Almost Cost-Free Communication in Federated Best Arm Identification

Workshop on Information Theory and Data Science
Institute for Mathematical Sciences
National University of Singapore

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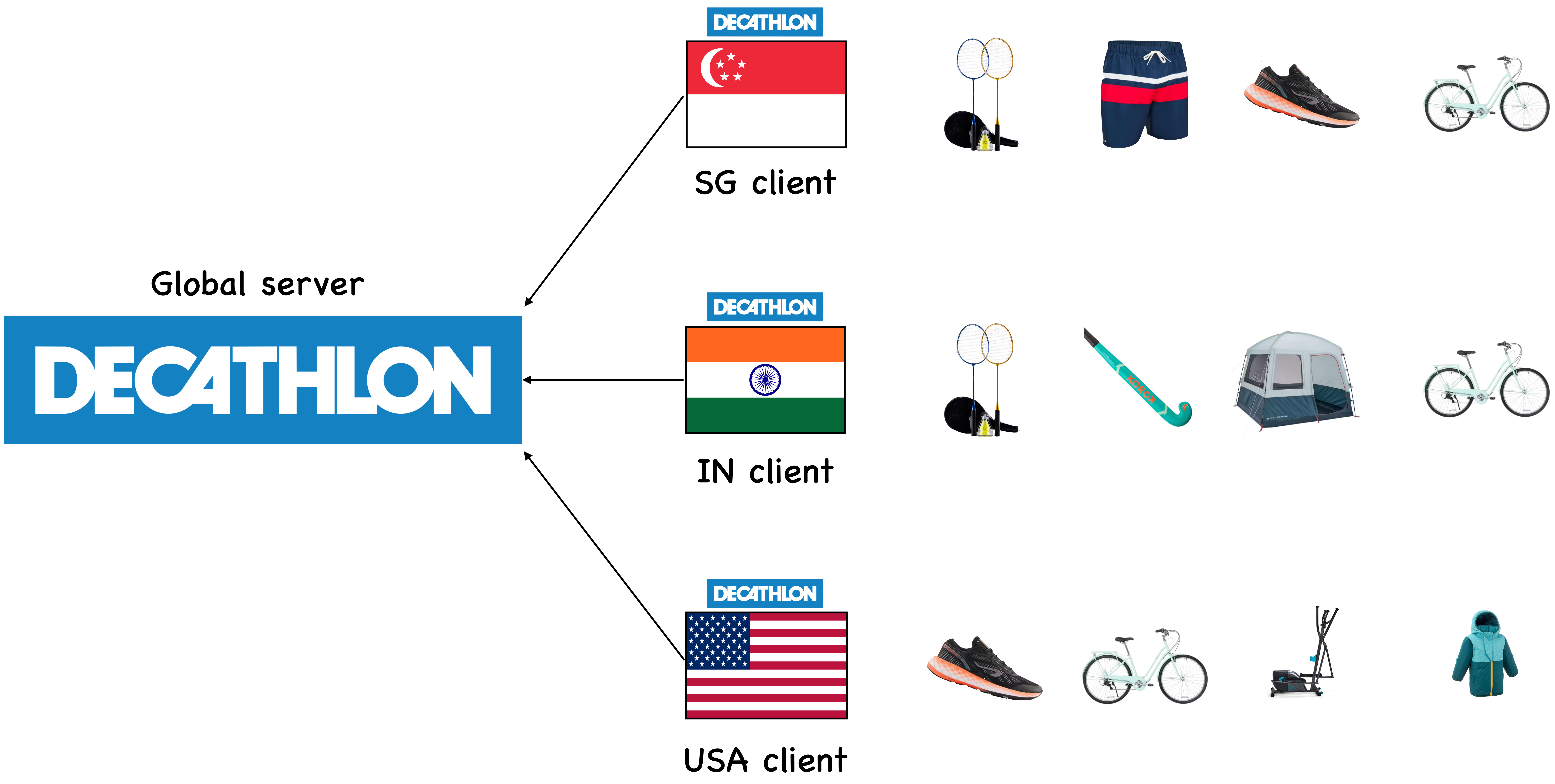
Joint Work with

Kota Srinivas Reddy

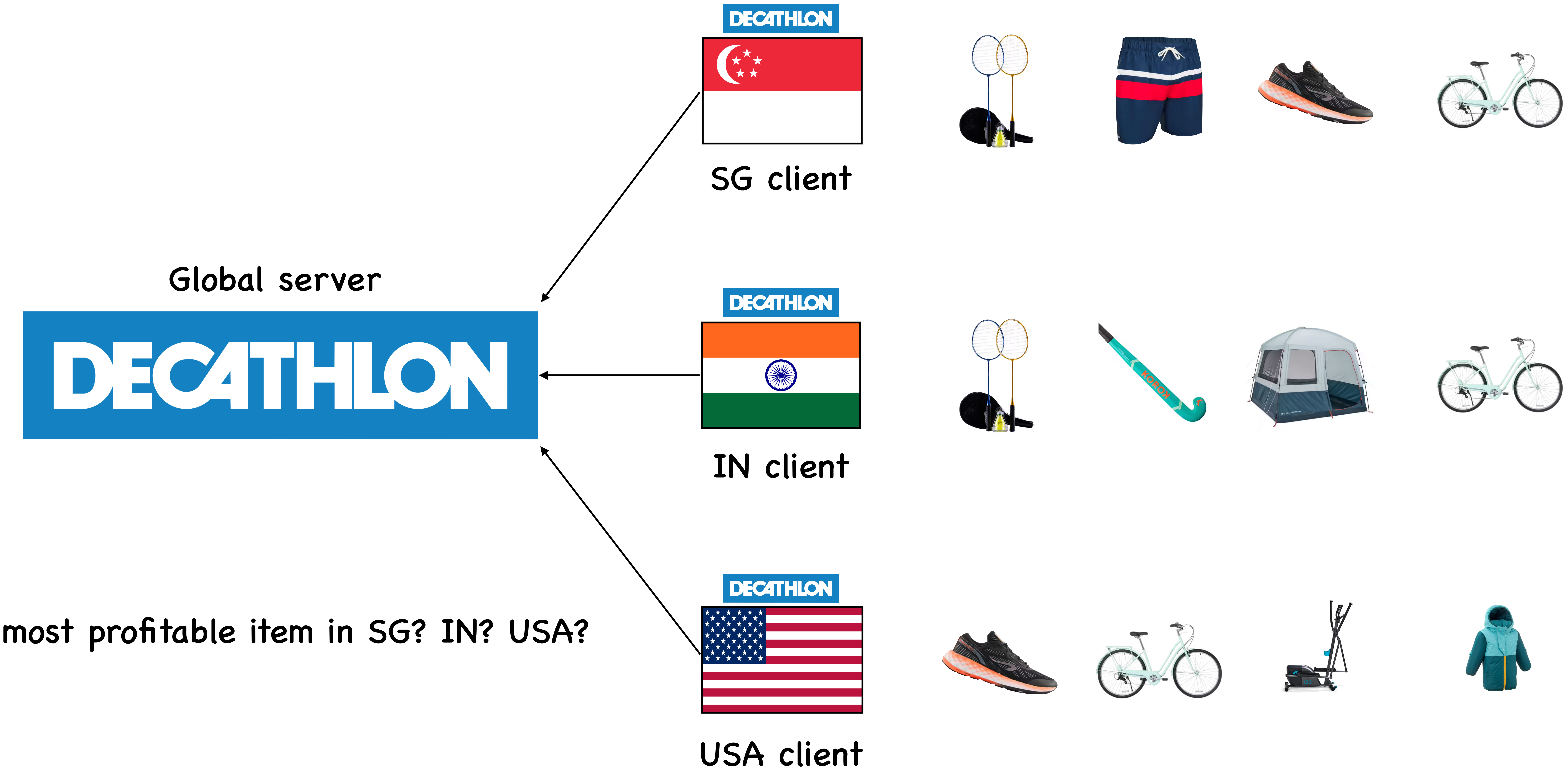
Vincent Y. F. Tan

Federated Best Arm Identification



* Pics only for demonstration, courtesy Google

Federated Best Arm Identification



Locally most profitable item in SG? IN? USA?

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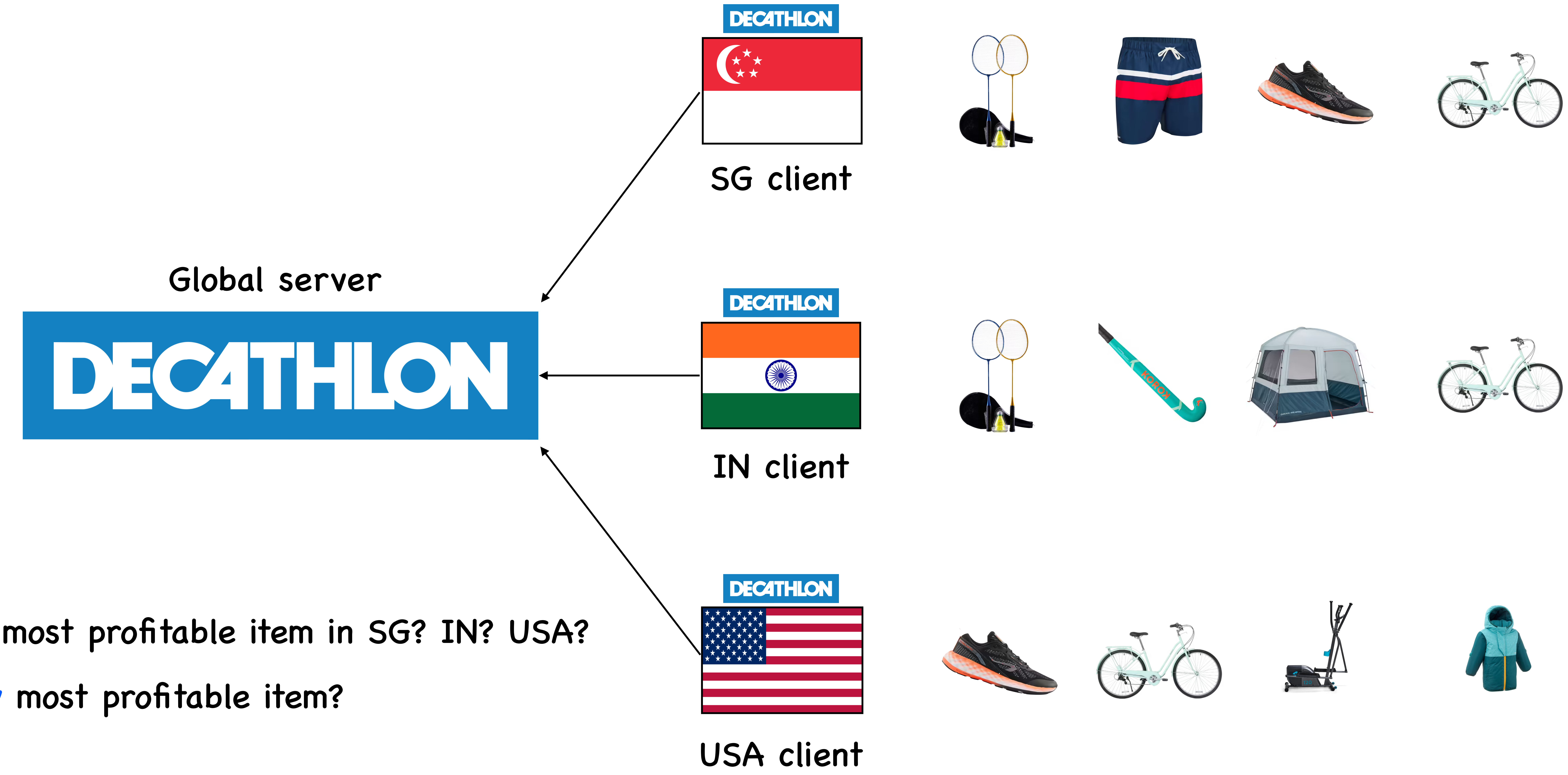

Locally most profitable item in SG? IN? USA?

Globally most profitable item?

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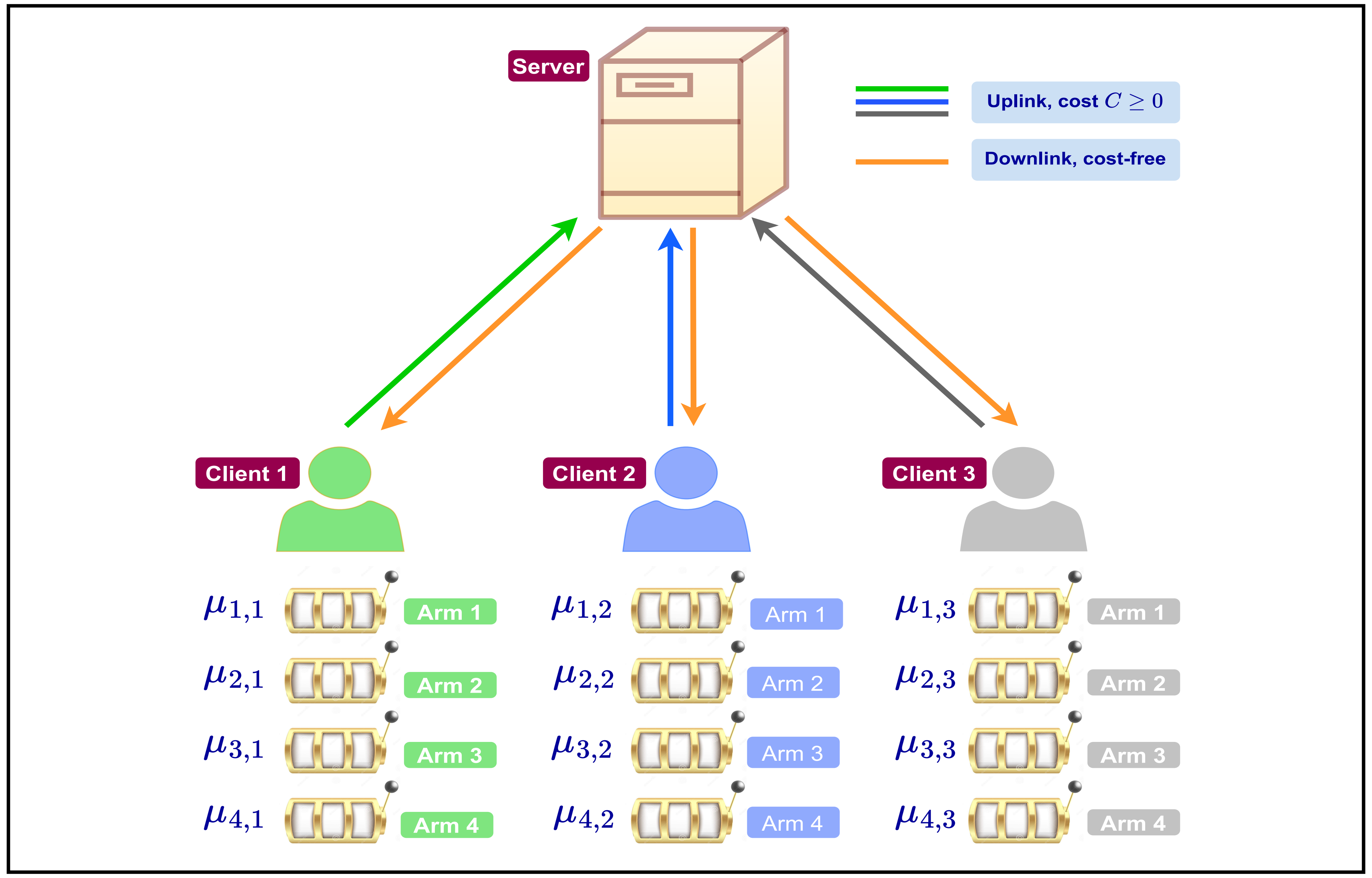
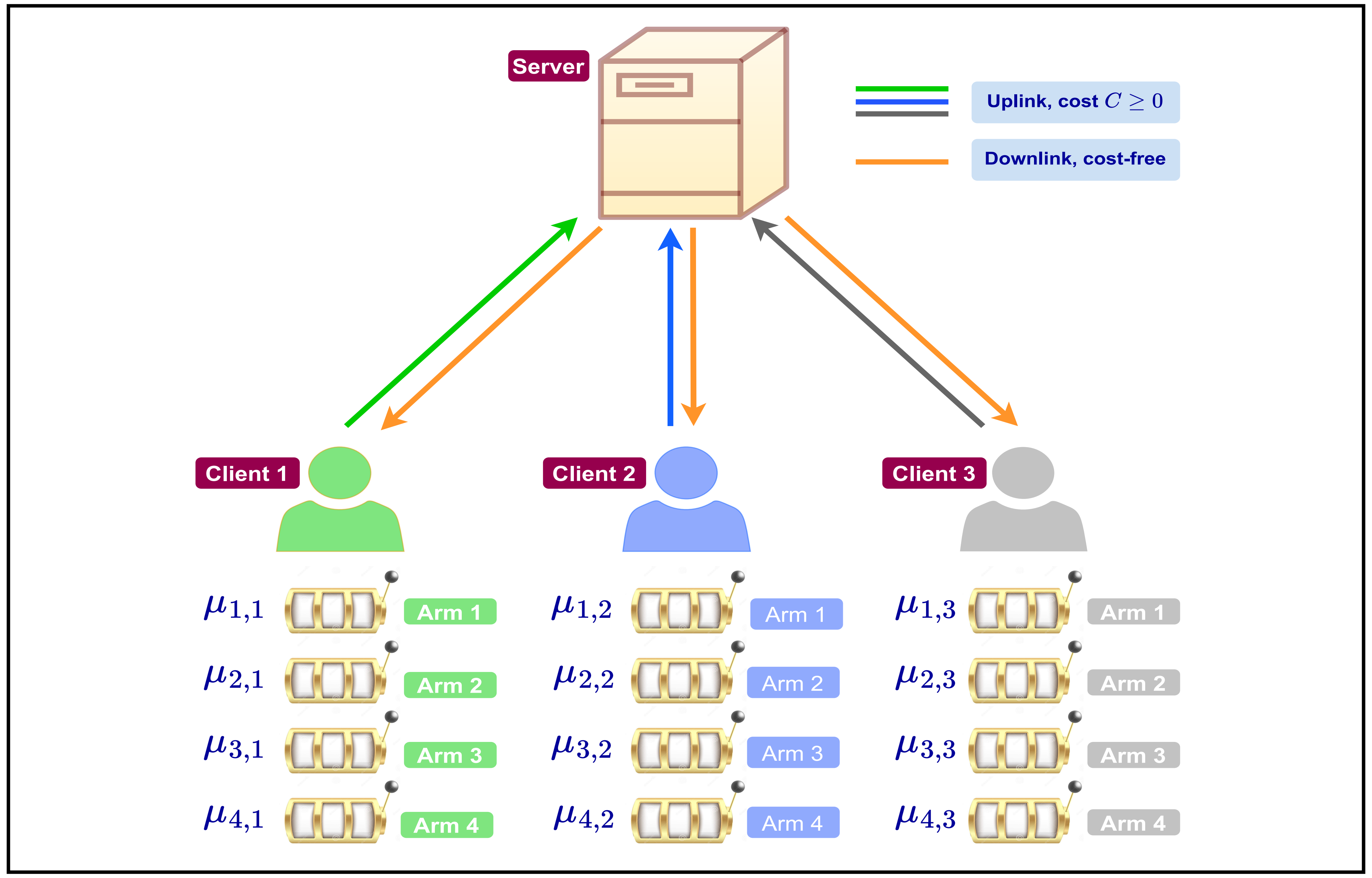
Problem Abstraction



Problem Abstraction

- Single server, M clients



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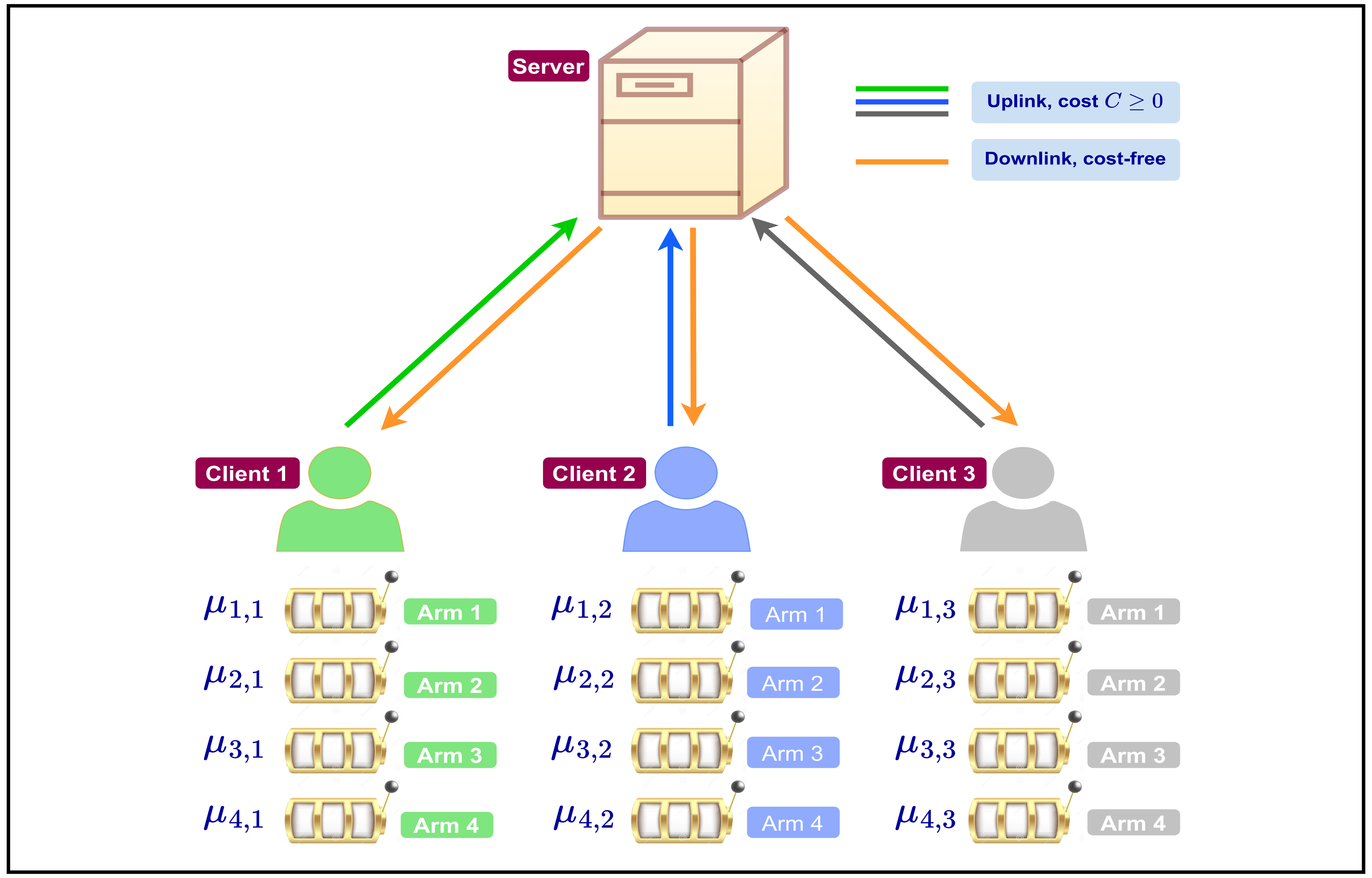

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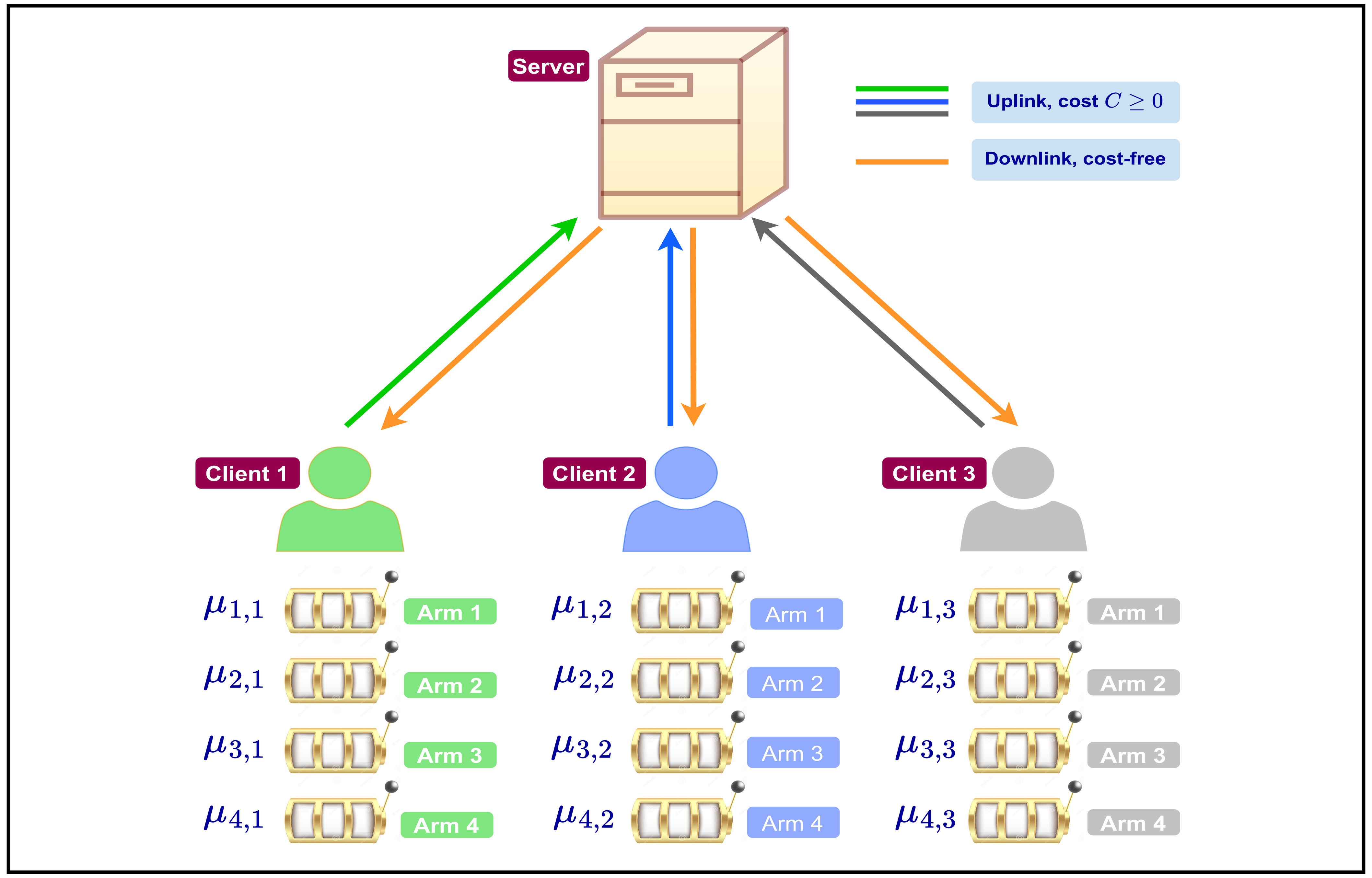

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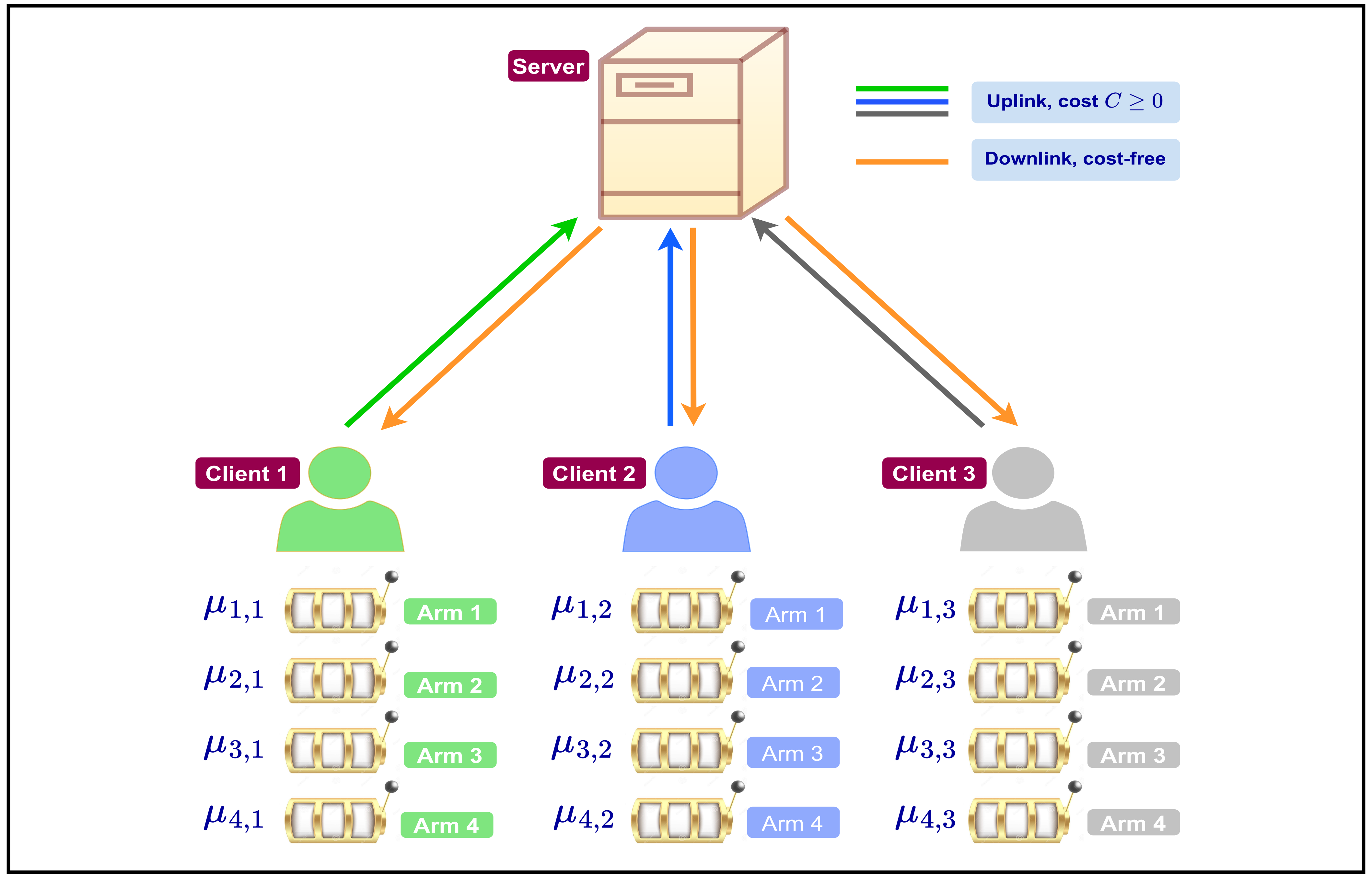

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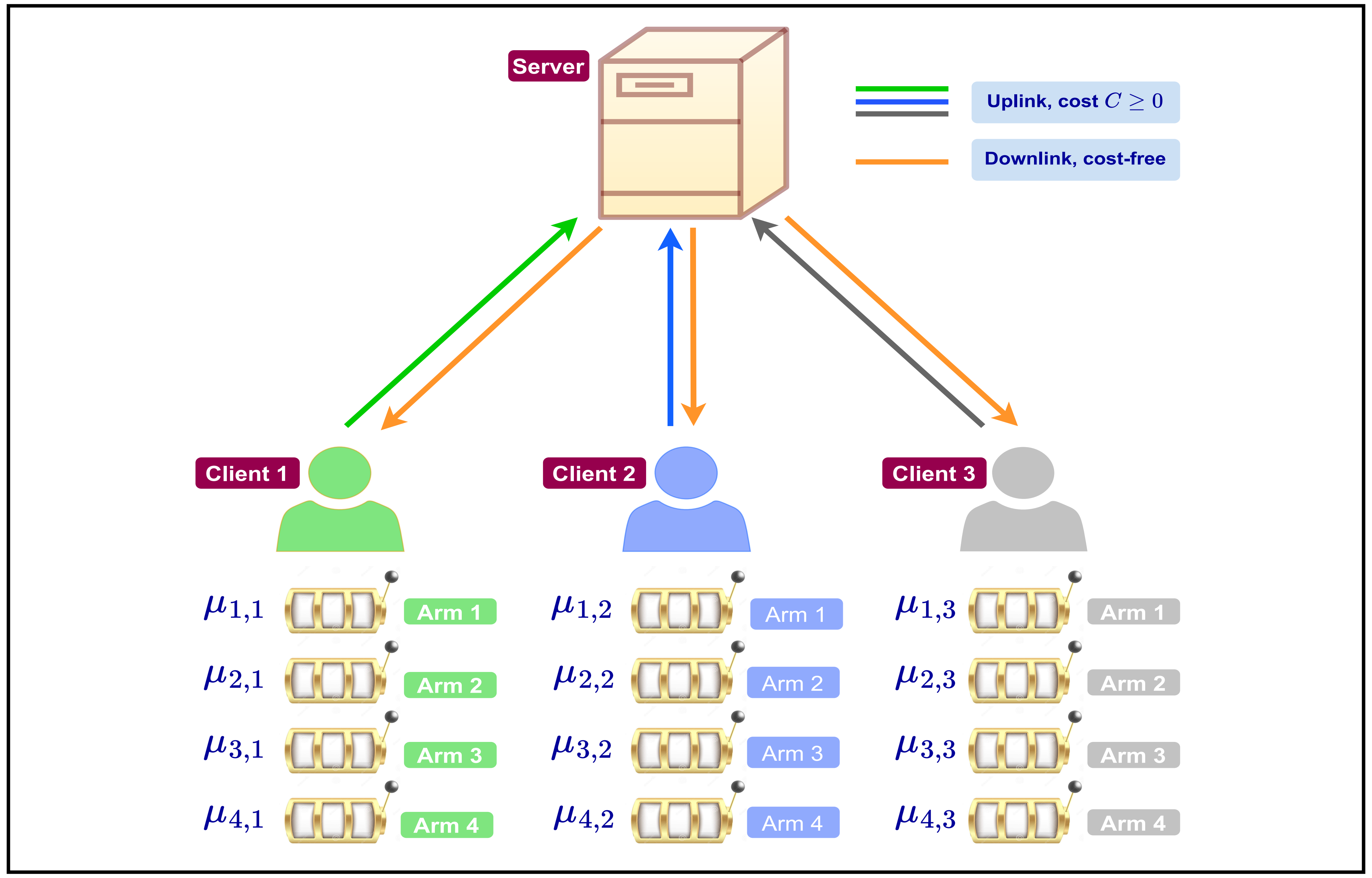

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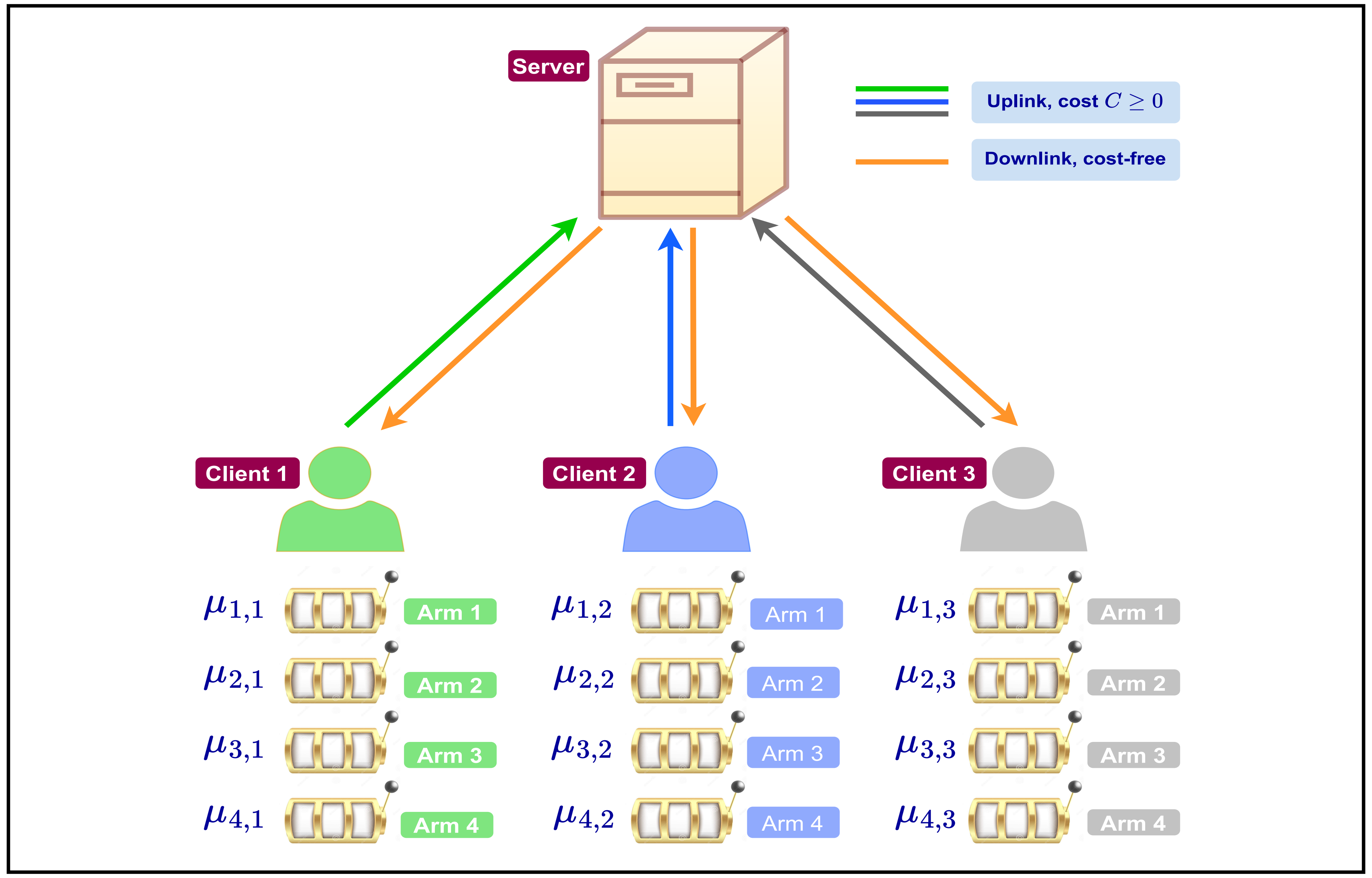

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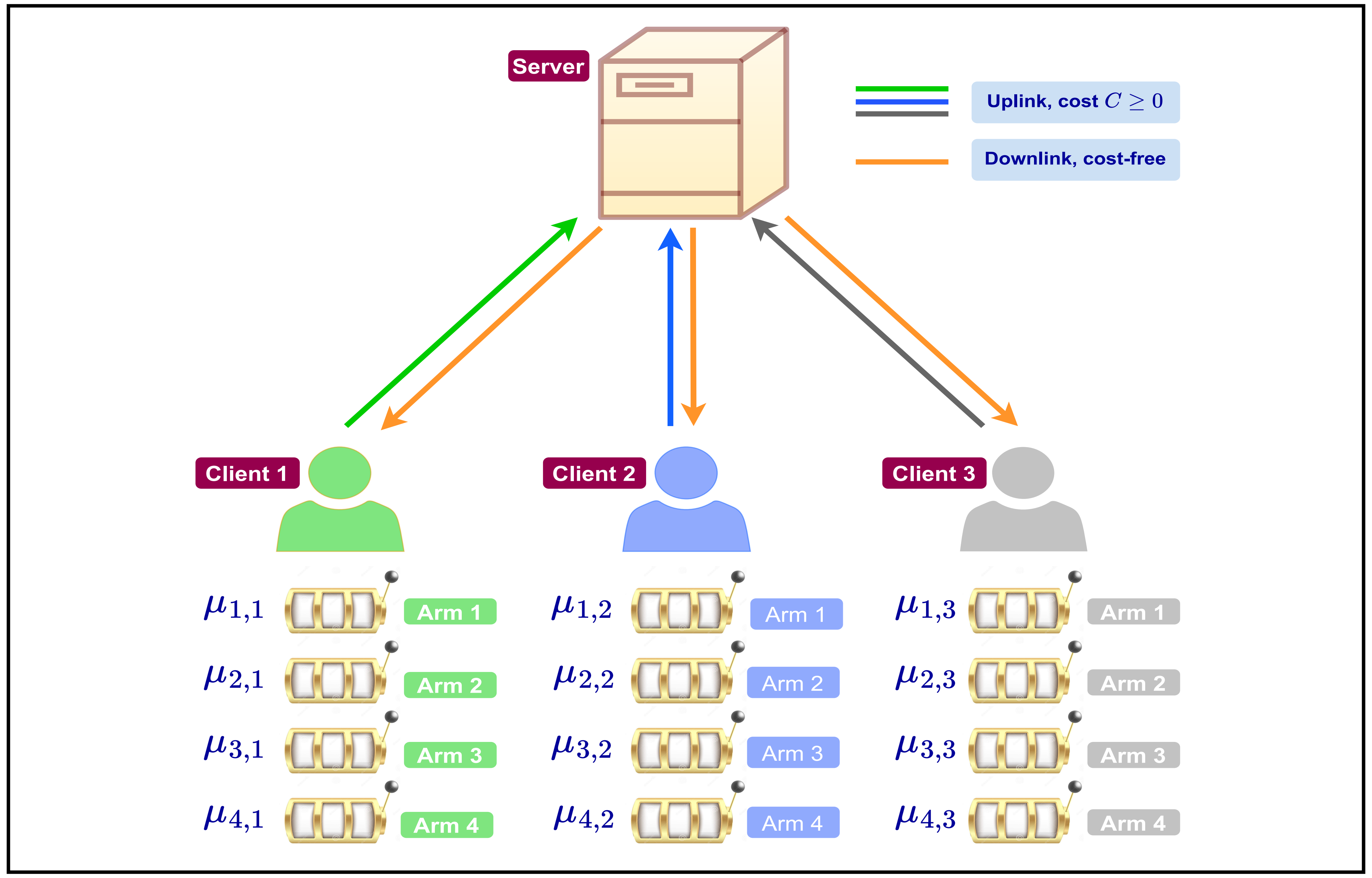
- Uplink communication cost: $C \geq 0$ per usage per client,

  Downlink cost: nil



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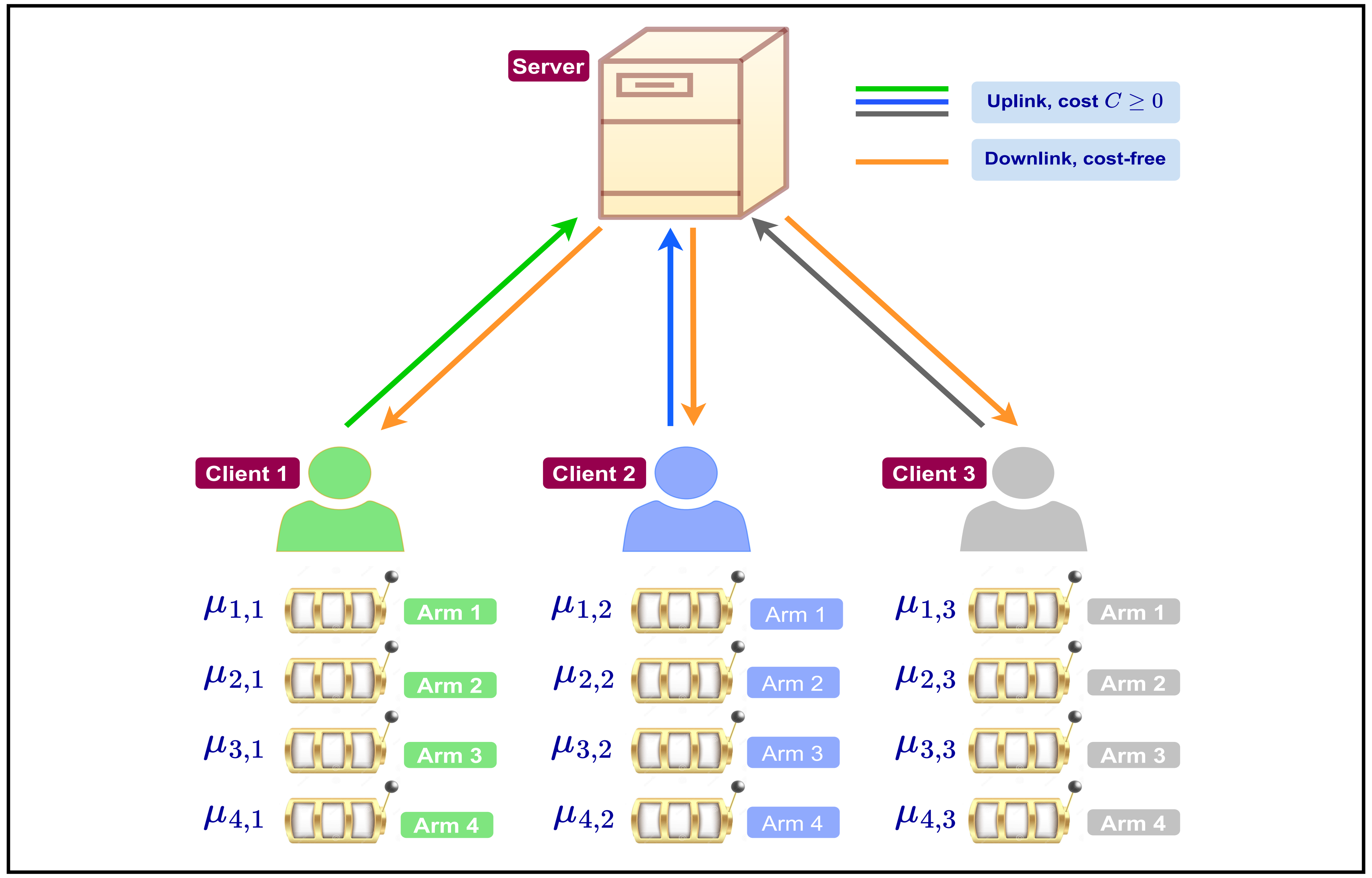

Find local (clients) & global (server) best arms

$$\min \quad \mathbb{E}\Big[\underbrace{\text{no. of arm pulls} + \text{communication cost}}_{=\ \text{total cost}}\Big]$$

$$\text{s.t.} \quad \text{error probability} \leq \delta$$

Results at a Glance — 1/3

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  - Based on successive elimination — FedElim
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Per(H) — Succ. elim. + comm. at $n \in \{1 + t\,\mathsf{H}\}_{t=0}^{\infty}$

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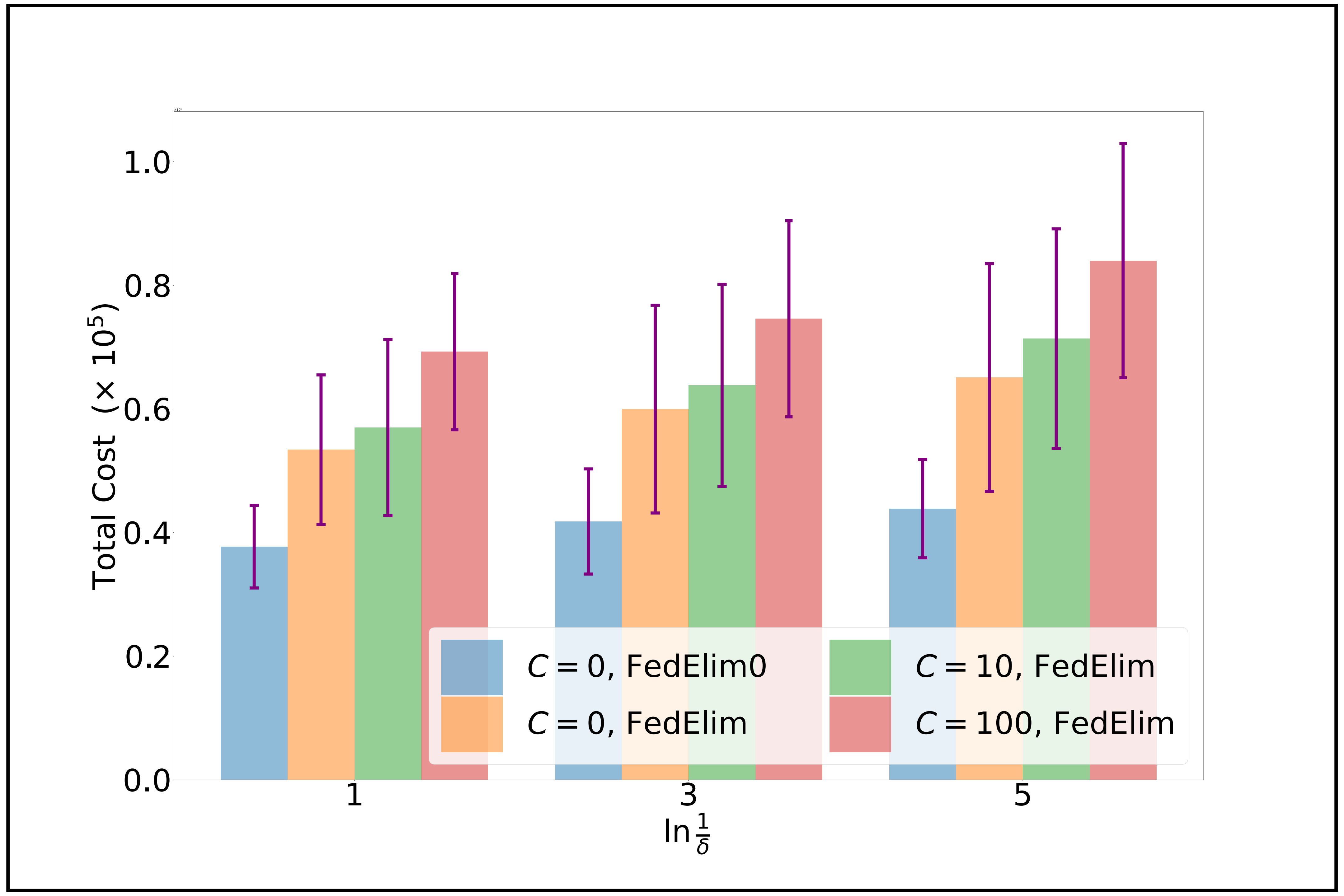
almost cost-free communication



For any $C > 0$, $\forall \delta$ small,

$$P\left(\text{total cost of FedElim} \lesssim 3 \times \text{total cost of Per(1) when } C = 0\right) \geq 1 - \delta$$

Key takeaway 1

Results at a Glance — 2/3

- A federated BAI algorithm
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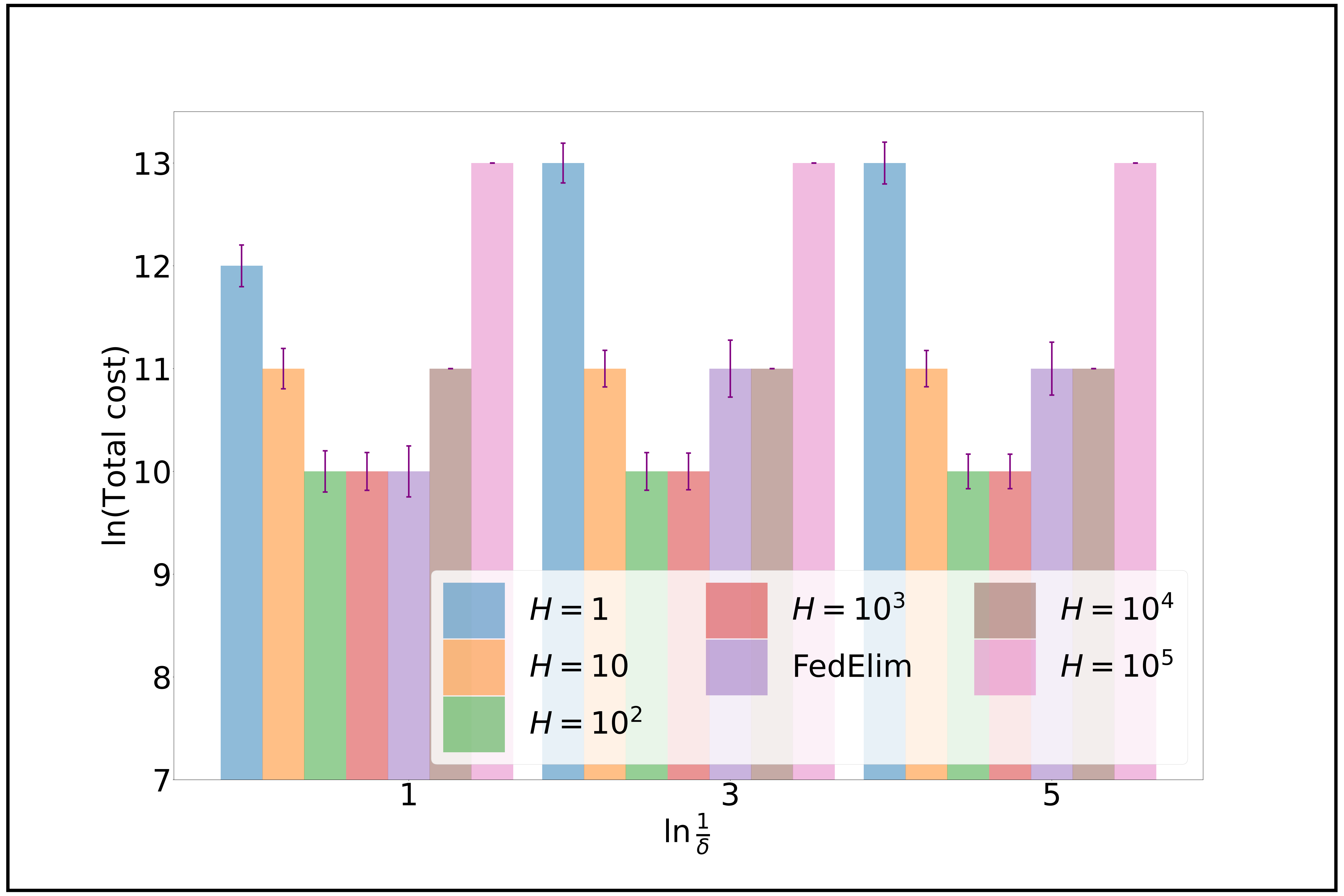

FedElim operates at a sweet spot

Key takeaway 2

Results at a Glance — 3/3

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Scheme	No. of arm pulls	Comm. cost	Total cost
Per(H)	O(T)	O(CT/H)	O(T+CT/H)
FedElim*	2T	O(C log(T))	3T
SupExp	O(T²)	O(C loglog(T))	O(T²)

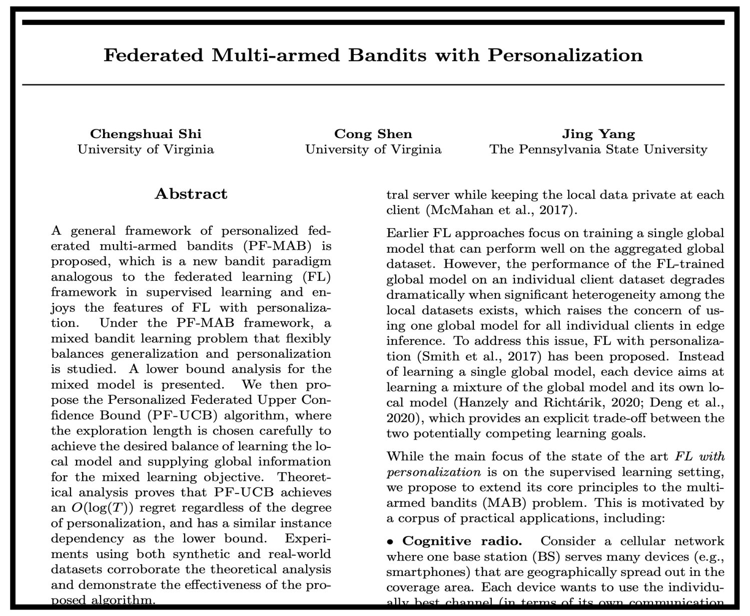
* for sufficiently small error probabilities

FedElim carefully balances no. of arm pulls and comm. cost

Key takeaway 3

Related Works

- Shi, Shen, and Yang (2021) — similar setting, regret minimisation
- Mitra, Hassani, and Pappas (2021)
    - Arm sets disjoint across clients
    - Only global best arm identification
    - Local best arms can be eliminated along the way
    - Periodic communication scheme
- Tao, Zhang, and Zhou (2019), Hillel et al. (2013)
    - Identical arm distributions at all the clients
    - Collaborative learning of the best arm



Federated Multi-armed Bandits with Personalization

Chengshuai Shi, University of Virginia
Cong Shen, University of Virginia
Jing Yang, The Pennsylvania State University

Abstract

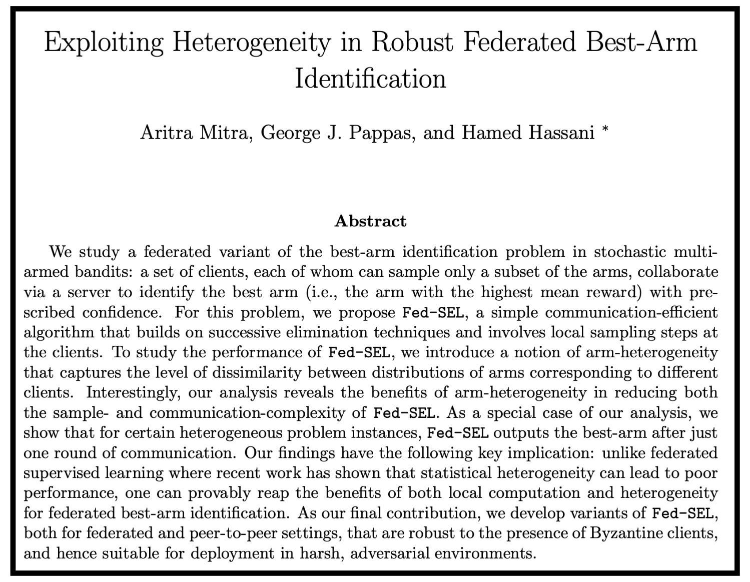
A general framework of personalized federated multi-armed bandits (PF-MAB) is proposed, which is a new bandit paradigm analogous to the federated learning (FL) framework in supervised learning and enjoys the features of FL with personalization. Under the PF-MAB framework, a mixed bandit learning problem that flexibly balances generalization and personalization is studied. A lower bound analysis for the mixed model is presented. We then propose the Personalized Federated Upper Confidence Bound (PF-UCB) algorithm, where the exploration length is chosen carefully to achieve the desired balance of learning the local model and supplying global information for the mixed learning objective. Theoretical analysis proves that PF-UCB achieves an $O(\log(T))$ regret regardless of the degree of personalization, and has a similar instance dependency as the lower bound. Experiments using both synthetic and real-world datasets corroborate the theoretical analysis and demonstrate the effectiveness of the proposed algorithm.

tral server while keeping the local data private at each client (McMahan et al., 2017).

Earlier FL approaches focus on training a single global model that can perform well on the aggregated global dataset. However, the performance of the FL-trained global model on an individual client dataset degrades dramatically when significant heterogeneity among the local datasets exists, which raises the concern of using one global model for all individual clients in edge inference. To address this issue, FL with personalization (Smith et al., 2017) has been proposed. Instead of learning a single global model, each device aims at learning a mixture of the global model and its own local model (Hanzely and Richtárik, 2020; Deng et al., 2020), which provides an explicit trade-off between the two potentially competing learning goals.

While the main focus of the state of the art *FL with personalization* is on the supervised learning setting, we propose to extend its core principles to the multi-armed bandits (MAB) problem. This is motivated by a corpus of practical applications, including:

- **Cognitive radio.** Consider a cellular network where one base station (BS) serves many devices (e.g., smartphones) that are geographically spread out in the coverage area. Each device wants to use the individually best channel (in terms of its own communication

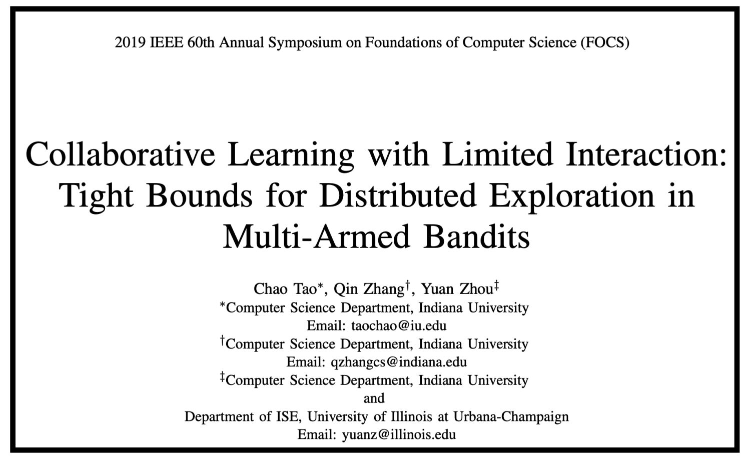


Exploiting Heterogeneity in Robust Federated Best-Arm Identification

Aritra Mitra, George J. Pappas, and Hamed Hassani *

Abstract

We study a federated variant of the best-arm identification problem in stochastic multi-armed bandits: a set of clients, each of whom can sample only a subset of the arms, collaborate via a server to identify the best arm (i.e., the arm with the highest mean reward) with prescribed confidence. For this problem, we propose `Fed-SEL`, a simple communication-efficient algorithm that builds on successive elimination techniques and involves local sampling steps at the clients. To study the performance of `Fed-SEL`, we introduce a notion of arm-heterogeneity that captures the level of dissimilarity between distributions of arms corresponding to different clients. Interestingly, our analysis reveals the benefits of arm-heterogeneity in reducing both the sample- and communication-complexity of `Fed-SEL`. As a special case of our analysis, we show that for certain heterogeneous problem instances, `Fed-SEL` outputs the best-arm after just one round of communication. Our findings have the following key implication: unlike federated supervised learning where recent work has shown that statistical heterogeneity can lead to poor performance, one can provably reap the benefits of both local computation and heterogeneity for federated best-arm identification. As our final contribution, we develop variants of `Fed-SEL`, both for federated and peer-to-peer settings, that are robust to the presence of Byzantine clients, and hence suitable for deployment in harsh, adversarial environments.



2019 IEEE 60th Annual Symposium on Foundations of Computer Science (FOCS)

Collaborative Learning with Limited Interaction: Tight Bounds for Distributed Exploration in Multi-Armed Bandits

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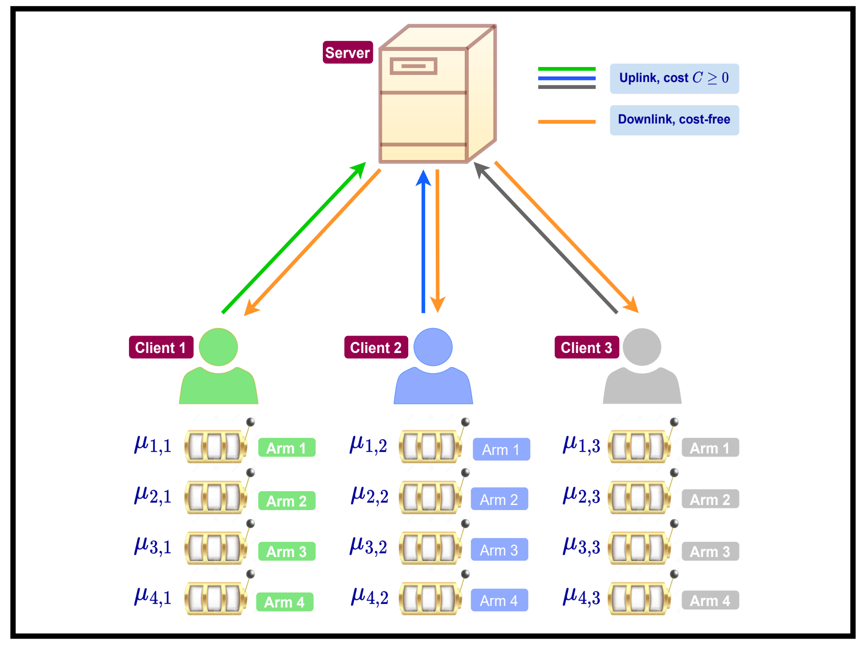
†Computer Science Department, Indiana University
Email: qzhangcs@indiana.edu
‡Computer Science Department, Indiana University
and
Department of ISE, University of Illinois at Urbana-Champaign
Email: yuanz@illinois.edu

The $C = 0$ Case

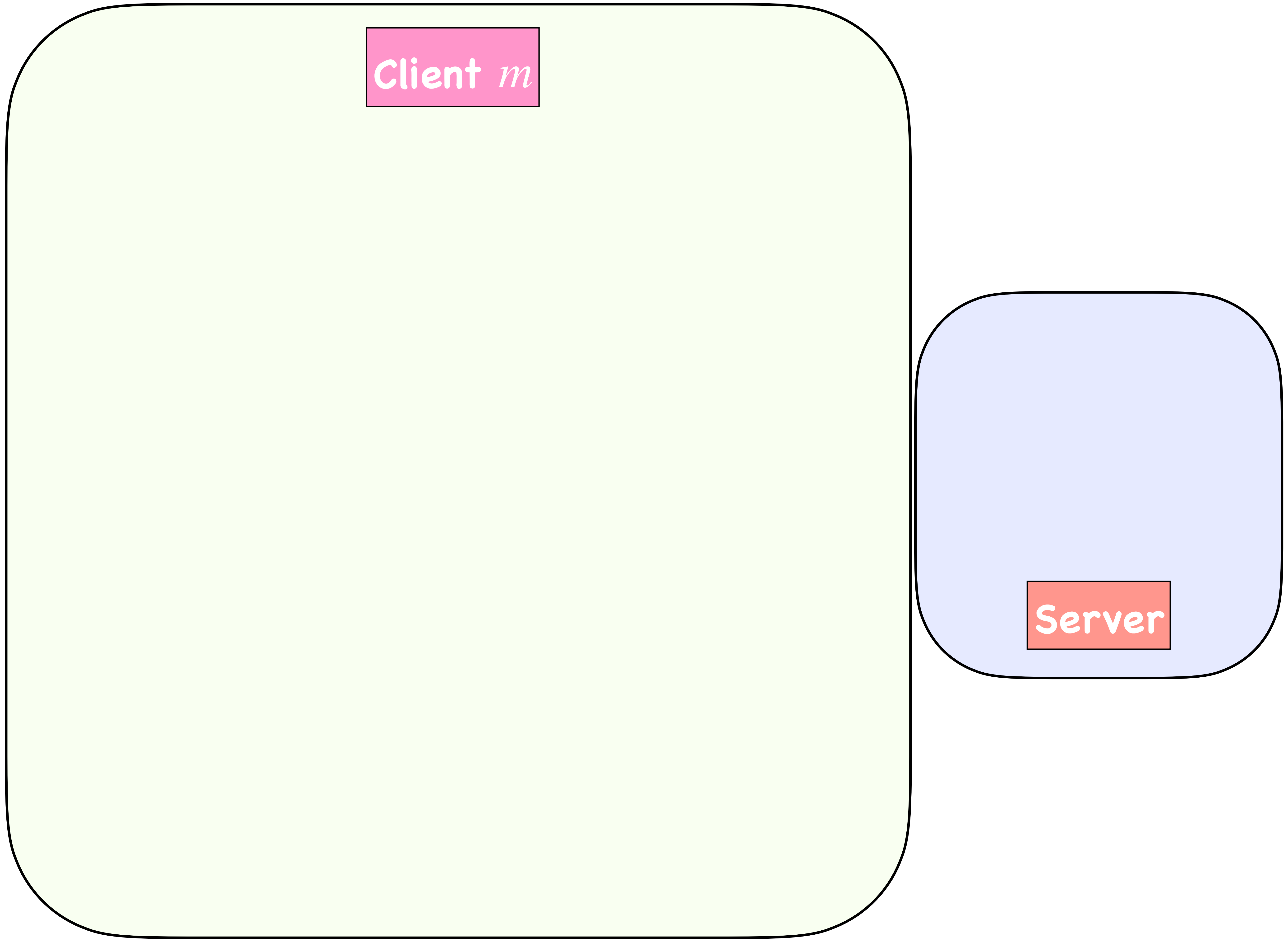
FedElim for $C = 0$ (FedElim0)

$$\min \quad \mathbb{E}\left[\underbrace{\text{no. of arm pulls} + \text{communication cost}}_{= \text{ total cost}}\right]$$

$$\text{s.t.} \quad \text{error probability} \leq \delta$$



Comm. at every time instant



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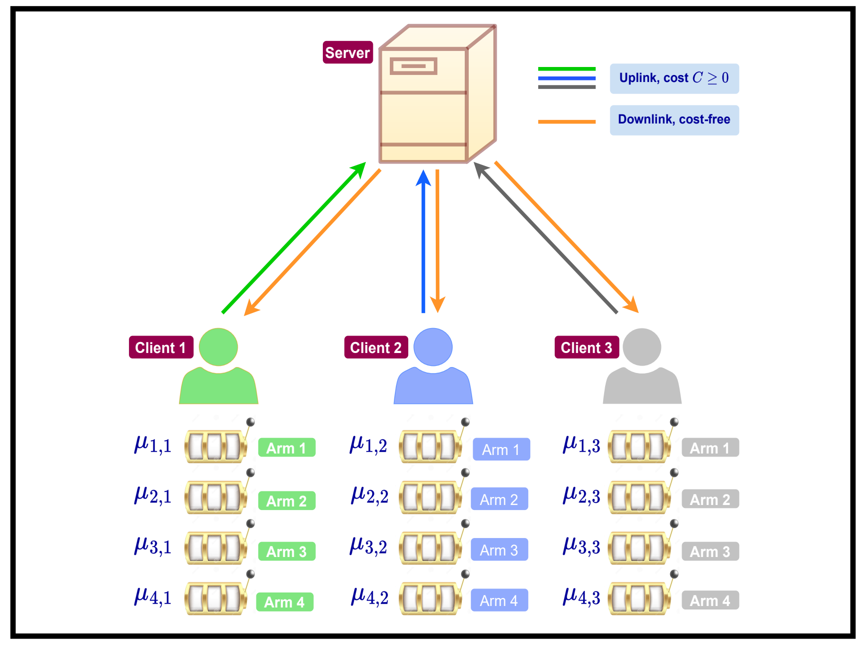

Comm. at every time instant

Client m

Server

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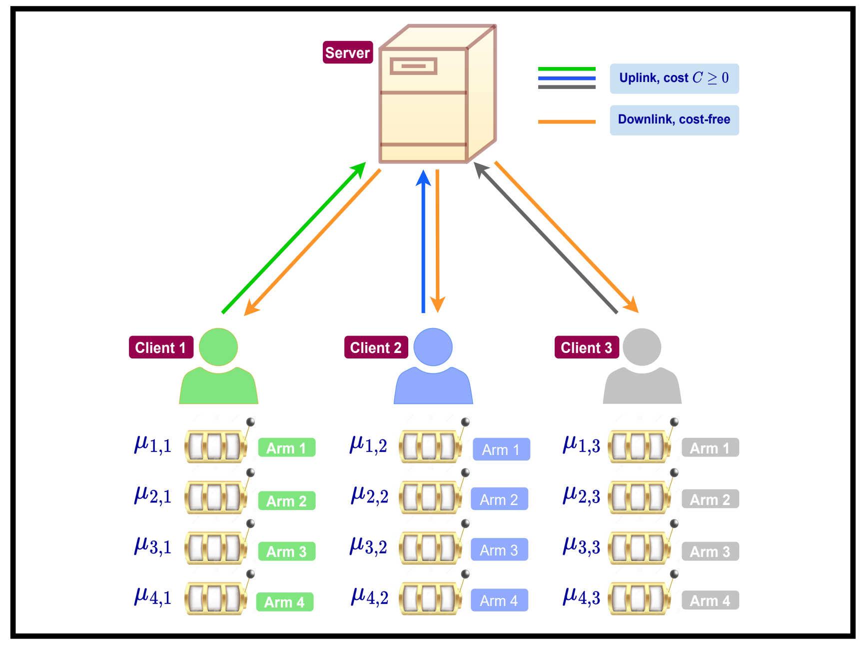

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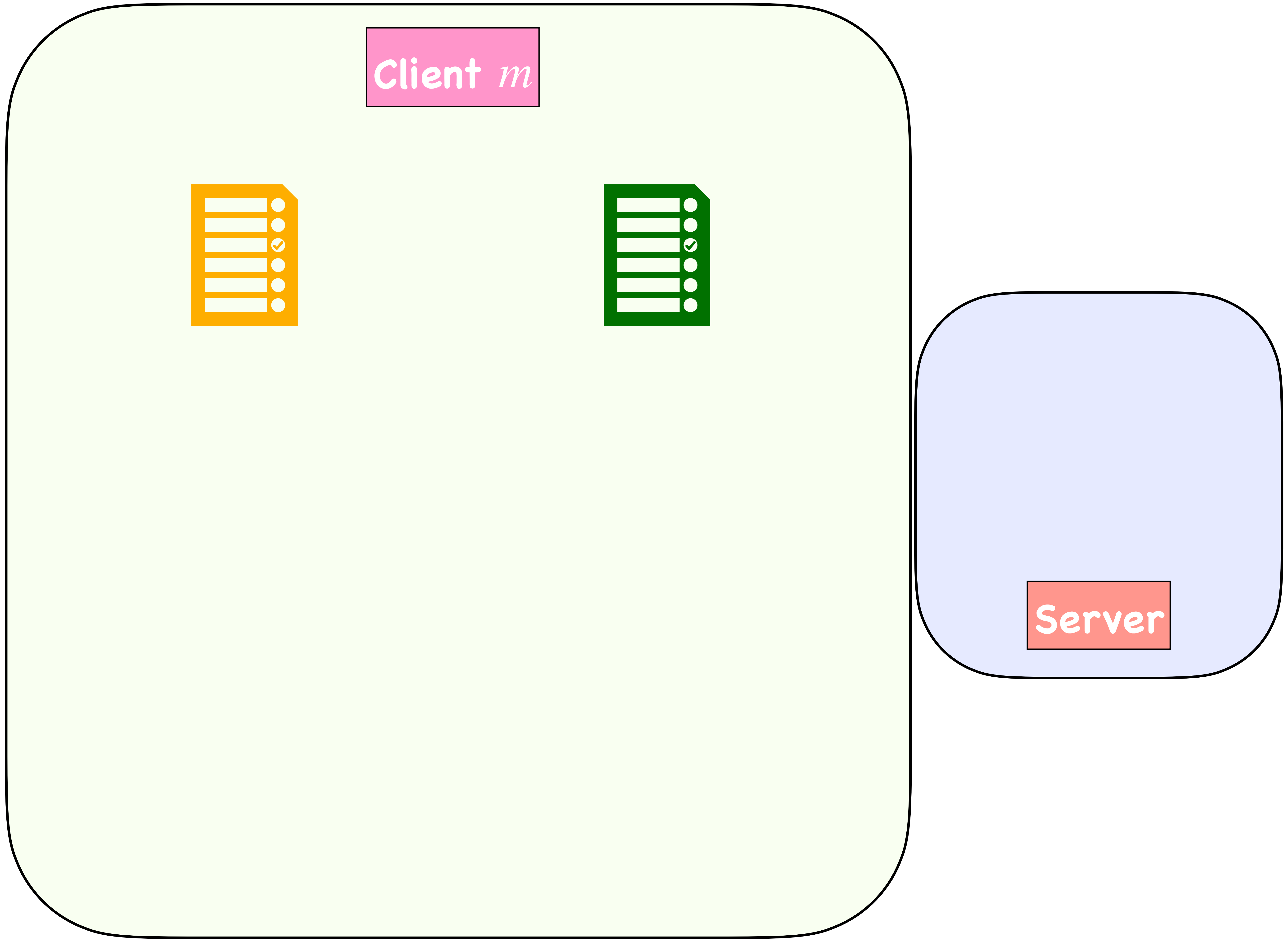

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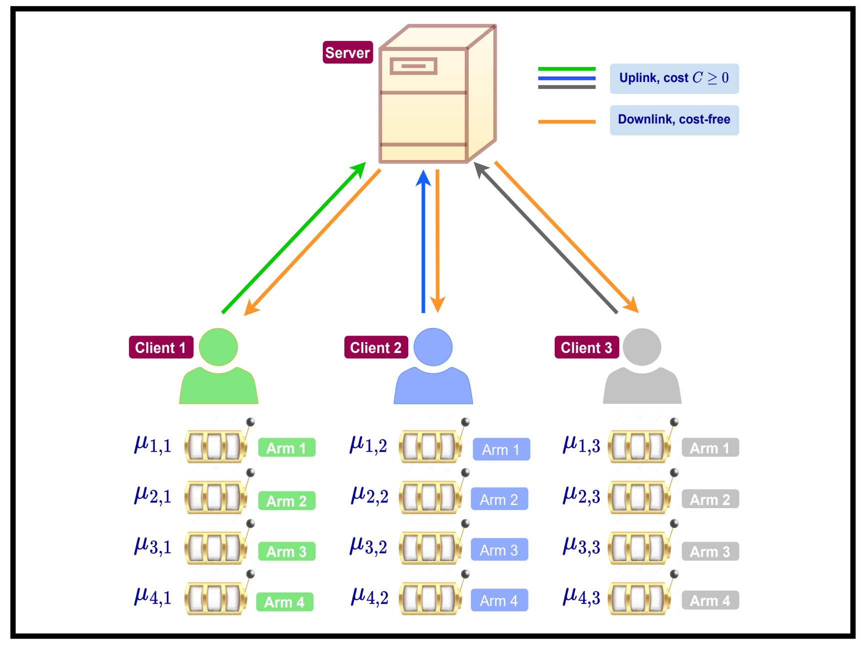

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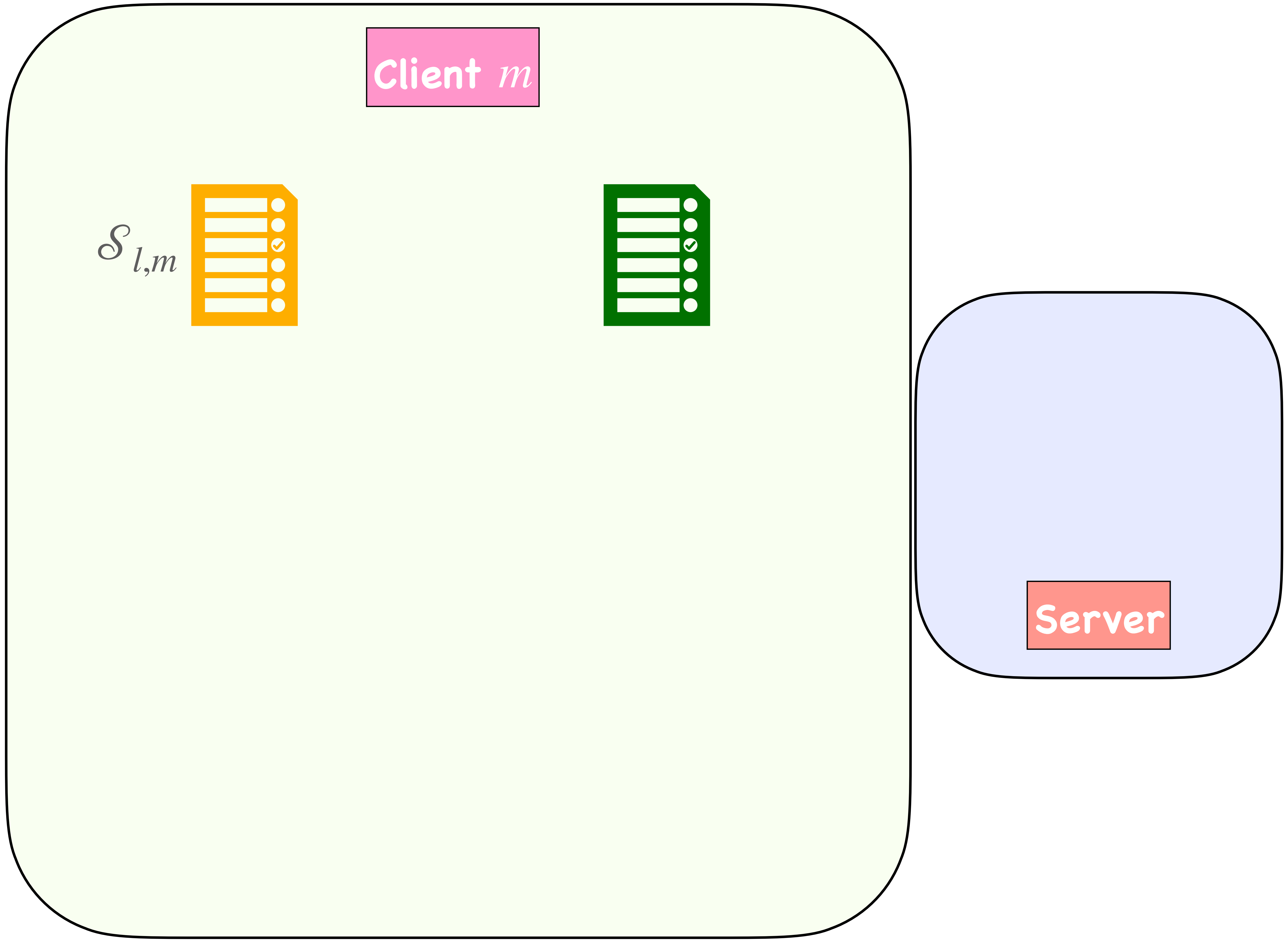

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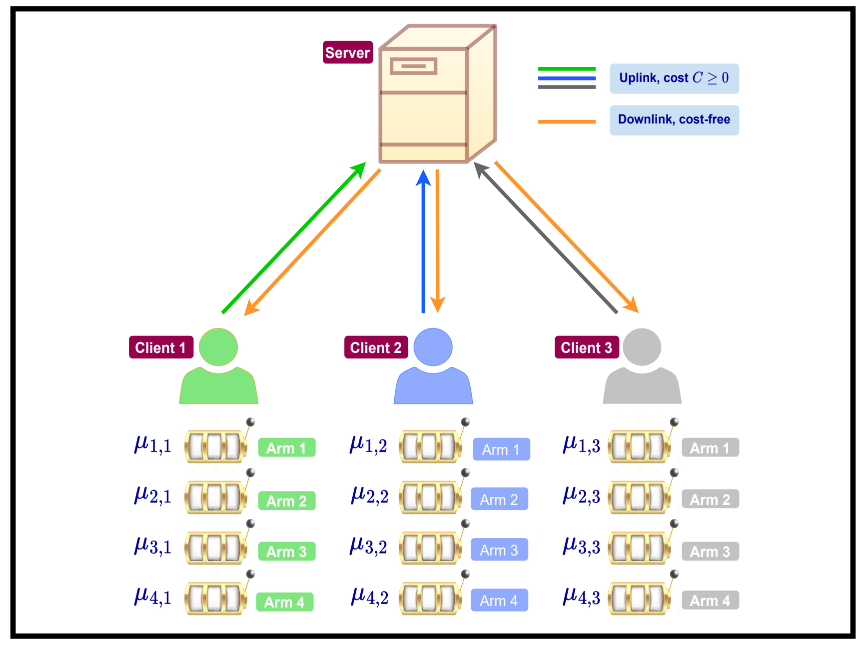

Comm. at every time instant

Client m

$\mathcal{S}_{l,m}$

$\mathcal{S}_g$

Server

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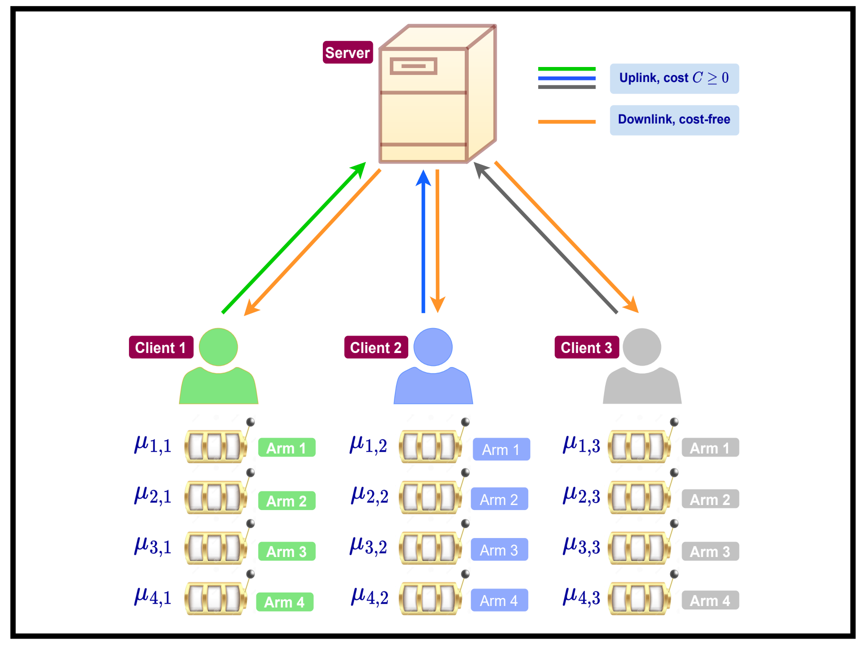

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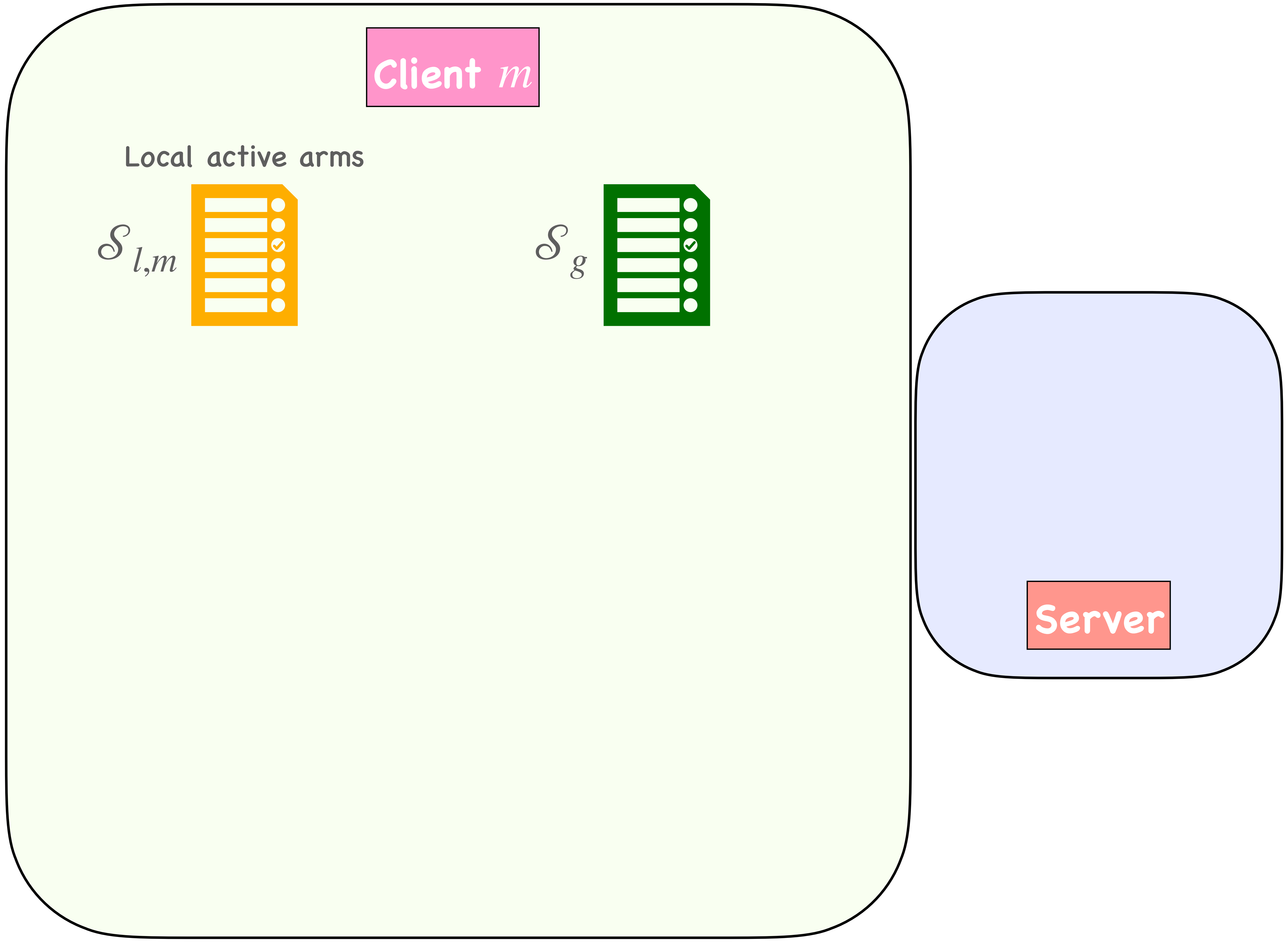

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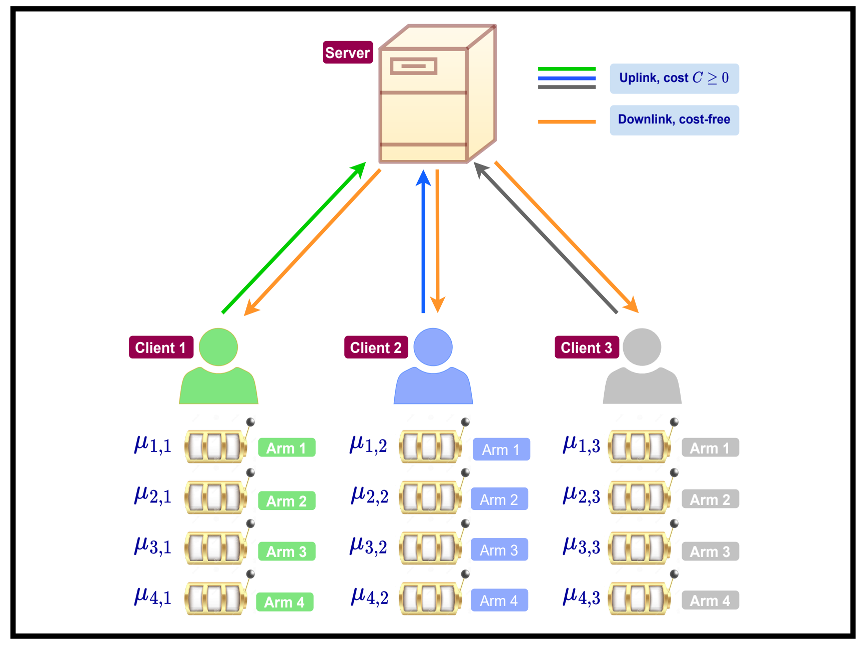

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Local active arms

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Global active arms

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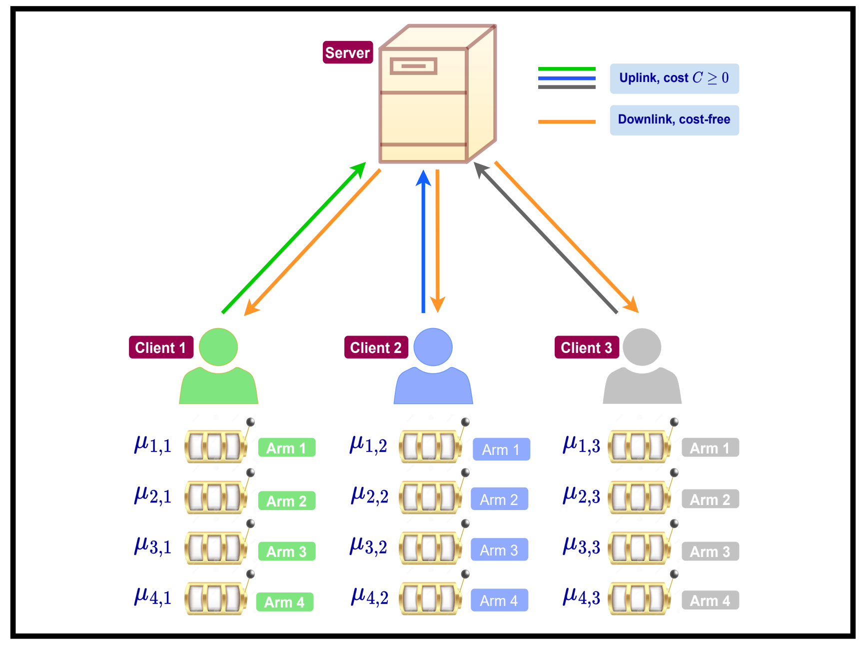

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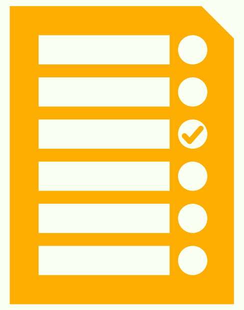
Client m

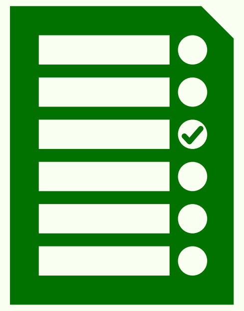
Local active arms

$\mathcal{S}_{l,m}$



Global active arms

$\mathcal{S}_g$



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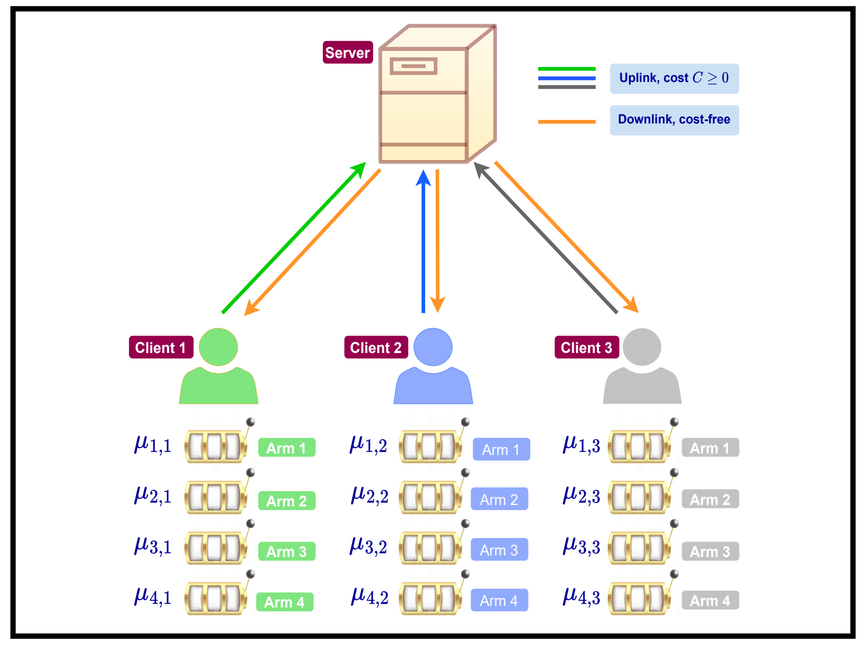

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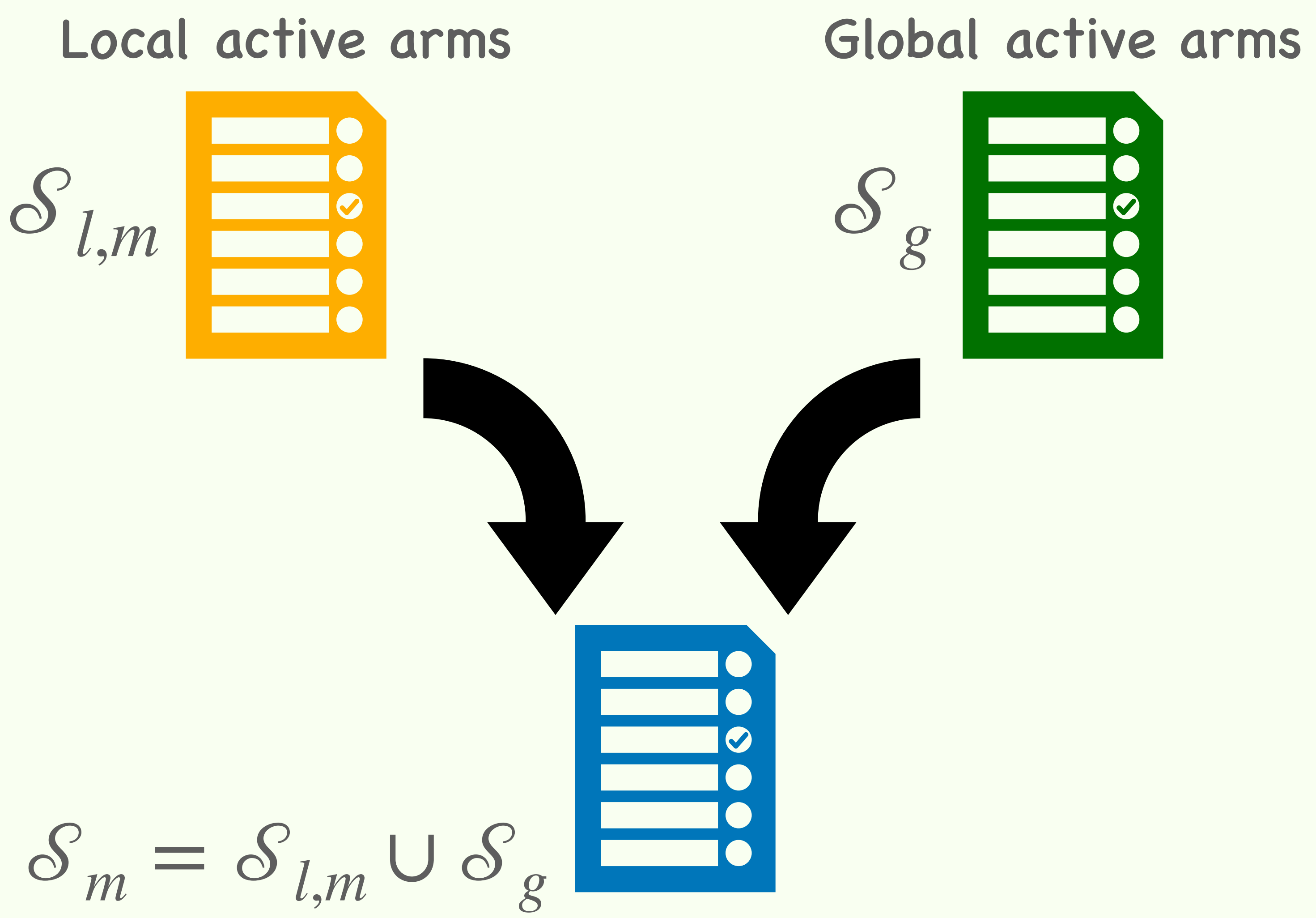

Server

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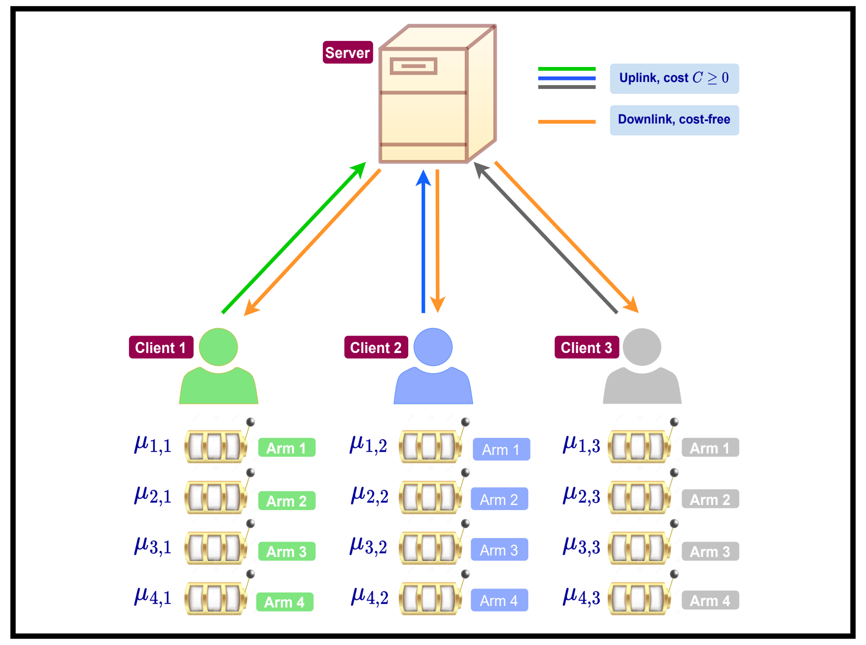

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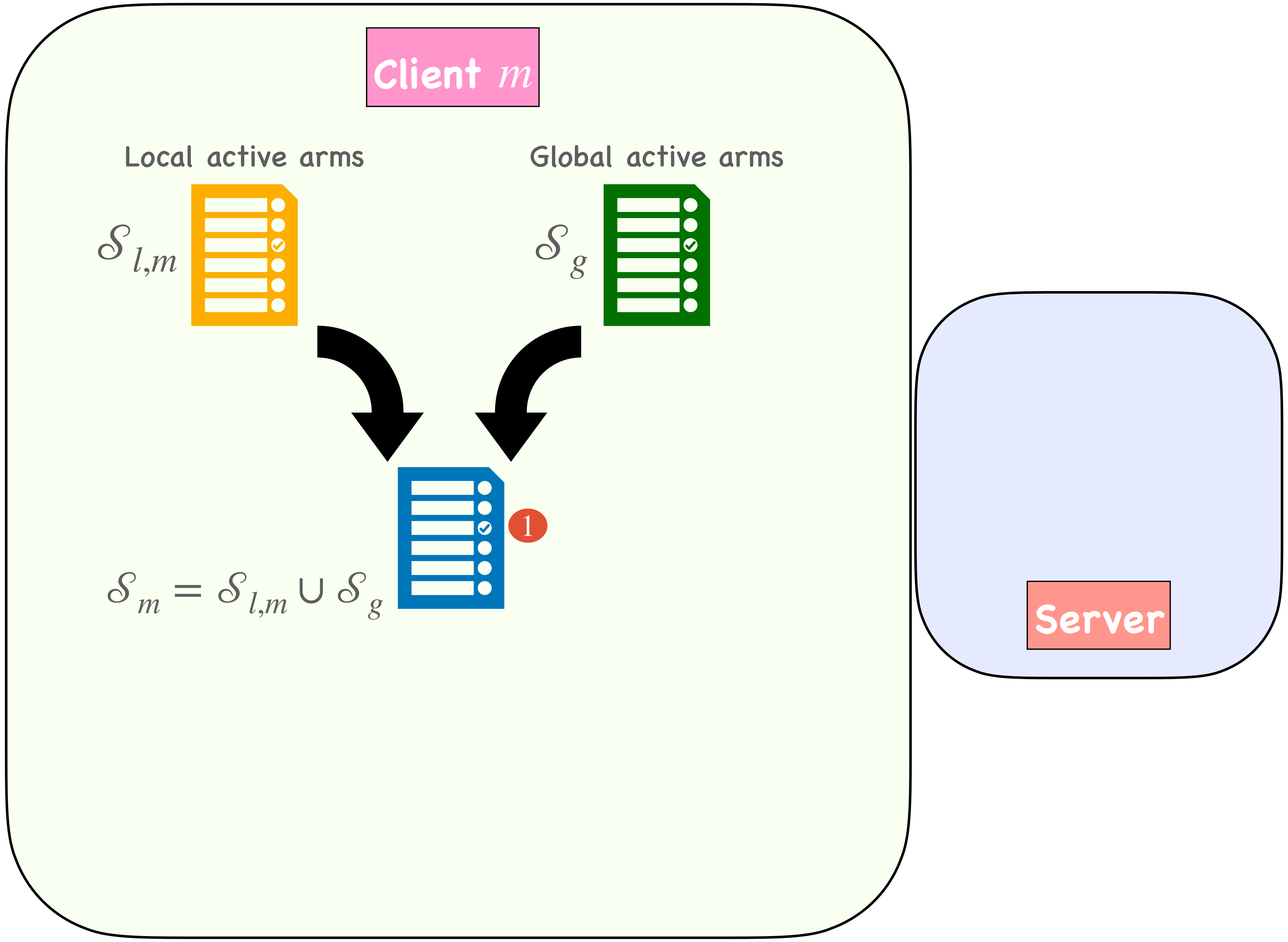

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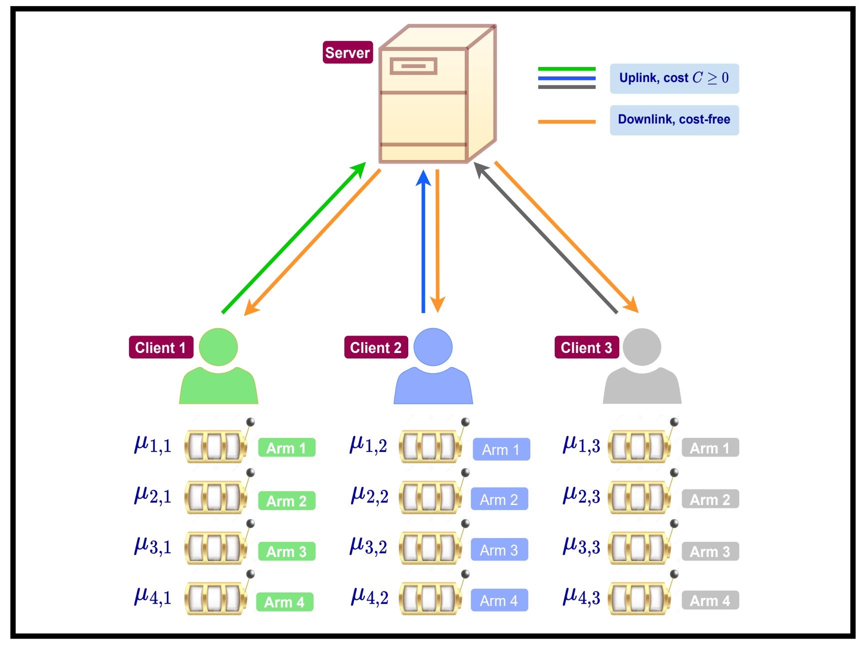

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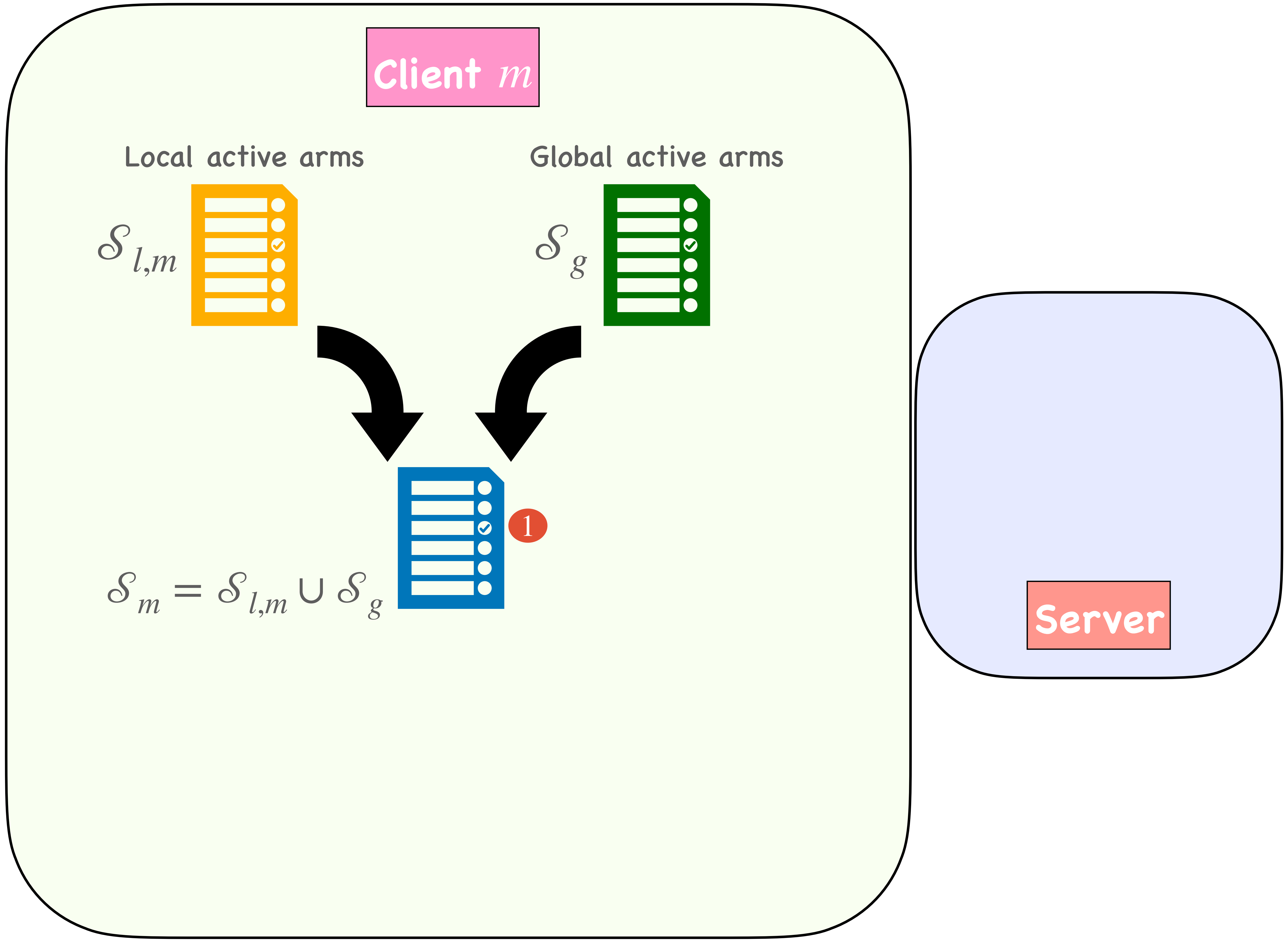

Initialise: $\mathcal{S}_{l,m} = [K]\ \forall m, \ \ \mathcal{S}_g = [K]$

1. Pull each arm $k \in \mathcal{S}_m$ and update its empirical mean $\hat{\mu}_{k,m}(n)$

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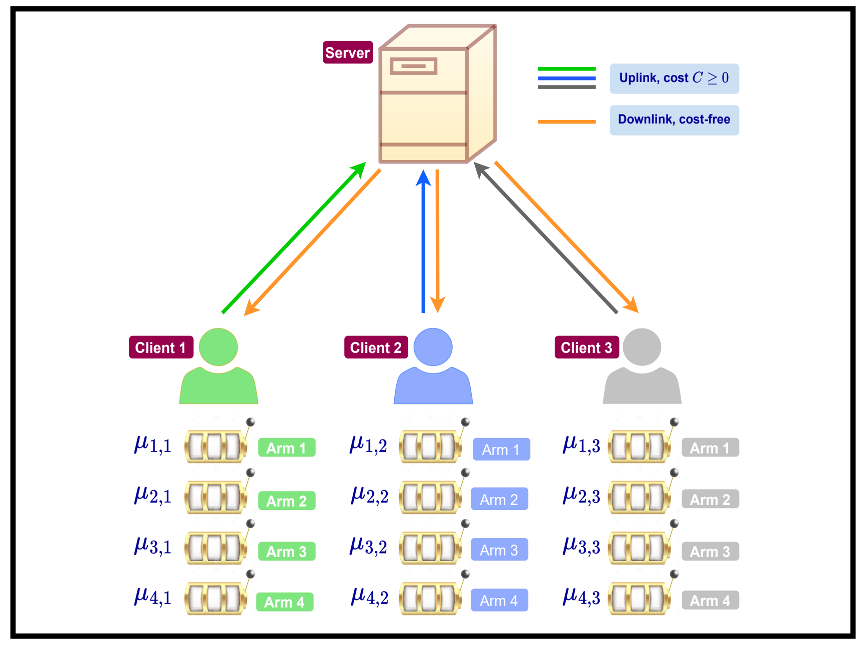

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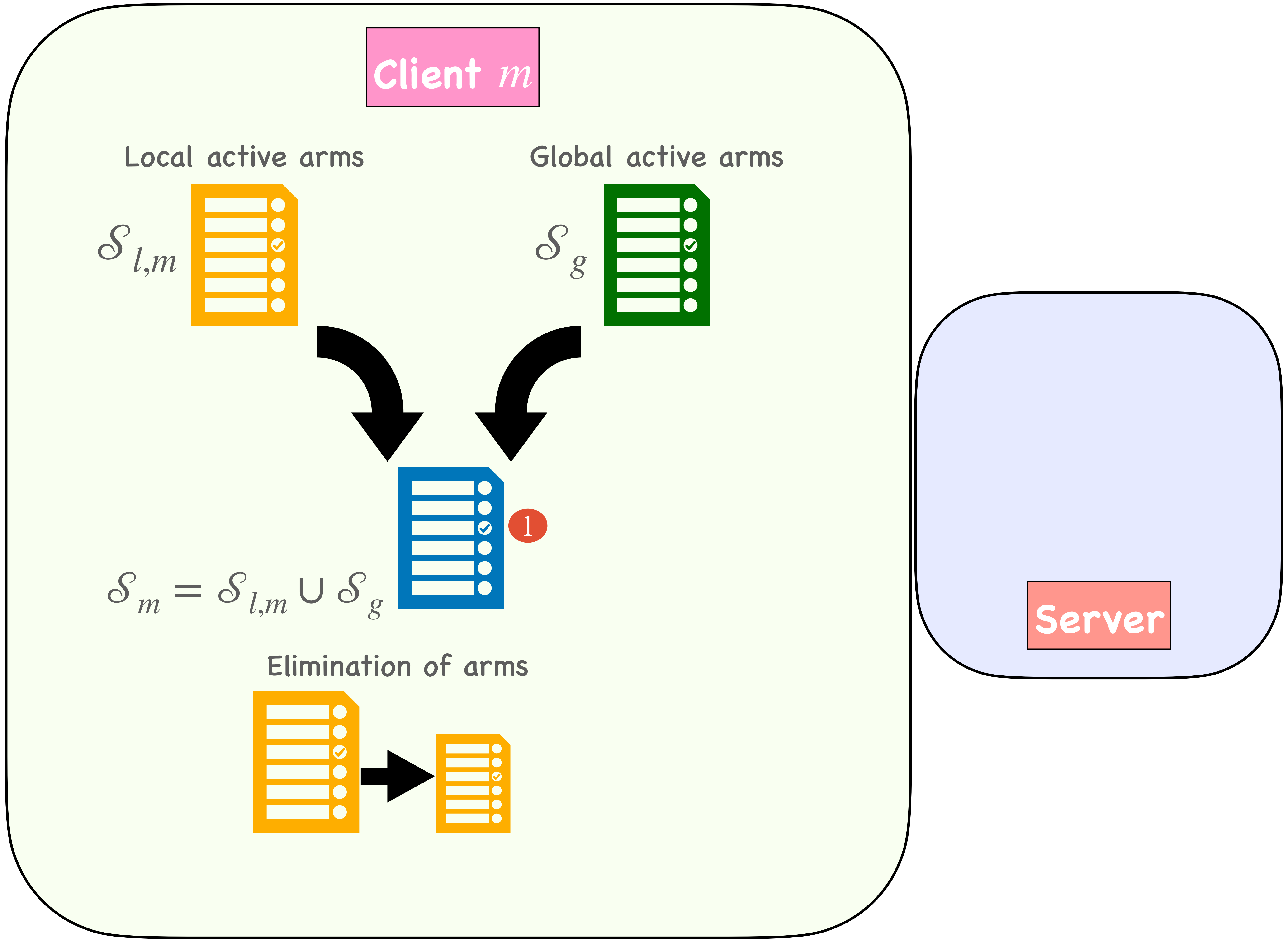

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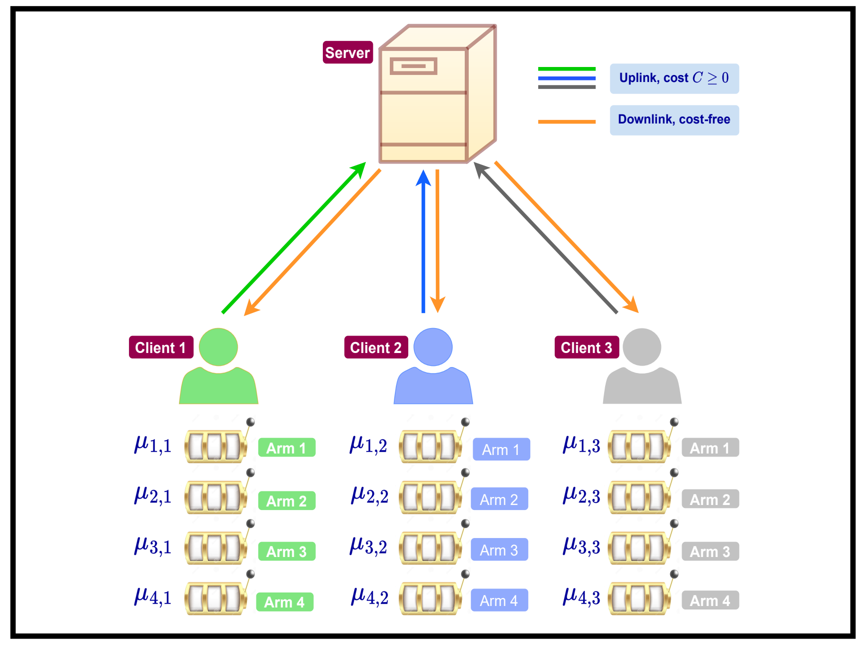

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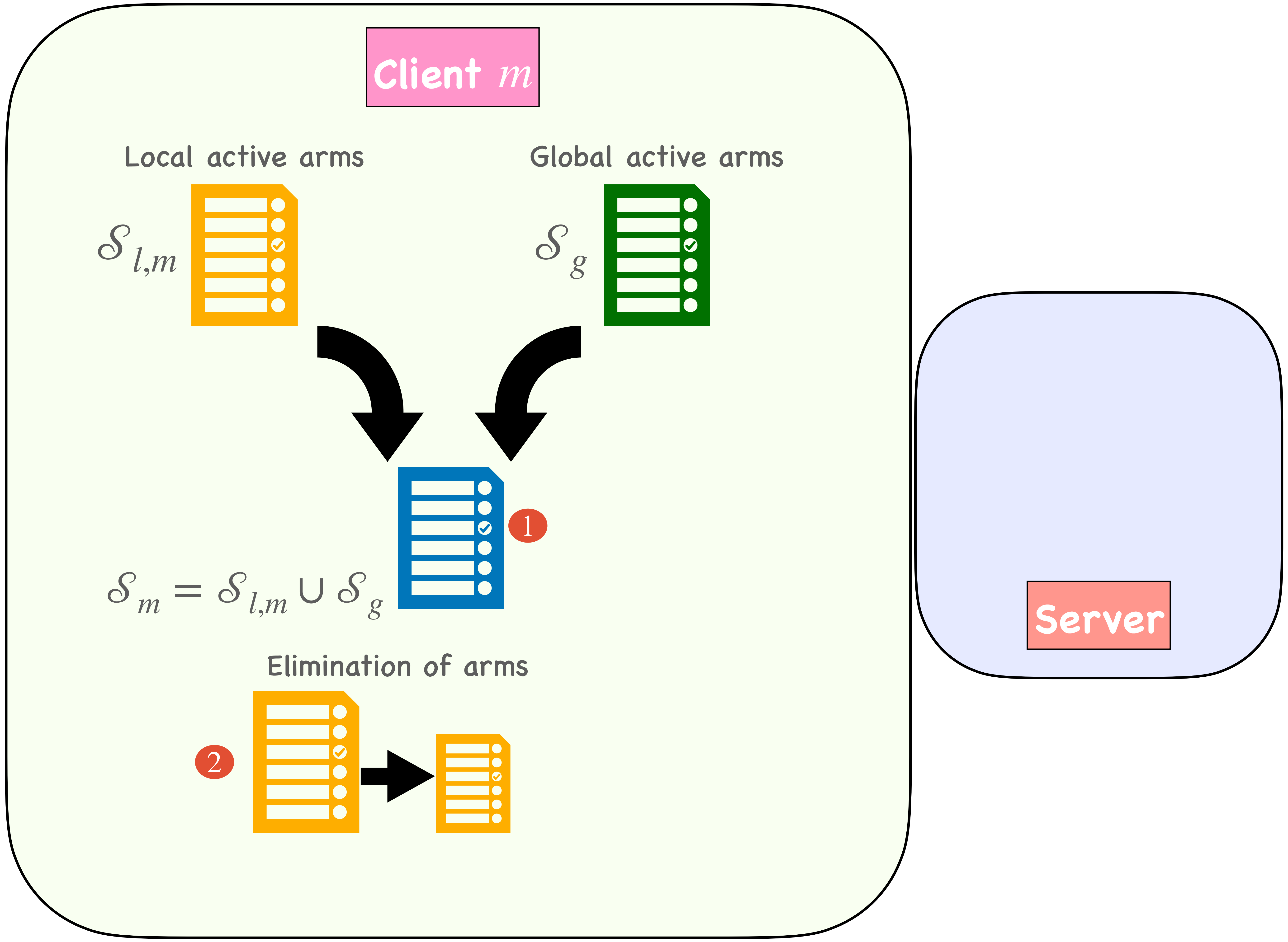

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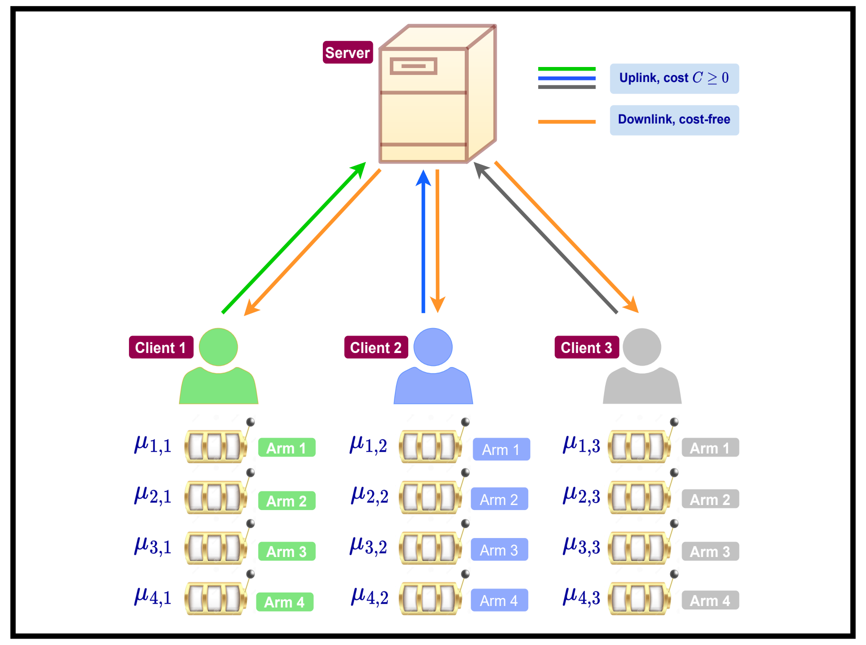

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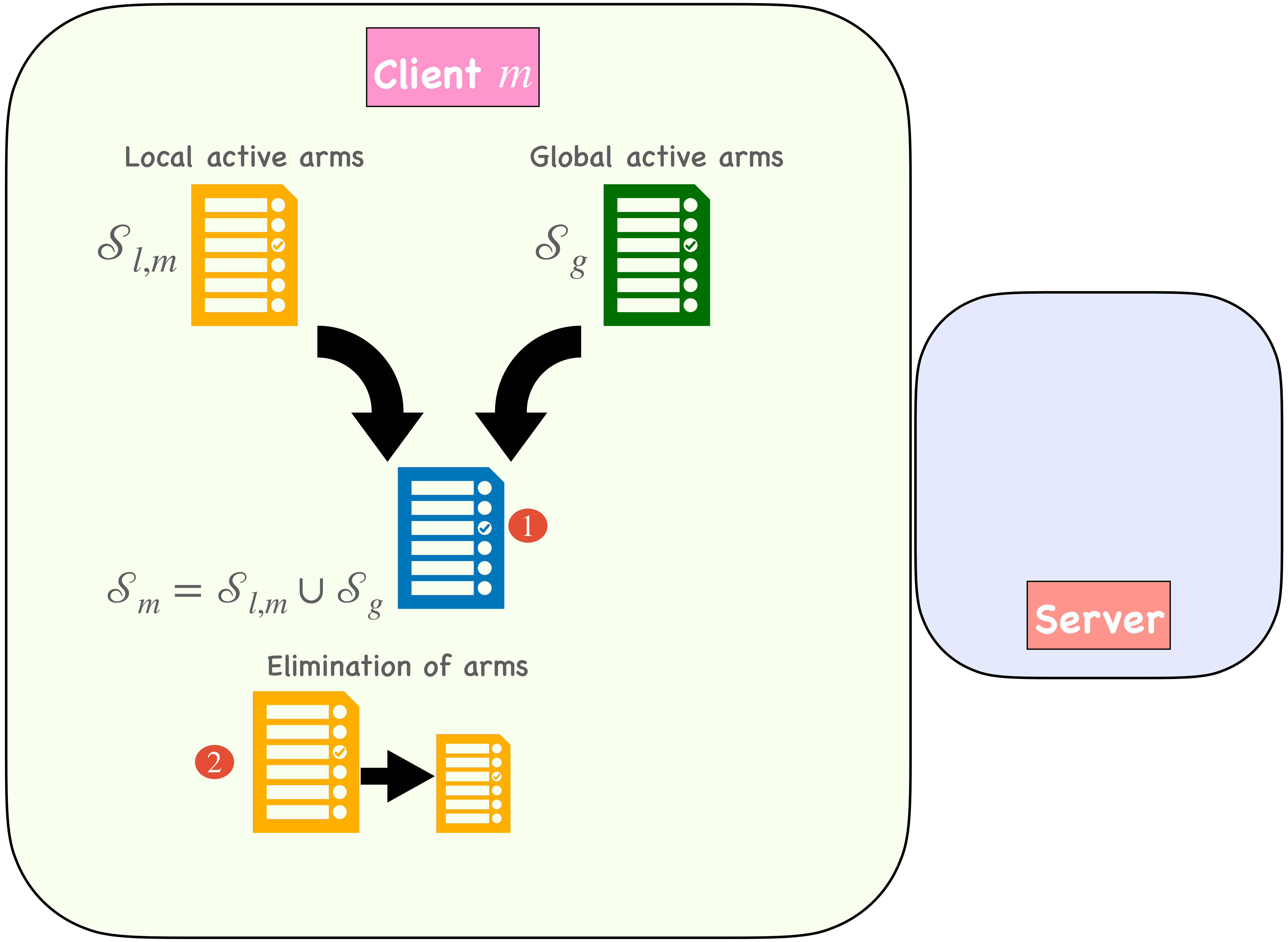

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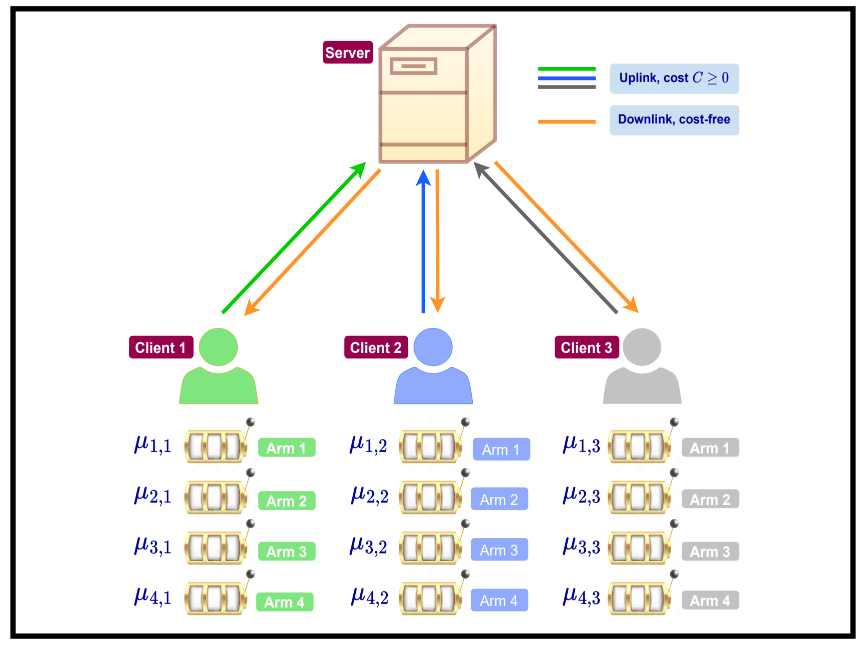

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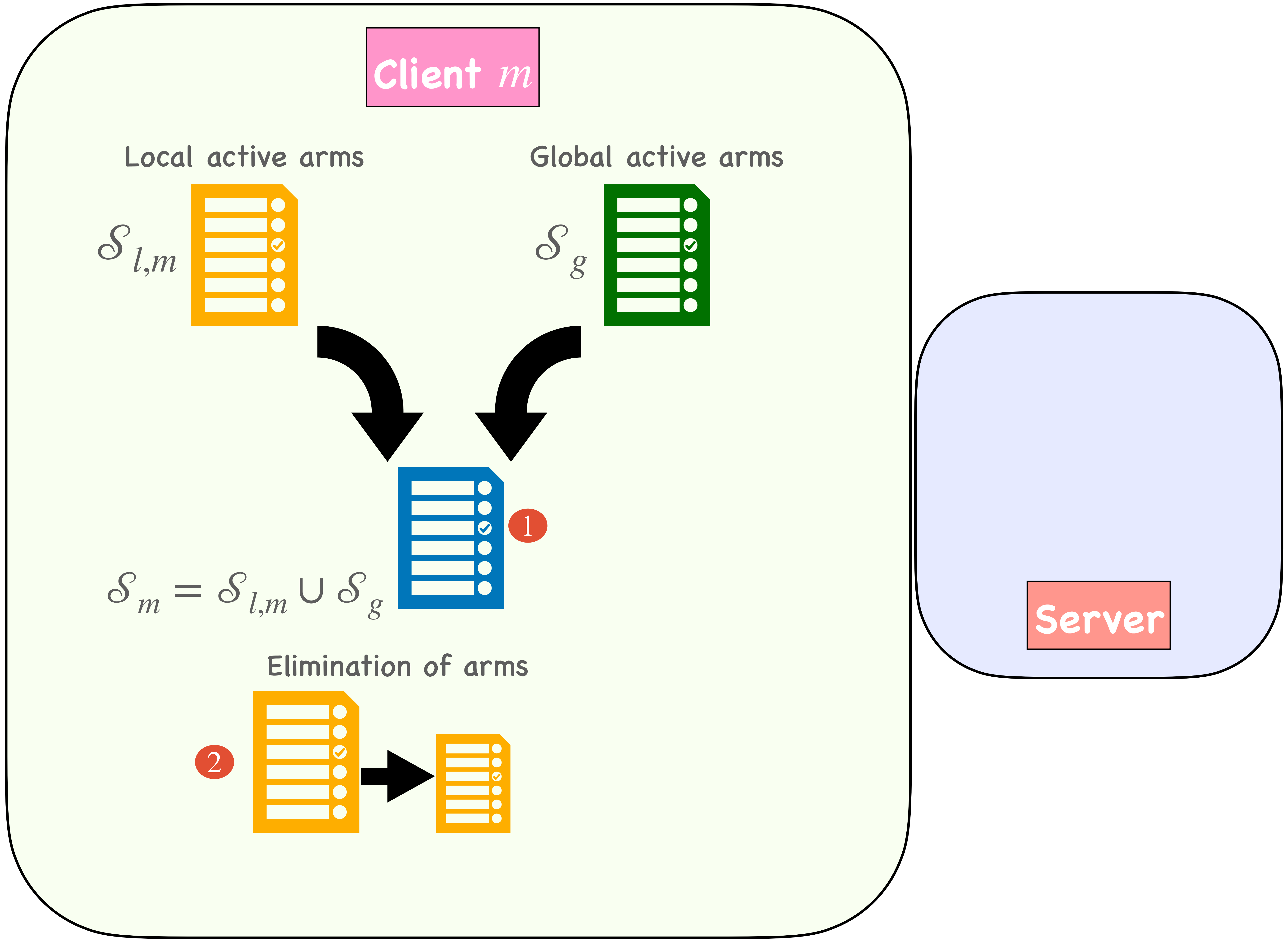

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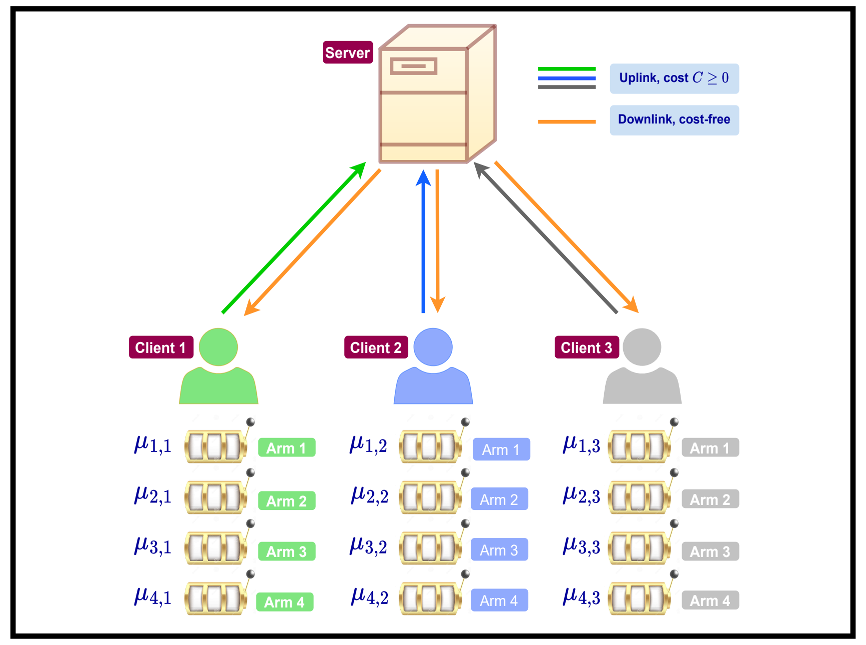

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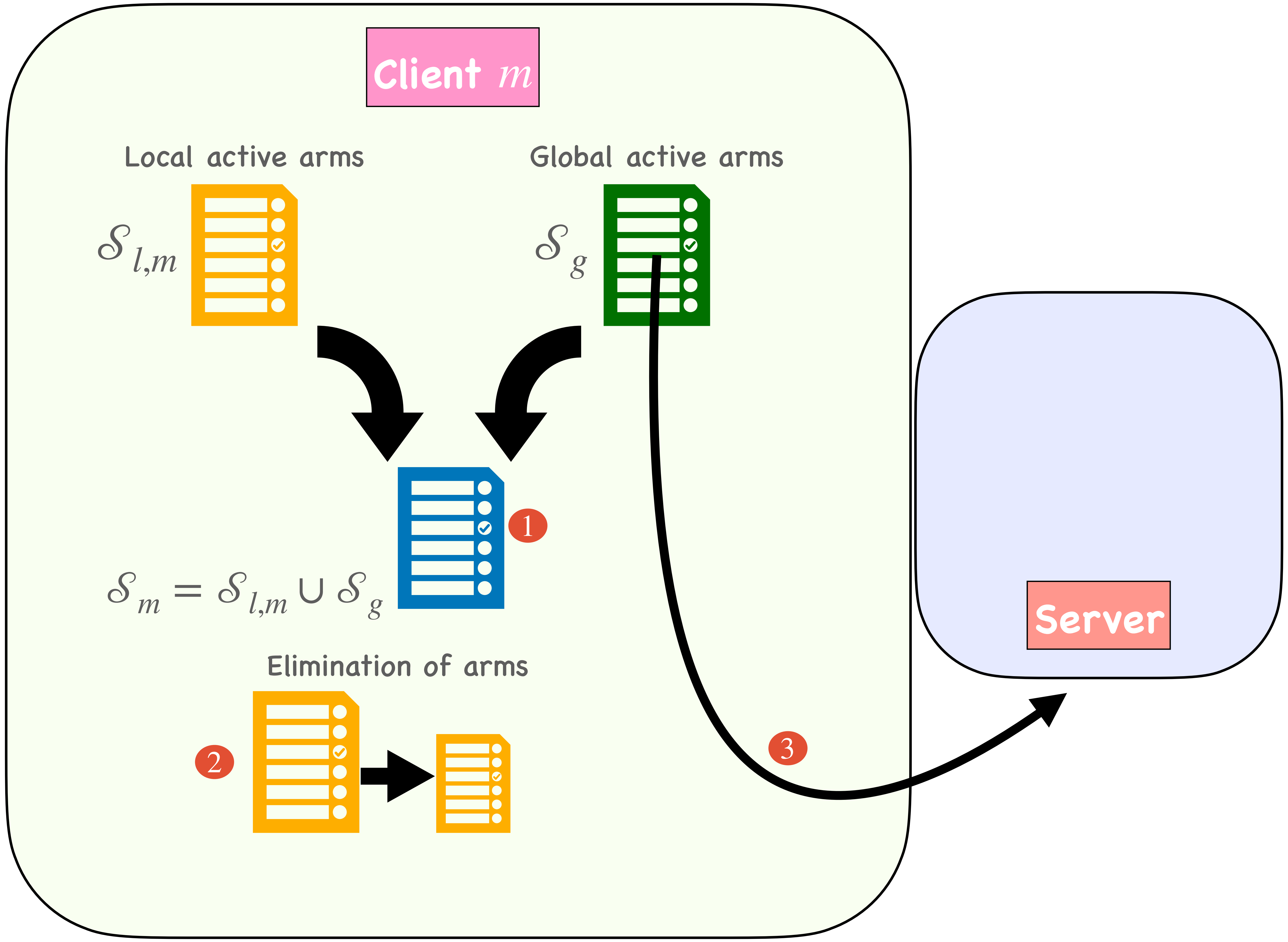

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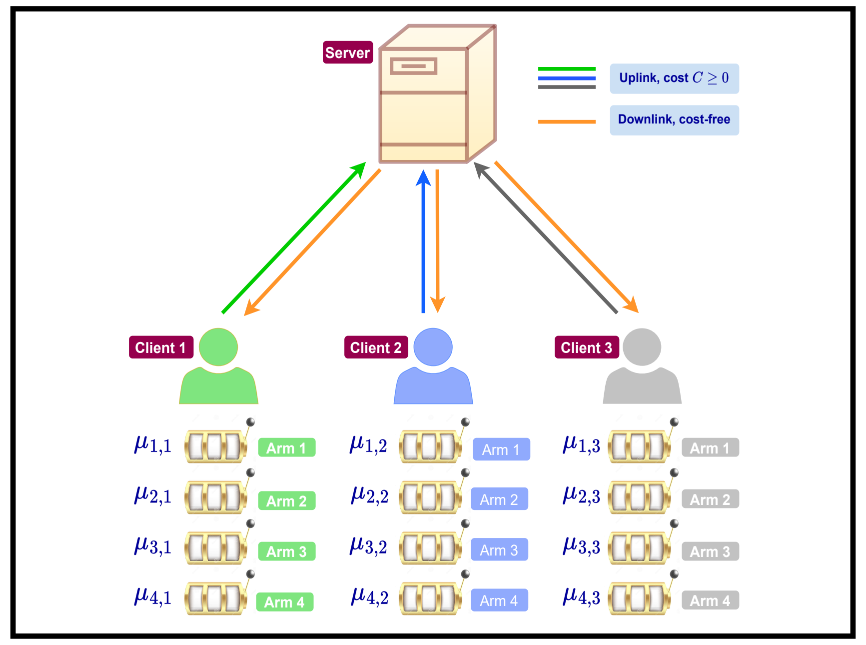

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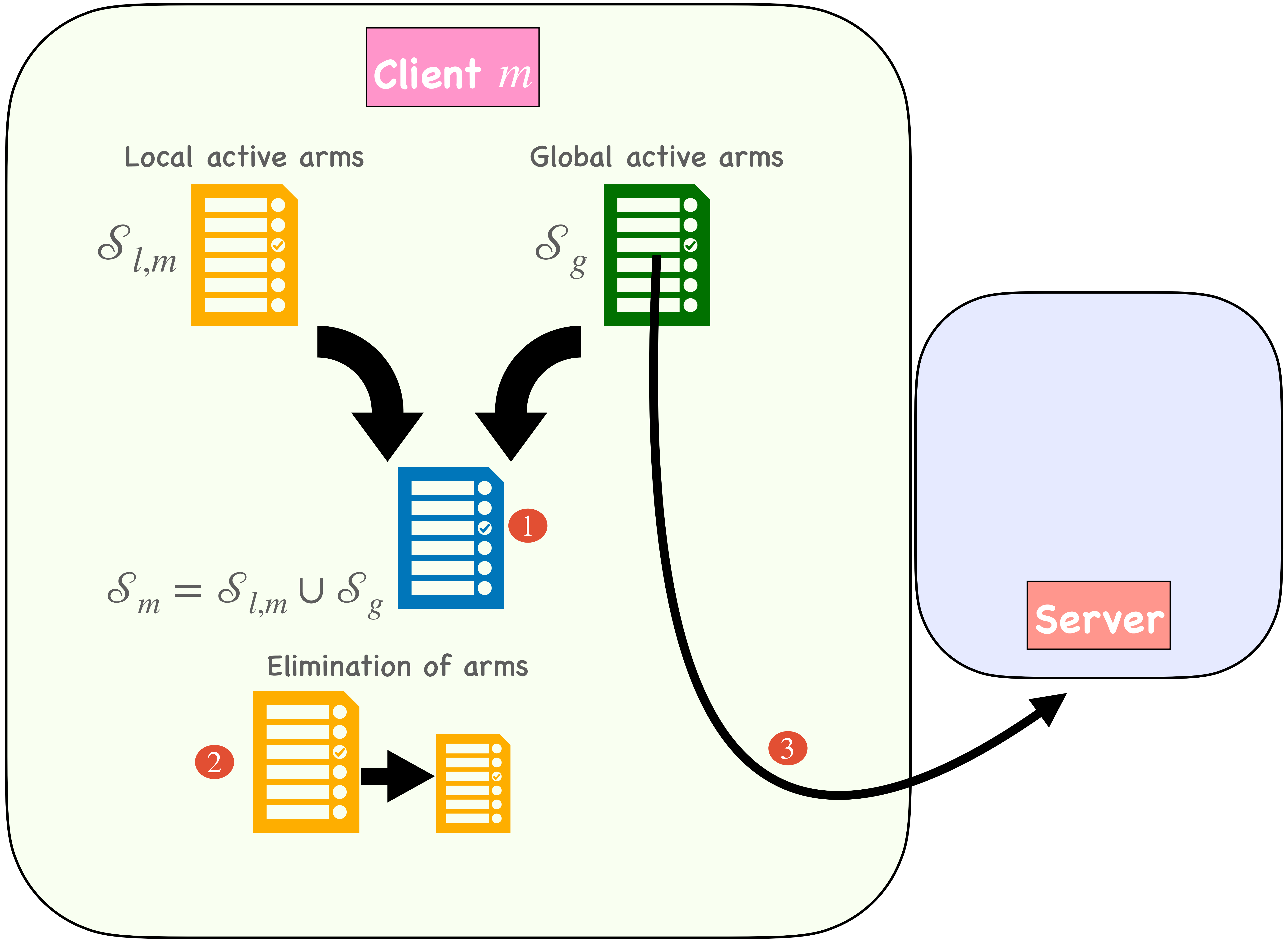

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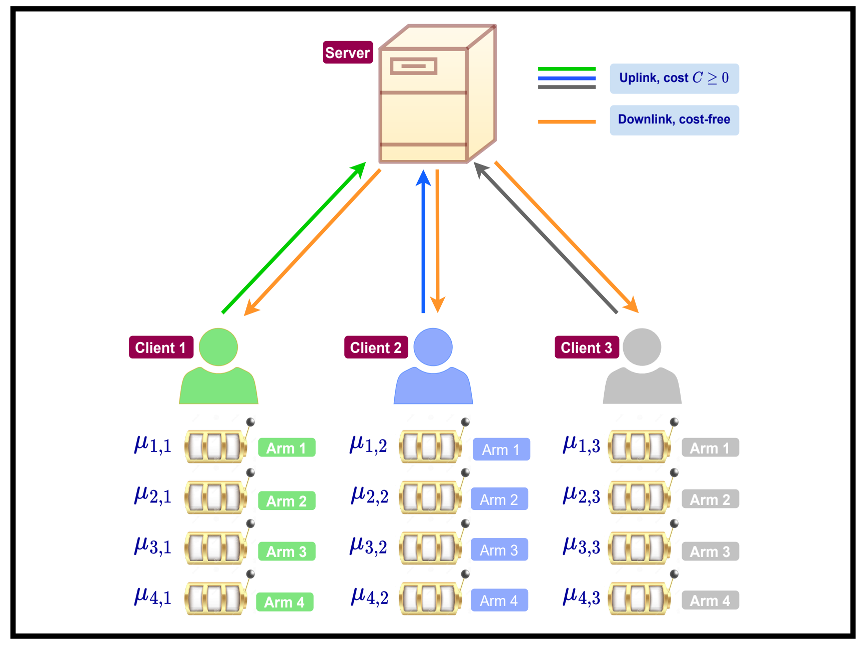

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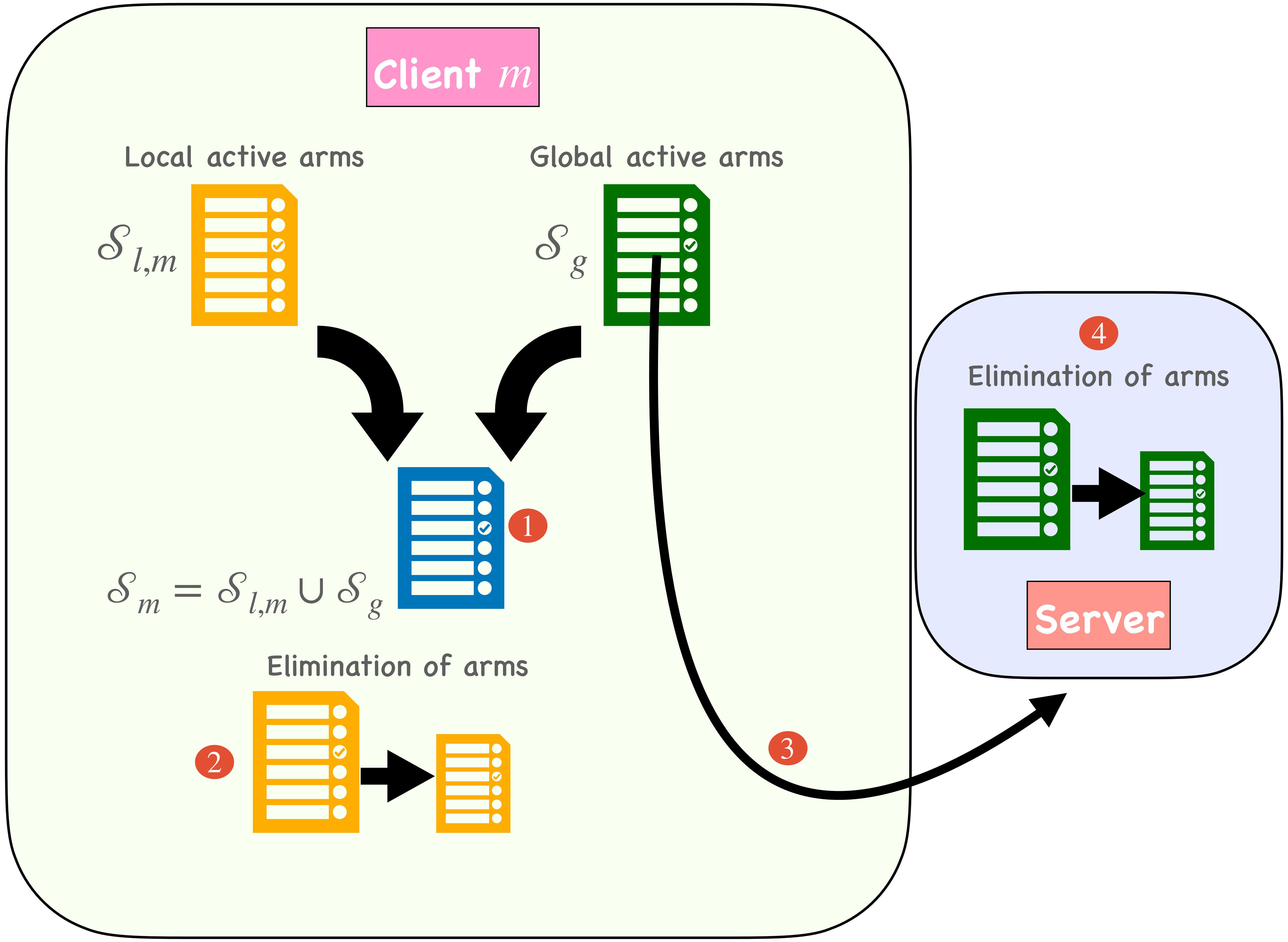

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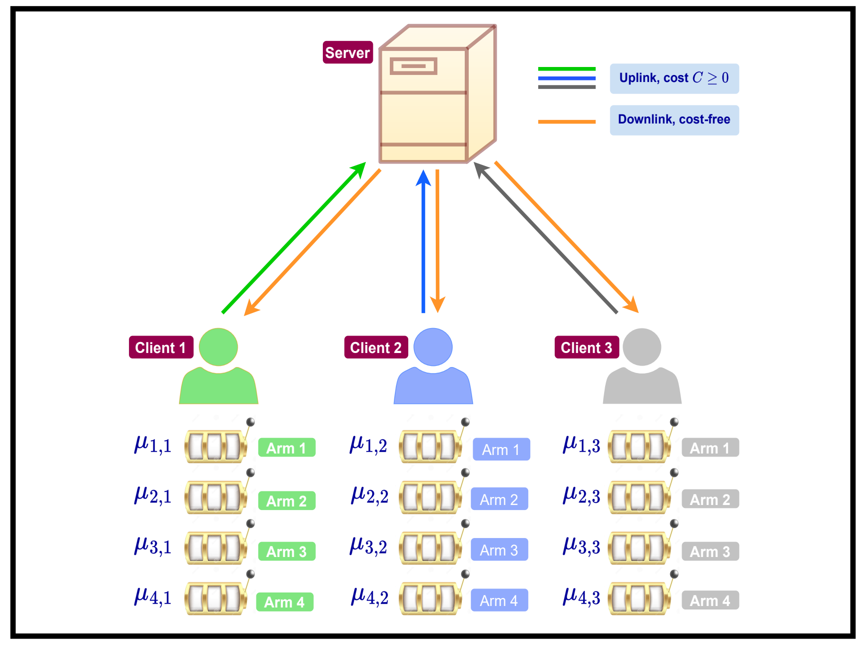

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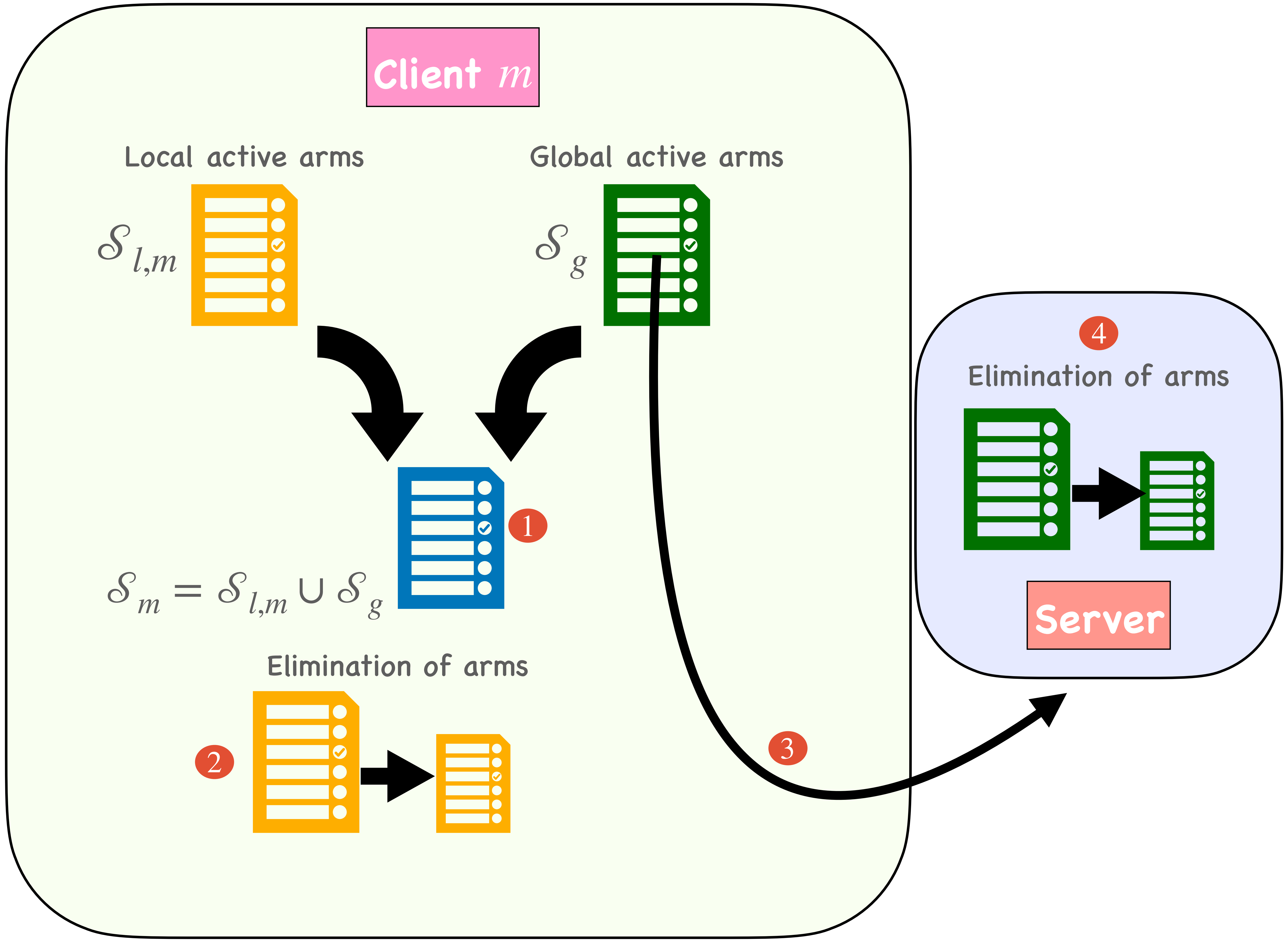

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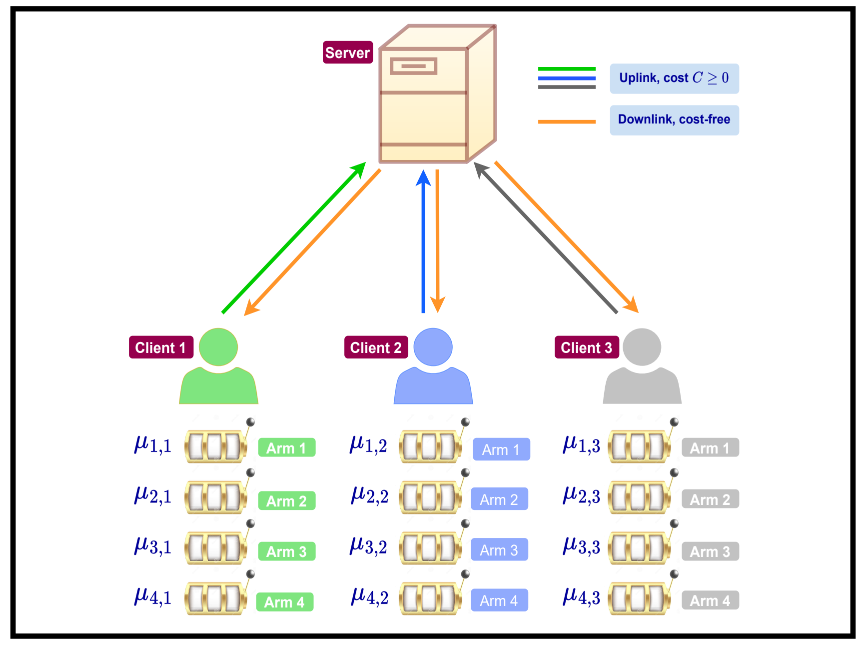

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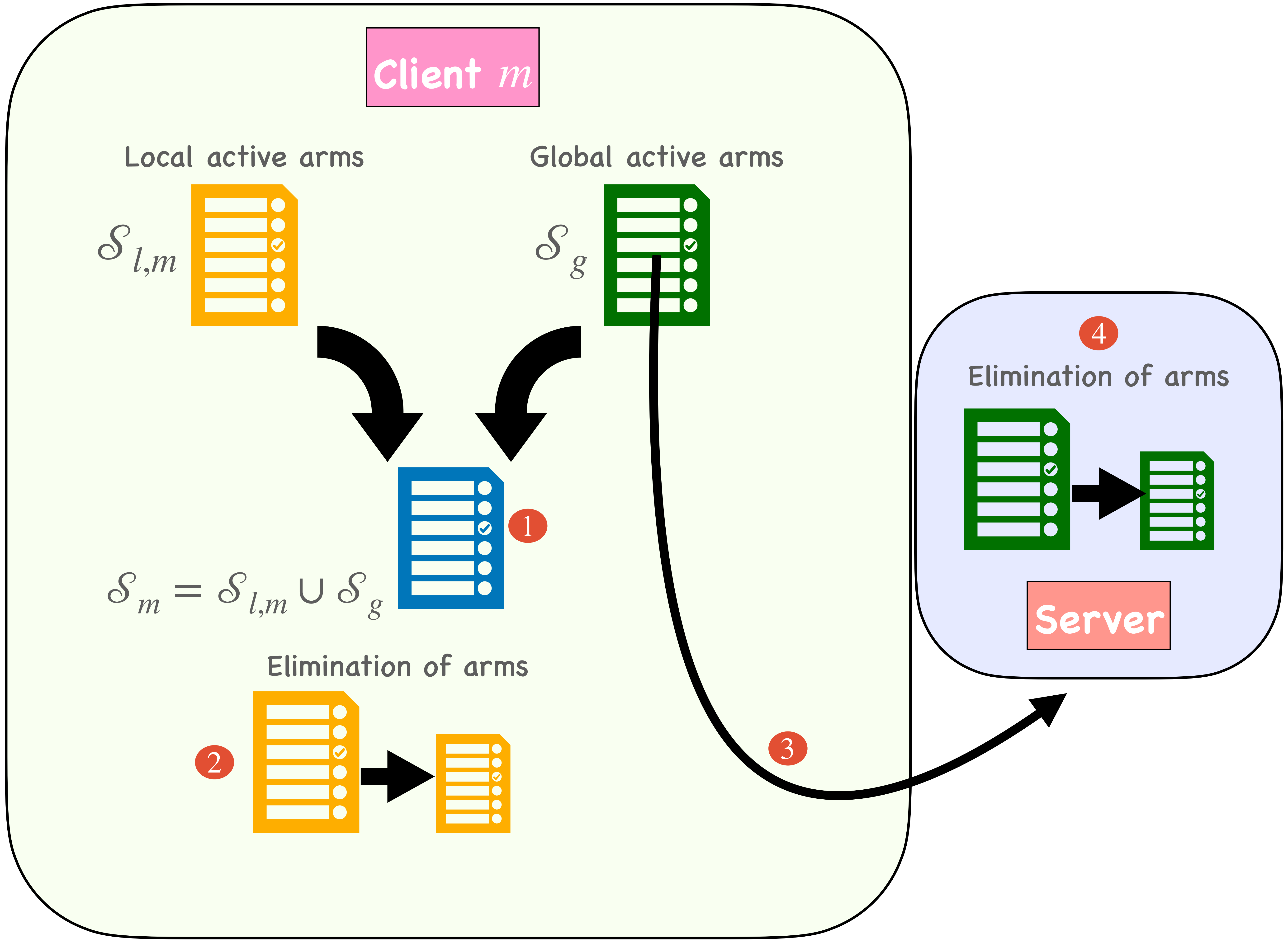

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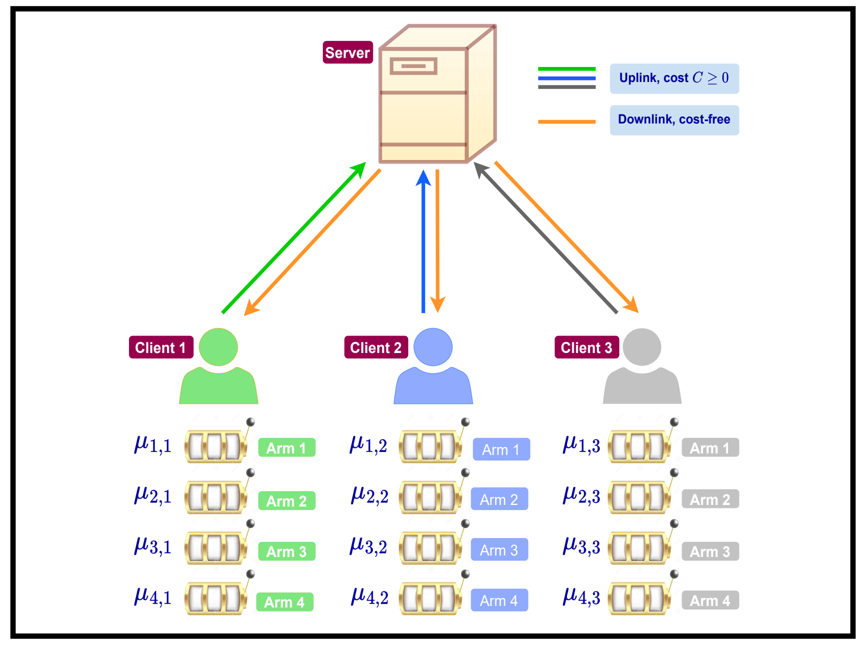

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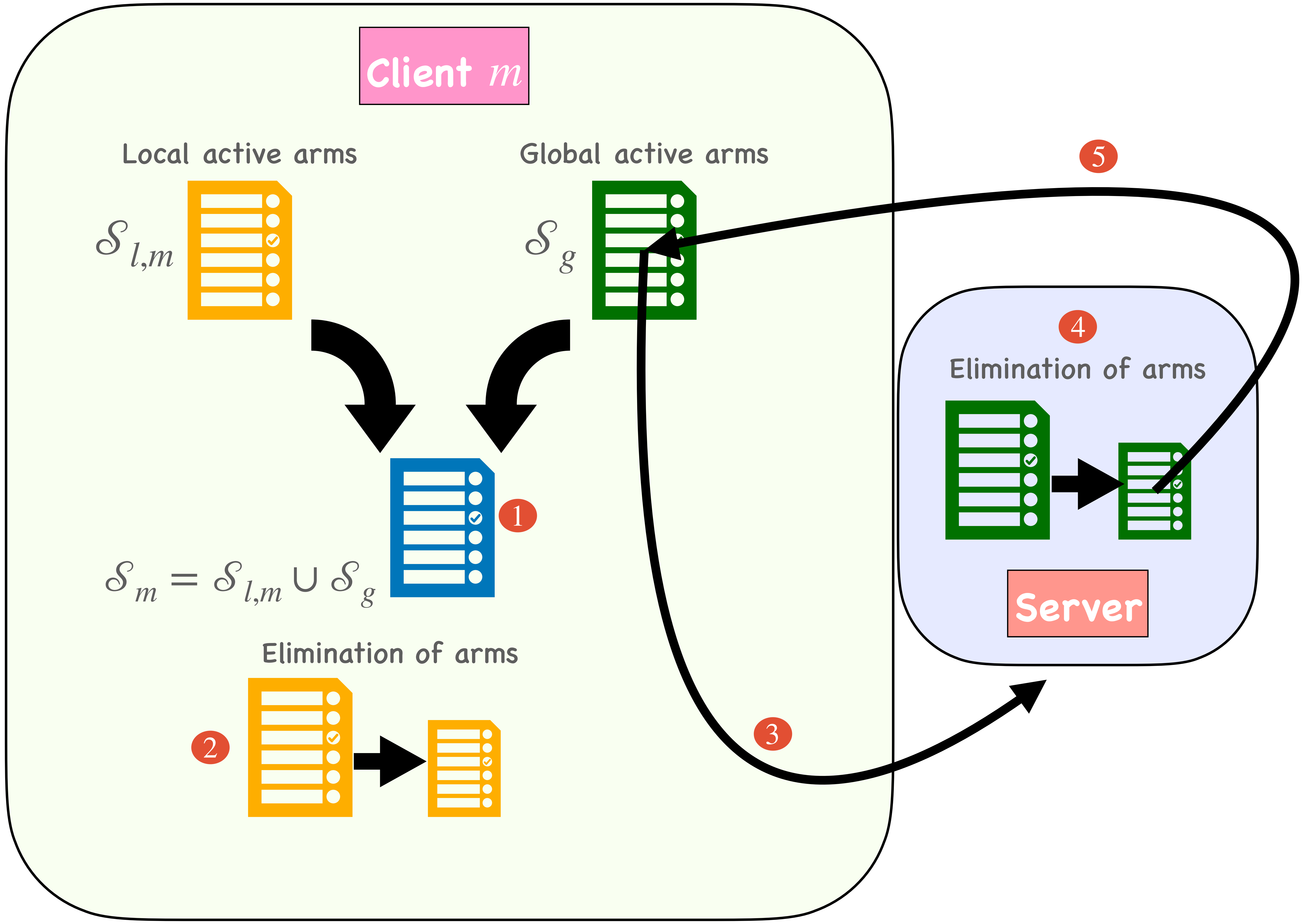

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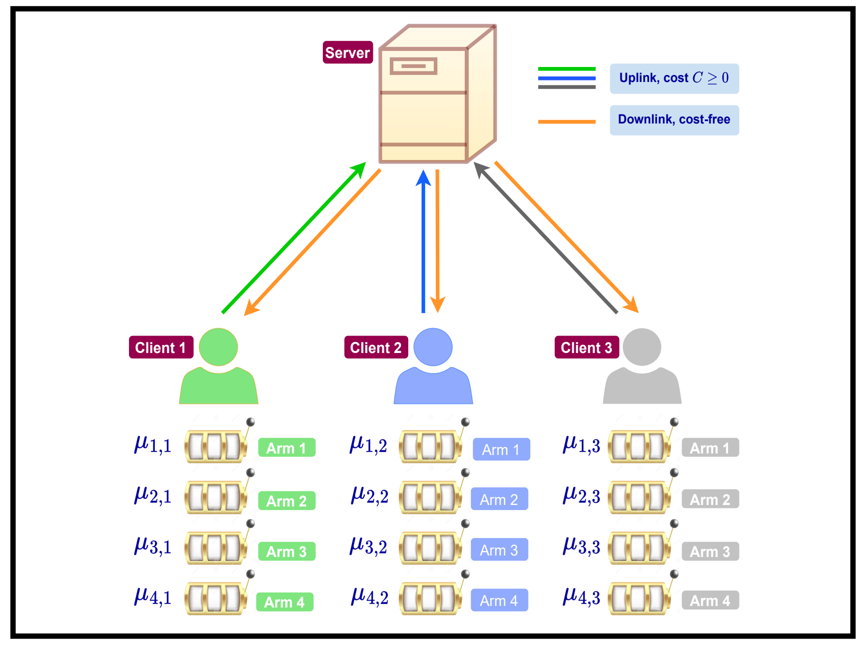

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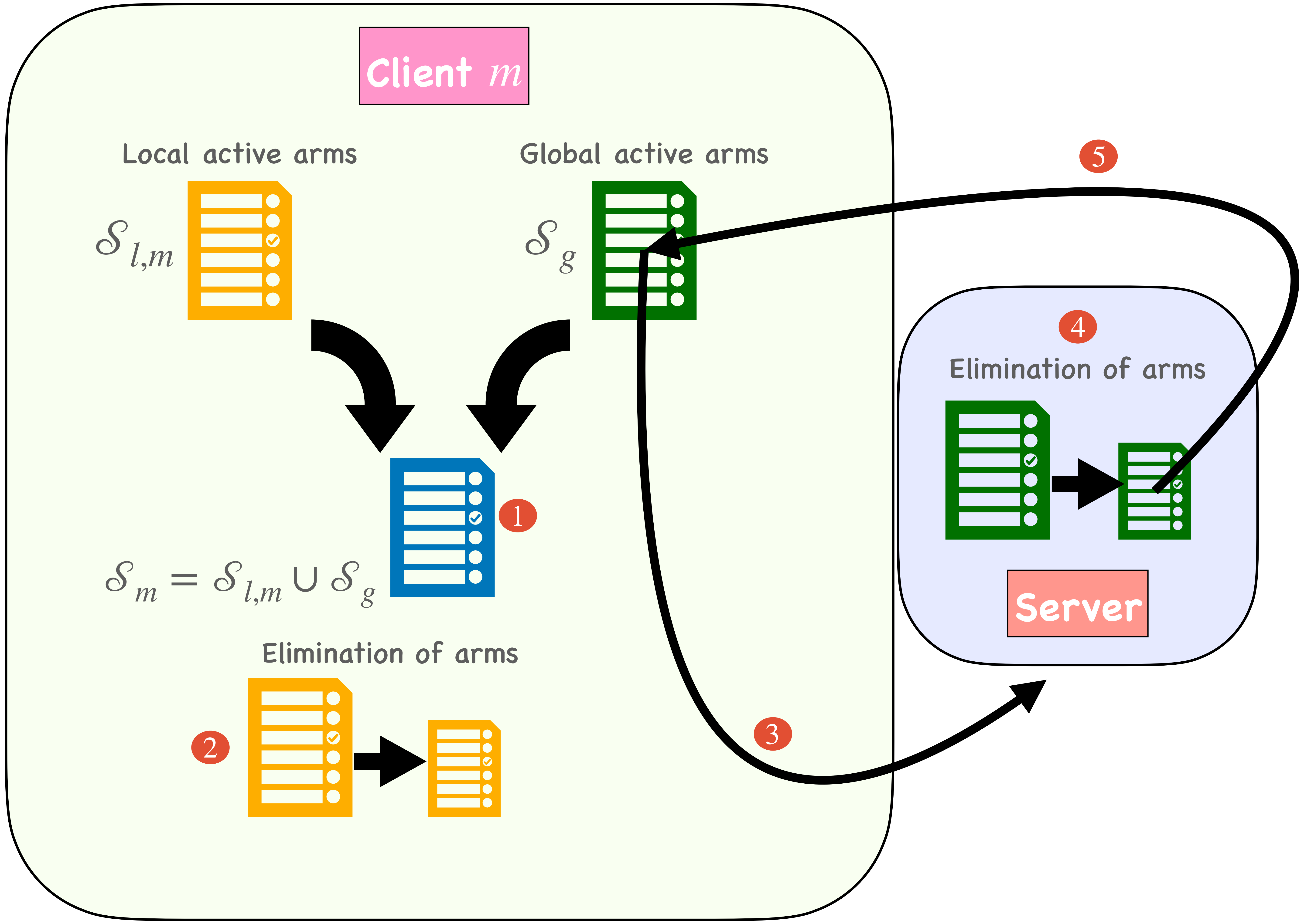

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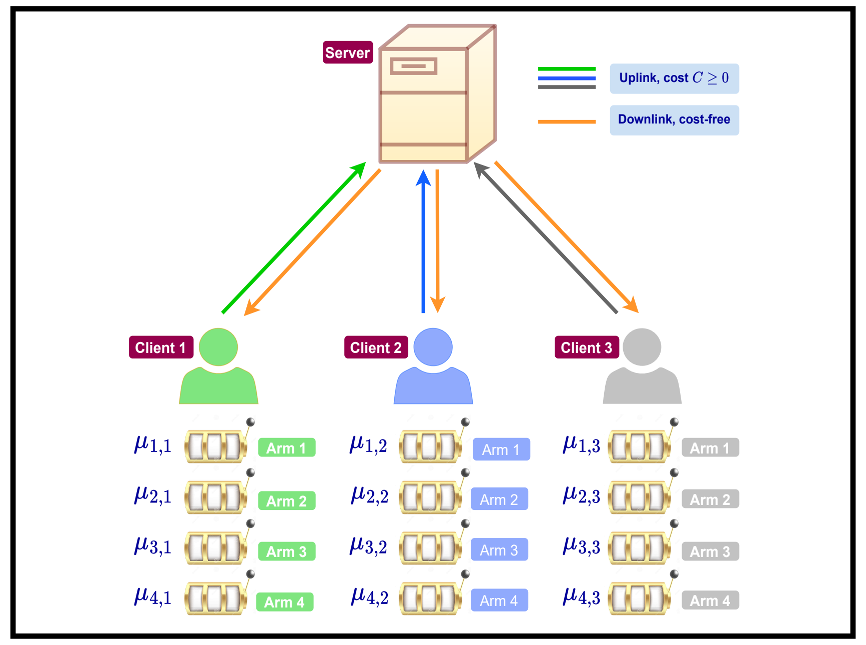

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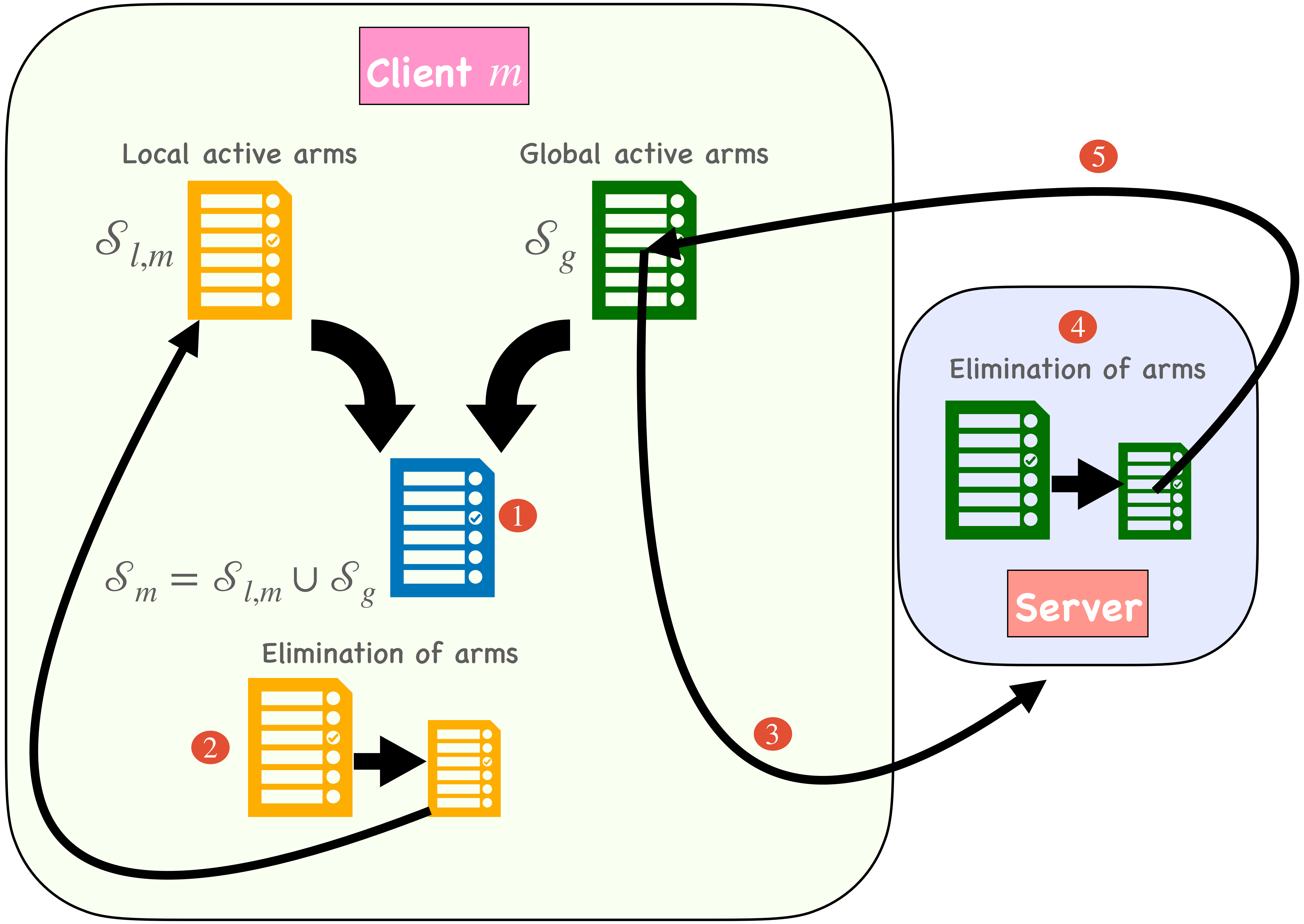

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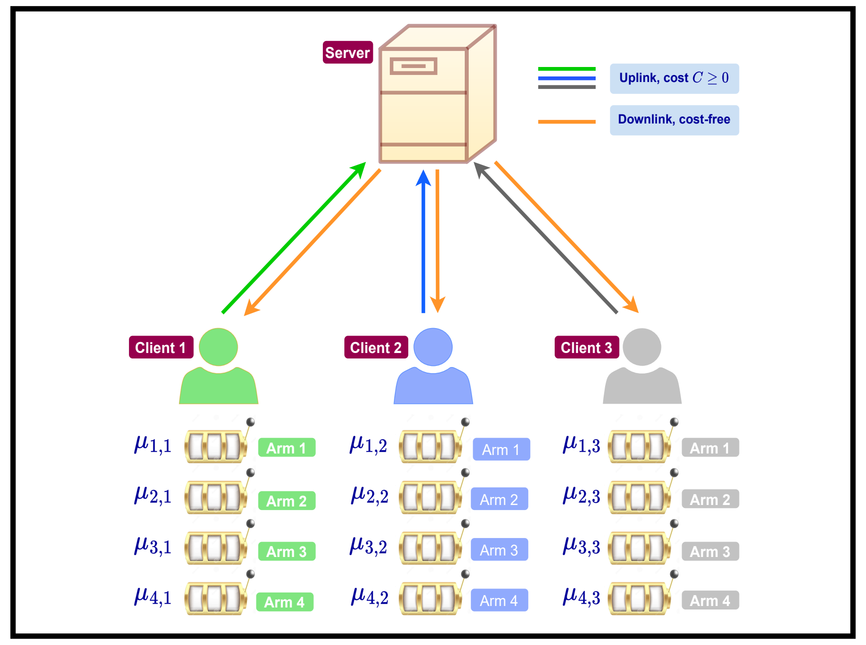

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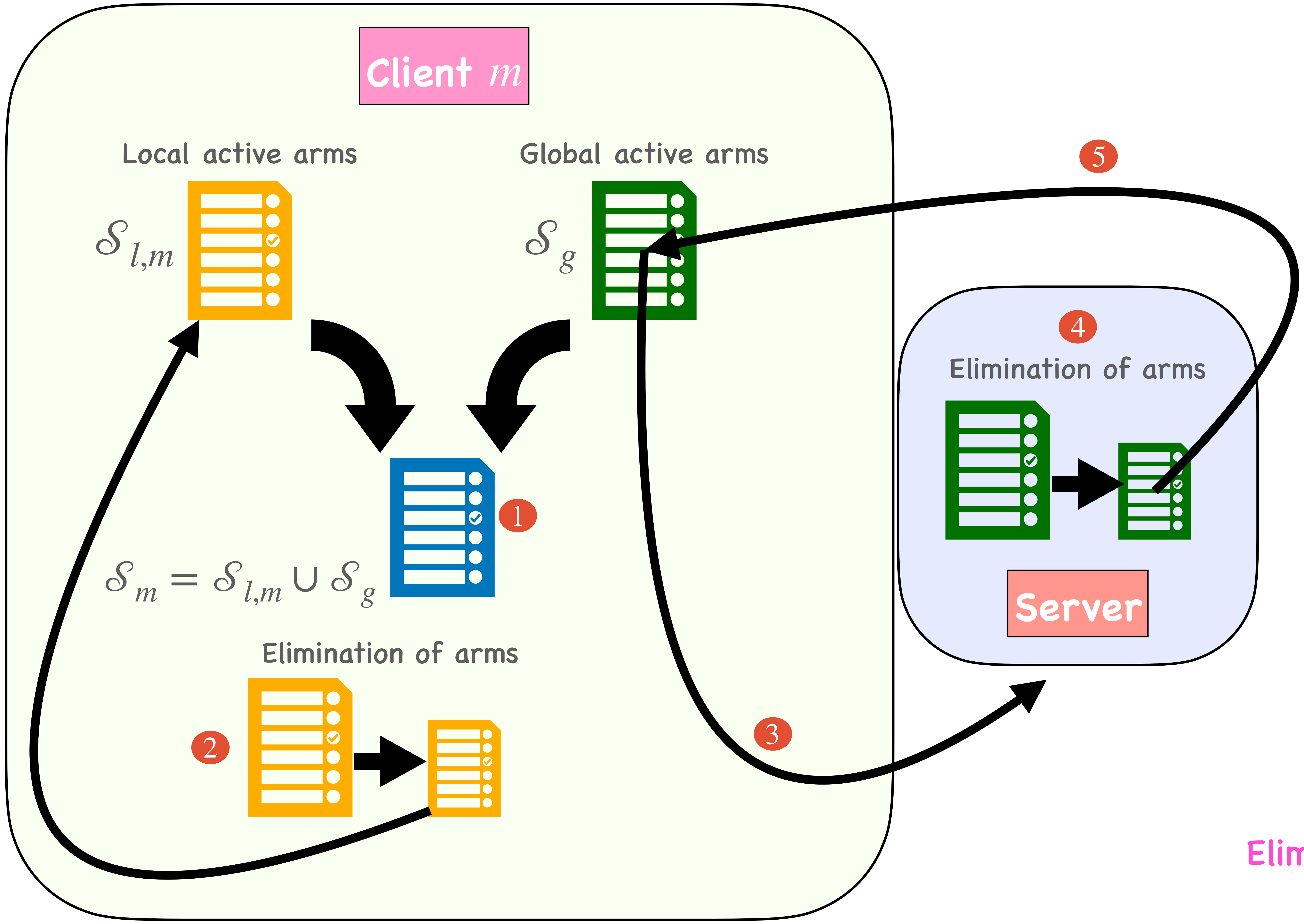

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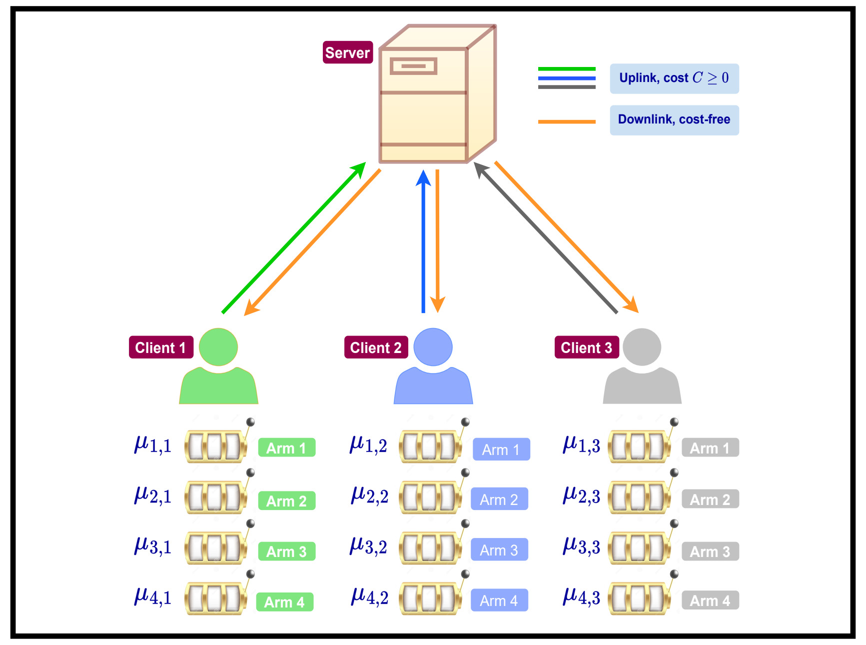

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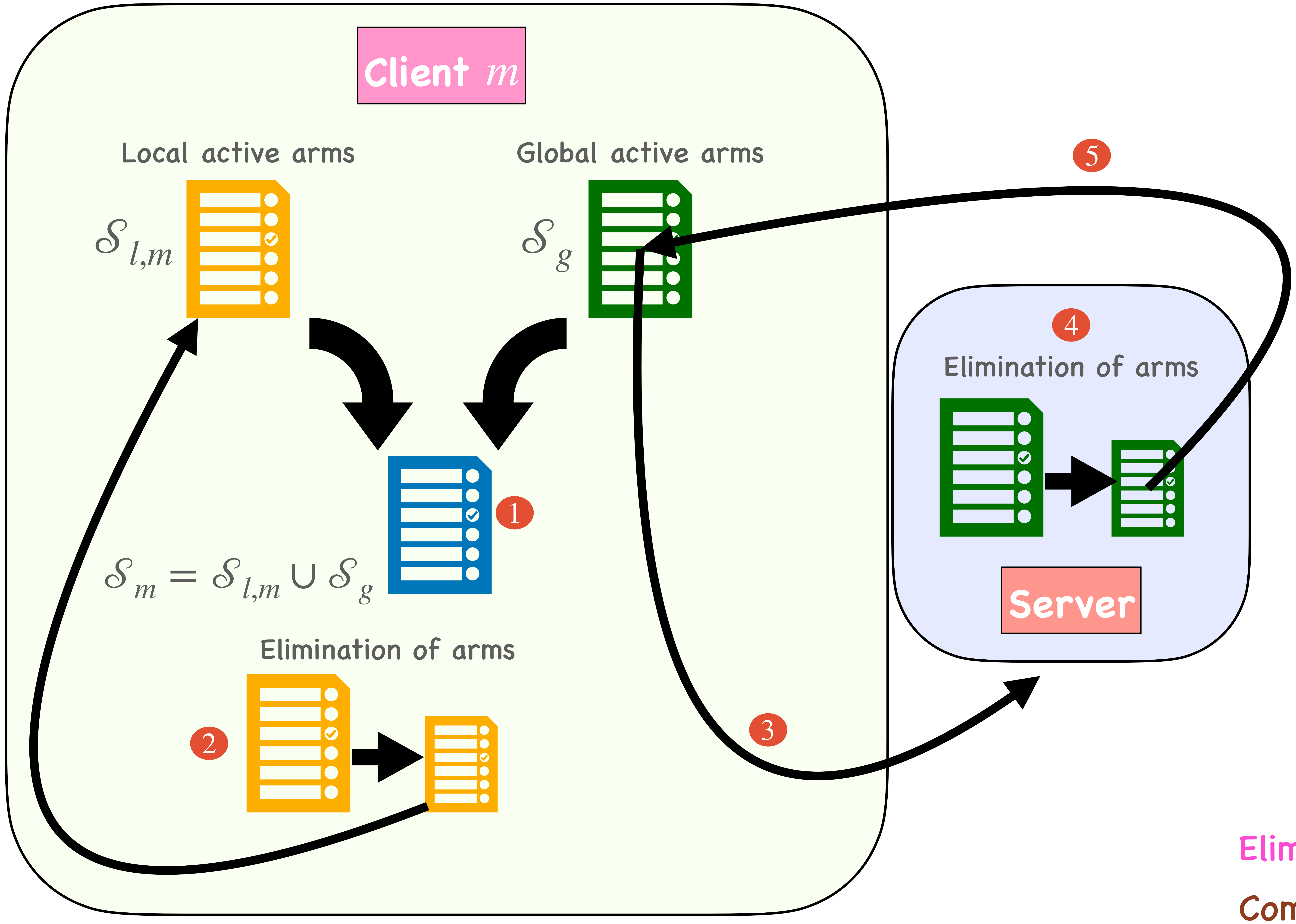

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Comm. with server stops when $|\mathcal{S}_g| = 1$ (global best arm)

Performance of FedElim0 ($C = 0$)

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- Total cost of FedElim0 ($\mathsf{T}_{\mathsf{FedElim0}}$) satisfies

$$P\left(\mathsf{T}_{\mathsf{FedElim0}} \leq \underbrace{\sum_{m=1}^{M} \sum_{k=1}^{K} \max\left\{T_{k,m},\, T_k\right\}}_{=\mathsf{T}}\right) \geq 1 - \delta,$$

$$T_{k,m} = 102 \cdot \frac{\ln\left(\frac{64\sqrt{\frac{8KM}{\delta}}}{\Delta_{k,m}^2}\right)}{\Delta_{k,m}^2} + 1, \qquad T_k = 102 \cdot \frac{\ln\left(\frac{64\sqrt{\frac{8K}{\delta}}}{M\Delta_k^2}\right)}{M\Delta_k^2} + 1,$$

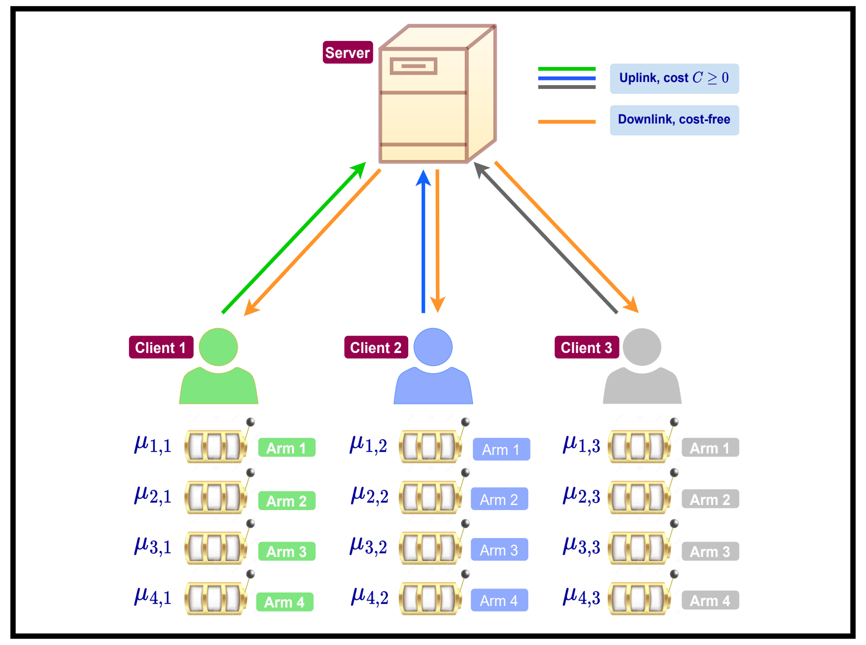
$$\Delta_{k,m} = \max_a \mu_{a,m} - \mu_{k,m} \text{ (local sub-optimality gap)}, \qquad \Delta_k = \max_a \frac{1}{M}\sum_{m=1}^{M} (\mu_{a,m} - \mu_{k,m}) \text{ (global sub-optimality gap)},$$

The $C > 0$ Case

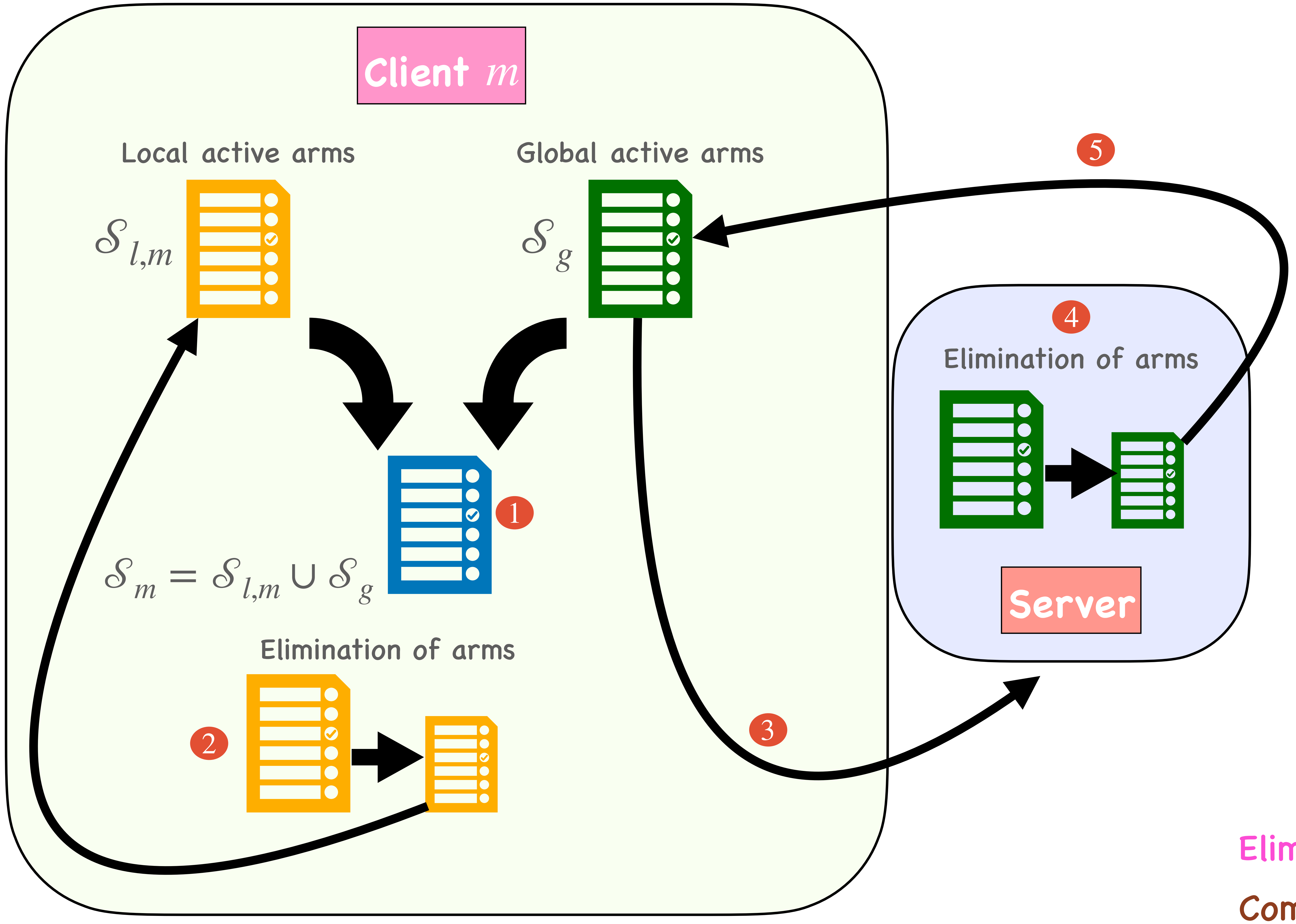
FedElim for $C = 0$ (FedElim0)

$$\min \quad \mathbb{E}\Big[\underbrace{\text{no. of arm pulls} + \text{communication cost}}_{=\ \text{total cost}}\Big]$$

$$\text{s.t.} \quad \text{error probability} \le \delta$$



Comm. only at exponential time instants $n \in \{2^t : t \geq 0\}$



Initialise: $\mathcal{S}_{l,m} = [K]$, $\mathcal{S}_g = [K]$

1. Pull each arm $k \in \mathcal{S}_m$ and update its empirical mean $\hat{\mu}_{k,m}(n)$
2. Eliminate $k \in \mathcal{S}_{l,m}$ if $\hat{\mu}_{k,m}(n) \leq \max_{a \in \mathcal{S}_{l,m}} \hat{\mu}_{a,m}(n) - 2\alpha_l(n)$
   Update $\mathcal{S}_{l,m}$
3. Client m sends $\{\hat{\mu}_{k,m}(n) : k \in \mathcal{S}_g\}$ and $\mathcal{S}_g$ to server if $n \in \{2^t\}_{t=0}^{\infty}$
4. Eliminate $k \in \mathcal{S}_g$ if $\frac{1}{M}\sum_{m=1}^{M} \hat{\mu}_{k,m}(n) \leq \max_{a \in \mathcal{S}_g} \frac{1}{M}\sum_{m=1}^{M} \hat{\mu}_{a,m}(n) - 2\alpha_g(n)$
5. Server sends updated S_g to all clients

$$\alpha_l(n) = \sqrt{\frac{2\ln(8KMn^2/\delta)}{n}} \qquad \alpha_g(n) = \sqrt{\frac{2\ln(8Kn^2/\delta)}{Mn}}$$

Elim. at client m continues until $|\mathcal{S}_{l,m}| = 1$ (local best arm)

Comm. with server stops when $|\mathcal{S}_g| = 1$ (global best arm)

Performance of FedElim ($C > 0$)

$$\mathsf{T} = \sum_{m=1}^{M} \sum_{k=1}^{K} \max\{T_{k,m}, T_k\}$$

Performance of FedElim ($C > 0$)

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- Stops in finite time w.p.1

Performance of FedElim ($C > 0$)

T $= \sum_{m=1}^{M} \sum_{k=1}^{K} \max\{T_{k,m}, T_k\}$

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- Error probability $\leq \delta$

Performance of FedElim ($C > 0$)

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- With prob. $1 - \delta$,

Performance of FedElim ($C > 0$)

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- Error probability $\leq \delta$
- With prob. $1 - \delta$,
  - Total no. of arm pulls ($\mathsf{T}_{\mathsf{FedElim}}$) satisfies $\mathsf{T}_{\mathsf{FedElim}} \leq \sum_{m=1}^{M} \sum_{k=1}^{K} \max\{T_{k,m}, 2T_k\} \leq 2\mathsf{T}$

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* for sufficiently small δ

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  - $\mathsf{T}_{\mathsf{FedElim}} + C^{\mathsf{comm}}_{\mathsf{FedElim}} \leq 3\mathsf{T}$

* for sufficiently small δ

Numerical Results

Synthetic Gaussian Data — 1/3

$$
\begin{array}{cc}
 & \\
\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} & \text{Arms} \\
\text{Clients} &
\end{array}
$$

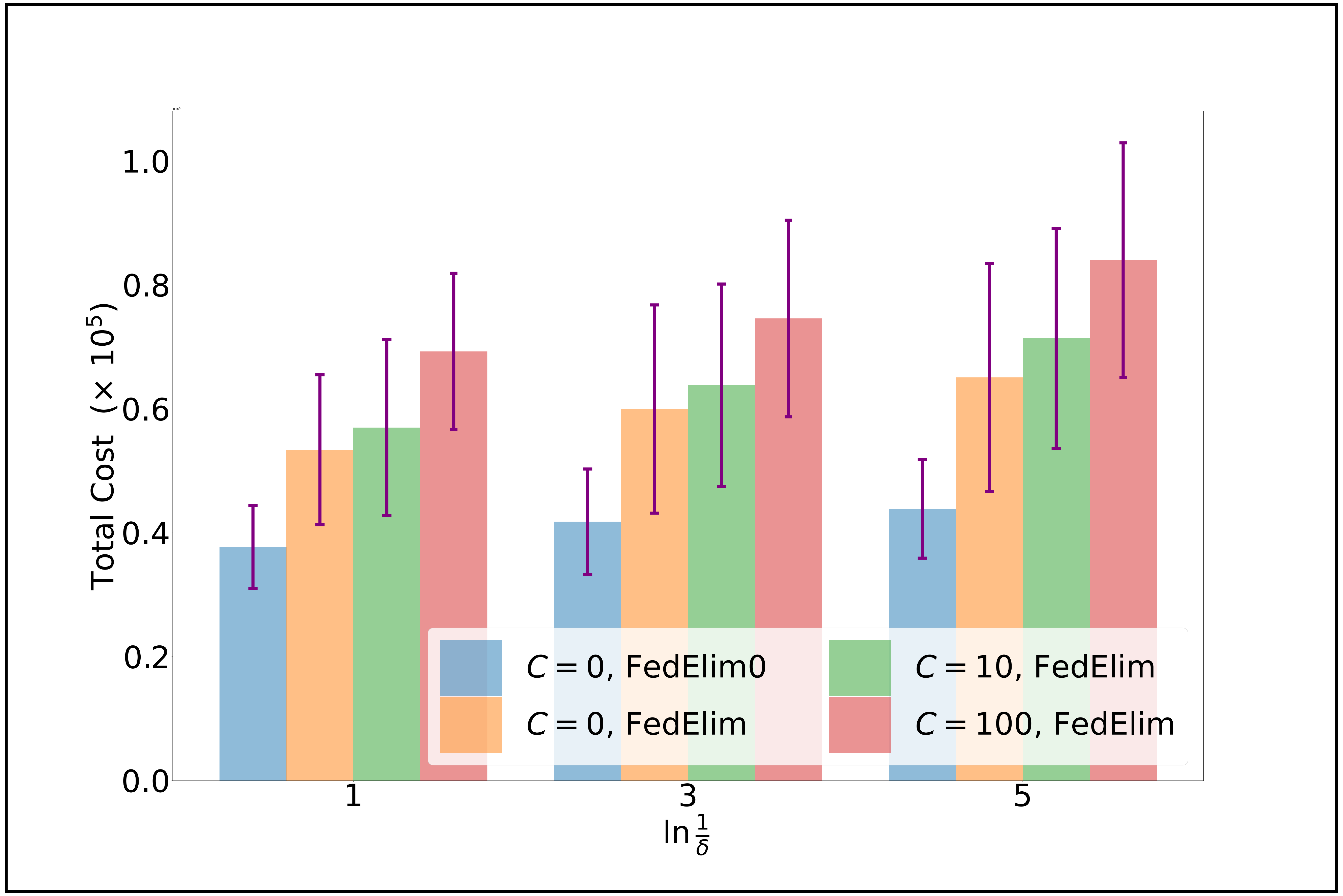
Local best arm of client 1 = 1

Local best arm of client 2 = 2

Local best arm of client 3 = 3

Global best arm = 4

Synthetic Gaussian Data — 1/3



$$\mu = \begin{array}{cc} \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} & \text{Arms} \\ \text{Clients} & \end{array}$$

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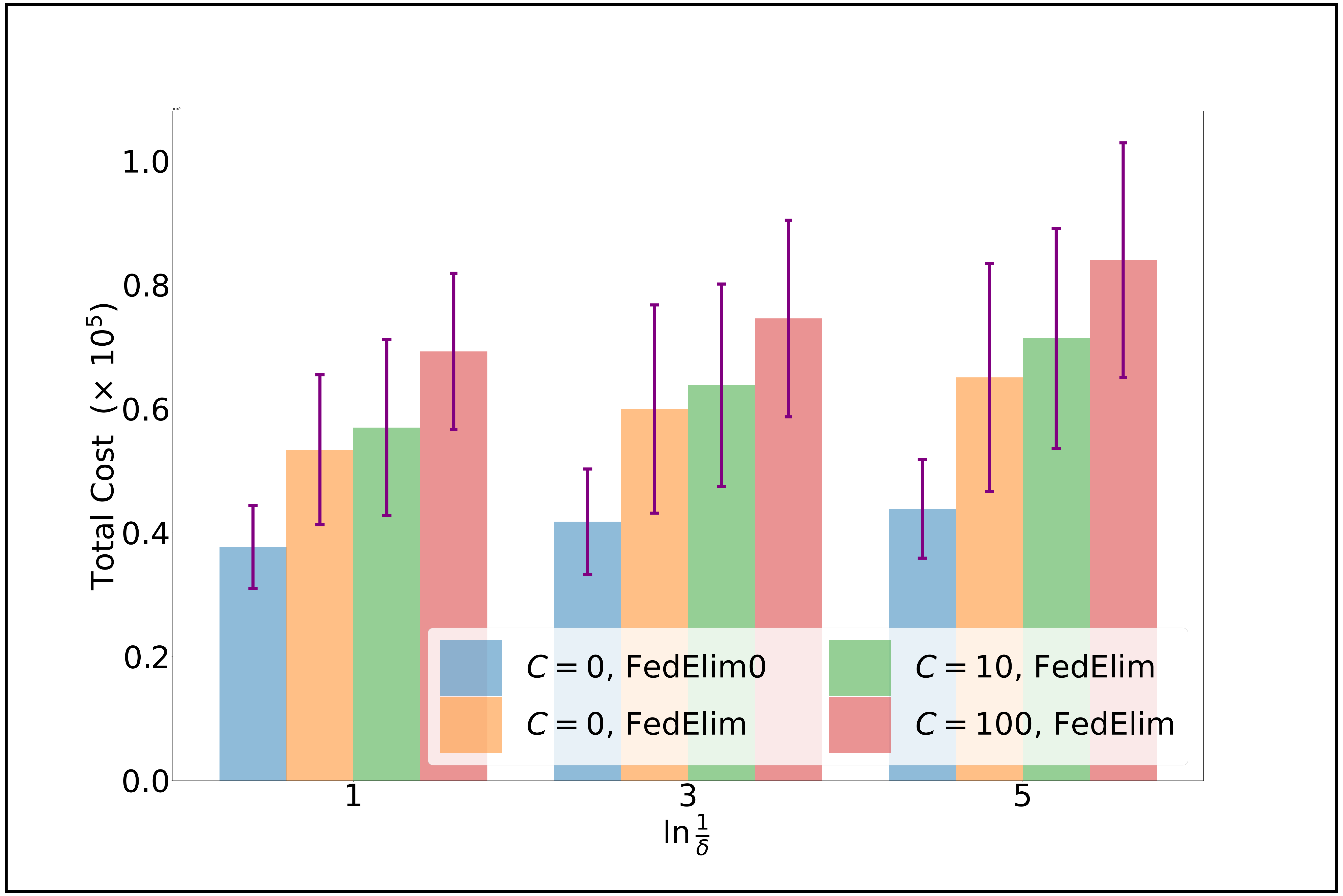

Length of blue bar $\leq$ T

$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Clients

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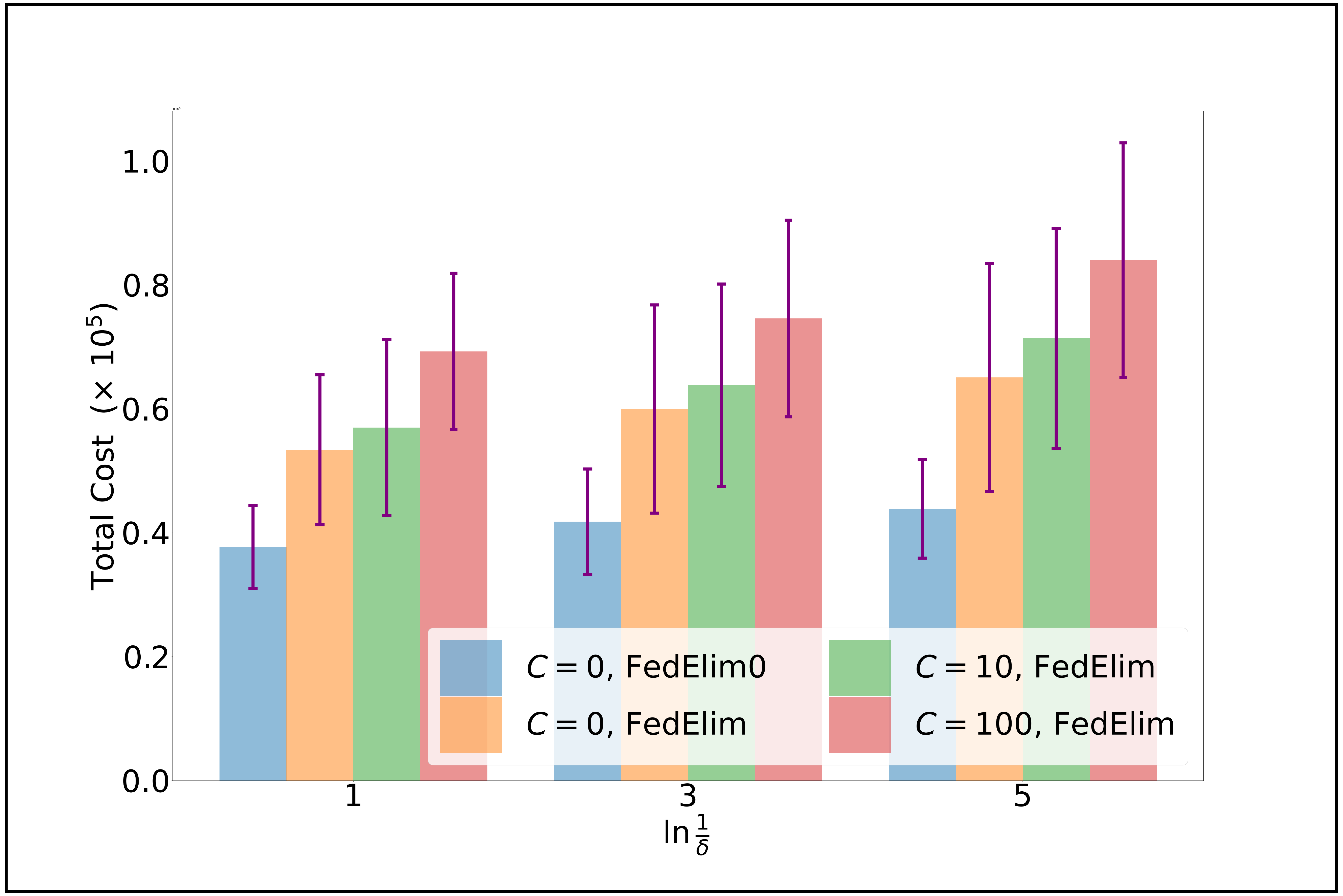

$$\mu = \begin{array}{cc} & \text{Arms} \\ \text{Clients} & \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \end{array}$$

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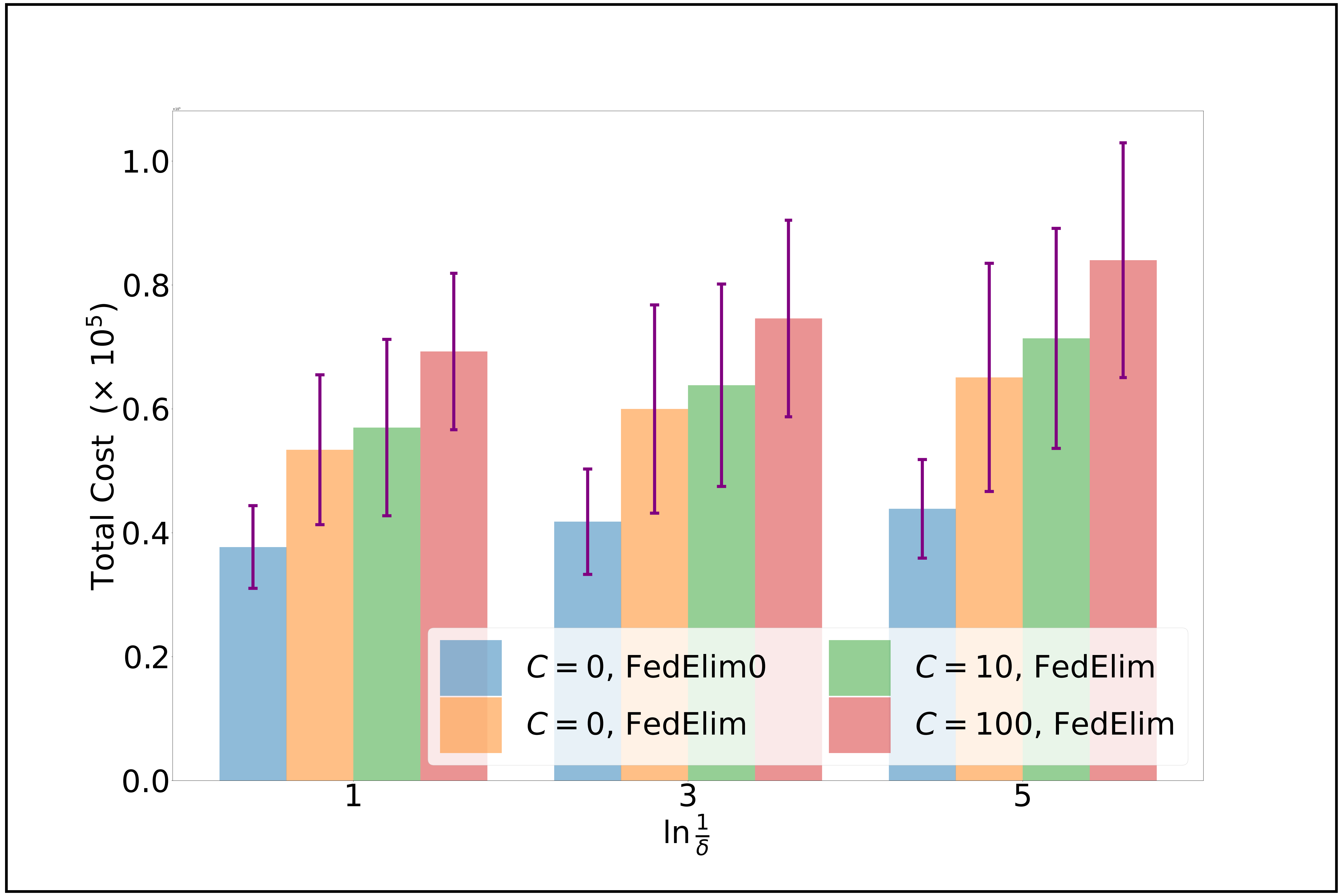
Length of blue bar $\leq$ T

Length of yellow bar $\leq$ 2T

Synthetic Gaussian Data — 1/3



Length of blue bar $\leq$ T

Length of yellow bar $\leq$ 2T

Length of any bar $\leq$ 3T

$$\mu = \begin{array}{cc} \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} & \text{Arms} \\ \text{Clients} & \end{array}$$

Local best arm of client 1 = 1

Local best arm of client 2 = 2

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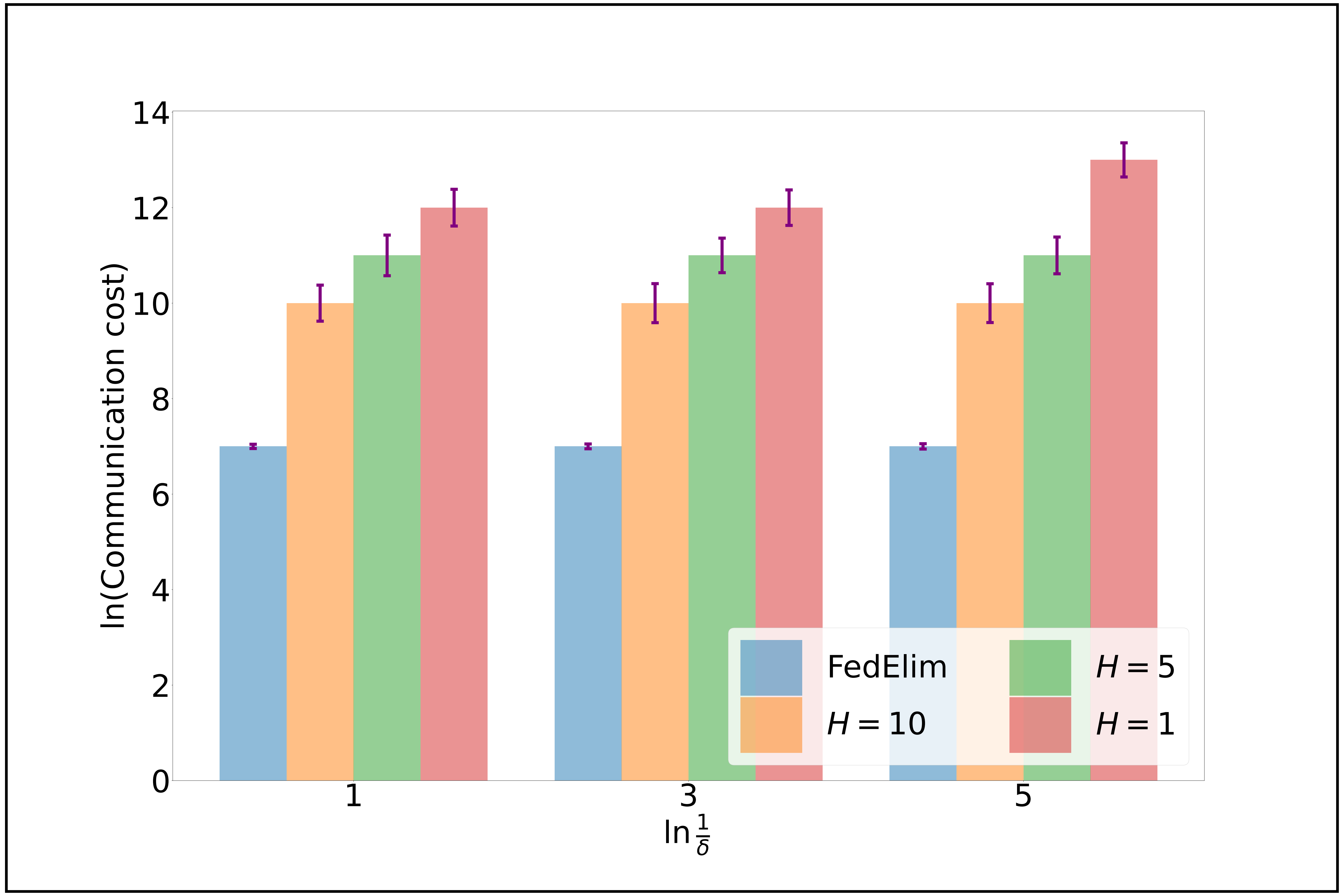
Global best arm = 4

Synthetic Gaussian Data — 2/3



$$\mu = \begin{array}{cc} \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} & \text{Arms} \\ \text{Clients} & \end{array}$$

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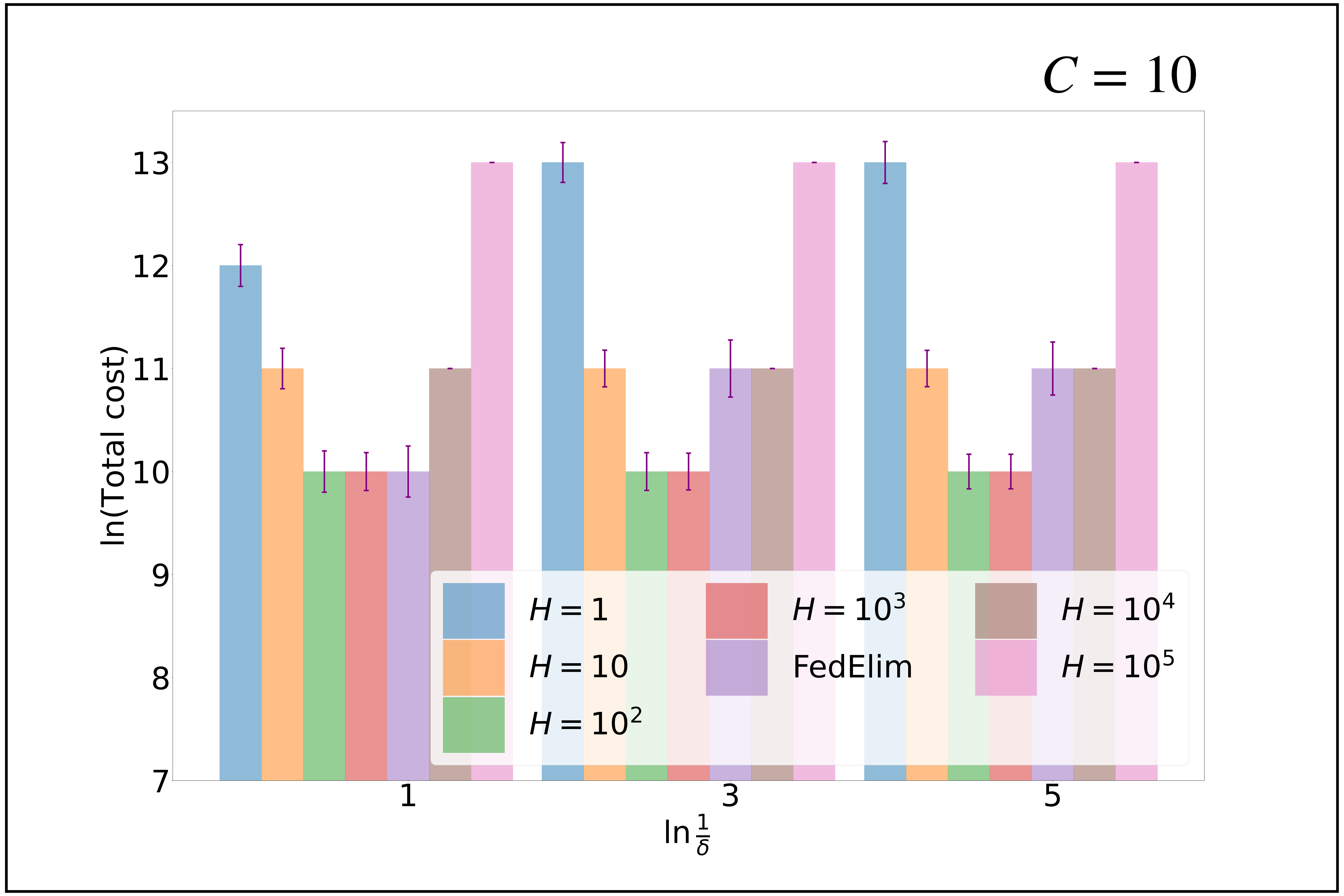
Synthetic Gaussian Data — 3/3



$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \quad \text{Arms}$$

Clients

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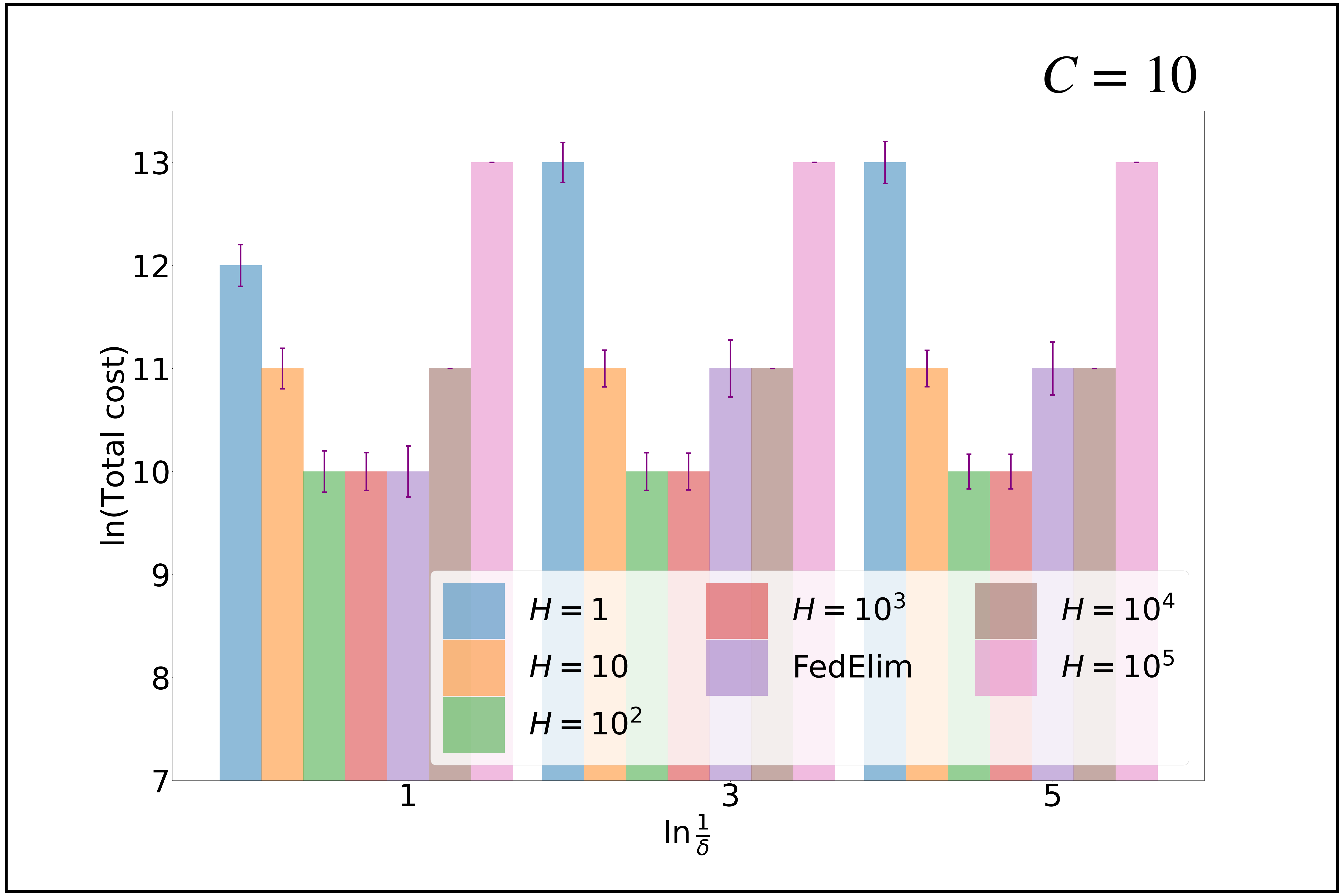
Synthetic Gaussian Data — 3/3



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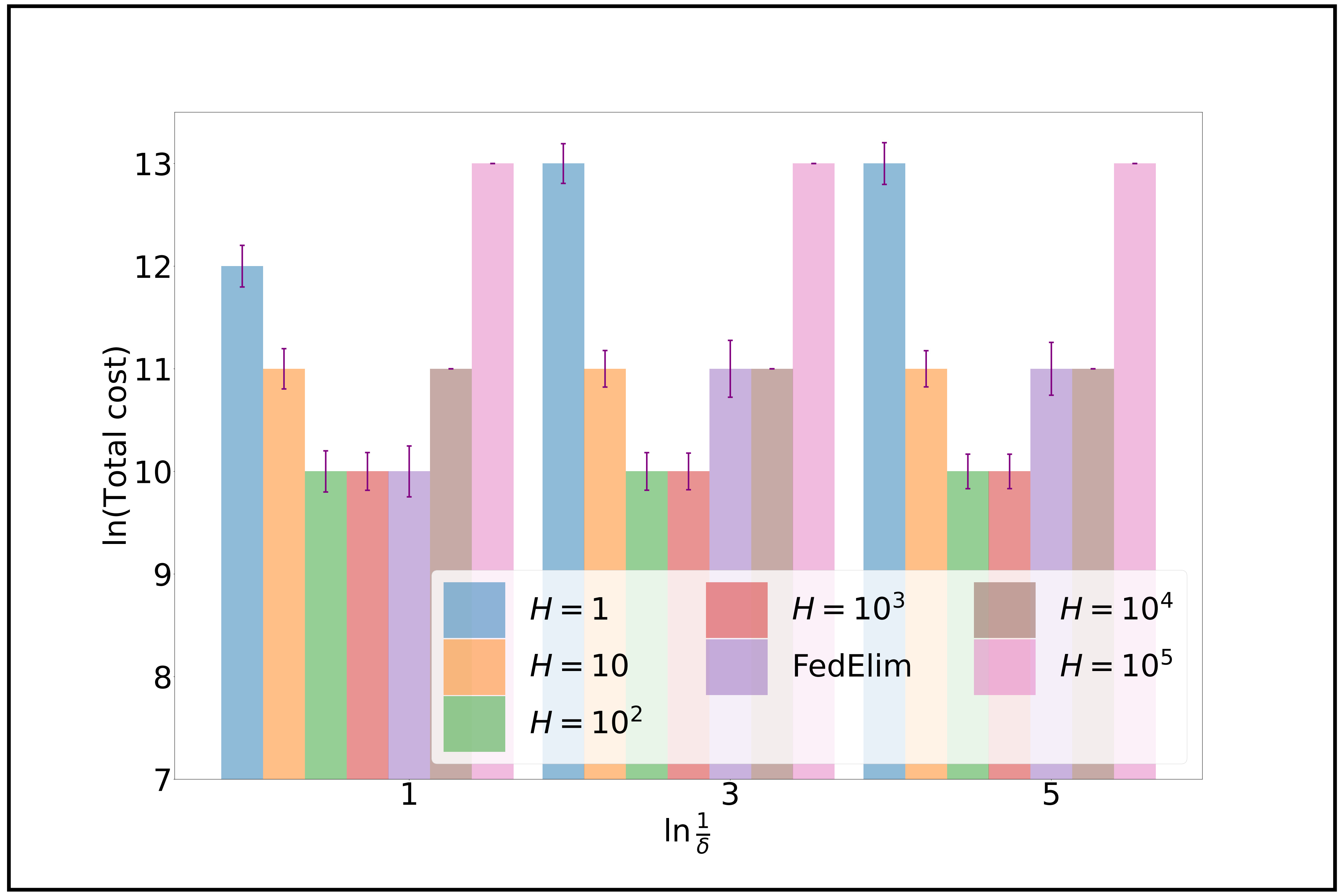
FedElim operates at a "sweet spot"

$C = 10$

T_{arm} = no. of pulls + $10 \times t$

$T_k = 1000$ (**depends on** δ)



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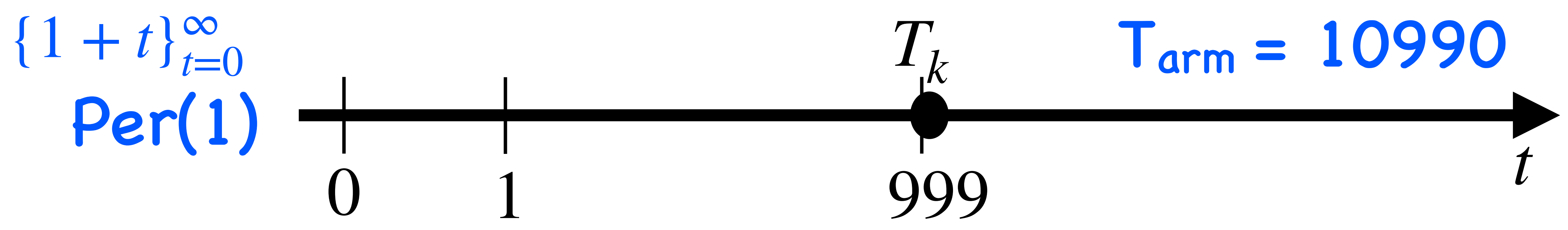

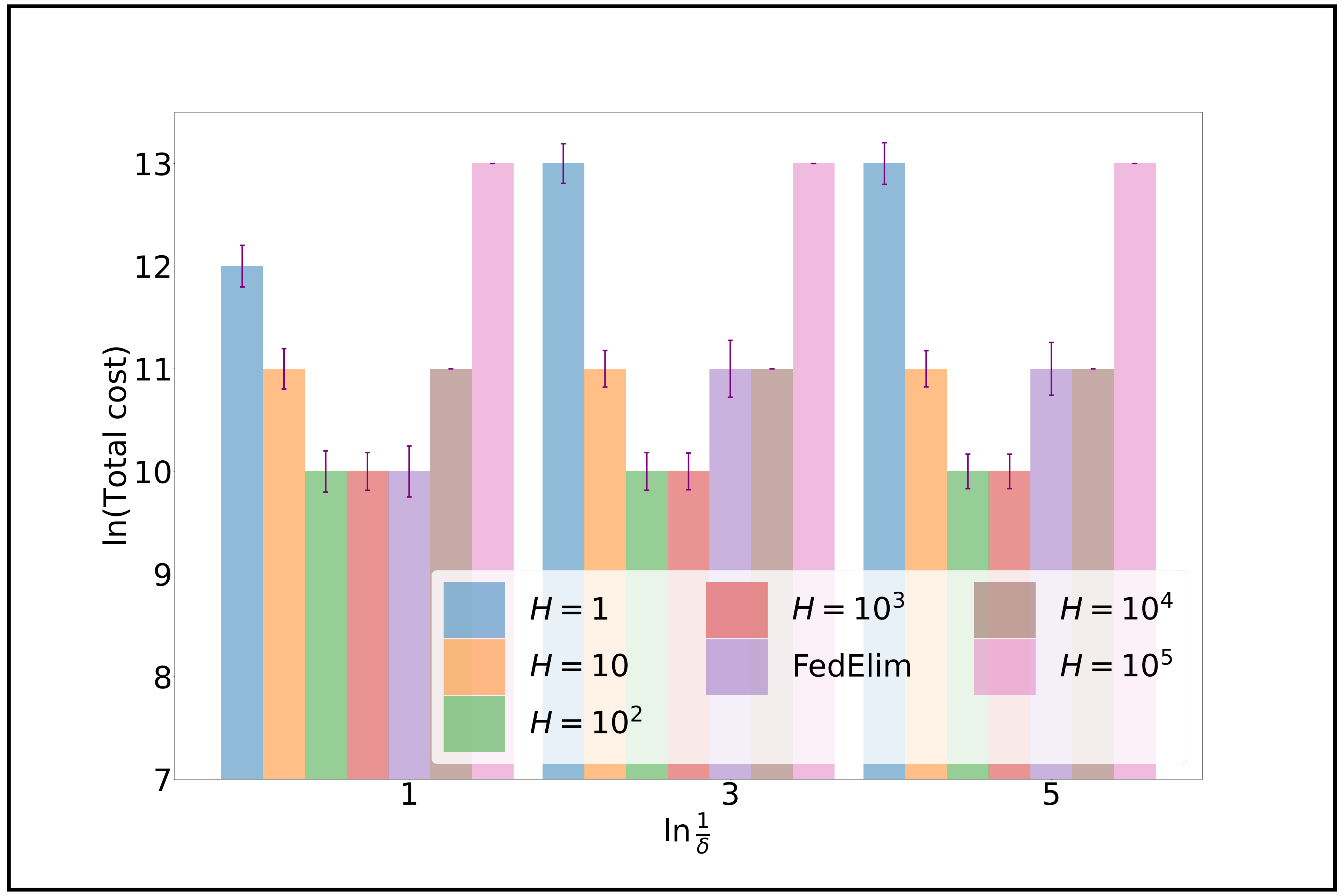

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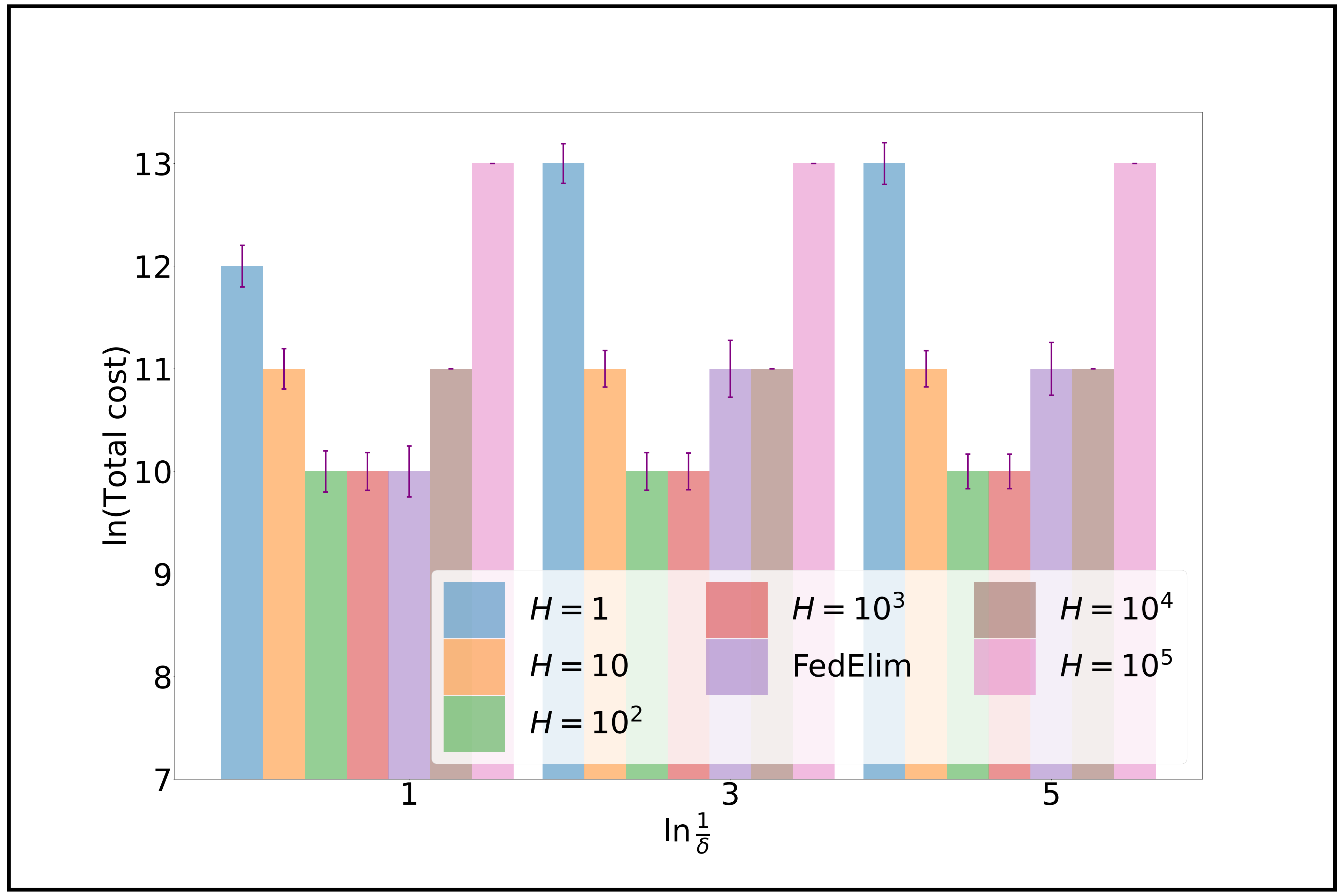

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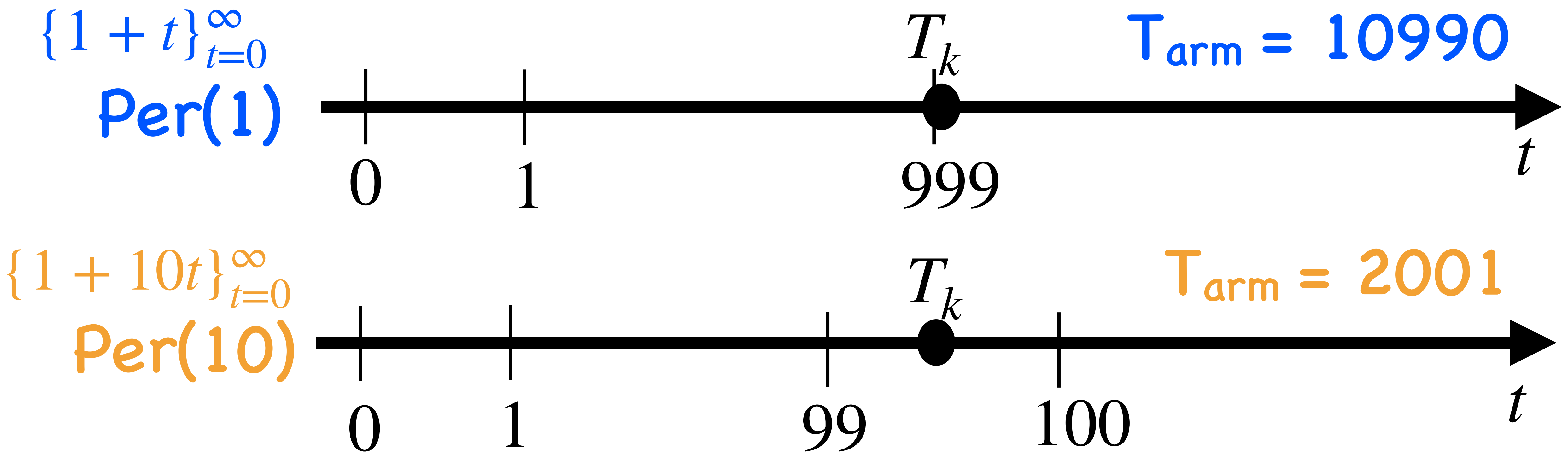

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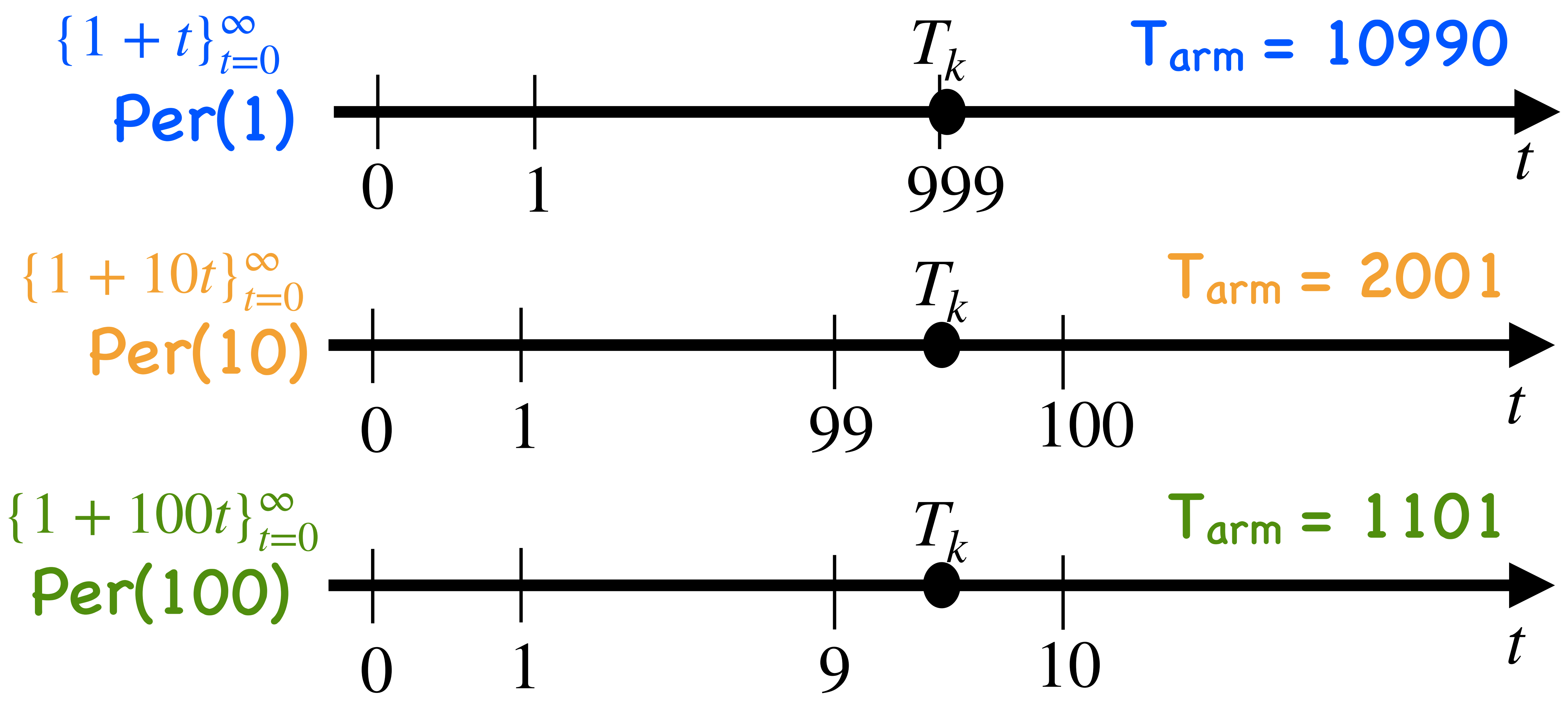

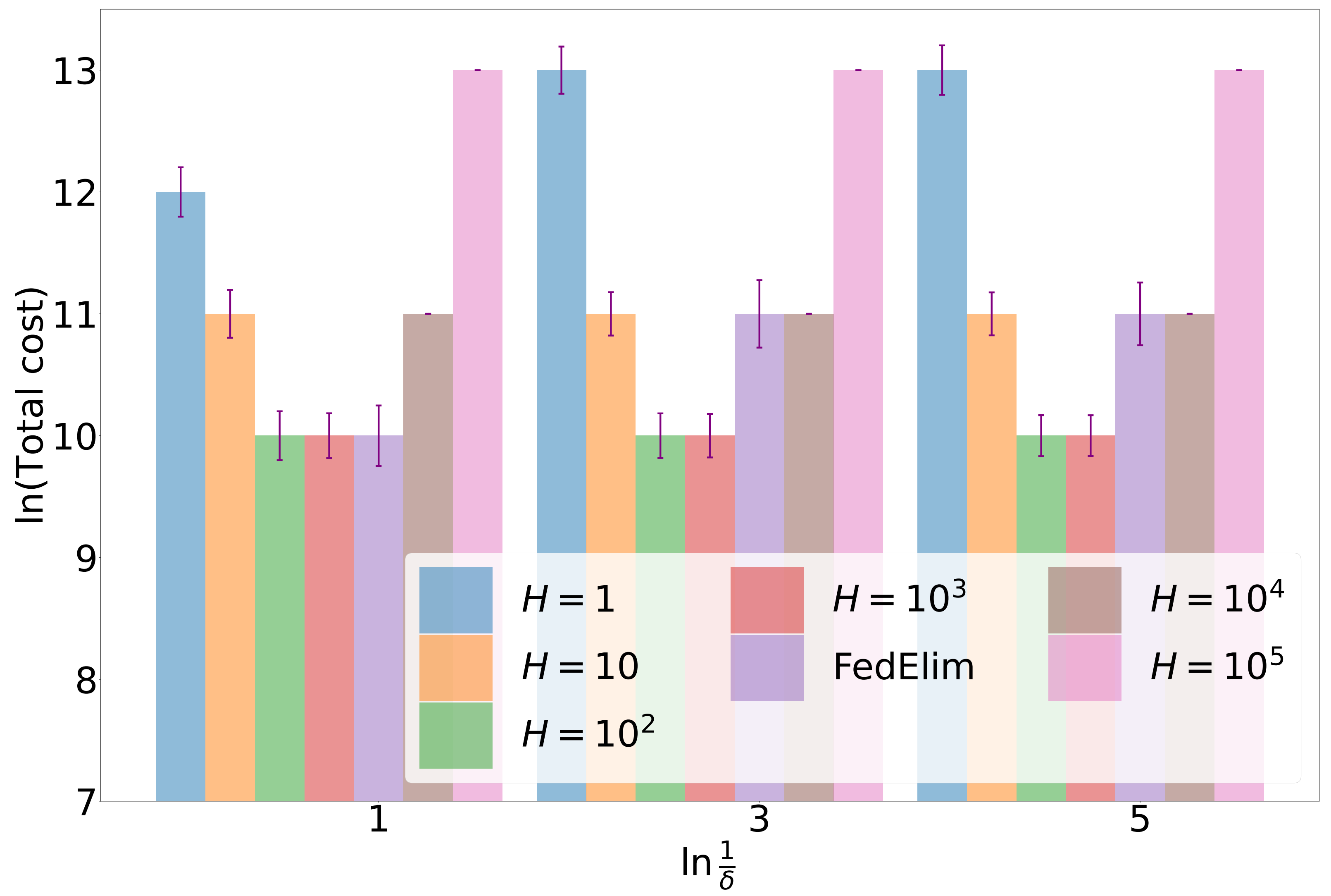

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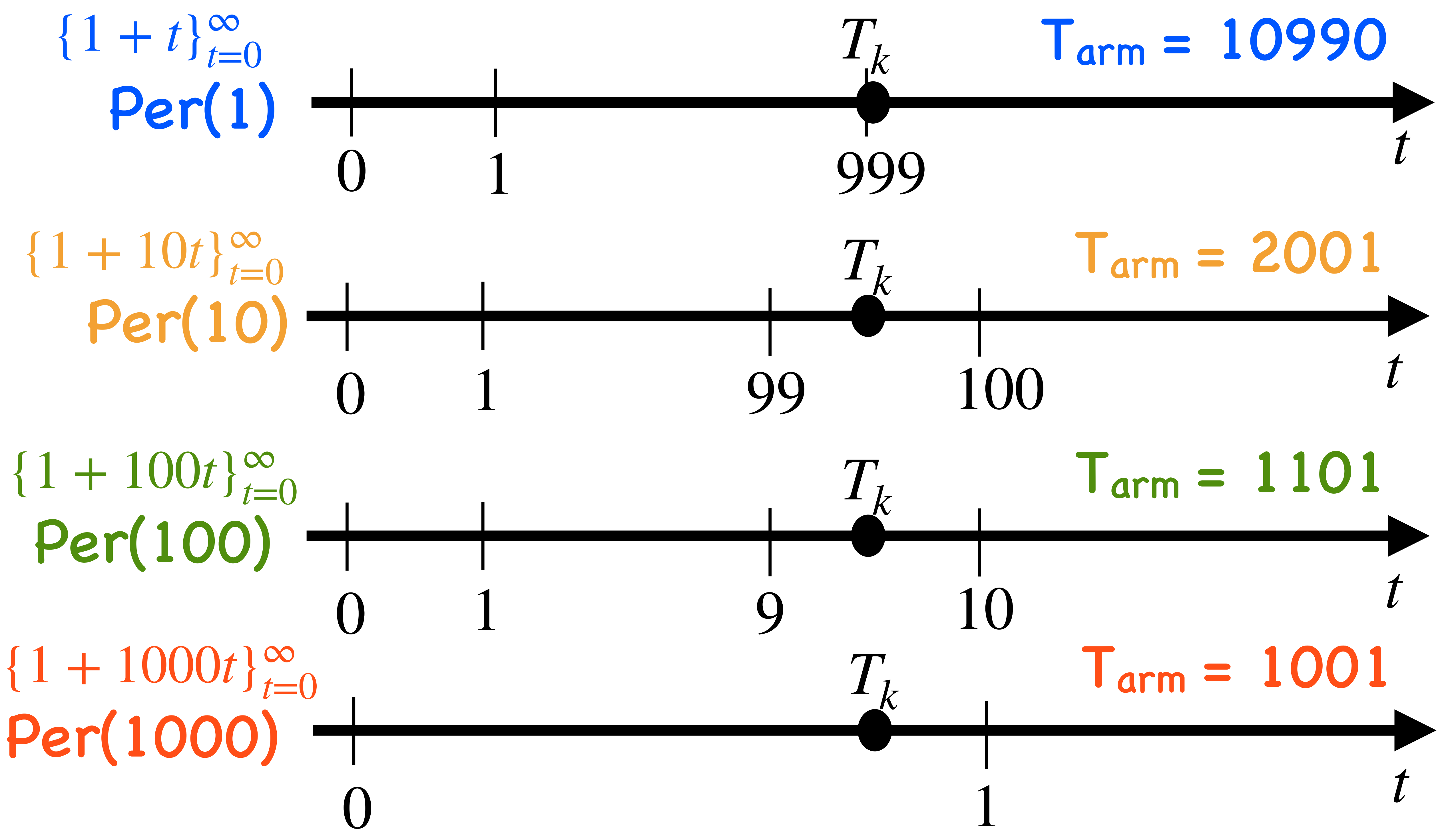

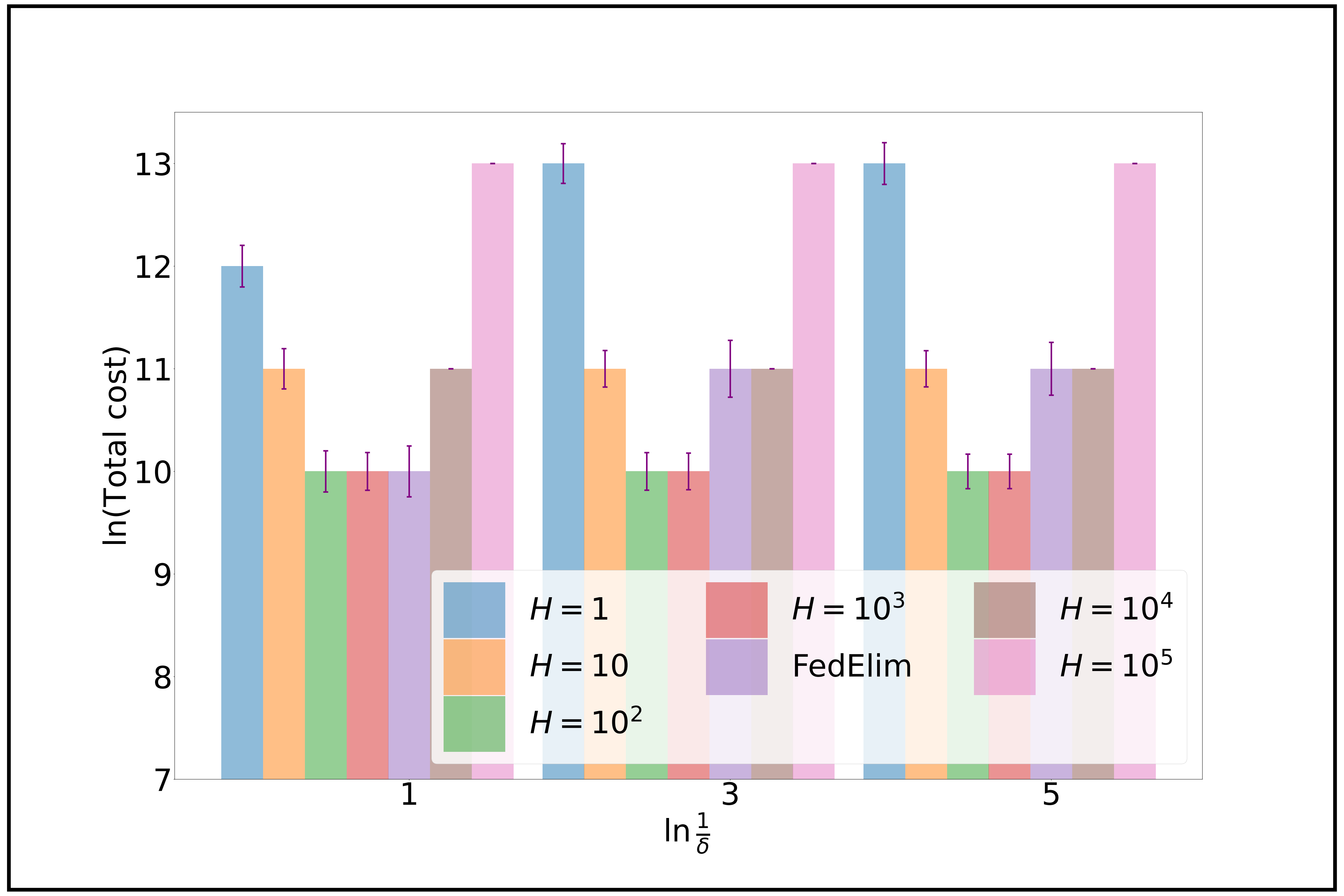

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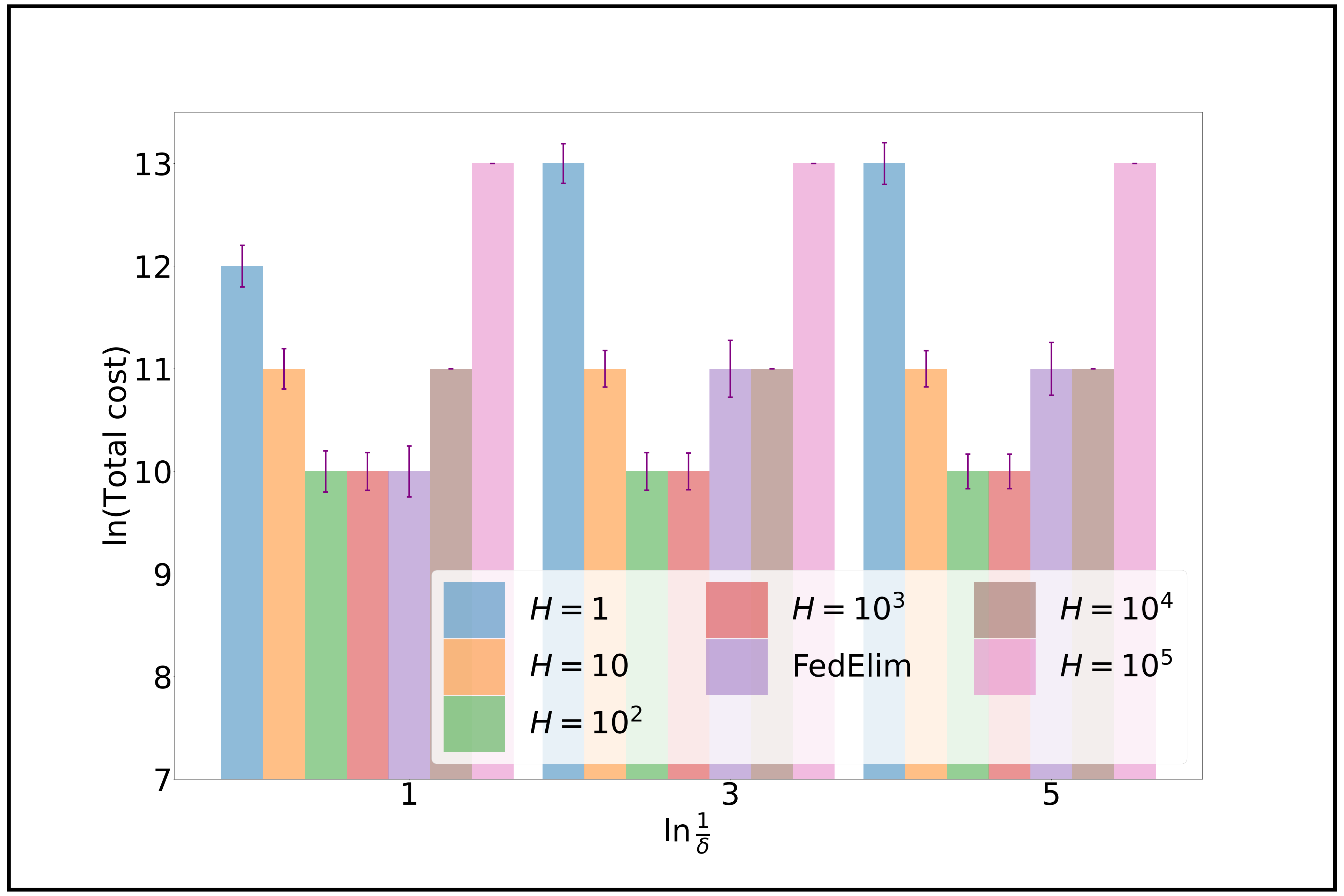

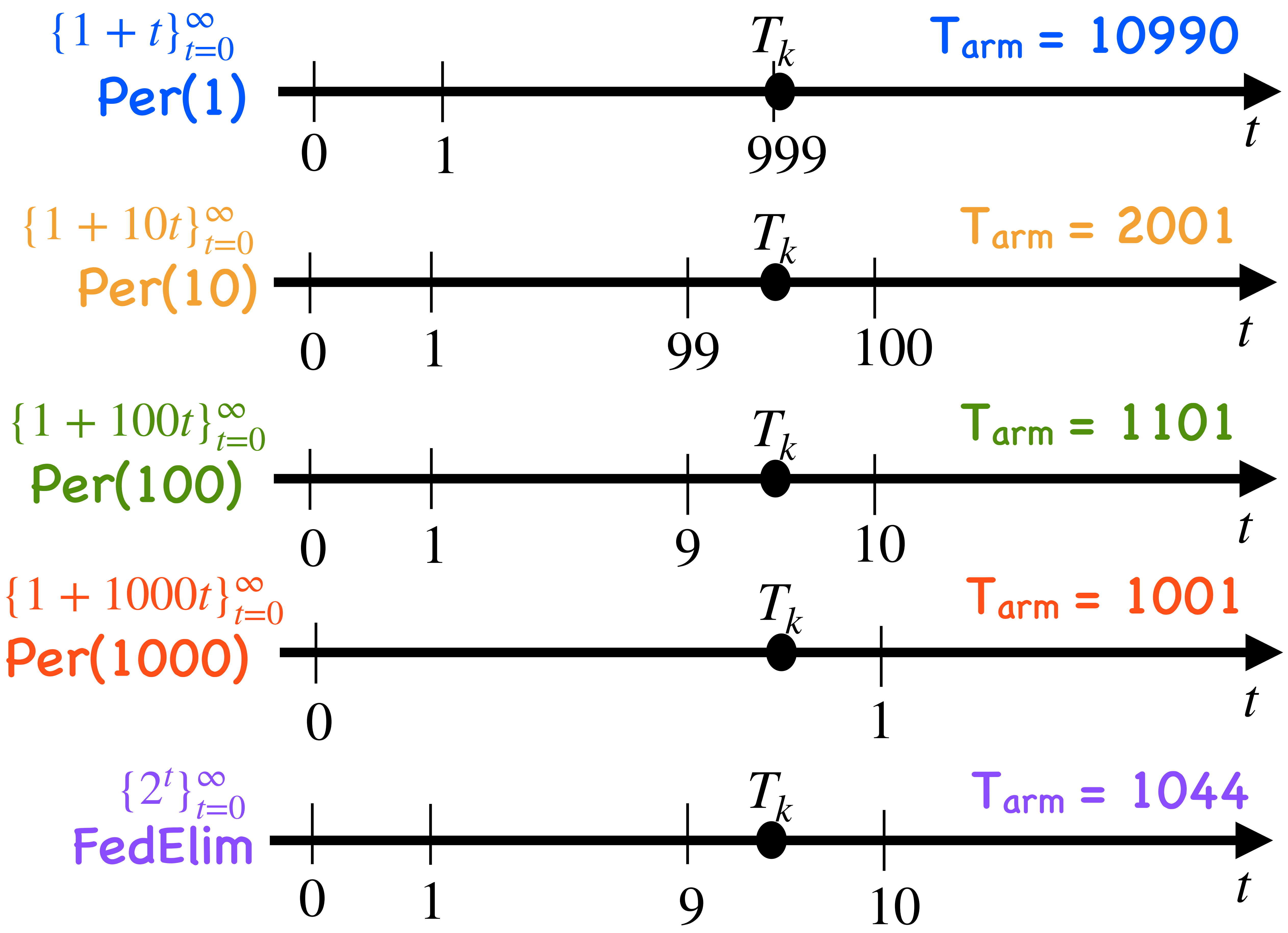

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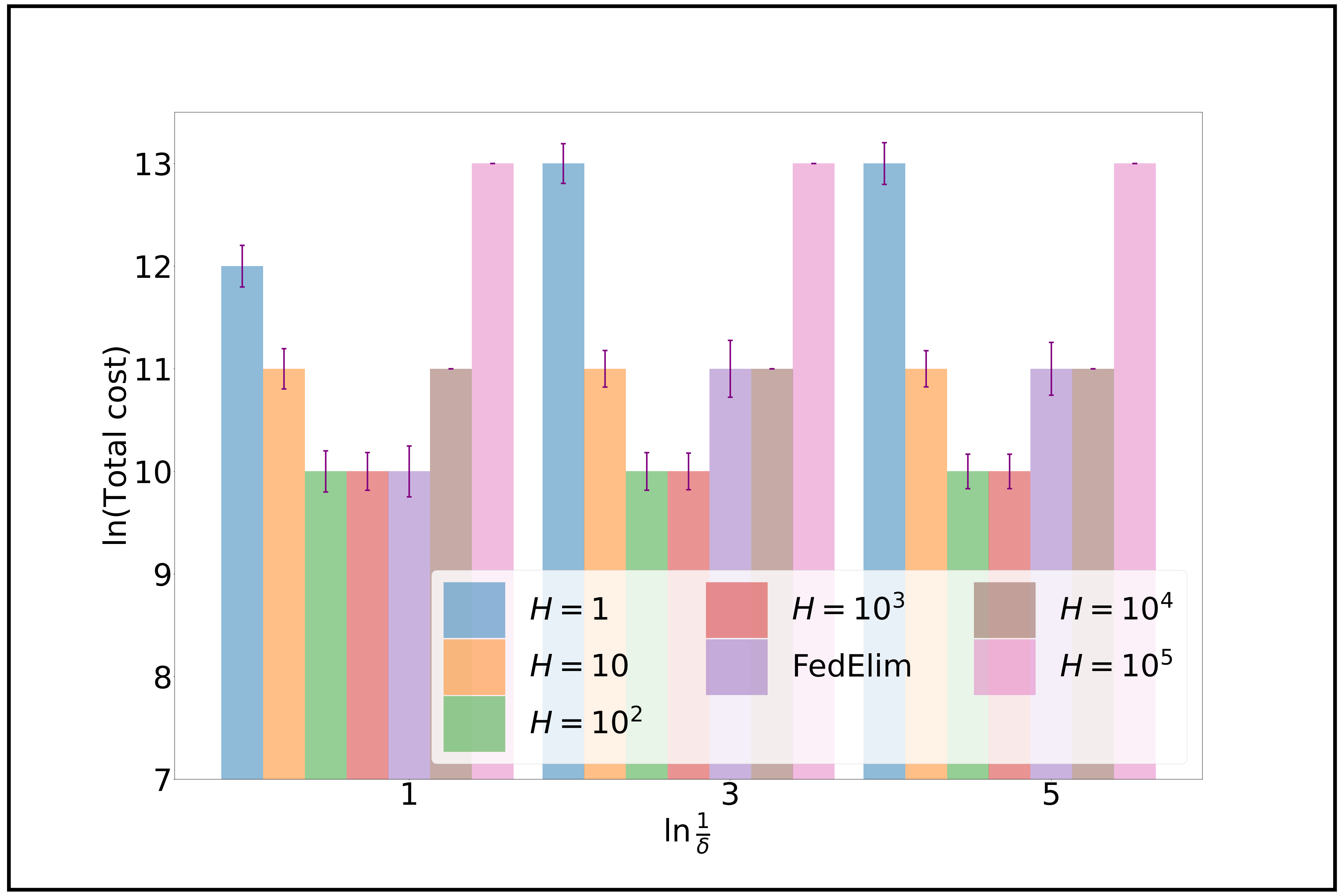
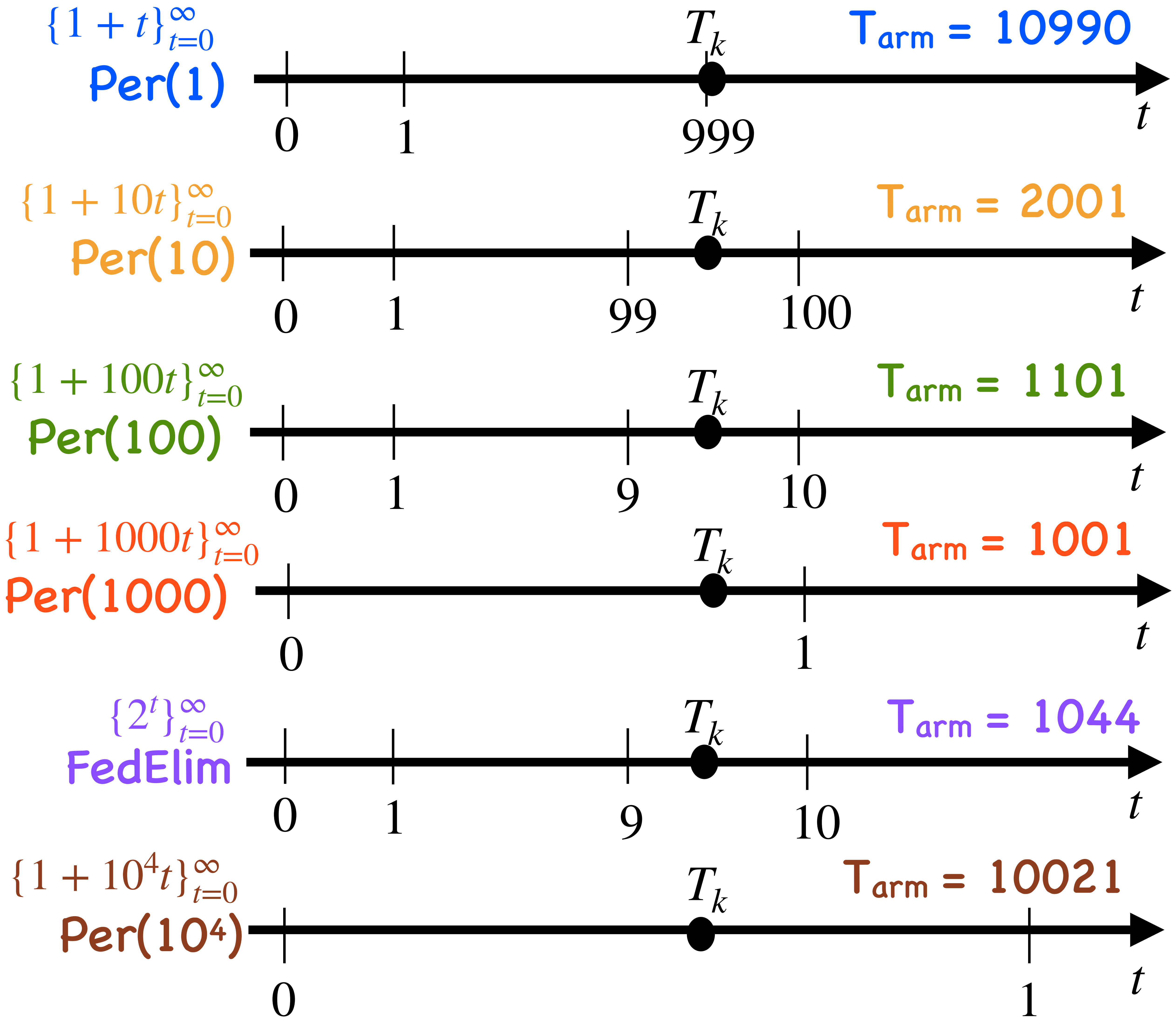
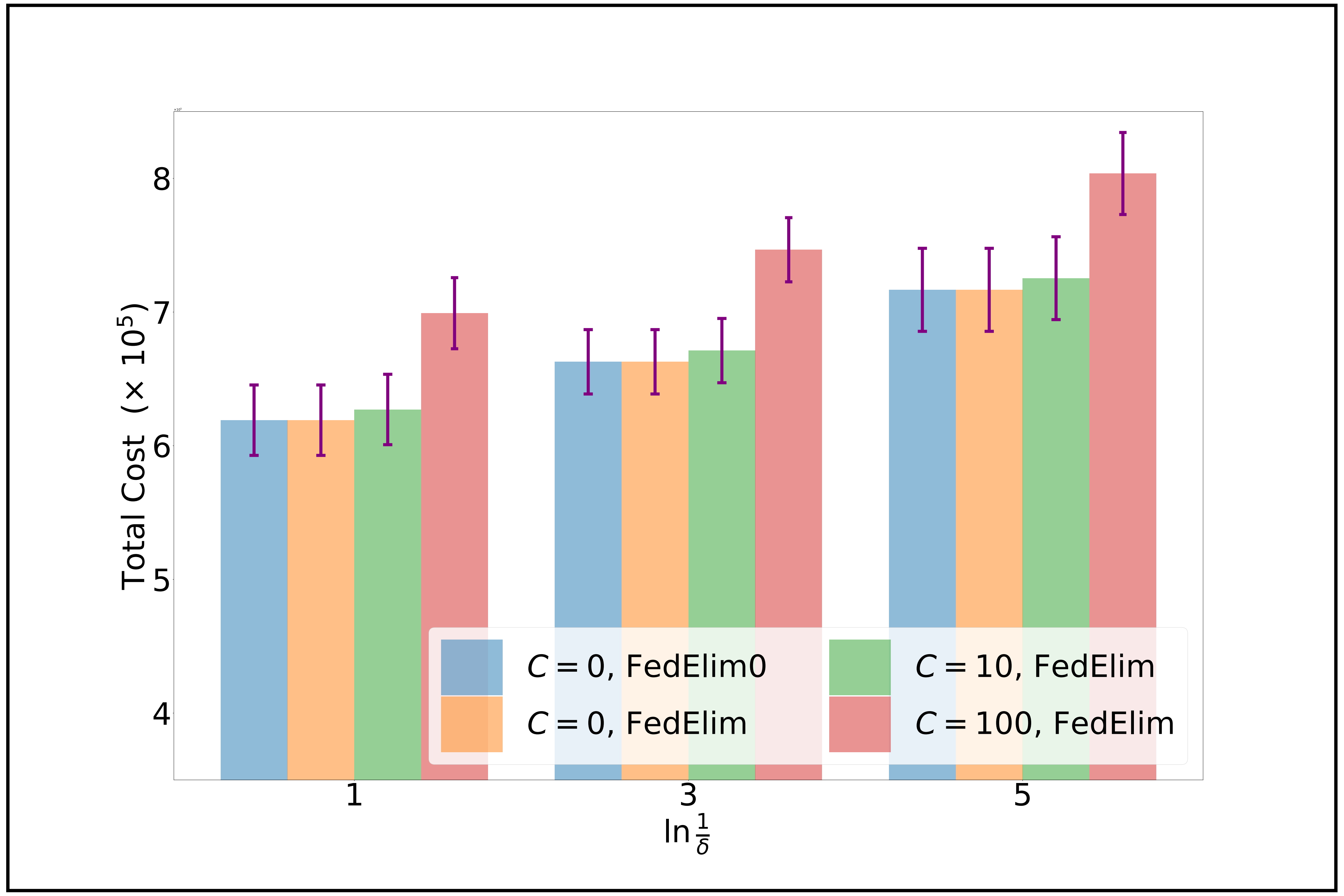
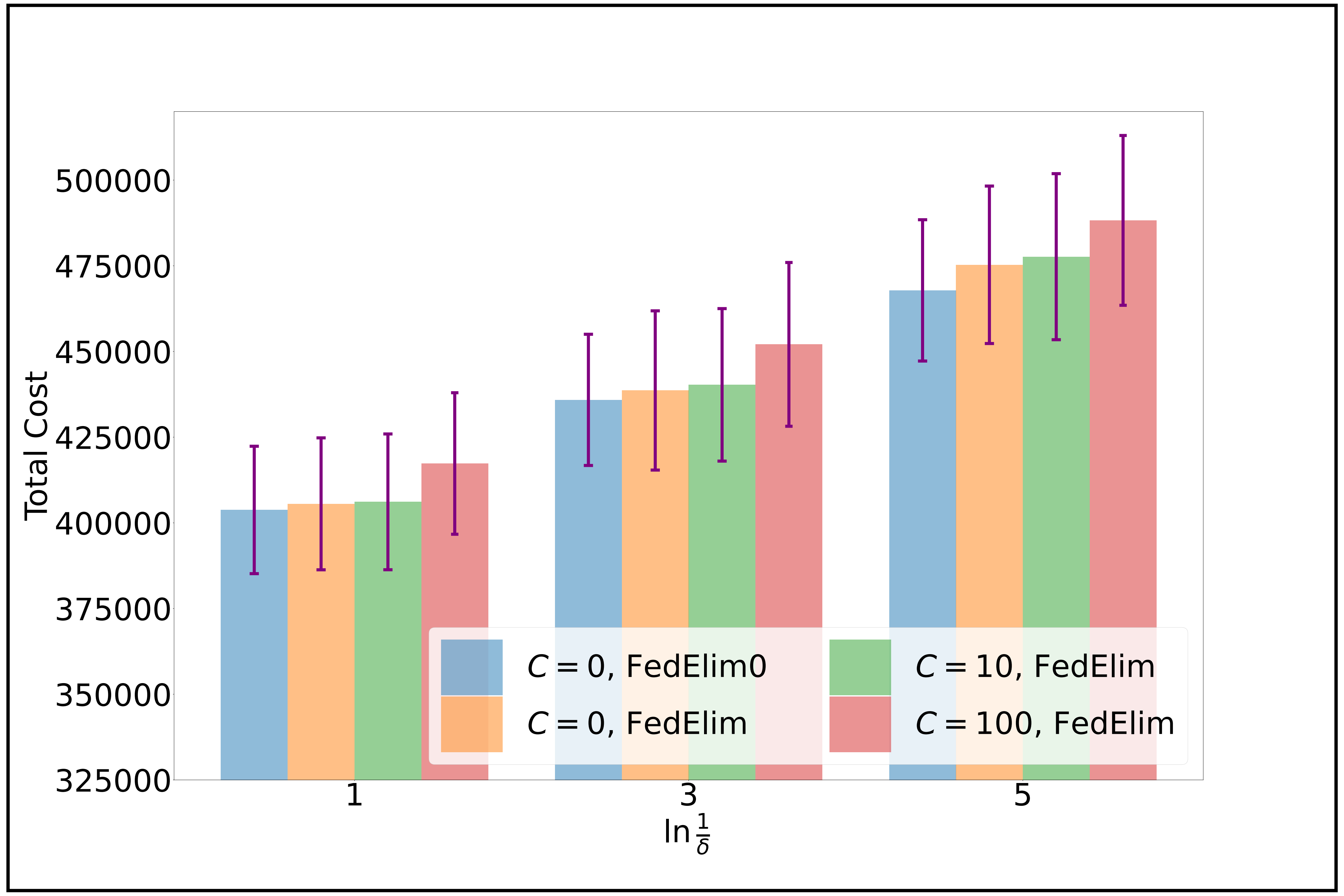




More Datasets — 1/3





MovieLens dataset (Cantador et al. (2011))

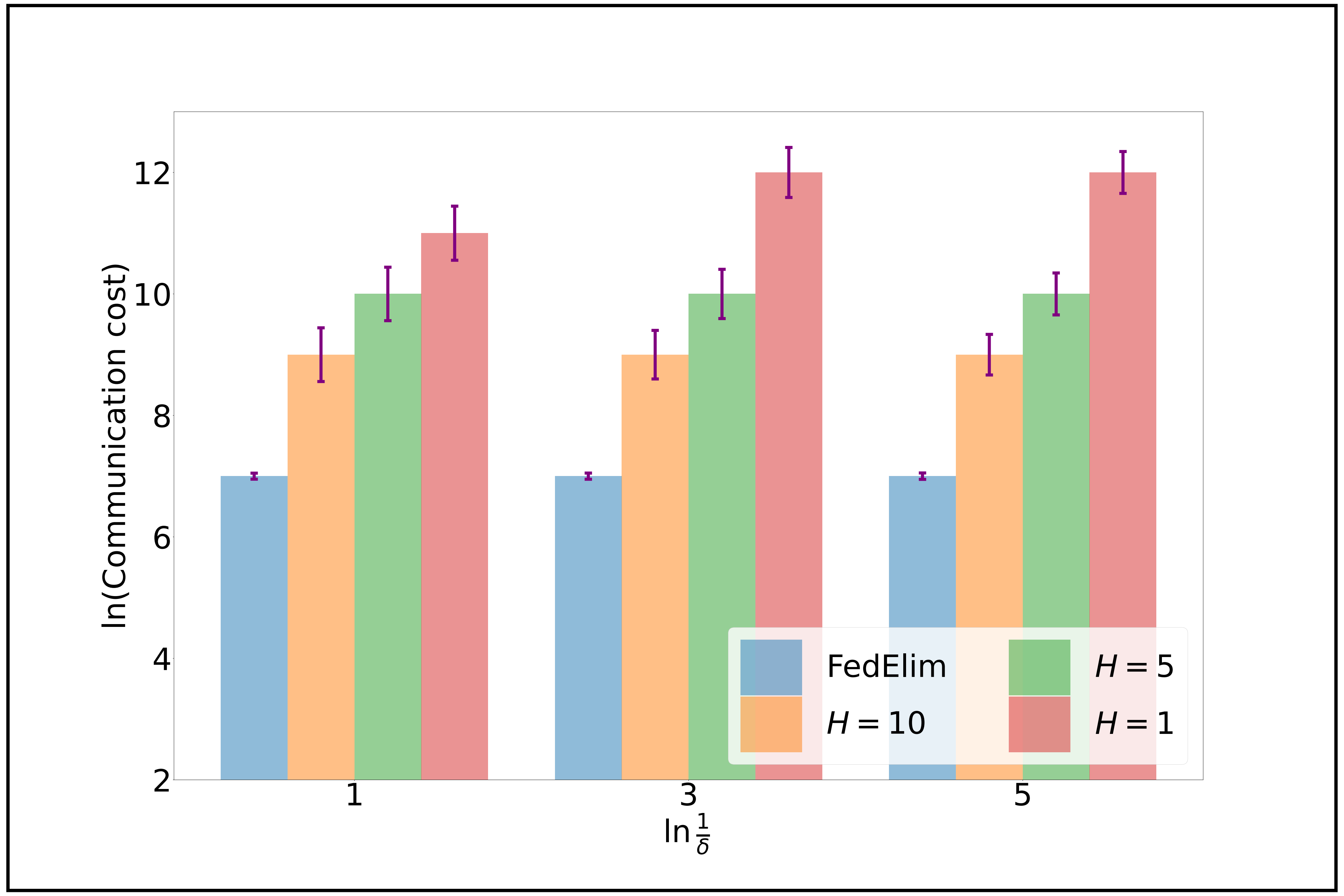
38 countries (clients), 20 genres (arms)

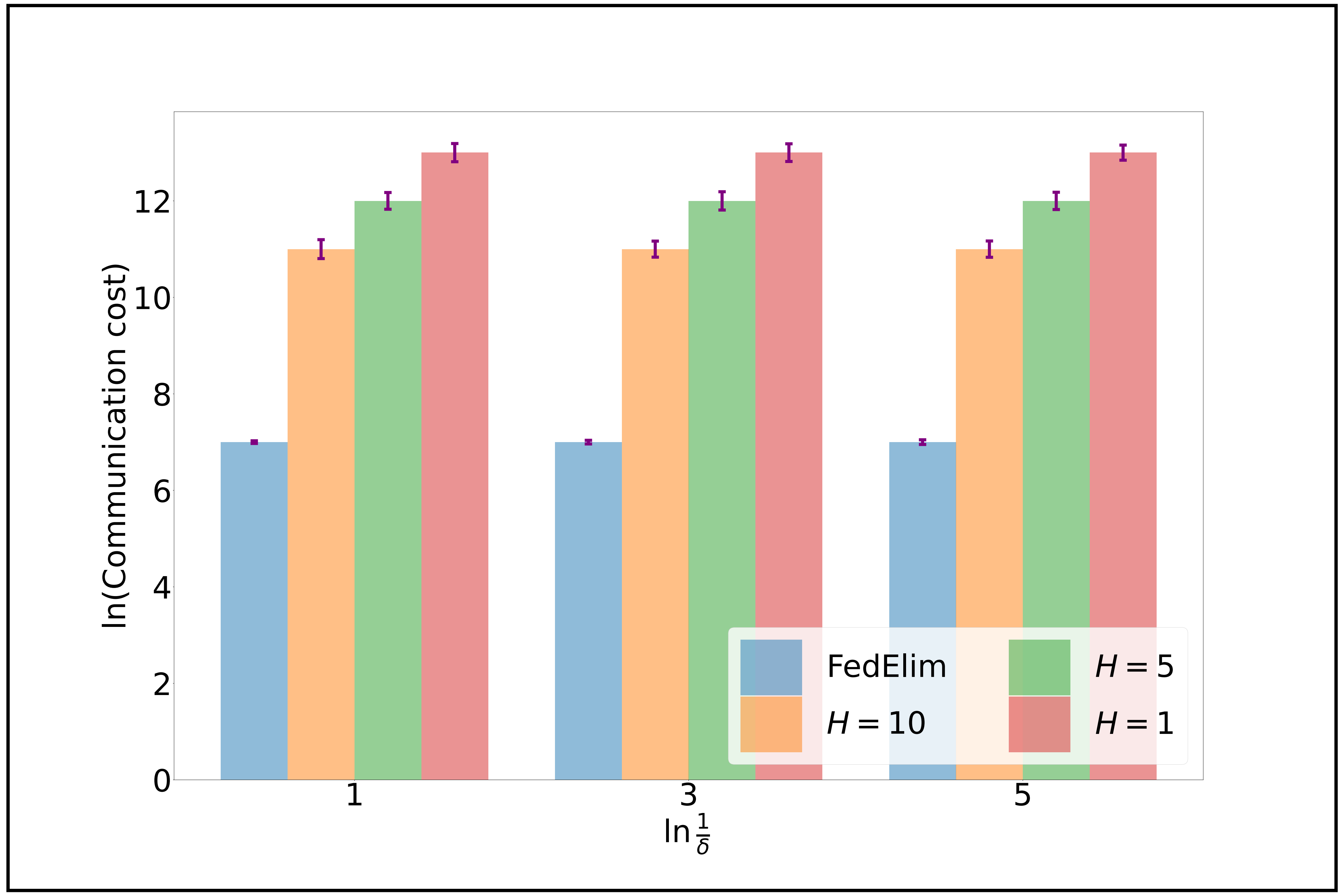
2.04M movie ratings

Bernoulli dataset from Mitra, Hassani, and Pappas (2021)

$$\mu = \begin{bmatrix} 0.9 & 0.85 & 0.1 \\ 0.85 & 0.8 & 0.3 \\ 0.7 & 0.6 & 0.5 \end{bmatrix}$$

More Datasets — 2/3





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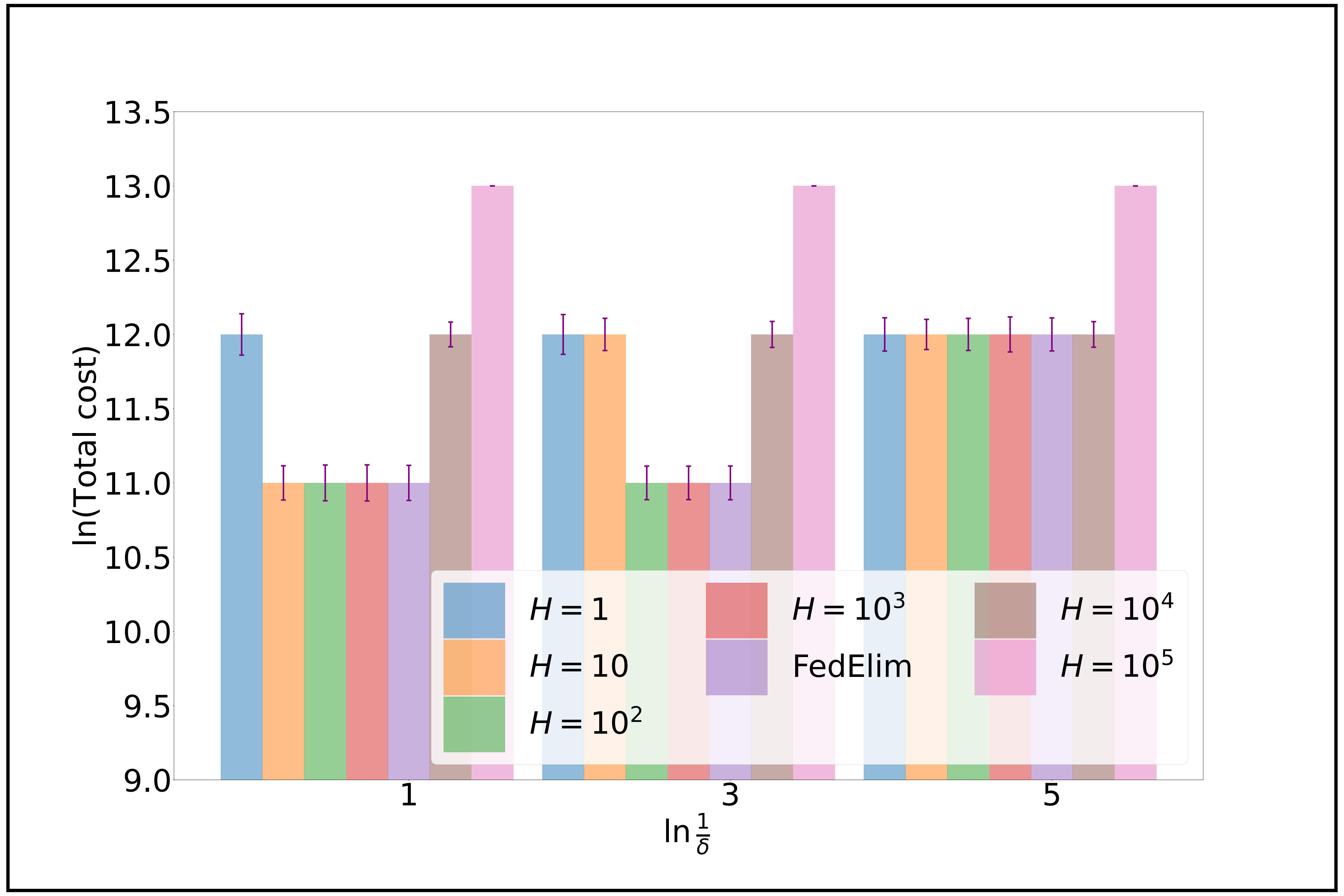

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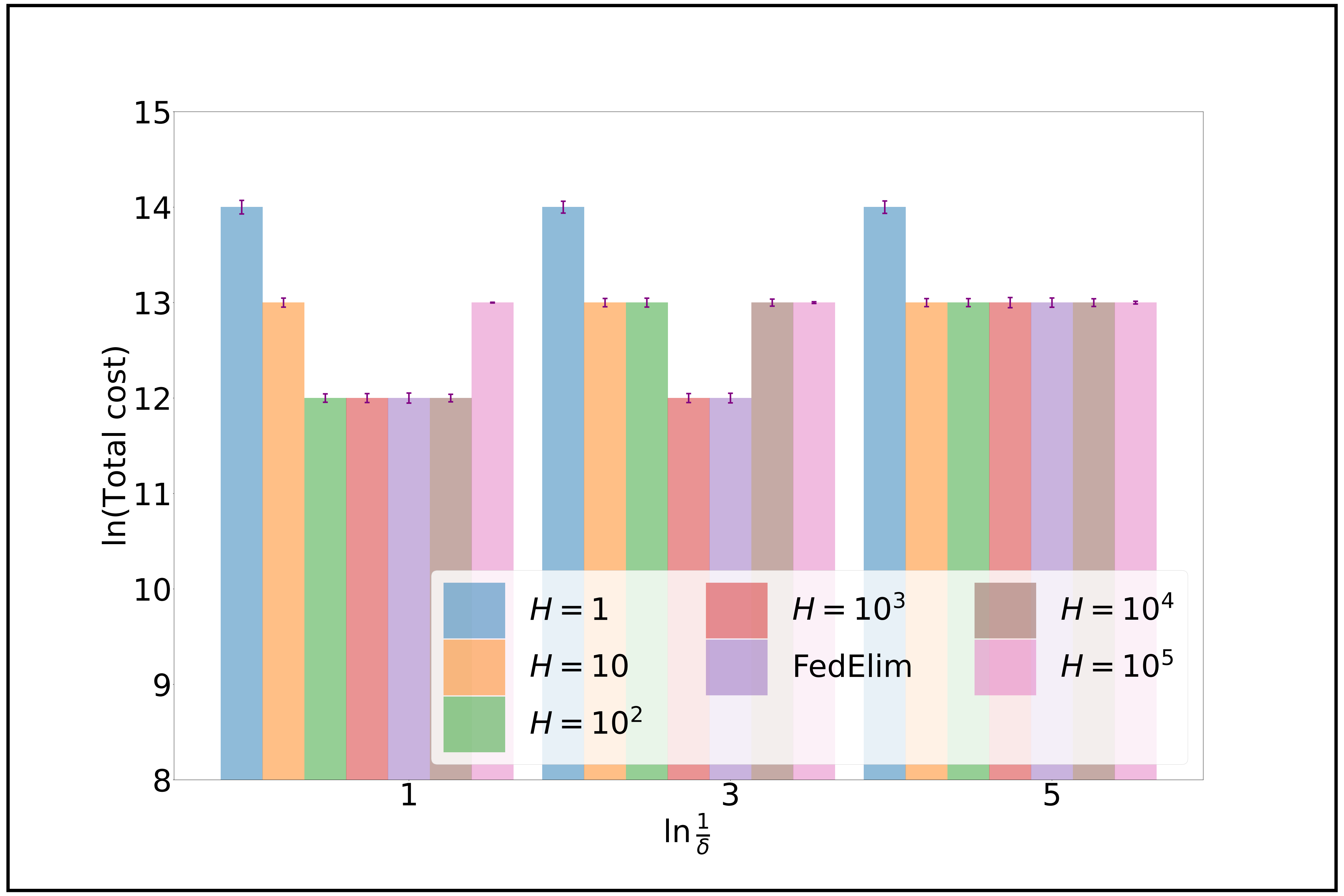

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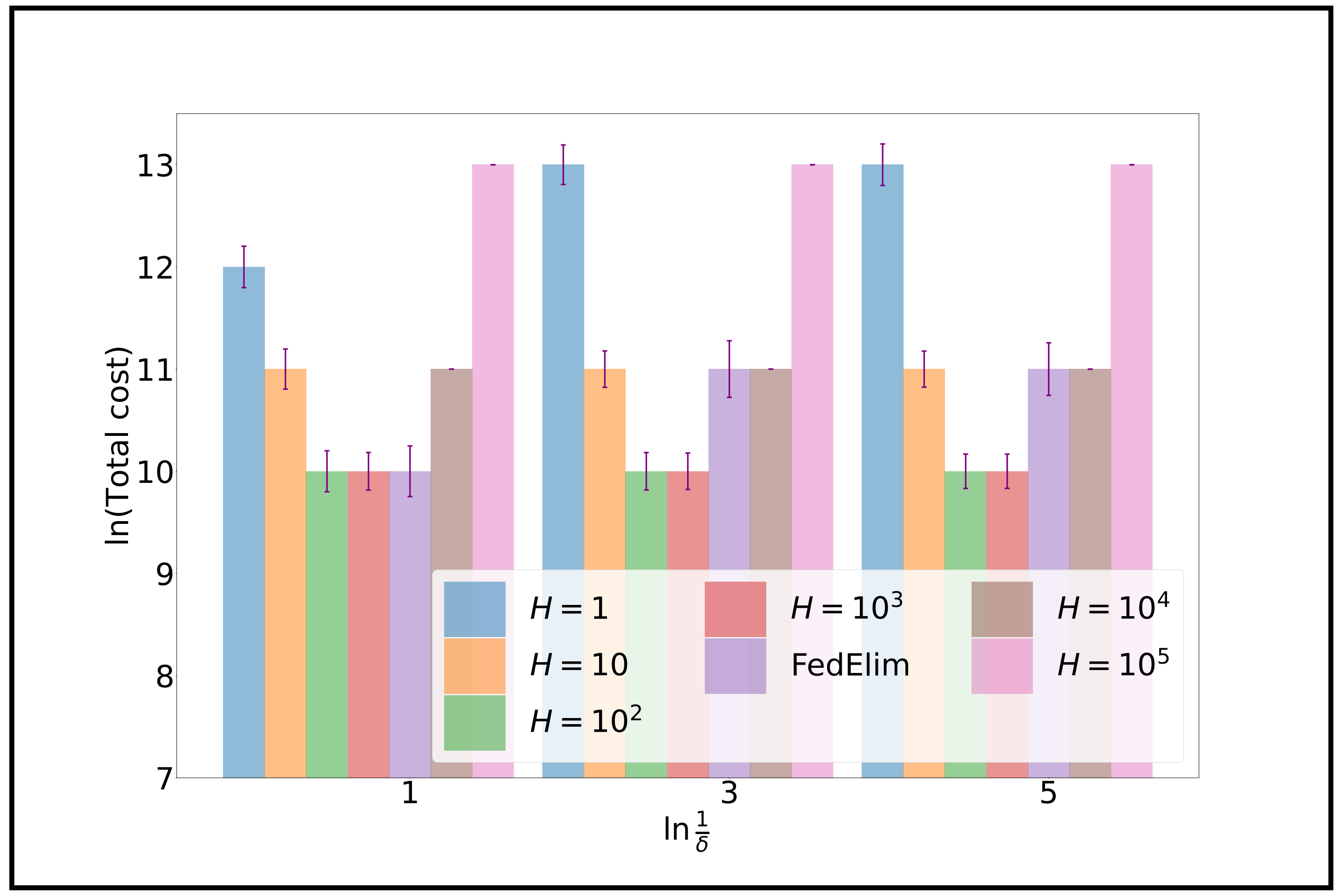
Comparison with Per(H) and SupExp

$$\mathsf{T} = \sum_{m=1}^{M} \sum_{k=1}^{K} \max\{T_{k,m}, T_k\}$$

- $\mathsf{T}_{\mathsf{Per(H)}} \leq \sum_{k=1}^{K} \sum_{m=1}^{M} \max\left\{T_{k,m}, \left\lceil \frac{T_k}{\mathsf{H}} \right\rceil \mathsf{H}\right\} = O\left(\mathsf{T} + (\mathsf{H} \times M \times K)\right)$
- $C^{\mathsf{comm}}_{\mathsf{Per(H)}} \leq C \times M \times \sum_{k=1}^{K} \left\lceil \frac{T_k}{\mathsf{H}} \right\rceil = O\left(C \times \frac{\mathsf{T}}{\mathsf{H}} + C \times M \times K\right)$



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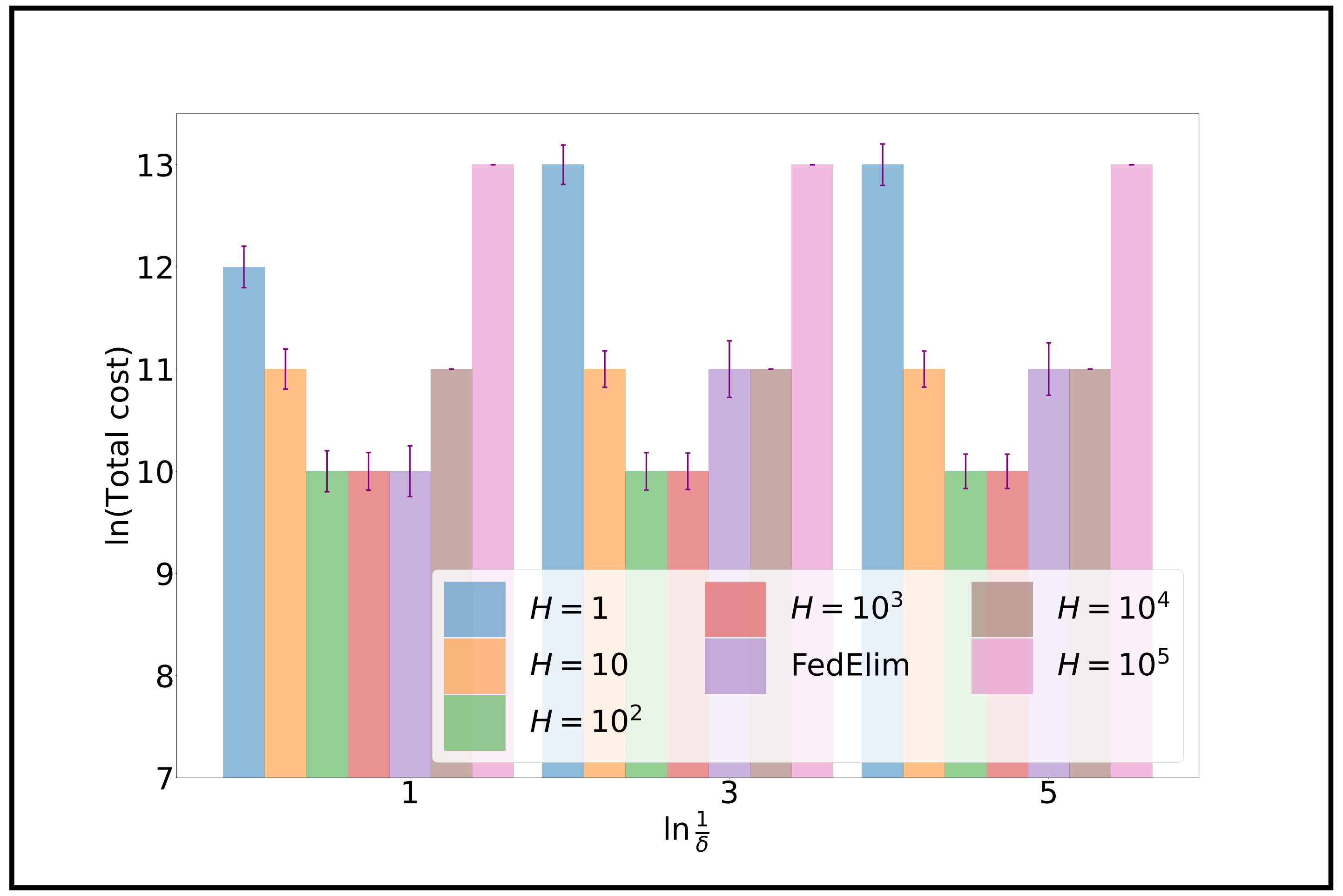

$$\mathsf{H}_{\mathsf{opt}} = \sqrt{\frac{C \cdot \mathsf{T}}{M \cdot K}}$$

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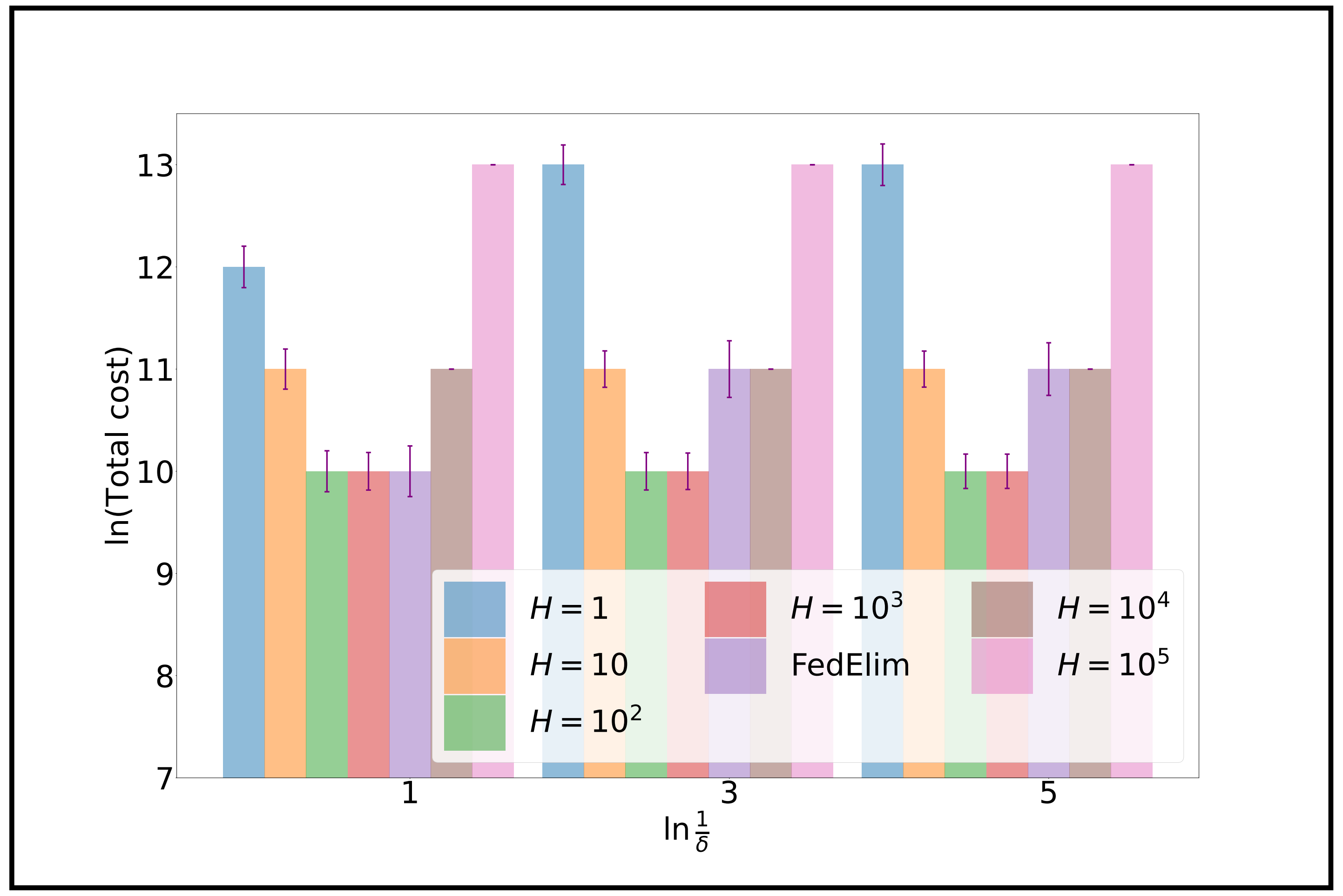

$$\mathsf{H}_{\mathrm{opt}} = \sqrt{\frac{C \cdot \mathsf{T}}{M \cdot K}}$$

Scheme	No. of arm pulls	Comm. cost	Total cost
Per(H)	O(T)	O(CT/H)	O(T + CT/H)
FedElim*	2T	O(C log(T))	3T
SupExp	O(T²)	O(C loglog(T))	O(T²)

High-probability upper bounds

Summary and Future Directions

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- For all δ small, $P(\mathsf{T}_{\mathsf{FedElim}} \lesssim 3\mathsf{T}_{\mathsf{FedElim0}}) \geq 1 - \delta$

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Thank You!

More details: arXiv:2208.09215

References

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