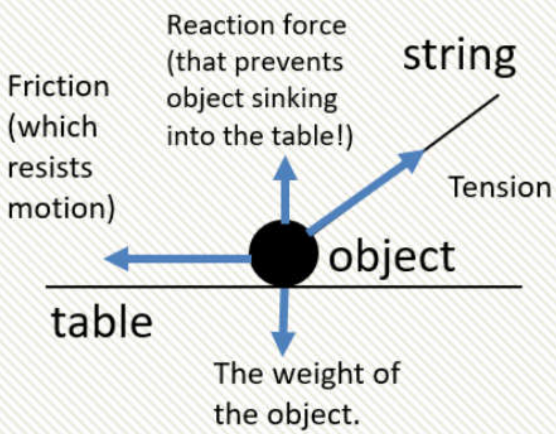
Mechanics - introduction

Mechanics, broadly speaking, concerns motion, forces, and how the two interrelate.

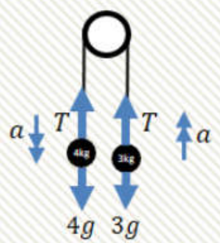
This chapter just gives you an overview of what you'll be covering in Year 1 and how it all links together.

Forces

You will later encounter force diagrams. This considers the forces acting at a particular point. Some forces you might consider…



- Forces can be considered as vectors.
- The **magnitude** of the force vector gives the 'size' of the force.
- We often **consider forces in a particular direction**. e.g. If the object above is stationary, the forces left must equal the force right, and forces up equal forces down (Newton's 1st Law).
- Often we need to consider the forces at multiple different points if objects are connected, e.g. with pulleys:



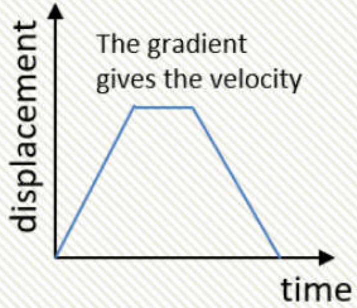
The bridge!

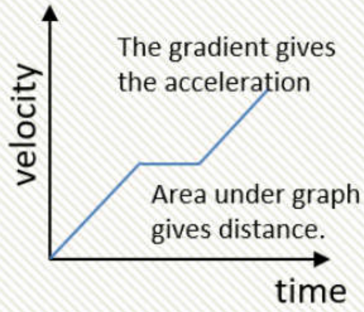
$$F = ma$$

Newton's 2nd Law allows us to connect the force world (F) with the motion world (acceleration a) if the object is moving.

Motion

At GCSE you may have encountered displacement-time and velocity-time graphs:





Given **constant acceleration** we have 5 quantities of motion ("$suvat$"):

$\boldsymbol{s}$ = displacement
$\boldsymbol{u}$ = initial velocity
$\boldsymbol{v}$ = final velocity
$\boldsymbol{a}$ = acceleration
$\boldsymbol{t}$ = time

which we will see are linked by various equations:

$$s = ut + \frac{1}{2}at^2$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$v^2 = u^2 + 2as$$
$$v = u + at$$

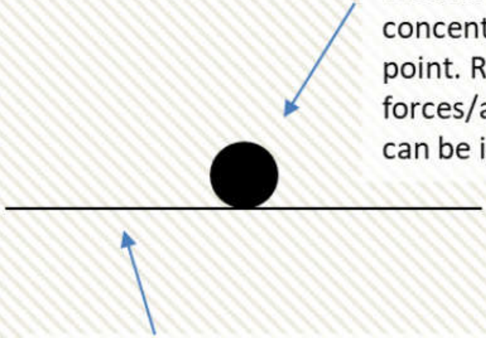
If the **acceleration is not constant**, we can specify displacement/velocity/acceleration as a function of time and differentiate/integrate to change between them.

$$s = 2t^3 + 3t \quad \rightarrow \quad v = \frac{ds}{dt} = 6t^2 + 3$$

Modelling Assumptions

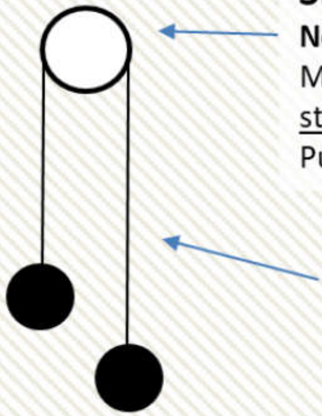
As with many areas of applied maths, we often have to make various modelling assumptions, to make the maths cleaner or to use well-known mathematical approaches.
Here are common modelling assumptions often made in Mechanics:

Particle
Dimensions of object are negligible
Means: Mass of object concentrated at single point. Rotational forces/air resistance can be ignored.



Smooth/light pulley
No friction.
Means: <u>Tension the same in string either side of pulley</u>. Pulley has no mass.

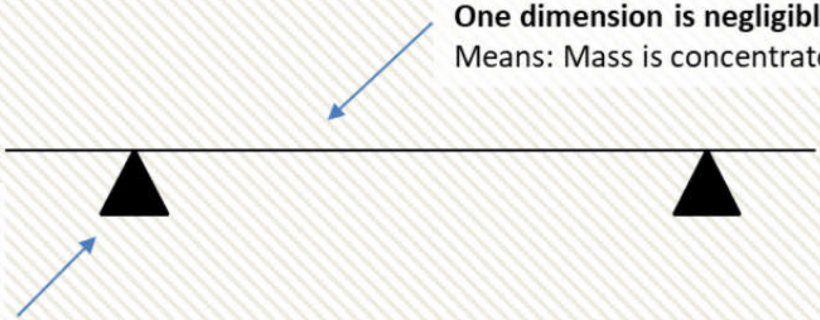
Fro Tip:
Particularly make note of underlined text!



Inextensible string
String does not stretch under load.
Means: <u>Acceleration the same</u> in any connected objects.

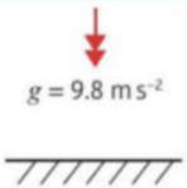
Rough/Smooth surface
Means: Objects in contact with surface does/does not experience friction.

Rod
One dimension is negligible, like a pole or beam.
Means: Mass is concentrated along line. Rigid.



Peg/Support
A support from which a body can be suspended or rested.
Means: Dimensionless and fixed. Can be rough or smooth depending on question.

These are some common models and modelling assumptions that you need to know.

Model	Modelling assumptions
Particle – Dimensions of the object are negligible.	• mass of the object is concentrated at a single point • rotational forces and air resistance can be ignored
Rod – All dimensions but one are negligible, like a pole or a beam.	• mass is concentrated along a line • no thickness • rigid (does not bend or buckle)
Lamina – Object with area but negligible thickness, like a sheet of paper.	• mass is distributed across a flat surface
Uniform body – Mass is distributed evenly.	• mass of the object is concentrated at a single point at the geometrical centre of the body – the **centre of mass**
Light object – Mass of the object is small compared to other masses, like a string or a pulley.	• treat object as having zero mass • tension the same at both ends of a light string
Inextensible string – A string that does not stretch under load.	• acceleration is the same in objects connected by a taut inextensible string
Smooth surface	• assume that there is no friction between the surface and any object on it
Rough surface – If a surface is not smooth, it is rough.	• objects in contact with the surface experience a frictional force if they are moving or are acted on by a force
Wire – Rigid thin length of metal.	• treated as one-dimensional
Smooth and light pulley – all pulleys you consider will be smooth and light.	• pulley has no mass • tension is the same on either side of the pulley
Bead – Particle with a hole in it for threading on a wire or string.	• moves freely along a wire or string • tension is the same on either side of the bead
Peg – A support from which a body can be suspended or rested.	• dimensionless and fixed • can be rough or smooth as specified in question
Air resistance – Resistance experienced as an object moves through the air.	• usually modelled as being negligible
Gravity – Force of attraction between all objects. Acceleration due to gravity is denoted by g. 	• assume that all objects with mass are attracted towards the Earth • Earth's gravity is uniform and acts vertically downwards • g is constant and is taken as $9.8\ \mathrm{m\,s^{-2}}$, unless otherwise stated in the question

SI units

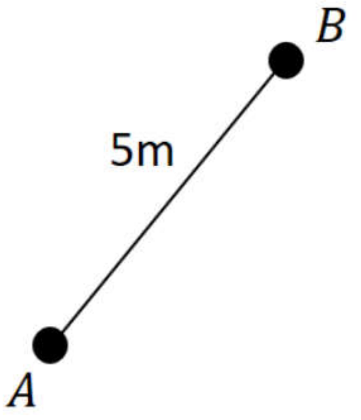
The SI units are a standard system of units, used internationally ("Système International d'unités"). These are the ones you will use:

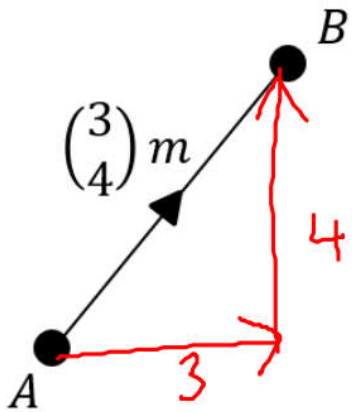
Quantity	Unit	Symbol
Mass	kilogram	kg
Length/Displacement	metre	m
Time	Seconds	s
Speed/Velocity	metres per second	$m\ s^{-1}$
Acceleration	metres per second per second	$m\ s^{-2}$
Force/Weight	Newton	N (= $kg\ m\ s^{-2}$)

This unit is consistent with force being mass × acceleration

Vectors ↔ Scalars

In Mechanics you will often need to convert to/from the scalar form of a quantity and the vector form.





A scalar quantity has magnitude (i.e. size) only.
The 5m is a distance.
The value is always positive.

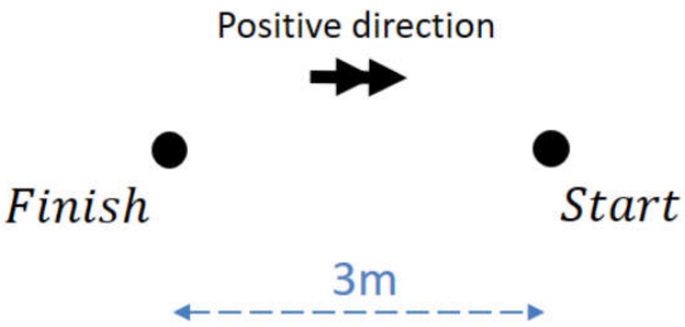
A vector quantity also has direction.
The vector equivalent of distance is displacement.

Scalar Form	Vector Form
Distance	displacement
Speed	velocity

Other quantities which can be vectors or scalars:
Force, acceleration

Quantities which can only be scalars:
Time, mass

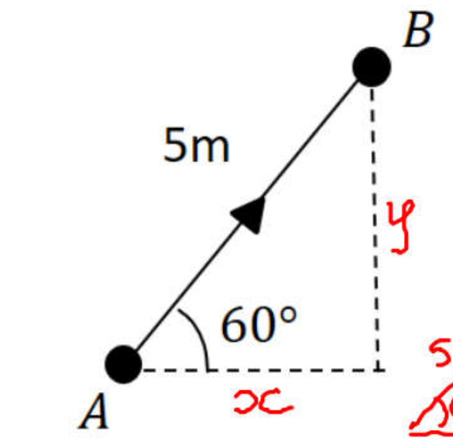
Note: 1-dimensional vectors are still different from scalars. Consider the displacement on a 1-dimensional line in a particular direction. If we'd gone backwards 3 units…

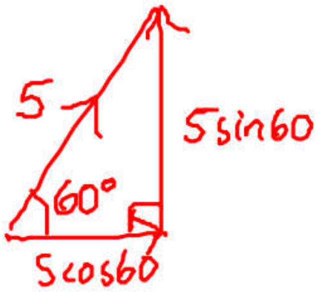


Distance: 3m
Displacement: −3m

Note: we don't write the brackets around 1D vectors. So 1D vectors look like scalars, except they're allowed to be positive or negative.

Scalar Form	Vector Form





$$\begin{pmatrix} 5\cos 60 \\ 5\sin 60 \end{pmatrix}$$

To convert to vector form, just use basic trigonometry to find the x-change and y-change.

Speed Tip: If F is the magnitude, use $F \cos\theta$ for the side adjacent to the angle and $F \sin\theta$ for the side opposite it.

$\cos 60 = \frac{x}{5}$

$x = 5\cos 60$

$\sin 60 = \frac{y}{5}$

$y = 5\sin 60$

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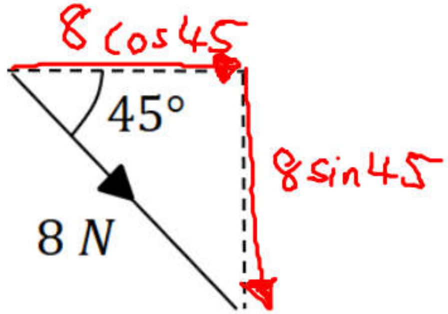
speed

$\sqrt{5^2+12^2} = \underline{13ms^{-1}}$

Velocity:

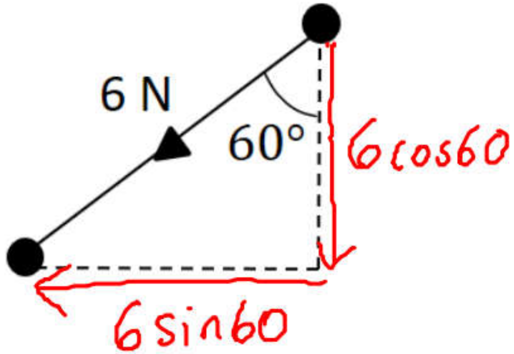
$$\begin{pmatrix} 5 \\ -12 \end{pmatrix} ms^{-1}$$

To convert scalar form, just find the **magnitude** of the vector using Pythagoras.

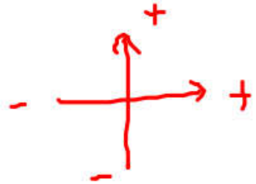


$$\begin{pmatrix} 8\cos 45 \\ -8\sin 45 \end{pmatrix} N$$

Scalar Form	Vector Form



$$\begin{pmatrix} -6\sin 60 \\ -6\cos 60 \end{pmatrix} N$$

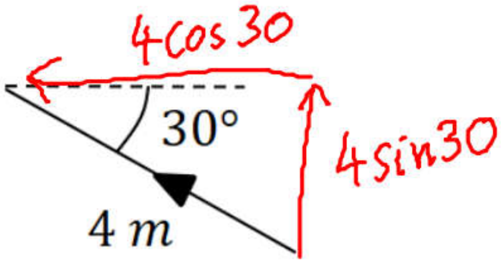


$$a = \sqrt{6^2+8^2}$$
$$= \underline{10\ ms^{-2}}$$

acceleration

$$(6\boldsymbol{i} - 8\boldsymbol{j})\ ms^{-2}$$

$$\begin{pmatrix} 6 \\ -8 \end{pmatrix}$$



$$\begin{pmatrix} -4\cos 30 \\ 4\sin 30 \end{pmatrix} m$$

Recall from Pure Year 1 that $6\boldsymbol{i} - 8\boldsymbol{j}$ is another way of writing $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the positive x and y directions.