Operational Amplifiers

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Topics

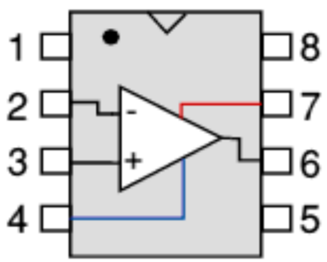
- What is an Operational Amplifier (Op Amp) ?
- Ideal and Non-ideal Op Amp.
- Inverting and Non-inverting Amplifier.
- Voltage Follower.
- Summing and Difference Amplifier.
- Cascaded Op Amp Circuits.

Introduction to Operational Amplifier

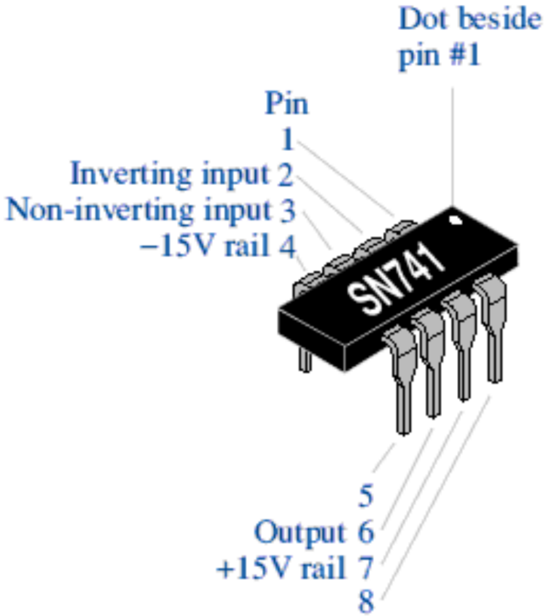
- The op amp is an electronic circuit that acts like a voltage-controlled voltage source (VCVS).
- An op amp may also be treated as a voltage amplifier with very high gain.
- Initially, we will discuss the ideal op amp prior to considering the non-ideal one. By using the nodal analysis as a tool, ideal op amp circuits such as an inverter, voltage follower, summer and difference amplifier will be studied.
- An input applied to the <u>non-inverting</u> terminal of an op amp will produce an output with the same polarity. Conversely, an input applied to the <u>inverting</u> terminal will produce an output with the opposite (inverted) polarity.

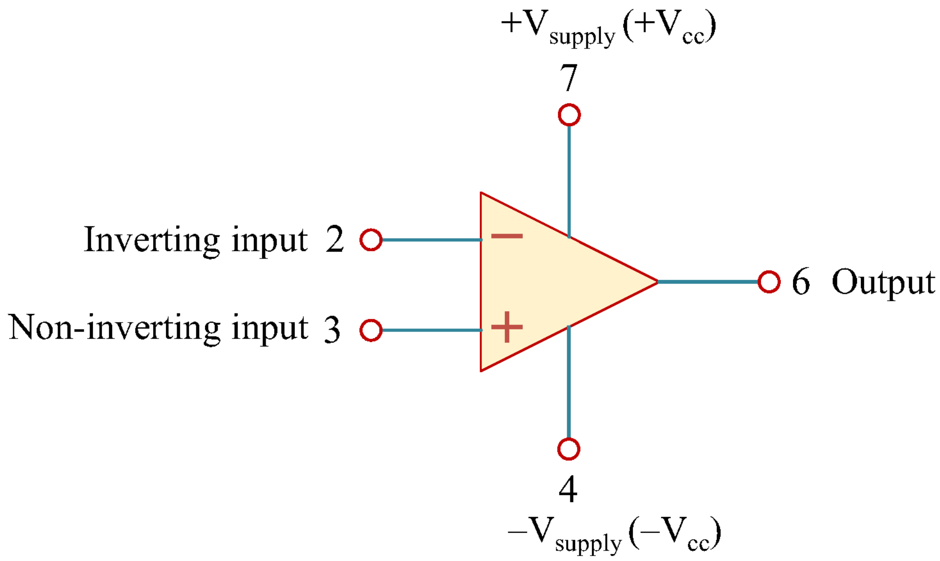
Introduction to Operational Amplifier (cont'd)

741 Op Amp
8-pin DIP



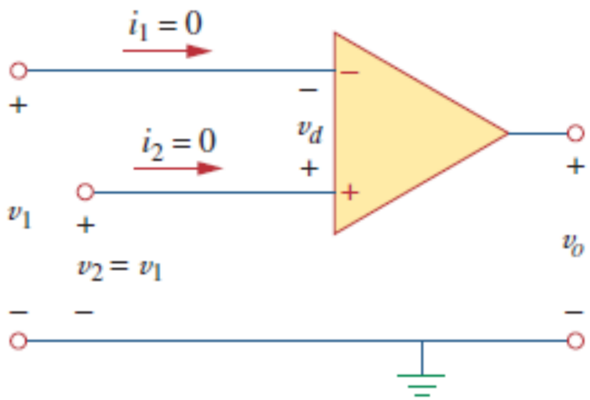
Top view





Pin 1 and 5 are offset null pins (they are used to eliminate the offset voltages and balance the input voltages.)
Pin 8 is NC (NC means No Connect. It should be left floating.).

Ideal Op Amp



- For ease of understanding, an ideal op amp is used here.
- An ideal op amp has the following characteristics:
  - Infinite open-loop gain $(A = \infty)$.
  - Infinite input resistance $(R_i = \infty)$.
  - Zero output resistance $(R_o = 0)$.
- Although the analysis using an ideal op amp is only an approximation, the result is accurate enough for general design purpose.
- The ideal op amp model is shown in the circuit on the top right.
- Two major properties of an ideal op amp are:
  - $i_1 = 0$ and $i_2 = 0$ (due to the infinite input resistance).
  - $v_1 = v_2$ (due to the infinite open-loop gain and assuming the op amp is not saturated).

Non-ideal Op Amp

- The op amp senses the difference between the two input voltages and then multiplies it by the gain A, which causes the resulting voltage to appear at the output.
- It can be modeled as shown in Fig. 1, where the input-output relationship is given by $v_o = Av_d = A(v_2 - v_1)$.
- In general, an op amp has the following characteristics:
  - Very high gain A (10^5 to 10^8).
  - High input resistance R_i (10^5 to $10^{13}\ \Omega$).
  - Low output resistance R_o (10 to 100 Ω).

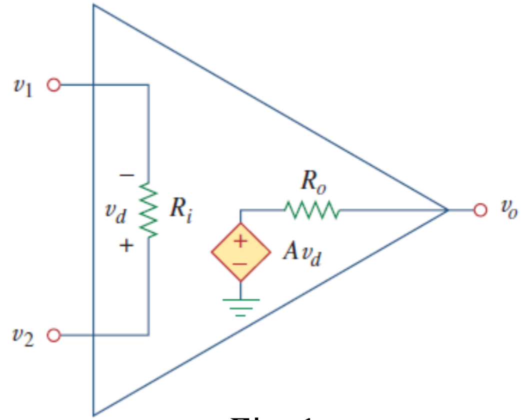
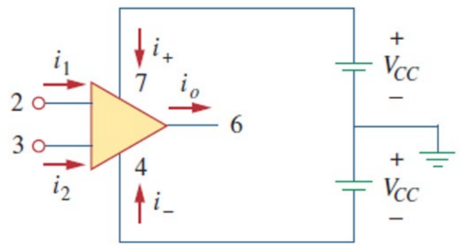
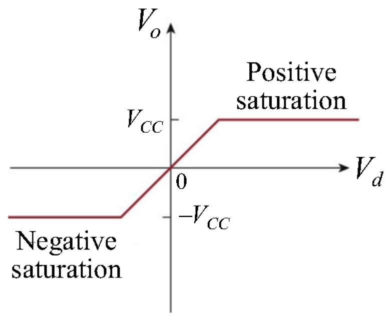


Fig. 1.

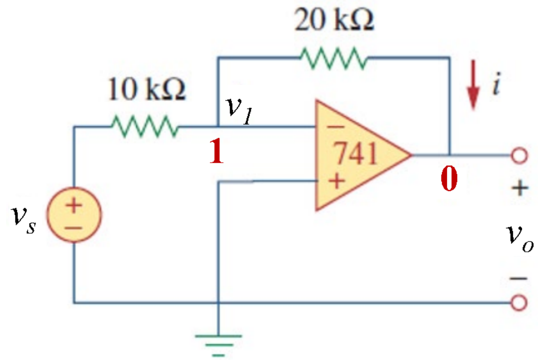
- The magnitude of the output voltage of an op amp cannot exceed $|V_{cc}|$, where V_{cc} is the power supply voltage (e.g. $|V_{cc}|$ = 12 V or 15 V).





Example (1): Non-ideal Op Amp

Question: Assume $A = 2\times10^5$, $R_i = 2\ \text{M}\Omega$, $R_o = 50\ \Omega$, and the op amp is connected as shown. Find v_o/v_s. Also, determine i when $v_s = 2$ V.



Solution:

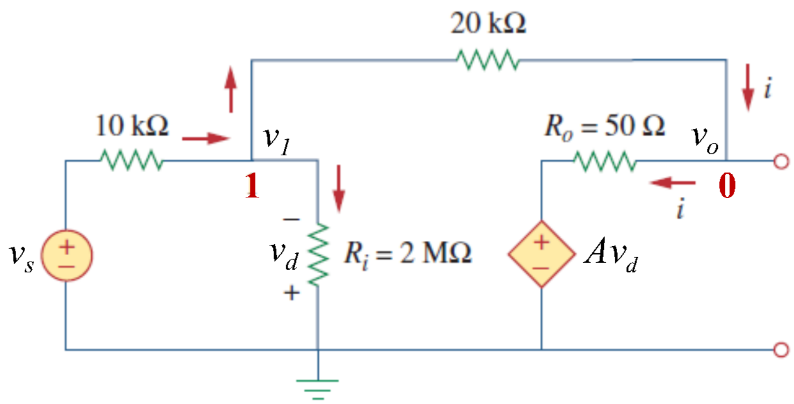
1. Use the simplified input-output model of op-amp shown in the previous slide; and
2. Apply nodal analysis by using KCL at node 1 and node 0.

At node 1: $$\frac{v_s - v_1}{10\times10^3} = \frac{v_1}{2000\times10^3} + \frac{v_1 - v_o}{20\times10^3}$$

$$200v_s = 301v_1 - 100v_o \qquad (1)$$

At node 0: $$\frac{v_1 - v_o}{20\times10^3} = \frac{v_o - Av_d}{50}$$

$$v_1 = -\frac{401}{80{,}000{,}000}v_o \qquad (2)$$

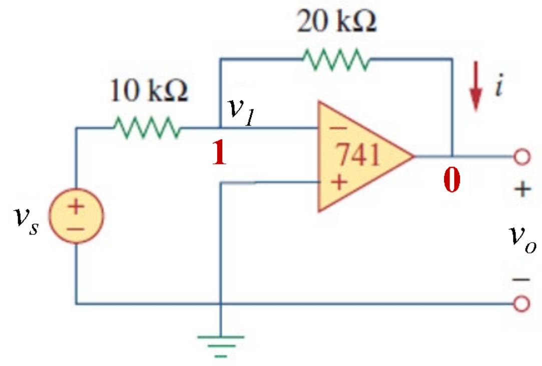


Example (1): Non-ideal Op Amp (cont'd)

Solution (cont'd):

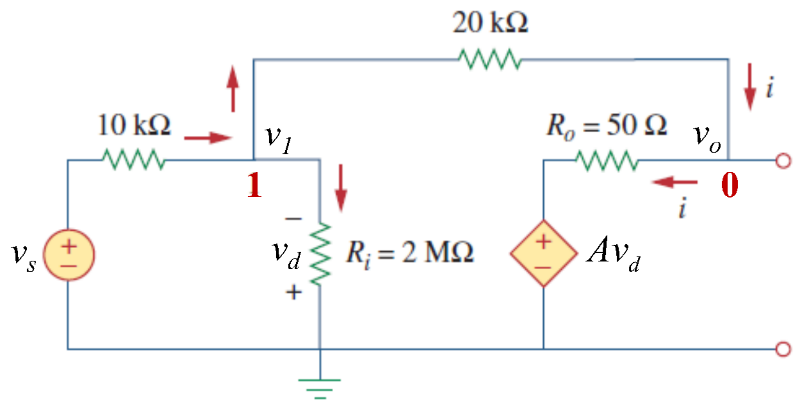
$$200v_s = 301v_1 - 100v_o \qquad (1)$$

$$v_1 = -\frac{401}{80{,}000{,}000}v_o \qquad (2)$$



By substituting (2) into (1) and re-arranging, we have

$$200v_s = -301\times\frac{401}{80{,}000{,}000}v_o - 100v_o \quad => \quad \frac{v_o}{v_s} = -1.999969825$$



When $v_s = 2$ V, $v_o = -3.99993965$ V

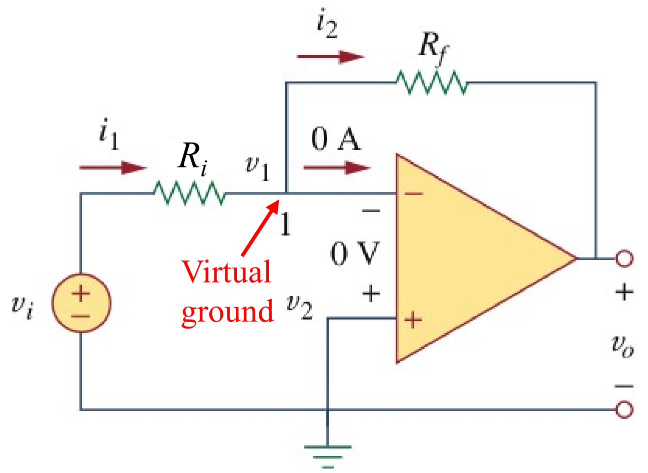
From (2), $v_1 = 20.05\ \mu$V

Hence, $i = \dfrac{v_1 - v_o}{20\times 10^3} \approx 200\ \mu$A

Inverting Amplifier

- Consider a general case of an inverting amplifier based on the ideal op amp model as shown in the figure below.
- This op-amp configuration is called an inverting amplifier.
- Apply KCL at node 1, we have $i_1 = i_2$ or equivalently,

$$\frac{v_i - v_1}{R_i} = \frac{v_1 - v_o}{R_f}$$



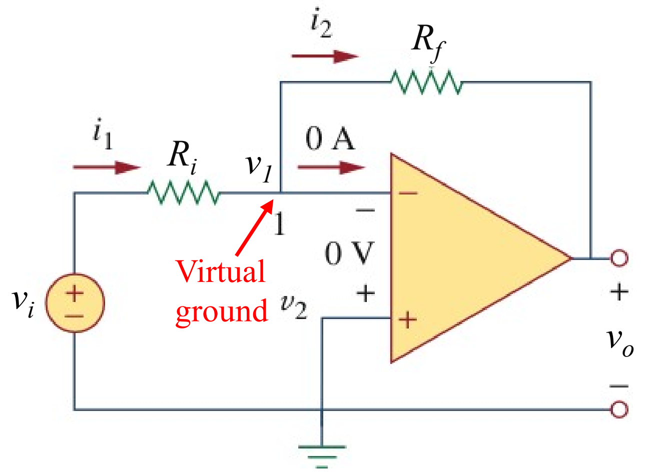
- Note that $v_1 = v_2 = 0$ for an ideal op amp. Hence, $\dfrac{v_i}{R_i} = -\dfrac{v_o}{R_f}$
- In other words, the closed-loop gain is given by:

$$\boxed{\frac{v_o}{v_i} = -\frac{R_f}{R_i}}$$

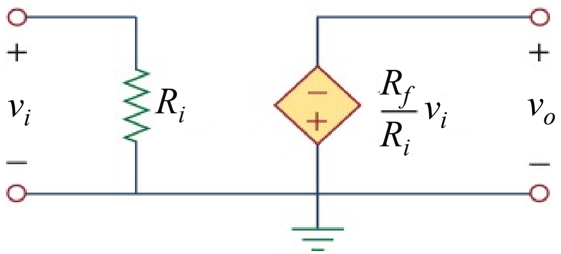
Inverting Amplifier (cont'd)

- Suppose R_1 = 10 kΩ and R_f = 20 kΩ. The closed-loop gain is given by:

$$\frac{v_o}{v_i} = -\frac{R_f}{R_i} = -\frac{20}{10} = -2$$



- The use of ideal op amp model greatly simplifies the analysis.
- The gain of the amplifier depends on the feedback resistance (R_f) divided by the input resistance (R_i).
- In an inverting amplifier, the polarity of the output signal is opposite to that of the input signal.
- The equivalent model of an inverting amplifier is shown below.



Example (2): Inverting Amplifier

Question: Determine v_o in the op-amp circuit shown in Fig. 2.

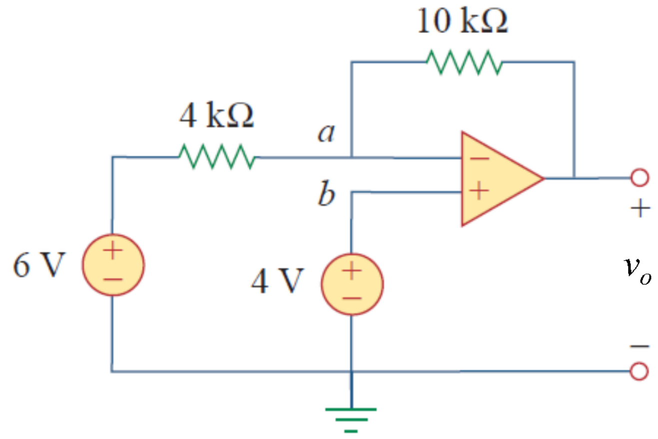


Fig. 2

<u>Method 1: Use superposition</u>

$v_o = v_{o1} + v_{o2}$, where v_{o1} = f(6 V), and v_{o12} = f(4 V).

To obtain v_{o1}, set the 4-V source to zero. That is, we treat the op-amp circuit as though it is powered by the 6-V voltage source only.

$$v_{o1} = -\frac{10}{4}(6) = -15\text{ V}$$

To obtain v_{o2}, set the 6-V source to zero. That is, we treat the op-amp circuit as though it is powered by the 4-V voltage source only.

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14\text{ V}$$

Hence, $v_o = v_{o1} + v_{o2} = -15 + 14 = -1$ V

Notice that the polarity of v_o is <u>negative</u>, which is opposite to that of the two input voltages, i.e., +6 V and +4 V.

Example (2): Inverting Amplifier (cont'd)

Method 2: Apply KCL to node *a*

Sum of current flowing into node *a* = Sum of current flowing out of node *a*

$$i_1 = i_2$$

Since $i_1 = \dfrac{6 - v_a}{4k}$ and $i_2 = \dfrac{v_a - v_o}{10k}$, we have

$$\frac{6 - v_a}{4k} = \frac{v_a - v_o}{10k} \qquad (3)$$

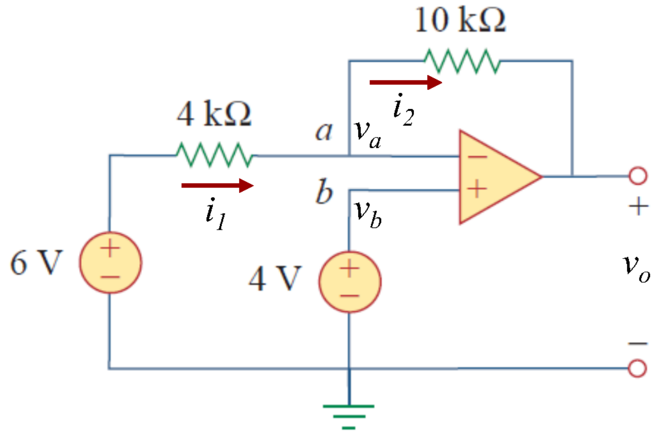


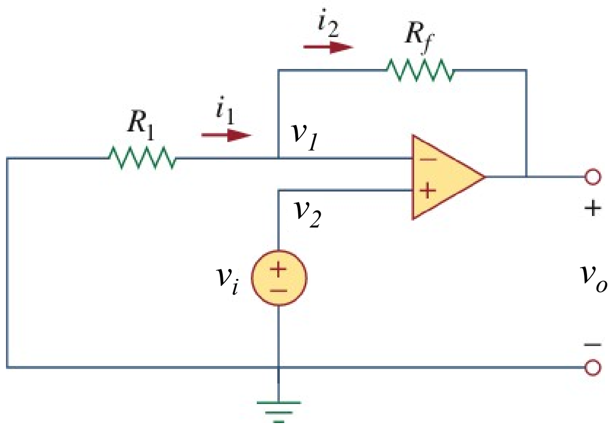
Fig. 2

Note that $v_a = v_b = 4$ V. By substituting these values into (3), we have

$$\frac{6-4}{4k} = \frac{4 - v_o}{10k} \Rightarrow 5 = 4 - v_o \Rightarrow v_o = -1$$

Non-inverting Amplifier

- Non-inverting amplifier is another important configuration of an op amp.
- The input voltage (v_i) is applied directly at the non-inverting input as shown in the circuit on the right.



- By applying KCL at the inverting terminal, we can write

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \qquad (4)$$

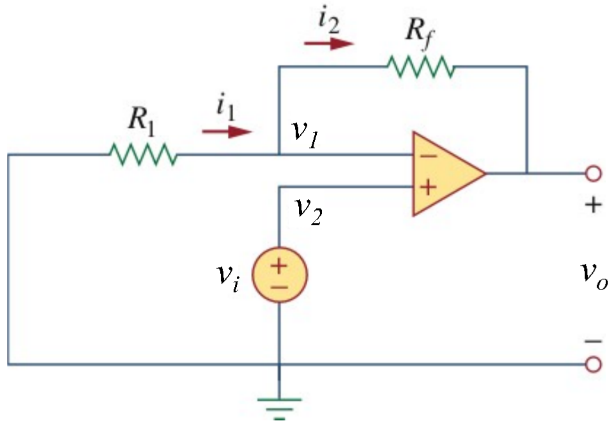
- But, $v_1 = v_2 = v_i$. Hence, by substituting $v_1 = v_i$ into (4), we have

$$\boxed{\frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}}$$

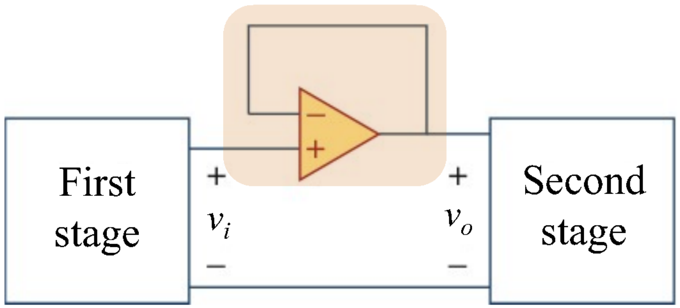
- Notice that the gain of the non-inverting op amp depends only on the external resistors.

Non-inverting Amplifier (cont'd)

- Special case: if $R_f = 0$ or $R_1 = \infty$ or both, the gain becomes 1.
- The resulting configuration is called a **voltage follower** because the output voltage *follows* the input voltage.
- It is used as a buffer amplifier to isolate one circuit from another as illustrated below.



$$\frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$



- Such circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another.

Example (3): Voltage Follower

Question: Determine v_L for the circuits shown in Fig. 3(a) and 3(b).

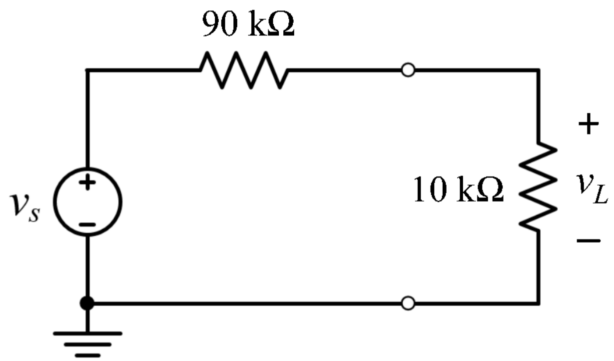


Fig. 3(a)

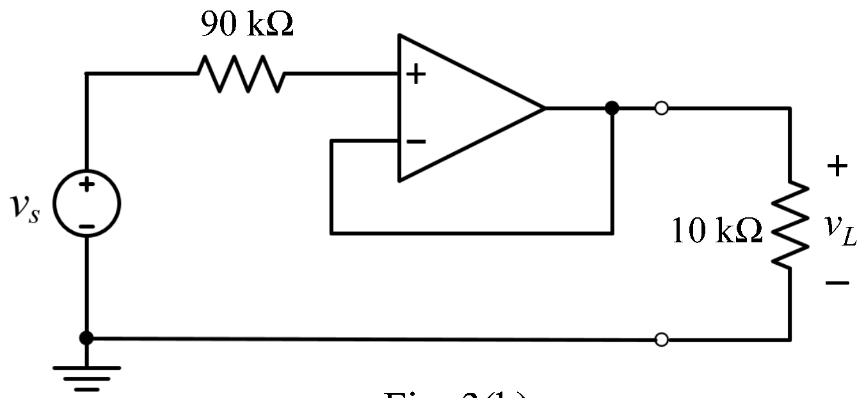


Fig. 3(b)

- In Fig. 3(a), by using voltage division, we can write $v_L = \{10/(10 + 90)\}v_s = 0.1v_s$
- In Fig. 3(b), no current flows across the 90 kΩ resistor due to the very large input resistance of the op amp. By Ohm's Law, zero current means zero voltage drop across this resistor.

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Example (3): Voltage Follower (cont'd)

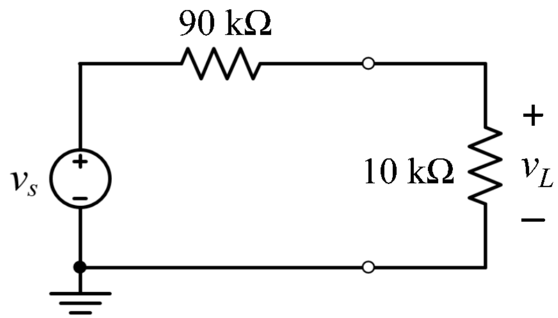


Fig. 3(a)

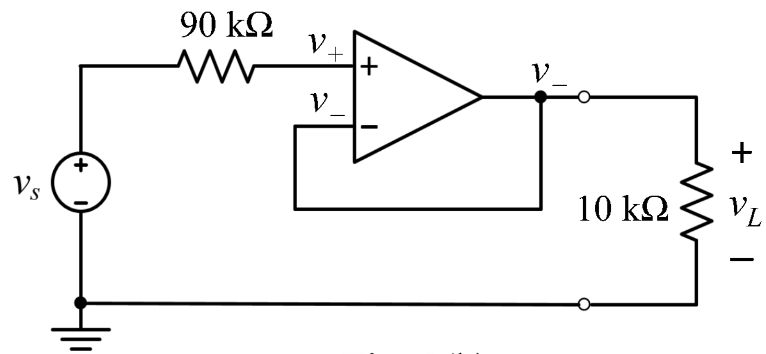
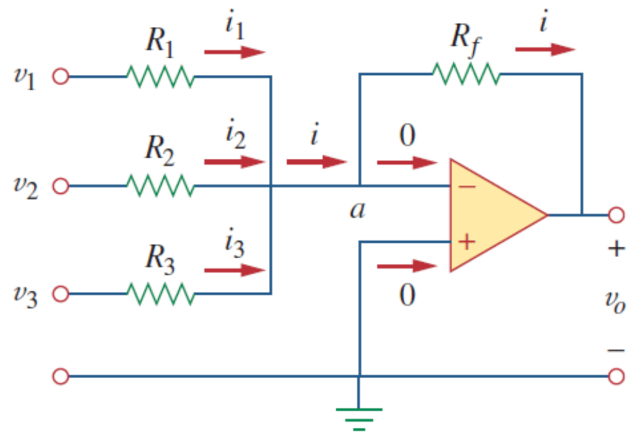


Fig. 3(b)

- The entire source voltage (v_s) appears at the non-inverting (positive) input terminal of the op amp, i.e., $v_+ = v_s$.
- Since $v_+ = v_-$ and $v_- = v_L$, the output voltage of the op amp (v_L) is equal to v_+ (or v_s). In other words, the output voltage of the op amp is equal to the source voltage, i.e., $v_L = v_s$.
- Due to the very low output resistance of the op amp, we can assume that $v_- = v_L$.
- Fig. 3(a) and 3(b) clearly show that the use of a voltage follower results in an increase in the output (load) voltage from $0.1v_s$ to v_s.

Summing Amplifier

- Op amp can also be configured to perform summation.
- Careful examination reveals that this is actually a variation of an inverting amplifier.
- By applying KCL at node a, we can write



$$i = i_1 + i_2 + i_3 \quad => \quad \frac{v_a - v_o}{R_f} = \frac{v_1 - v_a}{R_1} + \frac{v_2 - v_a}{R_2} + \frac{v_3 - v_a}{R_3}$$

- Notice that v_a is at virtual ground (i.e., $v_a = 0$), the above equation can be re-expressed as

$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)} \qquad (5)$$

- In other words, the output voltage is the <u>weighted sum</u> of the three input voltages.
- In general, the summer can be extended to more than three inputs.

Example (4): Summing Amplifier

Question: Calculate v_o and i_o in the op-amp circuit shown in Fig. 4.

- In Fig. 4, observe that this is a summer with two inputs. Suppose v_1 = 2 V and v_2 = 1 V.

- Apply equation (5) in the previous slide, we have

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4+4) = -8 \text{ V}$$

- The output current (i_o) is the sum of the currents flowing across the 10 kΩ and 2 kΩ resistors. Since $v_a = v_b = 0$, the voltage across these two resistors are the same (i.e., $v_o = -8$ V). Hence, i_o can be expressed as follows.

$$i_o = \frac{v_o - 0}{10000} + \frac{v_o - 0}{2000} = (-0.8 - 4) \text{ mA} = -4.8 \text{ mA}$$

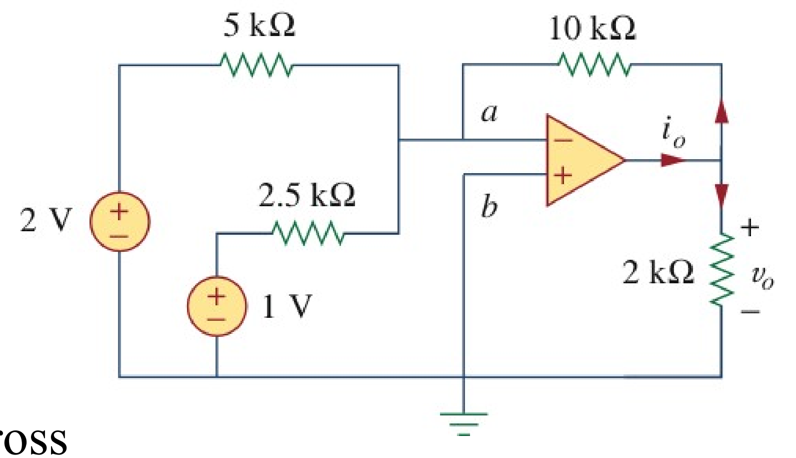
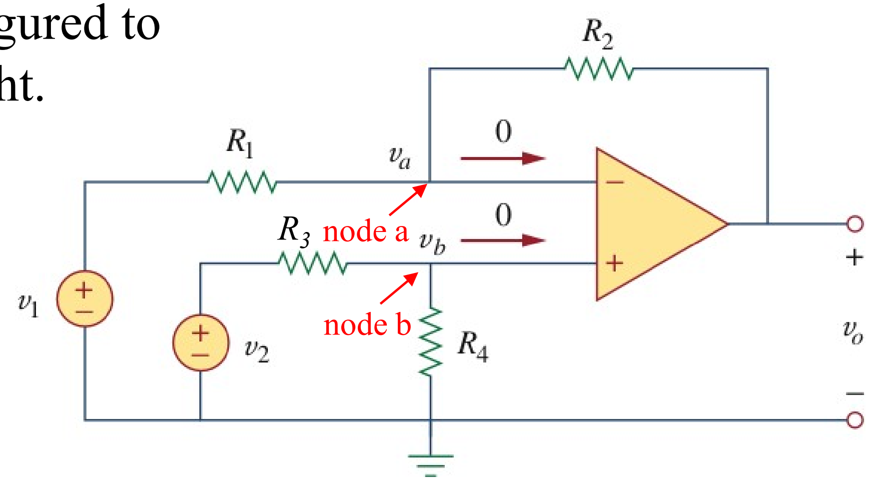


Fig. 4

Difference Amplifier

- Besides addition, the op-amp circuit can also be configured to perform subtraction, as shown in the circuit on the right.



- By applying KCL to node a, we have

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \quad \Rightarrow \quad v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1 \qquad \text{(6a)}$$

- By applying KCL to node b, we have

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4} \quad \Rightarrow \quad v_b = \frac{R_4}{R_3 + R_4} v_2 \qquad \text{(6b)}$$

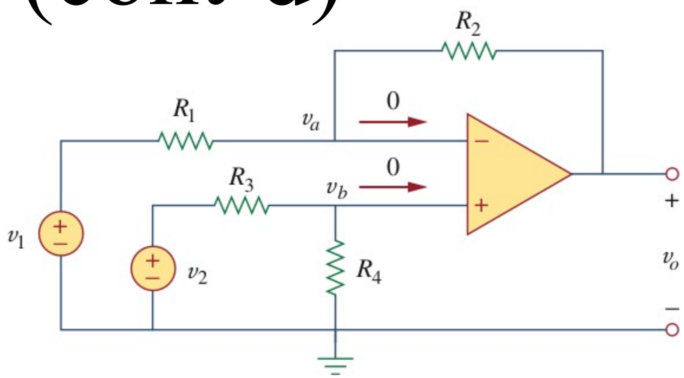
- Since $v_a = v_b$, substituting (6b) into (6a) yields

$$\boxed{v_o = \left[\frac{R_2\left(1 + \frac{R_1}{R_2}\right)}{R_1\left(1 + \frac{R_3}{R_4}\right)}\right] v_2 - \left(\frac{R_2}{R_1}\right) v_1}$$

This circuit performs weighted subtraction.

Difference Amplifier (cont'd)

$$v_o = \left[\frac{R_2\left(1+\frac{R_1}{R_2}\right)}{R_1\left(1+\frac{R_3}{R_4}\right)}\right]v_2 - \left(\frac{R_2}{R_1}\right)v_1$$



- Suppose we want the circuit to perform simple subtraction (e.g. $v_o = v_2 - v_1$). Obviously, $v_o = 0$ when $v_1 = v_2$. This condition is satisfied when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

- Therefore, the input-output relationship becomes $v_o = \frac{R_2}{R_1}(v_2 - v_1)$.
- In particular, by setting $R_1 = R_2$, we have $v_o = (v_2 - v_1)$.
- In short, a difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Example (5): Difference Amplifier

Question: Design an op-amp circuit with input v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Approach 1: Use only a single op amp [see Fig. 5(a)]

Recall this expression:

$$v_o = \left[\frac{R_2\left(1+\frac{R_1}{R_2}\right)}{R_1\left(1+\frac{R_3}{R_4}\right)} \right] v_2 - \left(\frac{R_2}{R_1}\right) v_1$$

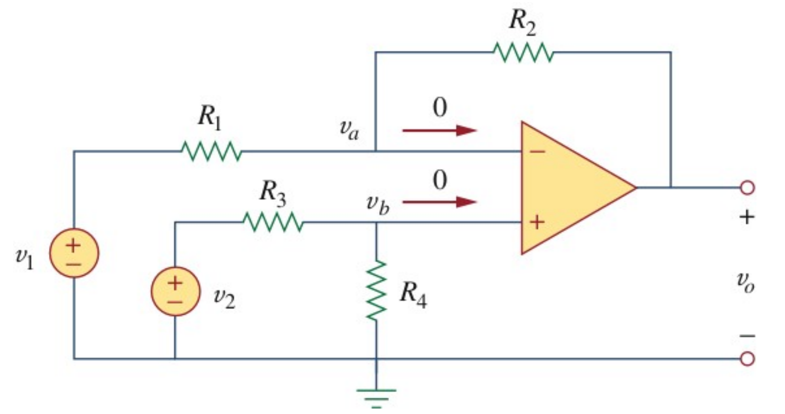


Fig. 5(a)

Compare the above expression with $v_o = 3v_2 - 5v_1$, we have

$$\left.\begin{cases} \dfrac{R_2}{R_1} = 5 \\[2ex] \dfrac{R_2\left(1+\frac{R_1}{R_2}\right)}{R_1\left(1+\frac{R_3}{R_4}\right)} = 3 \end{cases} \Rightarrow \begin{cases} R_2 = 5R_1 \\ R_3 = R_4 \end{cases}\right\}$$

Suppose we choose R_1 = 10 kΩ, R_3 = 20 kΩ.
Then, R_2 = 50 kΩ and R_4 = 20 kΩ.

Example (5): Difference Amplifier (cont'd)

Question: Design an op-amp circuit with input v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Approach 2: Use two op amps [see Fig. 5(b)]

We may cascade an inverting op amp with a two-input inverting summer as shown below.

- For the summer, $v_o = -v_a - 5v_1$.
- For the inverter, $v_a = -3v_2$.
- Combining the above two equations yields $v_o = 3v_2 - 5v_1$.
- For actual implementation, we may select $R_1 = R_3 = 10\ \text{k}\Omega$.

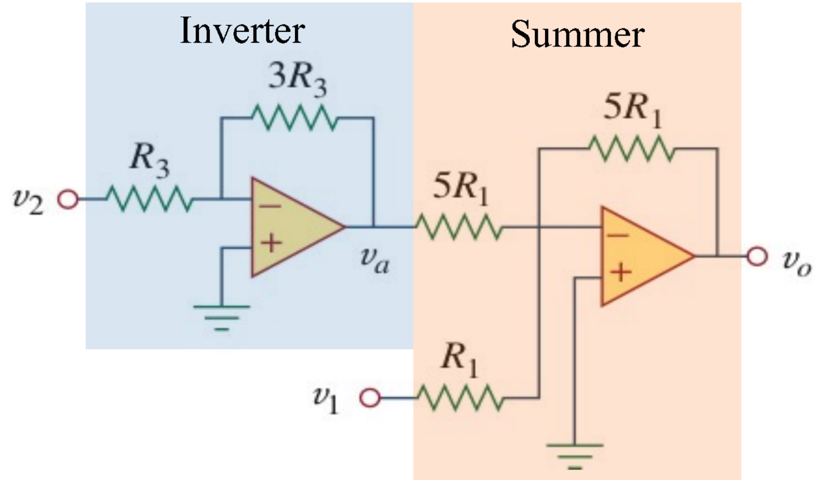


Fig. 5(b)

Example (6): Instrumentation Amplifier

Question: An instrumentation is a type of differential amplifier that has been outfitted with input buffer amplifiers, which eliminate the need for input impedance matching and thus make it particularly suitable for use in measurement applications and test equipment. Given the circuit in Fig. 6, show that

$$v_o = \frac{R_2}{R_1}\left(1+\frac{2R_3}{R_4}\right)(v_2 - v_1)$$

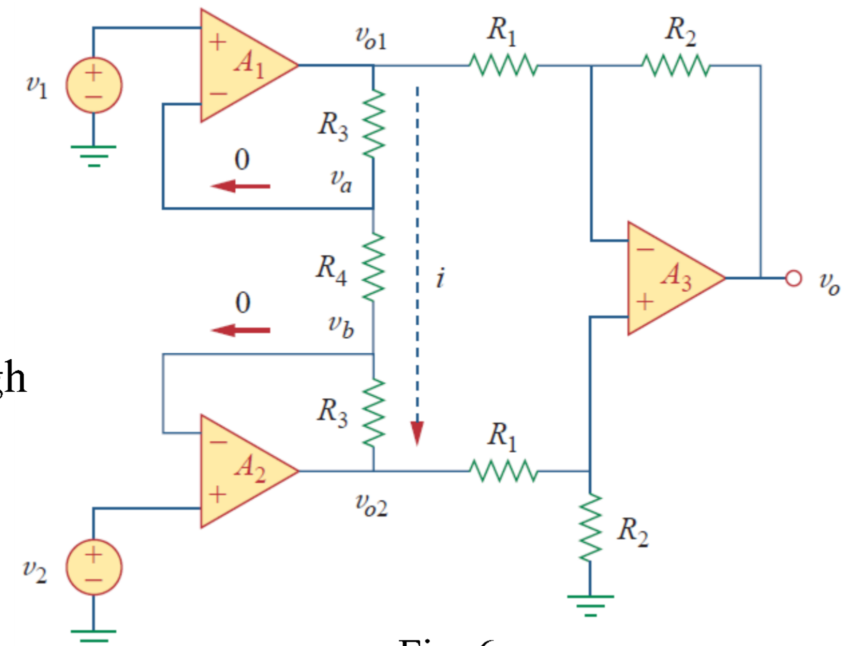


Fig. 6

- First, notice that A3 is a difference amplifier whose equation is given by:

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1}) \qquad (7a)$$

- Second, since A1 and A2 draw no current, current I flows through the three resistors as though they are connected in series:

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4) \qquad (7b)$$

- Since $v_1 = v_a$, $v_2 = v_b$, and $i = \dfrac{v_a - v_b}{R_4}$, $\quad i = \dfrac{v_1 - v_2}{R_4}$ (7c)

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Example (6): Instrumentation Amplifier (cont'd)

- By substituting equation (7c) into (7b), we have

$$v_{o1} - v_{o2} = \left(\frac{v_1 - v_2}{R_4} \right)(2R_3 + R_4) \quad (7d)$$

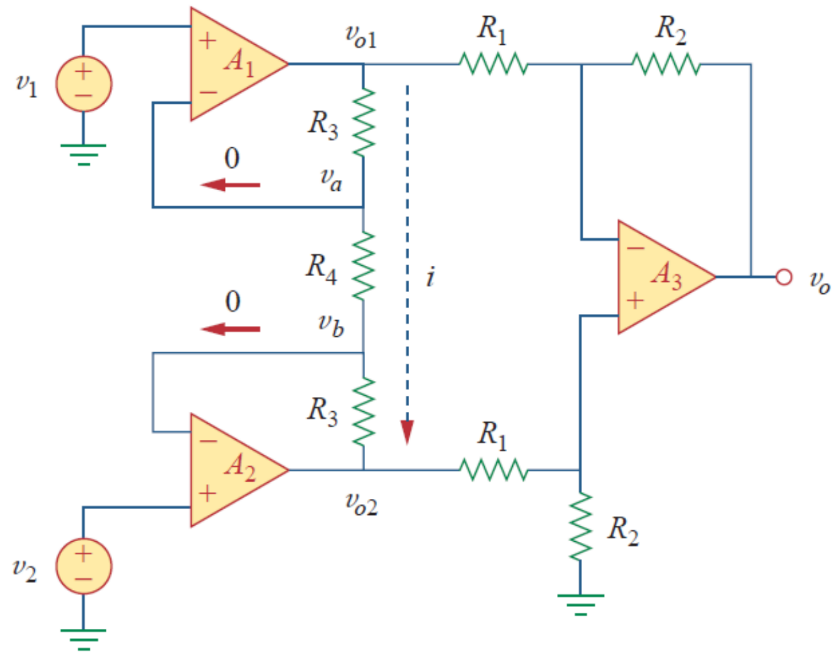
- By substituting equation (7d) into (7a), we have

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1}) = \frac{R_2}{R_1}\left(\frac{v_2 - v_1}{R_4} \right)(2R_3 + R_4) \quad (7e)$$

- By re-arranging equation (7e), we can write

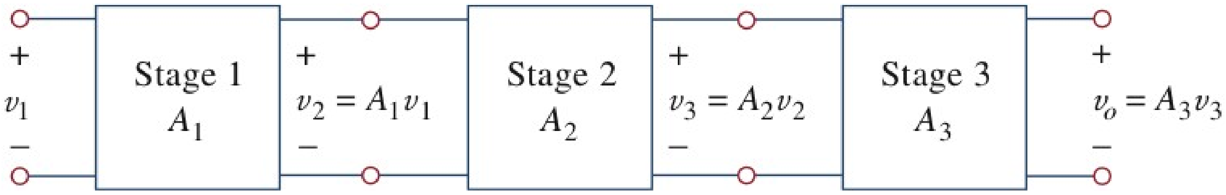
$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4} \right)(v_2 - v_1)$$

Question: Show that $v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4} \right)(v_2 - v_1)$



Cascaded Op Amp Circuits

- Op amp circuits can be cascaded to produce a large overall gain.



- Op amp circuits carry the advantage that they can be cascaded without changing their input-output relationships.
- This is attributed to the fact that each (ideal) op amp circuit has an <u>infinite input resistance</u> and <u>zero output resistance</u>.
- For the three-stage op-amp circuit shown above, the total gain (A) given by: $A = A_1 A_2 A_3$.

Example (7): Cascaded Op Amp Circuit

Question: Determine v_o and i_o in the op-amp circuit, as shown in Fig. 7.

Realize that this is a cascade of two non-inverting amplifiers.

$$\boxed{\frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}}$$

$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}, \quad v_o = \left(1 + \frac{10}{4}\right) v_a = (3.5)(100) = 350 \text{ mV}$$

$$i_o = \frac{v_o - v_b}{10000} = \frac{v_o - v_a}{10000} = \frac{0.350 - 0.100}{10000} = 25\ \mu\text{A}$$

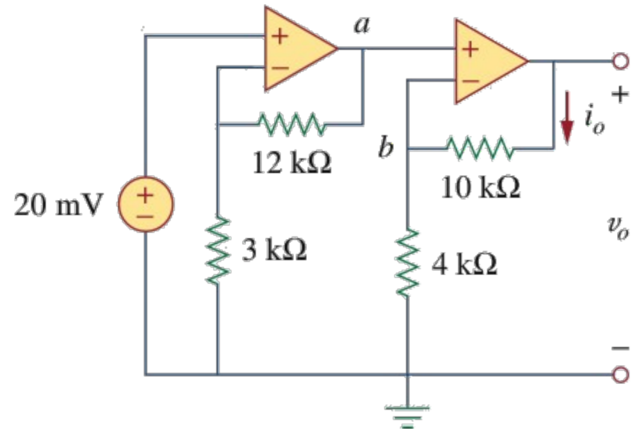


Fig. 7

Example (8): Cascaded Op Amp Circuit

Question: Given $v_1 = 1$ V and $v_2 = 2$ V, as shown in Fig. 8.

Notice that A and B are inverting amplifiers. Hence, we have

$$v_a = -\frac{6}{2}v_1 = -3v_1$$

$$v_b = -\frac{8}{4}v_2 = -2v_2$$

In addition, C is a summing circuit. Thus, we can write

$$v_o = -\left(\frac{10}{5}v_a + \frac{10}{15}v_b\right) = -\left[2(-3v_1) + \frac{2}{3}(-2v_2)\right] = 6v_1 + \left(\frac{4}{3}\right)v_2$$

By substituting $v_1 = 1$ V and $v_2 = 2$ V,
$v_o \approx 8.667$ V.

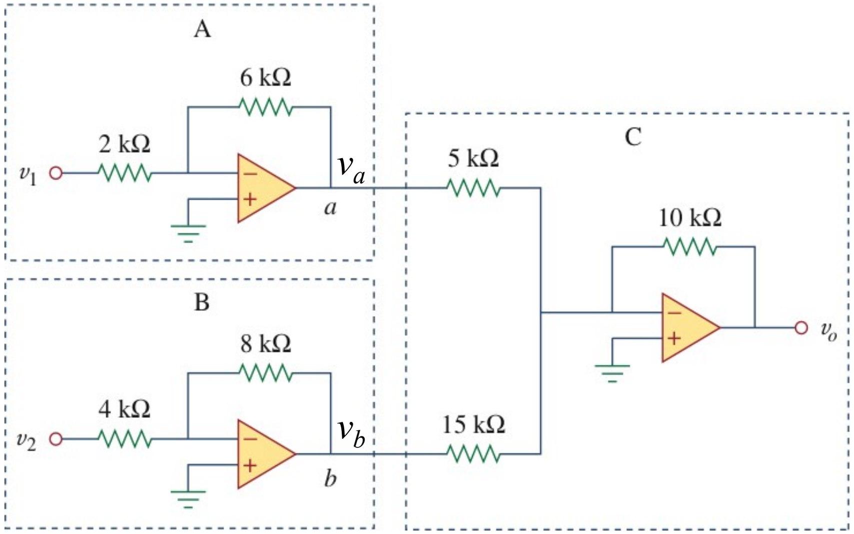
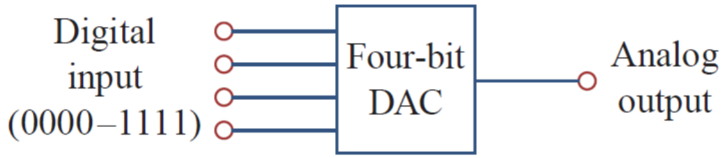
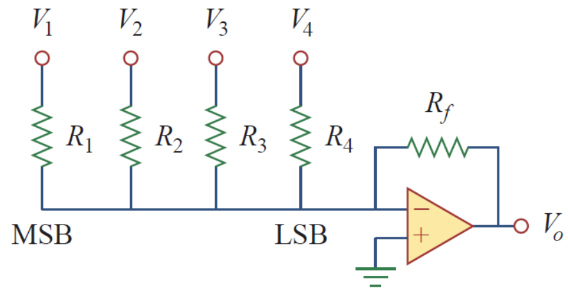


Fig. 8

Application: Digital-to-Analog Converter

- The digital-to-analog converter (DAC) is an IC or system that converts a digital signal into an analog signal. It can be implemented by an inverting summing amplifier.





Inverting summer amplifier

- Input V_1 is called the most significant bit (MSB) and V_4 is the least significant bit (LSB).
- Each of the four binary inputs (V_1, V_2, V_3, V_4) can assume only two voltage levels (either 0 V or 1 V). By choosing proper values for the input and feedback resistors, the DAC provides a single output that is proportional to the inputs.

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

Example (9): Digital-to-Analog Converter

Question: In the op-amp circuit shown in Fig. 9, let R_f = 10 kΩ, R_1 = 10 kΩ, R_2 = 20 kΩ, R_3 = 40 kΩ, and R_4 = 80 kΩ. Determine the voltage levels of the analog output corresponding to binary inputs (0000), (0001), (0010),…, (1111), respectively.

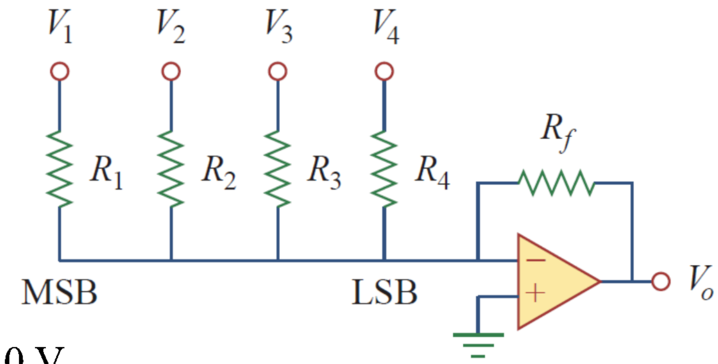


Fig. 9

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4 = V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

$$V_o = -V_1 - 0.5V_2 - 0.25V_3 - 0.125V_4$$

A digital input $(V_1V_2V_3V_4)$ = (0000) produces an analog output voltage of V_o = 0 V

$(V_1V_2V_3V_4)$ = (0001) => V_o = –0.125 V

$(V_1V_2V_3V_4)$ = (0010) => V_o = –0.25 V

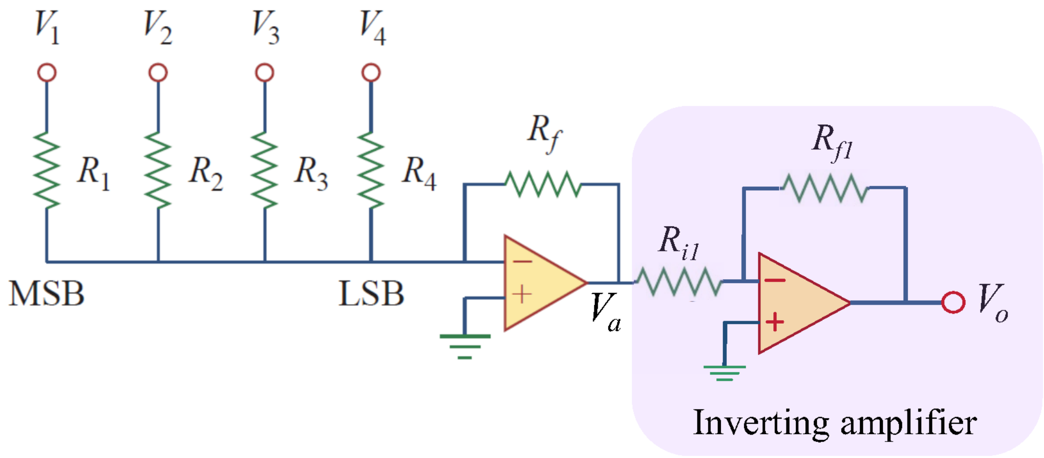
$(V_1V_2V_3V_4)$ = (0011) =>V_o = –0.25 – 0.125 = –0.375 V

$(V_1V_2V_3V_4)$ = (0100) => V_o = –0.50 V

⋮

$(V_1V_2V_3V_4)$ = (1111) => V_o = –1.875 V

Example (9): Digital-to-Analog Converter (cont'd)



$$R_{i1} = R_{f1}$$

$$V_o = -V_a$$

$$V_o = V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

Binary input $[V_1V_2V_3V_4]$	**Decimal value**	**Output** V_o
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875