Name:
Roll Number:

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

CS6660: Mathematical Foundations of Data Science (Probability)

Quiz 2

Date: 14 September 2024

Question	1	2(a)	2(b)	Total
Marks Scored				

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

1. **(1 Mark)**

   Suppose that two batteries are chosen simultaneously and uniformly at random from the following group of 12 batteries : 3 new, 4 used (yet working), 5 defective. You may assume that all batteries within a particular group are identical.
   Let X be the number of **new** batteries chosen, and let Y be the number of **defective** batteries chosen.
   If the value of $\mathbb{P}(\{X \geq Y\})$ is expressed as $\frac{\alpha}{\beta}$, determine the value of $\frac{\alpha+\beta}{\beta-\alpha}$. Give your answer up to 1 decimal place only.

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2. Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.
Assume that all random variables appearing below are defined with respect to $\mathscr{F}$.

Numbers from $[0, 1]$ are picked uniformly, independently, and sequentially over time.
Let X_n denote the number picked at time n, where $n \in \{0, 1, 2, \ldots\}$. Let N be the random variable defined as

$$N = \min\{n \geq 1 : X_n > X_0\}.$$

That is, N denotes the first time index n at which the value of X_n exceeds the value of X_0.

(a) **(3 Marks)**
For any fixed $n \in \mathbb{N}$, determine $\mathbb{P}(\{N = n\})$.
Hint: The event that $N = n$ is identical to the event that $X_1 \leq X_0, \ldots, X_{n-1} \leq X_0,\ X_n > X_0$.

(b) **(1 Mark)**
Compute $\mathbb{P}(\{N > 2\})$.