Methods of Analysis

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I. Nodal Analysis

II. Mesh Analysis

Introduction

In the last lecture, we have learned the fundamental laws of circuit theory (i.e., Ohm's law and Kirchhoff's laws).

In this lecture, we will apply these laws to develop two powerful techniques for performing circuit analysis.

- **Nodal analysis**: based on a systematic application of KCL; and
- **Mesh analysis**: based on a systematic application of KVL.

With these two techniques, we can analyze any linear circuit by obtaining a set of simultaneous equations.

Nodal Analysis

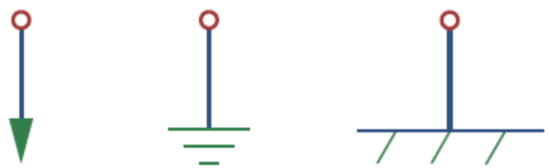
Steps to determine node voltages:

1. Select a node as the <u>reference node</u>. Assign voltages v_1, v_2,…, v_{n-1} to the remaining (n−1) nodes. The voltages are referenced with respect to the reference node.

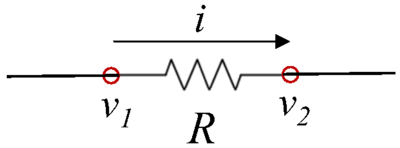
2. Apply KCL to each of the (n−1) <u>non-reference nodes</u>. Use Ohm's law to express the the branch currents in terms of node voltages.

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

The reference node is commonly called the ground since it is assumed to have zero potential (0 V).

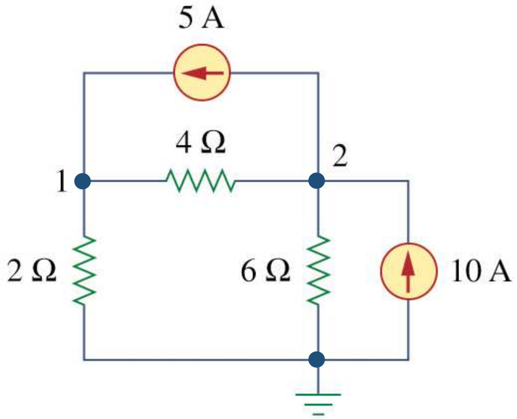


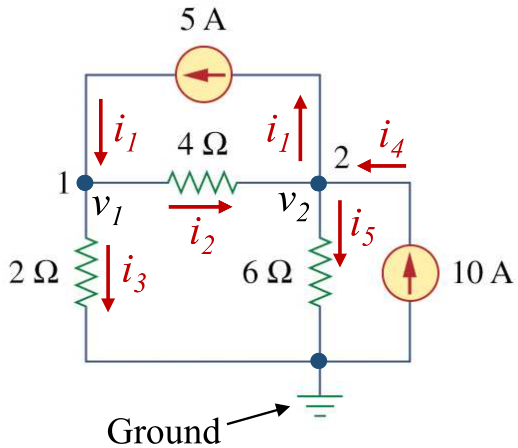
Current flows form a higher potential to a lower potential across a resistor.



$$i = \frac{v_1 - v_2}{R}, \text{ where } v_1 > v_2$$

Example (1): Nodal Analysis





Question: Calculate the node voltages in the circuit shown on the left.

Step 1: The ground is selected as the reference. Assign the voltage at node 1 as v_1, and the voltage at node 2 as v_2, where v_1 and v_2 are unknowns.

Step 2: Apply KCL at node 1 and node 2.

At node 1, we have $i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$ (1)

At node 2, we have $i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$ (2)

Note: Except for the branches with current sources, the direction of the current is arbitrary but needs to be consistent.

By simplifying (1) and (2) and re-arranging, we have

$$3v_1 - v_2 = 20 \qquad (3)$$

$$-3v_1 + 5v_2 = 60 \qquad (4)$$

Example (1): Nodal Analysis (cont'd)

Step 3: Solve the simultaneous equations.

One simple method is elimination technique: By adding (3) and (4), we have $4v_2 = 80 \Rightarrow v_2 = 20$ V. Plugging in the value of v_2 back into (3), we have $3v_1 - 20 = 20$, which gives $v_1 = 40/3 = 13.333$ V.

Another more systematic approach is to treat the simultaneous equations as a **matrix equation** and solve it using **Cramer's rule**.

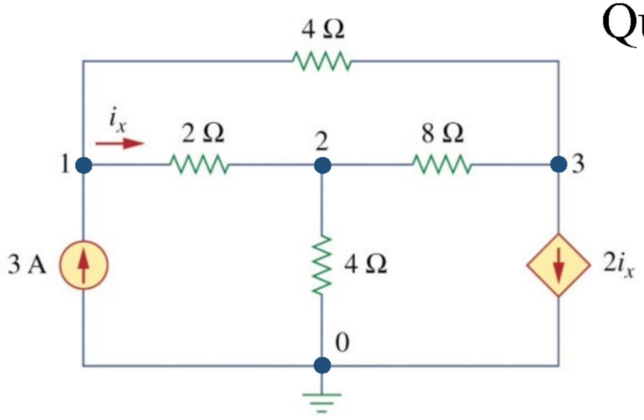
By re-writing (3) and (4) into matrix form and invoking Cramer's rule, we have

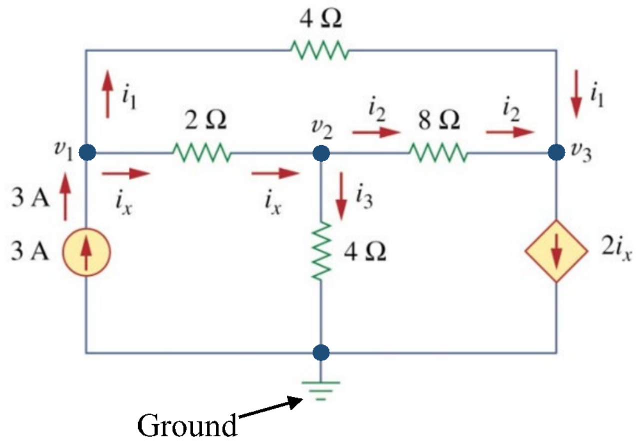
$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix}} = \frac{100-(-60)}{15-3} = 13.333\text{V}, \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix}} = \frac{180-(-60)}{15-3} = 20\text{V}$$

A software called MATLAB can also be used for solving the matrix equation.

Example (2): Nodal Analysis





Question: Determine all node voltages in the circuit shown on the left.

Step 1: Choose the ground as the reference node. Assign voltages at node 1, 2, and 3 as v_1, v_2 and v_3, respectively.

Step 2: Apply KCL at node 1, 2, and 3.

At node 1, we have $3 = \dfrac{v_1 - v_3}{4} + \dfrac{v_1 - v_2}{2}$

At node 2, we have $\dfrac{v_1 - v_2}{2} = \dfrac{v_2 - v_3}{8} + \dfrac{v_2 - 0}{4}$

At node 3, we have $\dfrac{v_1 - v_3}{4} + \dfrac{v_2 - v_3}{8} = \dfrac{2(v_1 - v_2)}{2}$

By simplifying the above three equations and re-arranging, we have

$$\begin{cases} 3v_1 - 2v_2 - v_3 = 12 & (1) \\ -4v_1 + 7v_2 - v_3 = 0 & (2) \\ 2v_1 - 3v_2 + v_3 = 0 & (3) \end{cases}$$

Example (2): Nodal Analysis (cont'd)

Step 3: Solve the set of equations, i.e., (1), (2) and (3).

Method 1–Elimination technique

Adding (1) and (3), we have $v_1 - v_2 = 2.4$

Adding (2) and (3), we have $v_1 = 2v_2$

By solving the above two equations, we have $v_2 = 2.4$ V and $v_1 = 4.8$ V

Plugging the values of v_1 and v_2 back into (3), the value of v_3 can be obtained as: $v_3 = -2.4$ V

Method 2–Cramer's rule (Rewrite the simultaneous equations into matrix equation)

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \qquad \Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = 10 \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

The solution is given by $v_1 = \Delta_1/\Delta$, $v_2 = \Delta_2/\Delta$ and $v_3 = \Delta_3/\Delta$.

Example (2): Nodal Analysis (cont'd)

Method 2: Cramer's rule (cont'd)

The solution is given by $v_1 = \Delta_1/\Delta$, $v_2 = \Delta_2/\Delta$ and $v_3 = \Delta_3/\Delta$.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix} = 12 \begin{vmatrix} 7 & -1 \\ -3 & 1 \end{vmatrix} = 48$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix} = -12 \begin{vmatrix} -4 & -1 \\ 2 & 1 \end{vmatrix} = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} = 12 \begin{vmatrix} -4 & 7 \\ 2 & -3 \end{vmatrix} = -24$$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = 10$$

$$\boxed{V_1 = \frac{48}{10} = 4.8\ \text{V}} \qquad \boxed{V_2 = \frac{24}{10} = 2.4\ \text{V}} \qquad \boxed{V_3 = \frac{-24}{10} = -2.4\ \text{V}}$$

Example (2): Nodal Analysis (cont'd)

$$v_1 = \frac{\Delta_1}{\Delta}, \qquad v_2 = \frac{\Delta_2}{\Delta}, \qquad v_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix} \begin{matrix} 12 & -2 & -1 \\ 0 & 7 & -1 \end{matrix} = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} \begin{matrix} 3 & -2 & -1 \\ -4 & 7 & -1 \end{matrix}$$

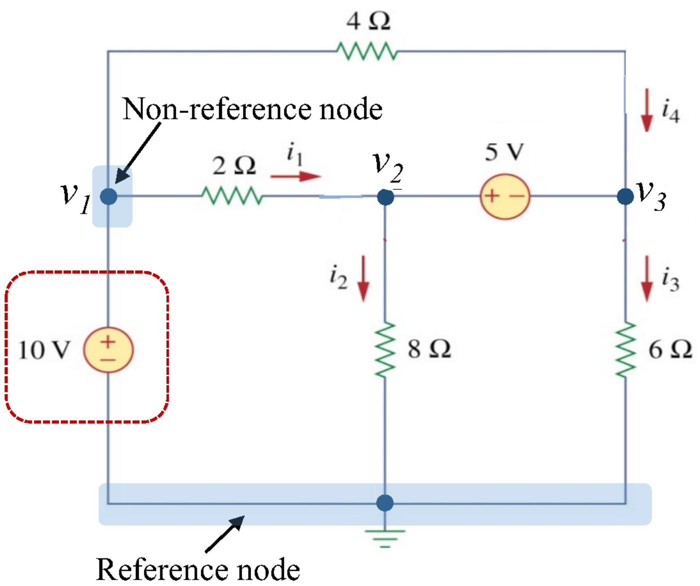
$$= 21 - 12 + 4 + 14 - 9 - 8 = 10$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix} \begin{matrix} 3 & 12 & -1 \\ -4 & 0 & -1 \end{matrix} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} \begin{matrix} 3 & -2 & 12 \\ -4 & 7 & 0 \end{matrix} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

Nodal analysis with voltage sources

- Up until now, we have *not* considered a circuit with voltage source in nodal analysis.
- The reason is that in general, there is no way of knowing the current through a voltage source in advance => difficult to apply KCL.
- How do we use nodal analysis if the circuit contains voltage sources ?

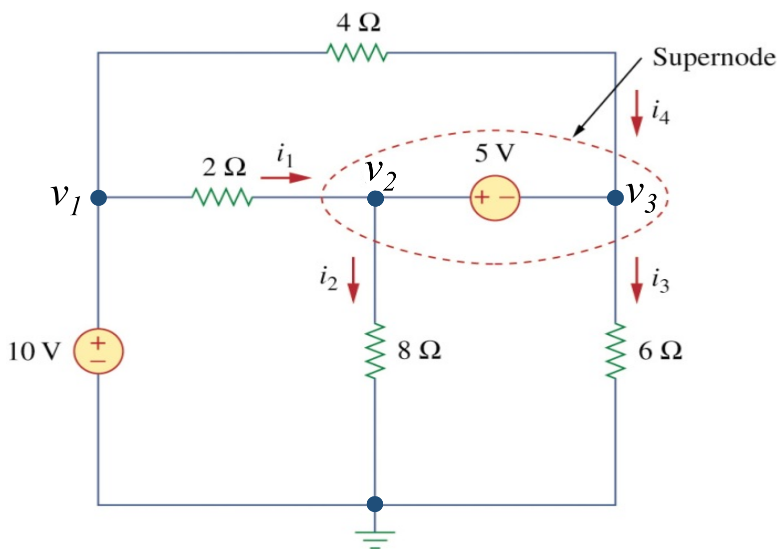


Case 1: Voltage source is connected between the reference node and a non-reference node.

Technique: Set the voltage at the non-reference node equal to the voltage of the voltage source.

Example: In the circuit on the left, $v_1 = 10$ V

Nodal analysis with voltage sources (cont'd)



Case 2: Voltage source (regardless of dependent or independent) is connected between two non-reference nodes.

Technique: Combine these two nodes into a supernode.

Example: In the circuit on the left, a supernode is formed by by enclosing the independent voltage source.

- In general, a supernode is formed by enclosing a voltage source connected between two non-reference nodes and any elements connected in parallel to it.
- KCL can still be applied at a supernode (Example: $i_1 + i_4 = i_2 + i_3$).
- The relationship between v_2 and v_3 is known, i.e., $v_2 - v_3 = 5$.

Example (3): Nodal Analysis

Question: Find the node voltages (v_1, v_2) in the circuit shown in Fig. 1

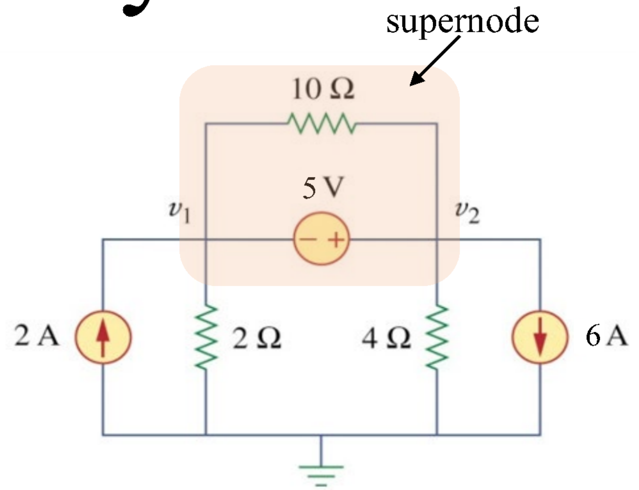


Fig. 1(a)

Ans:

- Assign a supernode across v_1 and v_2 (i.e., enclosing the 5 V voltage source and 10 Ω resistor), as shown in Fig. 1(b).
- Apply KCL to the supernode, we can write: $2 = i_1 + i_2 + 6$.
- Since $i_1 = \dfrac{v_1}{2}$ and $i_2 = \dfrac{v_2}{4}$,

  we have $2 = \dfrac{v_1}{2} + \dfrac{v_2}{4} + 6 \Rightarrow v_2 = -16 - 2v_1 \quad (1)$

- From Fig. 1(a), $v_2 = 5 + v_1 \quad (2)$
- Then, by solving (1) & (2), we have $v_1 = -7$ V and $v_2 = -2$ V.

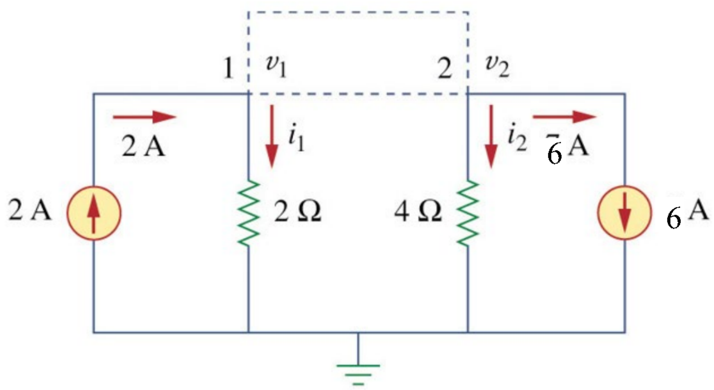


Fig. 1(b)

Example (4): Nodal Analysis

Question: Find the node voltages (v_1, v_2, v_3, v_4) in the circuit shown in Fig. 2(a).

Ans:

- Nodes 1 and 2 form a supernode. Likewise, nodes 3 and 4 form another supernode.
- Redraw the circuit with supernodes and then apply KCL, which results in Fig. 2(b).
- At supernode 1-2, $i_3 + 10 = i_1 + i_2$. By re-expressing this equation in terms of the node voltages, we have

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2} \qquad (1)$$

- At supernode 3-4, $i_1 = i_3 + i_4 + i_5$, which can be re-expressed as

$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4} \qquad (2)$$

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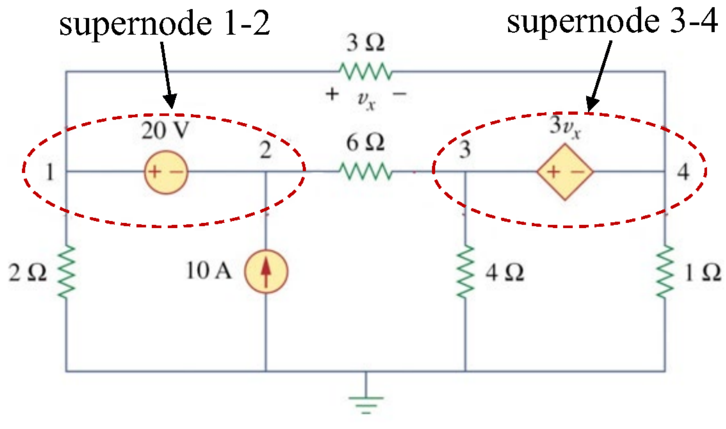

Fig. 2(a)

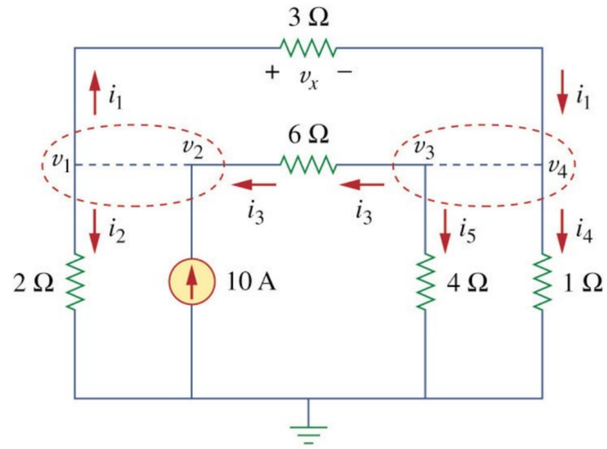

Fig. 2(b)

Example (4): Nodal Analysis (cont'd)

- From Fig. 2(a), we have

$$v_1 - v_2 = 20 \qquad (3)$$

$$v_3 - v_4 = 3v_x = 3(v_1 - v_4) \qquad (4)$$

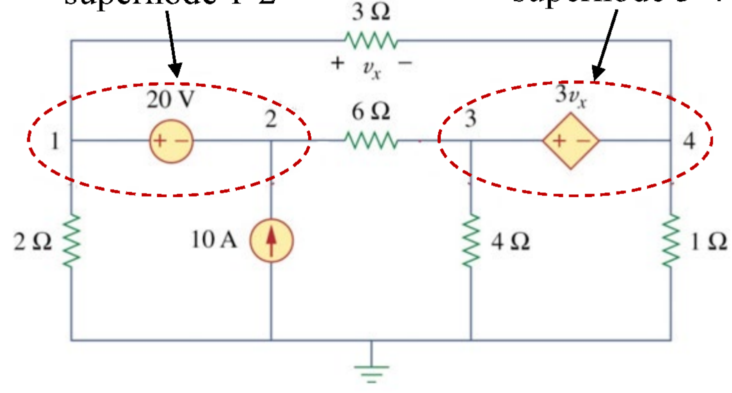

Fig. 2(a)

- Hence, we have four independent equations with four unknowns. After simplification, the four equations can be re-written as follows.

$$\begin{cases} 5v_1 + v_2 - v_3 - 2v_4 = 60 & (5) \\ 4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 & (6) \\ v_1 - v_2 = 20 & (7) \\ 3v_1 - v_3 - 2v_4 = 0 & (8) \end{cases}$$

This system of equations can be easily solved using MATLAB as the hand calculations are somewhat tedious. Fortunately, we can easily eliminate one variable (v_2) from this set of equations by making use of equation (7), which can be re-written as: $v_2 = v_1 - 20$.

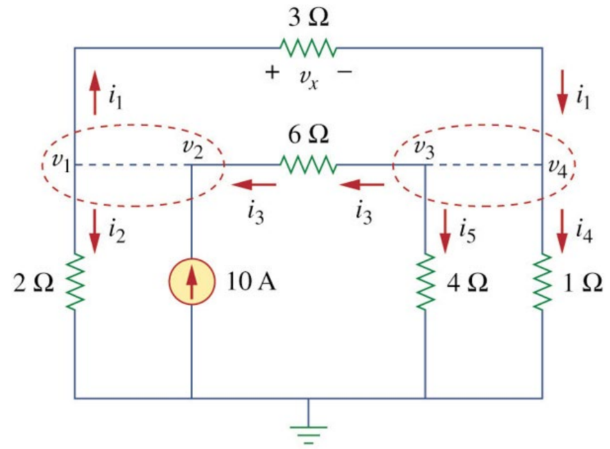

Fig. 2(b)

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Example (4): Nodal Analysis (cont'd)

$$v_2 = v_1 - 20 \qquad (9)$$

By substituting (9) into (5) and (6) and simplifying, a reduced set of equations can be obtained as follows.

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Using Cramer's rule, we have

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18, \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120, \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

The final answer is: $v_1 = -480/-18 = 26.67$ V, $v_3 = -3120/-18 = 173.33$ V, $v_4 = 840/-18 = -46.67$ V, and $v_2 = v_1 - 20 = 6.67$ V.

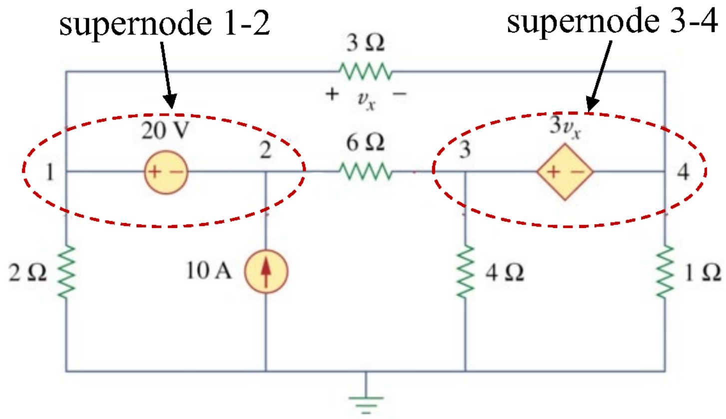


Fig. 2(a)

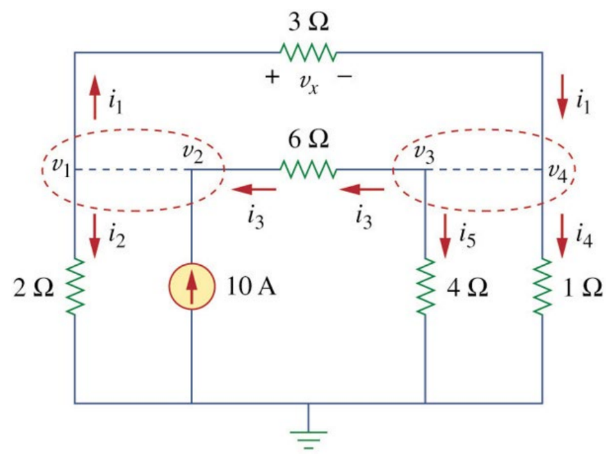
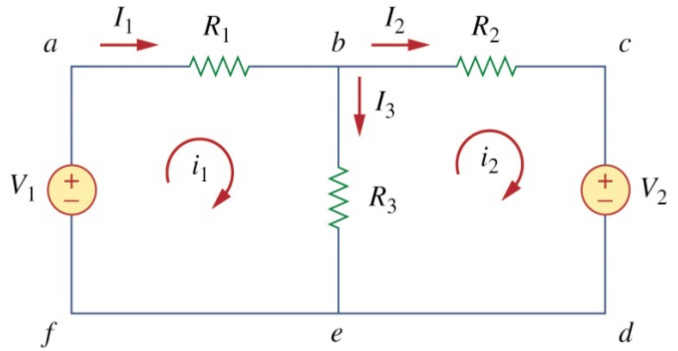


Fig. 2(b)

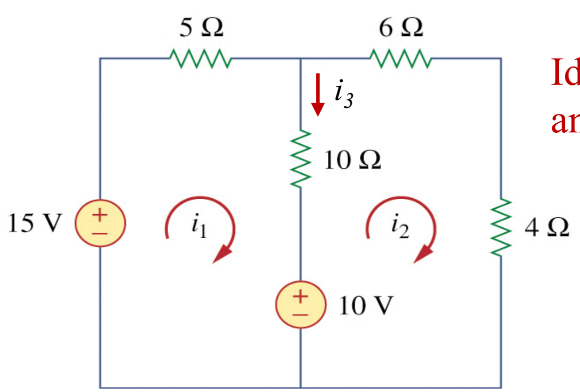
Mesh Analysis

- Mesh analysis is another well-organized method for solving a circuit.
- Mesh analysis uses mesh currents as the unknown variables.
- A mesh is a loop which does not contain any other loops within it. For example, the figure at the bottom shows a circuit with two meshes.
- Steps to determine mesh currents
  1. Assign mesh currents i_1, i_2, …, i_n to the n meshes;
  2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents;
  3. Solve the resulting n simultaneous equations to get the mesh currents.

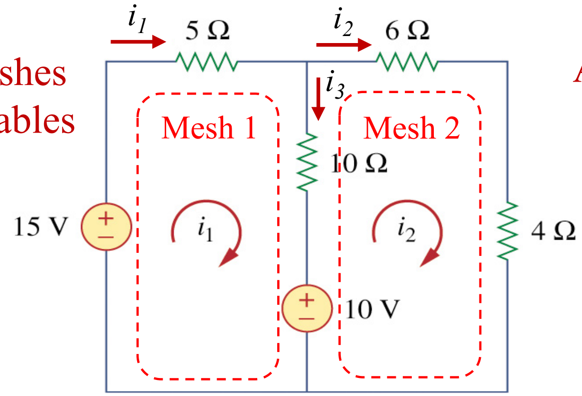


Example (5): Mesh Analysis

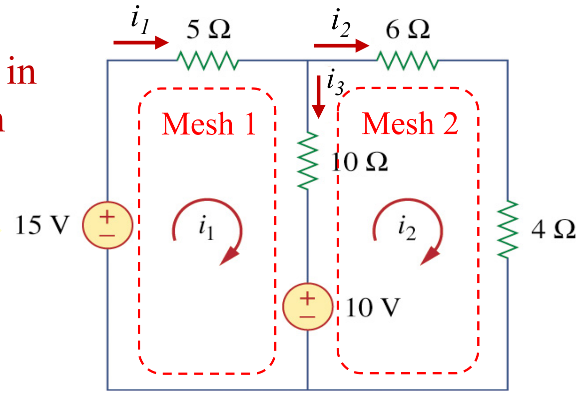
Question: Find the branch currents (i_1, i_2, and i_3) in the following circuit using mesh analysis.



Step 1
Identify the meshes and assign variables



Step 2
Apply KVL in each mesh



For mesh 1, $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$
For mesh 2, $6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$

Notice $i_3 = i_1 - i_2$

After simplifying the above two equations, we have

$$\begin{cases} 3i_1 - 2i_2 = 1 & (1) \\ i_1 - 2i_2 = -1 & (2) \end{cases}$$

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Example (5): Mesh Analysis (cont'd)

There are two methods of solving the following two independent equations.

$$\begin{cases} 3i_1 - 2i_2 = 1 & (1) \\ i_1 - 2i_2 = -1 & (2) \end{cases}$$

Method 1–Elimination technique

Subtracting (2) from (1) gives $2i_1 = 2 \Rightarrow i_1 = 1$ A. So, by plugging the value of i_1 into (2), we have $i_2 = 1$ A.

$i_3 = i_1 - i_2 = 1 - 1 = 0$ A. Hence, $i_1 = 1$ A, $i_2 = 1$ A, and $i_3 = 0$ A.

Method 2–Cramer's rule

Re-write equation (1) and (2) in matrix form.

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 4, \quad \Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 4, \quad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 4$$

$i_1 = \Delta_1/\Delta = 1$ A, $i_2 = \Delta_2/\Delta = 1$ A, and $i_3 = i_1 - i_2 = 0$ A.

Example (6): Mesh Analysis

Question: Consider the circuit shown in Fig. 3. Determine the branch current I_o.

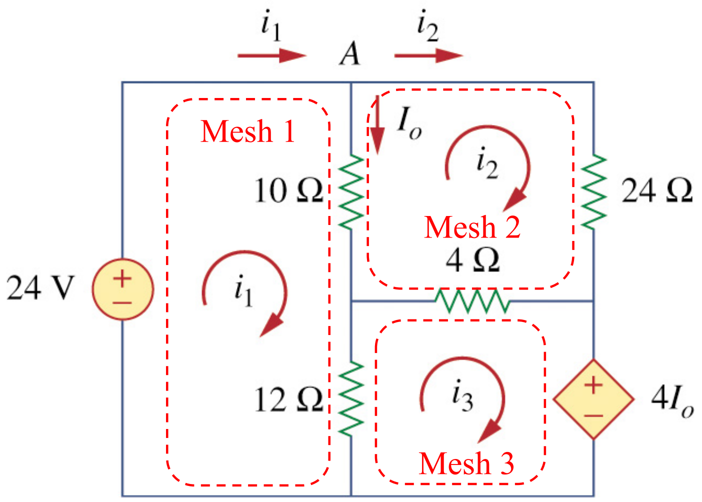

Fig. 3

Step 1: Identify meshes and assign variables

Step 2: Apply KVL to each mesh

For mesh 1, $-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$ (1)

For mesh 2, $24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$ (2)

For mesh 3, $4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$ (3)

Since $I_o = i_1 - i_2$, equation (3) becomes

$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$ (4)

By simplifying (1), (2) and (4), we can express them in matrix form as follows.

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

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Example (6): Mesh Analysis (cont'd)

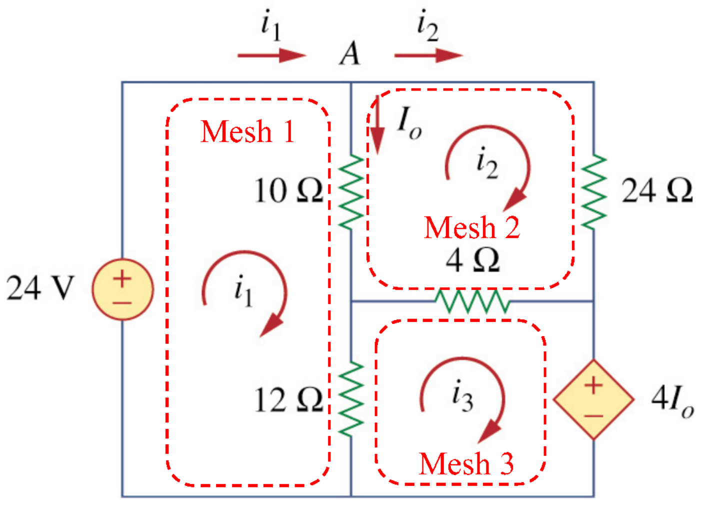


Fig. 3

Step 2 (cont'd):

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

To use Cramer's rule, we first need to calculate the determinants Δ, Δ_1, and Δ_2.

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{vmatrix} = 192 \qquad \Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{vmatrix} = (12)\begin{vmatrix} 19 & -2 \\ -1 & 2 \end{vmatrix} = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{vmatrix} = -(12)\begin{vmatrix} -5 & -2 \\ -1 & 2 \end{vmatrix} = 144$$

Note: Since we do not need i_3, Δ_3 is not calculated.

This yields $i_1 = \Delta_1/\Delta = 432/192 = 2.25$ A, $i_2 = \Delta_2/\Delta = 144/192 = 0.75$ A.

Thus, $I_o = i_1 - i_2 = 1.5$ A

Example (6): Mesh Analysis (cont'd)

$$\Delta = \begin{array}{c|ccc|c} & 11 & -5 & -6 & \\ & -5 & 19 & -2 & \\ & -1 & -1 & 2 & \\ - & 11 & -5 & -6 & + \\ - & -5 & 19 & -2 & + \\ - & & & & + \end{array}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{array}{c|ccc|c} & 12 & -5 & -6 & \\ & 0 & 19 & -2 & \\ & 0 & -1 & 2 & \\ - & 12 & -5 & -6 & + \\ - & 0 & 19 & -2 & + \\ - & & & & + \end{array} = 456 - 24 = 432$$

$$\Delta_2 = \begin{array}{c|ccc|c} & 11 & 12 & -6 & \\ & -5 & 0 & -2 & \\ & -1 & 0 & 2 & \\ - & 11 & 12 & -6 & + \\ - & -5 & 0 & -2 & + \\ - & & & & + \end{array} = 24 + 120 = 144$$

$$\Delta_3 = \begin{array}{c|ccc|c} & 11 & -5 & 12 & \\ & -5 & 19 & 0 & \\ & -1 & -1 & 0 & \\ - & 11 & -5 & 12 & + \\ - & -5 & 19 & 0 & + \\ - & & & & + \end{array} = 60 + 228 = 288$$

Mesh Analysis with Current Source

- Up until now, we have not yet applied mesh analysis to circuits containing current sources (either dependent or independent).
- The reason is that KVL requires that we know the voltage across each branch, but the voltage across a current source is *not* known in advance.
- Two possible scenarios:
  - Case 1: A current source exists **only in one mesh**. For example, the 5 A current source exists only in mesh 2 in Fig. 4. We can immediately infer that $i_2 = -5$ A. By applying KVL in mesh 1, we have $-10 + 4i_1 + 6(i_1 - i_2) = 0$ => $i_1 = -2$ A.

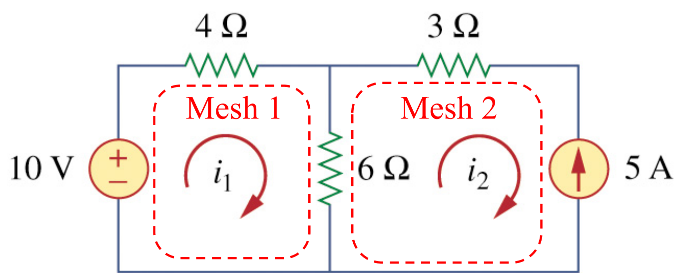


Fig. 4

Mesh Analysis with Current Source (cont'd)

➢ Case 2: A current source exists **between two meshes**. For example, in the original circuit shown in Fig. 5(a), a supermesh is created as shown in Fig. 5(b).

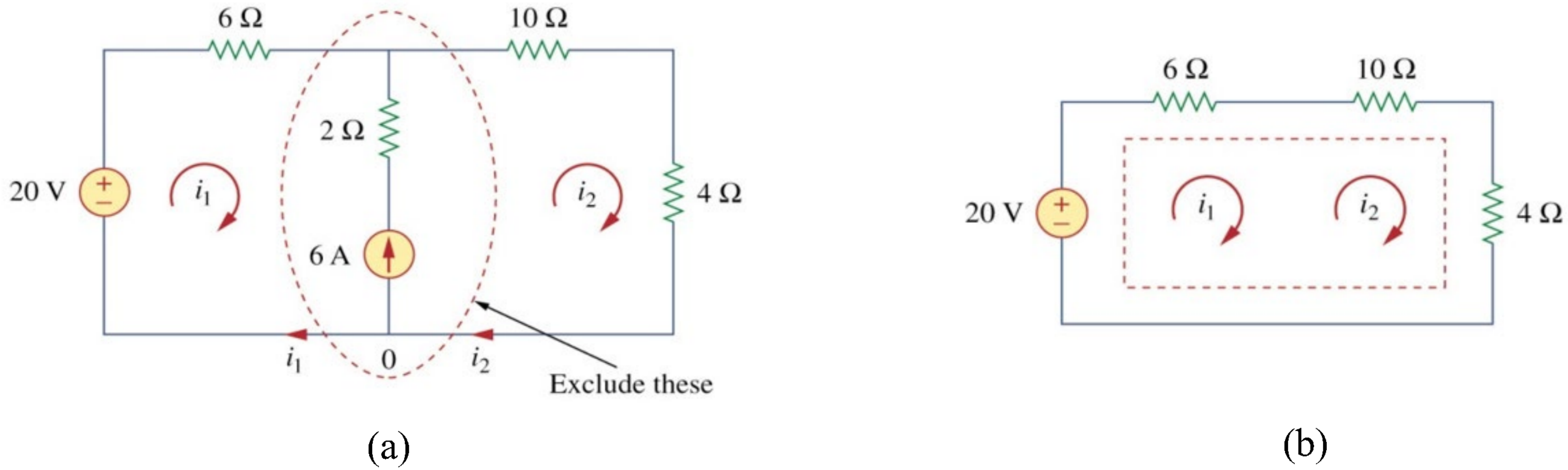


Fig. 5. (a) Original circuit with a current source between two meshes; (b) construction of a supermesh.

Supermesh

- A supermesh is created by *excluding* the current source and any elements connected in series with it.
- Supermesh must also satisfy KVL like any other meshes.
- Therefore, by applying KVL to the supermesh in Fig. 5(a) in the previous slide, we have $-20 + 6i_1 + 10i_2 + 4i_2 = 0$ (or equivalently, $6i_1 + 14i_2 = 20$).
- However, we only get one equation, but we have two unknowns!!
- Notice that at node 0, we can obtain another equation $i_2 = i_1 + 6$ by using KCL. Hence, by solving both equations, we have $i_1 = -3.2$ A, $i_2 = 2.8$ A.

Properties of Supermesh

- The current source *not* included in the supermesh provides the constraint equation necessary to solve for the mesh currents.
- A supermesh has no current of its own.
- A supermesh requires the application of both KCL and KVL.
- If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.

Example (7): Supermesh

Question: Consider the circuit shown in Fig. 6. Find i_1 to i_4 using mesh analysis.

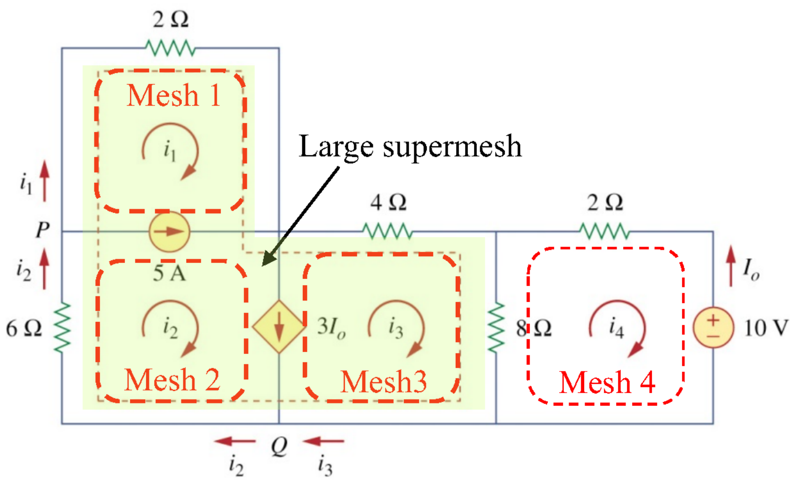


Fig. 6

- Mesh 1 and 2 form a supermesh.
- Mesh 2 and 3 form another supermesh.
- The two supermeshes intersect and form a large supermesh, as shown in Fig. 6.
- Apply KVL in the larger supermesh, we have
  $2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$ (1)
- Apply KVL in mesh 4, we have $2i_4 + 8(i_4 - i_3) + 10 = 0$ (2)
- Apply KCL at node P, $i_2 = i_1 + 5$ (3)
- Apply KCL at node Q, $i_2 = i_3 + 3I_o$. But, $I_o = -i_4 \Rightarrow i_2 = i_3 - 3i_4$ (4)

(continued on next slide)

Example (7): Supermesh (cont'd)

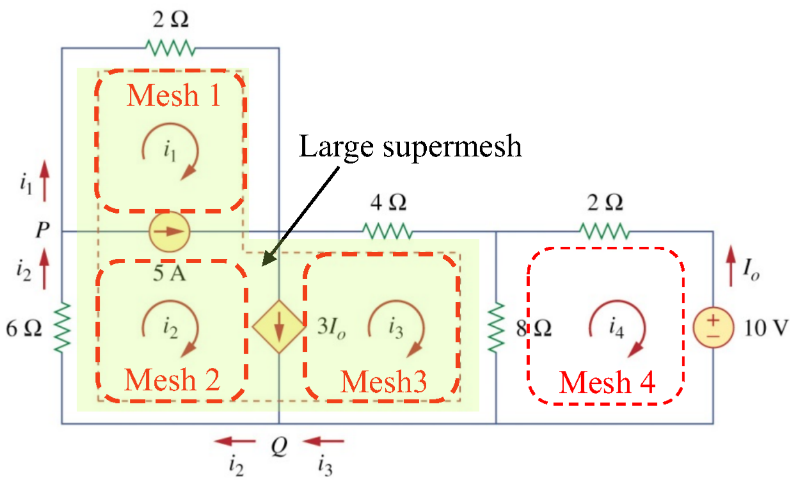


Fig. 6

- By simplifying equation (1)–(4), we have

$$\begin{cases} i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \\ 5i_4 - 4i_3 = -5 \\ i_2 = i_1 + 5 \\ i_2 = i_3 - 3i_4 \end{cases}$$

- By solving the above set of equations, we have

$i_1 = -7.5$ A
$i_2 = -2.5$ A
$i_3 = 3.93$ A
$i_4 = 2.143$ A

Nodal Analysis by Inspection

- In this section, a generalized procedure for nodal (or mesh) analysis will be presented.
- It is a shortcut approach of **writing node-voltage equations** by mere inspection of a circuit.
- When all sources in a circuit are **independent current sources**, we do *not* need to apply KCL to each node to obtain the node-voltage equations.
- In general, if a circuit with independent current sources has N non-reference nodes, the node-voltage equations can be written in terms of the conductances as

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

Note: This is valid for circuits with only independent current sources and linear resistors.

Nodal Analysis by Inspection (cont'd)

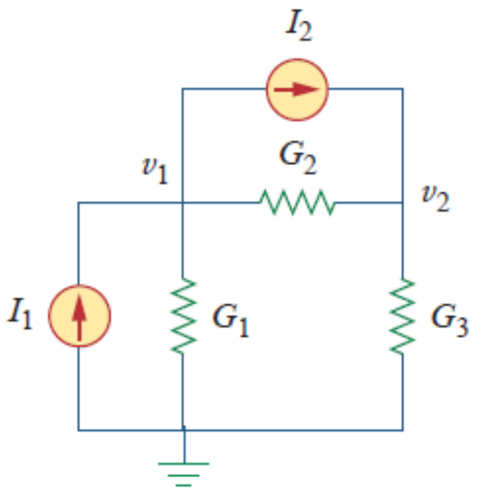
$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

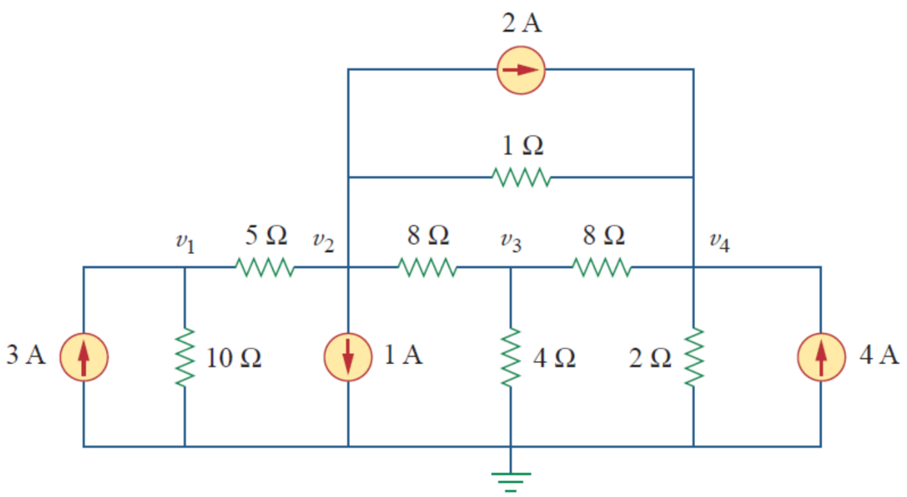
i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive



$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Example (8): Nodal Analysis by Inspection

Question: Write the node-voltage matrix equations for the following circuit by inspection.



Observe that the circuit contains four non-reference nodes, which implies that the size of the conductance matrix is 4×4.

Remember:

1. G_{kk} : Sum of the conductances connected to node k.
2. G_{kj} : Negative of the sum of the conductances directly connecting nodes k and j, where $k \neq j$.
3. i_k : Sum of all independent current sources directly connected to node k.

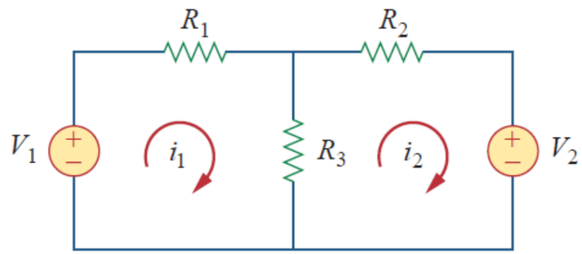
The node-voltage matrix equations are:

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

Mesh Analysis by Inspection

- We can obtain **mesh-current** equations by inspection when a linear resistive circuit has only **independent voltage sources**.
- In general, if the circuit has N meshes, the mesh-current equations can be expressed in terms of the resistances as

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$



R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

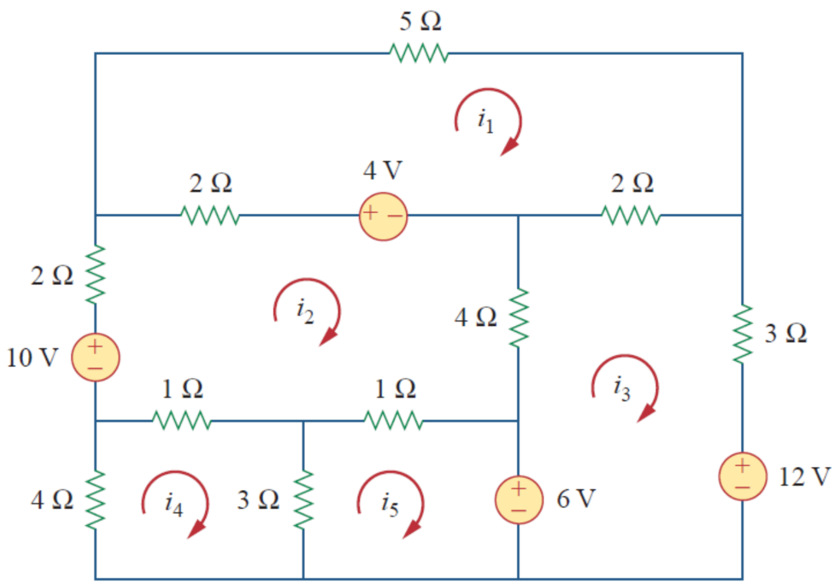
i_k = Unknown mesh current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

Example (9): Mesh Analysis by Inspection

Question: Write the mesh-current matrix equations for the following circuit by inspection.



Observe that the circuit contains five meshes. Hence, the size of the resistance matrix is 5×5.

Remember:

1. R_{kk} : Sum of the resistances in mesh k.
2. R_{kj} : Negative of the sum of the resistances in common with mesh k and mesh j, where $k \neq j$.
3. v_k : Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive.

The mesh-current matrix equations are:

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

Nodal vs. Mesh Analysis

- Both nodal and mesh analyses provide a systematic way of analyzing a complex network
- Which one is better? The answer depends on the particular network.
- Use mesh analysis if
  - the network contains many series-connected elements, voltage sources, or supermeshes.
  - the circuit has fewer meshes than nodes.
- Use nodal analysis if
  - The network contains many parallel-connected elements, current sources, or supernodes.
  - The circuit has fewer nodes than meshes.
- Of course, if node voltages are required, it may be more convenient to use nodal analysis. Likewise, if branch or mesh currents are required, it may be better to use mesh analysis.

Summary

I. Nodal Analysis

- ✓ Systematic application of KCL.
- ✓ Select one node (e.g. ground) as the reference node. The other nodes become non-reference nodes. Apply KCL to each of the non-reference nodes.
- ✓ Determine the values of the unknown node voltages.

II. Mesh Analysis

- ✓ Systematic application of KVL.
- ✓ Identify the meshes (loop currents) and apply KVL to each mesh.
- ✓ Determine the values of the mesh currents.
- ✓ Be able to use supermesh to solve for the mesh currents.

Summary

III. Nodal Analysis by Inspection

- ✓ Only applicable to circuits with only independent current sources and resistors.
- ✓ Construct an $N \times N$ matrix whose elements are the conductances (G_{kk} ,G_{kj} ,G_{jk}), where N is the total number of non-reference nodes.
- ✓ G_{kk} is the sum of conductances connected to node k.
- ✓ G_{kj} is the negative of the sum of conductances directly connected to node k and j, where $k \neq j$ (diagonal elements), and $G_{kj} = G_{jk}$.

IV. Mesh Analysis by Inspection

- ✓ Only applicable to circuits with only independent voltage sources and resistors.
- ✓ Construct an $N \times N$ matrix whose elements are the resistances (R_{kk} ,R_{kj} ,R_{jk}),
- ✓ R_{kk} is the sum of resistances in mesh node k.
- ✓ R_{kj} is the negative of the sum of resistances shared by mesh k and j.