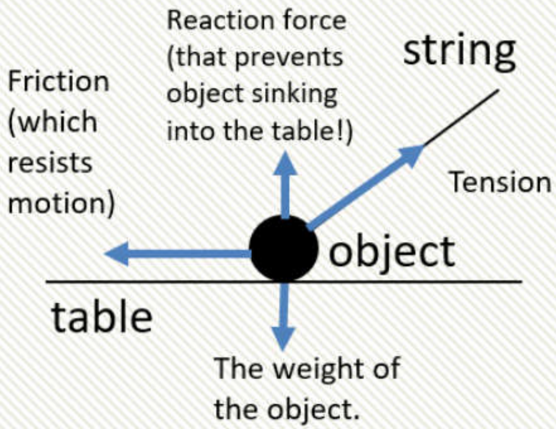
Mechanics - introduction

Mechanics, broadly speaking, concerns motion, forces, and how the two interrelate.

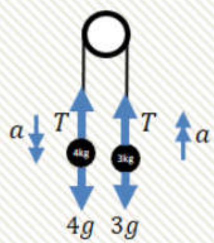
This chapter just gives you an overview of what you'll be covering in Year 1 and how it all links together.

Forces

You will later encounter force diagrams. This considers the forces acting at a particular point. Some forces you might consider…



- Forces can be considered as vectors.
- The **magnitude** of the force vector gives the 'size' of the force.
- We often **consider forces in a particular direction**. e.g. If the object above is stationary, the forces left must equal the force right, and forces up equal forces down (Newton's 1st Law).
- Often we need to consider the forces at multiple different points if objects are connected, e.g. with pulleys:



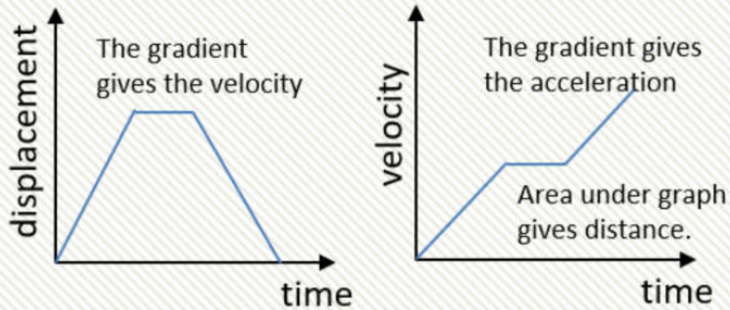
The bridge!

$$F = ma$$

Newton's 2nd Law allows us to connect the force world (F) with the motion world (acceleration a) if the object is moving.

Motion

At GCSE you may have encountered displacement-time and velocity-time graphs:



Given **constant acceleration** we have 5 quantities of motion ("*suvat*"):

$\boldsymbol{s}$ = displacement
$\boldsymbol{u}$ = initial velocity
$\boldsymbol{v}$ = final velocity
$\boldsymbol{a}$ = acceleration
$\boldsymbol{t}$ = time

which we will see are linked by various equations:

$$s = ut + \frac{1}{2}at^2$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$v^2 = u^2 + 2as$$
$$v = u + at$$

If the **acceleration is not constant**, we can specify displacement/velocity/acceleration as a function of time and differentiate/integrate to change between them.

$$s = 2t^3 + 3t \quad \rightarrow \quad v = \frac{ds}{dt} = 6t^2 + 3$$

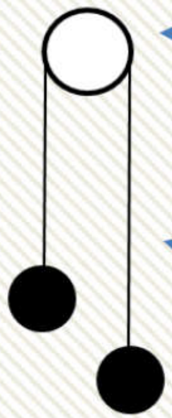
Modelling Assumptions

As with many areas of applied maths, we often have to make various modelling assumptions, to make the maths cleaner or to use well-known mathematical approaches.
Here are common modelling assumptions often made in Mechanics:

Particle
Dimensions of object are negligible
Means: Mass of object concentrated at single point. Rotational forces/air resistance can be ignored.

Smooth/light pulley
No friction.
Means: <u>Tension the same in string either side of pulley</u>. Pulley has no mass.

Fro Tip:
Particularly make note of underlined text!



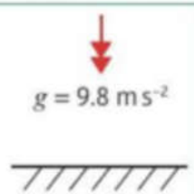
Inextensible string
String does not stretch under load.
Means: <u>Acceleration the same</u> in any connected objects.

Rough/Smooth surface
Means: Objects in contact with surface does/does not experience friction.

Rod
One dimension is negligible, like a pole or beam.
Means: Mass is concentrated along line. Rigid.

Peg/Support
A support from which a body can be suspended or rested.
Means: Dimensionless and fixed. Can be rough or smooth depending on question.

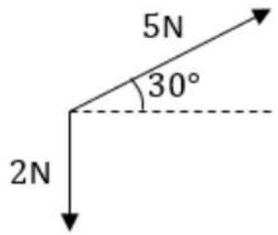
These are some common models and modelling assumptions that you need to know.

Model	Modelling assumptions
Particle – Dimensions of the object are negligible.	• mass of the object is concentrated at a single point • rotational forces and air resistance can be ignored
Rod – All dimensions but one are negligible, like a pole or a beam.	• mass is concentrated along a line • no thickness • rigid (does not bend or buckle)
Lamina – Object with area but negligible thickness, like a sheet of paper.	• mass is distributed across a flat surface
Uniform body – Mass is distributed evenly.	• mass of the object is concentrated at a single point at the geometrical centre of the body – the **centre of mass**
Light object – Mass of the object is small compared to other masses, like a string or a pulley.	• treat object as having zero mass • tension the same at both ends of a light string
Inextensible string – A string that does not stretch under load.	• acceleration is the same in objects connected by a taut inextensible string
Smooth surface	• assume that there is no friction between the surface and any object on it
Rough surface – If a surface is not smooth, it is rough.	• objects in contact with the surface experience a frictional force if they are moving or are acted on by a force
Wire – Rigid thin length of metal.	• treated as one-dimensional
Smooth and light pulley – all pulleys you consider will be smooth and light.	• pulley has no mass • tension is the same on either side of the pulley
Bead – Particle with a hole in it for threading on a wire or string.	• moves freely along a wire or string • tension is the same on either side of the bead
Peg – A support from which a body can be suspended or rested.	• dimensionless and fixed • can be rough or smooth as specified in question
Air resistance – Resistance experienced as an object moves through the air.	• usually modelled as being negligible
Gravity – Force of attraction between all objects. Acceleration due to gravity is denoted by g. 	• assume that all objects with mass are attracted towards the Earth • Earth's gravity is uniform and acts vertically downwards • g is constant and is taken as $9.8\ \text{m s}^{-2}$, unless otherwise stated in the question

Forces (Year 2)

1:: Resolving components

"Determine the magnitude and direction of the resultant force."

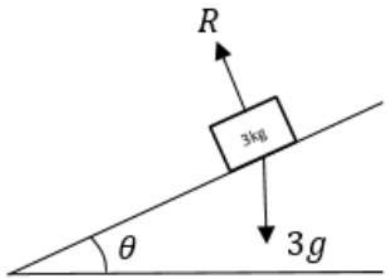


3:: $F \leq \mu R$

Understand that the maximum friction is μR, where μ is the coefficient of friction of the surface, and R is the normal reaction force of the surface on the object. Use to solve inclined plane problems when the surface is rough.

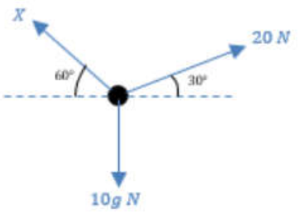
2:: Inclined Planes

"A block of mass 3kg is placed on a smooth slope with angle of inclination θ where $\tan\theta = \frac{3}{4}$. Determine the acceleration of the block down the slope."



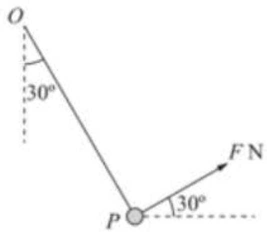
4:: Unknown forces for bodies in equilibrium.

"If the particle is in equilibrium, determine the magnitude of the force X."



5:: Static problem involving weight, tension and pulleys

A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.



Find

(i) the value of F,

(ii) the tension in the string.

(8)

Figure 3

6:: Objects in motion on inclined planes

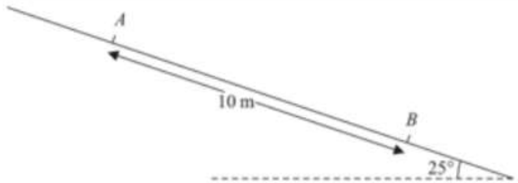


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B, **(3)**

(b) the acceleration of P, **(2)**

(c) the coefficient of friction between P and the plane. **(5)**

7:: Connected particles requiring resolution of forces.

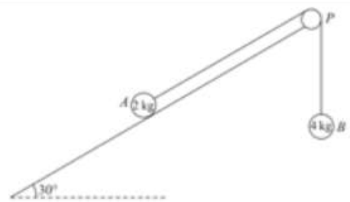


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

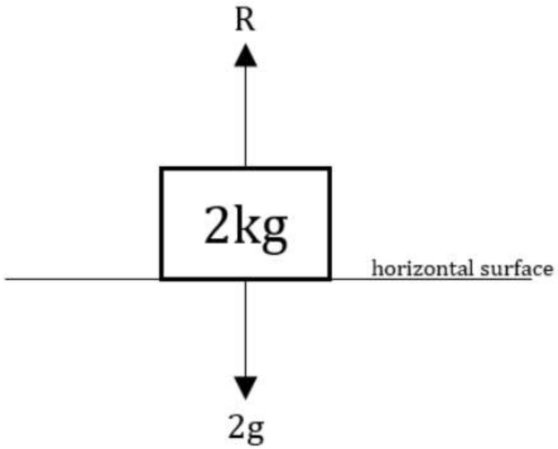
Find the tension in the string immediately after the particles are released.

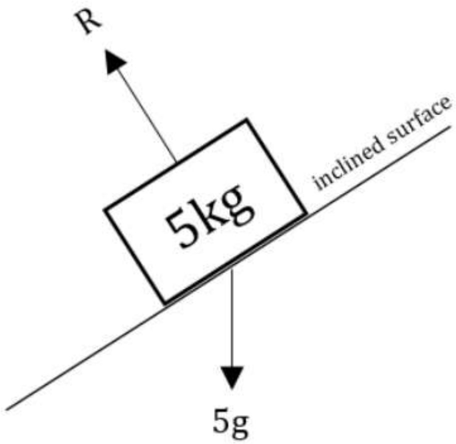
(9)

Mechanics essentials

weight = mass $\times g$ (where g is the acceleration due to gravity, $g = 9.8ms^{-2}$
$W = mg$
Weight acts vertically downwards (obviously)

The normal reaction (sometimes called the contact force) is the force which acts on a box/particle from the surface that it is on.
It is called a **normal** reaction because it acts normal (perpendicular) to the surface.
It is called a normal **reaction** because it has reacted to the forces in the opposing direction.
For example, when you are sat on a chair, your weight acts down, but the chair (surface) has a reaction force upwards which stops you falling to the floor. This is the normal reaction.
We use the letter R for the normal reaction.





Note that the weight acts vertically downwards, but the normal reaction is perpendicular to the slope

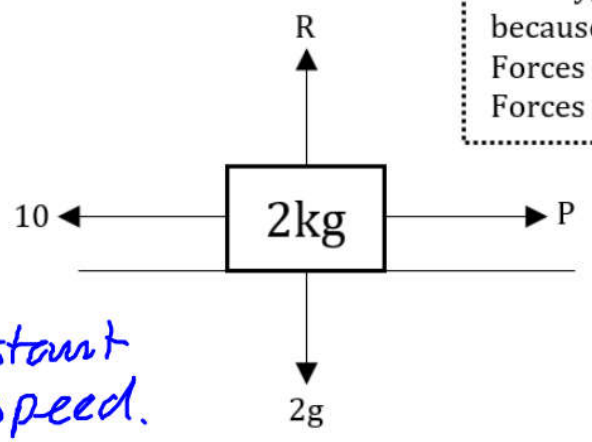
<u>Newton's First Law</u>

"An object will remain at rest or will continue to move with constant velocity unless acted upon by an external force"

Essentially, this means that something will not move, or move with no acceleration if there is no overall resultant force. It means that all the forces are balanced.

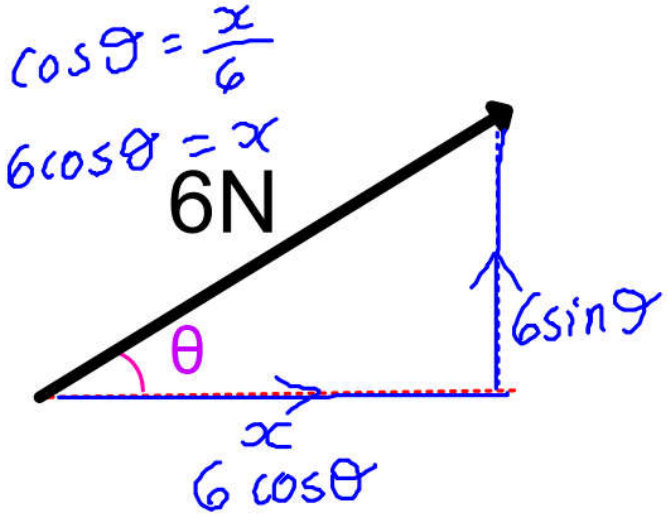
We call this **equilibrium** (think of the word 'equal')

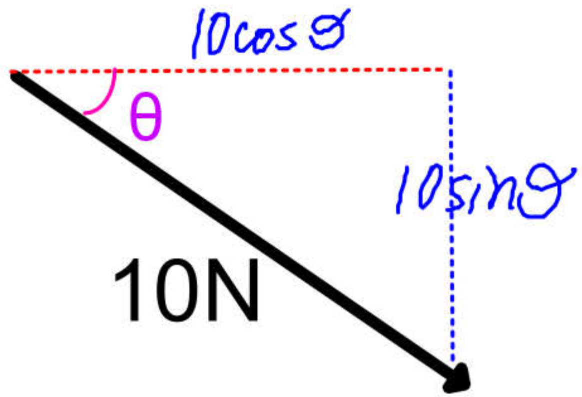
constant speed.



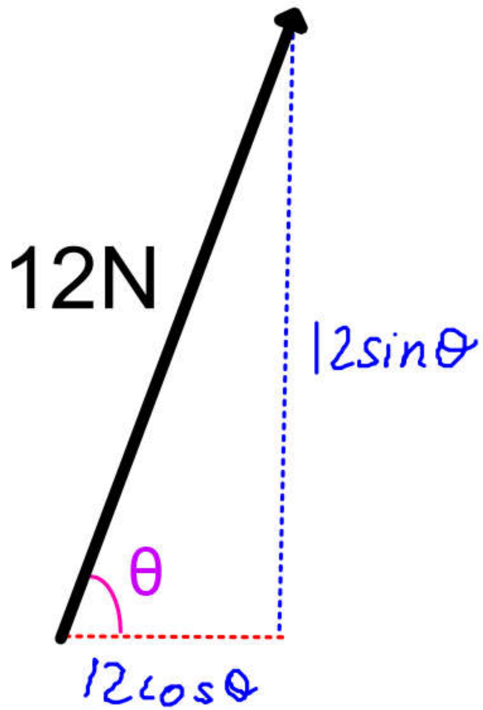
Given that this particle is rest, work out the value of P and R.
Clearly, R = 2g and P = 10, because it is in equilibrium.
Forces left = forces right.
Forces up = forces down

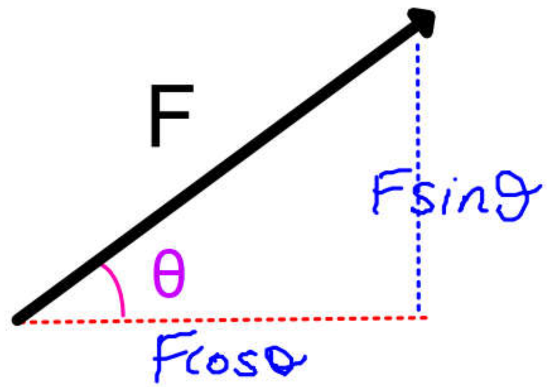
Resolving Forces into x and y components





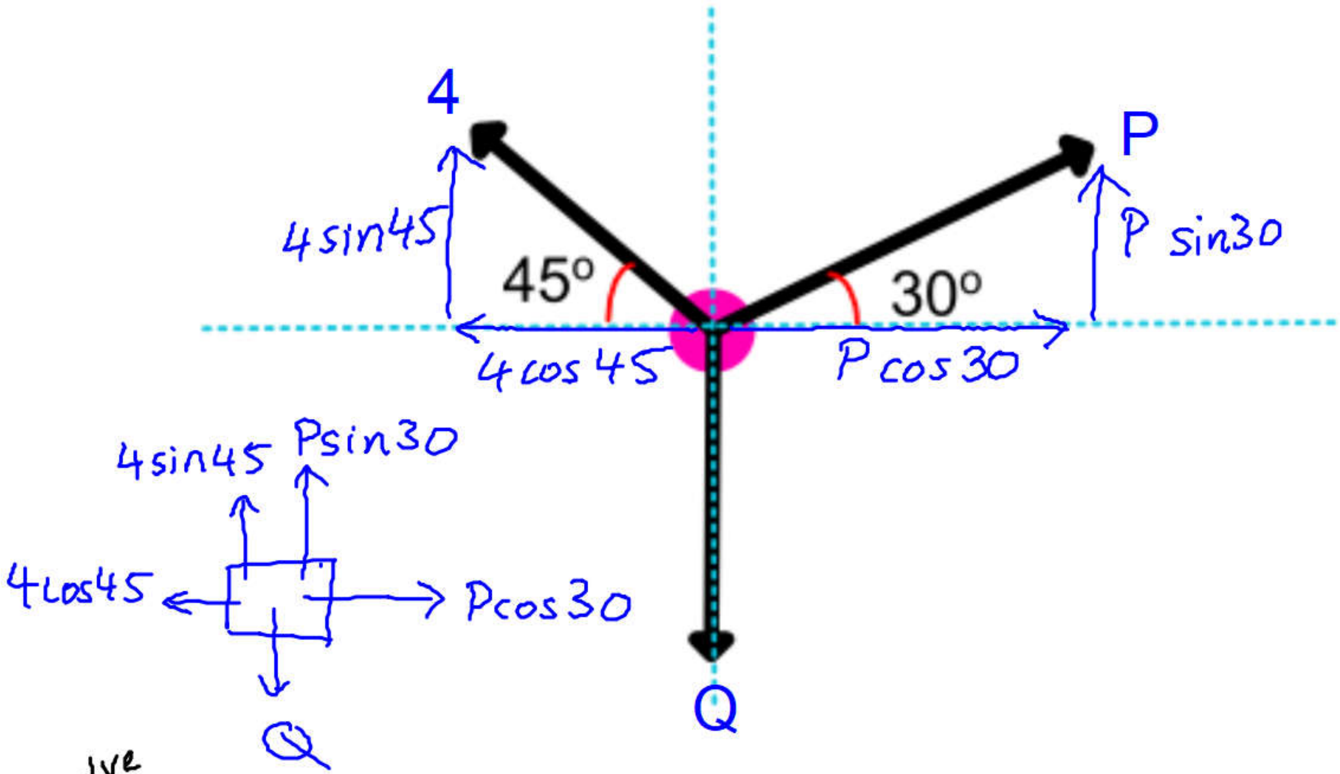
Forces can be considered by resolving them (splitting them) into 2 perpendicular forces.
Perpendicular forces do not interact with each other.
These triangles obey the rules of SOH CAH TOA and Pythagoras





Tip: If F is the magnitude/the hypotenuse, use $F\cos\theta$ for the side adjacent to the angle and $F\sin\theta$ for the side opposite it.

Statics - particles not moving



resolve

R ↔

$4\cos 45 = P\cos 30$

$P = \frac{4\sqrt{6}}{3} = \underline{3.27\text{ N}}$ (3sf)   3.26598..

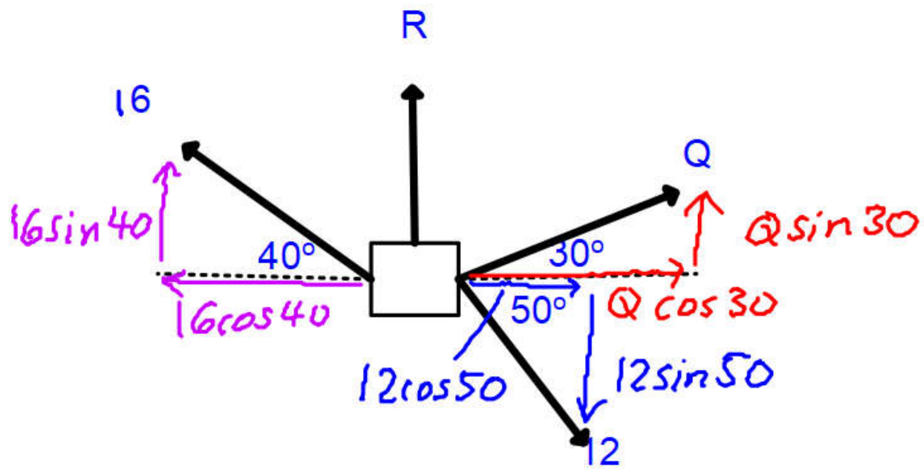
R ↕ $Q = 4\sin 45 + P\sin 30$

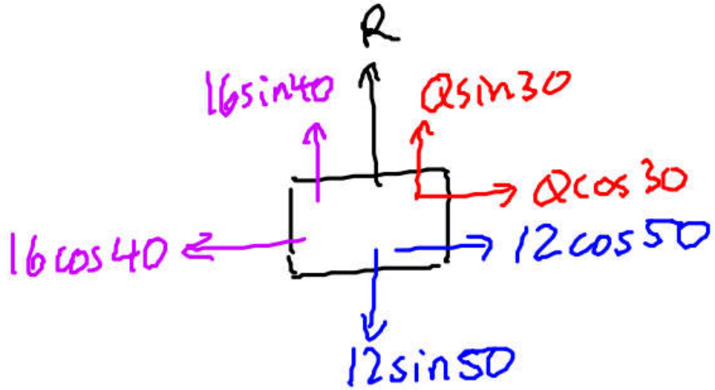
$Q = 4\sin 45 + 3.2659\ldots \sin 30$

$Q = \underline{4.46\text{ N}}$ (3sf)

'Static' means there is no movement.
This means there is no acceleration, so the particle is in *equilibrium*.
All the forces are balanced in *any direction*.
Forces left = forces right
Forces up = forces down

Further Example





$R \leftrightarrow$

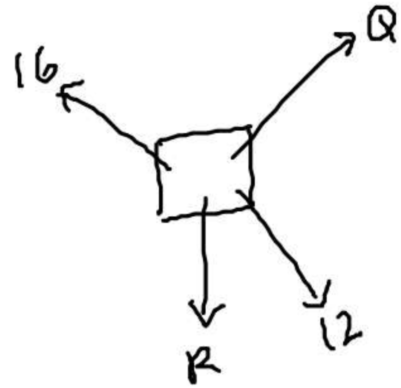
$$16\cos 40 = 12\cos 50 + Q\cos 30$$

$$\frac{16\cos 40 - 12\cos 50}{\cos 30} = Q \qquad 5.24610\ldots$$

$$Q = 5.25\text{ N}$$

$R \updownarrow$

$$R + 16\sin 40 + Q\sin 30 = 12\sin 50$$

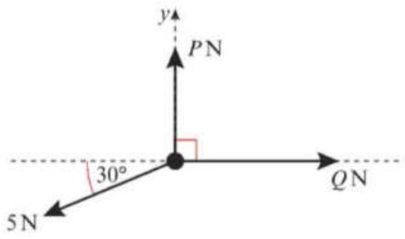


$$R = -3.715\ldots$$

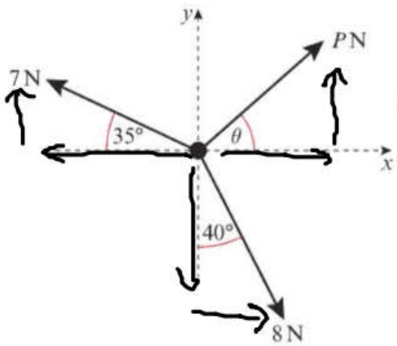
$$R = -3.72\text{ N}$$

Draw a second diagram

a



b



÷

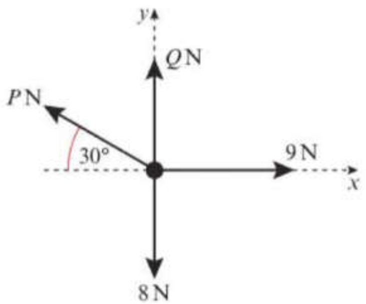
$P\cos\theta =$

$P\sin\theta =$

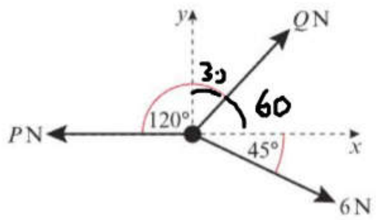
$$\frac{\cancel{P}\sin\theta}{\cancel{P}\cos\theta} = \_\_\_\_\_$$

$\tan\theta =$

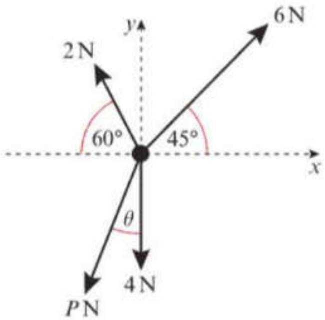
c



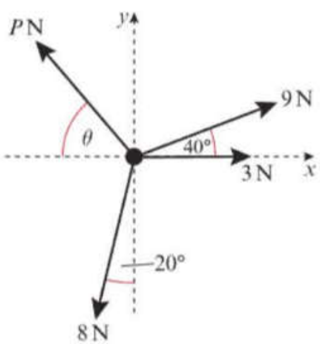
d



e



f



a **i** $Q - 5\cos 30° = 0$ **ii** $P - 5\sin 30° = 0$

iii $Q = 4.33\,\text{N}$ $P = 2.5\,\text{N}$

b **i** $P\cos\theta + 8\sin 40° - 7\cos 35° = 0$

ii $P\sin\theta + 7\sin 35° - 8\cos 40° = 0$

iii $\theta = 74.4°$ (allow $74.3°$) $P = 2.20\,\text{N}$ (allow 2.19)

c **i** $9 - P\cos 30° = 0$

ii $Q + P\sin 30° - 8 = 0$

iii $Q = 2.80\,\text{N}$ $P = 10.4\,\text{N}$

d **i** $Q\cos 60° + 6\cos 45° - P = 0$

ii $Q\sin 60° - 6\sin 45° = 0$

iii $Q = 4.90\,\text{N}$ $P = 6.69\,\text{N}$

e **i** $6\cos 45° - 2\cos 60° - P\sin\theta = 0$

ii $6\sin 45° + 2\sin 60° - P\cos\theta - 4 = 0$

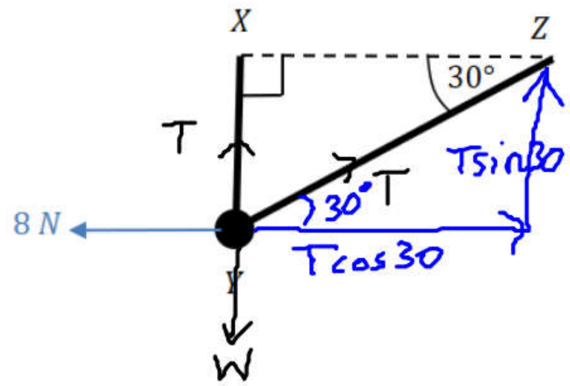
iii $\theta = 58.7°$ $P = 3.80\,\text{N}$

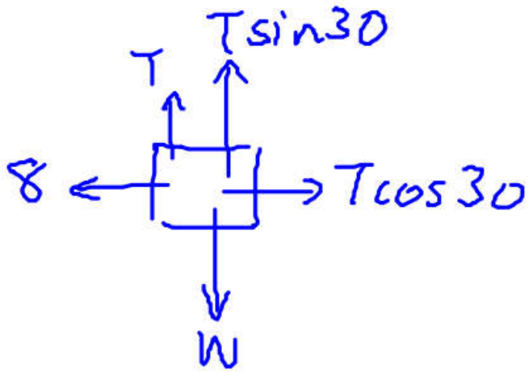
f **i** $9\cos 40° + 3 - P\cos\theta - 8\sin 20° = 0$

ii $P\sin\theta + 9\sin 40° - 8\cos 20° = 0$

iii $\theta = 13.6°$ $P = 7.36\,\text{N}$

A smooth bead Y is threaded on a light inextensible string. The ends of the string are attached to two fixed points, X and Z, on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 8 N acting parallel to ZX. The bead Y is vertically below X and $\angle XZY = 30°$ as shown in the diagram.
Find the tension in the string and the weight of the bead.





As the bead is smooth, the two parts of the string can be considered as a single piece of string, and therefore the tension is the same throughout.

$R(\leftrightarrow)$ $\boxed{T\cos 30 = 8}$

$$T = \frac{8}{\cos 30} = \frac{16\sqrt{3}}{3} = 9.24\text{ N} \quad (3sf)$$

$R(\updownarrow)$ $\boxed{W = T + T\sin 30}$

$$W = 9.24.. + 9.24\sin 30$$
$$= 8\sqrt{3}$$
$$= \underline{\underline{13.9\text{ N}}} \quad (3sf)$$

Edexcel M1(Old) May 2013(R) Q2

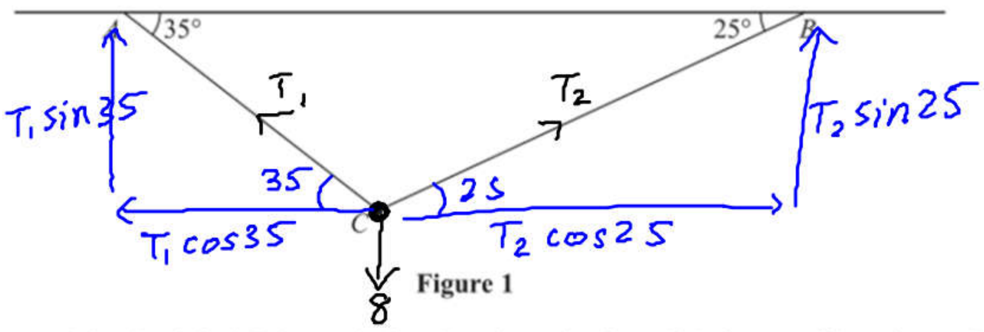


Figure 1

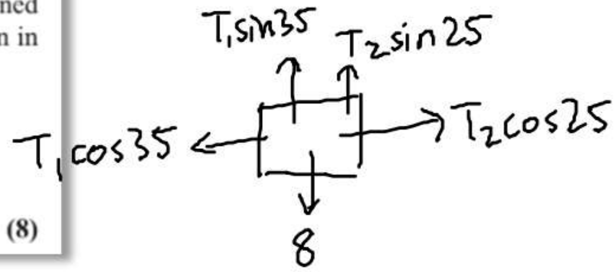
A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

(i) the tension in the string AC,

(ii) the tension in the string BC.

(8)

The particle can't move along the string, so we have two separate strings with separate tensions. Introduce suitable variables for the tensions of each, e.g. T_1 and T_2.



$R(\leftrightarrow)$ $T_1 \cos 35 = T_2 \cos 25$ ✓✓ → $T_1 \cos 35 - T_2 \cos 25 = 0$

$$T_1 = T_2 \frac{\cos 25}{\cos 35}$$

$R(\updownarrow)$ $T_1 \sin 35 + T_2 \sin 25 = 8$ ✓✓✓

$$T_2 \frac{\cos 25}{\cos 35} \times \sin 35 + T_2 \sin 25 = 8$$

$$0.63460..T_2 + 0.4226..T_2 = 8$$

$$1.057 T_2 = 8$$

$$T_2 = 7.5676...$$

$$= \underline{\underline{7.6 \text{ N}}} \ (2sf) \ ✓$$

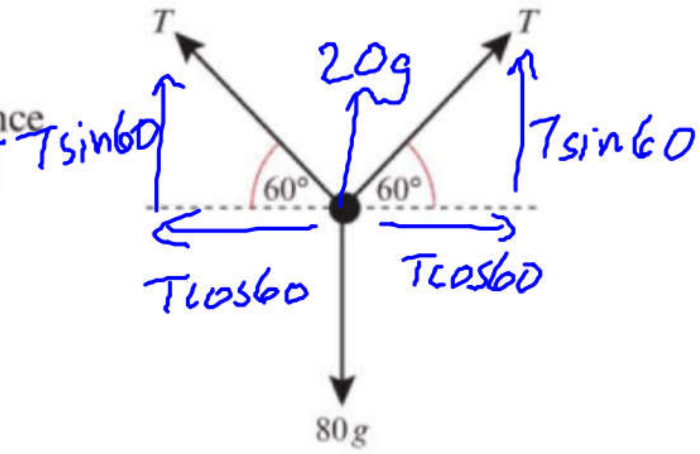
$$T_1 = T_2 \frac{\cos 25}{\cos 35}$$

$$= 7.56... \times \frac{\cos 25}{\cos 35}$$

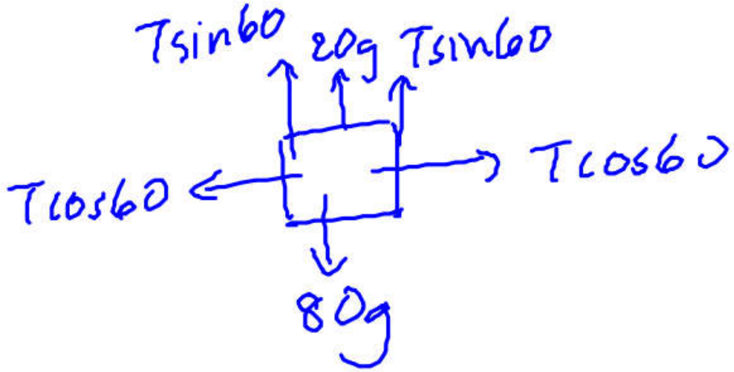
$$= 8.37...$$

$$✓ = \underline{\underline{8.4 \text{ N}}} \ (2sf)$$

8 A parachutist of mass 80 kg is attached to a canopy by two lines, each with tension T. The parachutist is falling with constant velocity, and experiences a resistance to motion due to air resistance equal to one quarter of her weight. Show that the tension in each line, T, is $20\sqrt{3}\,g$ N.



(3 marks)



$$2T\sin 60 + 20g = 80g$$

$$2T\sin 60 = 60g$$

$$T\sin 60 = 30g$$

$$\frac{\sqrt{3}}{2}T = 30g$$

$$T = \frac{60g}{\sqrt{3}}$$

$$T = \underline{\underline{20\sqrt{3}\,g}}$$

Applied Year 2
Ex 7B
Q1-7