OCR A Level Physics

Your notes

Circular Motion

Contents

Radians

Your notes

Radians

- In circular motion, it is more convenient to measure angular displacement in units of **radians** rather than units of degrees
- The **angular displacement** (θ) of a body in circular motion is defined as:

  The change in angle, in radians, of a body as it rotates around a circle

- The **angular displacement** is the ratio of:

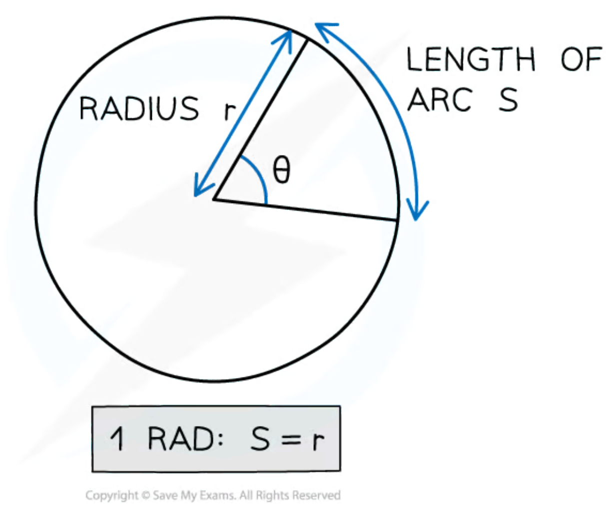
$$\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}}$$

- **Note**: both distances must be measured in the same units e.g. metres
- A **radian** (rad) is defined as:

  The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

- Angular displacement can be calculated using the equation:

$$\theta = \frac{S}{r}$$



When the angle is equal to one radian, the length of the arc (Δs) is equal to the radius (r) of the circle

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- Where:
  - θ = angular displacement, or angle of rotation (radians)
  - S = arc length, or the distance travelled around the circle (m)
  - r = radius of the circle (m)

- Radians are commonly written in terms of π (Pi)
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{\text{circumference of circle}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$

Radian Conversions

- If an angle of 360° = 2π radians, then 1 radian in degrees is equal to:

$$\frac{360}{2\pi} = \frac{180}{\pi} \approx 57.3^{\circ}$$

- Use the following equation to convert from degrees to radians:

$$\theta^{\circ} \times \frac{\pi}{180} = \theta \text{ rad}$$

Table of common degrees to radians conversions

Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

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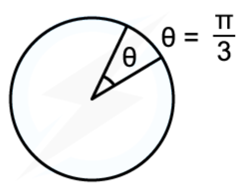
Worked Example

Convert the following angular displacement into degrees:



Answer:

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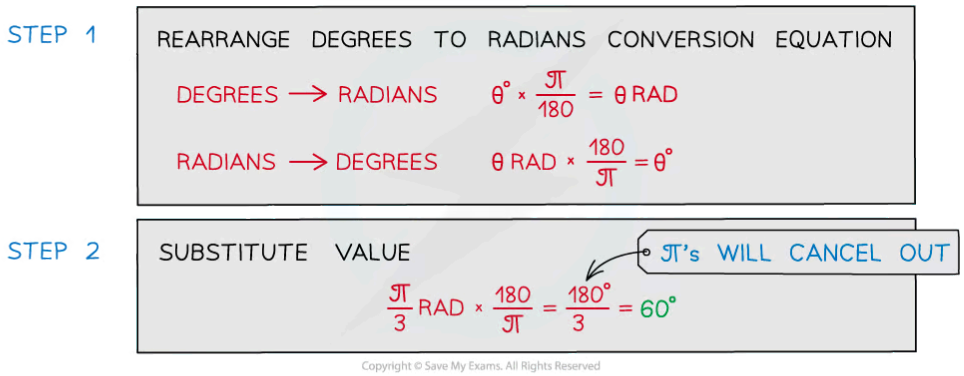
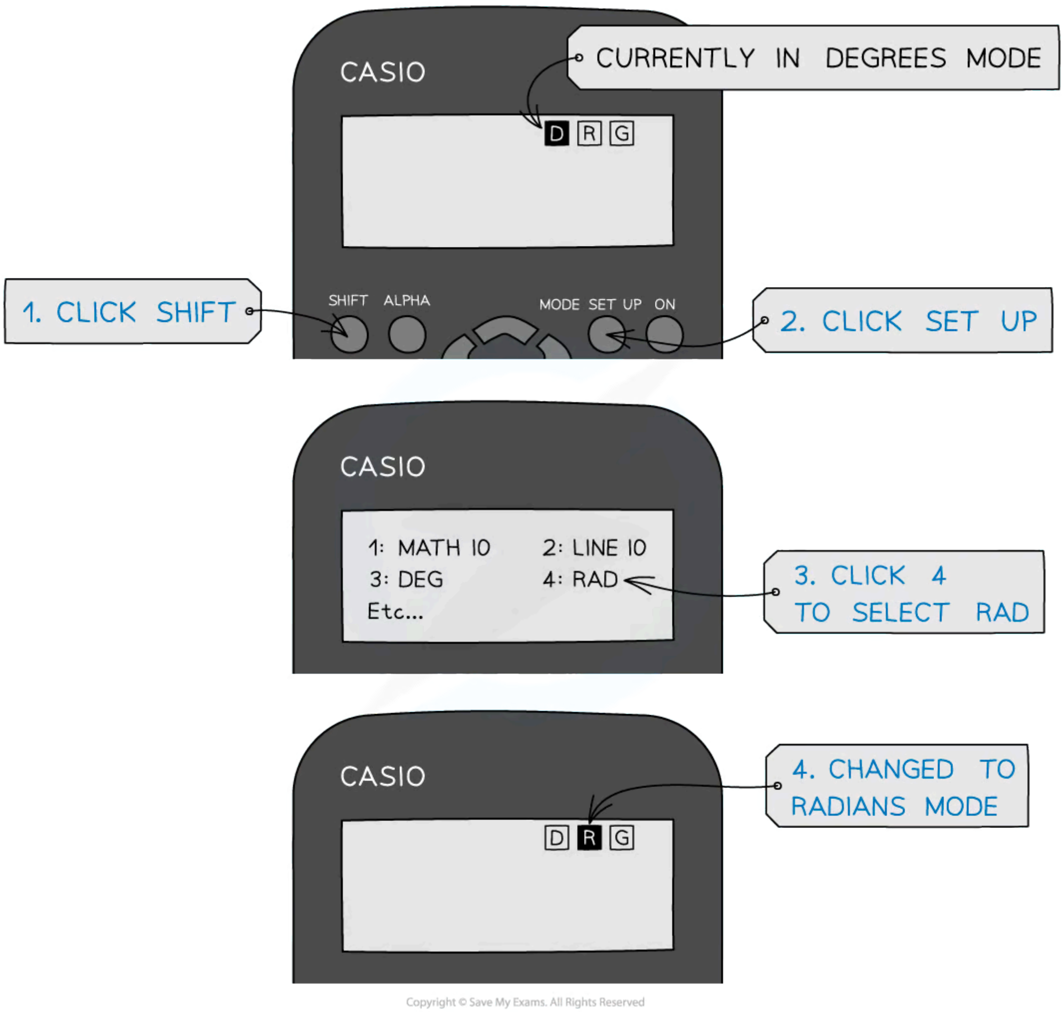


Examiner Tips and Tricks

- You will notice your calculator has a degree (Deg) and radians (Rad) mode
- This is shown by the "D" or "R" highlighted at the top of the screen
- Remember to make sure it's in the right mode when using **trigonometric** functions (sin, cos, tan) depending on whether the answer is required in **degrees** or **radians**
- It is extremely common for students to get the wrong answer (and lose marks) because their calculator is in the wrong mode - make sure this doesn't happen to you!


CASIO
CURRENTLY IN DEGREES MODE
D R G
1. CLICK SHIFT
SHIFT ALPHA
MODE SET UP ON
2. CLICK SET UP
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1: MATH IO 2: LINE IO
3: DEG 4: RAD
Etc...
3. CLICK 4 TO SELECT RAD
CASIO
4. CHANGED TO RADIANS MODE
D R G
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Time period & Frequency

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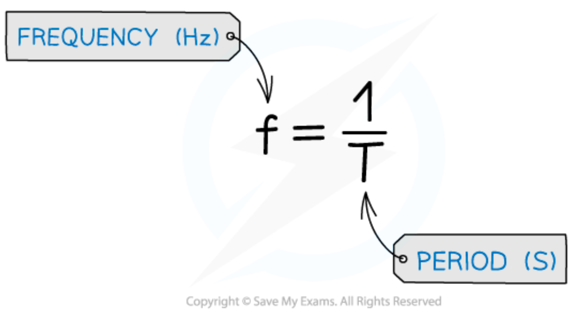
Period & Frequency in Circular Motion

- Frequency, *f*, is defined as:

  The number of complete oscillations per unit time

- It is measured in Hertz (Hz) and is defined by the equation



- Time period, *T*, is defined as:

  The time taken for one complete oscillation, in seconds

- One complete oscillation is defined as:

  The time taken for the oscillator to pass the equilibrium from one side and back again fully from the other side

- If a circle has a radius *r*, then the distance through which an object moves as it completes one rotation is equal to the circumference of the circle = $2\pi r$
- The speed of an object moving in a circle is therefore equal to:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T}$$

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Angular Velocity

Angular Velocity

Angular Displacement

- In circular motion, it is more convenient to measure angular displacement in units of radians rather than units of degrees
- Angular displacement is defined as:

  The change in angle, in radians, of a body as it rotates around a circle

- This can be summarised in equation form:

$$\Delta\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}} = \frac{S}{r}$$

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - S = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)
- Note: both distances must be measured in the same units e.g. metres

Angular Speed

- Any object travelling in a uniform circular motion at the same speed travels with a **constantly changing velocity**
  - This is because it is **constantly changing direction**, and is therefore accelerating
- The **angular speed** (ω) of a body in circular motion is defined as:

  The rate of change in angular displacement with respect to time

- Angular speed is a **scalar quantity** and is measured in rad s^{-1}
- It can be calculated using:

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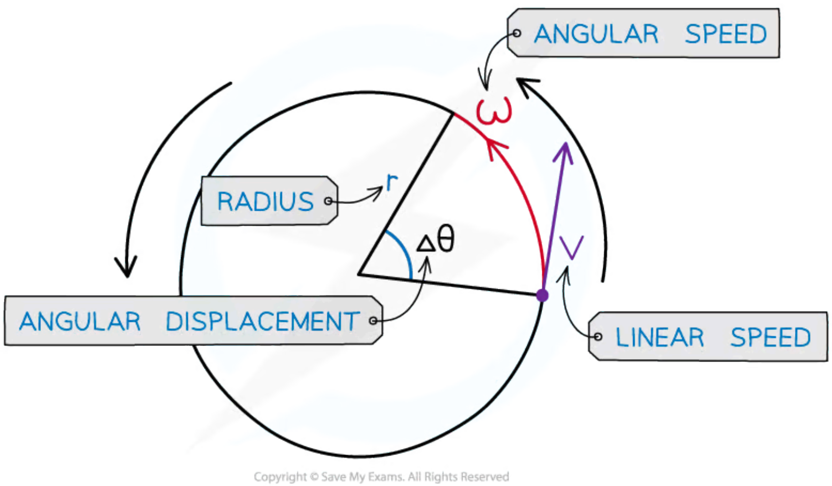
$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Where:
  - $\Delta\theta$ = change in angular displacement (radians)
  - Δt = time interval (s)



When an object is in uniform circular motion, velocity constantly changes direction, but the speed stays the same

- Taking the angular displacement of a complete cycle as 2π, the angular speed ω can be calculated using the equation:

$$\omega = \frac{v}{r} = 2\pi f = \frac{2\pi}{T}$$

- Where:
  - v = linear speed (m s^{-1})
  - r = radius of orbit (m)
  - T = the time period (s)

- f = frequency (Hz)

- Angular velocity is the same as angular speed, but it is a **vector quantity**
- This equation shows that:
  - The greater the rotation angle θ in a given amount of time, the greater the angular velocity ω
  - An object rotating further from the centre of the circle (larger r) moves with a smaller angular velocity (smaller ω)

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Centripetal Force

Your notes

Net Force on an Object in a Circular Path

- For an object moving in a circle, it will have the following properties:
  - Period
  - Frequency
  - Angular displacement
  - Angular velocity
- These properties can be inferred from the properties of objects moving in a straight line combined with the geometry of a circle

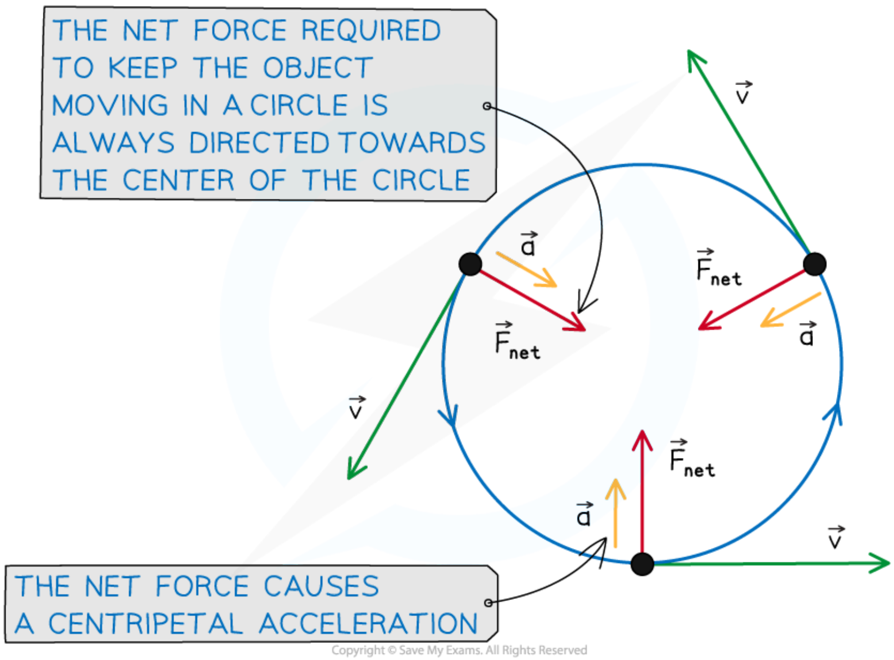
Motion in a Straight Line

- When an object moves in a straight line at a constant speed its motion can be described as follows:
  - The object moves at a constant velocity, v
  - Constant velocity means zero acceleration, a
  - Newton's First Law of motion says the object will continue to travel in a straight line at a constant speed unless acted on by another force
  - Newton's Second Law of motion says for zero acceleration that there is no net or resultant force
- For example, an ice hockey puck moving across a flat frictionless ice rink

Motion in a Circle

- If one end of a string was attached to the puck, and the other attached to a fixed point, it would no longer travel in a straight line, it would begin to travel in a circle
- The motion of the puck can now be described as follows:
  - As the puck moves it stretches the string a little to a length r
  - The stretched string applies a force to the puck pulling it so that it moves in a circle of radius r around the fixed point
- The force acts at 90° to the velocity so there is no force component in the direction of velocity
  - As a result, the **magnitude** of the velocity is constant
  - However, the **direction** of the velocity **changes**

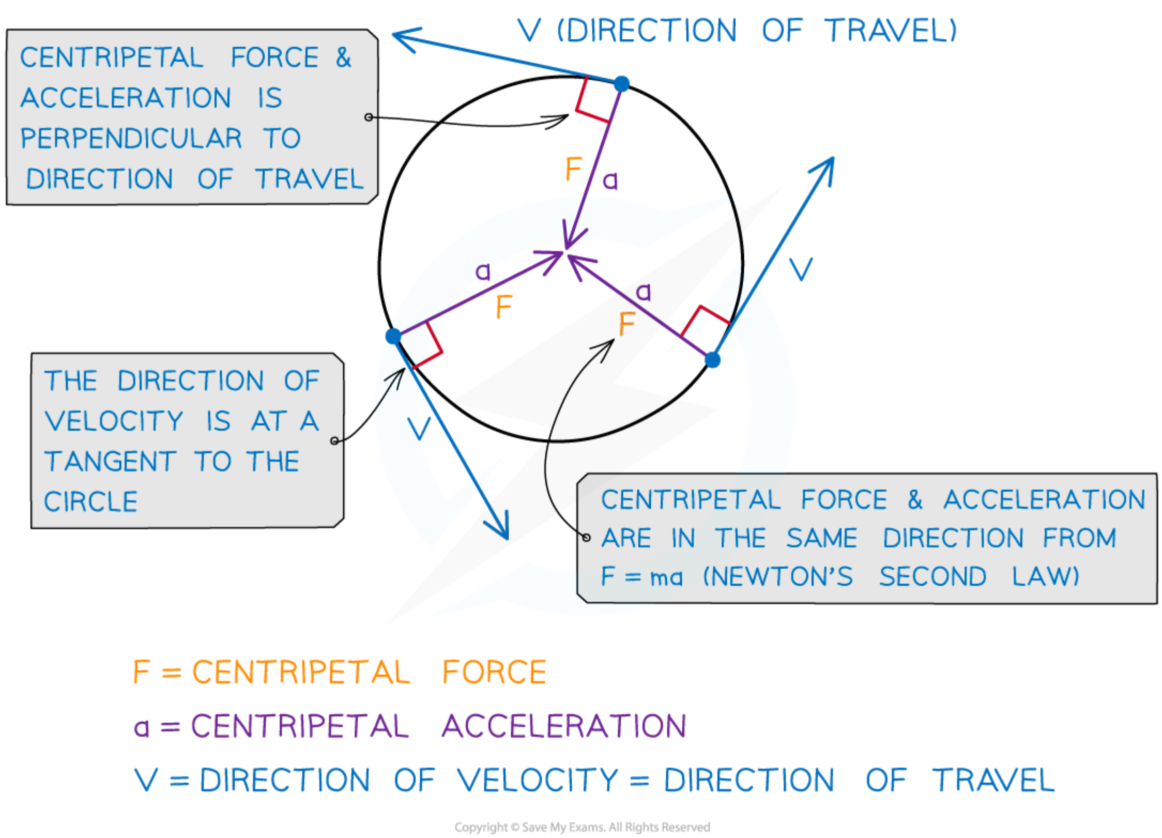
- This means there is **acceleration** present in the circular motion, so there must be a **net force**



- As it starts to move in a circle the tension of the string:
  - Continues to pull the puck at 90° to the linear velocity
  - Acts towards the centre of the circle
  - Is the only force acting on the puck
  - Hence, the **net** or **overall force** is towards the centre of the circle
- So, the **net force** acting on the puck is called the **centripetal force**

Your notes

Your notes



Centripetal force and acceleration are always directed towards the centre of the circle

- The centripetal force is **not** a separate force of its own
  - It can be any type of force, depending on the situation, which keeps an object moving in a circular path

Examples of centripetal force

Your notes

Situation	Centripetal force
Car travelling around a roundabout	Friction between car tyres and the road
Ball attached to a rope moving in a circle	Tension in the rope
Earth orbiting the Sun	Gravitational force

Examiner Tips and Tricks

The term centripetal force should not be confused with "centrifugal force" as this is something that is thought to act away from the centre of a circle - this is the opposite of what is happening in circular motion

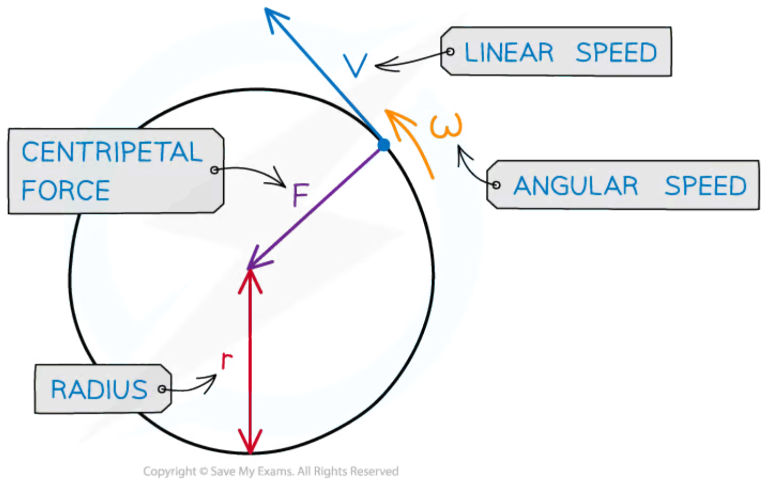
Centripetal Force

- The centripetal force, *F*, is defined as:

  The resultant force towards the centre of the circle required to keep a body in uniform circular motion. It is always directed towards the centre of the body's rotation.

- Centripetal force can be calculated using:

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$

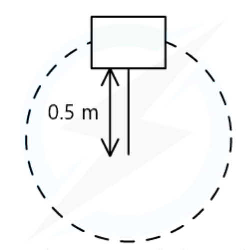


Centripetal force is always perpendicular to the direction of travel

- Where:
  - F = centripetal force (N)
  - v = linear velocity (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)

Worked Example

A bucket of mass 8.0 kg is filled with water is attached to a string of length 0.5 m. What is the minimum speed the bucket must have at the top of the circle so no water spills out?



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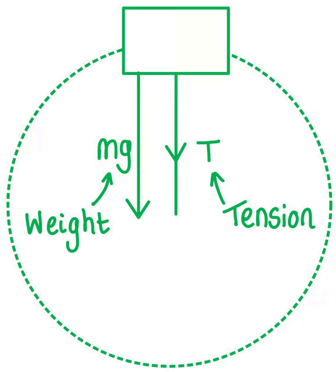
Answer:

Step 1: Draw the forces on the bucket at the top



Step 2: Calculate the centripetal force

- The weight of the bucket = mg
- This is equal to the centripetal force since it is directed towards the centre of the circle

$$mg = \frac{mv^2}{r}$$

Step 3: Rearrange for velocity *v*

- *m* cancels from both sides

$$v = \sqrt{gr}$$

Step 4: Substitute in values

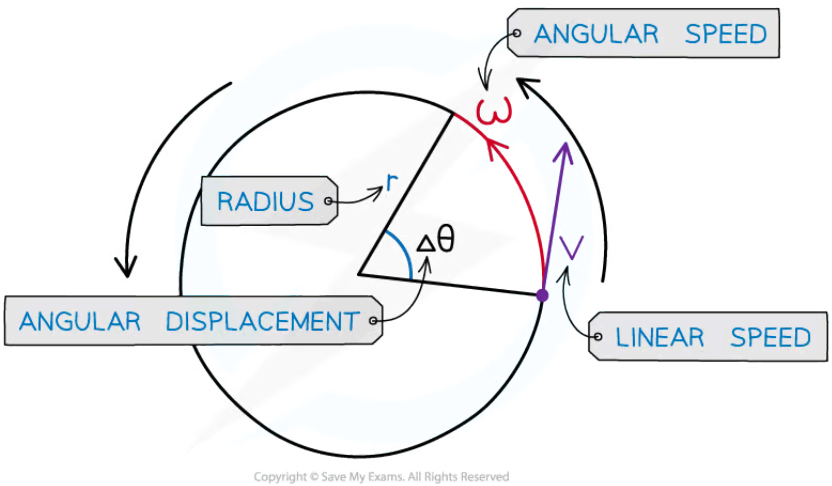
$$v = \sqrt{9.81 \times 0.5} = 2.21 \text{ m s}^{-1}$$

Your notes

Linear Speed

Constant Speed in a Circle

- An object moving in a **straight line** at **constant** speed has an **acceleration** of zero
  - When that object has a **net force** applied at **right angles** to its constant linear speed then it begins to move in a **uniform circular motion**
- Whilst the **linear speed** remains **constant** the **direction** of motion is **constantly changing**
  - So, there is acceleration towards the centre of the circle and the direction of the **net force**



When an object is in uniform circular motion, velocity constantly changes direction, but the speed stays the same

- Linear velocity is defined as:

  The velocity of an object rotating in uniform circular motion, with respect to a linear displacement, as opposed to angular displacement

- The linear speed, v, is related to the angular speed, ω, by the equation:

$$v = r\omega$$

- Where:
  - v = linear speed (m s^{-1})
  - r = radius of circle (m)

Your notes

- ω = angular speed (rad s^{-1})
- The equation for linear speed can be derived from the definitions for angular displacement and angular velocity
- Linear velocity can be described using the equation:

$$v = \frac{\text{distance travelled}}{\text{time taken}}$$

- If the distance travelled in a time, *t*, is equal to the length of an arc of a circle, $r\theta$:

$$v = \frac{r\theta}{t}$$

- Where:
  - θ = angular displacement (°)
- Since angular velocity is given by:

$$\omega = \frac{\theta}{t}$$

- The equation for linear speed of an object in circular motion can be written as

$$v = r\left(\frac{\theta}{t}\right) = r\omega$$

$$\boldsymbol{v = r\omega}$$

Angular Velocity	Linear Velocity
$\omega = \frac{\theta}{t}$ ω in radians per unit time θ in radians	$v = \frac{s}{t}$ $v = \frac{r\theta}{t}$ $v = r\omega$

- This equation shows:

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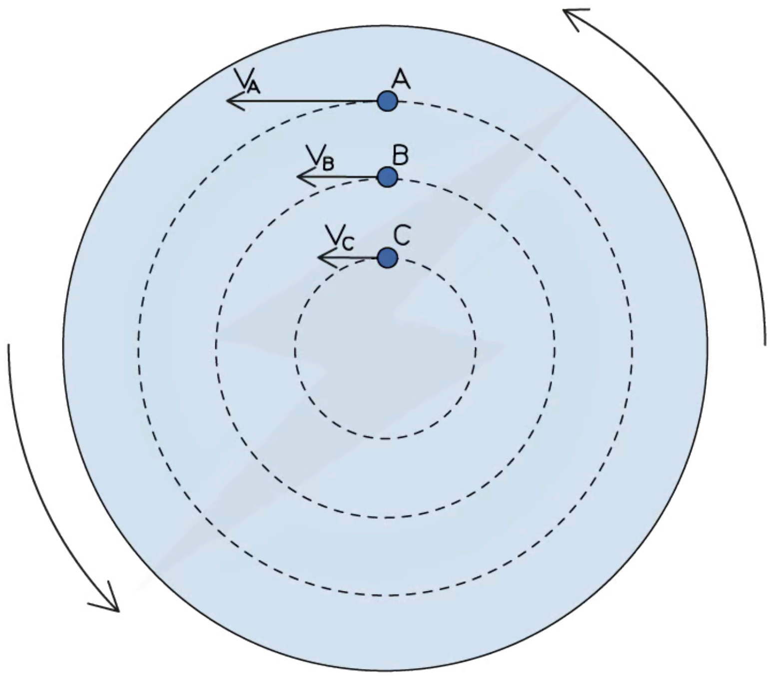
- As the **radius**, *r*, of the path **increases** so does the **linear velocity**, *v*



- Therefore, it can be seen:
  - The **angular** speed and velocity do **not** depend on the **radius** of the circle
  - The **linear** speed, however, **does** depend on the **radius** of the circle

Worked Example

A bird flies in a horizontal circle with an angular speed of 5.25 rad s^{-1} of radius 650 m.

Calculate:

a) The linear speed of the bird

b) The frequency of the bird flying in a complete circle

Answer:

Your notes

a) STEP 1 — LINEAR SPEED EQUATION

$v = r\omega$

STEP 2 — SUBSTITUTE IN VALUES

$V = 650 \times 5.25 = 3412.5 = 3410\ \text{ms}^{-1}$ (3 s.f.)

b) STEP 1 — ANGULAR SPEED WITH FREQUENCY EQUATION

$\omega = 2\pi f$

STEP 2 — REARRANGE FOR FREQUENCY

$f = \dfrac{\omega}{2\pi}$

STEP 3 — SUBSTITUTE IN VALUES

$f = \dfrac{5.25}{2\pi} = 0.83556... = 0.836\ \text{Hz}$ (3 s.f.)

Examiner Tips and Tricks

Try not to be confused by similar sounding terms like "angular velocity" and "angular speed". Just like in regular linear motion, you have linear velocity and linear speed: one is a scalar (speed) and the other is a vector (velocity).

Finally, you may sometimes come across ω being labelled as 'angular frequency', because of its relationship to linear frequency f as given by the alternative equation $\omega = 2\pi f$. Remember, the units of ω are rad s^{-1}, whereas the units of f are Hz.

Centripetal Acceleration

Your notes

Centripetal Acceleration

- Centripetal acceleration is defined as:

  The acceleration of an object towards the centre of a circle when an object is in motion (rotating) around a circle at a constant speed

- It can be defined using the radius *r* and linear speed *v*:

$$a = \frac{v^2}{r}$$

- Using the equation relating angular speed ω and linear speed *v*:

$$v = r\omega$$

- These equations can be combined to give another form of the centripetal acceleration equation:

$$a = \frac{(r\omega)^2}{r}$$

$$\mathbf{a = r\omega^2}$$

- This equation shows that centripetal acceleration is equal to the radius times the square of the angular speed
- Alternatively, rearrange for *r:*

$$r = \frac{v}{\omega}$$

- This equation can be combined with the first one to give us another form of the centripetal acceleration equation:

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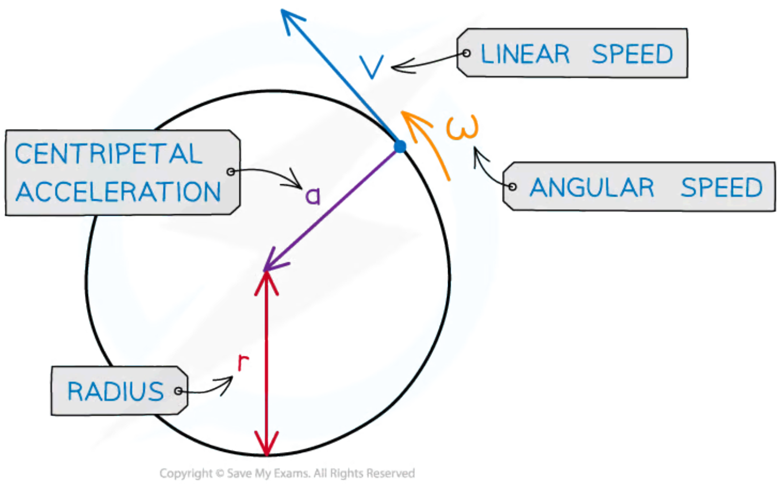
$$a = \frac{v^2}{\left(\frac{v}{\omega}\right)}$$

$$a = v\omega$$

- This equation shows how the centripetal acceleration relates to the linear speed and the angular speed



Centripetal acceleration is always directed toward the centre of the circle, and is perpendicular to the object's velocity

- Where:
  - a = centripetal acceleration ($m\ s^{-2}$)
  - v = linear speed ($m\ s^{-1}$)
  - ω = angular speed ($rad\ s^{-1}$)
  - r = radius of the orbit (m)

Worked Example

A ball tied to a string is rotating in a horizontal circle with a radius of 1.5 m and an angular speed of 3.5 rad s^{-1}.

Calculate its centripetal acceleration if the radius was twice as large and angular speed was twice as fast.

Answer:

STEP 1

ANGULAR ACCELERATION EQUATION WITH ANGULAR SPEED

$$a = r\omega^2$$

STEP 2

CHANGE IN ANGULAR ACCELERATION WITH TWICE THE RADIUS AND ANGULAR SPEED

$$a = (2r) \times (2\omega)^2 = 2r \times 4\omega^2 = 8r\omega^2$$

THE CENTRIPETAL ACCELERATION WILL BE 8x BIGGER

STEP 3

SUBSTITUTE IN VALUES OF RADIUS AND ANGULAR SPEED

$$a = 8r\omega^2 = 8 \times 1.5 \times 3.5^2 = 147 \text{ ms}^{-2}$$

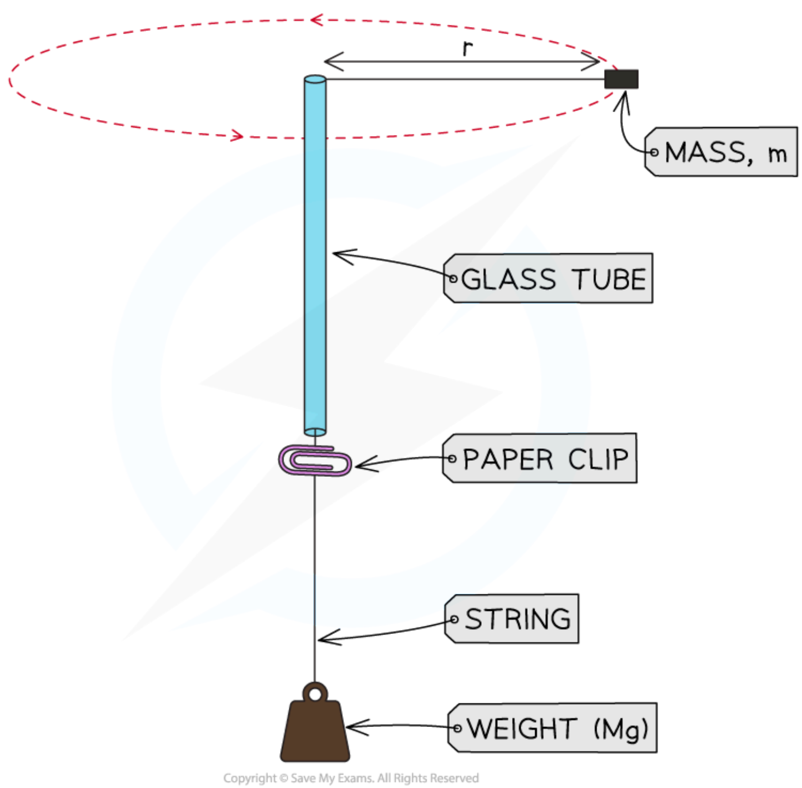
Investigating Circular Motion

Your notes

Investigating Circular Motion

Equipment & Method

- Circular motion can be investigated using the following setup:
  - Tie a bung of mass *m,* to a piece of string, which sits horizontally
  - Thread it though a glass tube and a paper clip, which sits vertically
  - At the other end of the string a heavier mass, *M* is suspended vertically
  - This provides the centripetal force, $F = Mg$ when the tension in the string is constant
- The string is spun in a circle:
  - The time taken for several rotations is recorded and repeated to remove any random errors
- The masses in the experimental set up are changed before the experiment is repeated again



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Explanation

- As the bung is swung in a circle the bigger suspended mass, *M* will remain stationary
  - When the force it provides is equal to the **centripetal force**, *Mg*
  - This is the centripetal force required to make the bung travel in a circular path
- The **weight**, and hence the **centripetal force**, required for different masses, radii and speeds can be investigated
- The forces acting on the bung are:
  - The **tension** in the string
  - The **weight** of the bung downwards
- If the **centripetal force** required is **greater** than its **weight** then the suspended mass moves **upwards**
- If the **centripetal force** required is **less** than its **weight** then the suspended mass moves **downwards**
  - The paperclip will move accordingly to make this movement clearer
- As the bung moves around the circle, the **direction** of the tension will change continuously
- The **magnitude** of the tension will also vary continuously, reaching a **maximum** value at the **bottom** and a **minimum** value at the **top**
  - This is because the direction of the weight of the bung never changes, so the resultant force will vary depending on the position of the bung in the circle
- At the bottom of the circle, the tension must overcome the weight, this can be written as:

$$T_{max} = \frac{mv^2}{r} + mg$$

- As a result, the acceleration, and hence, the **speed** of the bung will be **slower** at the top
- At the top of the circle, the tension and weight act in the same direction, this can be written as:

$$T_{min} = \frac{mv^2}{r} - mg$$

- As a result, the acceleration, and hence, the **speed** of the bung will be **faster** at the bottom