Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090/EE5817: Probability and Stochastic Processes

Quiz 01

Date: 12 August 2025

Question	1	2	Total
Marks Scored			

You may use any result covered in class without proof. Do not use any result from the homework unless suggested as a hint.

1. Suppose sets A and B are equicardinal.

   (a) **(1 Mark)**
   Prove or give a counterexample: every injective function from A to B must also be surjective.

   (b) **(1 Mark)**
   Prove or give a counterexample: every surjective function from A to B must also be injective.

2. For each $k \in \mathbb{N}$, let $\mathscr{C}_k \subset \{0,1\}^{\mathbb{N}}$ denote the set of all infinite binary strings containing **exactly** k ones in them. Furthermore, let $\mathscr{C} \subset \{0,1\}^{\mathbb{N}}$ denote the set of all infinite binary strings containing **finitely** many ones in them.

   (a) **(2 Marks)**
   Show that $\mathscr{C}_k$ is countably infinite for every $k \in \mathbb{N}$.
   You may use the fact that $\mathbb{N}^k := \mathbb{N} \underbrace{\times \cdots \times}_{k \text{ times}} \mathbb{N}$ is countably infinite for every $k \in \mathbb{N}$.

   (b) **(1 Mark)**
   Express $\mathscr{C}$ in terms of $\mathscr{C}_1, \mathscr{C}_2, \ldots$, and argue that $\mathscr{C}$ is countably infinite.