Solutions to Exercises from Algebra, by Michael Artin, 2nd ed.

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Contents

1 Matrices

1.1 The Basic Operations

1.1 What are the entries a_{21} and a_{23} of the matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 7 & 8 \\ 0 & 9 & 4 \end{bmatrix}$?

Solution: The a_{ij} entry of a matrix refers to the entry in the i-th row and j-th column. So $a_{21} = 2$ and $a_{23} = 8$.

1.2 Determine the products AB and BA for the following values of A and B.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} -8 & -4 \\ 9 & -5 \\ -3 & -2 \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 6 & -4 \\ 3 & 2 \end{bmatrix}$$

Solution: For the first product,

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} -8 & -4 \\ 9 & -5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} (1)(-8) + (2)(9) + (3)(-3) & (1)(-4) + (2)(-4) + (3)(-2) \\ (3)(-8) + (3)(9) + (1)(-3) & (3)(-4) + (3)(-4) + (1)(-2) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -18 \\ 0 & -26 \end{bmatrix}$$

1.3 Let $A = [a_1 ... a_n]$ be a row vector and $B = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ be a column vector. Compute the product AB and BA. **Solution:** The product AB is analagous to the dot product of the two vectors:

$$AB = [a_1 ... a_n] \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = [a_1 b_1 + ... + a_n b_n]$$

1.4 Verify the associative product for the matrix product

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

Solution:

$$\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 8 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 38 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 14 \end{bmatrix} = \begin{bmatrix} 38 \\ 14 \end{bmatrix}$$