Basic Circuit Laws

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Basic Circuit Laws

I. Ohm's Law

II. Kirchhoff's Laws

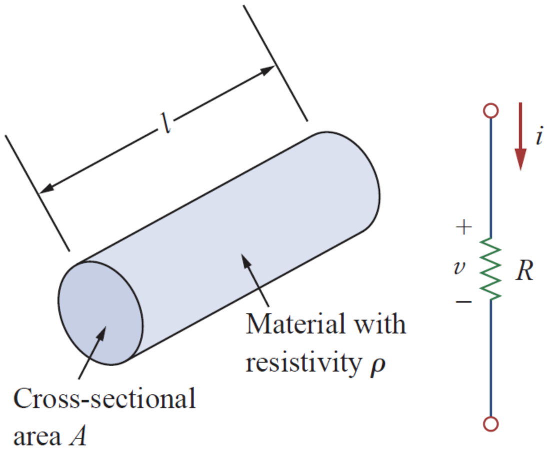
III. Voltage and Current Division Formulas

IV. Wye-Delta Transformations

Resistance

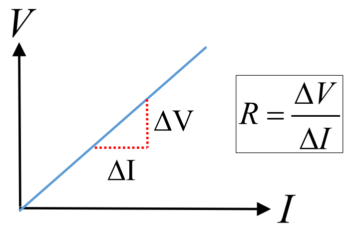
- In general, materials have a characteristic behaviour of resisting the flow of electric charge.
- This physical property, or ability to resist current, is called resistance and is represented by the symbol R.
- The resistance of any material with a uniform cross-sectional area A depends on A, its length l, and the resistivity ρ of the material.
- Mathematically, the resistance R is given by:

$$R = \rho \frac{l}{A}$$

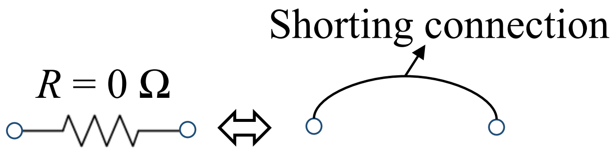


Ohm's Law

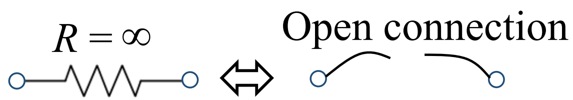
- Ohm's law states that the voltage V across a resistor R is directly proportional to the current I flowing through the resistor R.



- Mathematically, $V = IR$. The unit of R is ohm (Ω).
- The proportionality constant R (resistance) denotes the ability to resist the flow of electric current.
- $R = 0$ means short circuit.



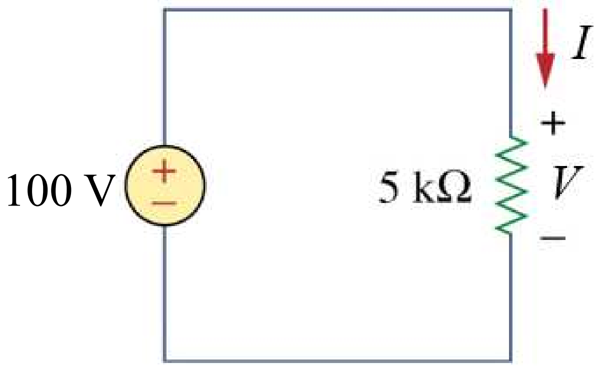
- $R = \infty$ means open circuit.



- The power dissipated by a resistor R is: $P = VI = I^2 R = \dfrac{V^2}{R}$
- Conductance is defined as $G = \dfrac{1}{R}$. The unit of G is siemens (S).

Example (1)

(1) Calculate the current *I*, the conductance *G* and the power *P* dissipated by the resistor.



(a) $I = \dfrac{V}{R} = \dfrac{100}{5000} = 20\text{ mA}$

(b) $G = \dfrac{1}{R} = \dfrac{1}{5000} = 0.2\text{ mS}$

(c) $P = VI = 100(0.02) = 2\text{ W}$

(2) The DC voltage source is now replaced by an AC voltage source of $v(t) = 50\sin\pi t$ V. Find the current across the resistor and the power dissipated.

(a) $i(t) = \dfrac{v(t)}{R} = \dfrac{50\sin\pi t}{5000} = 10\sin\pi t\text{ mA}$

(b) $p(t) = v(t)i(t) = 0.5\sin^2\pi t\text{ W}$

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Branch, Node and Loop

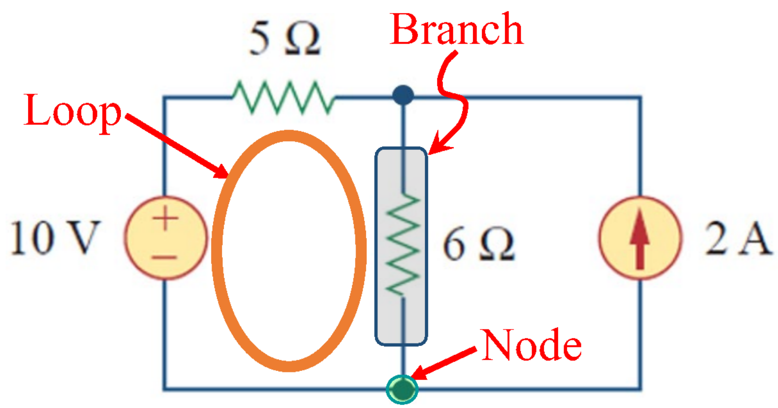
Definitions:

Branch – A branch is an element (e.g. source, resistor, inductor, capacitor, etc.). The number of branches in a circuit is equal to the number of elements.

Node – A node is a junction where two or more elements connect.

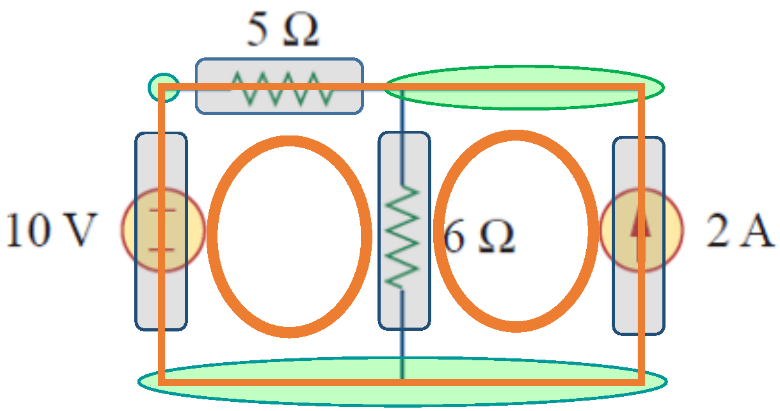
Loop – A loop is any closed path going through circuit elements.

In particular, a mesh is a loop that has no other loops inside it.



Branch, Node and Loop

How many branches, nodes and loops does the following circuit have ?



(a) Since there are 4 elements, there are **4 branches**.

(b) Since there are 3 junctions, there are **3 nodes**.

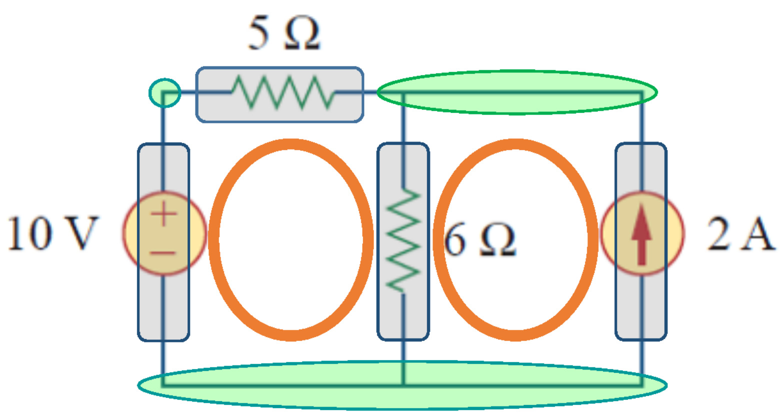
(c) Since there are 3 closed paths, there are **3 loops**.

Question: How many mesh loops does this circuit have ?

Branch, Node and Loop

- A loop is said to be independent if it contains at least one branch which is **not** a part of any other independent loop.
- A network with b branches, n nodes and l independent loops will satisfy the fundamental theory of network topology:

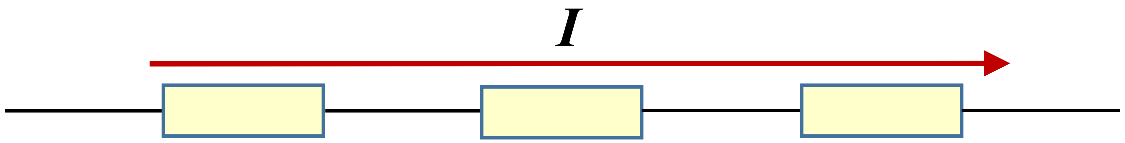
$$b = l + n - 1$$



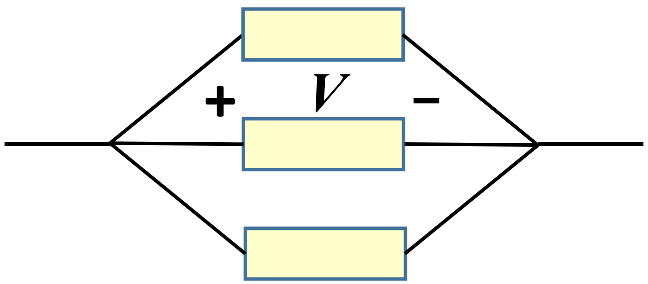
$$b = 4$$
$$n = 3$$
$$l = 2$$

Series or Parallel Connection

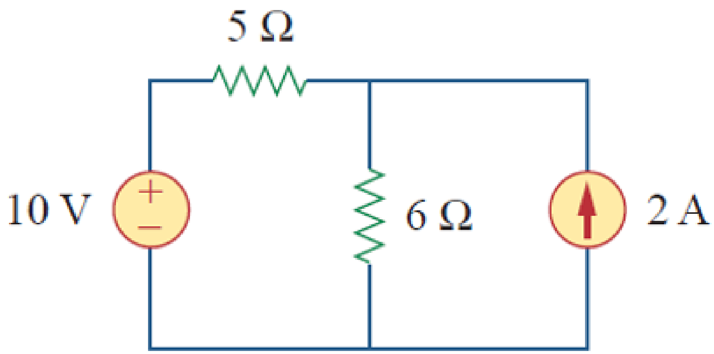
- Two or more elements are in **series** if they are cascaded or connected sequentially and consequently carry the *same current*.



- Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the *same voltage* across them.

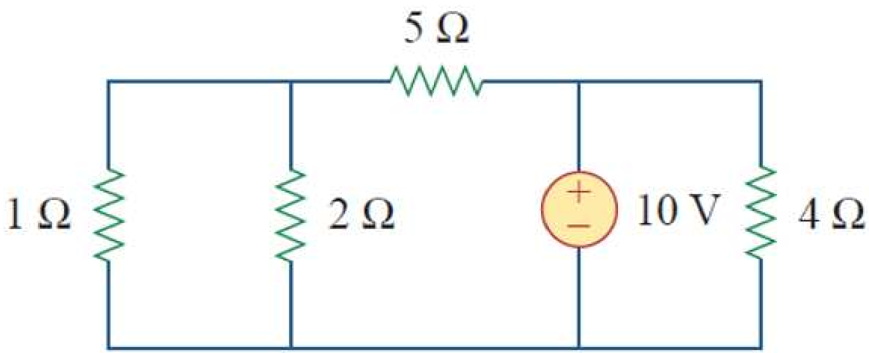


Examples of Series/Parallel Connection



The 5-Ω resistor is in **series** with the 10-V voltage source.

The 6-Ω resistor is in **parallel** with the 2-A current source.



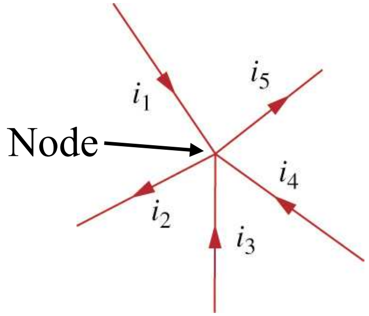
1-Ω and 2-Ω resistors are connected in **parallel**.

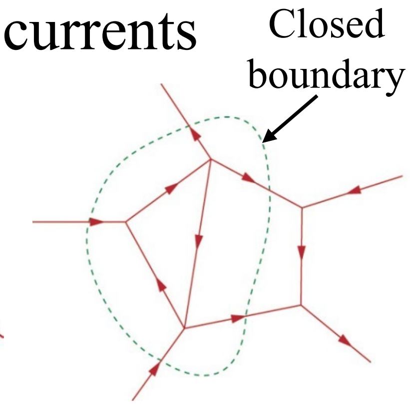
The 4-Ω resistor and 10-V source are also connected in **parallel**.

Kirchhoff's Current Law (KCL)

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a **node** (or **a closed boundary**) is zero.

$$\sum_{n=1}^{N} i_n = 0$$

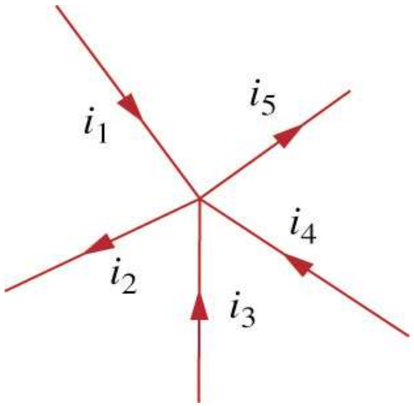




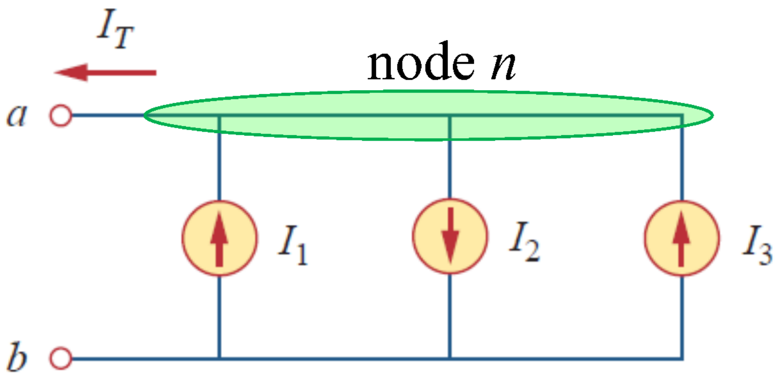
- Alternative form of KCL: The sum of the currents **entering** a node is equal to the sum of the currents **leaving** the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

$$\Rightarrow \sum_{n=1}^{5} i_n = i_1 - i_2 + i_3 + i_4 - i_5 = 0$$



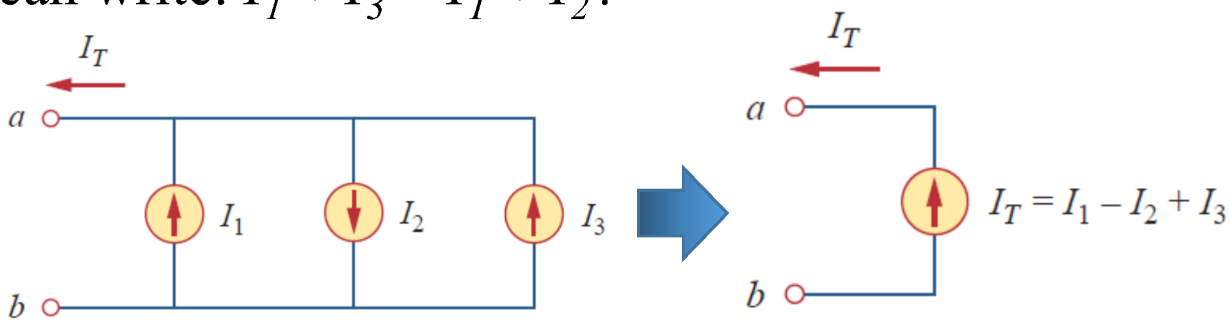
Simple Application of KCL



The sum of the currents entering node n is: $I_1 + I_3$ whereas the sum of the currents leaving node n is: $I_T + I_2$.

By applying KCL to node n, we can write: $I_1 + I_3 = I_T + I_2$.

Hence, $I_T = I_1 - I_2 + I_3$

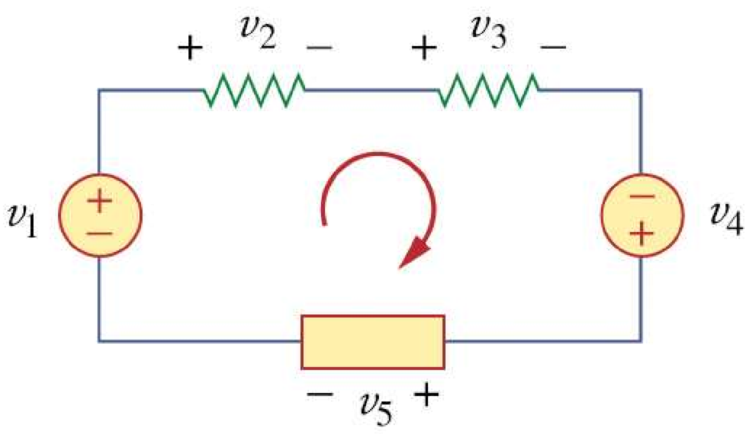


Kirchhoff's Voltage Law (KVL)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0$$

- Alternative form of KVL: Sum of the voltage drops = sum of voltage rises



Sum of voltage drops = $v_2 + v_3 + v_5$

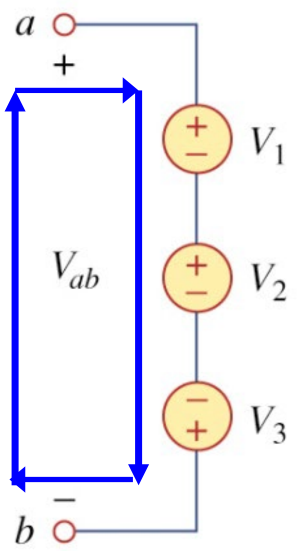
Sum of voltage rises = $v_1 + v_4$

Hence, by applying KVL, we have

$v_2 + v_3 + v_5 = v_1 + v_4$

$$\sum_{m=1}^{5} v_m = -v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Simple Application of KVL



Rule of Thumb:
Before applying KVL (or KCL), we need to define what is positive and what is negative. Then we stick with the convention for the sake of consistency.

Step 1: Define positive for voltage drop and negative for voltage rise across an element.

Step 2: Choose a consistent direction (either clockwise or counterclockwise) to go around a loop.

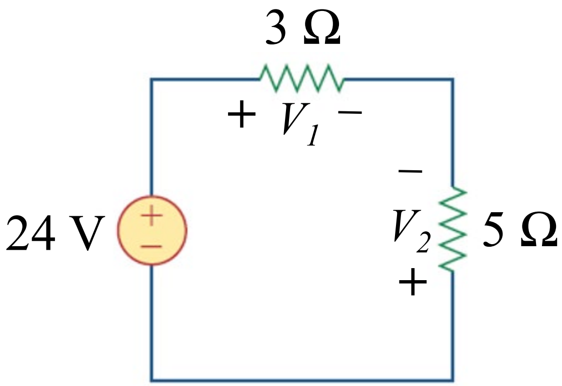
Suppose we choose a clockwise direction around the loop as shown in the above circuit.

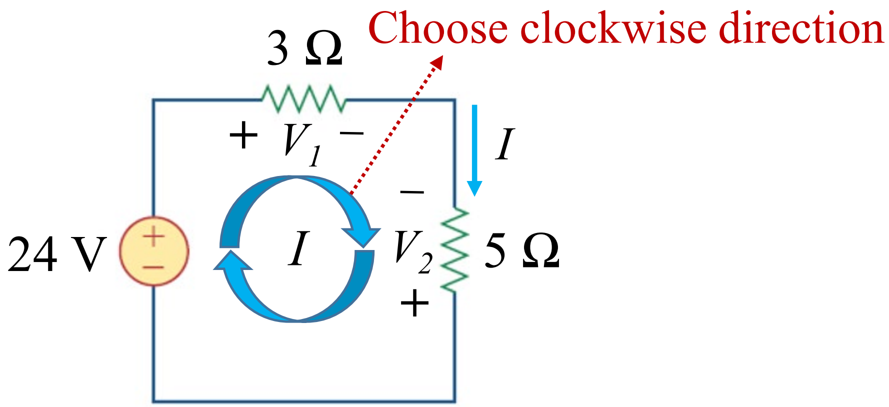
By applying KVL, we have $-V_{ab} + V_1 + V_2 - V_3 = 0$

By re-arranging the terms, $V_{ab} = V_1 + V_2 - V_3$

Example (2)

Find the voltages V_1 and V_2 in the following circuit.



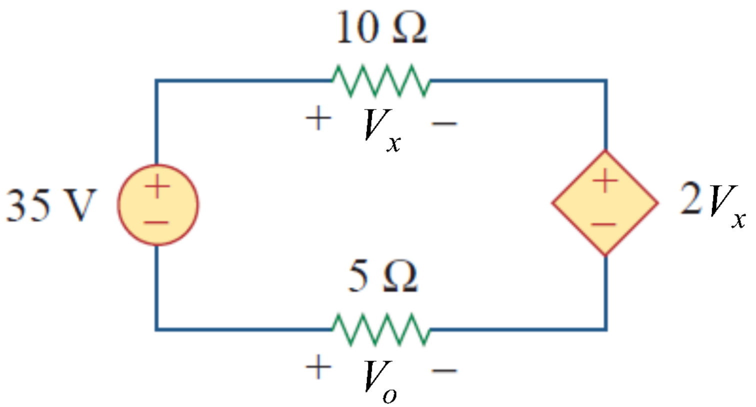


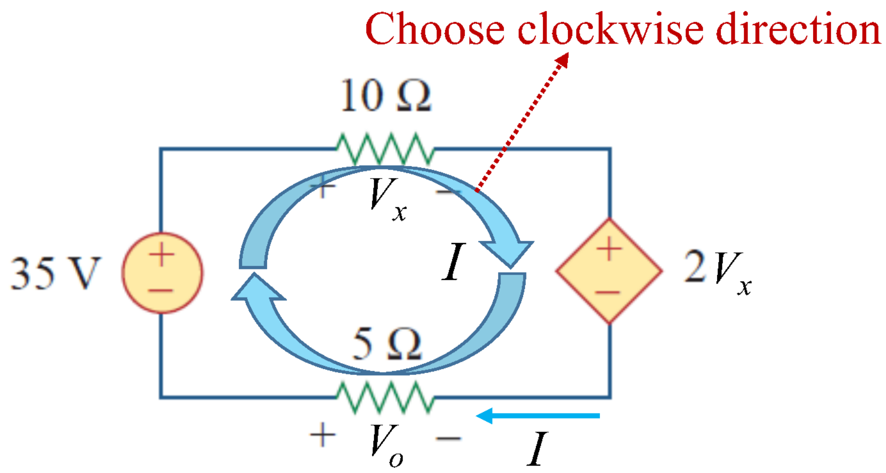
1. By applying KVL (in a clockwise direction) around the loop, we have
   $-24 + V_1 - V_2 = 0$

2. By using Ohm's law, $V_1 = 3I$; $V_2 = -5I$.
   $-24 + 3I - (-5I) = 0 \Rightarrow I = 3$ A
   Hence, $\boldsymbol{V_1 = 9}$ **V** and $\boldsymbol{V_2 = -15}$ **V**

Example (3)

Determine V_o and V_x in the circuit below.



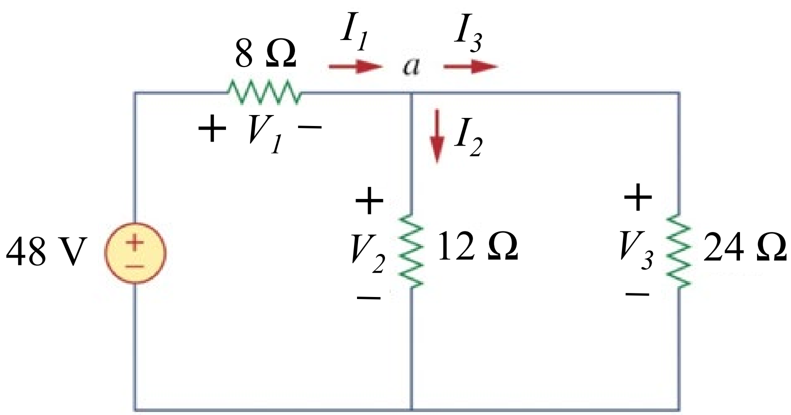


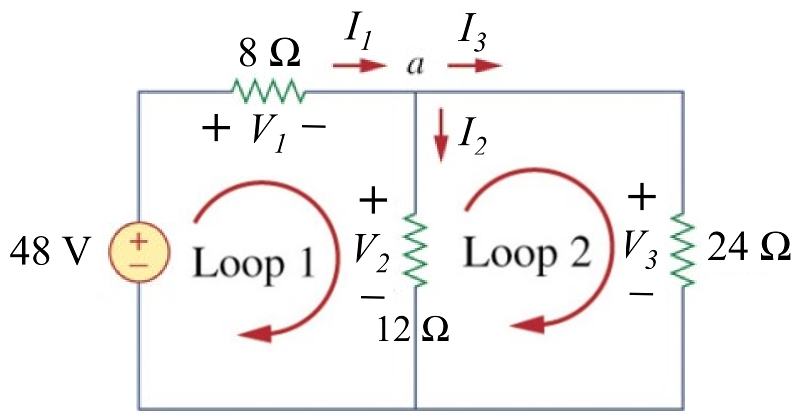
1. By applying KVL (in a clockwise direction) around the loop, we have
   $-35 + V_x + 2\,V_x - V_o = 0$

2. By using Ohm's law, $V_x = 10I$; $V_o = -5I$.
   $-35 + 10I + 2(10I) - (-5I) = 0 \;\Rightarrow I = 1\text{ A}$
   Hence, $\mathbf{V_x = 10\ V}$ and $\mathbf{V_o = -5\ V}$

Example (4)

Find the currents (I_1, I_2, I_3) and voltages (V_1, V_2, V_3) in the following circuit.





1. Apply KVL to loop 1: $-48 + 8I_1 + 12I_2 = 0$ (1)
2. Apply KVL to loop 2: $-12I_2 + 24I_3 = 0$ (2)
3. Apply KCL to node a: $I_1 = I_2 + I_3$ (3)
4. By solving (1)–(3), we have $I_1 = 3$ A, $I_2 = 2$ A, and $I_3 = 1$ A
5. Hence, $V_1 = 8I_1 = 24$ V, $V_2 = 12I_2 = 24$ V, and $V_3 = 24I_3 = 24$ V

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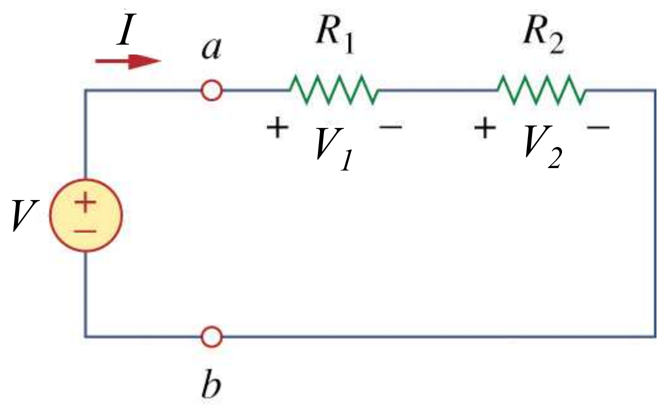
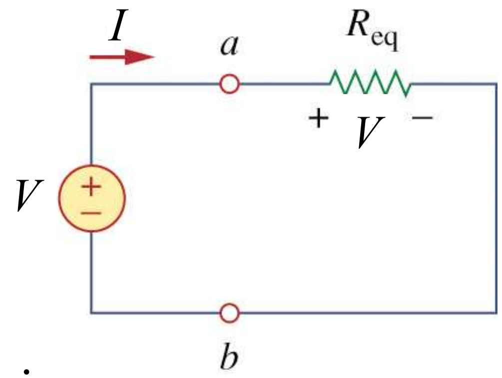
Series Resistors and Voltage Division

Consider a simple series circuit as shown.



- Apply KVL to the loop, we have $V = IR_1 + IR_2 = I(R_1+R_2)$
- Define $R_{eq} = R_1 + R_2$. We can write: $V = IR_{eq}$
- Therefore, the two resistors (R_1, R_2) can be replaced by an equivalent resistor (R_{eq}).



- Since $I = \dfrac{V}{(R_1 + R_2)}$, $V_1 = IR_1 = \left(\dfrac{R_1}{R_1 + R_2}\right)V; V_2 = IR_2 = \left(\dfrac{R_2}{R_1 + R_2}\right)V$

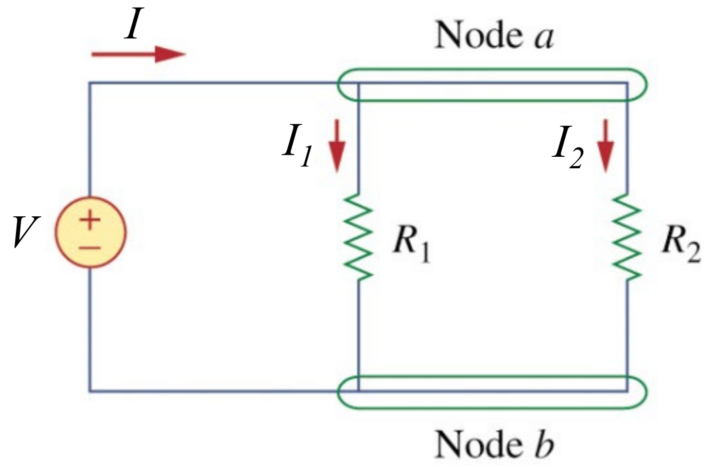
  This is the voltage division formula!

- In general, for N resistors in series, the voltage across the N^{th} resistor is:

$$V_N = \left(\frac{R_N}{R_1 + R_2 + \ldots + R_N}\right)V, \text{ where } R_{eq} = R_1 + R_2 + \ldots + R_N.$$

Parallel Resistors and Current Division

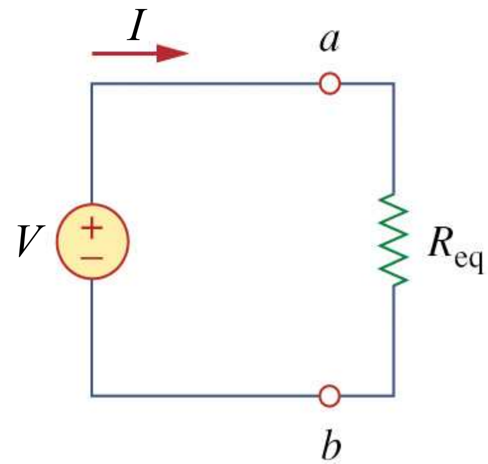
Consider a simple parallel circuit as shown.



- Apply KCL to node a, we have $I = \dfrac{V}{R_1} + \dfrac{V}{R_2}$
- Hence, $I = V\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) = V\left(\dfrac{R_1 + R_2}{R_1 R_2}\right) = \dfrac{V}{R_{eq}}$
- In general, for N resistors in parallel, we can write

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

- Special case 1: Two resistors in parallel, $R_{eq} = \dfrac{\overbrace{R_1 R_2}^{\text{Product}}}{\underbrace{R_1 + R_2}_{\text{Sum}}}$
- Special case 2: N resistors in parallel, $R_{eq} = \dfrac{R}{N}$



Parallel Resistors and Current Division

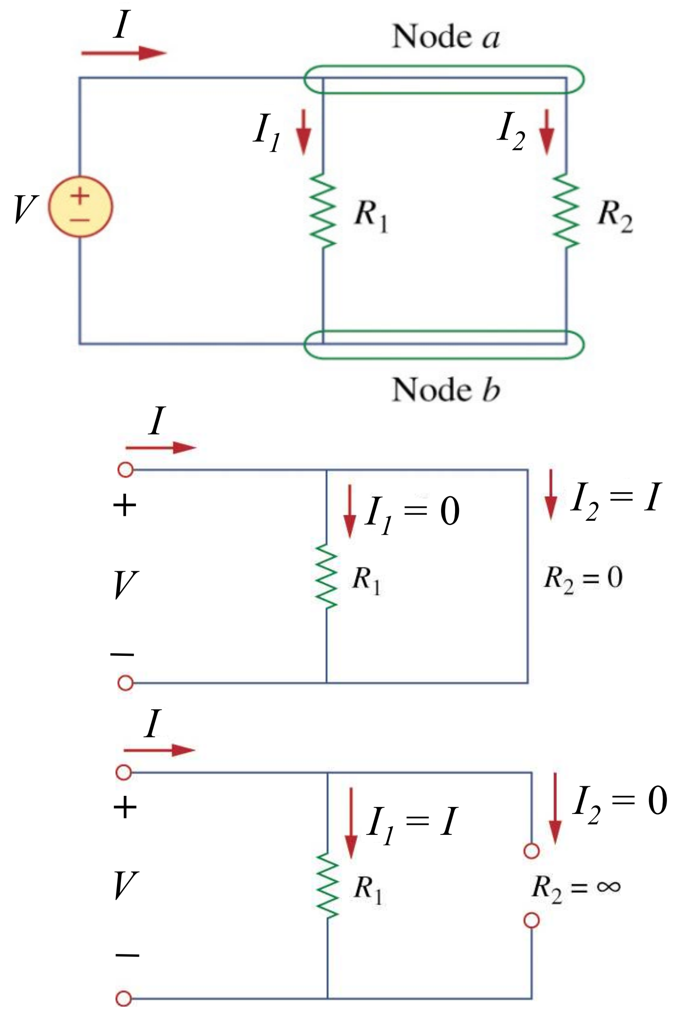
Consider the same circuit from the previous slide,

$$I_1 = \frac{V}{R_1} \text{ and } I_2 = \frac{V}{R_2}$$

- From previous slide, note that $V = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$
- Hence, we have $I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I$ and $I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$

  This is the current division formula!

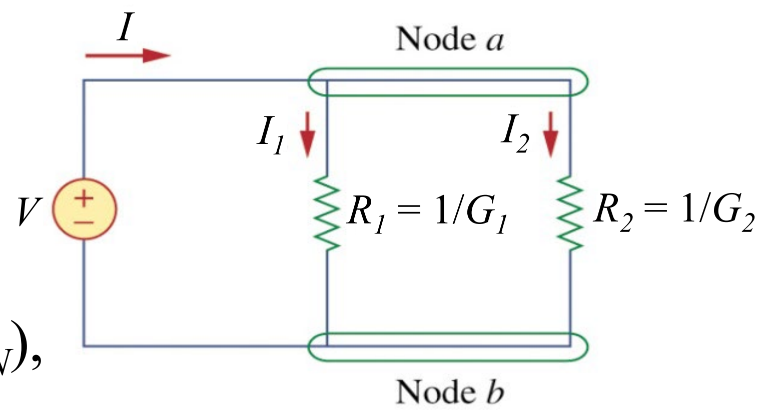
- Extreme case 1: $R_2 = 0 \Rightarrow I_1 = 0, I_2 = I$, and $R_{eq} = 0$.
- Extreme case 2: $R_2 = \infty \Rightarrow I_1 = I, I_2 = 0$, and $R_{eq} = R_1$.



Parallel Resistors and Current Division

- It is often more convenient to use conductance (G) when dealing with resistors in parallel because G is the reciprocal of R.
- Since $G = 1/R$, for N conductors in parallel, we have $G_{eq} = G_1 + G_2 + \ldots + G_N$.
- For *two* conductors in parallel, we have

$$I_1 = \left(\frac{G_1}{G_1 + G_2}\right) I \quad \text{and} \quad I_2 = \left(\frac{G_2}{G_1 + G_2}\right) I$$



- In general, for N conductors in parallel (G_1, G_2, …, G_N), the current across the N^{th} conductor is given by:

$$I_N = \left(\frac{G_N}{G_1 + G_2 + \ldots + G_N}\right) I$$

Example (5): Equivalent Resistance

Find the equivalent resistance R_{ab} in this circuit.

(1) The 3 Ω and 6 Ω resistors are in parallel.
 => Equivalent resistance is 2 Ω.

(2) The 12 Ω and 4 Ω resistors are in parallel.
 => Equivalent resistance is 3 Ω.

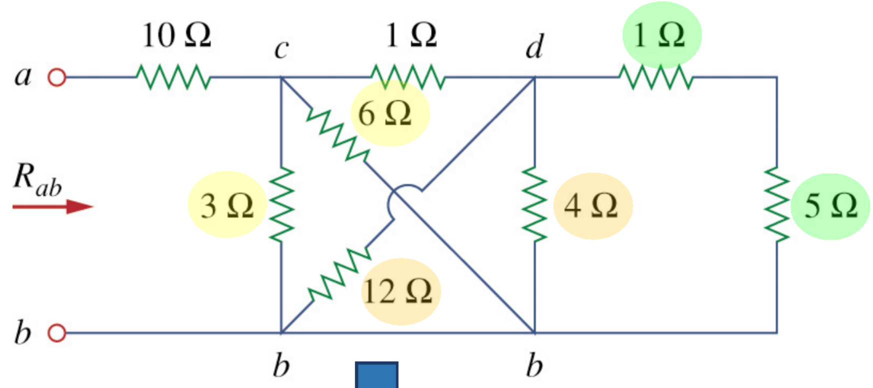
(3) The 1 Ω and 5 Ω resistors are in series.
 => Equivalent resistance is 6 Ω.

(4) The original circuit can be reduced to that of Fig. 1.

(5) Now, the 3 Ω and 6 Ω resistors are in parallel.
 => Equivalent resistance is 2 Ω.

(6) The 1 Ω and the equivalent 2 Ω are in series.
 => Equivalent resistance is 3 Ω (see Fig. 2).

(7) Finally, $R_{ab} = 10\ \Omega + (2\ \Omega \parallel 3\ \Omega) = \mathbf{11.2\ \Omega}$.



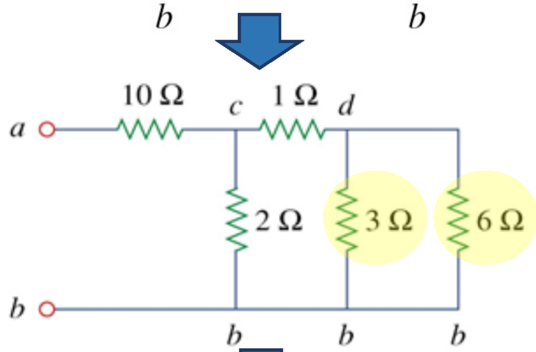


Fig. 1

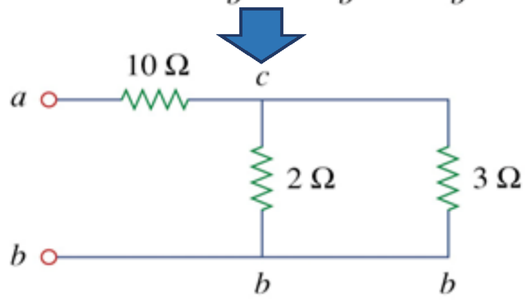


Fig. 2

Example (6): Equivalent Conductance

Find the equivalent conductance G_{eq} in this circuit.

(1) The 8 S and 12 S are in parallel.
=> Equivalent conductance is: 8 + 12 = 20 S.

(2) Now, the 5 S is in series with the equivalent 20 S (see Fig. 2).
Hence, the combined conductance is: (5)(20) / (20 + 5) = 4 S.

(3) This 4 S is in parallel with 6 S.
Hence, $G_{eq} = 4 + 6 = \mathbf{10\ S}$.

Note: Since $G = 1/R$, the circuit in Fig. 1 is the same as that in Fig. 3, where the conductances are expressed in resistances.

$$R_{eq} = \frac{1}{6} \| \left(\frac{1}{5} + \frac{1}{8} \| \frac{1}{12} \right) = \frac{1}{6} \| \left(\frac{1}{5} + \frac{1}{20} \right) = \frac{1}{6} \| \frac{1}{4} = \frac{1}{10} \Omega$$

Therefore, $G_{eq} = 1/R_{eq} = 10$ S

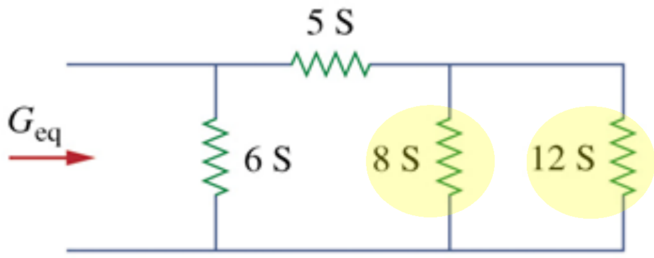


Fig. 1

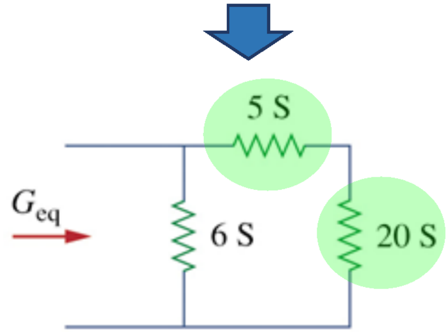


Fig. 2

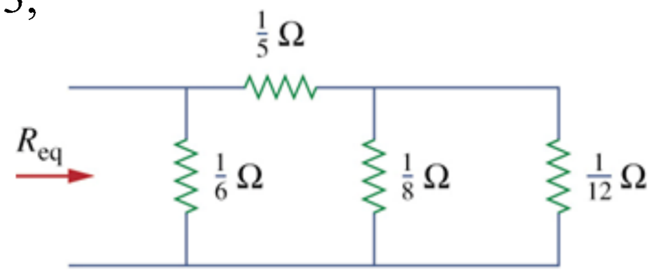
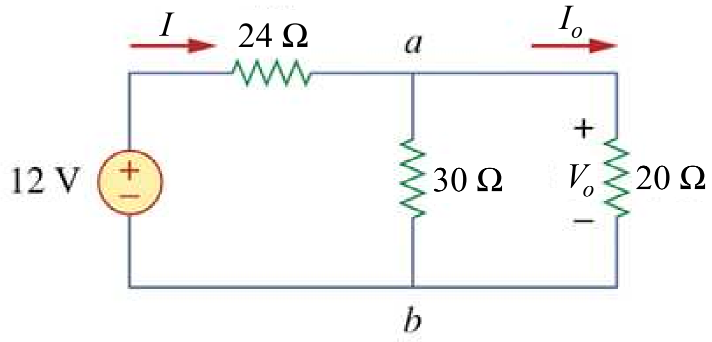


Fig. 3

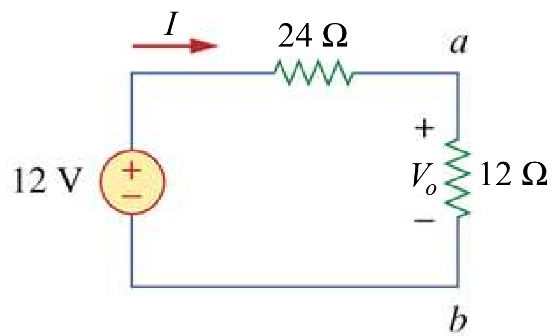
Example (7): Voltage and Current Division Formulas

Find V_o, I_o, and the power dissipated in the 20 Ω resistor.

- The 30 Ω and 20 Ω resistors are connected in parallel.
  So, their combined resistance is: (30)(20) / (30 + 20) = 12 Ω.
- The circuit in (a) is reduced to that in (b).
- There are two ways of obtaining V_o.
  (a) Ohm's law: $I = 12/(24+12) = 333.33$ mA => $V_o = 12I = 4$ V
  (b) Voltage division formula: $$V_o = (12)\left(\frac{12}{24+12}\right) = 4\,\text{V}$$
- Likewise, there are two ways of obtaining I_o.
  (a) Ohm's law: $V_o = 20I_o$ => $I_o = V_o/20 = 4/20 = 1/5$ A
  (b) Current division formula: $$I_o = I\left(\frac{30}{30+20}\right) = \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) = \frac{1}{5}\,\text{A}$$
- Power dissipated in the 20 Ω resistor is: $P_o = V_o I_o = (4)(1/5) = 0.8$ W



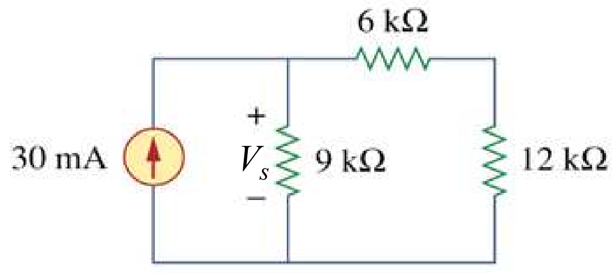
(a)



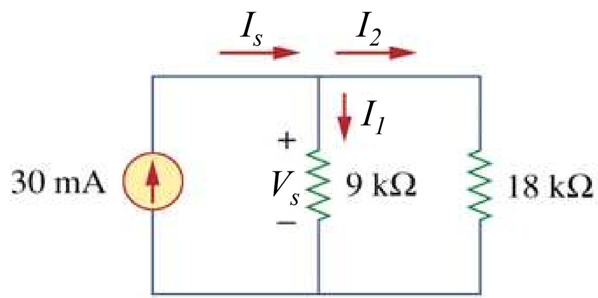
(b)

Example (8): Voltage and Current Division Formulas

Determine V_s, the power supplied by the current source, and the power dissipated by each resistor in Fig. (a).



(a)



(b)

- The 6 kΩ and 12 kΩ resistors are connected in series. So, their combined resistance is: (6 + 12) kΩ = 18 kΩ.
- The circuit in (a) is reduced to that in (b).
- Apply the current division, we have

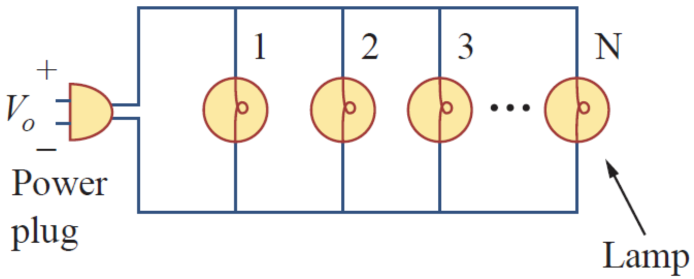
$$I_1 = \left(\frac{18000}{9000+18000}\right)(30\text{mA}) = 20\text{ mA} \Rightarrow V_s = I_1(9000) = 180\text{ V}$$

$$I_2 = \left(\frac{9000}{9000+18000}\right)(30\text{mA}) = 10\text{ mA}$$

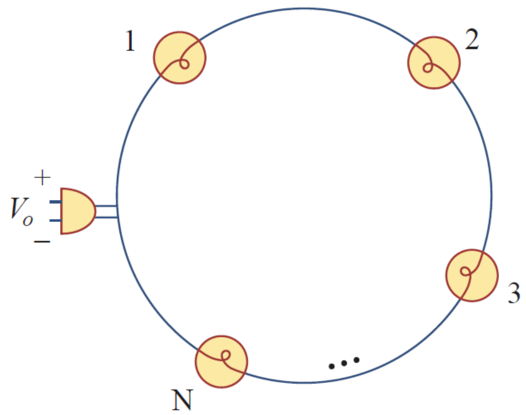
- Power supplied by the 30 mA current source: $P_s = V_s I_s = (180)(30\times10^{-3})$ W = 5.4 W
- Power dissipated by the 9 kΩ resistor: $P = I_1^2 R = (20\times10^{-3})^2(9000)$ = 3.6 W
- Power dissipated by the 6 kΩ resistor: $P = I_2^2 R = (10\times10^{-3})^2(6000)$ = 0.6 W
- Power dissipated by the 12 kΩ resistor: $P = I_2^2 R = (10\times10^{-3})^2(12000)$ = 1.2 W

Note: Power supplied = Power dissipated

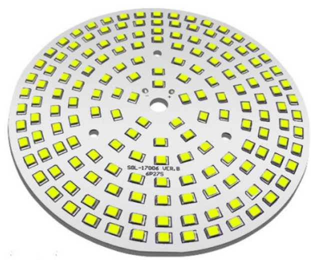
Application: Lighting Systems

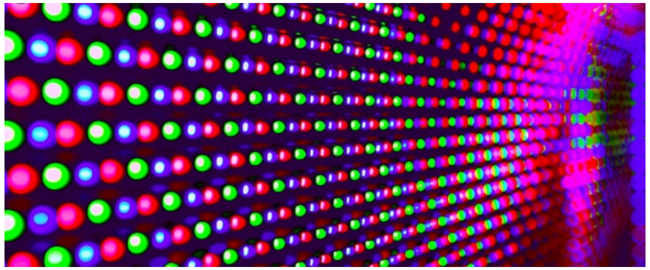


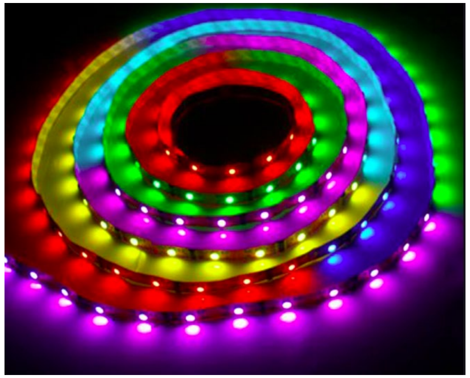
Parallel connection of light bulbs



Series connection of light bulbs

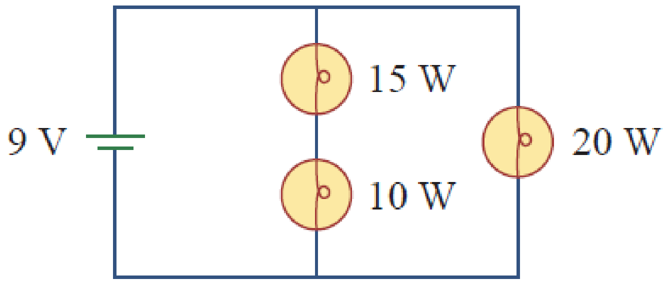






Example (9): Lighting Systems

In the following lighting system, three light bulbs are connected to a 9 V battery.



Calculate (a) the total current supplied by the battery, (b) the current through each bulb, (c) the resistance of each bulb.

(Hint: The total power supplied by the battery is equal to the total power absorbed by all light bulbs.)

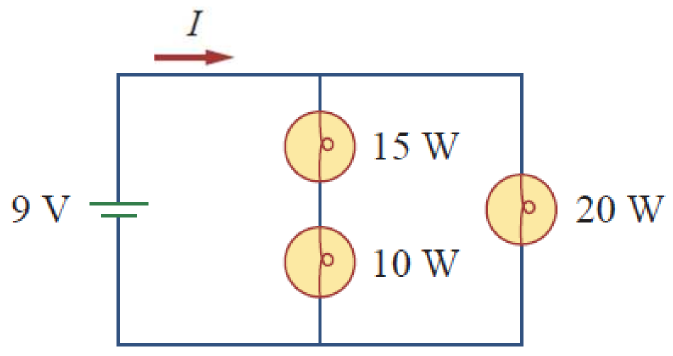
Example (9): Lighting Systems (cont'd)

Ans: (a) Total power absorbed by all light bulbs:

$P = (15 + 10 + 20)\ \text{W} = 45\ \text{W}$

Since the total power supplied by the battery is the same as that absorbed by all bulbs, the battery supplies a total power of 45 W.

Hence, the total current supplied by the battery is: $I = \dfrac{P}{V} = \dfrac{45}{9} = 5\ \text{A}$



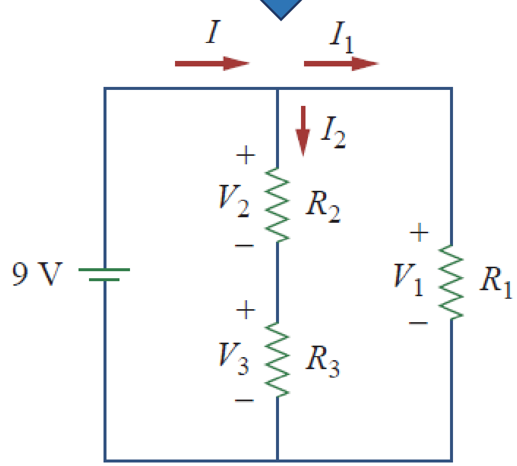
(b) Since R_1 (20-W bulb) is in parallel with the battery, $V_1 = 9$ V.

$$I_1 = \frac{P_1}{V_1} = \frac{20}{9} = 2.222\ \text{A}$$

By KCL, $I = I_1 + I_2 \Rightarrow I_2 = I - I_1 = (5 - 2.222)\ \text{A} = 2.778\ \text{A}$

(c) $R_1 = \dfrac{P_1}{I_1^2} = \dfrac{20}{2.222^2} = 4.051\ \Omega$ $\quad R_3 = \dfrac{P_3}{I_2^2} = \dfrac{10}{2.778^2} = 1.296\ \Omega$

$R_2 = \dfrac{P_2}{I_2^2} = \dfrac{15}{2.778^2} = 1.944\ \Omega$



I. Ohm's Law

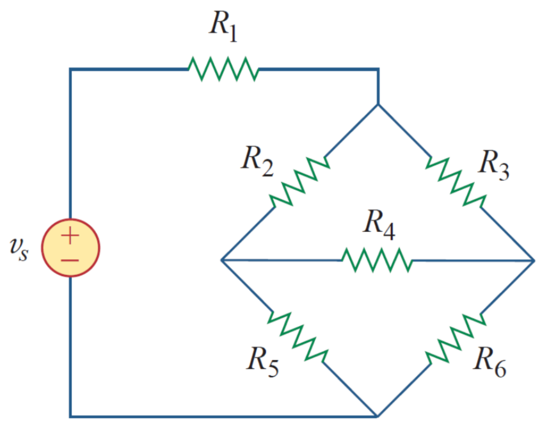
II. Kirchhoff's Laws

III. Voltage and Current Division Formulas

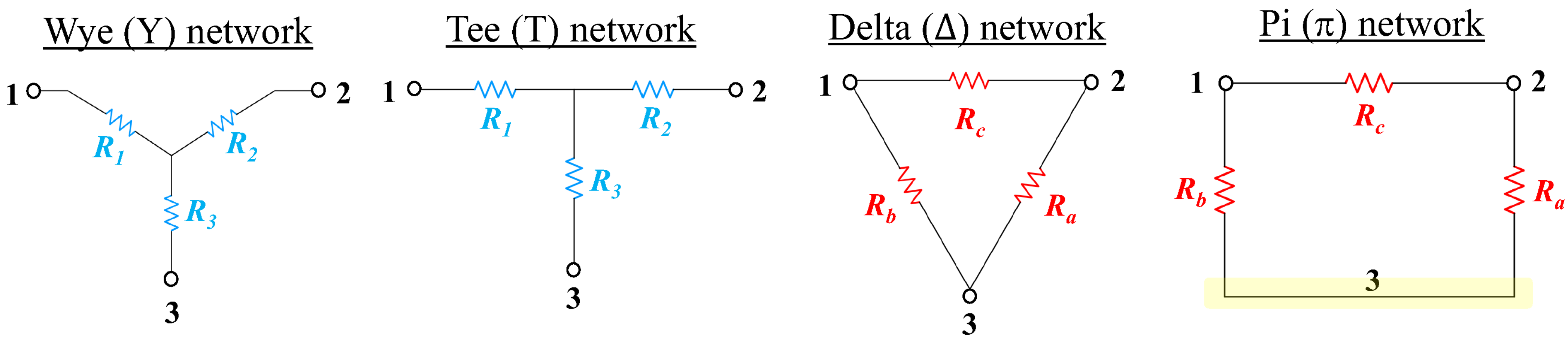
IV. Wye-Delta Transformations

Wye-Delta Transformations

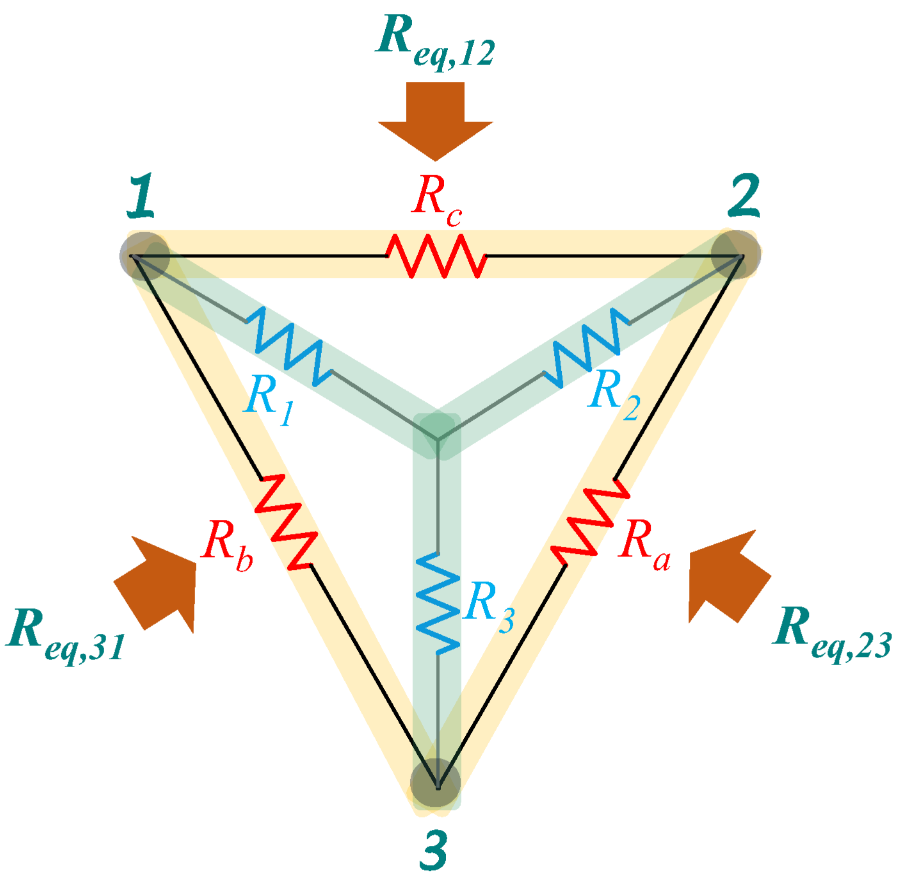
- In a real circuit, the resistors are neither in parallel nor in series.
- So, how do we combine resistors when they are neither in series nor in parallel ?



Three-terminal equivalent networks



Proof of Delta to Wye Conversion



	Δ		Y
$R_{eq,12}$:	$\dfrac{R_c(R_a+R_b)}{R_a+R_b+R_c}$	=	R_1+R_2
$R_{eq,23}$:	$\dfrac{R_a(R_b+R_c)}{R_a+R_b+R_c}$	=	R_2+R_3
$R_{eq,31}$:	$\dfrac{R_b(R_a+R_c)}{R_a+R_b+R_c}$	=	R_1+R_3

Proof of Delta to Wye Conversion

Y	Δ
(1) $R_1 + R_2 =$	$\dfrac{R_c\,(R_a + R_b)}{R_a + R_b + R_c}$
(2) $R_2 + R_3 =$	$\dfrac{R_a\,(R_b + R_c)}{R_a + R_b + R_c}$
(3) $R_1 + R_3 =$	$\dfrac{R_b\,(R_a + R_c)}{R_a + R_b + R_c}$

Solve for R_1: By adding equation (1) and (3), we have

$$2R_1 + R_2 + R_3 = \frac{R_c(R_a + R_b) + R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$2R_1 + (R_2 + R_3) = \frac{R_c(R_a + R_b) + R_b(R_a + R_c)}{R_a + R_b + R_c}$$

Note that $(R_2 + R_3)$ is given by equation (2). Hence, we can write:

$$2R_1 + \frac{R_a\,(R_b + R_c)}{R_a + R_b + R_c} = \frac{R_c(R_a + R_b) + R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$\Rightarrow\ 2R_1 = \frac{R_bR_c + R_bR_c}{R_a + R_b + R_c}$$

$$\Rightarrow\ \boxed{R_1 = \frac{R_bR_c}{R_a + R_b + R_c}}$$

Proof of Delta to Wye Conversion

Y	Δ
(1) $R_1 + R_2 =$	$\dfrac{R_c(R_a + R_b)}{R_a + R_b + R_c}$
(2) $R_2 + R_3 =$	$\dfrac{R_a(R_b + R_c)}{R_a + R_b + R_c}$
(3) $R_1 + R_3 =$	$\dfrac{R_b(R_a + R_c)}{R_a + R_b + R_c}$

Solve for R_2: By adding equation (1) and (2), we have

$$2R_2 + R_1 + R_3 = \frac{R_c(R_a + R_b) + R_a(R_b + R_c)}{R_a + R_b + R_c}$$

$$2R_2 + (R_1 + R_3) = \frac{R_c(R_a + R_b) + R_a(R_b + R_c)}{R_a + R_b + R_c}$$

Note that $(R_1 + R_3)$ is given by equation (3). Hence, we can write:

$$2R_2 + \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} = \frac{R_c(R_a + R_b) + R_a(R_b + R_c)}{R_a + R_b + R_c}$$

=> $$2R_2 = \frac{R_aR_c + R_aR_c}{R_a + R_b + R_c}$$

=> $$\boxed{\boldsymbol{R_2 = \frac{R_aR_c}{R_a + R_b + R_c}}}$$

Proof of Delta to Wye Conversion

Y	Δ
(1) $R_1 + R_2 =$	$\dfrac{R_c(R_a + R_b)}{R_a + R_b + R_c}$
(2) $R_2 + R_3 =$	$\dfrac{R_a(R_b + R_c)}{R_a + R_b + R_c}$
(3) $R_1 + R_3 =$	$\dfrac{R_b(R_a + R_c)}{R_a + R_b + R_c}$

Solve for R_3: By adding equation (2) and (3), we have

$$2R_3 + R_1 + R_2 = \frac{R_a(R_b + R_c) + R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$2R_3 + (R_1 + R_2) = \frac{R_a(R_b + R_c) + R_b(R_a + R_c)}{R_a + R_b + R_c}$$

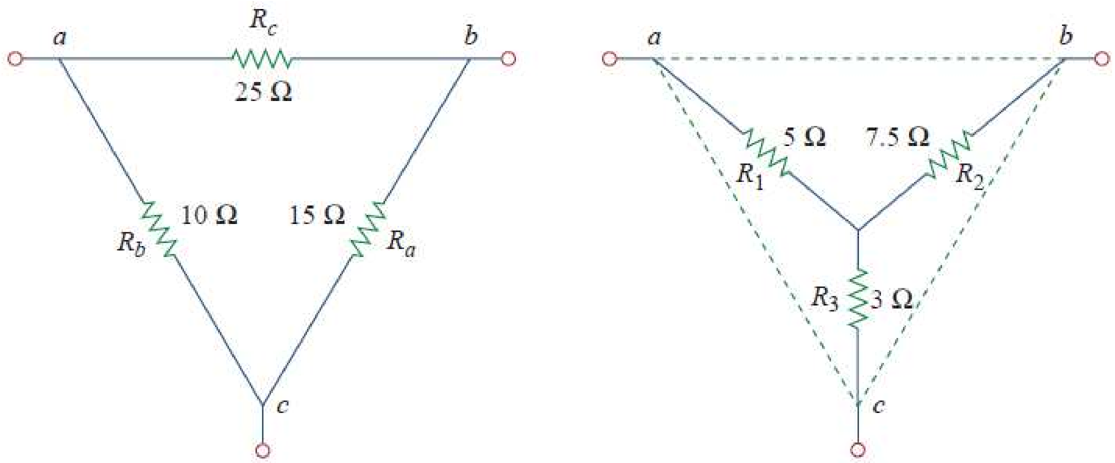
Note that $(R_1 + R_2)$ is given by equation (1). Hence, we can write:

$$2R_3 + \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = \frac{R_a(R_b + R_c) + R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$\Rightarrow \quad 2R_3 = \frac{R_aR_b + R_aR_b}{R_a + R_b + R_c}$$

$$\Rightarrow \quad \boxed{R_3 = \frac{R_aR_b}{R_a + R_b + R_c}}$$

Example (10): Delta to Wye Conversion



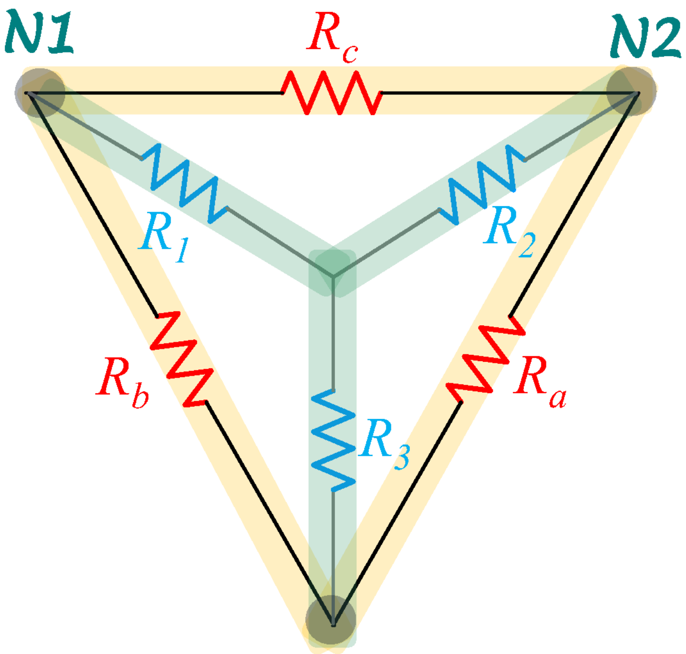
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{15 \times 25}{15 + 10 + 25} = \frac{375}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{15 + 10 + 25} = \frac{150}{50} = 3\ \Omega$$

Proof of Wye to Delta Conversion

$$\Delta \rightarrow Y: \quad R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \qquad R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \qquad R_3 = \frac{R_a R_b}{R_a+R_b+R_c}$$



Consider $R_1 \times R_2$:

$$R_1R_2 = \frac{R_aR_bR_c^2}{(R_a+R_b+R_c)^2} \quad (4)$$

Consider $R_2 \times R_3$:

$$R_2R_3 = \frac{R_a^2R_bR_c}{(R_a+R_b+R_c)^2} \quad (5)$$

Consider $R_1 \times R_3$:

$$R_1R_3 = \frac{R_aR_b^2R_c}{(R_a+R_b+R_c)^2} \quad (6)$$

By adding equation (4), (5) & (6), we have

$$R_1R_2 + R_2R_3 + R_1R_3$$

$$= \frac{R_aR_bR_c^2 + R_a^2R_bR_c + R_aR_b^2R_c}{(R_a+R_b+R_c)^2}$$

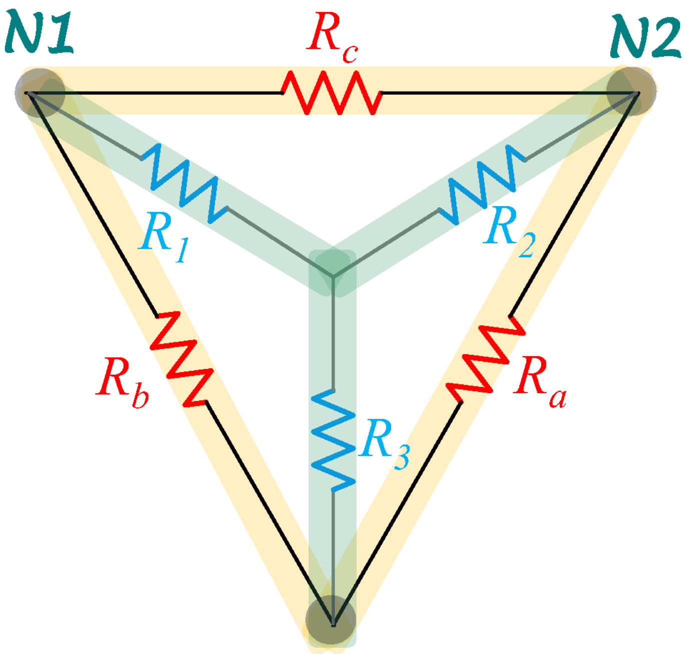
$$= \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a+R_b+R_c)^2}$$

$$= \frac{R_aR_bR_c}{(R_a+R_b+R_c)}$$

(*continued on next page*)

Proof of Wye to Delta Conversion

$$\Delta \rightarrow Y: \quad R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \qquad R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \qquad R_3 = \frac{R_a R_b}{R_a+R_b+R_c}$$



From previous slide, we have

$$R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_aR_bR_c}{(R_a+R_b+R_c)} \qquad (7)$$

Notice that $R_aR_1 = \dfrac{R_aR_bR_c}{(R_a+R_b+R_c)}$ (8)

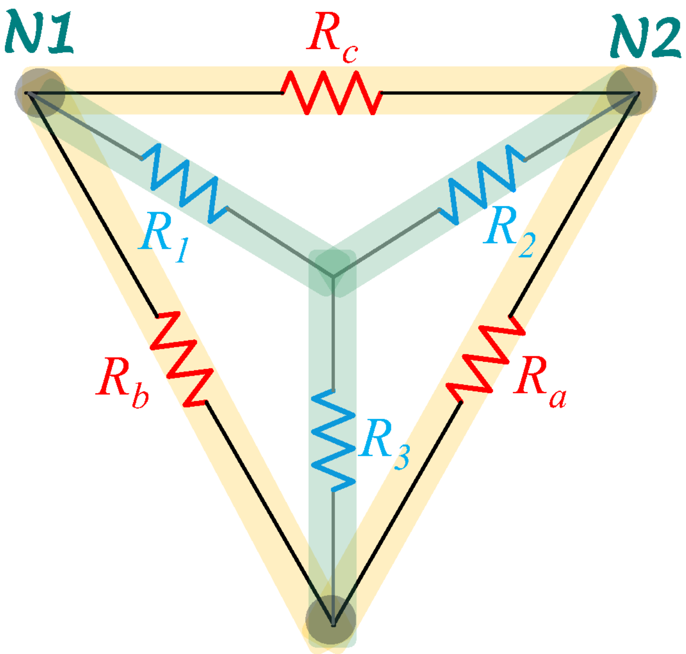
Hence, by combining equation (7) & (8), we have

$$R_aR_1 = R_1R_2 + R_2R_3 + R_1R_3 \qquad (9)$$

$$\Rightarrow \boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}}$$

Proof of Wye to Delta Conversion

$$\Delta \rightarrow Y: \quad R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

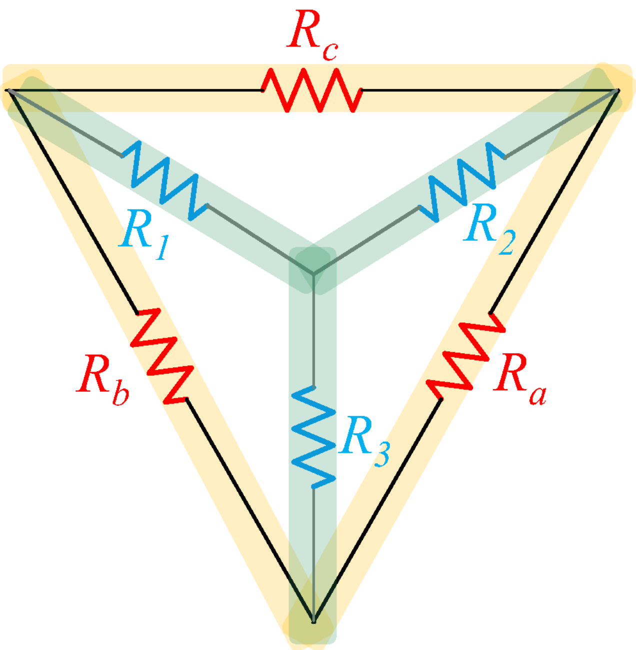


Likewise, we can obtain the expressions for R_b and R_c (Leave this as a take-home exercise).

$$\boxed{R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}}$$

$$\boxed{R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}}$$

Summary: Delta-to-Wye or Wye-to-Delta Conversion



$\Delta \rightarrow Y$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$Y \rightarrow \Delta$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Example (11): Equivalent Resistance

Determine the equivalent resistance R_{ab} for the circuit and use it to find the current I.

(1) Convert the Y network comprising the 5 Ω, 10 Ω and 20 Ω resistors to an equivalent Δ network.

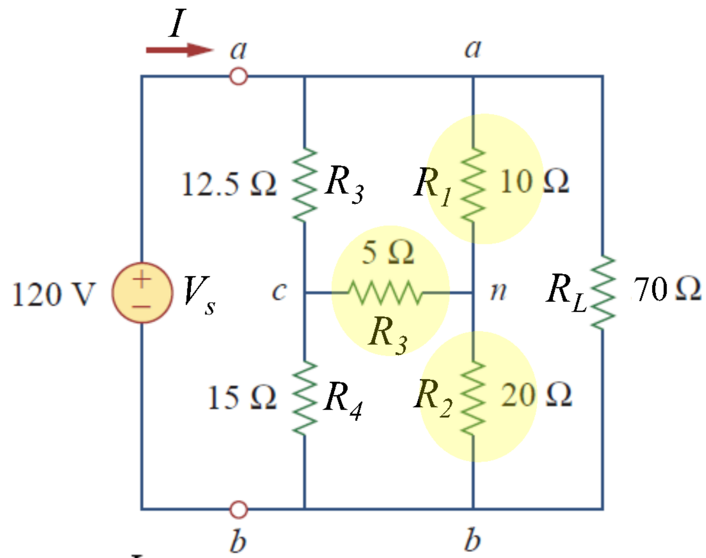
$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 5\ \Omega$

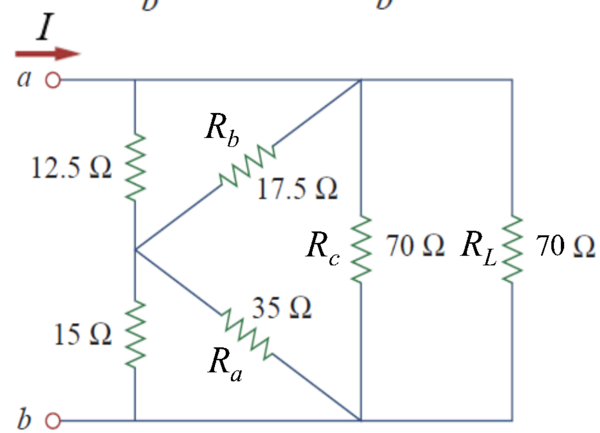
By using the Y-Δ conversion equations, we have

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$





Example (11): Equivalent Resistance (cont'd)

(2) Since $R_C \parallel R_L$, their equivalent resistance is:

$$R_{e1} = R_c \parallel R_L = \frac{R_c R_L}{(R_c + R_L)} = \frac{70 \times 70}{70 + 70} = 35\ \Omega$$

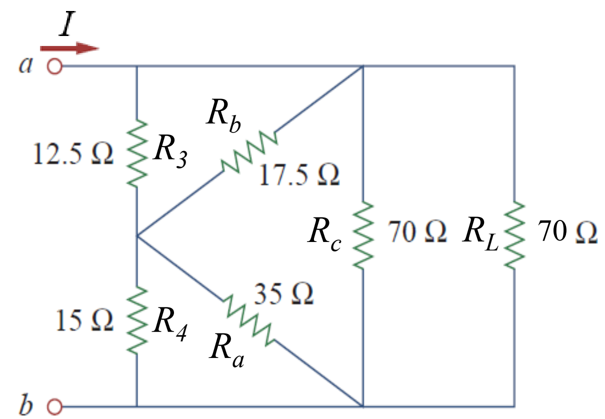
Similarly, since $R_3 \parallel R_b$ and $R_4 \parallel R_a$, their equivalent resistances are:

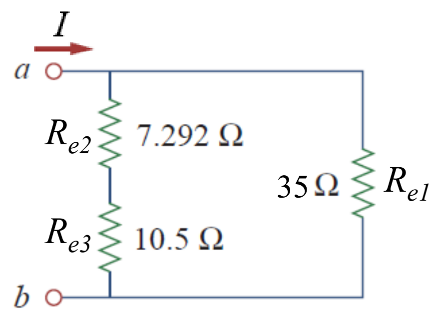
$$R_{e2} = R_3 \parallel R_b = \frac{R_3 R_b}{(R_3 + R_b)} = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$R_{e3} = R_4 \parallel R_a = \frac{R_4 R_a}{(R_4 + R_a)} = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

Hence, $R_{ab} = (R_{e2} + R_{e3}) \parallel R_{e1} = (7.292 + 10.5) \parallel 35 = \dfrac{17.792 \times 35}{17.792 + 35} = 11.796\ \Omega$

Finally, $I = \dfrac{V_s}{R_{ab}} = \dfrac{120}{11.796} = 10.173\ \text{A}$





Summary

I. Ohm's Law

- ✓ $V = IR$, where V is the voltage, I is the current, and R is the resistance.
- ✓ $G = 1/R$, where G is the conductance.
- ✓ Power dissipated by a resistor is: $P = VI = I^2 R = \frac{V^2}{R}$

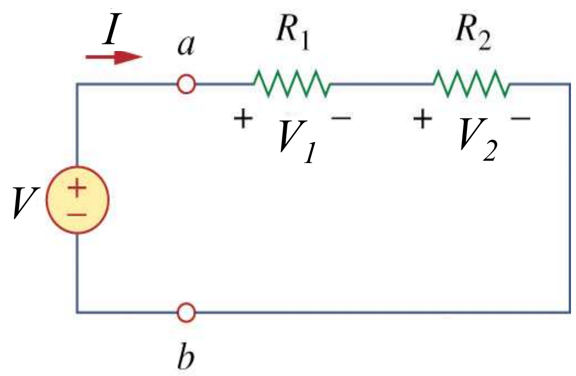
II. Kirchhoff's Laws

- ✓ KCL: $\sum_{n=1}^{N} I_n = 0$ Σ currents entering into a node = Σ currents leaving the same node
- ✓ KVL: $\sum_{m=1}^{M} V_n = 0$ Σ voltage drops around a loop = Σ voltage rises around the same loop

Summary (cont'd)

III. Voltage and Current Division Formulas

- ✓ Equivalent resistance for *N* resistors in series: $R_{eq} = \sum_{n=1}^{N} R_n$
- ✓ Equivalent resistance for *N* resistors in parallel: $\frac{1}{R_{eq}} = \sum_{n=1}^{N} \frac{1}{R_n}$
- ✓ Equivalent resistance for *two* resistors in parallel: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
- ✓ Voltage Division:

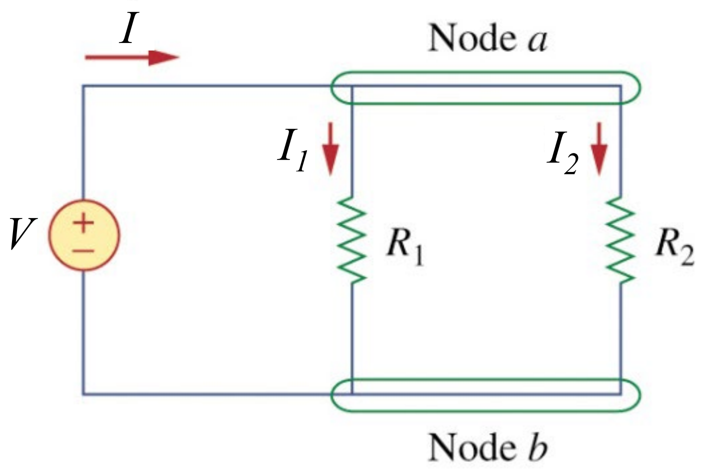


$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V ; V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V$$

Summary (cont'd)

III. Voltage and Current Division Formulas

✓ Current Division:



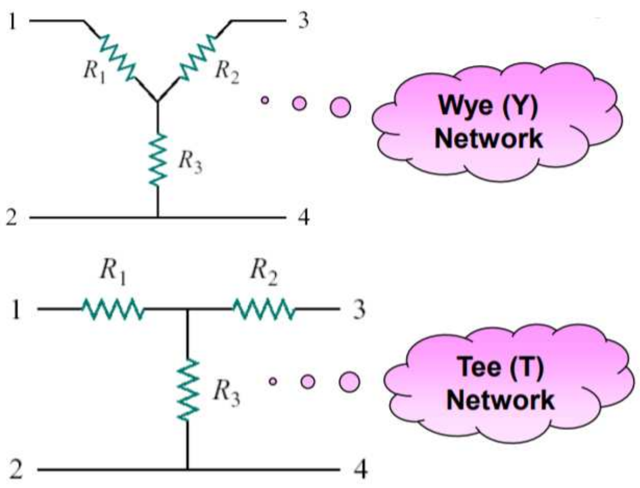
$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I$$

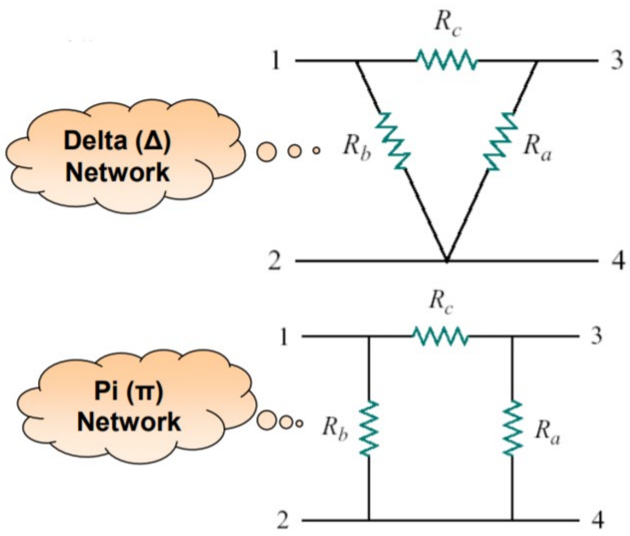
Summary (cont’d)

IV. Wye-Delta Transformations

Wye or Tee Network



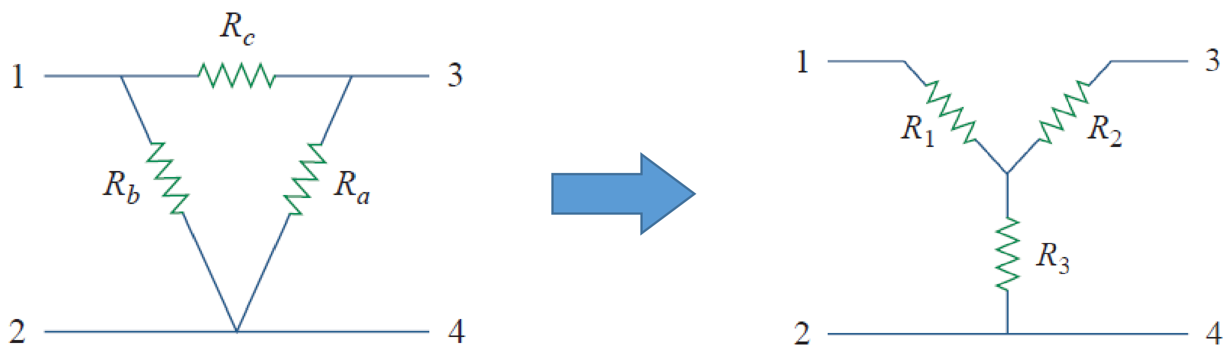
Delta or Pi Network



Summary (cont'd)

IV. Wye-Delta Transformations

✓ Delta to Wye (Δ–Y) Conversion:



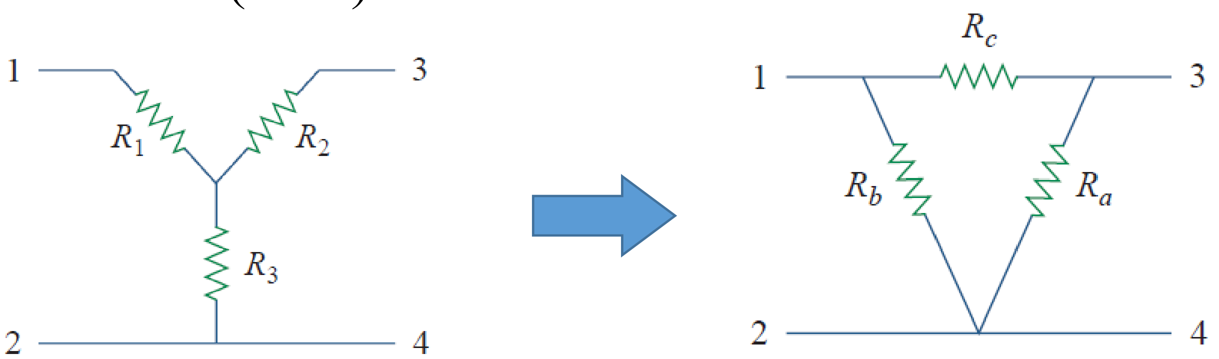
$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)} \qquad R_2 = \frac{R_a R_c}{(R_a + R_b + R_c)} \qquad R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Rule of Thumb:
Each resistor in the Y network is the *product* of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Summary (cont'd)

IV. Wye-Delta Transformations

✓ Wye to Delta (Y–Δ) Conversion:



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Rule of Thumb:
Each resistor in the Δ network is the *sum* of *all possible products* of Y resistors (two at a time), divided by the opposite Y resistor.