A Level · Edexcel · Maths

4 hours 42 questions

8.2 Further Integration (A Level only)

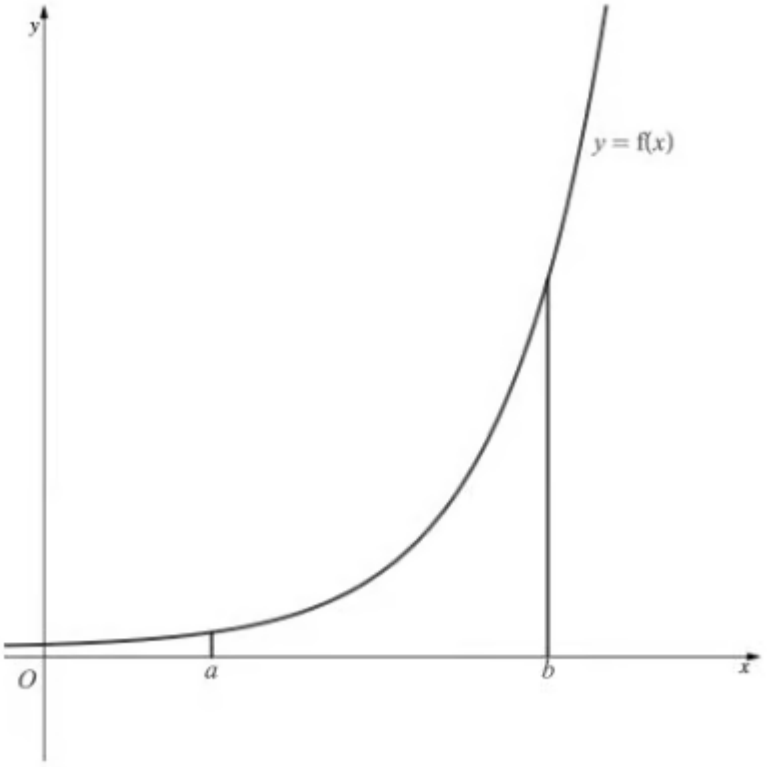
Easy (10 questions)	/53
Medium (11 questions)	/63
Hard (11 questions)	/67
Very Hard (10 questions)	/69
Total Marks	**/252**

Scan here to return to the course
or visit savemyexams.com

Easy Questions

1 A student is estimating the area bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$.

The student intends to estimate the area by using trapezia of equal width.



Add to the diagram above to show how the student can use 4 trapezia to estimate the area.

(2 marks)

2 (i) Find the integral

$$\int \frac{1}{x}\ dx.$$

(ii) Use calculus to evaluate

$$\int_0^1 e^x \ \mathrm{d}x.$$

(iii) Find an expression for y given that

$$y = \int 3\cos\theta \ \mathrm{d}\theta .$$

(6 marks)

3 (i) Integrate

$$\int 8(2x-1)^3 \ dx.$$

(ii) Use calculus to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sin 2x \ dx.$$

(iii) Find an expression for y given that

$$\frac{dy}{dx} = 3e^{3x}.$$

(8 marks)

4 (a) Given that $u = 3x + 2$ show that

(i) $\mathrm{d}u = 3\ \mathrm{d}x,$

(ii) $\int 3\cos(3x+2)\ \mathrm{d}x = \int \cos u\ \mathrm{d}u.$

(3 marks)

(b) Hence find an expression in terms of x for the integral

$$\int 3\cos(3x+2)\,\mathrm{d}x\,.$$

(3 marks)

5 (a) Given the identity $\cos 2A \equiv 1 - 2\sin^2 A$, show that

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A).$$

(2 marks)

(b) Hence find the exact value of

$$\int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx.$$

(3 marks)

6 Use the substitution $u = 4x + 1$ to find

$$\int_{2}^{6} 4(4x+1)^{\frac{1}{2}} dx$$

(4 marks)

7 (i) Given that $f(x) = 2x^2 + 5$, find $f'(x)$.

(ii)

Hence, or otherwise, find

$$\int \frac{4x}{2x^2+5}\,\mathrm{d}x.$$

(4 marks)

8 Use integration by parts to find an expression for

$$\int -3x\sin x\,\mathrm{d}x.$$

(6 marks)

9 Find the exact value of

$$\int_1^2 e^{3x+2}\,\mathrm{d}x$$

(5 marks)

10 (a) Show that

$$\frac{2x-1}{(x+1)(x-2)}$$

can be written in the form

$$\frac{A}{x+1}+\frac{B}{x-2}$$

(3 marks)

(b) Hence find

$$\int \frac{2x-1}{(x+1)(x-2)}\,\mathrm{d}x \qquad x>2$$

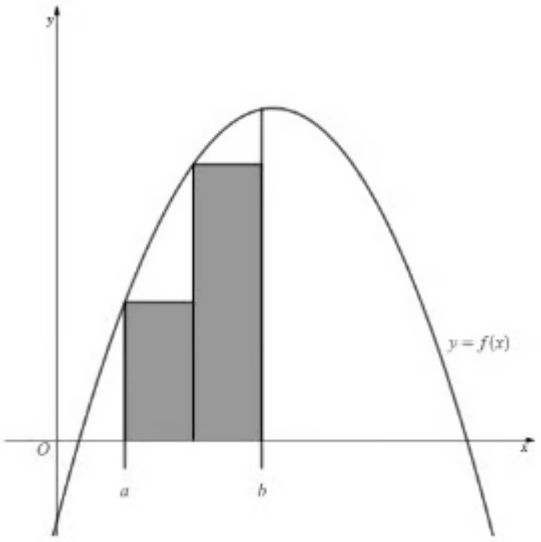
writing your answer as a single logarithm.

(4 marks)

Medium Questions

1 A student is estimating the area bounded by the curve y=fx, the x-axis and the lines $x = a$ and $x = b$.

The student intends to find the area of two rectangles of equal width in order to estimate the area as shown in the diagram below.



By drawing a sketch, show how the student's estimate of the area can be improved while still using rectangles of equal width.

(2 marks)

2 (a) Find the integral

$$\int \sin x \ \mathrm{d}x$$

(1 mark)

(b) Use calculus to evaluate

$$\int_1^4 \frac{1}{x} \mathrm{d}x$$

(3 marks)

(c) Find an expression for y given that

$$y = \int 7e^{7x} \, dx$$

(2 marks)

3 (a) Integrate

$$\int \cos 2x \ \mathrm{d}x$$

(2 marks)

(b) Use calculus to find the value of

$$\int_0^2 (3x-1)^3 \ \mathrm{d}x$$

(4 marks)

(c) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{5x}$$

(2 marks)

4 Use a suitable substitution to find

$$\int -15\sin(5x-2) \ \mathrm{d}x$$

(5 marks)

5 Use calculus and the substitution $u = x + 4$ to show that

$$\int_1^2 \frac{x}{x+4}\,\mathrm{d}x = 1 + 4\ln\frac{5}{6}$$

(7 marks)

6 Given that

$$\cos 2\theta \equiv 2\cos^2\theta - 1$$

use calculus to find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

(6 marks)

7 Find

$$\int \sqrt{1+cot^2 x}\ \mathrm{d}x$$

(3 marks)

8 (i) Given that $\mathrm{f}(x)=2x^3+4x$, find $\mathrm{f}'(x)$.

(ii) Hence, or otherwise, find

$$\int \frac{3x^2+2}{2x^3+4x}\ \mathrm{d}x$$

(4 marks)

9 Use integration by parts to find, in terms of e, the exact value of

$$\int_0^1 (5x-4)e^{3x}\ \mathrm{d}x$$

(6 marks)

10 (a) Show that

$$\frac{11}{(2x-3)(x+4)}$$

can be written in the form

$$\frac{A}{2x-3}+\frac{B}{x+4}$$

(3 marks)

(b) Hence find

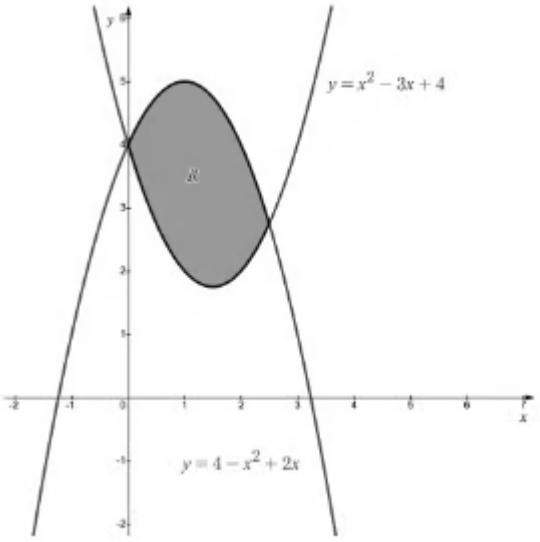
$$\int \frac{11}{(2x-3)(x+4)}\,\mathrm{d}x$$

writing your answer as a single logarithm (plus a constant of integration).

(4 marks)

11 (a) The diagram below shows a sketch of the curves with equations

$$y = x^2 - 3x + 4 \text{ and } y = 4 - x^2 + 2x$$



Find the x-coordinates of the intersections of the two graphs.

(2 marks)

(b) Show that the area of the shaded region labelled R is given by

$$\int_0^{\frac{5}{2}} (5x - 2x^2)\, \mathrm{d}x$$

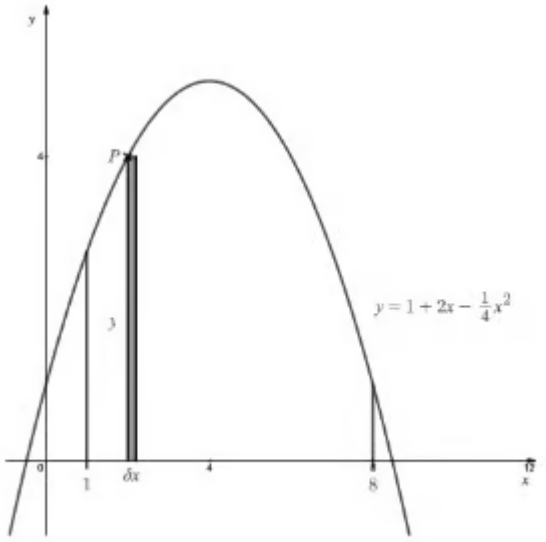
(2 marks)

(c) Use calculus to find the area of the shaded region labelled R.

(5 marks)

Hard Questions

1 The diagram below shows a sketch of the curve with equation $y = 1 + 2x - \frac{1}{4}x^2$.



The point $P(x, y)$ lies on the curve.

The shaded rectangle shown has width δx and height y.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=1}^{8} \left(1 + 2x - \frac{1}{4}x^2\right) \delta x$$

(3 marks)

2 (a) Find the integral

$$\int 5(e^{5x} - e^{-5x})\,\mathrm{d}x$$

(2 marks)

(b) Find an expression for y given that

$$y = \int (\sin x + \cos x)\ \mathrm{d}x$$

(2 marks)

(c) Use calculus to evaluate

$$\int_{-8}^{-2} \frac{1}{x}\,\mathrm{d}x$$

giving your answer as a single logarithm.

(3 marks)

3 (a) Integrate

$$\int 2\sin x \cos x \ \mathrm{d}x$$

(2 marks)

(b) Use calculus to find the value of

$$\int_1^3 (4x+1)^5 \ \mathrm{d}x$$

(4 marks)

4 Use a suitable substitution to find the following

$$\int 8x \sin\left(3x^2+1\right) \ \mathrm{d}x$$

(5 marks)

5 Use the substitution $u = 2 + \ln x$ to show that

$$\int \frac{1}{x(2+\ln x)^3}\,\mathrm{d}x = \frac{-1}{2(\ln x\ +2)^2} + c$$

where c is the constant of integration.

(5 marks)

6 (a) Show that, for $\theta \neq k\pi$ (where k is an integer),

$$\frac{2-2\cos^2\theta}{\sin 2\theta} = \tan\theta$$

(3 marks)

(b) Use calculus and your result from part (a) to show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2-2\cos^2\theta}{\sin 2\theta}\,\mathrm{d}\theta = \frac{1}{2}\ln 3$$

(4 marks)

7 Use calculus to find the exact value of

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \frac{2\cos x}{1-\cos 2x}\,\mathrm{d}x.$$

(6 marks)

8 Use calculus to show that

$$\int_0^1 \frac{3xe^{-3x^2}}{3-2e^{-3x^2}}\,\mathrm{d}x = \frac{1}{4}\ln\left(3-2e^{-3}\right).$$

(5 marks)

9 (a) Use integration by parts to find

$$\int (2x^2 - 1)e^x \, \mathrm{d}x$$

(5 marks)

(b) Show that

$$\int \ln x \ \mathrm{d}x = x\ln x - x + c$$

where c is the constant of integration.

(4 marks)

10 (a) Express

$$\frac{x^2-4x+7}{(x-1)(x-3)^2}$$

as partial fractions.

(4 marks)

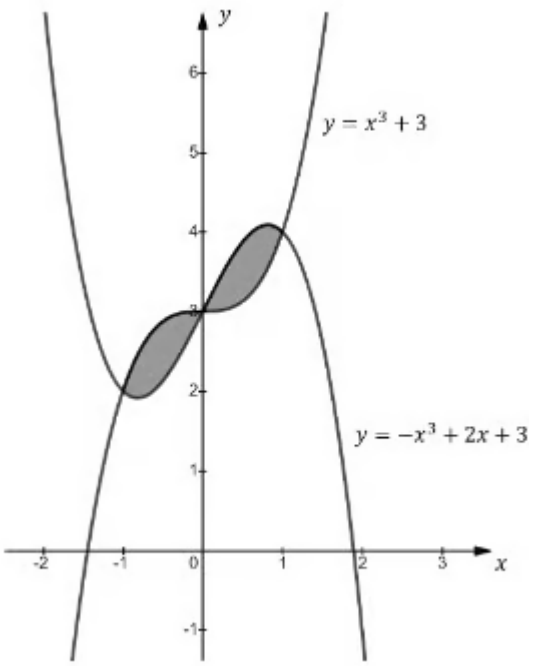
(b) Hence, or otherwise, find

$$\int \frac{x^2-4x+7}{(x-1)(x-3)^2}\,\mathrm{d}x$$

(3 marks)

11 (a) The diagram below shows a sketch of the curves with equations

$$y = x^3 + 3 \text{ and } y = -x^3 + 2x + 3$$



Find the x-coordinates of the points of intersection of the two graphs.

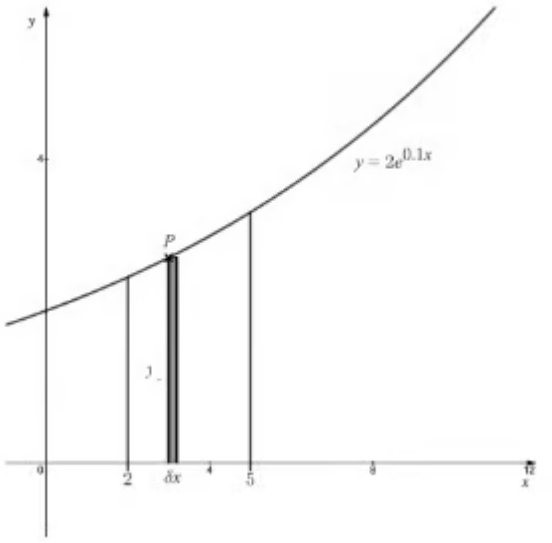
(2 marks)

(b) Use calculus to find the total shaded area enclosed by the two graphs.

(5 marks)

Very Hard Questions

1 The diagram below shows a sketch of the curve with equation $y = 2e^{0.1x}$.



A point $P(x, y)$ lies on the curve.

The shaded rectangle shown has width δx and height y.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=2}^{5} 2e^{0.1x}\, \delta x$$

(3 marks)

2 (a) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 5\cos^2 4x \sin 4x$$

(3 marks)

(b) Integrate

$$\int 3x(5x^2 + 4)^4 \, \mathrm{d}x$$

(3 marks)

3 (a) Find an expression for y given that

$$y = \int 6x^2 e^{x^3} \, \mathrm{d}x$$

(3 marks)

(b) Integrate

$$\int (16 - 32x)\sin(4x - 2)^2 \, \mathrm{d}x$$

(3 marks)

4 Use calculus and the substitution $x = \cos\theta$ to find the exact value of

$$\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1 - x^2}} \mathrm{d}x$$

(5 marks)

5 Find

$$\int (2\tan x + 3\sec x)^2 \, \mathrm{d}x$$

(5 marks)

6 (a) Show that

$$\int \tan kx \, \mathrm{d}x = \frac{1}{k} \ln |\sec kx| + c$$

where k is a constant, and c is the constant of integration.

(3 marks)

(b) Use calculus to find the exact value of

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{\operatorname{cosec}^2 3\theta}{3 \cot 3\theta} \, \mathrm{d}\theta$$

writing your answer in the form $a \ln b$, where a and b are rational numbers to be found.

(5 marks)

7 (a) Find

$$\int x^2 \sin 3x \ \mathrm{d}x$$

(6 marks)

(b) Find

$$\int \frac{\ln x}{x^3} \ \mathrm{d}x$$

(4 marks)

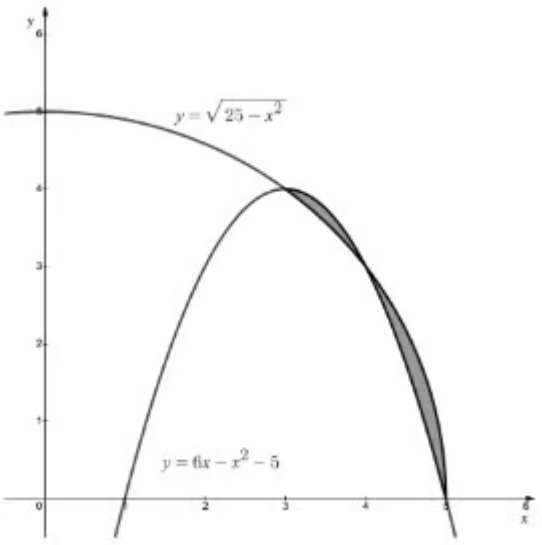
8 Find the integral

$$\int \frac{8x^2 - 8x - 1}{(4x^2 - 1)(x - 2)} \ \mathrm{d}x$$

(6 marks)

9 (a) The diagram below shows a sketch of the curves with equations

$$y = \sqrt{25 - x^2} \quad \text{and} \quad y = 6x - x^2 - 5$$



Show that the two curves intersect at the points (3, 4), (4, 3) and (5, 0).

(1 mark)

(b) By using the substitution $x = 5\sin u$, show that

$$\int \sqrt{25 - x^2}\, \mathrm{d}x = \frac{25\arcsin\left(\frac{x}{5}\right) + x\sqrt{25 - x^2}}{2} + c$$

where c is the constant of integration.

(7 marks)

(c) Using calculus, and your results from parts (a) and (b), show that the total shaded area enclosed by the two curves in the diagram is equal to

$$\frac{25\pi}{4} - 4 - \frac{25}{2}\left(2\arcsin\left(\frac{4}{5}\right) - \arcsin\left(\frac{3}{5}\right)\right) \text{units}^2$$

(5 marks)

10 Use integration by parts to show that

$$\int e^x \sin x \ \mathrm{d}x = \frac{1}{2}e^x(\sin x - \cos x) + c$$

where c is the constant of integration.

(7 marks)