Paul MP - A1 - V00712982

2.1.1 $y = ax^2 + bx + c$

$\frac{\partial y}{\partial x} = 2ax + b$

2.1.2 $y = \sin x \cos x$

$\frac{\partial y}{\partial x} = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$

2.1.3 $y = \frac{1}{1 + e^{-x}}$ $g = 1$, $h = 1 + e^{-x}$

$\frac{\partial y}{\partial x} = \frac{g'h - gh'}{h^2}$ $g' = 0$ $h' = -e^{-x}$

$= \frac{-(-e^{-x})}{(1 + e^{-x})^2}$

$\frac{\partial y}{\partial x} = \frac{e^{-x}}{(1 + e^{-x})^2}$

2.1.4 $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$

$\frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$

$y = \frac{\sinh x}{\cosh x} = \tanh x$

$\frac{\partial y}{\partial x} = (\tanh x)' = \operatorname{sech}^2 x$

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2.2.1

$y = e^{ax+b}$

$x=0$: $f(x) = e^{ax+b} \Rightarrow f(0) = e^{b}$

$f'(x) = ae^{ax+b} \Rightarrow f'(0) = ae^{b}$

$f''(x) = a^2 e^{ax+b} \Rightarrow f''(0) = a^2 e^{b}$

$y = e^{b} + \frac{ae^{b}}{1!}(x) + \frac{a^2e^{b}}{2!}(x)^2$

$\underline{y = e^{b} + a.e^{b}x + \frac{1}{2}a^2 e^{b} x^2}$

2.2.2

$y = \sin(ax+b)$

$x=0$: $f(x) = \sin(ax+b) \Rightarrow f(0) = \sin b$

$f'(x) = a\cos(ax+b) \Rightarrow f'(0) = a\cos b$

$f''(x) = -a^2\sin(ax+b) \Rightarrow f''(0) = -a^2\sin b$

$\underline{y = \sin b + (a\cos b)x - \frac{1}{2}(a^2 \sin b)x^2}$

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2.3.1 $$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1+4-3 \\ -4+10-6 \\ -7+16-9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2.3.2 $$\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \begin{bmatrix} 1, & -2, & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

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2.4 $y = \|A^T x - b\|_2^2$, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$A^T x - b = \begin{bmatrix} a_{11}x_1 + a_{21}x_2 + a_{31}x_3 - b_1 \\ a_{12}x_2 + a_{22}x_2 + a_{32}x_3 - b_2 \\ a_{13}x_3 + a_{23}x_3 + a_{33}x_3 - b_3 \end{bmatrix} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

$$y = \|A^T x - b\|_2^2 = \left(\sqrt{l_1^2 + l_2^2 + l_3^2}\right)^2$$

$$y = l_1^2 + l_2^2 + l_3^2$$

$$\frac{\partial}{\partial x} l_1^2 = \langle 2a_{11}l_1 , 2a_{21}l_1 , 2a_{31}l_1 \rangle$$

$$\frac{\partial}{\partial x} l_2^2 = \langle 2a_{12}l_2 ; 2a_{22}l_2 , 2a_{32}l_2 \rangle$$

$$\frac{\partial}{\partial x} l_3^2 = \langle 2a_{13}l_3 , 2a_{23}l_3 , 2a_{33}l_3 \rangle$$

$$\nabla y(x) = \frac{\partial}{\partial x} l_1^2 + \frac{\partial}{\partial x} l_2^2 + \frac{\partial}{\partial x} l_3^2$$

$$= \begin{bmatrix} 2a_{11}l_1 + 2a_{12}l_2 + 2a_{13}l_3 \\ 2a_{21}l_1 + 2a_{22}l_2 + 2a_{23}l_3 \\ 2a_{31}l_1 + 2a_{32}l_3 + 2a_{33}l_3 \end{bmatrix}$$

$$= 2 \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = 2Al$$

$$= \underline{2A(A^T x - b)}$$