Figure 2 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3} \sin 2t$$
, $y = 4 \cos^2 t$, $0 \le t \le \pi$.

- (a) Show that $\frac{dy}{dx} = k\sqrt{3} \tan 2t$, where k is a constant to be determined.
- (b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form y = ax + b, where a and b are constants.

(a)
$$\frac{dx}{dt} = 2\sqrt{3}\cos 2t$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -8\cos t \sin t$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$$

$$= -\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\frac{2}{3}\right) \quad | \quad \text{A1}$$

(b) When
$$t = \frac{\pi}{3}$$
 $x = \frac{3}{2}$, $y = 1$ can be implied B1
$$m = -\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right) \quad (=2)$$
 M1
$$y - 1 = 2\left(x - \frac{3}{2}\right)$$
 M1
$$y = 2x - 2$$
 A1

Differentiate with respect to x

$$f(x) = \ln(\sin x) \qquad f'(x) = \cot x \qquad \int \cot x \, dx = \ln|\sin x| + c$$

$$g(x) = \sec(x^3)$$
 $g'(x) = 3x^2 \sec(x^3)\tan(x^3)$

$$h(x) = 2xe^{x} \quad h'(x) = 2e^{x} + 2xe^{x}$$

$$u = 2x \quad v = e^{x}$$

$$u' = 2 \quad v' = e^{x}$$

$$u' = 2 \quad v' = e^{x}$$

$$x' = 2 \quad v' = e^{x}$$

$$m(x) = (x^2 + 1)\cot x$$
 $m(x) = (x^2 + 1)\cot x$
 $m'(x) = 2x\cot x - (x^2 + 1)\cot x^2 x$
 $m'(x) = 2x\cot x - (x^2 + 1)\cot x^2 x$

$$n(x) = e^{(2x)} \ln x \qquad \text{where } x = \ln x \qquad \text{where } x = \frac{1}{x}$$

$$n'(x) = e^{(2x)} \ln x \qquad \text{where } x = \frac{1}{x}$$

Implicit Differentiation

Implicit Functions

$$x^{2}+y^{2}=16$$

$$2y+15x-xy=0$$

$$-3sin(xy)+15x=y$$

$$y=2x+5$$

$$y=sin^{2}(x+3)$$

$$y=sin^{2}(x+3)$$

$$y=13-(nx)$$
We can already to find the final time of the final

There is a mixture of x and y terms. y is not the subject.

Explicit Functions

$$y = 2x + 5$$
 $y = \sin^2(x + 3)$

We can already already the subject

 $y = 13 - \ln x$

We can already the subject

 $y = \sin^2(x + 3)$

We can already the subject

$$y = x^{2}$$

$$\frac{d}{dx} \left(\frac{dy}{dx} = 2x \right) \frac{d}{dx}$$

day -> dy

To differentiate implicitly you only need to know 2 things:

- Differentiate each side of the equation (using chain rule if necessary).
- Remember that y differentiated with respect to x is, by definition, $\frac{dy}{dx}$

Try
$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \frac{dy}{dx} \quad \text{and then multiply by } \frac{dy}{dx}$$

$$x = y^2$$

$$1 = 2y \frac{dy}{dx}$$

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \frac{dy}{dx} \quad \text{and then multiply by } \frac{dy}{dx}$$

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$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2)\frac{dy}{dx}$$
$$= 2y\frac{dy}{dx}$$

$$\frac{d}{dx}(x^2 + \cos y) = 2x - \sin y \times \frac{dy}{dx}$$

$$\frac{d}{dx}(\sin y) = \frac{d}{dy}(\sin y) \frac{dy}{dx}$$

$$= (\cos y) \frac{dy}{dx}$$

$$\frac{d}{dx}(e^y) = \mathcal{L}^y \frac{dy}{dx}$$

$$\frac{d}{dx}(e^{x^2y}) = e^{x^2y} \times \frac{d}{dx}(x^2y)$$

$$u = x^2 + x \frac{dy}{dx}$$

$$u = x^2 + y = e^{x^2y}(2xy + x^2 \frac{dy}{dx})$$

$$u = x + y = y$$

$$u' = 1x + y' = \frac{dy}{dx}$$

$$u'' = 1 + y' = \frac{dy}{dx}$$

$$\frac{d}{dx}(xy) = y + x \frac{dy}{dx}$$

$$u = x \quad \forall = y$$

$$u' = | \quad \forall' = dy$$

$$\frac{dx}{dx}$$

$$\frac{d}{dx}(\tan(x+y)) = \sec^{2}(x+y) \times (1+\frac{dy}{dx})$$

$$= (1+\frac{dy}{dx})\sec^{2}(x+y)$$

In general, when differentiating a function of y, but with respect to x, slap a $\frac{dy}{dx}$ on the end. i.e. $\frac{d}{dx}(f(y)) = f'(y)\frac{dy}{dx}$

$$\frac{d}{dx}(f(y)) = f'(y)\frac{dy}{dx}$$

Find $\frac{dy}{dx}$ in terms of x and y where $x^3 + x + y^3 + 3y = 6$

$$3x^{2} + 1 + 3y^{2} \frac{dy}{dx} + 3\frac{dy}{dx} = 0$$
 rearranged
 $3x^{2} + 1 + 3y^{2} \frac{dy}{dx} + 3\frac{dy}{dx} = 0$ so dy tems were so dy tems side.

$$\frac{dy}{dx}(3y^{2} + 3) = -3x^{2} - 1$$
 on one side.

$$\frac{dy}{dx} \text{ terms}$$

$$\frac{dy}{dx} = \frac{-3x^{2} - 1}{3y^{2} + 3}$$

Note: to find the gradient, you require an x AND y value.

Find the value of $\frac{dy}{dx}$ at the point (1,1), where $e^{2x} \ln y = x + y - 2$

$$e^{2x} \ln y = x + y - 2$$

$$2e^{2x} \ln y + e^{2x} \frac{1}{y} dy = 1 + \frac{dy}{dx}$$

$$2e^{2x} \ln y - 1 = \frac{dy}{dx} - e^{2x} \frac{1}{y} \frac{dy}{dx}$$

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$$2e^{2x} \ln y - 1 = \frac{dy}{dx} \left(1 - e^{2x} \frac{1}{y}\right)$$

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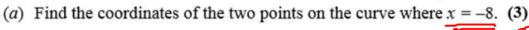
$$2e^{2x} \ln y - 1 = \frac{dy}{dx}$$

$$2e^{2x} \ln y - 1$$

A curve is described by the equation

$$x^3 - 4y^2 = 12xy$$
.





(b) Find the gradient of the curve at each of these points.

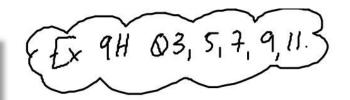
$$(-8)^{3} - 4y^{2} = 12(-8)y$$

$$-512 - 4y^{2} = -96y$$

$$0 = 4y^{2} - 96y + 512$$

$$y = 16 \quad y = 8$$

$$(-8, 16) \quad (-8, 8)$$



Then solve

(6)
$$x^3 - 4y^2 = 12xy$$
 $u = 12x$ $y^2 = 4y$ $3x^2 - 8y \frac{dy}{dx} = 12y + 12x \frac{dy}{dx}$ $3x^2 - 12y = 12x \frac{dy}{dx} + 8y \frac{dy}{dx}$ $3x^2 - 12y = \frac{dy}{dx} (12x + 8y)$ $x = -8, y = 16$ $x = -3$

C4 June 2014(R) Q3

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer. (5)
- (b) Find the values of y for which $\frac{dy}{dx} = 0$. (5)

Hint for (b): Solve simultaneously with original equation.

1	x = -8y = 8	ď
1	dy =0.	
	dr	

8	$ x^{2} + y^{2} + 10x + 2y - 4xy = 10$ $\left \frac{dy}{dx} \times \right = 2x + 2y \frac{dy}{dx} + 10 + 2\frac{dy}{dx} - \left(4y + 4x \frac{dy}{dx}\right) - \frac{dy}{dx}$	Scr pates	MI AJ NO
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{dy}{dx} = 0$	Dependent on the first ML mark	dan
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Al cones
5	$\begin{cases} \frac{dy}{dx} = 0 \Rightarrow \\ 0 \Rightarrow -2y = 0 \end{cases}$ 50 $y = 2y = 3$.		30 ps
	$(7y - 5)^2 + y^2 + 10(3y - 5) + 2y - 4(5y - 5)y + 10$ $4y^2 - 20y + 25 + y^2 + 20y - 93 + 2y - 8y^2 + 20y - 10$		M
	gives $-3y^2 + 22y - 35 = 0$ oc $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35 (= 0)$ see notes	Al se
	(2y - 7)(y - 5) = 0 and $y = 1$.	Method mark for solving a quadratic equation	84D41
	$p = \frac{7}{3}, 5$	$(y \rightarrow \frac{7}{3}, 3)$	Al me