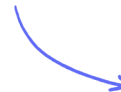


# 2.11 Partial Fractions (A Level only)

Easy (8 questions)	/36
Medium (9 questions)	/51
Hard (9 questions)	/53
Very Hard (9 questions)	/55
<b>Total Marks</b>	<b>/195</b>

Scan here to return to the course  
or visit [savemyexams.com](https://www.savemyexams.com)



# Easy Questions

1 Express

$$\frac{1}{x(x+1)}$$

as partial fractions.

(3 marks)

2 (i) Factorise  $x^2 + 5x - 6$ .

(ii) Hence, or otherwise, express

$$\frac{(5x+16)}{x^2+5x-6}$$

as partial fractions.

(4 marks)

3 Express

$$\frac{6x-13}{x^2-5x+6}$$

as partial fractions.

(4 marks)

4 Write

$$\frac{2(x-11)}{x^2+2x-15}$$

in the form

$$\frac{A}{x+5} + \frac{B}{x-3}$$

where  $A$  and  $B$  are integers to be found.

(3 marks)

5 Show that

$$\frac{3x+8}{(x+1)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

where  $A$  and  $B$  are integers to be found.

(3 marks)

6 Express

$$\frac{5x^2 + 10x + 8}{x(x+2)^2}$$

in partial fractions.

(5 marks)

7 Write

$$\frac{3x^2 - 6x + 2}{x^3 - 3x^2 + 2x}$$

in the form

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

where  $A, B$  and  $C$  are integers to be found.

(5 marks)

**8 (a)** (i) Briefly explain why

$$\frac{x^2 + x - 4}{x^2 + 4x + 3}$$

is an improper (algebraic) fraction.

(ii) Show that

$$(x^2 + x - 4) \div (x^2 + 4x + 3) = 1 - \frac{(3x + 7)}{x^2 + 4x + 3}$$

**(4 marks)**

**(b)** Express

$$\frac{3x + 7}{x^2 + 4x + 3}$$

as partial fractions.

**(3 marks)**

**(c)** Hence, show that

$$\frac{x^2 + x - 4}{x^2 + 4x + 3} = 1 - \frac{2}{x + 1} - \frac{1}{x + 3}$$

(2 marks)

# Medium Questions

- 1 Express  $\frac{2}{x(x+2)}$  as partial fractions.

(3 marks)

- 2 Write  $\frac{x-10}{(x+2)(x-1)}$  in the form  $\frac{A}{x+2} + \frac{B}{x-1}$ , where  $A$  and  $B$  are integers to be found.

(3 marks)



**3 (a)** It is given that

$$f(x) = x^3 - 13x + 12.$$

- (i) Show that  $f(1) = 0$ .
- (ii) Hence write down a factor of  $f(x)$ .

**(3 marks)**

- (b)** (i) Show that  $f(3) = 0$ .
- (ii) Hence write down another factor of  $f(x)$ .

**(3 marks)**

- (c)** Fully factorise  $f(x)$ .

**(2 marks)**

- (d)** Express  $\frac{3x^2 - 13}{x^3 - 13x + 12}$  as partial fractions.

(4 marks)

4 Express  $\frac{7x + 34}{x^2 + 9x + 14}$  as partial fractions.

(4 marks)

**5 (a)** Use algebraic division to work out  $(x^2 + 4x - 6) \div (x^2 - x - 6)$ .

**(2 marks)**

**(b)** Write  $\frac{5x}{(x+2)(x-3)}$  in the form  $\frac{A}{x+2} + \frac{B}{x-3}$

**(3 marks)**

**(c)** Hence, or otherwise, show that  $\frac{x^2 + 4x - 6}{x^2 - x - 6} = 1 + \frac{2}{x+2} + \frac{3}{x-3}$

**(3 marks)**

**6** Express  $\frac{2x}{(x+1)(x-2)}$  as partial fractions.

**(4 marks)**

**7 (a)** Use the factor theorem to show that  $(x + 1)$  is a factor of  $x^3 - 3x^2 + 4$ .

**(2 marks)**

**(b)** Hence, or otherwise, fully factorise  $x^3 - 3x^2 + 4$ .

**(2 marks)**

**(c)** Show that  $\frac{19 - 8x}{x^3 - 3x^2 + 4}$  can be written in the form  $\frac{A}{x + 1} + \frac{B}{x - 2} + \frac{C}{(x - 2)^2}$ , where  $A, B$  and  $C$  are integers to be found.

**(4 marks)**

**8** Express  $\frac{3x + 11}{x^2 + 6x + 9}$  as partial fractions.

**(4 marks)**

9 Express  $\frac{x^2 - 5}{x^3 - x^2 - 17x - 15}$  as partial fractions.

(5 marks)

# Hard Questions

- 1 Express  $\frac{12}{x^2 + 4x - 5}$  as partial fractions.

(3 marks)

- 2 Write  $\frac{12x - 6}{(x - 3)(x + 2)(x - 1)}$  in the form  $\frac{A}{x - 3} + \frac{B}{x + 2} + \frac{C}{x - 1}$ , where  $A, B$  and  $C$  are integers to be found.

(4 marks)

**3 (a)** It is given that

$$f(x) = x^3 + 3x^2 - 4x - 12.$$

- (i) Show that  $f(-3) = 0$
- (ii) Hence write down a factor of  $f(x)$ .

**(3 marks)**

**(b)** Fully factorise  $f(x)$ .

**(4 marks)**

**(c)** Express  $\frac{3x^2 - 8x - 36}{x^3 + 3x^2 - 4x - 12}$  as partial fractions.

**(3 marks)**

**4** Express  $\frac{2x^2 + 16x - 24}{x^3 - 2x^2 - 8x}$  as partial fractions.

**(4 marks)**



- 5 (a)** Show that  $\frac{x^2 + x + 5}{x^2 + 3x + 2}$  can be written in the form  $A - \frac{2x - 3}{(x + 1)(x + 2)}$ , where  $A$  is an integer to be found.

**(2 marks)**

- (b)** Express  $\frac{2x - 3}{(x + 1)(x + 2)}$  in the form  $\frac{B}{x + 1} + \frac{C}{x + 2}$  where  $B$  and  $C$  are integers to be found.

**(3 marks)**

- (c)** Using your values for  $A, B$  and  $C$  write  $\frac{x^2 + x + 5}{x^2 + 3x + 2}$  in the form  $A + \frac{B}{x + 1} + \frac{C}{x + 2}$

**(3 marks)**

- 6** Express  $\frac{2x}{x^2 - 9x - 52}$  as partial fractions.

(5 marks)

**7 (a)** Use the factor theorem to show that  $(x - 2)$  is a factor of  $x^3 + 4x^2 - 3x - 18$ .

**(2 marks)**

**(b)** Hence, or otherwise, fully factorise  $x^3 + 4x^2 - 3x - 18$ .

**(3 marks)**

**(c)** Show that  $\frac{7x^2 + 32x + 8}{x^3 + 4x^2 - 3x - 18}$  can be written in the form  $\frac{A}{x - 2} + \frac{B}{x + 3} + \frac{C}{(x + 3)^2}$ ,  
where  $A, B$  and  $C$  are integers to be found.

**(4 marks)**

**8** Express  $\frac{3x^2 - 22x + 25}{x^3 - 7x^2 + 8x + 16}$  as partial fractions.

(5 marks)

9 Express  $\frac{2x^2 - 3}{x^3 + 4x^2 - 3x - 18}$  as partial fractions.

(5 marks)

# Very Hard Questions

- 1 Express  $\frac{8}{x^3 - 6x^2 + 8x}$  as partial fractions.

(5 marks)

- 2 Write  $\frac{13x^2 - 10}{x^3 - x^2 - 2x}$  in the form  $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2}$ , where  $A, B$  and  $C$  are integers to be found.

(5 marks)

- 3 Show that  $\frac{34 - 3x^2 - x}{x^3 - 5x^2 + 3x + 9}$  can be written in the form  $\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ , where  $A, B$  and  $C$  are integers to be found.

(5 marks)

- 4 Express  $\frac{x^2 - 3}{(x - 3)(x + 2)^2}$  as partial fractions.

(4 marks)

- 5 Write  $\frac{x^2 + 3x + 10}{x^2 + 8x + 15}$  in the form  $A + \frac{B}{x + 3} + \frac{C}{x + 5}$ , where  $A, B$  and  $C$  are integers to be found.

(4 marks)

**6 (a)** It is given that

$$f(x) = x^3 + 3x^2 - 4x - 12.$$

- (i) Show that  $f(-3) = 0$ .
- (ii) Hence, fully factorise  $f(x)$ .

**(6 marks)**

**(b)** Express  $\frac{3x^2 - 8x - 36}{f(x)}$  as partial fractions.

**(4 marks)**

**7** Express  $\frac{2x - 5}{x^3 - 3x^2 - 13x + 15}$  as partial fractions.

(10 marks)

- 8 Express  $\frac{2(3x^3 + x^2 - 17x + 4)}{x(x-1)(x-4)(x+2)}$  as partial fractions.

(5 marks)

- 9 Given that  $x^4 - 4x^3 - 2x^2 + 12x + 9 = (x^2 - 6x + 9)(x^2 + 2x + 1)$ , express  $\frac{5x^3 - 15x^2 + 19x + 7}{x^4 - 4x^3 - 2x^2 + 12x + 9}$  as partial fractions.



**(7 marks)**