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Solution of Exercisesheet 3

Exercise 3-1

Trying some values:

$k = 2$	$a = 27$	$b = 72$	$c = \|b - a\| = 72 - 27 = 45$	$d = 45$	$e = 54$
$k = 3$	$a = 398$	$b = 983$	$c = \|983 - 398\| = 585$	$d = 585$	$e = 558$
$k = 3$	$a = 398$	$b = 938$	$c = \|983 - 938\| = 441$	$d = 441$	$e = 144$
$k = 3$	$a = 321$	$b = 213$	$c = \|321 - 213\| = 108$	$d = 18$	$e = 81$

In the table above one can see that the sum of the digits of d, respectivly e is always a multiple of 9.

The reason for this lies in the construction of the 'pseudorandom generator'. b has the same digits like a only in an other order. To get c we subtract the greater number of those from the smaller, so c is always positive. Then we remove all 0-digits to get d and scramble the letters again for e. The sum of digits didn't change after the computation of c, so we look at c to argue that this sum is always a multiple of 9.

If $a > b$ we compute for the sum of the digits of $c = a - b$: $(a_1 - b_1) + (a_2 - b_2) + ... + (a_n - b_n) = a_1 + a_2 + ... + a_n - (b_1 + b_2 + ... + b_n)$. But this only hold if $a_n > b_n$.

If $a_i < b_i$ then it is $(a_1 - b_1) + (a_2 - b_2) + ... + (a_{i-1} - b_{i-1} - 1) + (a_i - b_i + 10) + ... + (a_n - b_n) = a_1 + a_2 + ... + a_n - (b_1 + b_2 + ... + b_n) + 9$

The sum of the digit of a has to be the same like the sum of the digits from b, because b has the same digits like a only in an other order, so $a_1 + a_2 + ... + a_n - (b_1 + b_2 + ... + b_n) = 0$.

If $a_i < b_i$ holds for y positions the sum of the digits is $9 \cdot y$. a and b have the same digits, so $a_i < b_i$ holds at least for one position, so $y \geq 1$.

This argumentation is the same for $b > a$, so the sum of the digits from c, respectively d or e, is always a multiple from 9.

If now all but the last digit of e are given one can always determine the last digit, because the sum of all digits has to be a multiple of 9. If the sum of the digits is already 9, the last bit has to be 9 as well, because all 0-digits had erased. If this is not the case, we could not determine if the last digit has to be 0 or 9.

So the described generator does not pass the next-character test.

Exercise 3-2