Your Turn

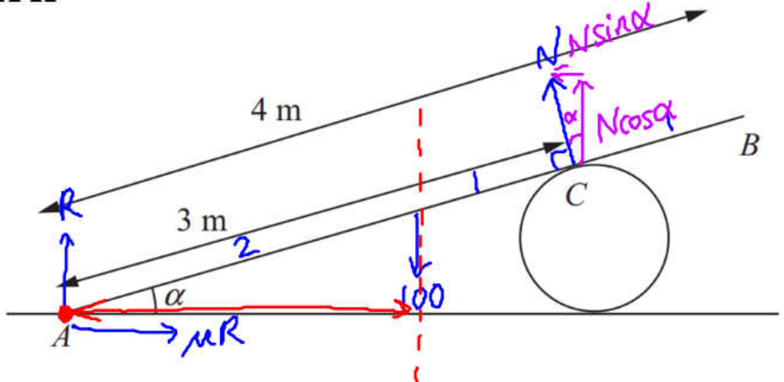
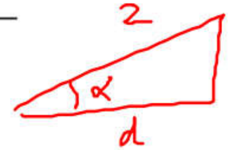


Figure 4

$\cos\alpha = \frac{2\sqrt{2}}{3}$



A uniform plank *AB*, of weight 100 N and length 4 m, rests in equilibrium with the end *A* on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is *C*, where $AC = 3$ m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin\alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ. Modelling the plank as a rod, find the least possible value of μ. → limiting equilibrium

(10)

M(A) $3N = 100 \times 2\cos\alpha$

$3N = 200\cos\alpha$

$N = \frac{200\cos\alpha}{3}$

$N = \frac{200}{3} \times \frac{2\sqrt{2}}{3}$

$= \frac{400\sqrt{2}}{9}$

R↕ $R + N\cos\alpha = 100$

R↔ $\mu R = N\sin\alpha$

$\mu R = \frac{1}{3}N$

$R = 100 - \frac{400\sqrt{2}}{9} \times \frac{2\sqrt{2}}{3}$

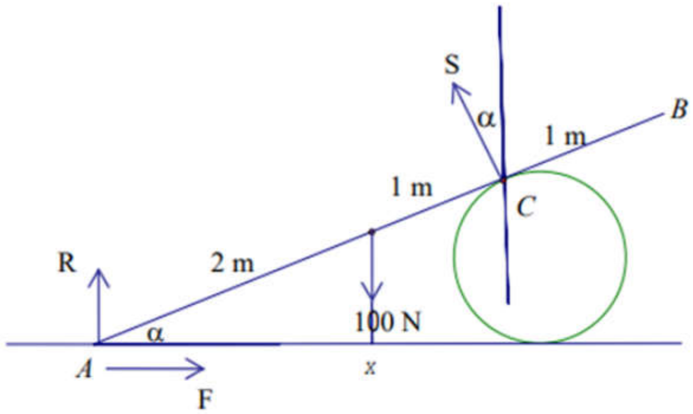
$R = \frac{1100}{27}$

$\mu = \frac{\frac{1}{3}N}{R}$

$\mu = \frac{\frac{1}{3} \times \frac{400\sqrt{2}}{9}}{\frac{1100}{27}} = 0.5142$

$= 0.51$ (2sf)

7.



Taking moments about A:

$3S = 100 \times 2 \times \cos\alpha$ **M1 A1**

Resolving vertically:

$R + S\cos\alpha = 100$ **M1 A1**

Resolving horizontally:

$S\sin\alpha = F$ **M1 A1**

(Most alternative methods need 3 independent equations, each one worth M1A1. Can be done in 2 e.g. if they resolve horizontally and take moments about X then $R \times 2 \times \cos\alpha = S \times (3 - 2 \times \cos^2\alpha)$ scores M2A2)

Substitute trig values to obtain correct values for F and R (exact or decimal equivalent). **DM1**

$\left(S = \dfrac{200\sqrt{8}}{9}\right)$, $R = 100 - \dfrac{1600}{27} = \dfrac{1100}{27} \approx 40.74$, $F = \dfrac{200\sqrt{8}}{27} \approx 20.95...$ **A1**

$F \le \mu R$, $200\sqrt{8} \le \mu \times 1100$, $\mu \ge \dfrac{200\sqrt{8}}{1100} = \dfrac{2\sqrt{8}}{11}$. **M1**

Least possible μ is 0.514 (3sf), or exact. **A1**

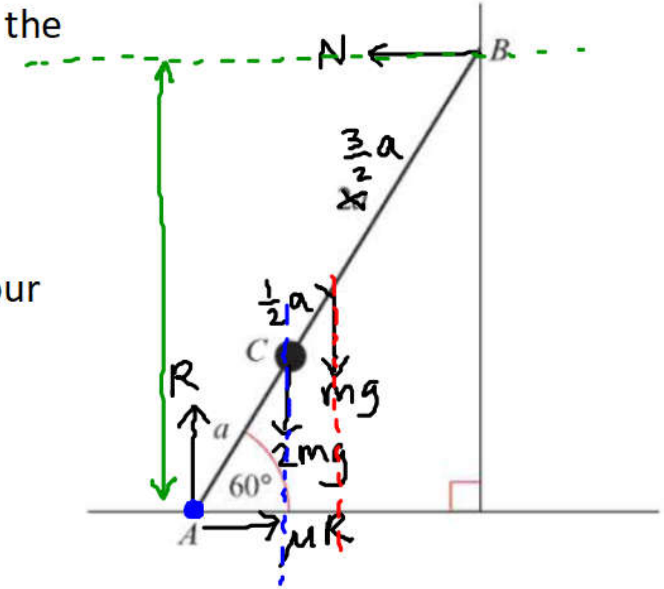
[10]

Static Rigid Bodies - ladders

Hints:

- Resolve vertically
- Resolve horizontally
- Take moments about A (usually the floor)
- Remember to find perpendicular distances!

A ladder AB, of mass m and length 3a, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall.
A load of mass 2m is fixed on the ladder at the point C, where AC = a.
The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle.
The ladder rests in limiting equilibrium at an angle of 60° with the ground.



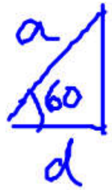
a) Find the coefficient of friction between the ladder and the ground.
b) State how you have used the assumption that the ladder is uniform in your calculations. weight acts at centre of ladder.

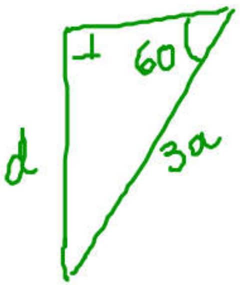
$R \leftrightarrow \quad N = \mu R$

$R \updownarrow \quad R = 2mg + mg$

$R = 3mg$

$M(A)$





$2mg \times \cancel{a}\cos 60 + mg \times \frac{3}{2}\cancel{a}\cos 60 = N \times 3\cancel{a}\sin 60$

$2\cancel{mg}\cos 60 + \frac{3}{2}\cancel{mg}\cos 60 = 3\sin 60 \times \mu \times 3\cancel{mg}$

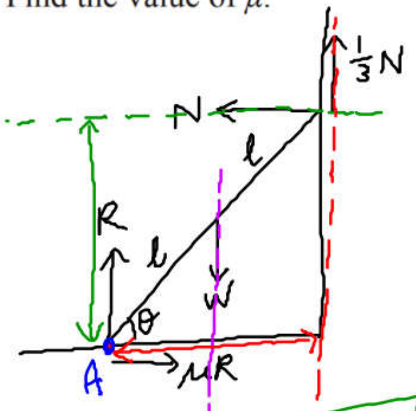
$\frac{7}{2}\cos 60 = 9\sin 60 \times \mu \quad (\div \cos 60)$

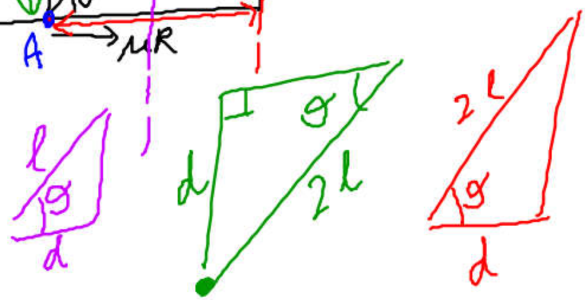
$\frac{7}{2} = 9\tan 60 \times \mu$

$\mu = \frac{7/2}{9\tan 60} = \frac{7\sqrt{3}}{54} = 0.22$ (2sf)

A ladder AB, of weight W and length $2l$, has one end A resting on rough horizontal ground. The other end B rests against a rough vertical wall. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ. Friction is limiting at both A and B. The ladder is at an angle θ to the ground, where $\tan\theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

Find the value of μ. **(9)**





$(R \leftrightarrow)$ $N = \mu R$ ①

$(R \updownarrow)$ $R + \frac{1}{3}N = W$ ②

$(\div \cos\theta)$

$m(A)$ $W \times \cancel{l}\cos\theta = N \times 2\cancel{l}\sin\theta + \frac{1}{3}N \times 2\cancel{l}\cos\theta$

$W = 2N\tan\theta + \frac{2}{3}N$

$W = \frac{10}{3}N + \frac{2}{3}N$

$W = 4N$ ③

Sub ③ into ②

$R + \frac{1}{3}N = 4N$

$R = \frac{11}{3}N$

Sub into ①

$N = \mu \times \frac{11}{3}N$

$\frac{3}{11} = \mu$.

Your Turn

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point C, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

(9)

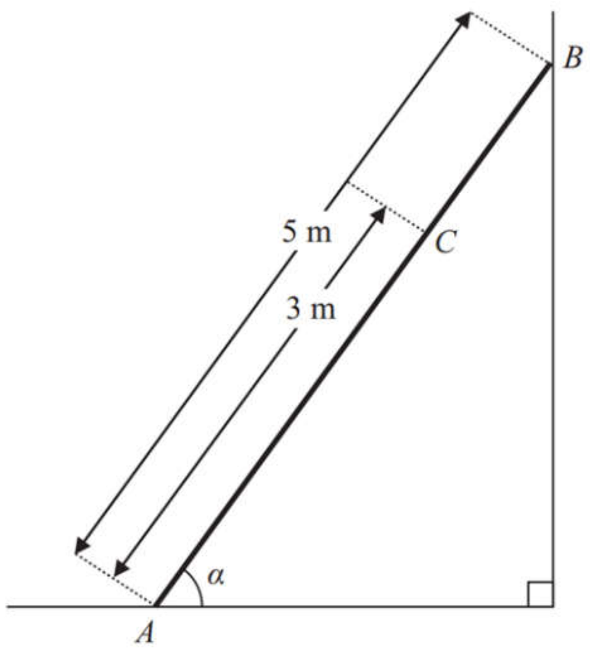
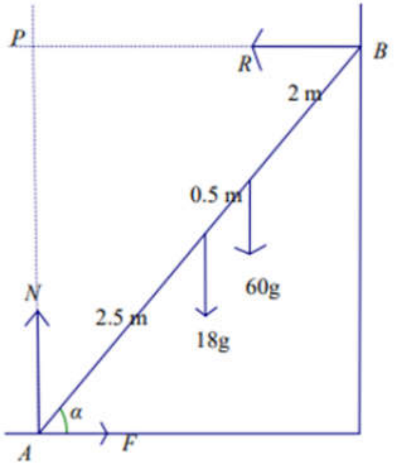


Figure 1

3			
	$F = \mu N$	B1	Used. Condone an inequality.
	R$(\uparrow)$ $\quad 18g + 60g = N$	M1	Resolve vertically
	$=78g$	A1	
	R$(\rightarrow)$ $\quad R = F = \mu N$		
P	$2.5 \times 18g\cos\alpha + 3 \times 60g\cos\alpha = 5F\sin\alpha$	M1A2	Moments equation. Condone sign errors. Condone sin/cos confusion
A	$18g \times 2.5\cos\alpha + 60g \times 3\cos\alpha = R \times 5\sin\alpha$		-1 each error
C	$\frac{1}{2}\cos\alpha \times 18g + 3\sin\alpha F + 2\sin\alpha R = 3\cos\alpha N$		
B	$5\cos\alpha N = 5\sin\alpha F + 2.5\cos\alpha \times 18g + 2\cos\alpha \times 60$		
W	$60g \times \frac{1}{2}\cos\alpha + 2.5N\cos\alpha = 2.5R\sin\alpha + 2.5F\sin\alpha$		
	$45 \times \frac{3}{5}g + 180 \times \frac{3}{5}g = 4R$	DM1	Eliminate α. Dependent on the second M1.
	$R = \frac{135}{4}g$		
	$78g\mu = \frac{135}{4}g$	DM1	Equation in μ only. (Dependent on the first two M marks.)
	$\mu = \frac{135}{4 \times 78} = \frac{135}{312} = 0.432... = 0.43$	A1	NB g cancels. 0.43269..., $\frac{225}{520}$, $\frac{45}{104}$, awrt 0.433 Do not accept an inequality.
	NB If use just two moments equations, M1A2 for the better attempt, M1A1 for the other. Remaining marks as above.		

Your Turn

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

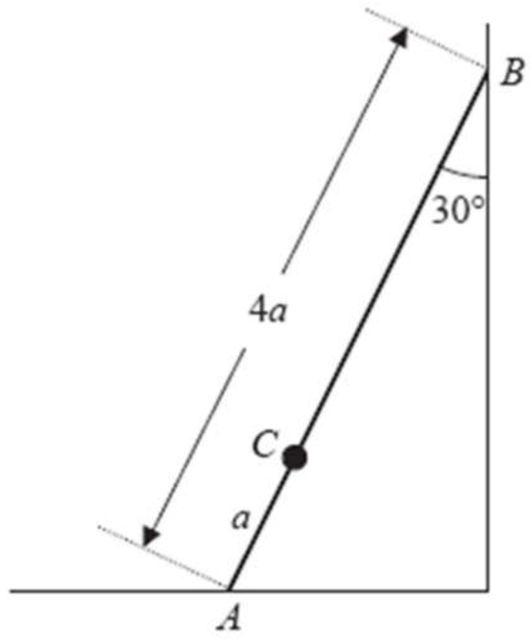
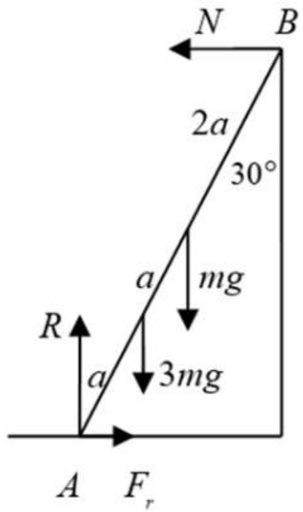


Figure 2

5.	(a) 	
	M(A) $N\times 4a\cos 30° = 3mg\times a\sin 30° + mg\times 2a\sin 30°$	M1 A2(1,0)
	$N = \frac{5}{4}mg\tan 30°$ $\left(= \frac{5}{4\sqrt{3}}mg = 7.07\ldots m\right)$	DM1 A1
	$\rightarrow\ F_r = N$, $\uparrow\ R = 4mg$	B1, B1
	Using $F_r = \mu R$	B1
	$\frac{5}{4\sqrt{3}}mg = \mu R$ for their R	M1
	$\mu = \frac{5}{16\sqrt{3}}$ awrt 0.18	A1 **(10)**
		[10]

Your Turn

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

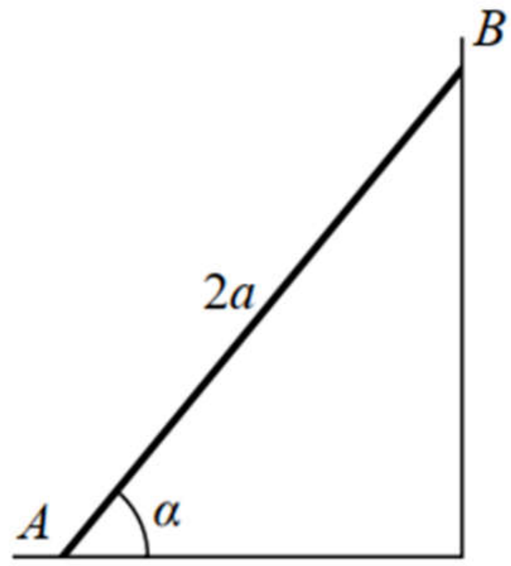


Figure 1

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

Question	Scheme	Marks	AOs
9(a)	Take moments about A (or any other complete method to produce an equation in S , W and α only)	M1	3.3
	$Wa\cos\alpha \; + \; 7W2a\cos\alpha \; = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha \; = \; \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}\,R\,(= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Static Rigid Bodies - hinges

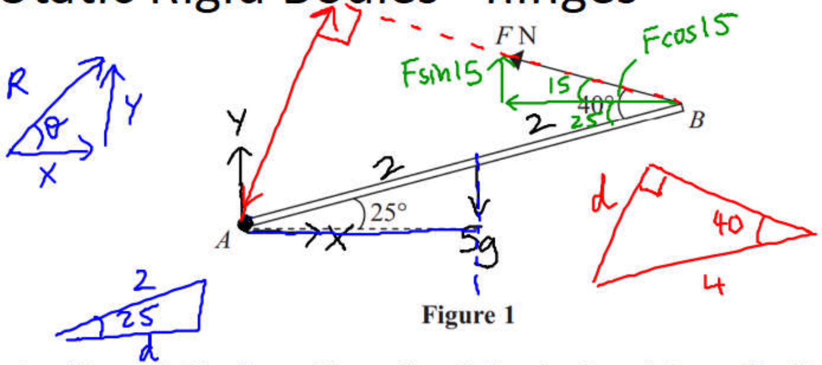


Figure 1

The reaction at the hinge will have a horizontal (X) and vertical (Y) component.

Hints:
- Resolve vertically
- Resolve horizontally
- Take moments about A (usually the floor)
- Remember to find perpendicular distances!

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(a) Find the value of F.

(4)

(b) Find the magnitude and direction of the vertical component of the force acting on the rod at A. Y

(4)

c) Find the magnitude of the normal reaction at the hinge.

a) $M(A)$ $\quad 5g \times 2\cos 25 = F \times 4\sin 40$

$F = 34.544133\ldots$

$= \underline{34.5\text{ N}}\ (3sf)$

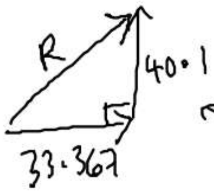
b) $R\updownarrow \quad Y + F\sin 15 = 5g$

$Y = 5g - 34.54\ldots\sin 15$

$Y = \underline{40.1\text{ N}}\ (3sf)$ because this is +ve, Y is going vertically upwards.

c) $R\leftrightarrow \quad X = F\cos 15$

$= 33.367\ldots$



Pythag. $\quad R = 52.16\ldots$

$= \underline{52\text{ N}}\ (2sf)$

A beam AB, of mass 20 kg and length 3 m, is smoothly hinged to a vertical wall at one end A.

The beam is held in equilibrium in a horizontal position by a rope of length 1 m. One end of the rope is fixed to a point C on the wall which is vertically above A. The other end of the rope is fixed to the point D on the beam so that angle ACD is 30°, as shown in Figure 2. The beam is modelled as a uniform rod and the rope is modelled as a light inextensible string.

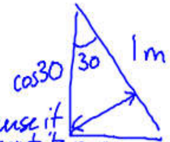
Using the model, find

(a) the tension in the rope,

(4)

(b) the direction of the force exerted by the wall on the beam at A.

(6)

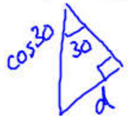


(c) If the rope were not modelled as being light, state how this would affect the tension in the rope, explaining your answer carefully.

(2)

The tension will not be equal throughout - it will increase towards the top of the rope, because it has to support its own weight too.

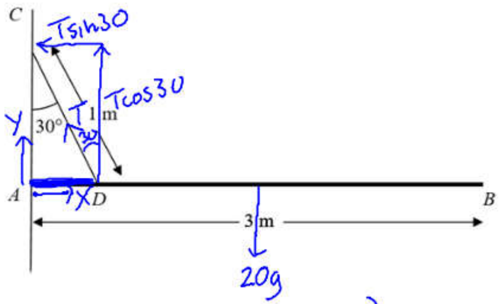
The rope is now removed and replaced by a longer rope which is still attached to the wall at C but is now attached to the beam at G, where G is the midpoint of AB. The beam AB remains in equilibrium in a horizontal position.



(d) Show that the force exerted by the wall on the beam at A now acts horizontally.

(2)

(Total 14 marks)



$M(A)$

$20g \times 1.5 = T\cos30 \sin30$

$T = 678.9639..$

$= 679\text{N}$ (3sf).

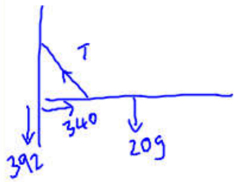
b) $R \leftrightarrow$ $X = T\sin30$

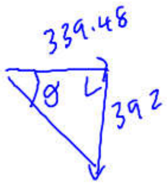
$= 339.48\ldots$

$R \updownarrow$ $Y + T\cos30 = 20g$

$Y = 20g - 678.96..\cos30$

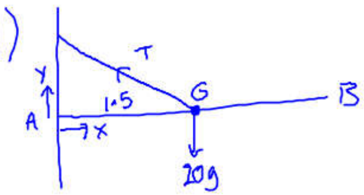
$Y = -392$





$\theta = \tan^{-1}\left(\frac{392}{339.48}\right) = 41°$ below horizontal (2sf).

d)



$M(G)$ $1.5Y = 0$

$Y = 0$

Hence reaction force is horizontal.

Your Turn

A uniform rod AB of weight W has its end A freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle θ to the horizontal, by a force of magnitude P. The force acts perpendicular to the rod at B and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at A is Y.

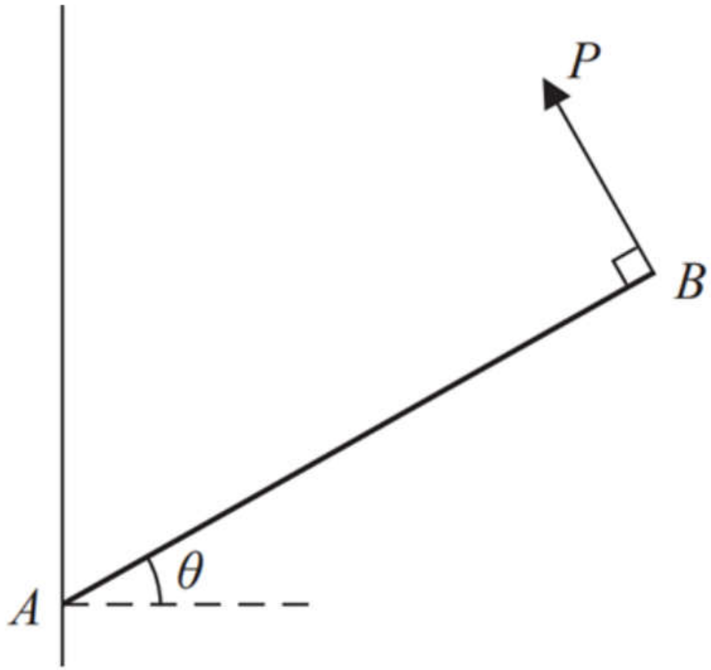
(a) Show that $Y = \frac{W}{2}(2 - \cos^2\theta)$.

(6)

Given that $\theta = 45°$

(b) find the magnitude of the force exerted on the rod by the wall at A, giving your answer in terms of W.

(6)



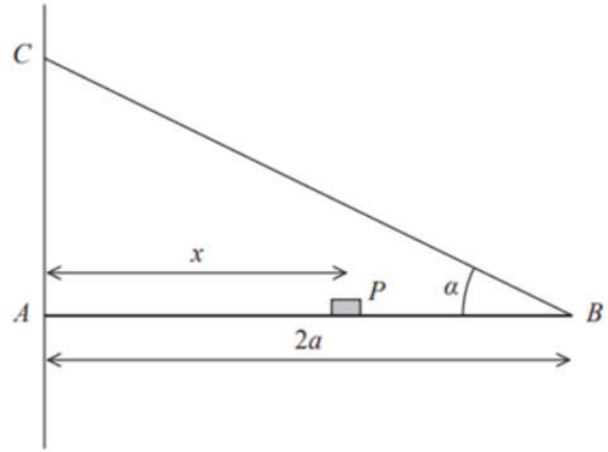
7(a)	Resolving vertically: $Y + P\cos\theta = W$	M1 A1	Needs all 3 terms. Condone sign errors and sin/cos confusion. Condone Wg
	Moments about A: $Wl\cos\theta = 2lP$	M1 A1	Terms need to be of the correct structure, but condone l implied if not seen.
	$P = \frac{W\cos\theta}{2} \Rightarrow Y = W - \frac{W\cos^2\theta}{2} = \frac{W}{2}(2-\cos^2\theta)$ **	DM1 A1 (6)	Substitute for P to obtain simplified Y Requires both preceding M marks Obtain **given result** correctly.
	NB $W + Y = P\cos\theta$ with correct conclusion is possible		
	They need to find two independent equations that do not include X. If they have equations involving X they need to attempt to eliminate X before they score any marks		
(b)	$\theta = 45° \Rightarrow Y = \frac{3W}{4}$	B1	
	$X = P\sin 45$	M1	Resolving horizontally. Accept in terms of θ.
	$= \frac{W\cos 45}{2}.\sin 45\left(= \frac{W}{4}\right)$	DM1 A1	Express X in terms of W. Accept in terms of θ. Requires preceding M mark. Correct unsimplified but substituted.
	Resultant at $A = \frac{W}{4}\sqrt{3^2+1^2} = \frac{W\sqrt{10}}{4}$ $(0.79W)$	DM1 A1 (6)	Use of Pythagoras with X, Y in terms of W only. Dependent on the first M1 Or equivalent $(0.79W$ or better)

Alternative moments equations: about the centre $Pl + X\sin\theta l = y\cos\theta l$

About the point where the lines of action of P and X intersect $Y \times \frac{2l}{\cos\theta} = W\left(\frac{2l}{\cos\theta} - l\cos\theta\right)$

Your Turn

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.



A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \frac{Mg(a+3x)}{2a \times \frac{3}{5}} = \frac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\frac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \frac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \frac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\frac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \frac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \frac{Y}{X}$ or $\frac{R\sin\beta}{R\cos\beta} = \frac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \frac{5}{4}$	A1	2.2a
		(5)	
(d)	$\frac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \frac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\frac{5a}{3}$ from A oe Or just: $x \le \frac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 A1	2.4
		(3)	

Your Turn

A uniform rod AB, of mass $3m$ and length $4a$, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A, where $AD = 3a$. A particle of mass $3m$ is attached to the rod at C, where $AC = x$. The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is $\frac{25}{4}mg$.

Show that

(a) $x = 3a$,

(5)

(b) the horizontal component of the force exerted by the wall on the rod has magnitude $5mg$.

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is about to slip,

(c) find the value of μ.

(5)

Even though not hinged, this will behave in the same way – there will be a reaction force and frictional force instead.

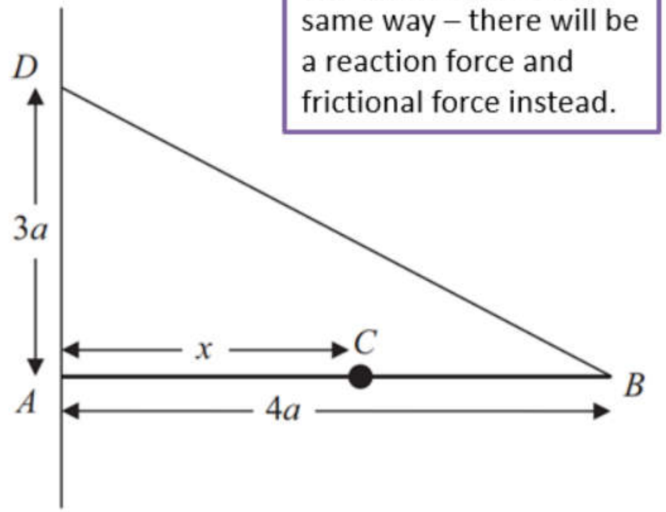
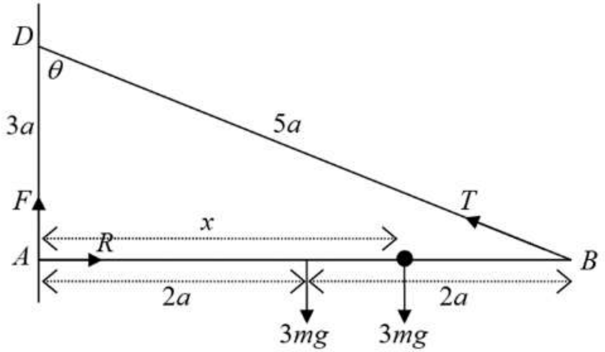


Figure 3

7. (a)		
	M(A) $3mg \times 2a + 3mgx = T\cos\theta \times 4a$	M1 A2,1,0
	$= \frac{12}{5}aT$	
	$\frac{12}{5}aT = 6mga + 3mgx$	
	$T = \frac{25}{4}mg \qquad \frac{12}{5}a \times \frac{25}{4}mg = 6mga + 3mgx$	M1
	$15a = 6a + 3x$	
	$x = 3a$ **	A1
		(5)

(b)	R$(\rightarrow)$ $R = T\sin\theta$	M1
	$= \frac{25}{4}mg \times \frac{4}{5}$	A1
	$= 5mg$ **	A1
		(3)
(c)	R$(\uparrow)$ $F + \frac{25}{4}mg \times \frac{3}{5} = 3mg + 3mg$	M1 A2,1,0
	$F = 6mg - \frac{15}{4}mg = \frac{9}{4}mg$	
	$\mu = \frac{F}{R} = \frac{\frac{9}{4}mg}{5mg} = \frac{9}{20}$	DM1 A1
		(5)
		13

Extra Questions:

Ex 4C
Hinges/ladders: Q10, 11

Ex 7D
Pegs/floor: Q1, 5, 7, 10
Ladders: Q2, 3, 4, 6, 8, 9, 11
Hinges: none in the textbook...

Mixed Exercise 7 has many more ladder questions