Mathematical Foundations for Data Science (Probability)

Problem Set 01

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Problems

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

- Suppose $A, B \in \mathscr{F}$ are such that $\mathbb{P}(A) = \frac{3}{4}$ and $\mathbb{P}(B) = \frac{1}{3}$. Show that

$$\frac{1}{12} \leq \mathbb{P}(A \cap B) \leq \frac{3}{4}.$$

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Problems

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

- Suppose $A_1, A_2, \ldots \in \mathscr{F}$ are such that $\mathbb{P}(A_i) = 1$ for all $i \in \mathbb{N}$. Show that

$$\mathbb{P}\left(\bigcap_{i=1}^{\infty} A_i\right) = 1.$$

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Problems

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

- Let $\Omega = \{1, 2, \ldots, p\}$, where p is a prime number. Let $\mathscr{F} = 2^{\Omega}$, let

$$\mathbb{P}(A) = \frac{|A|}{p}, \qquad A \in \mathscr{F}.$$

  Produce an example of two events A and B which are independent of each other.

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Problems

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

- Suppose X is a discrete random variable defined with respect to $\mathscr{F}$ and having the PMF

$$p_X(100) = 0.65, \qquad p_X(-10) = 0.35.$$

Let $Y = \max\{X, 0\}$.

1. Show that Y is a discrete random variable with respect to $\mathscr{F}$.
2. Sketch the CDFs of X and Y.

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Problems

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

- Let X be a random variable with CDF F_X as follows:

$$F_X(x) = \begin{cases} 0, & x < 0, \\ 0.5, & 0 \leq x < 1, \\ \frac{x}{2}, & 1 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$$

Sketch the above CDF, and compute $\mathbb{P}(\{X \leq 0.8\})$, $\mathbb{P}(\{X \geq 0.25\})$, $\mathbb{P}(\{X = 0\})$, $\mathbb{P}(\{X \in (1, 3)\})$.