ACT Math Preparation Guide

I. General Test Taking Tips for the Math Portion

- 60 questions – 60 minutes (33 algebra questions, 23 geometry questions, 4 trig questions)
- Know the directions before you go take the test – it will save valuable time!
- Easy questions and hard questions count the same! Go through the test working all the problems you know how to do. Circle the numbers of the questions you skip so you will know where you need to return. As you go back through, try to eliminate answer choices in case you have to guess. Try to think logically if you have to guess!
- Be sure you mark your answers on the correct line if you are skipping questions!
- Answer the question asked! The ACT sometimes contains partial answers.
- Do not leave any question blank. An unanswered question is just as wrong as a wrong answer!
- The answers are usually in numerical order. If you are working backwards, begin with the middle answer choice to see if you need a larger or smaller number.
- Draw and label pictures to help you see the problem.

II. Definitions, Explanations, and Formulas

A) Angles

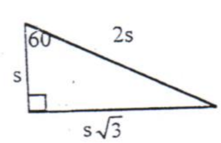
Complementary angles: two angles whose measures sum to 90º (**c**omplementary – **c**orner)
Supplementary angles: two angles whose measures sum to 180º (**s**upplementary – **s**traight)
Acute Angle: angle whose measure is between 0º and 90º
Right Angle: angle whose measure is equal to 90º
Obtuse Angle: angle whose measure is between 90º and 180º
Straight Angle: angle whose measure is equal to 180º

B) Triangles

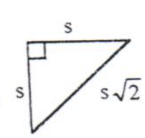
Equiangular: all angles are congruent and equal to 60º
Equilateral: all sides are congruent (If a triangle is equilateral, it is also equiangular)
Scalene: no sides are congruent
Isosceles: at least two sides are congruent
Triangle: p = sum of side lengths; A = $\frac{1}{2}bh$;
Heron's Formula for Area, where s = semi-perimeter = $\frac{p}{2}$; $A = \sqrt{s(s-a)(s-b)(s-c)}$
Area of an equilateral triangle; $A = \frac{s^2\sqrt{3}}{4}$
Sum of the measures of the interior angles is 180º
Similar triangles have corresponding equal angles and corresponding proportional sides.
Pythagorean Theorem: $\boldsymbol{a^2 + b^2 = c^2}$, where a and b are legs of a right triangle and c is the hypotenuse
Common Pythagorean Triples: 3, 4, 5   5, 12, 13   7, 24, 25   8, 15, 17   9,12,15
6, 8, 10   10 ,24, 26

Special Right Triangles

30-60-90 Triangles



45-45-90 Triangles



C) Quadrilaterals

Any Quadrilateral: sum of the interior angles = 360°

Properties of a trapezoid:

1. Exactly one pair of sides ∥ called bases
2. Legs: Non ∥ sides
3. Median: line connecting the midpoint of the legs, always ∥ to the bases (= average of bases)

$$A = mh \text{ or } \frac{1}{2}h(b_1 + b_2);$$
$$P = sum\ of\ side\ lengths$$

Properties of a parallelogram:

1. Opposite sides are ∥
2. Opposite sides ≅
3. Opposite angles are ≅
4. Adjacent angles are supplementary
5. Diagonals bisect each other
6. Diagonals divides the parallelogram into 2 ≅ Δs

$$A = bh\ ; \quad P = 2l + 2w$$

Properties of a rhombus

1. All sides are ≅
2. Diagonals are ⊥
3. Diagonals bisect the $\angle s$

$$A = \frac{1}{2}d_1d_2; \ \ A = bh$$
$$p = 4s$$

Properties of a rectangle

1. Measure of all angles is equal to 90°
2. Diagonals are ≅

$$A = lw \quad P = 2l + 2w$$

Properties of a square:

ALL OF THE ABOVE

$$A = s^2 \quad or\ A = \frac{1}{2}d^2$$
$$P = 4s$$

D) Polygons *(The below polygons are assumed to be convex)*

For Any Polygon:

Sum of the Interior angles of any polygon = (n-2)180°, where n is the number of sides.
Sum of the exterior angles of any polygon = 360°.
Number of Diagonals in a polygon = $\frac{n(n-3)}{2}$.

For a Regular Polygon:

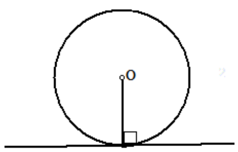
Regular Polygon – A polygon in which all sides and all angles are congruent.
$A = \frac{1}{2}ap$ where a is the apothem and p is the perimeter.
Measure of one interior angle = $\frac{180(n-2)}{n}$ Measure of one exterior angle = $\frac{360}{n}$.

A regular polygon inscribed in a circle can be cut into the same number of isosceles triangles as the number of sides. A hexagon will break up into six equilateral triangles.

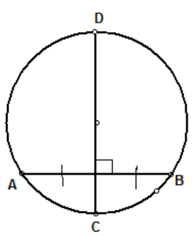
Remember to use proportions to solve problems with similar polygons.

Names of Polygons:
3 – triangle; 4 – quadrilateral; 5 – pentagon; 6-hexagon; 7-septagon (heptagon;) 8 – octagon; 9-nonagon; 10-decagon; 11-undecagon; 12-dodecagon; 15-pentadecagon; 20-icosagon

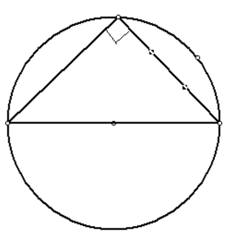
E) Circles
The radius of the circle can be drawn from the center to any point on the circle.
Area: $A = \pi r^2 \ or \ A = \frac{\pi d^2}{4}$ Circumference: $C = \pi d \ or \ C = 2\pi r$
Equation of a circle: $x^2 + y^2 = r^2$; $with\ center\ at\ (0,0) and\ radius\ of\ r$
General form for equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$; center at (h,k); radius = r



The radius drawn to the point of contact of a tangent is perpendicular to the tangent at that point.

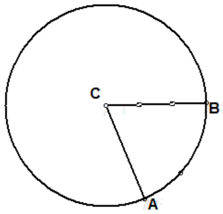


A diameter that is perpendicular to a chord bisects the chord and its arc: $\overline{CD} \perp \overline{AB}$ and $\overline{CD}\ bisects\ \overline{AB}$

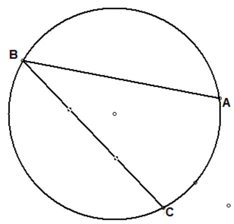


If a triangle is inscribed in a circle and the diameter is one of the sides, it is a *right* triangle. *(The diameter is equal to the hypotenuse)*

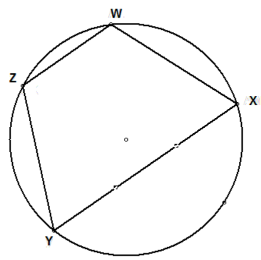
Angles and Arcs of Circles:



The measure of a central angle is equal to the measure of its intercepted arc: $m\angle C = m\widehat{AB}$

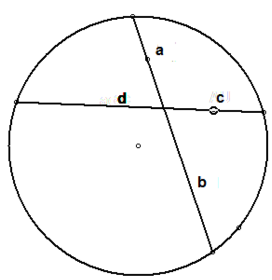


The measure of an inscribed angle is equal to half the measure of its intercepted arc: $m\angle B = \frac{1}{2} m\widehat{AC}$

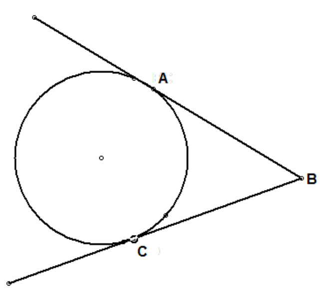


If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. $m\angle W + m\angle Y = 180^{o}$; $m\angle X + m\angle Z = 180^{o}$

Chords, Secants, and Tangents of Circles



When two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord. $a \cdot b = c \cdot d$



AB=BC

All Circles are similar, so proportions can be formed to determine unknown quantities of a circle:

$$\frac{r_1}{r_2} = \frac{d_1}{d_2} = \frac{C_1}{C_2};\quad \frac{(r_1)^2}{(r_2)^2} = \frac{(A_1)}{(A_2)}$$

F) Solids

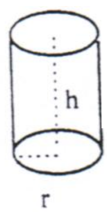
B = Base area of the polygon which is perpendicular (⊥) to the height (h) in a solid
side length = s; length = l; width = w; perimeter = p; radius = r; Total Surface Area = T.A.
volume = v; height = h; slant height = l; diagonal = d

Prism (all): TA = 2B + ph V = Bh

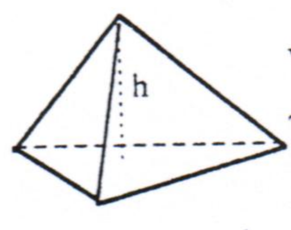
Rectangular Prism: TA = 2B + ph = 2(lw) + 2(lh) + 2(wh)
V = Bh = lwh
$d = \sqrt{l^2 + w^2 + h^2}$

Sphere: $TA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

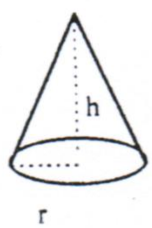
Cube : $TA = 6s^2$ $V = s^3$ $d = s\sqrt{3}$



Right Circular Cylinder
$V = \pi r^2 h;$
$TA = 2\pi r^2 + 2\pi rh$



Right Pyramid
$V = \frac{1}{3}Bh$
$TA = B + \frac{1}{2}pl$



Right Circular Cone
$V = \frac{1}{3}\pi r^2 h$
$TA = \pi rl + \pi r^2$

G) Coordinate Geometry

Linear Equation: An equation with <u>one or two variables</u> raised to the <u>first power</u> & will always graph a line.

Slope: if $x_1 \neq x_2$, then slope $= m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

If $x_1 = x_2$, then the slope is <u>undefined</u> (zero in the denominator!)

$\nearrow$ *positive slope*; $\searrow$ *negative slope*; $\leftrightarrow$ *zero slope*; $\updownarrow$ *undefined or no slope*

Standard (general) form: $\mathrm{Ax + By = C}$, where A and B are not both zero,
A is nonnegative integer, and B and C are integers
slope $= -\frac{A}{B}$; x − intercept$(x, 0) = \frac{C}{A}$; y − intercept $(0, y) = \frac{C}{B}$

Slope-Intercept Form: $y = mx + b$, where m is the slope and b is the y − intercept

Point-Slope Form: given the slope m, and a point (x_1, y_1), then $y - y_1 = m(x - x_1)$

Parallel Lines: same slopes, but different y-intercepts; *Perpendicular Lines*: negative reciprocal slopes

Midpoint Formula: given two points a and b(similar to a number line), the midpoint is $\frac{1}{2}(a + b)$

given two points $((x_1, y_1)$ *and* (x_2, y_2), then $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Formula: given two points $((x_1, y_1)$ *and* (x_2, y_2), then $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Most graphing questions can be worked by changing them to $y = mx + b$. Change equations to this form to find the slope or the y-intercept. Graphs which have the form $y = Ax^2 \text{ or } x = Ay^2$ are parabolas.

H) Algebra

Order of operations: PEMDAS (Please Excuse My Dear Aunt Sally);
[Parenthesis, Exponents, Multiplication and Division(left to right), Addition and Subtraction(left to right)]

You can only add and subtract like terms, but you can multiply and divide terms that are unlike

$x^5 \cdot x^3 = x^{5+3} = x^8$ Add exponents when multiplying like bases.
$(x^3)^5 = x^{3 \cdot 5} = x^{15}$ Multiply exponents to raise an exponential term to a power.
$\frac{x^5}{x^3} = x^{5-3} = x^2$ Subtract exponents to divide like bases.
$x^{-5} = \frac{1}{x^5}$ A negative exponent inverts the base and becomes positive.

Solving Linear Equations: What you do to one side of an equation you must do to the other.
Solving Linear Inequalities: Similar to solving a linear equation except that when you multiply or divide by a negative number, you must reverse the inequality sign.

Solving a System of Equations:

Solve for one variable, then substitute $\begin{cases} x - 2y = 4 & so\ x = 4 + 2y \\ x + y = 7 & and\ (4 + 2y) + y = 7; \end{cases}$ $\quad y = 1, x = 6 \rightarrow (6,1)$

Solve by elimination, then substitute $\begin{cases} 2x + y = 14 & multiply\ by\ 3 \rightarrow 6x + 3y = 42 \\ x - 3y = -7 & x - 3y = -7 \end{cases}$
$7x = 35;\ x = 5, y = 4 \rightarrow (5,4)$

Function: a set of ordered pairs in which <u>no first component is repeated</u> (an x, abscissa, of the domain is paired with one distinct y, ordinate, of the range).

Domain: the set of all first components (x's or abscissas) of ordered pairs.
Range: the set of all second components (y's or ordinates) of ordered pairs.
The graph of a function will ALWAYS pass the vertical line test! Use this when looking at a piece-wise function.

Multiplying Binomials:
- *FOIL:* First + Outer + Inner + Last
- Sum & Difference: $(F + L)(F - L) = F^2 - L^2$
- Perfect Square Trinomial: $(F \pm L)^2 = F^2 \pm 2FL + L^2$

Factoring After Removing GCF
- To factor the trinomial $ax^2 + bx + c$, look for two numbers whose sum is b and whose product is ac.
- To identify <u>perfect square trinomials,</u> look for perfect squares in the first and last terms, and twice the product of their square roots in the middle term: $(F \pm L)^2 = F^2 \pm 2FL + L^2$;
- The only binomial that can factor into two binomials is the <u>difference of two perfect squares:</u> $x^2 - y^2 = (x + y)(x - y)$. The sum of two perfect squares, $x^2 + y^2$ is not factorable.
- Sum /Difference of two perfect Cubes: $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$
- Given a polynomial with 4 or more terms, rearrange and <u>factor by grouping</u>.

Absolute Value Equations:

$for\ b > 0, if\ |a| = b, then\ a = b\ or\ a = -b$
$for\ b < 0, if\ |a| = b, then\ no\ solution$

Absolute Value Inequalities (for b>0):

$if\ |a| > b, then\ a > b\ OR\ a < -b$ (> great<u>OR</u>)
$if\ |a| < b, then\ a < b\ AND\ a > -b$ (*< less th<u>AN</u>*) which is $-b < a < b$
Remember that absolute value graphs are shaped like a V.

Quadratic Equations:

These are equations which can be written in the form $ax^2 + bx + c = 0; where\ a \neq 0$ and can be solved by factoring or applying the quadratic formula. In ACT questions where you are trying to determine the equation when given the roots, you can always go backwards.
Example: $x = -6; x = 1;$ $x + 6 = 0, x - 1 = 0;$ $(x + 6)(x - 1) = 0;$ $x^2 + 5x - 6 = 0$

Sum of the Roots = -b/a; Product of the Roots = c/a

Quadratic Formula: $where\ a \neq 0\ and\ ax^2 + bx + c = 0, then\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminate:

If $b^2 - 4ac > 0, then\ two\ distinct\ real\ roots\ exist$
If $b^2 - 4ac = 0, then\ one\ double\ (not\ distinct) real\ solution\ exists$
If $b^2 - 4ac < 0, then\ no\ real\ solutions\ exist, but\ there\ are\ 2\ complex\ solutions$.

Vertical Asymptotes

Example: $y = \frac{(x+1)}{(x-2)(x-3)}$ since 2&3 are zeros of the denominator, neither is zero of the numerator; then x= 2 & x = 3 are vertical asymptotes.

Example: $y = \frac{(x+4)}{(x-3)(x+4)}\ is\ same\ as\ \frac{1}{x-3}$ so vertical asymptote at x = 3

Rational Exponents:

$a^{\frac{1}{n}} = \sqrt[n]{a}$ $a^{-m} = \frac{1}{a^m}$ $a^{\frac{m}{n}} = \sqrt[n]{a^m} or \left(\sqrt[n]{a}\right)^m$ [ex: $64^{2/3} = \left(\sqrt[3]{64}\right)^2 = 16$

Logarithm:

If $y = b^x, then\ \log_b y = x;$ $Example\ \log_9 27 = x, then\ 9^x = 27; so\ 3^{2x} = 3^3; thus\ x = \frac{3}{2}$
Product Property; $\log_b MN = \log_b M + \log_b N$
Power Property: $\log_b m^x = x \log_b m$
Quotient Property; $\log_b \frac{M}{N} = \log_b M - \log_b N$
Example; $\log_a (xy)^2 = 2 \log_a (xy) = 2(\log_a x + \log_a y)$

Variation:

Direct Variation: any function defined by an equation of the form $y = kx$ $\left(\therefore \frac{y}{x} = k\right)$, where k is a nonzero constant.

Ex: Q varies directly as X (or Q is directly proportional to x) means Q = kx, $\frac{Q}{x} = k$; or as a Proportion $\frac{Q_1}{x_1} = \frac{Q_2}{x_2}$

Ex: y varies directly as x, and when y is 15, x is 3; find y when x is 7. y=kx, so 15=3k, k = 5 then y = 5 (7) = 35 . . or as a proportion $\frac{15}{3} = \frac{y}{7}$ $so\ y = 35$

Inverse Variation: any function defined by the equation of the form $xy = k\ or\ y = \frac{k}{x}$, where k is a nonzero constant.

Ex: Q varies inversely as x means $Q = \frac{k}{x}\ or\ Qx = k; and\ as\ a\ proprtion\ Q_1x_1 = Q_2x_2$

Ex: y varies inversely as x and when y is 6, x is 4; find x if y is 8.

Then $y = \frac{k}{x}, 6 = \frac{k}{4}, so\ 24 = k\ and\ 8 = \frac{24}{x}, x = 3$

$or\ yx = (6)(4) = 24\ and\ 24 = 8x, x = 3$

Joint Variation: when one variable varies directly as the product of two or more other variables, K is a nonzero constant.

Ex: Q varies jointly as x and y means $Q = kxy, or\ \frac{Q}{xy} = k; or\ \frac{Q_1}{x_1y_1} = \frac{Q_2}{x_2y_2}$

Combined Variation: when a variable varies directly as one variable and inversely as another variable where k is a nonzero constant.

Ex: Q varies directly as x and inversely as y means $\frac{Qy}{x} = k; or\ \frac{Q_1y_1}{x_1} = \frac{Q_2y_2}{x_2}$

Complex Numbers

$i = \sqrt{-1}$; $i^2 = -1$; $i^3 = -\sqrt{-1} = -1$; $i^4 = 1$; Standard Form: $a + bi$

Example Problem: $(i+1)^2 = i^2 + 2i + 1$; $since\ i^2 = -1, then\ (i+1)^2 = 2i$

Sequences and Series

Arithmetic Sequences: With first term a_1 and a common difference d, and nth term a_n, with a series sum S_n

The nth term of the sequence is $a_n = a_1 + (n-1)d$

The sum of the first nth terms is $S_n = \frac{n}{2}(2a_1 + (n-1)d)\ or\ S_n = \frac{n}{2}(a_1 + a_n)$

Geometry Sequence: with first term a_1, and common ratio r, with a series sum S_n

The nth term of the sequence, $a_n = a_1r^{n-1}$

The sum of the first nth terms in a series, $S_n = \frac{a_1(r^n-1)}{r-1}$

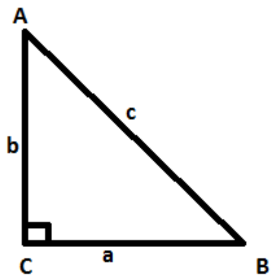
The sum of an infinite geometric series with r<1; $S_n = \frac{a_1}{1-r}$

Other Algebraic Formulas

<u>Work Problems:</u> using two workers (jobs) at a time, $\frac{product}{sum} = working\ together = \frac{XY}{X+Y}$

<u>Average Speed:</u> when <u>distance</u> is the <u>same</u> , average speed = $\frac{2XY}{X+Y}$

I) Trigonometry



When you need to determine the missing side or angle of a right triangle, use SOH-CAH-TOA to help you set up the trig ratio

$$sinA = \frac{opp}{hyp} = \frac{a}{c}; \;\; cosA = \frac{adj}{hyp} = \frac{b}{c}; \;\; tanA = \frac{opp}{adj} = \frac{a}{b}$$

Reciprocal Identities

cosecant x = cscx = $\frac{1}{sinx}$; secant x = secx = $\frac{1}{cosx}$;

cotangent x = cotx = $\frac{1}{tanx}$; tangent x = tanx = $\frac{sinx}{cosx}$

Pythagorean Identities

$$sin^2\theta + cos^2\theta = 1; \;\; 1 + cot^2\theta = csc^2\theta; \;\; 1 + tan^2\theta = sec^2\theta$$

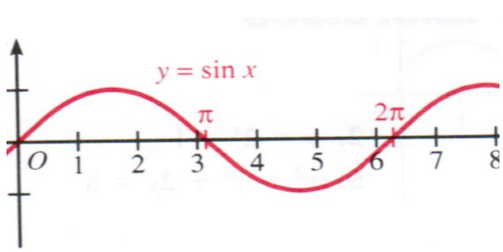
Law of Sines: $\frac{sina}{a} = \frac{sinb}{b} = \frac{sinc}{c}$ *Law of Cosines:* $a^2 = b^2 + c^2 - 2bc(cosa)$

Unit Circle – A circle with a radius of 1 unit and center at origin of the coordinate plane

A circle is 360° or 2π radians. To convert between degrees and radians or radians and degrees, use $\frac{x}{180}$ or $\frac{180}{x}$.

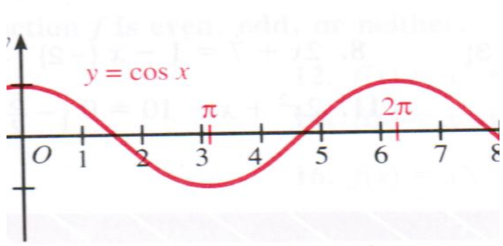
To determine if a trig function will be positive, use: All Students Take Calculus.

In Quadrant I All trig functions are positive, In quadrant II Sin is positive,
In quadrant III Tan is positive, In quadrant IV Cos is positive.



For $y = sinx$
Amplitude = 1
Period = 2π

For $y = asinb\theta$
$|a| = amplitude$
$\frac{2\pi}{b} = period$



For $y = cosx$
Amplitude = 1
Period = 2π

For $y = acosb\theta$
$|a| = amplitude$
$\frac{2\pi}{b} = period$

J) Statistics

Mean is the average.

Median is the middle number when the numbers are in order(if an even number, the average of the two middle numbers).

Mode is the number(s) that occur(s) the most often.

Range is the positive difference in the largest and smallest numbers.

K) Probability

If you have cards numbered 1-10 and cards lettered A-D and you draw one of each, there are 40 possible outcomes (10 X 4).

If you have 3 shirts, 5 pants, and 3 belts, how many different outfits can you make? 3 X 5 X 3 = 45 outfits

If you have 5 books to arrange on a shelf, there are 5 X 4 X 3 X 2 X 1 = 120 possible arrangements.

Probability General Formulas:

$P = \frac{favored\ outcomes}{total\ number\ of\ possible\ outcomes}$ Example: Probability of rolling a 6 with a single die = $\frac{1}{6}$

$P(A\ and\ B) = P(A) \cdot P(B)$ Example: What is the probability of drawing an 8 out of a deck of 52 cards and then rolling a six on a die? $\frac{4}{52} \cdot \frac{1}{6} = \frac{1}{78}$

Permutation: The number of ordered arrangements of n distinct things taken r at a time, where $r \leq n$ is $P(n,r) = {}_{n}P_{r} = \frac{n!}{(n-r)!} = n(n-1)(n-2) \ldots to\ r\ factors$

Combinations: (order is not important) – the number of subsets of *n* distinct things taken r at a time, where $r \leq n$ Is $C(n,r) = {}_{n}C_{r} = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$

L) Conics

Parabola (Vertical): $y = ax^2 + bx + c$ *or* $y = a(x-h)^2 + k$ with vertex at (h,k).
For a vertical parabola, if $a > 0, parabola\ opens\ up\quad or\ if\ a < 0, parabola\ opens\ down.$

Circle: $x^2 + y^2 = r^2$ or $(x-h)^2 + (y-k)^2 = r^2$
Center (0,0); r = radius — center (h,k); r = radius

Ellipse (Horizontal): $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Center: (0,0) — Center: (h,k)

Hyperbola (Horizontal): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Center: (0,0) — Center: (h,k)