

Introduction to Deep Learning for Computer Vision

Assignment 3: Simple Linear Classifier II

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Abstract

TODO

1 Derivation of the gradient for cross entropy

1.1 The cross entropy function

$$L_i = -\log \frac{\sum_j t_{ij} e^{s_{ij}}}{\sum_k e^{s_{ik}}} \quad (1)$$

I assume \log base e for (1).

$$t_{ij} = \delta_{ij} = \begin{cases} 1 & j = y_i \\ 0 & j \neq y_i \end{cases} \quad (2)$$

Softmax P_{ij}

$$P_{ij} = \frac{e^{s_{ij}}}{\sum_k e^{s_{ik}}} \quad (3)$$

Among the k classes, $j = y_i$ for exactly one of them. Therefore, we can ignore the other $k - 1$ terms in the sum of (1).

$$L_i = -\ln \frac{e^{s_{ij}}}{\sum_k e^{s_{ik}}} = -\ln P_{ij} \quad (4)$$

1.2 Gradient of W

Computing the gradient $\frac{\partial L_i}{\partial W}$ via chain rule.

$$\frac{\partial L_i}{\partial W} = \frac{\partial L_i}{\partial P_{ij}} \frac{\partial P_{ij}}{\partial s_{ij}} \frac{\partial s_{ij}}{\partial W}$$

Linear classifier $f(x_i, W) = s_{ij}$.

$$s_{ij} = Wx_i + b_i \quad (5)$$

The partial derivative of s_{ij} wrt W is nonzero when $j = y_i$ and zero everywhere else. This condition is expressed with δ_{ij} (2).

$$\frac{\partial s_{ij}}{\partial W} = \delta_{ij} x_i$$

The partial derivative of P_{ij} (3) wrt W by applying the chain rule.

$$\begin{aligned} \frac{\partial P_{ij}}{\partial W} &= \frac{\partial}{\partial W} \frac{e^{s_{ij}}}{\sum_k e^{s_{ik}}} = \frac{\delta_{ij} x_i e^{s_{ij}} - e^{s_{ij}} x_i e^{s_{ij}}}{(\sum_k e^{s_{ik}})^2} \\ &= \left(\frac{e^{s_{ij}}}{\sum_k e^{s_{ik}}} \right) \left(\frac{\delta_{ij} x_i \sum_k e^{s_{ik}} - x_i e^{s_{ij}}}{\sum_k e^{s_{ik}}} \right) \\ &= P_{ij} (\delta_{ij} x_i - P_{ij} x_i) \\ &= P_{ij} x_i (\delta_{ij} - P_{ij}) \end{aligned}$$

Finally the partial derivative of L_i (4) wrt W .

$$\begin{aligned} \frac{\partial L_i}{\partial W} &= \frac{\partial}{\partial W} (-\ln P_{ij}) \\ &= -\frac{1}{P_{ij}} P_{ij} x_i (\delta_{ij} - P_{ij}) \\ &= (\delta_{ij} - P_{ij}) x_i \end{aligned} \quad (6)$$

1.3 Gradient of b_i

Computing the gradient $\frac{\partial L_i}{\partial b_i}$ via chain rule.

$$\frac{\partial L_i}{\partial b_i} = \frac{\partial L_i}{\partial P_{ij}} \frac{\partial P_{ij}}{\partial s_{ij}} \frac{\partial s_{ij}}{\partial b_i}$$

The partial derivative of s_{ij} (5) wrt b_i is nonzero when $j = y_i$ and zero everywhere else. This condition is expressed with δ_{ij} (2).

$$\frac{\partial s_{ij}}{\partial W} = \delta_{ij}$$

The partial derivative of P_{ij} (3) wrt b_i by applying the chain rule.

$$\frac{\partial P_{ij}}{\partial b_i} = \frac{\partial}{\partial W} \frac{e^{s_{ij}}}{\sum_k e^{s_{ik}}} = \frac{-e^{s_{ij}} e^{s_{ij}}}{(\sum_k e^{s_{ik}})^2} = -P_{ij}^2$$

Finally the partial derivative of L_i (4) wrt b_i .

$$\begin{aligned} \frac{\partial L_i}{\partial b_i} &= \frac{\partial}{\partial b_i} (-\ln P_{ij}) \\ &= -\frac{1}{P_{ij}} (-P_{ij}^2) \\ &= P_{ij} \end{aligned} \tag{7}$$



Figure 1: Example caption.

Example Section.

1.4 Example Subsection

Example Sub Section.

Example paragraph.

1.5 Example Equation

This is how a equation looks like.

$$y = ax^2 + bx + c \quad , \quad (8)$$

where an inline equation looks like $a = b$.

1.6 Example Figure

To put a figure, you can do as shown in Fig. 1.