Edexcel C4 June 2009 Q3

$f(ax+b)$

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \int \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(*a*) Find the values of the constants *A*, *B* and *C*. **(4)**

$$= \left[\frac{A}{2}\ln|2x+1| + B\ln|x+1| + C\ln|x+3| \right]_0^2 + \ln k$$

(*b*) (i) Hence find $\int f(x)\ dx$. **(3)**

(ii) Find $\int_0^2 f(x)\ dx$ in the form $\ln k$, where k is a constant. **(3)**

SKILL #8: Integrating top-heavy algebraic fractions

$$\int \frac{x^2}{x+1}\, dx = ?$$

How would we deal with this? (the clue's in the title)

$$\begin{array}{r|l} & x - 1 \\ \hline x+1 & x^2 \\ & x^2 + x \\ \hline & -x \\ & -x - 1 \\ \hline & 1 \end{array}$$

$$\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$$

$$\int \frac{x^2}{x+1}\, dx = \int \left(x - 1 + \frac{1}{x+1}\right) dx$$

$$= \frac{1}{2}x^2 - x + \ln|x+1| + c$$

$$\int \frac{x}{x-1}\,dx = \int \left(1 + \frac{1}{x-1}\right) dx = x + \ln|x-1| + c.$$

$$\begin{array}{r} 1 \\ x-1 \,\big)\, x \\ x - 1 \\ \hline 1 \end{array}$$

Let $u = x-1$

$u+1 = x$

$\frac{du}{dx} = 1$

$du = dx$

$$\int \frac{x}{x-1}\,dx = \int \frac{u+1}{u}\,du$$

$$= \int \left(1 + \frac{1}{u}\right) du$$

$$= u + \ln|u| + c$$

$$= x - 1 + \ln|x-1| + c$$

Contrast this with $\int \frac{x-1}{x}\,dx$ which can be integrated more simply

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$$\int \frac{x^3+2}{x+1}\,dx = \int \left(x^2 - x + 1 + \frac{1}{x+1}\right) dx$$

$$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + x + \ln|x+1| + c.$$

$$\begin{array}{r} x^2 - x + 1 \\ x+1 \,\big)\, x^3 + 0x^2 + 0x + 2 \\ x^3 + x^2 \\ \hline -x^2 + 0x + 2 \\ -x^2 - x \\ \hline x + 2 \\ x + 1 \\ \hline 1 \end{array}$$

Q2a c

Q5

Q7

Ex 11G