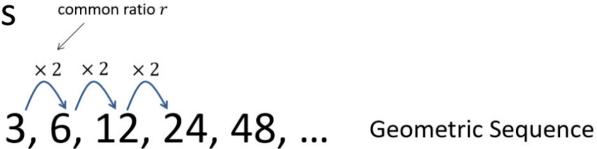
Geometric Sequences



Identify the common ratio r:

A geometric sequence is one in which there is a common ratio between terms.

2 27, 18, 12, 8,
$$\frac{16}{3}$$
 $r = \frac{18}{27} = \frac{2}{3}$

3 10, 5, 2.5, 1.25, ...
$$r = \frac{1}{2}$$
 or $r = 0.5$

$$5, -5, 5, -5, 5, -5, \dots$$
 $r = -1$

$$-2x^2, 4x^3$$
 $r = -2x$ $-2x^2$ $-2x^3$

$$[6] 1, p, p^2, p^3, ...$$

7
$$4, -1, 0.25, -0.0625, \dots$$

An alternating sequence is one which oscillates between positive and negative.

nth term of an geometric sequence

1st Term2nd Term3rd Term... n^{th} term

a ar ar^2 \cdots ar^{n-1}

 ${\mathscr N}\,n^{\mathsf{th}}$ term of geometric sequence:

$$u_n = ar^{n-1}$$

The second term of a geometric sequence is 4 and the 4th term is 8. The common ratio is positive. Find the exact values of:

- a) The common ratio.
- b) The first term.
- c) The 10th term.

a)
$$u_{1} = 4$$
 $u_{2} = 8$

$$4 = ar^{2-1}$$

$$4 = ar^{2}$$

$$4 = ar^{3}$$

$$4 = ar^{3}$$

$$4 = a\sqrt{2}$$

$$a = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$ar^{3} = 8$$

$$ar^{3} = 8$$

$$4 = a\sqrt{2}$$

$$5 = a\sqrt{2}$$

$$6 = a\sqrt{2}$$

$$\frac{ar^{3}}{ar} = \frac{8}{4}$$

$$c) u_{10} = ar^{9}$$

$$= 2\sqrt{2} \times (\sqrt{2})^{9}$$

$$r = \sqrt{2}$$

$$= 64$$

The numbers 3, x and x + 6 form the first three terms of a positive geometric sequence. Find:

- a) The value of x.
- b) The 10th term in the sequence.

$$r = \frac{6}{3} = 2$$

nth term with inequalities

What is the first term in the geometric progression 3, 6, 12, 24, ... to exceed 1 million?

$$\alpha = 3 \qquad u_{n} = 3 \times 2^{n-1}$$

$$7 = 2$$

$$3 \times 2^{n-1} > 1{,}000{,}000$$

$$2^{n-1} > 1{,}000{,}000$$

$$(n-1) \ln 2 > \ln \left(\frac{1000000}{3}\right)$$

$$(n-1) > \frac{\ln \frac{1000000}{3}}{2}$$

$$\ln 2$$

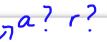
$$1 = 786, +32$$

$$1 = 786, +32$$

$$1 = 1,572864$$

Your Turn

All the terms in a geometric sequence are positive.



The third term of the sequence is 20 and the fifth term 80. What is the 20th term?

$$u_3 = 20$$
 $u_5 = 80$

$$20 = ar^2 6 80 = ar^4 0$$

$$20-0\times2$$

$$0 \div 2 = \frac{80}{20} = \frac{ar^4}{ar^2}$$

$$20 = a \times 2^{1}$$

$$u_{26} = 5 \times 2^{19}$$

$$= 2,621,440$$

The second, third and fourth term of a geometric sequence are the following:

$$x, x + 6, 5x - 6$$

- a) Determine the possible values of x.
- b) Given the common ratio is positive, find the common ratio.
- c) Hence determine the possible values for the first term of the sequence.



$$r = \underbrace{x+6}_{x} = \underbrace{5x-6}_{x+6}$$

b) If
$$x=6$$
 6, 12, 24
 $x=-1.5$ -1.5, 4.5, -13.5

$$(3c+4)^{2} = 53c^{2} - 63c^{2}$$

$$3c^{2} + 123c + 36 = 53c^{2} - 63c^{2}$$

Sum of the first n terms of a geometric series

Geometric Series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{\alpha(r^n - 1)}{r - 1}$$

Proof:

Exam Note: This once came up in an exam.

$$S_{n} = \alpha + \alpha r + \alpha r^{2} + ... + \alpha r^{n-2} + \alpha r^{n-1}$$

$$rS_{n} = \alpha + \alpha r^{2} + ... + \alpha r^{n-2} + \alpha r^{n-1} + \alpha r^{n}$$

$$S_{n} - rS_{n} = \alpha - \alpha r^{n}$$

$$S_{n} (1-r) = \alpha(1-r^{n})$$

$$S_{n} = \alpha(1-r^{n})$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Find the sum of the first 10 terms.

$$S_{1b} = \underbrace{3(1-2^{1b})}_{1-2} = \underbrace{3069}_{1-2}$$

$$a=3$$
 $n=10$ $r=2$

 $r = \frac{1}{2}$

4, 2, 1,
$$\frac{1}{2}$$
, $\frac{1}{4}$, $\frac{1}{8}$, ...

$$S_{10} = \frac{4(1 - (\frac{1}{2})^{16})}{1 - \frac{1}{2}} = 7.9921875$$

$$a = 4 \qquad h = 10$$

Find the least value of n such that the sum of $1+2+4+8+\cdots$ to n terms would exceed 2 000 000.

$$S_{n} = \frac{1(1-2^{n})}{1-2}$$

$$C = 2$$

$$= \frac{1-2^{n}}{-1}$$

$$S_{n} = 2^{n}-1$$

$$S_n > 2000000$$
 $2^n - 1 > 2000000$
 $2^n > 20000001$
 $1 = 1 = 2000001$
 $1 = 1 = 2000001$
 $1 = 1 = 2000001$
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 $1 = 1 = 2000001$

Your Turn

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256, 192, 144

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

- (a) the common ratio,
- (b) the first term,
- (d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

a)
$$r = \frac{144}{192} = 0.75$$

c)
$$S_n = \frac{256(1-0.75^n)}{1-0.75}$$

$$S_n = 1024(1-0.75^n)$$

$$1024(1-0.75^{\circ}) > 1006$$
 $1-0.75^{\circ} > \frac{1006}{1024}$
 $1-\frac{1006}{1024} > 0.75^{\circ}$

$$\frac{3}{128} > 0.75^{n}$$

$$\ln \frac{3}{128} > n \ln 0.75$$

$$\ln \frac{3}{128} < n$$

$$\ln 0.75$$

$$\ln 0.75$$

$$\ln 0.75$$

$$\ln 0.75$$

$$\ln 0.75$$

(2)

(2)