MODULE 5

Randomness

WHY RANDOMNESS?

- Data scientists should be able to understand **randomness**.
- For example, they should be able to assign individuals to **treatment** and **control** groups **at random**, and then try to say whether any observed differences in the outcomes of the two groups are simply due to the random assignment or genuinely due to the treatment.

COMPARISON AND BOOLEANS

RANDOMNESS AND BOOLEANS

- A fundamental question about random events is **whether or not** they occur. For example:
  - Did an individual get assigned to the treatment group, or not?
- Once the event has occurred, you can answer "yes" or "no" to these questions.
- In programming, it is conventional to do this by labeling statements as True or False.
- In Python, Boolean values, named for the logician George Boole, represent **truth** and take only two possible values: True and False.

COMPARISON OPERATORS

The result of a comparison expression is a **bool** value

```
x = 2                y = 3
```

Assignment statements

```
x > 1                x > y                y >= 3

x == y               x != 2               2 < x < 5
```

Comparison expressions

AGGREGATING COMPARISONS

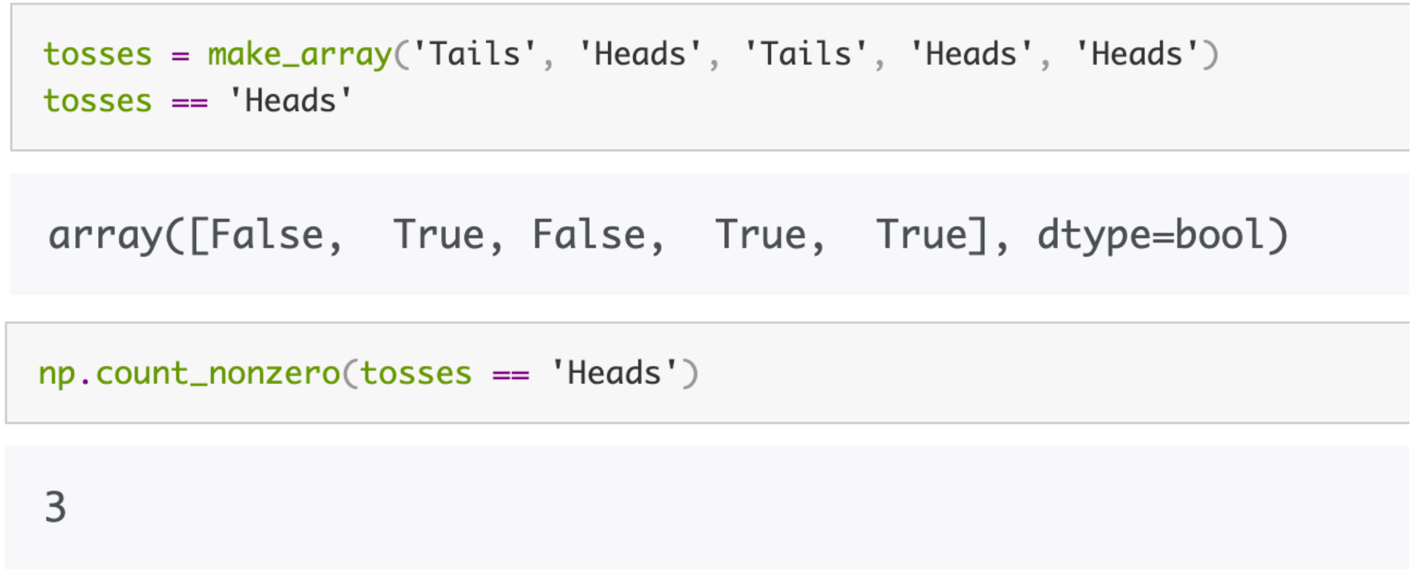
Summing an array or list of bool values will count the True values only.

```
1    + 0     + 1              == 2
True + False + True           == 2
sum([1   , 0     , 1   ])    == 2
sum([True, False, True])     == 2
```

(Demo – notebook 5.1, Aggregating comparisons)

NP.COUNT_NONZERO()

- The ***numpy*** method ***count_nonzero*** evaluates to the number of non-zero (that is, True) elements of the array.



```
tosses = make_array('Tails', 'Heads', 'Tails', 'Heads', 'Heads')
tosses == 'Heads'
```

```
array([False,  True, False,  True,  True], dtype=bool)
```

```
np.count_nonzero(tosses == 'Heads')
```

```
3
```

CONDITIONAL STATEMENTS

RANDOMNESS AND CONDITIONAL STATEMENTS

- In many situations, actions and results depend on a specific set of conditions being satisfied.
  - For example, individuals in randomized controlled trials receive the treatment if they have been assigned to the treatment group.
- A *conditional statement* is a multi-line statement that allows Python to choose among different alternatives **based on the truth value** of an expression.

CONDITIONAL STATEMENTS

- A conditional statement always begins with an if header, which is a single line followed by an indented body.
  - The purpose of **if** is to define functions that choose different behavior based on their arguments
- The body is only executed if the expression directly following if (called the *if expression*) evaluates to a true value.
- If the *if expression* evaluates to a false value, then the body of the if is skipped.

GENERAL FORM OF CONDITIONAL STATEMENTS

- A conditional statement can have **multiple clauses** with **multiple bodies**, and only one of those bodies can ever be executed.

```
if <if expression>:
    <if body>
elif <elif expression 0>:
    <elif body 0>
elif <elif expression 1>:
    <elif body 1>
...
else:
    <else body>
```

(Demo – notebook 5.1, Conditional statements)

RANDOM SELECTION

RANDOM SELECTION IN PYTHON

np.random.choice

- Selects uniformly at random
- with replacement
- from an array,
- a specified number of times

np.random.choice(some_array, sample_size)

Demo – notebook 5.1, Random selection

ITERATION

RANDOMNESS AND ITERATION

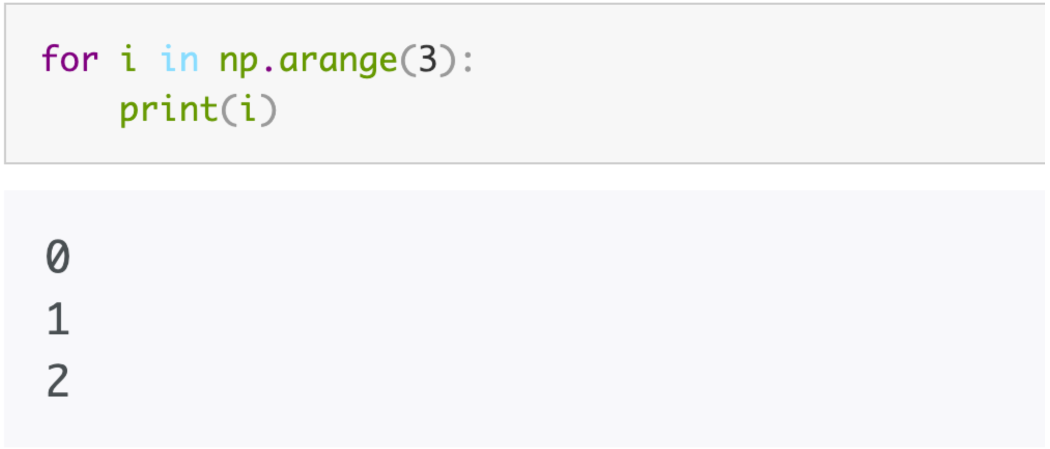
- In programming – especially when dealing with randomness – we often want to repeat a process multiple times.
- For example, we might want to assign each person in a study to the treatment group or to control, based on tossing a coin.
- We could run np.random.choice(make_array('Heads', 'Tails'))
  - However, we would need to copy and paste and run it for each of the participants in the study, even if they are 1000!
- Better strategy? Iteration

ITERATION

- A more automated solution is to use a for statement to loop over the contents of a sequence. This is called *iteration*.
- A for statement:
  - begins with the word for,
  - followed by a name we want to give each item in the sequence,
  - followed by the word in,
  - and ending with an expression that evaluates to a sequence.
- The indented body of the for statement is executed once *for each item in that sequence*.

FOR STATEMENTS

- **for** is a keyword that begins a control statement
- The purpose of **for** is to perform a computation for every element in a list or array
- Example:



```python
for i in np.arange(3):
    print(i)
```

```
0
1
2
```

(Demo – notebook 5.1, For statements)

APPENDING ARRAYS

STORING THE RESULTS OF A ITERATION

- The *for statement* in the previous slide simply prints the output.
- This output is NOT in a form that we can use for computation.
- A typical use of a *for statement* is to **create an array of results**, ***by augmenting it each time***.

- The append method in numpy helps us do this.

APPENDING TO AN ARRAY

- **np.append(array_1, value)**
  - new array with value appended to array_1
  - value has to be of the same type as elements of array_1
- **np.append(array_1, array_2)**
  - new array with array_2 appended to array_1
  - array_2 elements must have the same type as array_1 elements

(Demo – notebook 5.1,
Appending arrays)

SIMULATION

DEFINITION AND STEPS

Simulation is the process of using a computer to mimic a physical experiment.

- **Step 1: What to Simulate**
  - For example, you might decide that you want to simulate the outcomes of tosses of a coin.
- **Step 2: Simulating One Value**
  - In our example, figure out how to simulate the outcome of *one* toss of a coin.
- **Step 3: Number of Repetitions**
  - Decide how many times you want to simulate the quantity, then repeat Step 2 that many times. E.g., 1000
- **Step 4: Coding the Simulation**
  - Put it all together in code.

SIMULATION STEP 4

Step 4: Coding the Simulation

1. Create an empty array in which to collect all the simulated values. We will call this the **collection array.**

2. Create a "repetitions sequence," - a sequence whose length is the number of repetitions you specified in Step 3.
    1. For n repetitions we will often use the sequence np.arange(n).

3. Create a for loop. For each element of the repetitions sequence:
    1. Simulate *one* value based on the code you developed in Step 2.
    2. Augment the collection array with this simulated value.

EX 1: BIGGER NUMBER = $1

- Let's play a game: we each roll a die.
  - If my number is bigger: you pay me a dollar.
  - If they're the same: we do nothing.
  - If your number is bigger: I pay you a dollar.
- Steps:
  1. **What to simulate**: two dice rolls.
  2. **Simulate one value**: compute how much money we win/lose based on the result.
  3. **Number of repetitions**: Do steps 1 and 2 10,000 times.
  4. **Put it all in code**

(Demo – notebook 5.1, Simulation)

EX 2: NUMBER OF HEADS IN 100 TOSSES

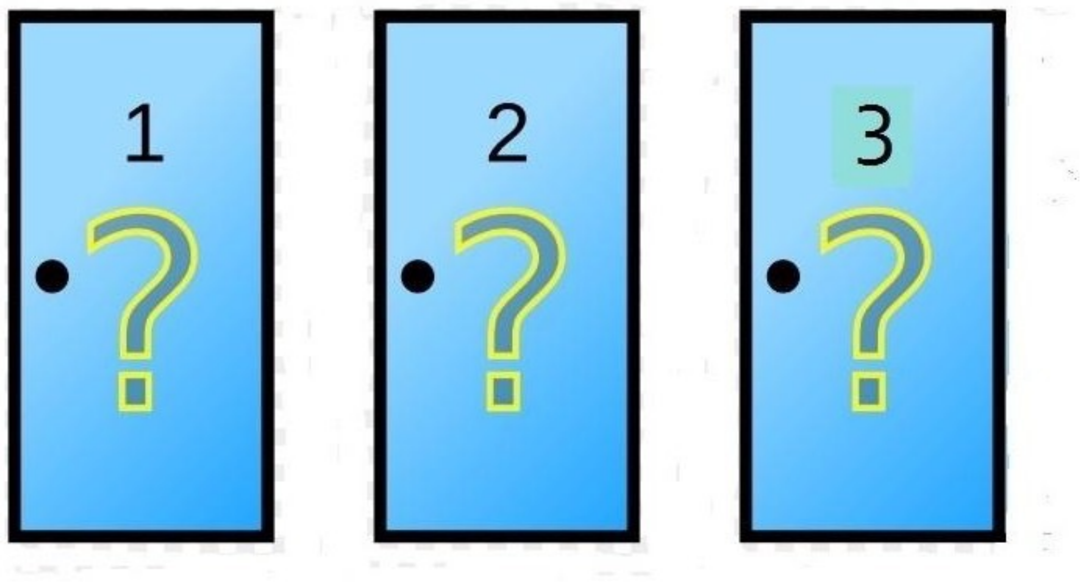
- In this example we will simulate the number of heads in 100 tosses of a coin.
- Steps:
  1. **What to stimulate**: outcomes of tosses of a coin.
  2. **Simulate one value**: make ***one* set of 100 tosses** and **count the number of heads**
  3. **Number of repetitions**: Do steps 1 and 2 10,000 times.
  4. **Put it all in code**

(Demo – notebook 5.1, Simulation)

CHANCE AND PROBABILITY

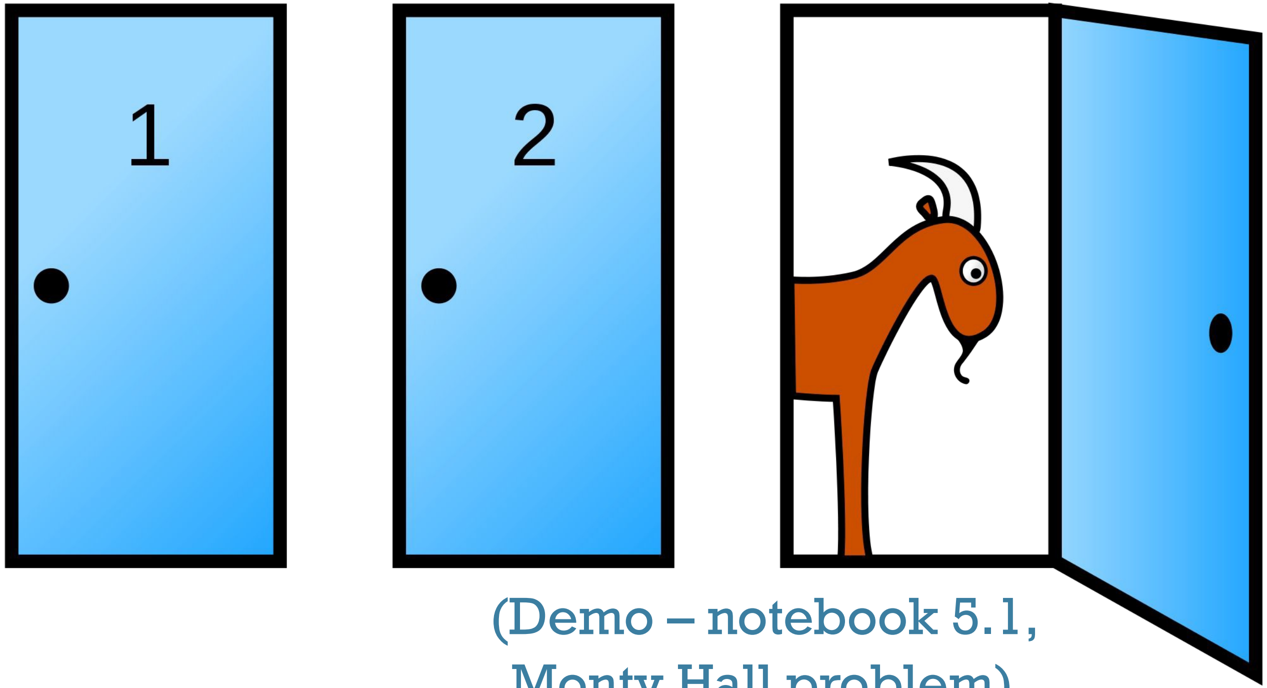
THE MONTY HALL PROBLEM

MONTY HALL PROBLEM



https://probabilityandstats.files.wordpress.com/2017/05/monty-hall-pic-1.jpg

THE FINAL CHOICE



(Demo – notebook 5.1, Monty Hall problem)

https://en.wikipedia.org/wiki/Monty_Hall_problem

Still can't wrap your head around it? Here's another intuitive way to think about it

PROBABILITY

DEFINITIONS AND NOTATIONS

- Most probabilities will be **relative frequencies**

- By convention, probabilities are numbers between 0 and 1, or, equivalently, 0% and 100%.

- Standard notation: **P(event)** denotes the **probability that "event" happens**

BASICS

- **Lowest value**: 0
  - Chance of event that is impossible
- **Highest value**: 1 (or 100%)
  - Chance of event that is certain
- **Complement**: P(an event doesn't happen)
  - = 1−P(the event happens)
  - If an event has chance 70%, then the chance that it doesn't happen is
    - 100% - 70% = 30%, or, 1 - 0.7 = 0.3

EQUALLY LIKELY OUTCOMES

Assuming all outcomes are equally likely, the chance of an event A is:

$$P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}}$$

A QUESTION

- I have three cards: **ace of hearts**, **king of diamonds**, and **queen of spades**.
- I shuffle them and draw two cards *at random without replacement.*
- What is the chance that I get the Queen followed by the King?

MULTIPLICATION RULE

Chance that two events *A* and *B* both happen
= P(*A* happens) x P(*B* happens given that *A* has happened)

- The answer is *less than or equal to* each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

ANOTHER QUESTION

- I have three cards: **ace of hearts**, **king of diamonds**, and **queen of spades**.

- I shuffle them and draw two cards *at random without replacement.*

- What is the chance that one of the cards I draw is a King and the other is Queen?

AK AQ KA [✓ KQ] QA [✓ QK]

ADDITION RULE

If event *A* can happen in *exactly one* of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way

COMPLEMENT: E.G., AT LEAST ONE HEAD

- In 3 tosses:
  - Any outcome *except* TTT
  - P(T) = ½
  - P(TTT) = (1/2) x (1/2) x (1/2) = (1/2)**3 = 1/8
  - P(at least one head) = 1 - P(TTT) = 1 - (1/8) = 87.5%
- In 10 tosses:
  - 1 - (1/2)**10 ≅ 99.9%

DISCUSSION QUESTION

A population has 100 people, including Rick and Morty. We sample two people at random **without replacement.**

(a) P(both Rick and Morty are in the sample)
= P(first Rick, then Morty) + P(first Morty, then Rick)
= (1/100) * (1/99) + (1/100) * (1/99) = 0.0002

(b) P(neither Rick nor Morty is in the sample)
= (98/100) * (97/99) = 0.9602

SAMPLING

RANDOM SAMPLES

- Deterministic sample:
  - Sampling scheme doesn't involve chance

- Random sample:
  - Before the sample is drawn, you have to know the selection probability of every group of people in the population
  - NOTE: Not all individuals / groups have to have equal chance of being selected

(Demo – notebook 5.2, Random sampling)

SAMPLE OF CONVENIENCE

- Example: sample consists of whoever walks by
- Just because you think you're sampling "randomly", doesn't mean you have a random sample.
- If you can't figure out ahead of time
  - what's the population
  - what's the chance of selection, for each group in the population

  then you don't have a random sample

DISTRIBUTIONS

PROBABILITY DISTRIBUTION

- Random quantity with various possible values
- “Probability distribution”:
  - All the possible values of the quantity
  - The probability of each of those values
- If you can do the math, you can work out the probability distribution without ever simulating it
- But… simulation is often easier!

EMPIRICAL DISTRIBUTION

- “Empirical”: based on observations
- Observations can be from repetitions of an experiment
- “Empirical Distribution”
  - All observed values
  - The proportion of times each value appears

(Demo – notebook 5.2, Distributions)

LARGE RANDOM SAMPLES

LAW OF AVERAGES / LAW OF LARGE NUMBERS

If a chance experiment is repeated many times, independently and under the same conditions, then the proportion of times that an event occurs

gets closer to the theoretical probability of the event

As you increase the number of rolls of a die, the proportion of times you see the face with five spots gets closer to 1/6

EMPIRICAL DISTRIBUTION OF A SAMPLE

If the sample size is large,

then the empirical distribution of a uniform random

sample resembles the distribution of the population,

with high probability

(Demo - – notebook 5.2,
Large Random Samples)

A STATISTIC

INFERENCE

- **Statistical Inference:**

  Making conclusions based on data in random samples
- **Example**:

fixed

Use the data to guess the value of an unknown number

depends on the random sample

Create an estimate of the unknown quantity

TERMINOLOGY

- Parameter
  - A number associated with the population
- Statistic
  - A number calculated from the sample

A statistic can be used as an estimate of a parameter

PROBABILITY DISTRIBUTION OF A STATISTIC

- Values of a statistic vary because random samples vary

- "Sampling distribution" or "probability distribution" of the statistic:
    - All possible values of the statistic,
    - and all the corresponding probabilities
- Can be hard to calculate
    - Either have to do the math
    - Or have to generate all possible samples and calculate the statistic based on each sample

(Demo – notebook 5.2, Simulating statistics)

EMPIRICAL DISTRIBUTION OF A STATISTIC

- Empirical distribution of the statistic:
  - Based on simulated values of the statistic
  - Consists of all the observed values of the statistic,
  - and the proportion of times each value appeared

- Good approximation to the probability distribution of the statistic
  - if the number of repetitions in the simulation is large

(Demo – notebook 5.2,
Simulating statistics)

QUESTIONS?