Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090: Stochastic Processes

Quiz 1

Date: 08 February 2025

Question	1	2	Total
Marks Scored			

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

1. Fix $a, b \in \mathbb{R}$ such that $a < b$.
   For each $n \in \mathbb{N}$, let
   $$A_n = \left(a - \frac{1}{n},\ b - \frac{1}{n}\right].$$
   (a) **(1 Mark)**
   For each $n \in \mathbb{N}$, compute $B_n = \bigcup_{k=n}^{\infty} A_k$.
   (b) **(1 Mark)**
   Using the result in part (a) above, compute A_n i.o..

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2. Let $(\Omega, \mathscr{F}, \mathbb{P}) = ((0,1), \mathscr{B}((0,1)), \text{Unif})$.
   For each $n \in \mathbb{N}$, let

$$X_n(\omega) = n\omega - \lfloor n\omega \rfloor, \quad \omega \in \Omega.$$

   (a) **(2 Marks)**
   Fix $\omega \in (0,1)$.
   For the fixed ω, plot $X_n(\omega)$ as a function of n.
   From the plot, what can you infer about $\lim_{n\to\infty} X_n(\omega)$?

   (b) **(1 Mark)**
   Does X_n converge almost surely? Justify.