Differentiating Vectors

Suppose that $\underset{\sim}{v} = \begin{pmatrix} t^2 \\ \sin t \end{pmatrix}$. What would be the acceleration?

$$\underset{\sim}{a} = \begin{pmatrix} 2t \\ \cos t \end{pmatrix}$$

If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ then $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}} = \dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j}$

and $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d^2\boldsymbol{r}}{dt} = \ddot{\boldsymbol{r}} = \ddot{x}\boldsymbol{i} + \ddot{y}\boldsymbol{j}$

$$\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \underset{\sim}{v} = \dot{\underset{\sim}{r}} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

A particle P of mass 0.8kg is acted on by a single force $\mathbf{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$

Find:

(a) the speed of P when $t = 4$

(b) the acceleration of P as a vector when $t = 2$

(c) $\mathbf{F}$ when $t = 2$.

mag.

$$\underline{r} = \begin{pmatrix} 2t^3 \\ 50t^{-\frac{1}{2}} \end{pmatrix}$$

a) $\underline{v} = \begin{pmatrix} 6t^2 \\ -25t^{-3/2} \end{pmatrix}$

$t = 4$

$$\underline{v} = \begin{pmatrix} 6 \times 16 \\ -25 \times 4^{-3/2} \end{pmatrix} = \begin{pmatrix} 96 \\ -\frac{25}{8} \end{pmatrix}$$

$$v = \sqrt{96^2 + \left(\frac{25}{8}\right)^2} = \underline{96.1 \text{ ms}^{-1}}$$

(3sf)

b) $\underline{a} = \begin{pmatrix} 12t \\ \frac{75}{2}t^{-5/2} \end{pmatrix}$

$t = 2 \qquad \underline{a} = \underline{\begin{pmatrix} 24 \\ 6.63 \end{pmatrix}}$ ms^{-2} (3sf)

c) $\underline{F} = m\underline{a}$

$$= 0.8 \begin{pmatrix} 24 \\ 6.63 \end{pmatrix} = \underline{\begin{pmatrix} 19.2 \\ 5.30 \end{pmatrix} \text{N}}$$

Integrating Vectors

A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by

$$\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}, \qquad t \geq 0$$

When $t = 0$, the position vector of P with respect to a fixed O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector of P at time t seconds.

$$\underline{v} = \begin{pmatrix} 3t \\ \frac{1}{2}t^2 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} \frac{3}{2}t^2 \\ \frac{1}{6}t^3 \end{pmatrix} + \underline{c}$$

$$t = 0 \quad \underline{r} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underline{c}$$

$$\underline{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} \frac{3}{2}t^2 \\ \frac{1}{6}t^3 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} \frac{3}{2}t^2 + 2 \\ \frac{1}{6}t^3 - 3 \end{pmatrix} \text{ m}$$

A particle P is moving in a plane so that, at time t seconds, its acceleration is $(4\boldsymbol{i} - 2t\boldsymbol{j})$ ms^{-2}. When $t = 3$, the velocity of P is $6\boldsymbol{i}$ ms^{-1} and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find:

(a) the angle between the direction of motion of P and $\boldsymbol{i}$ when $t = 2$

(b) the distance of P from O when $t = 0$.

magnitude

v

a)

$$\underline{a} = \begin{pmatrix} 4 \\ -2t \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} 4t \\ -t^2 \end{pmatrix} + \underline{c}$$

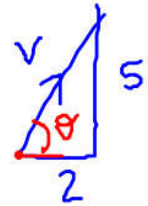
$$t=3 \quad \underline{v} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ -9 \end{pmatrix} + \underline{c}$$

$$\underline{c} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} 4t - 6 \\ -t^2 + 9 \end{pmatrix}$$

$$t=2 \quad \underline{v} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$



$$\theta = \tan^{-1}\left(\frac{5}{2}\right) = \underline{\underline{68.2^\circ}}$$

b)

$$\underline{r} = \begin{pmatrix} 2t^2 - 6t \\ -\frac{1}{3}t^3 + 9t \end{pmatrix} + \underline{d}$$

$$t=3, \ \underline{r} = \begin{pmatrix} 20 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 20 \\ 3 \end{pmatrix} = \begin{pmatrix} 18 - 18 \\ -9 + 27 \end{pmatrix} + \underline{d}$$

$$\begin{pmatrix} 20 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 18 \end{pmatrix} + \underline{d}$$

$$\underline{d} = \begin{pmatrix} 20 \\ -15 \end{pmatrix}$$

when $t=0$

$$\underline{r} = \begin{pmatrix} 20 \\ -15 \end{pmatrix} \quad \text{dist} = \sqrt{20^2 + 15^2}$$

$$= \underline{\underline{25\text{m}}}$$

Ex 8E Evens

Edexcel M2(Old) Jan 2013 Q4

At time t seconds the velocity of a particle P is $[(4t-5)\mathbf{i}+3\mathbf{j}]$ m s^{-1}. When $t=0$, the position vector of P is $(2\mathbf{i}+5\mathbf{j})$ m, relative to a fixed origin O.

(*a*) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(*b*) Find an expression for the position vector of P at time t seconds.

(4)

A second particle Q moves with constant velocity $(-2\mathbf{i}+c\mathbf{j})$ m s^{-1}. When $t=0$, the position vector of Q is $(11\mathbf{i}+2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i}+14\mathbf{j})$ m.

(*c*) Find

(i) the value of c,

(ii) the value of d.

(a)	$t=\dfrac{5}{4}$	M1
(b)	$\mathbf{r}=\left(2t^2-5t\right)\mathbf{i}+3t\mathbf{j}(+\mathbf{c})$	
		A1
		DM1
	$t=0\quad 2\mathbf{i}+5\mathbf{j}=\mathbf{c}$	A1
	$\mathbf{r}=\left(2t^2-5t\right)\mathbf{i}+3t\mathbf{j}+\left(2\mathbf{i}+5\mathbf{j}\right)$	
	$(2t^2-5t+2)\mathbf{i}+(3t+5)\mathbf{j}$	B1

(c)	$\mathbf{r}_Q=11\mathbf{i}+2\mathbf{j}-2t\mathbf{i}+ct\mathbf{j}$		
	$(11-2t)\mathbf{i}+(2+ct)\mathbf{j}$		
	$\mathbf{r}_P=\left(2t^2-5t+2\right)\mathbf{i}+\left(3t+5\right)\mathbf{j}$		
	$\mathbf{r}_Q=\mathbf{r}_P=d\mathbf{i}+14\mathbf{j}$		$2t^2-5t$
	$3t+5=14$	$2t^2-3t-9$ $(2t+3)(t-3)=0$ $t=3$	M1
	$t=3$	A1 ft	A1
	$2+ct=14\Rightarrow c=4$		A1 ft
	$d=11-2\times3=5$ or		
	$d=2\times3^2-5\times3+2\Rightarrow d=5$		
	Alt: $2t^2-5t+2=11-2t=d\Rightarrow t=\dfrac{11-d}{2}$		
	$2\left(\dfrac{11-d}{2}\right)^2-5\left(\dfrac{11-d}{2}\right)+2=d,$		
	$d^2-19d+70=0=(d-5)(d-14)$		

Exam Questions

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

Question	Scheme	Marks	AOs
6	Integrate $\mathbf{a}$ w.r.t. time	M1	1.1a
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of $\mathbf{C}$)	A1	1.1b
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\,\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)

7. A particle, P, moves under the action of a single force in such a way that at time t seconds, where $t \geq 0$, its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = (t^2 - 3t)\,\mathbf{i} - 12t\,\mathbf{j}$$

The mass of P is 0.5 kg.

Find the time at which the magnitude of the force acting on P is 6.5 N.

(7)

$$\underline{v} = \begin{pmatrix} t^2 - 3t \\ -12t \end{pmatrix}$$

$$\underline{a} = \begin{pmatrix} 2t - 3 \\ -12 \end{pmatrix}$$

$$a = \sqrt{(2t-3)^2 + 12^2}$$

$$\underline{F} = m\underline{a}$$

$$F = ma$$

$$6.5 = 0.5\left(\sqrt{(2t-3)^2 + 12^2}\right)$$

$$13 = \sqrt{(2t-3)^2 + 12^2}$$

$$169 = (2t-3)^2 + 144$$

$$25 = (2t-3)^2$$

$$\pm 5 = 2t - 3$$

$$3 \pm 5 = 2t$$

$$t = \frac{3 \pm 5}{2}$$

$$\underline{\underline{t = 4}} \text{ or } t = \cancel{-1}$$

3. [*In this question position vectors are given relative to a fixed origin O*]

A particle P moves under the action of a single force $\mathbf{F}$ newtons.

At time t seconds, where $t \geqslant 0$, the position vector of P, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = (t^3 - 5t)\mathbf{i} + (5t^2 + 6t)\mathbf{j}$$

The mass of P is 0.5 kg.

At time T seconds, P is moving in the direction of the vector $(\mathbf{i} + 2\mathbf{j})$.

(a) Find the value of T.

(5)

(b) Find the magnitude of $\mathbf{F}$ when $t = 2$

(4)

Question	Scheme	Marks	AOs
3(a)	$\mathbf{v} = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{r})$	M1	1.1b
	$\mathbf{v} = (3t^2 - 5)\mathbf{i} + (10t + 6)\mathbf{j}$	A1	1.1b
	Parallel to $(\mathbf{i} + 2\mathbf{j}) \Rightarrow (10T + 6) = 2(3T^2 - 5)$	M1	3.1a
	$6T^2 - 10T - 16 = 0$	A1	1.1b
	$T = \frac{8}{3}$	A1	2.2a
		(5)	
(b)	$\mathbf{a} = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{v})$. $(\mathbf{a} = 6t\mathbf{i} + 10\mathbf{j})$	M1	1.1b
	$\mathbf{F} = 0.5(12\mathbf{i} + 10\mathbf{j})(= 6\mathbf{i} + 5\mathbf{j})$	M1	2.1
	$\lvert\mathbf{F}\rvert = \sqrt{6^2 + 5^2}$	M1	1.1b
	$= \sqrt{61}\,(= 7.8(1....))$	A1	1.1b
		(4)	
		(9 marks)	