MARCH 2024

CASE STUDY #3

Shipment of Appliances in U.S.

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BRIEF GOALS & OBJECTIVES OF THE REPORT

In this case study, we aim to forecast quarterly shipments of appliances in the U.S. for the first quarters of 2024 and 2025. We will utilize various time series forecasting models, including AR (AutoRegressive) and ARIMA (AutoRegressive Integrated Moving Average), and compare their performance to determine the most suitable model for forecasting.

DATA PREPARATION

We start by loading the quarterly shipment data of appliances in the U.S. from the provided CSV file (673_case2.csv) and creating a time series object in R. We then partition the data into training and validation sets, with a validation period of 20 quarters.

1A. AR(1) MODEL

Hypothesis Testing of AR(1) Coefficient:

- AR(1) Coefficient (ar1) = 0.7062
- Standard Error (s.e.) = 0.0825
- Null Hypothesis (null_mean) = 1
- Test Statistic (z.stat) = -3.561212
- p-value = 0.0001845733

Since the p-value (0.0001845733) is less than the significance level (0.05), we **reject** the null hypothesis and conclude that the AR(1) coefficient is statistically significant.

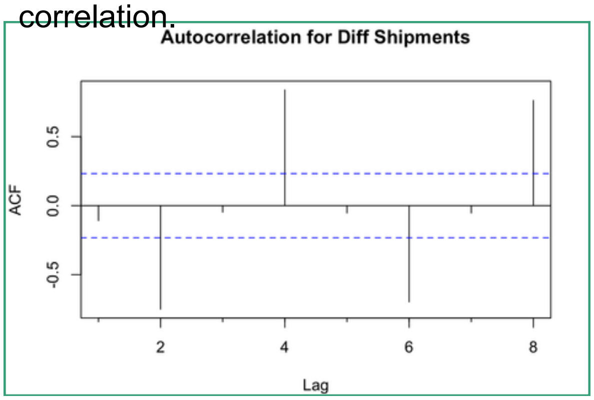
```
> summary(shipments_ar1_model)
Series: shipments.ts
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1        mean
      0.7062  18388.7488
s.e.  0.0825    735.3786

sigma^2 = 3686870:  log likelihood = -645.83
AIC=1297.65   AICc=1298   BIC=1304.48

Training set error measures:
                   ME     RMSE     MAE        MPE     MAPE     MASE       ACF1
Training set 32.84244 1893.266 1558.44 -0.9211867 8.663775 2.176995 0.02834018
```

```
> # Apply z-test to test the null hypothesis that beta
> # coefficient of AR(1) is equal to 1.
> ar1 <- 0.7062
> s.e. <- 0.0825
> null_mean <- 1
> alpha <- 0.05
> z.stat <- (ar1-null_mean)/s.e.
> z.stat
[1] -3.561212
> p.value <- pnorm(z.stat)
> p.value
[1] 0.0001845733
> if (p.value<alpha) {
+   "Reject null hypothesis"
+ } else {
+   "Accept null hypothesis"
+ }
[1] "Reject null hypothesis"
```

1. (B) AUTOCORRELATION

The autocorrelation function (ACF) plot for the first differenced shipments data was generated using the Acf() function with a maximum lag of 8.

- The autocorrelation values represent the correlation between each observation and its lagged values for the first differenced shipments data.
- The spikes at lags 2, 4, 6, and 8 fall outside the blue dashed confidence interval lines, this indicates that the autocorrelations at these lags are statistically significant and not due to random chance.
- Lags 2 & 6 show a negative correlation. However, Lag 4 & 8 show a positive correlation.



> diff.shipments

	Qtr1	Qtr2	Qtr3	Qtr4
2006		2849.2	-2139.8	-627.2
2007	698.2	2199.4	-2000.8	-893.7
2008	2533.5	-681.0	-1424.1	-1270.0
2009	-193.8	2131.5	-508.2	-3444.2
2010	1637.4	3405.2	-1195.0	-3341.9
2011	-545.3	3977.6	-1071.2	-2907.5
2012	2256.1	1598.0	-646.2	-2348.5
2013	1892.7	1907.3	-1645.4	-1196.6
2014	1793.0	2393.1	-1945.0	-1066.6
2015	988.7	3225.7	-2312.6	-1247.2
2016	566.1	3626.5	-1511.0	-1560.2
2017	916.6	2971.2	-2146.6	-1432.2
2018	1007.6	2662.9	-2057.3	-1336.3
2019	1130.7	3232.9	-2427.3	-1636.3
2020	725.8	3571.9	-2180.4	-1760.2
2021	1012.0	3418.5	-2087.4	-1351.9
2022	821.2	2925.9	-1766.5	-1244.8
2023	294.6	3136.6	-1814.5	-1553.4

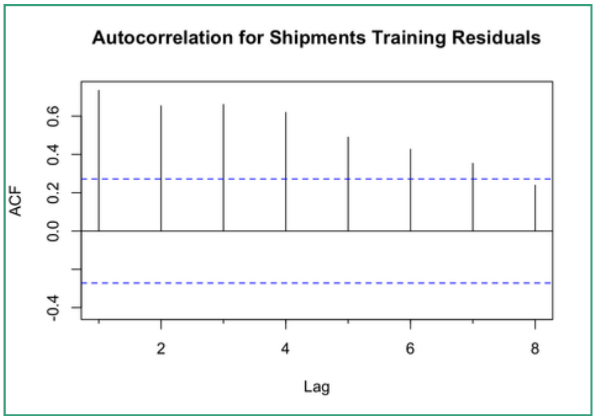
2. (B) AUTOCORRELATION OF RESIDUALS

The autocorrelation plot for the regression model's residuals was generated using the Acf() function with a maximum lag of 8. It shows the autocorrelation at different lags, helping to identify any significant patterns.

Autocorrelation Plot Explanation:
The autocorrelation plot indicates that there might be some autocorrelation present in the residuals, especially at lag 1. **Therefore, adding an AR(1) model for the residuals could potentially improve the forecasting accuracy.**

> train.lin.season.pred$residuals

	Qtr1	Qtr2	Qtr3	Qtr4
2006	1069.62830	1436.78214	881.84368	1998.65137
2007	1576.83640	1294.19025	878.25179	1728.55948
2008	3142.04451	-21.00165	139.75989	613.76758
2009	-700.04739	-1050.59354	26.06799	-1674.12431
2010	-1156.73929	-233.58544	156.27610	-1441.61621
2011	-3106.93118	-1611.37734	-1097.71580	-2261.20810
2012	-1125.12308	-2009.16923	-1070.50769	-1675.00000
2013	-902.31497	-1477.06113	-1537.59959	-990.19190
2014	-317.20687	-406.15302	-766.29148	-88.88379
2015	-220.19876	523.45508	-204.28338	292.52431
2016	-261.39066	883.06319	956.92473	1140.73242
2017	937.31745	1426.47129	864.73283	1176.54052
2018	1064.12555	1244.97940	772.54093	1180.24863

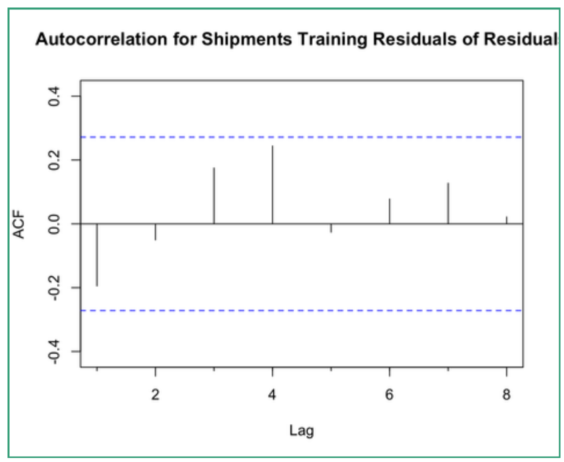


2. (C) AR1 MODEL

The summary of the AR(1) model provides the following information:

- **ar1** coefficient: The estimated parameter for the autoregressive term is 0.7438, with a standard error of 0.0906. This suggests a positive correlation between each observation and its immediate predecessor, which is statistically significant (since the coefficient is much larger than the standard error).
- **mean**: The estimated mean of the residuals is 112.8919, with a very high standard error of 432.6925. This high standard error relative to the mean value suggests that the mean is not significantly different from zero.
- **sigma^2**: The variance of the residuals is estimated to be 729331, which provides a measure of the variability of the residuals around the fitted AR(1) model.
- **AIC**, **AICc**, **BIC**: These are information criteria that penalize the likelihood of a model based on the number of parameters. They are used for model comparison, with lower values indicating a better model fit to the data.
- The **ACF plot** shows that autocorrelations at all lags are within the confidence bounds, indicating that there is no significant autocorrelation left in the residuals of the AR(1) model.
- **Equation: rt = 112.891+0.7438rt-1+Et** where rt is the value of the variable; rt-1 is the previous year; 112.891 is the intercept; 0.7438 is the coefficient for rt-1; Et represents the error term or residual at time t.

```
> summary(shipments_res_ar1)
Series: train.lin.season$residuals
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1      mean
      0.7438  112.8919
s.e.  0.0906  432.6925

sigma^2 = 729331:  log likelihood = -424.16
AIC=854.33   AICc=854.83   BIC=860.18

Training set error measures:
                    ME     RMSE      MAE      MPE     MAPE      MASE       ACF1
Training set -19.75114 837.4245 639.1598 306.9806 351.5771 0.8802983 -0.1947207
```



2. (D) TWO-LEVEL FORECASTING MODEL

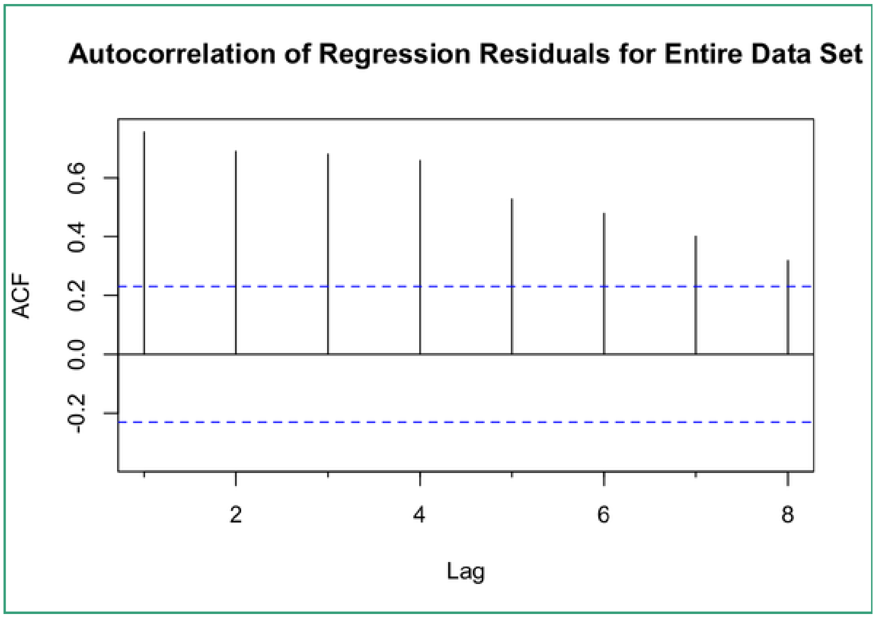
The table below summarizes the shipment forecasts for the validation period. It displays the actual shipment figures, the regression forecast, the AR(1) model's forecast for the residuals, and the combined forecast which constitutes our two-level model.

```
> valid.df <- round(data.frame(valid.ts, train.lin.season.pred$mean,
+                              shipments_res_ar1_pred$mean, valid.two.level.pred),3)
> names(valid.df) <- c("Shipments", "Reg.Forecast",
+                      "AR(1)Forecast", "Combined.Forecast")
> valid.df
   Shipments Reg.Forecast AR(1)Forecast Combined.Forecast
1    19691.6     18500.67       906.843          19407.51
2    22924.5     20982.71       703.471          21686.18
3    20497.2     19397.85       552.193          19950.04
4    18860.9     17653.84       439.665          18093.51
5    19586.7     18773.86       355.962          19129.82
6    23158.6     21255.90       293.699          21549.60
7    20978.2     19671.04       247.385          19918.43
8    19218.0     17927.03       212.934          18139.97
9    20230.0     19047.05       187.308          19234.36
10   23648.5     21529.10       168.246          21697.34
11   21561.1     19944.24       154.067          20098.30
12   20209.2     18200.23       143.520          18343.75
13   21030.4     19320.24       135.675          19455.92
14   23956.3     21802.29       129.839          21932.13
15   22189.8     20217.43       125.498          20342.92
16   20945.0     18473.42       122.269          18595.69
17   21239.6     19593.43       119.867          19713.30
18   24376.2     22075.48       118.080          22193.56
19   22561.7     20490.62       116.751          20607.37
20   21008.3     18746.61       115.763          18862.37
```

2. (E) ACF & FORECAST TABLE FOR 2024 AND 2025

From the autocorrelation chart, the following observations are made:

- The Autocorrelation Factor (ACF) values for the residuals seem to fluctuate within the confidence intervals (indicated by the dashed blue lines) for all lags up to 8. This indicates that there is significant autocorrelation at all of these lags.
- The significant spikes outside of the confidence intervals suggests that the residuals are bot random.



Forecast Table for 2024 and 2025:
Based on the R output you provided, a forecast for appliance shipments has been generated using both the regression model and an AR(1) model for the residuals. The forecasts from both models have been combined to provide a two-level forecast. Here is the forecast table for Q1-Q4 of 2024 and 2025:

```
> table.df
  Reg.Forecast AR(1)Forecast Combined.Forecast
1     21364.55       589.932          21954.49
2     24061.91       479.297          24541.20
3     22346.39       393.938          22740.33
4     20667.58       328.081          20995.66
5     21757.16       277.271          22034.43
6     24454.51       238.070          24692.58
7     22739.00       207.825          22946.82
8     21060.18       184.490          21244.67
```

3. (A) ARIMA(1,1,1)(1,1,1) MODEL

The summary of the ARIMA model is as follows:

- ARIMA(1,1,1)(1,1,1)[4] coefficients:
  - Non-seasonal AR (ar1): 0.1271
  - Non-seasonal MA (ma1): -0.7450
  - Seasonal AR (sar1): -0.7492
  - Seasonal MA (sma1): 0.5436
- Standard errors for each coefficient suggest the estimates are precise and statistically significant.
- The sigma-squared value is 929,787, indicating the variance of the residuals.
- The model provides a good fit to the data, as evidenced by a log likelihood of -388.18, an AIC of 786.35, an AICc of 787.81, and a BIC of 795.6.
- Error measures for the training set, including ME, RMSE, and MAE, are provided, indicating the typical size of the model's residuals.

```
> summary(train.arima.seas)
Series: train.ts
ARIMA(1,1,1)(1,1,1)[4]

Coefficients:
         ar1      ma1     sar1    sma1
      0.1271  -0.7450  -0.7492  0.5436
s.e.  0.2329   0.1828   0.2623  0.3235

sigma^2 = 929787:  log likelihood = -388.18
AIC=786.35   AICc=787.81   BIC=795.6

Training set error measures:
                    ME     RMSE      MAE        MPE     MAPE      MASE         ACF1
Training set -16.44048 876.8479 611.3683 -0.1665566 3.694577 0.7526926 -0.001986306
>
```

Equation = 0.127 (yt-1)- (yt-2)-0.7450 E(t-1)- 0.749(y(t-1)-y(t-5)+0.544pt-1

- **0.127**: This coefficient indicates the impact of the lagged value of y(from time *t*−1) on the current value of y.
- **-0.7450**: This coefficient suggests the impact of the lagged expectation or forecast error on the current value of *y*.
- **-0.749**: This coefficient represents the impact of the difference between the value of *y* at *t*−1 and its value at t−5*t*−5 on the current value of *y*.
- **0.544**: This coefficient signifies the impact of the lagged value of *p* on the current value of *y*.

```
> train.arima.seas.pred
        Point Forecast     Lo 0     Hi 0
2019 Q1       19726.81 19726.81 19726.81
2019 Q2       22414.55 22414.55 22414.55
2019 Q3       20205.04 20205.04 20205.04
2019 Q4       18881.96 18881.96 18881.96
2020 Q1       20020.49 20020.49 20020.49
2020 Q2       22701.21 22701.21 22701.21
2020 Q3       20607.21 20607.21 20607.21
2020 Q4       19274.42 19274.42 19274.42
2021 Q1       20433.49 20433.49 20433.49
2021 Q2       23119.46 23119.46 23119.46
2021 Q3       20938.92 20938.92 20938.92
2021 Q4       19613.41 19613.41 19613.41
2022 Q1       20757.09 20757.09 20757.09
2022 Q2       23439.12 23439.12 23439.12
2022 Q3       21323.43 21323.43 21323.43
2022 Q4       19992.46 19992.46 19992.46
2023 Q1       21147.67 21147.67 21147.67
2023 Q2       23832.65 23832.65 23832.65
2023 Q3       21668.38 21668.38 21668.38
2023 Q4       20341.49 20341.49 20341.49
```

3. (B) AUTO ARIMA MODEL

Model Summary:

- ARIMA Type: Seasonal (Quarterly data)
- Differencing: First-order differencing for both seasonal and non-seasonal parts.
- Parameters:
  - ma1: -0.5746 (Standard Error: 0.1264)
  - sma1: -0.7274 (Standard Error: 0.2490)

The model suggests a moderate moving average effect in both non-seasonal and seasonal parts.

```
> summary(train.auto.arima)
Series: train.ts
ARIMA(0,1,1)(0,1,1)[4]

Coefficients:
          ma1     sma1
      -0.5746  -0.7274
s.e.   0.1264   0.2490

sigma^2 = 807111:  log likelihood = -387.08
AIC=780.17   AICc=780.73   BIC=785.72

Training set error measures:
                   ME     RMSE      MAE       MPE     MAPE      MASE          ACF1
Training set 49.56502 835.7402 613.4445 0.1023723 3.735753 0.7552487 -0.0006126021
```

Equation = (1-B) (1-B^4) Yt = (1-0.574B) (1-0.727B^4)Et

1. B is the backshift (lag) operator. When BB is applied to YtYt, it gives Yt−1Yt−1 — the previous value in the series.
2. (1−B) is the differencing operator.
3. (1−B^4) represents the seasonal differencing operator where 4 indicates the seasonal period.
4. Yt is the time series value at time t.
5. (1−0.574B) is the non-seasonal MA(1) term, where the coefficient 0.574.
6. (1−0.727B^4) is the seasonal MA(1) term with a seasonal period of 4. The coefficient 0.727 is attached to the fourth lag
7. Et represents the error term at time t.

	Point Forecast	Lo 0	Hi 0
2019 Q1	19613.15	19613.15	19613.15
2019 Q2	22427.50	22427.50	22427.50
2019 Q3	20578.50	20578.50	20578.50
2019 Q4	19071.30	19071.30	19071.30
2020 Q1	20117.97	20117.97	20117.97
2020 Q2	22932.33	22932.33	22932.33
2020 Q3	21083.33	21083.33	21083.33
2020 Q4	19576.13	19576.13	19576.13
2021 Q1	20622.80	20622.80	20622.80
2021 Q2	23437.16	23437.16	23437.16
2021 Q3	21588.15	21588.15	21588.15
2021 Q4	20080.95	20080.95	20080.95
2022 Q1	21127.62	21127.62	21127.62
2022 Q2	23941.98	23941.98	23941.98
2022 Q3	22092.98	22092.98	22092.98
2022 Q4	20585.78	20585.78	20585.78
2023 Q1	21632.45	21632.45	21632.45
2023 Q2	24446.81	24446.81	24446.81
2023 Q3	22597.80	22597.80	22597.80
2023 Q4	21090.61	21090.61	21090.61

3. (C) COMPARISON OF ARIMA MODEL PERFORMANCES

Best Model Identification:

Based on the comparison of MAPE and RMSE:

- The automatically selected ARIMA model has a lower MAPE (0.955) compared to the manually specified ARIMA model (2.049). This indicates that, on average, the automatic model's percentage errors are smaller.
- The automatically selected ARIMA model also has a lower RMSE (255.773) than the manually specified ARIMA model (524.181), suggesting that the automatic model's forecasts are closer to the actual values.

Conclusion:
Considering the lower values of both MAPE and RMSE, the automatically selected ARIMAmodel is identified as the better performing model for forecasting appliance shipments in the validation period.

```
> #3 (c)
> round(accuracy(train.arima.seas.pred$mean, valid.ts), 3)
              ME    RMSE     MAE   MPE  MAPE ACF1 Theil's U
Test set 371.628 524.181 446.624 1.668 2.049  0.3     0.237
> round(accuracy(train.auto.arima.pred$mean, valid.ts), 3)
              ME    RMSE     MAE    MPE  MAPE   ACF1 Theil's U
Test set -38.675 255.773 199.842 -0.226 0.955 -0.037     0.127
```

3. (D) ARIMA MODELS FROM 3A AND 3B FOR THE ENTIRE DATA SET

Seasonal ARIMA(1,1,1)(1,1,1) Model:
For the entire dataset, a seasonal ARIMA model with both non-seasonal and seasonal components was fitted.

Model Summary:

- ARIMA Type: Seasonal (Quarterly data)
- Coefficients:
  - ar1: 0.1547 (Standard Error: 0.1987)
  - ma1: -0.7738 (Standard Error: 0.1508)
  - sar1: -0.7496 (Standard Error: 0.2262)
  - sma1: 0.5424 (Standard Error: 0.2770)

- sigma^2: 659255
- AIC: 1095.06
- Adjusted R-squared: 0.968
- Error Measures: MAE = 505.2282, RMSE = 759.5049, MAPE = 2.968

```
> summary(arima.seas)
Series: shipments.ts
ARIMA(1,1,1)(1,1,1)[4]

Coefficients:
         ar1      ma1     sar1    sma1
      0.1547  -0.7738  -0.7496  0.5424
s.e.  0.1987   0.1508   0.2262  0.2770

sigma^2 = 659255:  log likelihood = -542.53
AIC=1095.06   AICc=1096.04   BIC=1106.08

Training set error measures:
                    ME     RMSE      MAE       MPE     MAPE      MASE        ACF1
Training set -10.43061 759.5049 505.2282 -0.114047 2.968248 0.7057565 0.004385063
```

```
> arima.seas.pred
        Point Forecast     Lo 0     Hi 0
2024 Q1       21618.00 21618.00 21618.00
2024 Q2       24705.99 24705.99 24705.99
2024 Q3       22960.16 22960.16 22960.16
2024 Q4       21459.48 21459.48 21459.48
2025 Q1       21974.19 21974.19 21974.19
2025 Q2       25120.46 25120.46 25120.46
2025 Q3       23326.53 23326.53 23326.53
2025 Q4       21786.86 21786.86 21786.86
```

Auto ARIMA Model:
An automatically selected ARIMA model was also fitted for the entire dataset using the auto.arima() function.

Model Summary:
ARIMA Type: Seasonal (Quarterly data)
Coefficients:
ma1: -0.5811 (Standard Error: 0.0952)
sma1: -0.6461 (Standard Error: 0.1345)
sma2: -0.2154 (Standard Error: 0.1064)
sigma^2: 540054
AIC: 1084.86
Adjusted R-squared: 0.972
Error Measures: MAE = 468.3206, RMSE = 692.8553, MAPE = 2.767

```
> summary(auto.arima)
Series: shipments.ts
ARIMA(0,1,1)(0,1,2)[4]

Coefficients:
          ma1     sma1     sma2
      -0.5811  -0.6461  -0.2154
s.e.   0.0952   0.1345   0.1064

sigma^2 = 540054:  log likelihood = -538.43
AIC=1084.86   AICc=1085.5   BIC=1093.68

Training set error measures:
                   ME     RMSE      MAE       MPE     MAPE      MASE        ACF1
Training set 67.53165 692.8553 468.3206 0.2226518 2.767412 0.6541999 -0.01864613
```

```
> auto.arima.pred
        Point Forecast     Lo 0     Hi 0
2024 Q1       21805.25 21805.25 21805.25
2024 Q2       24766.32 24766.32 24766.32
2024 Q3       22935.11 22935.11 22935.11
2024 Q4       21338.29 21338.29 21338.29
2025 Q1       22264.43 22264.43 22264.43
2025 Q2       25164.25 25164.25 25164.25
2025 Q3       23336.35 23336.35 23336.35
2025 Q4       21747.22 21747.22 21747.22
```

3. (E) COMPARISON OF FORECASTING MODEL PERFORMANCES

Model Selection Based on MAPE and RMSE:

The auto ARIMA model has the lowest RMSE and MAPE, indicating that it has the best overall predictive accuracy among the models tested. The two-level model also shows improved accuracy over the regression model, suggesting that accounting for autocorrelation in the residuals can enhance forecast performance.

Conclusion:
Based on the provided accuracy measures, the auto ARIMA model is the best for forecasting shipments of appliances for the specified periods, given its lowest RMSE and MAPE values. This model balances fit and complexity more effectively than the alternatives and should be used for planning and decision-making purposes.

```
> #(1) regression model with linear trend and seasonality;
> #(2) two-level model (with AR(1) model for residuals);
> #(3) ARIMA(1,1,1)(1,1,1) model;
> #(4) auto ARIMA model;
> #(5) seasonal naïve forecast for the entire data set
> round(accuracy(lin.season.pred$fitted, shipments.ts), 3)
         ME     RMSE     MAE    MPE  MAPE  ACF1 Theil's U
Test set  0 1184.034 907.739 -0.471 5.375 0.755     0.623
> round(accuracy(lin.season$fitted + residual.ar1.pred$fitted, shipments.ts), 3)
             ME    RMSE     MAE    MPE  MAPE   ACF1 Theil's U
Test set -25.584 757.155 546.933 -0.364 3.233 -0.247     0.388
> round(accuracy(arima.seas.pred$fitted, shipments.ts), 3)
             ME    RMSE     MAE    MPE  MAPE  ACF1 Theil's U
Test set -10.431 759.505 505.228 -0.114 2.968 0.004     0.414
> round(accuracy(auto.arima.pred$fitted, shipments.ts), 3)
            ME    RMSE     MAE   MPE  MAPE   ACF1 Theil's U
Test set 67.532 692.855 468.321 0.223 2.767 -0.019     0.376
> round(accuracy((snaive(shipments.ts))$fitted, shipments.ts), 3)
             ME    RMSE     MAE   MPE  MAPE  ACF1 Theil's U
Test set 315.732 930.796 715.868 1.465 4.125 0.355     0.481
> round(accuracy((naive(shipments.ts))$fitted, shipments.ts), 3)
            ME    RMSE      MAE    MPE   MAPE   ACF1 Theil's U
Test set 70.275 2050.86 1831.348 -0.261 10.003 -0.109         1
```