Quantum Algorithms: Quiz 5 2020-02-20

Name: _____

1. **(i)** Define Σ_k.

(ii) Define Π_k.

Solution: Define

$$\begin{aligned}
\Sigma_0 &= \Pi_0 = P, \\
\Sigma_{k+1} &= \exists \Pi_k \\
&= \Big\{ L \mid L = \{x \mid \exists y \; M(y,x)\} \text{ where } M \in \Pi_k \Big\} \\
&= \Big\{ L \mid L = \{x \mid \exists y_1 \forall y_2 \exists y_3 \cdots \; M(y_1, \ldots, y_{k+1}, x)\} \text{ where } M \in P \Big\}, \\
\Pi_{k+1} &= \forall \Sigma_k \\
&= \Big\{ L \mid L = \{x \mid \forall y \; M(y,x)\} \text{ where } M \in \Sigma_k \Big\} \\
&= \Big\{ L \mid L = \{x \mid \forall y_1 \exists y_2 \forall y_3 \cdots \; M(y_1, \ldots, y_{k+1}, x)\} \text{ where } M \in P \Big\}.
\end{aligned}$$

All variables $y, y_1, y_2, \ldots, y_k$ are taken to be of length at most $p(|x|)$, where p is some polynomial.