



Modeling Green Data-Centers and Jobs Balancing with Energy Packet Networks and Interrupted Poisson Energy Arrivals

Jean-Michel Fourneau¹

Received: 22 July 2019 / Accepted: 2 September 2019 / Published online: 12 October 2019
© Springer Nature Singapore Pte Ltd 2019

Abstract

We investigate micro data-centers systems with intermittent energy (due to solar panels or price of energy) which balance their jobs to mitigate these fluctuations. The model is based on energy packet networks recently proposed by E. Gelenbe and his colleagues. These models explicitly represent both energy and data processing in a combined stochastic process. We prove that under some technical conditions on the rates and with a suitable control of the migration rates, the leakage rates and the service rates, the steady state distribution has a product form steady-state distribution.

Keywords Energy packet network · Product form equilibrium distribution · Interrupted Poisson process · Clouds with harvesting energy

Introduction

Geographical load balancing is an appealing strategy to reduce the energy consumption or cost for data-centers (see for instance [19] or [20]). We consider micro data-centers powered by renewable energy sources like photovoltaic panels [2]. These sources of energy are intermittent and random. Hence we want to ideally adapt the data-center loads to the energy harvesting rate. Here we develop a model based on Energy Packets networks to prove the equilibrium distributions of a set of micro-grids with intermittent energy harvesting and an ideal geographical load balancing mechanism.

Energy packet networks (EPNs), recently introduced by Gelenbe and his colleagues [8–10, 13], are used to model the flow of intermittent sources of energy like batteries and solar- or wind-based generators and study their interactions with IT devices consuming energy like sensors, cpu, storage systems and networking elements.

The key idea of EPNs is to represent energy with packets of discrete units called energy packets (EPs). Each EP

models a certain number of Joules. Since the EPs are produced by an intermittent source of energy (typically solar, tides and wind), the flow of EPs is associated with some random processes. EPs are consumed by some devices after some random duration to perform requested works or can also be stored in a battery from which they can also leak after a random delay. Note that an independent approach to the EPNs has been presented in the electrical engineering literature under the name “power packet”, see [21]. In this approach, power packets consist in a pulse of current and are associated with a header and a protocol to control the routing using some hardware switching.

In one of the first EPN models [13], the authors represent the energy as EPs and the workload as data packets (DPs). Each element in the network is associated with a server queue to store the DPs and a battery (the EP queue) to keep the energy as depicted in Fig. 1. To transmit a DP between two cells, one must use one EP (this is the definition of the EP). More precisely in this paper (i.e., [13]), the EPs are sent to the DP queue and triggers the customer movement between workload queues in the network. When an EP arrives at a DP queue which is not backlogged, the energy is lost. EPN models allow to optimize the energy distribution and design networks with energy harvesting. Since then, this model has been generalized in several directions:

This article is part of the topical collection “Modelling methods in Computer Systems, Networks and Bioinformatics” guest edited by Erol Gelenbe.

✉ Jean-Michel Fourneau
jean-michel.fourneau@uvsq.fr

¹ DAVID, UVSQ, Université Paris-Saclay, Versailles, France

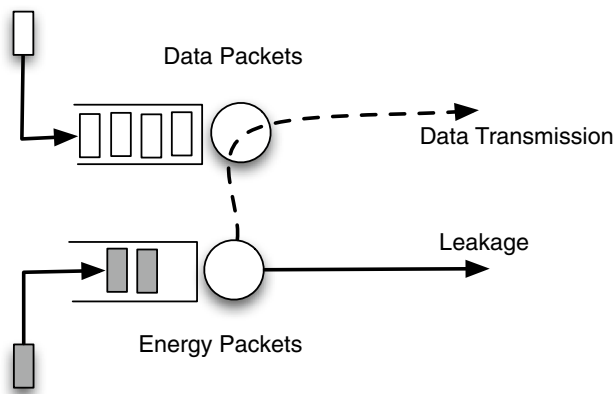


Fig. 1 The basic element of an EPN

- Quantity of energy: One EP may be not sufficient to send a DP as in [11]. In [17] the number of EP needed to transfer a DP is a constant K . This energy can even be a discrete random variable like in [15] or associated with a continuous process in [1].
- Master: in [18], the DP queue is the initiator of the transfer. The arrival of a DP at the battery triggers the dele-

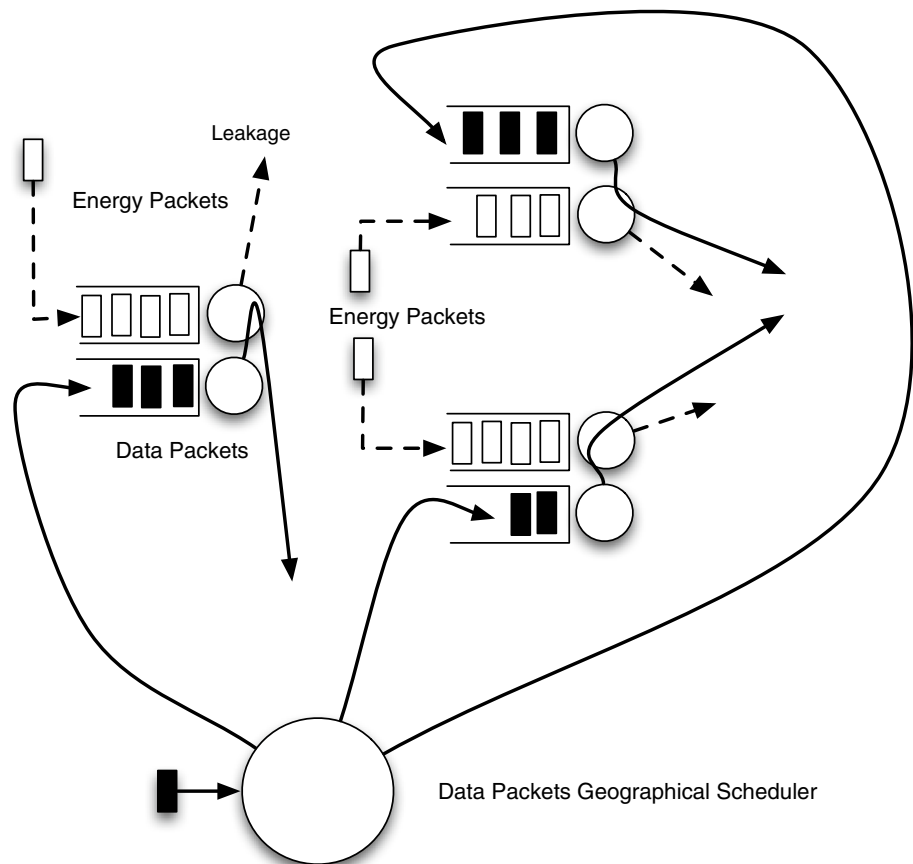
tion of a an EP and the movement of the data. If a data packet does not find an energy packet, it is lost. Thus we represent losses of DP, while the initial model (i.e., [13]) represents packets delayed due to the lack of energy.

- Classes of data packets: in [3], networks with multiple classes of DP are proved to also have a product form steady state distribution. The routing and the energy needed for a DP movement are class-dependent.

Some of these EPN models (not all of them) are G-networks of queues, introduced by the seminal papers by Gelenbe on networks of queues with positive and negative customers [6], and queues with triggers [7]. As G-networks have a product form for the steady-state distribution of jobs in the queues, they allow to optimize the system for some utility function like losses or response time [4, 12–15]. Note, however, that EPN models are not always related to G-networks but may be associated to other stochastic models (for instance, in [1] the authors use a diffusion process).

Here, we consider a network of EPN (as depicted in Fig. 2) modeling a set of micro data-centers (or instance several clouds) which are spread worldwide to insure that globally enough energy is always available. The energy parts of these systems are explicitly represented by EP queues while the DP

Fig. 2 Three data-centers with their energy harvesting parts and the scheduling of jobs



queues are the data-centers where the jobs receive service. The EP queues are used to store the energy as it is clear that energy storage greatly improves the efficiency of such systems [16]. Both EP and DP queues have an infinite capacity. However, the energy may be too expensive during some period of the day or the energy harvesting mechanisms may be interrupted during some periods of the day (for instance the night/day alternance for solar systems, the tides, the wind). Thus we assume that a control mechanism has been included into the system to balance the tasks to the cloud which is currently operating.

Each data-center may be in two modes: ON when it receives energy and OFF where the energy harvesting is impossible (or the cost of energy is too high). During their OFF state, the data-centers are in a sleep mode where the transitions are blocked. The evolution between ON state and OFF state of each data-center is governed by a global Markov chain called as a modulating phase. We show that the steady-state distribution of jobs in the queues and the energy has a product form provided that a stable solution of a fixed point problem exists. The remainder of the article is organized as follows. In the next section, we describe the model and we prove the main result of the article. In the following section an example is given before we present the conclusions.

Model and Results

Each micro-cloud is modeled by a DP queue and an associated EP queue such as depicted in Fig. 1. The data-centers are linked by the scheduler which balances the jobs between the queues. Once they have been served the jobs leave the system. Moreover the scheduler is not based on the queues description (like a Join Shortest Queue discipline). We build a multi dimensional Markov chain model, the first component of which is a modulating phase. We consider that the phase evolution is governed by a continuous time Markov chain. We further assume that this Markov chain is finite and primitive. Therefore it admits a steady-state distribution (say α_M where M is the transition rate matrix of the Markov chain). Each micro data-center is represented by the state of the DP queue (the number of jobs) and the state of the EP queue (the battery). The transition rates of these queues are phase-dependent. Of course we have:

$$\alpha_M(\phi) = \sum_{\psi} \alpha_M(\psi) M(\psi, \phi). \quad (1)$$

Let γ_i^ϕ be the arrival rate of energy packets at the battery of data-center i during phase ϕ . Similarly, λ_i^ϕ will be the rate of jobs arrival at DP queue i during phase ϕ and μ_i^ϕ will be the job service rate while ω_i^ϕ will denote the EP leakage rate.

Let us now describe the modeling assumptions. First we consider that the clusters are spread in several

locations such that the arrivals of EP are independent processes and that their global arrival rate does not depend on the phase even if each arrival rate is phase dependent:

$$\forall \phi, \quad \sum_i \gamma_i^\phi = \Gamma. \quad (2)$$

We similarly assume that the total arrival of jobs is not phase-dependent. Moreover it is a Poisson process with rate Λ :

$$\forall \phi, \quad \sum_i \lambda_i^\phi = \Lambda. \quad (3)$$

Thus the controller sends a fraction of the total load to each data center. This fraction is equal to $\frac{\lambda_i^\phi}{\Lambda}$ for DP queue i during phase ϕ .

Second, we assume that the arrivals of EP at each station follow interrupted Poisson processes. Typically, for all station index i , the set of phases \mathcal{S} is partitioned into two subsets $\mathcal{S}_i^{\text{ON}}$ and $\mathcal{S}_i^{\text{OFF}}$:

- $\mathcal{S}_i^{\text{ON}} \cup \mathcal{S}_i^{\text{OFF}} = \mathcal{S}$,
- and $\mathcal{S}_i^{\text{ON}} \cap \mathcal{S}_i^{\text{OFF}} = \emptyset$,
- and $\forall \phi \in \mathcal{S}_i^{\text{OFF}}, \quad \gamma_i^\phi = 0$,
- while $\gamma_i^\phi = \Gamma_i, \quad \forall \phi \in \mathcal{S}_i^{\text{ON}}$.

Note that this ON–OFF behavior for the arrivals of EP is a simple approximation for the daily evolution of the efficiency of solar panels.

Finally we consider the following assumptions for the scheduler, the battery and the jobs server (i.e., the DP queue).

- The services consist in the consumption of an energy packet (if available) at the EP queue. Once the EP have been consumed, a DP is instantaneously removed from the DP queue (i.e., the jobs is completed). If the DP queue is empty, the energy packet is lost. This model represents two important features of the model: first without energy, the jobs are not served, and second even if there are no jobs in the server, there is still some energy consumption.

The services are exponential with rate μ_i when the data-center is in ON mode while we have:

$$\forall \phi \in \mathcal{S}_i^{\text{OFF}}, \quad \omega_i^\phi = 0 \quad \text{and} \quad \mu_i^\phi = 0. \quad (4)$$

Similarly, the leakages follow a Poisson process with rate Ω_i when the data-center is in ON mode.

- The arrivals of jobs are only sent by the scheduler to the data-center which are in ON mode such that:

$$\forall \phi \in \mathcal{S}_i^{\text{ON}}, \quad \lambda_i^\phi = \Lambda_i \quad \text{and} \quad \forall \phi \in \mathcal{S}_i^{\text{OFF}}, \quad \lambda_i^\phi = 0. \quad (5)$$

Note that unlike [19], we do not assume that all the data centers have similar transition rates. They may have distinct rates. We only need that the sums of the rates do not depend on the phase. Let N be the number of data-centers.

Let Φ be the modulating phase, X_i the number of jobs at the DP queue of data-center i and Y_i the number of energy packets at the battery of data-center i . Let us note $\mathbf{X} = (X_1, \dots, X_N)$ and $\mathbf{Y} = (Y_1, \dots, Y_N)$. Under the assumptions about the arrivals, the leakages and the services, $(\Phi, \mathbf{X}, \mathbf{Y})_t$ is a continuous time Markov chain. In the next theorem, we prove that this Markov chain has a product form steady-state distribution if the flow equation has a solution which satisfies the stability constraints. Let us first introduce the flow equations:

$$\beta_i = \frac{\Gamma_i}{\Omega_i + \mu_i}, \quad (6)$$

and,

$$\rho_i = \frac{\Lambda_i}{\beta_i \mu_i}. \quad (7)$$

Theorem 1 Assume that Markov chain $(\Phi, \mathbf{X}, \mathbf{Y})_t$ is ergodic. If the following flow Eqs. 1, 6 and 7 have a fixed point solution (ρ_i, β_i) such that $\rho_i < 1$ and $\beta_i < 1$ for all i , then the steady-state distribution is:

$$\pi(\phi, \mathbf{X}, \mathbf{Y}) = \alpha_M(\phi) \prod_{i=1}^N (1 - \rho_i)(1 - \beta_i)(\rho_i)^{X_i}(\beta_i)^{Y_i} \quad (8)$$

The proof is based on the analysis of the global balance equation. First let us introduce some notations. As usual \mathbf{e}_i will denote a vector whose components are all 0 except component with index i which is 1. Moreover 1_C is the step function equal to 1 when condition C is true and 0 otherwise. Let us now write the Chaman Kolmogorov equation at steady-state:

$$\begin{aligned} \pi(\phi, \mathbf{X}, \mathbf{Y}) & \left[\sum_{\psi} M(\phi, \psi) + \sum_i \lambda_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} + \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \right. \\ & \left. + \sum_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} + \sum_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} \right] \\ & = \sum_{\psi} M(\psi, \phi) \pi(\psi, \mathbf{X}, \mathbf{Y}) \\ & + \sum_i \lambda_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \pi(\phi, \mathbf{X} - \mathbf{e}_i, \mathbf{Y}) 1_{X_i > 0} \\ & + \sum_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \pi(\phi, \mathbf{X}, \mathbf{Y} + \mathbf{e}_i) \\ & + \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \pi(\phi, \mathbf{X}, \mathbf{Y} - \mathbf{e}_i) 1_{Y_i > 0} \\ & + \sum_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \pi(\phi, \mathbf{X} + \mathbf{e}_i, \mathbf{Y} + \mathbf{e}_i) \\ & + \sum_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \pi(\phi, \mathbf{X}, \mathbf{Y} + \mathbf{e}_i) 1_{X_i = 0} \end{aligned} \quad (9)$$

Before proceeding with the algebraic manipulation of terms, let us describe the right hand side of the equation. The first term describes the evolution of the modulation phase. The

second and the fourth terms describe, respectively, the arrival of a job and the arrival of an energy packet when the data-center is in an ON phase. Similarly, the third term models the energy leakage which only happens when the battery is not in sleep mode (i.e., associated with the OFF state). Terms 5 and 6 represent the service of a job associated to the consumption of an EP (Term 5) and the consumption of an EP while the DP queue is empty (Term 6).

We divide both sides of the equation by $\pi(\phi, \mathbf{X}, \mathbf{Y})$ and take into account the multiplicative form of the solution. After cancellation of terms, we obtain for all phase ϕ :

$$\begin{aligned} \sum_{\psi} M(\phi, \psi) + \sum_i \lambda_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} + \sum_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} \\ + \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} + \sum_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} \\ = \sum_{\psi} M(\psi, \phi) \frac{\alpha_M(\psi)}{\alpha_M(\phi)} \\ + \sum_i \lambda_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \frac{1}{\rho_i} 1_{X_i > 0} \\ + \sum_i \beta_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \\ + \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \frac{1}{\beta_i} 1_{Y_i > 0} \\ + \sum_i \rho_i \beta_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \\ + \sum_i \beta_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{X_i = 0} \end{aligned}$$

First due to the definition of the distribution α_M the first term of the left hand side cancel with the first term of the right hand side. Moreover, in the last term of the r.h.s, we substitute $1_{X_i = 0}$ with $1 - 1_{X_i > 0}$ and we move the negative term in the l.h.s. to get:

$$\begin{aligned} \sum_i \lambda_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} + \sum_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} + \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \\ + \sum_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} + \sum_i \beta_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{X_i > 0} \\ = \sum_i \lambda_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \frac{1}{\rho_i} 1_{X_i > 0} \\ + \sum_i \beta_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \\ + \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \frac{1}{\beta_i} 1_{Y_i > 0} \\ + \sum_i \rho_i \beta_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \\ + \sum_i \beta_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \end{aligned}$$

This equation is decomposed into three equations based on the step functions they exhibit.

$$\begin{aligned} \sum_i \omega_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} + \sum_i \mu_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} 1_{Y_i > 0} \\ = \sum_i \gamma_i^{\phi} 1_{\phi \in S_i^{\text{ON}}} \frac{1}{\beta_i} 1_{Y_i > 0}, \end{aligned} \quad (10)$$

and

$$\sum_i \beta_i \mu_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} 1_{X_i > 0} = \sum_i \lambda_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} \frac{1}{\rho_i} 1_{X_i > 0}, \quad (11)$$

and finally,

$$\begin{aligned} \sum_i \lambda_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} + \sum_i \gamma_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} &= \sum_i \beta_i \omega_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} \\ &+ \sum_i \rho_i \beta_i \mu_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} \\ &+ \sum_i \beta_i \mu_i^\phi 1_{\phi \in \mathcal{S}_i^{\text{ON}}} \end{aligned} \quad (12)$$

First note that when the step function is 0 (i.e., the condition $\phi \in \mathcal{S}_i^{\text{ON}}$ does not hold) then all these equations are satisfied because we get $0 = 0$ which is clearly true. Thus we only have to check these equations when the condition $\phi \in \mathcal{S}_i^{\text{ON}}$ holds. Let us consider Eq. 10. We know that for all ϕ in $\mathcal{S}_i^{\text{ON}}$ we have by assumptions of the model $\gamma_i^\phi = \Gamma_i$, $\omega_i^\phi = \Omega_i$ and $\mu_i^\phi = \mu_i$. Thus after factorisation we get:

$$\sum_i 1_{Y_i > 0} 1_{\phi \in \mathcal{S}_i^{\text{ON}}} [\Omega_i + \mu_i - \frac{\Gamma_i}{\beta_i}] = 0,$$

which is obviously true due to the flow equation on energy (i.e., Eq. 6).

Let us now consider Eq. 11 (also when condition $\phi \in \mathcal{S}_i^{\text{ON}}$ holds). By assumptions, $\lambda_i^\phi = \Lambda_i$ when ϕ is in $\mathcal{S}_i^{\text{ON}}$. After factorization we obtain:

$$\sum_i 1_{\phi \in \mathcal{S}_i^{\text{ON}}} 1_{X_i > 0} \left[\beta_i \mu_i - \frac{\Lambda_i}{\rho_i} \right] = 0.$$

Again this is a simple consequence of the flow equation on jobs (i.e., Eq. 7).

Finally, we have to study Eq. 12 taking into account the same simplifications we already done for the two former equations. We get:

$$\sum_i 1_{\phi \in \mathcal{S}_i^{\text{ON}}} [\Lambda_i + \Gamma_i - \beta_i \Omega_i - \rho_i \beta_i \mu_i - \beta_i \mu_i] = 0$$

Due to Eq. 6 we have $\beta_i \Omega_i + \rho_i \beta_i \mu_i = \Gamma_i$ while due to Eq. 7 the following relation holds: $\Lambda_i = \rho_i \beta_i \mu_i$. Hence Eq. 12 clearly holds. To finish the proof, we just have to verify that the steady-state probabilities sum up to 1. This is clearly the case and the proof is complete. \square

Note that solving the flow equations is a rather simple task. We do not need the complex algorithm presented in [5]. It is only needed for all i to compute β_i with Eq. 6 and then compute ρ_i with Eq. 7 once β_i is known. Thus checking the stability is an easy task for that model.

Table 1 ON–OFF periods for six data-centers

	1	2	3	4
A	ON	ON	OFF	OFF
B	OFF	ON	ON	OFF
C	OFF	OFF	ON	ON
D	ON	OFF	OFF	ON
E	OFF	OFF	ON	ON
F	OFF	OFF	ON	ON

Table 2 Routing probability during each phase for each data-center

	1	2	3	4
A	1/2	1/2	0	0
B	0	1/2	1/2	0
C	0	0	1/4	1/4
D	1/2	0	0	1/2
E	0	0	1/8	1/8
F	0	0	1/8	1/8

A Simple Example

We consider a system with six data-centers (say A, B, C, D, E and F and four phases noted from 1 to 4. The ON–OFF periods are given in Table 1. We also assume that $\Lambda_A = \Lambda_B = \Lambda_D = 1$, $\Lambda_C = 1/2$ and $\Lambda_E = \Lambda_F = 1/4$. The routing probabilities for the scheduler during each phase are given in Table 2.

Therefore one can check easily that $\sum_i \lambda_i^\phi = 1.5$ for all phase ϕ from 1 to 4. We must also check the equality for the sum of rates for the energy harvesting. From Table 1, one readily found that we must have:

$$\begin{aligned} \Gamma_A + \Gamma_D &= \Gamma_A + \Gamma_B = \Gamma_B + \Gamma_C + \Gamma_E + \Gamma_F \\ &= \Gamma_D + \Gamma_C + \Gamma_E + \Gamma_F \end{aligned}$$

which clearly holds if the following constraints are satisfied:

$$\Gamma_D = \Gamma_B \quad \text{and} \quad \Gamma_A = \Gamma_C + \Gamma_E + \Gamma_F$$

In this example, we assume that $\Gamma_D = 3 = \Gamma_B$, $\Gamma_C = \Gamma_E = \Gamma_F = 2$ and $\Gamma_A = 6$.

The service rates and the leakage rates are let as parameters. Thus we have:

$$\begin{cases} \beta_A = \frac{6}{\Omega_A + \mu_A} \\ \beta_B = \frac{3}{\Omega_B + \mu_B} \\ \beta_C = \frac{2}{\Omega_C + \mu_C} \\ \beta_D = \frac{3}{\Omega_D + \mu_D} \\ \beta_E = \frac{2}{\Omega_E + \mu_E} \\ \beta_F = \frac{2}{\Omega_F + \mu_F} \end{cases},$$

and,

$$\begin{cases} \rho_A = \frac{\Omega_A + \mu_A}{6\mu_A} \\ \rho_B = \frac{\Omega_B + \mu_B}{3\mu_B} \\ \rho_C = \frac{\Omega_C + \mu_C}{4\mu_C} \\ \rho_D = \frac{\Omega_D + \mu_D}{3\mu_D} \\ \rho_E = \frac{\Omega_E + \mu_E}{8\mu_E} \\ \rho_F = \frac{\Omega_F + \mu_F}{8\mu_F} \end{cases}$$

Furthermore we assume that $M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Thus the steady-state distribution for matrix M is $\alpha = (1/4, 1/4, 1/4, 1/4)$.

Concluding Remarks

To the best of our knowledge, this paper is the first attempt to have a non constant arrival rate of energy packets in an EPN model. Note that it is straightforward to modify the proof to consider a single data-center with energy harvesting where the center stops (or go to sleep mode) when the energy rate goes to zero. We hope that this result will open new research to deal with time varying energy arrivals and load balancing of jobs.

References

1. Abdelrahman OH, Gelenbe E. A diffusion model for energy harvesting sensor nodes. In: 24th IEEE international symposium on modeling, analysis and simulation of computer and telecommunication systems, MASCOTS 2016, London. IEEE Computer Society; 2016. p. 154–58.
2. Bianchini R. Leveraging renewable energy in data centers: present and future. In: The 21st international symposium on high-performance parallel and distributed computing, HPDC'12, Delft. ACM; 2012. p. 135–136.
3. Doncel J, Fourneau JM. Energy packet networks with multiple energy packet requirements. *Probab Eng Inf Sci*. 2019;. <https://doi.org/10.1017/S0269964819000226>.
4. Fourneau J, Marin A, Balsamo S. Modeling energy packets networks in the presence of failures. In: 24th IEEE international symposium on modeling, analysis and simulation of computer and telecommunication systems, MASCOTS 2016, London. 2016. p. 144–53.
5. Fourneau JM, Quessette F. Computing the steady-state distribution of G-networks with synchronized partial flushing. In: Levi A, Savas E, Yenigün H, Balcisoy S, Saygin Y, editors. 21st international symposium on computer and information sciences—ISCIS 2006, Istanbul, Lecture notes in computer science, vol 4263. Berlin, Heidelberg: Springer; 2006. p. 887–96.
6. Gelenbe E. Product-form queuing networks with negative and positive customers. *J Appl Probab*. 1991;28:656–63.
7. Gelenbe E. G-networks with instantaneous customer movement. *J Appl Probab*. 1993;30(3):742–8.
8. Gelenbe E. Energy packet networks: Ict based energy allocation and storage (invited paper). In: *GreenNets*. 2011. p. 186–95.
9. Gelenbe E. Energy packet networks: smart electricity storage to meet surges in demand. In: International ICST conference on simulation tools and techniques, SIMUTOOLS '12, Sirmione-Desenzano, March 19–23, 2012. Brussels: ICST; 2012. p. 1–7.
10. Gelenbe E. A sensor node with energy harvesting. *SIGMETRICS Perform Eval Rev*. 2014;42(2):37–9.
11. Gelenbe E. Synchronising energy harvesting and data packets in a wireless sensor. *Energies*. 2015;8(1):356–69.
12. Gelenbe E, Abdelrahman OH. An energy packet network model for mobile networks with energy harvesting. *Nonlinear Theory Appl IEICE*. 2018;9(3):322–36.
13. Gelenbe E, Ceran ET. Central or distributed energy storage for processors with energy harvesting. In: 2015 Sustainable internet and ICT for sustainability (SustainIT). Madrid: IEEE; 2015. p. 1–3.
14. Gelenbe E, Ceran ET. Energy packet networks with energy harvesting. *IEEE Access*. 2016;4:1321–31.
15. Gelenbe E, Zhang Y. Performance optimization with energy packets. *IEEE Syst J*. 2019. <https://doi.org/10.1109/JSYST.2019.2912013>
16. Guo Y, Ding Z, Fang Y, Wu D. Cutting down electricity cost in internet data centers by using energy storage. In: Proceedings of the global communications conference, GLOBECOM 2011, 5–9 December 2011, Houston. Kathmandu: IEEE; 2011. p. 1–5.
17. Kadioglu YM, Gelenbe E. Packet transmission with K energy packets in an energy harvesting sensor. In: 2nd ACM international workshop on energy-aware simulation. 2016. p. 1–6.
18. Kadioglu YM, Gelenbe E. Product-form solution for cascade networks with intermittent energy. *IEEE Syst J*. 2019;13(1):918–27.
19. Neglia G, Sereno M, Bianchi G. Geographical load balancing across green datacenters: a mean field analysis. *SIGMETRICS Perform Eval Rev*. 2016;44(2):64–9.
20. Rahman A, Liu X, Kong F. A survey on geographic load balancing based data center power management in the smart grid environment. *IEEE Commun Surv Tutor*. 2014;16(1):214–33.
21. Takahashi R, Takuno T, Hikihara T. Estimation of power packet transfer properties on indoor power line channel. *Energies*. 2012;5(7):2141.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.