

Perspectives on Neural Network Verification

Patrick Musau

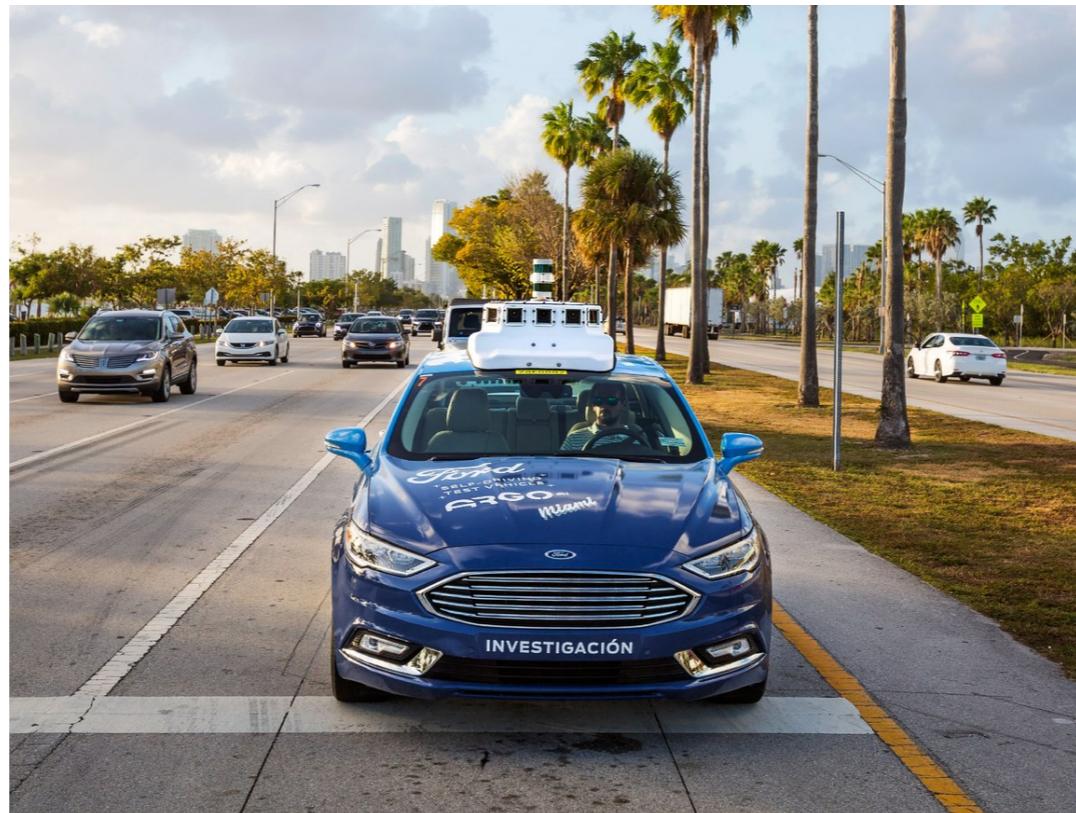
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Context and Origins

Artificial Neural Networks have demonstrated remarkable utility in numerous application domains including:

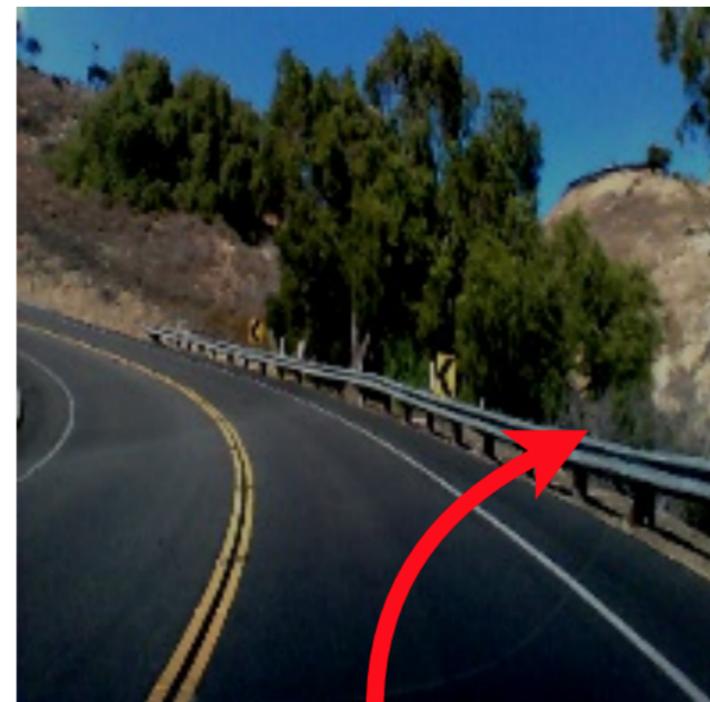
- ▶ Autonomous Vehicles
- ▶ Image and Pattern Recognition
- ▶ Machine Translation
- ▶ Non-linear system identification and control
- ▶ Natural Language Processing



Uber Self-Driving Car (Marshall, 2018 [1])

Adversarial Examples [3]

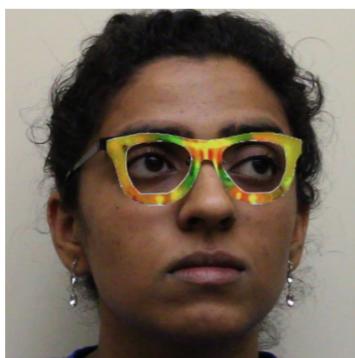
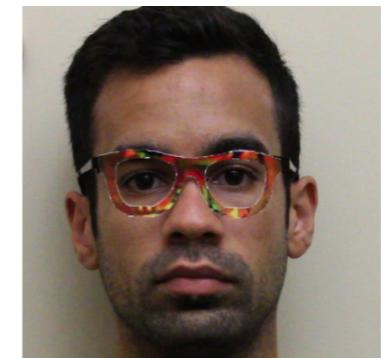
- ▶ Susceptible to errant behavior via a slight perturbation to their inputs
 - ▶ These inputs are known as *Adversarial Examples*
 - ▶ Can be tailored to cause a misclassification to a specific class



Adversarial Example discovered for self-driving car applications (Pei et al. 2018 [2])

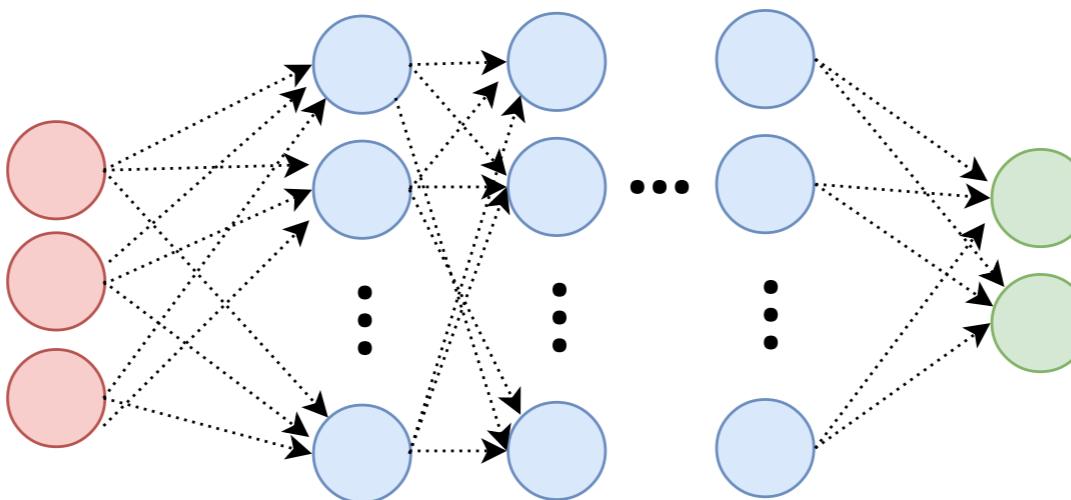
Impersonation Attacks, Shariff et al.

- ▶ Sharif et al. fooled a facial recognition system using a printed a pair of eyeglass frames
- ▶ Students were able to impersonate celebrities as well as evade being recognized



Physical impersonation attacks designed for state-of-the-art facial biometric systems. (Sharif, 2016 [4])

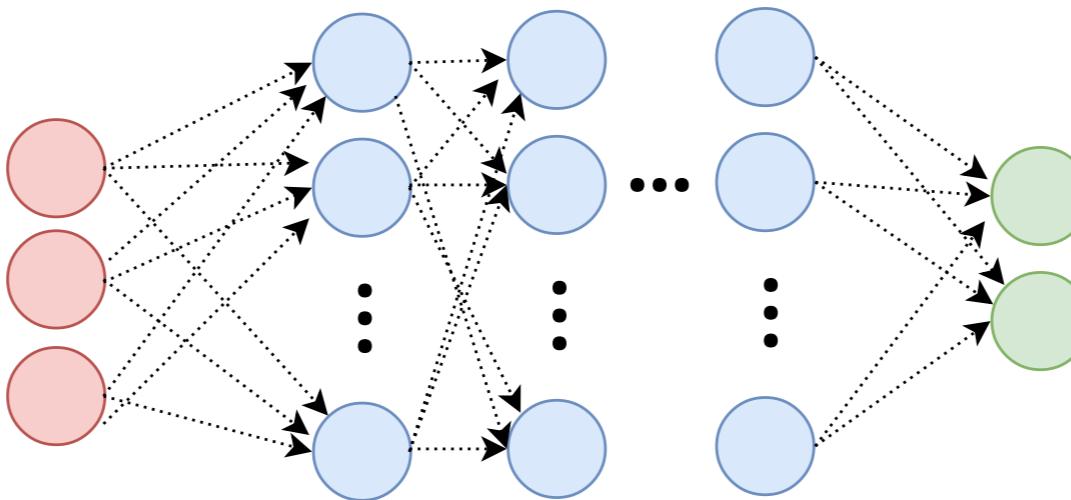
Verification Challenge



- ▶ Given a neural network, N and a property P , we wish to demonstrate:

$$N \models P$$

Verification Challenge

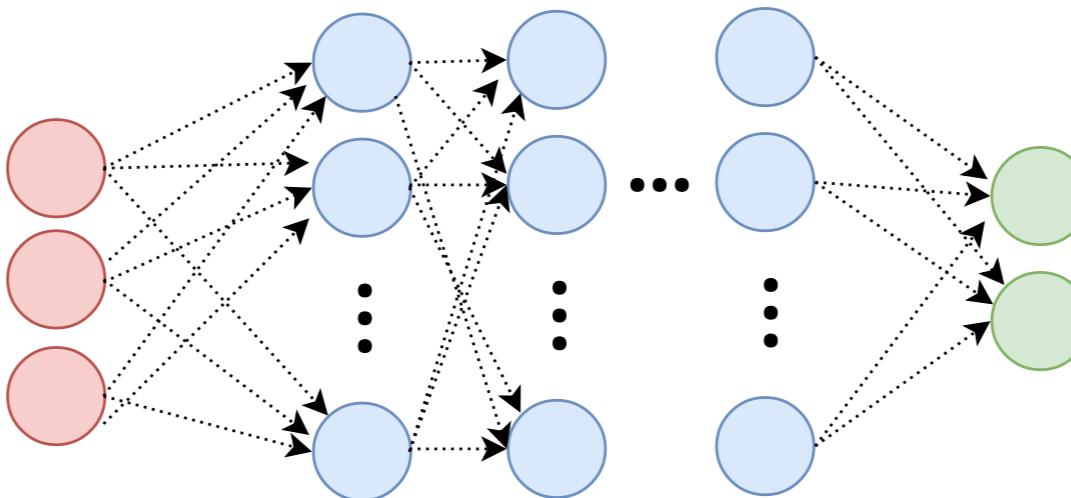


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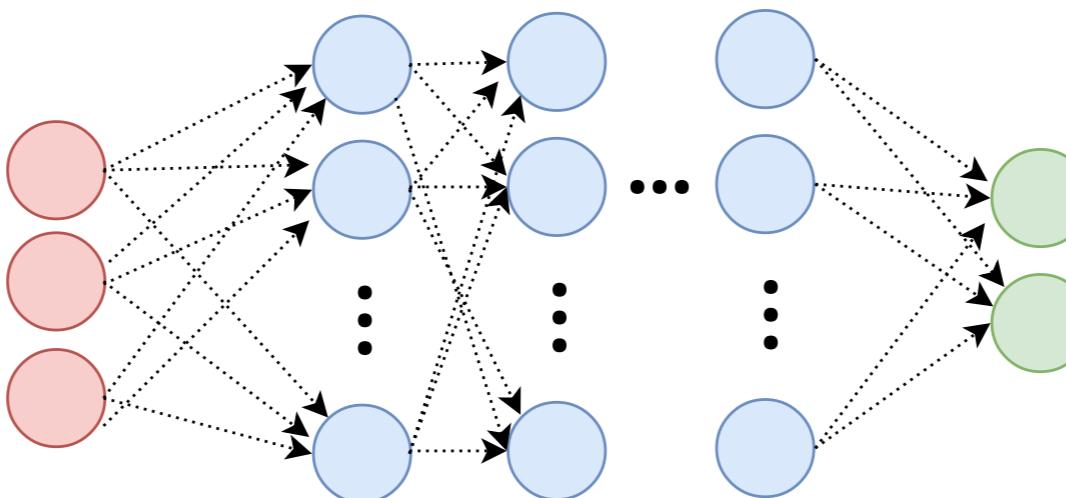


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- ▶ Notoriously difficult problem

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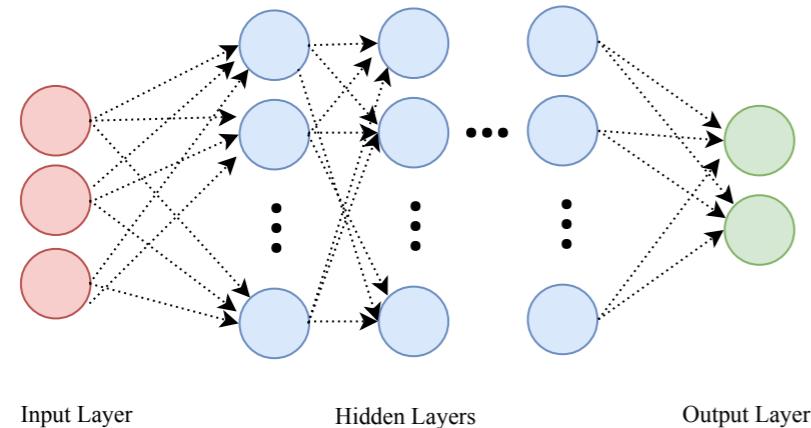


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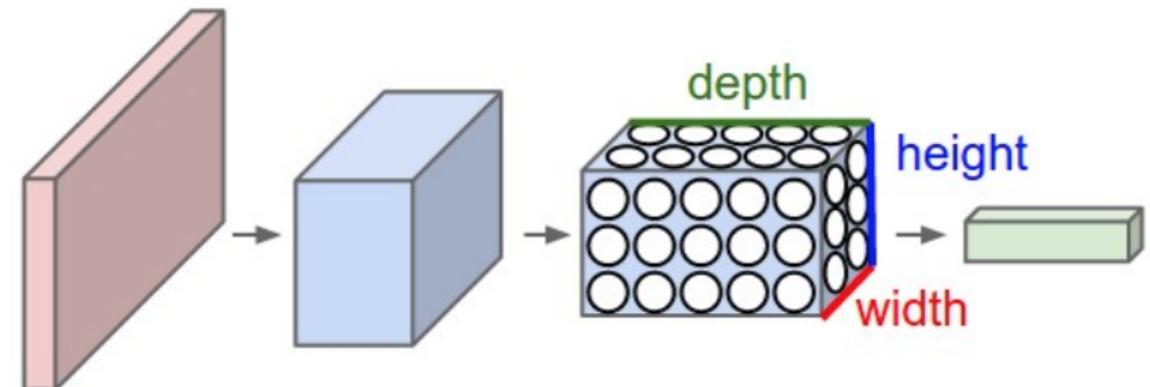
$$N \models P$$

- ▶ Otherwise demonstrate why $N \not\models P$ by generating a counterexample
- ▶ Notoriously difficult problem
 - ▶ Verifying even simple properties is NP Complete [5]

Neural Network Preliminaries



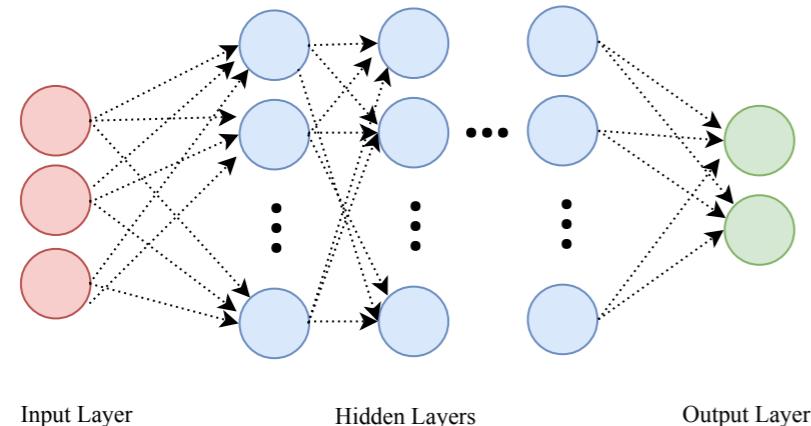
Feed-forward Architecture. (Karpathy, 2017 [6])



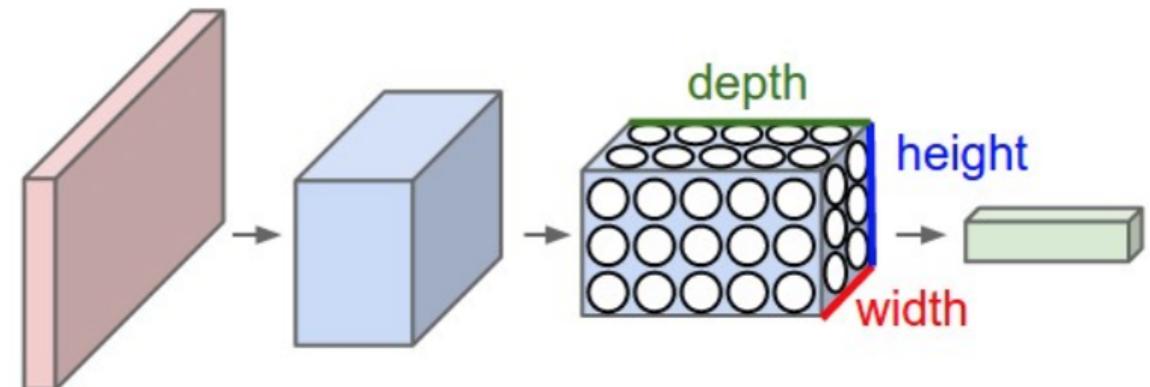
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- ▶ Organized into three types of layers

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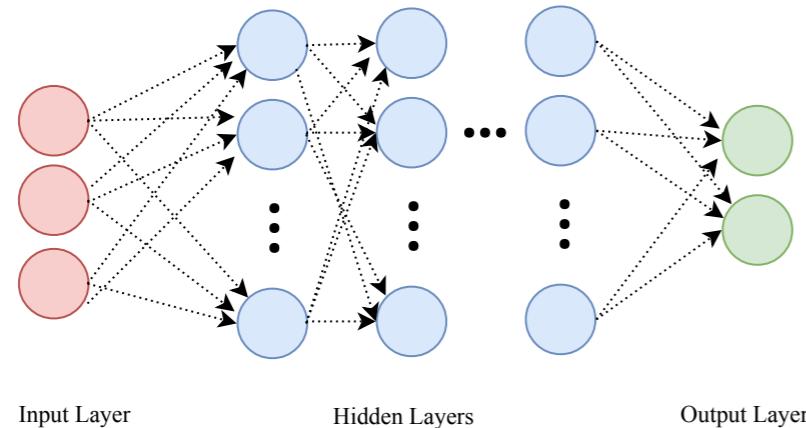
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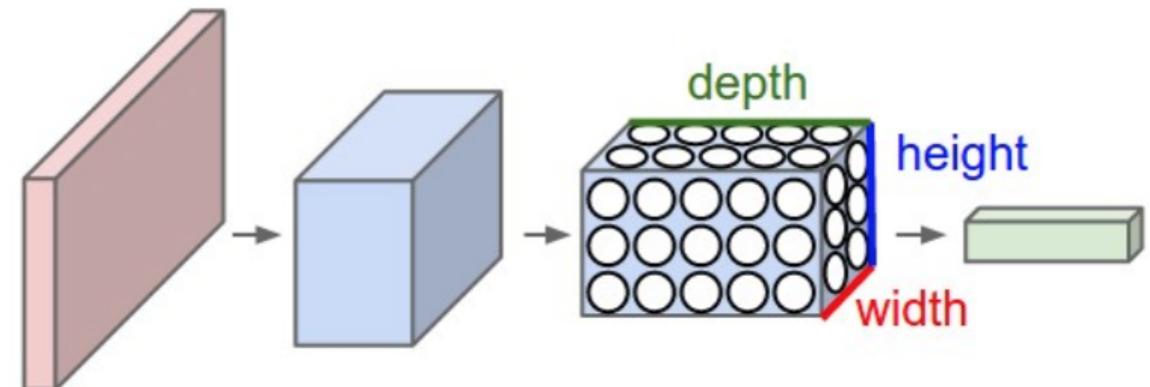
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- ▶ Variety of different architectures
 - ▶ Feed-forward, Convolutional

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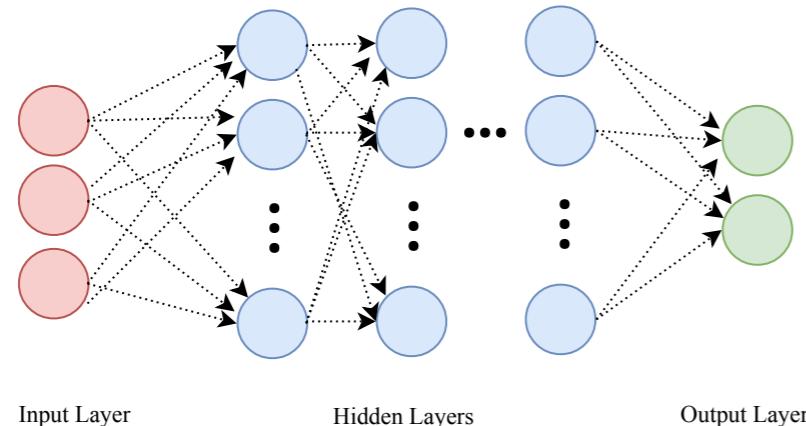
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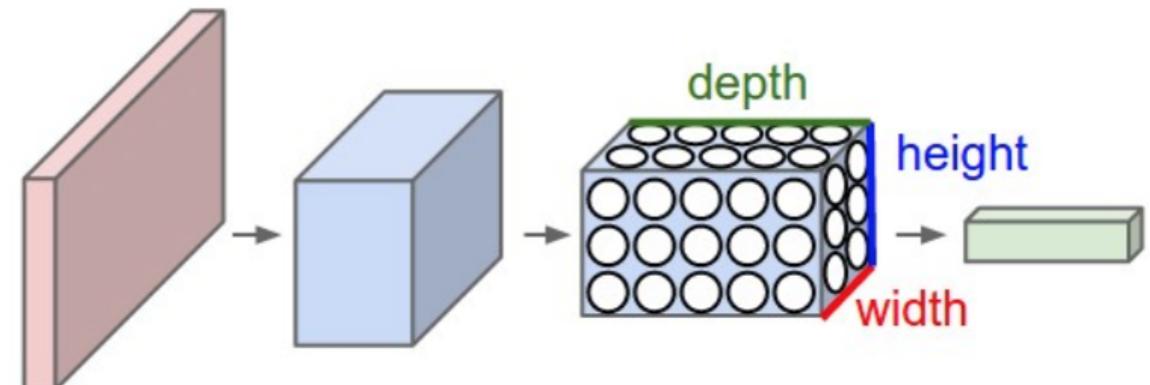
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 - ▶ weights define the network's behavior and are determined during a training phase

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- ▶ Each connection is assigned a weight
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- ▶ Trained on finite set of inputs expected to generalize

Neural Network Preliminaries

- ▶ Output of the the network is determined layer by layer

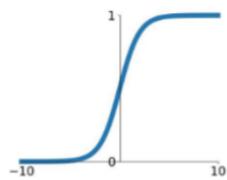
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 - ▶ Each layer's output is determined from its predecessor, using the edge weights and a non-linear activation function

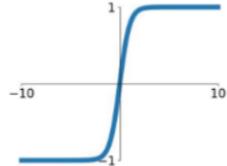
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- ▶ Activation Functions introduce non-linearities that allow neural networks to learn complex behavior

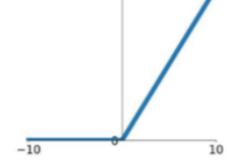
Sigmoid
 $\sigma(x) = \frac{1}{1+e^{-x}}$



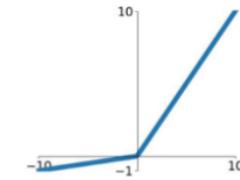
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 $\tanh(x)$



ReLU
 $\max(0, x)$

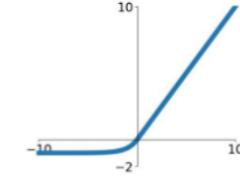


Leaky ReLU
 $\max(0.1x, x)$



Maxout
 $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$

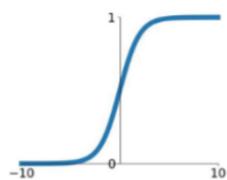


Various Activation Functions. (Jadon, 2018 [7])

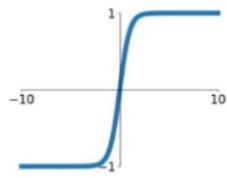
Neural Network Preliminaries

- ▶ Output of the network is determined layer by layer
 - ▶ Each layer's output is determined from its predecessor, using the edge weights and a non-linear activation function
- ▶ Activation Functions introduce non-linearities that allow neural networks to learn complex behavior
- ▶ Can be viewed as a function, $v : I^n \rightarrow O^m$, mapping an n -dimensional input, I^n , to an m -dimensional output, O^m

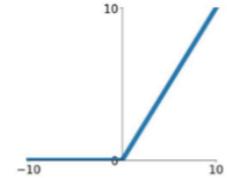
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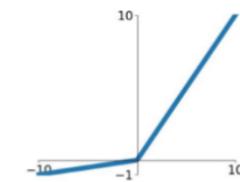
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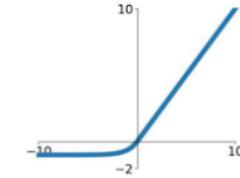


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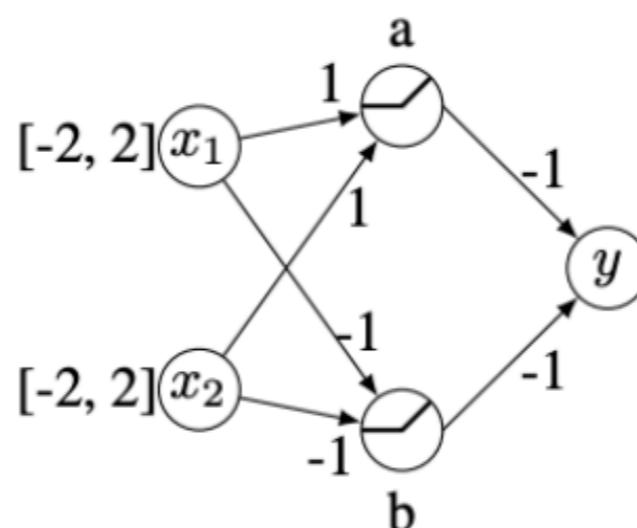
Various Activation Functions. (Jadon, 2018 [7])

Neural Network Verification [8]

- Given an input domain, C , and a predicate, P , we wish to prove:

$$\forall x_1 \in C, u_1 = v(x_1) \Rightarrow P(u_1)$$

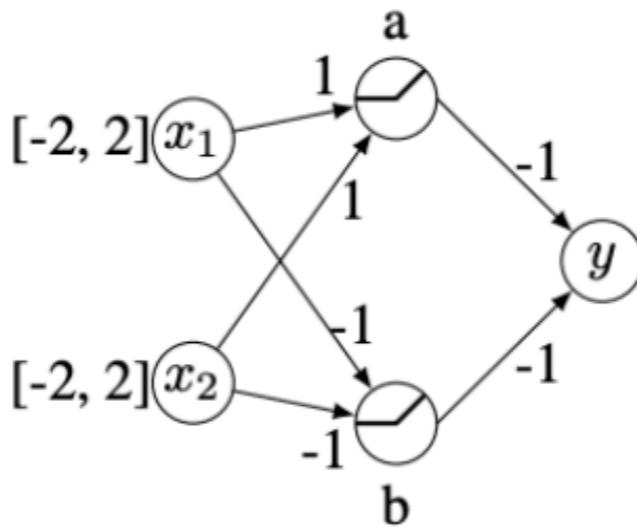
- Does there exist an example x_1 such that $\neg P(u_1)$
 - If we cannot find such an input we can conclude $N \models P$
 - Otherwise $N \not\models P$ and x_1 is a counterexample



Prove that $y > -5$

Example Neural Network Verification Problem. (Bunel, 2018 [8])

Encoding The Network [8]



Prove that $y > -5$

Example Neural Network Verification Problem. (Bunel, 2018 [8])

- ▶ Goal: Find $\mathbf{x} \in C = [-2, 2] \times [-2, 2]$ such that $y \leq -5$

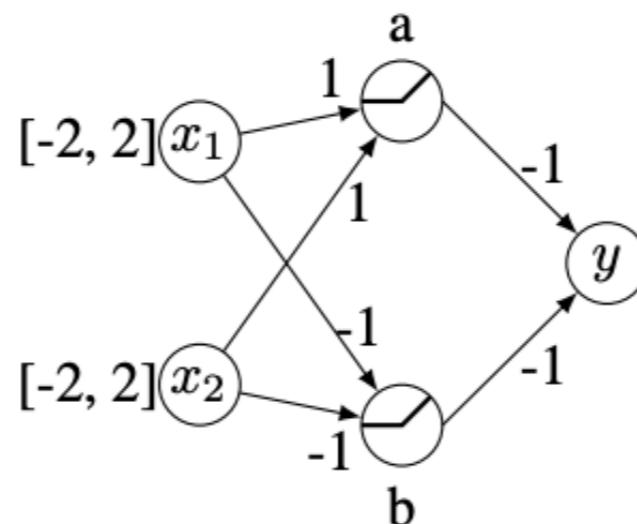
$$-2 \leq x_1 \leq 2 \quad -2 \leq x_2 \leq 2$$

$$v_1 = x_1 + x_2 \quad v_2 = -x_1 - x_2$$

$$a = \max(v_1, 0) \quad b = \max(v_2, 0)$$

$$y = -a - b$$

Decision Procedures [9]

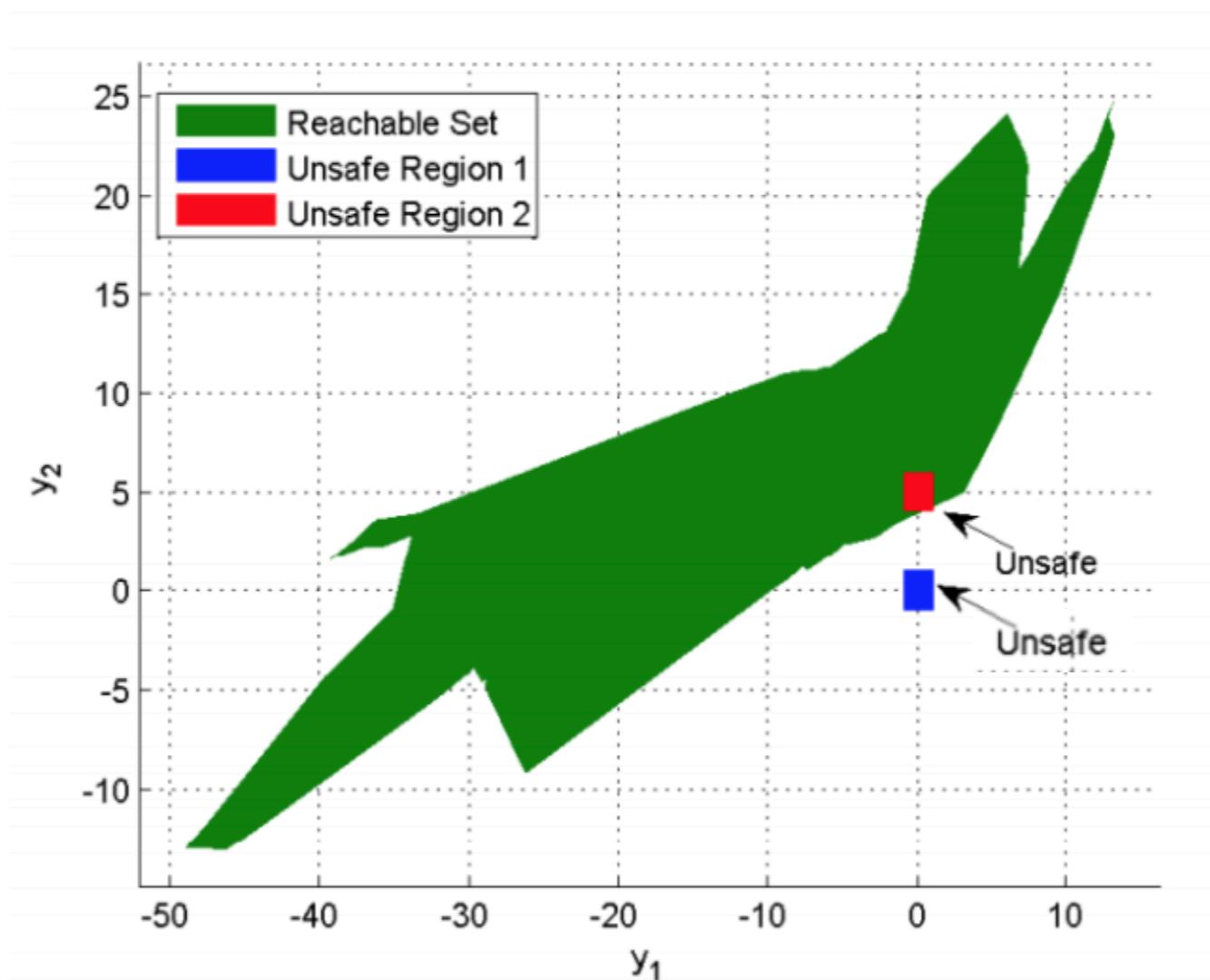


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Example Neural Network Verification Problem. (Bunel, 2018 [8])

- ▶ Satisfiability Modulo Theories (SMT)
- ▶ Mixed Integer Programming (MIP)

Reachability Analysis Methods [10]



Reachable Set Estimation. (Xiang, 2018 [10])

Leofante et al. [9]

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Leofante et al. [9]

- ▶ Categorized verification approaches into three distinct classes
 - ▶ Invariance: Global and Local
 - ▶ Invertibility
 - ▶ Equivalence
- ▶ Majority of verification approaches deal with some notion of invariance
- ▶ Very few approaches deal with network equivalence
 - ▶ Leofante et el. cite this as an interesting problem for further investigation.

Rudy Bunel et al. [8]

- ▶ Posed the verification of neural networks with piecewise-linear activation functions as a Branch and Bound Optimization problem

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 - ▶ Reluplex [5]: Did not provide Scalability Analysis

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 - ▶ can return "I don't know"
 - ▶ Work well in practice

Summary of Verification Tools

- ▶ Majority of verification methods are for FNNs with piece-wise linear activation functions
- ▶ State-of-the-art image classification networks are currently out of reach.
 - ▶ VGGNet ([12]): 1.38×10^8 parameters
 - ▶ AlexNet ([13]): 6.0×10^7 parameters
 - ▶ GoogLeNet ([14]): 4.0×10^6 parameters

Tool Name	Network Type	Verification Approach	Largest Network Considered	Completeness	Type of Property
Reluplex [5]	FNN	SMT	1800 neurons	Complete	Invariance
Planet [10]	FNN, CNN	SMT, MIP	1341 neurons	Complete	Invariance, Invertibility
PLNN [8]	FNN, CNN	Branch and Bound	1485	Complete	Invariance
Sherlock [15]	FNN	MIP	3822	Complete	Invariance
VeriDeep/DLV[16]	FNN, CNN	SMT	1.25×10^6 parameters	Incomplete	Invariance
NNAF [17]	CNN	LP	60000 parameters	Incomplete	Invariance
MIPVerify.jl [18]	FNN, CNN	MIP	500 neurons	Incomplete	Invariance, Invertibility
AI ² [19]	FNN, CNN	Abstract Interpretation	53000 neurons	Incomplete	Invariance

Summary of several available software tools for network verification present in the research literature. A more comprehensive list of tools can be found in [20].

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 - ▶ Verification specifications considered
 - ▶ Benchmark sets and tool input formats

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- ▶ Shortage of methods for correcting erroneous behavior detected in neural network models

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 - ▶ Interdisciplinary studies from domain experts within the Automated Verification and Artificial Intelligence communities
 - ▶ Development of explainable neural network models
 - ▶ Consideration of other verification strategies such as Test Coverage Methods, Run-time Monitoring

Research Direction

- ▶ Currently working on creating a translation tool for neural network models³



What is ONNX?

ONNX is a open format to represent deep learning models. With ONNX, AI developers can more easily move models between state-of-the-art tools and choose the combination that is best for them. ONNX is developed and supported by a community of partners.



Open Neural Network Exchange Format. ([22])

³<https://github.com/verivital/NeuralNetworkToolParsers>

Research Direction

- ▶ Currently working on creating a translation tool for neural network models³
- ▶ Considering Verification of
 - ▶ Recurrent Neural Networks
 - ▶ Feed-forward networks with more general activation functions



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- ▶ Numerous promising verification approaches have been proposed
 - ▶ Majority of these methods deal with feed-forward neural network with piece-wise linear activation functions
- ▶ Main challenge is scalability but there are several other open problems
- ▶ Field is growing rapidly
 - ▶ Obtaining successful methods will have a large impact in numerous application domains

Thank you!

Questions?



References

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