DANIEL KUDUM.

Given the matrix A

$$= \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

1. from the definition of the eigenvector v corresponding to the eigenvalue λ we have $Av = \lambda v$

then: $Av - \lambda v = (A - \lambda I) \cdot v = 0$

Equation has none zero if and if only $\det(A - \lambda I) = 0$

$$\det(A - \lambda I) = \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix} = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 \approx$$

$$(\lambda + (21.125)) \cdot (\lambda + 5.604)) \cdot (\lambda - (2.675)) \cdot (\lambda - (11.054)) = 0$$

1. $\lambda_1 = -21.125$
2. $\lambda_2 = 5.604$
3. $\lambda_3 = 2.675$
4. $\lambda_4 = 11.054$

for every λ we find its own vector:

1. $\lambda_1 = 21.125$

$$A - \lambda_1 = \begin{pmatrix} 25.125 & 8 & -1 & -2 \\ -2 & -12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix}$$

$Av = \lambda v$

$(A - \lambda I) \cdot v = 0$

so let solve this linear equation by Gaussian Elimination:

$$\left(\begin{array}{cccc|c} 25.125 & 8 & -1 & -2 & 0 \\ -2 & 12.125 & -2 & -4 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{array}\right) \times (0.040) \quad \left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ -2 & 12.125 & -2 & -4 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{array}\right) \times (2)$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 12.761 & -2.080 & -4.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{array}\right) \times 1$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 12.761 & -2.080 & -4.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array}\right) \times (0.078)$$

$A = \lambda v$

$(A - \lambda I) \cdot v = 0$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array}\right) \times (-10)$$

$$= \left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 29.754 & -6.741 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array}\right) \times (12.682)$$

$=$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 27.754 & -6.741 & 0 \\ 0 & 0 & -16.106 & 3.912 & 0 \end{array}\right) \times (0.036)$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & -16.106 & 3.912 & 0 \end{array}\right) \times (16.106)$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (0.163)$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & 0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (0.040)$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & 0 & -0.089 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (-0.318)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0.027 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 + 0.027 \cdot x_4 = 0 \\ x_2 - 0.365 \cdot x_4 = 0 \\ x_3 - 0.243 \cdot x_4 = 0 \end{cases}$$

Calculation of matrix determinant by Bareiss algorithm

the Δ of matrix = elements in the last line

$$\begin{matrix} a_{11} & \cdots & a_{1j} \\ \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} \end{matrix}$$

$$\begin{pmatrix} -\lambda+4 & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & -\lambda+5 & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{pmatrix}$$

pivot element $P = a_{1,1} = \lambda+4$

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{1,3} & a_{1,4} \end{pmatrix} \to a_{ij}$$

$$\begin{pmatrix} -\lambda+4 & 8 & -1 & -2 \\ 0 & \lambda^2+5\lambda-20 & 2\lambda-10 & 4\lambda-20 \\ 0 & -10\lambda+40 & \lambda^2-\lambda+20 & 10\lambda-40 \\ 0 & 13\lambda-44 & 14\lambda-57 & \lambda^2+9\lambda-54 \end{pmatrix}$$

pivot element

$P_2 = a_{2,2}\lambda^2+5\lambda-20$

$$\begin{pmatrix} a_{2,2} & a_{2,3} \\ a_{2,4} & a_{2,4} \end{pmatrix} \to P_2$$

$$\begin{pmatrix} \lambda^2+5\lambda-20 & 0 & \lambda+25 & 2\lambda+50 \\ 0 & \lambda^2+5\lambda-20 & 2\lambda-10 & 4\lambda-20 \\ 0 & 0 & -\lambda^3+25\lambda & -10\lambda^2-90\lambda+400 \\ 0 & 0 & -14\lambda^2-43\lambda+175 & -\lambda^3-18\lambda^2+9\lambda+50 \end{pmatrix}$$

pivot element:

$P_3 = a_{3,3} = \lambda^3+25\lambda$

$$\begin{pmatrix} a_{3,3} & a_{3,4} \\ a_{1,3} & a_{2,3} \end{pmatrix} \to a_{ij}$$

P_3

$$\begin{pmatrix} -\lambda^3+25\lambda & 0 & 0 & -2\lambda^2-30\lambda+500 \\ 0 & -\lambda^3+25\lambda & 0 & -4\lambda^2+60\lambda-200 \\ 0 & 0 & -\lambda^3+25\lambda & -10\lambda^2-90\lambda+400 \\ 0 & 0 & 0 & \lambda^4+13\lambda^3-219\lambda^2-835\lambda+3500 \end{pmatrix}$$

$P_4 = a_{4,4} = \lambda^4+13\lambda^3-219\lambda-835\lambda+3500$

$$\begin{pmatrix} a_{4,4} & a_{4,2} \\ a_{1,4} & a_{3,4} \end{pmatrix} \to a_{ij}$$

P_2

$$\begin{pmatrix} \lambda^4+13\lambda^3-219\lambda^2-835\lambda+3500 & 0 & 0 & 0 \\ 0 & \lambda^4+13\lambda^3-219\lambda^2-835\lambda+3500 & 0 & 0 \\ 0 & 0 & \lambda^4+13\lambda^3-219\lambda^2-835\lambda+3500 & 0 \\ 0 & 0 & 0 & \lambda^4+13\lambda^3-219\lambda^2-835\lambda+3500 \end{pmatrix}$$

$x_1 + 0.027 \cdot x_2 = 0$

$x_2 - 0.365 \cdot x_4 = 0$ (1)

$x_3 - 0.243 \cdot x_4 = 0$

find x_3 from eqn 3 (1) $x_3 = 0.243 x_4$

find x_2 from eqn 2 (1) $x_2 = 0.365 x_4$

find x_1 from eqn 1 (1) $x_1 = 0.027 x_4$.

Ans

$x_1 = 0.027 x_4$

$x_2 = 0.365 x_4$

$x_3 = 0.243 x_4$

$x_4 = x_4$

$$x = \begin{pmatrix} -0.027 x_4 \\ 0.365 \cdot x_4 \\ 0.243 x_4 \\ x_4 \end{pmatrix} \quad x_4 \begin{pmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{pmatrix} =$$

let $x_4 = 1$ $V_1 = \begin{pmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{pmatrix}$

MUGABO Patricia.

Finding Eigen value using $\lambda_2 = -5.604$

① $A - \lambda_2 \simeq \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix}$

$AV = \lambda V$

$(A - \lambda I) \cdot V = 0$

$$\left(\begin{array}{cccc|c} 9.604 & 8 & -1 & -2 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right) \times(0.104)$$

$$\left(\begin{array}{cccc|c} 1 & 0833 & -0.104 & -0.208 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right) \times(2)$$

(2)

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & -1.730 & -2.208 & -4.416 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & 13 & -14.[illegible] & -7.396 & 0 \end{array}\right) \times (1)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & -1.730 & -2.208 & -4.416 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right) \times (-0.578)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right) \times (-10)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & -2.160 & -35.529 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right) \times (12.167)$$

(3)

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & -2.160 & -35.529 & 0 \\ 0 & 0 & 1.426 & 23.457 & 0 \end{array}\right) \times (-0.463)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 1.426 & 23.457 & 0 \end{array}\right) \times (-1.426)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (-1.276)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (0.104)$$

(4)
$$\left(\begin{array}{cccc|c} 1 & 0.833 & 0 & 1.504 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (-0.833)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 16.864 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 + 16.864 \cdot x_4 = 0 \\ x_2 - 18.439 \cdot x_4 = 0 \\ x_3 + 16.446 \cdot x_4 = 0 \end{cases}$$

$x_3 + 16.446 \cdot x_4 = 0$

$x_3 = 16.446 x_4$ $\quad x_2 = 18.439 x_4$

$x_1 = 16.864 x_4$

$x_1 = 16.864 x_4$

$x_2 = 18.439 x_4$

$x_3 = 16.446 x_4$

$x_4 = x_4$

$$X = \begin{pmatrix} -16.864 x_4 \\ 18.439 x_4 \\ -16.446 x_4 \\ x_4 \end{pmatrix}$$

Let $x_4 = 1$

(5) $v_2 \simeq \begin{pmatrix} -16.864 \\ 18.439 \\ -16.446 \\ 1 \end{pmatrix}$

21 Wednesday 172-193

$\lambda_3 = 2.675$

$$A - \lambda_3 I \approx \begin{pmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{pmatrix}$$

$Av = \lambda v$

$(A - \lambda I) \cdot v = 0$

Gaussian Elimination

$$\left(\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right) \times (0.754)$$

22 Thursday 173-192

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right) \times 2$$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right) \times 1$$

7 Wednesday 158-207

7.00

$$\begin{pmatrix} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{pmatrix} \times$$

8.00

9.00

$$\begin{pmatrix} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{pmatrix} \times -10$$

10.00

11.00

$$\begin{pmatrix} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 90.669 & 1.838 & 0 \\ 0 & -6.964 & -14.754 & -140.231 & 0 \end{pmatrix} \times 6.964$$

12.00

1.00

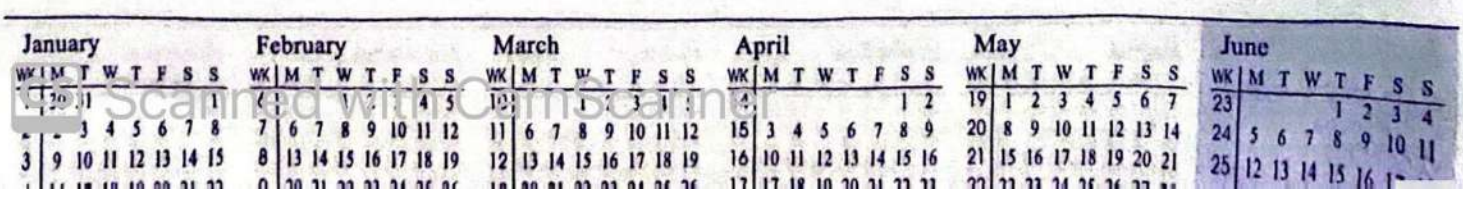
2.00

$$\begin{pmatrix} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 90.669 & 166.688 & 0 \\ 0 & 0 & -76.278 & -140.231 & 0 \end{pmatrix} \times 0.011$$

3.00

4.00

5.00



$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & -76.278 & -140.231 & 0 \end{array}\right)$$

$\times 76.278$

$$\left(\begin{array}{ccc|c} 1 & 6.036 & -0.754 & 0 \\ 0 & 1 & -8.834 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & 0 & -0.122 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

7.00
8.00
9.00
10.00
11.00
12.00
1.00
2.00
3.00
4.00
5.00

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 8.494 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 + 8.494 x_4 = 0 \\ x_2 - 1.428 x_4 = 0 \\ x_3 + 1.838 x_4 = 0 \end{cases}$$

$$\begin{cases} x_1 = -8.494 x_4 \\ x_2 = +1.428 x_4 \\ x_3 = -1.838 x_4 \end{cases}$$

$$x_4 = x_4$$

$$X = \begin{pmatrix} -8.494 x_4 \\ 1.428 x_4 \\ -1.838 x_4 \\ x_4 \end{pmatrix}$$

10 Saturday 161-204

Solution Set :

$$x_4 \begin{pmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{pmatrix}$$

Let $x_4 = 1$

$$V_3 = \begin{pmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{pmatrix}$$

11 Sunday 162-203

Done by ASHIMWE Carine

Mike Manzi

$\lambda_4 = 11.054$

$$A - \lambda_4 I = \begin{pmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{pmatrix}$$

$Av = \lambda v$

$(A - \lambda I).v = 0$

Gaussian Elimination

$$\left(\begin{array}{cccc|c} -7.054 & 8 & -1 & -2 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times(-0.142)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times(2)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times(1)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times (0.045)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times (-10)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times (14.134)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array}\right) \times (-0.147)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array}\right) \times (12.771)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (-0.077)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-0.142)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0 & 0.044 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(1.134)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0.071 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 + 0.071 \cdot x_4 = 0 \\ x_2 + 0.024 \cdot x_4 = 0 \\ x_3 + 1.691 x_4 = 0 \end{cases}$$

$$\begin{cases} x_1 = -0.071 x_4 \\ x_2 = -0.024 x_4 \\ x_3 = -1.691 x_4 \\ x_4 = x_4 \end{cases} \qquad X = \begin{pmatrix} -0.071 x_4 \\ -0.024 x_4 \\ -1.691 x_4 \\ x_4 \end{pmatrix}$$

Solution Set: $\left\{ x_4 \cdot \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix} \right\}$

$$\lambda_4 = 1, \quad v_4 = \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix}$$

Importance of each eigenvalue in %

$\lambda_1 = -21.125$, $\lambda_2 = -5.604$, $\lambda_3 = 2.675$, $\lambda_4 = 11.05$

$|\lambda_1| = 21.125$, $|\lambda_2| = 5.604$, $|\lambda_3| = 2.675$, $|\lambda_4| = 11.05$

Total $= 21.125 + 5.604 + 2.675 + 11.054 = 40.458$

$$\text{Importance} = \frac{|\lambda_i|}{\text{Total}} \times 100\,\%$$

λ_1: $\frac{21.125}{40.458} \times 100\% = \underline{\underline{52.23\,\%}}$

λ_2: $\frac{5.604}{40.458} \times 100\% = \underline{\underline{13.86\,\%}}$

λ_3: $\frac{2.675}{40.458} \times 100\% = \underline{\underline{6.61\,\%}}$

λ_4: $\frac{11.054}{40.458} \times 100\% = \underline{\underline{27.31\,\%}}$