MEASUREMENT AND DATA PROCESSING

11

CORE

11.1 UNCERTAINTY & ERROR IN MEASUREMENT

11.1.1 Describe and give examples of random uncertainties and systematic errors.

11.1.2 Distinguish between precision and accuracy.

11.1.3 Describe how the effects of random uncertainties may be reduced.

11.1.4 State random uncertainty as an uncertainty range (±).

11.1.5 State the results of calculations to the appropriate number of significant figures.

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In practical science, the results of experiments are never completely reliable as there are always **experimental errors** and **uncertainties** involved. It is therefore important, especially in quantitative work, to be able to assess the magnitude of these and their effect on the reliability of the final result. It is important to differentiate between the **accuracy** of a result and the **precision** of a result. The accuracy of the result is a **measure** of how close the result is to some accepted, or literature value for the quantity being determined. For example, an experiment that gives a value of 8.317 J mol^{-1} K^{-1} for the ideal gas constant (accepted value 8.314 J mol^{-1} K^{-1}) is obviously more accurate than one that gives a value of 8.103 J mol^{-1} K^{-1}. The accuracy is usually measured as the percentage deviation from the accepted value using the expression:

Percentage deviation =

$$\left| \frac{\text{Experimental value} - \text{Accepted value}}{\text{Accepted value}} \right| \times 100$$

It can be seen that the percentage deviation of the two values quoted for the (ideal) gas constant are 0.04% and 2.54% respectively:

$$\text{Percentage deviation} = \left| \frac{8.317 - 8.314}{8.314} \right| \times 100$$

$$= 0.03608$$
$$= 0.04 \ (\textit{to 1 sig. fig. precision of top line})$$

$$\text{Percentage deviation} = \left| \frac{8.103 - 8.314}{8.314} \right| \times 100$$

$$= 2.53788$$
$$= 2.54 \ (\textit{to 3 sig. fig. precision of top line})$$

The precision of the result is a measure of the certainty of the value determined, usually quoted as a ± value. Again, with regard to the gas constant, one group might quote a result of 8.34 ± 0.03 J mol^{-1} K^{-1}, whilst another group gives a value of 8.513 ± 0.006 J mol^{-1} K^{-1}. Whilst the result of the former group is the more accurate (i.e. closer to

the accepted value), that of the latter group is the more precise (i.e. has the smallest uncertainty). The way that uncertainties are calculated is discussed extensively later in the chapter.

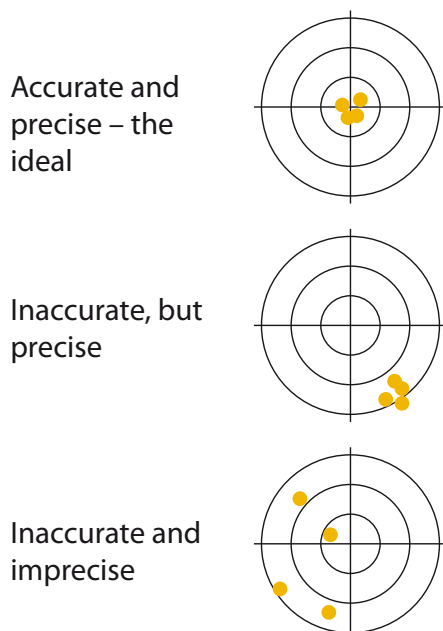The relationship between accuracy and precision is illustrated in Figure 1101.

Accurate and precise – the ideal

Inaccurate, but precise

Inaccurate and imprecise

Figure 1101   The relationship between accuracy and precision

Random uncertainties make a **measurement** less precise, but not in any particular direction, in other words the actual value may be either greater or smaller than the value you record. Random uncertainties (or errors) arise mostly from inadequacies or limitations in the instrument. This may be a result of how precisely you can read a meter or scale of, for example, the burette in Figure 1102. We would probably take the reading as 43.6, but in so doing we are saying that it is nearer to 43.6 than it is to 43.5 or 43.7, in other words it is greater than 43.55 (had it been less we would have recorded it as 43.5) and smaller than 43.65 (had it been greater we would have recorded it as 43.7), hence we should record this value as 43.6 ± 0.05. See Figure 1102.

Figure 1102   A sample reading

Similarly if we are reading a digital instrument, such as a balance, then we should record the uncertainty as being half of the last digit. For example the reading **37.361** on a digital readout should be recorded as 37.361 ± 0.0005, assuming the readout is steady. If the reading is fluctuating, then you have to try to estimate the degree of certainty you feel you have in the value you record. If the last digit keeps going up or down by one, you may feel you can say the value is definitely greater than 37.359 and less than 37.363, so you record it as 37.361 ± 0.001.

In some cases, such as many thermometers, it is only possible to read a scale to the nearest 0.2 (that is, one would record 23.0, 23.2, 23.4 etc., but never an odd final digit such as 23.3). In this case the uncertainty would be ±0.1, because a reading of 23.2 means it is greater than 23.0, but less than 23.4.

Quantitative apparatus usually has a nominal random uncertainty which reflects the tolerances used in its manufacture. With volumetric apparatus it is possible to purchase both A-grade and B-grade apparatus. For example, with a 25 cm^3 bulb pipette the manufacturer's uncertainty for A-grade is ±0.03 cm^3, whereas for B-grade it is ±0.06 cm^3.

Large random uncertainties obviously decrease the precision of the values obtained and can also lead to inconsistent results if the procedure is repeated. If only one reading is taken, then a large random uncertainty can lead to an inaccurate result. Repeating experimental determinations should, however, increase the precision of the final result as the random variations statistically cancel out.

Systematic errors always affect a result in a particular direction and hence the accuracy of the experiment. They arise from flaws or defects in the instrument or from errors in the way that the measurement was taken. If, for example, a student takes the mass of an empty weighing bottle to calculate the mass of solid used, rather than re-weighing it after tipping the contents into a beaker, then this would be a systematic error because the mass of solid in the beaker will always be slightly less than the value the student uses (because some solid may have been left in the weighing bottle). It could never be higher. It is often difficult to allow for such errors quantitatively, but the direction in which it would affect the final result can always be determined. For example, in this case, if the solid were used to prepare a solution that was then titrated with a standard solution from a burette, then it might explain why the burette reading was less than the expected value, but it could not explain a higher reading. Often the order of magnitude is estimated. In this example, (assuming the mass taken was 10.000 g) 'solid left in the bottle' would not be able to explain why the titre was 10% less than expected – you would surely have noticed 1 g of remaining solid.).

Taking the initial reading of a burette when it was well above head height would also give rise to a systematic error. Parallax error, caused by not reading the scale

perpendicularly would, in this case, give a value for the initial reading that was greater than the true reading. Hence the values obtained for the titre would be consistently lower than the true value. Another example of a systematic error would be to read the top rather than the bottom of the meniscus in measuring cylinders and pipettes, which would consistently deliver a smaller volume than that recorded. Errors in the calibration of instruments, for example pH meters, also lead to systematic errors. The most familiar systematic error is heat exchange to the surroundings which, in calorimetric experiments, inevitably leads to smaller temperature changes than would be observed in a perfectly insulated system.

A large systematic error can lead to the result being inaccurate and, if it is a variable systematic error (i.e. it always has an effect in a particular direction, but the size of the error varies), it can lead to inconsistent results.

If you are involved in calculations involving experimental data then these should be combined so as to reflect all of the uncertainties involved and that is covered in the next section. If, however, you are doing calculations involving data that you have been given, then it is important to quote the final result to a precision that reflects that of these data. The general rules to apply are:

- For multiplication and division, the result should have the same number of **significant figures** as the least precise piece of data.

- For addition and subtraction the result should not have more **decimal places** than the least precise piece of data.

Suppose you wish to calculate the formula of a fluoride of uranium given that 5.84 g of uranium produced 8.637 g of the fluoride and that the molar masses of uranium and fluorine are 238.03 and 19.00 g mol^{-1} respectively. If you were initially asked for the mass of fluorine present in the compound, this should be quoted as 2.80 g (rather than 2.797 g as there are only 2 d.p. in the mass data for uranium; note that the final zero in the mass of fluorine should be written as it is significant). The moles of uranium ($\frac{5.84}{238.03}$ = 0.0245347) should be quoted as 0.0245 as the mass of uranium is only known to 3 significant figures. Likewise, the moles of fluorine ($\frac{2.797}{19.00}$ = 0.1472105) should be quoted as 0.147 and the F:U ratio ($\frac{0.147...}{0.0245...}$ = 6.0000937) as 6.00. It is good practice (especially with modern calculators) to keep intermediate results to the maximum precision (in the memory of the calculator) and then to round off the final result to the appropriate precision.

EXTENSION

Another factor related to experimental data is consistency. The **consistency** of a result is a measure of how reproducible the result is when the experiment is repeated. For example, the results of two groups for determining the value of the gas constant (literature value 8.314 J mol^{-1} K^{-1}) are given:

Group A – 8.537; 8.487; 8.598; 8.492; 8.472
Mean = 8.517 J mol^{-1} K^{-1} Range = 0.126

Group B – 8.13; 7.94; 8.44; 8.54; 8.22
Mean = 8.25 J mol^{-1} K^{-1} Range = 0.60

It can be seen that those obtained by Group A are far more consistent and more precise than those obtained by Group B, even though those of Group B are more accurate. The consistency is indicated by the range (maximum value – minimum value), though more sophisticated indicators, such as standard deviation, could be used to compare consistency. The uncertainty of a set of repeated measurements should reflect the consistency of the results. There are many statistical ways of estimating the uncertainty of repeated measurements, but probably one of the simplest is to divide the range of the results (X_{max} – X_{min}) by twice the square root of the number of readings taken (N):

$$\Delta X = \frac{X_{max} - X_{min}}{2\sqrt{N}}$$

Applying this, the results of Group A should be quoted as 8.517 ± 0.028 J mol^{-1} K^{-1} (or perhaps more correctly 8.52 ± 0.03 J mol^{-1} K^{-1}) and that of group B as 8.25 ± 0.13 J mol^{-1} K^{-1}. Note that the results obtained by Group B confirm the literature value (it is within the uncertainty) whereas those of Group A do not. Which group performed the better experiment?

Figure 1103 extends the relationship between accuracy and precision shown in Figure 1101 to include the concept of consistency.
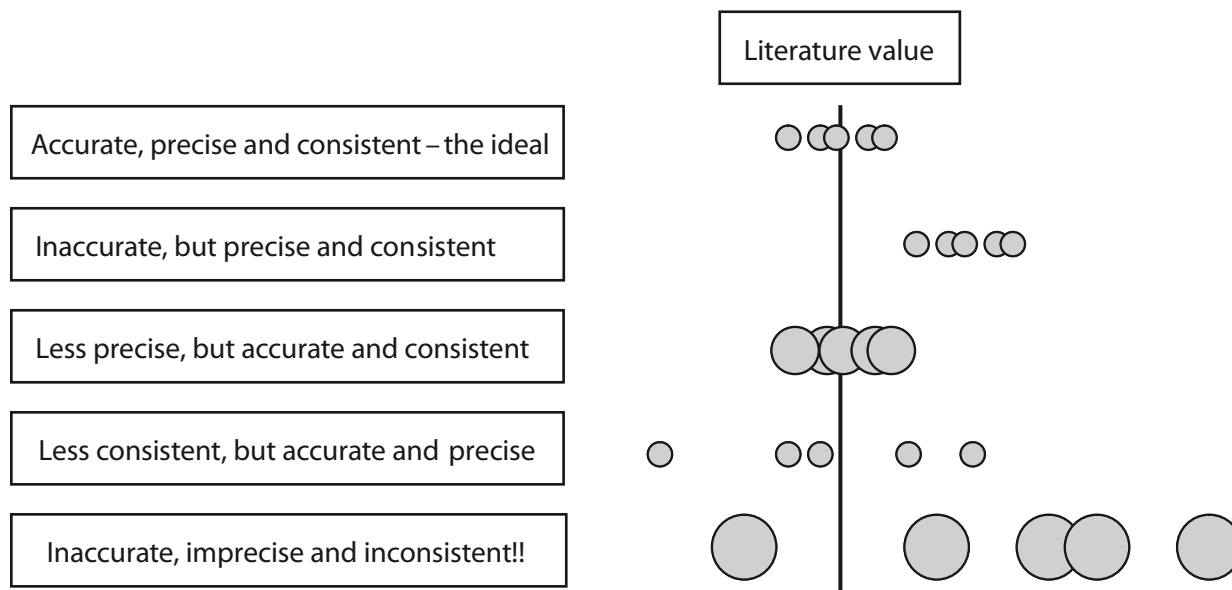
EXTENSION

CORE

Literature value

Accurate, precise and consistent – the ideal

Inaccurate, but precise and consistent

Less precise, but accurate and consistent

Less consistent, but accurate and precise

Inaccurate, imprecise and inconsistent!!

Figure 1103   The relationship between accuracy, precision and consistency

11.2 UNCERTAINTIES IN CALCULATED RESULTS

11.2.1 State uncertainties as absolute and percentage uncertainties.

11.2.2 Determine the uncertainties in results.

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The uncertainties in individual measurements can be combined to calculate the uncertainty in the final value of the quantity being determined. One way to estimate this would be to assume all the uncertainties were in the direction that would give the largest value of the final quantity (that is taking the largest possible number if the value is added or used to multiply and the smallest value if it is subtracted or used to divide) and recalculate the result with these data.

An alternative that is often easier to apply is to use the following simple rules:

- add **absolute** uncertainties when adding or subtracting numbers

- add **percentage** uncertainties when multiplying or dividing numbers

- [multiply **percentage** uncertainties by the exponential when raising to a power]

The **absolute uncertainty** is the **actual uncertainty** in the value, for example ±0.05 for a quantity that has the value 28.5 ± 0.05. The percentage uncertainty is the absolute uncertainty expressed as a percentage of the value. For example the percentage uncertainty of 28.5 ± 0.05 is ±0.18% ($100 \times \frac{0.05}{28.5}$). The third point is put in parentheses because it is a consequence of the second as $X^2 = X.X$. So applying the second rule the uncertainty of X^2 will be double the percentage uncertainty of X. Similarly the percentage uncertainty of $\sqrt{X}$ will be half the percentage uncertainty of X.

Suppose you want to calculate the value and uncertainty of X, where

$$X = A(B - C)$$

given the values:

$$A = 123 \pm 0.5; \quad B = 12.7 \pm 0.2; \quad C = 4.3 \pm 0.1$$

$$\begin{aligned} X &= 123 \times (12.7 - 4.3) \\ &= 1033.2 \quad \text{(note that this has not yet been rounded} \\ &\qquad\qquad\text{to an appropriate precision as the} \\ &\qquad\qquad\text{uncertainty has not been calculated)} \end{aligned}$$

Actual uncertainty in $(B - C) = \pm 0.3$ (add actual uncertainties, 0.2 + 0.1 = 0.3)

% uncertainty in A = 0.407% $(100 \times \frac{0.5}{123})$

% uncertainty in (B – C) = 3.571% $(100 \times \frac{0.3}{8.4})$

% uncertainty in X = 3.978%
    (add percentage uncertainties, 0.407 + 3.571)

Actual uncertainty in X = 41.1 $(1033.2 \times \frac{3.978}{100})$

Therefore X = 1033.2 ± 41.1

The usual practice is to only give the uncertainty to one significant figure and then to round off the value to a similar number of decimal places; hence the final; result should be quoted as

$$X = 1030 \pm 40$$

Note that, even though B and C were measured to greater precision, because they are subtracted they contribute much more to the final uncertainty than does A.

When using a literature value in calculations, assume its precision is limited to the number of digits given. For example, if the relative atomic mass of chlorine is given as 35.45 it should be taken as ±0.005. In some cases, however, the uncertainty of one quantity is much greater than that of all of the others, so the uncertainty of the final value can be considered to be due to only that factor and hence have the same percentage uncertainty. In these cases it is important to state that you are ignoring the minor uncertainties in other data. Consider a calorimetry experiment that gave the following results:

Initial temperature	21.6 ± 0.1°C
Final temperature	24.2 ± 0.1°C
Mass of water heated	200 ± 0.5 g
Amount of limiting reagent	0.0500 ± 0.00005 mol

Combined with the value below from a data book:

Specific heat capacity of water   4.183 ± 0.0005 J g^{-1} K^{-1}

It would be acceptable to state that the percentage uncertainty in the heat absorbed was being taken as being the same as the percentage uncertainty of the temperature change $(100 \times \frac{0.2}{2.6} = 7.7\%)$ as this was so much larger than all the other quantities involved (±0.25%, 0.1% and 0.12% respectively for Δm and ±0.01% for s). The error analysis for the experiment would then be:

$\Delta T = 2.6 \pm 0.2$°C (in addition and subtraction, the uncertainties are added)

% uncertainty $= 100 \times \frac{0.2}{2.6}$

$= 7.7\%$

$Q = m \, c \, \Delta T$

$= 200 \text{ g} \times 4.18 \text{ J g}^{-1} \times 2.6 \text{ °C}$

$= 2173.6 \text{ J}$

$\Delta H = -\dfrac{Q}{n}$

$= -\dfrac{2173.6}{0.0500}$

$= 43472 \text{ J mol}^{-1}$

As the uncertainty in the temperature change is so much greater than that of the other quantities, the percentage uncertainty in the final result will be taken as 7.7%:

Absolute uncertainty $= 43472 \times \dfrac{7.7}{100}$

$= 3347$

So the final result should be quoted as

$\Delta H = -43000 \pm 3000 \text{ J mol}^{-1}$

$= -43 \pm 3 \text{ kJ mol}^{-1}$

Suppose the literature value for the enthalpy change was –45.6 kJ mol^{-1}, the percentage discrepancy would be:

$\dfrac{45.6 - 43}{45.6} = 5.7\%$

In this case the percentage discrepancy is smaller than the uncertainty in the experimental value (7.7%), hence the result of the experiment is in agreement with the literature value. The major errors in the experiment are therefore those related to the precision, especially that of the temperature change, and suggested improvements should therefore concentrate on improving this, may be by using a more precise thermometer (perhaps one that reads to ±0.01 °C, if available) or more likely by increasing the temperature change, perhaps by using more concentrated reagents. Note that there is little point in trying to reduce the uncertainties in other values (such as using a more precise balance to weigh the water so that $\Delta m = $ ±0.05 g) until the uncertainty in the temperature has been significantly decreased.

CORE

If the literature value for the enthalpy change was −56.2 kJ mol^{-1}, the percentage discrepancy would be:
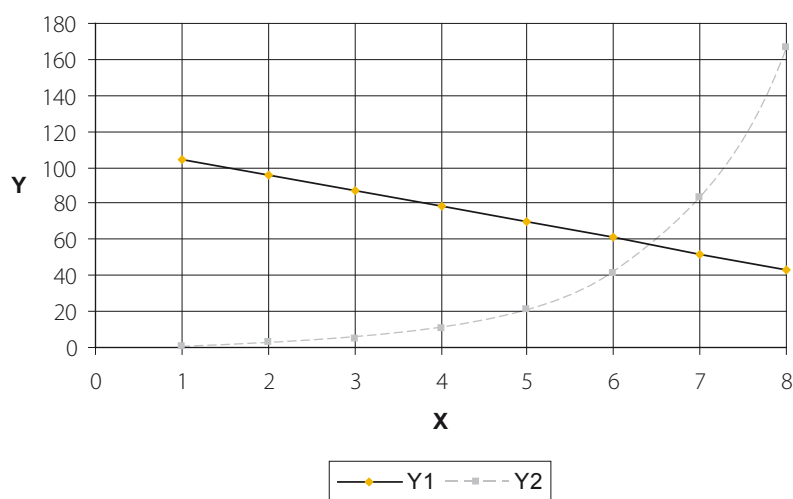
$$\frac{56.2 - 43}{56.2} \times 100 = 23.5\%$$

In this case the percentage discrepancy is much greater than could be accounted for by the uncertainties in the experimental value (7.7%), hence there must be some major systematic errors and the major focus of error analysis would now be to identify these. In this case, they would probably be factors such as heat loss to the surroundings, not taking into account the heat capacity of the calorimeter, assuming the heat capacity of the solution is equal to that of water etc. Improvements should then be suggested that minimise these such as covering the calorimeter to reduce heat loss, using Styrofoam calorimeter, measuring the heat capacity of the solutions used etc. The precision is poor, so suggesting ways to improve it would be appropriate, but this should not be the main focus.

TOK   Why are graphs helpful in providing powerful interpretations of reality?

"Seeing is believing." I suppose this sums up better than anything else the extent to which we rely on vision to interpret the world and probably that is why we find graphs, and indeed diagrams in general, such a useful way of "getting a feel" for things. I suppose another relevant quotation would be "A picture's meaning can express 10,000 words" (apparently a more accurate literal translation than the usual one of the anonymous Chinese proverb). Diagrams and graphs provide us with a more concrete and less abstract experience. If we had the data table:

Given a little time we could probably work out that the numbers represented a linear decrease (Y1) and an exponential increase (Y2), but it is immediately obvious if we look at the graph. Maybe that is because one requires the use of the logic circuits in our brain (is the difference between 104.3 and 95.6, the same as between 78.2 and 69.5?), whereas the pictorial representation seems to by-pass this - we immediately see a recognisable, familiar pattern, like the face of an old friend.

X	Y1	Y2
1	104.3	1.3
2	95.6	2.6
3	86.9	5.2
4	78.2	10.4
5	69.5	20.8
6	60.8	41.6
7	52.1	83.2
8	43.4	166.4

11.3 GRAPHICAL TECHNIQUES

11.3.1 Sketch graphs to represent dependences and interpret graph behaviour.

11.3.2 Construct graphs from experimental data.

11.3.3 Draw best-fit lines through data points on a graph.

11.3.4 Determine the values of physical quantities from graphs.

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Graphs are one of the most useful ways for interpreting scientific data because they allow for direct visual correlation between the data obtained and a particular scientific model or hypothesis. In its simplest form this involves organising data so that a linear graph is expected.

See Figures 1104 and 1105.
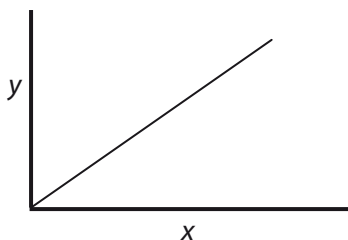
Figure 1104   Direct proportionality
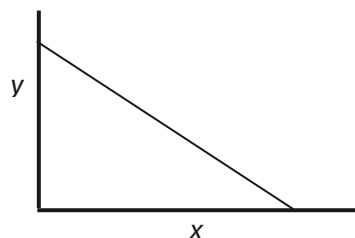
Figure 1105   A linear relationship

Sometimes it is possible to rearrange an equation that is not a linear relationship to give a modified equation that does have such a relationship. An example would be testing to see if the approximation of neglecting the effect of dissociation on the concentration of a weak acid is valid:

$$HA \rightleftharpoons H^+ + A^-$$

Initial Concentrations	a	0	0
Equilibrium concentrations	$a-x$	x	x

Substituting in the equilibrium expression:

$$K_a = \frac{[H^+].[A^-]}{[HA]} = \frac{x.x}{a-x} = \frac{x^2}{a-x} \approx \frac{x^2}{a}$$

$$K_a.a \approx x^2 \text{ so } x \approx \sqrt{K_a}.\sqrt{a}$$

Hence, if this approximation is valid, the $[H^+]$ (x) would be proportional to $\sqrt{[HA]}$ ($\sqrt{a}$) and this could be tested by drawing a graph of $[H^+]$ against $\sqrt{[HA]}$ and seeing if it was linear.

If manipulation cannot produce a linear function, sometimes there is an inverse proportionality or inverse linear relationship, which can be used to plot a graph that should give a straight line. See Figures 1106 and 1107.
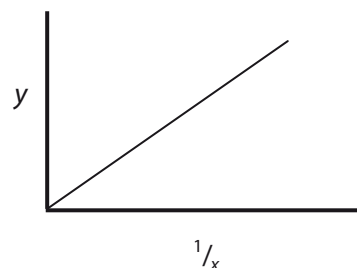

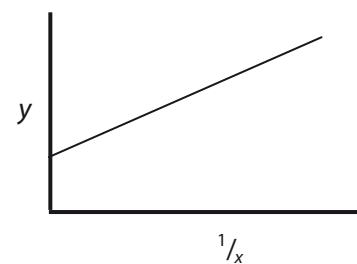Figure 1106   Inverse proportionality

Figure 1107   Inverse linear relationship

If a buffer solution is made by taking a solution of a weak acid and dissolving a solid salt of that weak acid in it, then the dependence of $[H^+]$ on the amount of a solid salt added would be an inverse linear relationship:

$$K_a = \frac{[H^+].[A^-]}{[HA]} \text{ so } [H^+] = K_a.\frac{[HA]}{[A^-]}$$

This could be tested by seeing if a graph of $[H^+]$ against $\frac{1}{\text{mass of salt}}$ (as this will be proportional to $\frac{1}{[A^-]}$) was linear, assuming the volume and concentration of the acid are kept constant.

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When drawing graphs it is usual to choose the axes so that the independent variable (frequently time) is plotted along the horizontal axis and the dependent variable on the vertical axis. The scale should be chosen so as to maximise the use of the graph area, taking into account any extrapolation of the data that may be required. A fairly simple scale should also be chosen (2, 4, 5, 8 or 10 squares equal to one unit) to facilitate the easy plotting of data. More complex scales (for example, 7 squares equal to one unit) may maximise the use of the graph area, but frequently lead to errors in the plotting of points.

Care must also be taken to have enough data points to ensure that the graph really is linear. For example it is questionable whether the data in Figure 1108 (a) really do represent a straight line rather than a curve, but the addition of extra data points, as shown in Figure 1108 (b), confirm that they do indeed describe a linear relationship, though if these points had been elsewhere, as in Figure 1108 (c), they could have indicated that the relationship is non-linear.
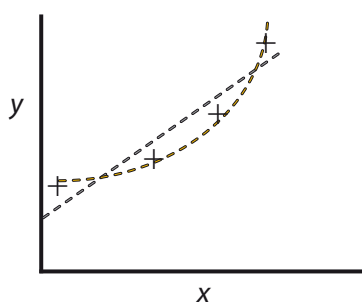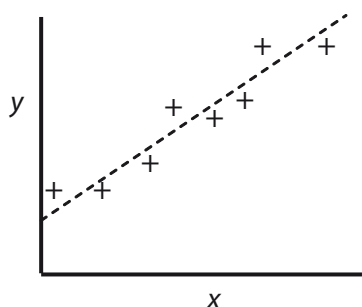

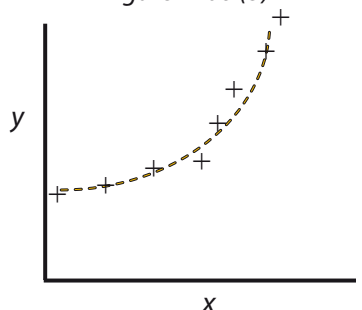*Figure 1108 (a)*

Figure 1108 (b)

Figure 1108 (c)

Graphs can be used to check the validity of a relationship, as in the examples above, or to determine some value from either the intercept (in which case the units are the same as those of the axis) or the gradient of the line (in which case the units are those of the vertical axis divided by those of the horizontal axis). Some of the more common graphs that are encountered in chemistry and their uses are considered:

Ideal gas law $\qquad P.V = n.R.T$

so $\qquad V = \dfrac{n.R.T}{P}$

or $\qquad P = \dfrac{n.R.T}{V}$

Where P is the pressure, V the volume, n the number of moles and T the absolute temperature of the gas sample. R is the ideal gas constant.

Most commonly P is plotted against $\left(\frac{1}{V}\right)$ at constant temperature (**Boyle-Mariotte Law**). A linear graph shows the validity of this relationship and, if the number of moles of gas can be determined (for example by knowing its mass) and the temperature is measured, then the value of the ideal gas constant (R) can be found from the gradient. See Figure 1109.
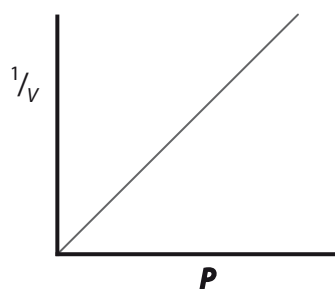

Figure 1109   Boyle's Law

Alternatively V can be plotted against T at constant pressure (**Charles's Law**). Again a linear graph shows the validity of this relationship and, if the number of moles of gas is known and the pressure is measured, then the value of the ideal gas constant (R) can be found from the gradient. The data can also be extrapolated back to $V = 0$ to find the value of absolute zero. See Figure 1110.
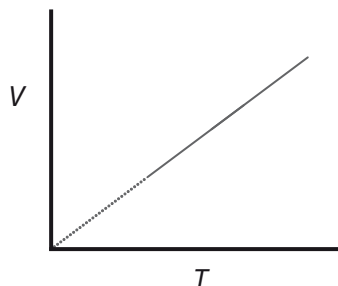
Figure 1110   Charles's Law

Cooling curves Graphs of temperature versus time during and just after a chemical reaction

The portion of these graphs following the maximum temperature are often extrapolated backwards in order to estimate the temperature rise that might have occurred if the reaction had been instantaneous and hence involved no heat loss. A linear extrapolation is often used even though there is little theoretical justification for this. See Figure 1111.
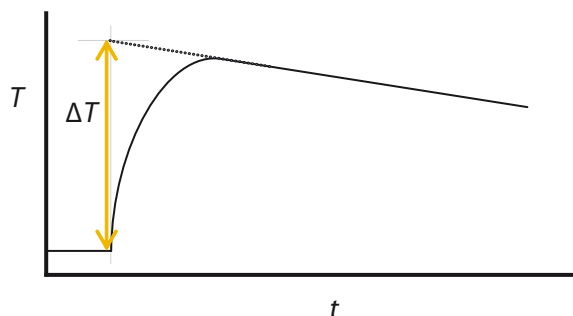

Figure 1111   A cooling curve

Reaction rate graphs Graphs of concentration (or some property proportional to it) versus time

The gradient of a tangent to the line at any time (with the sign reversed if the concentration of a reactant is being followed) is proportional to the rate of reaction. The units of rate are therefore those of concentration divided by time (i.e. mol dm^{-3} s^{-1}). See Figure 1112.
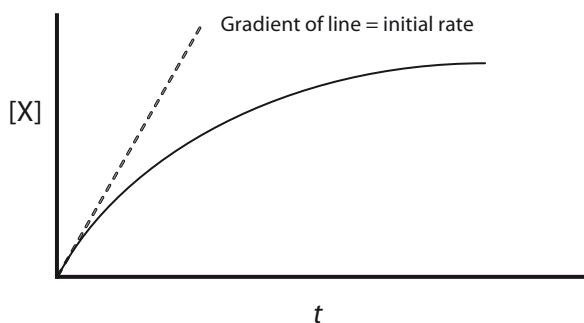

Figure 1112   A reaction rate curve

Reaction order graphs Graphs of reaction rate versus concentration of one reactant (others constant).

The shape of the graph indicates the reaction order with respect to that reactant:

Zero order - linear horizontal line
1st order   - linear
2nd order   - curved upwards
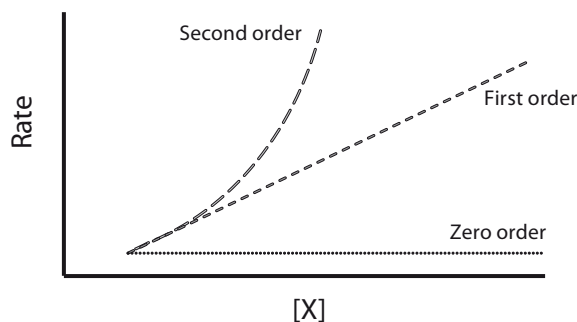See Figure 1113.

Figure 1113   A reaction order curve

Activation energy graphs   Graphs of the effect of temperature on reaction rate.

The Arrhenius relationship states:

$$k = Ae^{\left(\frac{-E_a}{RT}\right)}$$

Taking natural logarithms and rearranging:

$$\ln k = \ln A - \left(\frac{E_a}{R}\right)\left(\frac{1}{T}\right)$$

So a graph of ln(rate) (which is proportional to k if the concentrations remain constant) against $\frac{1}{T}$ (in Kelvin) is linear. The intercept is equal to ln$A$ and the gradient is equal to $-\frac{E_a}{R}$, so that knowing R the activation energy can be determined. See Figure 1114.
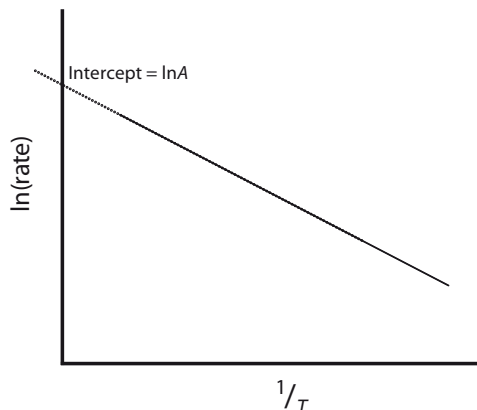

Figure 1114   An activation energy graph

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CORE

Exercise 11.1

Four groups have done a calorimetric experiment to determine the enthalpy change of a reaction in aqueous solution, for which the literature value is 38.73 kJ mol^{-1}. The values found by the groups (in kJ mol^{-1}) were:

A 35.1 ± 0.3   B   36.5 ± 0.5
C 33.2 ± 0.1   D   34.7 ± 0.2

1. Which result is the most precise?

2. Which result is the most accurate?

3. Do you think the major problem is

   A   Random error
   B   Systematic error
   C   Both are equally important
   D   It is not possible to tell from the data given

4. If you were to repeat the experiment which of the following is most likely to be the improvement that would most improve the result?

   A   Using a more precise thermometer.
   B   Not assuming that the specific heat of the solution equalled that of water.
   C   Using pipettes rather than measuring cylinders.
   D   Improving the insulation of the calorimeter.

5. Which of the following is the best way to report the mean result of the group?

   A   34.875 ± 0.100 kJ mol^{-1}.
   B   34.9 ± 0.5 kJ mol^{-1}.
   C   34.9 ± 0.8 kJ mol^{-1}.
   D   35 ± 2 kJ mol^{-1}.

6. Repeating an experiment a number of times will lead to a decrease in:

   A   the random error.
   B   the systematic error.
   C   both the random and the systematic error.
   D   neither the random and the systematic error.

Exercise 11.2

1. Which result has the lowest absolute uncertainty?

   A   34.875 ± 0.017
   B   749 ± 5
   C   0.0004 ± 0.0001
   D   87500 ± 200

2. Which result has the lowest percentage uncertainty?

   A   34.875 ± 0.017
   B   749 ± 5
   C   0.0004 ± 0.0001
   D   87500 ± 200

3. If x = 749 ± 5 and y = 34.8 ± 0.7, which one of the following will have the greatest percentage uncertainty?

   A   x + y
   B   x - y
   C   x/y
   D   x.$\sqrt{y}$

4. Give the result and the absolute uncertainty, to the correct precision, of the following calculations:

   a)   20.1 ± 0.1 + 2.75 ± 0.05
   b)   115.4 ± 0.2 × 8.137 ± 0.001
   c)   0.572 ± 0.001 (17.6 0.2 - 114 ± 1)
   d)   (1 - 0.276 ± 0.002)/0.024 ± 0.001
   e)   52800 ± 100/ $\sqrt{}$(17.2 ± 0.2 - 2.37 ± 0.01)

Exercise 11.3

1. Plot a graph of the following data. Construct the best straight line and use it to determine the value of the intercept and the gradient of the line.

x	1	2	3	4	5	6	7	8	9	10
y	7.6	10.7	13.2	17.1	19.8	23.0	25.7	28.7	32.9	35.4

2. Two variables, x and y, are related by the equation:

$$S\,x(T-y) = 5$$

where S and T are constants. Draw an appropriate graph to investigate whether the data below supports this and use it to find values for S and T.

x	1	2	3	4	5	6	7	8	9	10
y	2.52	3.78	4.11	4.35	4.52	4.59	4.62	4.71	4.73	4.74