## QUANTUM ALGORITHMS HOMEWORK 6 ADDITIONAL PROBLEMS

## PROF. MATTHEW MOORE

 $Due:\ 2021\text{-}03\text{-}16$ 

1. Prove that the inner product and the tensor product commute:

$$\langle \alpha \otimes \beta \mid \gamma \otimes \delta \rangle = \langle \alpha \mid \gamma \rangle \langle \beta \mid \delta \rangle.$$

This is asserted on page 57 of the textbook.

**2.** An *n*-ary function  $f: \mathbb{B}^n \to \mathbb{B}$  is idempotent if

$$f(0,...,0) = 0$$
 and  $f(1,...,1) = 1$ .

Find a basis  $\mathcal{A}$  so that every idempotent Boolean function is representable as a circuit over  $\mathcal{A}$ . Prove your answer is correct. [Hint 1: Post's Lattice.] [Hint 2: ?:.]