Solution of Exercisesheet 7

Exercise 7-1

This modified encryption does not achieve CPA-security. $H(m)$ is only collision-resistant and doesn't have to hide the message m. If $H(m)$ leaks the message m and this is concatenated with $Enc(k, m)$ the resulting scheme $Enc'(k, m)$ can't be CPA-secure.

Exercise 7-2

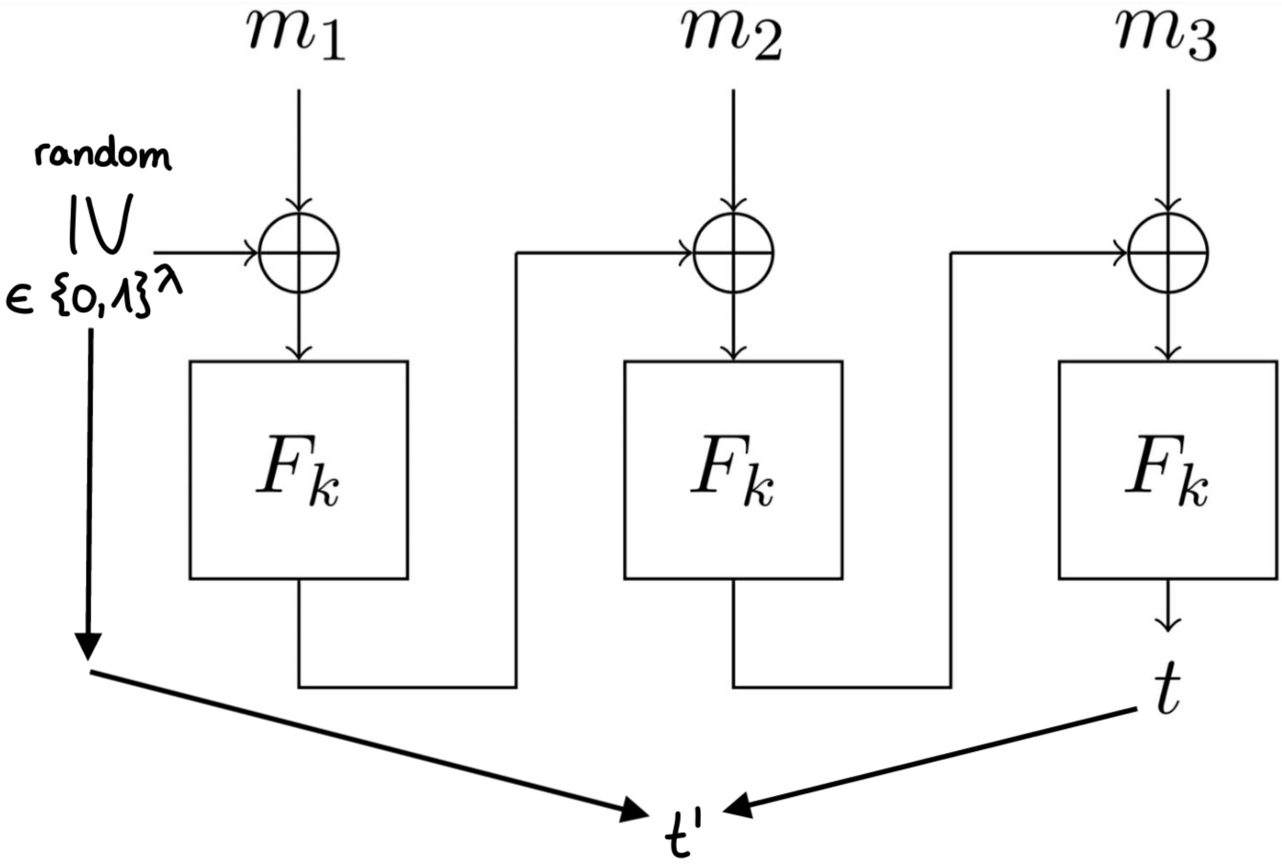
(a) not secure:
The adversary $\mathcal{A}$ makes two queries to the oracle:
$m^1 = m_1||m_2 \Rightarrow t^1 = t_1^1||t_2^1 = F(K, m_1)||F(K, F(K, m_2))$
$m^2 = F(K, m_1)||m_2 \Rightarrow t^2 = t_1^2||t_2^2 = F(K, F(K, m_1))||F(K, F(K, m_2))$
Then he knows the tag for the message $m^* = m_1||m_1$ which is $t^* = F(K, m1)||F(K, F(K, m1)) = t_1^1||t_1^2$. Because $m^* \neq m^1$ and $m^* \neq m^2$, (m^*, t^*) is a valid attack.

(b) not secure:
The adversary $\mathcal{A}$ makes one query to the oracle:
$m^1 = m_1||m_2 \Rightarrow t^1 = F(K, m_1) \oplus F(K, m_2)$
Then he knows the tag for the message $m^* = m_2||m_1$ which is $t^* = F(K, m_2) \oplus F(K, m_1) = F(K, m_1) \oplus F(K, m_2) = t^1$. Because $m^* \neq m^1$, (m^*, t^*) is a valid attack.

(c) not secure:
The adversary $\mathcal{A}$ makes one query to the oracle:
$m^1 = m_1||m_2 \Rightarrow t^1 = (r \oplus (F(K, m_1) \oplus F(K, m_2)), r)$
Then he knows the tag for the message $m^* = m_2||m_1$ which is $t^* = (r \oplus (F(K, m_2) \oplus F(K, m_1)), r) = (r \oplus (F(K, m_1) \oplus F(K, m_2)), r) = t^1$. Because $m^* \neq m^1$, (m^*, t^*) is a valid attack.

Solution of Exercisesheet 7

Exercise 7-3



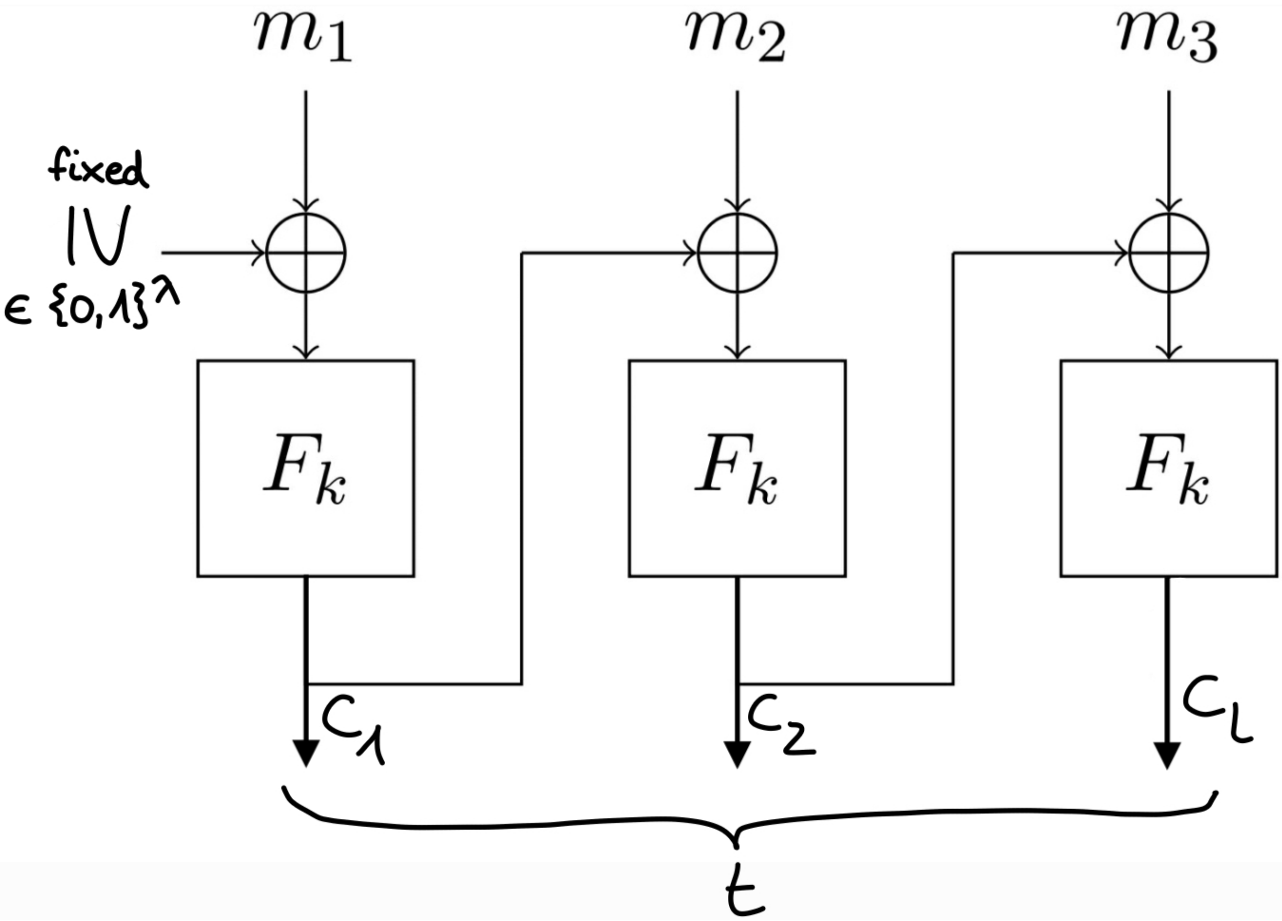
(a) The adversary $\mathcal{A}$ can choose the message m* of length $\ell(\lambda) \cdot \lambda$. We set $\ell(\lambda) = l$
$m^* = m_1^* \,||\, ... \,||\, m_l^*$. We set $m_i^* = 0^\lambda$.
We query m* to our $Mac_k(\cdot)$ oracle and recieve the tag $t^{*'} = (IV^*, t^*)$. $IV^* \in \{0,1\}^\lambda$ is a randomly generated vector

$c_1^* = F_k(IV^* \oplus m_1^*) = F_k(IV^* \oplus 0^\lambda) = F_k(IV^*)$
$c_2^* = F_k(c_1^* \oplus m_2^*) = F_k(F_k(IV^*) \oplus 0^\lambda) = F_k(\ F_k(IV^*))$
...
$c_l^* = F_k(\ ...\ F_k(IV^*)\ ...) = t^*$

To break the security of the MAC the adversary $\mathcal{A}$ can construct the following forgery:
$m = 0^{\lambda-1}1 \,||\, m_2^*) \,||\, ... \,||\, m_l^*)$ with $IV = IV^* \oplus 0^{\lambda-1}1$
meaning we flip the last bit of the first block of our message as well as the last bit of our IV*.
This causes that $IV^* \oplus m_1^* = IV \oplus m_1$ and therefore
$c_1^* = F_k(IV^* \oplus m_1^*) = F_k(IV \oplus m_1) = c_1$ as well as all other $c_i^* = c_i$ of the chain for i ℓ {2,l}.

We can conclude that $t^* = t$ but $m^* \neq m$ and therefore m was not queried to Mac_k before.
$\Rightarrow$ The adversary $\mathcal{A}$ can now create the valid forgery $t' = (IV, t)$ and can break the unforgability of the Mac.

Solution of Exercisesheet 7



(b) The adversary $\mathcal{A}$ can choose the messages m of length $\ell(\lambda) \cdot \lambda$. We set $\ell(\lambda) = l$. We choose
$m^1 = m_1^1 \,||\, ... \,||\, m_l^1$ and
$m^2 = m_1^2 \,||\, ... \,||\, m_l^2$
with $m_i^1 = m_i^2 = 0^\lambda$ for $2 \leq i \leq l$.

$c_1^i = F_k(IV \oplus m_1^i)$ with $IV^* \in \{0,1\}^\lambda$ as a fixed vector
$c_2^i = F_k(c_1^i \oplus m_2^i)$
...
$c_l^i = F_k(\,...\, F_k(IV \oplus m_1^i) \,...)$
$t = (c_1^i, ..., c_l^i)$ for $i \in \{1,2\}$

To break the security of the MAC the adversary $\mathcal{A}$ can construct the following forgery:
$m^* = m_1^2 \,||\, m_2^* \,||\, m_3^1 \,||\, ... \,||\, m_l^1$
with $m_2^* = c_1^1 \oplus m_2^1 \oplus c_1^2$ which generates a vector that satisfies the following equation:
$\Rightarrow c_1^1 \oplus m_2^1 = c_1^2 \oplus m_2^*$
This helps us construct m^* such that $c_2^1 = F_k(c_1^1 \oplus m_2^1) = F_k(c_1^2 \oplus m_2^*) = c_2^*$ and $c_i^1 = c_i^*$ for $i \in \{3,l\}$.
Therefore the adversary $\mathcal{A}$ can create the valid forgery $t = (c_1^2, c_2^1, \ldots, c_l^1)$ for m^* which has not been queried before and can break the unforgability of the Mac.

Solution of Exercisesheet 7

Exercise 7-4