

Given the matrix A

$$= \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$$v = A$$

1. from the definition of the eigenvector v corresponding to the eigenvalue λ
we have $Av = \lambda v$

$$\text{then : } Av - \lambda v = (A - \lambda I) \cdot v = 0$$

Equation has none zero v if and only if $\det(A - \lambda I) = 0$

$$\det(A - \lambda I) = \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix} = \lambda^4 + 13\lambda^3 + 219\lambda^2 + 835\lambda + 3500 \approx (\lambda - 21.125)(\lambda + 5.604)(\lambda - 2.675)(\lambda - 11.054) = 0$$

$$1. \lambda_1 = 21.125$$

for every λ we find its own vector:

$$2. \lambda_2 = 5.604$$

$$1. \lambda_1 = 21.125$$

$$3. \lambda_3 = 2.675$$

$$A - \lambda_1 = \begin{pmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix}$$

$$4. \lambda_4 = 11.054$$

$$Av = \lambda v$$

$$(A - \lambda I) \cdot v = 0$$

so let solve this linear equation by Gaussian Elimination:

$$\left(\begin{array}{cccc|c} 25.125 & 8 & -1 & -2 & 0 \\ -2 & 12.125 & -2 & -4 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{array} \right) \times (0.040)$$

$$\left(\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 2 & 12.125 & -2 & -4 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{array} \right) \times (2)$$

$$\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 12.761 & -2.080 & -4.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{array} \times 1 \quad \begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 12.761 & -2.080 & -4.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array} \times (0.078),$$

$$A = \lambda V$$

$$(A - \lambda I) \cdot v = 0$$

$$\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array} \times (-10)$$

$$= \begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 27.754 & -6.741 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array} \times (12.682)$$

$$\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 27.754 & -6.741 & 0 \\ 0 & 0 & -16.106 & 3.912 & 0 \end{array} \times (0.036) \quad \begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & -16.106 & 3.912 & 0 \end{array} \times (16.106)$$

$$\begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \times (0.163) \quad \begin{array}{cccc|c} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & 0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \times (0.040)$$

$$\begin{array}{cccc|c} 1 & 0.318 & 0 & -0.089 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \times (-0.318) \quad \begin{array}{cccc|c} 1 & 0 & 0 & 0.027 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$x_1 + 0.027 \cdot x_4 = 0$$

$$-0.365 \cdot x_4 = 0$$

$$x_3 - 0.243 \cdot x_4 = 0$$

Calculation of matrix determinant by Berezin's algorithm
 the A of matrix = elements in the last line

~~are~~ $\dots a_{ij}$

$$\begin{pmatrix} -\lambda + 4 & 8 & -1 & -2 \\ -2 & -7\lambda - 9 & -2 & -4 \\ 0 & 10 & -7\lambda + 5 & -10 \\ -1 & -13 & -14 & -7\lambda - 13 \end{pmatrix} \xrightarrow{\text{pivot element } p = a_{1,1} = \lambda + 4} \begin{pmatrix} -\lambda + 4 & 8 & -1 & -2 \\ 0 & \lambda^2 + 5\lambda - 20 & 2\lambda - 10 & 4\lambda - 20 \\ 0 & -10\lambda + 40 & \lambda^2 - 9\lambda + 20 & 10\lambda - 40 \\ 0 & 13\lambda - 44 & (14\lambda - 57) & \lambda^2 + 9\lambda - 84 \end{pmatrix}$$

$\frac{a_{13}}{a_{11}} \xrightarrow{\lambda} a_{1j}$

$$\begin{pmatrix} p_1 & a_{12} \\ a_{22} & a_{23} \\ a_{32} & a_{33} \\ p_2 & a_{11} \end{pmatrix}$$

$$\begin{pmatrix} \lambda^2 + 5\lambda - 20 & 0 & \lambda + 25 & 2\lambda + 50 \\ 0 & \lambda^2 + 5\lambda - 20 & 2\lambda - 10 & 4\lambda - 20 \\ 0 & 0 & -\lambda^3 + 25\lambda & -10\lambda^2 - 90\lambda + 400 \\ 0 & 0 & -14\lambda^2 - 43\lambda + 175 & -\lambda^3 - 18\lambda^2 + 9\lambda + 50 \end{pmatrix} \xrightarrow{\text{pivot element: } p_3 = a_{3,3} = \lambda^3 + 25\lambda}$$

$$\begin{pmatrix} a_{3,3} & a_{31} \\ a_{1,3} & a_{2,3} \end{pmatrix} \xrightarrow{\lambda} p_3$$

$$\begin{pmatrix} -\lambda^3 + 25\lambda & 0 & 0 & -2\lambda^2 - 30\lambda + 500 \\ 0 & -\lambda^3 + 25\lambda & 0 & -4\lambda^2 + 60\lambda - 200 \\ 0 & 0 & -\lambda^3 + 25\lambda & -10\lambda^2 - 90\lambda + 400 \\ 0 & 0 & 0 & (\lambda^3 + 13\lambda^2 - 219\lambda^2 - 835\lambda + 3500) \end{pmatrix} \xrightarrow{= p_4 = a_{4,4} = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500}$$

$$\begin{pmatrix} a_{44} & a_{42} \\ a_{14} & a_{34} \end{pmatrix} \xrightarrow{\lambda} p_2 = a_{1,j}$$

$$\begin{pmatrix} \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 & 0 & 0 & 0 \\ 0 & \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 & 0 & 0 \\ 0 & 0 & \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 & 0 \\ 0 & 0 & 0 & \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 \end{pmatrix}$$

$$x_1 + 0.027 \cdot x_2 = 0$$

$$x_2 - 0.365 \cdot x_4 = 0 \quad (1)$$

$$x_3 - 0.243 \cdot x_4 = 0$$

find x_3 from eqn 3 (1) $x_3 = 0.243 x_4$

find x_2 from eqn 2 (1) $x_2 = 0.365 x_4$

find x_1 from eqn 1 (1) $x_1 = 0.027 x_4$.

Ans

$$x_1 = 0.027 x_4$$

$$x_2 = 0.365 x_4$$

$$x_3 = 0.243 x_4$$

$$x_4 = x_4$$

$$x = \begin{pmatrix} -0.027 x_4 \\ 0.365 x_4 \\ 0.243 x_4 \\ x_4 \end{pmatrix}$$

$$\begin{pmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{pmatrix}$$

$$\text{let } x_4 = 1, v_1 = \begin{pmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{pmatrix}$$

MUGABO Patricia.

Finding eigenvalue using $\lambda_2 = -5.604$

$$\textcircled{1} \quad A - \lambda_2 \approx \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix}$$

$$Ax = \lambda x$$

$$(A - \lambda I) \cdot v = 0$$

$$\left(\begin{array}{cccc|c} 9.604 & 8 & -1 & -2 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array} \right) \times (0.104)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array} \right) \times (2)$$

②

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & 0.208 & 0 \\ 0 & -1.730 & -2.208 & -4.416 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & 13 & -11.104 & -7.396 & 0 \end{array} \right) \times (-1)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & -1.730 & -2.208 & -4.416 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array} \right) \times (-0.578)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array} \right) \times (-10)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & -2.160 & -35.529 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array} \right) \times (12.167)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & -2.160 & -35.529 & 0 \\ 0 & 0 & 1.426 & 23.457 & 0 \end{array} \right) \times (-0.463)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 1.426 & 23.457 & 0 \end{array} \right) \times (-1.0426)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \times (-1.276)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \times (0.104)$$

$$(4) \left(\begin{array}{cccc|c} 1 & 0.833 & 0 & 1.504 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \times (-0.833)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 16.864 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left\{ \begin{array}{l} x_1 + 16.864 \cdot x_4 = 0 \\ x_2 - 18.439 \cdot x_4 = 0 \\ x_3 + 16.446 \cdot x_4 = 0 \end{array} \right.$$

$$x_3 + 16.446 \cdot x_4 = 0$$

$$x_3 = 16.446 x_4 \quad x_2 = 18.439 x_4$$

$$x_1 = 16.864 x_4$$

$$x_1 = 16.864 x_4$$

$$x_2 = 18.439 x_4$$

$$x_3 = 16.446 x_4$$

$$x_4 = x_4$$

$$\chi = \begin{pmatrix} -16.864 x_4 \\ 18.439 x_4 \\ -16.446 x_4 \\ x_4 \end{pmatrix}$$

Let $x_4 = 1$

⑤

$$v_2 \approx \begin{pmatrix} -16.864 \\ 18.439 \\ -16.446 \\ 1 \end{pmatrix}$$

21

Wednesday
172-193

$$\lambda_3 = 2.675$$

$$A - \lambda_3 I \approx \begin{pmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{pmatrix}$$

$$Av = \lambda v$$

$$(A - \lambda I) \cdot v = 0$$

Gaussian Elimination

$$\left(\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array} \right) \times (0.754)$$

22

Thursday
173-192

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array} \right) \times 2$$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array} \right) \times 1$$

January							February							March							April							May							June												
W	M	T	W	T	F	S	W	M	T	W	T	F	S	W	M	T	W	T	F	S	W	M	T	W	F	S	W	M	T	W	F	S	W	M	T	W	F	S									
1	30	31					6		1	2	3	4	5	10		1	2	3	4	5	14		1	2	3	4	5	6	7	19	1	2	3	4	23		1	2	3	4							
2	2	3	4	5	6	7	8	7	6	7	8	9	10	11	12	11	6	7	8	9	10	11	12	15	3	4	5	6	7	8	9	20	8	9	10	11	12	13	14	24	5	6	7	8	9	10	11
3	9	10	11	12	13	14	15	8	13	14	15	16	17	18	19	12	13	14	15	16	17	18	19	16	10	11	12	13	14	15	16	21	15	16	17	18	19	20	21	25	12	13	14	15	16	17	18
4	16	17	18	19	20	21	22	9	20	21	22	23	24	25	26	13	20	21	22	23	24	25	26	17	17	18	19	20	21	22	23	22	23	24	25	26	27	28	26	19	20	21	22	23	24	25	
5	23	24	25	26	27	28	29	10	27	28	29	30	31			14	27	28	29	30	31			18	24	25	26	27	28	29	30	23	29	30	31				27	26	27	28	29	30			

7

Wednesday
158-207

Wednesday 158-207				
7.00				
	1	6.036	-0.754	-1.509
	0	0.397	-3.509	-7.018
	0	10	2.325	-10
8.00				
	0	-6.964	-14.754	-7.184

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 6 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 10 & 2.325 & \cancel{-10} & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array} \right)$$

$$\begin{array}{c} 11.00 \\ \hline 1 & 6.036 & -0.754 & -1.509 & 0 \\ \hline 12.00 & 0 & 1 & -8.834 & -17.669 & 0 \\ \hline 1.00 & 0 & 0 & 90.669 & 1.838 & 0 & 6.96 \\ \hline & 0 & -6.964 & -14.754 & -140.231 & 0 \end{array}$$

$$\begin{array}{c} 2.00 \\ \hline 1 & 6.036 & -0.754 & -1.509 & | 0 \\ \hline 0 & 1 & -8.834 & -17.669 & | 0 \quad x \\ \hline 3.00 \\ \hline 0 & 0 & 90.669 & 166.688 & | 0 \quad 0.61 \\ \hline 0 & 0 & -76.278 & -140.231 & | 0 \end{array}$$

Year	Series 1 (Top)	Series 2 (Bottom)
2000	4.00	4.00
2001	4.05	4.05
2002	4.10	4.10
2003	4.15	4.15
2004	4.20	4.20
2005	4.25	4.25
2006	4.30	4.30
2007	4.85	4.65

January		February		March		April		May		June	
W	K	W	K	W	K	W	K	W	K	W	K
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	1	2	3	4	5
6	7	8	9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26	27	28	29
30	31	1	2	3	4	5	6	7	8	9	10

8

Thursday
159-206

1	6.036	-0.754	-1.509	0	
0	1	-8.834	-17.669	0	
0	0	1	1.838	0	
0	0	-76.278	-140.231	0	8.00

X 76.278 7.00

1	6.036	-0.754	0		9.00
0	1	-8.834	0		
0	0	1	0		10.00
0	0	0	0		

11.00

1	6.036	-0.754	-1.509	0	
0	1	0	-1.428	0	
0	0	1	1.838	0	
0	0	0	0	0	1.00

12.00

1	6.036	0	-0.122	0	
0	1	0	-1.428	0	
0	0	1	1.838	0	
0	0	0	0	0	4.00

2.00

3.00

4.00

5.00



Scanned with CamScanner

August

W M T W T F S S

September

W M T W T F S S

October

W M T W T F S S

November

W M T W T F S S

December

W M T W T F S S

7.00

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 8.494 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

9.00

$$x_1 + 8.494 x_4 = 0$$

10.00

$$x_2 - 1.428 x_4 = 0$$

11.00

$$x_3 + 1.838 x_4 = 0$$

12.00

$$\left\{ \begin{array}{l} x_1 = -8.494 x_4 \\ x_2 = +1.428 x_4 \end{array} \right.$$

1.00

$$\left\{ \begin{array}{l} x_3 = -1.838 x_4 \\ x_4 = x_4 \end{array} \right.$$

2.00

$$\left\{ \begin{array}{l} x_3 = -1.838 x_4 \\ x_4 = x_4 \end{array} \right.$$

3.00

$$x_4 = x_4$$

4.00

$$x = \begin{pmatrix} -8.494 x_4 \\ 1.428 x_4 \\ -1.838 x_4 \\ x_4 \end{pmatrix}$$

5.00

January

M	T	W	T	F	S	S
30	31		1	2	3	4
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

February

W	K	M	T	W	T	F	S	S
10	11	12	13	14	15	16	17	18
17	18	19	20	21	22	23	24	25
24	25	26	27	28	29	30		
31								

March

W	K	M	T	W	T	F	S	S
11	12	13	14	15	16	17	18	19
22	23	24	25	26	27	28	29	30
31								

April

W	K	M	T	W	T	F	S	S
12	13	14	15	16	17	18	19	20
29	30	31						

May

W	K	M	T	W	T	F	S	S
19	20	21	22	23	24	25	26	27
28	29	30	31					

June

W	K	M	T	W	T	F	S	S
23	24	25	26	27	28	29	30	31
1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31					



Scanned with CamScanner

10

Saturday
161-204Solution Set :

$$x_1 \begin{pmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{pmatrix}$$

Let $x_4 = 1$

$$\therefore V_3 = \begin{pmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{pmatrix}$$

11

Sunday
162-203Done by ASHIMWE Canine

July	August	September	October	November	December
W	M	W	T	W	T
1 2	3 4 5 6 7 8 9	10 11 12 13 14 15 16	17 18 19 20 21 22 23	24 25 26 27 28 29 30	31
28	29	30	31	1	49
30	31	1 2 3 4 5 6	7 8 9 10 11 12 13	14 15 16 17 18 19 1	2 3 4 5 6 7 8

Mike Manzi

$$\lambda_4 = 11.054$$

$$A - \lambda_4 I = \begin{pmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{pmatrix}$$

$$Av = \lambda v$$
$$(A - \lambda I) \cdot v = 0$$

Gaussian Elimination

$$\left(\begin{array}{cccc|c} -7.054 & 8 & -1 & -2 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array} \right) \times (-\frac{1}{10.054})$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array} \right) \times (2)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array} \right) \times (1)$$

$$\left(\begin{array}{ccccc} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.0716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array} \right) \times (-0.045)$$

$$\left(\begin{array}{ccccc} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array} \right) \times (-10)$$

$$\left(\begin{array}{ccccc} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array} \right) \times (14.134)$$

$$\left(\begin{array}{ccccc} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array} \right) \times (-0.147)$$

$$\left(\begin{array}{ccccc} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array} \right) \times (12.771)$$

$$\left(\begin{array}{ccccc} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right) \times (-0.077)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \times (-0.142)$$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0 & 0.044 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \times (1.134)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0.071 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left\{ \begin{array}{l} x_1 + 0.071x_4 = 0 \\ x_2 + 0.024x_4 = 0 \\ x_3 + 1.691x_4 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 = -0.071x_4 \\ x_2 = -0.024x_4 \\ x_3 = -1.691x_4 \\ x_4 = x_4 \end{array} \right. \quad X = \begin{pmatrix} -0.071x_4 \\ -0.024x_4 \\ -1.691x_4 \\ x_4 \end{pmatrix}$$

Solution Set : $\{ x_4 \cdot \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix} \}$

$$x_4 = 1, \quad v_4 = \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix}$$

Importance of each eigenvalue in %

$$\lambda_1 = -21.125, \lambda_2 = -5.604, \lambda_3 = 2.675, \lambda_4 = 11.054$$

$$|\lambda_1| = 21.125, |\lambda_2| = 5.604, |\lambda_3| = 2.675, |\lambda_4| = 11.054$$

$$\text{Total} = 21.125 + 5.604 + 2.675 + 11.054 = 40.458$$

$$\text{Importance} = \frac{|\lambda_i|}{\text{Total}} \times 100 \%$$

$$\lambda_1: \frac{21.125}{40.458} \times 100\% = \underline{\underline{52.23\%}}$$

$$\lambda_2: \frac{5.604}{40.458} \times 100\% = \underline{\underline{13.86\%}}$$

$$\lambda_3: \frac{2.675}{40.458} \times 100\% = \underline{\underline{6.61\%}}$$

$$\lambda_4: \frac{11.054}{40.458} \times 100\% = \underline{\underline{27.31\%}}$$