I affirm that all the work done on this exam is my own and that I have not given nor received any help during this exam. I understand that any indication of violation of this word of honour may lead to a zero grade on this exam and to a disciplinary action.

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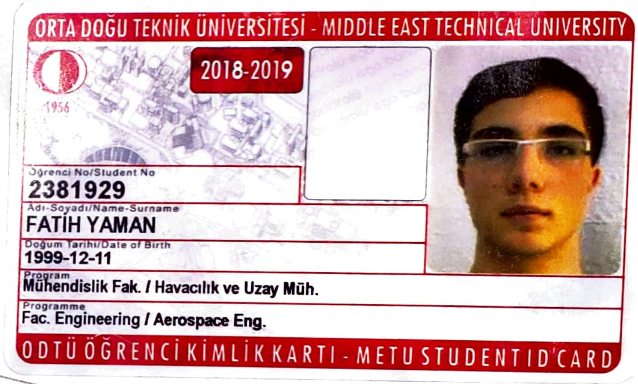
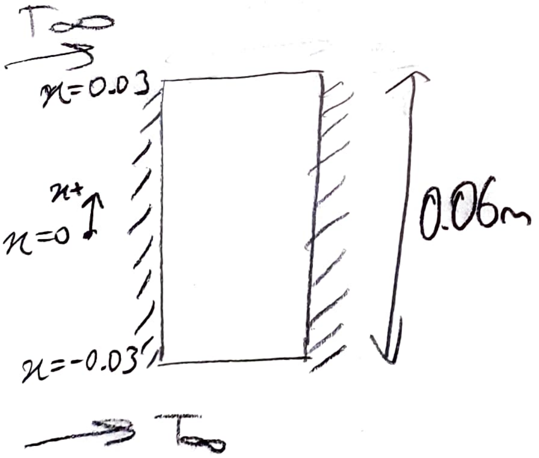
AEE311 Mid-Term 1



Question 2



$A_T = 1 m^2$

$A_B = 1 m^2$

$\dot{q} = 200 kW/m^3 = 200000 W/m^3$

$T_\infty = 300 K$ $\quad h = 35 W/m^2K$

$k = 15 W/mK$ $\quad \rho = 7500 kg/m^3$

$C_p = 420 J/kgK$

Assume:

1) 1D heat transfer

2) constant k

(a) use general heat eqn for 1D;

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\dot{q}=\rho c_p \cancel{\frac{\partial T}{\partial t}} \rightarrow \text{steady state}$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{-\dot{q}}{k} \qquad \frac{\partial T}{\partial x}=\frac{-\dot{q}x}{k}+C_1$$

$$T=\frac{-\dot{q}x^2}{2k}+C_1x+C_2$$

BC: $T\big|_{x=0.03}=340 \Rightarrow \frac{-200000(0.03)^2}{2(15)}+0.03C_1+C_2=340$

$T\big|_{x=-0.03}=340 \Rightarrow \frac{-200\,000(0.03)^2}{2(15)}-0.03C_1+C_2=340$

+

$$\frac{-200\,000(0.03)^2}{15}+2C_2=680 \Rightarrow C_2=346$$

$$C_1=0$$

$$T=\frac{-\dot{q}x^2}{2k}+346=\frac{-200\,000\,x^2}{30}+346$$

T is max at $x=0$;

$T\big|_{x=0}=346\,K \rightarrow$ max temperature across wall

(b)

$$Bi = \frac{hL}{k} = \frac{35 \times 0.03}{15} = 0.07 \ll 1$$

So we can use lumped capacitance method for part (b)

$T_i = 340\ K$

we will use case 2 of the general lumped capacitance analysis;

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}\left[1 - \exp(-at)\right] \quad (*)$$

$$a = \frac{hA}{\rho V C_p} \qquad b = \frac{q''_s A_n + \dot{E}_g}{\rho V C_p};$$

where; A is the total area of convection $A = 2m^2$

A_n is the area of heat flux. $q''_s = A_n = 0$

$\dot{E}_g$ is the heat generated $= \dot{q}V = \dot{q}A0.06 = 200000 \times 0.06$

$$a = \frac{35 \times 2}{7500 \times 1 \times 0.06 \times 420} = 3.70 \times 10^{-4}$$

$$b = \frac{200000 \times 0.06}{7500 \times 1 \times 0.06 \times 420} = 0.06349$$

$T = 350\ K$

put it into (*)

$$\frac{350-300}{340-300} = \exp(-3.70\times10^{-4}\times t) + \frac{0.06349/3.70\times10^{-4}}{340-300}\left[1-\exp(-3.70\times10^{-4}t)\right]$$

$$1.25 = \exp(-3.70\times10^{-4}t) + 4.285714\left[1-\exp(-3.70\times10^{-4}t)\right]$$

$$\exp(-3.70\times10^{-4}t) = \frac{1.25-4.285714}{(1-4.285714)}$$

$$t = \frac{\ln\left(\frac{1.25-4.285714}{1-4.285714}\right)}{-3.70\times10^{-4}}$$

$$t = 213.88\,s$$

→ this is the time required to reach 350K