Normalizing Distant Lights - In Depth

Recall that when normalizing a light, we divide it's luminance, L_V, by a $sizeFactor$ which depends on the shape properties of the light - ie, assuming that the exposure is zero:

$$L_V = Intensity/sizeFactor$$

For distant lights, we defined $sizeFactor$ to be:

$$sizeFactor(\theta_{max}) = \begin{cases} 1, & \text{if} \quad \theta_{max} = 0 \\ sin(\theta_{max})^2 * \pi, & \text{if} \quad 0 < \theta_{max} \leq \pi/2 \\ (2 - sin(\theta_{max})^2) * \pi, & \text{if} \quad \pi/2 < \theta_{max} \leq \pi \end{cases}$$

...where we define θ_{max} as:

$$\theta_{max} = toRadians(distantLightAngle)/2$$

We claimed that using this formula meant the following two properties held:

- **Property 1: Intensity = Illuminance** When normalize is enabled, the received illuminance from this light on a surface normal to the light's primary direction is held constant when angle changes, and the "intensity" property becomes a measure of the illuminance, expressed in lux, for a light with 0 exposure.

- **Property 2: Proportional to surface area of a distant sphere** If we assume that our distant light is an approximation for a "very far" sphere light (like the sun), then (for $0 < \theta_{max} \leq \pi/2$), this definition agrees with the definition used for area lights - ie, the total power of this distant sphere light is constant when the "size" (ie, $distantLightAngle$) changes, and our $sizeFactor$ is proportional to the total surface area of this sphere.

This section demonstrates why these properties hold, and how these formulas were derived.

Property 1: Intensity = Illuminance

We wish to demonstrate that when normalize is on and our $sizeFactor$ is defined as above, then intensity becomes a measure of incoming illuminance, E_V:

$$Intensity = E_V$$

To calculate E_V, we integrate the luminance, L_V, over all the solid angles Ω incoming from the light on a surface normal to the light - in this case, the spherical cap defined by θ_{max}, denoted $\Omega_{\theta_{max}}$:

$$E_V = \int\limits_{\Omega_{\theta_{max}}} L_V \, cos(\theta) \, d\Omega$$

By defining our spherical cap $\Omega_{\theta_{max}}$ in terms of polar coordinates (θ, ϕ), where $\theta \leq \theta_{max}$ (and assuming $0 < \theta_{max} \leq \pi/2$), we have:

$$E_V = \underset{\phi}{\int_0^{2\pi}} \underset{\theta}{\int_0^{\theta_{max}}} L_V \, cos(\theta) \, sin(\theta) \, d\theta \, d\phi$$

Our distant light is defined such that it is a constant value for all incoming directions on $\Omega_{\theta_{max}}$, and nothing is dependent on ϕ, so:

$$E_V = L_V \, 2\pi \underset{\theta}{\int_0^{\theta_{max}}} cos(\theta) \, sin(\theta) \, d\theta$$

$$E_V = L_V \, 2\pi \left[\frac{sin^2(\theta)}{2} \right]_0^{\theta_{max}}$$

$$E_V = L_V \, \pi sin^2(\theta_{max})$$

Next, we substitute our formula for L_V for a normalized light:

$$L_V = Intensity / sizeFactor(\theta_{max})$$

to get:

$$E_v * sizeFactor(\theta_{max}) = Intensity * \pi sin^2(\theta_{max})$$

...and we can see that if we use our desired definition of $sizeFactor(\theta_{max}) = \pi \, sin^2(\theta_{max})$, then

$$E_V = Intensity$$

Property 2: Proportional to surface area of a distant sphere

To see that the same formula for $sizeFactor$ is equivalent to that used for a sphere area light, consider that for a sphere light with radius r that is distance D away from a point P the relationship between θ_{max} (the half-angle of the circular arc subtended by the sphere) r and D is:

$$sin(\theta_{max}) = r/D$$

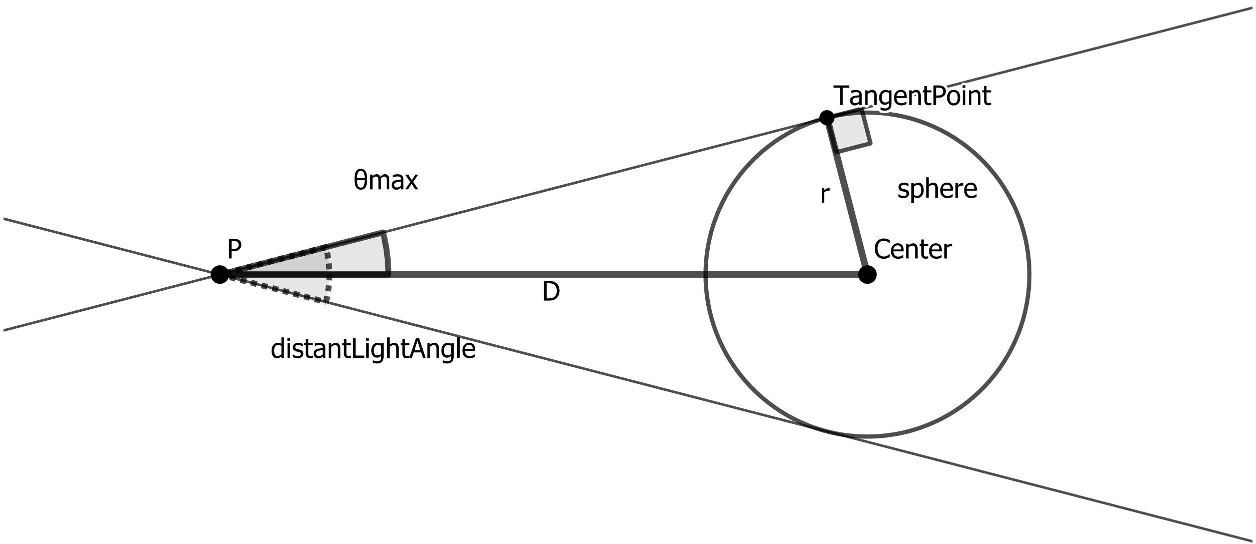

Figure 1: image

So, since the surface area of a sphere is $4\pi r^2$, for a "normal" sphere area light we have:

$$sizeFactor(Light) = surfaceArea(light) = 4\pi D^2 sin^2(\theta_{max})$$

Thus, for a sphere light, the $sizeFactor$ is proportional to $sin^2(\theta_{max})$, with a scaling factor which is a property of the distance of the light.

For our distant light, we assume that the distance D is so much larger than our local coordinate system measurements that, when calculating the direction between a local point $\mathbf{P}$ and the center of our distant sphere, $\mathbf{C}$, we can effectively ignore the local coordinates $\vec{P}_{Local}$, and assume the direction is constant.

However, if we assume when we change θ_{max} what we are "really" doing is scaling up the radius r of our sphere while keeping it's center in the same position, we can apply the above equation. In our distant light case, since we do not have a definite value for D, we cannot use this to give us a complete formula for $sizeFactor(\theta_{max})$ - however, it does demonstrate that any formula of the form

$$sizeFactor(\theta_{max}) = K\, sin^2(\theta_{max})$$

for some constant K will be proportional to the surface area of such a sphere, and therefore also hold power constant for this sphere.

A note on distant lights with $\theta_{max} > \pi/2$

To this point, I've ignored the possibility of distant lights that emit from more than a hemisphere - ie, where $distantLightAngle > 180°$, or $\theta_{max} > \pi/2$.

Property 1 To derive a formula for such angles via **Property 1** we first assume that rays coming from behind the receiving plane still contribute to total illuminance - transparent and translucent materials are possible, after all! Then the "true" formula for E_V becomes:

$$E_V = \int\limits_{\Omega_{\theta_{max}}} L_V \, |cos(\theta)| \, d\Omega$$

For $0 < \theta_{max} \leq \pi/2$, this is identical to the above version, as $cos(\theta) = |cos(\theta)|$ on this domain. For $\pi/2 < \theta_{max} \leq \pi$, we need to split the polar-coordinate integral at $\theta = \pi/2$, and we get:

$$E_V = \underset{\phi}{\int_0^{2\pi}} \underset{\theta}{\int_0^{\pi/2}} L_V \, cos(\theta) \, sin(\theta) \, d\theta \, d\phi + \underset{\phi}{\int_0^{2\pi}} \underset{\theta}{\int_{\pi/2}^{\theta_{max}}} L_V \, (-cos(\theta)) \, sin(\theta) \, d\theta \, d\phi$$

$$E_V = L_V \, \pi - L_V \, 2\pi \underset{\theta}{\int_{\pi/2}^{\theta_{max}}} cos(\theta) \, sin(\theta) \, d\theta$$

$$E_V = L_V \, \pi \left(1 - 2 \left[\frac{sin^2(\theta)}{2} \right]_{\pi/2}^{\theta_{max}} \right)$$

$$E_V = L_V \, \pi \left(2 - sin^2(\theta_{max}) \right)$$

Thus, if $E_V = Intensity$ and $L_V = Intensity/sizeFactor(\theta_{max})$,

$$sizeFactor(\theta_{max}) = \pi \left(2 - sin^2(\theta_{max}) \right), \quad \text{if } \pi/2 < \theta \leq \pi$$

Property 2 **Property 2** doesn't really apply when $\theta_{max} > \pi/2$, as our assumption that a distant light is an approximation of far spherical light breaks down, because there's no way such a light would result in incident light coming in from more than a hempisphere, but less than the complete sphere.

But our $sizeFactor(\theta_{max})$ function is now at least well defined and continuous in this region, even if we no longer have an appropriate physical light analogy for this domain.