 0.4. A horizontal force P is applied to the block. Find the magnitude of the friction force acting on the block when the magnitude of P is

a) 10N
b) 19.6N
c) 30N

A particle of mass 5kg is pulled along a rough horizontal surface by a horizontal force of magnitude 20N. The coefficient of friction between the particle and the floor is 0.2. Calculate:

(a) the magnitude of frictional force
(b) the acceleration of the particle.

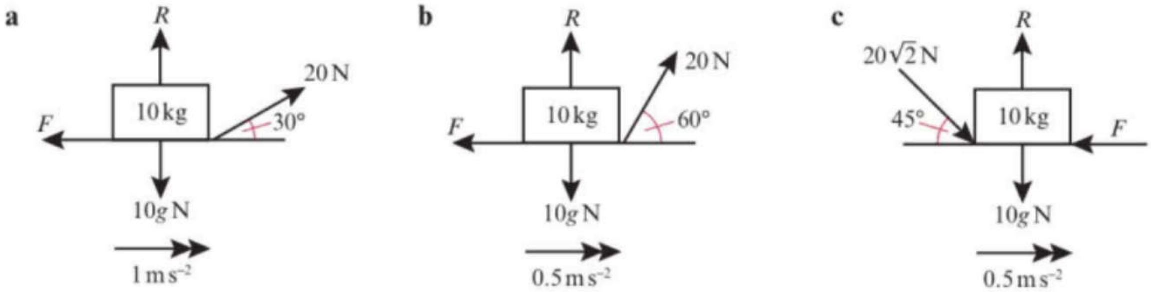
Friction - slopes

A particle of mass 2kg is sliding down a rough slope that is inclined at 30° to the horizontal. Given that the acceleration of the particle is $1ms^{-2}$, find the coefficient of friction μ between the particle and the slope.

Recall that friction acts in opposite direction to motion.

Your Turn

2 In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. R is the normal reaction and F is the frictional force. Find the normal reaction and the coefficient of friction in each case.



3 A particle of mass 0.5 kg is sliding down a rough slope that is angled at 15° to the horizontal. The acceleration of the particle is $0.25\,\text{m s}^{-2}$. Calculate the coefficient of friction between the particle and the slope. **(3 marks)**

4 A particle of mass 2 kg is sliding down a rough slope that is angled at 20° to the horizontal. A force of magnitude P acts parallel to the slope and is attempting to pull the particle up the slope. The acceleration of the particle is $0.2\,\text{m s}^{-2}$ down the slope and the coefficient of friction between the particle and the slope is 0.3. Find the value of P. **(4 marks)**

5 A particle of mass 5 kg is being pushed up a rough slope that is angled at 30° to the horizontal by a horizontal force P. Given that the coefficient of friction is 0.2 and the acceleration of the particle is $2\,\text{m s}^{-2}$ calculate the value of P.

6 A sled of mass 10 kg is being pulled along a rough horizontal plane by a force P that acts at an angle of 45° to the horizontal. The coefficient of friction between the sled and the plane is 0.1. Given that the sled accelerates at $0.3\,\text{m s}^{-2}$, find the value of P.

(7 marks)

7 A train of mass m kg is travelling at $20\,\text{m s}^{-1}$ when it applies its brakes, causing the wheels to lock up. The train decelerates at a constant rate, coming to a complete stop in 30 seconds.

Problem-solving

Use the formulae for constant acceleration.

← **Year 1, Chapter 9**

a By modelling the train as a particle, show that the coefficient of friction between the railway track and the wheels of the train is $\mu = \frac{2}{3g}$. **(6 marks)**

The train is no longer modelled as a particle, so that the effects of air resistance can be taken into account.

b State, with a reason, whether the coefficient of friction between the track and the wheels will increase or decrease in this revised model. **(2 marks)**

Challenge

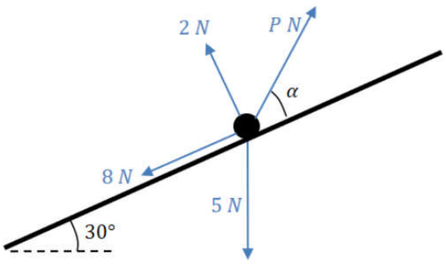
A particle of mass m kg is sliding down a rough slope that is angled at α to the horizontal. The coefficient of friction between the particle and the slope is μ. Show that the acceleration of the particle is independent of its mass.

2 **a** $R = 88\,\text{N}$, $\mu = 0.083$ (2 s.f.)
b $R = 80.679\,\text{N}$, $\mu = 0.062$ (2 s.f.)
c $R = 118\,\text{N}$, $\mu = 0.13$ (2 s.f.)
3 0.242 (3 s.f.)
4 0.778 N (3 s.f.)
5 56.1 N
6 16.5 N (3 s.f.)
7 **a** Use $v = u + at$ to find $a = -\frac{2}{3}\text{ms}^{-2}$
R(→): $-\mu mg = -\frac{2}{3}m$
$\mu = \frac{2}{3g}$
b The coefficient of friction remains unchanged. The air resistance has no effect on the coefficient of friction, which is dependent on the properties of the wheels and the rails.

Challenge
R(↗): $mg\sin\alpha - \mu mg\cos\alpha = ma$
$g\sin\alpha - \mu g\cos\alpha = a$

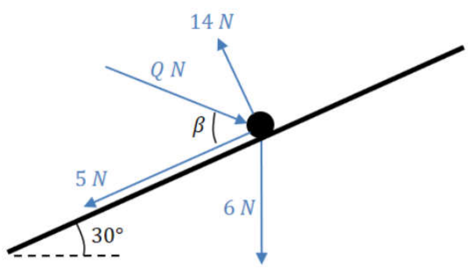
Statics - revisited

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force P and the size of the angle α.



Your Turn

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force Q and the size of the angle β.



Statics revisted - included connected particles

A mass of 3kg rests on the surface of a smooth plane which is inclined at an angle of $45°$ to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1kg freely suspended at the other end. The masses are modelled as particles, and the cable as a light inextensible string. There is a force of P N acting horizontally on the 3kg mass and the system is in equilibrium.

Calculate

(a) the magnitude of P

(b) the normal reaction between the mass and the plane

(c) State how you have used the assumption that the pulley is smooth in your calculations.

Ex 7B
Q8-12

Statics - including friction

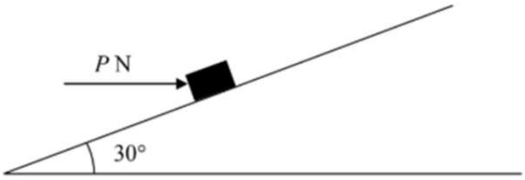
Earlier we saw that the frictional force $F \leq \mu R$, where $F = \mu R$ if the object on the plane is moving. Where the object is not moving, we saw that the **force of friction acts in a direction opposite** to that which it would be moving if the frictional force wasn't there.

A box of mass 8kg rests on a rough horizontal plane. The coefficient of friction between the mass and the plane is 0.5. Find the magnitude of the maximum force P N which acts on this mass without causing it to move if:
(a) The force P is horizontal
(b) The force P acts at an angle 60^o above the horizontal

A box of mass 10kg rests in limiting equilibrium on a rough plane inclined at $20°$ above the horizontal.
(a) Find the coefficient of friction between the box and the plane.
A horizontal force of magnitude P N is applied to the box. Given that the box remains in equilibrium,
(b) find the maximum possible value of P.

Ex 7C
odd

Edexcel M1(Old) Jan 2006 Q5



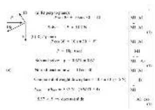
A parcel of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown in Figure 2. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18 N. The coefficient of friction between the parcel and the plane is μ. Find

(*a*) the value of P, **(4)**

(*b*) the value of μ. **(5)**

The horizontal force is removed.

(*c*) Determine whether or not the parcel moves. **(5)**



Dynamics - including friction

A particle is held at rest on a rough plane which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.5. The particle is released and slides down the plane. Find:

(a) the acceleration of the particle.

(b) the distance it slides in the first 2 seconds.

Ex 7E odd

Edexcel M1(Old) May 2013(R) Q5

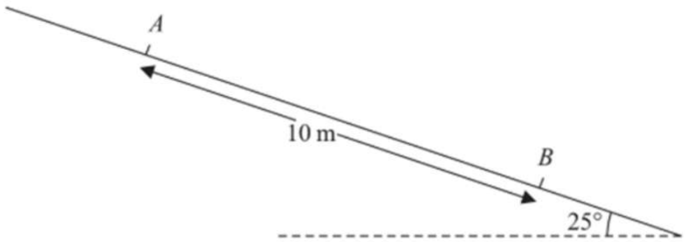


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B, **(3)**

(b) the acceleration of P, **(2)**

(c) the coefficient of friction between P and the plane. **(5)**

Jun 2014 M1

Hint:
Try drawing the diagram reflected!

A rough plane is inclined at 40° to the horizontal. Two points A and B are 3 metres apart and lie on a line of greatest slope of the inclined plane, with A above B, as shown in Figure 2. A particle P of mass m kg is held at rest on the plane at A. The coefficient of friction between P and the plane is $\frac{1}{2}$. The particle is released.

(*a*) Find the acceleration of P down the plane. **(5)**

(*b*) Find the speed of P at B. **(2)**

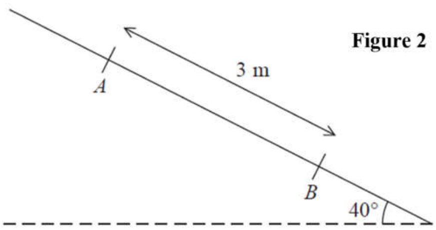


Figure 2

Jan 2013 M1

A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

(9)

Jun 2013 M1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α, where $\tan \alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)

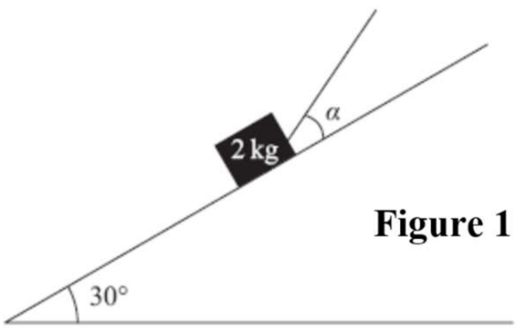


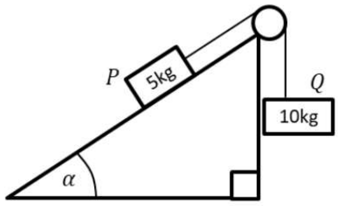
Figure 1

Friction - including connected particles on slopes

Two particles P and Q of masses 5kg and 10kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. P rests on the inclined plane and Q hangs on the edge of the plane with the string vertical and taut. The plane is inclined to the horizontal at an angle α where $\tan\alpha = 0.75$, as shown in the diagram. The coefficient of friction between P and the plane is 0.2. The system is released from rest.

(a) Find the acceleration of the system.

(b) Find the tension in the string.



Resultant force acting on pulley

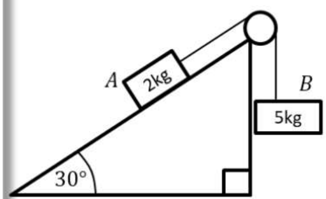
One end of a light inextensible string is attached to a block A of mass 2kg. The block A is held at rest on a **smooth** fixed plane which is inclined to the horizontal at an angle of $30°$. The string lies along the line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a block B of mass 5kg. The system is released from rest. By modelling the blocks as particles and ignoring air resistance,

(a)(i) show that the acceleration of block B is $\frac{4}{7}g$

(ii) find the tension in the string.

(b) State how you have used the fact that the string is inextensible in your calculations.

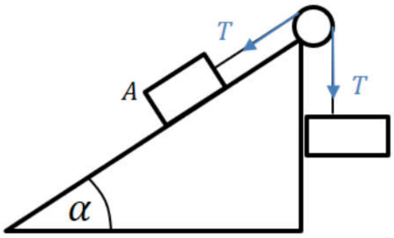
(c) Calculate the magnitude of the force exerted on the pulley by the string.



Part (c) on next slide

Resultant force is

$$2T\cos\left(\frac{90-\alpha}{2}\right)$$



Ex 7F odd

Edexcel M1(Old) May 2013(R) Q3

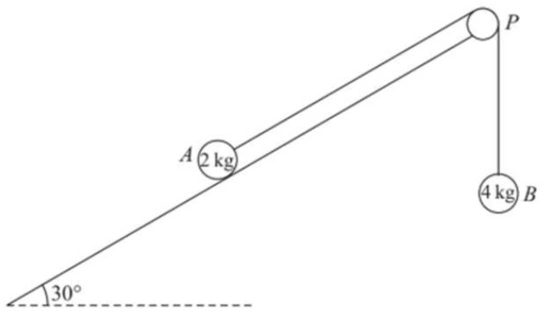


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)