Introduction to Sets

Linguist 130A/230A Section

January 12, 2015

1 What is a set?

1.1 Definition

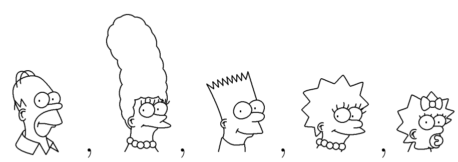
A **set** is simply a collection of objects. Any collection of objects can be thought of as a set, whether or not you can get your hands on it.

1.2 Some Examples

Example 1.1. The set containing every prime number less than 10:

$$\{2, 3, 5, 7\}$$

Example 1.2. The set containing the members of the Simpsons family:

$\{$  $\}$

Example 1.3. The set containing the first names of all U.S. presidents since 1990:

$$\{\textit{George, William, Barack}\}$$

Question: Why does *George* appear only once?

Example 1.4. The set containing Bart Simpson, Barack Obama's first name, and the set of prime numbers less than 10:

$\{$  $, \textit{Barack}, \{2, 3, 5, 7\}\}$

Example 1.5. The containing all of the trees on Stanford's campus and the colour green

As example 5 shows, we don't have to be able to "neatly" write down each member of a set for it to count as a set. The set of every apple you have ever eaten is a set (and is a different set for each person who reads this!) even though there's no hope of putting the elements of the set out in a line in front of us.

1.3 Notation

When we can write down the elements of a set, we typically enclose them in curly brackets like in examples 1-4 above, with commas in between the elements. Sometimes there are too many items in a set to write down, but we can use a shorthand called **predicate notation**:

Example 1.6. The set containing all even numbers:

$$\{x : x \text{ is an even number}\}$$

This is read as "the set of all xs such that x is an even number." The colon is sometimes represented as a vertical line: to the left of it, we put a symbol to represent an arbitrary object in the set (x is often used, but we could use any symbol at all), and to the right we put a rule for constructing the set.
The following are alternative ways we could represent the set in example 6:

(1) a. { ✦ | ✦ is an even number }
  b. $\{x : x \text{ is an integer divisible by } 2\}$

1.4 The empty set

Consider the following set:

(2) $\{\ y \mid y \text{ is a hamburger that Alec Baldwin ate in 2014}\}$

But Alec Baldwin is a vegan[1] and didn't eat any meat in 2014, so there are no such hamburgers! Is this still a set?

Yes! It's a very important set with nothing in it: **the empty set**. Here are some other ways of writing it down:

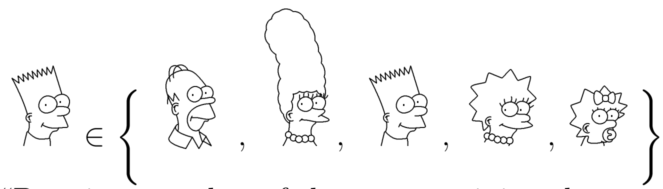
(3) a. $\emptyset$
  b. $\{\}$
  c. $\{n : n \text{ is an even prime number greater than } 2\}$
  d. $\{y : 2 \text{ is an odd number}\}$

(3)a and (3)b are the standard ways of writing the empty set, but all of the sets here are the same – there is only one empty set.

[1]This is true.

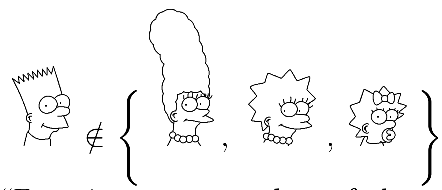
1.5 Set membership

The things in a given set are called its **elements**, or **members**. We use the Greek letter epsilon to represent set membership:

(4) 

"Bart is a member of the set containing the members of the Simpsons family."

We put a slash through the "is a member of" symbol to mean that something is not in a given set:

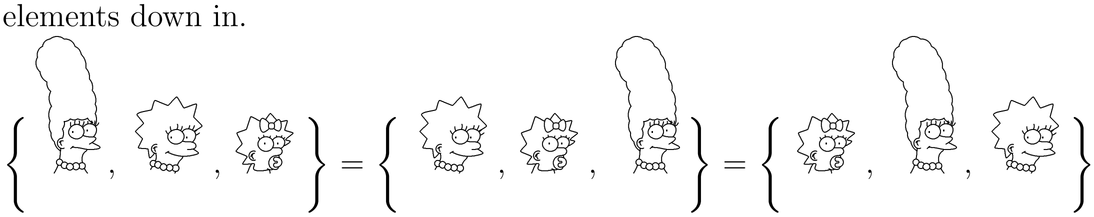
(5) 

"Bart is not a member of the set containing the female members of the Simpsons family."

1.6 Some important properties of sets

Here are some important things to keep in mind about sets:

- Sets are **unordered** collections of objects: it doesn't matter what order we write their elements down in.



- Sets have **no repetitions**: that is, they don't contain multiple copies of the same element. Writing down one element twice gives you the same set as writing it down once:

  (6) The set of first names of US president since 1990 could be written in any of the following ways (and a number of others)

  a. {*George, William, Barack*}

  b. {*George, William, George, Barack*}

  c. {*Barack, George, William, Barack*}

- Since the empty set is a set, it counts as an object. This means that:

$$\emptyset \neq \{\emptyset\}$$

2 Relationships between sets

Sets can have interesting relationships to one another, and we have some ways of talking about and representing these.

2.1 Union

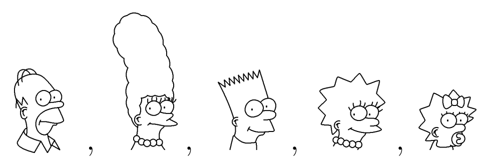
Supposing we have two sets, A and B. The **union** of A and B is the set C which contains all members of A as well as all members of B. We write this as $A \cup B$ and read it as "A union B":

Definition 2.1. $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Example 2.2. Let $A = \{x : x \text{ is a prime under } 10\}$ and $B = \{y : y \text{ is a multiple of 3 under } 14\}$.

$$A \cup B = \{2, 3, 5, 7\} \cup \{3, 6, 9, 12\} = \{2, 3, 5, 6, 7, 9, 12\}$$

Example 2.3. Let $A =$ *the set of male Simpsons* and $B =$ *the set of female Simpsons.*

$A \cup B = \{$  $\}$

2.2 Intersection

We can also think about the set of elements that occur belong to more than one set. The **intersection** of A and B is the set C which contains those objects that are in both A and B. We write this as $A \cap B$ and read it as "A intersect B":

Definition 2.4. $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Example 2.5. Let $A = \{x : x \text{ is a prime under } 10\}$ and $B = \{y : y \text{ is a multiple of 3 under } 14\}$.

$$A \cap B = \{2, 3, 5, 7\} \cap \{3, 6, 9, 12\} = \{3\}$$

Example 2.6. Let $A =$ *the set of male Simpsons* and $B =$ *the set of female Simpsons.*

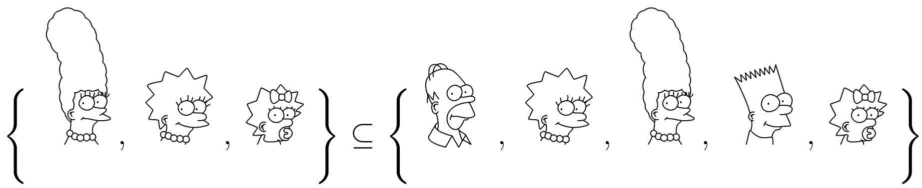
$$A \cap B = \emptyset$$

2.3 Subset

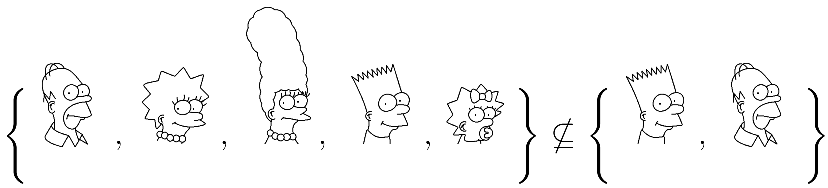
If all of the members of a set A are contained in a set B, we call A a **subset** of B. We write this as $A \subseteq B$:

Definition 2.7. $A \subseteq B$ if and only if for all x, $x \in A$ means that $x \in B$

Example 2.8. The set of female Simpsons is a subset of the set of Simpsons:



Example 2.9. The set of Simpsons is **not** a subset of the set of male Simpsons:



Note: This definition means that, for any set A, $A \subseteq A$, and also that $\emptyset \subseteq A$. (The line under the subset symbol means that the two sets could be equal. If you know that A is a subset of B but B has elements that A does not, you can also write $A \subset B$.)

2.4 Equality

Two sets are considered **equal** if they contain exactly the same set of elements (remember, repetitions don't count). Since we have the subset relation, we can define equality as follows:

Definition 2.10. Let A and B be sets. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Example 2.11. $\{x : x \text{ is an integer multiple of } 2\} = \{y : y \text{ is an even number}\}$

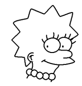
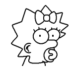
Example 2.12. Let $A =$ *the set of first names of presidents since 1990* and $B =$ *the set of first names of the last three US presidents.* Then $A = B$.

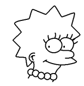
2.5 Set difference

We might also want to think about the set of things that are in one set but not in another. We write the **difference** between A and B as $A - B$ (you might also see it as $A \backslash B$):

Definition 2.13. $A - B = \{x : x \in A \text{ and } x \notin B\}$

Example 2.14. $\{2, 3, 5, 7\} - \{3, 5\} = \{2, 7\}$

Example 2.15. Let $A = \{$  ,  ,  $\}$ and let $B = \{$  ,  $\}$.

$A - B = \{$  ,  $\}$

$B - A = \{$  $\}$

2.6 Power set

We also sometimes need to think about sub collections of sets. The set of all subsets of a set A is called its **power set**, and is written $\wp(A)$:

Definition 2.16. Let A be a set. Then $\wp(A) = \{B : B \subseteq A\}$.

Example 2.17. Let $A = \{\textit{George, William, Barack}\}$.

$$\wp(A) =$$

$$\left\{ \begin{array}{ccc} & \{\textit{George, William, Barack}\}, & \\ \{\textit{George, William}\}, & \{\textit{George, Barack}\}, & \{\textit{William, Barack}\}, \\ \{\textit{George}\}, & \{\textit{William}\}, & \{\textit{Barack}\}, \\ & \emptyset & \end{array} \right\}$$

Note: $A \in \wp(A)$, but $A \nsubseteq \wp(A)$.