

4/4

Name: MADHU PEDURI

1. What does it mean for a predicate P on domain X to be *decidable*?

→ A predicate $P: A^* \rightarrow \{0,1\}$ is decidable if there exists a Turing machine M such that

$$P(x) = \varphi_M$$

→ If the function of Predicate, φ , is computable, then such predicate is Decidable.

4/4

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1. Define the class NP.

A predicate L belongs to class NP, if there exists a non deterministic TM that computes L in polynomial time

$$L(x) = \exists y \left((|y| \leq q(|x|)) \wedge R(x, y) \right)$$

or

$R(x, y)$ is a predicate $\in P$ having two variables) what is $q()$?

$L(x) = 1 \rightarrow$ there exists a computational path that gives yes in $\text{poly}(n)$

$L(x) = 0 \rightarrow$ there is no such property
or
there is no path of any length that gives 'yes'

4/4

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1. Define the complexity class BPP.

A predicate L belongs to class BPP if there exists a Probabilistic TM M and P , a polynomial function,

for an input string x , it gives answer in $P(|x|)$ steps (Polynomial time) such that

$L(x) = 1$ M gives yes with probability $\geq (1-\epsilon)$

$L(x) = 0$ M gives No with probability $\geq 1-\epsilon$

where ϵ is the admissible error

$$0 < \epsilon < 1/2$$

↑
9000 in back

Name: MADHU PEDURI

1. State Fermat's Little Theorem.

If a, p are coprime then p primefor an integer $a \in \mathbb{Z}$ $a \neq 0$; $p \neq 0$

$$a^{p-1} \equiv 1 \pmod{p}$$

Name: Madhu peduri

1. Let $\mathcal{B} = \{|0\rangle, |1\rangle, |2\rangle\}$ be an ordered basis for the vector space \mathbb{V} . Let $\varphi = |0\rangle\langle 0| - 2|1\rangle\langle 2|$ be a linear transformation. Find the matrix for φ relative to the basis \mathcal{B} , $(\varphi)_{\mathcal{B} \rightarrow \mathcal{B}}$.

$$\varphi = \begin{bmatrix} 1/2 & 2/3 \\ 1 & -1/3 \end{bmatrix} X$$

Why $2/3$?

4/4

Name: MADHU PEDURTI

1. Show that if $U^{-1} = U^\dagger$ then U preserves the bracket. That is, if $U|\alpha\rangle = |\gamma\rangle$ and $U|\beta\rangle = |\delta\rangle$ then $\langle\alpha|\beta\rangle = \langle\gamma|\delta\rangle$.

$$U|\alpha\rangle = |\gamma\rangle$$

$$U|\beta\rangle = |\delta\rangle$$

$$\langle\alpha|\beta\rangle = \langle\alpha|U^\dagger U|\beta\rangle$$

$$= \langle\gamma|U^\dagger U|\delta\rangle$$

$$= \langle\gamma|I|\delta\rangle$$

$$= \langle\gamma|\delta\rangle \quad (\because U^\dagger U = I)$$

$$= \langle\gamma|\delta\rangle$$

9/4

Name: MADHU PEARL

1. Let U be a unitary matrix. Using the definition of the operator norm, show that $\|U\| = 1$.

Definition of
operator
norm

$$\|X\| = \sup_{\|\alpha\| \neq 0} \frac{\|X|\alpha\rangle\|}{\|\alpha\|}$$

$$= \sup_{\|\alpha\| \neq 0} \frac{\|U|\alpha\rangle\|}{\|\alpha\|}$$

\because Unitary matrix (operator)

Preserve geometry of the Vector ✓

$$\|U\| = \frac{\|U|\alpha\rangle\|}{\|\alpha\|} = 1$$

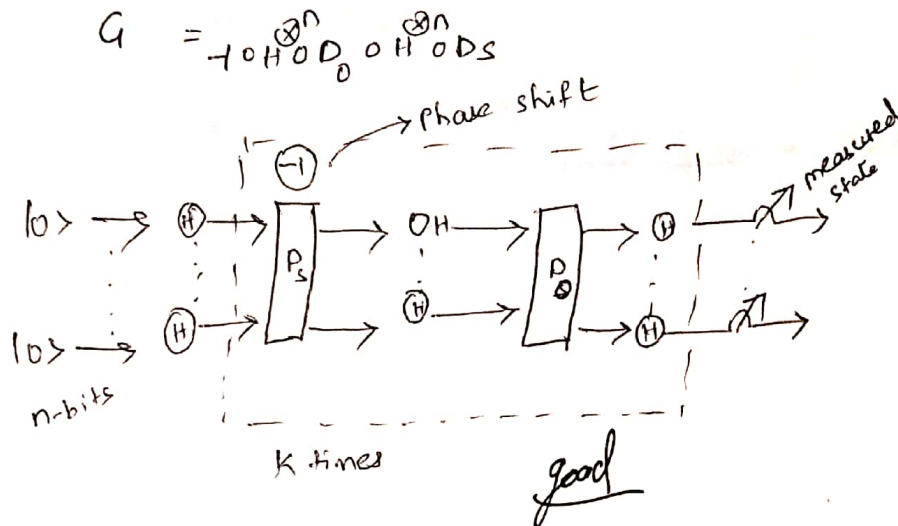
$$\therefore \|U|\alpha\rangle\| = \|\alpha\|$$

$$\therefore \|U|\alpha\rangle\| = \|\alpha\|$$

✓

Name: MAOHU PEDURI

1. Draw the circuit for Grover's algorithm



2/4

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1. Let A be a subspace of the vector space V and let $|\alpha\rangle \in V$. Define Π_α and Π_A .

$$\Pi_\alpha = |\alpha\rangle\langle\alpha|$$

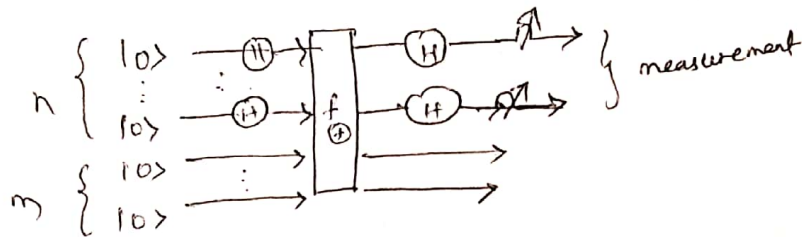
$$\Pi_A = \underbrace{|\alpha\rangle\langle\alpha| \otimes B^{n-m}}_{\text{vector tensor a space?}}$$

$$\Pi_A = \sum_K \Pi_{\alpha_K}$$

4/4

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1. Draw the circuit that solves Simon's Problem. Be sure to indicate on which qubits measurement is performed.



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1. Draw the eigenvalue approximation circuit for unitary U with eigenvector $|\alpha\rangle$.

$$U|\alpha_k\rangle = \lambda_k |\alpha_k\rangle$$

