IAL M1 Jan 2022

5.

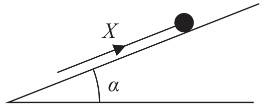


Figure 3

Try this Q whilst we wait for everyone to arrive! We'll go over it at the start of the session!

A particle of mass m rests in equilibrium on a fixed rough plane under the action of a force of magnitude X. ~~The force acts up a line of greatest slope of the plane,~~ as shown in Figure 3.

The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$ $\frac{\text{opp}}{\text{adj}}$

$\sin\alpha = \frac{3}{5} = 0.6$

$\cos\alpha = \frac{4}{5} = 0.8$

The coefficient of friction between the particle and the plane is μ.

- When $X = 2P$, the particle is on the point of sliding up the plane.
- When $X = P$, the particle is on the point of sliding down the plane.

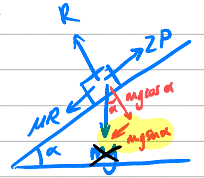
Find the value of μ.

$F_r = \mu R$

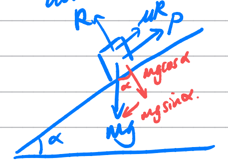
$F_r \leq \mu R$

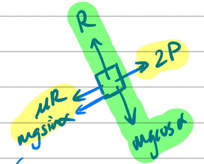
(8)

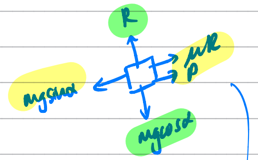
up



down







resolve $R = mg\cos\alpha$

$R = 0.8mg$

Equilibrium

$2P = \mu R + mg\sin\alpha$

$2P = \mu 0.8mg + 0.6mg$

Equilibrium

$P + \mu R = mg\sin\alpha$

$P + \mu 0.8mg = 0.6mg$

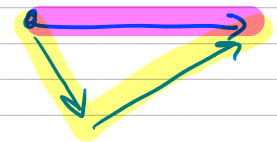
$P = 0.6mg - \mu 0.8mg$

$2(0.6mg - \mu 0.8mg) = \mu 0.8mg + 0.6mg$

$1.2\cancel{mg} - \mu 1.6\cancel{mg} = \mu 0.8\cancel{mg} + 0.6\cancel{mg}$

$0.6 = 2.4\mu$

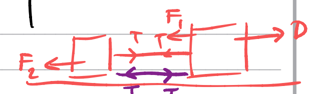
$\mu = \frac{0.6}{2.4} = \frac{1}{4}$

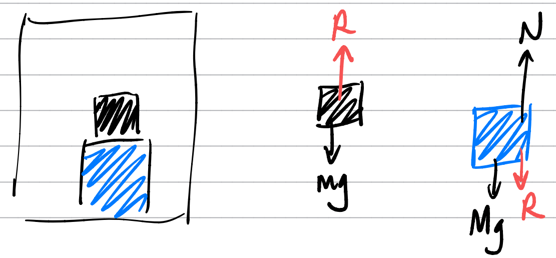


parallel to $\underline{i}$ $k\begin{pmatrix}1\\0\end{pmatrix}$

$\underline{j}$ $k\begin{pmatrix}0\\1\end{pmatrix}$

If the car was braking tension becomes thrust





If it is moving/accelerating/on the point of moving $F = \mu R$.

If not, $F < \mu R$

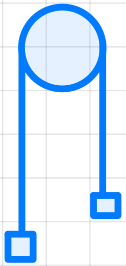
Exam Patterns / Predictions

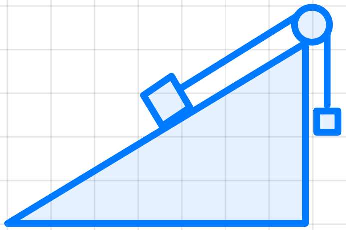
- Moments – ladder, beam, resting on a drum etc.
- Forces : either – connected particles
  – one on a slope
- Projectiles
- Variable acceleration } potentially combined
- Vectors

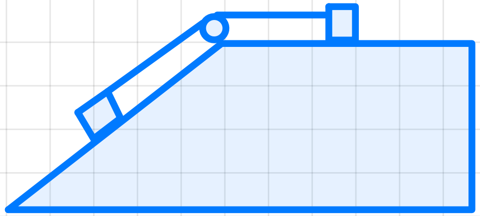
Possible but niche:

- no connected particles, but <u>lift</u> Q
- subsequent motion with connected particles
- SUVAT – sim. eq. or vertical motion.

Connected Particles







I think less likely in A2, more AS style

unlikely/unusual but possible.

"Equation of motion for A" → F=ma for A

smooth pulley → (no friction) → tension same either side of the pulley

inextensible string → (doesn't stretch) → acceleration/speed of both particles is equal

light string → (has no mass) → tension in each section of string equal throughout

Improvements

↳ read through their model, tell them not to make one of these assumptions!

Things other than air resistance...

→ more accurate value for g

→ include dimensions of particles

→ resistance should <u>vary</u> with speed/not be constant.

7.

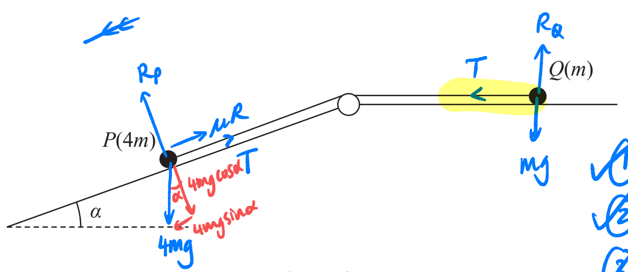


Figure 4

① F=ma
② F=ma
③ sim. eq.

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

$\sin\alpha = \frac{3}{5} = 0.6$

$\cos\alpha = \frac{4}{5} = 0.8$

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

~~The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.~~

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string.

(7)

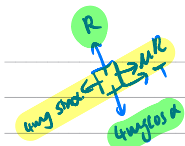
(c) State where in your calculations you have used the fact that the pulley is smooth. → tensions either side of the pulley are equal.

(d) Suggest an improvement for the model. → model the string as extensible.

a) $F = ma$ ←

$T = ma$ Q

b)



$R = 4mg\cos\alpha$

$= 4mg \times 0.8$

$= 3.2mg$

F=ma ↙

$\mu = \frac{1}{4}$

P

$4mg\sin\alpha - \mu R - T = 4m \times a$

$R = 3.2mg$

$4mg \times 0.6 - \frac{1}{4} \times 3.2mg - T = 4T$

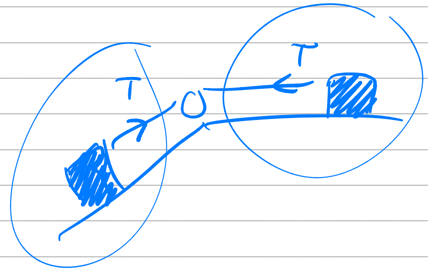
$T = ma$ Q

$2.4mg - 0.8mg = 5T$

$1.6mg = 5T$

$T = \frac{8}{25}mg$

→ OK for m to be in the answer



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3.

$\frac{1}{5}g$

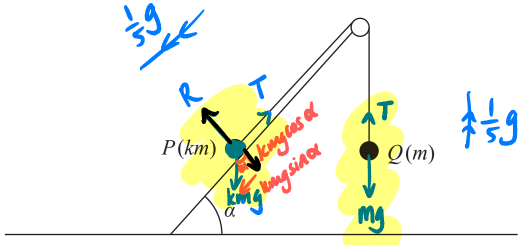
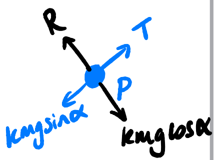


Figure 1



One end of a light inextensible string is attached to a particle P of mass km, where $k > 1.25$

The other end of the string is attached to a particle Q of mass m.

The string passes over a small smooth light pulley that is fixed at the top of a plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{4}{3}$ $\sin\alpha = \frac{4}{5}$ 0.8 $\cos\alpha = \frac{3}{5}$ 0.6

Particle P is held at rest on the plane and particle Q hangs at rest with the string taut, as shown in Figure 1.

The part of the string from P to the pulley lies along a line of greatest slope of the plane.
The two particles and the pulley all lie in the same vertical plane.
The particle P is released from rest.

In an initial model,

- the plane is modelled as being smooth
- P slides down the plane with acceleration $\frac{1}{5}g$

Using this model,

(a) write down an equation of motion for P **(2)**

(b) find the value of k. **(4)**

In a second model,

- the plane is modelled as being rough
- the coefficient of friction between P and the plane is μ
- P remains at rest but is on the point of slipping down the plane

F_r max
$F_r = \mu R$

Using this model,

(c) find, in terms of k, m and g, the magnitude of the normal reaction exerted by the plane on P. **(2)**

(d) find, in terms of k, the value of μ. **(6)**

Question 3 continued

a) $F = ma$ ↙

$kmg\sin\alpha - T = kma$

$0.8kmg - T = kma$

b) $0.8kmg - T = km \times \frac{1}{5}g$

$0.8kmg - T = \frac{1}{5}kmg$

Q $F = ma$ ↑

$T - mg = m \times \frac{1}{5}g$

$T - mg = \frac{1}{5}mg$

$T = 1.2mg$

$0.8k\cancel{mg} - 1.2\cancel{mg} = 0.2k\cancel{mg}$

$0.8k - 1.2 = 0.2k$

$0.6k = 1.2$

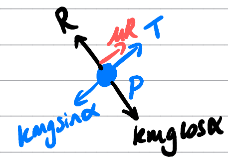
$k = \frac{1.2}{0.6} = \underline{\underline{2}}$

c) $R = kmg\cos\alpha$

$R = 0.6kmg$

Q 

$T = mg$

d) 

$kmg\sin\alpha = T + \mu R$

$0.8k\cancel{mg} = 1\cancel{mg} + \mu 0.6k\cancel{mg}$

$0.8k = 1 + \mu 0.6k$

$0.8k - 1 = \mu 0.6k$

$\frac{0.8k - 1}{0.6k} = \mu$

$\underline{\underline{\frac{4k-5}{3k} = \mu}}$

Moments / Rigid Bodies : ladders and drums

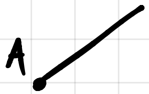
Moment is force × perp. dist → I always use this one!

OR

perp force × dist → but this is OK if you use this!

3 things:

1. Ⓘ Moments (usually about A)

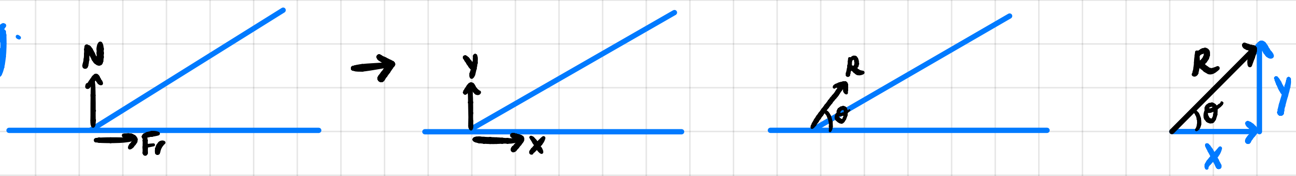


2. Ⓘⓘ Resolve ↕
3. Ⓘⓘⓘ Resolve ↔

... then combine!

If there are 2 forces acting on the end of the rod, they may describe this as a single reaction force

eg.



$R = \sqrt{x^2 + y^2}$

$\tan\theta = \dfrac{y}{x}$

Leave blank

6.

① Moments
② ↕
③ ↔

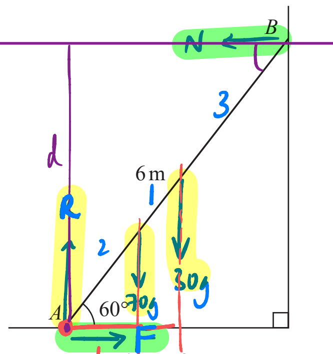
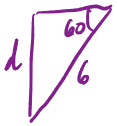
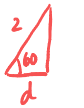


Figure 3



$d = 6\sin 60$

$F = \mu R$
$F \leq \mu R$



$d = 2\cos 60$

$d = 3\cos 60$

A ladder AB has length 6 m and mass 30 kg. The ladder rests in equilibrium at 60° to the horizontal with the end A on rough horizontal ground and the end B against a smooth vertical wall, as shown in Figure 3.

A man of mass 70 kg stands on the ladder at the point C, where $AC = 2$ m, and the ladder remains in equilibrium. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall. The man is modelled as a particle.

(a) Find the magnitude of the force exerted on the ladder by the ground.
(6)

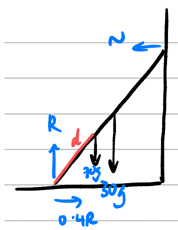
The man climbs further up the ladder. When he is at the point D on the ladder, the ladder is about to slip. $F = \mu R = 0.4R$

Given that the coefficient of friction between the ladder and the ground is 0.4

(b) find the distance AD.
(4)

(c) State how you have used the modelling assumption that the ladder is a rod.
(1)

↳ rigid and so does not bend.



$N = 0.4R$
$R = 100g$

Moments (A)

$70g \times d\cos 60 + 30g \times 3\cos 60 = N \times 6\sin 60$

$35gd + 45g = 0.4R \times 6\frac{\sqrt{3}}{2}$

$35\cancel{g}d + 45\cancel{g} = 0.4 \times 100\cancel{g} \times 6\frac{\sqrt{3}}{2}$

$d = 4.65\text{ m}$ (3sf)

P 6 0 7 0 6 A 0 1 6 2 8

DO NOT WRITE IN THIS AREA

Resolve ↕

$R = 70g + 30g$

$R = 100g$

$R = 100g$
$F = ?$

Resolve ↔

$N = F$

Moments (A)

$70g \times 2\cos 60 + 30g \times 3\cos 60 = N \times 6\sin 60$

$70g + 45g = F \times 3\sqrt{3}$

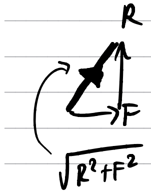
$F = \frac{115g}{3\sqrt{3}} = 216.89\ldots\text{ N}$

Overall Reaction force $= \sqrt{R^2 + F^2}$

$= \sqrt{(100g)^2 + (216.89\ldots)^2}$

$= 1003.7\ldots\text{ N}$

$= 1000\text{ N}$ (3sf)



YOUR TURN

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3.

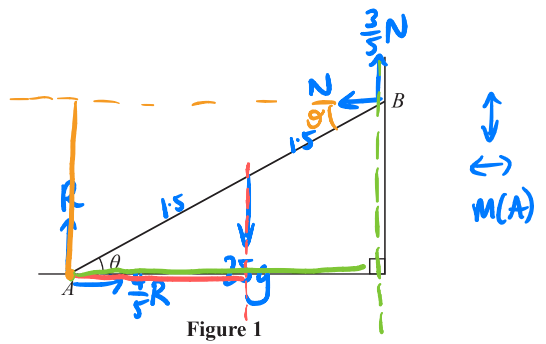


Figure 1

A uniform rod AB, of mass 25 kg and length 3 m, has end A resting on rough horizontal ground. The end B rests against a rough vertical wall.

The rod is in a vertical plane perpendicular to the wall.

The coefficient of friction between the rod and the ground is $\frac{4}{5}$

The coefficient of friction between the rod and the wall is $\frac{3}{5}$

The rod rests in limiting equilibrium.

The rod is at an angle of θ to the ground, as shown in Figure 1.

Find the exact value of $\tan\theta$.

(9)

$\updownarrow \quad R + \frac{3}{5}N = 25g$

$\leftrightarrow \quad \frac{4}{5}R = N$

$R + \frac{3}{5} \times \frac{4}{5}R = 25g$

$R + \frac{12}{25}R = 25g$

$\frac{37}{25}R = 25g$

$R = \frac{625}{37}g \qquad N = \frac{4}{5}R = \frac{500}{37}g$

P 6 5 2 6 5 A 0 8 3 2

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

$N = \frac{500}{37}g$

M(A)

$25g \times 1.5\cancel{\cos\theta} = \frac{3}{5}N \times 3\cancel{\cos\theta} + N \times 3\sin\theta \rightarrow \tan\theta$

$\cancel{\cos\theta} \qquad \cancel{\cos\theta} \qquad \cos\theta$

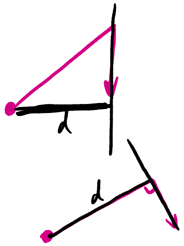
$37.5g = 1.8N + 3N\tan\theta$

$37.5\cancel{g} = 1.8 \times \frac{500}{37}\cancel{g} + 3 \times \frac{500}{37}\cancel{g}\tan\theta$

$\frac{975}{74} = \frac{1500}{37}\tan\theta$

$\frac{13}{40} = \tan\theta$

4.



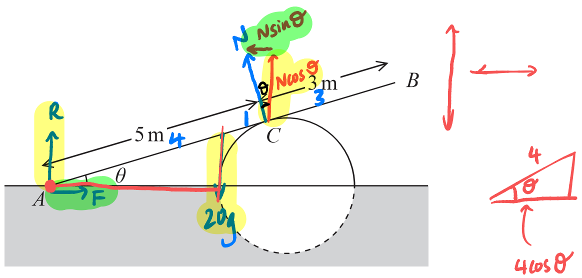


Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where AC = 5 m, as shown in Figure 2.

~~The ramp is resting in a vertical plane which is perpendicular to the axis of the drum,~~ at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

$\sin\theta = \frac{7}{25} = 0.28$ $\quad \cos\theta = \frac{24}{25} = 0.96$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp. **(1)**

The ramp is a tangent to the drum circle, so is perp.. There is no friction as the drum is smooth.

(b) Find the magnitude of the resultant force acting on the ramp at A. **(9)**

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C. **(1)**

~~explain~~ it will decrease.

P 6 3 3 5 9 A 0 1 0 2 0

Question 4 continued

R ↕

$R + N\cos\theta = 20g$

$R + 0.96N = 20g$

R = ?

F = ?

R ↔

$F = N\sin\theta$

$F = 0.28N$

Moments (A)

$20g \times 4\cos\theta = 5N$

$\frac{20g \times 4 \times 0.96}{5} = N$

$N = 150.528$

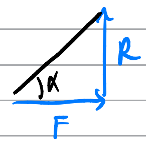
$F = 0.28 \times 150.528$

$= 42.14784$

$R = 20g - 0.96N$

$= 20g - 0.96 \times 150.528$

$= 51.49312$



$\sqrt{R^2 + F^2} = 66.5N$ (3sf)

$\tan\alpha = \frac{R}{F}$

P 6 3 3 5 9 A 0 1 1 2 0

5.

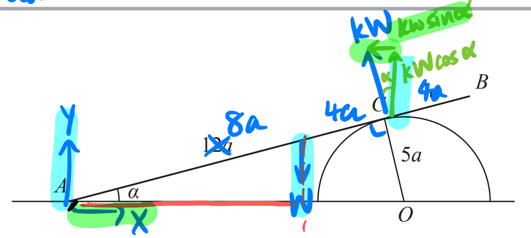
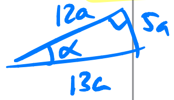


Figure 1

$\tan\alpha = \frac{5}{12}$

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$



A smooth solid hemisphere is fixed with its flat surface in contact with rough horizontal ground. The hemisphere has centre O and radius $5a$.
A uniform rod AB, of length $16a$ and weight W, rests in equilibrium on the hemisphere with end A on the ground. The rod rests on the hemisphere at the point C, where $AC = 12a$ and angle $CAO = \alpha$, as shown in Figure 1.

Points A, C, B and O all lie in the same vertical plane.

(a) Explain why $AO = 13a$ **(1)**

AB is a tangent hence a right angle, meaning $AO = \sqrt{(12a)^2+(5a)^2} = 13a$

The normal reaction on the rod at C has magnitude kW

(b) Show that $k = \frac{8}{13}$ **(3)**

The resultant force acting on the rod at A has magnitude R and acts upwards at $\theta°$ to the horizontal.

(c) Find

(i) an expression for R in terms of W

(ii) the value of θ **(8)**

b) M(A)

$\cancel{W} \times 8\cancel{a}\cos\alpha = k\cancel{W} \times 12\cancel{a}$

$8\cos\alpha = 12k$

$\frac{2}{3}\cos\alpha = k$

$\frac{2}{3} \times \frac{12}{13} = k$

$k = \frac{8}{13}$

c) R↕

$Y + kW\cos\alpha = W$

$Y + \frac{8}{13} \times \frac{12}{13} W = W$

$Y + \frac{96}{169} W = W$

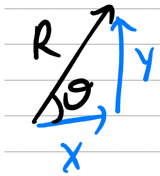
$Y = \frac{73}{169} W$

R↔

$X = kW\sin\alpha$

$= \frac{8}{13} \times \frac{5}{13} W$

$= \frac{40}{169} W$



$R = \sqrt{X^2+Y^2}$

$R = W\sqrt{\left(\frac{73}{169}\right)^2 + \left(\frac{40}{169}\right)^2}$

$R = W\frac{\sqrt{41}}{3} = 0.49W$

$\tan\theta = \frac{Y}{X} = \frac{\frac{73}{\cancel{169}}\cancel{W}}{\frac{40}{\cancel{169}}\cancel{W}}$

$\tan\theta = \frac{73}{40}$

$\theta = 61.3°$

P 6 9 2 8 9 A 0 1 6 2 8

Projectiles

2 things (I) Horizontal Motion → horizontal speed never changes

→ use dist = speed × time (or SUVAT with $a=0$)

(II) Vertical Speed → vertical speed varies with $-g$ acceleration

→ use SUVAT with $a=-g$

- time (t) is the "bridging variable" that lets you jump between I and II

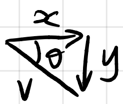
Some sneaky questions and how to deal with them...

- Find when the particle has min. speed / find the min speed

↳ vertical speed $=0$ → remember, it will have horizontal speed.

- Find the speed of the particle after eg. 2 seconds
  direction of motion

↳



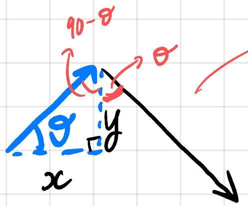
$v=\sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$

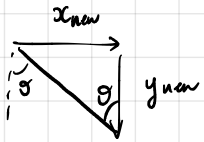
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

- ★ Find when the particle is travelling perp. to its angle of projection

v. unlikely but here to stretch!



$m = ? = \frac{y}{x}$ ↓ perp. $-\frac{x}{y}$



$\tan\theta = \frac{x}{y}$

5.

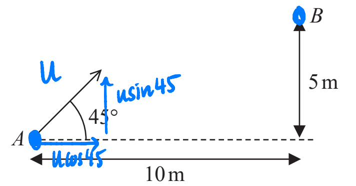


Figure 3

In a game, a small ball is thrown from a point A to a point B.
The ball is thrown at 45° to the horizontal.
The point B is 10 m horizontally from A and 5 m vertically from A, as shown in Figure 3.

In an initial model,

- the ball is modelled as a particle moving freely under gravity
- the initial speed of the ball is $U \text{ m s}^{-1}$
- $g = 9.8 \text{ m s}^{-2}$

Using this model,

(a) find the value of U.
(6)

(b) find the speed of the ball at B.
(3)

One limitation of this model is that the air resistance on the ball is ignored.

(c) State one other limitation of this model.
(1)

spin of the ball
wind direction
dimensions of ball
more accurate g.

In a refinement of the model,

- the ball is modelled as a particle
- air resistance on the ball is included
- the initial speed of the ball is $V \text{ m s}^{-1}$
- $g = 9.8 \text{ m s}^{-2}$

(d) State how the value of V compares with the value of U, giving a reason for your answer.
(1)

V is greater than u.
This is because the air resistance will slow it down, and it still reaches the same point.

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Horizontal motion

dist = speed × time

$10 = u\cos 45 \times t \rightarrow \frac{10}{u\cos 45}$

bridging variable

Vertical motion ↑+

$u = u\sin 45$
$s = 5$
$a = -g = -9.8$
$t = \frac{10}{u\cos 45}$

$s = ut + \frac{1}{2}at^2$

$5 = \cancel{u}\sin 45 \times \frac{10}{\cancel{u\cos 45}} - 4.9\left(\frac{10}{u\cos 45}\right)^2$

$5 = 10 - 4.9\left(\frac{100}{u^2 \times \frac{1}{2}}\right)$

$4.9\left(\frac{100}{\frac{1}{2}u^2}\right) = 10 - 5$

$4.9\left(\frac{100}{\frac{1}{2}u^2}\right) = 5$

$\frac{100}{\frac{1}{2}u^2} = \frac{50}{49}$

$\frac{\frac{1}{2}u^2}{100} = \frac{49}{50}$

$u^2 = 196$

$\underline{u = 14}$

b) horiz. speed at B

speed $= u\cos 45 = 14 \times \cos 45$
$= \underline{7\sqrt{2}}$

vert. speed at B
$u = 14\sin 45$
$s = 5$
$a = -9.8$
$v = ?$

$v^2 = u^2 + 2as$
$= (14\sin 45)^2 + 2(-9.8)5$
$\underline{= 0}$

Speed at B $= 7\sqrt{2}$
$= 9.90 \text{ ms}^{-1}$ (3sf).

4. [*In this question the unit vectors* **i** *and* **j** *are directed horizontally and vertically upwards respectively.*]

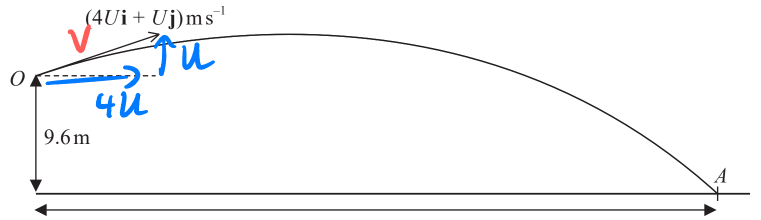


Figure 2

The point O is 9.6 m above horizontal ground.

A small ball is projected with velocity $(4U\mathbf{i} + U\mathbf{j})\,\text{m s}^{-1}$, where U is a positive constant, from the point O

The ball first hits the ground T seconds later, at the point A

The point A is at a horizontal distance of 64 m from O, as shown in Figure 2.

In an initial model

- the ball is modelled as a particle moving under gravity
- air resistance is ignored
- the ball has an initial speed of $V\,\text{m s}^{-1}$

Using this model,

(a) show that $UT = 16$ **(2)**

(b) find the value of V **(6)**

(c) State two improvements to the model, other than including air resistance, that would make the model more realistic. **(2)**

horizontal

d / $s \quad t$

dist = 64
speed = 4u
time = T

$d = s \times t$

$64 = 4u \times T$

$64 = 4uT$

$\underline{16 = uT}$ → $T = \frac{16}{u}$

vertical $\uparrow +$

$u = u$

$a = -9.8$

$s = -9.6$

$t = \frac{16}{u}$

$s = ut + \frac{1}{2}at^2$

$-9.6 = \cancel{u} \times \frac{16}{\cancel{u}} - 4.9\left(\frac{16}{u}\right)^2$

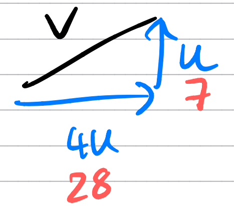
$-9.6 = 16 - \frac{1254.4}{u^2}$

$\frac{1254.4}{u^2} = 25.6$

$\frac{1254.4}{25.6} = u^2$

$u^2 = 49$

$\underline{u = 7}$



$V = \sqrt{28^2 + 7^2}$

$= \underline{28.9\,\text{ms}^{-1}}$

c) spin of the ball
wind direction
more accurate value for g.

... Vectors?

TOP TIP: identify the type of question FIRST

Ⓘ constant acceleration   Ⓘⅈ variable acceleration   (Ⓘⅈⅈ constant velocity / no acceleration) → less likely in my view.

↳ dist = speed × time

$\underline{r} = \underline{v}t$

... and don't forget $\underline{F} = m\underline{a}$ if they mention mass!

Ⓘ use $\underline{v} = \underline{u} + \underline{a}t$

$\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$ or $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$ if it doesn't start at origin

etc.

Ⓘⅈ Calculus!

$$\begin{matrix} s/x/\underline{r} \\ \underline{v} \\ \underline{a} \end{matrix}$$

int. ↑ diff ↓

Key points to remember:

→ moving/travelling in direction of $\begin{pmatrix} a \\ b \end{pmatrix}$

↳ $\underline{v} = k\begin{pmatrix} a \\ b \end{pmatrix}$ ($\underline{F} = k\begin{pmatrix} a \\ b \end{pmatrix}$ $\underline{a} = k\begin{pmatrix} a \\ b \end{pmatrix}$)

(could be $\underline{a}$ or $\underline{F}$ if particle started from rest and if F and a are constant)

→ "speed", "magnitude" → Pythagoras.

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6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$ N and $(2q\mathbf{i} + p\mathbf{j})$ N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\text{m s}^{-2}$

(a) find the value of p and the value of q. **(5)**

(b) Find the size of the angle between the direction of the acceleration and the vector $\mathbf{j}$. **(2)**

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T. **(5)**

$m = 2$ $\quad \underline{F}_1 = \begin{pmatrix} p \\ q \end{pmatrix}$ $\quad \underline{F}_2 = \begin{pmatrix} 2q \\ p \end{pmatrix}$ $\quad \underline{a} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\underline{F}_1 + \underline{F}_2 = \begin{pmatrix} p+2q \\ p+q \end{pmatrix}$

Use $\underline{F} = m\underline{a}$

$\begin{pmatrix} p+2q \\ p+q \end{pmatrix} = 2\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} p+2q \\ p+q \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$p + 2q = 2$

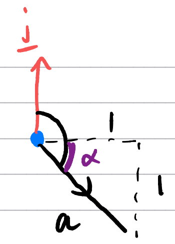
$p + q = -2$

$q = 4$

$p + 4 = -2$

$p = -6$

b)



$\alpha = 45$

angle between $\underline{j}$ and $\underline{a}$ $= 90 + 45$

$= 135°$

c) $\underline{a} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$t = T$

$\underline{u} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$\underline{v} = k\begin{pmatrix} 11 \\ -13 \end{pmatrix}$

$\underline{v} = \underline{u} + \underline{a}t$

$\begin{pmatrix} 11k \\ -13k \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} T$

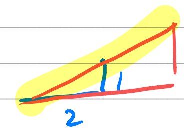
$\begin{pmatrix} 11k \\ -13k \end{pmatrix} = \begin{pmatrix} 3 + T \\ -4 - T \end{pmatrix}$

$11k = 3 + T \rightarrow 11k - T = 3$

$-13k = -4 - T \rightarrow -13k + T = -4$

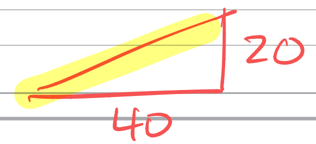
$k = 0.5$ $\quad T = 2.5$

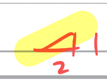
direction of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$



$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $\quad$ Same direction $k\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 40 \\ 20 \end{pmatrix}$





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3. [*In this question*, **i** *and* **j** *are horizontal unit vectors*.]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})$ N, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that ↳ v (a if started from rest)

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

$M = 4$

$\underline{F} = \underline{F}_1 + \underline{F}_2$ $\underline{F} = m\underline{a}$

$\underline{F} = \begin{pmatrix}4\\-1\end{pmatrix} + \begin{pmatrix}\lambda\\\mu\end{pmatrix}$

$\underline{F} = \begin{pmatrix}4+\lambda\\\mu-1\end{pmatrix}$ ↳ resultant force will be in the direction $\begin{pmatrix}3\\1\end{pmatrix}$

$\begin{pmatrix}4+\lambda\\\mu-1\end{pmatrix} = k\begin{pmatrix}3\\1\end{pmatrix}$

$4+\lambda = 3k$
$\mu - 1 = k$

$4+\lambda = 3(\mu-1)$
$4+\lambda = 3\mu - 3$
$\lambda - 3\mu + 7 = 0$

b) $\lambda = 2$ $2 - 3\mu + 7 = 0$
$9 - 3\mu = 0$
$9 = 3\mu$
$\mu = 3$

$\underline{F} = \begin{pmatrix}4+\lambda\\\mu-1\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$

Question 3 continued

$\underline{F} = m\underline{a}$

$\begin{pmatrix}6\\2\end{pmatrix} = 4\underline{a}$

$\begin{pmatrix}1.5\\0.5\end{pmatrix} = \underline{a}$ $\underline{u} = \begin{pmatrix}0\\0\end{pmatrix}$ $t = 4$ $\underline{r} = ?$

$\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{r} = \begin{pmatrix}0\\0\end{pmatrix}4 + \frac{1}{2}\begin{pmatrix}1.5\\0.5\end{pmatrix}4^2$

$\underline{r} = 8\begin{pmatrix}1.5\\0.5\end{pmatrix} = \begin{pmatrix}12\\4\end{pmatrix}$

dist AB $= \sqrt{12^2+4^2} = 12.6$ (3sf)

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2. A particle P of mass 1.5 kg moves under the action of a single force $\mathbf{F}$ newtons.

At time t seconds, $t \geqslant 0$, P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

(a) Find $\mathbf{F}$ when $t = 2$ **(4)**

$\underline{F} = m\underline{a}$

$s/\underline{r}$, v, a

At time $t = 0$, P is at the origin O.

(b) Find the position vector of P relative to O at the instant when P is moving in the direction of the vector $\mathbf{j}$ **(4)**

velocity

$k\begin{pmatrix}0\\1\end{pmatrix}$

$\uparrow \underline{j} = \begin{pmatrix}0\\1\end{pmatrix}$

$m = 1.5$

$\underline{v} = \begin{pmatrix}5t^2 - t^3\\2t^3 - 8t\end{pmatrix}$ diff.

$\underline{a} = \begin{pmatrix}10t - 3t^2\\6t^2 - 8\end{pmatrix}$

$t = 2$

$\underline{a} = \begin{pmatrix}20 - 3\times 2^2\\6\times 2^2 - 8\end{pmatrix}$

$\underline{a} = \begin{pmatrix}8\\16\end{pmatrix}$

$\underline{F} = 1.5\begin{pmatrix}8\\16\end{pmatrix}$

$\underline{F} = \begin{pmatrix}12\\24\end{pmatrix}$ N

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b) integrate v to find $\underline{r}$

direction of j

$\underline{v} = \begin{pmatrix}5t^2 - t^3\\2t^3 - 8t\end{pmatrix}$ integration.

$\begin{pmatrix}5t^2 - t^3\\2t^3 - 8t\end{pmatrix} = k\begin{pmatrix}0\\1\end{pmatrix}$

$\underline{r} = \begin{pmatrix}\frac{5}{3}t^3 - \frac{1}{4}t^4 + c\\\frac{1}{2}t^4 - 4t^2 + d\end{pmatrix}$

$5t^2 - t^3 = 0$
$5 - t = 0$
$t = 5$

when $t = 5$, it is moving in $\underline{j}$ direction.

$t = 0$, $\underline{r} = \begin{pmatrix}0\\0\end{pmatrix}$

If $t = 5$ $\underline{v} = \begin{pmatrix}0\\218\end{pmatrix}$ $\uparrow \underline{j}$

$\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0 - 0 + c\\0 - 0 + d\end{pmatrix}$

$c = 0$
$d = 0$

$\underline{r} = \begin{pmatrix}\frac{5}{3}t^3 - \frac{1}{4}t^4\\\frac{1}{2}t^4 - 4t^2\end{pmatrix}$

$t = 5$

$r = \begin{pmatrix}\frac{5}{3}(5)^3 - \frac{1}{4}(5)^4\\\frac{1}{2}(5)^4 - 4(5)^2\end{pmatrix}$ position vector

$= \begin{pmatrix}625/12\\425/2\end{pmatrix} = \begin{pmatrix}52.083\\212.5\end{pmatrix}$

$= \begin{pmatrix}52.1\\213\end{pmatrix}$ 3sf

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5. At time t seconds, $t \geqslant 0$, a particle P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (5t^2 - 12t + 15)\mathbf{i} + (t^2 + 8t - 10)\mathbf{j}$$

When $t = 0$, P is at the origin O.

At time T seconds, P is moving in the direction of $(\mathbf{i} + \mathbf{j})$. $\underline{v} = k\begin{pmatrix}1\\1\end{pmatrix}$

(a) Find the value of T.

(3)

When $t = 3$, P is at the point A.

(b) Find the magnitude of the acceleration of P as it passes through A.

Find $\underline{a}$, $t=3$

(4)

(c) Find the position vector of A.

(4)

a) $\begin{pmatrix}5t^2-12t+15\\t^2+8t-10\end{pmatrix} = k\begin{pmatrix}1\\1\end{pmatrix}$

$5t^2-12t+15 = k$
$t^2+8t-10 = k$

$5t^2-12t+15 = t^2+8t-10$
$4t^2-20t+25=0$

$\underline{\underline{t=2.5}}$ $\quad \underline{\underline{T=2.5}}$

$\uparrow s/r$
$v \downarrow$
a

b) $\underline{v} = \begin{pmatrix}5t^2-12t+15\\t^2+8t-10\end{pmatrix}$

$\underline{a} = \begin{pmatrix}10t-12\\2t+8\end{pmatrix}$ $\quad t=3 \quad \underline{a} = \begin{pmatrix}30-12\\6+8\end{pmatrix} = \begin{pmatrix}18\\14\end{pmatrix}$

$|\underline{a}| = \sqrt{18^2+14^2}$

$= \underline{22.8}$ ms^{-2}

$\underline{v} = \begin{pmatrix}5t^2-12t+15\\t^2+8t-10\end{pmatrix}$

$\underline{r} = \begin{pmatrix}\frac{5}{3}t^3-6t^2+15t+c\\\frac{1}{3}t^3+4t^2-10t+d\end{pmatrix}$

When $t=0$, $\underline{r} = \begin{pmatrix}0\\0\end{pmatrix}$ $\quad c=0$, $d=0$

$\underline{r} = \begin{pmatrix}5/3\,t^3-6t^2+15t\\1/3\,t^3+4t^2-10t\end{pmatrix}$

$t=3 \quad \underline{r} = \begin{pmatrix}36\\15\end{pmatrix}$ m

$\underline{r} = (36\underline{i} + 15\underline{j})$ m

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