Exercises on relations and functions

Linguist 130A/230A Section

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1. For each of the following representations of relations, determine whether it is a function, and if so, whether it is (i) total; (ii) one-to-one; (iii) onto.

   (a)
   $$\left|\begin{array}{ccc} x & \longrightarrow & a \\ y & \longrightarrow & b \\ y & \longrightarrow & c \\ z & \longrightarrow & d \end{array}\right|$$

   (b)
   $$\left|\begin{array}{ccc} x & \longrightarrow & a \\ y & \longrightarrow & b \\ & & c \\ z & \longrightarrow & d \end{array}\right|$$

   (c)
   $$\left|\begin{array}{ccc} x & & a \\ & & b \\ & & c \\ y & \longrightarrow & d \\ z & \longrightarrow & d \end{array}\right|$$

   (d)
   $$\left|\begin{array}{ccc} x & \longrightarrow & c \\ y & \longrightarrow & b \\ z & \longrightarrow & d \\ w & \longrightarrow & a \end{array}\right|$$

2. Let $\mathbb{R}$ be the set of all real numbers.

   (a) Let $R \subseteq \mathbb{R} \times \mathbb{R}$, where $R = \{\langle x, y\rangle : y = x\}$.
   Is R a function? If so, is it onto? One-to-one? Explain.

   (b) Let $Q \subseteq \mathbb{R} \times \mathbb{R}$ where $Q = \{\langle x, y\rangle : y = x^2\}$.
   Is Q a function? If so, is it onto? One-to-one? Explain.

   (c) Let $P \subseteq \mathbb{R} \times \mathbb{R}$, where $P = \{\langle x, y\rangle : x = y^2\}$.
   Is P a function? If so, is it onto? One-to-one? Explain.

3. Hawkinsites are an alien species that never die. So every Hawkinsite who has ever lived is still alive. Let's assume that every Hawkinsite was born of another (just one) other Hawkinsite (no chicken-or-egg problem).[1] Let $H = \{h : h \text{ is a Hawkinsite alive right now}\}$.

   (a) Let $R \subseteq H \times H$, where $R = \{\langle x, y\rangle : x \text{ is } y\text{'s parent}\}$.
   Is R a function? If so, is it onto? One-to-one? Explain.

   (b) Let $Q \subseteq H \times H$, where $Q = \{\langle x, y\rangle : y \text{ is } x\text{'s parent}\}$.
   Is Q a function? If so, is it onto? One-to-one? Explain.

[1] Apologies to Isaac Asimov.

4. Suppose we have a model M of Middle-Earth, and the set F of individuals in this model is given by $\{f,\ s,\ m,\ p,\ g\}$.[2] P is a **property** if $P \subseteq D$.

   We speak a fragment of Middle-English where the only property-denoting expressions are *comical, hobbit, jovial, mortal, cantankerous, wise, wizard* and *ring bearer*. These are defined:

   $[\![\text{comical}]\!]^M = \{s, m, p\}$

   $[\![\text{hobbit}]\!]^M = \{f, s, m, p\}$

   $[\![\text{jovial}]\!]^M = \{m, p, g\}$

   $[\![\text{mortal}]\!]^M = \{f, s, m, p, g\}$

   $[\![\text{cantankerous}]\!]^M = \{f, g\}$

   $[\![\text{wise}]\!]^M = \{f, s, g\}$

   $[\![\text{wizard}]\!]^M = \{g\}$

   $[\![\text{ringbearer}]\!]^M = \{f, s\}$

   Let's assume that noun phrases (expressions of the form [determiner + noun]) denote sets of properties, so that, for instance, for any noun N:

   $[\![\text{every } N]\!]^M = \{P : P \text{ is a property and } [\![N]\!]^M \subseteq P\}$

   So, for instance, $[\![\text{every hobbit}]\!]^M = \{\{f, s, m, p\}, \{f, s, m, p, g\}\}$.

   A property P is true of a noun phrase Y if and only if $P \in Y$. So, for example, the properties $[\![\text{hobbit}]\!]^M$ (trivially) and $[\![\text{mortal}]\!]^M$ are true of $[\![\text{every hobbit}]\!]^M$.

   Problems:

   (a) Suppose $[\![X \text{ ringbearers}]\!]^M = \{\emptyset, \{m\}, \{p\}, \{g, \}, \{m, p\}, \{m, g\}, \{p, g\}, \{m, p, g\}\}$. What determiner could X be?

   (b) Give a definition of $[\![\text{some } N]\!]^M$, where N is any noun.
   Start with: $[\![\text{some } N]\!]^M = \{P : P \text{ is a property and} \ldots\}$

   (c) What is the meaning of *at least two hobbits*?
   Start with: $[\![\text{at least two hobbits}]\!]^M = \{\ldots\}$

   (d) True or False? $[\![\text{at least two mortals}]\!]^M \subseteq [\![\text{every mortal}]\!]^M$.

[2] If it helps, you can think: f = Frodo, s = Sam, m = Merry, p = Pippin, and g = Gandalf.