

2.1 Laws of Indices & Surds

2.1.1 Laws of Indices / 2.1.2 Manipulating Surds / 2.1.3 Surds - Rationalising the Denominator

Easy (10 questions)	/27
Medium (9 questions)	/40
Hard (9 questions)	/42
Very Hard (8 questions)	/49
Total Marks	/158

Scan here to return to the course
or visit [savemyexams.com](https://www.savemyexams.com)



Easy Questions

1 Write down the value of

(i) $81^{\frac{1}{4}}$

(ii) $8^{\frac{2}{3}}$

(iii) 5^{-2}

(3 marks)

2 Simplify

(i) $x^3 \times x^5$

(ii) $\frac{a^2}{a^4}$

(iii) $\frac{y^{\frac{5}{7}} \times y^{\frac{1}{7}}}{y^{\frac{3}{7}}}$

(3 marks)

3 Write the following in the form $a\sqrt{b}$

(i) $\sqrt{2} + \sqrt{8}$

(ii) $4\sqrt{3} - \sqrt{12} + 4\sqrt{48}$

(iii) $(2\sqrt{2})^3 + 3\sqrt{2}$

(3 marks)

4 Rationalise the denominator of $\frac{3}{\sqrt{2}}$.

(2 marks)

5 Rationalise the denominator of $\frac{2}{3+\sqrt{5}}$.

(2 marks)

6 Write $\frac{6x^2 + 5x^3}{x^4}$ in the form $ax^m + bx^n$ where a , b , m and n are constants to be found.

(2 marks)

7 Show that $(p + 2\sqrt{5})(p - 2\sqrt{5}) = p^2 - 20$.

(3 marks)

8 Simplify

$$\frac{28x^2(2x^2 + 3)}{7x^{\frac{1}{2}}}$$

(3 marks)

9 Simplify

$$\frac{3 - 2\sqrt{3}}{4 - \sqrt{3}}$$

(3 marks)

10 Solve the equation

$$\frac{1}{9}x^{\frac{4}{5}} = 9$$

(3 marks)

Medium Questions

1 (a) Write down the value of $27^{\frac{1}{3}}$

(1 mark)

(b) Use your answer to part (a) to show that $27^{\frac{2}{3}} = 9$.

(2 marks)

2 (a) Given that $a^{\frac{1}{3}} = 2$, find the value of a .

(2 marks)

(b) Simplify $x^2 \div x^{\frac{5}{3}}$

(2 marks)

3 (a) Simplify the following expressions:

$$4x^2 \times 3x^{-1}$$

(2 marks)

(b) $24x^3 \div 8x^{\frac{11}{5}}$

(2 marks)

(c) $15x^{-\frac{2}{3}} \div 10x^{-\frac{2}{3}}$

(2 marks)

- 4 (a)** Given that $y = \frac{1}{16}x^4$, express each of the following in the form ax^n , where a and n are constants.

$y^{\frac{1}{2}}$

(1 mark)

(b) y^{-1}

(1 mark)

(c) $y^{-\frac{3}{2}}$

(2 marks)

5 (a) Simplify $3\sqrt{5} + 4\sqrt{3} - 7\sqrt{3} + \sqrt{5}$

(2 marks)

(b) By expanding and simplifying, show that

$$(\sqrt{3} - 5)(\sqrt{3} + 5) = -22$$

(2 marks)

6 (a) Give an example to show that $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$ is not true in general.

(1 mark)

(b) Show that $\frac{14}{4 + \sqrt{2}} = 4 - \sqrt{2}$.

(3 marks)

7 Solve the equation $6 - x\sqrt{7} = \frac{2x}{\sqrt{7}}$, giving your answer in the form $\frac{a\sqrt{7}}{b}$ where a and b are integers.

(4 marks)

8 A rectangle has an area of 14 m^2 and a length of $(3 - \sqrt{2}) \text{ m}$. Find the width of the rectangle, showing clear algebraic working. Give your answer as an exact value.

(4 marks)

9 (a) Show that $\frac{(3 - \sqrt{x})^2}{x}$ can be written as $9x^{-1} - 6x^{-\frac{1}{2}} + 1$.

(2 marks)

(b) Given that $128\sqrt{2} = 2^a$, find the value of a .

(3 marks)

(c) Show that $\frac{x(2x^4 - \sqrt{x})}{\sqrt{x}}$ can be written as $2x^a - x^b$, where a and b are rational numbers to be found.

(2 marks)

Hard Questions

1 (a) Write down the value of $64^{\frac{1}{3}}$.

(1 mark)

(b) Use your answer to part (a) to show that $64^{-\frac{2}{3}} = \frac{1}{16}$

(2 marks)

2 (a) Given that $a^{\frac{1}{3}} = -4$, find the value of a .

(2 marks)

(b) Simplify $x^{\frac{11}{6}} \div x^{\frac{4}{3}}$

(2 marks)

- 3 (a)** Simplify the following expressions, giving your answers in the form ax^n where a and n are rational numbers and any fractions are in lowest terms.

$$7x^{-3} \div 2x^{-\frac{1}{3}}$$

(2 marks)

(b) $\frac{2}{3}x^{\frac{7}{6}} \times \frac{5}{2}x^{-\frac{5}{3}}$

(2 marks)

(c) $\left(3x^{-\frac{2}{3}}\right)^2 \div 6x^{-\frac{1}{6}}$

(3 marks)

- 4 (a)** Given that $y = \frac{8}{27}x^6$, express each of the following in the form ax^n , where a and b are constants.

$y^{\frac{1}{3}}$

(1 mark)

(b) y^{-1}

(1 mark)

(c) $y^{-\frac{4}{3}}$

(2 marks)

5 (a) Show that $2\sqrt{3} + 4\sqrt{12} - 3\sqrt{75} = -5\sqrt{3}$

(3 marks)

(b) By expanding and simplifying, show that

$$(\sqrt{3} - 5)(5 - \sqrt{3}) = 10\sqrt{3} - 28$$

(2 marks)

- 6 (a) $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$ is not true in general. Give an example of an a and a b for which it is true.

(1 mark)

- (b) Show that $\frac{3}{1+\sqrt{7}} = a + b\sqrt{7}$, where a and b are rational numbers to be found.

(3 marks)

- 7 Solve the equation $66 + x\sqrt{75} = \frac{4x}{\sqrt{3}}$, giving your answer in the form $a\sqrt{b}$ where a and b are integers.

(4 marks)

- 8 The two parallel sides of a trapezium have lengths of $(4 + \sqrt{3})$ cm and $(6 - 3\sqrt{3})$ cm, and the area of the trapezium is $(10\sqrt{3} - 6)$ cm². Showing clear algebraic working, determine the perpendicular distance between the two parallel sides of the trapezium. Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4 marks)

9 (a) Show that $\frac{(5 + 2\sqrt{x})^2}{\sqrt{x}}$ can be written as $25x^{-\frac{1}{2}} + 20 + 4x^{\frac{1}{2}}$.

(2 marks)

(b) Show that $\frac{729}{4\sqrt{3} - \sqrt{27}} = 3^a$, where a is a rational number to be found.

(3 marks)

(c) Show that $\frac{2\sqrt{x}}{\sqrt{2}}(\sqrt{8x} + 6x)$ can be written as $ay^2 + by^3$, where $y = \sqrt{2x}$ and a and b are integers to be found.

(2 marks)

Very Hard Questions

1 (a) Write down the value of $256^{\frac{1}{4}}$

(1 mark)

(b) Use your answer to part (a) to show that $1 \div 256^{-\frac{3}{4}} = 64$.

(2 marks)

2 (a) Given that $a^{\frac{2}{3}} = 16$, find the possible values of a .

(3 marks)

(b) Simplify $x^{-\frac{2}{3}} \div x^{-\frac{3}{4}}$

(2 marks)

- 3 (a)** Simplify the following expressions, giving your answers in the form ax^n where a and n are rational numbers and any fractions are in lowest terms.

$$(8x^2)^{-\frac{1}{3}} \times \frac{1}{4}x^{-\frac{1}{3}}$$

(3 marks)

(b)
$$\left(\frac{2}{9}x^{\frac{1}{2}} \times \frac{1}{18}x^{-\frac{3}{4}} \right)^{-\frac{1}{4}}$$

(3 marks)

(c)
$$\frac{\left(8x^{-\frac{2}{3}} \right)^{\frac{2}{3}}}{\left(64x^{-\frac{1}{3}} \right)^{\frac{1}{3}}}$$

(3 marks)

- 4 (a)** Given that $y = \frac{81}{16}x^{-12}$, express each of the following in the form ax^n , where a and n are constants.

$$y^{\frac{3}{4}}$$

(1 mark)

(b) $y^{-\frac{1}{2}}$

(1 mark)

(c) $\left(y^{\frac{1}{2}}\right)^{-3}$

(2 marks)

5 (a) Show that $2\sqrt{18} + \sqrt{50} - 5\sqrt{32} = a\sqrt{b}$, where a and b are integers.

(3 marks)

(b) By expanding and simplifying, show that

$$(\sqrt{12} - 3)(2 - \sqrt{75}) = 19\sqrt{3} - 36$$

(3 marks)

- 6 (a)** $\sqrt{a} - \sqrt{b} = \sqrt{a-b}$ is not true in general. Give an example of an a and a b for which it is true.

(1 mark)

- (b)** Show that $\frac{2-\sqrt{3}}{1+\sqrt{3}} = a + b\sqrt{3}$, where a and b are rational numbers.

(4 marks)

- 7** Solve the equation $\sqrt{20} + \frac{\sqrt{5}}{2x} = \frac{1}{x\sqrt{45}}$

(5 marks)

8 (a) Expand $(a + b\sqrt{5})^2$.

(2 marks)

(b) A square has an area of $(49 + 12\sqrt{5}) \text{ m}^2$ and a side length of $(a + b\sqrt{5}) \text{ m}$.

Show that $ab = 6$, and explain why this proves that a and b must both be non-negative.

(2 marks)

(c) Show that $a^4 - 49a^2 + 180 = 0$.

(3 marks)

(d) By using the substitution $y = a^2$ or otherwise, solve the equation $a^4 - 49a^2 + 180 = 0$.
Hence determine the side length of the square.

(5 marks)