BILAL HASSAN — AE 331

2350015 — MIDTERM

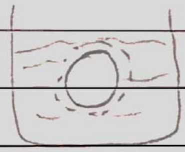
Q2. $T_\infty = 305K$, $D = 0.005m$, $k = 25\ W/mK$, $\rho = 8015\ kg/m^3$

$c_p = 515\ J/kgK$, $h = 515\ W/m^2K$, $\dot{q} = 5\times10^8\ W/m^3$.

(a) $Bi = \frac{hL_c}{k}$, $L_c = \frac{V_s}{A_s} = \frac{4/3\,\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{d}{6} = 8.33\times10^{-4}$

$Bi = \frac{515\times8.33\times10^{-4}}{25} = 0.0172 < 0.1$ so Lumped capacitance method can be used

(b) Applying cons. of energy over the volume of sphere



$$\cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$0 \quad -q_{conv} + \dot{q}V = \rho V c_p \frac{dT}{dt}$$

$$-hA_s(T-T_\infty) + \dot{q}V = \rho V c_p \frac{dT}{dt} \qquad \theta = T - T_\infty \qquad \frac{d\theta}{dt} = \frac{dT}{dt}$$

$$-hA_s\theta + \dot{q}V = \rho V c_p \frac{d\theta}{dt}$$

$$\frac{\dot{q}V - \rho V c_p \frac{d\theta}{dt}}{hA_s} = \theta \qquad = \frac{\dot{q}L_c}{h} - \frac{\rho L_c c_p}{h}\frac{d\theta}{dt} = \theta$$

$$= 808.74 - 6.6765\frac{d\theta}{dt} = \theta$$

$$\frac{808.74-\theta}{6.6765} = \frac{d\theta}{dt}$$

$$\int_0^t \frac{1}{6.6765}\,dt = \int_{\theta_i}^{\theta} \frac{d\theta}{808.74-\theta}$$

$$\rightarrow \frac{t}{6.6765} = -\ln(808.74-\theta) + \ln(808.74-\theta_i) = \ln\left(\frac{808.74-\theta_i}{808.74-\theta}\right)$$

$$e^{\frac{-t}{6.6765}} = \frac{808.74-\theta_i}{808.74-\theta} \qquad \theta = T - T_\infty$$

$$\theta_i = T_i - T_\infty$$

$$\frac{808.74-\theta}{808.74-\theta_i} = \exp\left(-\frac{1}{6.6765}t\right)$$

As $t\to\infty$, $808.74 = T - T_\infty = \theta$

$$T = 808.74 + 305 = \boxed{1113.74K}$$

(c) $\dfrac{t}{6.6765} = \ln\left(\dfrac{808.74 - \theta_i}{808.74 - \theta}\right)$

t for $T = 1113.74\,K - 1 = 1112.74\,K$ [within 1K]

$\theta = 1112.74 - 305 = 807.74$.

$\dfrac{t}{6.6765} = \ln\left(\dfrac{808.74 - \theta_i}{\quad}\right)$ Assuming $\theta_i = 305\ K$ (water Temp) initial

$\boxed{t = 41.542\,s}$