Basic Circuit Principles

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Basic Concepts

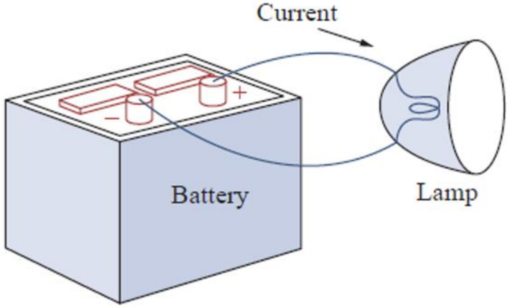
I. Charge and Current

II. Voltage

III. Power and Energy

IV. Circuit Element

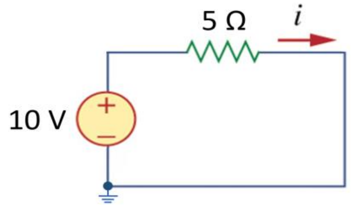
Motivation



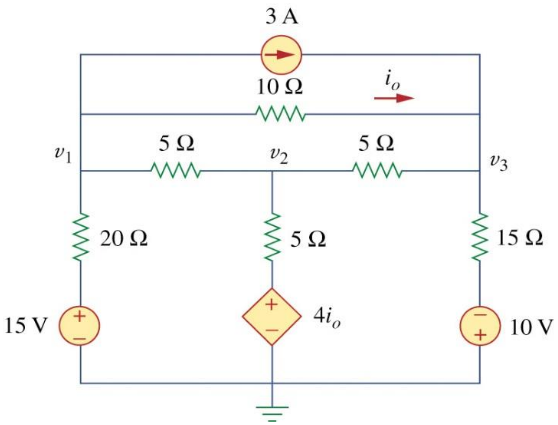
- An electric circuit is an interconnection of electrical elements.
- What is circuit analysis ?

  To predict how a circuit behaves without implementing it
- For a simple circuit, it is trivial

  I = 10 V / 5 Ω = 2 A



- How about this ?



International Systems of Units (SI)

- Universal language of measurement
- The metric system is often referred to as the SI unit which is based on the number 10 and multiples of 10.

Example: 100,000,000 mm 100,000 m 100 km

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

I. Charge and Current

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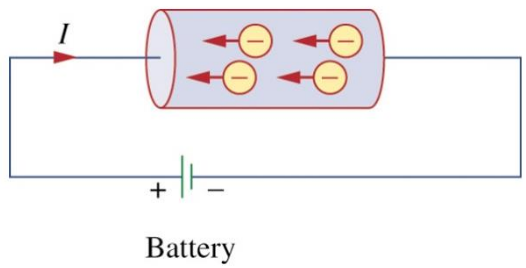
Charge and Current

- Electrons have a negative charge (Q) measured in coulombs (C).
  - charge on a proton is $+1.602 \times 10^{-19}$ C
  - charge on an electron is -1.602×10^{-19} C

What is current?

- Net movement of electrons carrying negative charges.
- By convention, the direction of electric current is the direction in which a positive charge would move from positive to negative charged terminal in a closed circuit.
- Current and electrons move in opposite directions.



Charge and Current

- Electric current is the amount of charge flowing per second (rate of change of charge) and is given the unit ampere (A).
- Mathematically, it is given by

$$i(t) = \frac{dq(t)}{dt}$$

Charge transferred in coulombs (C)

Current in amperes (A)

Time duration in seconds (s)

- A current of 1 A means 1 C of charge moving across a fixed surface in 1 s.
- The charge transferred between time t_0 and t_1 is given by the inverse relationship:

$$Q = \int_{t_0}^{t_1} i(t)dt$$

Example (1)

(1) The total charge entering a terminal is given by:

$$q(t) = 5t\sin\left(4\pi t\right)\ \text{mC}$$

Calculate the current at $t = 0.5$ s.

By applying the Product Rule, we have

$$\frac{d}{dx}\left(u\cdot v\right) = u\cdot\frac{dv}{dx} + v\cdot\frac{du}{dx}$$

Ans:

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}\left[5t\sin\left(4\pi t\right)\right] = 5\sin\left(4\pi t\right) + 20\pi t\cos\left(4\pi t\right)$$

$$i(0.5) = 5\sin\left(2\pi\right) + 10\pi\cos\left(2\pi\right) \approx 31.42\ \text{mA}$$

Example (2)

(1) The charge profile entering a certain element across time is shown in Fig. 1. Find the current at (a) t = 1 ms, (b) t = 6 ms, and (c) t = 10 ms.
(Hint: Current is the *slope* of the curve.)

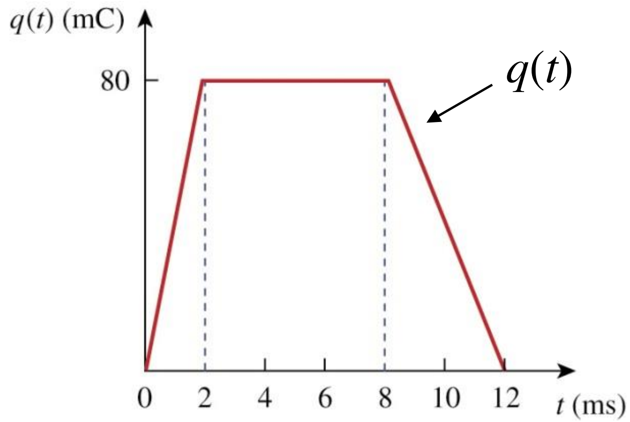


Fig. 1

Ans: (a) i(1 ms) = 80/2 = 40 A
(b) i(6 ms) = 0 A
(c) i(10 ms) = -80/4 = -20 A

(2) Determine the total charge entering a terminal between t = 1 s and t = 2 s if the current across the terminal is given by: $i(t) = (6t^2 - t)$ A.

Ans: $$Q = \int_1^2 i(t)dt = \int_1^2 \left(6t^2 - t\right)dt = \left(2t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (16-2) - \left(2 - \frac{1}{2}\right) = 12.5 \text{ C}$$

I. Charge and Current

II. Voltage

III. Power and Energy

IV. Circuit Element

Voltage

- Electric current does not flow on its own (free electrons in conductors are in random motions with zero net flow).
- To move an electron from point *a* to point *b*, an external electromotive force (EMF), e.g. battery, is needed.
- The strength of the EMF is expressed in **voltage** and is measured in **volts** (**V**).
- The voltage v_{ab} between two points *a* and *b* is the energy required to move a unit charge from *a* to *b*. Mathematically,

$$v_{ab} = \frac{dw}{dq}$$

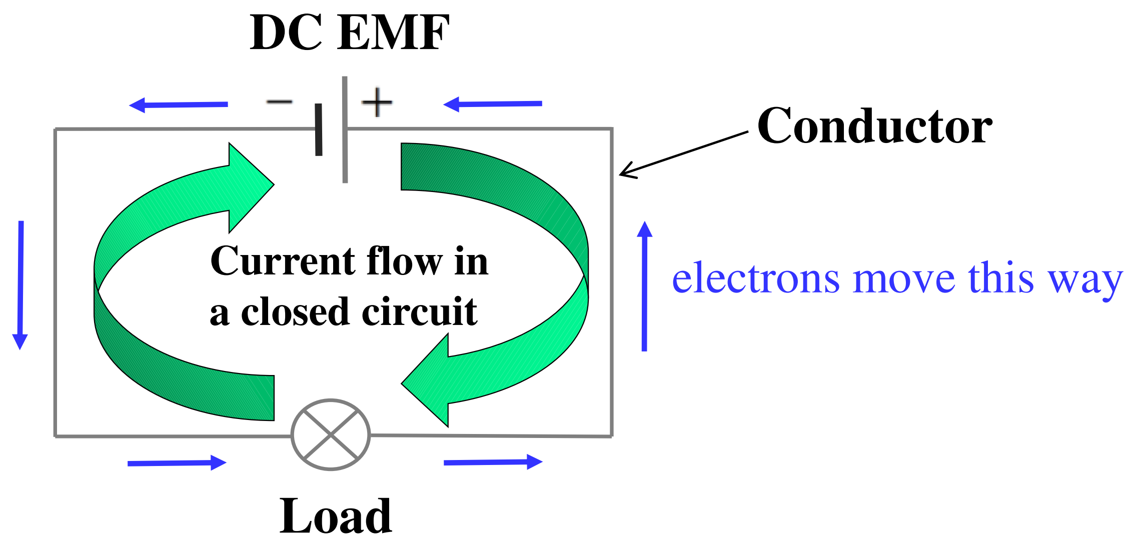
Energy in joules (J)

Charge transferred in coulombs (C)

Voltage in volts (V)

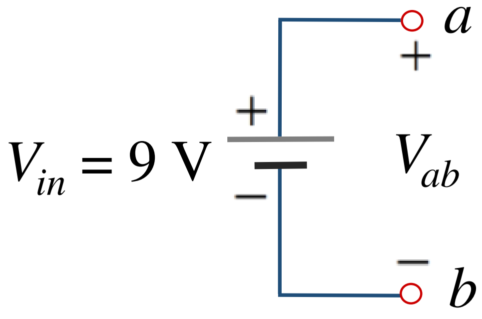
- 1 volt = 1 joule / coulomb (1 V = 1 J C^{-1}).

Voltage

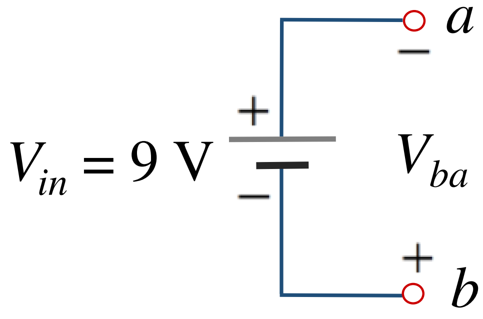


- If the voltage of the DC EMF is maintained between two points of a circuit, ***current will flow in one direction*** – from high (+) to low (–) potential.
- This is called a ***direct current*** (***DC***) ***circuit***.
- Battery-powered circuits are DC circuits.

Voltage



(a)



(b)

Without loss of generality, assume point b is connected to ground. So, $V_b = 0$ V.

Case (a): $V_{in} = V_{ab} = V_a - V_b = 9\text{ V} - 0\text{ V} = +9\text{ V}$

Case (b): $V_{in} = -V_{ba} = -(V_b - V_a) = -(0\text{ V} - 9\text{ V}) = +9\text{ V}$

Hence, case (a) and (b) are equivalent, i.e., $V_{ab} = -V_{ba}$.

Voltage (or potential difference), measured in volts (V), is defined as the energy required to move a unit charge through an element.

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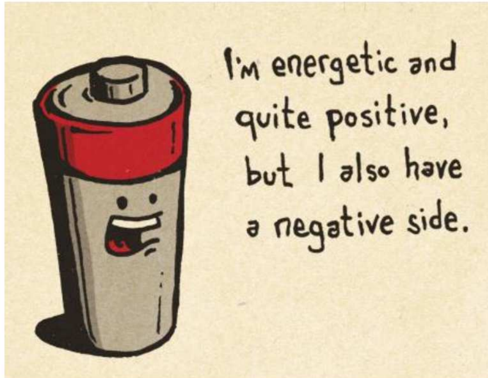
Power and Energy



- Power is the rate (w.r.t. time) of expending or absorbing energy, measured in **watts** (**W**).
- Mathematically,

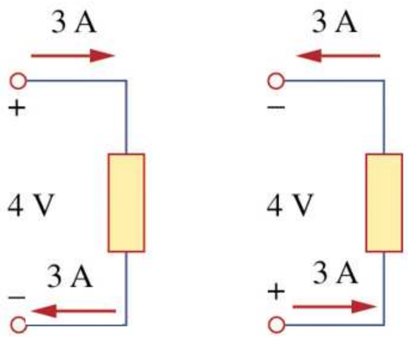
$$p(t) = \frac{dw(t)}{dt} \Rightarrow p(t) = \overbrace{\frac{dw(t)}{dq(t)}}^{v(t)} \overbrace{\frac{dq(t)}{dt}}^{i(t)} = v(t)i(t)$$

- The sign of power across a circuit element depends on the current direction and voltage difference across that element.

- In DC circuits, since $p(t)$, $v(t)$ and $i(t)$ are constants, $p(t) = P = VI$.

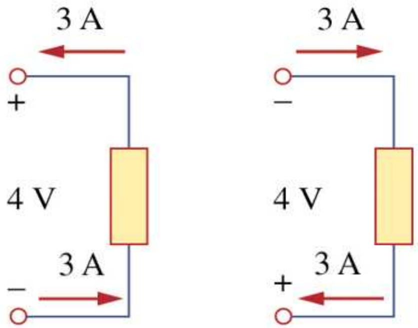
Power and Energy

- The element absorbs (consumes) a power of +12 W.



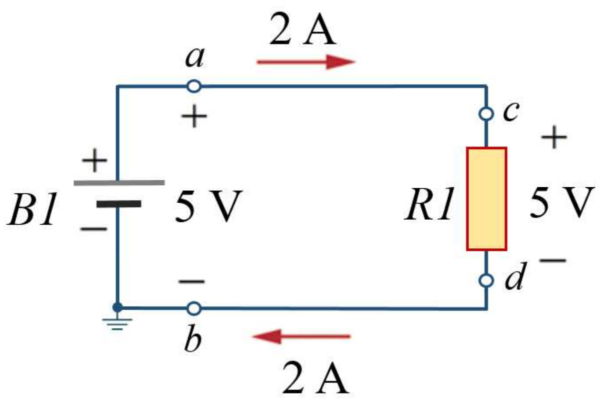
The 3 A current flows into the **positive** terminal of the element, i.e., it goes from a higher potential to lower potential.

- The element absorbs a power of –12 W (or supplies a power of +12 W).



The 3 A current flows into the **negative** terminal of the element, i.e., it goes from a lower potential to a higher potential.

Power and Energy



- Based on the current direction, the voltage across the battery *B1* (i.e., from point *b* to point *a*):
  $V_{ba} = V_b - V_a = 0\text{ V} - 5\text{ V} = -5\text{ V}$
- Likewise, the voltage across the load *R1* (i.e., from point *c* to point *d*):
  $V_{cd} = V_c - V_d = 5\text{ V} - 0\text{ V} = 5\text{ V}$ (Note: $V_{cd} = -V_{ba} = 5\text{ V}$)
- Power absorbed by *B1*: $P_{B1} = V_{ba} I = -5\text{ V} \times 2\text{ A} = -10\text{ W}$
  (or equivalently, *B1* supplies a power of +10 W.)
- Power absorbed by *R1*: $P_{R1} = V_{cd} I = 5\text{ V} \times 2\text{ A} = 10\text{ W}$

Power and Energy

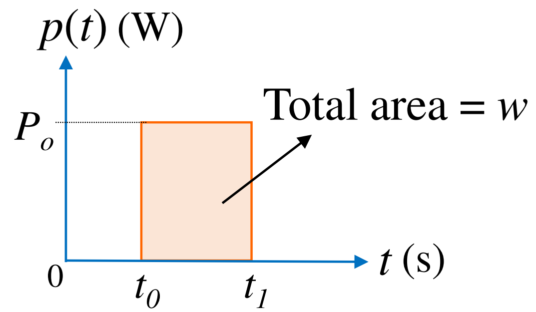
Recall: $p(t) = \dfrac{dw(t)}{dt}$

Energy absorbed (or supplied) by a circuit element from time t_0 to t_1:

$$w = \int_{t_o}^{t_1} p(t)dt = \int_{t_o}^{t_1} v(t)i(t)dt$$

In particular, when $p(t)$ is a constant (i.e., $p(t) = P_o$), we have

$$w = P_o\left(t_1 - t_0\right)$$



Example (3)

(1) Find the power delivered to an electronic load at t = 10 ms if the current entering its positive terminal is $i(t) = 5\cos(2\pi t)$ A and the voltage is: a) $v = 3i$; b) $v = 3di/dt$.

Ans: (a) $p(t) = v(t)i(t) = 3[i(t)]^2 = 75\cos^2 2\pi t$ W

$\Rightarrow p(0.01) = 75\cos^2(0.02\pi) \approx 74.7$ W

(b) $v(t) = 3di/dt = -30\pi[\sin(2\pi t)]$

$\Rightarrow p(t) = v(t)i(t) = -150\pi[\sin(2\pi t)][\cos(2\pi t)]$ W

$\Rightarrow p(0.01) = -150\pi[\sin(0.02\pi)][\cos(0.02\pi)] = -29.53$ W

(2) How much energy does a 100 W LED light bulb consume in two hours ?

Ans: Since p is a constant (i.e., $p(t) = P_{LED} = 100$ W), $w = P_{LED}\, t = 100 \times 2 \times 3600 = 720$ kJ

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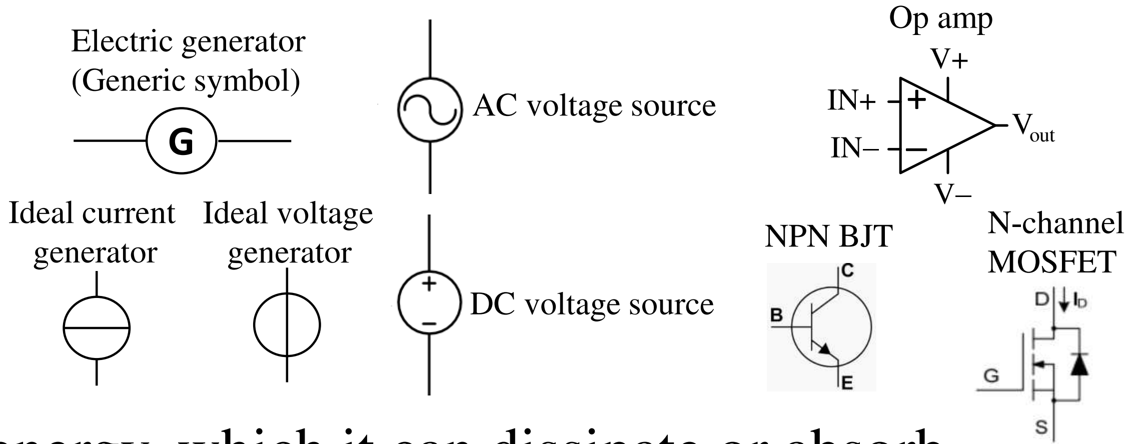
Circuit Element

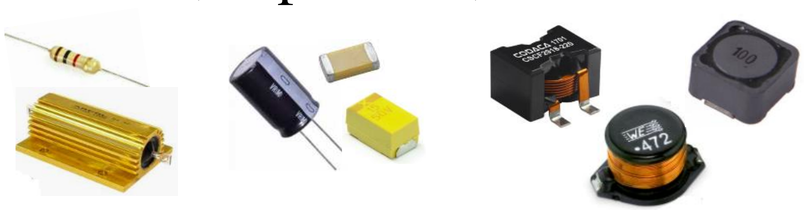
- **Active elements** are capable of generating or supplying energy to a circuit.

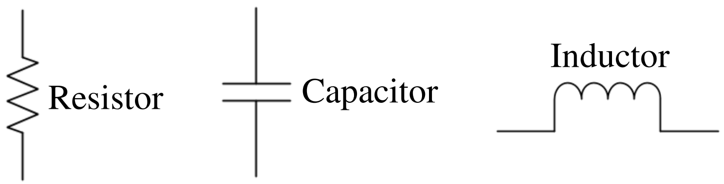
  e.g. generator, battery, operational amplifier (op amp), and transistors.



- **Passive elements** can only receive energy, which it can dissipate or absorb.

  e.g. resistors, capacitors, and inductors.





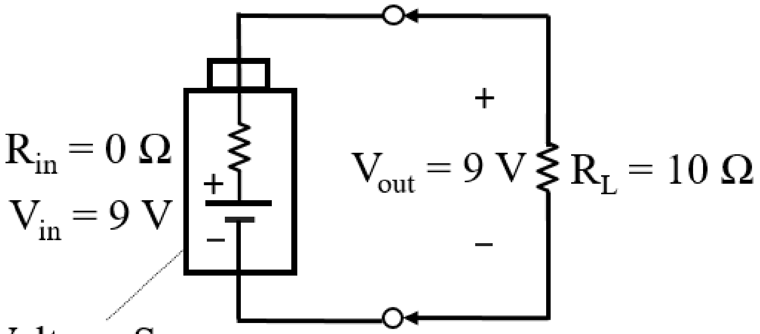
Voltage Source & Current Source

There are two important active elements: (a) voltage source and (b) current source.

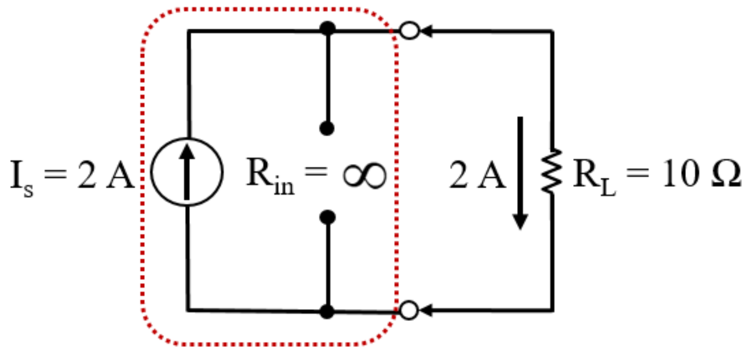
Definition:

(i) **Ideal voltage source** is a voltage source that supplies constant voltage to a circuit despite the current which the circuit draws. It has zero internal resistance.

(ii) **Ideal current source** is a current source that supplies constant current to a circuit despite any other conditions present in the circuit. It has infinite internal resistance.



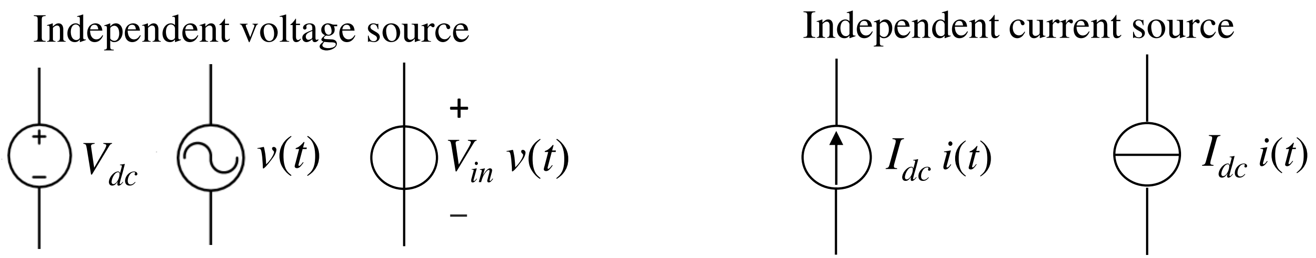
Ideal Voltage Source



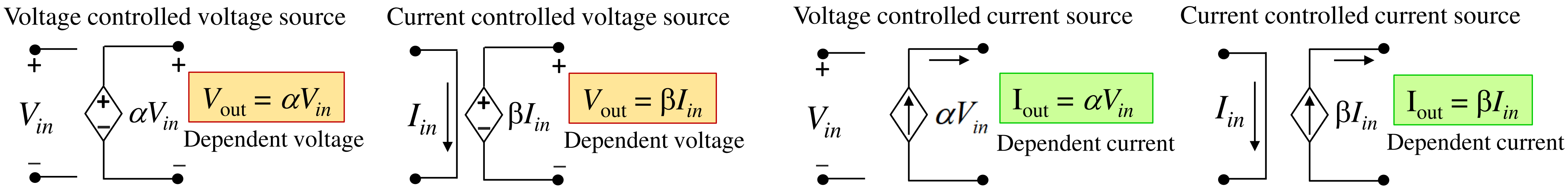
Ideal Current Source

Independent vs Dependent Source

- **Independent voltage (current) source** maintains a specified voltage (current) whose value is *not* affected by other circuit elements.

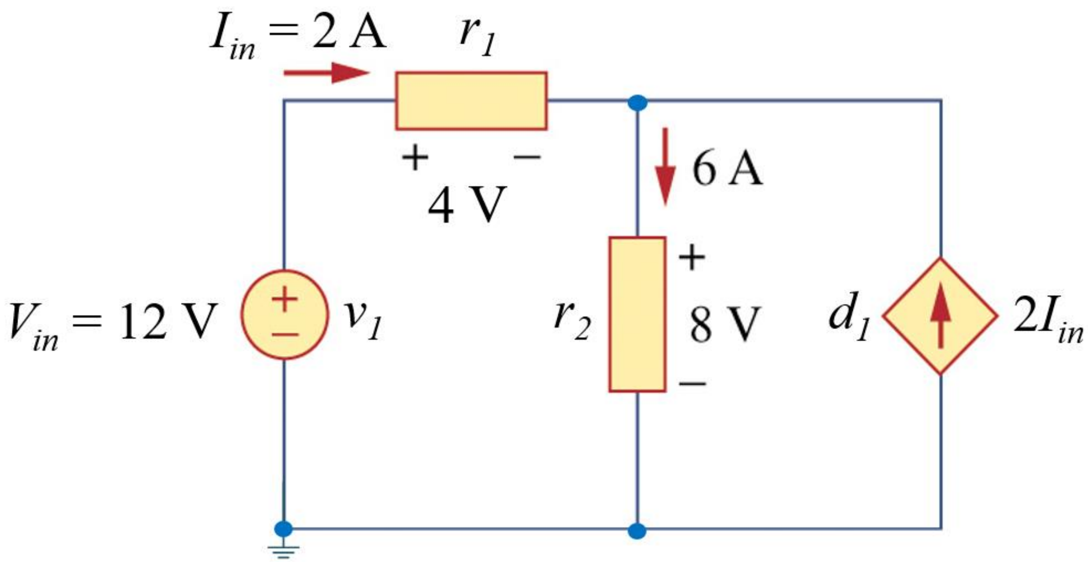


- **Dependent voltage (current) source** has its voltage (current) value varying with another variable (or a set of variables). It is designated by a diamond-shaped symbol.



Example (4)

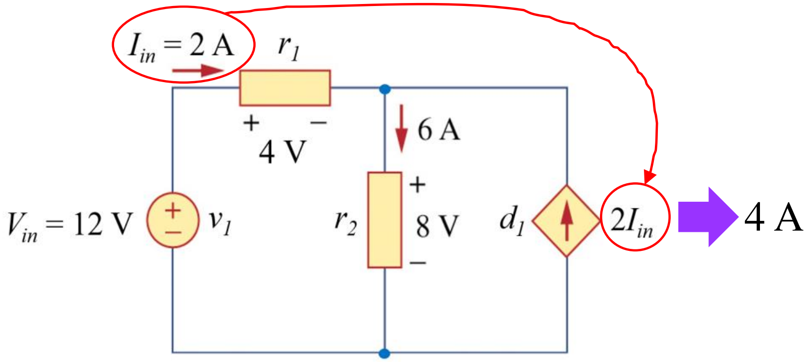
(1) Calculate the power absorbed (dissipated) by each element in the following circuit.



(Hint: Consider the direction of the current across each element.)

Example (4) cont'd

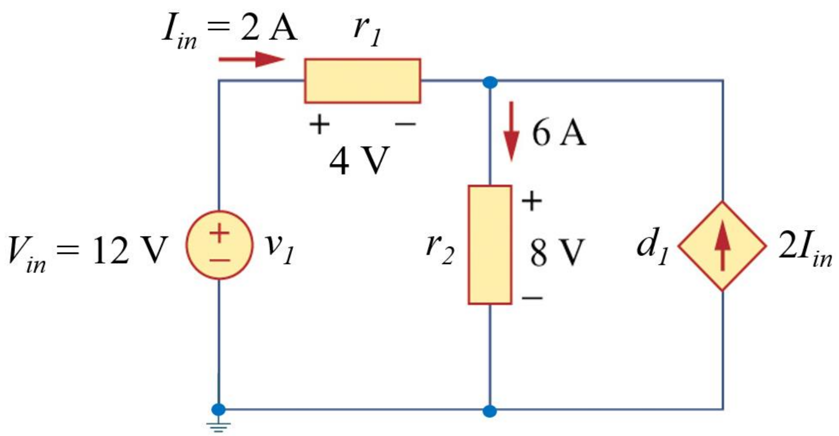
Ans:



Technique:
First determine the values of the dependent sources wherever possible.

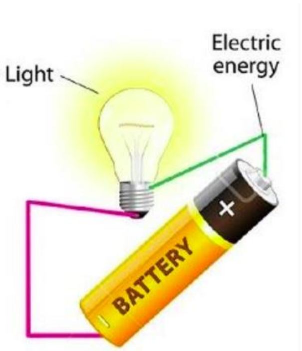
- For v_1: Since the 2 A current flows out of the positive terminal of v_1, the power absorbed by v_1 is:
  $P_{v1} = -V_{in} I_{in} = -12(2) = \mathbf{-24\ W}$ (In other words, v_1 supplies +24 W of power).
- For r_1: Since the 2 A current flows into the positive terminal of r_1, the power absorbed by r_1 is:
  $P_{r1} = V_{r1} I_{r1} = 4(2) = \mathbf{8\ W}$
- For r_2: Since the 6 A current flows into the positive terminal of r_2, the power absorbed by r_2 is:
  $P_{r2} = V_{r2} I_{r2} = 8(6) = \mathbf{48\ W}$
- For d_1: Since the 4 A current flows out of the positive terminal of d_1, the power absorbed by d_1 is:
  $P_{d1} = -V_{d1} I_{d1} = -8(4) = \mathbf{-32\ W}$

Example (4) cont'd



Observations:

- The independent voltage source (v_1) and the current-controlled current source (d_1) are supplying power.
- The two passive elements (r_1, r_2) are absorbing (consuming) power.
- Note that the total power of *all* elements is equal to zero,
  i.e., $P_{v1} + P_{d1} + P_{r1} + P_{r2} = -24 - 32 + 8 + 48 = 0$ W.
  This is *the law of conservation of energy*!



Summary

I. Charge and Current

- ✓ Current and electrons move in opposite direction.
- ✓ Current flows from the positive terminal to the negative terminal in a closed circuit.
- ✓ The mathematical relationship between charge and current is defined as

$$i(t) = \frac{dq(t)}{dt} \quad \text{or} \quad Q = \int_{t_0}^{t_1} i(t)dt$$

II. Voltage

- ✓ The mathematical relationship between energy and voltage is defined as

$$v_{ab} = \frac{dw}{dq}$$

- ✓ The voltage (potential difference) between two points *a* and *b* is expressed as $v_{ab} = v_a - v_b$. In general, $v_{ab} = -v_{ba}$.

Summary (cont'd)

III. Power and Energy

- ✓ The mathematical relationship between power and energy is defined as

$$p(t) = \frac{dw(t)}{dt} = v(t)i(t)$$

- ✓ By convention, when the current flows from the +ve terminal to −ve terminal of an element, the element absorbs (dissipates) power. The sign of power is positive.

- ✓ Likewise, when the current flows from the −ve terminal to +ve terminal of an element, the element supplies (generates) power. The sign of power is negative.

Summary (cont'd)

IV. Circuit Element

- ✓ Know the difference between active element and passive element.
- ✓ Understand the meaning of ideal voltage (current) source.
- ✓ Distinguish between independent and dependent voltage (current) source.
- ✓ Be able to determine the power absorbed (or supplied) by each element.
- ✓ Remember that all closed circuits obey the law of conservation of energy.