OCR A Level Physics

Your notes

Scalars & Vectors

Contents

Scalars & Vectors

Your notes

Scalars & Vectors

- All quantities can be one of two types:
  - a **scalar**
  - a **vector**

Scalars

- Scalars are quantities that have **magnitude** but not direction
  - For example, **mass** is a **scalar** quantity because it has **magnitude** but no direction

Vectors

- Vectors are quantities that have both **magnitude** and **direction**
  - For example, **weight** is a **vector** quantity because it is a force and has **both** magnitude and direction

Distance and displacement

- **Distance** is a measure of how far an object has travelled, **regardless of direction**
  - Distance is the total length of the path taken
  - Distance, therefore, has a **magnitude** but **no direction**
  - So, distance is a **scalar** quantity
- **Displacement** is a measure of how far it is between two points in space, **including the direction**
  - Displacement is the **length** and **direction** of a **straight line** drawn from the starting point to the finishing point
  - Displacement, therefore, has a **magnitude** and a **direction**
  - So, displacement is a **vector** quantity

What is the difference between distance and displacement?

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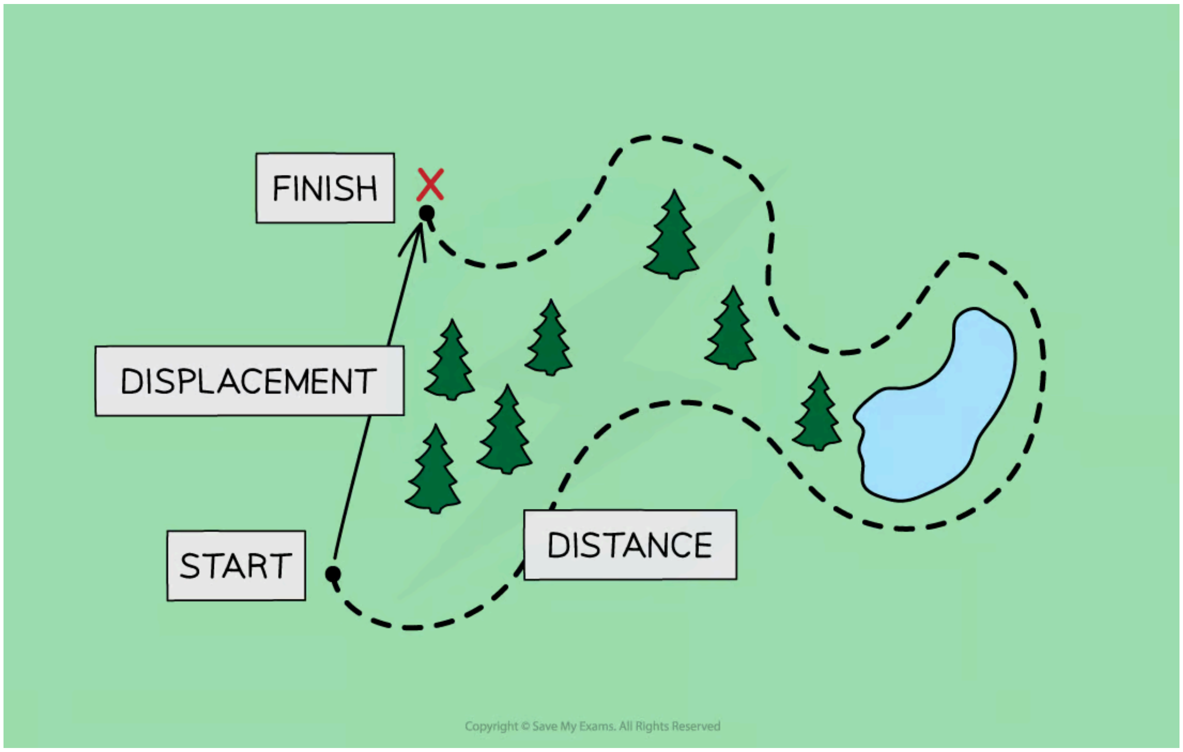

Displacement is a vector quantity while distance is a scalar quantity

- When a student travels to school, there will probably be a **difference** in the distance they travel and their displacement
  - The **overall distance** they travel includes the total lengths of all the roads, including any twists and turns
  - The **overall displacement** of the student would be a straight line between their home and school, regardless of any obstacles, such as buildings, lakes or motorways, along the way

Speed and velocity

- **Speed** is a measure of the **distance** travelled by an object per unit time, **regardless of the direction**
  - The speed of an object describes how fast it is moving, but **not** the direction it is travelling in
  - Speed, therefore, has **magnitude** but **no direction**
  - So, speed is a **scalar** quantity
- **Velocity** is a measure of the **displacement** of an object per unit time, **including the direction**
  - The velocity of an object describes how fast it is moving **and** which direction it is travelling in

- An object can have a **constant speed** but a **changing velocity** if the object is **changing direction**
- Velocity, therefore, has **magnitude and direction**
- So, velocity is a **vector** quantity

Examples of scalars & vectors

- The table below lists some common examples of scalar and vector quantities

Table of scalars and vectors

Scalars	Vectors
distance	displacement
speed	velocity
mass	acceleration
time	force
energy	momentum
volume	
density	
pressure	
electric charge	
temperature	

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Combining Vectors

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Combining Vectors

- **Vectors** are represented by an arrow
  - The arrowhead indicates the **direction** of the vector
  - The length of the arrow represents the **magnitude**
- Vectors can be combined by **adding** or **subtracting** them to produce the resultant vector
  - The resultant vector is sometimes known as the 'net' vector (eg. the net force)
- There are two methods that can be used to add vectors
  - **Calculation** – if the vectors are perpendicular
  - **Scale drawing** – if the vectors are not perpendicular

Vector Calculation

- Vector calculations will be limited to two vectors at right angles
- This means the combined vectors produce a right-angled triangle and the magnitude (length) of the resultant vector is found using **Pythagoras' theorem**

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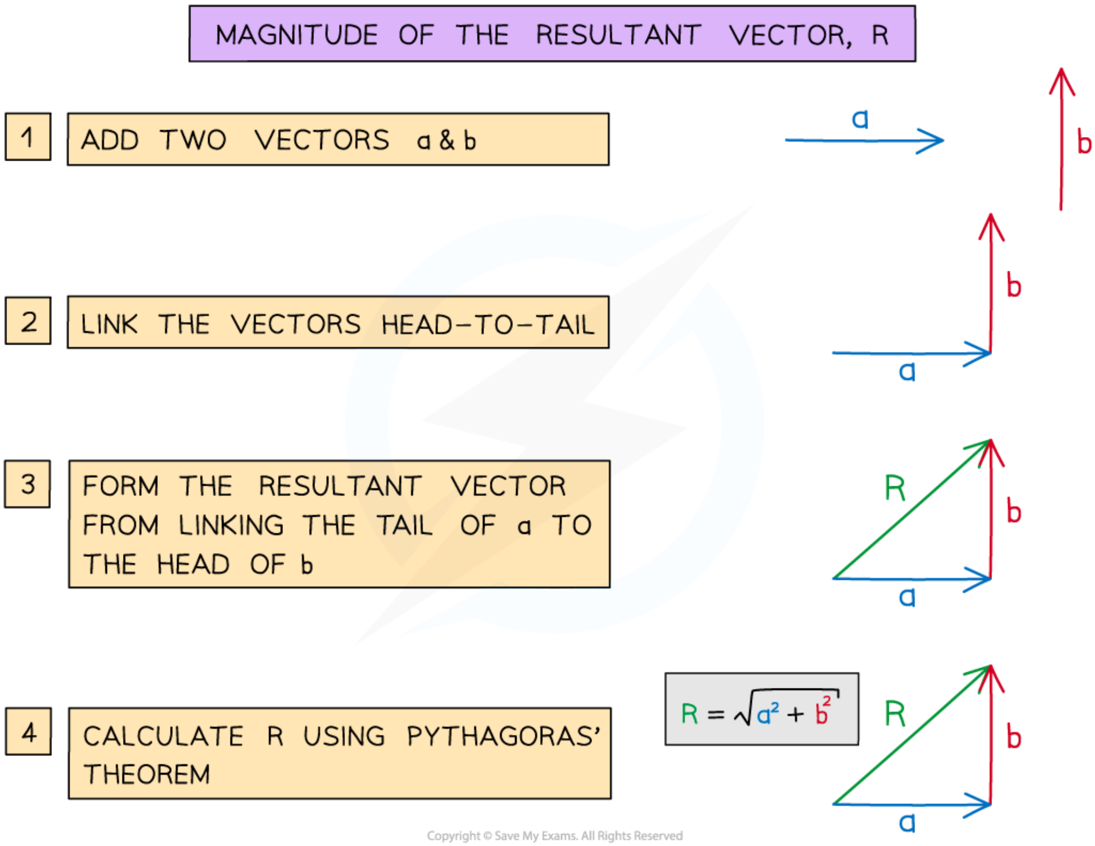

The magnitude of the resultant vector is found by using Pythagoras' Theorem

- The direction of the resultant vector is found from the angle it makes with the horizontal or vertical
  - The question should imply which angle it is referring to (ie. Calculate the angle from the x-axis)
- Calculating the angle of this resultant vector from the horizontal or vertical can be done using **trigonometry**
  - Either the sine, cosine or tangent formula can be used depending on which vector magnitudes are calculated

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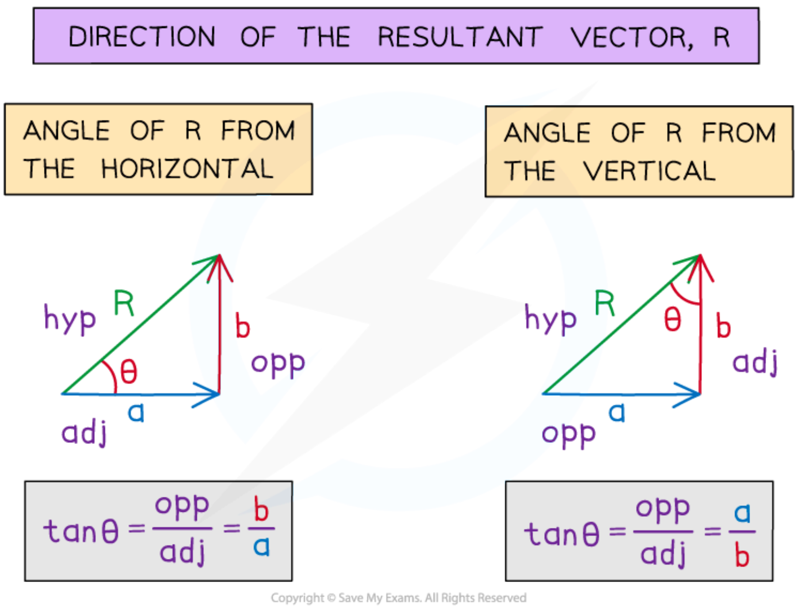
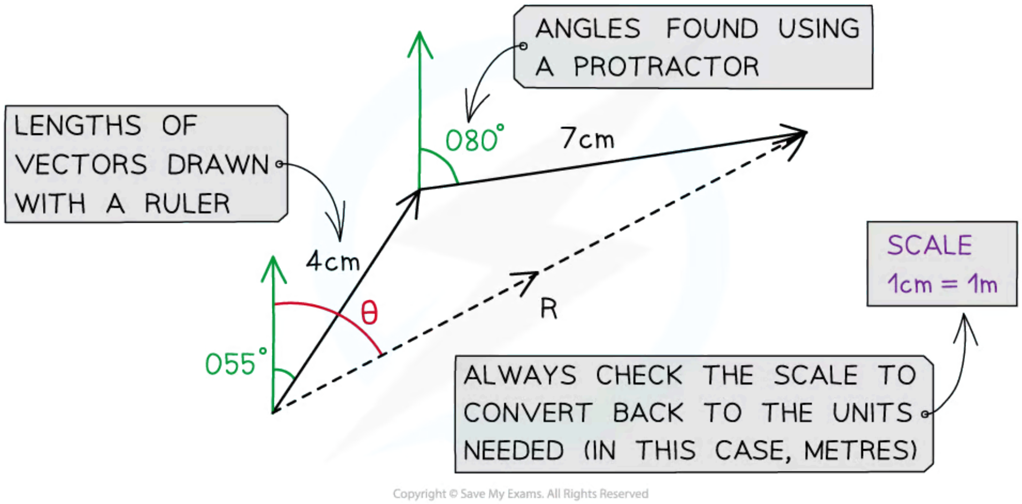


The direction of vectors is found by using trigonometry

Scale Drawing

- When two vectors are not at right angles, the resultant vector can be calculated using a scale drawing
  - **Step 1:** Link the vectors head-to-tail if they aren't already
  - **Step 2:** Draw the resultant vector using the triangle or parallelogram method
  - **Step 3:** Measure the length of the resultant vector using a ruler
  - **Step 4:** Measure the angle of the resultant vector (from North if it is a bearing) using a protractor



A scale drawing of two vector additions. The magnitude of resultant vector R is found using a rule and its direction is found using a protractor

- Note that with scale drawings, a scale may be given for the diagram such as 1 cm = 1 km since only limited lengths can be measured using a ruler
- The final answer is always converted back to the units needed in the diagram
  - Eg. For a scale of 1 cm = 2 km, a resultant vector with a length of 5 cm measured on your ruler is actually 10 km in the scenario
- There are two methods that can be used to combine vectors: the **triangle method** and the **parallelogram method**
- To combine vectors using the triangle method:
  - **Step 1:** link the vectors head-to-tail
  - **Step 2:** the resultant vector is formed by connecting the tail of the first vector to the head of the second vector
- To combine vectors using the parallelogram method:
  - **Step 1:** link the vectors tail-to-tail
  - **Step 2:** complete the resulting parallelogram
  - **Step 3:** the resultant vector is the diagonal of the parallelogram

Vector Addition

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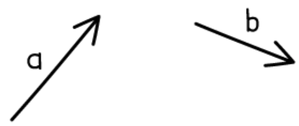
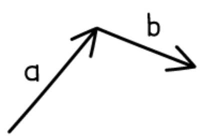
Draw the vector $c = a + b$

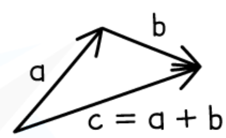


TRIANGLE METHOD

STEP 1: LINK THE VECTORS HEAD-TO-TAIL

STEP 2: FORM THE RESULTANT VECTOR FROM LINKING THE TAIL OF a TO THE HEAD OF b

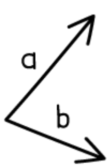


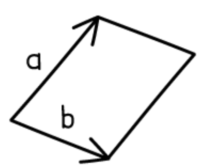


PARALLELOGRAM METHOD

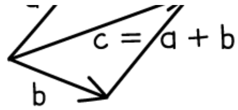
STEP 1: LINK THE VECTORS TAIL-TO-TAIL

STEP 2: COMPLETE THE RESULTING PARALLELOGRAM





STEP 3: THE RESULTANT VECTOR IS THE DIAGONAL OF THE PARALLELOGRAM



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Vector Subtraction

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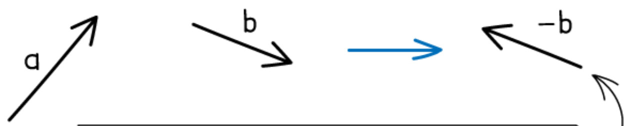
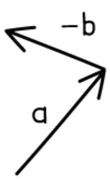
Draw the vector c = a - b



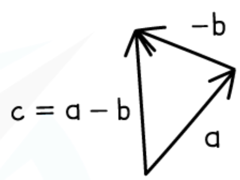
FIRST REVERSE THE DIRECTION OF VECTOR b TO MAKE −b

TRIANGLE METHOD

STEP 1: LINK THE VECTORS HEAD–TO–TAIL

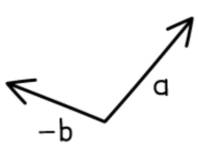


STEP 2: FORM THE RESULTANT VECTOR BY LINKING THE TAIL OF a TO THE HEAD OF −b

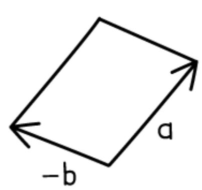


PARALLELOGRAM METHOD

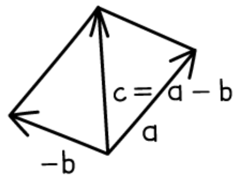
STEP 1: LINK THE VECTORS TAIL–TO–TAIL



STEP 2: COMPLETE THE RESULTING PARALLELOGRAM



STEP 3: THE RESULTANT VECTOR IS THE DIAGONAL OF THE PARALLELOGRAM



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Worked Example

A hiker walks a distance of 6 km due east and 10 km due north. Calculate the magnitude of their displacement and its direction from the horizontal

Answer:

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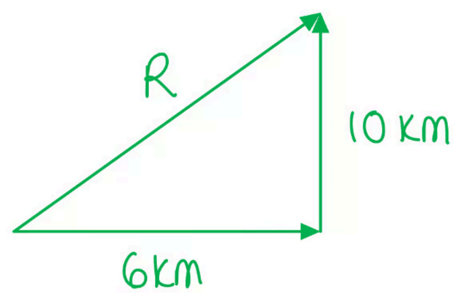
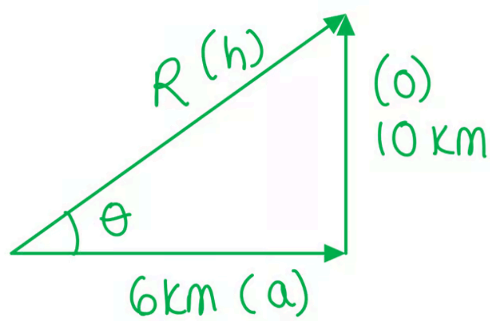
Step 1: Draw a vector diagram



Step 2: Calculate the magnitude of the resultant vector using Pythagoras' Theorem

$$R = \sqrt{6^2 + 10^2}$$

Step 3: Calculate the direction of the resultant vector using trigonometry



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{6}$$

$$\theta = \tan^{-1}\left(\frac{10}{6}\right) = 59°$$

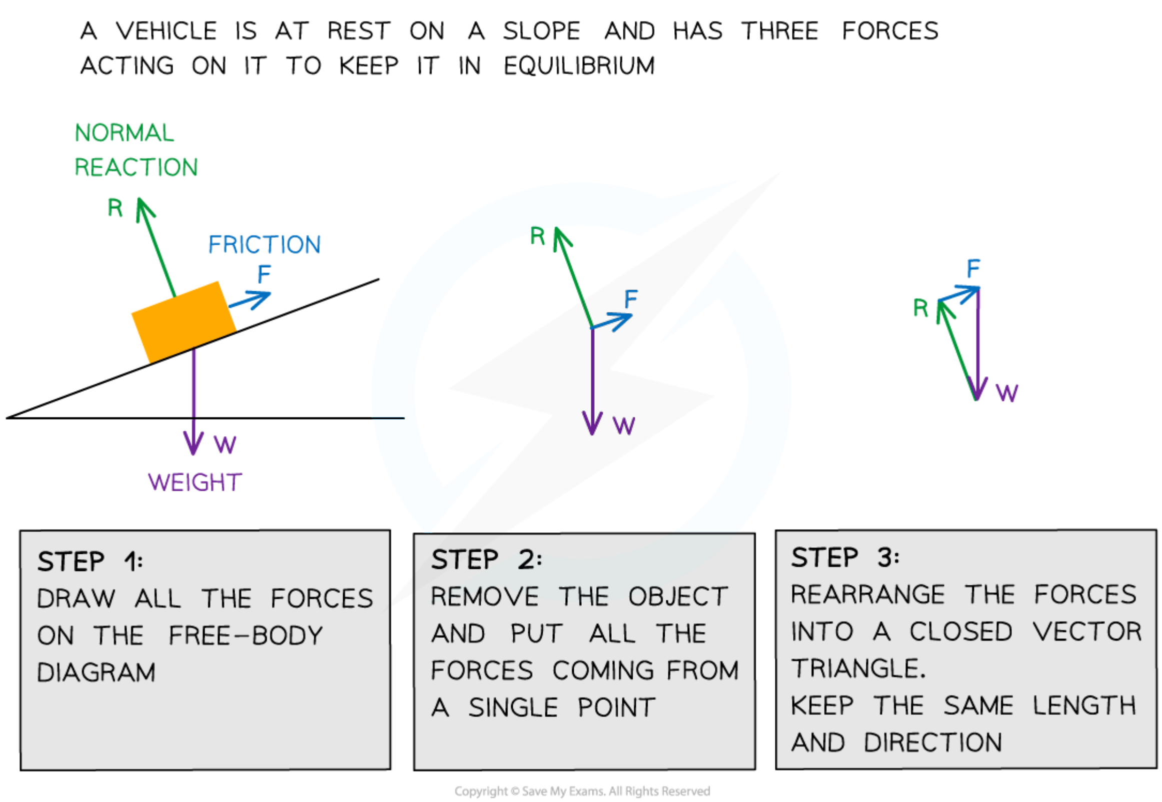
Examiner Tips and Tricks

Pythagoras' Theorem and trigonometry are consistently used in vector addition, so make sure you're fully confident with the maths here!

Combining Vectors with a Vector Triangle

- Coplanar forces can be represented by the resultant of any two vector triangles

- Forces are in equilibrium if an object is either
  - At rest
  - Moving at **constant** velocity
- In equilibrium, coplanar forces are represented by **closed** vector triangles
  - The vectors, when joined together, form a closed path
- The most common forces on objects are
  - Weight
  - Normal reaction force
  - Tension (from cords and strings)
  - Friction
- The forces on a body in equilibrium are demonstrated below:



Three forces on an object in equilibrium form a closed vector triangle

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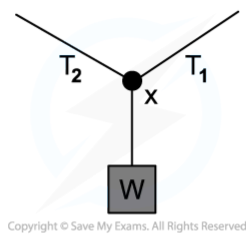
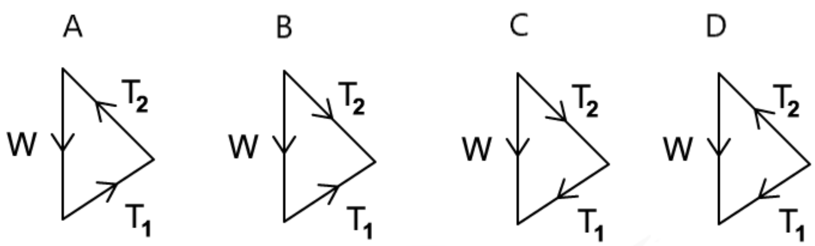
Worked Example

A weight hangs in equilibrium from a cable at point **X**. The tensions in the cables are T_1 and T_2 as shown.



Which diagram correctly represents the forces acting at point **X**?



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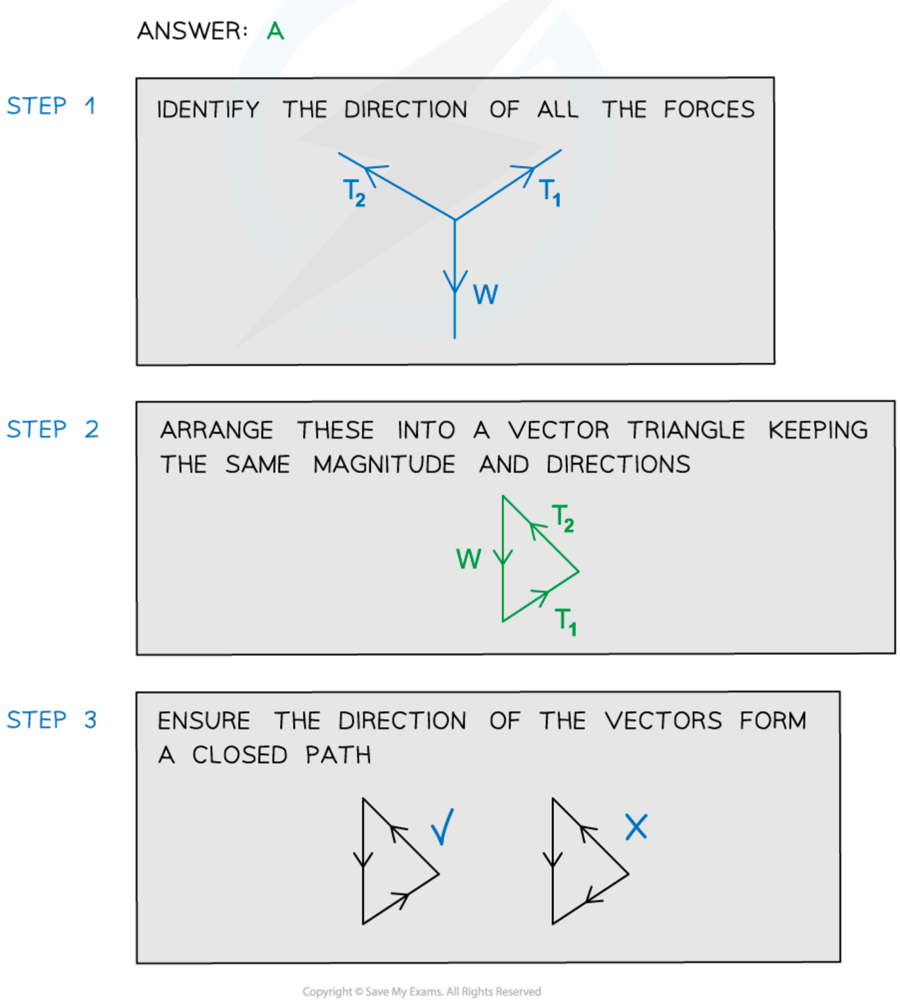


Examiner Tips and Tricks

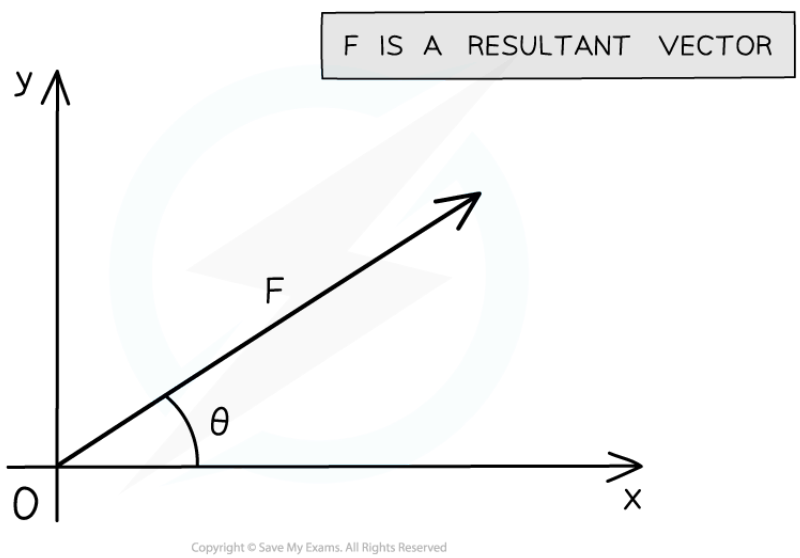
The diagrams in exam questions about this topic tend to be drawn to scale, so make sure you have a ruler handy!

Your notes

Resolving Vectors

Resolving Vectors

- Two vectors can be represented by a single **resultant vector**
  - Resolving a vector is the opposite of adding vectors
- A single resultant vector can be resolved
  - This means it can be represented by **two** vectors, which in combination have the same effect as the original one
- When a single resultant vector is broken down into its **parts**, those parts are called **components**
- For example, a force vector of magnitude *F* and an angle of *θ* to the horizontal is shown below



The resultant force F at an angle θ to the horizontal

- It is possible to **resolve** this vector into its **horizontal** and **vertical** components using trigonometry

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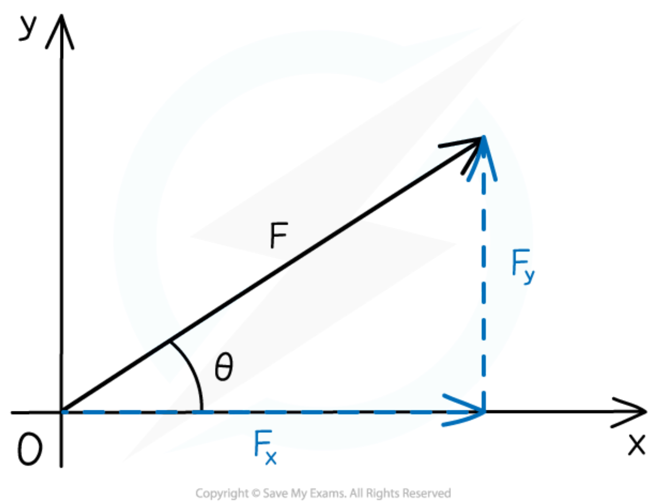


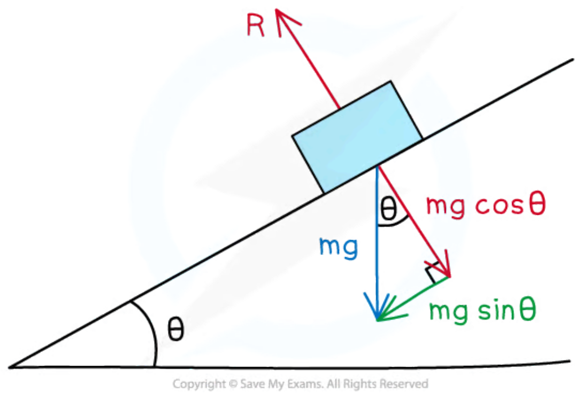
The resultant force F can be split into its horizontal and vertical components

- For the **horizontal** component, $F_x = F\cos\theta$
- For the **vertical** component, $F_y = F\sin\theta$

Example: Forces on an Inclined Plane

- Objects on an inclined plane is a common scenario in which vectors need to be resolved
  - An inclined plane, or a slope, is a flat surface tilted at an angle, θ
- Instead of thinking of the component of the forces as horizontal and vertical, it is easier to think of them as **parallel** or **perpendicular** to the slope
- The **weight** of the object is vertically downwards and the **normal** (or reaction) force, *R* is always vertically up from the object
- The weight *W* is a vector and can be split into the following components:
  - *W* cos (θ) perpendicular to the slope
  - *W* sin (θ) parallel to the slope
- If there is no friction, the force *W* sin (θ) causes the object to move down the slope
- The object is not moving perpendicular to the slope, therefore, the normal force *R* = *W* cos (θ)

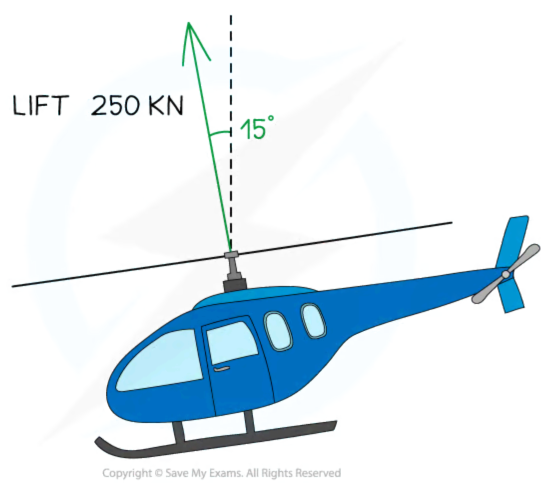
Your notes



The weight vector of an object on an inclined plane can be split into its components parallel and perpendicular to the slope

Worked Example

A helicopter provides a lift of 250 kN when the blades are tilted at 15° from the vertical.



Calculate the horizontal and vertical components of the lift force.

Answer:

Step 1: Draw a vector triangle of the resolved forces

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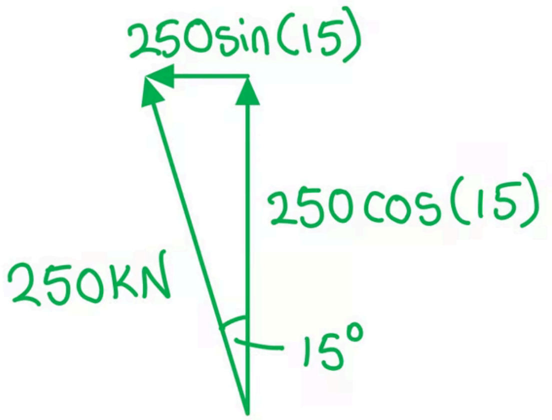

Step 2: Calculate the vertical component of the lift force

$$\text{Vertical} = 250 \times \cos(15) = 242 \text{ kN}$$

Step 3: Calculate the horizontal component of the lift force

$$\text{Horizontal} = 250 \times \sin(15) = 64.7 \text{ kN}$$

Examiner Tips and Tricks

If you're unsure as to which component of the force is cos θ or sin θ, just remember that the cos θ is always the adjacent side of the right-angled triangle AKA, making a 'cos sandwich'