

Name: \_\_\_\_\_

1. Show that if  $U^{-1} = U^\dagger$  then  $U$  preserves the bracket. That is, if  $U|\alpha\rangle = |\gamma\rangle$  and  $U|\beta\rangle = |\delta\rangle$  then  $\langle\alpha|\beta\rangle = \langle\gamma|\delta\rangle$ .

**Solution:** We have

$$\langle\gamma|\delta\rangle = \langle\gamma||\delta\rangle = (|\gamma\rangle)^\dagger|\delta\rangle = (U|\alpha\rangle)^\dagger U|\beta\rangle = (|\alpha\rangle)^\dagger U^\dagger U|\beta\rangle = \langle\alpha||\beta\rangle = \langle\alpha|\beta\rangle.$$