Solution of Exercisesheet 5

Exercise 5-1

Exercise 5-2

Exercise 5-3

1. To show: If f is pseudorandom, than Π_{CTR} is CPA-secure.
Proof by contradiction. We assume there exists an adversary $\mathcal{A}$, which can break Π_{CTR}. Then we construct the distinguisher $\mathcal{B}$, who can distinguish f from a truly random function and invokes $\mathcal{A}$ as follows:
$\mathcal{B}$ has access to an oracle $\mathcal{O}_B$ that runs either the pseudorandom permutation function f or a randomly choosen permutation function f^*. $\mathcal{B}$ has to give $\mathcal{A}$ access to an encryption oracle $\mathcal{O}_{Enc}$. $\mathcal{O}_{Enc}$ is realised by answering with $Enc_k(m)$ on the input m, where f_k is replaced with the oracle $\mathcal{O}_B$. Thus, c looks like $c = (IV, m \oplus s)$, where $s = \mathcal{O}_B(IV)||\mathcal{O}_B(IV+1)||...||\mathcal{O}_B\left(IV + \left\lceil \frac{|m|}{n} \right\rceil\right)$ with the last bits truncated so $|s| = |m|$. $\mathcal{B}$ than samples a bit $b \leftarrow \$\{0,1\}$ and forwards $c_b \leftarrow Enc(k, m_b)$ to $\mathcal{A}$. $\mathcal{B}$ then outputs the same bit b' which $\mathcal{A}$ outputs.
$\mathcal{B}$ is efficient, because he only forwards messages which can be done in constant time and invokes $\mathcal{A}$ which is efficient.
To analyse the success distuiguish two cases: If $\mathcal{O}_B$ runs a pseudorandom permutation function f then $\mathcal{B}$ perfectly simulates Π_{CTR} to $\mathcal{A}$. $\Rightarrow Pr[\mathcal{B}^{f(\cdot)}(1^\lambda) = 1] = Pr[PrivK^{CPA}_{\Pi^{CTR},\mathcal{A}} = 1] = \frac{1}{2} + non - negl(\lambda)$, because $\mathcal{A}$ is an efficient adversary against die CPA-security of Π_{CTR}
If the oracle runs a randomly choosen function f^* and $\mathcal{A}$ queries the encryption oracle at least q times we have $Pr[\mathcal{B}^{f^*(\cdot)}(1^\lambda) = 1] = \frac{1}{2} + \frac{q(\lambda)}{2^\lambda}$.
Now we subtract those two cases:
$|Pr[\mathcal{B}^{f(\cdot)}(1^\lambda) = 1] - Pr[\mathcal{B}^{f^*(\cdot)}(1^\lambda) = 1]| = \left| \frac{1}{2} + non - negl(\lambda) - \frac{1}{2} - \frac{q(\lambda)}{2^\lambda} \right| = non - negl(\lambda) - \frac{q(\lambda)}{2^\lambda} = non - negl(\lambda)$. So the distiguisher $\mathcal{B}$ can distinguish between f and f^* with a non-negligible gab which is a contradiction to the pseudorandomness of f. Therefore such an adversary $\mathcal{A}$ against the CPA-security of Π_{CTR4} cannot exist.

2. To show: Π_{CTR} is not CCA-secure.
In the game for CCA-security the adversary $\mathcal{A}$ has access to an encryption oracle $\mathcal{O}_{Enc}$ and a decryption oracle $\mathcal{O}_{Dec}$.
$\mathcal{A}$ gives the challenger two messages m_0 and m_1 with $|m_0| = |m_1|$ and gets the ciphertext $c_b = (IV, c'_b)$ back. Then $\mathcal{A}$ askes the decryption oracle $\mathcal{O}_{Dec}$ for the decoding of $c_b = (IV, c'_b)$. If the result is m_0 he returns $b' = 0$, if the result is m_1 $b' = 1$. Since $Dec_k(Enc_k(m)) = Dec_k((IV, m \oplus s)) = m \oplus s \oplus s = m$ one of this cases has to hold.

Exercise 5-4