Jan 29, 2015

LINGUIST 130A / 230A – Sample derivations

(using the "semantic composition" handout)

General points

- when we talk about "a derivation", we are usually looking at a tree: we want to use the meanings of the lexical items and the composition rules to work out the combined meanings of the top nodes
- a rule like

```
    S
   / \
  PN  VP
```

$\rightsquigarrow$ $[\![S]\!] = [\![VP]\!]([\![PN]\!])$

tells us how to put things together, but it's not especially enlightening until we put lexical items or phrases in for $[\![PN]\!]$ and $[\![VP]\!]$

Examples

(~~23~~ 24)

```
     S
    / \
  PN   VP
  |    |
 Bart  V
       |
   skateboards
```

- start by looking up lexical items:

$[\![Bart]\!] =$ 

$[\![skateboards]\!] = \lambda x\,(T \text{ if } x \in \{y \mid y \text{ skateboards}\})$

(this is written out explicitly ~~as~~ on the handout)

- next, we work out the nodes dominating the lexical items using rule (NB) for nonbranching structures:

$[\![PN]\!] = [\![Bart]\!]$ *, $[\![VP]\!] = [\![V]\!] = [\![skateboards]\!]$

- now we can use rule (S) to work out the meaning of the sentence:

$[\![S]\!] = [\![VP]\!]([\![PN]\!]) = [\![skateboards]\!]([\![Bart]\!])$

$[\![S]\!] = \lambda x(T \text{ if } x \in \{y \mid y \text{ skateboards}\})([\![\text{Bart}]\!])$

- now, we use λ-substitution to reduce this: "λx" tells us that $[\![\text{skateboards}]\!]$ is looking for an x, and we have fed it $[\![\text{Bart}]\!]$

$[\![S]\!] = T \text{ if } [\![\text{Bart}]\!] \in \{y \mid y \text{ skateboards}\}$

- at this point, we can just go look up whether or not $[\![\text{Bart}]\!]$ is in the skateboard set, to figure out whether S is true or false.

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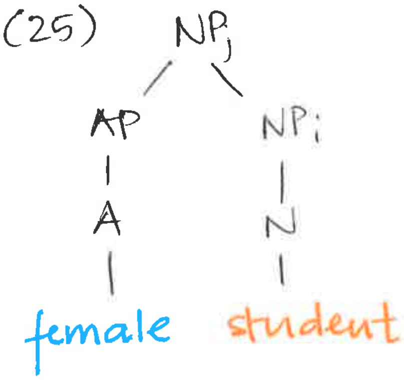
(25)



$[\![\text{female}]\!] = \lambda f(\lambda x(T \text{ if } x \in \{[\![\text{Maggie}]\!], [\![\text{Lisa}]\!]\}$ and $f(x) = T))$

$(\{y \mid y \text{ is female}\})$

$[\![\text{student}]\!] = \lambda x(T \text{ if } x \in \{[\![\text{Lisa}]\!], [\![\text{Bart}]\!]\})$

$(\rightarrow \{y \mid y \text{ is a student}\})$

By rule (NB), $[\![AP]\!] = [\![A]\!] = [\![\text{female}]\!]$, and

$[\![NP_i]\!] = [\![N]\!] = [\![\text{student}]\!]$

Finally, we use rule (A) to work out the meaning of the top NP:

$$[\![NP_j]\!] = [\![AP]\!]([\![NP_i]\!]) = [\![\text{female}]\!]([\![\text{student}]\!])$$

$$= \lambda f(\lambda x(T \text{ if } x \in \{[\![\text{Maggie}]\!], [\![\text{Lisa}]\!]\} \text{ and } f(x) = T))(\lambda x(T \text{ if } x \in \{[\![\text{Lisa}]\!], [\![\text{Bart}]\!]\}))$$

Notice that $[\![\text{female}]\!]$ starts with "λf", which means it's looking for an f, supplied by $[\![\text{student}]\!]$

When we substitute, we only substitute for f (This got confusing in section, because $[\![female]\!]$ has x in it, and $[\![student]\!]$ has x in it - but these are different xs, as the colours are meant to show. As Chris said in class today, you can avoid this problem w/out color-coding by using a different variable name, e.g. z, instead of the x in $[\![student]\!]$)

- Result of substitution

$$[\![NP_j]\!] = (\lambda x(T \text{ if } x \in \{[\![Maggie]\!], [\![Lisa]\!]\} \text{ and } \underbrace{(\lambda x(T \text{ if } x \in \{[\![Lisa]\!], [\![Bart]\!]\})}_{\text{formerly } f}(x) = T))$$

- Notice that the expression that used to just read $f(x)=T$ now has a reducible λ-term on the left side. We reduce by "plugging in" x for x (see why careful naming matters?)

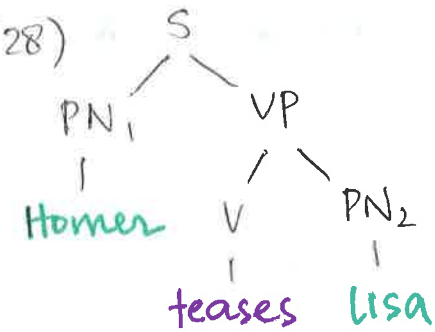
$$[\![NP_j]\!] = (\lambda x (T \text{ if } x \in \{[\![Maggie]\!], [\![Lisa]\!]\} \text{ and } (T \text{ if } x \in \{[\![Lisa]\!], [\![Bart]\!]\}) = T))$$

- we pretty much have to stop here, but can "clean up" some of the stuff that looks redundant

$$[\![NP_j]\!] = \lambda x (T \text{ if } x \in \{[\![Maggie]\!], [\![Lisa]\!]\} \text{ and } x \in \{[\![Lisa]\!], [\![Bart]\!]\}))$$

- the colour coding isn't doing much at this point, but it gives you a sense of where the pieces came from.

A few more examples, with less annotation:

(28) 

$\llbracket \text{Homer} \rrbracket =$  , $\llbracket \text{Lisa} \rrbracket =$ 

$$\llbracket \text{teases} \rrbracket = \lambda y(\lambda x(T \text{ if } \langle x, y\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\}))$$

- By (NB),

$$\llbracket \text{PN}_1 \rrbracket = \llbracket \text{Homer} \rrbracket,\quad \llbracket \text{PN}_2 \rrbracket = \llbracket \text{Lisa} \rrbracket,\quad \llbracket \text{V} \rrbracket = \llbracket \text{teases} \rrbracket$$

- By (TV),

$$\llbracket \text{VP} \rrbracket = \llbracket \text{V} \rrbracket(\llbracket \text{PN}_2 \rrbracket) = \llbracket \text{teases} \rrbracket(\llbracket \text{Lisa} \rrbracket)$$

$$= \lambda y(\lambda x(T \text{ if } \langle x, y\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\}))(\llbracket \text{Lisa} \rrbracket)$$

$$= \lambda x(T \text{ if } \langle x, \llbracket \text{Lisa} \rrbracket\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\})$$

(notice this is the same "sort" of meaning as $\llbracket \text{skateboards} \rrbracket$ from (24) – it wants an x)

- By (S),

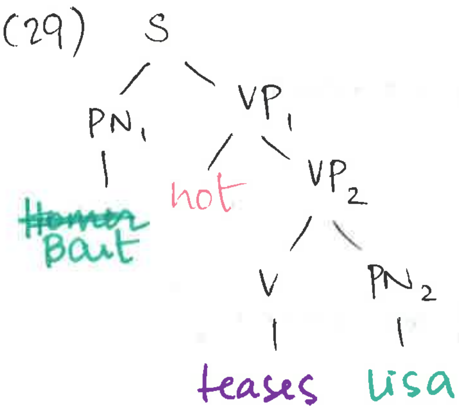
$$\llbracket \text{S} \rrbracket = \llbracket \text{VP} \rrbracket(\llbracket \text{PN}_1 \rrbracket) = \llbracket \text{teases} \rrbracket(\llbracket \text{Lisa} \rrbracket)(\llbracket \overset{\text{Homer}}{\cancel{\text{Bart}}} \rrbracket)$$

since we worked out $\llbracket \text{teases} \rrbracket(\llbracket \text{Lisa} \rrbracket)$ already, we get

$$\llbracket \text{S} \rrbracket = \lambda x(T \text{ if } \langle x, \llbracket \text{Lisa} \rrbracket\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\})(\llbracket \text{Homer} \rrbracket)$$

$$= T \text{ if } \langle \llbracket \text{Homer} \rrbracket, \llbracket \text{Lisa} \rrbracket\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\})$$

(29)



• we worked out $[\![VP_2]\!]$ in (28), so lets skip doing it again:

$$[\![VP_2]\!] = \lambda x\,(T \text{ if } \langle x, [\![\text{Lisa}]\!]\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\})$$

• the new piece here is $[\![\text{not}]\!]$.

$$[\![\text{not}]\!] = \lambda f\,(\lambda x\,(T \text{ if } f(x) = F, \text{ else } F))$$

• By rule (N), we get

(note the different coloured x's, again!)

$$[\![VP_1]\!] = [\![\text{not}]\!]([\![VP_2]\!])$$

$$= \lambda f\,(\lambda x\,(T \text{ if } f(x) = F, \text{ else } F))\,(\lambda x\,(T \text{ if } \langle x, [\![\text{Lisa}]\!]\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\}))$$

$$= \lambda x\,(T \text{ if } \underbrace{\lambda x\,(T \text{ if } \langle x, [\![\text{Lisa}]\!]\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\})}_{\text{formerly } f}(x) = F, \text{ else } F)$$

$$= \lambda x\,(T \text{ if } (T \text{ if } \langle x, [\![\text{Lisa}]\!]\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\}) = F, \text{ else } F)$$

• By rule (S), we get:

$$[\![S]\!] = [\![VP_1]\!]([\![PN_1]\!]) = [\![\text{not}]\!]([\![\text{teases}]\!]([\![\text{Lisa}]\!]))([\![\text{Bart}]\!])$$

$$= \lambda x\,(T \text{ if } (T \text{ if } \langle x, [\![\text{Lisa}]\!]\rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\}) = F, \text{ else } F)\,([\![\text{Bart}]\!])$$

(even though it looks more complicated, $[\![VP_1]\!]$ is the same "sort" of thing as $[\![\text{skateboards}]\!]$ - it just wants an x)

$[\![S]\!] = T \text{ if } (T \text{ if } \langle [\![\text{Bart}]\!], [\![\text{Lisa}]\!] \rangle \in \{\langle a,b\rangle \mid a \text{ teases } b\}) = F, \text{ else } F$

and if we want to write that down more simply, we have

$[\![S]\!] = T \text{ if } \langle [\![\text{Bart}]\!], [\![\text{Lisa}]\!] \rangle \notin \{\langle a,b\rangle \mid a \text{ teases } b\})$

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(30) $[_{QP}\ [_{D}\ \text{a}]\ [_{NP}\ \text{Simpson}]]$

- since everyone is a Simpson, we can "cheat" (not cheating) and write

$[\![\text{Simpson}]\!] = \lambda x (T \text{ if } x \in U)$

we have

$[\![a]\!] = \lambda f(\lambda g(T \text{ if } \{x \mid f(x) = T\} \cap \{x \mid g(x) = T\} \neq \emptyset))$

(assuming "a" and "some" are the same, for now)

- Rule (NB) gives us

$[\![NP]\!] = [\![\text{Simpson}]\!]$, $[\![D]\!] = [\![a]\!]$

- Rule (Q1) tells us

$[\![QP]\!] = [\![D]\!]([\![NP]\!]) = [\![a]\!]([\![\text{Simpson}]\!])$

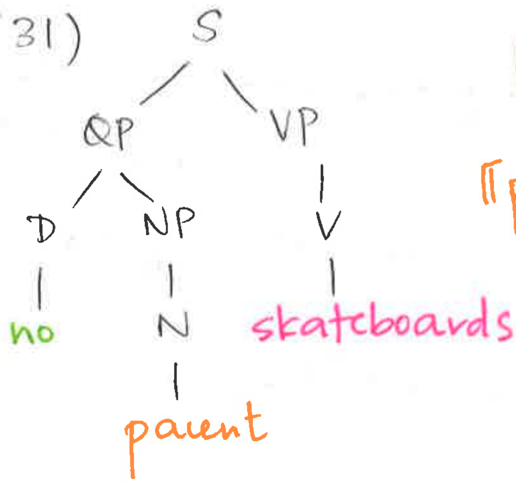
$= \lambda f(\lambda g(T \text{ if } \{x \mid f(x) = T\} \cap \{x \mid g(x) = T\} \neq \emptyset))$
$(\lambda x (T \text{ if } x \in U))$

$= (\lambda g(T \text{ if } \{x \mid \overbrace{\lambda x (T \text{ if } x \in U)}^{\text{formerly } f}(x) = T\} \cap \{x \mid g(x) = T\} \neq \emptyset))$

$= \lambda g(T \text{ if } \{x \mid (T \text{ if } x \in U) = T\} \cap \{x \mid g(x) = T\} \neq \emptyset)$

$= \lambda g(T \text{ if } \{x \mid x \in U\} \cap \{x \mid g(x) = T\} \neq \emptyset)$

(31)



$[\![no]\!] = \lambda f(\lambda g(T \text{ if } \{x \mid f(x)=T\} \cap \{x \mid g(x)=T\} = \emptyset))$

$[\![parent]\!] = \lambda x(T \text{ if } x \in \overset{\{y \mid y \text{ is a parent}\} =}{\{[\![Homer]\!]\}})$

$[\![skateboards]\!] = \lambda x(T \text{ if } x \in \{y \mid y \text{ skateboards}\})$

- from (NB),

$[\![D]\!] = [\![no]\!]$, $[\![NP]\!] = [\![parent]\!]$, $[\![VP]\!] = [\![skateboards]\!]$

- from (Q1),

$[\![QP]\!] = [\![D]\!]([\![NP]\!]) = [\![no]\!]([\![parent]\!])$

$$= \lambda f(\lambda g(T \text{ if } \{x \mid f(x)=T\} \cap \{x \mid g(x)=T\} = \emptyset))(\lambda x(T \text{ if } x \in \{[\![Homer]\!]\}))$$

$$= \lambda g(T \text{ if } \{x \mid \overbrace{\lambda x(T \text{ if } x \in \{[\![Homer]\!]\})}^{\text{formerly } f}(x) = T\} \cap \{x \mid g(x)=T\} = \emptyset)\cancel{)}$$

$$= \lambda g(T \text{ if } \{x \mid \cancel{\lambda x}(T \text{ if } x \in \{[\![Homer]\!]\}) = T\} \cap \{x \mid g(x)=T\} = \emptyset)\cancel{)}$$

$$= \lambda g(T \text{ if } \{x \mid x \in \{[\![Homer]\!]\}\} \cap \{x \mid g(x)=T\} = \emptyset)\cancel{)}$$

- and from (Q2), we have

$[\![S]\!] = [\![QP]\!]([\![VP]\!]) = [\![no]\!]([\![parent]\!])([\![skateboards]\!])$

$$= \lambda g(T \text{ if } \{x \mid x \in \{[\![Homer]\!]\}\} \cap \{x \mid g(x)=T\} = \emptyset)\cancel{)}(\lambda x(T \text{ if } x \in \{y \mid y \text{ skateboards}\}))$$

$$= T \text{ if } \{x \mid x \in \{[\![\text{Homer}]\!]\}\} \cap \{x \mid \overbrace{\lambda x(T \text{ if } x \in \{y \mid y \text{ skateboards}\})}^{\text{formerly } g}(x) = T\} = \emptyset$$

$$= T \text{ if } \{x \mid x \in \{[\![\text{Homer}]\!]\}\} \cap \{x \mid (T \text{ if } x \in \{y \mid y \text{ skateboards}\}) = T\} = \emptyset$$

$$= T \text{ if } \{x \mid x \in \{[\![\text{Homer}]\!]\}\} \cap \{x \mid x \in \{y \mid y \text{ skateboards}\}\} = \emptyset$$

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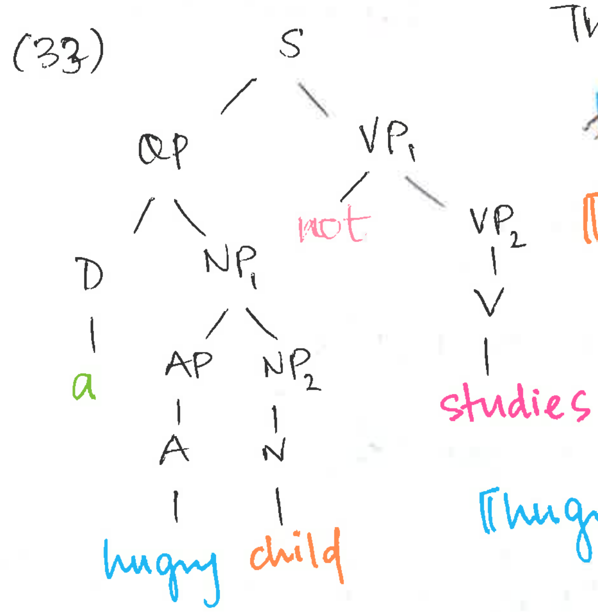
(33)



The new items are

$[\![\text{child}]\!] = \lambda x(T \text{ if } x \in \{y \mid y \text{ is a child}\})$

$[\![\text{studies}]\!] = \lambda x(T \text{ if } x \in \{y \mid y \text{ studies}\})$

$[\![\text{hungry}]\!] = \lambda f(\lambda x(T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } f(x) = T))$

- By (NB),

$[\![D]\!] = [\![a]\!]$, $[\![AP]\!] = [\![A]\!] = [\![\text{hungry}]\!]$, $[\![NP_2]\!] = [\![N]\!] = [\![\text{child}]\!]$,

$[\![VP_2]\!] = [\![V]\!] = [\![\text{studies}]\!]$

- By (A),

$[\![NP_1]\!] = [\![AP]\!]([\![NP_2]\!]) = [\![\text{hungry}]\!]([\![\text{child}]\!])$

$$= \lambda f(\lambda x(T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } f(x) = T))(\lambda x(T \text{ if } x \in \{y \mid y \text{ is a child}\}))$$

$$= \lambda x(T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } \lambda x(T \text{ if } x \in \{y \mid y \text{ is a child}\})(x) = T)$$

$$= \lambda x(T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\})$$

By (Q1),

$$[\![QP]\!] = [\![D]\!]([\![NP_1]\!]) = [\![a]\!]([\![\text{hungry}]\!]([\![\text{child}]\!]))$$

$$= \lambda f(\lambda g(T \text{ if } \{x \mid f(x) = T\} \cap \{x \mid g(x) = T\} \neq \emptyset))$$
$$(\lambda x(T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}))$$

$$= \lambda g(T \text{ if } \{x \mid \overbrace{\lambda x(T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\})}^{\text{former } f}(x) = T\}$$
$$\cap \{x \mid g(x) = T\} \neq \emptyset)$$

$$= \lambda g(T \text{ if } \{x \mid (T \text{ if } x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}) = T\}$$
$$\cap \{x \mid g(x) = T\} \neq \emptyset)$$

$$\lambda g(T \text{ if } \{x \mid x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}\}$$
$$\cap \{x \mid g(x) = T\} \neq \emptyset)$$

By (N),

$$[\![VP_1]\!] = [\![\text{not}]\!]([\![VP_2]\!]) = [\![\text{not}]\!]([\![\text{studies}]\!])$$

$$= \lambda f(\lambda x(T \text{ if } f(x) = F, \text{ else } F))(\lambda x(T \text{ if } x \in \{y \mid y \text{ studies}\}))$$

$$= \lambda x(T \text{ if } \lambda x(T \text{ if } x \in \{y \mid y \text{ studies}\})(x) = F, \text{ else } F)$$

$$= \lambda x(T \text{ if } x \notin \{y \mid y \text{ studies}\}, \text{ else } F)$$

and by (Q2),

$$[\![S]\!] = [\![QP]\!]([\![VP,1]\!]) = [\![a]\!]([\![hungry]\!]([\![child]\!]))([\![not]\!]([\![studies]\!]))$$

$$= \lambda g(T \text{ if } \{x \mid x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}\} \cap \{x \mid g(x) = T\} \neq \emptyset)$$

$$(\lambda x\ (T \text{ if } x \notin \{y \mid y \text{ studies}\}, \text{ else } F))$$

$$= T \text{ if } \{x \mid x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}\} \cap$$
$$\{x \mid \lambda x(T \text{ if } x \notin \{y \mid y \text{ studies}\}, \text{ else } F)(x) = T\} \neq \emptyset$$

$$= T \text{ if } \{x \mid x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}\} \cap$$
$$\{x \mid (T \text{ if } x \notin \{y \mid y \text{ studies}\}, \text{ else } F) = T\} \neq \emptyset$$

$$= T \text{ if } \{x \mid x \in \{y \mid y \text{ is hungry}\} \text{ and } x \in \{y \mid y \text{ is a child}\}\} \cap$$
$$\{x \mid x \notin \{y \mid y \text{ studies}\}\} \neq \emptyset$$

So, at the end of all of that, we have to check whether anyone who is in both the set of hungry people and the set of children is also in the set of those who don't study: if so, $[\![S]\!] = T$, if not $[\![S]\!] = F$.