Bubble Rise Dynamics in a Field of Varying Gravity

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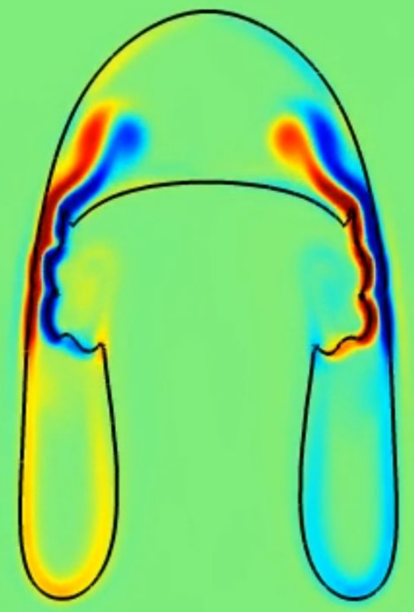

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CONTEXTUALIZATION

Reorientation Studies

Performance optimization of liquid propulsion rocket systems.
Reorientation accelerations are used to collect the propellant at the inlet of the engine.

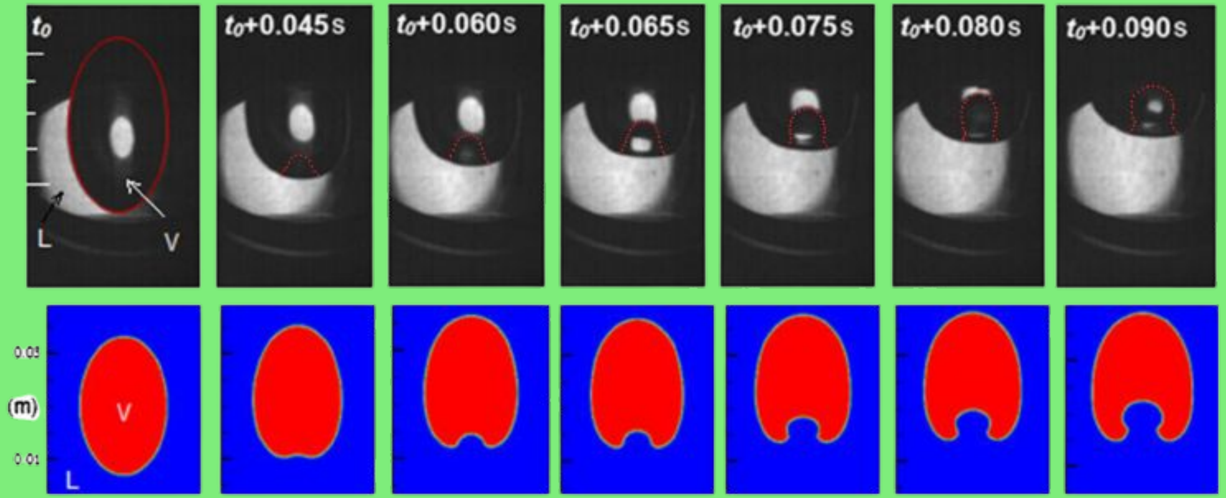
Configuration

The bubble occupies around 20-80 % the container volume.
Elliptical shape and varying accelerations.

Phenomena

Formation of a geyser (unstable concavity).
Under severe acceleration conditions it reach the fore-end of the tank.

CONTEXTUALIZATION



Gandikota et al. Work

Experimental and numerical studies regarding bubble ascension in a confined environment under varying accelerations

Experimental Setup

Experiments are carried out using the facility OLGA, capable of realizing experiments with a very fast variation of gravity

OBJECTIVES

CONTEXTUALIZATION

VALIDATION

Acquire confidence and solidity in Basilisk as a skillful tool to study bubble rise in different scenarios

COMPARISON TO GANDIKOTA

Validate and discuss the work done by the author.

TRANSIENT BEHAVIOR

Study how the added mass force varies under confinement effects and different bubble eccentricities.

MATHEMATICAL FORMULATION

METHODOLOGY

Incompressible Navier-Stokes:

$$\overrightarrow{\nabla} \cdot \overrightarrow{\mathbf{u}} = 0$$

$$\rho \left[\frac{\partial(\overrightarrow{\mathbf{u}})}{\partial t} + (\overrightarrow{\mathbf{u}} \cdot \overrightarrow{\nabla})\overrightarrow{\mathbf{u}} \right] = -\overrightarrow{\nabla} p + \eta \Delta \overrightarrow{\mathbf{u}} - \rho g(t) \overrightarrow{\mathbf{e}_z}$$

$$g(t) = g_0 f(t)$$

+

Young-Laplace Equation

$$\sigma_1 - \sigma_2 = \gamma \kappa \cdot \overrightarrow{\mathbf{n}}$$

MATHEMATICAL FORMULATION

Nondimensionalization

$$\begin{cases} x = x^* \ R \\ u = u^* \ \sqrt{g_0 R} \end{cases} \qquad \begin{cases} t = t^* \ \dfrac{\sqrt{R}}{g_0} \\ p = p^* \ p_0 \end{cases}$$

$$\begin{cases} \rho_1^* = 1 \\ \eta_1^* = \dfrac{1}{Ga} \\ g^*(t) = f(t) \end{cases} \qquad \begin{cases} \rho_2^* = \dfrac{1}{\rho_R} \\ \eta_2^* = \dfrac{1}{\eta_R} \dfrac{1}{Ga} \\ g^*(t) = f(t) \end{cases}$$

$$Bo = \frac{\rho_1 R^2 g_0}{\gamma}$$

$$\rho_1^* \left[\frac{\partial(\overrightarrow{\mathbf{u}^*})}{\partial t^*} + (\overrightarrow{\mathbf{u}^*} \cdot \overrightarrow{\nabla^*})\overrightarrow{\mathbf{u}^*} \right] = -\overrightarrow{\nabla^* p^*} + \eta_1^* \Delta^* \overrightarrow{\mathbf{u}^*} - \rho_1^* g^*(t) \overrightarrow{\mathbf{e}_z}$$

$$\rho_2^* \left[\frac{\partial(\overrightarrow{\mathbf{u}^*})}{\partial t^*} + (\overrightarrow{\mathbf{u}^*} \cdot \overrightarrow{\nabla^*})\overrightarrow{\mathbf{u}^*} \right] = -\overrightarrow{\nabla^* p^*} + \eta_2^* \Delta^* \overrightarrow{\mathbf{u}^*} - \rho_2^* g^*(t) \overrightarrow{\mathbf{e}_z}$$

MATHEMATICAL FORMULATION

METHODOLOGY

Nondimensional Parameters

$$\begin{cases} Ga = \dfrac{\rho_1 \sqrt{R g_0} R}{\eta_1} \\ Bo = \dfrac{\rho_1 R^2 g_0}{\gamma} \end{cases} \qquad \begin{cases} \rho_R = \dfrac{\rho_1}{\rho_2} \\ \eta_R = \dfrac{\eta_1}{\eta_2} \end{cases}$$

METHODOLOGY

Bond Number

$$Bo = \frac{\rho_1 R^2 g_0}{\gamma}$$

Gravity VS Surface Tension

Galilei Number

$$Ga = \frac{\rho_1 \sqrt{R g_0} R}{\eta_1}$$

Gravity VS Viscosity

GALILEI AND BOND NUMBERS

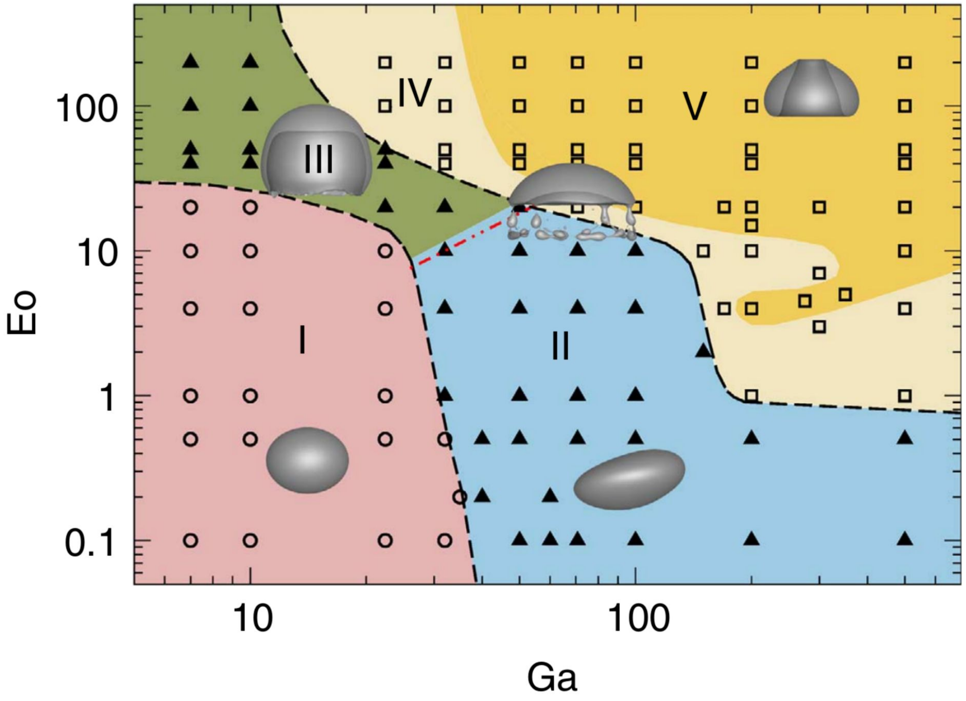
METHODOLOGY

Challenge :
Gandikota's Setup

→

High Galilei and Bond Numbers

- Densities ratio $\rho_R = 581.763$
- Viscosities $\mu_R = 28$
- Galilei numbers $Ga_1 = 11418.8$ and $Ga_2 = 18489.9$
- Bond numbers $Bo_1 = 33.4671$ and $Bo_2 = 63.6357$



TRIPATHI'S DIAGRAM

Different regimes of bubble shape and behaviour varying with Galilei Number and Bond Number.

BREAKUP REGIMES

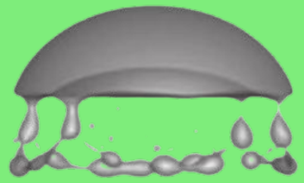
Peripheral Breakup

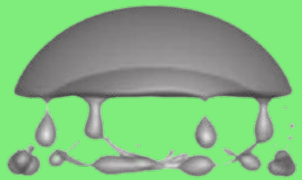
Central Breakup

METHODOLOGY

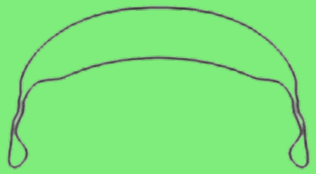
BREAKUP REGIMES

METHODOLOGY





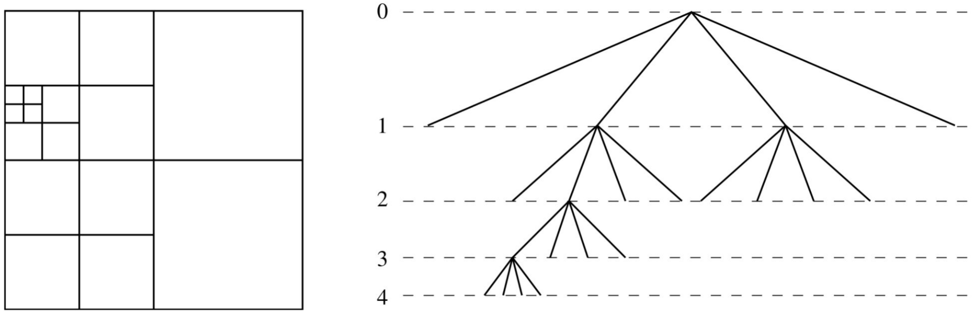
3D Simulations



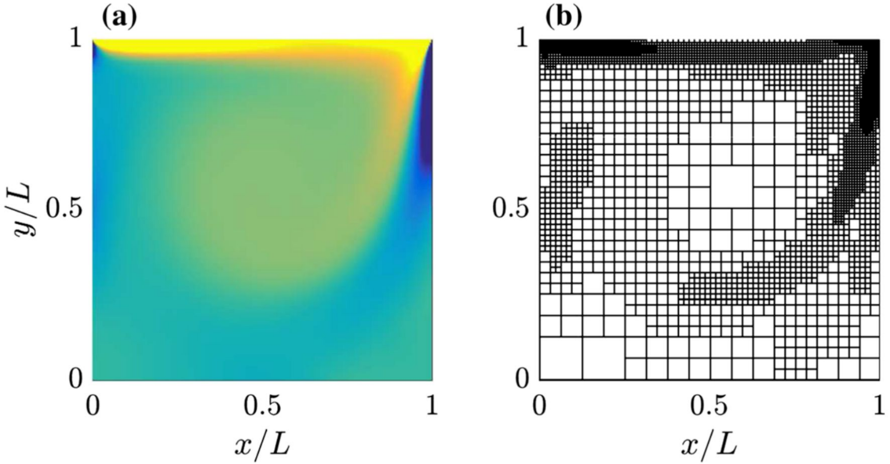
2D Simulations

BASILISK

METHODOLOGY



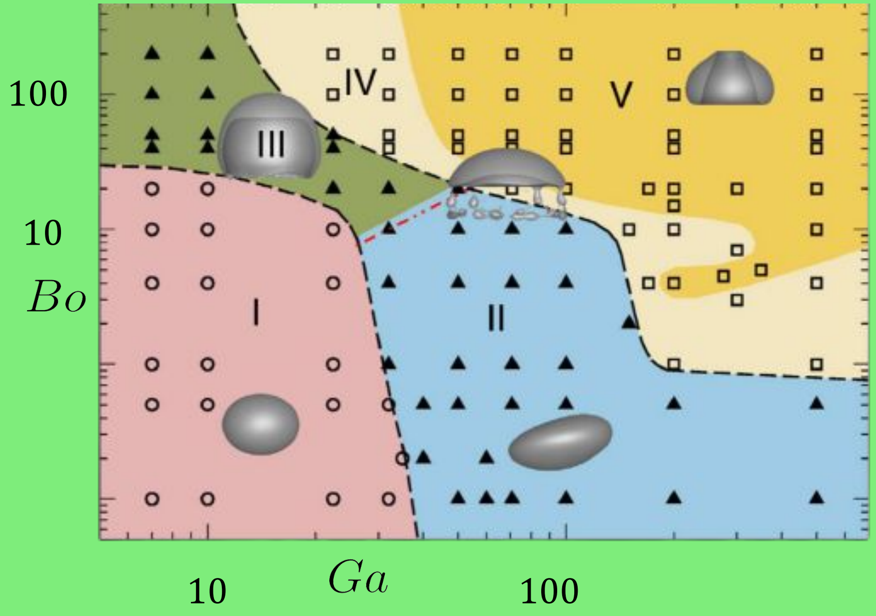
Adaptive mesh refinement based on octrees discretization



Example application to lid-driven cavity simulation

TRIPATHI'S DIAGRAM

VALIDATION



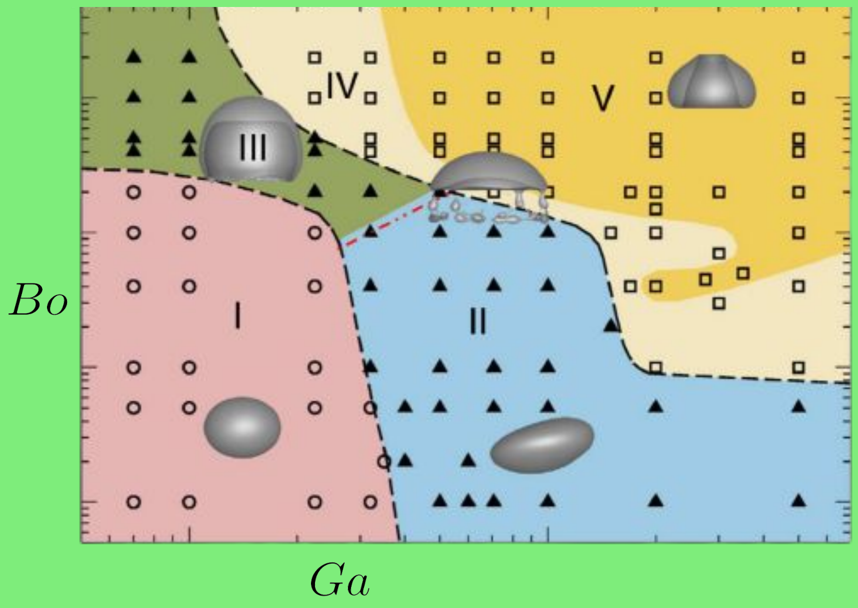
Nondimensional Parameters:

$$\rho_R = 1000$$

$$\mu_R = 100$$

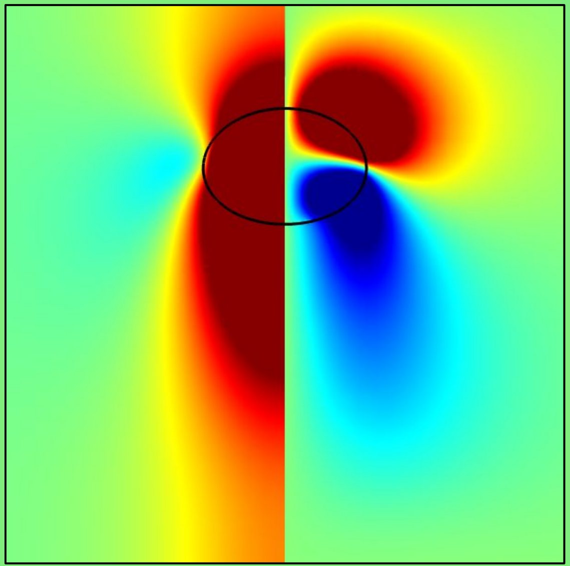
	Ga	*Bo*
Region I	10	1
Region II	100	5
Region III	10	100
Region IV	30	100
Region V	200	100

TRIPATHI'S DIAGRAM



Region I

Axisymmetric, terminal, elliptical shape

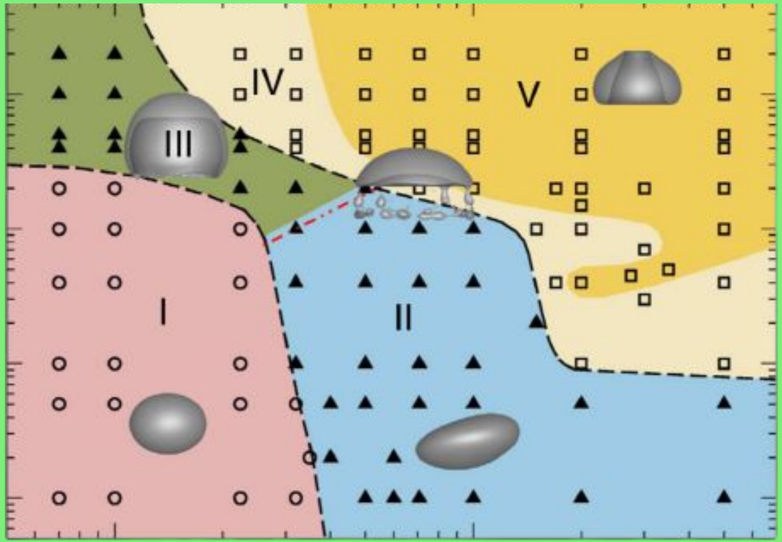


VALIDATION

TRIPATHI'S DIAGRAM

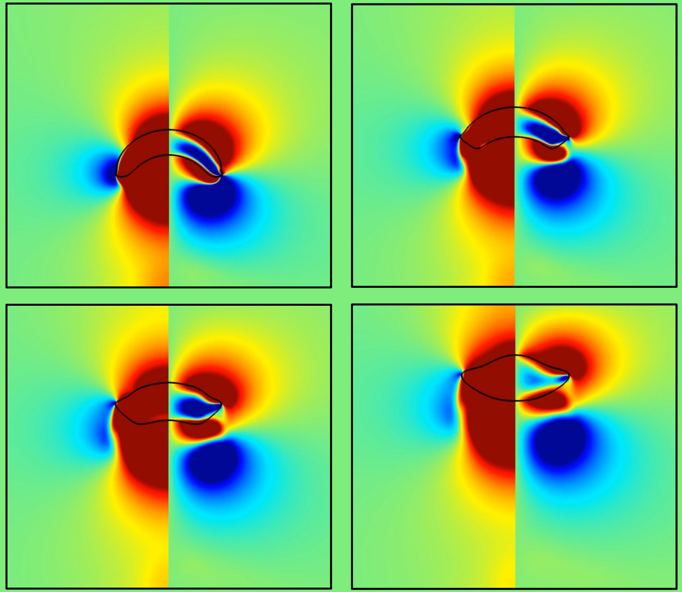
Region II

Asymmetric, oscillating



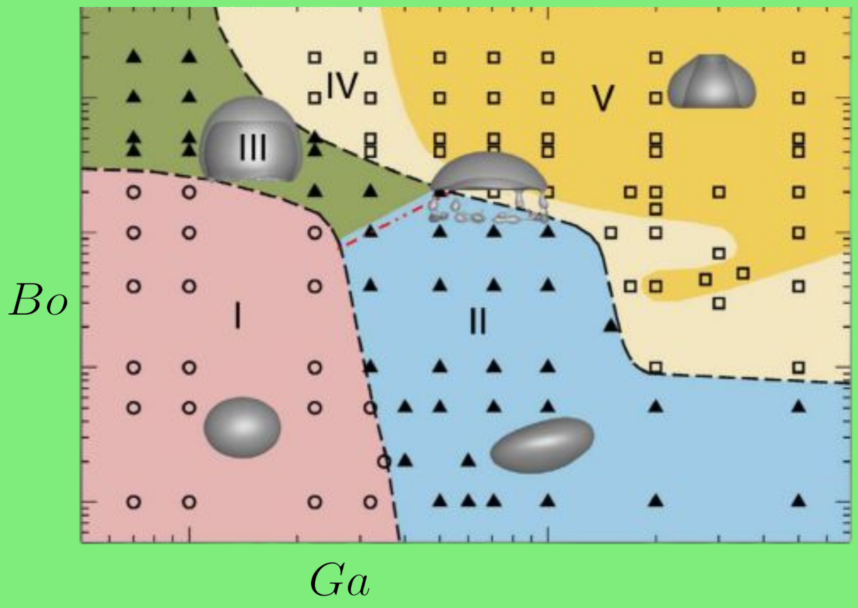
Bo

Ga



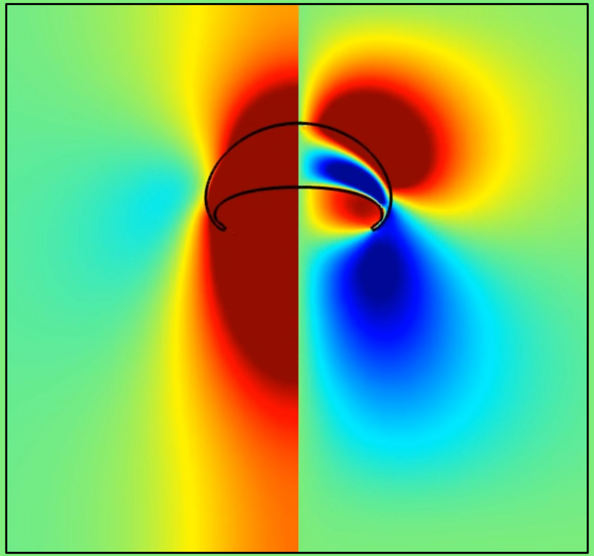
VALIDATION

TRIPATHI'S DIAGRAM



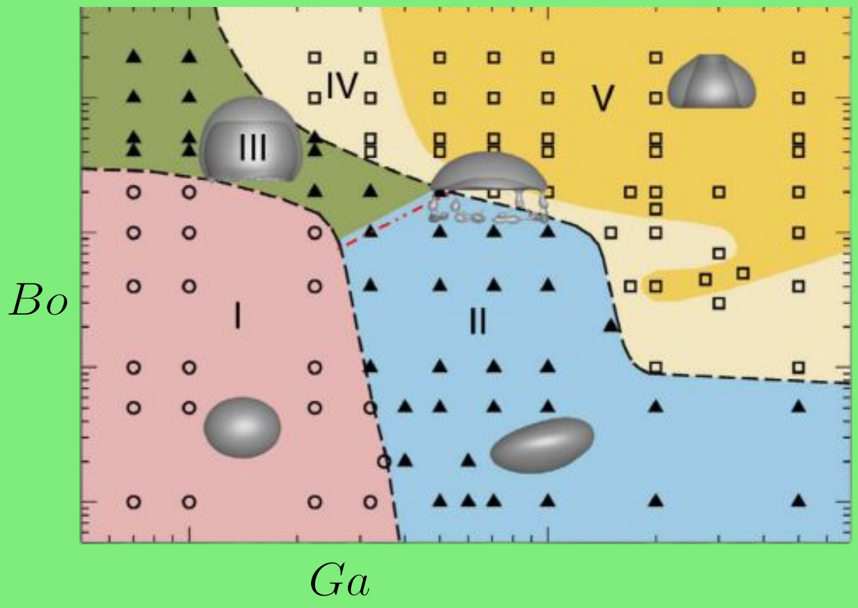
Region III

Asymmetric, geyser formation, path instability



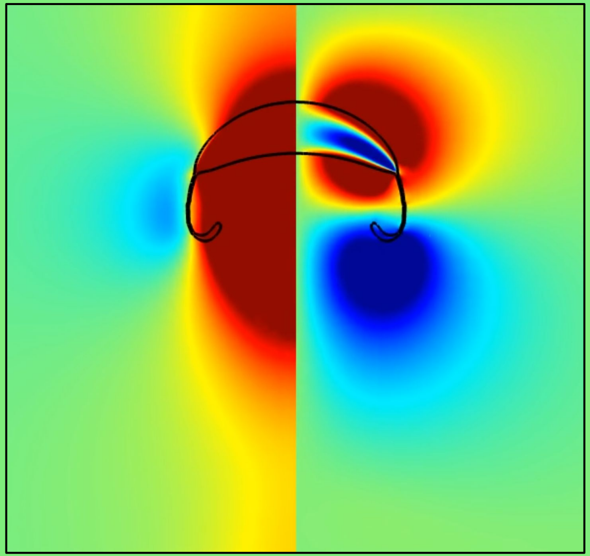
VALIDATION

TRIPATHI'S DIAGRAM



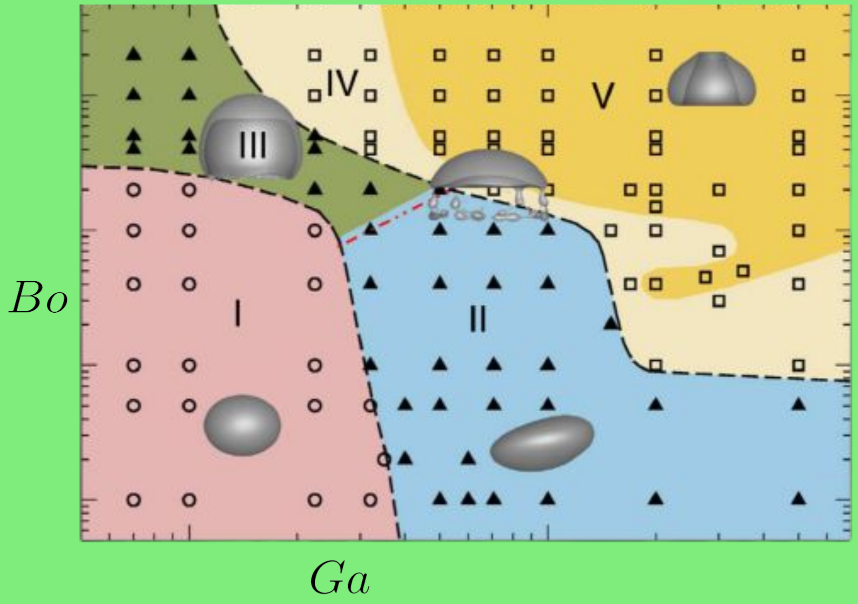
Region IV

Asymmetric, geyser formation, peripheral breakup



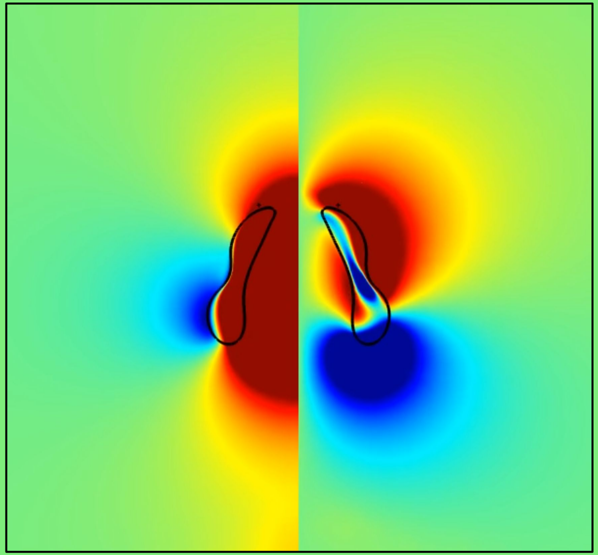
VALIDATION

TRIPATHI'S DIAGRAM



Region V

Axisymmetric, geyser formation, central breakup



VALIDATION

TAYLOR BUBBLES

VALIDATION

Terminal Velocity for Ascending Bubble:

$$U = \frac{2}{3}\sqrt{gR_C}$$

Validated bubble regime for Taylor's studies

Volume [cm]	Radius [cm]	*Ga*	*Bo*
1.5	0.71	2099.62	6.76
2	0.78	2424.43	8.20
2.5	0.84	2710.6	9.52
10	1.33	5421.2	23.98
50	2.28	12122.2	70.11
100	2.88	17143.3	111.29
200	3.63	24244.3	176.673

Nondimensional Parameters:

$$\rho_R = 813.878$$

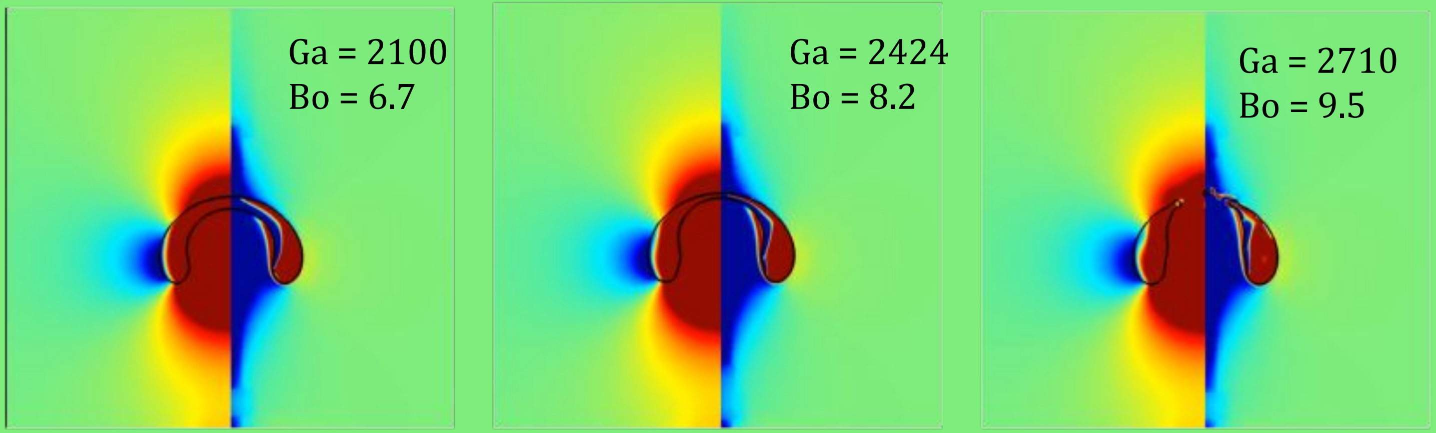
$$\mu_R = 49.171$$

TAYLOR BUBBLES

VALIDATION

Central Breakup Problem

Snapshots of instant of greatest deformation

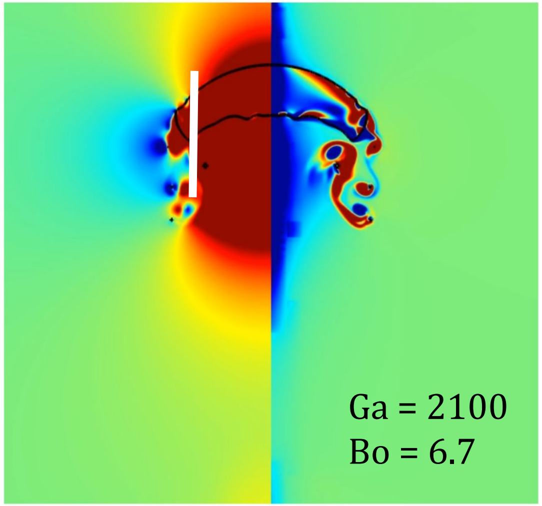


(a) **Volume** = 1.5 cm^3 (b) **Volume** = 2.0 cm^3 (c) **Volume** = 2.5 cm^3

Bubble 1 **Bubble 2** **Bubble 3**

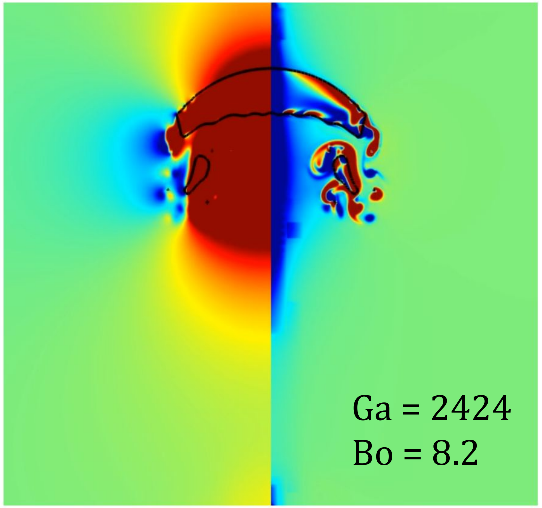
TAYLOR BUBBLES

VALIDATION



(a) **Volume** = 1.5 cm^3

Bubble 1



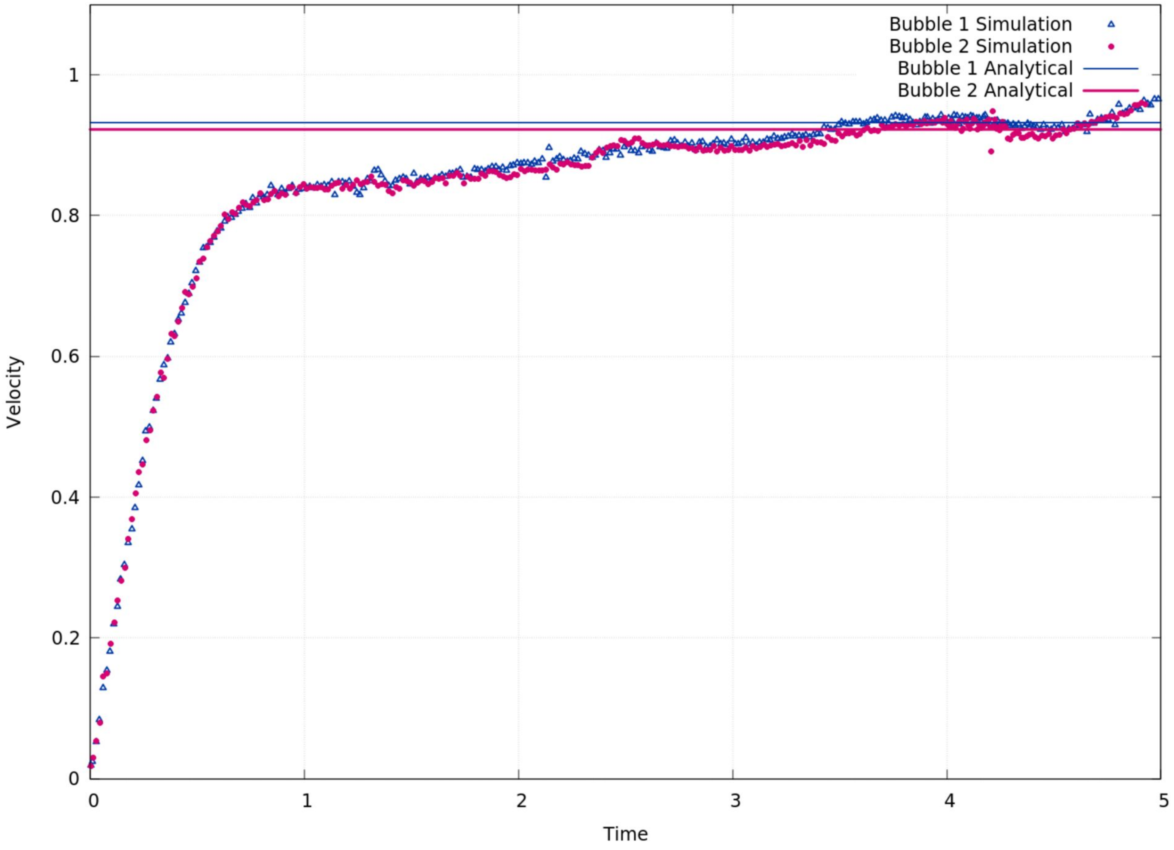
(b) **Volume** = 2.0 cm^3

Bubble 2

Terminal shape for successful Taylor bubbles simulation

TAYLOR BUBBLES

VALIDATION



Comparison between analytical and numerical solution for Taylor bubbles velocities

TAYLOR BUBBLES

RESULTS

Central Breakup Problem

How to circumvent it?

TAYLOR BUBBLES

RESULTS

Central Breakup Problem

How to circumvent it? → **Varying gravity approach**

$$g(\mathbf{t}) = \begin{cases} \frac{g_0}{2}\left(1 - \cos\left(\frac{\pi\,\mathbf{t}}{t_u}\right)\right) & \text{if } \mathbf{t} \leq t_u \\ g_0 & \text{if } \mathbf{t} > t_u \end{cases}$$

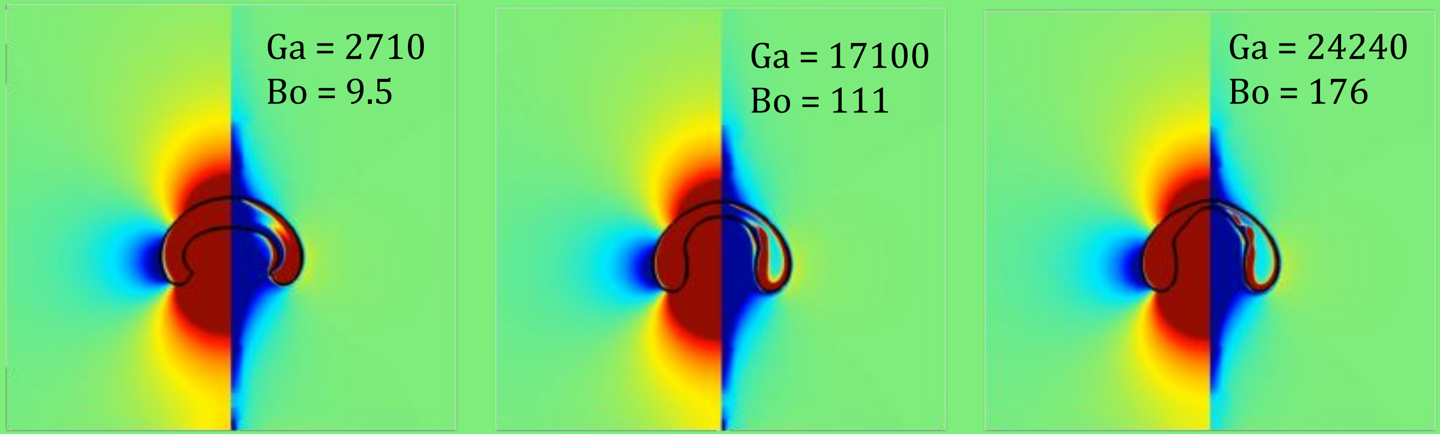
t_u Breakup instant in the original simulation

TAYLOR BUBBLES

RESULTS

Central Breakup Problem

Snapshots of instant of greatest deformation
With varying gravity approach

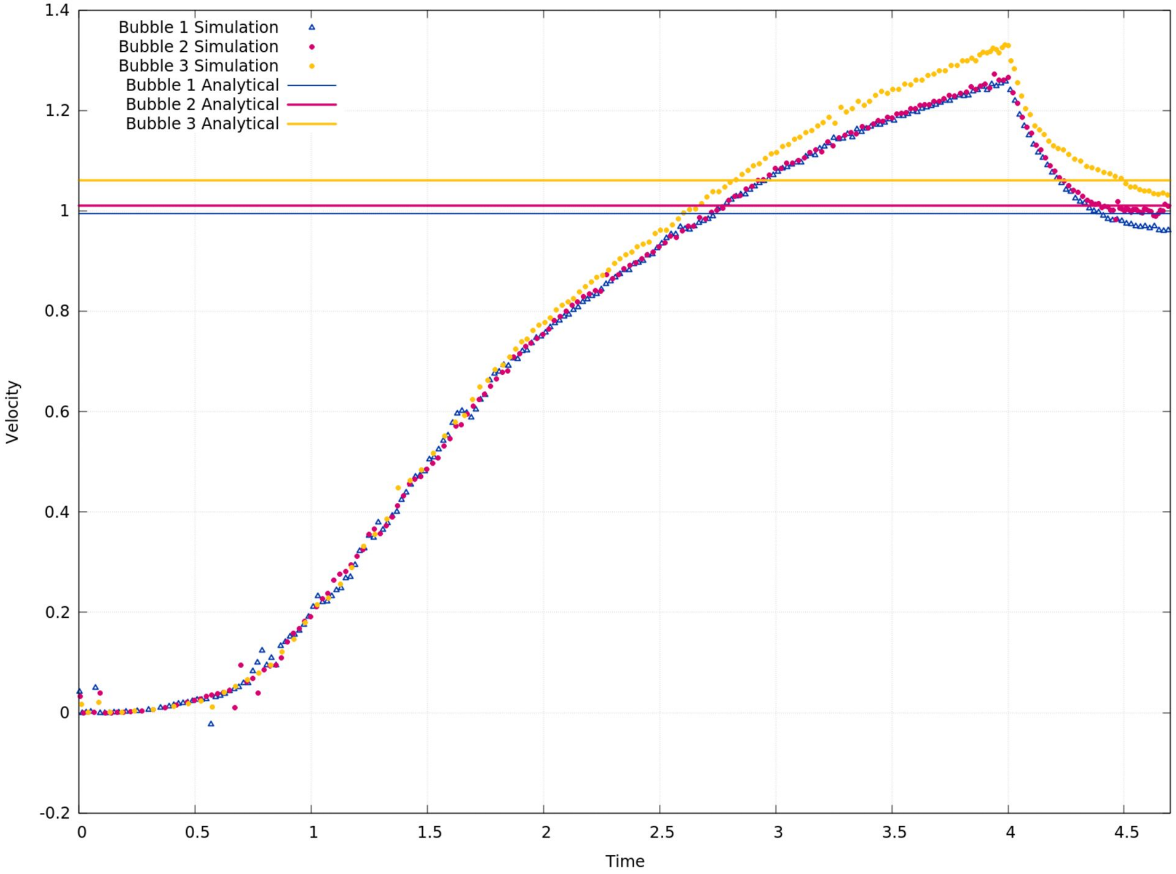


(a) **Volume** = 2.5 cm^3 (b) **Volume** = 100 cm^3 (c) **Volume** = 200 cm^3

Bubble 1 **Bubble 2** **Bubble 3**

TAYLOR BUBBLES

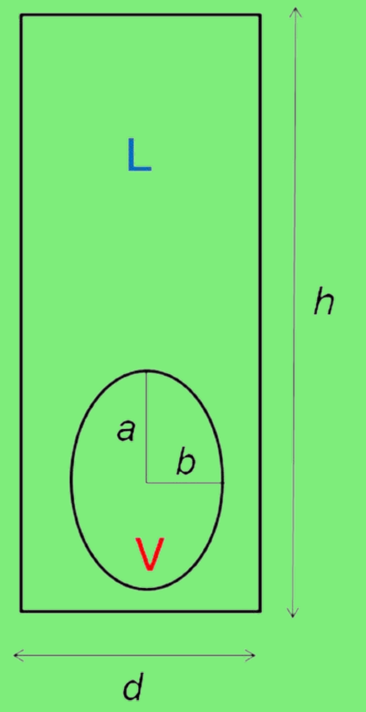
RESULTS



Comparison between analytical and numerical solution for Taylor bubbles velocities **with varying gravity approach**

GANDIKOTA COMPARISON

- Densities ratio $\rho_R = 581.763$
- Viscosities $\mu_R = 28$
- Galilei numbers $Ga_1 = 11418.8$ and $Ga_2 = 18489.9$
- Bond numbers $Bo_1 = 33.4671$ and $Bo_2 = 63.6357$



	Semi-major axis (mm)	Semi-minor axis (mm)	Bubble Volume (mm^3)	Fill Ratio (%)
Bubble 1	13.5	8.5	4100	5.9
Bubble 2	16.5	12.5	10750	15.3

GANDIKOTA COMPARISON

RESULTS

2D Simulation

- Densities ratio $\rho_R = 581.763$
- Viscosities $\mu_R = 28$
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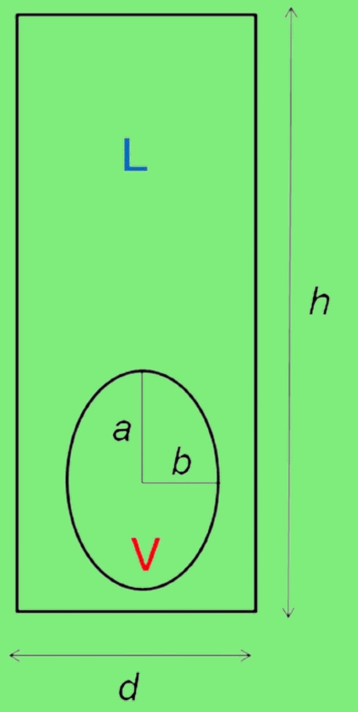

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GANDIKOTA COMPARISON

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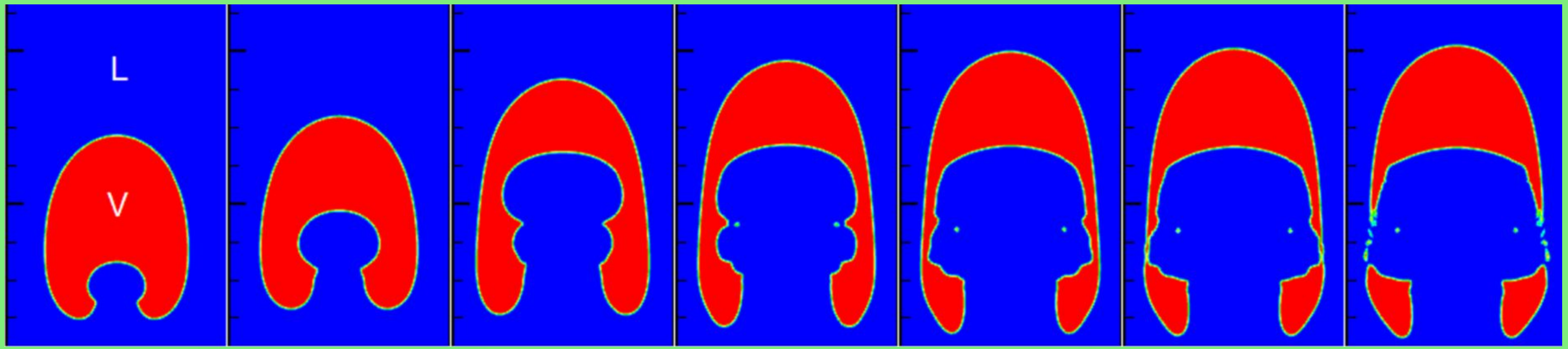

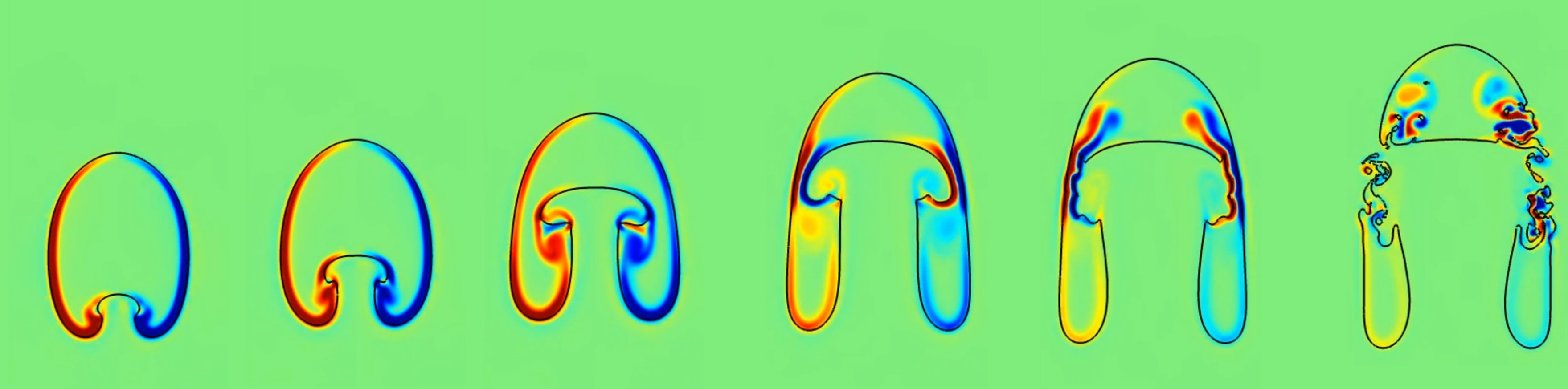

ADDED MASS

$$\underbrace{-F\,\vec{\mathbf{u}}}_{\text{Rate of Additional Work}} = \underbrace{\frac{dK}{dt}}_{\text{Rate of Kinectic Energy}}$$

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$$m\frac{d\overrightarrow{\mathbf{u}(t)}}{dt} = F_{Buoyancy} - F_{Weight} - F_{Drag} - F_{Added\ Mass}$$

ADDED MASS

RESULTS

$$\underbrace{-F\vec{\mathbf{u}}}_{\text{Rate of Additional Work}} = \underbrace{\frac{dK}{dt}}_{\text{Rate of Kinectic Energy}}$$

$$m\frac{d\overrightarrow{\mathbf{u}(t)}}{dt} = F_{Buoyancy} - F_{Weight} - F_{Drag} - F_{Added\ Mass}$$

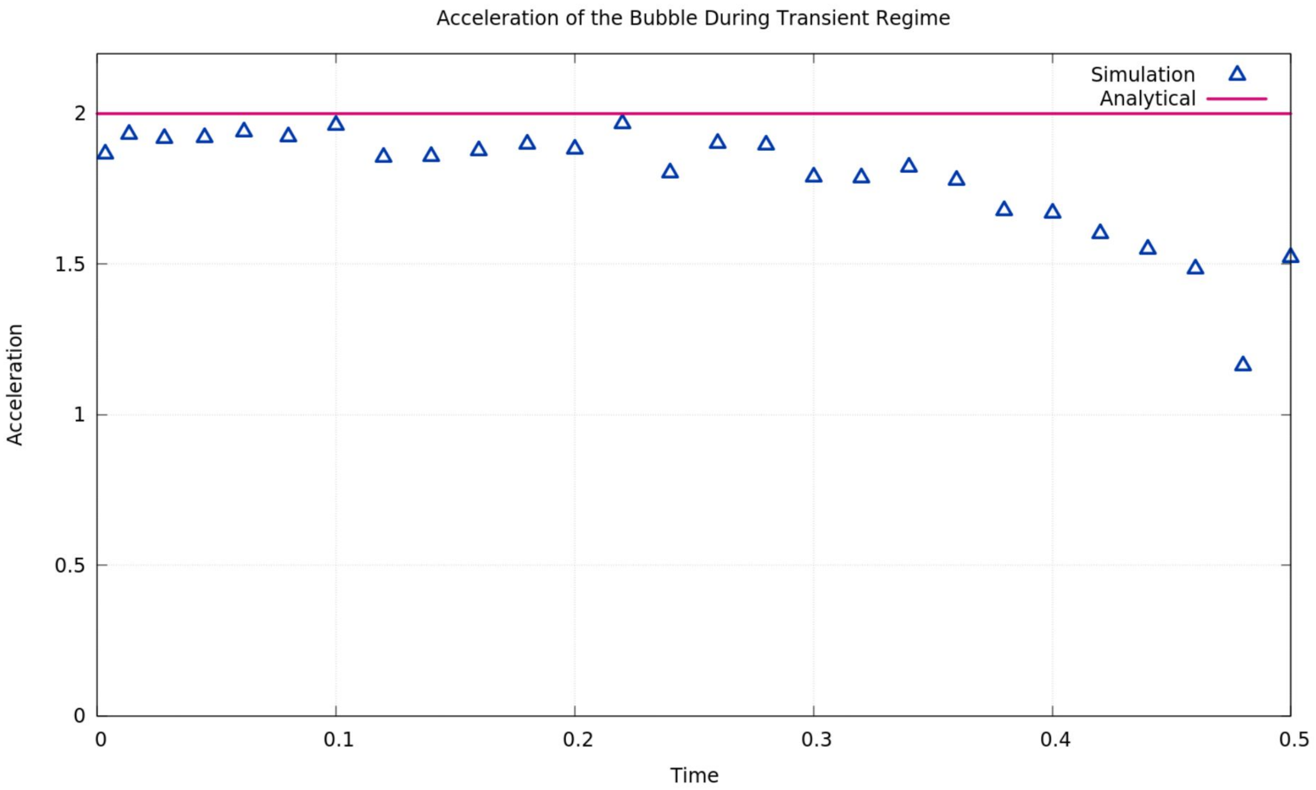
$$\frac{4}{3}\pi R_b^3(\rho_2 + C_m\rho_1)\frac{d\overrightarrow{\mathbf{u}(t)}}{dt} = \frac{4}{3}\pi R_b^3(\rho_1 - \rho_2)g(t) - \frac{1}{2}\rho_1\pi R_b^2 C_D \overrightarrow{\mathbf{u}(t)}^2$$

$$\boxed{\frac{d\overrightarrow{\mathbf{u}(t)}}{dt} = \frac{1}{C_m}g(t)}$$

$$C_m = 0.5$$

For spherical bubbles without confinement effects

ADDED MASS



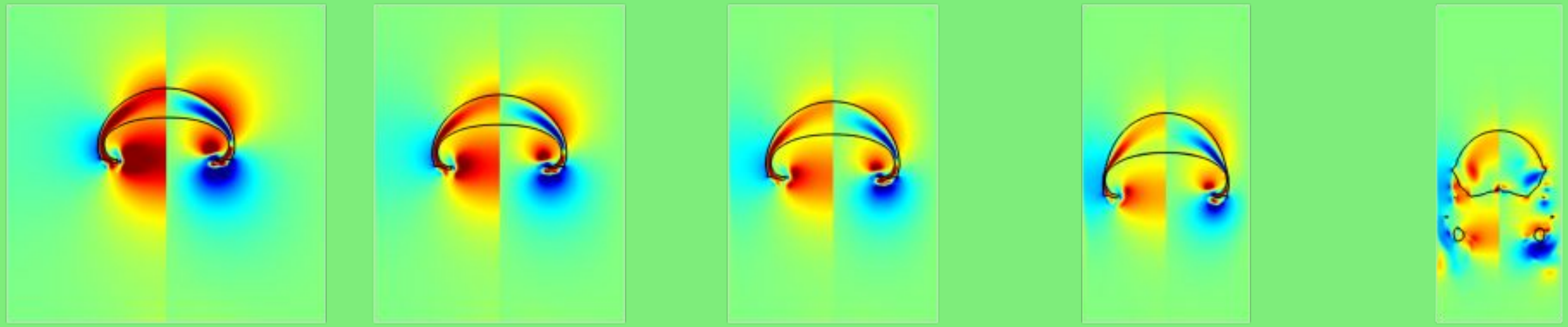
RESULTS

Acceleration of simple spherical bubble in infinite domain

ADDED MASS

Domain Ratio Study

	Radius	Ratio $\frac{r}{h}$
Bubble 1	4	0.5
Bubble 2	3.5	0.4375
Bubble 3	3	0.375
Bubble 4	2.5	0.3125
Bubble 5	2	0.25



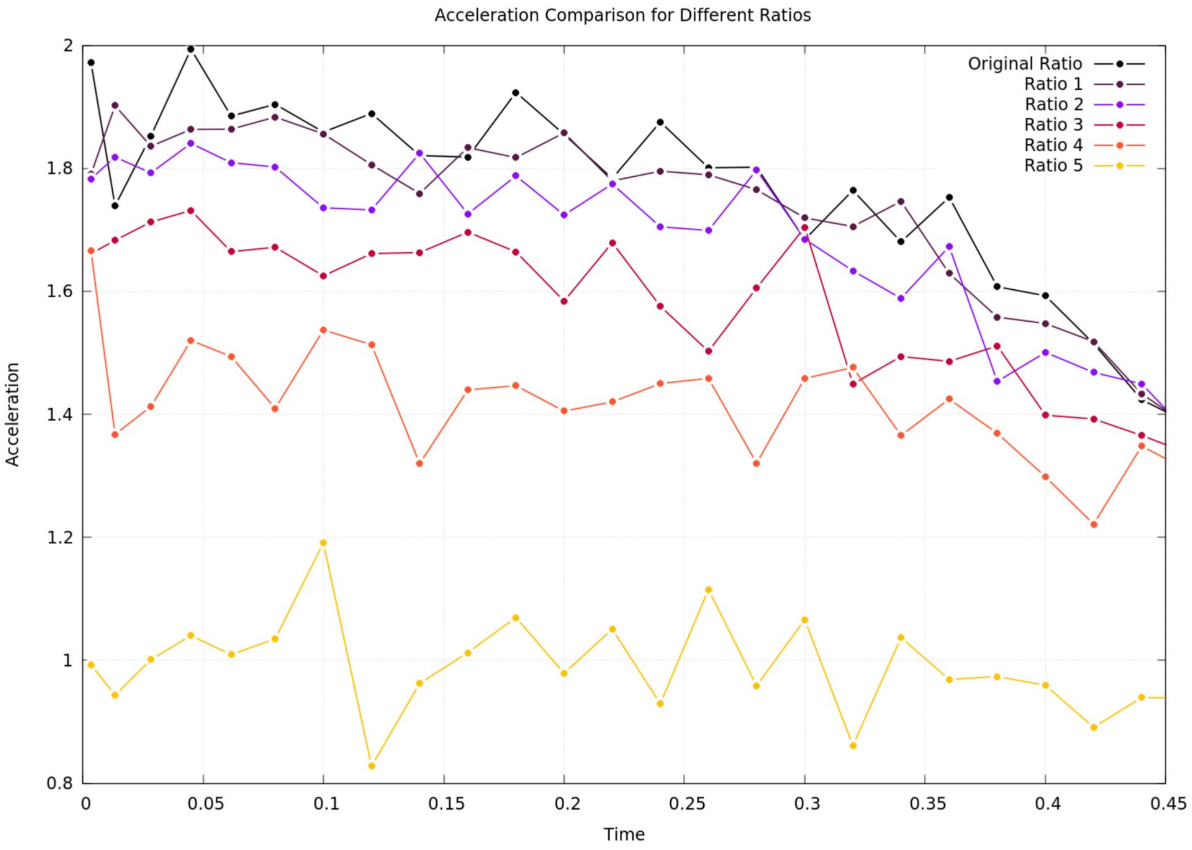
(a) Bubble 1 (b) Bubble 2 (c) Bubble 3 (d) Bubble 4 (e) Bubble 5

RESULTS

ADDED MASS

Domain Ratio Study

RESULTS

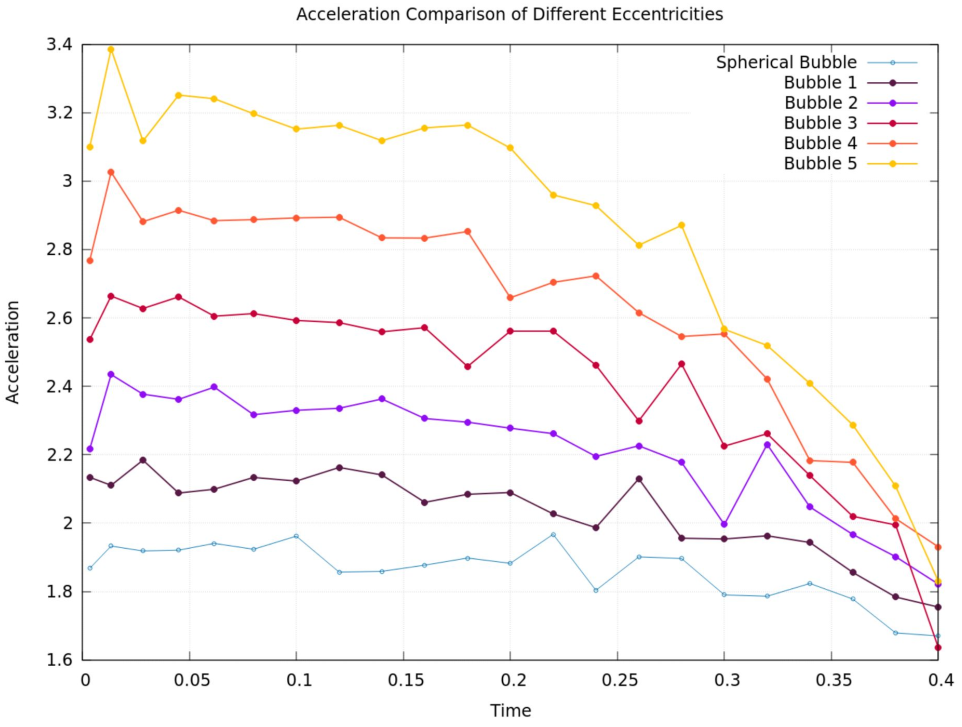


Nondimensional acceleration comparison between bubble rises at different confinement ratios.

ADDED MASS

Eccentricity Study

RESULTS



	Major Axis	Minor Axis	Eccentricity
Bubble 1	1.052	0.958	0.413
Bubble 2	1.104	0.916	0.558
Bubble 3	1.156	0.874	0.654
Bubble 4	1.208	0.832	0.725
Bubble 5	1.26	0.79	0.779

Nondimensional acceleration comparison between bubble rises at different eccentricities.

OVERVIEW

- Basilisk as a powerful tool to study interfacial dynamics
- Non-convergence of high Galilei and high Bond axisymmetric simulations
- Development of an approach to circumvent this problem
- Successful validation of Gandikota's results
- Study of bubble transient behavior in a classic reorientation study setup

PERSPECTIVES

- 3D Simulations for high Galilei and high Bond numbers
- More profound study of non-convergence in axisymmetric simulations

THANKS

Does anyone have any questions?

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