A-Level Mathematics

Edexcel

2024 Predicted Paper

Paper 2

Pure Mathematics

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Name:...

Date:..

2 hours allowed

You may use a calculator

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These do not guarantee you the same mark in the exam.

A* - 75%
A - 55%
B - 45%
C - 35%
D - 25%
E - 15%

Mark scored	
Total	**100**

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01 Prove that there are an infinite number of prime numbers.

[4 marks]

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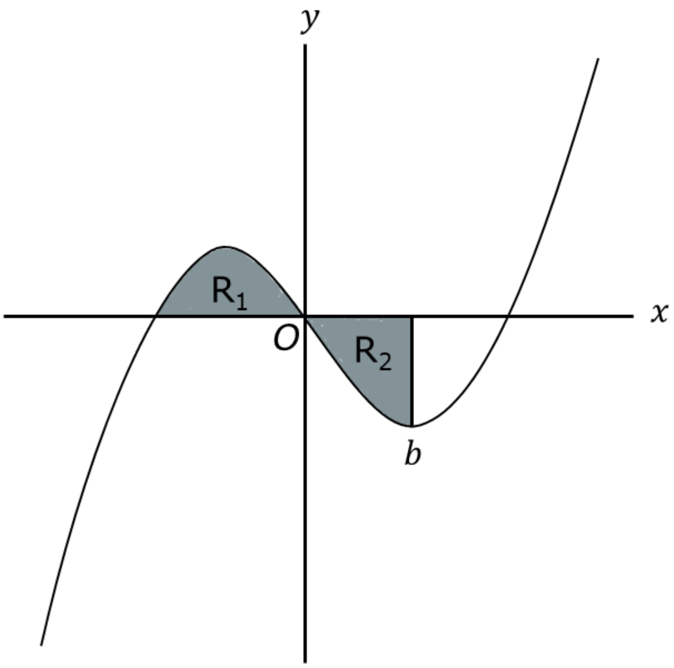
02 The diagram below shows a sketch of part of the curve with equation:

$$y = x(x+2)(x-3)$$



The region R_1 shown in the diagram is bounded by the curve and the negative x-axis.

a) Show that the exact area of R_1 is $\frac{16}{3}$

[4 marks]

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Point b corresponds to the minimum of the curve.

b) Find the x coordinate of b.

[3 marks]

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03 $f(x) = ax^3 - 3x^2 + bx + 20,\ x \epsilon R$

$(x+4)$ is a factor of $f(x)$.

When $f(x)$ is divided by $(x-2)$ it leaves a remainder of -54.

a) Find the values of a and b.

[4 marks]

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b) Write $f(x)$ as a product of three linear factors.

[3 marks]

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04 For a small angle θ, where θ is in radians, show that:

$$1 + cos\theta - 2cos^2\theta \approx \frac{3}{2}\theta^2$$

[4 marks]

05 Solve the logarithmic equation:

$$\log_3(2x+5) - 2\log_3(x-1) = 2$$

[4 marks]

06

a) Solve, for $-180° \leq \theta \leq 180°$, the equation:

$$9sin^2x - 3 = 2cos^2x + sin\ x$$

Give your answers to 2 decimal places.

[6 marks]

b) Hence find the smallest positive solution of the equation:

$$18sin^2(2\theta + 30^o) - 2sin\,(2\theta + 30^o) - 6 = 4cos^2(2\theta + 30^o)$$

Give your answer to 2 decimal places.

[2 marks]

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07 Using the substitution $x = \sin u$, show that,

$$\int_{0}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{4-4x^2}}\,dx = \frac{\pi}{6}$$

[6 marks]

08

a) Prove that the sum of n terms of an arithmetic progression a_n with first term a and common difference d is:

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$

[4 marks]

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The fifth term of an arithmetic progression is 24 and the tenth term is 120.

b) Find the common difference.

[3 marks]

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c) Find the first term.

[2 marks]

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d) Find:

$$\sum_{n=5}^{10} a_n$$

[4 marks]

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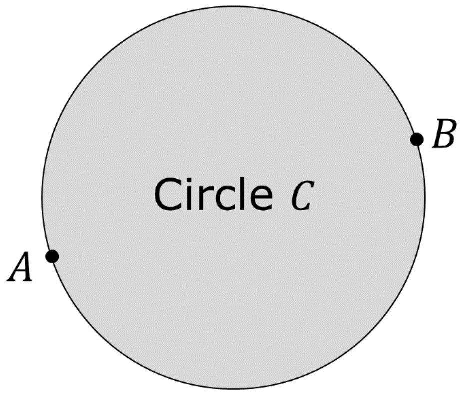
09 A circle C has two points A and B at opposite sides of its diameter.



Point A has coordinates (a, -2) and point B has coordinates (7, 5).

The tangent to the circle C at point A has the equation $y = -\frac{1}{2}x - 3$.

Find the equation of the circle C.

[7 marks]

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a) Express $\frac{1}{P(13-2P)}$ in partial fractions.

[3 marks]

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The population of mice living on a island was modelled using the differential equation:

$$\frac{dP}{dt} = \frac{1}{13}P(13 - 2P), \qquad t \geq 0, \qquad 0 < P < 6.5$$

P is the population of mice, in thousands, and t is the number of years measured from 1st January 2023.

On 1st January 2023 the population of mice on the island was found to be 2000.

b) Determine the time taken, to the nearest year, for this population of mice to increase by 250%.

[6 marks]

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c) Show that $P = \frac{A}{4 + Be^{-t}}$ where *A* and *B* are integers to be found.

[3 marks]

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d) Find the maximum population of mice on the island.

[2 marks]

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e) State a limitation of this model.

[1 mark]

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11 The equation $3x^3 + x^2 - 1 = 0$ has exactly one real root.

a) Show that, for this equation, the Newton-Raphson formula can be written:

$$x_{n+1} = \frac{6x^3{}_n + x^2{}_n + 1}{9x^2{}_n + 2x_n}$$

[3 marks]

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b) Using the formula with $x_1 = 1$, find the values of x_2 and x_3

[2 marks]

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c) In this case, we are not able to use $x_1 = 0$ in the Newton-Raphson method.
Suggest why.

[1 mark]

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12 Given that:

$$y = \frac{5sin\theta}{2sin\theta + 2cos\theta}$$

Show that:

$$\frac{dy}{d\theta} = \frac{A}{1 + sin2\theta}$$

Where A is a rational constant to be found.

[5 marks]

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13 A curve is defined by two parametric equations:

$$x = 2\cos 2\theta$$

$$y = 4 + \cos\theta$$

a) Find $\frac{dy}{dx}$ in terms of $k\sec\theta$, where k is a constant to be found.

[5 marks]

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b) Find $\int_0^1 y\, dx$.

Give your answer in simplified surd form.

[7 marks]

c) Find a cartesian equation for the curve in the form $y = f(x)$.

[2 marks]

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END OF QUESTIONS

MARKING GUIDANCE

Question	Solution
1	A1M assume a finite number of prime numbers: p_1 to p_n A1M let P = p_1 x p_2 x p_3 x … x pn A1M P + 1 has no factors and so must be prime A1M this contradicts the assumption there is a finite number of primes so there must be an infinite number of primes
2 (a)	A1M for expansion $y = x^3 - x^2 - 6x$ A1M for integration $\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$ A1M for $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2\right]\begin{matrix} 0 \\ -2 \end{matrix}$ A1M for $\frac{16}{3}$
2 (b)	A1M for $\frac{dy}{dx} = 3x^2 - 2x - 6$ A1M for $\frac{dy}{dx} = 3x^2 - 2x - 6 = 0$ A1M for $x = \frac{1+\sqrt{19}}{3}$
3 (a)	A1M for $a(-4)^3 - 3(-4)^2 + (-4)b + 20 = 0$ A1M for $a(2^3) - 3(2^2) + 2b + 20 = -54$ A1M for a = 2 A1M for b = -39
3 (b)	A1M for suitable method to find other factors e.g. algebraic division A1M for $(x+4)(2x^2 - 11x + 5)$ A1M for $(x+4)(2x-1)(x-5)$
4	A1M for using $cos\,\theta \approx 1 - \frac{1}{2}\theta^2$ A1M for substitution $1 + (1 - \frac{1}{2}\theta^2) - 2(1 - \frac{1}{2}\theta^2)^2$ A1M for expansion $\frac{3}{2}\theta^2 - \frac{1}{2}\theta^4$ A1M for explaining that since θ is small, we can leave off the higher order term θ^4 Allow alternative method using $\sin^2\theta + \cos^2\theta = 1$ and small angle approximation for $\sin\theta$

5	A1M for correct use of log power rule $$\log_3(2x+5) - \log_3(x-1)^2 = 2$$ A1M for correct use of log subtraction rule $$\log_3 \frac{2x+5}{(x-1)^2} = 2$$ A1M for $9x^2 - 20x + 4 = 0$ A1M for x = 2
6 (a)	A1M for use of trig identity $8sin^2x - sin\,x - 3 = 2(1 - sin^2x)$ A1M for rearrangement $11sin^2x - sin\,x - 5 = 0$ A1M for use of quadratic formula $$sin\,x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 11 \times -5}}{2 \times 11}$$ A1M for $sin\,x = \frac{1 \pm \sqrt{221}}{22}$ A1M for $x = 46.15^o, -39.07^\circ$ A1M for $x = 133.85^o, -140.93^o$
6 (b)	A1M for $2x + 30^o = 46.15^o$ A1M for 8.075^0
7	A1M for $\frac{dx}{du} = \cos u$ A1M for correctly changing limits $u = \frac{\pi}{3}$ and $u = 0$ A1M for correct use of $\sin^2 x + \cos^2 x = 1$ identity to find $4 - 4\sin^2 x = 4\cos^2 x$ A1M for correct substitution and simplification $$\int_0^{\frac{\pi}{3}} \frac{1}{2\cos u} \times \cos u\, du = \int_0^{\frac{\pi}{3}} \frac{1}{2} du$$ A1M for correct integration $$\left[\frac{1}{2}u\right]_0^{\frac{\pi}{3}}$$ A1M Correct substitution of limits $$\left[\frac{\pi}{6}\right] - [0] = \frac{\pi}{6}$$

8 (a)	A1M for $S_n = a + (a+d) + \ldots \quad \ldots + a + (n-1)d$ A1M for $2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \ldots \quad \ldots + (2a + (n-1)d$ A1M for $2S_n = n(2a + (n-1)d)$ A1M for $S_n = \frac{n}{2}[2a + (n-1)d]$
8 (b)	A1M for a + 4d = 24 A1M for a + 9d = 120 A1M for d = 19.2
8 (c)	A1M for substitution $a + 4 \times 19.2 = 24$ A1M for solving for $a = -52.8$
8 (d)	A1M for correct substitution into S_n A1M for $$\sum_{n=5}^{10} a_n = S_{10} - S_4$$ A1M for 336 – (-96) A1M for 432
9	A1M for correct substitution $-2 = -\frac{1}{2}a - 3$ A1M for a = -2 A1M for correct method to find centre e.g. $\left(\frac{-2+7}{2}, \frac{-2+5}{2}\right)$ A1M for (5/2, 3/2) oe A1M for correct method to find diameter e.g. $\sqrt{(7-(-2))^2 + (5-(-2))^2}$ A1M radius = $\frac{\sqrt{130}}{2}$ A1M for $\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{65}{2}$
10 (a)	A1M for $\frac{1}{P(13-2P)} = \frac{A}{P} + \frac{B}{(13-2P)}$ A1M for substitution of P=0 and P=6.5 into $1 = A(13 - 2P) + BP$ A1M for $\frac{1}{P(13-2P)} = \frac{1}{13P} + \frac{2}{13(13-2P)}$

10 (b)	A1M for separating the variables $\int \frac{13}{P(13-2P)} dP = \int 1\, dt$ A1M for $\int \frac{1}{P} + \frac{2}{(13-2P)} dP = t + c$ A1M for integration $ln\|P\| - ln\|13-2P\| = t + c$ A1M for substitution of $P = 2$ and $t = 0$ to find $c = ln\left(\frac{2}{9}\right)$ A1M for substitution of $P = 5$ into $$t = ln\|P\| - ln\|13-2P\| - ln\left(\frac{2}{9}\right)$$ A1M for $t = 2.0149$ years So the population will have tripled in year 2026.
10 (c)	A1M for using laws of logs $$t = ln(P) - ln(13-2P) - ln\left(\frac{2}{9}\right)$$ $$t = ln\left(\frac{9P}{26-4P}\right)$$ A1M for $$e^t = \frac{9P}{26-4P}$$ $$9P = (26-4P)e^t$$ A1M for $A = 26$, $B = 9$ $P = \frac{26}{4+9e^{-t}}$
10 (d)	A1M for correct use of $t \to \infty$ $$P = \frac{26}{4 + 9e^{-\infty}}$$ $$P = \frac{13}{2}$$ A1M for P = 6500
10 (e)	A1M It doesn't consider factors such as disease etc which may cause fluctuations in the population.
11 (a)	A1M for differentiation of $3x^3 + x^2 - 1$ to find $9x^2 + 2x$ A1M for substitutions $$x_{n+1} = x_n - \frac{3x^3{}_n + x^2{}_n - 1}{9x^2{}_n + 2x_n}$$ $$= \frac{x_n(9x^2{}_n + 2x_n)}{9x^2{}_n + 2x_n} - \frac{3x^3{}_n + x^2{}_n - 1}{9x^2{}_n + 2x_n}$$ A1M for simplification to $x_{n+1} = \frac{6x^3{}_n + x^2{}_n + 1}{9x^2{}_n + 2x_n}$

11 (b)	A1M for $$x_2 = \frac{6(1)^3 + (1)^2 + 1}{9(1)^2 + 2(1)} = \frac{8}{11}$$ A1M for $$x_3 = \frac{6\left(\frac{8}{11}\right)^3 + \left(\frac{8}{11}\right)^2 + 1}{9\left(\frac{8}{11}\right)^2 + 2\left(\frac{8}{11}\right)} = \frac{5107}{8272} = \sim 0.617$$
11 (c)	A1M for stating that there is a stationary point at $x = 0$
12	A2M for $\frac{dy}{d\theta} = \frac{(2sin\theta+2cos\theta)5cos\theta-5sin\theta(2cos\theta-2sin\theta)}{(2sin\theta+2cos\theta)^2}$ A1M for expansion and using $sin^2\theta+cos^2\theta = 1$ A1M for expansion and using $2sin\theta cos\theta = sin2\theta$ A1M for $\frac{dy}{d\theta} = \frac{2.5}{1+sin2\theta}$
13 (a)	A1M for $\frac{dx}{d\theta} = -4\sin 2\theta$ A1M for $\frac{dy}{d\theta} = -\sin\theta$ A1M for $\frac{dy}{dx} = \frac{-\sin\theta}{-4\sin 2\theta}$ A1M for $\frac{dy}{dx} = \frac{-\sin\theta}{-8\sin\theta\cos\theta}$ A1M for $\frac{dy}{dx} = \frac{1}{8\cos\theta} = \frac{1}{8}\sec\theta$
13 (b)	A1M for $\int(4+\cos\theta) \times -4\sin 2\theta\, d\theta$ A1M for $\int(-16\sin 2\theta - 8\sin\theta\cos^2\theta)d\theta$ A1M for Change limits $x = 0 \;\rightarrow\; \theta = \frac{\pi}{4}$ $x = 1 \rightarrow \theta = \frac{\pi}{6}$ A2M $\left[8\cos 2\theta + \frac{8}{3}\cos^3\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{6}}$ A1M $(8\cos\left(2\times\frac{\pi}{6}\right) + \frac{8}{3}\cos^3\frac{\pi}{6}) - (8\cos\left(2\times\frac{\pi}{4}\right) + \frac{8}{3}\cos^3\frac{\pi}{4})$ A1M $4 + \sqrt{3} - \frac{2\sqrt{2}}{3}$

13 (c)	A1M for correct substitution into $\cos 2\theta = 2\cos^2\theta - 1$ e.g. $$\frac{x}{2} = 2(y-4)^2 - 1$$ A1M for correct rearranging: $$y = \sqrt{\frac{x}{4} + \frac{1}{2}} + 4$$
Total	**100**