Odd Arm Identification in Multi-armed Bandits with Markov Observations

EECS Research Students Symposium - 2020

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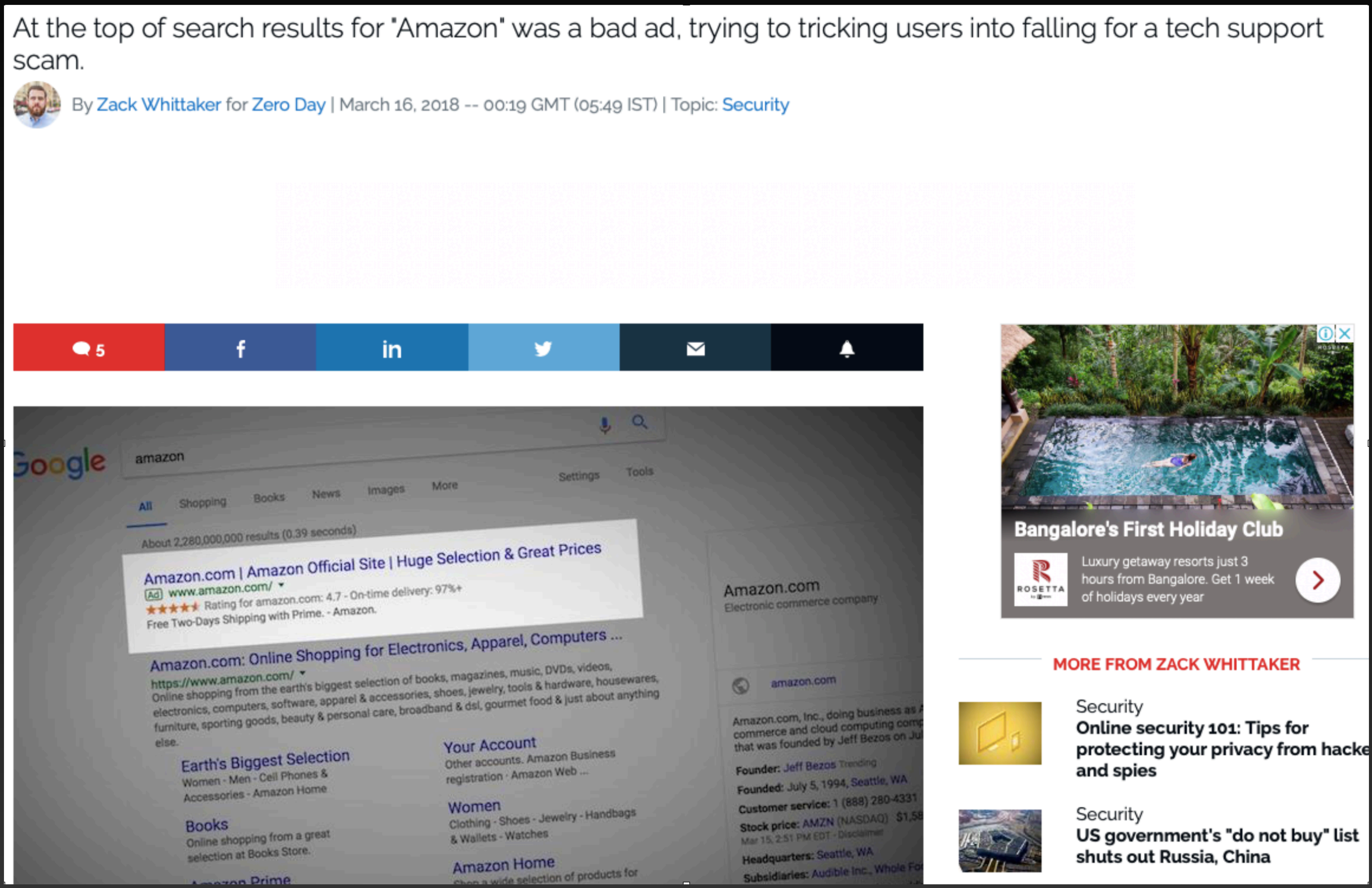
Department of Electrical Communication Engineering

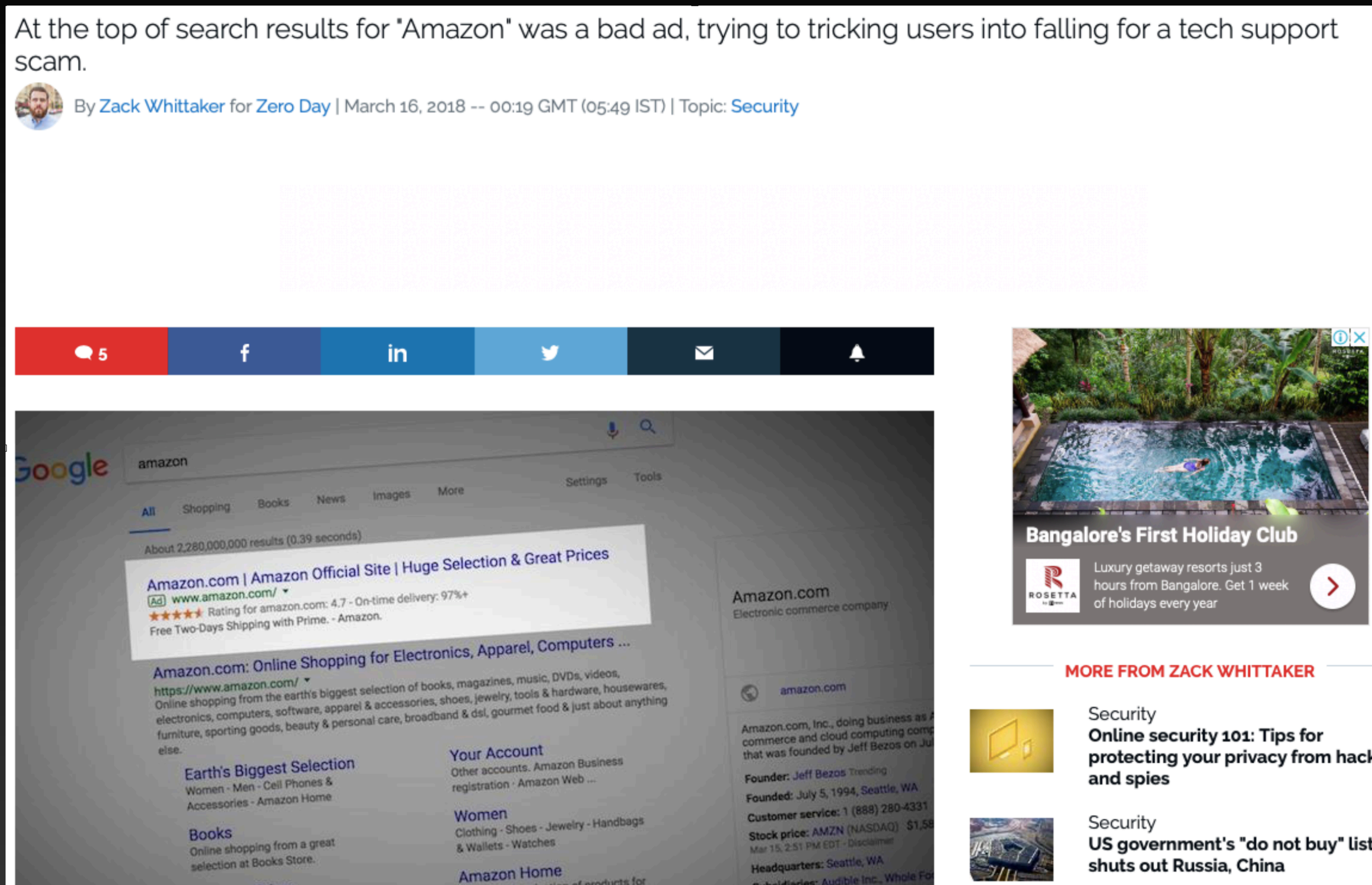
This work was supported by

Identification of a Malicious Ad



An online post bringing to light a malicious ad on Google.

Identification of a Malicious Ad



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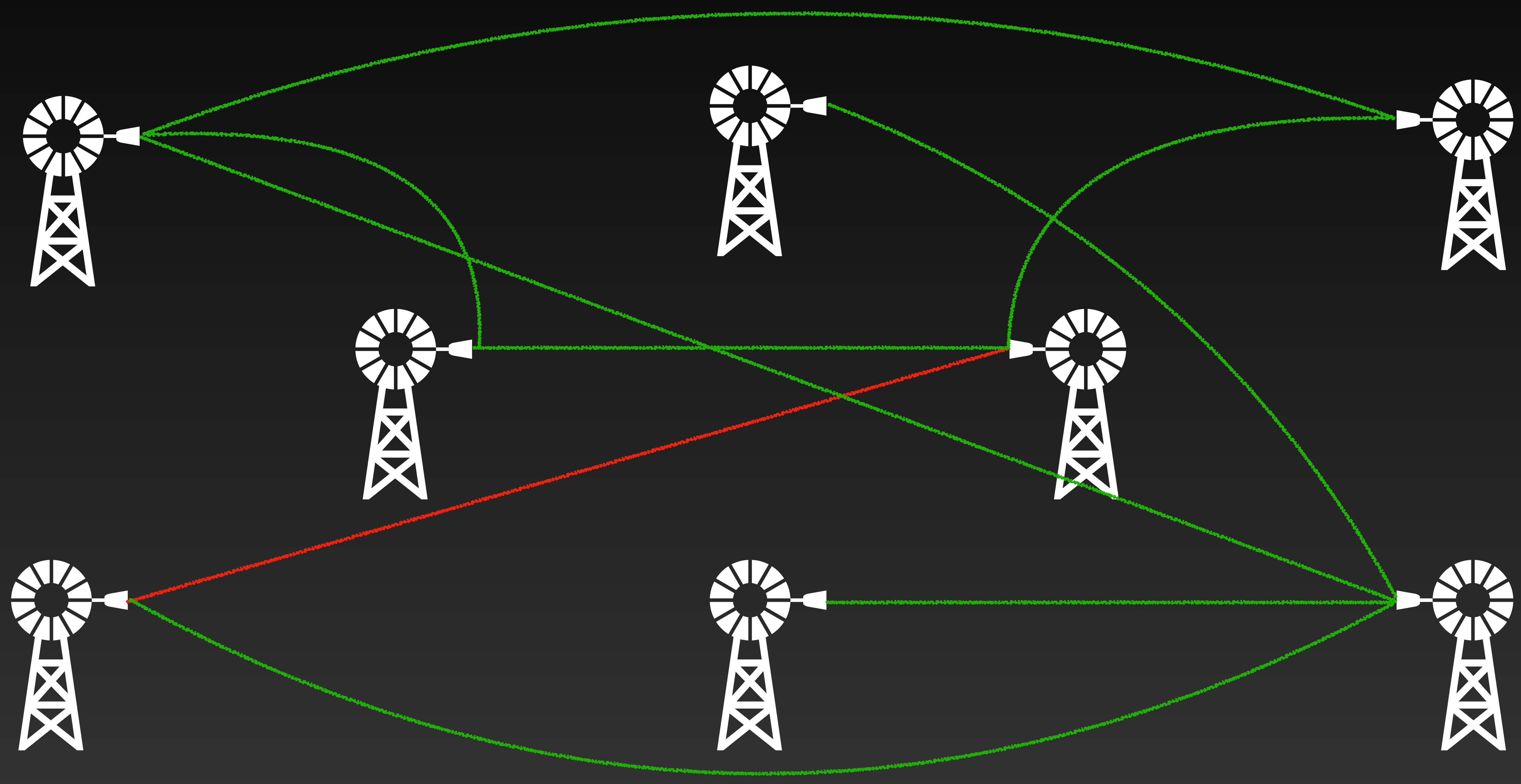
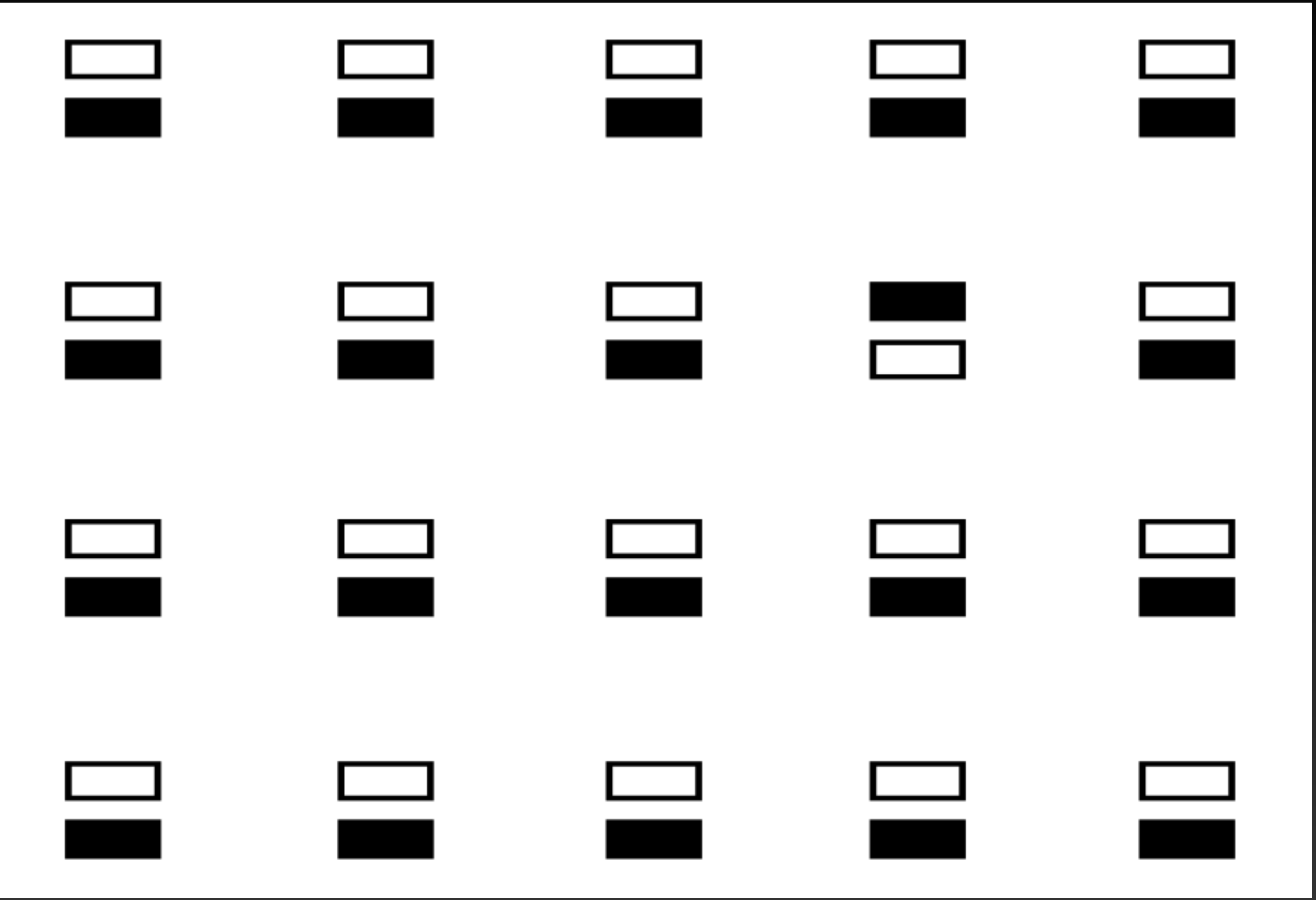
Identification of a Line in Outage in a Power Grid



Line working fine

Line in outage

Odd Image Identification



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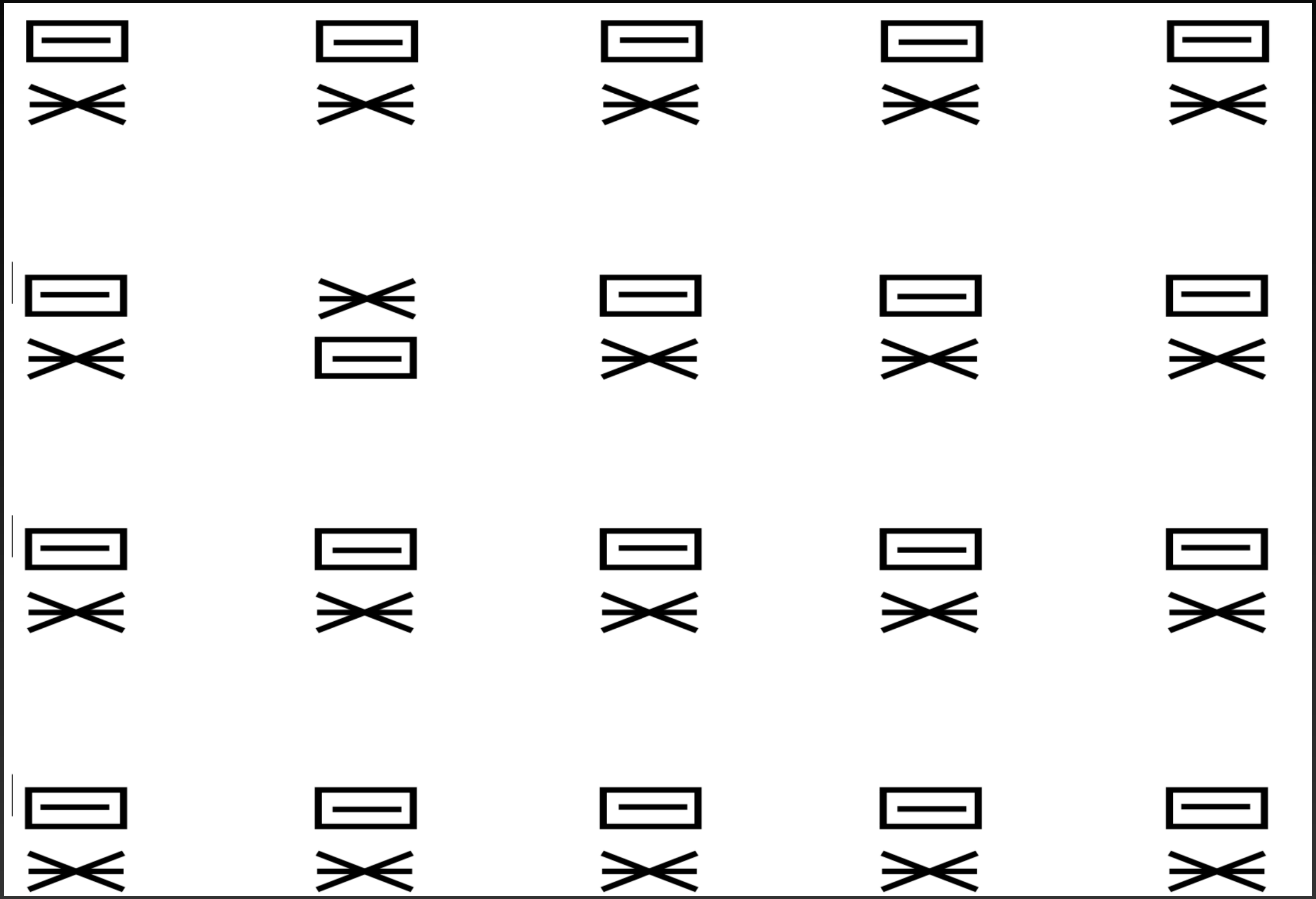
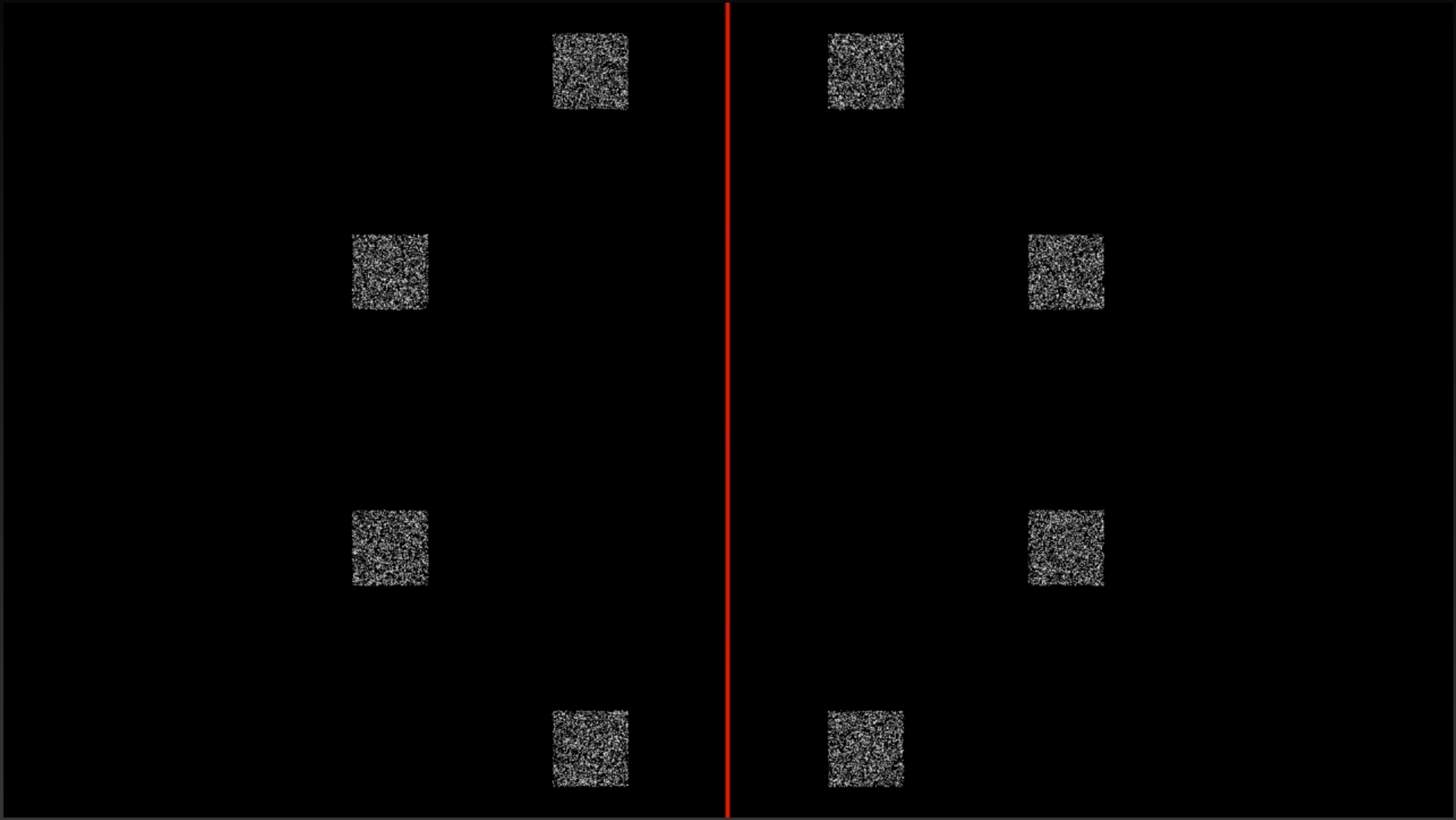


Identify the ‘odd’ image.

Odd Movie Identification



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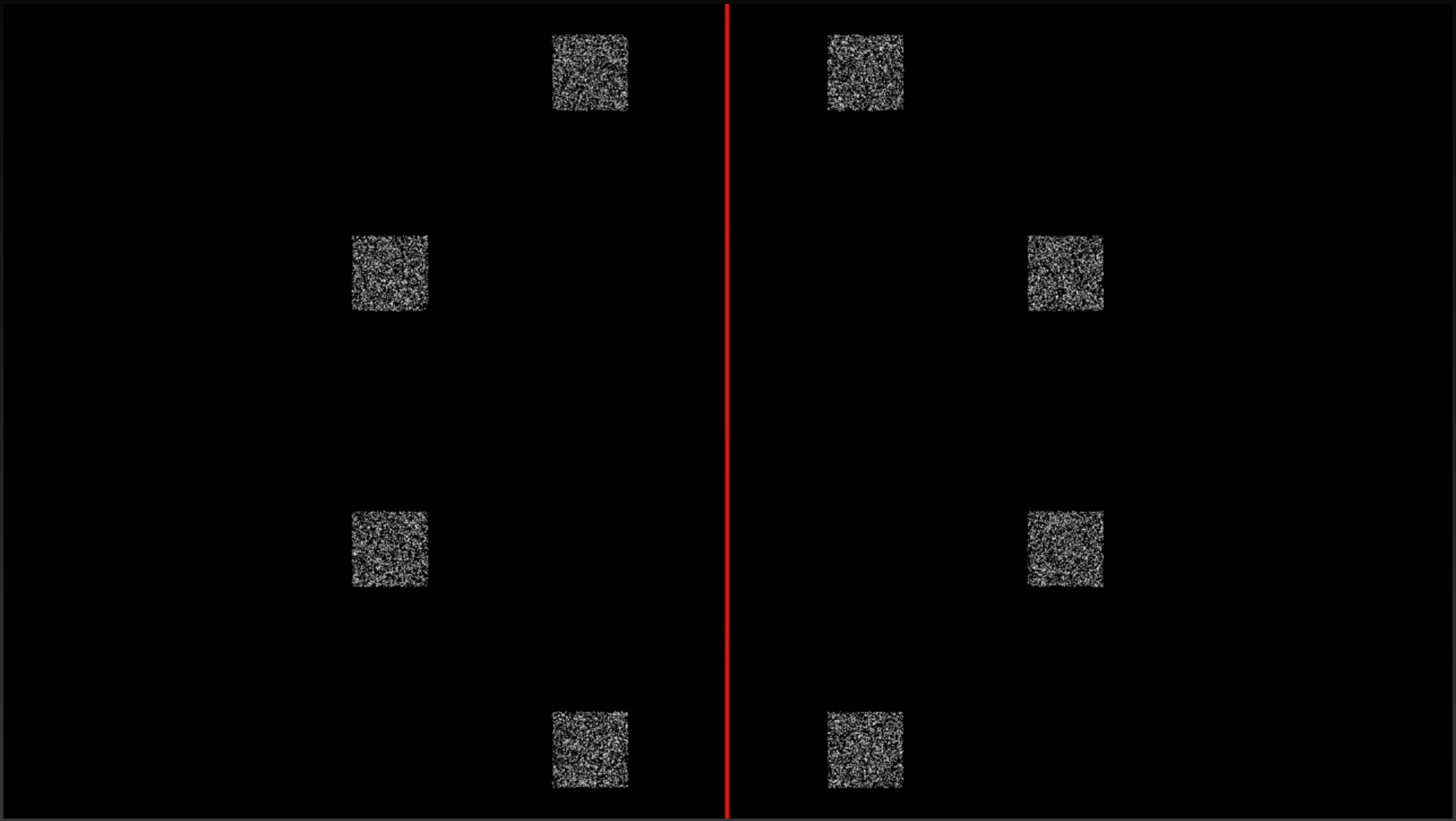
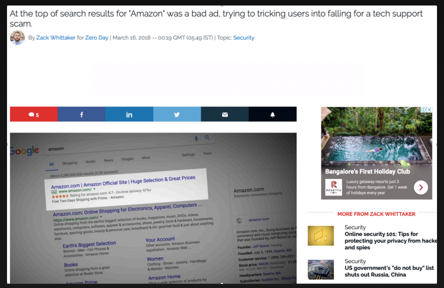
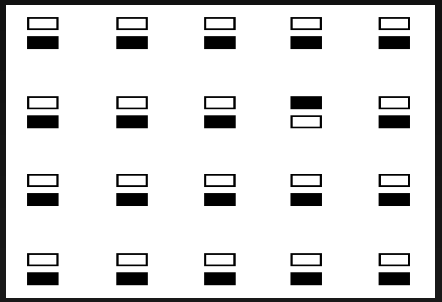
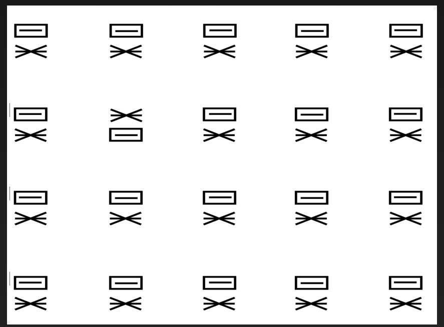
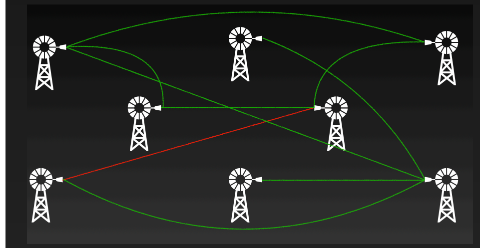


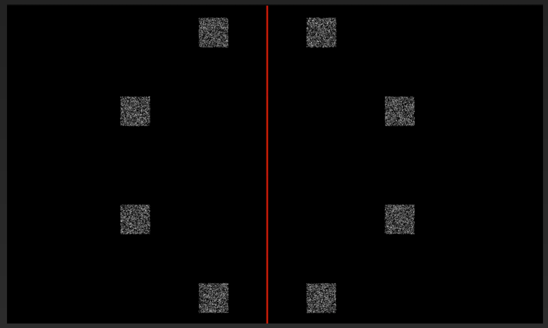
A Closer Look at the Examples





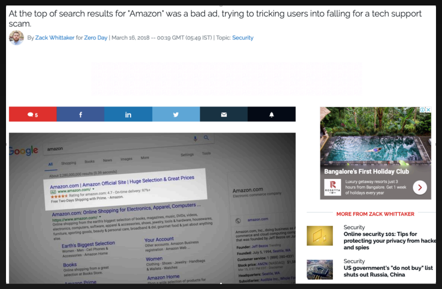


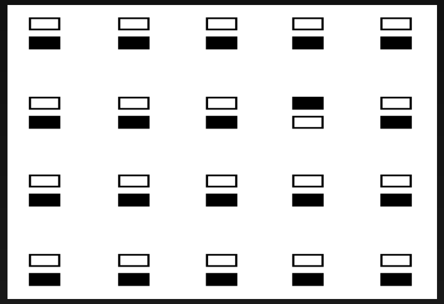


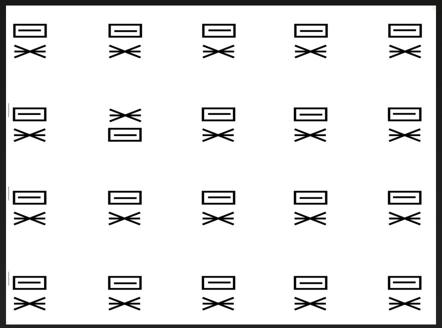


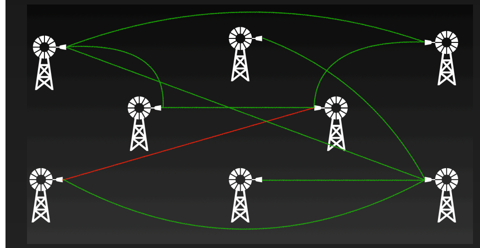
A Closer Look at the Examples

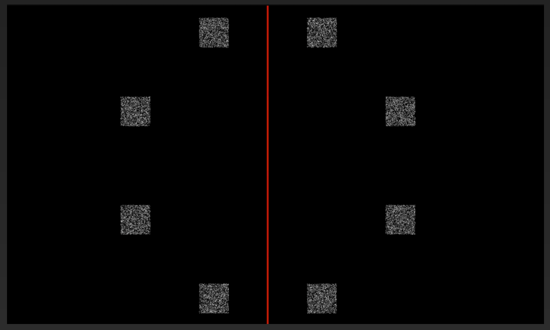
Anomalous or “odd” behaviour











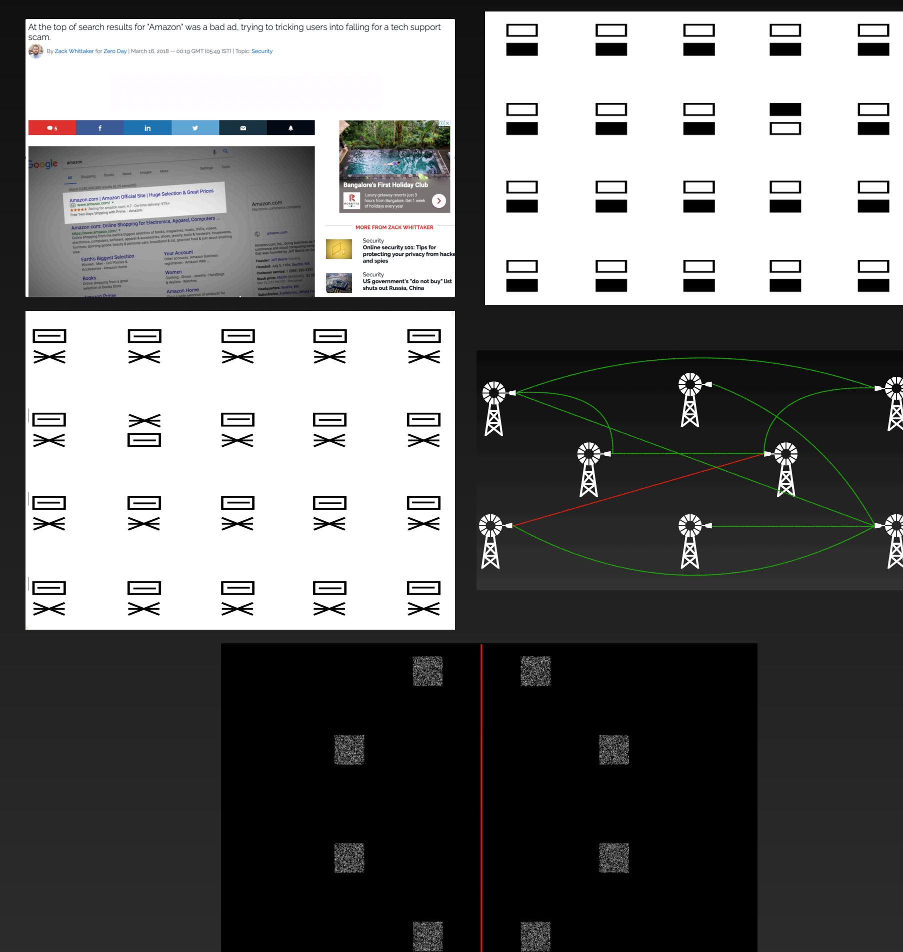
A Closer Look at the Examples

Anomalous or “odd” behaviour

Two quantities of interest

Time to identify the anomaly

Error in reporting the anomaly correctly



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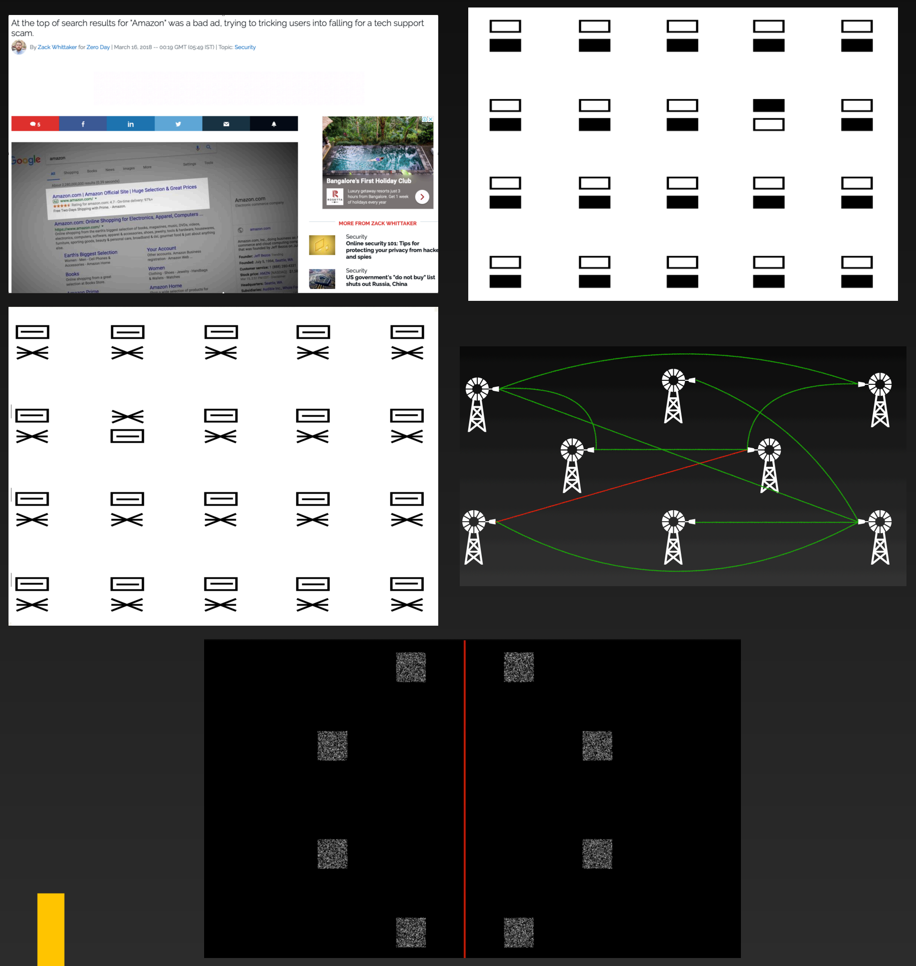

A study of the prior works reveals that

$$\text{Time to identify the anomaly} \approx \left(\log \frac{1}{\text{Error in reporting the anomaly correctly}} \right) \frac{1}{D^*}$$

D^* captures the hardness of the problem

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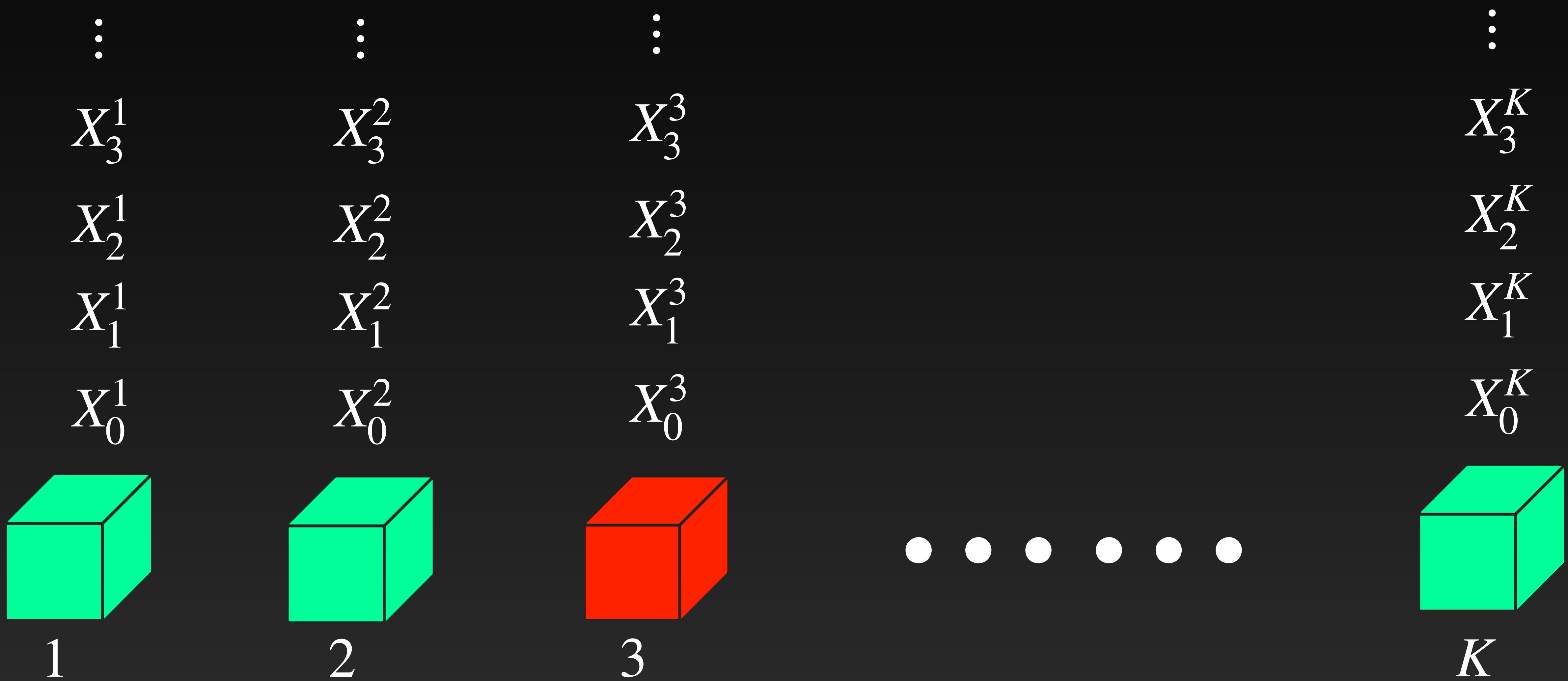
Multi-armed Bandits

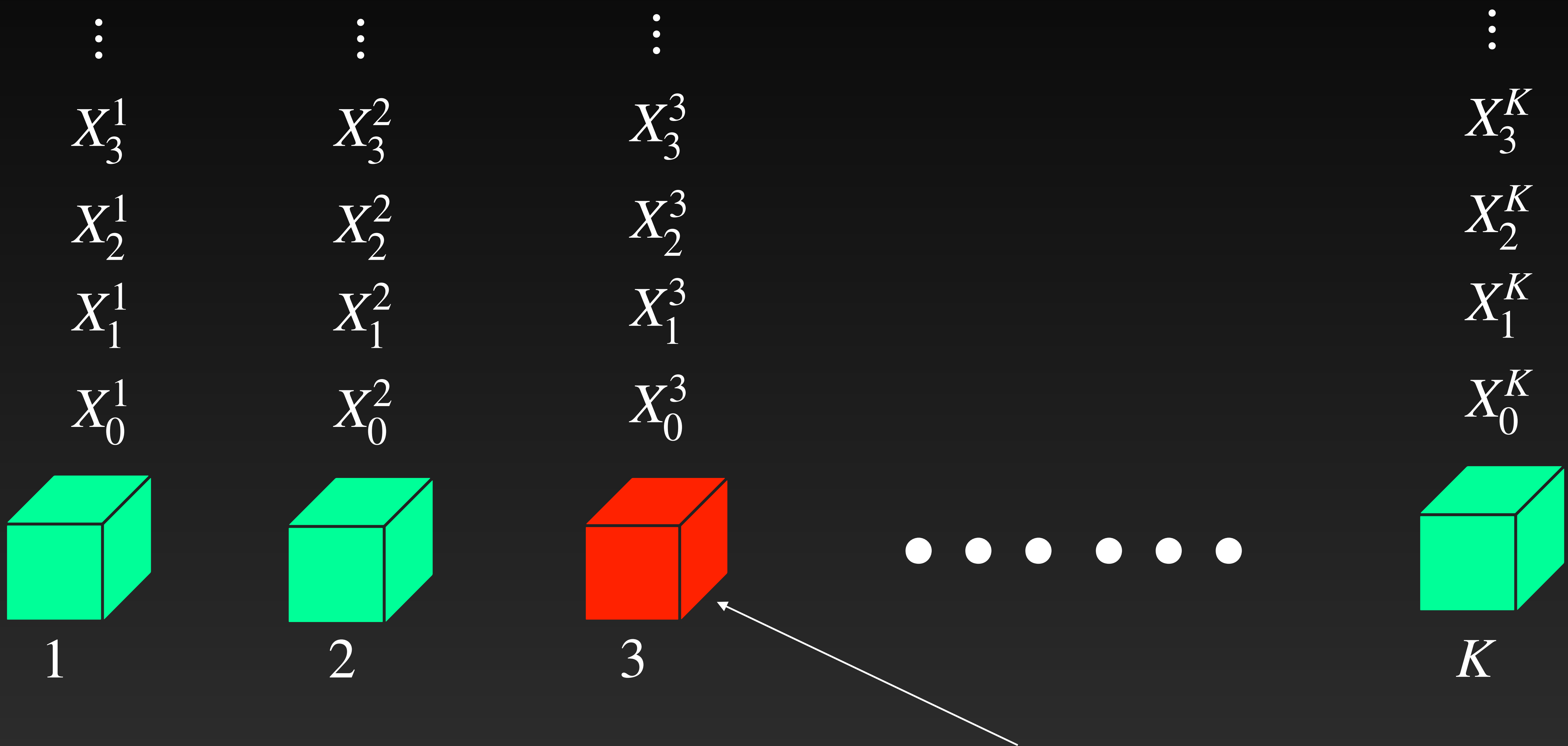


X_t^a

a ← Arm index

t ← Time (starts from 0)

Multi-armed Bandits



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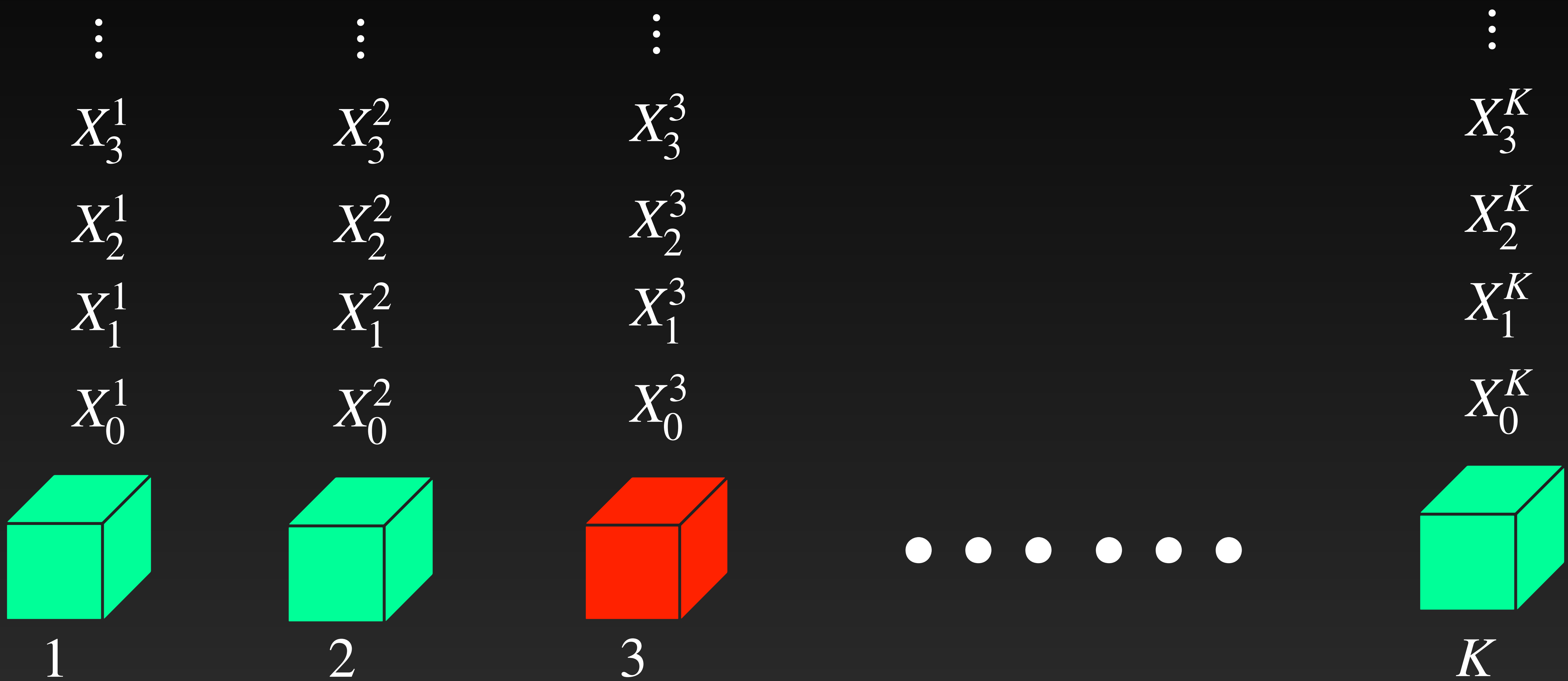

Prior works: $\{X_t^a : t \geq 0\}$ is an iid process

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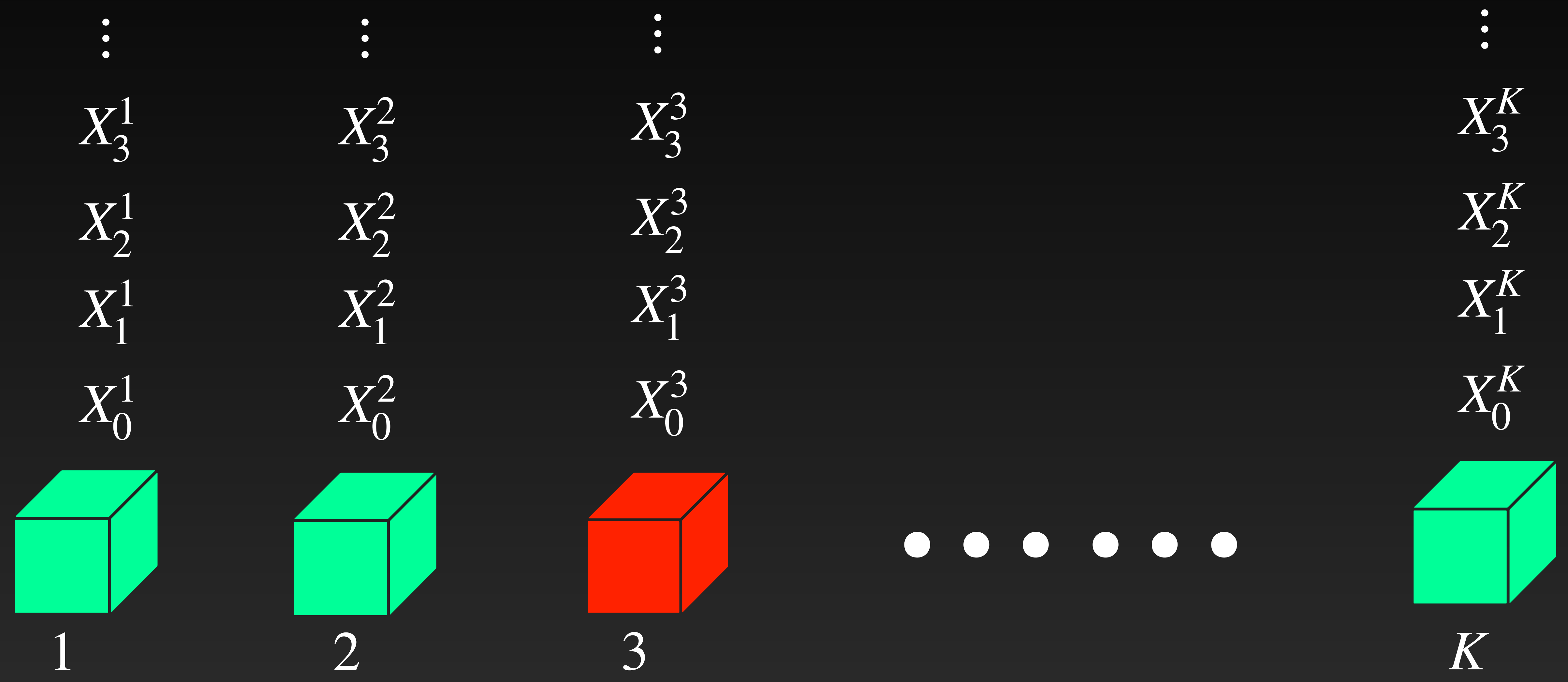
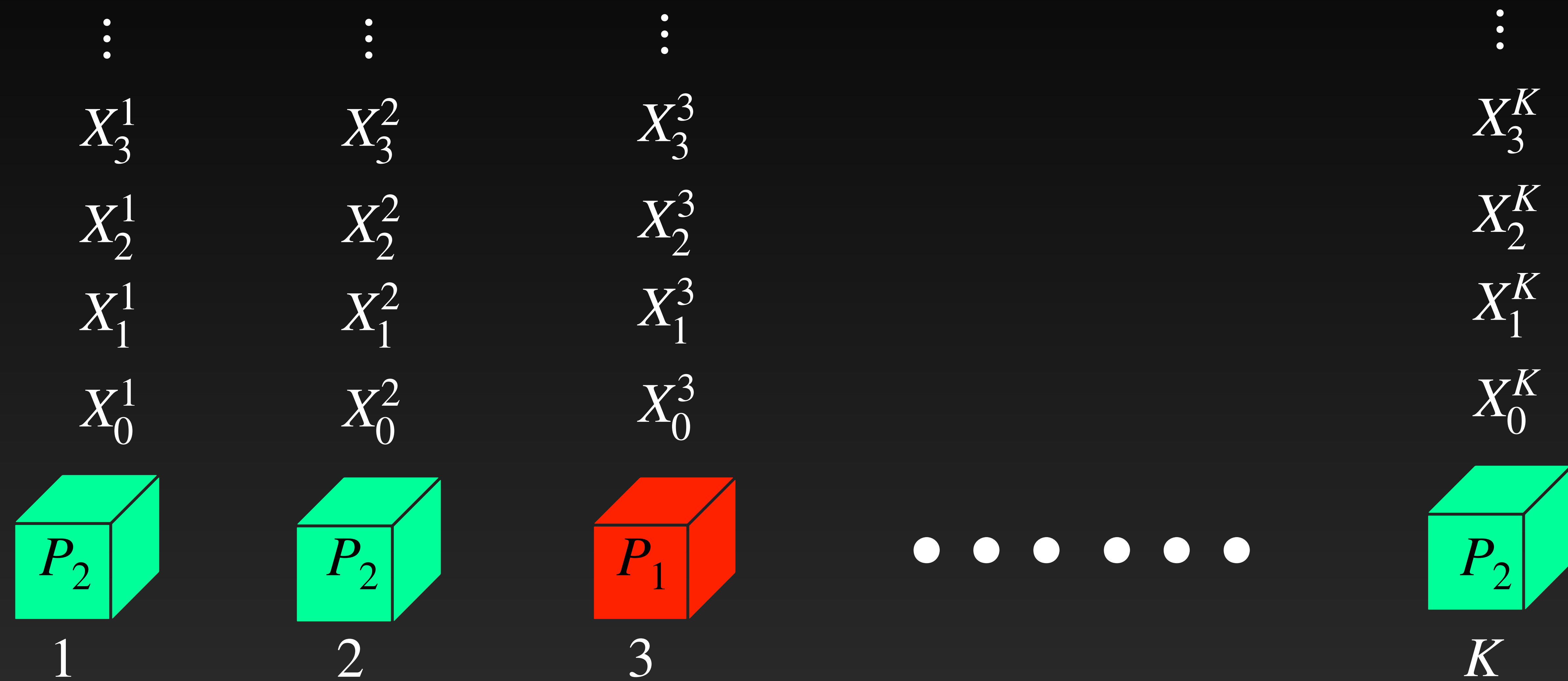


Prior works: $\{X_t^a : t \geq 0\}$ is an iid process

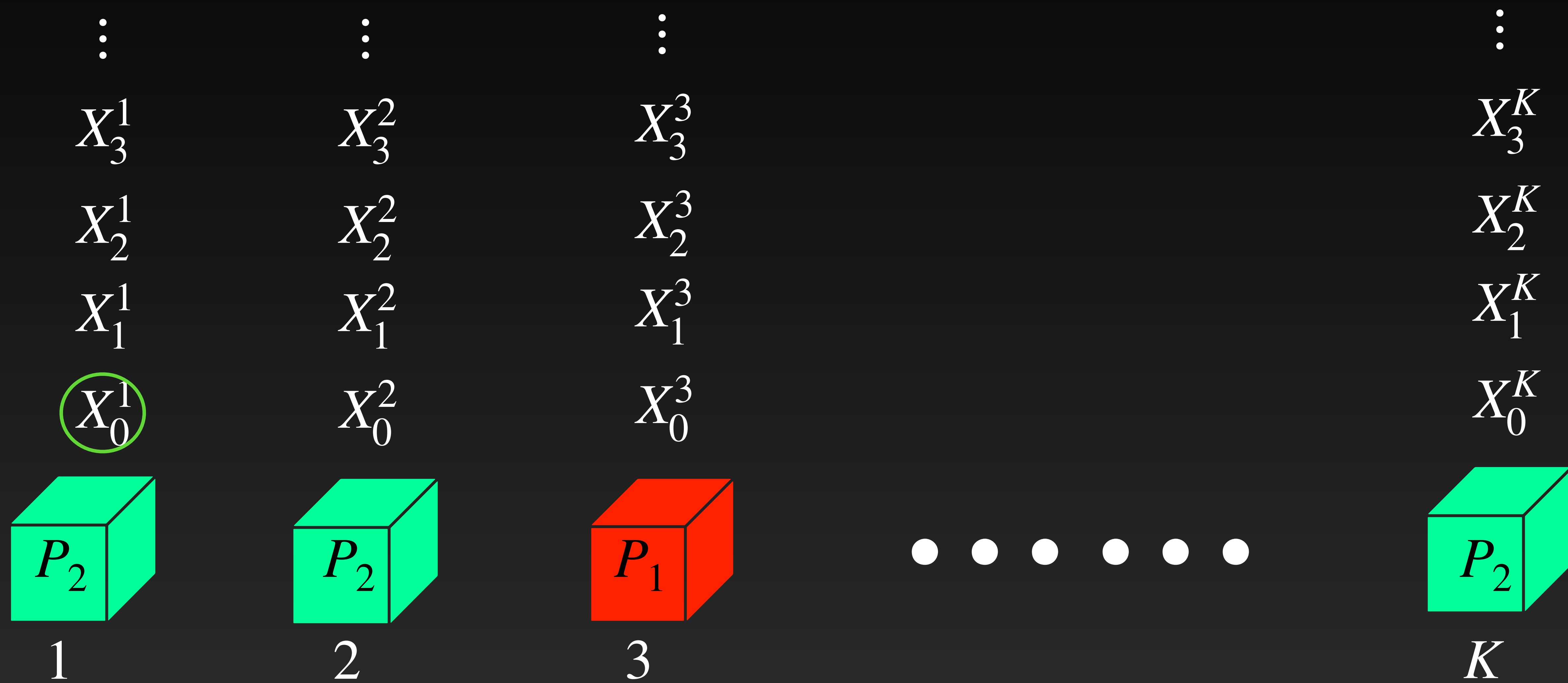
Our work: $\{X_t^a : t \geq 0\}$ is a Markov process

Markov Observations: Rested Case



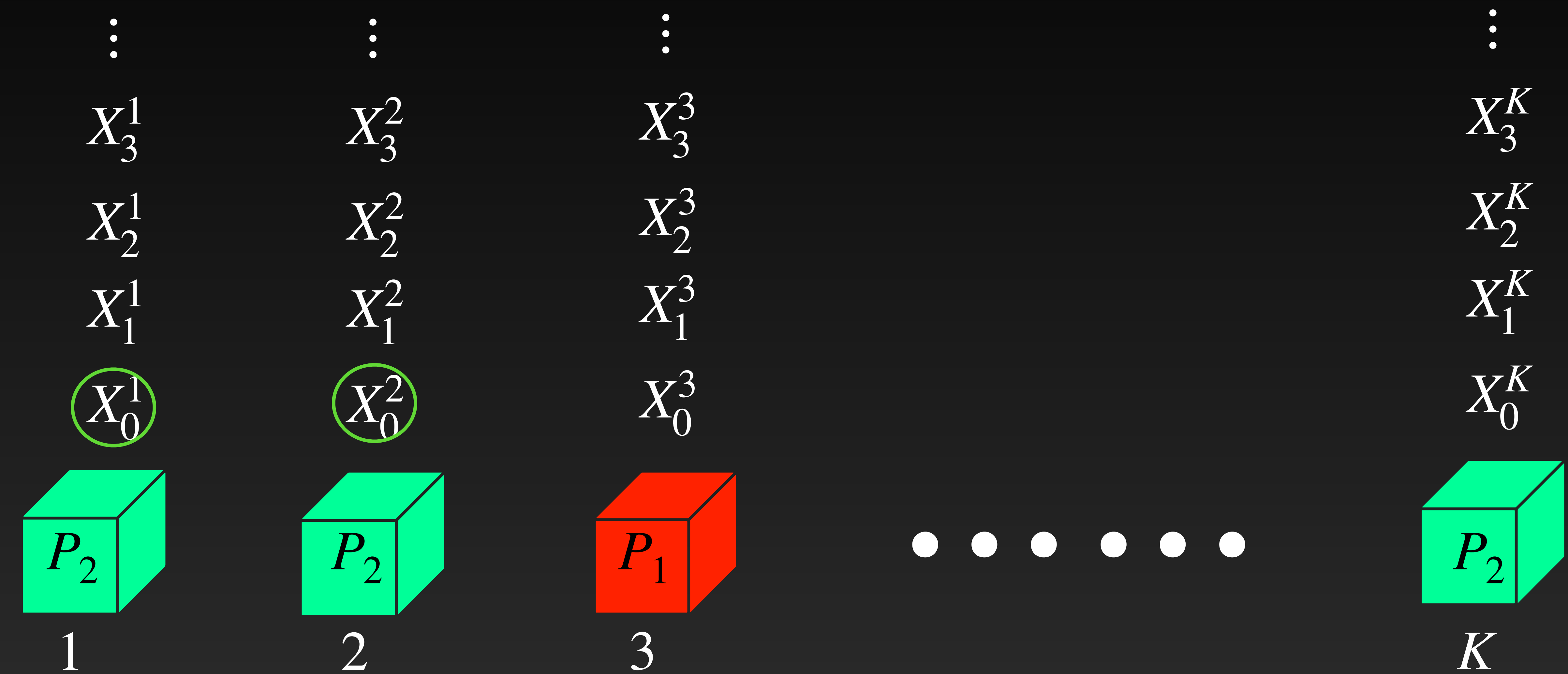
t	**0**	**1**	**2**	**3**	**4**	**5**
Arm	1	2	3	K	3	K
Obs						

Markov Observations: Rested Case



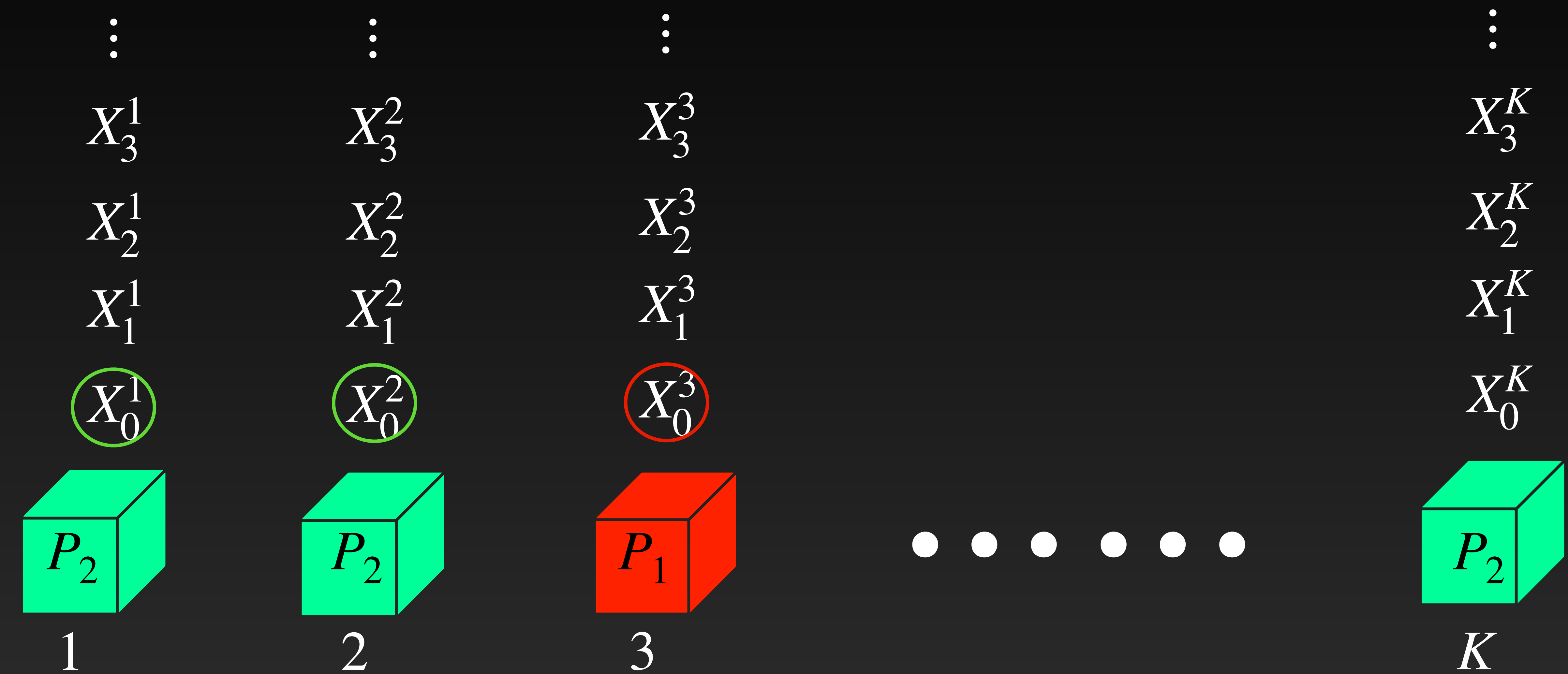
t	0	1	2	3	4	5
Arm	1	2	3	K	3	K
Obs	X_0^1					

Markov Observations: Rested Case



t	0	1	2	3	4	5
Arm	1	2	3	K	3	K
Obs	X_0^1	X_0^2				

Markov Observations: Rested Case



t	**0**	**1**	**2**	**3**	**4**	**5**
Arm	1	2	3	K	3	K
Obs	X_0^1	X_0^2	X_0^3			

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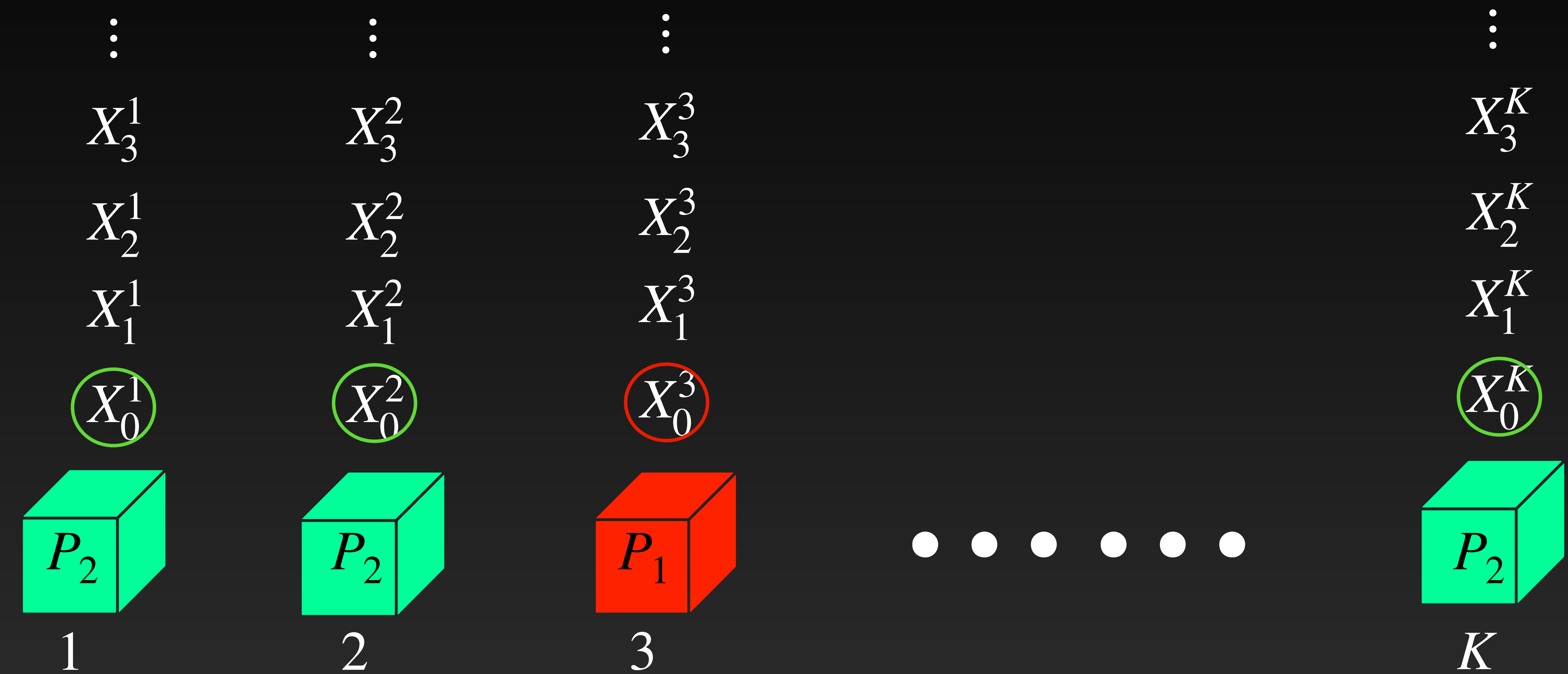
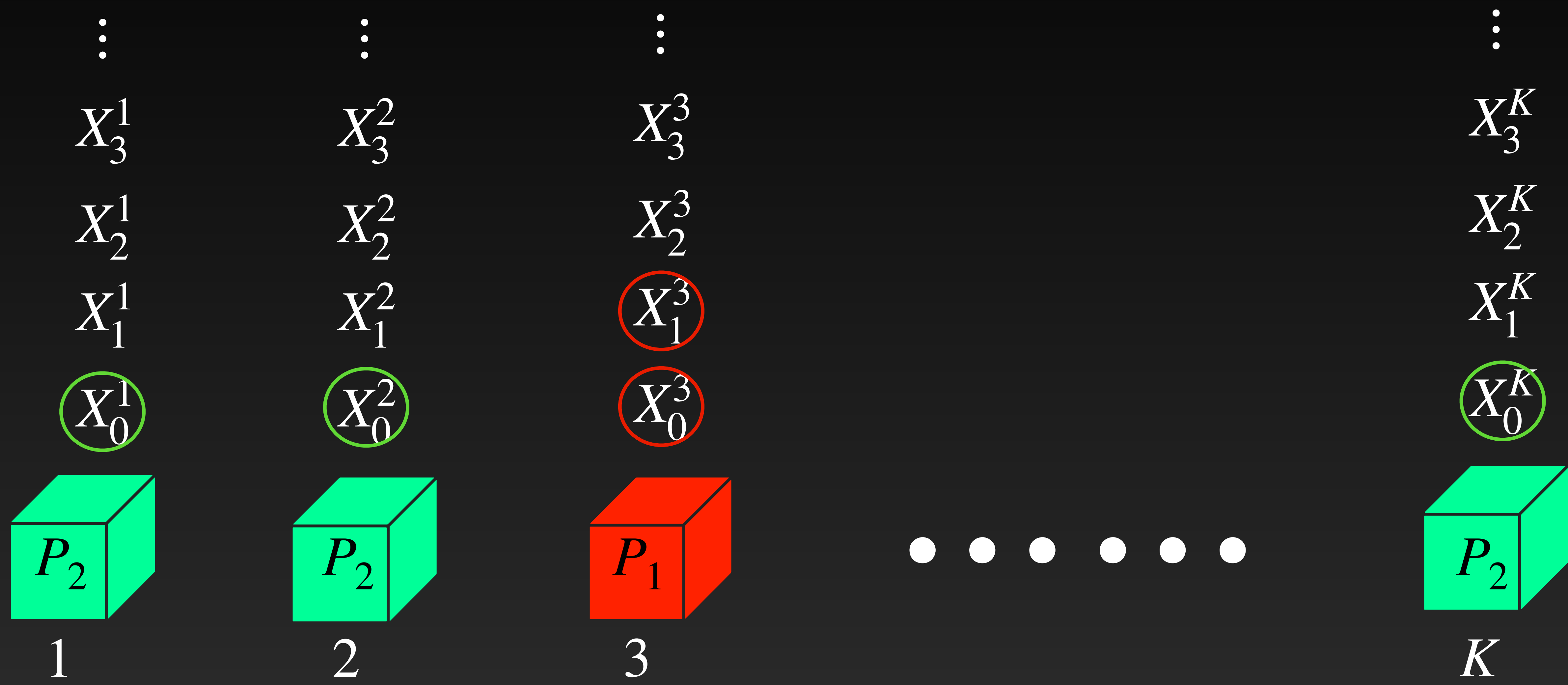


t	0	1	2	3	4	5
Arm	1	2	3	K	3	K
Obs	X_0^1	X_0^2	X_0^3	X_0^K		

Markov Observations: Rested Case



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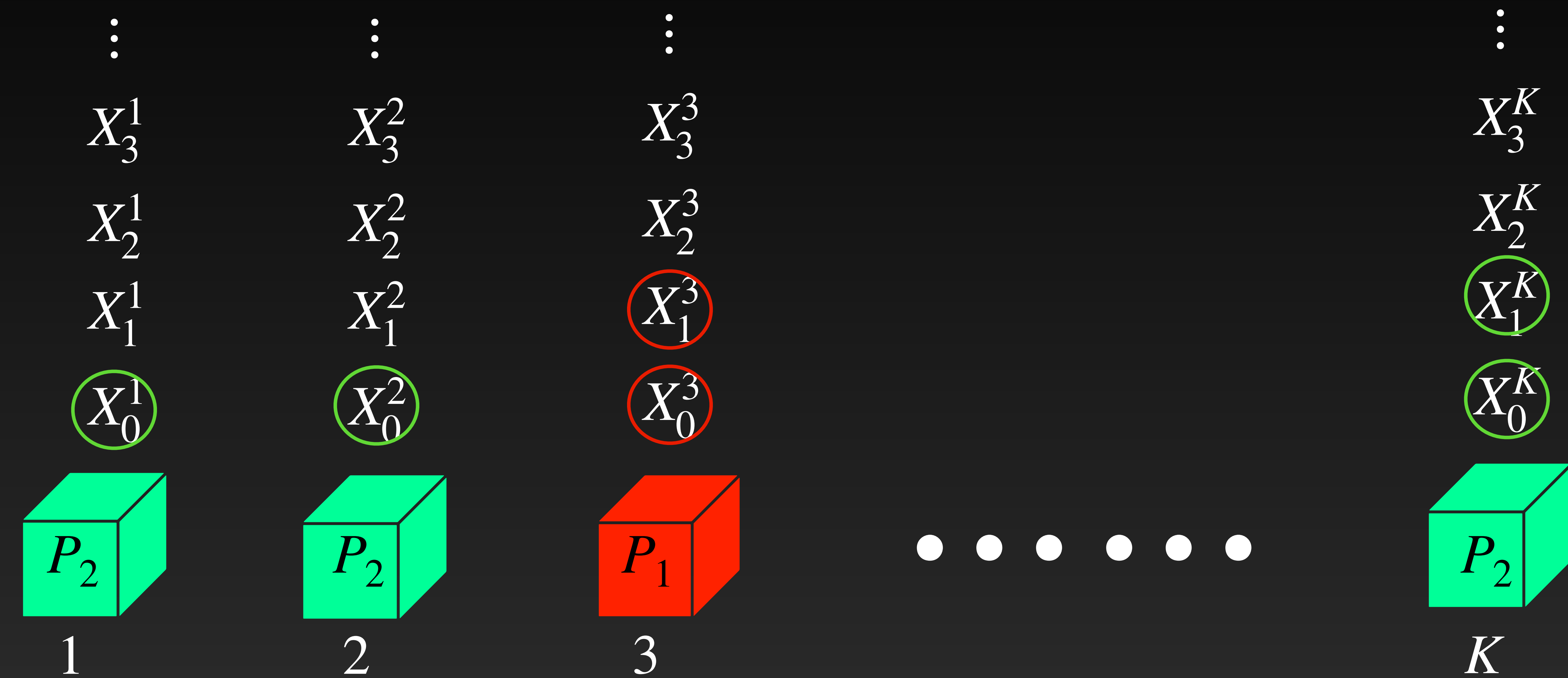
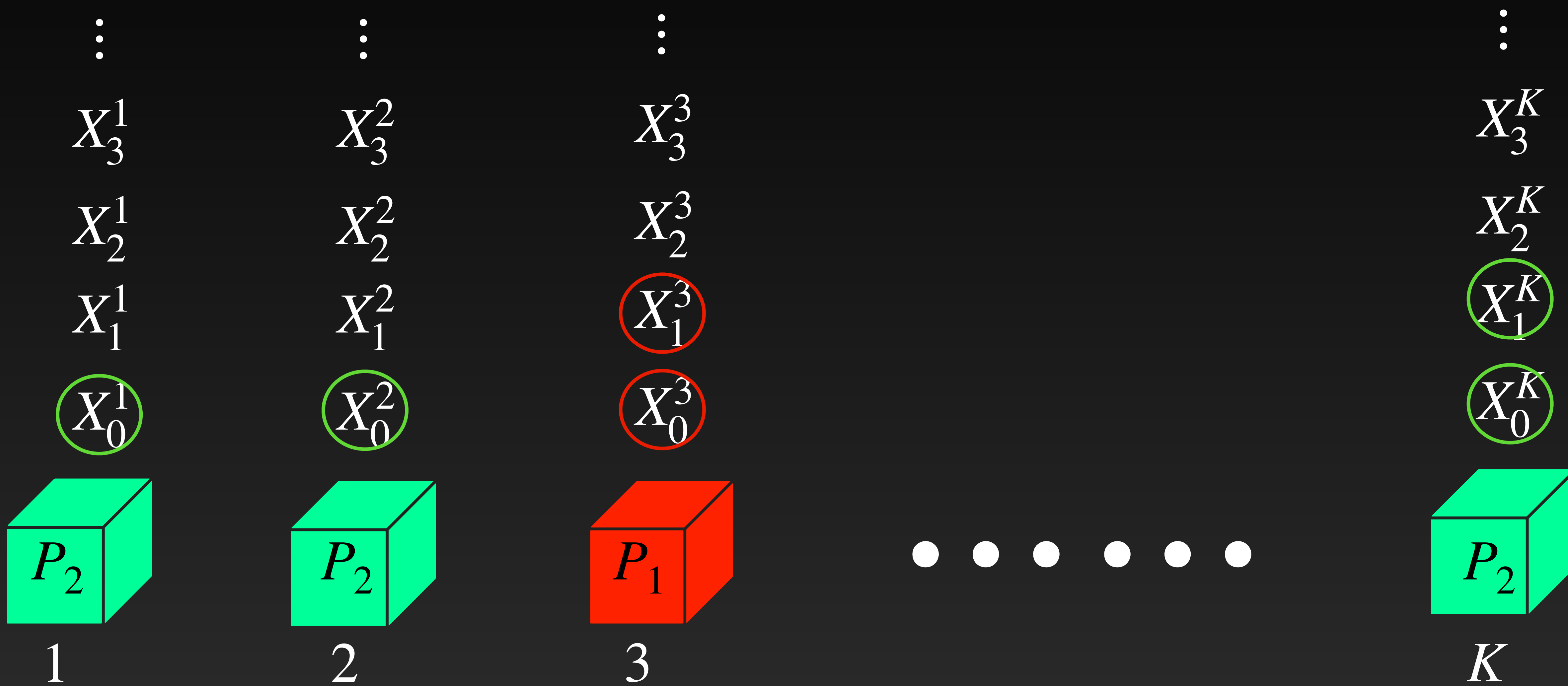
Markov Observations: Rested Case



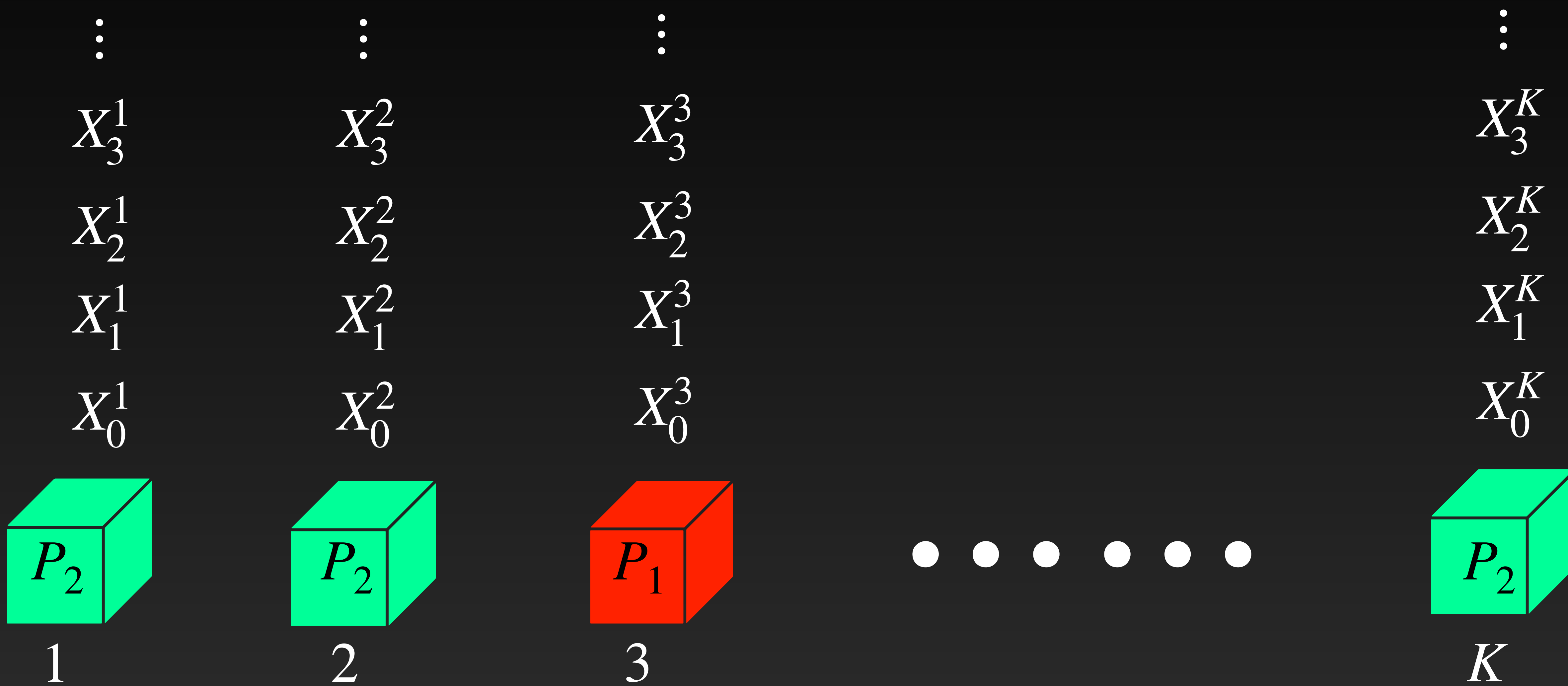
t	0	1	2	3	4	5
Arm	1	2	3	K	3	K
Obs	X_0^1	X_0^2	X_0^3	X_0^K	X_1^3	X_1^K

Markov Observations: Rested Case



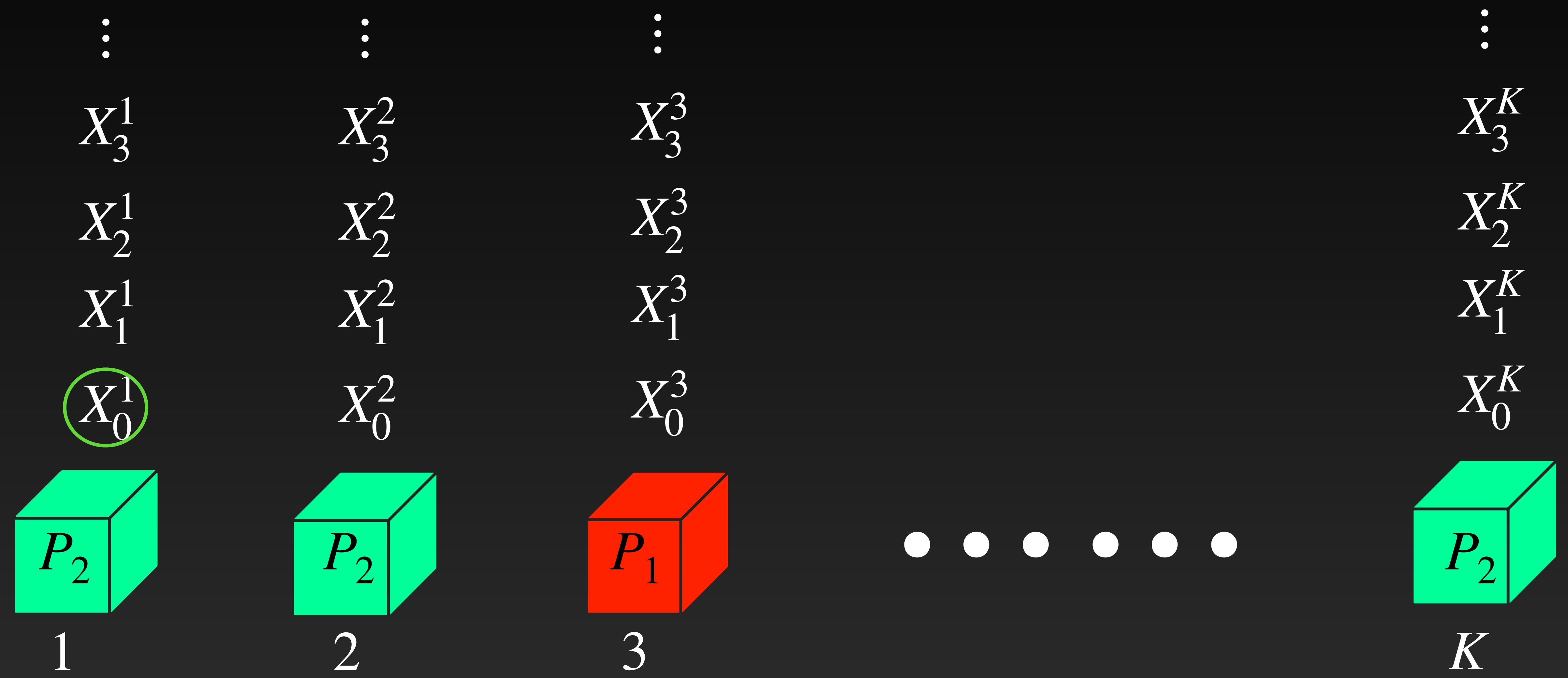
t	**0**	**1**	**2**	**3**	**4**	**5**
Arm	1	2	3	K	3	K
Obs	X_0^1	X_0^2	X_0^3	X_0^K	X_1^3	X_1^K

Markov Observations: Restless Case



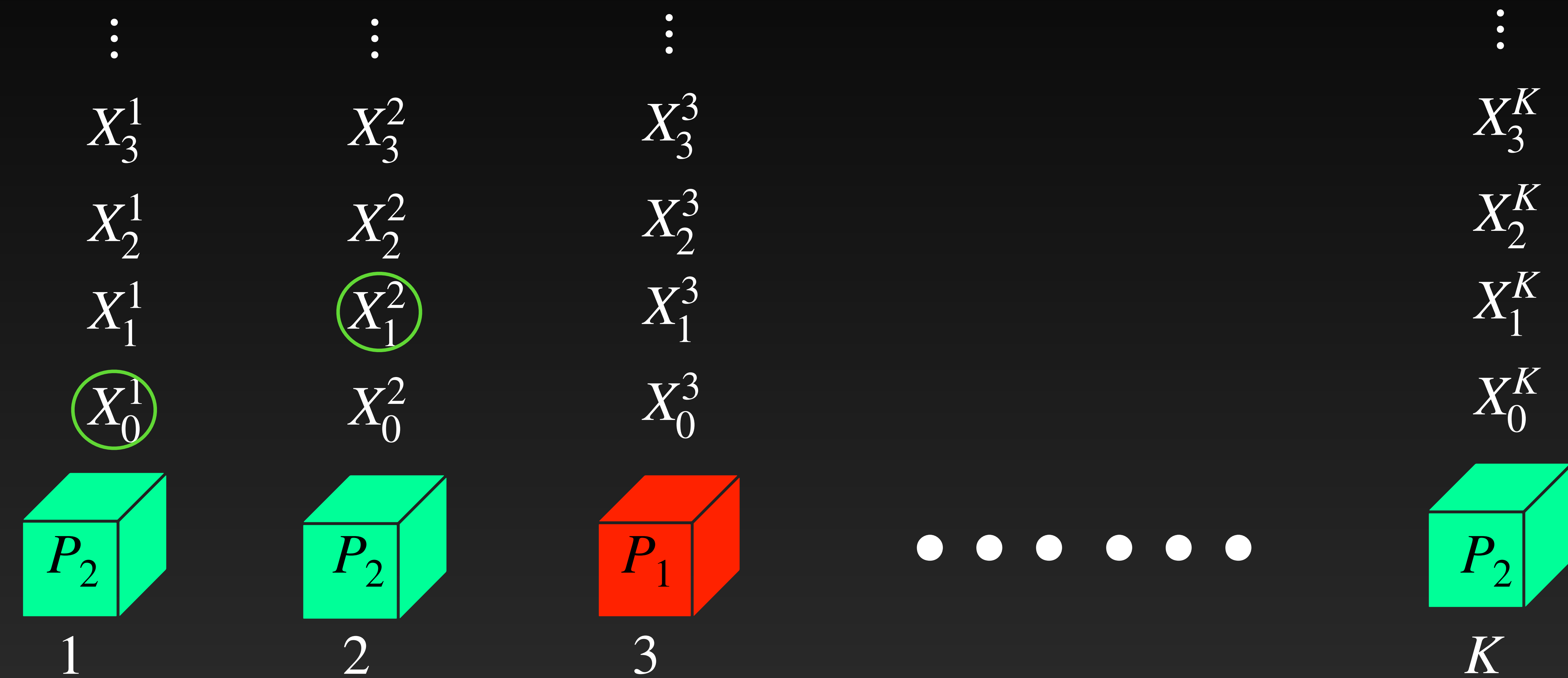
t	**0**	**1**	**2**	**3**	**4**	**5**
Arm	1	2	3	K	3	K
Obs						

Markov Observations: Restless Case



t	0	1	2	3	4	5
Arm	1	2	3	K	3	K
Obs	X_0^1					

Markov Observations: Restless Case



t	0	1	2	3	4	5
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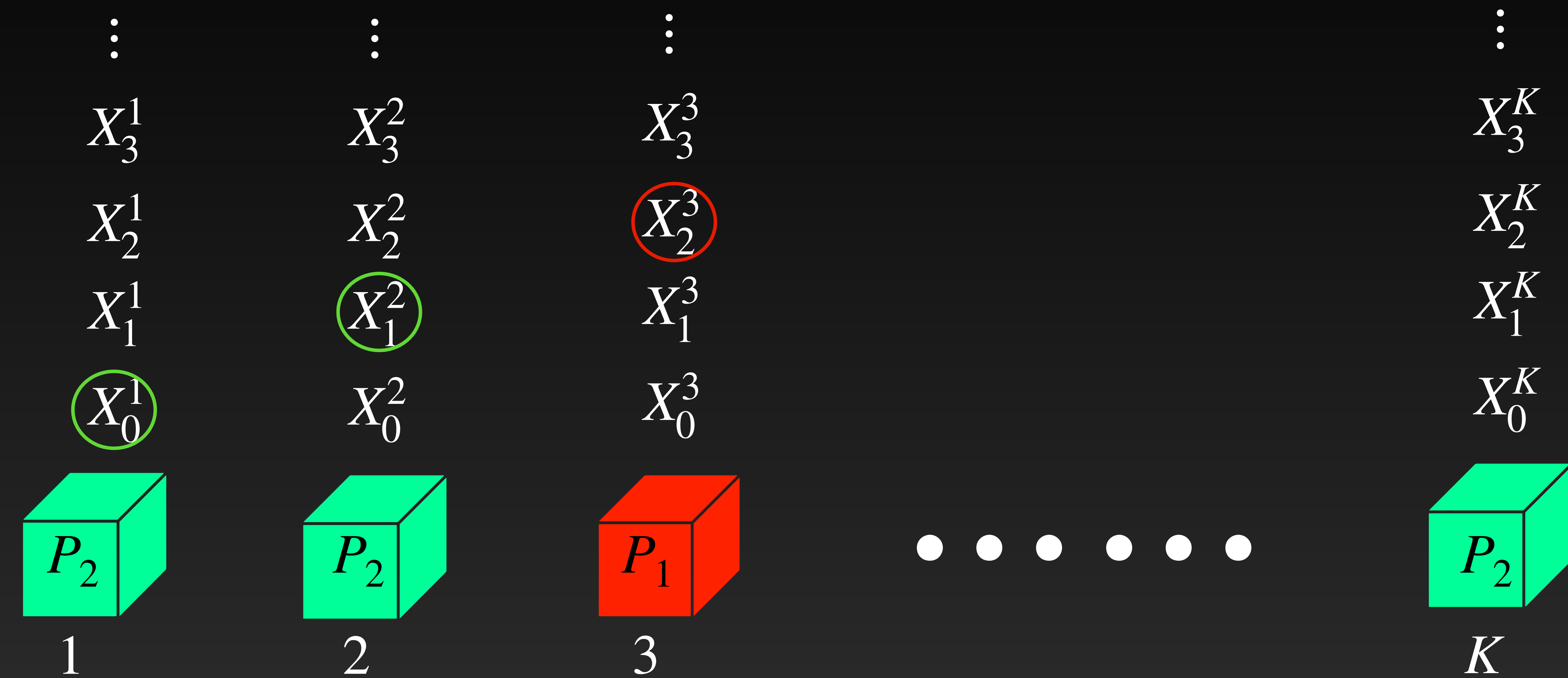
Obs	X_0^1	X_1^2				

Markov Observations: Restless Case



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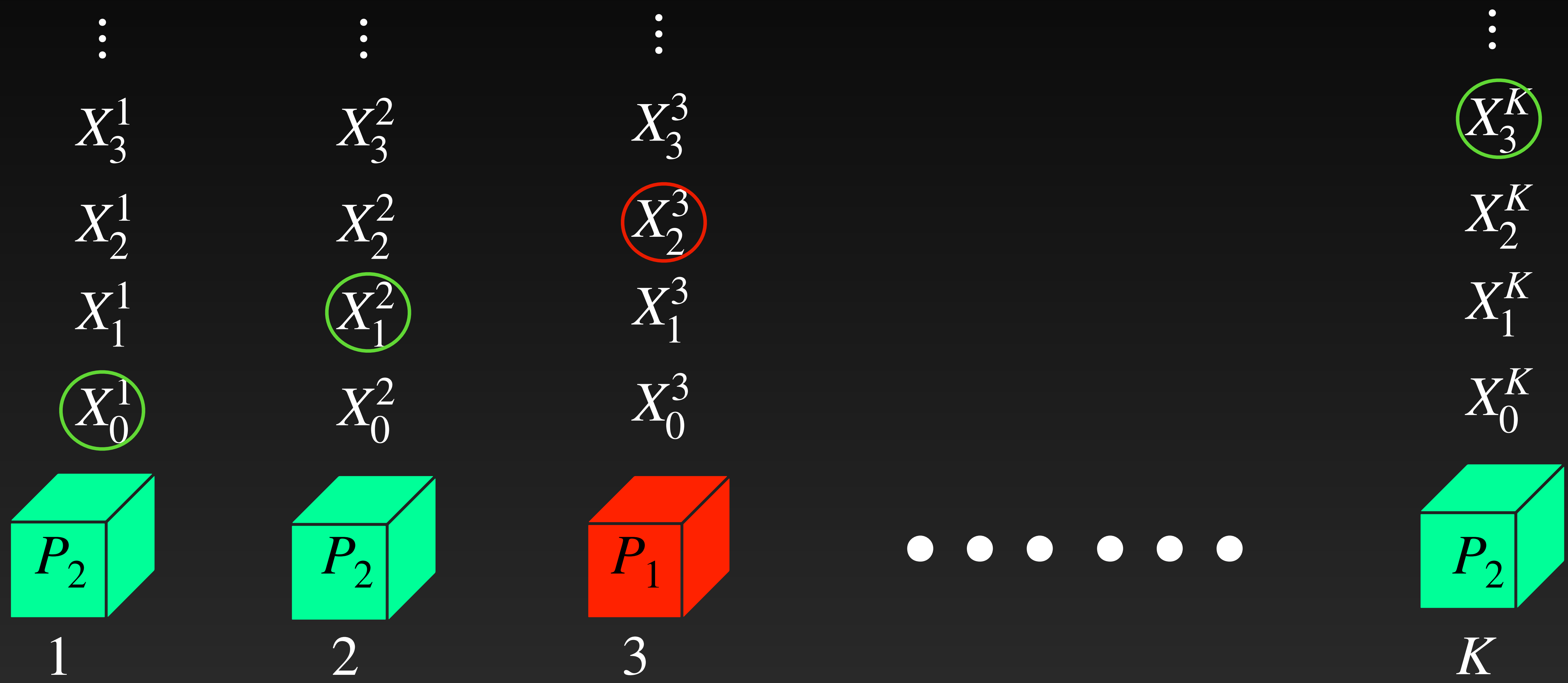

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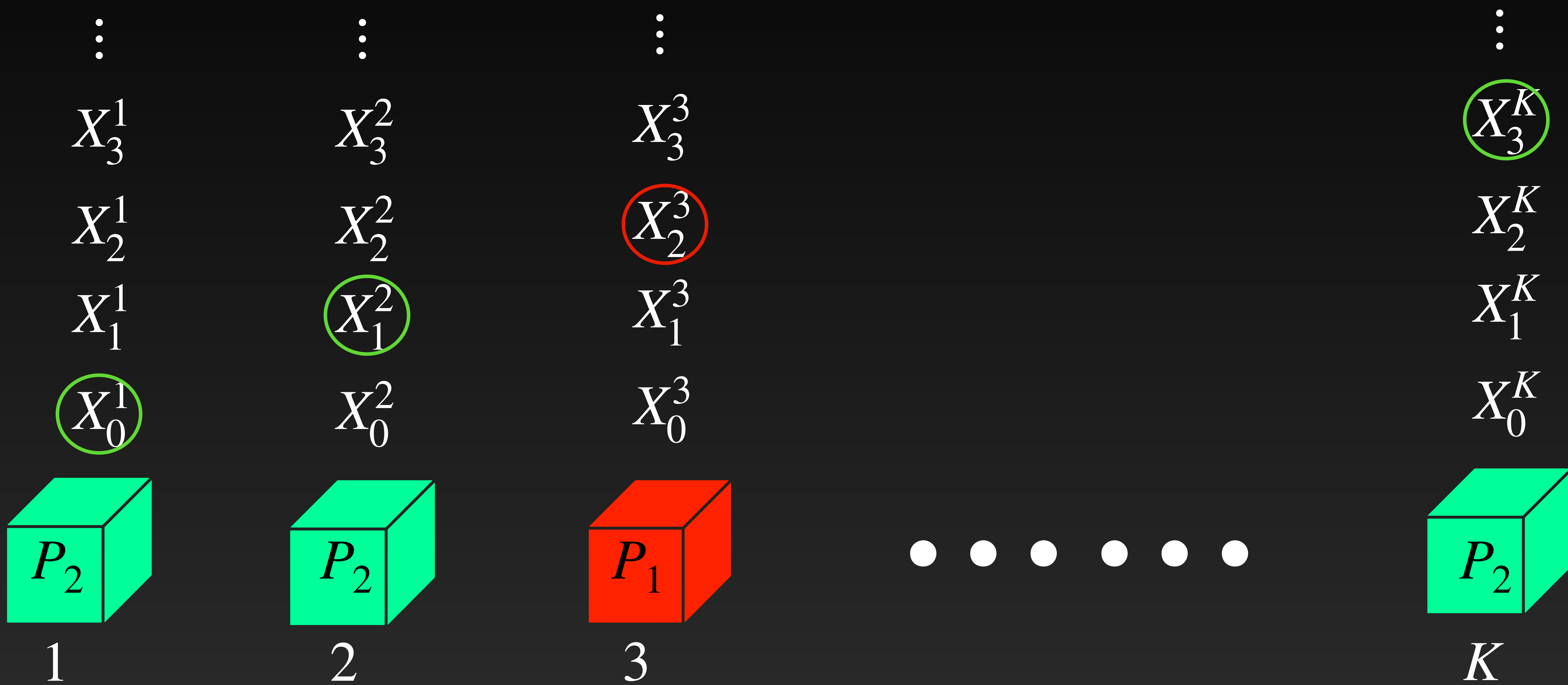

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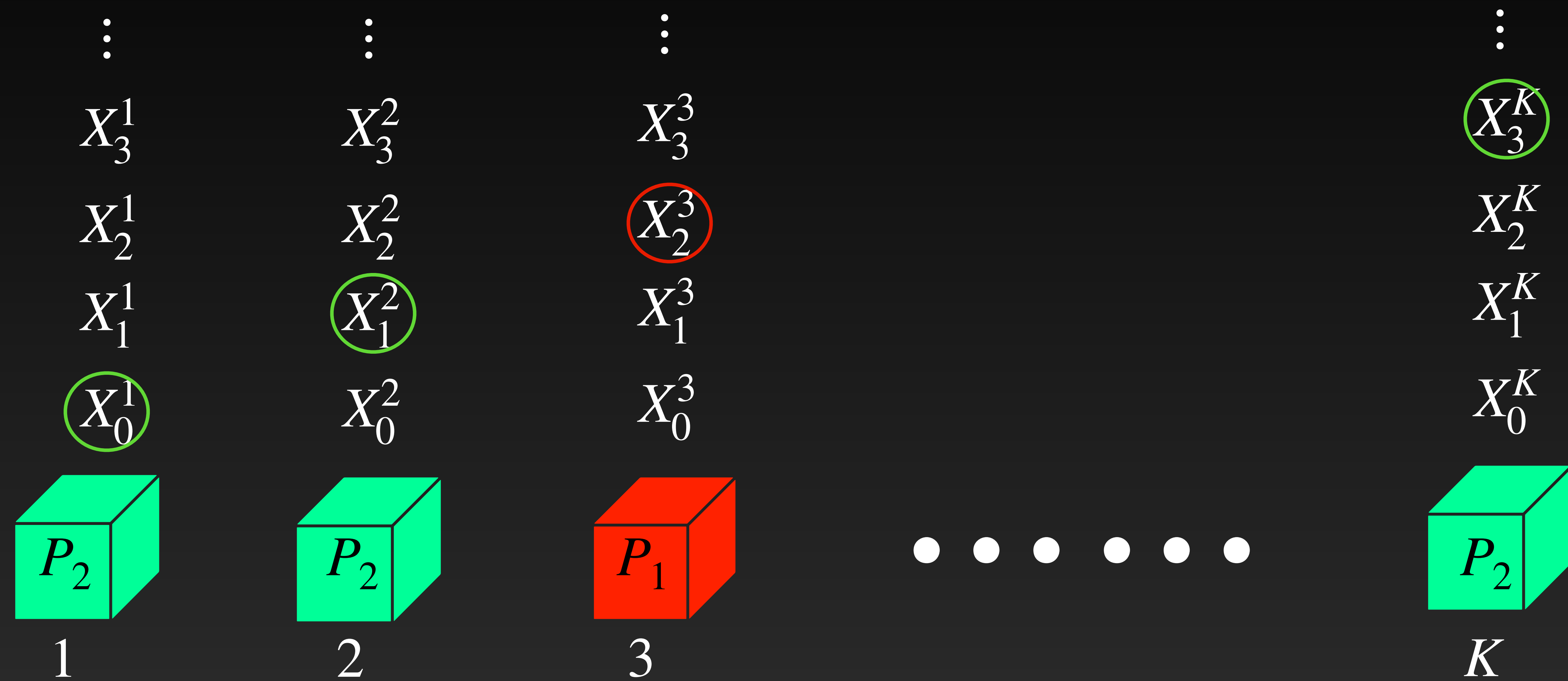

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When the observations from each arm form a Markov process, does the following still hold?

Exact in the asymptotic limit as error vanishes

$$\text{Time to identify the anomaly} \;\boxed{\approx}\; \left(\log \frac{1}{\text{Error in reporting the anomaly correctly}} \right) \frac{1}{D^*}$$

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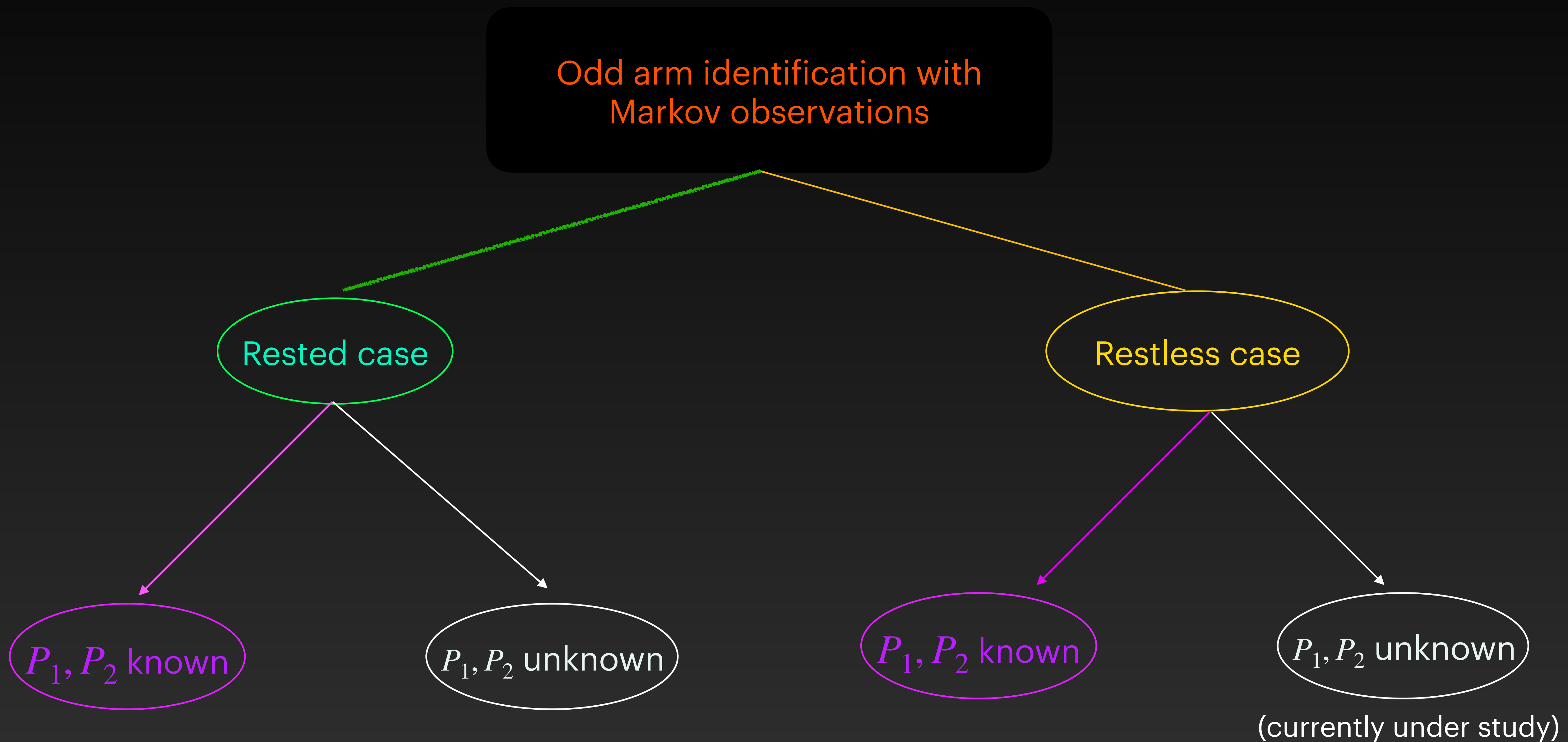
Time to identify the anomaly $\boxed{\approx}$ $\left(\log \dfrac{1}{\text{Error in reporting the anomaly correctly}} \right) \dfrac{1}{D^*}$

YES!

We characterise D^* explicitly

Organisation



Organisation

Odd arm identification with Markov observations

- Rested case
  - P_1, P_2 known
  - P_1, P_2 unknown
- Restless case
  - P_1, P_2 known
  - P_1, P_2 unknown (currently under study)

$$\log \frac{1}{\text{Error in reporting the anomaly correctly}} \cdot \frac{1}{D^*} \underset{\text{Asymptotic lower bound}}{\overset{\text{Data processing inequality}}{\lesssim}} \text{Time to identify the anomaly}$$

Organisation

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P_1, P_2 known

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Restless case

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(currently under study)

$$\log \frac{1}{\text{Error in reporting the anomaly correctly}} \cdot \frac{1}{D^*} \underset{\text{Asymptotic lower bound}}{\overset{\text{Data processing inequality}}{\lesssim}} \text{Time to identify the anomaly} \underset{\text{Asymptotic upper bound}}{\overset{\text{Policy design}}{\lesssim}} \log \frac{1}{\text{Error in reporting the anomaly correctly}} \cdot \frac{1}{D^*}$$

Key Ideas and Challenges

When arm h is the odd arm and the arms are rested:

When P_1 and P_2 are known:

$$D_h^* = \sup_{\lambda(\cdot)} \min_{h' \neq h} \cdots\cdots\cdots$$

Key Ideas and Challenges

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Keep an estimate $\hat{h}(t)$ of h at time t, and sample according to $\frac{\eta}{K} + (1-\eta)\ \lambda^*_{\hat{h}(t)}(\,\cdot\,)$ for some $\eta > 0$

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Trembling hand model; ensures sufficient exploration

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$\hat{h}(t) \to h$ as $t \to \infty$, and so $\lambda_{\hat{h}(t)}^* \to \lambda_h^*$

Trembling hand model; ensures sufficient exploration

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When arm h is the odd arm and the arms are rested:

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Principle of certainty equivalence

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Keep estimates $(\hat{h}(t), \hat{P}^t_1, \hat{P}^t_2)$ of (h, P_1, P_2) at time t, and sample according to $\lambda^*_{\hat{h}(t),\hat{P}^t_1,\hat{P}^t_2}$ with η-trembling

Principle of certainty equivalence

$(\hat{h}(t), \hat{P}^t_1, \hat{P}^t_2) \to (h, P_1, P_2)$ as $t \to \infty$

Continuity of λ^* (e.g., Berge's maximum theorem) will give the result

Key Ideas and Challenges

When arm h is the odd arm and the arms are restless:

A key MDP formulation makes the problem amenable to analysis. The MDP viewpoint is new and does not appear in the prior works. Trembling hand ensures ergodicity.

When P_1 and P_2 are known:

$$D_h^* = \sup_{\lambda(\cdot|\cdot)} \min_{h' \neq h} \cdots\cdots\cdots$$

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Stitching the solutions for various δ does the job

Some Future Directions

- General state space for each arm (our work deals with finite state space)
- More than one odd arm
- Number of arms $\rightarrow \infty$
- Time is continuous rather than discrete

Publications

- P. N. Karthik, R. Sundaresan, “Detecting an Odd Markov Arm with a Trembling Hand”, proceedings of the 2020 IEEE International Symposium on Information Theory (virtual).

- P. N. Karthik, R. Sundaresan, “Learning to Detect an Odd Markov Arm”, proceedings of the 2019 IEEE International Symposium on Information Theory, Paris, France.

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