Name:
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Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090: Stochastic Processes

Quiz 5

Date: 15 April 2025

Question	1	2	Total
Marks Scored			

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Assume that all random variables appearing in the problems below are defined w.r.t. $\mathscr{F}$.

1. Consider a random walk on the integer line $\mathbb{Z}$ with IID uniform $\{\pm 1\}$ step size at every time instant, that starts at 0.

   (a) **(1 Mark)**
   Classify the states of the random walk as transient, positive recurrent, or null recurrent.

   (b) **(2 Marks)**
   If P denotes the TPM of the random walk, identify a left eigenvector of P corresponding to the eigenvalue $\lambda = 1$.

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2. **(2 Marks)**

Consider a game in which a biased coin $C \sim \text{Ber}(0.9)$ is tossed repeatedly and independently. **At the start of the game, you are given exactly 2 lives**. Each toss of the coin results in the following outcomes: if the coin lands heads, you gain one life, up to a maximum of 3 lives. In particular, if you already have 3 lives and the coin lands heads, your lives remain unchanged. Every time the coin lands tails, you lose a life. The game continues until you lose all your lives. Determine the probability that the game terminates without you ever reaching the state of 3 lives at any point during the gameplay.