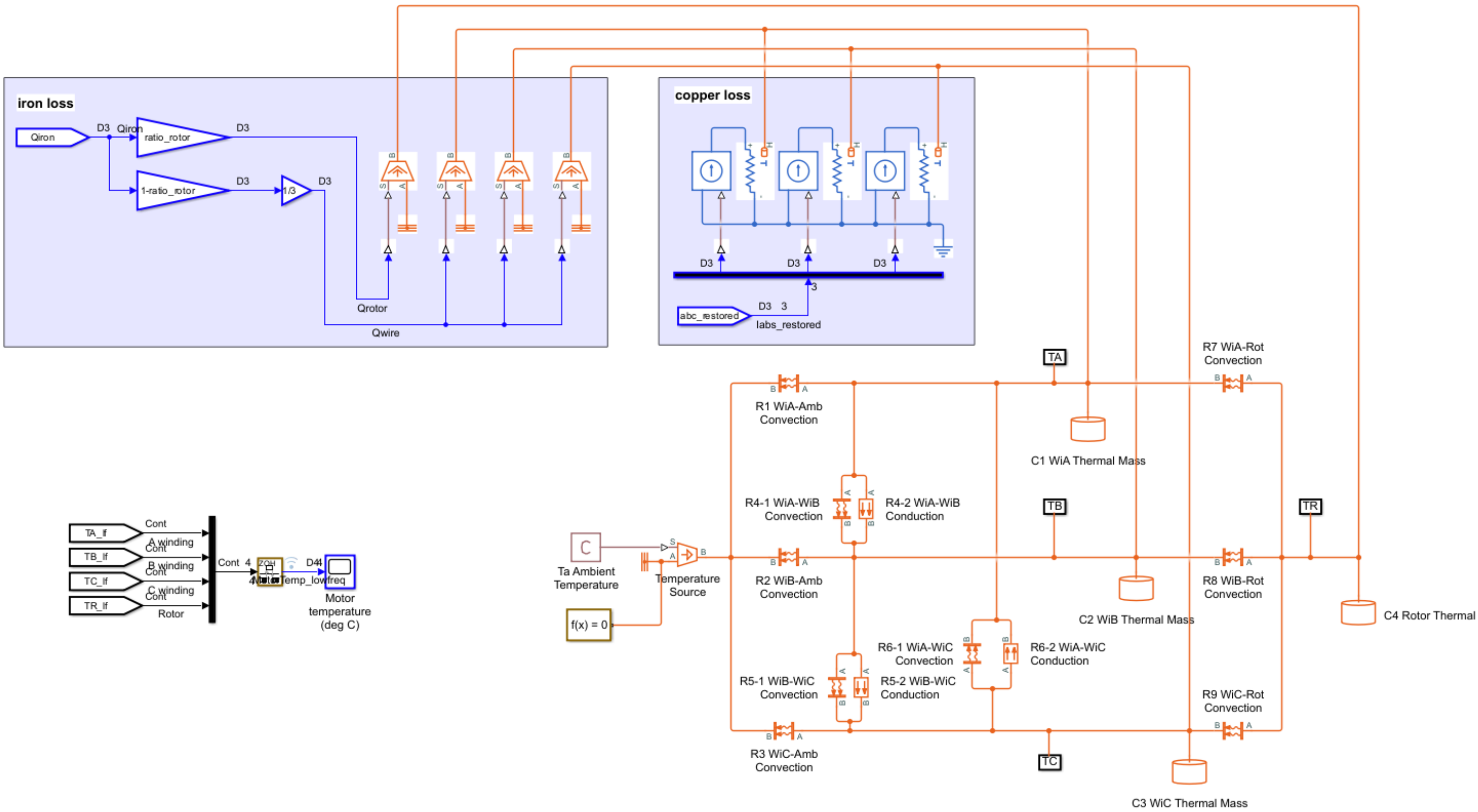
# Derivation of state equation for thermal network

```
close all;
clear;
clc;
```




## define variables

### state

$x_1$ wire A temperature $T_A[\mathrm{K}]$

$x_2$ wire B temperature $T_B[\mathrm{K}]$

$x_3$ wire C temperature $T_C[\mathrm{K}]$

$x_4$ rotor temperature $T_R[\mathrm{K}]$

### input

$u_1$ wire A current $I_A[\mathrm{A}]$

$u_2$ wire B current $I_B[\mathrm{A}]$

$u_3$ wire C current $I_C[\mathrm{A}]$

$u_4$ ambient temperature $T_a[\mathrm{K}]$

$u_5$ iron loss $Q_{iron}[\mathrm{W}]$

```
% states
```

```
syms x [4,1] real
T_A = x(1);
T_B = x(2);
T_C = x(3);
T_R = x(4);

% inputs
syms u [5,1] real
I_A = u(1);
I_B = u(2);
I_C = u(3);
T_a = u(4);
Q_iron = u(5);

% params
syms R [9,1] real
syms C [4,1] real
syms T_0 real
syms R_0 real
syms alpha real
syms ratio_rotor real
```

## heat flux

### copper loss

```
Q_copper_A = I_A^2*R_0*(1+alpha*(T_A-T_0));
Q_copper_B = I_B^2*R_0*(1+alpha*(T_B-T_0));
Q_copper_C = I_C^2*R_0*(1+alpha*(T_C-T_0));
```

### iron loss

#### wire

```
Q_iron_wire = Q_iron*(1-ratio_rotor);
Q_iron_A = Q_iron_wire/3;
Q_iron_B = Q_iron_wire/3;
Q_iron_C = Q_iron_wire/3;
```

#### rotor

```
Q_iron_rotor = Q_iron*ratio_rotor;
```

### through thermal resistance

#### wire to environment

```
q(1) = (T_A-T_a)/R(1);
q(2) = (T_B-T_a)/R(2);
q(3) = (T_C-T_a)/R(3);
```

#### wire to wire

```
q(4) = (T_A-T_B)/R(4);
q(5) = (T_B-T_C)/R(5);
q(6) = (T_C-T_A)/R(6);
```

### rotor to wire

```
q(7) = (T_R-T_A)/R(7);
q(8) = (T_R-T_B)/R(8);
q(9) = (T_R-T_C)/R(9);
```

## time derivative of temperature

### state equation

```
f(1) = (Q_copper_A+Q_iron_A-q(1)-q(4)+q(6)+q(7))/C(1);
f(2) = (Q_copper_B+Q_iron_B-q(2)-q(5)+q(4)+q(8))/C(2);
f(3) = (Q_copper_C+Q_iron_C-q(3)-q(6)+q(5)+q(9))/C(3);
f(4) = (Q_iron_rotor-q(7)-q(8)-q(9))/C(4);

f = f.'
```

f =

$$\begin{pmatrix} -\dfrac{\sigma_1 - \dfrac{u_4 - x_1}{R_1} + \dfrac{x_1 - x_2}{R_4} + \dfrac{x_1 - x_3}{R_6} + \dfrac{x_1 - x_4}{R_7} + R_0\,{u_1}^2\,\left(\alpha\,\left(T_0 - x_1\right) - 1\right)}{C_1} \\ -\dfrac{\sigma_1 - \dfrac{u_4 - x_2}{R_2} - \dfrac{x_1 - x_2}{R_4} + \dfrac{x_2 - x_3}{R_5} + \dfrac{x_2 - x_4}{R_8} + R_0\,{u_2}^2\,\left(\alpha\,\left(T_0 - x_2\right) - 1\right)}{C_2} \\ -\dfrac{\sigma_1 - \dfrac{u_4 - x_3}{R_3} - \dfrac{x_2 - x_3}{R_5} - \dfrac{x_1 - x_3}{R_6} + \dfrac{x_3 - x_4}{R_9} + R_0\,{u_3}^2\,\left(\alpha\,\left(T_0 - x_3\right) - 1\right)}{C_3} \\ \dfrac{\mathrm{ratio}_{\mathrm{rotor}}\,u_5 + \dfrac{x_1 - x_4}{R_7} + \dfrac{x_2 - x_4}{R_8} + \dfrac{x_3 - x_4}{R_9}}{C_4} \end{pmatrix}$$

where

$$\sigma_1 = \frac{u_5\,\left(\mathrm{ratio}_{\mathrm{rotor}} - 1\right)}{3}$$

### output equation

```
h(1) = T_R;

h = h.'
```

h = $x_4$

## substitute parameters

```
controller_parameters();
f = subs(f,C,C_);
f = subs(f,R,R_);
f = subs(f,[T_0,R_0,alpha], [T_0_,R_0_,alpha_]);
f = subs(f,ratio_rotor,ratio_rotor_);
f
```

f =

$$\begin{pmatrix} \frac{u_4}{50}+\frac{u_5}{1200}-\frac{27\,x_1}{250}+\frac{3\,x_2}{125}+\frac{3\,x_3}{125}+\frac{x_4}{25}+\frac{13\,{u_1}^2\left(\frac{393\,x_1}{100000}-\frac{343459}{2000000}\right)}{100000} \\ \frac{u_4}{50}+\frac{u_5}{1200}+\frac{3\,x_1}{125}-\frac{27\,x_2}{250}+\frac{3\,x_3}{125}+\frac{x_4}{25}+\frac{13\,{u_2}^2\left(\frac{393\,x_2}{100000}-\frac{343459}{2000000}\right)}{100000} \\ \frac{u_4}{50}+\frac{u_5}{1200}+\frac{3\,x_1}{125}+\frac{3\,x_2}{125}-\frac{27\,x_3}{250}+\frac{x_4}{25}+\frac{13\,{u_3}^2\left(\frac{393\,x_3}{100000}-\frac{343459}{2000000}\right)}{100000} \\ \frac{3\,u_5}{800}+\frac{x_1}{50}+\frac{x_2}{50}+\frac{x_3}{50}-\frac{3\,x_4}{50} \end{pmatrix}$$

### export to file

```
matlabFunction(f,'File','nonlinear_model/state_equation.m','Vars',{x,u});
matlabFunction(h,'File','nonlinear_model/output_equation.m','Vars',{x});
matlabFunction(x+f*Ts,'File','nonlinear_model/state_equation_discrete.m','Vars',
{x,u});
```

## linearization

### Jacobi matrix

```
A = jacobian(f,x)
```

A =

$$\begin{pmatrix} \frac{5109\,{u_1}^2}{10000000000}-\frac{27}{250} & \frac{3}{125} & \frac{3}{125} & \frac{1}{25} \\ \frac{3}{125} & \frac{5109\,{u_2}^2}{10000000000}-\frac{27}{250} & \frac{3}{125} & \frac{1}{25} \\ \frac{3}{125} & \frac{3}{125} & \frac{5109\,{u_3}^2}{10000000000}-\frac{27}{250} & \frac{1}{25} \\ \frac{1}{50} & \frac{1}{50} & \frac{1}{50} & -\frac{3}{50} \end{pmatrix}$$

```
B = jacobian(f,u)
```

B =

$$\begin{pmatrix} \frac{13\,u_1\left(\frac{393\,x_1}{100000}-\frac{343459}{2000000}\right)}{50000} & 0 & 0 & \frac{1}{50} & \frac{1}{1200} \\ 0 & \frac{13\,u_2\left(\frac{393\,x_2}{100000}-\frac{343459}{2000000}\right)}{50000} & 0 & \frac{1}{50} & \frac{1}{1200} \\ 0 & 0 & \frac{13\,u_3\left(\frac{393\,x_3}{100000}-\frac{343459}{2000000}\right)}{50000} & \frac{1}{50} & \frac{1}{1200} \\ 0 & 0 & 0 & 0 & \frac{3}{800} \end{pmatrix}$$

```
C = jacobian(h,x)
```

C = $\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$

```
matlabFunction(A,'File','Jacobian/dfdx.m','Vars',{x,u});
matlabFunction(C,'File','Jacobian/dhdx.m','Vars',{x});
```

### equilibrium point to check observability

```
xe = [298.15,298.15,298.15,298.15]';
ue = [1.0,2.0,3.0,300,0.0]';

A_lin = double(subs(A,[x;u],[xe;ue]));
B_lin = double(subs(B,[x;u],[xe;ue]));
C_lin = double(subs(C,[x;u],[xe;ue]));
```

### observability

```
Go = obsv(A_lin,C_lin);
rank(Go)
```

```
ans =
     4
```