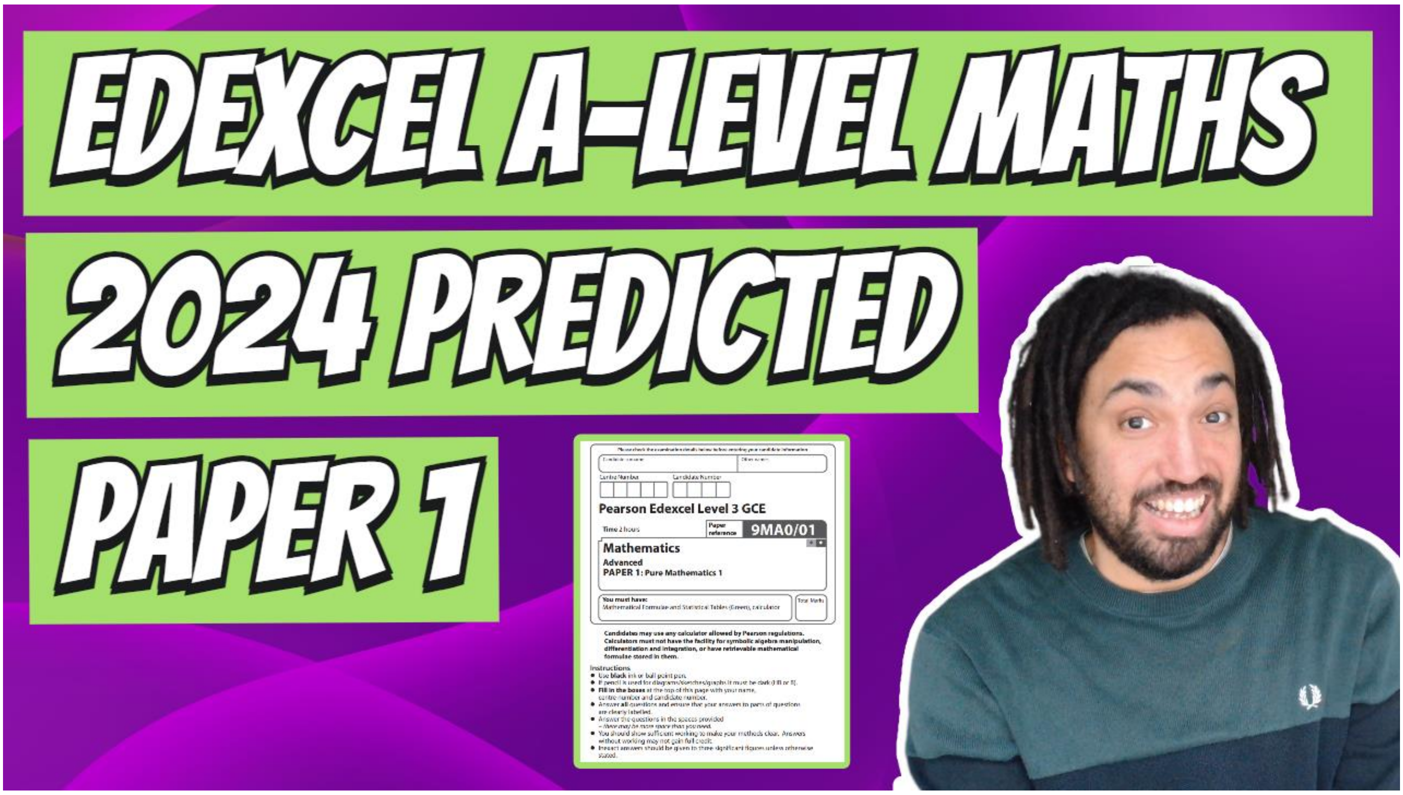

These topics could be in paper 1 or paper 2, or not at all! Revise thoroughly and use this paper as practice.

<u>Topics</u>

Q1 Circles

Q2 Vectors

Q3 Exponential Modelling

Q4 Proof

Q5 Functions

Q6 Binomial Expansion

Q7 Geometric Sequences

Q8 Trigonometric Equations

Q9 Differentiation Y1

Q10 Differentiation Y2

Q11 Implicit Differentiation

Q12 Iteration

Q13 Integration

Q14 Differential Equations

Q15 Parametric Calculus

Marks: 126, Time: 2.5 hours

<u>Video Solutions</u>

#1 Circles

The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

#2 Vectors

Relative to a fixed origin O,

the point A has position vector $\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$,
the point B has position vector $4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$,
and the point C has position vector $2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$.

Given that $ABCD$ is a parallelogram,

(a) find the position vector of point D.

(2)

The vector $\overrightarrow{AX}$ has the same direction as $\overrightarrow{AB}$.

Given that $\left|\overrightarrow{AX}\right| = 10\sqrt{2}$,

(b) find the position vector of X.

(3)

#3 Exponential Modelling

The growth of a weed on the surface of a pond is being studied.

The surface area of the pond covered by the weed, A m^2, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

where p is a positive constant and t is the number of days after the start of the study.

Given that

- 30 m^2 of the surface of the pond was covered by the weed at the start of the study
- 50 m^2 of the surface of the pond was covered by the weed T days after the start of the study

(a) show that $p = 2.4$

(2)

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on graphical or numerical methods are not acceptable.*)

(4)

#4 Proof

(i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

#5 Functions

The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, \quad x \neq -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, \quad x \neq \pm 1$$

where a is an integer to be found.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of $\mathrm{fg}(2)$.

(2)

(d) Find the range of g.

(3)

(e) Explain why the function g does not have an inverse.

(1)

#6 Binomial Expansion

(a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

#7 Geometric Sequences

A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

#8 Trig

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

#9 Differentiation Year 1

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

#10 Differentiation Year 2

Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where n, A and B are constants to be found.

(4)

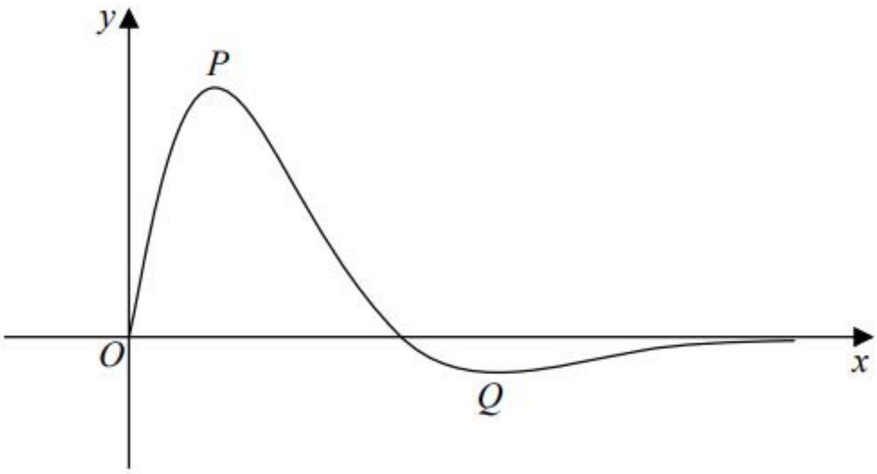

Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

#11 Implicit Differentiation

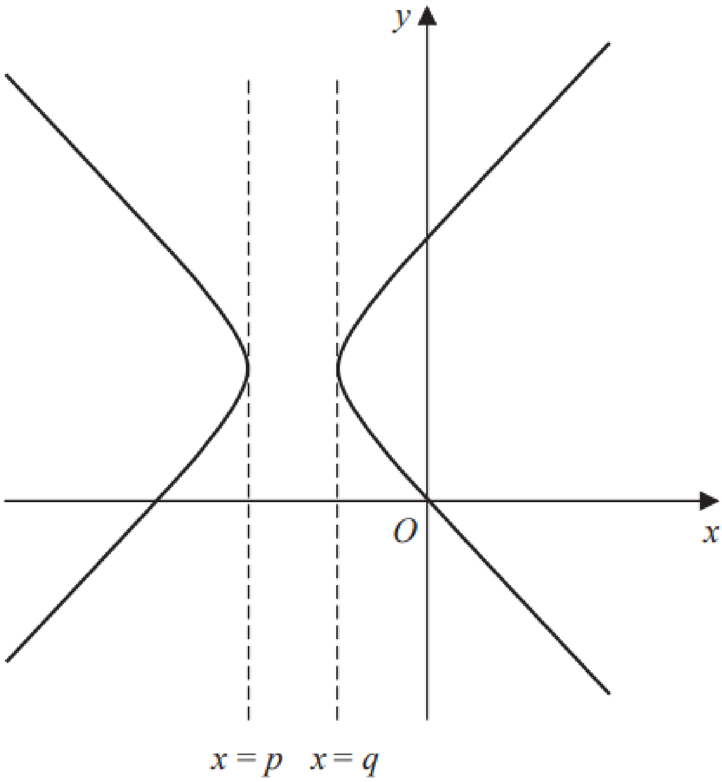

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

#12 Iteration

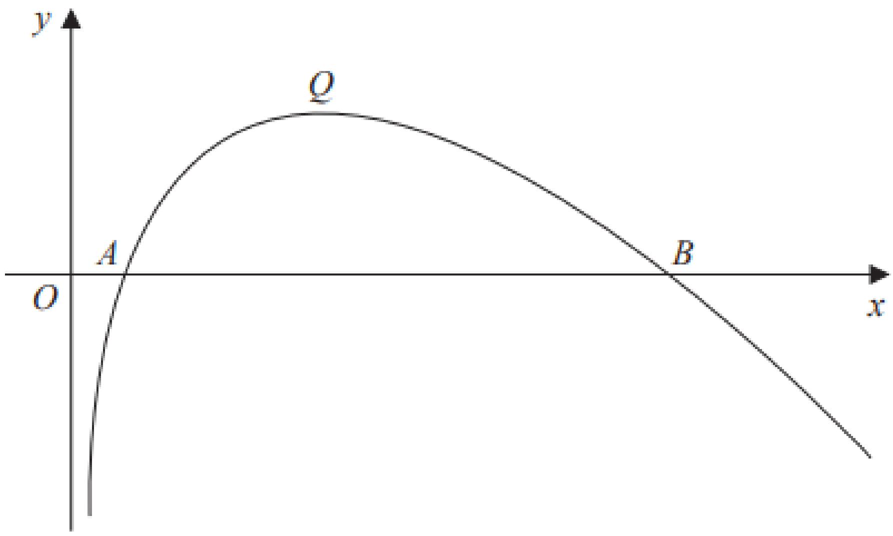

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x coordinate of Q satisfies

$$x = \frac{8}{1 + \ln x}$$

(4)

(c) Show that the x coordinate of Q lies between 3.5 and 3.6

(2)

(d) Use the iterative formula

$$x_{n+1} = \frac{8}{1 + \ln x_n} \qquad n \in \mathbb{N}$$

with $x_1 = 3.5$ to

(i) find the value of x_5 to 4 decimal places,

(ii) find the x coordinate of Q accurate to 2 decimal places.

(2)

#13 Integration

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x}\, dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2}\, dx = a + \ln b$$

where a and b are constants to be found.

(7)

#14 Differential Equations

(a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11 - 2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

#15 Parametric Calculus

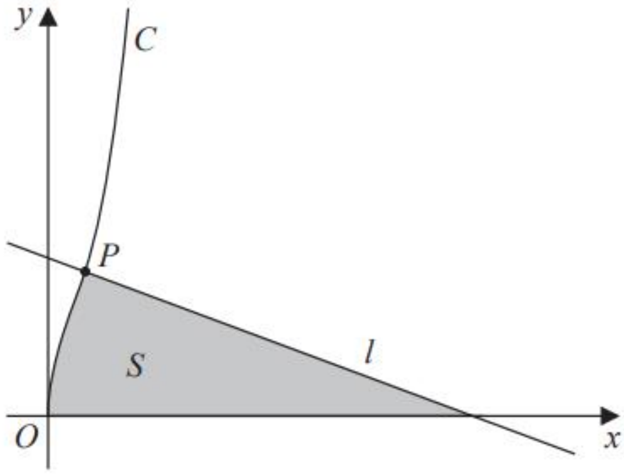

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

(5)

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

(6)