Exponential Distribution Simulation

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Project Objective

The project is meant to answer to this particular question:

```
The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate paramet

Illustrate via simulation and associated explanatory text the properties of the distribution of the mea

- Show where the distribution is centered at and compare it to the theoretical center of the distributio
- Show how variable it is and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.
- Evaluate the coverage of the confidence interval for 1/lambda:: $\bar X \pm 1.96 \frac{S}{\sqrt{n}}$.
```

Simulation

The next code runs a thousand simulations of 40 exponential(0.2)s and store the values in a matrix with 1000 columns and 40 rows. Each matrix element corresponds to a value of an exponential(0.2).

```
set.seed(1994)
lambda <- 0.2
numTests <- 1000
testCount <- 40
data <- matrix(rexp(numTests * testCount, rate=lambda), nrow = numTests)
means <- rowMeans(data)

# Compute the mean of each row of the generated data.
meanDist <- apply(data, 1, mean)
```

Question 1 & 2:

1. Show where the distribution is centered at and compare it to the theoretical center of the distribution.
2. Show how variable it is and compare it to the theoretical variance of the distribution.

We can now concetrate on some particular values:

```
centre <- round(mean(meanDist), 3)
theoreticalmean <- round(1/lambda, 3)

SD <- round(sd(meanDist), 3)
theoreticalSD <- round(1/(lambda * sqrt(testCount)), 3)

var <- round(var(meanDist), 3)
theoreticalvar <- round((1/(lambda * sqrt(testCount)))^2, 3)
```

After the computation of the values we can quickly compare them:

- mean: 5.012 and theoreticalmean:5

- variance: 0.608 and theoreticalvariance = 0.625

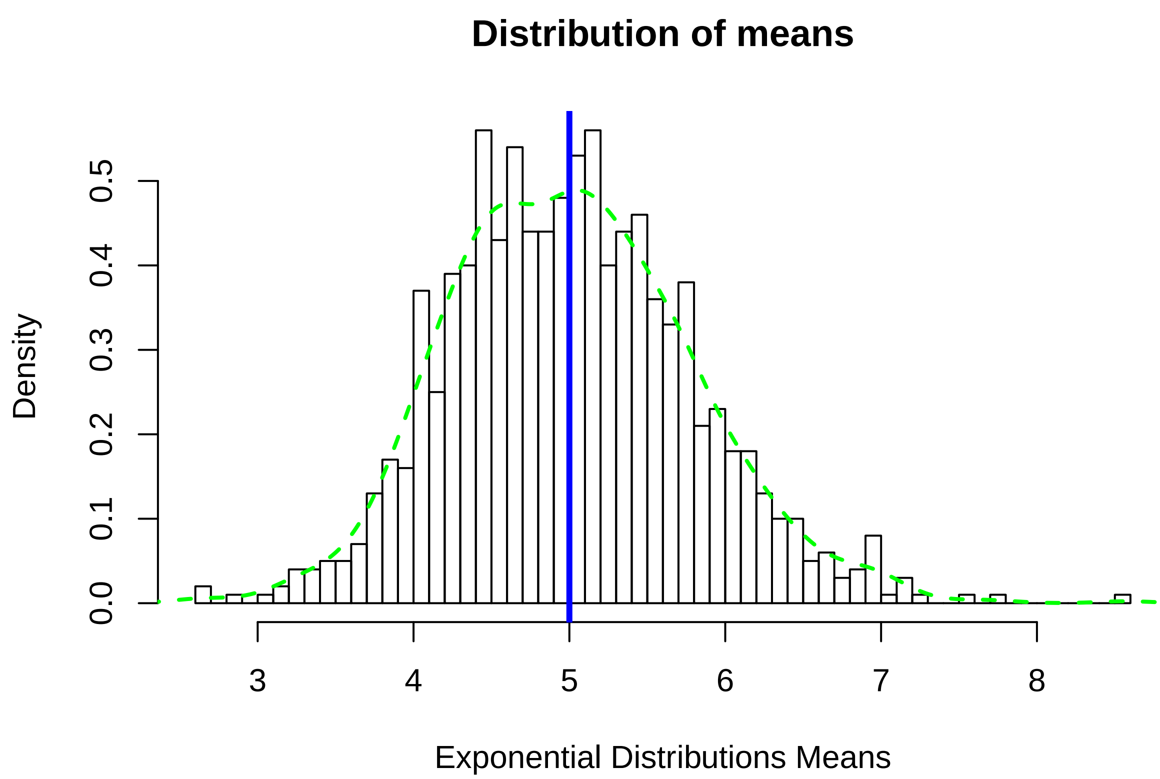
- mean: |centre-theoreticalmean| = 0.012
- variance: |var-theoreticalvar| = 0.017

As we can see, the values are not only comparable but almost the same. Computed Mean and Varaince are almost similar to the Theoretical mean and varainace

Question 3 :Show that the distribution is approximately normal.

We can quickly provide a graphical rappresentation by plotting the data.

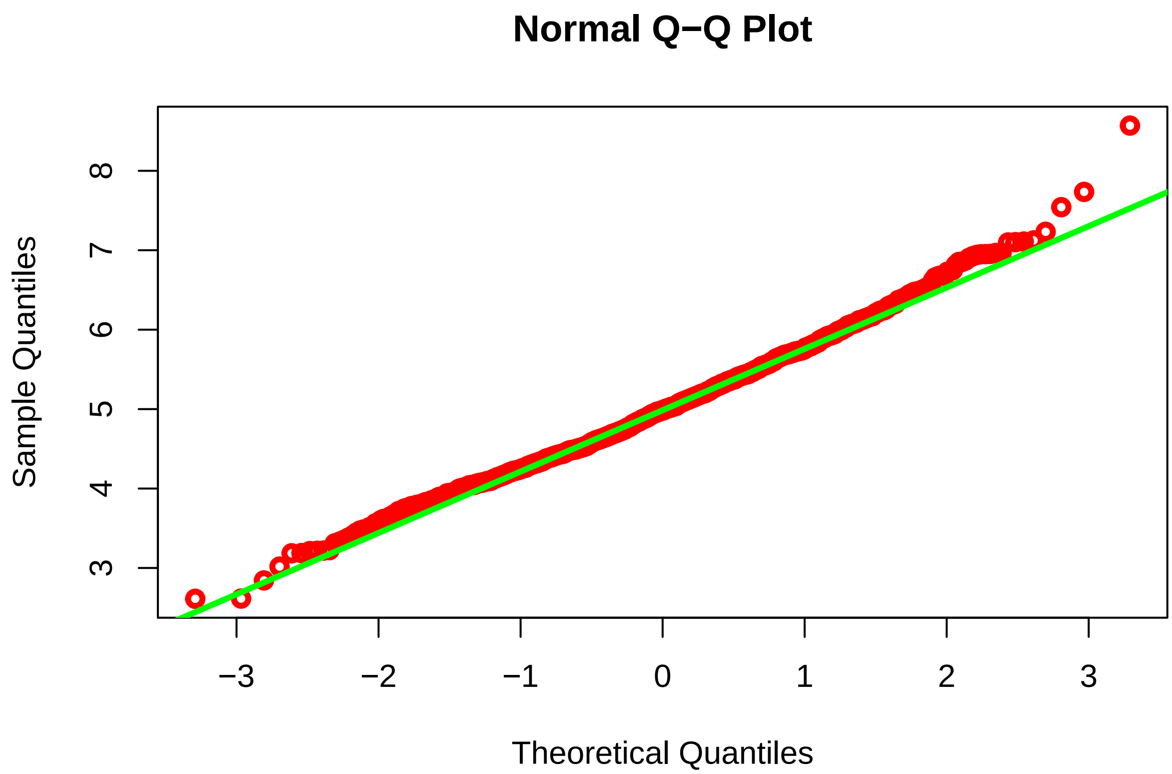
```r
# plot the histogram of the means
hist(means, breaks=50, prob=TRUE,
     main="Distribution of means",
     xlab="Exponential Distributions Means",
     ylab="Density")
lines(density(means), col="green", lwd=2, lty=2)
# show the mean of distribution
abline(v=1/lambda, col="blue", lwd=3)
```



The plot shows that the distribution get close to the normal distribution.

Using a QQplot we can show that the distribution is very similar to the normal, apart from the tails that tends to diverge from the line.

```
qqnorm(means, col="red", lwd=3)
qqline(means, col="green", lwd=3)
```



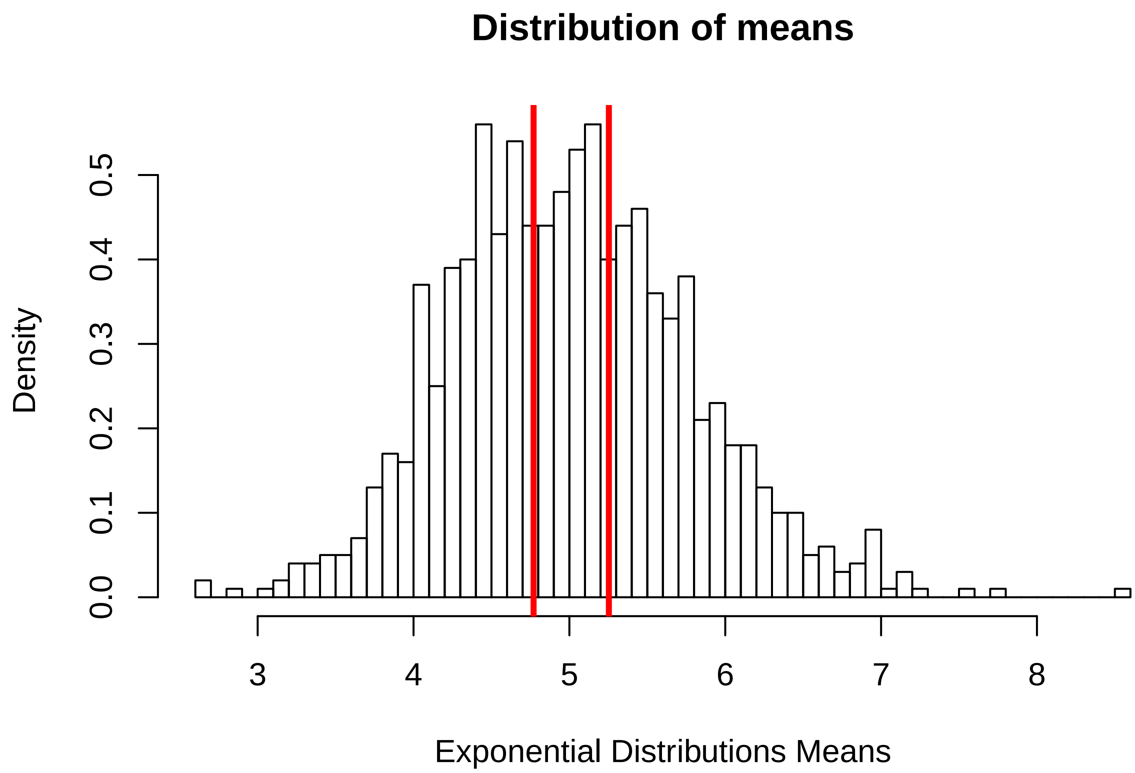
Question 4: Evaluate the coverage of the confidence interval for 1/lambda:: $\bar{X} \pm 1.96 \frac{S}{\sqrt{n}}$.

We can finally compute the 95% confidence intervals:

```
CI <- 1.96
ll <- mean(means) - CI * (sd(means)/sqrt(testCount))
ul <- mean(means) + CI * (sd(means)/sqrt(testCount))
```

Lower interval 4.77 and upper interval 5.253.

```
# plot the histogram of the means
hist(means, breaks=50, prob=TRUE,
     main="Distribution of means",
     xlab="Exponential Distributions Means",
     ylab="Density")
# show the two CIs
abline(v=c(ll, ul), col="red", lwd=3)
```


Distribution of means
Density
0.0
0.1
0.2
0.3
0.4
0.5
3
4
5
6
7
8
Exponential Distributions Means