Lambdas: a (quick) reference guide

Linguist 130A/230A

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The λ calculus was developed by Alonzo Church in the 1930s. It's a simple and powerful way of notating and handling functions and computations (it might be familiar from programming). This handout is a brief non-comprehensive reference.

1 What is a λ(-term)?

- a variable (x, y, f, P)
- a combination of a λ-term t and a variable x: $\lambda x[t]$ or $\lambda x.t$
- a combination of two λ-terms where one is interpreted as a function t and the other is an input s to the function: $t(s)$
- nothing else is a λ-term

2 How to interpret a λ-term

$$\lambda x.f(x)$$

"Give me an x and get back f applied to x"

$\lambda x.f(x)(y)$	$\rightsquigarrow$	$f(y)$
"Apply the function $\lambda x.f(x)$ to the input y"	*reduces to*	"f applied to the input y"

3 Some correspondences

Example 3.1. Two ways of writing the function that always returns 2:

(a) $f(x) = 2$

(b) $\lambda y.2$

Example 3.2. Two ways of writing the squaring function:

(a) $f(x) = x^2$
$f(2) = 2^2 = 4$

(b) $\lambda y.y^2$
$\lambda y.y^2(2) = 2^2 = 4$

Example 3.3. Two ways of writing the $[\![\text{even}]\!]$ function:

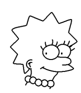
(a)

$$[\![\text{even}]\!](x) = \begin{cases} \text{T} & \text{if } x\%2 = 0 \\ \text{F} & \text{else} \end{cases}$$

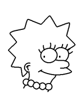
(b) $\lambda y.(\text{T if } x\%2 = 0, \text{ else F})$

NB: "$x\%2$" gives you the remainder when you divide x by 2

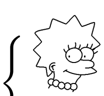
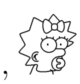
Example 3.4. Three ways of writing the $[\![\text{daughter}]\!]$ function on the Simpson children:

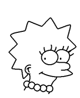
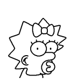
$U = \{$  ,  ,  $\}$

(a)

$[\![\text{daughter}]\!] =$
[
 $\mapsto$ T
 $\mapsto$ F
 $\mapsto$ T
]

(b)

$[\![\text{daughter}]\!](x) =$ {
T if $x \in \{$  ,  $\}$
F else

(c) $\lambda x.\Big($T if $x \in \{$  ,  $\}$, else F$\Big)$