A Level · Edexcel · Maths

3 hours 42 questions

6.2 Laws of Logarithms

6.2.1 Laws of Logarithms / 6.2.2 Exponential Equations

Easy (12 questions)	/36
Medium (9 questions)	/41
Hard (11 questions)	/45
Very Hard (10 questions)	/40
Total Marks	**/162**

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Easy Questions

1 Evaluate

(i) $\log_3 27$

(ii) $\log_5 625$

(iii) $\log_2 \frac{1}{4}$

(iv) $\log_a a$

(4 marks)

2 Write the following in the form $a + b \ln 2$, where a and b are integers to be found.

(i) $3^2 + \ln 4$

(ii) $\ln e^7 + \ln 8$

(iii) $\log 1000 + 3 \ln 16$

(iv) $5(3^2 + \ln 64)$

(4 marks)

3 Solve the following equations, giving your answer in exact form.

(i) $e^{2x} = 5$

(ii) $3e^{\frac{1}{3}x} = 27$

(4 marks)

4 Show that

$$3\log_a 4 + 2\log_a 256 = 22\log_a 2$$

(3 marks)

5 Solve the equation

$$\log_x 16 = 2$$

(2 marks)

6 A square has side length $3\ln 4$

Show that the perimeter of the square is $24 \ln 2$.

(2 marks)

7 Write the following in the form $a \ln b$, where a and b are integers to be found.

$$4 \ln 9 + 2 \ln 81 - 3 \ln 27$$

(3 marks)

8 Solve the equation

$$7^{2x-1} = 343$$

(2 marks)

9 Write down the value of

(i) $\log_3 3$

(ii) $\ln e^6$

(iii) $\log_a 1$

(iv) $\log 1000$

(4 marks)

10 Show that

$$4\log\left(\frac{27}{16}\right) = 12\log 3 - 16\log 2$$

(3 marks)

11 (a) Express 42 as a product of its prime factors.

(1 mark)

(b) Hence, or otherwise, show that

$$\ln 42 = \ln 7 + \ln 3 + \ln 2$$

(2 marks)

12 Sketch the graph of $y = e^x$, marking clearly the coordinates of any points where the graph intersects the coordinate axes and stating the equation of any asymptotes.

(2 marks)

Medium Questions

1 (a) Evaluate

$$\log_2 4 + \log_3 27 - \log_4 4$$

(2 marks)

(b) Evaluate

$$3 \ln 2 + \frac{1}{2} \ln 81 - 2 \ln 3$$

giving your answer in the form $\ln q$, where q is an integer to be found.

(3 marks)

2 (a) Solve the following equations, giving your answers in exact form.

$e^x = 5$

(2 marks)

(b) $3e^{2x} = 9$

(3 marks)

(c) $e^{2x-1} = 4$

(3 marks)

3 By writing $1 = \log_a a$, show that

$$1 + 2\log_a b + 3\log_a c = \log_a ab^2c^3$$

(3 marks)

4 (a) Write the following as a single logarithm

$$2\log_a 6 + 3\log_a 2 - \log_a 4$$

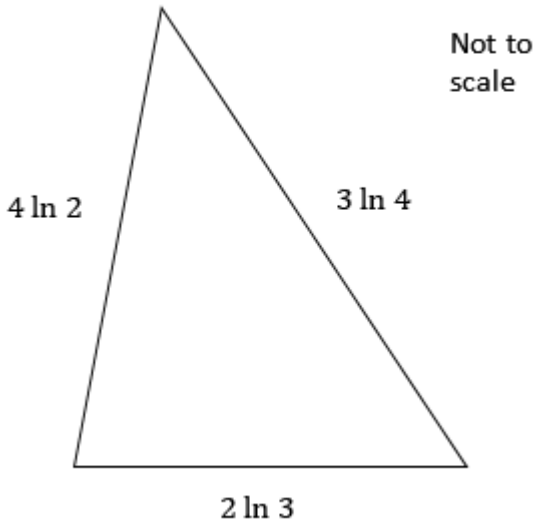
(3 marks)

(b) Write the following in the form $a \ln b$, where *a* and *b* are integers to be found.

$$2\ln 3^4 + \ln 3^3 - \ln 9$$

(3 marks)

5 The diagram below shows the length of three sides of a triangle, with each side measured in centimetres.



Work out the perimeter of the triangle, giving your answer in the form $2 \ln b$, where b is an integer to be found.

(4 marks)

6 Show that

$$2 \ln x^3 - 3 \ln x^2 = 0$$

(3 marks)

7 (a) Solve the equation

$$5^{2x} - 25 = 0$$

(2 marks)

(b) Solve the equation

$$3^{2x-1} = 4^3 + 4^2 + 1$$

(2 marks)

8 (i) On the same axes, sketch the graphs of $y = e^x$ and $y = e^{-x}$.
Label any points of intersection between each graph and the coordinate axes.
Write down the equations of any asymptotes.

(ii) Write down the equation of the line of reflection between the graphs of $y = e^x$ and $y = e^{-x}$

(5 marks)

9 Solve the equation

$$\log_x(5x - 6) = 2$$

(3 marks)

Hard Questions

1 (a) Evaluate

$$\log_2 8^2 + 3\log_2 16 - 2\log_2 2^5$$

(2 marks)

(b) Evaluate

$$3\ln 2 + 2\ln 5 - \frac{1}{2}\ln 10000$$

giving your answer in the form $\ln p$.

(3 marks)

2 (a) Solve the equation

$$4^{3x+2} = 16^{x+6}$$

(2 marks)

(b) Solve the equation

$$4^{2x+3} - 8 = 92$$

giving your answer to 3 significant figures.

(3 marks)

3 (a) Solve the following equations, giving your answers in exact form.

$4e^{3x-2} = 12$

(2 marks)

(b) $3e^{2x} + 8 = 14e^{x}$

(3 marks)

4 (a) Simplify

$$2 \ln 3^4 + \ln 3^3 - \ln 9$$

giving your answer in the form $a \ln b$, where a and b are integers to be found.

(2 marks)

(b) Write

$$2 \log_a x + 3 \log_a (x+1) - \log_a 4(x+2)$$

as a single logarithm.

(2 marks)

5 (i) On the same axes, sketch the graphs of $y = e^x$ and $y = \ln x$
On each graph, label any points where the graph intersects the coordinate axes.
Write down the equations of any asymptotes for each graph.

(ii) Write down the line of reflection between the graphs $y = e^x$ and $y = \ln x$.

(5 marks)

6 Solve the equation

$$5^{2x} - 8 \times 5^{x} + 12 = 0$$

giving your answers in the form $\log_a b$.

(3 marks)

7 Solve the equation

$$6 \times 3^{x-1} = 6^{2x}$$

giving your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers to be found.

(5 marks)

8 A ship sets sail from a harbour.
After some time, the ship's position is $(4 \ln 3)$ east of the harbour and $(3 \ln 3)$ north of the harbour.
Find the direct distance between the ship and the harbour at this time giving your answer in the form $(p \ln 3)$km.

(4 marks)

9 By writing 5 as $5 \ln e$, show that

$$5 \ln 2 + 5$$

can be written as $5 \ln 2e$

(3 marks)

10 Solve the equation

$$\log_3(x+4) = 4 + 2 \log_3 x$$

giving your answers correct to 3 significant figures.

(3 marks)

11 Solve the equation

$$2 \log_x(x+2) = 3$$

giving your answer correct to 3 significant figures.

(3 marks)

Very Hard Questions

1 (a) Evaluate

$$4\log_3 729 + 3\log_2 64^2 - 3\log 100 + \ln e^6$$

(2 marks)

(b) Evaluate

$$\frac{1}{2}\ln 196 + \frac{1}{3}\ln 125 + \frac{1}{4}\ln 81 + \frac{1}{5}\ln 32$$

giving your answer in the form $\ln q$.

(3 marks)

2 Solve the equation

$$2 \times 5^{2x+1} + 21 = 41 \times 5^x$$

giving your answers in the form $\log_a b$, where a and b are rational numbers to be found.

(4 marks)

3 (a) Solve the following equations, giving your answers correct to 3 significant figures.

$$8e^{3x^2-1} = 12$$

(3 marks)

(b) $e^{3x} - 42 = 2e^{x}(6e^{x} - 7)$

(3 marks)

4 Show that

$$2\log_3 x + \log_3(x^2-1) - 2\log_3(x+1) \equiv \log_3 \frac{x^2(x-1)}{(x+1)}$$

(3 marks)

5 Write the following as a single logarithm

$$2\log_p(x+1) + 3\log_p(x-1) - \log_p(x^2-1)$$

(3 marks)

6 On the same axes, sketch the graphs of $y = e^{2x}$ and $y = \frac{1}{2}\ln x$

On each graph, label any points where the graph intersects the coordinate axes.

Write down the equations of any asymptotes for each graph.

Explain the significance of the line $y = x$.

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(5 marks)

7 Show that $4 - \ln 16$ can be written in the form $4\ln\left(\frac{e}{2}\right)$

(3 marks)

8 A triangle is drawn inside a circle such that one side of the triangle is the diameter and all three vertices of the triangle lie on the circumference.

The radius of the circle is $(3\ln 2)$ cm.

The two smallest angles in the triangle are α and β respectively where $\beta = 2\alpha$.

Find all three sides of the triangle, giving your answers in the form $\alpha\ln 2$.

(5 marks)

9 How many real solutions does the equation have? Justify your answer.

$$3\log_x(x+1) = \ln e^3$$

(3 marks)

10 Without using a calculator, show that

$$\log_4 8 = \log_9 27$$

(3 marks)