A Level · Edexcel · Maths

3 hours · 40 questions

# 5.8 Trigonometric Proof (A Level only)

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|---|---|
| Easy (10 questions) | /37 |
| Medium (10 questions) | /44 |
| Hard (10 questions) | /50 |
| Very Hard (10 questions) | /65 |
| **Total Marks** | **/196** |

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# Easy Questions

**1** Show that

$$\cot\theta \equiv \frac{\cos\theta}{\sin\theta}$$

**(2 marks)**

**2 (a)** Use the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to show that

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

**(2 marks)**

**(b)** Show by counter-example that

$$\cos 2\theta \not\equiv \cos\theta + \cos\theta$$

**(3 marks)**

**3 (a)** Given that is $\theta$ small and measured in radians, use an appropriate approximation to show that

$$3\sin\theta - 2\cos\theta \approx \theta^2 + 3\theta - 2$$

**(3 marks)**

**(b)** Use the result in part (a) to find an approximation to $3\sin(0.2) - 2\cos(0.2)$.

**(1 mark)**

**4** Prove the identity

$$\frac{\sin 2\theta}{2\sin\theta} \equiv \cos\theta, \qquad \theta \neq k\pi$$

**(2 marks)**

**5** Show that

$$\sin^2\theta(\sec^2\theta + \operatorname{cosec}^2\theta) \equiv \sec^2\theta$$

**(4 marks)**

**6** (i) Use the quotient rule to show that

$$\frac{d}{dx}[\operatorname{cosec} x] = \frac{-\cos x}{\sin^2 x}$$

(ii) Hence show that

$$\frac{d}{dx}[\operatorname{cosec} x] = -\cot x \operatorname{cosec} x$$

**(5 marks)**

**7** Show that

$$3 \sin 2\theta - 2 \sin \theta \equiv 2 \sin \theta (3 \cos \theta - 1)$$

**(3 marks)**

**8** Prove the identity

$$2 \operatorname{cosec} 2x \cot x \equiv \operatorname{cosec}^2 x, \qquad x \neq \frac{k\pi}{2}$$

**(5 marks)**

**9 (a)** Find the value of

(i) $\arccos(\cos(150°))$

(ii) $\arcsin(\sin(210°))$

**(2 marks)**

**(b)** Explain why the answer to part (a) (ii) is not $210°$.

**(2 marks)**

**10** Use the identity

$$R\sin(\theta+\alpha) \equiv R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$$

to show that

$$4\sin\left(\theta+\frac{\pi}{4}\right) \equiv 2\sqrt{2}(\sin\theta+\cos\theta)$$

**(3 marks)**

# Medium Questions

**1** Given the identity

$$\sin^2\theta + \cos^2\theta \equiv 1$$

prove the following identities:

(i) $\sec^2\theta \equiv 1 + \tan^2\theta$

(ii) $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$

**(4 marks)**

**2** (i) By using the double angle formula for cosine, prove the identity

$$\cos 4\theta \equiv 8\cos^4\theta - 8\cos^2\theta + 1$$

(ii) Show by counter-example that

$$\sin 4\theta \not\equiv 8\sin^4\theta - 8\sin^2\theta + 1$$

**(5 marks)**

**3 (a)** Given that $\theta$ is small, and that terms involving $\theta^3$ or higher powers of $\theta$ can be ignored, use an appropriate approximation to show that

$$4\cos 4\theta - 2\cos^2 2\theta \approx 2 - 24\theta^2$$

**(3 marks)**

**(b)** Show that the result in part (a) gives a percentage error of 0.583%, to 3 significant figures, when used to approximate

$$4\cos\frac{\pi}{6} - 2\cos^2\frac{\pi}{12}$$

**(3 marks)**

**4** Prove the identity

$$\frac{4\sin^4\theta}{\sin^2 2\theta} \equiv \tan^2\theta \qquad \theta \neq k\pi$$

**(3 marks)**

**5** Show that

$$\sin\theta(\operatorname{cosec}^2\theta - 2) \equiv \frac{\cos 2\theta}{\sin\theta}$$

**(4 marks)**

**6** Use the quotient rule to show that

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

**(5 marks)**

**7** Show that

$$\sin 3\theta + \sin \theta \equiv 4 \sin \theta - 4 \sin^3 \theta$$

**(5 marks)**

**8** Prove the identity

$$\frac{4 \cot x \cos 2x}{\sin 4x} \equiv \operatorname{cosec}^2 x \qquad x \neq \frac{k\pi}{4}$$

**(5 marks)**

**9 (a)** Show that

$$\sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \sqrt{3}\,\sin\left(\frac{\pi}{6}\right)$$

**(2 marks)**

**(b)** Show that

$$\arcsin\left(\cos\frac{3\pi}{4}\right) = -\arcsin\left(\cos\frac{\pi}{4}\right)$$

**(2 marks)**

**10** Show that

$$\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) \equiv \sin\theta - \cos\theta$$

**(3 marks)**

# Hard Questions

**1** Given the identity

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

prove the following identities:

(i) $\cos 2\theta \equiv \cos^2 \theta - \sin^2 \theta$

(ii) $\cos 2\theta \equiv 1 - 2\sin^2 \theta$

(iii) $\cos 2\theta \equiv 2\cos^2 \theta - 1$

**(4 marks)**

**2** (i) Prove the identity

$$\sin 3\theta \equiv 3\sin \theta - 4\sin^3 \theta$$

(ii) Show by counter-example that

$$\cos 3\theta \not\equiv 3\cos \theta - 4\cos^3 \theta$$

**(5 marks)**

**3 (a)** Given that $\theta$ is small, and that terms involving $\theta^3$ or higher powers of $\theta$ can be ignored, show that

$$\frac{1}{\operatorname{cosec}^2\left(\frac{\theta}{2}\right)} + \frac{1}{\sec^2\left(\frac{\theta}{4}\right)} \approx 1 + \frac{3}{16}\theta^2$$

**(3 marks)**

**(b)** Determine the percentage error when the result in part (a) is used to approximate

$$\frac{1}{\operatorname{cosec}^2\left(\frac{7}{20}\right)} + \frac{1}{\sec^2\left(\frac{7}{40}\right)}$$

giving your answer correct to 3 significant figures.

**(3 marks)**

**4** Show that

$$\cos 4\theta + \cos\frac{\pi}{3} \equiv 8\sin^4\theta - 8\sin^2\theta + \frac{3}{2}$$

**(5 marks)**

**5** Prove that

$$\cot^2\theta - \tan^2\theta \equiv 4\cot 2\theta \operatorname{cosec} 2\theta.$$

**(5 marks)**

**6** Prove the identity

$$\frac{1-\tan^2 x}{\cos 2x} \equiv \sec^2 x \qquad x \neq \frac{2k+1}{4}\pi$$

**(5 marks)**

**7** Prove the identity

$$\operatorname{cosec} x \equiv \frac{\frac{1}{2}\sec^2\frac{x}{2}}{\tan\frac{x}{2}}$$

**(4 marks)**

**8** Show that

$$\tan\frac{x}{2} \equiv \frac{1}{\operatorname{cosec} x + \cot x} \qquad x \neq 2k\pi$$

**(5 marks)**

**9 (a)** Given that $y = \arcsin(kx)$, where $k$ is a constant, show that $x = \frac{1}{k}\cos\left(\frac{\pi}{2} - y\right)$.

**(3 marks)**

**(b)** Hence show that the value of $\arcsin kx + \arccos kx$ is constant and independent of $k$. Find the value of this constant.

**(3 marks)**

**10** Show that

$$\frac{10}{4\cos\theta + 3\sin\theta} \equiv 2\sec(\theta - \alpha)$$

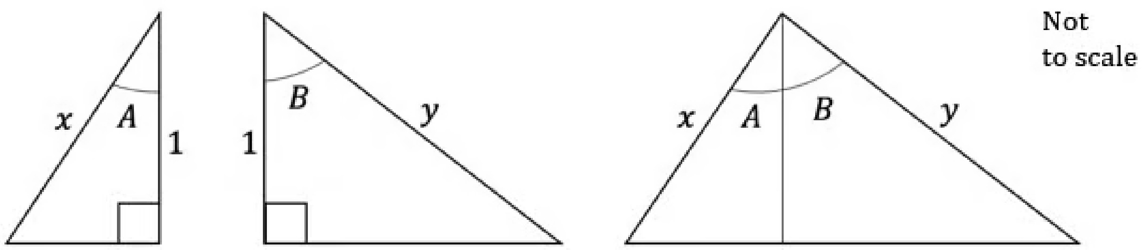
where

$$\alpha = \arctan\left(\frac{3}{4}\right)$$

**(5 marks)**

# Very Hard Questions

**1** Consider the three triangles, all of height 1, as shown below.




By applying the area of a triangle formula $A = \frac{1}{2}\text{ab} \sin C$ to each one, prove that,

$$\sin(A + B) \equiv \sin A \cos B + \sin B \cos A$$

Briefly explain why this only proves the result for $A$ and $B$ being acute angles.

**(6 marks)**

**2** Prove the identity

$$\tan 4\theta \equiv \frac{4 \tan \theta (1 - \tan^2 \theta)}{1 - 6 \tan^2 \theta + \tan^4 \theta}$$

**(4 marks)**

**3 (a)** Use the small angle approximations for sine and cosine to confirm the following two limit results:

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \qquad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

Be sure to explain why use of the small angle approximations is justified here.

**(4 marks)**

**(b)** Hence prove from first principles that

$$\frac{d}{dx}[\sin x] = \cos x$$

**(5 marks)**

**4** Prove the identity

$$-16 \cot 2\theta \operatorname{cosec}^3 2\theta \equiv \sec^4 \theta - \operatorname{cosec}^4 \theta$$

**(5 marks)**

**5** Show that

$$\frac{\sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)}{\sin\left(\theta - \frac{\pi}{2}\right)} \equiv \tan\theta - 1$$

**(4 marks)**

**6 (a)** Show that

$$\sin 3\theta \equiv 3 \sin \theta \cos^2 \theta - \sin^3 \theta$$

**(4 marks)**

**(b)** Hence, or otherwise, show that

$$\frac{\cos 3\theta - \cos \theta}{\sin 3\theta \sin \theta} \equiv \frac{4 \cos \theta}{1 - 4 \cos^2 \theta} \qquad \theta \neq k\pi$$

**(5 marks)**

**7** Show that

$$4\cos^2\left(x - \frac{\pi}{6}\right) \equiv 3 - 2\sin^2 x + \sqrt{3} \sin 2x$$

**(5 marks)**

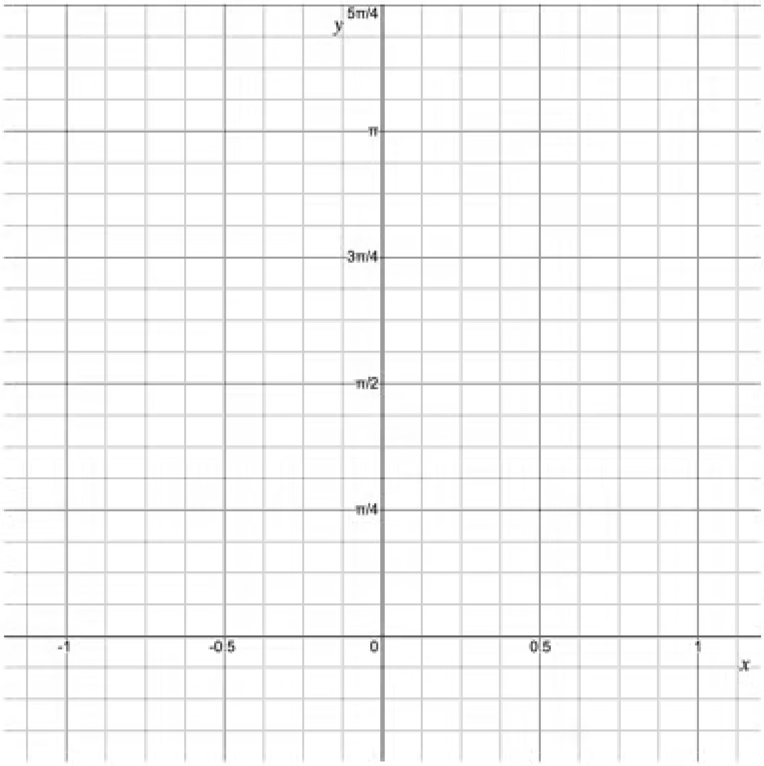
**8** Show that

$$\tan\left(\frac{2x+\pi}{4}\right) \equiv \sec x + \tan x$$

**(6 marks)**

**9 (a)** On the axes below sketch the graphs of

$$y = \arccos(-x) \quad \text{and} \quad y = |\arcsin x|$$




**(4 marks)**

**(b)** With the help of your sketch, determine the exact solution(s) to the equation

$$\arccos(-x) = |\arcsin x|$$

**(2 marks)**

**(c)** What can you say about the solution(s) to the equation

$$|\arccos x| = \arcsin(-x)?$$

Justify your answer.

**(2 marks)**

**10** Show that

$$\frac{1}{\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right)^2} \equiv 4\,\mathrm{cosec}^2\left(2\theta + \frac{\pi}{3}\right)$$

**(9 marks)**