Forming the equations yourself

[Textbook] A colony of 1000 mole-rats is made up of adult males, adult females and youngsters. Originally there were 100 more adult females than adult males.
After one year:

- The number of adult males had increased by 2%
- The number of adult females had increased by 3%
- The number of youngsters had decreased by 4%
- The total number of mole-rats had decreased by 20

Form and solve a matrix equation to find out how many of each type of mole-rat were in the original colony.

Let $x =$ number of adult males
$y =$ number of adult females
$z =$ number of youngsters

$$\begin{aligned} x + y + z &= 1000 \\ x - y &= -100 \\ 1.02x + 1.03y + 0.96z &= 980 \end{aligned}$$

"1000 mole-rats"

"Originally 100 more adult females than adult males"

"Total mole-rats after 1 year decreased by 20."

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.02 & 1.03 & 0.96 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1000 \\ -100 \\ 980 \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{13}\begin{pmatrix} -96 & 7 & 100 \\ -96 & -6 & 100 \\ 205 & -1 & -200 \end{pmatrix}\begin{pmatrix} 1000 \\ -100 \\ 980 \end{pmatrix} = \begin{pmatrix} 100 \\ 200 \\ 700 \end{pmatrix}$$

100 adult males, 200 adult females, 700 youngsters in the original colony.

Ex 6F

Q3, 4

Mixed Exercise

Q12

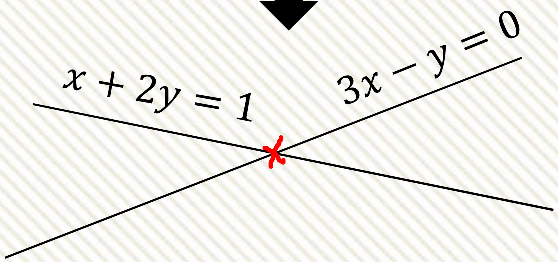
Review Exercise 2 (p.210)

Q11

Consistency of linear equations

From Pure Year 1 you are already familiar with the idea that the solution of a system of two equations (with two unknowns) can be visualised by finding the point of intersection of two lines.
A system of linear equations is known as consistent if there is at least one set of values that satisfies all the equations simultaneously (i.e. at least one point of intersection).

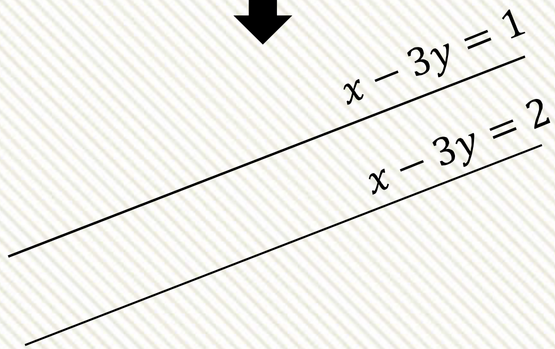
$$x + 2y = 1$$
$$3x - y = 0$$



System of equations is **consistent.** It has **one solution.**

The corresponding matrix $\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ is **non-singular**.

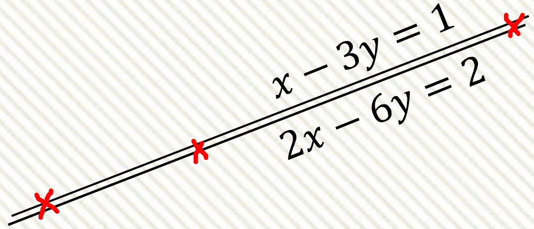
$$x - 3y = 1$$
$$x - 3y = 2$$



System of equations is **inconsistent.** It has **no solutions.**

Matrix $\begin{pmatrix}1 & -3\\ 1 & -3\end{pmatrix}$ is **singular.**

$$x - 3y = 1$$
$$2x - 6y = 2$$



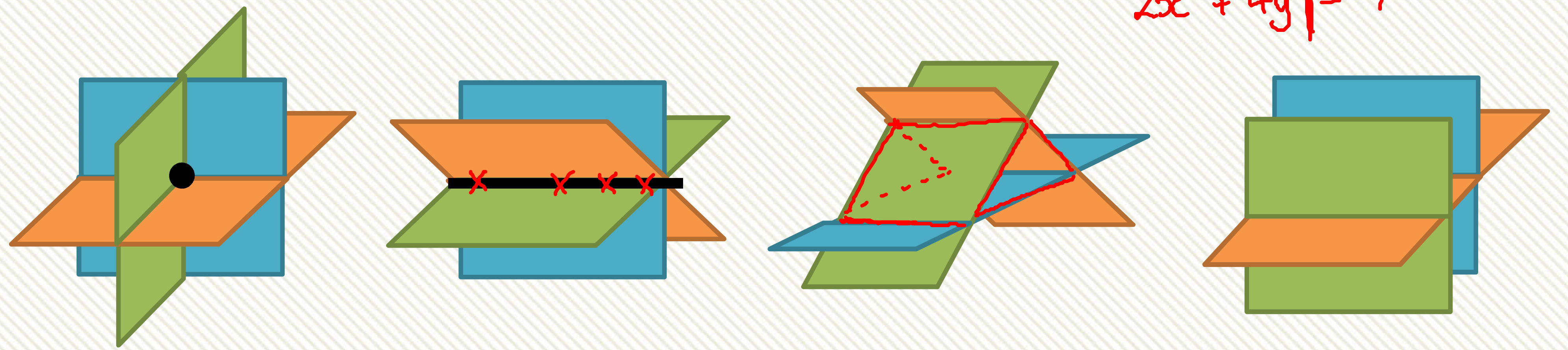
System of equations is **consistent.** It has **infinitely many solutions.**

Matrix $\begin{pmatrix}1 & -3\\ 2 & -6\end{pmatrix}$ is **singular.**

Extending consistency to 3 variables

In Chapter 9 you will learn that just as $ax + by = c$ gives the equation of a straight line, $ax + by + cz = d$ gives the equation of a plane.
Again, we get solutions to the system of linear equations **when <u>all</u> of the planes intersect:**

$$\begin{array}{c|c} x + 2y & = 3 \\ 2x + 4y & = 7 \end{array}$$



Scenario 1: Planes all meet at a single point. System of equations consistent, and one solution.

non-singular

Scenario 2: Planes form a sheaf. They have a line of intersection consisting of infinitely many points. System of equations consistent and infinitely many solutions.

singular

Scenario 3: Planes form a prism. While planes intersect in pairs, they don't all intersect at any point. System of equations is inconsistent.

singular

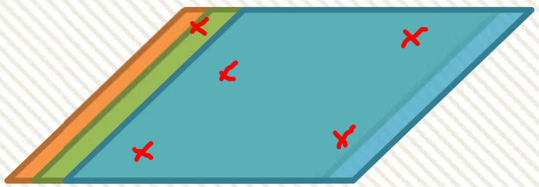
Scenario 4: Two ~~of~~ or more planes parallel and non-identical. Again, inconsistent, as the parallel planes never intersect, and thus all equations can't be satisfied.

singular

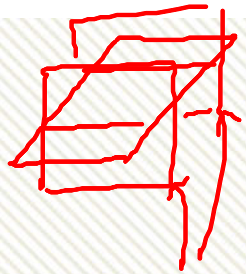
Any rows in the corresponding matrix which are multiples of each other will be parallel.

Extending consistency to 3 variables

In Chapter 9 you will learn that just as $ax + by = c$ gives the equation of a straight line, $ax + by + cz = d$ gives the equation of a plane.
Again, we get solutions to the system of linear equations **when <u>all</u> of the planes intersect:**



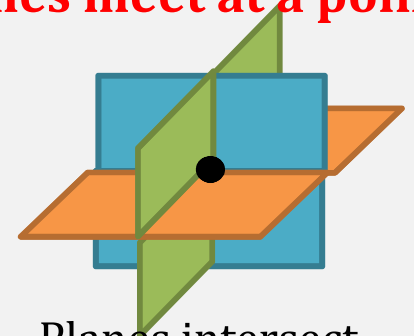
$$\begin{cases} x + 2y + z = 1 \\ 2x + 4y + 2z = 2 \\ -x - 2y - z = -1 \end{cases}$$



Scenario 5: Planes represented by equations are equivalent. System of equations consistent, and infinitely many solutions.

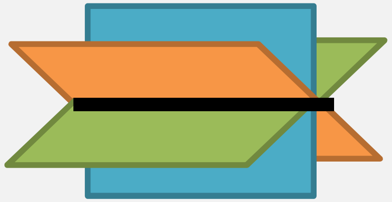
So, **CONSISTENT** means that there is at least one solution of all three equations – i.e. all the planes are intersecting. This could be:

Planes meet at a point



Planes intersect
Only thing that can happen if matrix is non-singular,
i.e. det is not 0

A sheaf



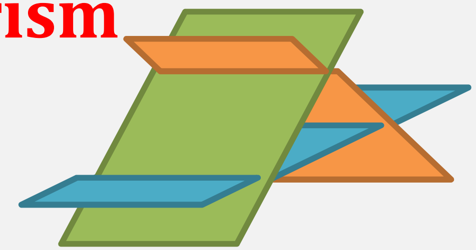
Planes intersect along a line,
Infinitely many solutions

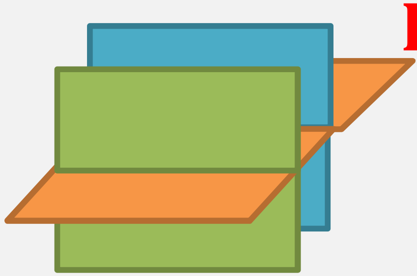
Same plane

All equations of are multiples of each other – they are the same equation.
Infinitely many solutions

So, **INCONSISTENT** means that there are NO SOLUTIONS for **<u>all three</u>** equations at the same time – i.e. they are not all intersecting together. This could be:

A prism





2 or all rows of matrix are multiples of each other

Parallel planes

Decide what geometric situation each set of equations represents.

1) Check what the determinant of the matrix is. If it is NOT zero, they meet at a point
2) Solve the equation on your calculator to decide if it has one solution (meet at a point), infinite solutions (sheaf or same plane) or no solutions (parallel planes or prism)
3) Use algebra to see if they are consistent or inconsistent

$$\begin{aligned} 3x + 4y + z &= 2 \\ 6x + 8y + 2z &= 4 \\ 9x + 12y + 3z &= 6 \end{aligned}$$

$\times 2$ $\times 3$

same plane

$$\begin{aligned} 4x + 3y - 2z &= 5 \\ 2x + 4y - 3z &= 8 \\ 8x + 6y - 4z &= 9 \end{aligned}$$

$$\begin{aligned} x + 2y - z &= 5 \\ 2x + 3y - 3z &= 18 \\ x + 5y + z &= 10 \end{aligned}$$

$$\begin{aligned} x - 2y + 3z &= -2 \\ 2x - 3y + 5z &= -3 \\ x + 3y - 2z &= 3 \end{aligned}$$

$$\begin{aligned} x - 2y - 3z &= -2 \\ 2x - 3y + 5z &= -3 \\ x + 3y - 2z &= 3 \end{aligned}$$

$$\begin{aligned} x - 2y - 11z &= -2 \\ 2x + 11y + 5z &= 11 \\ x + 3y - 2z &= -11 \end{aligned}$$

ActivInspire - Studio

File Edit View Insert Tools Help Untitled * CLASSFLOW Connect Page 1 of 1 Best Fit

$$x - 2y - 11z = -2 \quad ①$$
$$2x + 11y + 5z = 11 \quad ②$$
$$x + 3y - 2z = -11 \quad ③$$

Consistent or inconsistent?

We know from our calc they are inconsistent.

Use equation ① and ③ to eliminate x

$x - 2y - 11z = -2$
$x + 3y - 2z = -11$
$\boxed{-5y - 9z = 9}$

Use equation ① and ② to eliminate x

① × 2

$2x - 4y - 22z = -4$
$2x + 11y + 5z = 11$
$\boxed{-15y - 27z = -15} \longrightarrow -5y - 9z = -5$

But, $-5y - 9z = 9$ when we used ① and ③,

So system is inconsistent.
No planes are parallel, so a prism.

10:47 02/12/2019