Name:
Roll Number:

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

CS6660: Mathematical Foundations of Data Science (Probability)

Quiz 3

Date: 14 September 2024

Question	1	2(a)	2(b)	Total
Marks Scored				

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

1. **(1 Mark)**
   Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Let X and Y be jointly continuous random variables with the joint PDF

$$f_{X,Y}(x,y) = \frac{1}{x}, \qquad 0 \leq y \leq x \leq 1.$$

   Let $Z = X + Y$.
   If the value of $\mathbb{P}(\{0 \leq Z \leq 1\})$ is expressed as $\log \alpha$, where the logarithm is the natural logarithm, then what is the value of α?

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2. Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Assume that all random variables appearing below are defined with respect to $\mathscr{F}$.

   Let $X, Y \overset{\text{i.i.d.}}{\sim}$ Exponential(λ).

   (a) **(3 Marks)**
   Determine the joint PDF of $Z = X + Y$ and $W = \frac{X}{X+Y}$.
   Clearly specify the range of Z and the range of W in the joint PDF expression.

   (b) **(1 Mark)**
   Compute $\mathbb{P}(\{W \leq \frac{1}{3}\})$.