Solution of Exercisesheet 2

Exercise 2-1

Test

Exercise 2-2

Exercise 2-3

f and g are negligible functions and q be a positive polynomial.

(a) Is e^{-x} negligible?
For any polynomial x^c, choose $N = c$, then for all $x > N$ holds:
$e^{-x} < \frac{1}{x^c}$, because $e^x > x^c$ for all $x > N = c$.
$\Rightarrow e^{-x}$ is negligible.

(b) Is $\frac{1}{x^{2021}} + 1$ negligible?
For the polynomial x^{2022} there is no N, that for all $x > N$ holds:
$\frac{1}{x^{2021}} + 1 < \frac{1}{x^{2022}}$, because x^{2021} is always smaller than x^{2022}.
$\Rightarrow \frac{1}{x^{2021}} + 1$ is not negligible.

(c) Is $h(x)$ negligible, when $h(x)$ is a positive function such that $h(x) < f(x)$ for all x?
For $f(x)$ holds: $f(x) < \frac{1}{p(x)}$ (Definition 0.1).
Because of $h(x) < f(x) < \frac{1}{p(x)}$ for all x, $h(x)$ is also negligible.

(d) Is $f(x) + g(x)$ negligible?
$f(x)$ negligible $\Rightarrow f(x) < \frac{1}{p(x)}$ (Definition 0.1).
$g(x)$ negligible $\Rightarrow g(x) < \frac{1}{p'(x)}$ (Definition 0.1).

$$\begin{aligned}\Rightarrow f(x) + g(x) &< \frac{1}{p(x)} + \frac{1}{p'(x)} \\ &= \frac{p'(x) + p(x)}{p(x) \cdot p'(x)} \\ &= \frac{1}{\frac{p(x) \cdot p'(x)}{p'(x) + p(x)}}\end{aligned}$$

Addition, multiplication and division of two polynomials results in another polynomial. Because of that the denominator ($\frac{p(x) \cdot p'(x)}{p'(x)+p(x)}$) can also be any polynomial.
$\Rightarrow f(x) + g(x)$ is negligible.

(e)

(f)

(g)

Solution of Exercisesheet 2

(h)

(i)

Exercise 2-4

Exercise 2-5