

AI5090/EE5817: PROBABILITY AND STOCHASTIC PROCESSES

QUIZ 05

DATE: 06 NOVEMBER 2025

Question	1	2	Total
Marks Scored			

1. (2 Marks)

Fix $b > 0$. Suppose that X has the PDF

$$f_X(x) = \frac{1}{2b} e^{-\frac{|x|}{2b}}, \quad x \in \mathbb{R}.$$

Determine the PDF of $Y = b|X|$, defined as $Y(\omega) = b|X(\omega)|$ for every $\omega \in \Omega$.

2. Suppose that each elevator in IIT Hyderabad has a maximum capacity of 1080 kg. A large group of students try to enter one such elevator in the CSE building during an afternoon lecture slot. The security personnel deployed in the CSE building decides to allow students to enter the elevator sequentially, one at a time, until the total weight of all students inside the elevator does not exceed 1080 kg.

Let N denote the (random) number of students who are allowed to enter the elevator in one upward trip. Let $X_1, X_2, X_3, \dots \stackrel{\text{IID}}{\sim} \text{Exp}(1/72)$ denote respectively the weights of the first, second, third, ... students entering the elevator. It is easy to see that

$$N = \sup\{n \geq 1 : \sum_{i=1}^n X_i \leq 1080\} = \sup\{n \geq 1 : S_n \leq 1080\}, \quad S_n := \sum_{i=1}^n X_i.$$

(a) (2 Marks)

Compute $\mathbb{P}(\{N = n\})$ for any $n \in \mathbb{N}$.

Hint: Notice that

$$\mathbb{P}(\{N = n\}) = \mathbb{P}(\{S_n \leq 1080, X_{n+1} + S_n > 1080\}).$$

Use the fact that X_{n+1} is independent of S_n to simplify the above probability.

You may use the fact that for each n , the PDF of S_n is given by

$$f_{S_n}(x) = \frac{\mu^n e^{-\mu x} x^{n-1}}{(n-1)!}, \quad x > 0,$$

where $\mu = 1/72$.

(b) (1 Mark)

Compute $\mathbb{E}[N]$.