HW5 - Common Probability Distributions

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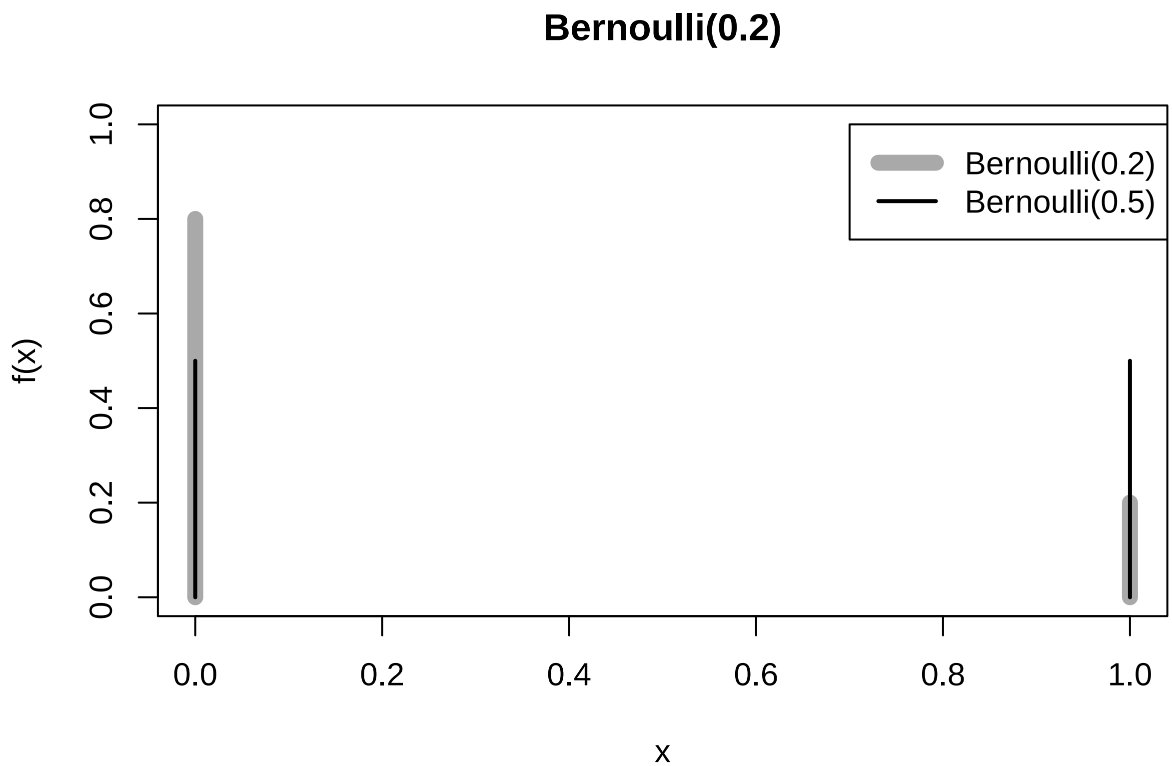
0.1 Discrete Distributions

0.1.1 Bernoulli

The **Bernoulli distribution**, named for Jacob Bernoulli, assigns probability to the outcomes of a single Bernoulli experiment—one where the only possible outcomes can be thought of as a "success" or a "failure" (e.g., a coin toss). Here, the random variable x can take on the values 1 (success) with probability p, or 0 (failure) with probability $q = 1 - p$. The plot below contains the pmf of two Bernoulli distributions. The first (in gray) has a probability of success $p = 0.2$ and the second (in black) has a probability of success $p = 0.5$.

```r
x <- 0:1
plot(x, dbinom(x, 1, 0.2), type = "h", ylab = "f(x)", ylim = c(0,
    1), lwd = 8, col = "dark gray", main = "Bernoulli(0.2)")
lines(x, dbinom(x, 1, 0.5), type = "h", lwd = 2, col = "black")
legend(0.7, 1, c("Bernoulli(0.2)", "Bernoulli(0.5)"), col = c("dark gray",
    "black"), lwd = c(8, 2))
```

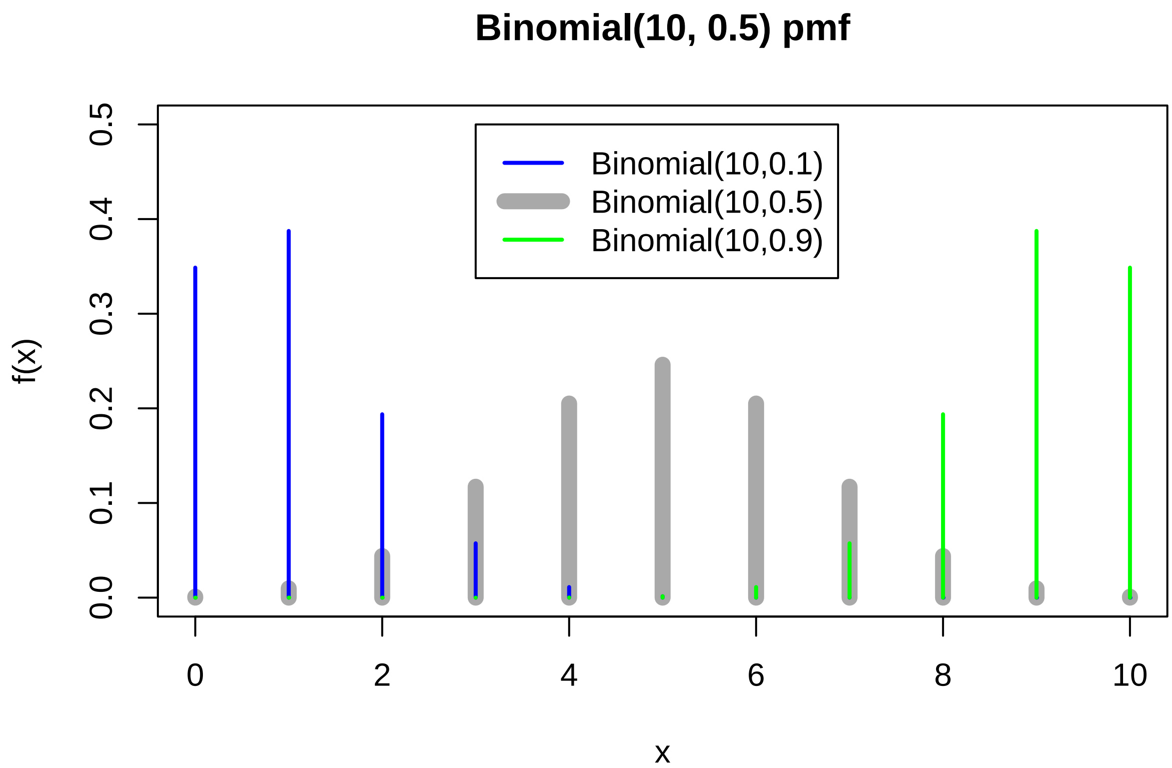


The Bernoulli experiment forms the foundation for many of the next discrete distributions.

0.1.2 Binomial

The **binomial distribution** applies when we perform n Bernoulli experiments and are interested in the total number of "successes" observed. The outcome here, $y = \sum x_i$, where $P(x_i = 1) = p$ and $P(x_i = 0) = 1 - p$. The plot below displays three binomial distributions, all for $n = 10$ Bernoulli trials: in gray, $p = 0.5$; in blue, $p = 0.1$; and in green, $p = 0.9$.

```r
x <- seq(0, 10, 1)
plot(x, dbinom(x, 10, 0.5), type = "h", ylab = "f(x)", lwd = 8,
    col = "dark gray", ylim = c(0, 0.5), main = "Binomial(10, 0.5) pmf")
lines(x, dbinom(x, 10, 0.1), type = "h", lwd = 2, col = "blue")
lines(x, dbinom(x, 10, 0.9), type = "h", lwd = 2, col = "green")
legend(3, 0.5, c("Binomial(10,0.1)", "Binomial(10,0.5)", "Binomial(10,0.9)"),
    col = c("blue", "dark gray", "green"), lwd = c(2, 8, 2))
```



We can see the shifting of probability from low values for $p = 0.1$ to high values for $p = 0.9$. This makes sense, as it becomes more likely with $p = 0.9$ to observe a success for an individual trial. Thus, in 10 trials, more successes (e.g., 8, 9, or 10) are likely. For $p = 0.5$, the number of successes are likely to be around 5 (e.g., half of the 10 trials).

0.1.3 Hypergeometric

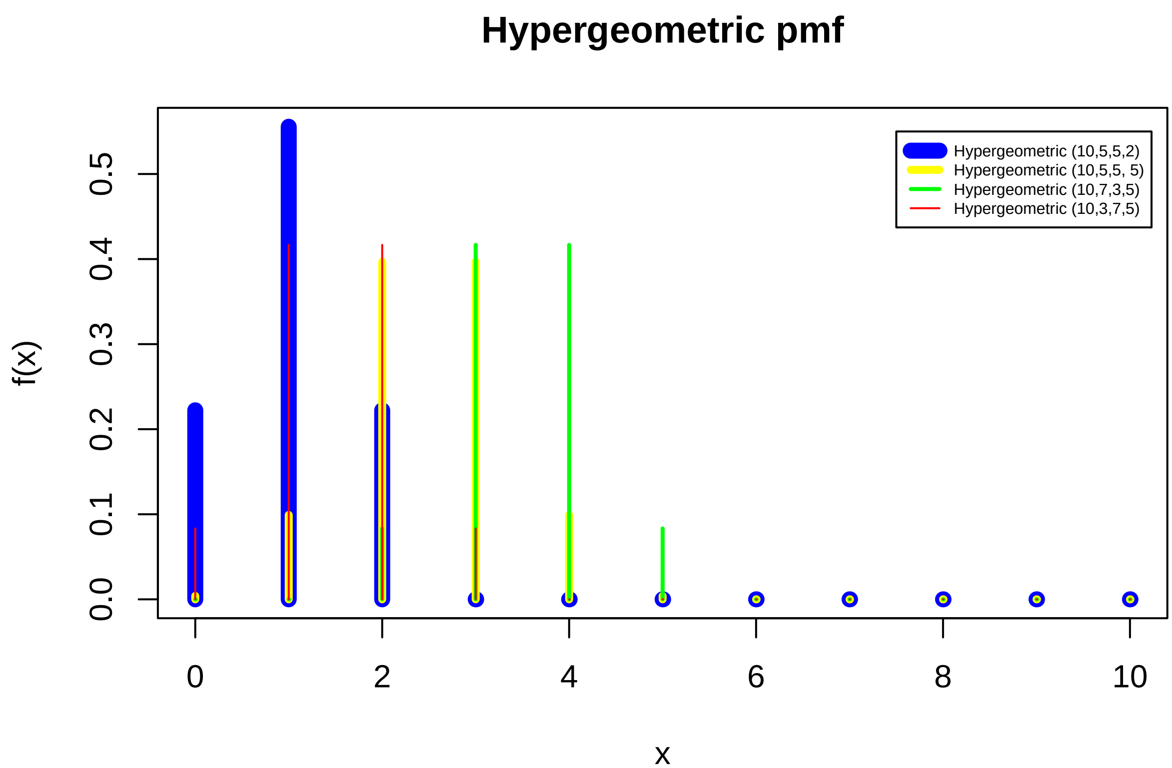
The **Hypergeometric distribution** is a discrete distribution that describes the probability of x successes in n draws without replacement wherein each draw is either success or failure. In contrast, the **binomial distribution** describes the probability of x successes in n draws with replacement. We can represent this using below notation

$$p(x) = \frac{choose(m, x).choose(n, k - x)}{choose(m + n, k)}$$

- In the example below, we have different set of parameters.
- If we choose 2 balls from the urn - $x = (0, 2), (1, 1), (2, 0)$.
- If we choose 5 balls from the urn with different number of colored balls - (7,3),$x = (2, 3)(3, 2), (4, 1), (5, 0)$.
- Probability distribution shifts as the number of success or failures changes.
- Probability distribution is symmetric.

```
x <- seq(0, 10, 1)
plot(x, dhyper(x, 5, 5, 2), type = "h", ylab = "f(x)", lwd = 8,
    col = "blue", main = "Hypergeometric pmf")
lines(x, dhyper(x, 5, 5, 5), type = "h", ylab = "f(x)", lwd = 4,
    col = "yellow")
lines(x, dhyper(x, 7, 3, 5), type = "h", ylab = "f(x)", lwd = 2,
```

```r
    col = "green")
lines(x, dhyper(x, 3, 7, 5), type = "h", ylab = "f(x)", lwd = 1,
    col = "red")
par(cex = 0.5)
legend(7.5, 0.55, c("Hypergeometric (10,5,5,2)", "Hypergeometric (10,5,5, 5)",
    "Hypergeometric (10,7,3,5)", "Hypergeometric (10,3,7,5)"),
    col = c("blue", "yellow", " green", "red"), lwd = c(8, 4,
        2, 1))
```



0.1.4 Poisson

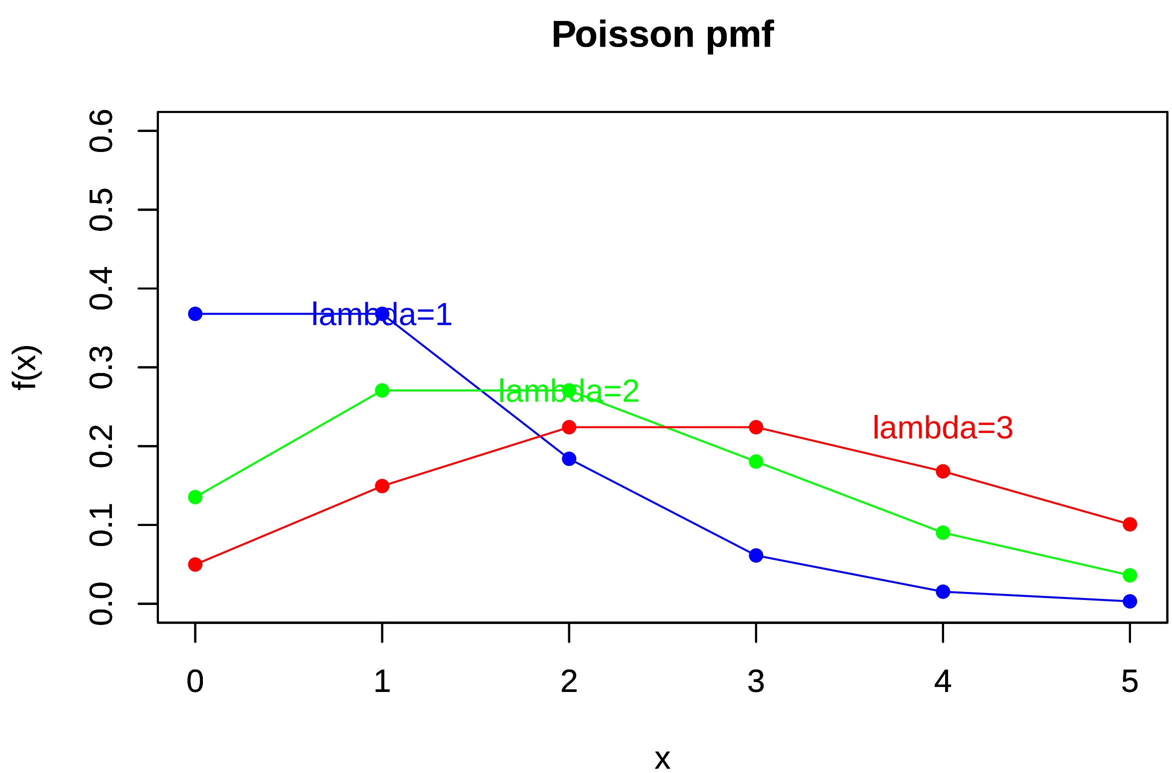
The **Poisson distribution** expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event. The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

We use below function to derive the poisson distribution $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad for \quad x = 0, 1, 2 \dots n$

- For poission distribution, we use lambda as the parameter. It denotes the number of successes that we get with in the given time frame.
- We can observe from the below distributions that, as the lamda increases, we get the maximum probability towards the right. That is, to have more number of success events, number of trials should be more.

```r
x <- seq(0, 5, 1)
poisl = dpois(x, 1)
plot(x, poisl, ylab = "f(x)", main = "Poisson pmf", pch = 16,
```

```
    ylim = c(0, 0.6), col = "blue")
lines(x, poisl, col = "blue")
text(x = which(poisl == max(poisl))[1], y = poisl[poisl == max(poisl)][1],
    "lambda=1", col = "blue")

par(new = TRUE)
poisl = dpois(x, 2)
plot(x, poisl, ylab = "f(x)", main = "Poisson pmf", pch = 16,
    ylim = c(0, 0.6), col = "green")
lines(x, poisl, col = "green")
text(x = which(poisl == max(poisl))[1], y = poisl[poisl == max(poisl)][1],
    "lambda=2", col = "green")

par(new = TRUE)
poisl = dpois(x, 3)
plot(x, poisl, ylab = "f(x)", main = "Poisson pmf", pch = 16,
    ylim = c(0, 0.6), col = "red")
lines(x, poisl, col = "red")
text(x = which(poisl == max(poisl))[1], y = poisl[poisl == max(poisl)][1],
    "lambda=3", col = "red")
```



0.1.5 Geometric

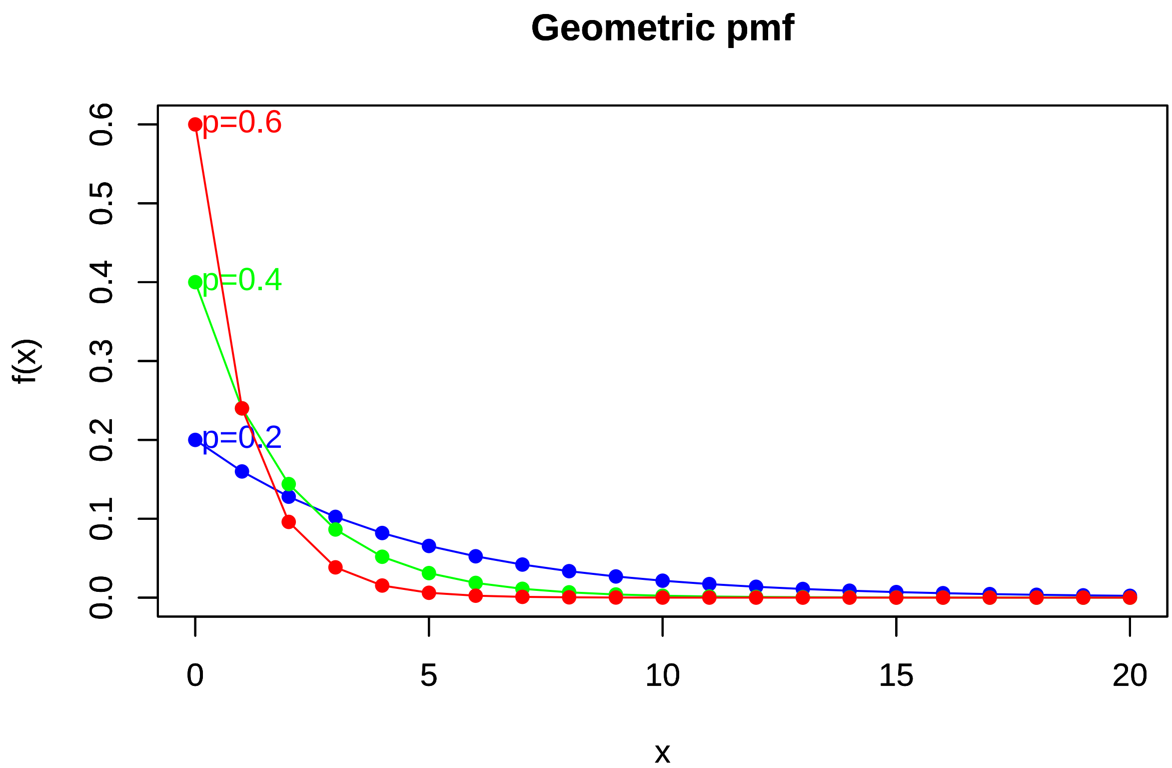
The **geometric distribution** gives the probability that the first occurrence of success requires k independent trials, each with success probability p. We use below function to derive the geometric distribution.

$Pr(X = k) = p(1 - p)^{k-1}$

We can make below observations from below plots,

- As the initial success probability p is high, the probability of getting a success event of certian failures decreases.
- For example, after 3 falilures (x=3), the probablity of observing a success event decreases from 0.14 to 0.05 as the initial probability p increases from 0.2 to 0.6.
- For a given success probability, as the number of failure trials in creases, the probability of observing a success decreases.

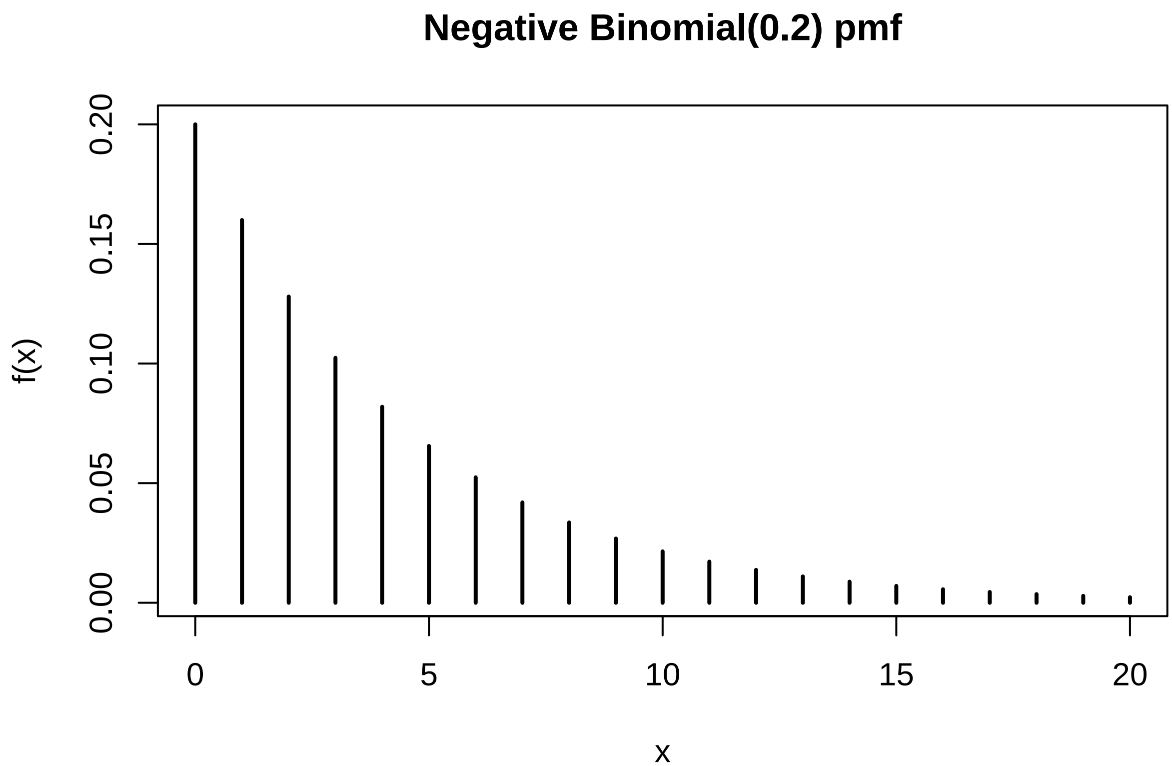
```
x <- seq(0, 20, 1)
# plot(x, dgeom(x, 0.2), type = 'h', ylab = 'f(x)', lwd =
# 2, main = 'Geometric(0.2) pmf')
geom = dgeom(x, 0.2)
plot(x, geom, ylab = "f(x)", main = "Geometric pmf", pch = 16,
    ylim = c(0, 0.6), col = "blue")
lines(x, geom, col = "blue")
text(x = which(geom == max(geom))[1], y = geom[geom == max(geom)][1],
    "p=0.2", col = "blue")

par(new = TRUE)
geom = dgeom(x, 0.4)
plot(x, geom, ylab = "f(x)", main = "Geometric pmf", pch = 16,
    ylim = c(0, 0.6), col = "green")
lines(x, geom, col = "green")
text(x = which(geom == max(geom))[1], y = geom[geom == max(geom)][1],
    "p=0.4", col = "green")

par(new = TRUE)
geom = dgeom(x, 0.6)
plot(x, geom, ylab = "f(x)", main = "Geometric pmf", pch = 16,
    ylim = c(0, 0.6), col = "red")
lines(x, geom, col = "red")
text(x = which(geom == max(geom))[1], y = geom[geom == max(geom)][1],
    "p=0.6", col = "red")
```



0.1.6 Negative Binomial

The negative binomial I have below has set $r = 1$, so it's identical to the geometric above. Play around with r and see how it changes.

```r
x <- seq(0, 20, 1)
plot(x, dnbinom(x, 1, 0.2), type = "h", ylab = "f(x)", lwd = 2,
    main = "Negative Binomial(0.2) pmf")
```

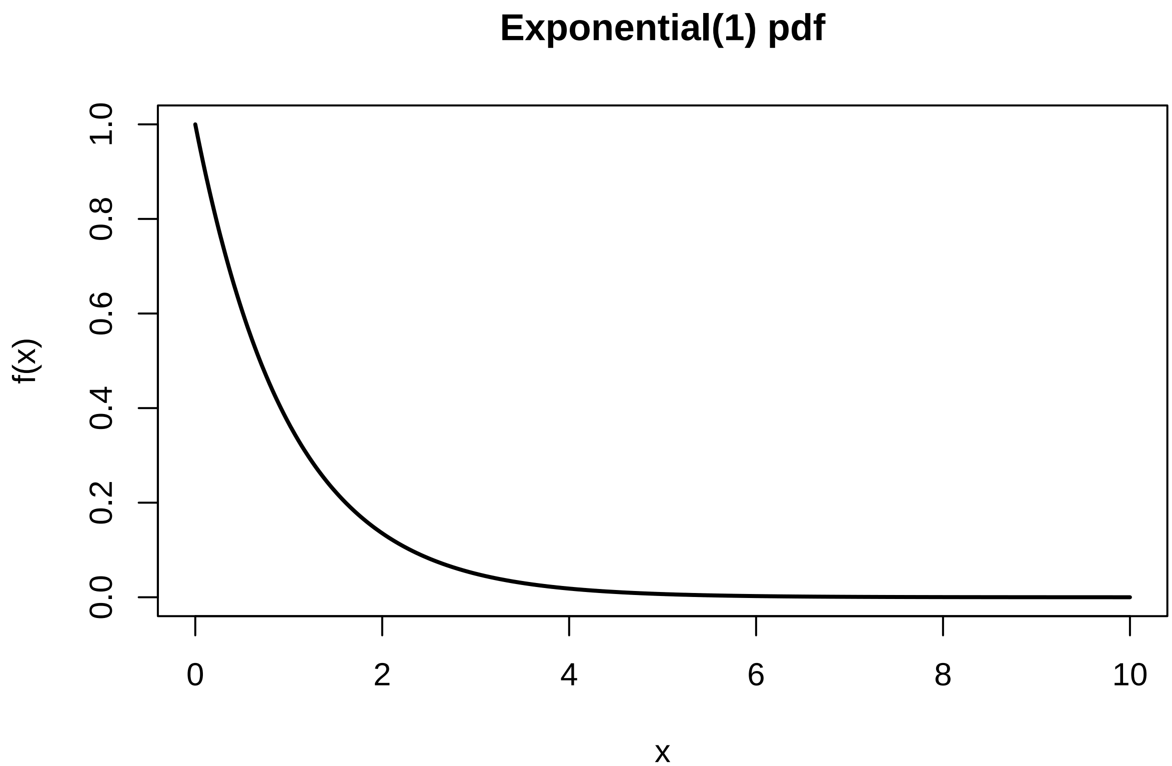


0.2 Continuous Distributions

0.2.1 Exponential

Vary λ and describe.

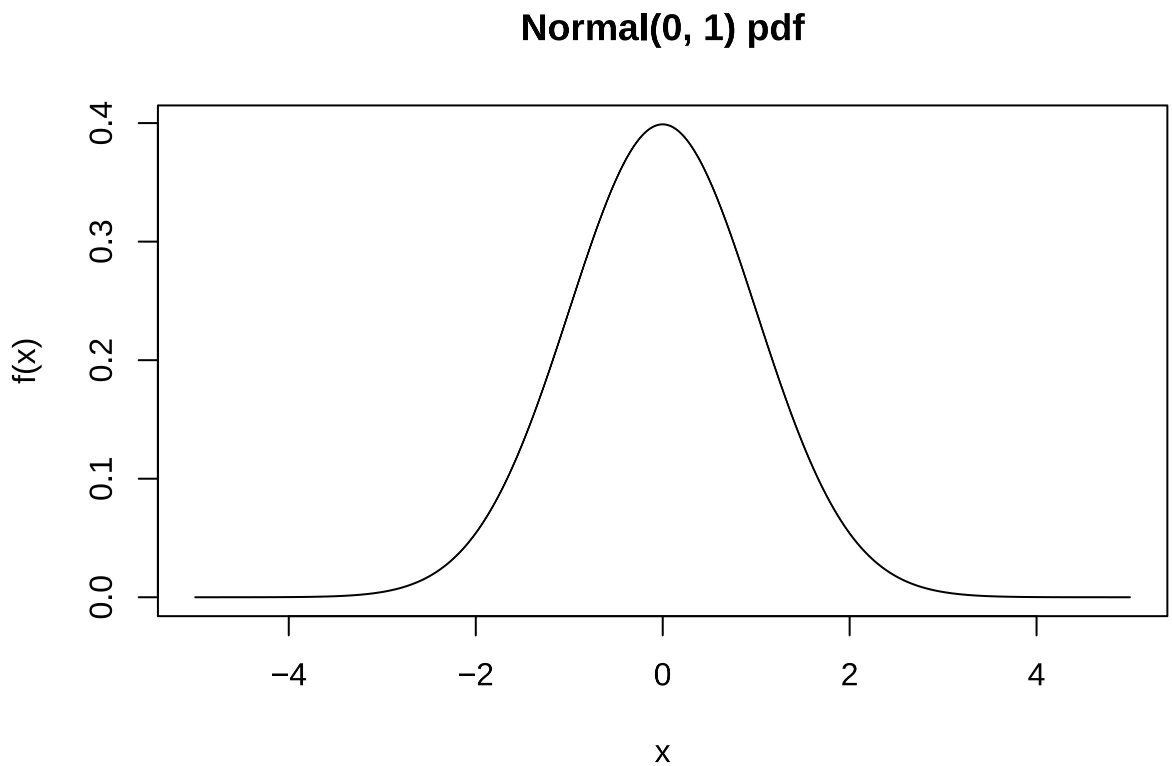
```
x <- seq(0, 10, 0.01)
plot(x, dexp(x, 1), type = "l", ylab = "f(x)", lwd = 2, main = "Exponential(1) pdf")
```



0.2.2 Normal

Vary σ and see how the distribution changes. If you make it too big, you may need to adjust the x-axis by making the sequence span a wider range than -5 to 5. You can use a trial-and-error approach to determing the proper limits for x for a given σ.

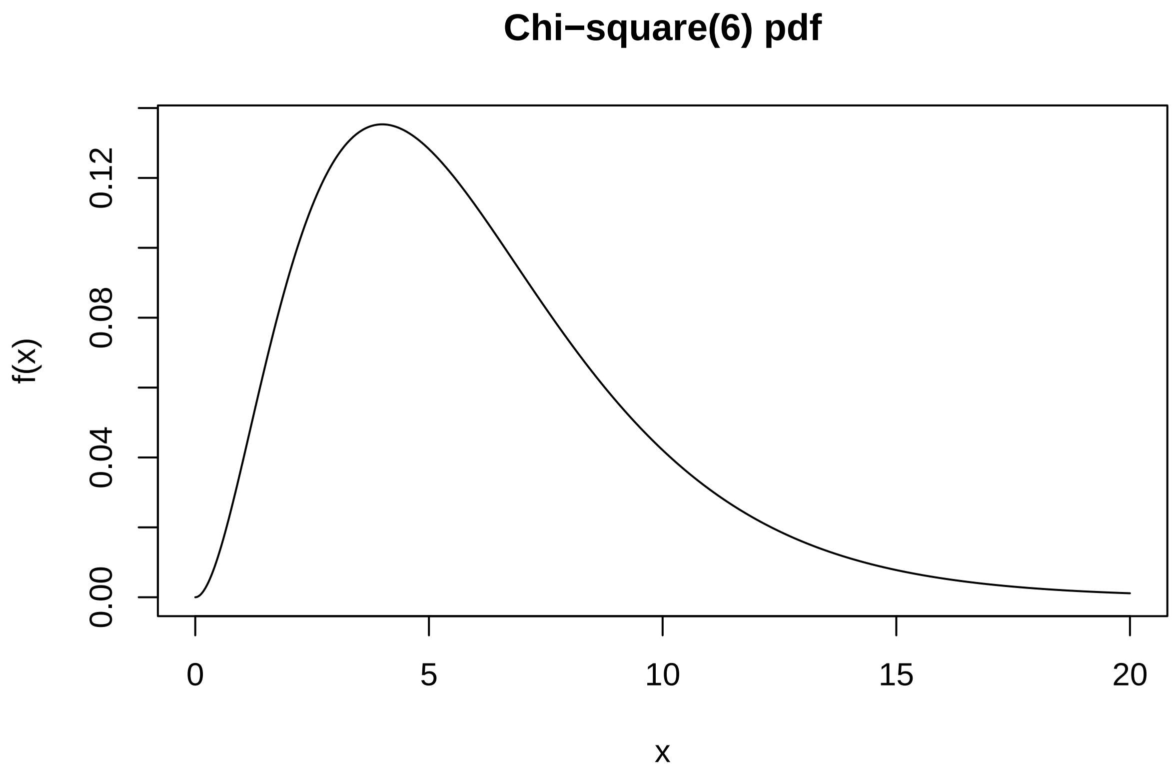
```
x <- seq(-5, 5, 0.01)
plot(x, dnorm(x, 0, 1), type = "l", ylab = "f(x)", main = "Normal(0, 1) pdf")
```



0.2.3 Chisquare

How do the degrees of freedom change the shape? Plot a few and explain.

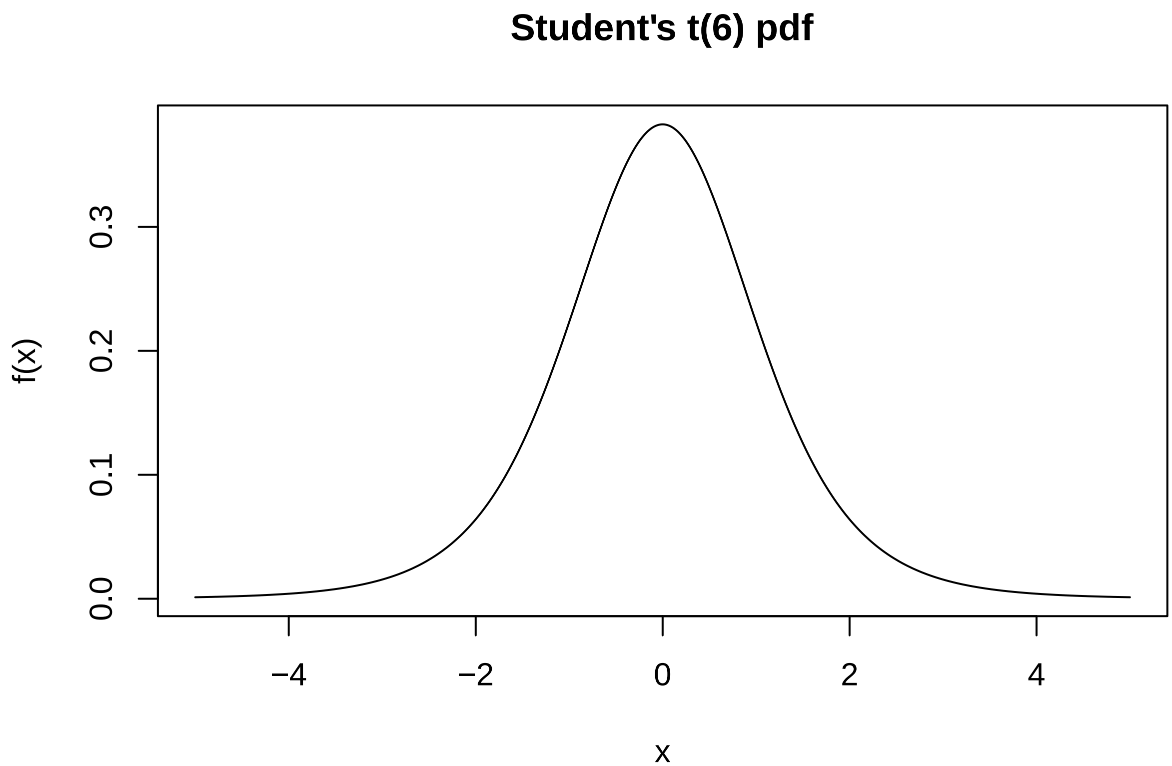
```
x <- seq(0, 20, 0.01)
plot(x, dchisq(x, 6), type = "l", ylab = "f(x)", main = "Chi-square(6) pdf")
```



0.2.4 Students t

How do the degrees of freedom change the shape? Plot a few and explain.

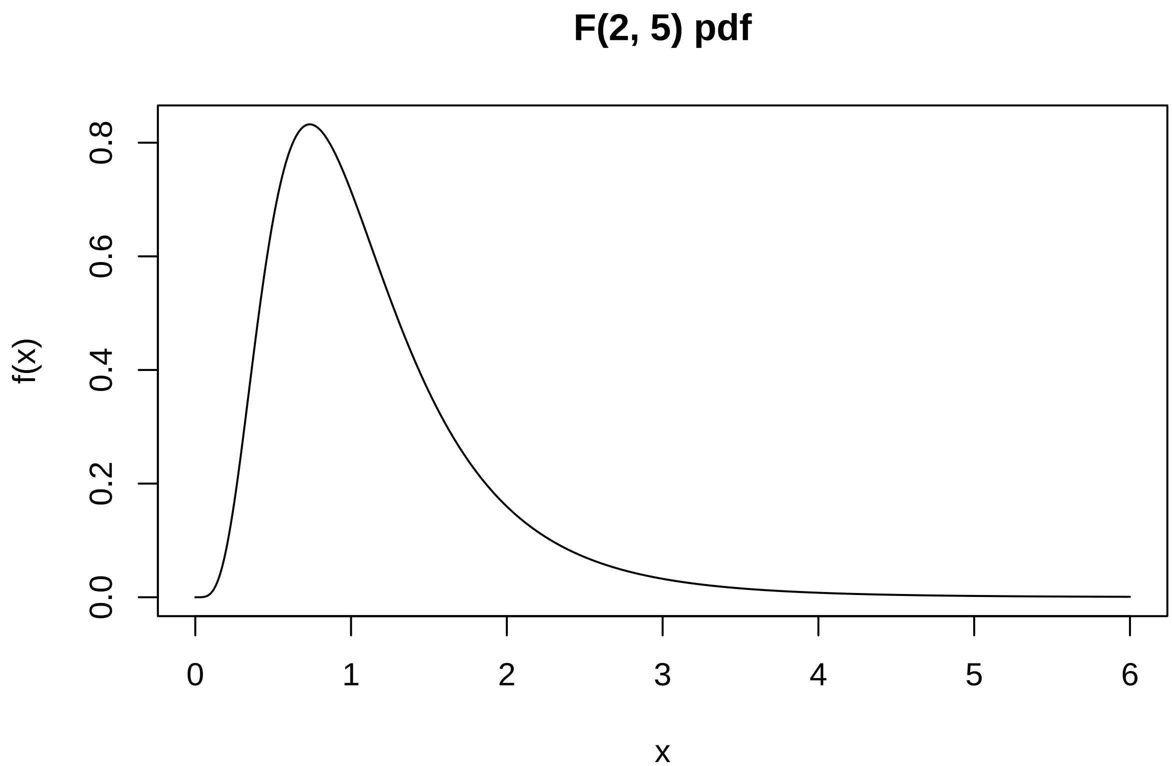
```
x <- seq(-5, 5, 0.01)
plot(x, dt(x, 6), type = "l", ylab = "f(x)", main = "Student's t(6) pdf")
```



0.2.5 F

How do the degrees of freedom (numerator and/or denominator) change the shape? Plot a few and explain.

```
x <- seq(0, 6, 0.01)
plot(x, df(x, 12, 15), type = "l", ylab = "f(x)", main = "F(2, 5) pdf")
```



0.3 Document Information.

All of the statistical analyses in this document will be performed using R version 4.1.0 (2021-05-18). R packages used will be maintained using the packrat dependency management system.

```r
sessionInfo()
```

```
## R version 4.1.0 (2021-05-18)
## Platform: x86_64-w64-mingw32/x64 (64-bit)
## Running under: Windows 10 x64 (build 19041)
##
## Matrix products: default
##
## locale:
## [1] LC_COLLATE=English_United States.1252
## [2] LC_CTYPE=English_United States.1252
## [3] LC_MONETARY=English_United States.1252
## [4] LC_NUMERIC=C
## [5] LC_TIME=English_United States.1252
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] rmarkdown_2.8 knitr_1.33
##
## loaded via a namespace (and not attached):
```

```
##  [1] compiler_4.1.0    magrittr_2.0.1    formatR_1.11      tools_4.1.0
##  [5] htmltools_0.5.1.1 yaml_2.2.1        stringi_1.6.1     highr_0.9
##  [9] stringr_1.4.0     xfun_0.23         digest_0.6.27     rlang_0.4.11
## [13] evaluate_0.14
```