Analytical solutions for the transient temperature field of the semi-infinite body subjected to 3-D power density moving heat sources.

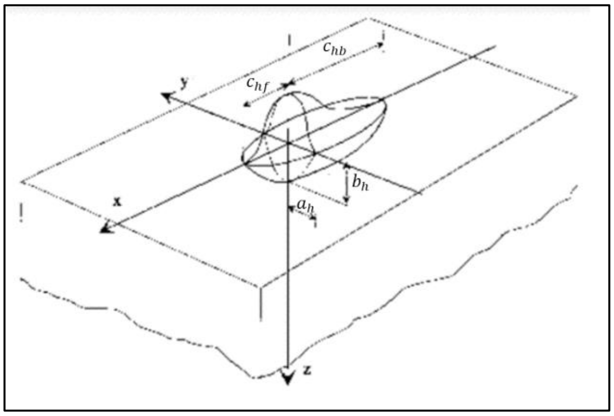

Figure 1. Double ellipsoidal density distributed heat source.

A Goldak double ellipsoidal heat source model is considered and the heat flux Q(x,y,z) at a point (x,y,z) within the ellipsoid is given by :

For a point (x,y,z) within the front portion of semi ellipsoid located in front portion of the welding arc, the heat flux is described as :

$$Q(x,y,z) = \frac{6\sqrt{3}r_f Q}{a_h b_h c_{hf}\pi\sqrt{\pi}} \exp\left(-\frac{3x^2}{c_{hf}^2} - \frac{3y^2}{a_h^2} - \frac{3z^2}{b_h^2}\right) \quad (1)$$

For point (x,y,z) in the rear portion of the semi ellipsoid covering the rear portion of the arc, as

$$Q(x,y,z) = \frac{6\sqrt{3}r_b Q}{a_h b_h c_{hb}\pi\sqrt{\pi}} \exp\left(-\frac{3x^2}{c_{hb}^2} - \frac{3y^2}{a_h^2} - \frac{3z^2}{b_h^2}\right) \quad (2)$$

where a_h,b_h,c_{hf}, c_{hb} = ellipsoidal heat source parameters as described in Fig. 1;
Q = arc heat input ($Q = \eta\ VI$); r_f, r_b = proportion coefficient representing heat in front and back of the heat source, respectively, where $r_f + r_b = 2$.

The solution for the temperature field of a semi-ellipsoidal heat source in a semi-infinite body is based on the solution for an instant point source that satisfies the following differential equation for *heat conduction* in fixed coordinates:

$$dT_{t'} = \frac{\delta Q dt'}{\rho c[4\pi a(t-t')]^{\frac{3}{2}}} . \exp\left(-\frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4a(t-t')}\right) \quad (3)$$

where a = thermal diffusivity ($a = k/c\rho$); c = specific heat; k = thermal conductivity; ρ = mass density; t, t' = time; $dT_{t'}$ = transient temperature due to the point heat source δQ at time t'; and (x',y',z') = location of the instant point heat source δQ at time t'.

The solution for the double ellipsoidal heat source is the superposition of a series of instant point heat sources over the volume of the distributed Gaussian heat source. Substitute Equation 1 for the point source in Equation 3 and take integration over the heat source volume for both front and back portion of the volume heat source.
This volume is corresponding to two quadrants of the front and back semi ellipsoids, respectively, as

$$dT_{t'} = \frac{1}{4}\frac{6\sqrt{3}Qdt'}{\rho c a_h b_h \pi \sqrt{pi}[4\pi a(t-t')]^{\frac{3}{2}}} \tag{4}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4a(t-t')}\right)$$

$$\left(\frac{r_f}{c_{hf}}.\exp\left(-\frac{3x'^2}{c_{hf}^2}-\frac{3y'^2}{a_h^2}-\frac{3z'^2}{b_h^2}\right)+\frac{r_b}{c_{hb}}.\exp\left(-\frac{3x'^2}{c_{hb}^2}-\frac{3y'^2}{a_h^2}-\frac{3z'^2}{b_h^2}\right)\right).dx'dy'dz'$$

When considering a moving heat source with a constant speed v from time *t' = 0* to time *t' = t*, the increase of the temperature during this time is equivalent to the sum of all the contributions of the moving heat source during the travelling time as:

moving along x axis : x' = vt' ; y' and z'=0;

$$T - T_0 = \int_0^t Equation\ (4) \tag{5}$$

The governing equation for heat transfer energy balance is:

$$\rho C\frac{dT}{dt} = -\nabla\cdot\mathbf{q}(\mathbf{r},t) + Q(\mathbf{r},t)$$

ρ is the material density, C is the specific heat capacity, T is the temperature, t is the time, Q is the heat source, **r** is the relative reference coordinate, and **q** is the heat flux vector, calculated as

$$q = -k\nabla T$$

where k is the thermal conductivity of the material.