The length of the day

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1 Introduction

The topic of my research is the length of the day. The length of the day is the time interval between the sunrise and the sunset, during which at least part of the solar disk is above the horizon. What does this value depend on? Is it possible to predict it? In my research I will answer these questions.

2 Aims for exploration

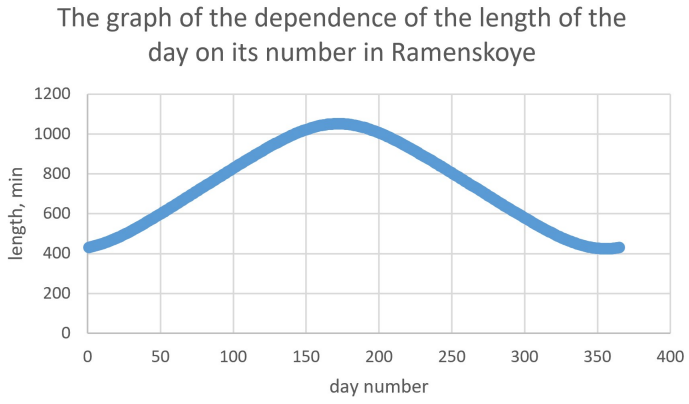
1. Investigate the dependence of the length of the day on its sequence number in the year.
2. Create a simple function, when substituting a sequence number of the day in the year, you can get a fairly close to the reality length of the day.

Here is the plan to achieve the aims:

1. Collect and analyze data in Ramenskoye.
2. Set a hypothesis about the type of the dependence.
3. Prove the hypothesis using spherical coordinates.
4. Approximate the dependence.

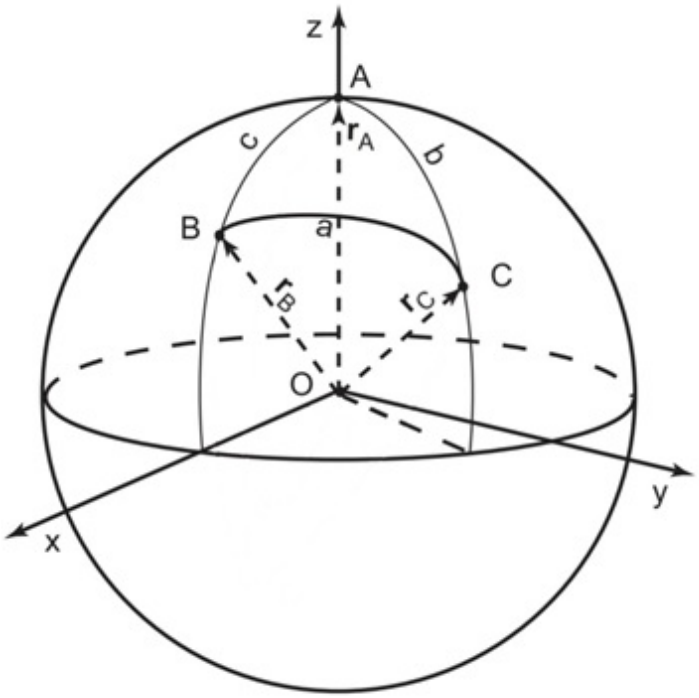
3 Data from Ramenskoye

The first thing I had to do in my research was collect data on the length of the day in my hometown using the weather archive [1]. The data table is presented in the appendix at the end of the article. Using Excel, I reflected on the graph the dependence of the length of the day on its sequence number in the year. Based on the graph, I have set a hypothesis: the function is a cosine.



4 Proof

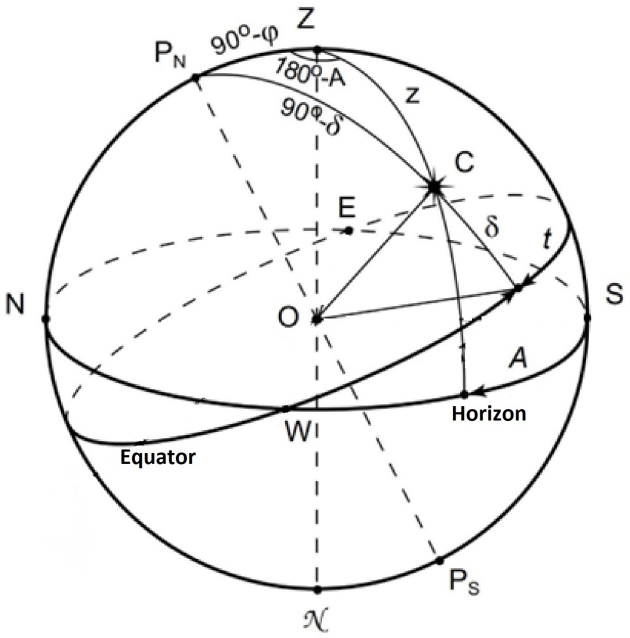
Let r_a, r_b, and r_c be the unit vectors from the center of the sphere to those corners of the triangle, so $r_a = (0,\ 0,\ 1)$, $r_b = (\sin c, 0, \cos c)$, $r_c = (\sin b \cos A, \sin b \sin A, \cos b)$.



So we have $r_a \cdot r_a = 1$, $r_b \cdot r_c = \cos a$, $r_a \cdot r_b = \cos c$, and $r_a \cdot r_c = \cos b$.
The cross product r_a x r_b is
$\left| \ r_a \text{ x } r_b \ \right| = \sqrt{(0-0)^2 + (0-0)^2 + (\sin c - 0)^2} = \sin c$
Similarly, $\left| \ r_a \text{ x } r_c \ \right| = \sin b$.
The angle between $\left| \ r_a \text{ x } r_b \ \right|$ and $\left| \ r_a \text{ x } r_c \ \right|$ is A, so
$\sin a \sin b \cos C = (r_a \times r_b) \cdot (r_a \times r_c) = (r_a \cdot r_a)(r_b \cdot r_c)(r_a \cdot r_b)(r_a \cdot r_c) = \cos c \cos a \cos b$

Hence,
$\cos c = \cos a \cos b + \sin a \sin b \cos C$

Let our planet be the sphere. With points P_n - north pole; Z - zenith of the sun; O - center of the planet, C - projection of the sun to the surface of the sphere.



Let ϕ be the angle between axis of rotation and the horizon plane, and δ be the geographical latitude.
Let t be the angle between the plane that contains C, P_n, and O and the plane that contains P_n, Z and O.
In the triangle $Pn - Z - C$, according to the cosine theory
$\cos z = \cos 90°\delta \cos 90°\phi + \sin 90°\delta \sin 90°\phi$
$\cos z = \sin\delta \sin\phi + \cos\delta \cos\phi \cos t$
At the moments of dusk and dawn $z = 90° =>$
$0 = \sin\delta \sin\phi + \cos\delta \cos\phi \cos t$
$-cos\delta \cos\phi \cos t = \sin\delta \sin\phi$
$\cos t = -\tan\delta \tan\phi$
It is obvious that Φ depends on the position of the earth relative to the sun and it changes during the year.

Knowing t and the angle $\angle P_nOZ$ we can calculate the length of the day as we can calculate the length of the arc z and as we know that speed of rotation of the earth is constant.

That is why our function is a part of a cosine function.

5 Approximation

I have proved that the dependence is similar to a cosine function. So, its formula is $y = a \times \cos kx + b + c$.
It is known from the data that the maximum of the function is at the point (172.5, 1051), and the minimum is (356, 423).

Hence $c = \frac{1051+423}{2} = 737$;

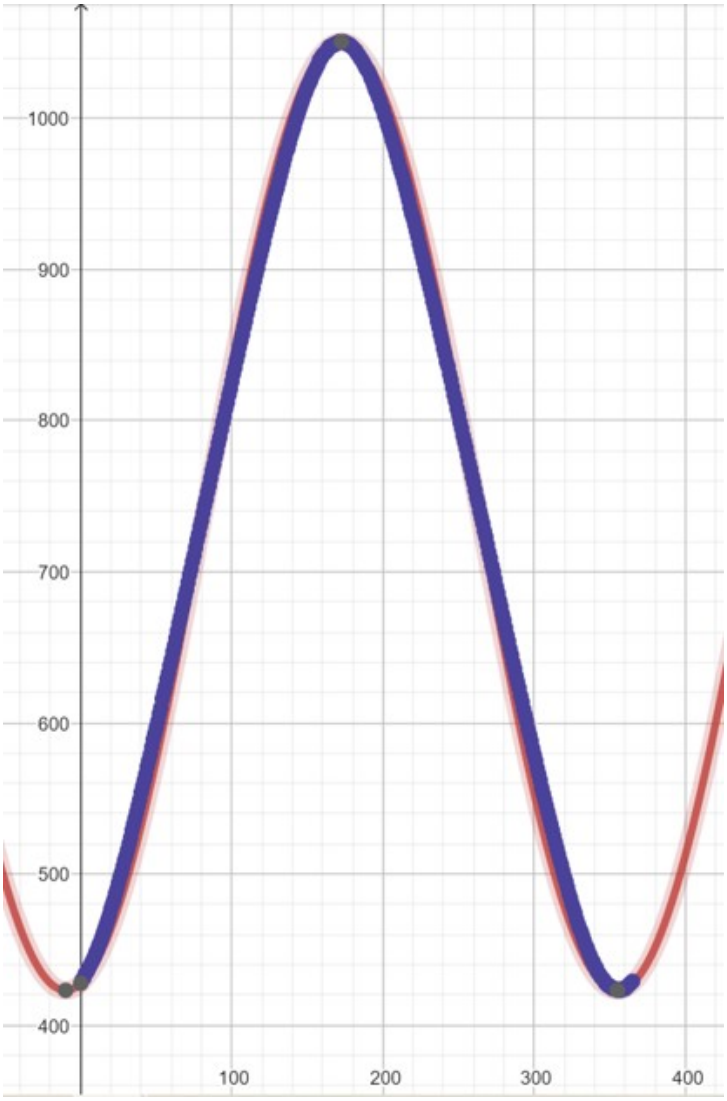
$a = 1051–c = 314$;

$k = \frac{2\pi}{365}$;

$b = -172.5k = -\frac{69\pi}{73}$.

Thus, we got the function $y = 314\cos\frac{2\pi\times x}{365} - \frac{69\pi}{73} + 737$

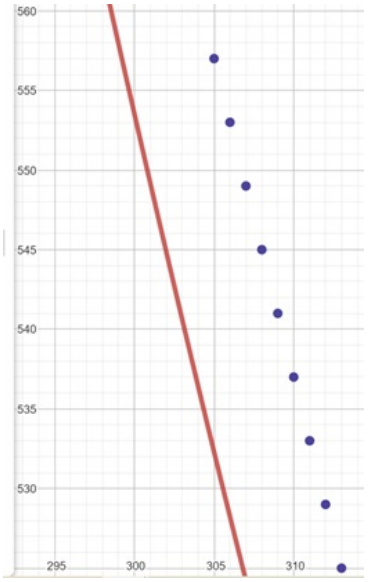
And below is combined chart of actual data for Ramenskoye (blue) and the function described above (red).



6 Reflection

Analysing done job we can see the following problems:

1. This function does not work properly for leap years
2. There is an error less than 30 minutes due to the imperfect shape of our planet and orbit (see the area with the maximum error on the picture below)



Hence I can suggest following possible enhancements and areas for further research:

1. Improve the error rate
2. Create a similar function for leap years
3. Investigate the influence of the latitude

To sum up, I have investigated length of the day in my hometown, understood what function can be used to approximate length of the day based on its number and hence all aims of the research were successfully completed in time.

7 Resources

1. https://arhivpogodi.ru/arhiv/ramenskoye/2023/10
2. Zharov V. E. Sfericheskaya astronomia. 4.2. Voskhod i zakhod nebesnykh tel. 2006
3. geogebra.org and Excel

8 Appendix

Table 1: Data for Ramenskoye

Begin of Table	
day number	duration of the day
1	430
2	431
3	434
4	435
5	437
6	439
7	441
8	443
9	445
10	448
11	450
12	453
13	455

Continuation of Table 1	
day number	duration of the day
14	458
15	461
16	464
17	467
18	470
19	473
20	477
21	480
22	483
23	487
24	490
25	494
26	498
27	501
28	505
29	509
30	512
31	516
32	520
33	524
34	529
35	533
36	537
37	541
38	545
39	549
40	554
41	558
42	562
43	566
44	571
45	575
46	579
47	584
48	589
49	593
50	597
51	601
52	606
53	610
54	616
55	620
56	624
57	629
58	633
59	638
60	642
61	647
62	651
63	656
64	661
65	666
66	670
67	675
68	679

Continuation of Table 1	
day number	duration of the day
69	684
70	688
71	693
72	698
73	702
74	707
75	711
76	716
77	721
78	725
79	730
80	734
81	739
82	744
83	748
84	753
85	757
86	762
87	767
88	771
89	776
90	780
91	785
92	789
93	794
94	799
95	803
96	808
97	812
98	817
99	821
100	826
101	831
102	835
103	840
104	844
105	849
106	853
107	857
108	862
109	866
110	871
111	875
112	880
113	884
114	888
115	893
116	897
117	901
118	906
119	910
120	914
121	918
122	923
123	927

Continuation of Table 1	
day number	duration of the day
124	931
125	935
126	939
127	943
128	947
129	951
130	954
131	958
132	962
133	966
134	970
135	974
136	978
137	980
138	984
139	988
140	991
141	994
142	998
143	1001
144	1004
145	1007
146	1010
147	1013
148	1016
149	1018
150	1021
151	1023
152	1025
153	1028
154	1030
155	1032
156	1034
157	1036
158	1039
159	1040
160	1042
161	1042
162	1044
163	1045
164	1047
165	1048
166	1048
167	1049
168	1049
169	1050
170	1050
171	1051
172	1051
173	1051
174	1051
175	1050
176	1050
177	1049
178	1049

Continuation of Table 1	
day number	duration of the day
179	1048
180	1047
181	1046
182	1045
183	1043
184	1042
185	1040
186	1039
187	1037
188	1035
189	1033
190	1032
191	1030
192	1027
193	1025
194	1022
195	1019
196	1017
197	1014
198	1012
199	1008
200	1006
201	1002
202	1000
203	996
204	993
205	990
206	986
207	983
208	980
209	976
210	972
211	968
212	965
213	962
214	958
215	954
216	950
217	946
218	941
219	937
220	934
221	930
222	926
223	922
224	917
225	913
226	909
227	905
228	900
229	896
230	892
231	888
232	884
233	879

Continuation of Table 1	
day number	duration of the day
234	875
235	871
236	866
237	862
238	857
239	853
240	848
241	844
242	839
243	836
244	831
245	827
246	822
247	817
248	813
249	808
250	804
251	799
252	795
253	790
254	786
255	782
256	777
257	772
258	768
259	763
260	759
261	754
262	749
263	745
264	740
265	736
266	732
267	727
268	723
269	718
270	713
271	709
272	704
273	700
274	695
275	690
276	686
277	681
278	677
279	672
280	668
281	663
282	658
283	654
284	649
285	645
286	640
287	636
288	631

Continuation of Table 1	
day number	duration of the day
289	627
290	623
291	618
292	614
293	609
294	605
295	601
296	596
297	592
298	588
299	583
300	579
301	575
302	570
303	566
304	562
305	557
306	553
307	549
308	545
309	541
310	537
311	533
312	529
313	525
314	521
315	517
316	513
317	509
318	506
319	502
320	498
321	495
322	491
323	487
324	484
325	480
326	477
327	474
328	470
329	467
330	464
331	462
332	459
333	456
334	453
335	451
336	448
337	446
338	443
339	442
340	439
341	438
342	435
343	434

Continuation of Table 1	
day number	duration of the day
344	432
345	431
346	430
347	428
348	427
349	426
350	426
351	425
352	424
353	424
354	424
355	423
356	423
357	423
358	424
359	424
360	424
361	425
362	426
363	427
364	428
365	429
End of Table	