A Level · Edexcel · Further Maths

39 mins 5 questions

5.2 Methods in Calculus

5.2.1 Improper Integrals / 5.2.2 Mean Value of a Function / 5.2.3 Integrating with Partial Fractions / 5.2.4 Calculus involving Inverse Trig / 5.2.5 Integration by Substitution

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Total Marks **/39**

1 (a)

$$f(x) = \frac{x+2}{x^2+9}$$

(a) Show that

$$\int f(x)\mathrm{d}x = A\ln(x^2 + 9) + B\arctan\left(\frac{x}{3}\right) + \mathrm{c}$$

where c is an arbitrary constant and A and B are constants to be found.

(4 marks)

(b) (b) Hence show that the mean value of $f(x)$ over the interval [0, 3] is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi$$

(3 marks)

(c) (c) Use the answer to part (b) to find the mean value, over the interval [0, 3], of

$$f(x) + \ln k$$

where k is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, where p and q are constants and q is in terms of k.

(2 marks)

2 (a) a) Explain why $\int_1^{\infty} \frac{1}{x(2x+5)} \mathrm{d}x$ is an improper integral.

(1 mark)

(b) (b) Prove that

$$\int_1^{\infty} \frac{1}{x(2x+5)} \mathrm{d}x = a\ln b$$

where a and b are rational numbers to be determined.

(6 marks)

3 (a) (a) $y = \tan^{-1} x$

Assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

(3 marks)

(b) $f(x) = x \tan^{-1} 4x$

(b) Show that

$$\int f(x)dx = Ax^2 \tan^{-1} 4x + Bx + C \tan^{-1} 4x + k$$

where k is an arbitrary constant and A, B and C are constants to be determined.

(5 marks)

(c) (c) Hence find, in exact form, the mean value of $f(x)$ over the interval $\left[0, \frac{\sqrt{3}}{4}\right]$

(2 marks)

4 Show that

$$\int_0^{\infty} \frac{8x-12}{(2x^2+3)(x+1)}\,\mathrm{d}x = \ln k$$

where k is a rational number to be found

(7 marks)

5 (a)

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)\mathrm{d}x = A \sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4 marks)

(b) (b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval [0, 3].

(2 marks)