Covariance Matrix Resolution Calculations

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INTRODUCTION

The purpose of this guide is to provide a brief review of the mathematics and relevant experimental context using the covariance matrix resolution program.

DERIVATION

Consider a set of m functions $\{f_1, ..., f_m\}$ which each depend on a set of n random variables $\{x_1, ..., x_n\}$. We define the Jacobian matrix of our system to be

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \quad (1)$$

If the assumption is made that each of our n random variables is normally distributed, not necessarily independently of the others, then we have a well-defined, although possibly difficult to compute, covariance matrix $\Sigma_{\mathbf{x}}$ which describes the joint distribution.

Again operating under simplifying assumptions, we assume the covariance matrix $\Sigma_{\mathbf{x}}$ describes linear deviations, and thus we compute an approximate covariance matrix, Σ, for the m variables described by our m functions f_k as follows:

$$\Sigma = J\Sigma_{\mathbf{x}}J^T \quad (2)$$

which yields an m x m covariance matrix for our system.

ARCS

Pertinent Experimental Variables:

- L_{sp} = (m) distance from sample to detector pixel
- L_{12} = (m) distance between beam monitors 1 and 2
- L_{ms} = (m) distance from moderator to sample position
- t_{12} = (s) time for a neutron to travel from beam monitor 1 to monitor 2
- t_{ms} = (s) time for neutron to travel from moderator to sample
- t_{sp} = (s) time for neutron to travel from sample to detector pixel
- $v_{i,f}$ = (m/s) initial (resp., final) neutron velocity
- $E_{i,f}$ = (m/s) initial (resp., final) neutron velocity
- $Q_{x,y,z}$ = x (resp. y, resp. z) component of neutron wavevector in instrumental beam coordinates (z along beam, y vertical, x completing right-hand coordinate system)

TABLE I. ARCS Instrument Parameters

Instrument Parameters	Values
L_{sp}	(event dependent)
L_{12}	6.67 m
L_{ms}	13.60 m