

## SOLUTION OF EXERCISESHEET 5

### Exercise 5-1

Given  $\{F_k\}_{k \in K}$  is a family of bit permutations.

Adversary  $A$  can choose two inputs  $x_1$  and  $x_2$ . Then the corresponding bit permutation with key  $k$  be  $F_k(x_1)$  and  $F_k(x_2)$ .

Let's take an example,  $F_k$  as the function that swaps the first two bits of its input. For this function, the distinguisher can easily distinguish between  $F_k(x_1)$  and  $F_k(x_2)$  with a function,  $f$  that also swaps first two bits of the output. Or this can also be easily distinguished just comparing the bits other than first two bits.

Similarly, for any such bit permutation function there exists a function  $f$  which can easily distinguish the between the inputs of a bit permutation  $F_k$ . Hence these family of bit permutations cannot be a pseudorandom permutation.

### Exercise 5-2

Assume  $n = 3$

The Adversary  $\mathcal{A}$  can choose the two messages  $m_0 = 001\ 001$  and  $m_1 = 110\ 011$

The challenger encrypted one of the messages.

Since  $f$  is a pseudorandom permutation,  $f$  is deterministic and has a reverse operation.

If  $m_0$  is encrypted:

$$\text{Enc}(m_0) = \text{Enc}(m_0^0 \parallel m_0^1) = \text{Enc}(001\ 001) = f(m_0^0) \parallel f(m_0^1) = f(001) \parallel f(001) = c^0 \parallel c^1 \\ \Rightarrow c^0 = c^1$$

If  $m_1$  is encrypted:

$$\text{Enc}(m_1) = \text{Enc}(m_1^0 \parallel m_1^1) = \text{Enc}(110\ 011) = f(m_1^0) \parallel f(m_1^1) = f(110) \parallel f(011) = c^0 \parallel c^1 \\ \Rightarrow c^0 \neq c^1$$

$$\text{If } b = 0: c_b = c^0 \parallel c^1, c^0 = c^1$$

$$\text{If } b = 1: c_b = c^0 \parallel c^1, c^0 \neq c^1$$

This shows that even when the adversary can't break the encryption of the pseudorandom permutation the adversary can distinguish what message was encrypted with a probability of 100%

$\Rightarrow$  ECB mode is not EAU-secure

### Exercise 5-3

1. To show: If  $f$  is pseudorandom, then  $\Pi_{CTR}$  is CPA-secure.

Proof by contradiction. We assume there exists an adversary  $\mathcal{A}$ , which can break  $\Pi_{CTR}$  with a non-negligible probability. Then we construct the distinguisher  $\mathcal{B}$ , who can distinguish  $f$  from a truly random function and invokes  $\mathcal{A}$  as follows:

$\mathcal{B}$  has access to an oracle  $\mathcal{O}_B$  that runs either the pseudorandom permutation function  $f$  or a randomly chosen permutation function  $f^*$ .  $\mathcal{B}$  has to give  $\mathcal{A}$  access to an encryption oracle  $\mathcal{O}_{Enc}$ .

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$\mathcal{O}_{Enc}$  is realised by answering with  $Enc_k(m)$  on the input  $m$ , where  $f_k$  is replaced with the oracle  $\mathcal{O}_B$ . Thus,  $c$  looks like  $c = (IV, m \oplus s)$ , where  $s = \mathcal{O}_B(IV) || \mathcal{O}_B(IV + 1) || \dots || \mathcal{O}_B\left(IV + \left\lceil \frac{|m|}{n} \right\rceil\right)$  with the last bits truncated so  $|s| = |m|$ .

$\mathcal{A}$  then asks for the encryption of one of the two messages  $m_0$  and  $m_1$  with  $|m_0| = |m_1|$ .  $\mathcal{B}$  then samples a bit  $b \leftarrow \{0, 1\}$  and forwards  $c_b \leftarrow Enc_k(m_b)$  to  $\mathcal{A}$ , where  $Enc_k(m_b)$  is realised like in the encryption oracle.  $\mathcal{B}$  then outputs the same bit  $b'$  which  $\mathcal{A}$  outputs.

$\mathcal{B}$  is efficient, because he only forwards messages which can be done in constant time and invokes  $\mathcal{A}$  which is efficient.

To analyse the success distinguish two cases: If  $\mathcal{O}_B$  runs a pseudorandom permutation function  $f$  then  $\mathcal{B}$  perfectly simulates  $\Pi_{CTR}$  to  $\mathcal{A}$ .  $\Rightarrow Pr[\mathcal{B}^{f(\cdot)}(1^\lambda) = 1] = Pr[PrivK_{\Pi_{CTR}, \mathcal{A}}^{CPA} = 1] = \frac{1}{2} + non - negl(\lambda)$ , because  $\mathcal{A}$  is an efficient adversary against the CPA-security of  $\Pi_{CTR}$ .

If the oracle runs a randomly chosen function  $f^*$  and  $\mathcal{A}$  queries the encryption oracle at least  $q$  times we have  $Pr[\mathcal{B}^{f^*(\cdot)}(1^\lambda) = 1] = \frac{1}{2} + \frac{q(\lambda)}{2^\lambda}$ .

Now we subtract those two cases:

$$|Pr[\mathcal{B}^{f(\cdot)}(1^\lambda) = 1] - Pr[\mathcal{B}^{f^*(\cdot)}(1^\lambda) = 1]| = \left| \frac{1}{2} + non - negl(\lambda) - \frac{1}{2} - \frac{q(\lambda)}{2^\lambda} \right| = non - negl(\lambda) - \frac{q(\lambda)}{2^\lambda} = non - negl(\lambda).$$

So the distinguisher  $\mathcal{B}$  can distinguish between  $f$  and  $f^*$  with a non-negligible gap which is a contradiction to the pseudorandomness of  $f$ . Therefore such an adversary  $\mathcal{A}$  against the CPA-security of  $\Pi_{CTR}$  cannot exist.

2. To show:  $\Pi_{CTR}$  is not CCA-secure.

In the game for CCA-security the adversary  $\mathcal{A}$  has access to an encryption oracle  $\mathcal{O}_{Enc}$  and a decryption oracle  $\mathcal{O}_{Dec}$ .

$\mathcal{A}$  gives the challenger the two messages  $m_0 = 0^\lambda$  and  $m_1 = 1^\lambda$  and gets the ciphertext  $c_b = (IV, c'_b)$  back. Then  $\mathcal{A}$  asks the decryption oracle  $\mathcal{O}_{Dec}$  for the decoding of  $c_b^* = (IV, c'_{b*})$ , where  $c'_{b*}$  is  $c'_b$  with the last bit flipped. Since  $c'_b$  is not equal to  $c_b$ ,  $\mathcal{O}_{Dec}$  will answer the query. The result is then either  $0^{\lambda-1}1$  or  $1^{\lambda-1}0$ , because the only difference to computation of  $c'_b \oplus s = m$  is the last bit of  $c'_b$ . If the result is  $0^{\lambda-1}1$  the adversary returns  $b' = 0$ , if the result is  $1^{\lambda-1}0$   $b' = 1$ .

### Exercise 5-4