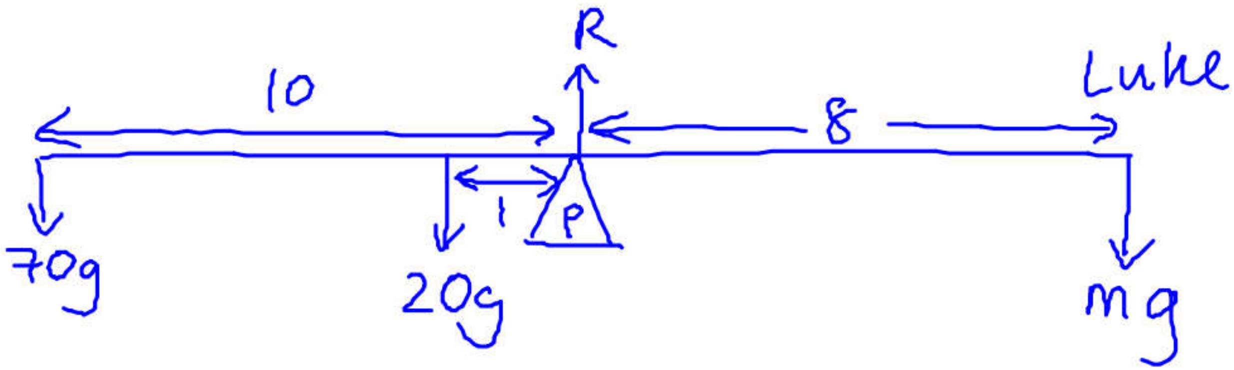
Rayhan and Luke are having fun on a **uniform** seesaw of mass 20kg. Rayhan weighs 70kg and is 10m from the pivot. Luke is 8m from the pivot. The seesaw remains horizontal.

a) Determine the reaction force at the pivot of the seesaw.

b) Determine Luke's mass.



$R \updownarrow \quad R = 70g + 20g + mg$

a) $M(L) \quad 70g \times 18 + 20g \times 9 = 8R$

$R = \underline{\underline{180g}}$

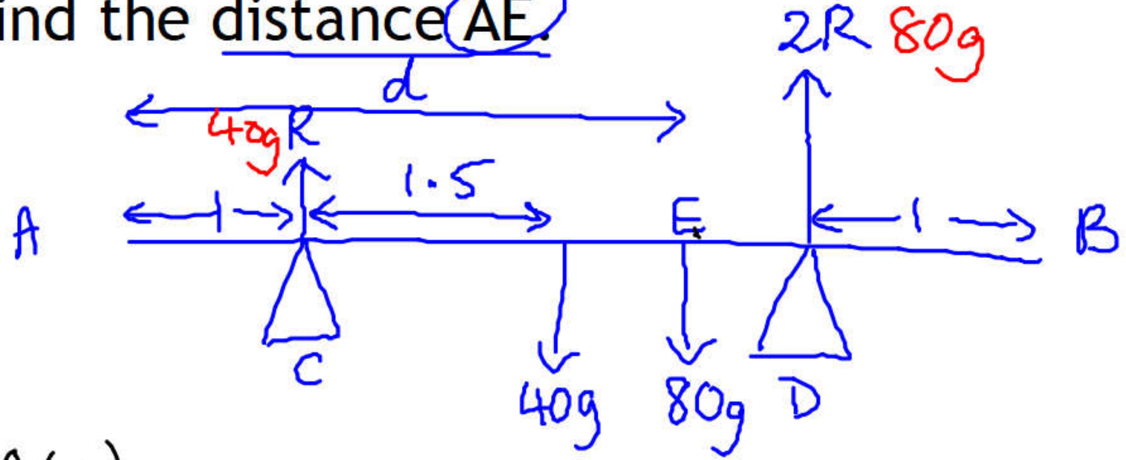
b) $180g = 90g + mg$

$\underline{\underline{m = 90}}$

Weights and reactions
Moments
Resolving vertically

e.g.

A uniform beam AB, of mass 40kg and length 5m, rests horizontally on supports at C and D, where AC=DB=1m.
When a man of 80kg stands on the beam at E the magnitude of the reaction at D is twice the magnitude of the reaction at C.
Find the distance AE.



$R \updownarrow$

$$R + 2R = 40g + 80g$$
$$3R = 120g$$
$$R = 40g$$

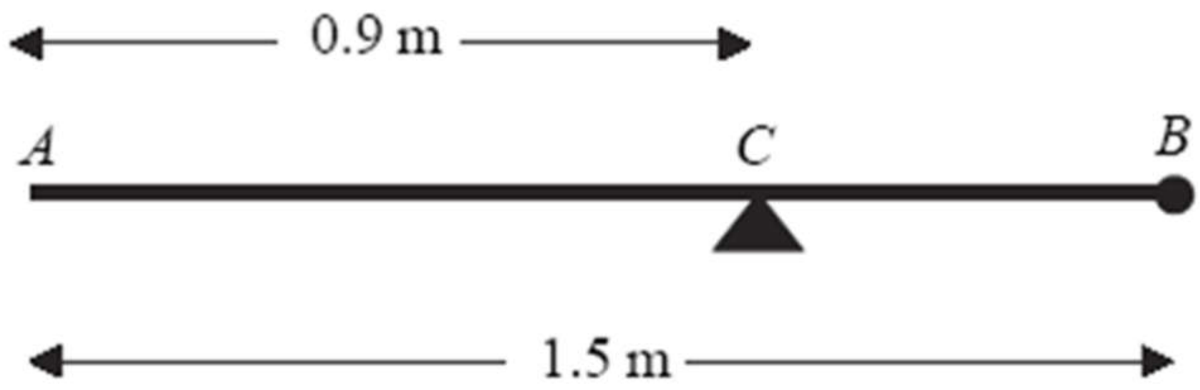
M(A)

$$1 \times R + 4 \times 2R = 40g \times 2.5 + 80g\,d$$
$$9R = 100g + 80gd$$
$$360\cancel{g} = 100\cancel{g} + 80\cancel{g}d$$
$$\frac{260}{80} = d \qquad d = \underline{3.25\text{m}}$$

Weights and reactions
Moments
Resolving vertically

June 2007

Your Turn



A uniform rod AB has length 1.5 m and mass 8 kg. A particle of mass m kg is attached to the rod at B. The rod is supported at the point C, where $AC = 0.9$ m, and the system is in equilibrium with AB horizontal, as shown in Figure 2.

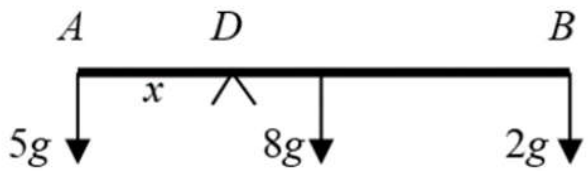
(*a*) Show that $m = 2$.

(4)

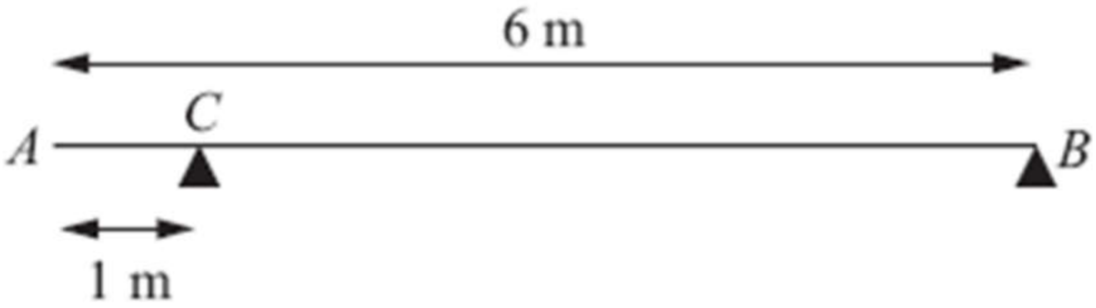
A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

(*b*) Find the distance AD.

(5)

3.	(a) M(C) $8g\times(0.9-0.75)=mg(1.5-0.9)$		M1 A1	
	Solving to $m=2$ ✱	cso	DM1 A1	**(4)**
	(b)			
				
	M(D) $5g\times x=8g\times(0.75-x)+2g(1.5-x)$		M1 A2(1, 0)	
	Solving to $x=0.6$ ($AD=0.6$ m)		DM1 A1	**(5)**
				[9]

Your Turn



A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in Figure 1. The beam is modelled as a rod.

(*a*) Find the magnitudes of the reactions on the beam at B and at C.

(5)

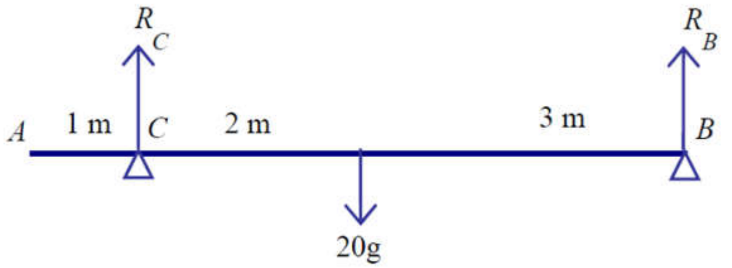
A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

(*b*) Find the distance AD.

(5)

3.

(a)



Taking moments about B: $5 \times R_C = 20g \times 3$ — M1A1

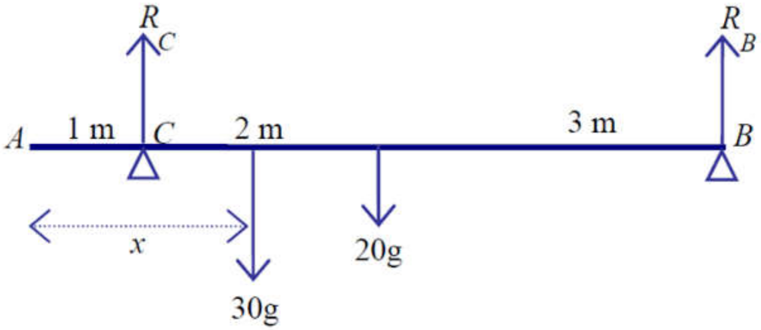
$R_C = 12g$ or $60g/5$ or 118 or 120 — A1

Resolving vertically: $R_C + R_B = 20g$ — M1

$R_B = 8g$ or 78.4 or 78 — A1

(5)

(b)



Resolving vertically: $50g = R + R$ — B1

Taking moments about B:

$$5 \times 25g = 3 \times 20g + (6 - x) \times 30g$$ — M1 A1 A1

$$30x = 115$$

$x = 3.8$ or better or 23/6 oe — A1

(5)

[10]

Your Turn

A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

(*a*) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

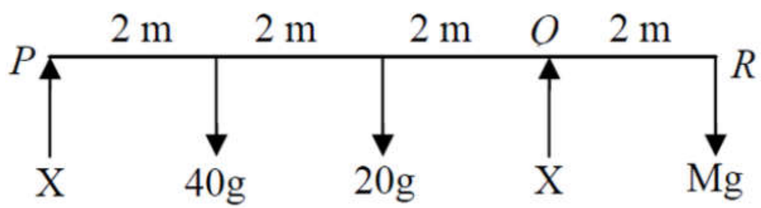
(10)

(*b*) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.

(1)

5.

(a)



(i)	**EITHER**	M(R), $8X + 2X = 40\text{g} \times 6 + 20\text{g} \times 4$	M1 A2
		solving for X, $X = 32\text{g} = 314$ or 310 N	M1 A1
(ii)		($\uparrow$) $X + X = 40\text{g} + 20\text{g} + M\text{g}$ (or another moments equation)	M1 A2
		solving for M, $M = 4$	M1 A1
(i)	**OR**	M(P), $6X = 40\text{g} \times 2 + 20\text{g} \times 4 + M\text{g} \times 8$	M1 A2
		solving for X, $X = 32\text{g} = 314$ or 310 N	M1 A1
		($\uparrow$) $X + X = 40\text{g} + 20\text{g} + M\text{g}$ (or another moments equation)	M1 A2
(ii)		solving for M, $M = 4$	M1 A1
			(10)

(b)

Masses concentrated at a point or weights act at a point	B1
	(1)

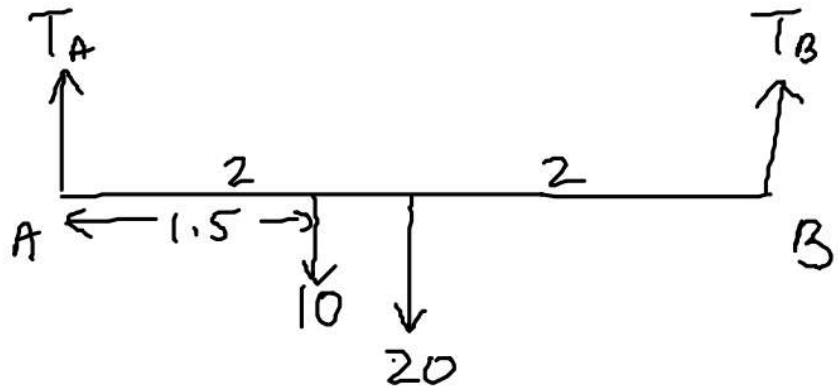
Your Turn

A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D, where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B.

(7)

Hanging Rods/Beams

A uniform rod AB of length 4 m and weight 20 N is suspended horizontally by two vertical strings attached at A and at B. A particle of weight 10 N is attached to the rod at point C, where $AC = 1.5$ m. Find the magnitudes of the tensions in the two strings.



m(A) $\quad 1.5 \times 10 + 2 \times 20 = 4T_B$

$15 + 40 = 4T_B$

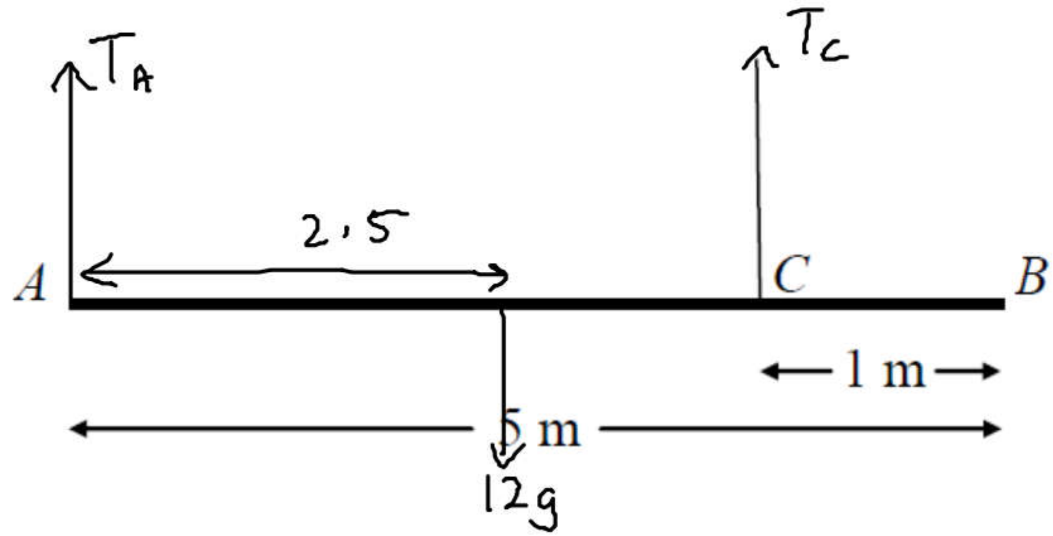
$\frac{55}{4} = T_B$

$T_B = 13.75 = \underline{13.8\text{N}}$ (3sf)

$T_A + T_B = 10 + 20$

$T_A = 30 - 13.75$

$= 16.25 \quad \underline{16.3\text{N}}$ (3sf).



A beam AB has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A, the other to the point C on the beam, where $BC = 1$ m, as shown in Figure 2. The beam is modelled as a uniform rod, and the ropes as light strings.

(a) Find

(i) the tension in the rope at C,

(ii) the tension in the rope at A.

(5)

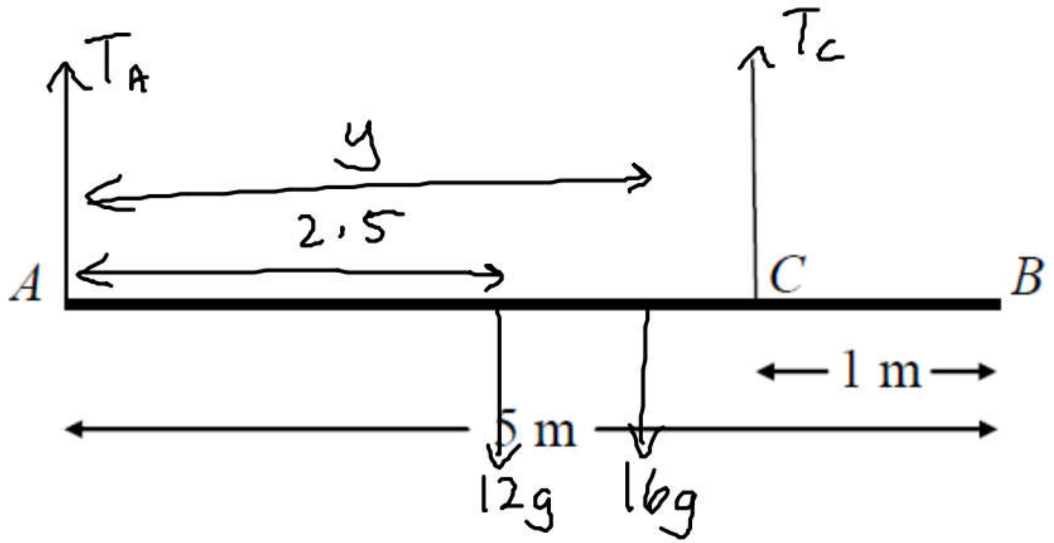
i) M(A) $\quad 2.5 \times 12g = 4 \times T_c$

$T_c = \underline{\underline{7.5g}}$

ii) R $\updownarrow$

$T_A + T_c = 12g$

$T_A = \underline{\underline{4.5g}}$



A small load of mass 16 kg is attached to the beam at a point which is y metres from A. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(*b*) find, in terms of y, an expression for the tension in the rope at C.

$M(A)$ $\quad 2.5 \times 12g + 16gy = 4T_C \qquad T_C = 7.5g + 4gy$ **(3)**

The rope at C will break if its tension exceeds 98 N. The rope at A cannot break.

(*c*) Find the range of possible positions on the beam where the load can be attached without the rope at C breaking.

(3)

$T_C \leq 98$

$7.5g + 4gy \leq 98$

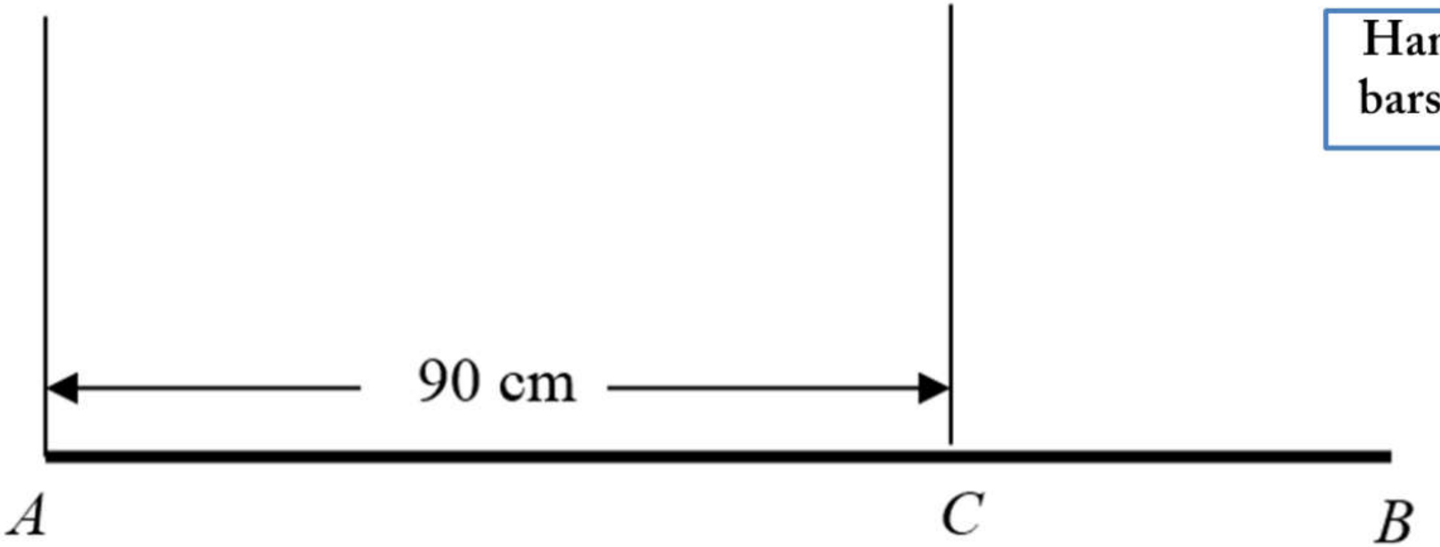
$y \leq \frac{5}{8}$

$y \leq \underline{0.63 \text{ m}}$ (2sf)

June 2006

Your Turn

Hanging bars/rods



A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other able is attached to the point C on the girder, where $AC = 90$ cm, as shown in Figure 3. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(a) the tension in the cable at A,

(2)

(b) show that $AB = 120$ cm.

(4)

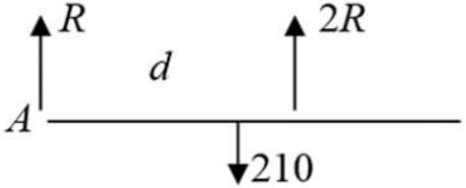
A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

(c) Find the value of W.

(7)

5

(a)



$R + 2R = 210 \Rightarrow R = \underline{70\text{ N}}$ M1 A1

(2)

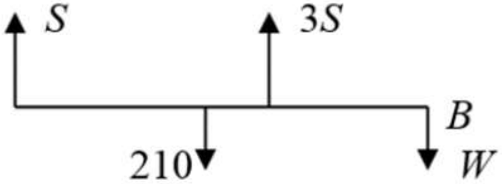
(b) e.g. M(A): $140 \times 90 = 210 \times d$ M1 A1√

↓

$\Rightarrow d = 60 \Rightarrow AB = \underline{120\text{ cm}}$ M1 A1

(4)

(c)



$4S = 210 + W$ M1 A1

e.g. M(B): $S \times 120 + 3S \times 30 = 210 \times 60$ M1 A2,1,0

↓ ↓

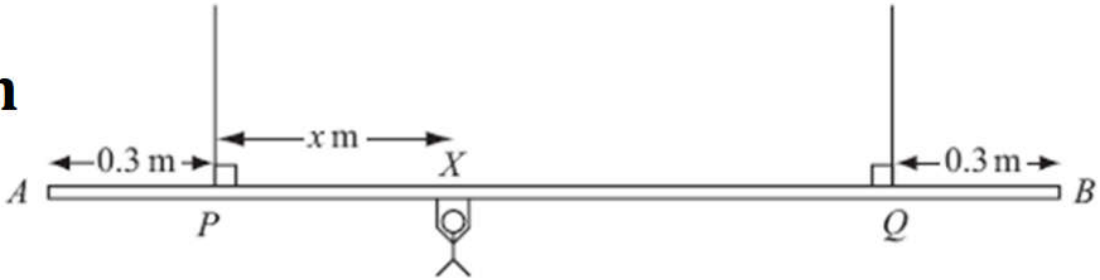
Solve $\to$ ($S = 60$ and) $W = \underline{30}$ M1 A1

(7)

May 2009

Your Turn

Hanging bars/rods



A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(*a*) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(*b*) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(*c*) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(*d*) find the value of x.

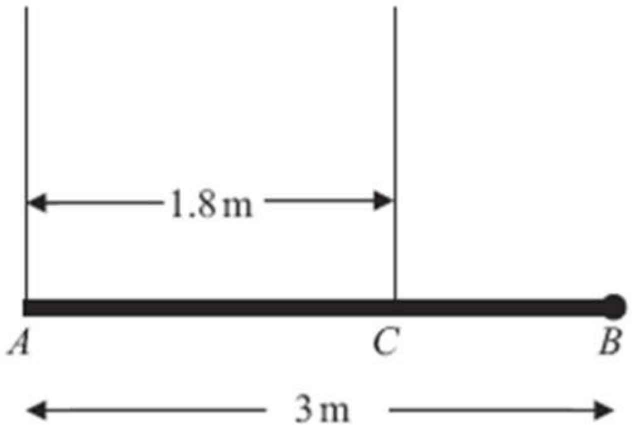
(3)

7. (a)	$M(Q),\quad 50g(1.4-x)+20g \text{ x } 0.7 = T_P \text{ x } 1.4$	M1 A1	
	$T_P = 588 - 350x$ Printed answer	A1	(3)
(b)	$M(P),\quad 50gx + 20g \text{ x } 0.7 = T_Q \text{ x } 1.4$ or R(↑), $T_P + T_Q = 70g$	M1 A1	
	$T_Q = 98 + 350x$	A1	(3)
(c)	Since $0 < x < 1.4$,	M1	
	$98 < T_P < 588$ and $98 < T_Q < 588$	A1 A1	(3)
(d)	$98 + 350x = 3\,(588 - 350x)$	M1	
	$x = 1.19$	M1 A1	(3)

Jan 2010

Your Turn

Hanging bars/rods



A pole AB has length 3 m and weight W newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points A and C where $AC = 1.8$ m, as shown in Figure 2. A load of weight 20 N is attached to the rod at B. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(*a*) Show that the tension in the rope attached to the pole at C is $\left(\frac{5}{6}W + \frac{100}{3}\right)$ N.

(4)

(*b*) Find, in terms of W, the tension in the rope attached to the pole at A.

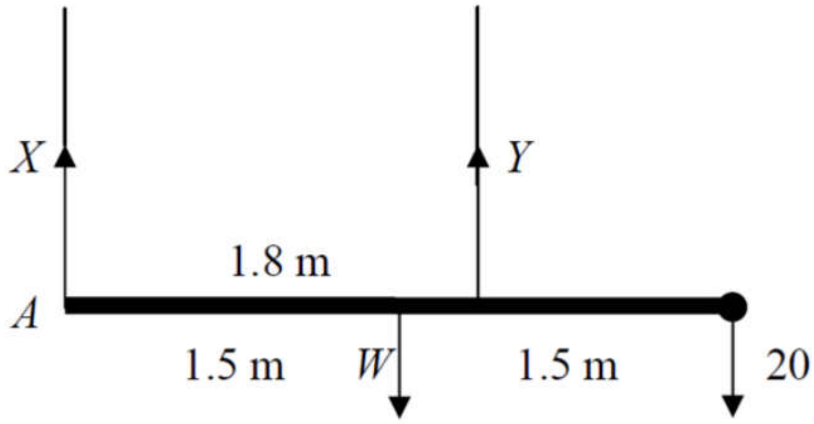
(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(*c*) find the value of W.

(3)

Q4. (a)



M (A)	$W \times 1.5 + 20 \times 3 = Y \times 1.8$		M1 A2 (1, 0)
	$Y = \frac{5}{6}W + \frac{100}{3}$ *	cso	A1 (4)
(b) $\uparrow$	$X + Y = W + 20$	or equivalent	M1 A1
	$X = \frac{1}{6}W - \frac{40}{3}$		A1 (3)
(c)	$\frac{5}{6}W + \frac{100}{3} = 8\left(\frac{1}{6}W - \frac{40}{3}\right)$		M1 A1 ft
	$W = 280$		A1 (3)
			[10]

Alternative to (b)

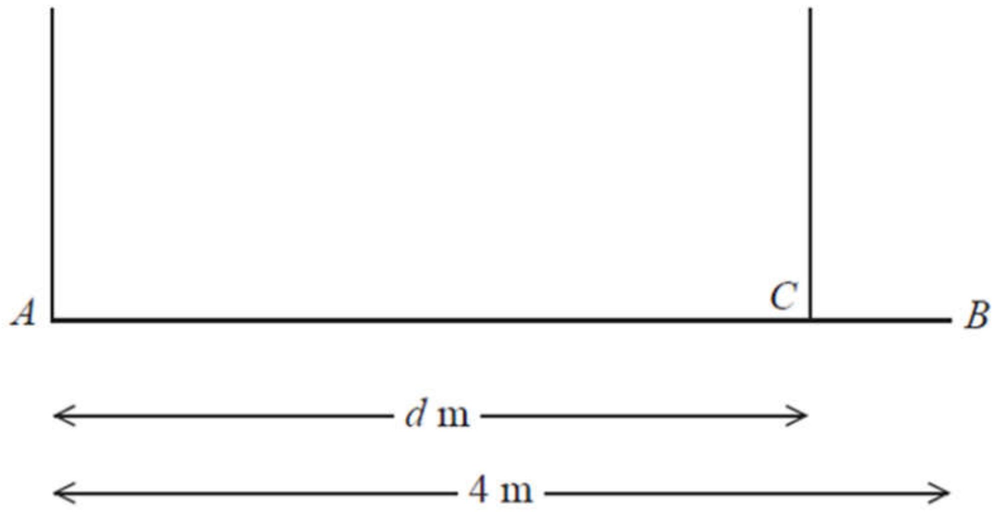
M(C) $X \times 1.8 + 20 \times 1.2 = W \times 0.3$ — M1 A1

$X = \frac{1}{6}W - \frac{40}{3}$ — A1

June 2014

Your Turn

Hanging bars/rods



A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(*a*) Find the value of d.

(6)

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(*b*) Find the value of k.

(6)

4a	Resolving vertically: $T + 2T\,(=3T) = W$	M1A1
	Moments about A: $2W = 2T \times d$	M1A1
	Substitute and solve: $2W = 2\dfrac{W}{3}d$	**DM1**
	$d = 3$	A1
		(6)
b	Resolving vertically: $T + 4T = W + kW \quad \left(5T = W(1+k)\right)$	M1A1 **ft**
	Moments about A: $2W + 4kW = 3 \times 4T$	M1A1 **ft**
	Substitute and solve: $2W + 4kW = \dfrac{12}{5}W(1+k)$	**DM1**
	$2 + 4k = \dfrac{12}{5} + \dfrac{12}{5}k$	
	$\dfrac{8}{5}k = \dfrac{2}{5}, \qquad k = \dfrac{1}{4}$	A1
		(6)
		[12]