# Connected Rates of Change/Relating Rates of Change

**Q** Determine the rate of change of the area $A$ of a circle when the radius $\boxed{r = 3\text{cm},}$ given that the radius is changing at a rate of $5\ cm\ s^{-1}$.

$\overline{dt}$ $cm s^{-1}$ $\frac{cm}{s}$ cm per second.

**Tip:** Whenever you see the word '<u>rate</u>', think $/dt$

**<u>Firstly, how would we represent...</u>**

"the rate of change of the area $A$" $\frac{dA}{dt}$

"the rate of change of the radius $r$ is 5" $\frac{dr^{cm}}{dt_{s}} = 5$

"the area $A$ of a circle" $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$$\frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \times 5$$

$$\frac{dA}{dt} = 10\pi r$$

$$\frac{dA^{cm^2}}{dt_{s}} = 10\pi \times 3$$

$$= 30\pi = 94.2\ cm^2 s^{-1}\ (3sf)$$

# Edexcel C4 June 2008 Q3

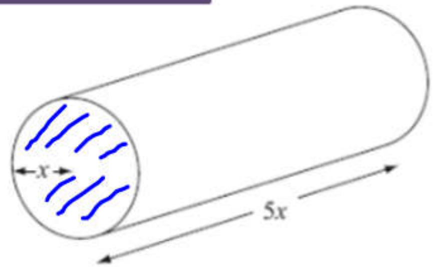


Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After $t$ seconds the radius of the rod is $x$ cm and the length of the rod is $5x$ cm.
The cross-sectional area of the rod is increasing at the constant rate of 0.032 cm$^2$ s$^{-1}$. $cm^2 s^{-1}$

$\frac{dA}{dt} = 0.032$

(*a*) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.
**(4)**

(*b*) Find the rate of increase of the volume of the rod when $x = 2$. **(4)**

$A = \pi x^2$

$\frac{dA}{dx} = 2\pi x$

a) $\frac{dx}{dt} = \frac{dx}{dA} \frac{dA}{dt}$

$= \frac{1}{2\pi x} \times 0.032$

$x = 2$

$= \frac{1}{4\pi} \times 0.032$

$= 0.00255 \text{ cm s}^{-1}$ (3sf)

b) $V = \pi x^2 \times 5x$

$V = 5\pi x^3$

$\frac{dV}{dx} = 15\pi x^2$ $\rightarrow x = 2$

$\frac{dV}{dt} = \frac{dV}{dx} \frac{dx}{dt}$

$= 15\pi x^2 \times \frac{0.032}{4\pi}$

$= 15\cancel{\pi} \times \cancel{4} \times \frac{0.032}{\cancel{4\pi}}$

$= 0.48 \text{ cm}^3 \text{s}^{-1}$

**June 11 Q3.**

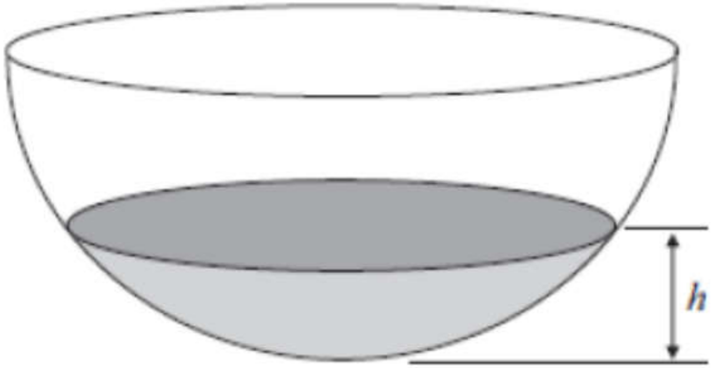


**Figure 1**

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.

When the depth of the water is $h$ m, the volume $V$ m$^3$ is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \leq h \leq 0.25.$$

($a$) Find, in terms of $\pi$, $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.1$.

**(4)**

Water flows into the bowl at a rate of $\dfrac{\pi}{800}$ m$^3$ s$^{-1}$.

($b$) Find the rate of change of $h$, in m s$^{-1}$, when $h = 0.1$.

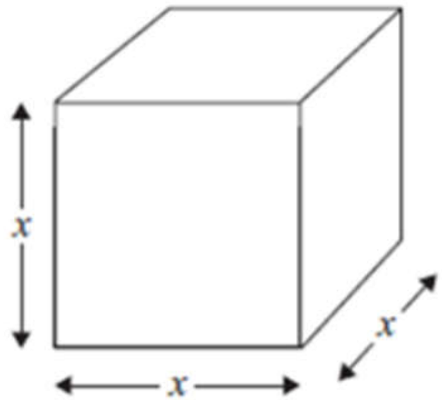


**Figure 1**

Ex 9J

Q1 – 4

Q10, 11

Exam Questions.

$V = x^3$

$\frac{dV}{dx} = 3x^2$

$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{1}{3 \times 8^2} \times 0.048$

$= 0.00025$

$A = 6x^2$

$\frac{dA}{dx} = 12x$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$= (12 \times 8) \times 0.00025 = 0.024$

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time $t$ seconds, the length of each edge of the cube is $x$ cm, and the volume of the cube is $V$ cm$^3$.

(*a*) Show that $\frac{dV}{dx} = 3x^2$.

**(1)**

Given that the volume, $V$ cm$^3$, increases at a constant rate of 0.048 cm$^3$ s$^{-1}$,

(*b*) find $\frac{dx}{dt}$ when $x = 8$,

**(2)**

(*c*) find the rate of increase of the total surface area of the cube, in cm$^2$ s$^{-1}$, when $x = 8$.

**(3)**

The volume of a hemisphere $V cm^3$ is related to its radius $r cm$ by the formula $V = \frac{2}{3}\pi r^3$ and the total surface area $S = 3\pi r^2$

Given that the rate of increase of volume, in $cm^3s^{-1}$, $dV/dt = 6$, find the rate of increase of surface area, $dS/dt$, when $r = 9cm$

$V = \frac{2}{3}\pi r^3$ $\quad S = 3\pi r^2$ $\quad \boxed{\frac{dV}{dt} = 6}$ $\quad r = 9$

$\boxed{\frac{dV}{dr} = 2\pi r^2}$ $\quad \boxed{\frac{dS}{dr} = 6\pi r}$

$\frac{dS}{dt} = \frac{dS}{dr}\frac{dr}{dV}\frac{dV}{dt}$

$= \cancel{6}^{3}\cancel{\pi r} \times \frac{1}{\cancel{2}\cancel{\pi}r^{\cancel{2}}} \times 6 = \frac{18}{r}$ $\quad \frac{dS}{dt} = \frac{18}{r}$

when $r = 9$

$\frac{dS}{dt} = \frac{18}{9} = \underline{2 cm^2 s^{-1}}$

## Exercise 9J

**Note**: I have skipped out the content on 'setting up differential equations' as I think it's better to do this in the Integration chapter. You therefore won't be able to yet do Q5, Q8, Q9, Q12, Q14.

Figure 4 shows a bowl with a circular cross-section.

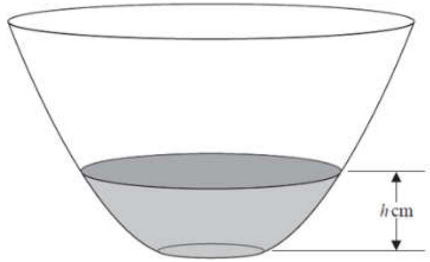


Figure 4

Initially the bowl is empty. Water begins to flow into the bowl.

At time $t$ seconds after the water begins to flow into the bowl, the height of the water in the bowl is $h$ cm.

The volume of water, $V$ cm³, in the bowl is modelled as

$$V = 4\pi h(h+6) \qquad 0 \leqslant h \leqslant 25$$

The water flows into the bowl at a constant rate of $80\pi$ cm³ s⁻¹ $\frac{dV}{dt} = 80\pi$

(a) Show that, according to the model, it takes 36 seconds to fill the bowl with water from empty to a height of 24 cm.

**(1)**

(b) Find, according to the model, the rate of change of the height of the water, in cm s⁻¹, when $t = 8$ $\frac{dh}{dt}$

**(8)**

a) $h = 24 \quad V = 4\pi \times 24(24+6)$
$= 2880\pi$

time $= \frac{2880\pi}{80\pi} = 36$ secs

b) $V = 4\pi h(h+6)$

$V = 4\pi h^2 + 24\pi h$

$\frac{dV}{dh} = 8\pi h + 24\pi$

$\frac{dV}{dt} = 80\pi$

$\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt}$

$= \frac{1}{\cancel{8\pi}h + \cancel{24\pi}_{3}} \times \cancel{80\pi}^{10} = \frac{10}{h+3}$

When $t = 8 \quad V = 80\pi \times 8 = 640\pi$

$V = 4\pi h^2 + 24\pi h$

$640\cancel{\pi} = 4\cancel{\pi}h^2 + 24\cancel{\pi}h$

$0 = 4h^2 + 24h - 640$

$h = 10, \cancel{-16}$

$\frac{dh}{dt} = \frac{10}{10+3} = \frac{10}{13}$

$= 0.77$ cms⁻¹