Edexcel A Level Further Maths: Decision Maths 1

Linear Programming (LP) problems

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Formulating a Linear Programming Problem

Introduction to Linear Programming

What is linear programming?

- **Linear programming,** often abbreviated to **LP,** is a means of solving problems that
  - involve working with a set of **constraints**
  - require a quantity to be **maximised** or **minimised**
- Typical uses of linear programming problems including finance and manufacturing
  - For example, a furniture manufacturer may make a mixture of chairs and tables
    - they would want to **minimise** their **costs** (of raw materials, manufacturing time)
    - they would want to **maximise** their **profit** (chairs may make more profit than tables)
    - but would be restricted **(constrained)** by things such as build time and the amount of raw materials available

Decision Variables in Linear Programming

What are decision variables?

- In a linear programming problem, **decision variables** are the quantities that can be **varied**
  - x, y, z are usually used as the decision variables
  - A furniture manufacturer, it could be that they produce x chairs and y tables (per day/week)
- Varying the **decision variables** will vary the quantity that is to be maximised or minimised
  - 12 chairs and 3 tables may lead to a profit of £200, whilst 3 chairs and 12 tables may lead to a profit of £160
- The values that the **decision variable** may take will depend upon the **constraints**
  - A furniture manufacturer can't make endless chairs and tables to gain unlimited profit !

What is the objective function in a linear programming?

- The **objective function** is the quantity in a linear programming problem that requires optimising
  - The objective of a furniture manufacturer may be to **maximise** its **profits**
  - they may also wish to **minimise** their **costs**
- The objective function is the aim of a linear programming problem
  - It is a function of the **decision variables**
  - P is usually used for **maximising** problems (P, profit)
  - C is usually used for **minimising** problems (C, costs)

Constraints & Inequalities in Linear Programming

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What are constraints in linear programming?

- The **constraints** in a linear programming problem are the **restrictions** the problem is contained within
  - These restrictions are called **constraints**
    - Mathematically they are represented by **inequalities** involving the **decision variables**
  - for a furniture manufacturer constraints could include glue drying time and the amount of paint/varnish available
- Decision variables are usually zero or positive as they represent a 'number of things'
  - the constraint $x, y \geq 0$ is usually included
  - this is called the **non-negativity** constraint

How do I formulate a linear programming problem?

- Formulating a linear programming problems involves
  - defining the **decision variables**
  - deducing the **constraints** as inequalities
  - determining the **objective function**
  - writing the problem out formally

- **STEP 1**
  Define the decision variables
  - Read the question carefully to gather what quantities can be varied
  - Typically these will be a 'number of things'

- **STEP 2**
  Write each constraint (given in words) as a mathematical inequality
  - Where possible, inequalities should be simplified, e.g. $2x + 4y \leq 8$ simplifies to $x + 2y \leq 4$
  - Each inequality will be in terms of the decision variables

- **STEP 3**
  Determine the objective function
  - This will be the quantity that is required to be maximised or minimised
  - Typically this would be maximising profit or minimising costs

- **STEP 4**
  Formulate the linear programming problem by writing it in a formal manner
  - This is of the form
    Maximise
      <objective function>
    subject to
      <constraints>
  - Include the non-negativity constraint, e.g. $x, y \geq 0$

Examiner Tip

- Whether specified or not, and where appropriate, always include the **non-negativity constraint** when writing formal linear programming problem
  - $x, y \geq 0$

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Worked example

A furniture manufacturer makes chairs and tables.

Due to the availability of quality timber, on any particular day the manufacturer cannot make more than a total of 10 chairs and tables.

A chair takes an hour to produce whilst a table takes 2 hours to produce. The manufacturer's factory can produce items for a maximum 18 hours per day.

The varnish on a chair takes 3 hours to dry whilst the varnish on a table takes 2 hours to dry. There are four drying zones within the factory, each able to provide 6 hours of drying time per day.

The manufacturer makes £30 profit on each chair it produces in a day, and £40 profit on each table. The manufacturer wants to maximise its daily profit.

Formulate the above as a linear programming problem, defining the decision variables, stating the objective function and listing the constraints.

- **STEP 1**
  Define the decision variables
  The 'things' that can be varied here are the number of chairs and the number of tables that are made per day

  Let x be the number of chairs made by the furniture manufacturer per day

  Let y be the number of tables made by the furniture manufacturer per day

- **STEP 2**
  Write each constraint (given in words) as an inequality
  The first constraint is the total amount of chairs and tables able to be made in a day

$$x + y \leq 10$$

  The second constraint is the production time - chairs take one hour, tables take two with a maximum 18 hours available per day

$$x + 2y \leq 18$$

  The third constraint is the varnish drying time - 3 hours for a chair, 2 hours for a table.
  The maximum drying time per day available is 4 x 6 = 24 hours

$$3x + 2y \leq 24$$

- **STEP 3**
  Determine the objective function
  Profit is to be maximised

$$P = 30x + 40y$$

- **STEP 4**
  Formulate the linear programming problem by writing it in a formal manner, including the non-negativity constraint

  Maximise

  $$P = 30x + 40y$$

  subject to

  $$\begin{aligned} x + y &\leq 10 \\ x + 2y &\leq 18 \\ 3x + 2y &\leq 24 \\ x,\ y &\geq 0 \end{aligned}$$

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