Probability Assignment —

Q1. Two dies rolled at once. Find out the probability for sum of numbers being even and one of die shows 6.

$P(A \cup B)$ = $P(A) + P(B) - P(A \cap B)$

A: Sum of numbers being even

B: shows 6

$= \frac{18}{36} + \frac{1}{36} - \frac{1}{36}$

$= 0.5$

$P(A \cap B)$ = $P(A) * P(B)$

A: Sum of no's being even

B: shows 6

and

$= \frac{18}{36} * \frac{1}{36} = 0.013$

P(Sum of no's being even and one of die shows 6) $= \frac{5}{36} = 0.138$

(26, 46, 62, 64, 66)

Total outcomes

2. Two dies are rolled at once. Find out the probability for sum of numbers being less than 7.

P(sum of numbers less than 7) $= \frac{15}{36} = 0.416$

(11, 12, 13, 14, 15, 21, 22, 23, 24, 31, 32, 33, 41, 42, 51)

Q3. You toss a fair coin 3 times. Given that you have observed one heads, what is the probability that you observe atleast 2 heads?

~~When calculating probability to have atleast 2 heads in three flips~~

~~Since we already observed one head, so we can calculate the probability of one head in 2 flips~~

~~P(atleast one head in 2 flips) = 3/4 = 0.75~~

$$P(2H/1H) = \frac{P(1H \cap 2H)}{P(1H)} = 4/7$$

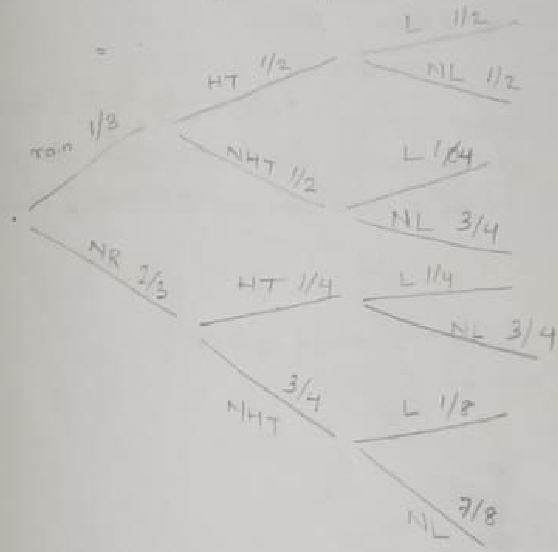
HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

Q4. In my town, it's rain one third of the days. Given that it is raining, there will be heavy traffic with probability 1/4. If it's raining and there is heavy traffic, I arrive late at work with probability 1/2. On the other hand, the probability of being late is 1/8, if it is not raining and there is no heavy traffic. In other situations (raining and no traffic, not raining and traffic), the probability of being late is 0.25. You pick a random day.

(a) What is the probability that it's not raining and there is heavy traffic and I am not late?

P (not raining ∩ Heavy traffic ∩ not late) = ?

=



$$P(NR \cap HT \cap NL) = P(NR) \times P(HT/NR) \times P(NL/HT \text{ and } NR)$$

$$= 2/3 \times 1/4 \times 3/4$$

$$= 1/8$$

(b) What is the probability that ~~it is not raining and there~~ I am late?

Tracing back the tree,

$$\begin{aligned} P(\text{Late}) = {} & P(\text{rain}) \times P(HT|R) \times P(L|\text{rain and } HT) \\ & + P(\text{rain}) \times P(NHT|R) \times P(L|R \text{ and } NHT) \\ & + P(NR) \times P(HT|NR) \times P(L|NR \text{ and } HT) \\ & + P(NR) \times P(NHT|NR) \times P(L|NR \text{ and } NHT) \end{aligned}$$

$$= 11/48$$

(c) Given that I arrive late at work, what is the probability that it rained that day?

$P(\text{rain} \mid \text{Late}) = ?$

$$= \frac{P(\text{rain} \cap \text{Late})}{P(\text{Late})}$$

→ We already know 11/48

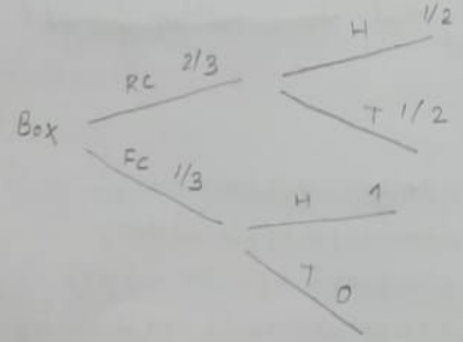
$$= \frac{P(\text{rain}) \times P(HT \mid \text{rain}) \times P(\text{Late} \mid \text{rain and } HT) + P(\text{rain}) \times P(\overline{HT} \mid \text{rain}) \times P(\text{Late} \mid \overline{HT} \text{ and rain})}{11/48}$$

$$= \frac{1/3 \times 1/2 \times 1/2 + 1/3 \times 1/2 \times 1/4}{11/48}$$

$$= \frac{1/8}{\frac{11}{48}} = 6/11$$

5) A box contains three coins. Two regular and one fake headed ($P(\text{Heads}) = 1$), you pick a coin at random and toss it.

(a) What is the probability that it lands heads up?



So,

$$P(H) = P(H \mid RC) \times P(RC) + P(H \mid FC) \times P(FC)$$
$$= 1/2 \times 2/3 + 1 \times 1/3$$
$$= 2/3 = 0.667$$

(b) You pick a coin at random and toss it and get heads. What is the probability that it is a 2 headed coin.

$$P(FC \mid H) = \frac{P(FC \cap H)}{P(H)}$$

$$= \frac{P(FC) \times P(H \mid FC)}{P(H)}$$

$$= \frac{1/3 \times 1}{2/3} = 1/2$$

(6) Suppose that, of all the customers at a coffee shop,

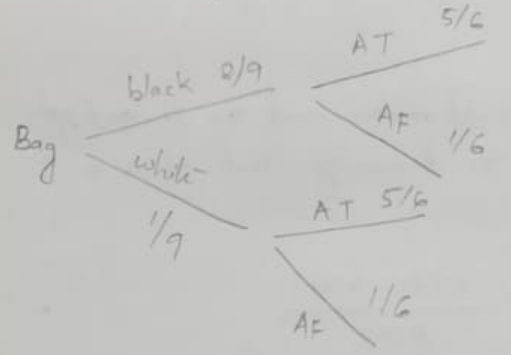
(A) 70% purchase a cup of coffee
(b) 40% purchase a piece of cake
(c) 20% purchase both coffee and cake.

Given that a randomly chosen customer has purchased a piece of cake, what is the probability he/she has also purchased a cup of coffee.

$$P(\text{coffee} \mid \text{cake}) = \frac{P(\text{coffee} \cap \text{cake})}{P(\text{cake})}$$

$$= \frac{0.2}{0.4} = 1/2 = 0.5$$
$$= 50\%$$

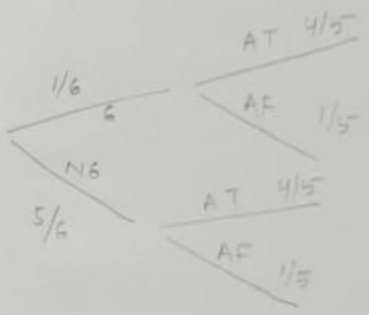
Q11. A is known to tell the truth on 5 cases out of 6. And he states that a white ball was drawn from a bag containing 8 blacks and 1 white ball. Find the probability that the white ball was drawn.



$$P(\text{white}) = \frac{P(\text{white ball was drawn and A says the truth})}{P(\text{white ball was drawn and A says the truth}) + P(\text{black ball was drawn and A says white ball was drawn})}$$

$$= \frac{5/6 \times 1/9}{5/6 \times 1/9 + 1/6 \times 8/9} = 5/13$$

12. A speaks 4 out of 5 times. A die is tossed. A reports that it is a 6. What are the chances that there actually was a 6.



$$P(6) = \frac{P(6 \text{ and A says truth})}{P(\text{Not 6 and A says false that it is a 6})}$$

$$= \frac{4/5 \times 1/6}{4/5 \times 1/6 + 1/5 \times 5/6} = 4/9 = 0.44$$