Probability and Stochastic Processes

Open Quiz 02

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Question

Suppose that $\mathbb{E}[X|Y] = Y$ and $\mathbb{E}[Y|X] = X$.

Compute the value of

$$\frac{\mathrm{Var}(X)}{2\,\mathrm{Var}(X) + 3\,\mathrm{Var}(Y)}.$$

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Question

Suppose $X, Y \overset{\text{IID}}{\sim} \mathcal{N}(0, 1)$.

Evaluate

$$\frac{\mathbb{E}\Big[|2X - 3Y|\Big]}{\mathbb{E}\Big[|2X + 3Y|\Big]}.$$

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Elevator

The number of people who enter an elevator on the ground floor is a Poisson random variable with parameter $\lambda = 10$. Suppose that there are N floors above the ground floor, and each person is equally likely to get off at any one of the N floors, independently of where the others get off.

Compute the expected number of stops that the elevator will make before dropping off all of its passengers.

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Exponential Families

A PMF/PDF is said to belong to a **single-parameter exponential family** if it can be expressed as

$$f_\theta(x) = h(x) \cdot \exp\Big(\theta\, T(x) - A(\theta)\Big), \qquad x \in \mathbb{R},$$

where $h, T : \mathbb{R} \to \mathbb{R}$ are real-valued functions, θ is a real-valued parameter, and $A(\theta)$ is a normalisation factor (to make sure the PMF sums to 1 or PDF integrates to 1)

Given $p \in [0, 1]$, express the $\text{Bernoulli}(p)$ PMF as a single-parameter exponential family distribution. Identify $h(\cdot)$, $T(\cdot)$, and θ for this distribution.

Question

Let the sample space Ω be partitioned as $\Omega = \Omega_1 \sqcup \Omega_2 \sqcup \Omega_3$. Let $\mathscr{F} = \sigma(\Omega_1, \Omega_2, \Omega_3)$.
Let $\mathcal{V}$ be the vector space of all random variables on $(\Omega, \mathscr{F})$, i.e.,

$$\mathcal{V} = \left\{ X : \ X \text{ is a random variable with respect to } \mathscr{F} \right\}.$$

- What is the dimension of $\mathcal{V}$?
- Identify a basis for $\mathcal{V}$.

Envelopes

Two envelopes, each containing a cheque, are placed in front of you.
The amounts in these cheques are **distinct and unknown** (say A and B, $A < B$ without loss of generality). You pick one of the two envelopes **uniformly at random** and open it. You can either accept the amount or exchange it for the amount on the unopened cheque. What should you do?

- **Strategy 1:** Accept the amount seen on the opened cheque.
- **Strategy 2:** Let F be a strictly increasing CDF of a continuous random variable.
  Accept the amount on the opened cheque with probability $F(\text{amount})$.
  Exchange with probability $1 - F(\text{amount})$.
- **Strategy 3:** Let X be a continuous random variable. Generate a sample of X on a computer. If the amount on the opened cheque is larger than X, accept it, else exchange for the unopened cheque.

Determine the expected reward you get under each strategy.

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Verifying Matrix Multiplications

Let A, B, C be $n \times n$ matrices (think $n \approx 20000$ or larger).
We want to verify whether $AB = C$ or not, without explicitly multiplying two matrices.

Multiplication of two $n \times n$ matrices takes $O(n^3)$ computations!

Let $\mathbf{X} \sim \text{Unif}\{0, 1\}^n$. Consider the following strategy.

If $(AB - C)\,\mathbf{X} = \mathbf{0}$, then we conclude $AB = C$. Else, we conclude $AB \neq C$.

$(AB - C)\mathbf{X} = A(B\mathbf{X}) - C\mathbf{X}$ takes only $O(n^2)$ computations!

What is the probability of making an error?

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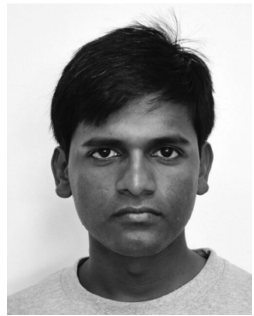

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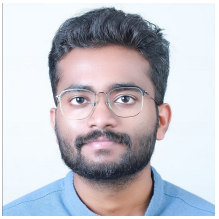

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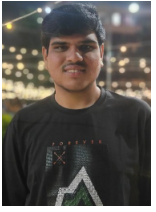

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