Diodes

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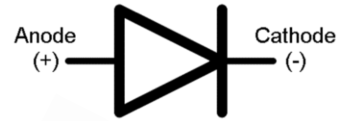
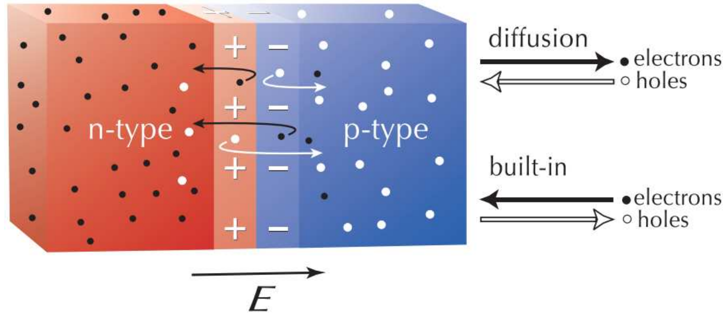
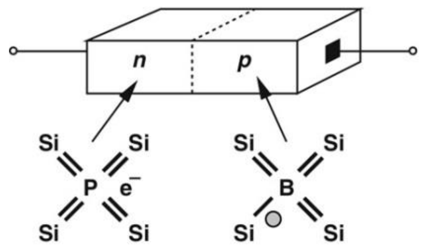
Introduction

- *Non-linear* circuit element
- 2-terminals: cathode and anode
- Implemented as semiconductor *p-n* junction







I-V Characteristics of Diodes

The mathematical relationship between applied voltage V_D across the *p-n* junction and its current I_D is

$$\boxed{I_D = I_S\left(\exp\frac{V_D}{V_T} - 1\right)}$$

Introduction

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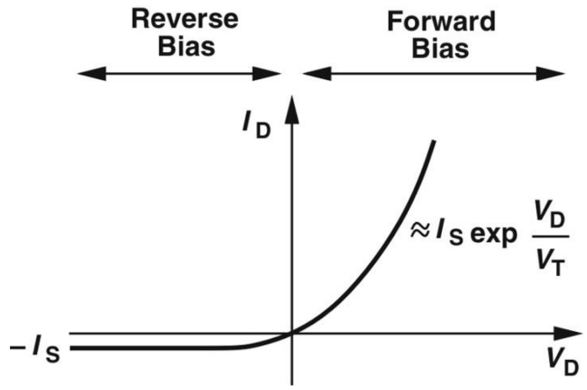
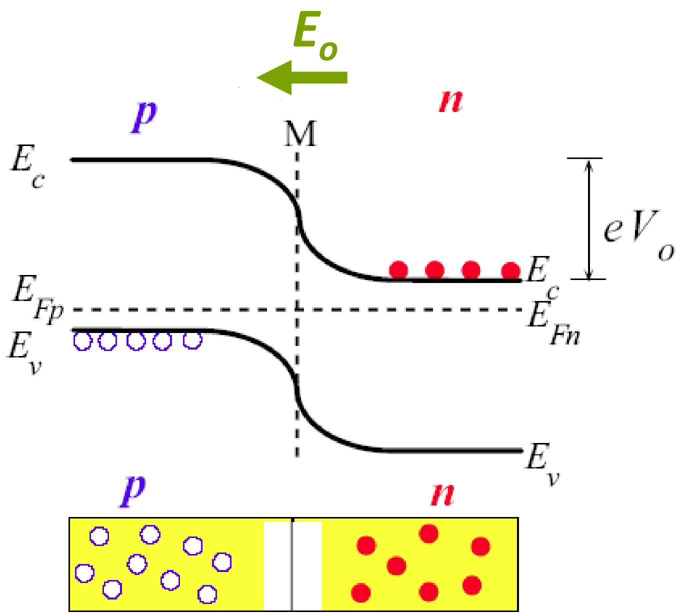
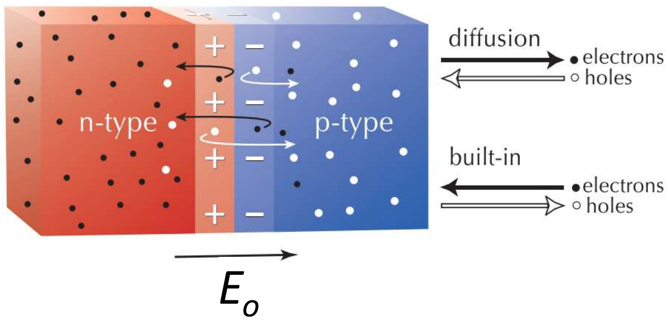


- I_S is the "reverse saturation current" and is proportional to the cross-section area of the diode
- $V_T = kT/q$ is the thermal voltage
  - k = Boltzmann's constant = 1.38×10^{-23} joules/kelvin
  - T = absolute temperature in kelvin
  - q = the amount of charge carried by an electron = 1.6×10^{-19} coulomb
- Note that both I_S and V_T can be considered to be constant for a given diode and operating environment

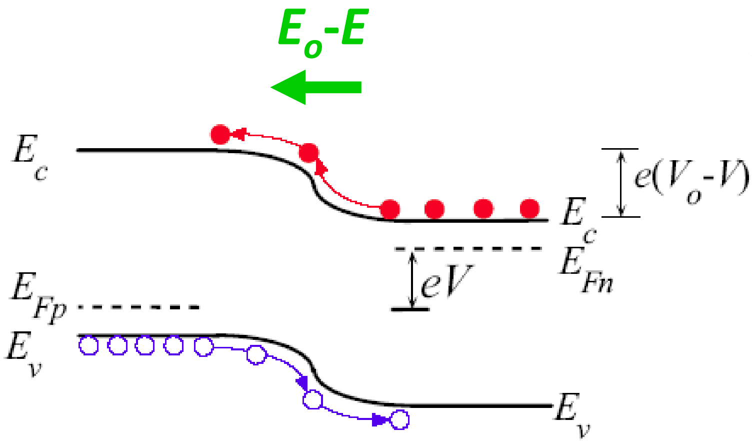
Open-circuited PN junction

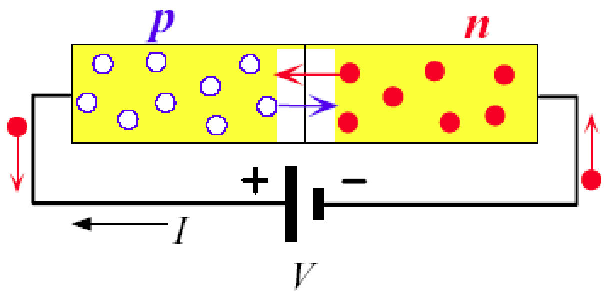


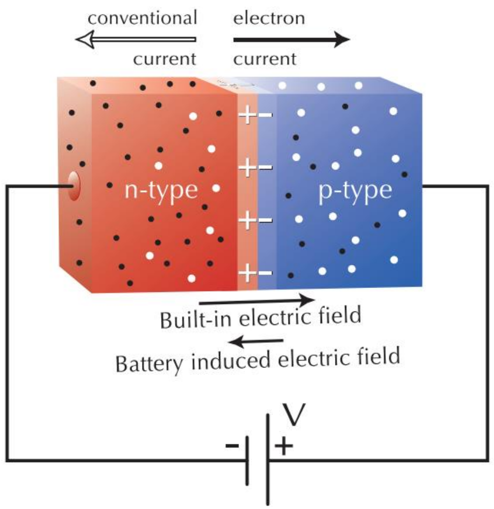


- The "built-in" electric field (E_o) causes some of the electrons and holes to flow in the opposite direction to the flow caused by diffusion.
- These opposing flows eventually reach a stable equilibrium with the number of electrons flowing due to diffusion exactly balancing the number of electrons flowing back due to the electric field.
- The net flow of electrons (or holes) across the junction is zero.
- Hence, there is no net current.

Forward biased PN junction

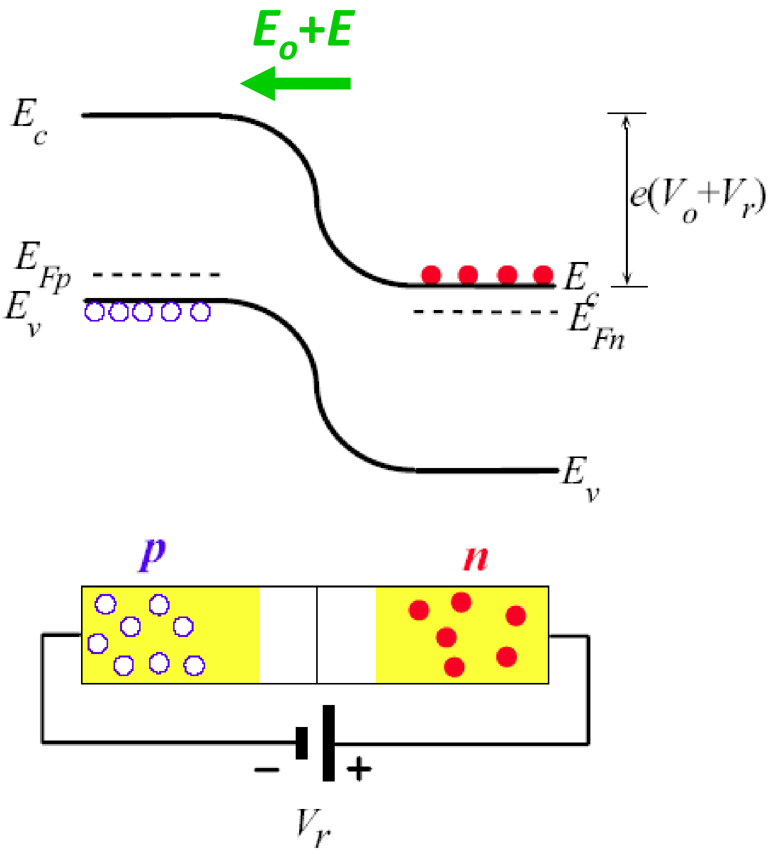


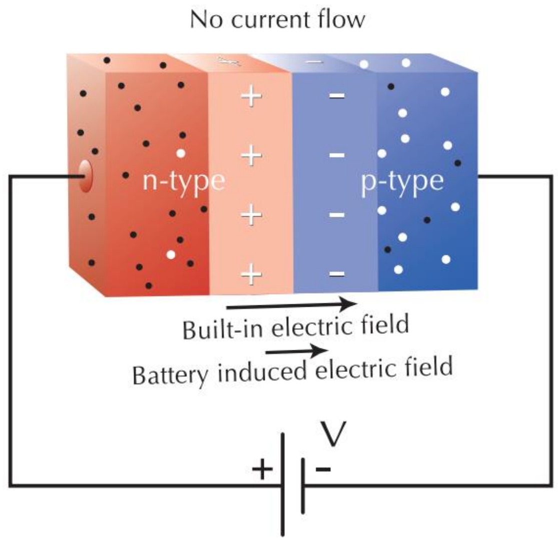




- The potential barrier is reduced with the applied voltage (V).
- Electrons at E_c on the n-side can now readily overcome the barrier and diffuse to the p-side.
- There is a net flow of electrons (or holes) across the junction.
- Hence, conventional current (I) flows from P side to N side.

Reverse biased PN junction

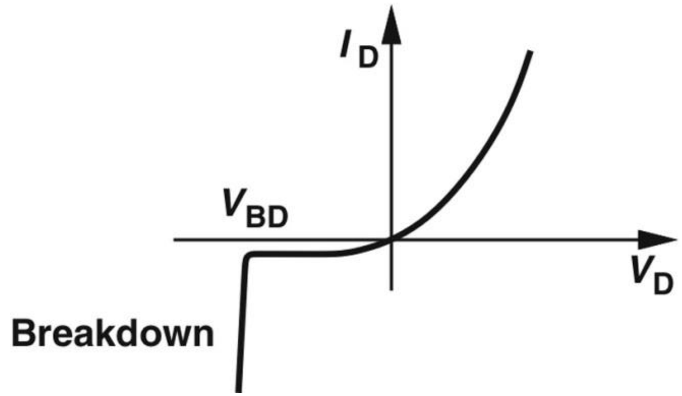




- The potential barrier is increased with the applied reverse voltage (V_r).
- Diffusion current due to electrons is now negligible.
- Hence, there is no (or exceptionally small) current.

Reverse Breakdown

- As we increase the reverse voltage across the p-n junction, the depletion region widens.
- The diffusion current I_D becomes smaller and smaller, eventually to zero.
- That is why the reverse current in the previous slide is $-I_S$.
- If the reverse voltage continues to increase, "breakdown" eventually occurs.
- Two breakdown mechanisms: zener effect, avalanche effect.
- Zener breakdown occurs because of the high electric field whereas avalanche breakdown occurs due to the collision of free electrons with atoms. Both these breakdowns can occur simultaneously.



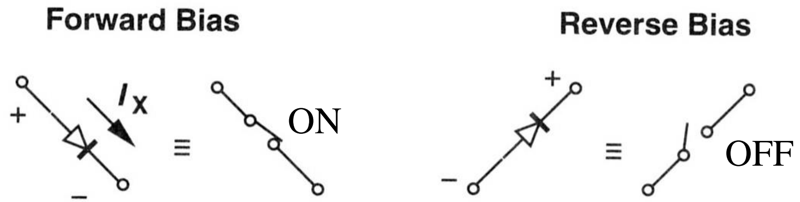
Diode Models

Circuit models for solving circuit problems:

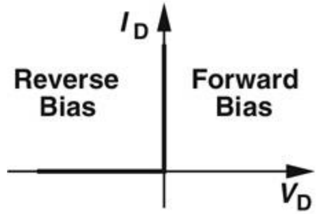
- Ideal diode model
- Constant voltage model
- Small-signal model

Ideal Diode Model

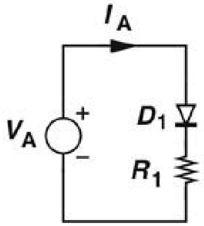
- Ideal diode has no resistance in the forward direction, but infinite resistance in the reverse direction.
- Equivalent circuit:

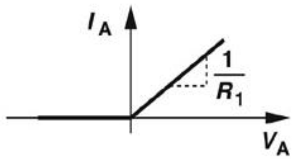


- I/V Characteristics:



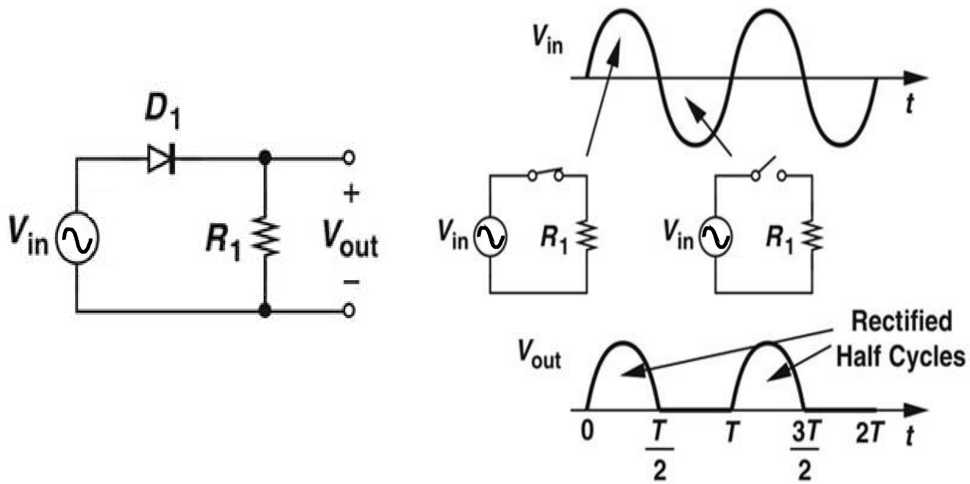
- Example: Consider the circuit on the right.
  - If $V_A > 0$, D_1 exhibits no resistance $\Rightarrow I_A = V_A/R_1$
  - If $V_A < 0$, D_1 behaves as an open circuit $\Rightarrow I_A = 0$





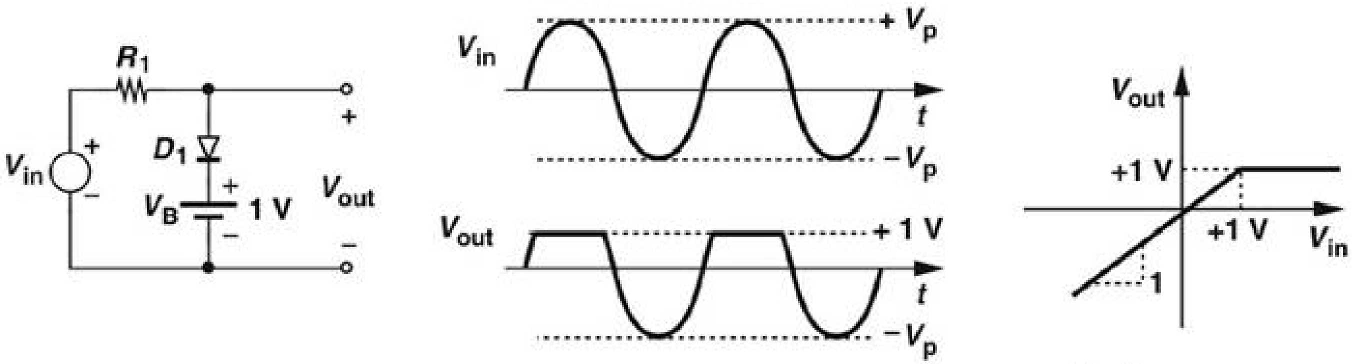
Simple Rectifying Circuit (Ideal Diode Model)

- As V_{in} rises, D_1 is forward biased, shorting the output to the input; this state hold for the positive half cycle.
- When V_{in} falls below zero, D_1 turns off and R_1 ensures that $V_{out} = 0$ because $I_d R_1 = 0$.
- The circuit is known as a rectifier.

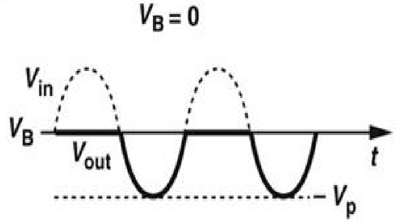


Voltage Rectifying Circuit (Ideal Diode Model)

- 1-V battery placed in series with an ideal diode
- D_1 turns on only when V_{out} approaches +1 V
- Circuit "clips" or "limits" at +1 V

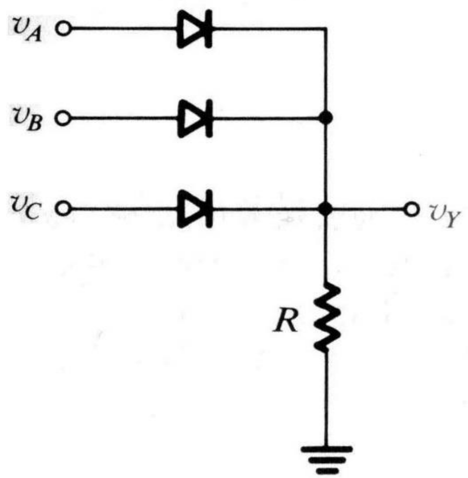


- What happens when $V_B = 0$ V ?



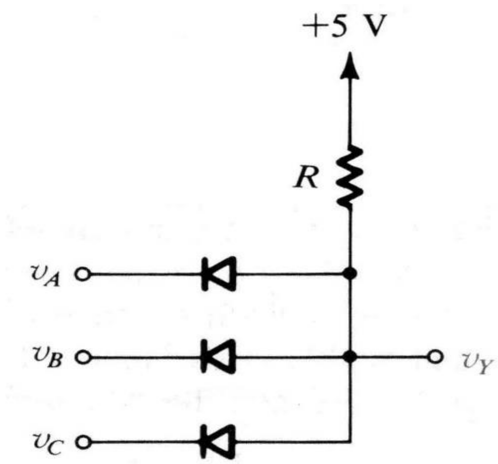
Diode Logic (OR function)

- Diodes together with resistors can be used to implement digital logic functions
  - Diodes connected to 5 V inputs (logic 1) will conduct (forward biased).
  - Current from source flow to resistor, so $v_Y = 5$ V and keep the diodes whose inputs are low (logic 0) in reverse bias.
  - Y=A OR B OR C



Diode Logic (AND function)

- When all input voltages are high (input logic 1), the voltage drops across the diodes are zero and these diode switches are open.
- The output voltage is high (output logic 1) since no current flows through the resistor and there is no voltage drop across it.
- If the voltage of some input voltage source is low (input logical 0), a current will flow from the power supply (+5 V) to the input source via the resistor (*R*) and diode.
- Y = A AND B AND C



Example (1)

Question: Find the values of I and V in Fig. 1 on the right.

- At first sight, it is not obvious which diodes (D_1, D_2) are ON.
- We can make assumption, proceed with assumption.
- Then check whether we end up with inconsistency.

– Assume both diodes are conducting.
– Then $V_B = 0$ V and $V = 0$ V
– By applying KCL at node B, we have $I + I_{D2} = I_B \Rightarrow I = I_B - I_{D2}$
– Since I_{D2} = 10/10k = 1 mA and I_B = 10/5k = 2 mA, we have I =1 mA
– This is consistent with the assumption that D_1 is conducting

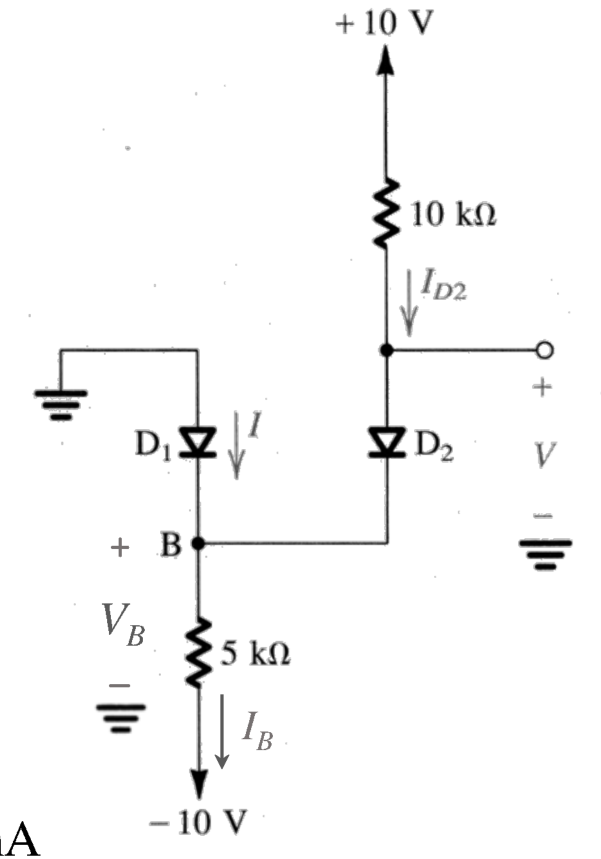


Fig. 1.

Example (2)

Consider the circuit in Fig. 2 on the right.

- If we assume both diodes (D_1, D_2) are conducting (ON), then
  - We still have $V_B = 0$ V and $V = 0$ V
  - By invoking KCL at node B, we have $I + I_{D2} = 10/10\text{k} = 1$ mA
  - I_{D2} is now $10/5\text{k} = 2$ mA
  - Then $I = -1$ mA, which is impossible

- If we assume D_1 is OFF and D_2 is ON
- Then $I = 0$, and $I_{D2} = [10 - (-10)] / 15\text{k} = 4/3$ mA
- Hence, $V = 10 - 5\text{k} \times (4/3\ \text{mA}) = 3.33$ V
- Since V_B is also 3.33 V, D_1 is indeed OFF.
- This validates the assumption that D_1 is OFF.

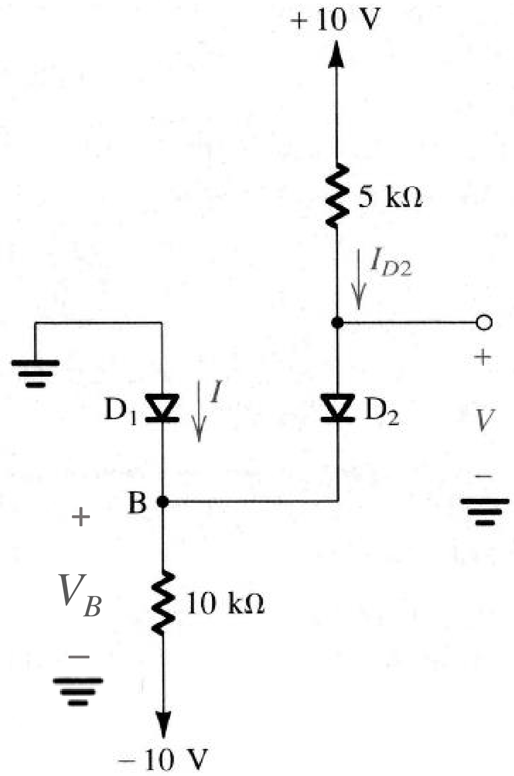
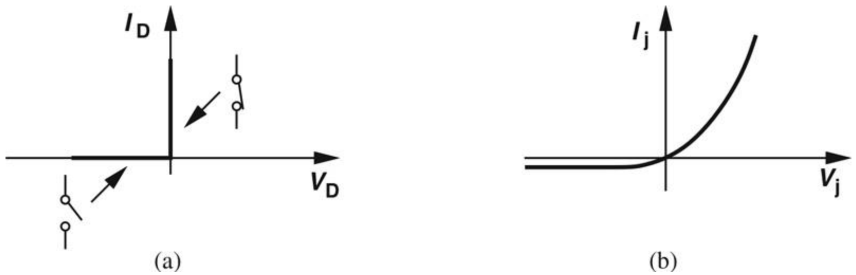


Fig. 2.

PN Junction as a Diode

- In practice, a diode is implemented using a p-n junction



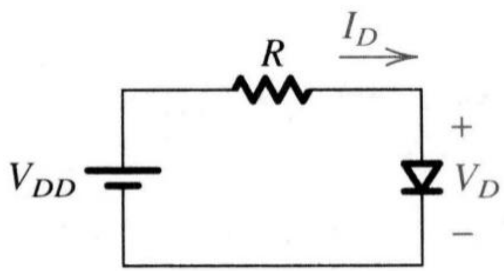
- Example: Determine V_D and I_D in the simple circuit. Use the exponential model of the p-n junction for the diode.

The I-V characteristic of the pn junction diode is

$$I_D \approx I_S \exp\left(V_D / V_T\right) \qquad (1)$$

On the other hand, the I-V equation for the resistor is

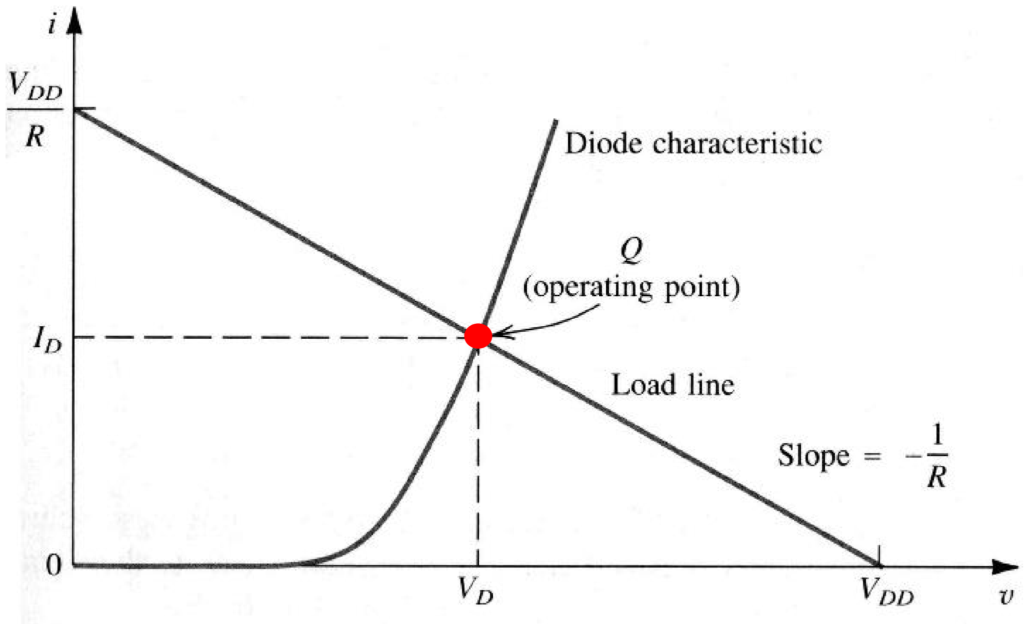
$$I_D = \frac{V_{DD} - V_D}{R} \qquad (2)$$



In both equations, we don't know V_D and I_D. Thus, we need to solve equation (1) and (2) for the two unknowns. But, one of them is nonlinear!

PN Junction as a Diode

One method is draw both lines on the I-V plane and determine the intersection point, which is called the intersection point or Q-point.



PN Junction as a Diode

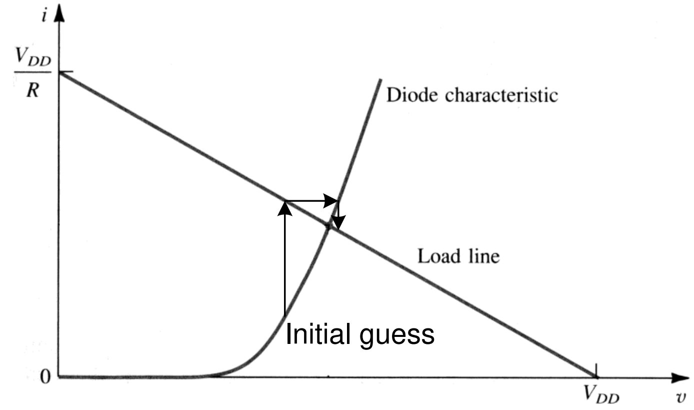
Another method is *iterative method:*

- Guess a V_D, then solve for I_D using equation (2).
- Using the obtained I_D to solve for a new V_D using equation (1).
- The process is repeated until the solutions from two consecutive iterations are very close to each other.

Iterative method in action:

Given: $V_{DD} = 5$ V, $R = 1$ kΩ, $I_s = 2.0298 \times 10^{-15}$ A, and $V_T = 26$ mV.

- Assume $V_D = 0.7$ V (or 700 mV).
- From (2), we have $I_D = (V_{DD} - V_D) / R = (5 - 0.7) / 1\text{k} \Rightarrow I_D = 4.3$ mA
- By substituting $I_D = 4.3$ mA into (1), we have
  $4.3 \times 10^{-3} = 2.0298 \times 10^{-15} \times \exp(V_D / 26 \times 10^{-3}) \Rightarrow V_D = 0.7379$ V
- From (2), $I_D = 4.262$ mA
- From (1), $V_D = 0.73769$ V (or 737.69 mV)



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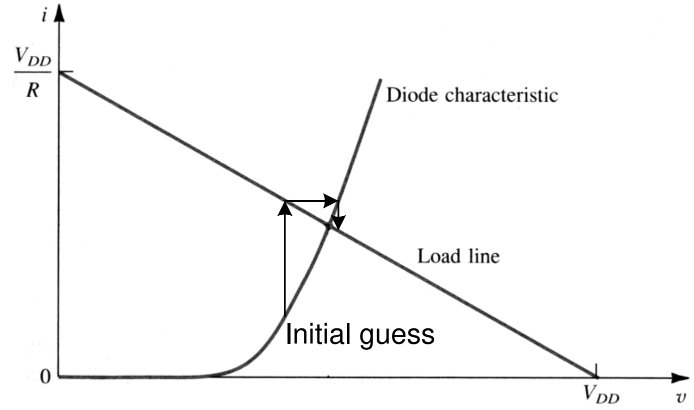
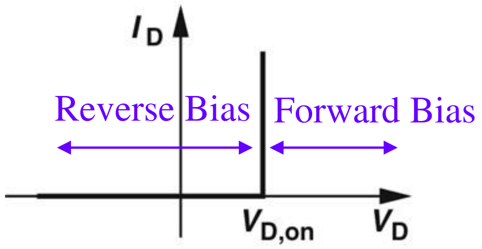


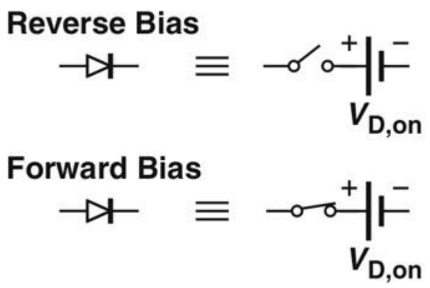
Constant-voltage Model

- The exponential I/V characteristic of the diode results in nonlinear equations, making analysis rather difficult.
- Notice that the diode voltage is a relatively weak function of the device current.
- Example: Suppose $I_D = I_S \exp V_D / V_T$. With two valid points on this curves, we can write

$$\frac{I_{D1}}{I_{D2}} = \exp\frac{V_{D1} - V_{D2}}{V_T} \quad \Rightarrow \quad V_{D1} - V_{D2} = V_T \ln(I_{D1} / I_{D2})$$

- If I_{D1} = 10 I_{D2}, the increase in voltage is only $V_T \ln 10 \approx$ 60mV, which verifies the claim above.
- This forms the basis for the constant-voltage model with a typical value of $V_{\mathrm{D,on}}$= 700 mV (or 0.7 V).





Constant-voltage Model

Plot the input/output characteristic for the circuit shown in (a). Assume a constant-voltage model for the diode.

- We begin with $V_{in} = -\infty$, then the circuit is shown in (b).
- D_1 is on, and $V_{out} = V_{in} + V_{D,on}$
- The currents through R_2 and R_1 are:

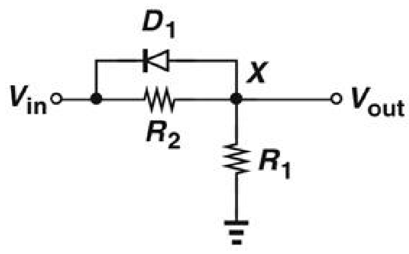
$$I_{R2} = \frac{V_{D,on}}{R_2}, \quad I_{R1} = \frac{0 - V_{out}}{R_1} = -\frac{(V_{in} + V_{D,on})}{R_1}$$

- As V_{in} increases, I_{R2} remains constant but I_{R1} decreases.
- At some point, $I_{R2} = I_{R1}$, and then D_1 is turned off. This occurs at

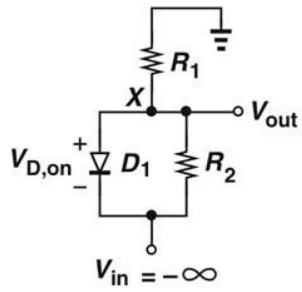
$$\frac{V_{D,on}}{R_2} = -\frac{(V_{in} + V_{D,on})}{R_1} \quad \Rightarrow \quad V_{in} = -\left(1 + \frac{R_1}{R_2}\right) V_{D,on}$$

- After this voltage, the circuit is shown in (c)

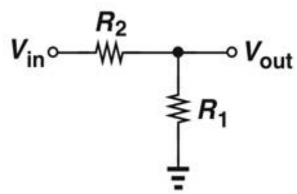
$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + R_2}$$



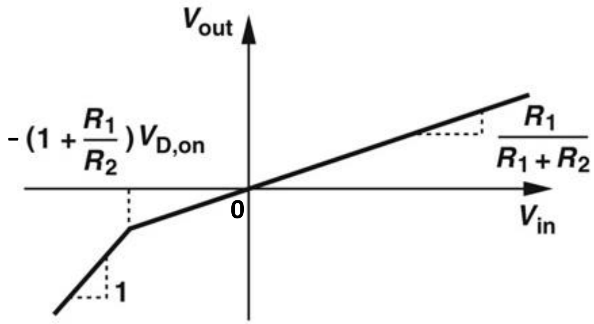
(a)



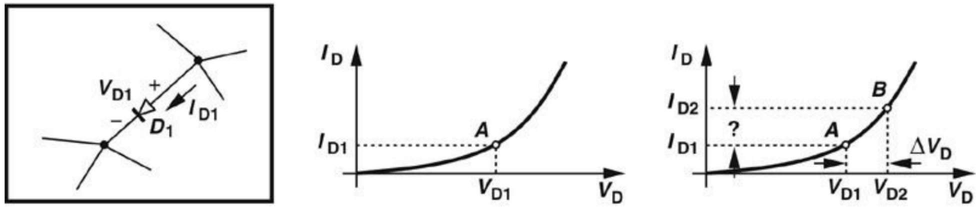
(b)



(c)



Small-signal Operations



- Suppose a diode operates at a DC operating point in the forward I/V curve (say point A).
- A small perturbation in the circuit changes the diode voltage by a small amount (ΔV_D), how can we predict the change in the diode current ?
- We begin by considering the exponential model

$$I_{D2} = I_S \exp\left(\frac{V_{D1} + \Delta V_D}{V_T}\right) = I_S \exp\left(\frac{V_{D1}}{V_T}\right) \exp\left(\frac{\Delta V_D}{V_T}\right)$$

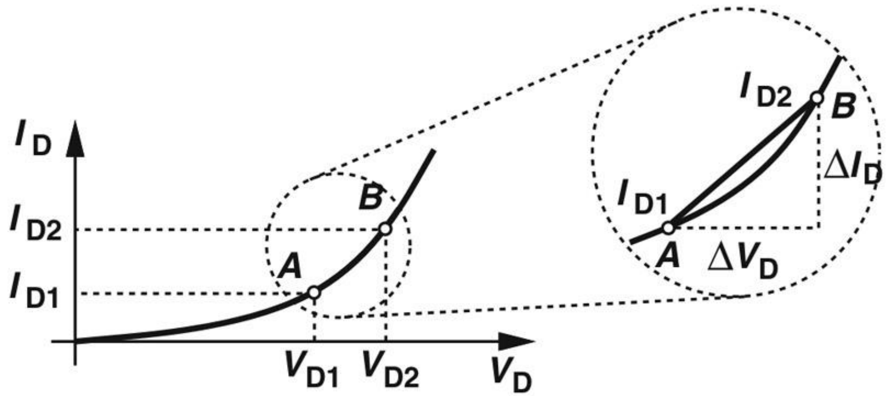
- If $\Delta V_D << V_T$, we have $\exp(\Delta V_D / V_T) \approx 1 + \Delta V_D / V_T$. Hence, we have

$$I_{D2} \approx I_S \exp\frac{V_{D1}}{V_T} + \frac{\Delta V_D}{V_T} I_S \exp\frac{V_{D1}}{V_T} = I_{D1} + \underbrace{\frac{I_{D1}}{V_T}\Delta V_D}_{=\Delta I_D}$$

Small-signal Operations

- Physical meaning:
  - If ΔV_D is small, the section A–B is approximately linear.
  - The slope is

$$\frac{\Delta I_D}{\Delta V_D} = \left.\frac{dI_D}{dV_D}\right|_{V_D = V_{D1}} = \frac{1}{V_T} \times I_s \exp\frac{V_{D1}}{V_T} = \frac{I_{D1}}{V_T}$$



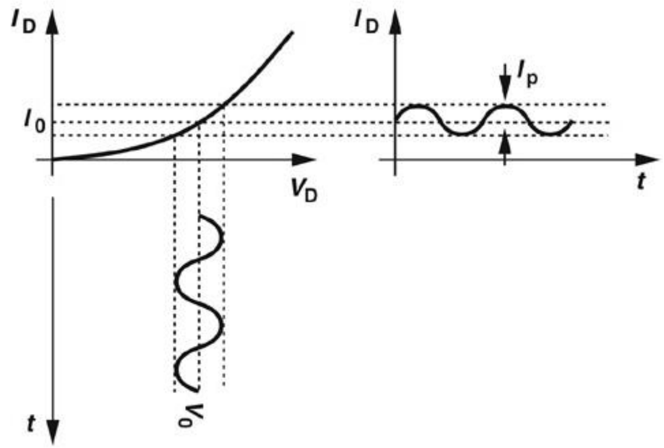
- Point A is the operating point / Q-point
- Hence, the small-signal resistance of the diode is

$$\boxed{r_d = \frac{V_T}{I_D}}$$

Small-signal Operations

A signal $V(t) = V_o + V_p\cos(\omega t)$ is applied to a diode with $V_p << V_T$, determine the diode current.

The operating point of the diode is (V_o, I_o): $I_o = I_s \exp\left(V_o / V_T\right)$



From the small-signal model, $r_d = V_T / I_o$, so the peak current is:

$$I_p = V_p / r_d = \frac{I_o}{V_T} V_p$$

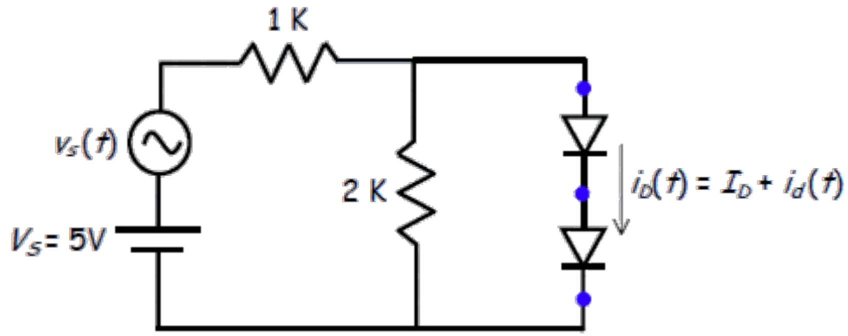
Hence, the total current is:

$$I_D(t) = I_o + I_p \cos\omega t = I_s \exp\frac{V_o}{V_T} + \frac{I_o}{V_T} V_p \cos\omega t$$

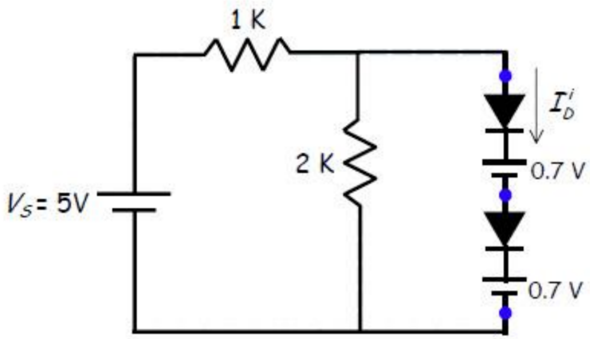
Notice that if V_p was large, we would need to solve $I_D(t) = I_s \exp\dfrac{V_o + V_p \cos\omega t}{V_T}$

Example (3)

Consider the circuit. Determine $i_d(t)$ if $v_s(t) = 0.01 sin\omega t$ by small-signal analysis.

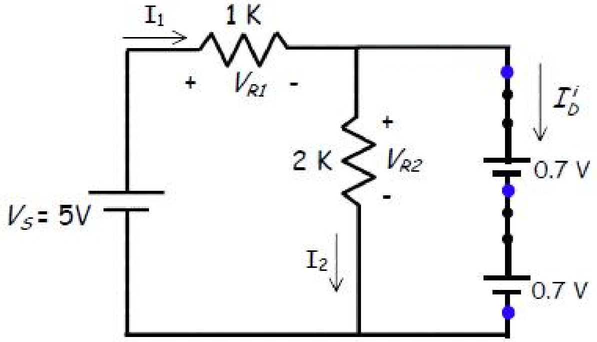


- Find r_d requires knowledge of the quiescent current I_D (DC).
- Therefore, both DC and AC analysis needs to be carried out.
- The DC and AC circuits can be singled out by superposition.
- For DC analysis, turn off the small-signal source and replace the diodes with the **constant voltage model.**



Example (3) (cont'd)

- DC analysis:

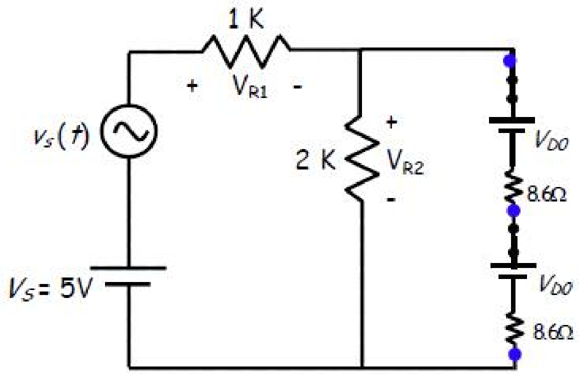


- By KVL, $V_{R2} = 0.7 + 0.7 = 1.4$ V
- Since $I_2 = V_{R2} / 2\text{k}$ and $V_{R2} = 1.4$ V, $I_2 = 0.7$ mA
- By KVL, $V_{R1} = 5.0 - V_{R2} = 3.6$ V
- By Ohm's law, $I_1 = V_{R1} / 1\text{k} = 3.6$ mA
- Hence, $I^i_D = I_1 - I_2 = 2.9$ mA > 0 (Quiescent current)
- Calculate the small-signal resistance r_d:

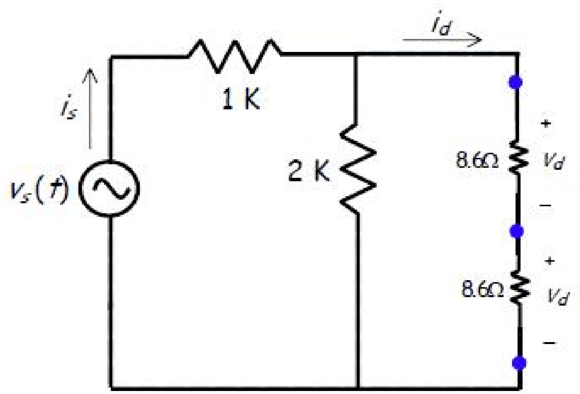
$$r_d = V_T / I_D = 0.025/0.0029 \approx 8.62\ \Omega$$

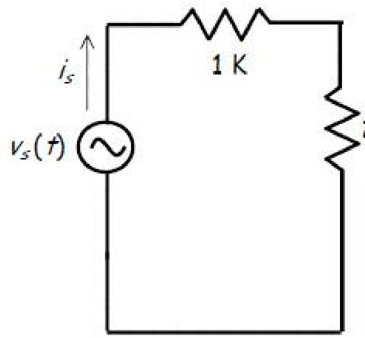
Example (3) (cont'd)

- Replace the diodes with the small-signal model:



- Carry out ac small-signal analysis by turning off all DC sources:





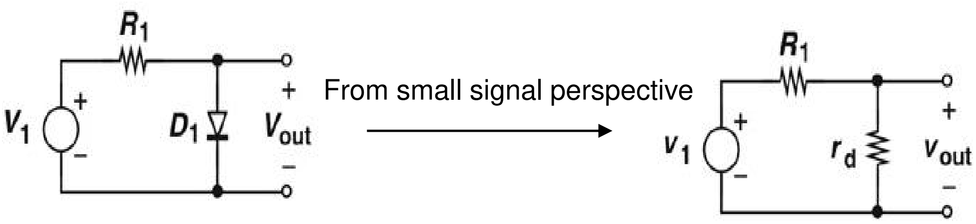
$2K \| (8.6 + 8.6) = \ 17\ \Omega$

Therefore, $i_s(t) = v_s(t)$ / (1k + 17)
$= 0.01 \sin \omega t / 1017$
$\approx 9.83 \sin \omega t\ \ \mu A$

Summary

How to systematically perform analysis of diode circuits using the small-signal model ?

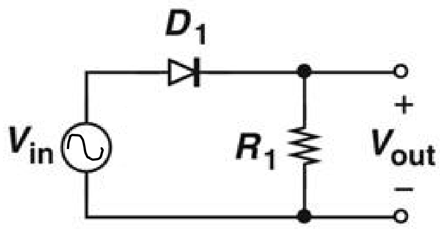
- Determine the operating point (initial voltage and current).
- Develop the small-signal model (i.e., calculate r_d).
- Replace each diode with its small-signal model.

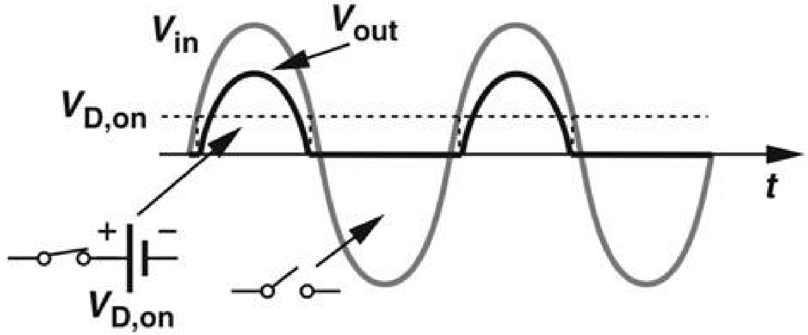


$$\boxed{r_d = V_T / I_D}$$

Application: Rectifying Circuits

- Half-wave rectifier with constant-voltage diode model:

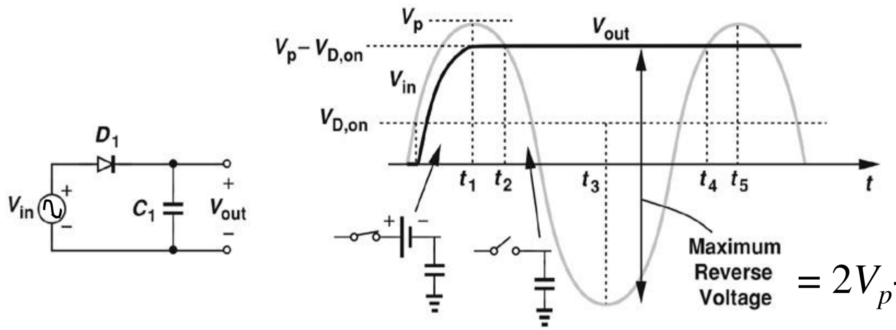




- The diode D_1 is no longer assumed ideal, use constant-voltage model instead.
- V_{out} remains zero until V_{in} exceeds $V_{D,on}$, <u>at which point</u> D_1 is turned on (i.e. forward biased).
- The circuit essentially operates as a half-wave rectifier.

Application: Rectifying Circuits (cont'd)

- Output amplitude of the "half-wave rectifier" varies too much for practical use. Now, replace the resistor with a capacitor.

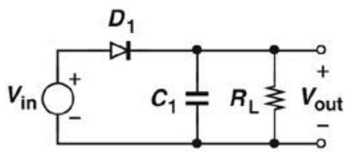


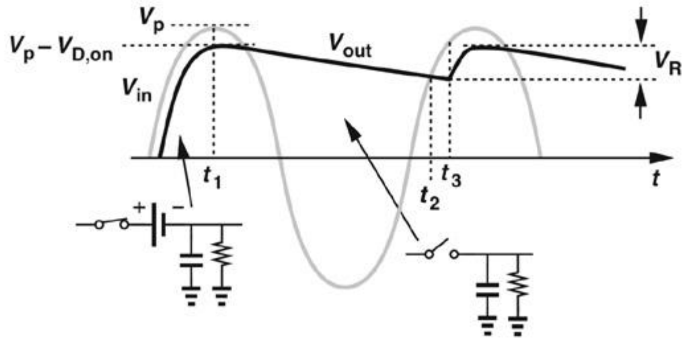
$= 2V_p\text{-}V_{D,on} \approx 2V_p$

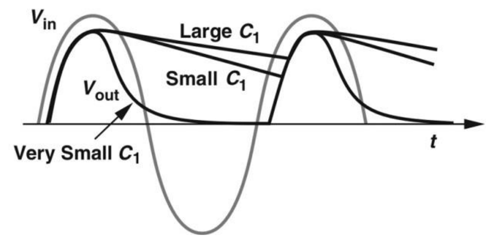
- As V_{in} rises from zero, the diode D_1 is OFF until $V_{in} > V_{D,on}$ at which point D_1 is turned ON (forward biased).
- At $t = t_1$, $V_{in} = V_p$, where V_p is the <u>peak value</u> of V_{in}, and $V_{out} = V_p - V_{D,on}$.
- As V_{in} begins to fall, V_{out} must remain constant because if V_{out} were to fall, C_1 would need to be discharged by a current flowing from its top plate through the cathode of D_1, which is impossible.
- Therefore, D_1 is OFF (reverse biased) after $t = t_1$.
- At $t = t_{2,}$ $V_{in} = V_p - V_{D,on} = V_{out}$ (D_1 experiences **zero** voltage difference).
- At $t > t_2$, $V_{in} < V_{out}$, D_1 experiences negative voltage.
- At $t = t_3$, $V_{in} = -V_p$ with a maximum reverse bias voltage of $V_{out} - V_{in} = 2V_p - V_{D,on}$ across the diode D_1.
- Hence, the diode in this rectifier must withstand a reverse voltage of $\sim 2V_p$.

Application: Rectifying Circuits (cont'd)

- In reality, the rectifying circuit has to provide a current to a load, which can be represented by a simple resistor.



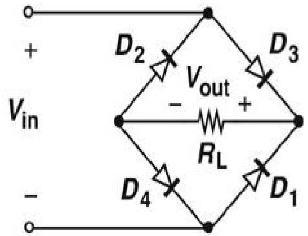




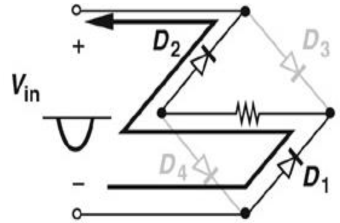
- V_{out} behaves as before until $t = t_1$.
- As V_{in} begins to fall after t_1, so does V_{out} because R_L provides a discharge path for C_1.
- Since too much variation in V_{out} is undesirable, C_1 must be large enough so that the current drawn by R_L does *not* reduce V_{out} significantly.
- Nonetheless, V_{out} continues to decrease while V_{in} goes through a negative excursion before returning to positive values again.
- At $t = t_3$, V_{in} exceeds V_{out} by $V_{D,on}$, thereby turning D_1 on and forcing $V_{out} = V_{in} - V_{D,on}$.
- Variation in V_{out} is called "ripple", and C_1 is the "smoothening" capacitor.
- Because this is an RC circuit, the ripple amplitude V_R depends on the value of C_1.

Full-wave Rectifier

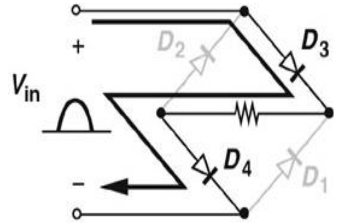
- Half-wave rectifier suffers from large ripple.
- Full-wave rectifier (also known as bridge rectifier), assuming ideal diodes:



(a)



(b)



(c)

- If $V_{in} < 0$, D_1 and D_2 are ON whereas D_3 and D_4 are OFF, reducing the circuit to (b) and $V_{out} = -V_{in}$.
- If $V_{in} > 0$, the bridge is simplified to (c) and $V_{out} = V_{in}$.
- However, with non-ideal diodes, $V_{out} = V_{in} - 2V_{D,on}$ for full-wave rectifier, compared to $V_{out} = V_{in} - V_{D,on}$ for half-wave rectifier.
- Less ripple than half-wave rectifier: