

HW10 - Matrix Approach to Linear Regression

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0.0.1 Problem 1(a)

```
A = matrix(c(1, 5, 0, 1, 0, 5, 1, 0, 5), byrow = TRUE, ncol = 3)
```

```
rankA <- rankMatrix(A)
sprintf("Rank of matrix A is %i", rankA)
```

```
## [1] "Rank of matrix A is 2"
```

```
detA <- det(A)
sprintf("Determinant of matrix A is %i", detA)
```

```
## [1] "Determinant of matrix A is 0"
```

```
traceA <- sum(diag(A))
sprintf("Trace of matrix A is %i", traceA)
```

```
## [1] "Trace of matrix A is 6"
```

```
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    5    0
## [2,]    1    0    5
## [3,]    1    0    5
```

(i) are the column vectors of A linearly dependent?

- Matrix A has row 2 and row 3 same, which are linearly dependent. We have determinant = 0, which makes it singular suggesting linear dependency.

(ii) What is the rank of A?

- Rank of a matrix A is 2

(iii) What must be the determinant of A?

- Determinant of Matrix A is zero

(iv) Find the trace of A.

- Trace of matrix A is 6

0.0.2 Problem 1(b)

```
B = matrix(c(5, 1, 3, 4, 0, 5, 1, 9, 6), byrow = TRUE, ncol = 3)
```

```
rankB <- rankMatrix(B)
sprintf("Rank of matrix B is %i", rankB)

## [1] "Rank of matrix B is 3"

detB <- det(B)
sprintf("Determinant of matrix B is %.2f", detB)

## [1] "Determinant of matrix B is -136.00"

traceB <- sum(diag(B))
sprintf("Trace of matrix B is %i", traceB)

## [1] "Trace of matrix B is 11"

inverseB <- round(solve(B), 3)
inverseB

##           [,1] [,2] [,3]
## [1,]  0.331 -0.154 -0.037
## [2,]  0.140 -0.199  0.096
## [3,] -0.265  0.324  0.029
```

(i) are the column vectors of B linearly dependent?

- Matrix B do not have any rows or columns that can be derived from one another. Determinant of matrix is not zero that makes it non-singular and of full-rank suggesting no linear dependency.

(ii) What is the rank of B?

- Rank of the matrix B is 3 which is equal to its largest dimension.

(iii) Calculate the determinant of B.

- Determinant of matrix B = -136.

(iv) Find the trace of B.

- Trace of matrix B = 11

(v) Find the inverse of B, $B^{-1} = \begin{bmatrix} 0.331 & -0.154 & -0.037 \\ 0.140 & -0.199 & 0.096 \\ -0.265 & 0.324 & 0.029 \end{bmatrix}$

0.0.3 Problem 2

```
c1 <- c(16, 5, 10, 15, 13, 22)
c2 <- c(4, 1, 2, 3, 3, 4)
data <- cbind(c1, c2)

Y <- matrix(c1, byrow = TRUE, ncol = 1)
X <- matrix(cbind(1, c2), ncol = 2)

# (i)
yty <- t(Y) %*% Y
yty

##           [,1]
## [1,] 1259
```

```

# (ii)
xtx <- t(X) %*% X
xtx

##      [,1] [,2]
## [1,]    6  17
## [2,]   17  55

# (iii)
xty <- t(X) %*% Y
xty

##      [,1]
## [1,]    81
## [2,]   261

# (iv)
xtxInv <- solve(xtx)
xtxInv

##      [,1] [,2]
## [1,]  1.341 -0.415
## [2,] -0.415  0.146

# (v)
coef_beta <- xtxInv %*% xty
coef_beta

##      [,1]
## [1,] 0.439
## [2,] 4.610

# (vi)
Yhat <- X %*% coef_beta
Yhat

##      [,1]
## [1,] 18.88
## [2,]  5.05
## [3,]  9.66
## [4,] 14.27
## [5,] 14.27
## [6,] 18.88

e_residual <- (Y - Yhat)
res_vec <- as.vector(e_residual)
res_vec

## [1] -2.8780 -0.0488  0.3415  0.7317 -1.2683  3.1220

# (vii)
er <- (Yhat - mean(Y))^2
SSR <- sum(er)
sprintf("Sum of squares due to regression is:%.3f", SSR)

## [1] "Sum of squares due to regression is:145.207"

# (viii)
er <- (Y - Yhat)^2

```

```
SSE <- sum(er)
sprintf("Sum of squares of residuals is:%.3f", SSE)
```

```
## [1] "Sum of squares of residuals is:20.293"
```

```
# (ix) Variance Covariance Matrix of coefficients
```

```
MSE <- SSE/(length(Y) - 2)
varCovarMatrix <- MSE * xtxInv
varCovarMatrix
```

```
##      [,1]      [,2]
## [1,]  6.81 -2.104
## [2,] -2.10  0.742
```

0.1 Document Information.

All of the statistical analyses in this document will be performed using R version 4.1.0 (2021-05-18). R packages used will be maintained using the packrat dependency management system.

```
sessionInfo()
```

```
## R version 4.1.0 (2021-05-18)
## Platform: x86_64-w64-mingw32/x64 (64-bit)
## Running under: Windows 10 x64 (build 19041)
##
## Matrix products: default
##
## locale:
## [1] LC_COLLATE=English_United States.1252
## [2] LC_CTYPE=English_United States.1252
## [3] LC_MONETARY=English_United States.1252
## [4] LC_NUMERIC=C
## [5] LC_TIME=English_United States.1252
##
## attached base packages:
## [1] grid      stats      graphics  grDevices  utils      datasets  methods
## [8] base
##
## other attached packages:
## [1] Matrix_1.3-4      psych_2.1.6      leaps_3.1        faraway_1.0.7
## [5] xtable_1.8-4      lmtest_0.9-38    zoo_1.8-9        PairedData_1.1.1
## [9] mvtnorm_1.1-2      gld_2.6.2        ggpubr_0.4.0     car_3.0-11
## [13] carData_3.0-4      mnormt_2.0.2     vcd_1.4-8        epiDisplay_3.5.0.1
## [17] nnet_7.3-16        foreign_0.8-81    Hmisc_4.5-0      Formula_1.2-4
## [21] survival_3.2-11    lattice_0.20-44  MASS_7.3-54      ggplot2_3.3.5
## [25] rmarkdown_2.8      knitr_1.33
##
## loaded via a namespace (and not attached):
## [1] nlme_3.1-152      RColorBrewer_1.1-2  tools_4.1.0
## [4] backports_1.2.1   utf8_1.2.1          R6_2.5.0
## [7] rpart_4.1-15      colorspace_2.0-1    withr_2.4.2
## [10] tidyselect_1.1.1  gridExtra_2.3       curl_4.3.1
## [13] compiler_4.1.0    formatR_1.11        htmlTable_2.2.1
## [16] scales_1.1.1      checkmate_2.0.0     proxy_0.4-26
## [19] stringr_1.4.0     digest_0.6.27       minqa_1.2.4
## [22] rio_0.5.27        base64enc_0.1-3     jpeg_0.1-8.1
```

## [25] pkgconfig_2.0.3	htmltools_0.5.1.1	lme4_1.1-27.1
## [28] htmlwidgets_1.5.3	rlang_0.4.11	readxl_1.3.1
## [31] rstudioapi_0.13	generics_0.1.0	dplyr_1.0.7
## [34] zip_2.2.0	magrittr_2.0.1	Rcpp_1.0.6
## [37] munsell_0.5.0	fansi_0.5.0	abind_1.4-5
## [40] lifecycle_1.0.0	stringi_1.6.1	yaml_2.2.1
## [43] parallel_4.1.0	forcats_0.5.1	crayon_1.4.1
## [46] lmom_2.8	haven_2.4.1	splines_4.1.0
## [49] hms_1.1.0	tmvnsim_1.0-2	pillar_1.6.1
## [52] boot_1.3-28	ggsignif_0.6.2	glue_1.4.2
## [55] evaluate_0.14	latticeExtra_0.6-29	data.table_1.14.0
## [58] nloptr_1.2.2.2	png_0.1-7	vcvtrs_0.3.8
## [61] cellranger_1.1.0	gtable_0.3.0	purrr_0.3.4
## [64] tidyr_1.1.3	xfun_0.23	openxlsx_4.2.4
## [67] broom_0.7.8	e1071_1.7-7	rstatix_0.7.0
## [70] class_7.3-19	tibble_3.1.2	cluster_2.1.2
## [73] ellipsis_0.3.2		