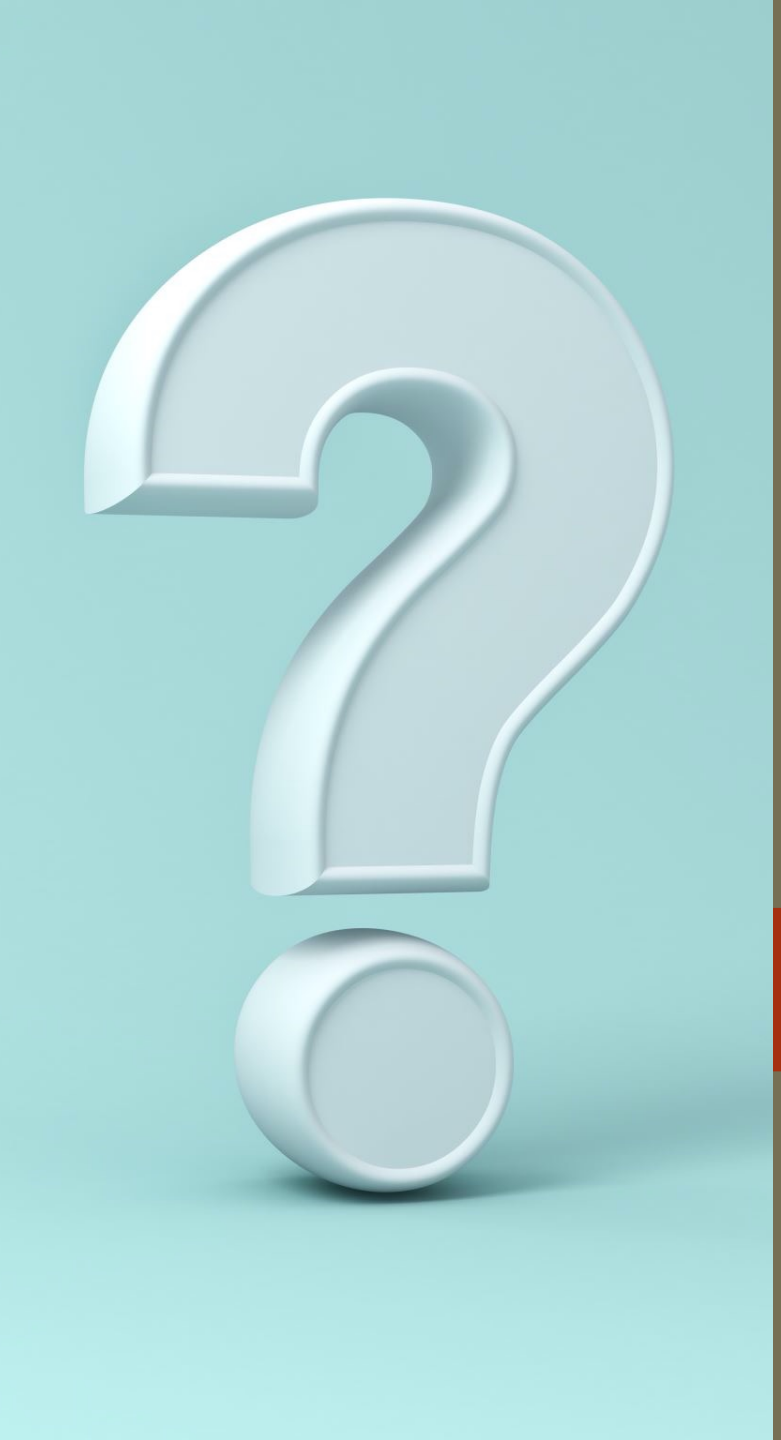
Inferential Statistics

Study from example problems



- Sales of the new promoted product as expectation: given in first 30 days, the daily average sales is only 8120 over 8500 as planned ?
- In a customer satisfaction survey, the score is 3.25 over 5, whether it's higher than average ?
- Two call center working in same process, but looks like call-length are different ?
- Can we manage the sigma for listing-to-sell time below 21 days, given in the recent 15 transactions it's 24.5 days ?
- Can we consider the difference in selling skills among year-1, year-2, year 3 experience agency, given average score in the selling skill assessment are difference ?

If you are asked the above...

Then you need to explore **Inferential Statistics** in addition to *Descriptive statistics*

What is Inferential Statistic

- Inferential Statistic is a way of making inferences about populations based on samples.
  - With inferential statistic, you take data from samples and make generalization about population.
- There are several type of inferential statistic examples in practices:
  - Hypothesis testing : where we use sample data to answer the assumption, the research question.
  - Confident interval estimation for the population by using samples.
  - Regression analysis : to predict the relationship between independent variables and dependent variables.

Problems statement

Hypothesis testing examples

P1: Is the new promoted product is under expectation ?

- A new product is built and financed with expectation of sales 8,500 items per day. In the first 30 days of promoting, daily average sales were 8,120 items. The management wonder whether the expectation was incorrect.

  Using 1% level of significance, test these hypothesis (assuming that the distribution of sales is normally distributed with standard deviation of 950).

Solution

P2: Customer Satisfaction Survey

- Head of Customer Service is interested in assessing customer sastifaction. To conduct the study, they asked 250 customers: rating the customer service as follow scale:

Much worse	Worse	Average	Better	Much better
1	2	3	4	5

The average rating was determined to be 3.25.

Based on the previous studies done by the company, it is assume sigma equal 1.5.

a) Is the customer satisfaction rating at or below the average ?

b) With another 250 samples, at which score they can consider the customer satisfaction higher than average.

Solution

P3: Call Center performance comparison

- Manager of two call centers would like to analysis the performance of her call center departments. She wishes to see if there is a difference in the average call-length between call centers.

  Random sample of 30 calls from 2 call centers were selected: their average call-length were measured at 11.91 minutes and 12.02 minutes; s_1 = 1.2 minutes and s_2 = 1.5 minutes.

Solution

P4: Listing-to-sell time goal

- A real estate company has set a time sales goal whereby the standard deviation of listing-to-sell time should be not longer than 21 days. Realizing this goal will help the company with sales forecasting and marketing.

  To conduct this analysis, sales records for 15 previously sold homes were randomly selected. Based on this sample the following information was obtained:

  Average listing-to-sell time is 162 days and standard deviation is 24.5.

Solution

P5: Agency Headcount

- Over the past 5 years, the number of agency headcount has changed at each level(Advisor, Unit Manager, Agency Manager, Senior Agency Manager, District Manager, District Director).

  Even though some headcount random variation is expected, is the variation beyond what we would expect due to chance alone ?

	2016	2017	2018	2019	2020
Advisor	560	552	540	580	632
Unit Manager	369	385	358	361	420
Agency Man	209	226	218	268	324
S.AM	132	180	170	223	259
District Manager	64	70	75	77	101
District Director	17	14	18	18	23

Solution

P6: Agency selling skill assessment

- Twenty-four sales agent were selected for an informal study about selling skills: they are 8 in first year, 8 in second year and 8 in third year.

  Participants were given a skill assessment with maximum score of 100. As studying objectives, we are interested in whether or not a difference exist somewhere between three different year level.

Year 1 scores	Year 2 scores	Year 3 scores
74	71	74
78	62	74
64	85	88
74	94	89
73	78	66
70	66	78
57	71	86
69	74	77
Avg	**69.88**	**75.13**	**79.00**

Solution

P6+ : Agency selling skill assessment –

additional questions

- Additional questions arised :
  a) Amongs each pairs (Y1-Y2), (Y1-Y3) and (Y2-Y3), ***which one is significant different between year level ?***
  b) Futher assessment were conducted with addition consideration: ***whether or not prior Financial experiences have an effect to selling skill assessmen***t. Twenty-four sales agents with prior finance experience and twenty-four sales agents without prior finance experience participated the assessment are selected. Their assessment scores are as below:

FIN experince	Year 1	Year 2	Year 3
W/o prior FIN	80 73 65 72 69 70 58 71	71 62 84 93 78 66 71 74	70 73 85 90 69 76 86 76
With prior FIN	82 90 68 74 73 71 63 72	73 64 86 94 78 70 74 75	71 74 87 91 71 78 87 77

Solution

Solutions illustration

Hypothesis testing examples

P1: Is the new promoted product is under expectation ?

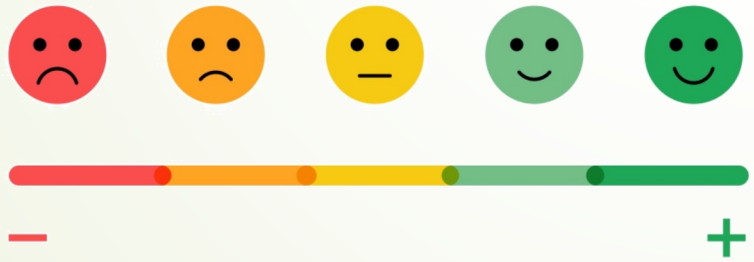
- Hypothesis test:
  - Ho : μ = 8500
  - Ha : μ ≠ 8500 (the expectation was incorrect)
  - *With significant level* $\alpha = .01$
- Our sample data:
  - $\bar{x} = 8120$, $\sigma = 950$, n = 30

- Calculate test statistic and critical value
  - $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ = -2.191
  - $Z_{crit} = \pm 2.576$
- Conclusion
  - As -2.576 < z < 2.576, our z value is in non rejection area, we do not reject null hypothesis.
  - With the first 30 days of sales data, there is **not enough evidence** at 1% of significance to say the daily sales **different from 8,500**. Further observation is needed.

Problem

P2: Customer Satisfaction Survey

- Our sample data
  - n = 250, x=3.25, σ = 1.5
- Hypothesis test
  - Ho : μ ≤ 3
  - Ha : μ > 3 (expecting customer satisfaction higher than average)
  - *Let's assume significant level α = .01*



a. The customer satisfaction is at or below the average ?
   - Calculate test statistic and critical value
     - With α = .01 then zcrit = 2.33
     - $z = \frac{x-\mu}{\sigma/\sqrt{n}}$ = 2.64
   - As z > 2.33, we would reject null hypothesis. We then accept the alternative that the customer satifation **higher than average.**

b. Threshold to consider customer satisfaction with 250 samples survey.
   - At critical value 2.33 , the average score is calculated as $\bar{x}$ **= 3.221**.
   - Any average score above threshold 3.221 would lead to rejection of null hypothesis. It means, in another 250 sample:
     - If we got a mean score **3.19** (<3.221), we can consider customer satisfaction calculated from this sample below the average.
     - But if we got a mean of **3.24** (>3.221), we can consider customer satisfaction is higher than average.

Problem

P3: Call Center performance comparison

- Hypothesis testing for difference between two sample means
  - Ho : The difference between the average call-length from 2 call-centers is Zero
  - Ha : The difference between the average call-length from 2 call-centers is NOT Zero
  - *Assume significance level* $\alpha = .05$
- Our sample data
  - n=30, $\bar{x}_1$=11.91, $\bar{x}_2$=12.05, s_1 =1.2, s_2=1.5

- Calculate test statistic and p-value
  - Difference between two sample means : d(11,91,12.05) = -0.11
  - Standard Error of the distribution of difference : SE = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ = 0.35. *It's also* ***standard deviation*** *of the sampling distribution.*
  - *95% of CI estimate for the difference: [-0.592, 8.120]*
  - *t-statistic = -0.11/0.35 = -0.314*
  - *p-value = .7549.*
- *Conclusion*
  - *Since p-value >* α*, we would fail to reject Ho. The sample results do not have sufficient evidence to conclude average call length is differ by call centers.*

Problem

P4: Listing-to-sell time goal

- Null & Alternate hypothesis
  - Ho : $\sigma < 21$ or $\sigma^2 \leq 441$ *(as the goal is standard deviation less than 21 days)*
  - Ha : $\sigma^2 > 441$
  - Let's assume significance level $\alpha = .05$
- Our gathering sample data
  - n = 15, $\bar{x}$ = 162 days, $s^2 = 600$, s=24.5
- Calculate test statistic and critical value
  - Critical value for Chi-square distribution with df =n-1 =14 , $\alpha = .05$:

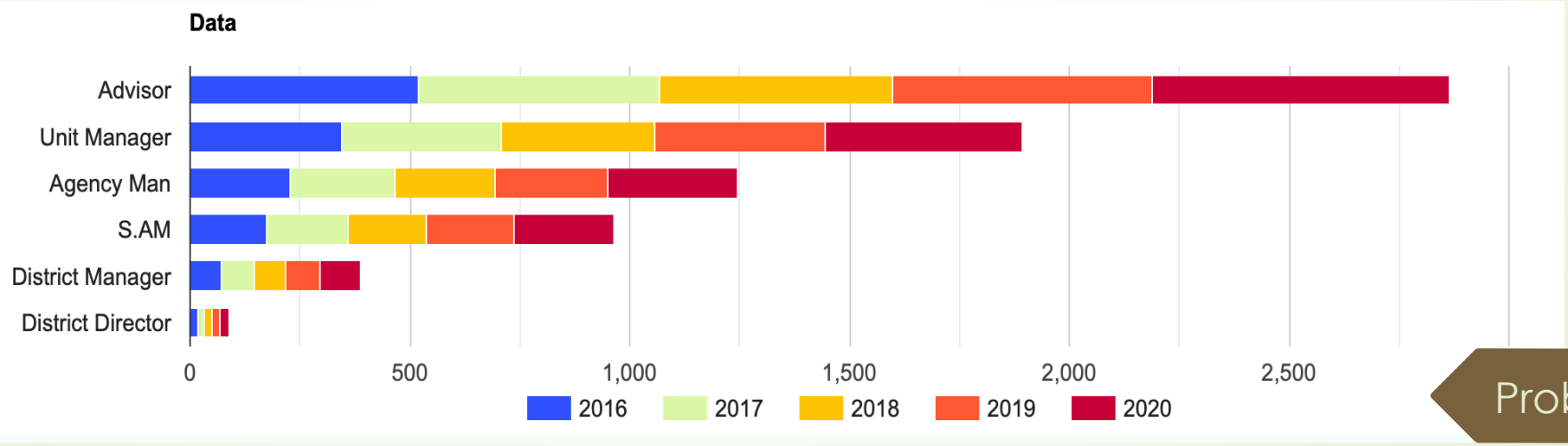
    $X^2_{critical} = 23.685$
  - Chi-square

    $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = 19.06$
  - p-value(x>19.06) = .163
- Conclusion
  - Since p-value>α, Ho is not rejected.
  - It means although sample variance looks too high, but sample data does not offer sufficient evidence to conclude variance of listing-to-sell time exceeds the upper limit set by the company

Problem

P5: Agency Headcount

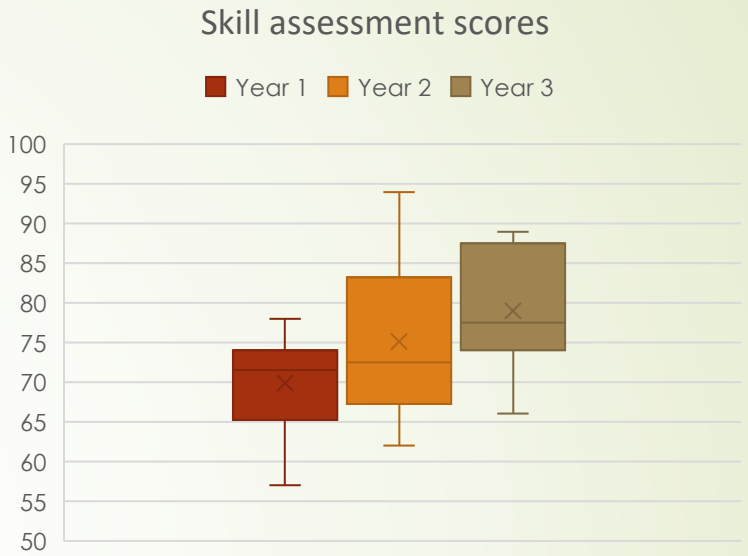
- Null & Alternative Hypothesis
  - Ho: The two categories (Year & Level) are independence
  - Ha: The two categories are related
  - Significance level α = .05
- Chi-square Test to compare Obeserved and Expected value
  - Test statistic χ^2 = 41.09, not in the 95% of acceptance region [-∞;31.41]
  - p-value p(x>41.09)= 0.00362
- Conclusion
  - Since p-value < α, we would reject Null hypothesis.
  - The two categories (year and level) are related in the variation of head count change.

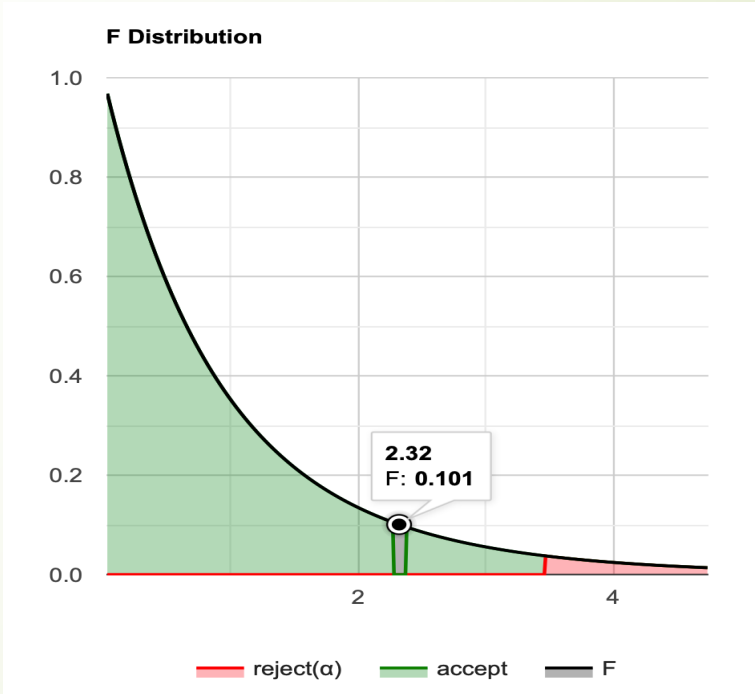


Problem

P6: Agency selling skill assessment

- Null hypothesis
  - Ho : μ1 = μ2 = μ3 (no difference between average of three group)
- The statistic and p-value
  - Test statistic F = 2.322, is in the 95% accepted region [-∞, 3,467]
  - *p-value* : P(x>F) = 0.122
- Conclusion
  - Since *p-value* > $\boldsymbol{\alpha}$ (.05), We would accept Ho.
  - The average of all groups considered to be equal. In other words, the difference between average of three group is not statistically significant.





Problem

P6+ : Agency selling skill assessment –

additional questions

a. Post hoc ANOVA

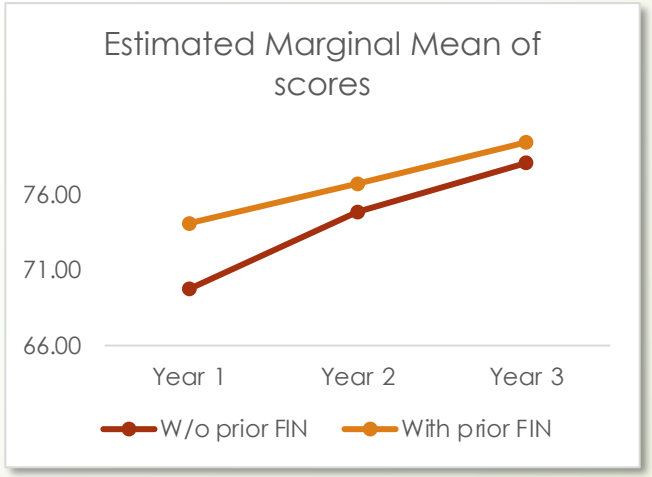
- Since p-value from ANOVA, is NOT less than significance level, the Post hoc test might not need as there is no significant difference between the means of any pair.
- Turkey's HSD shows the same results:

PAIR	Mean Diff.	$T_{critical}$	Significant at α = .05
Year 1 - Year 2	5.250	10.770	.446
Year 1 - Year 3	9.125	10.770	.104
Year 2 - Year 3	3.875	10.770	.639

- ANOVA test for two factors – Interaction

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
FIN Experience	77.521	1	77.521	1.103	0.300	4.073
Year	380.167	2	190.083	2.705	0.079	3.220
Interaction	20.667	2	10.333	0.147	0.864	3.220
Within	2951.625	42	70.277			
Total	3429.979	47				

- Conclusion
  - P-value > α, fail to reject Ho.
  - No Interaction between Year and prior financial experience in the selling skill assessment.



Problem

Thank you