Networks Seminar 2019

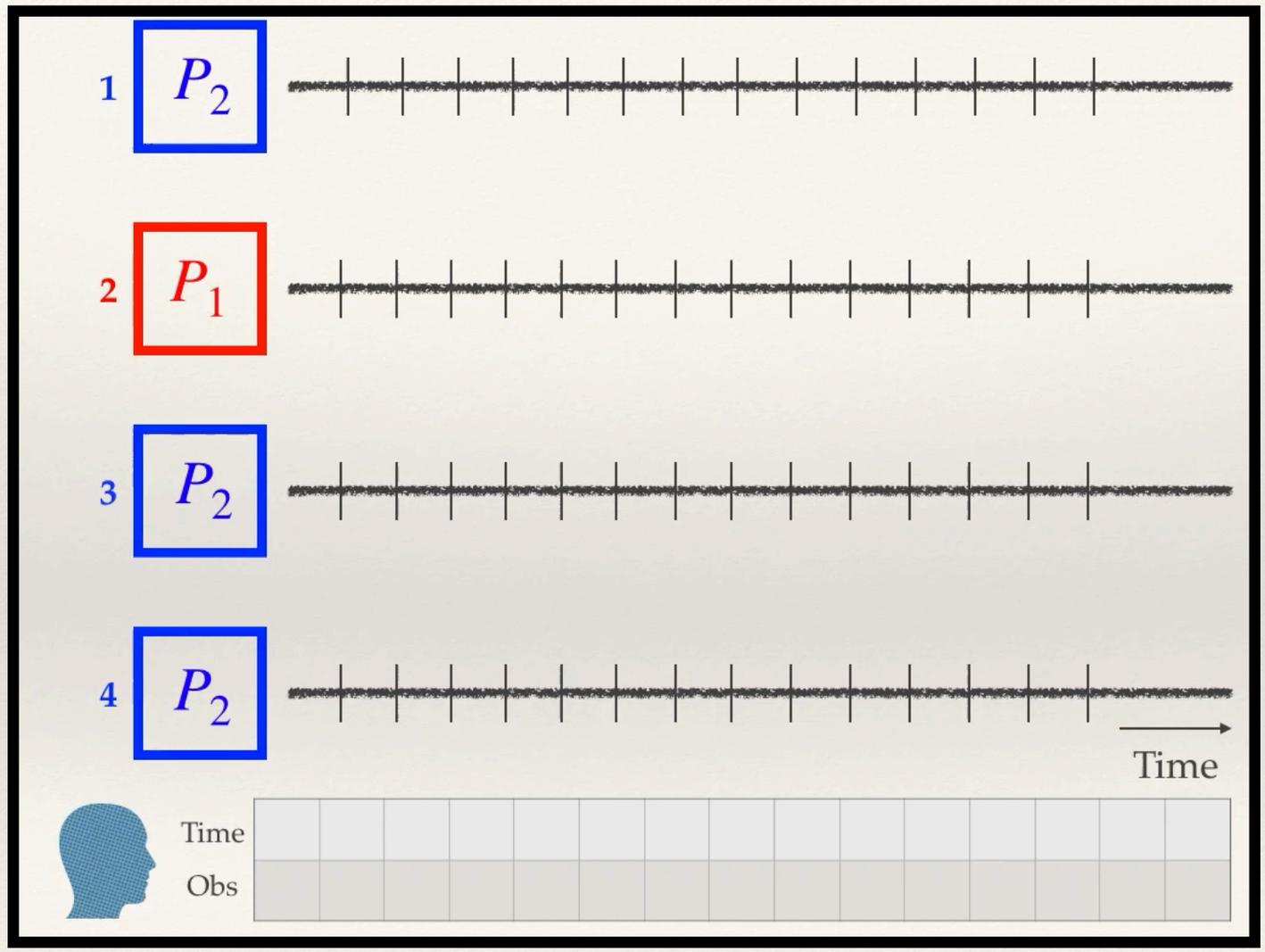
Nov 13 2019

On Detecting An Anomalous Arm in Multi-armed Bandits with Markov Observations

Karthik PN

Joint Work with Prof. Rajesh Sundaresan

Presented in part at the 2019 IEEE International Symposium on Information Theory (ISIT)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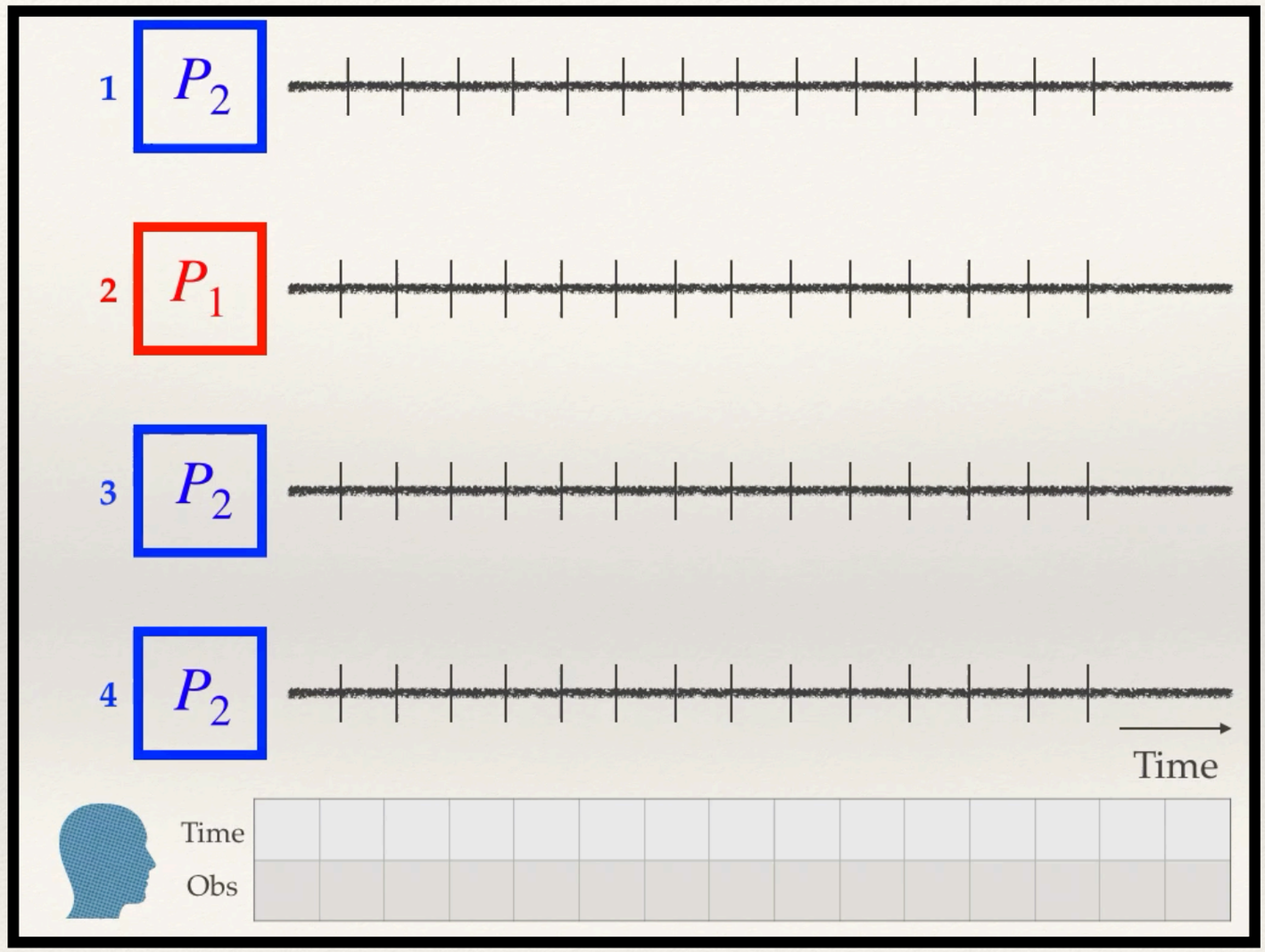
Obs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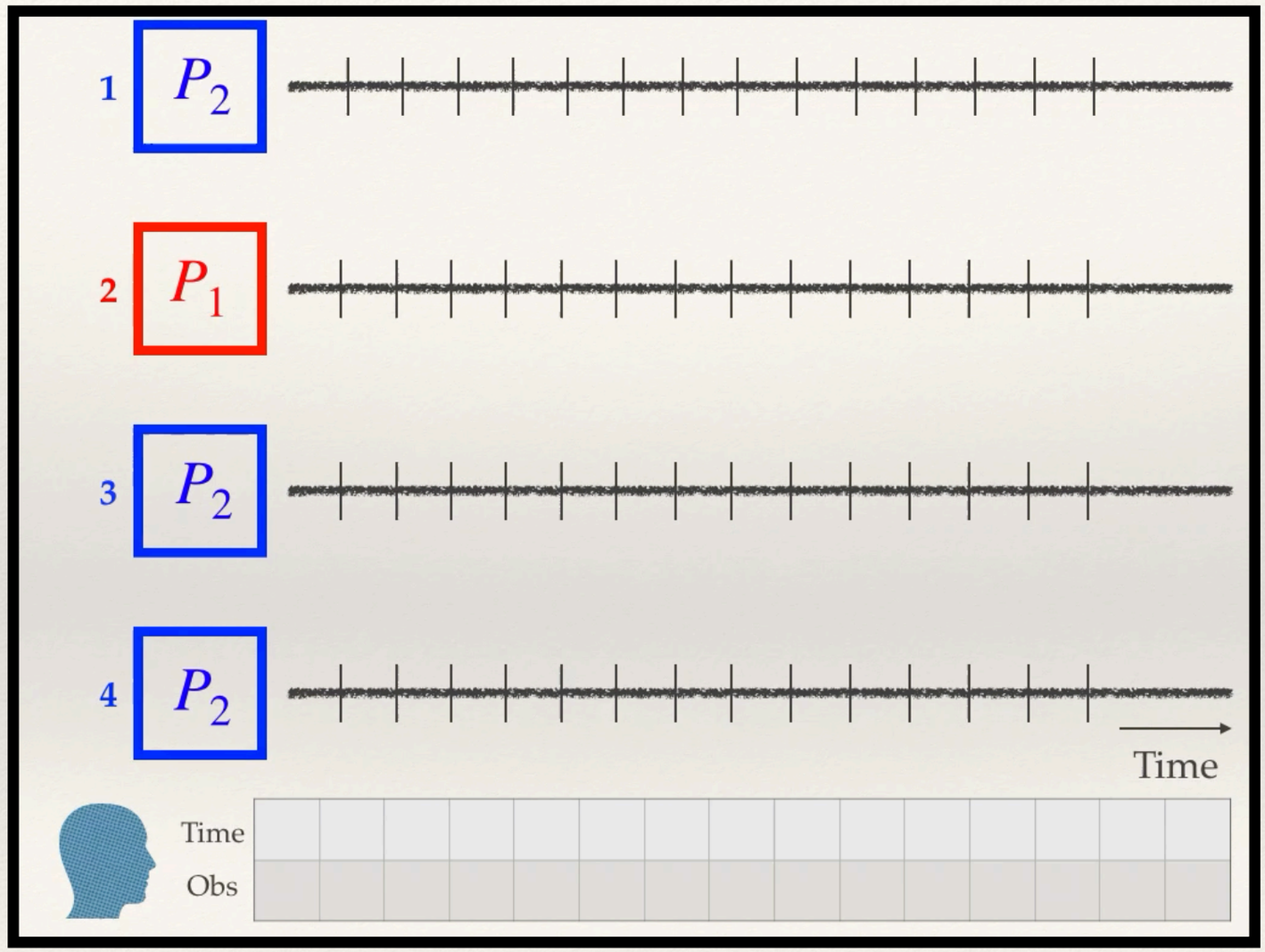

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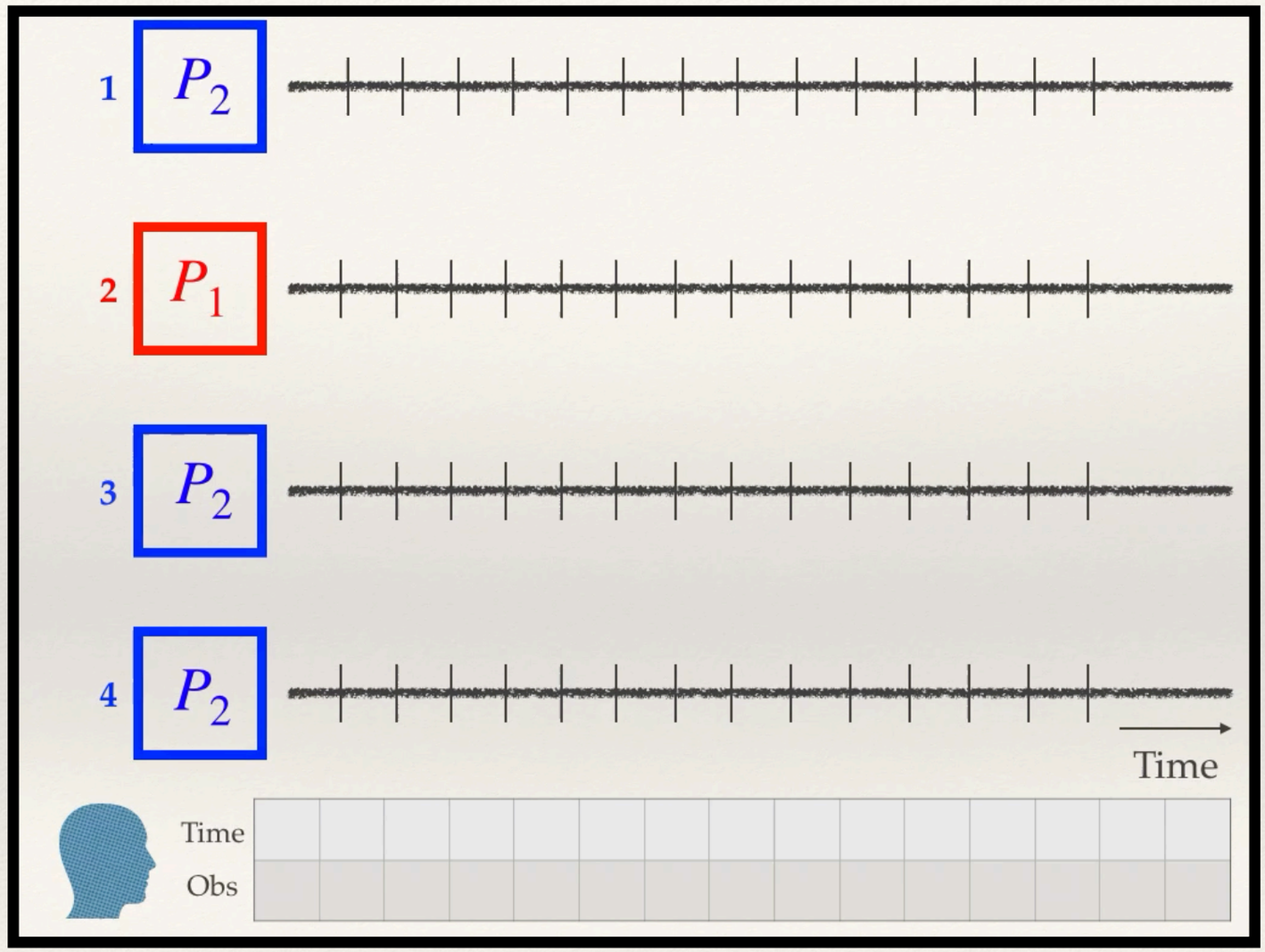

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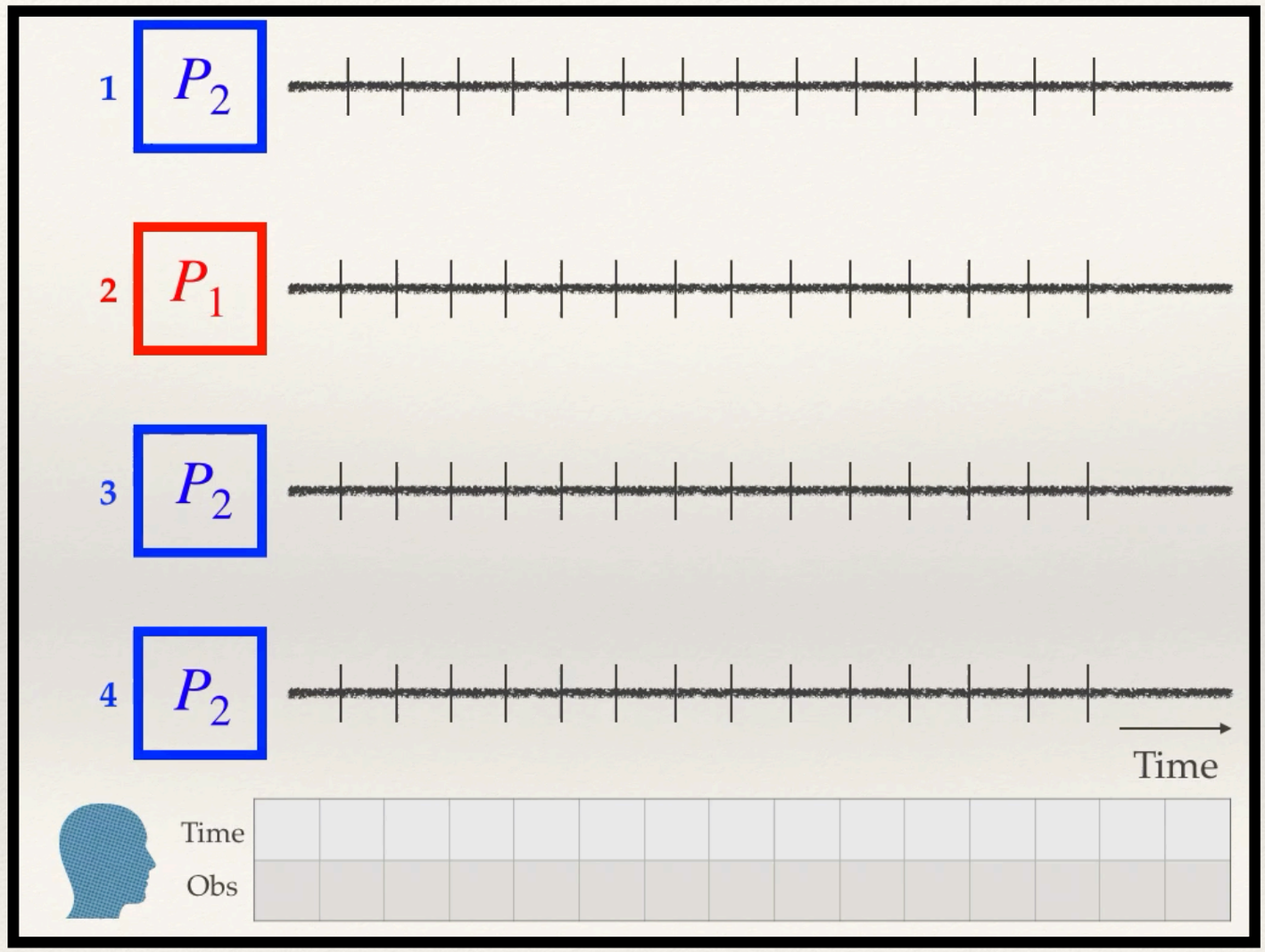

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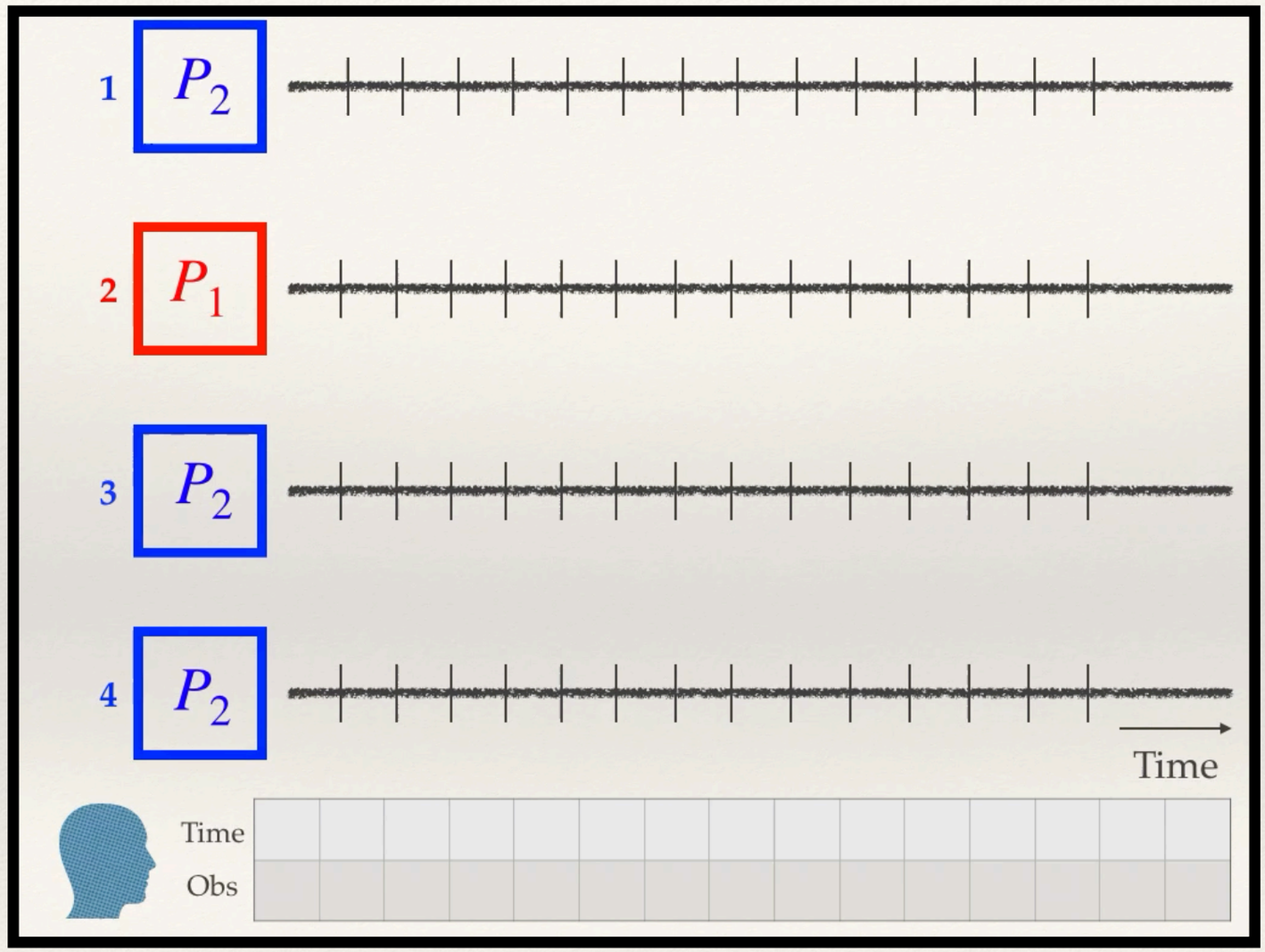

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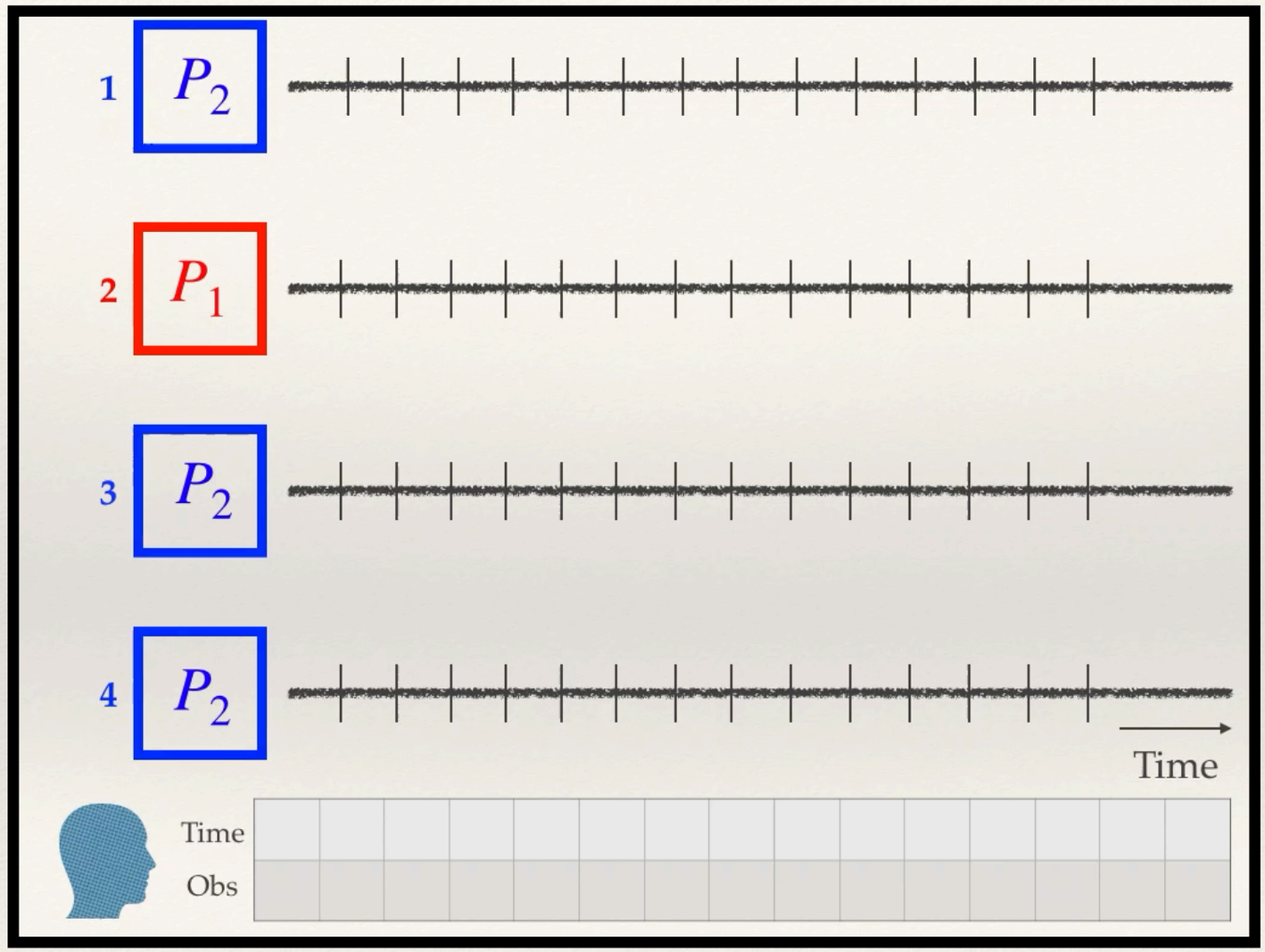

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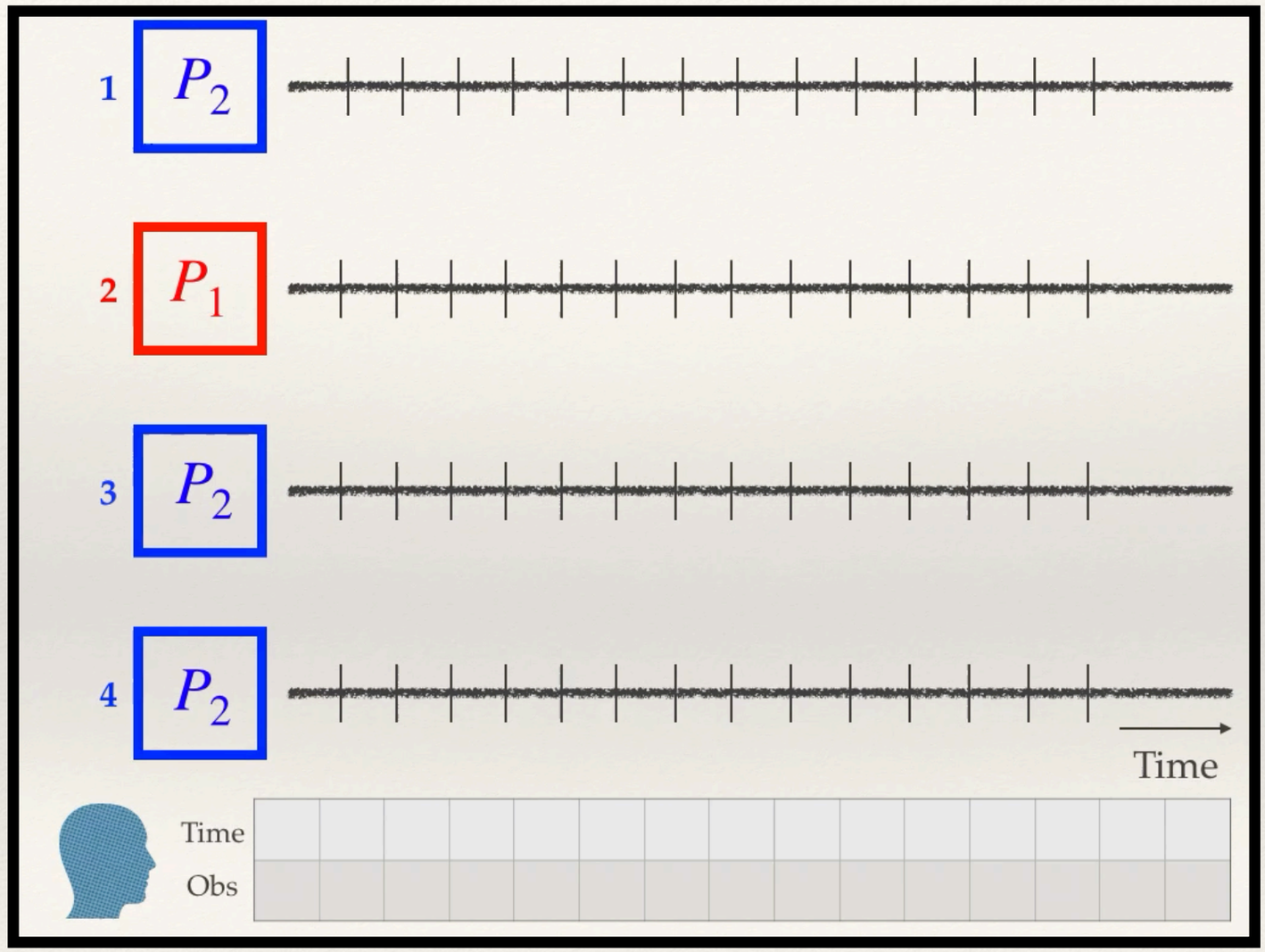

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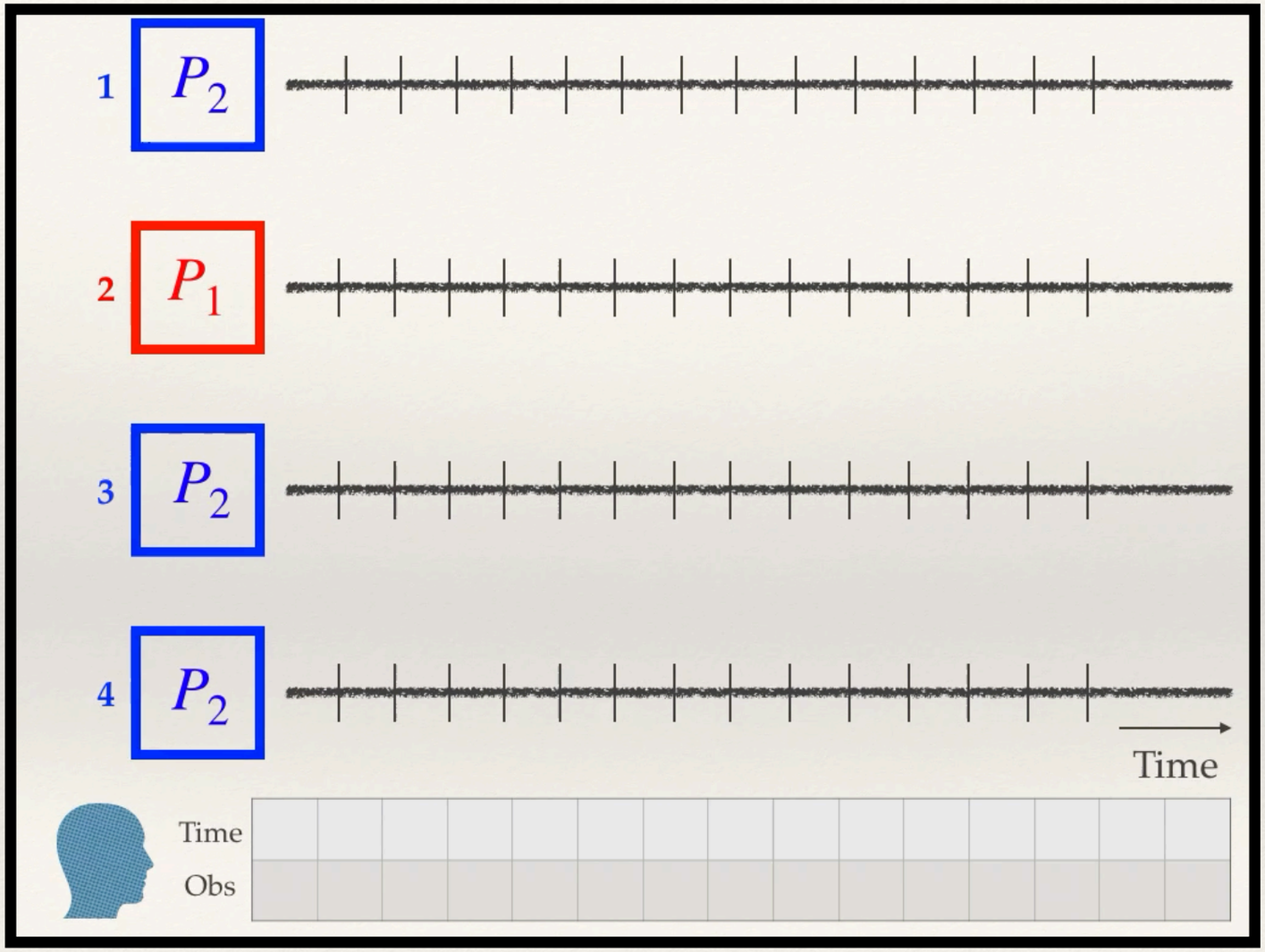

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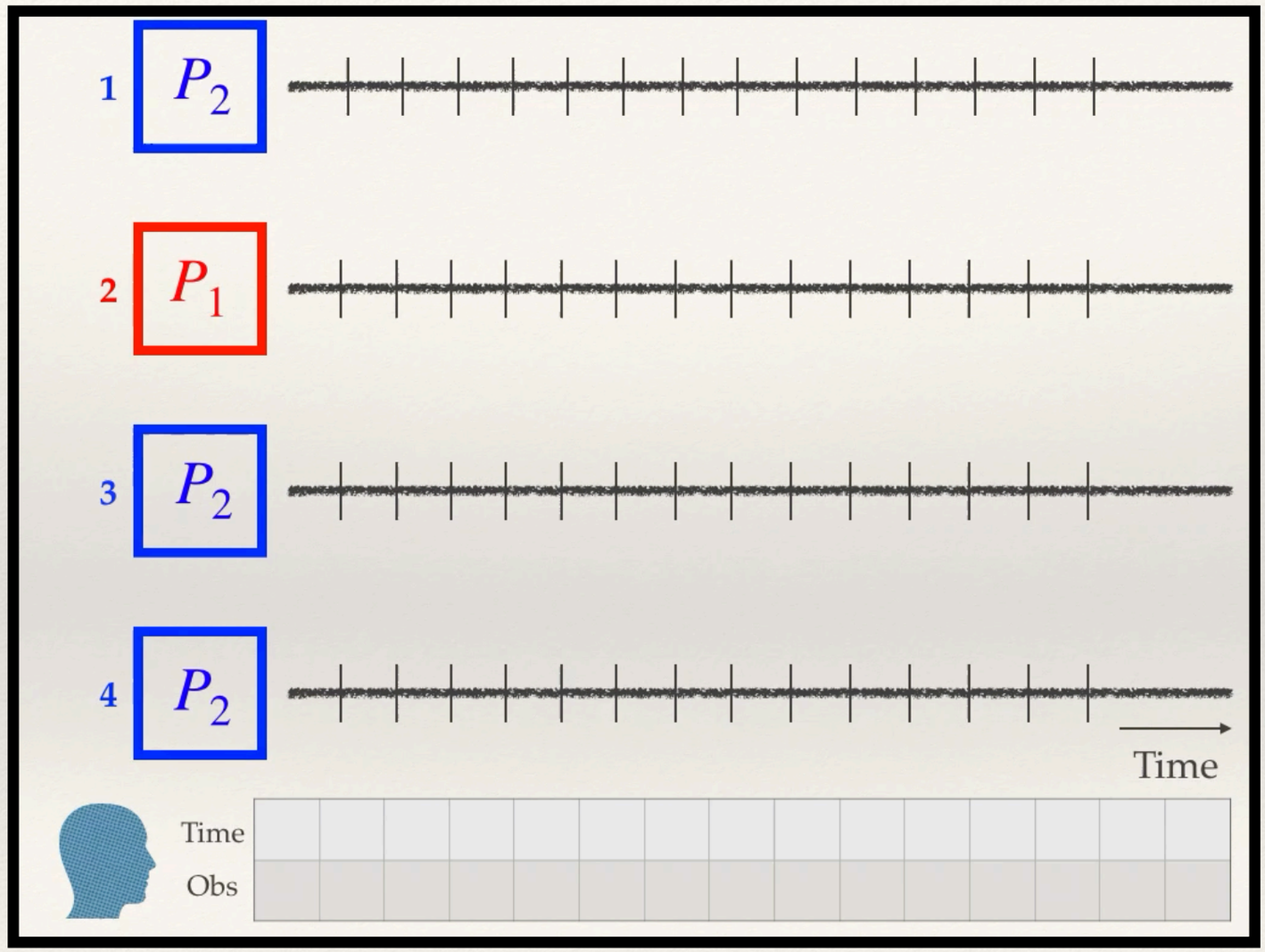

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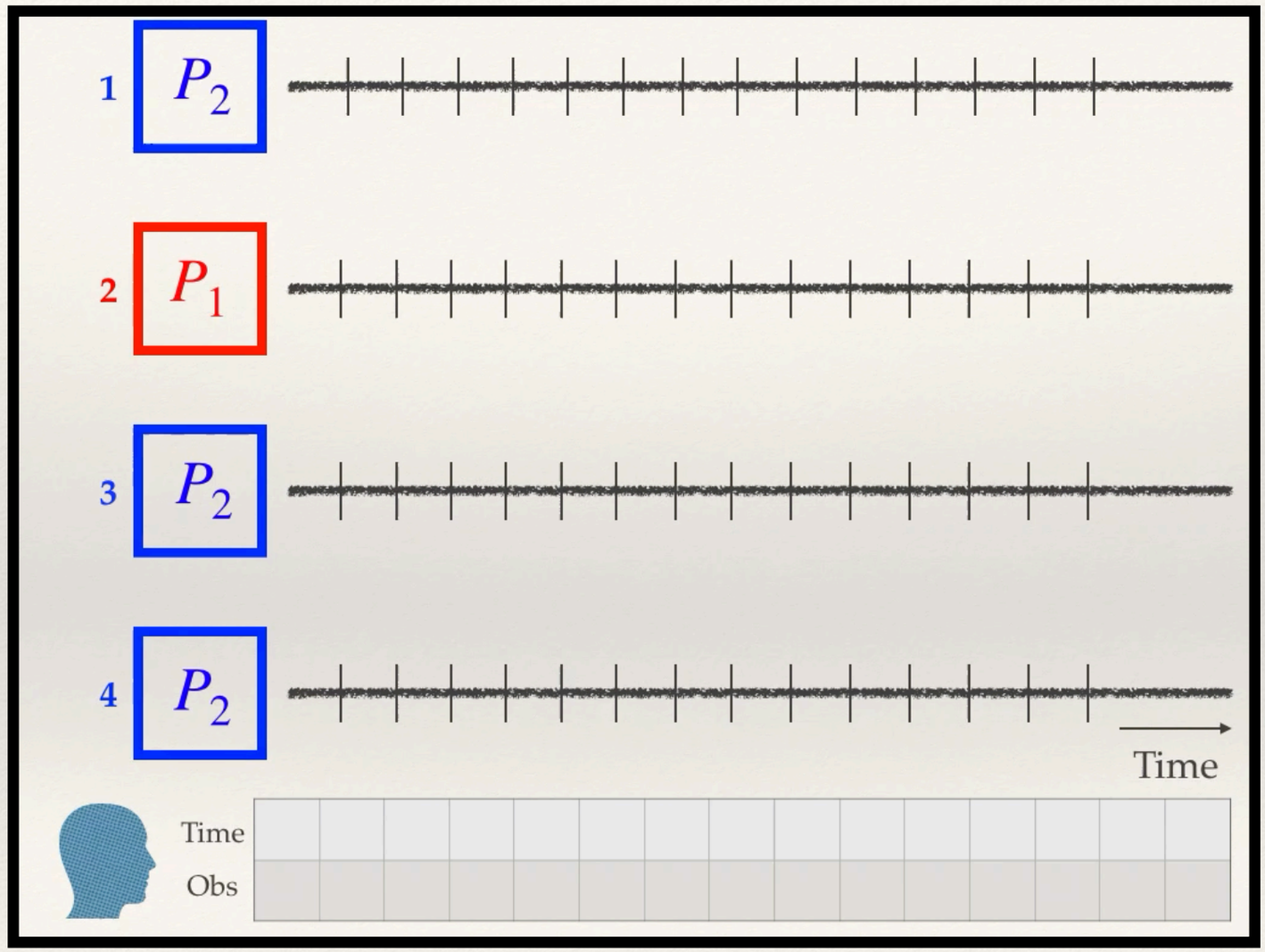

Odd arm identification (OAI) in multi-armed bandits with Markov Observations

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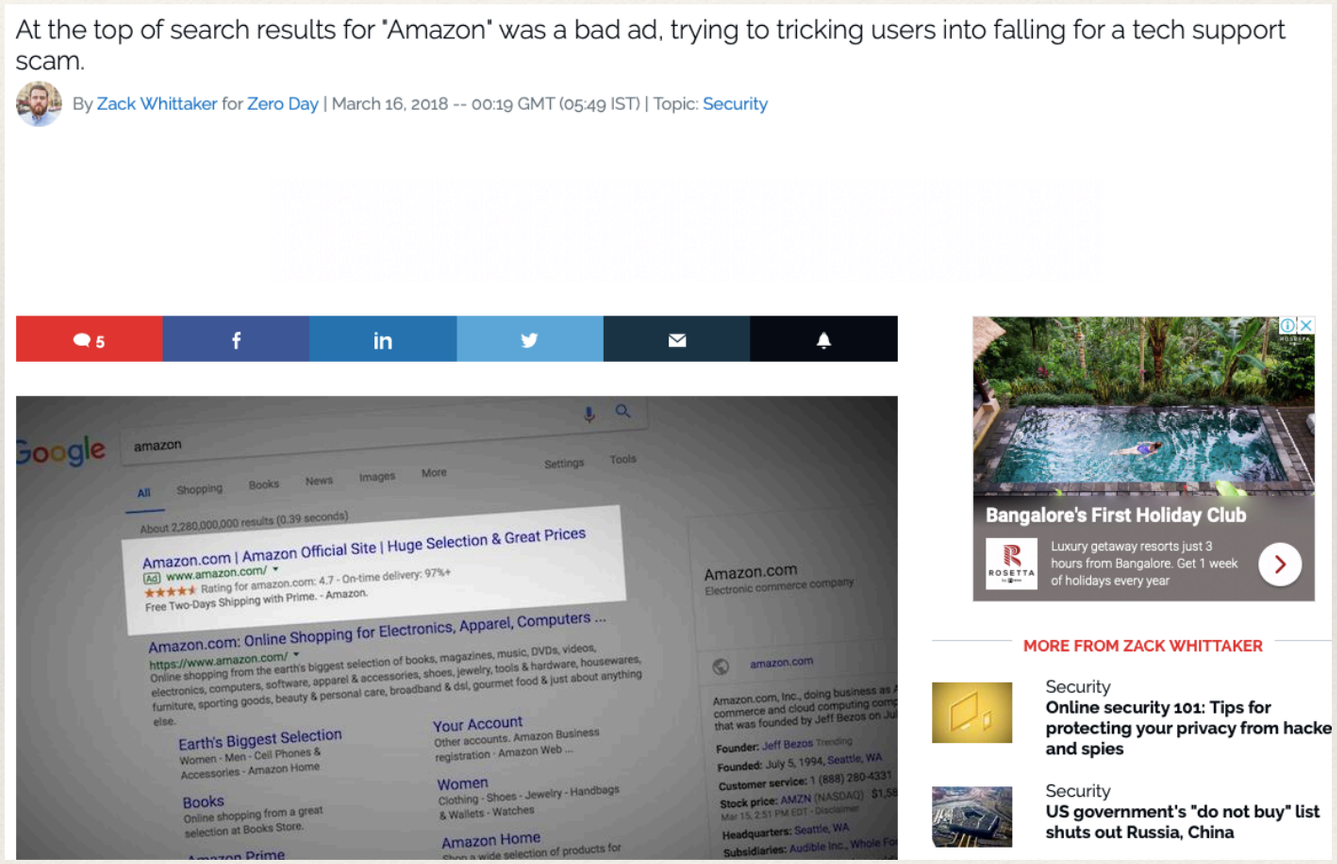

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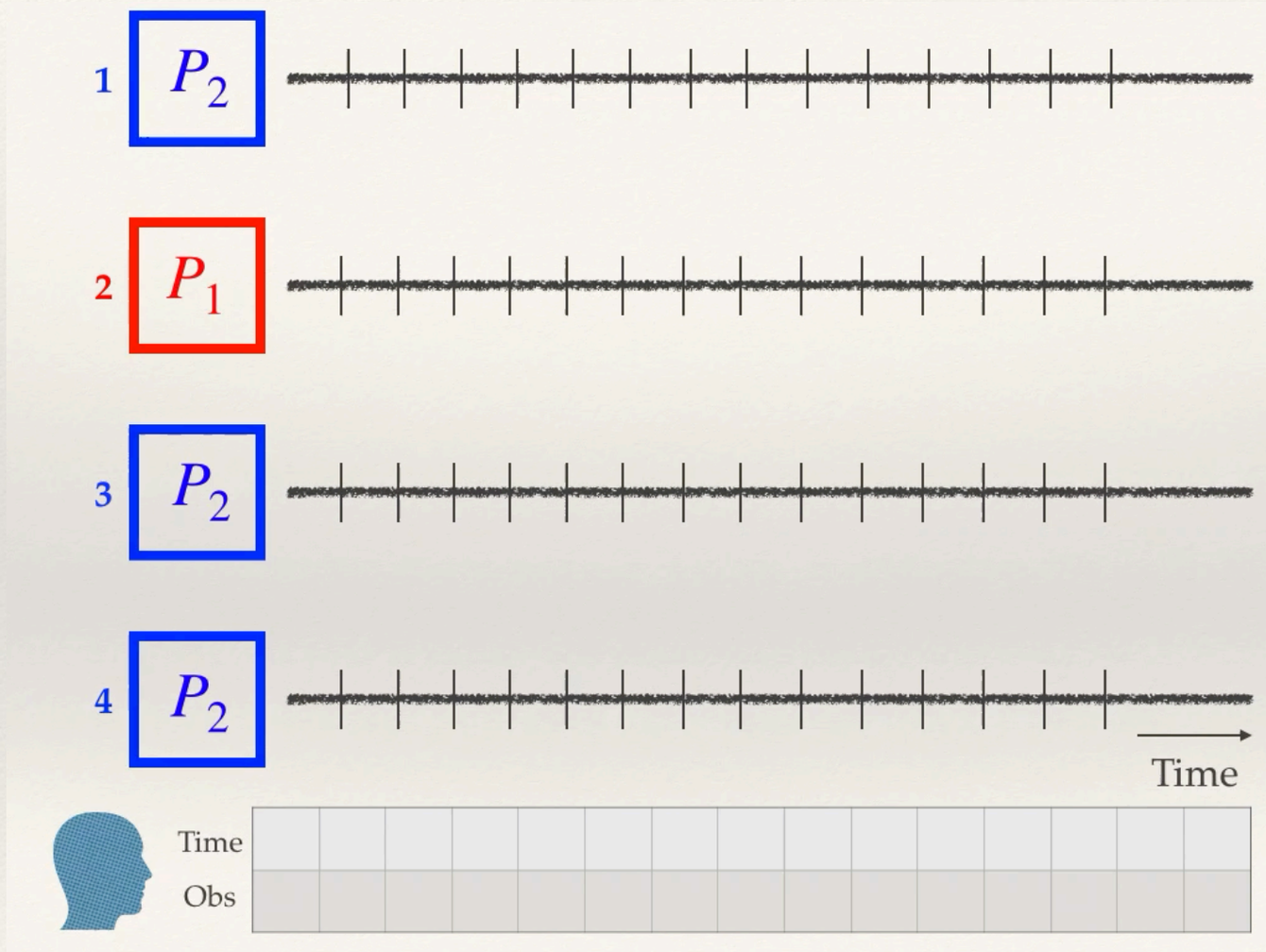

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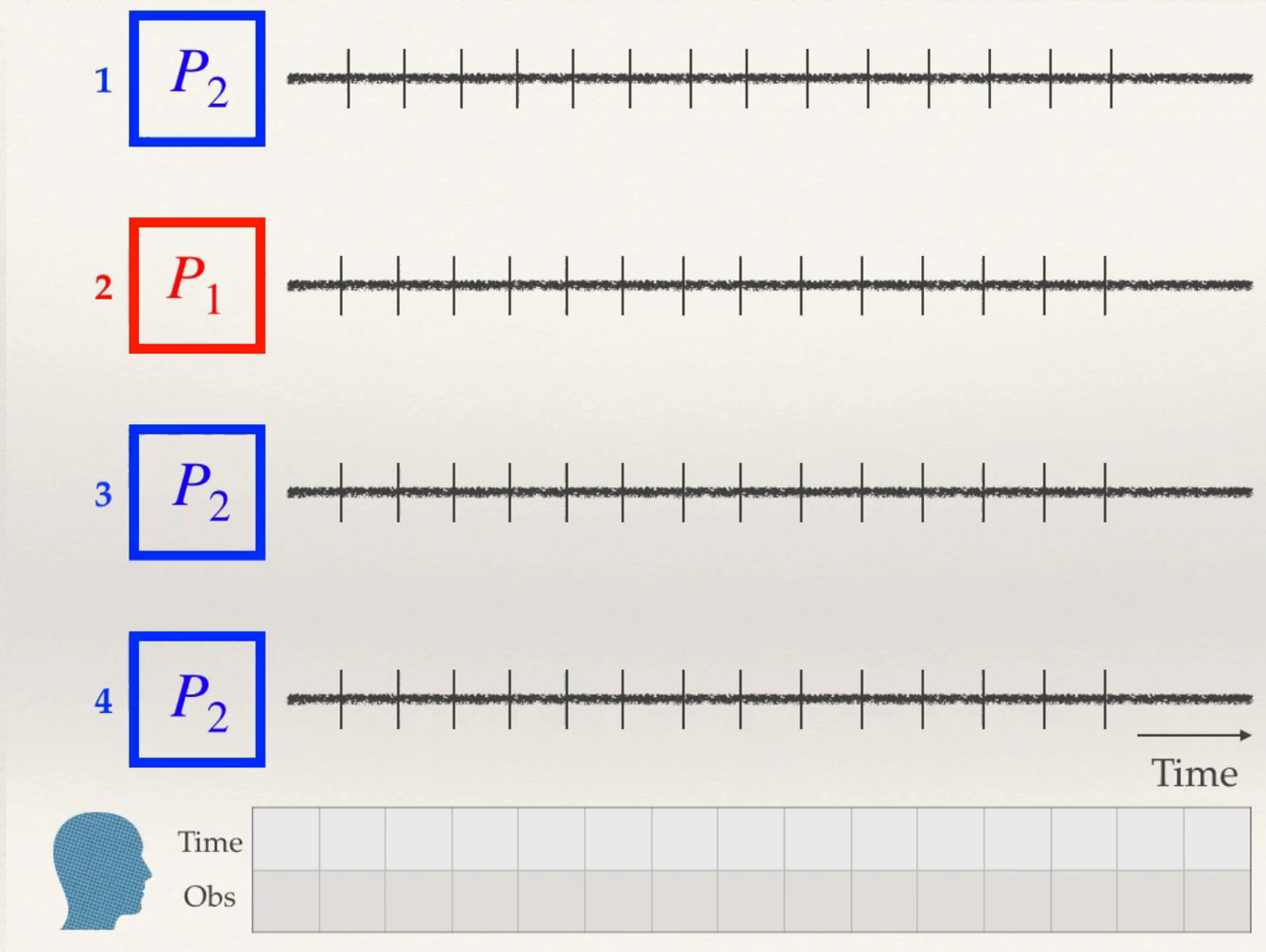

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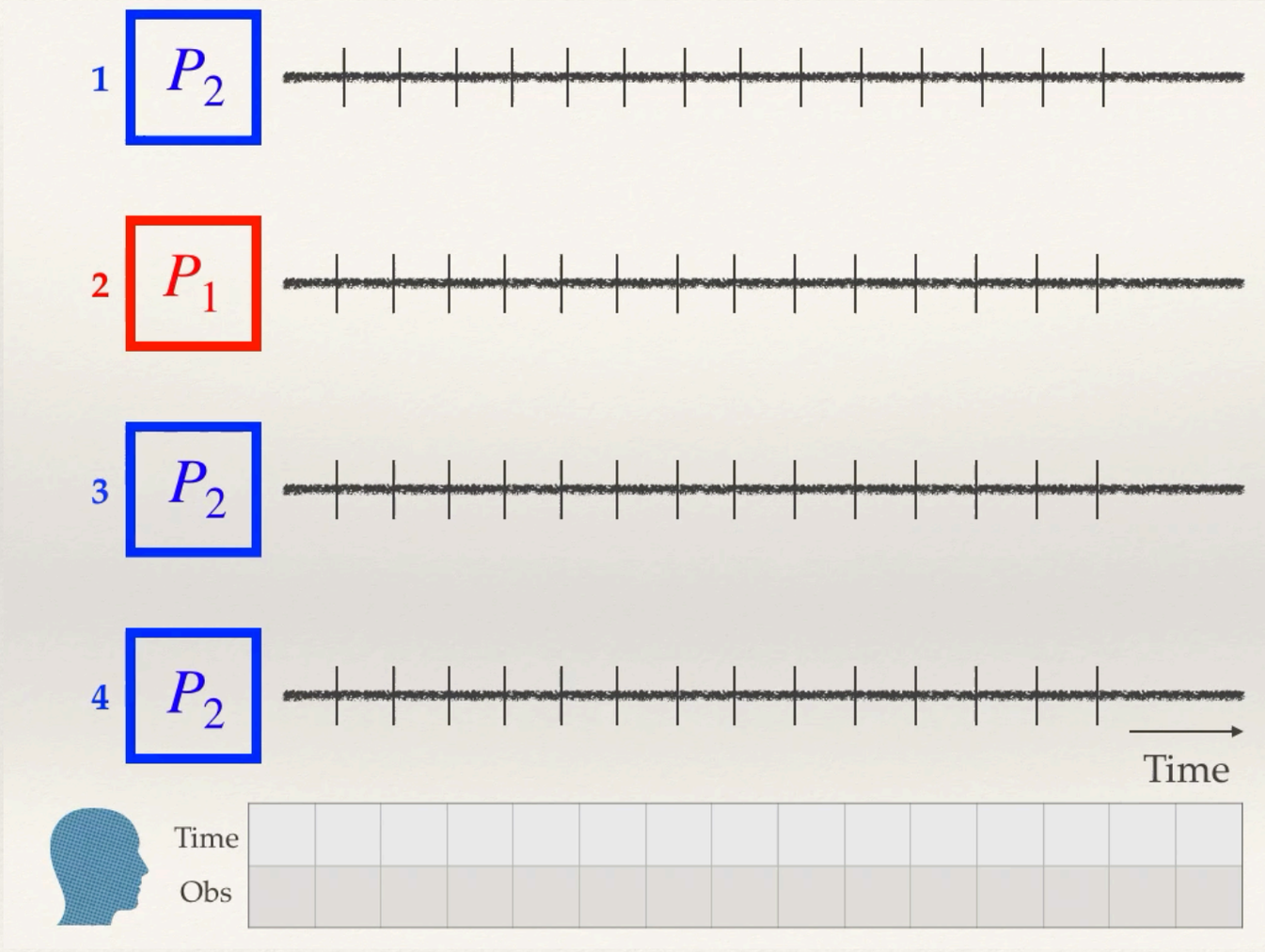
- Unobserved arms remain frozen at their last observed states



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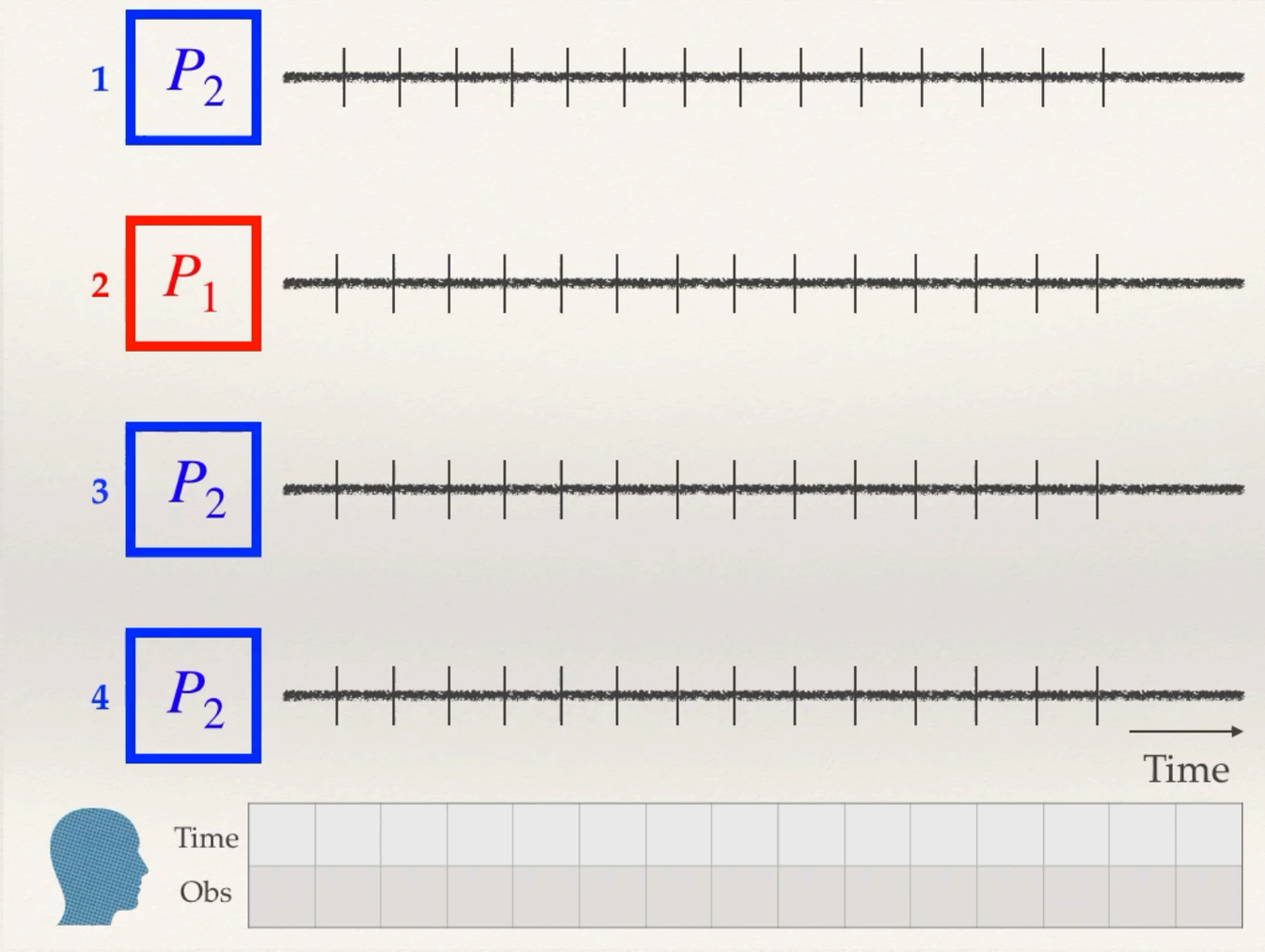
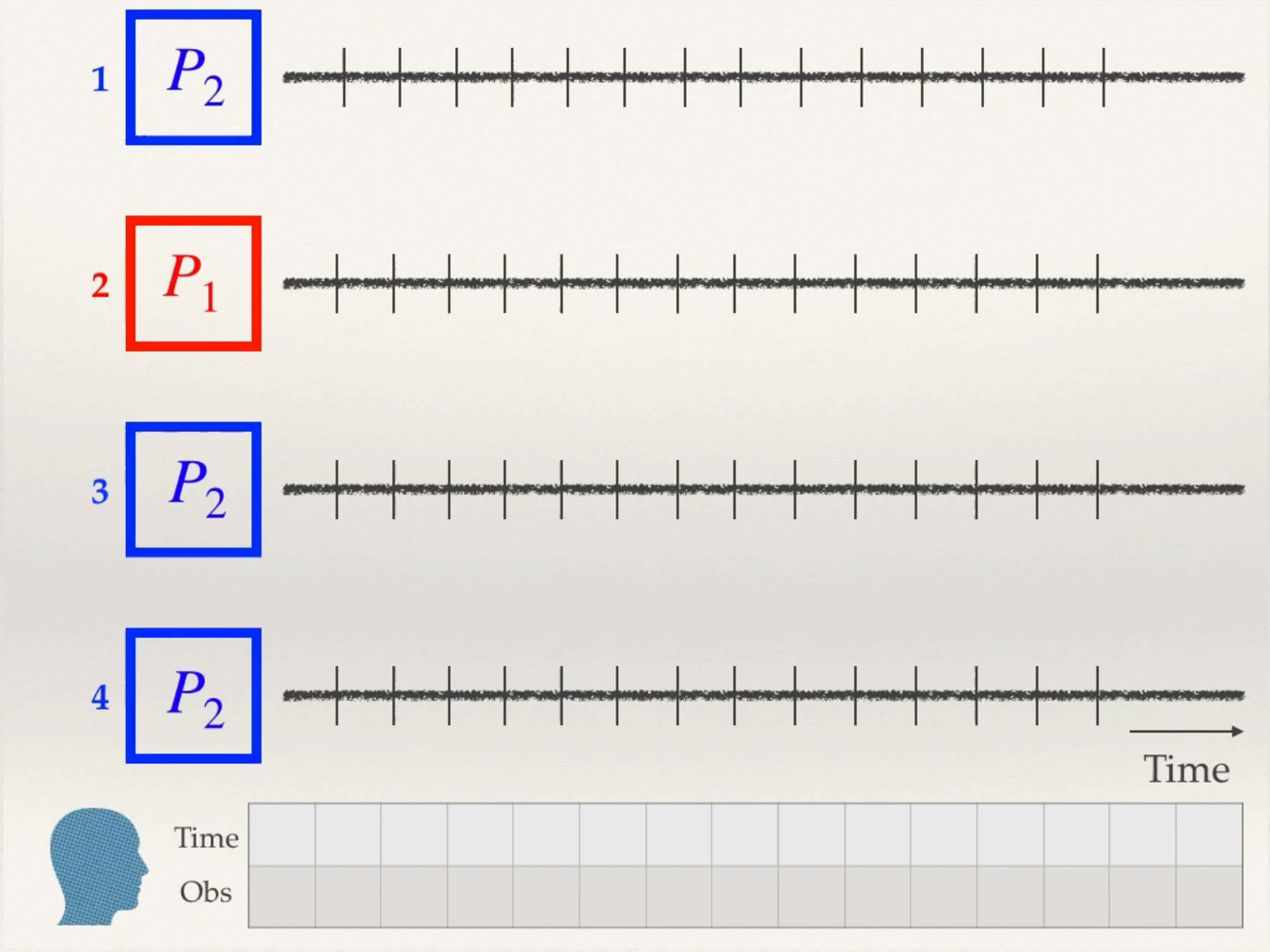


A Simplification: MAB with Rested Arms

- Unobserved arms remain frozen at their last observed states
- Simpler to analyse
- Insights derived may serve as pointers to solve the more difficult case of restless arms



Prior Works on Rested Markov Bandits and OAI

		Problem Setting: Regret Minimisation	Problem Setting: Optimal Stopping	Nature of observations: IID	Nature of observations: Markov	Arm distributions: Known	Arm distributions: Unknown	
Rested arms	Gittins[1]	✓	✗	✗	✓	✓	✗	
	Agarwal et al.[2] Anantharam et al.[3]	✓	✗	✗	✓	✗	✓	Finite parameter space
OAI	Vaidhiyan et al.[4] Prabhu et al.[5]	✗	✓	✓	✗	✗	✓	Uncountable parameter space
	Current work	✗	✓	✗	✓	✗	✓	

[1] J. C. Gittins, "Bandit processes and dynamic allocation indices," Journal of the Royal Statistical Society. Series B (Methodological), pp. 148–177, 1979.

[2] R. Agrawal, D. Teneketzis, and V. Anantharam, "Asymptotically efficient adaptive allocation schemes for controlled Markov chains: Finite parameter space," IEEE Trans. on Automatic Control, 1989.

[3] Anantharam V, Varaiya P, Walrand J. Asymptotically efficient allocation rules for the multi-armed bandit problem with multiple plays-Part II: Markovian rewards. IEEE Trans. on Automatic Control, 1987.

[4] N. K. Vaidhiyan and R. Sundaresan, "Learning to detect an oddball target," IEEE Trans. on Information Theory, vol. 64, no. 2, pp. 831–852, 2018.

[5] G. R. Prabhu, S. Bhashyam, A. Gopalan, and R. Sundaresan, "Learning to detect an oddball target with observations from an exponential family," 2017.

Humble Beginnings: Wald and Chernoff

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All's well that starts well

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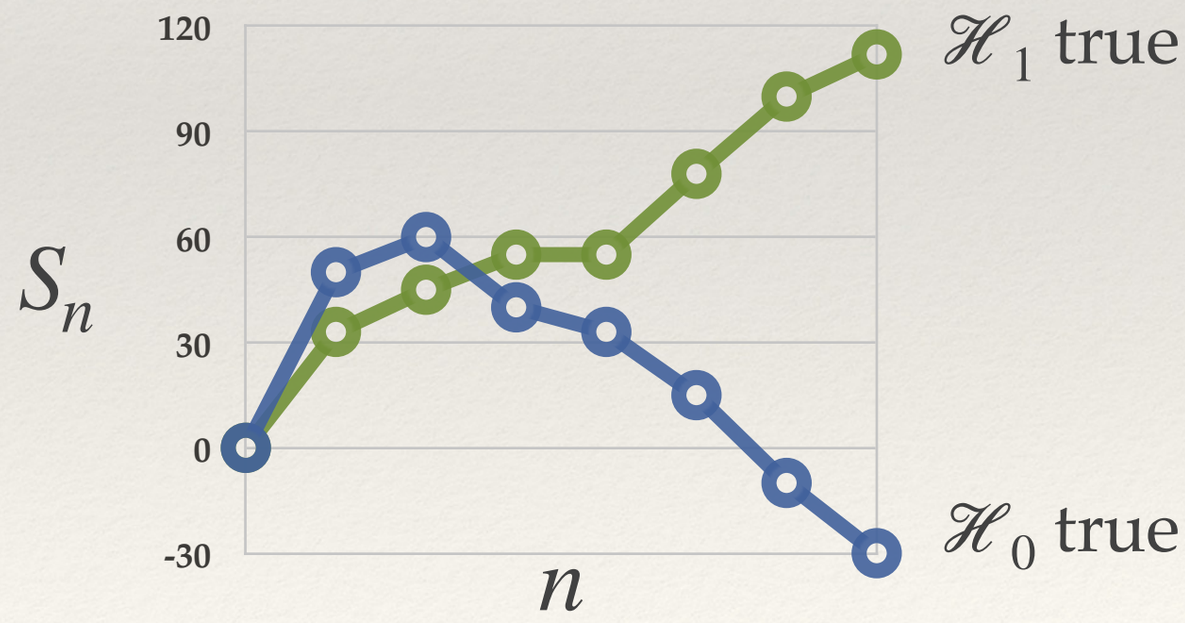

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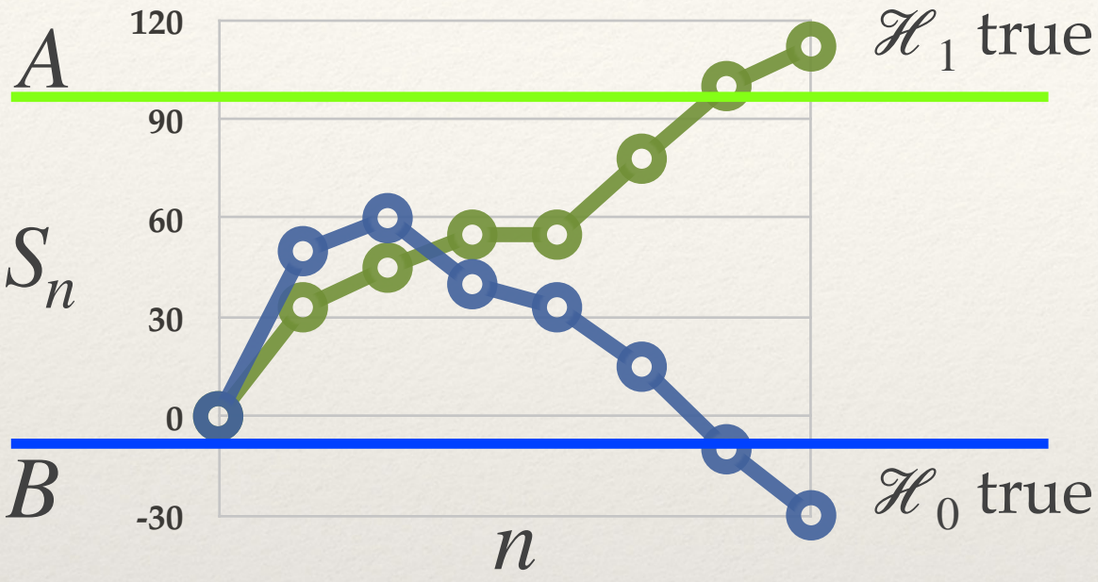

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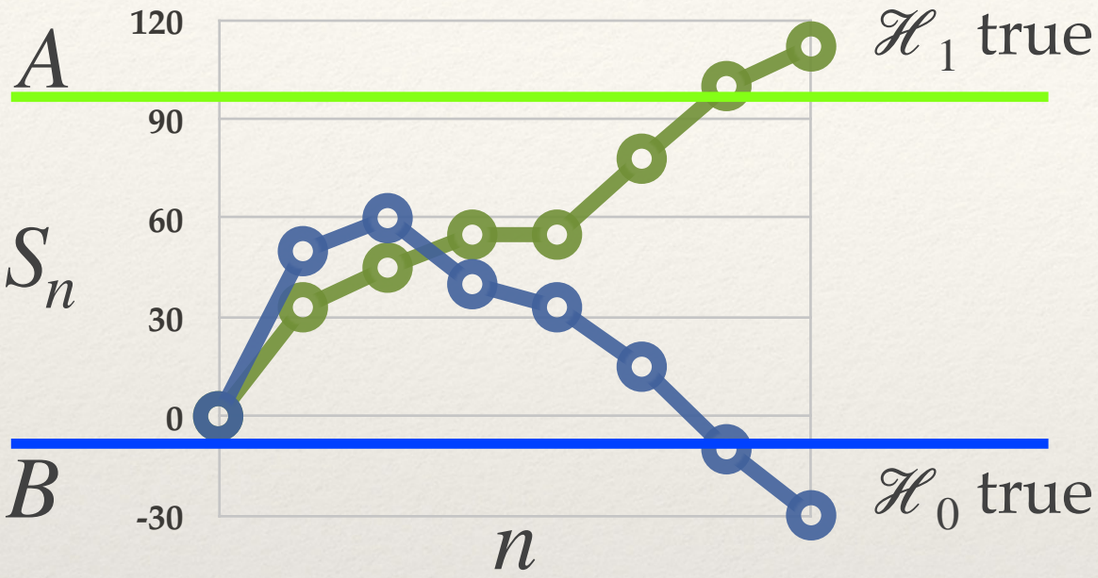

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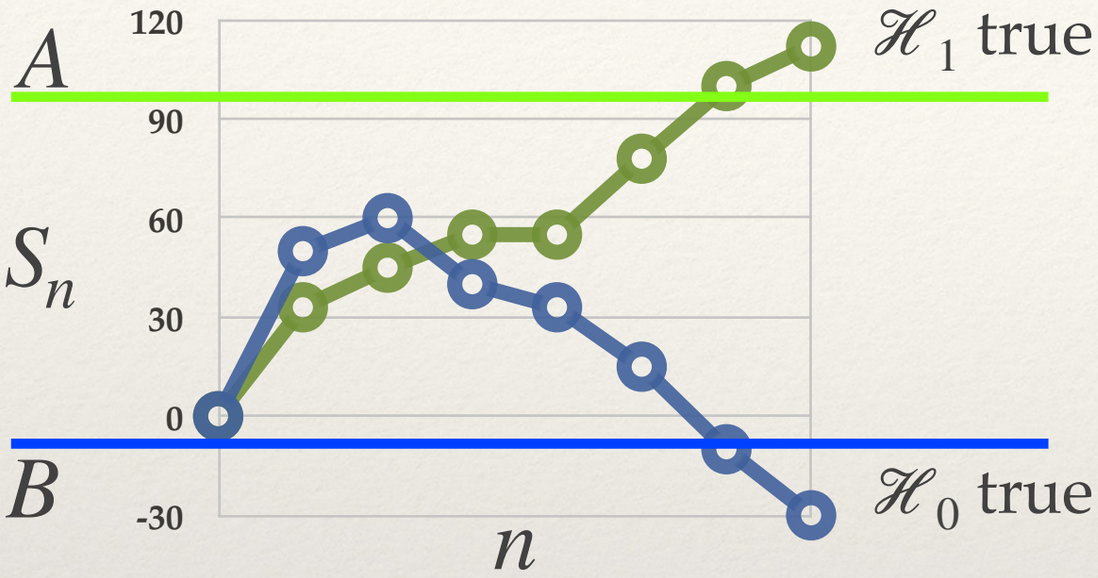

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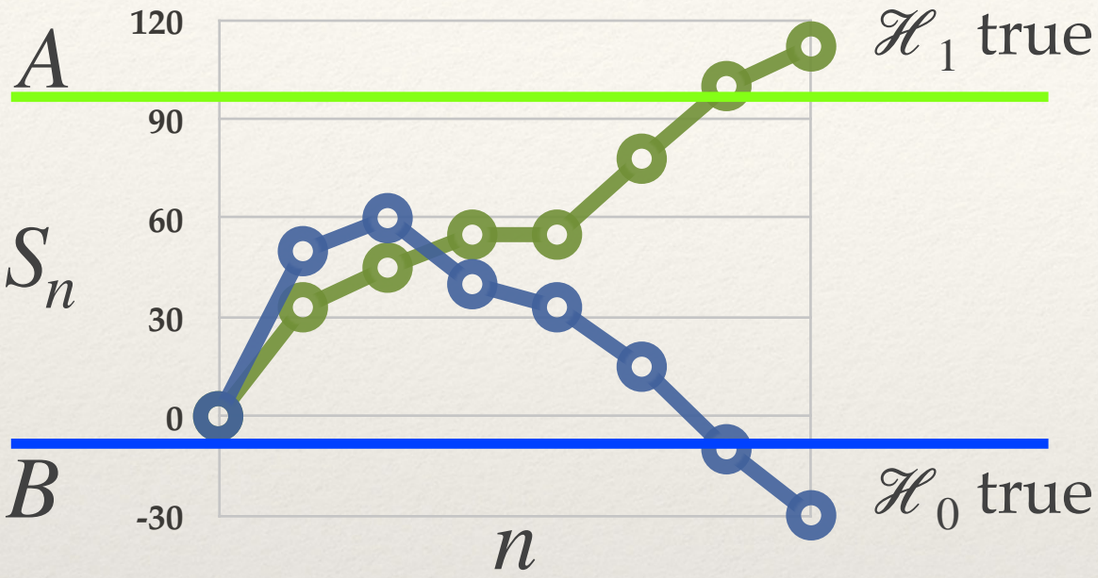

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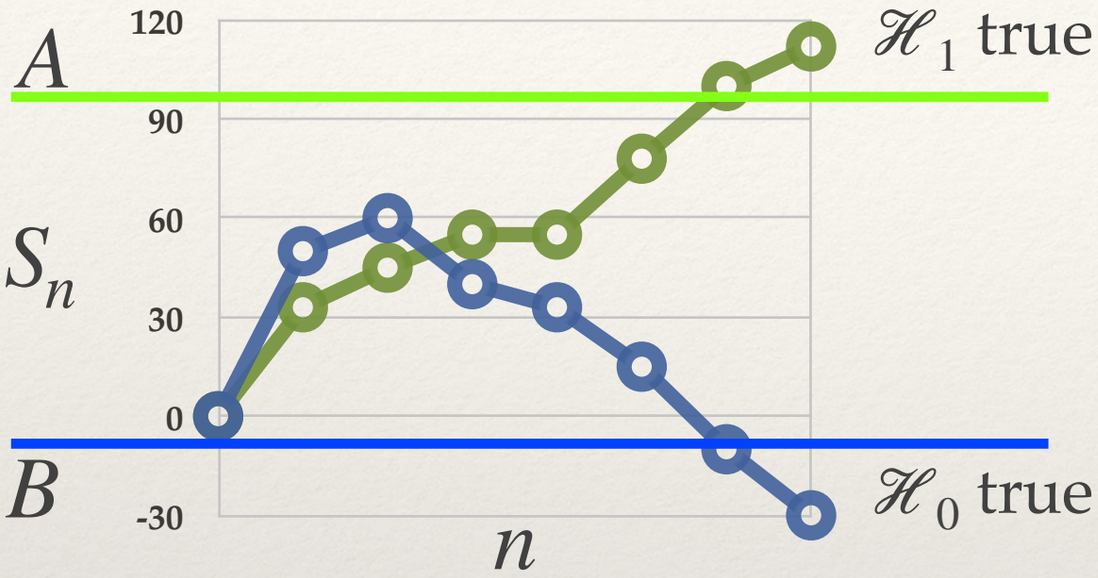

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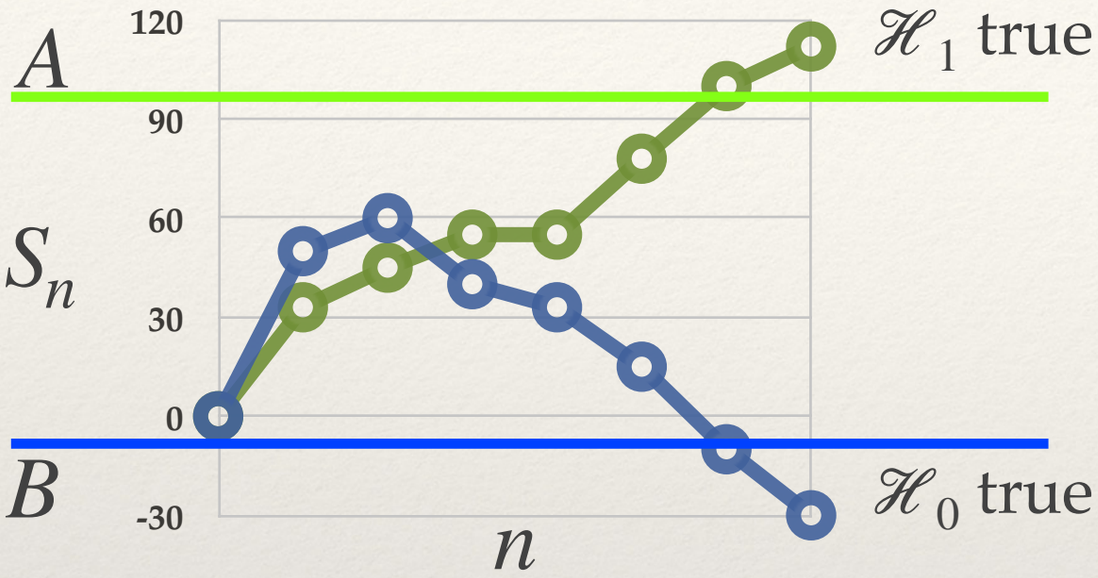

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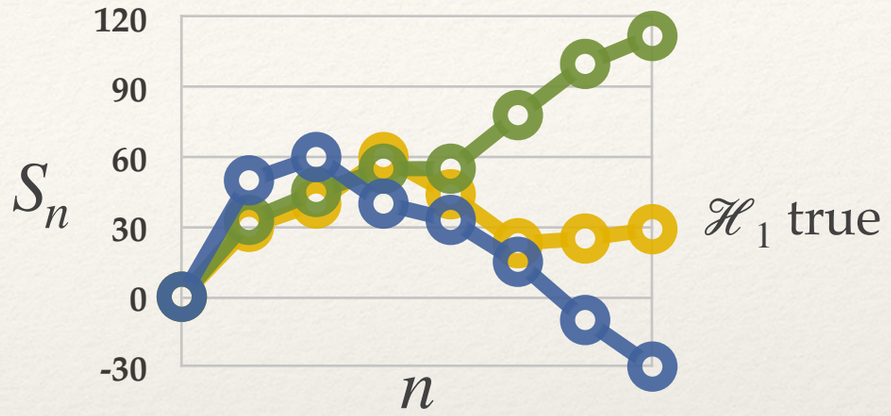

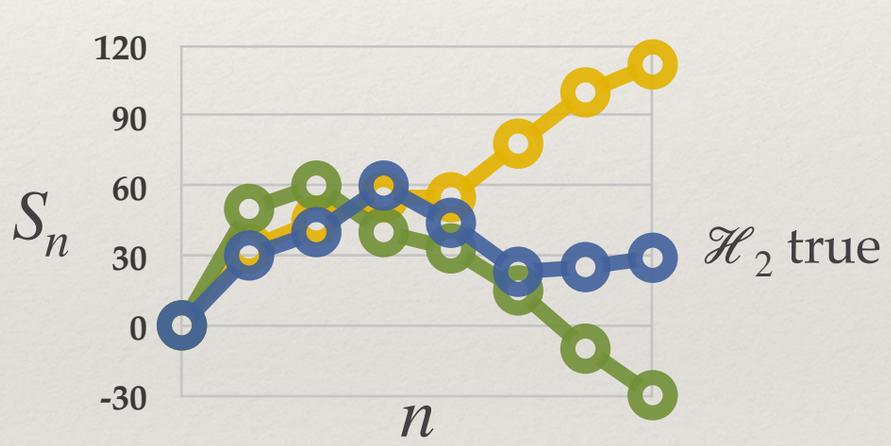

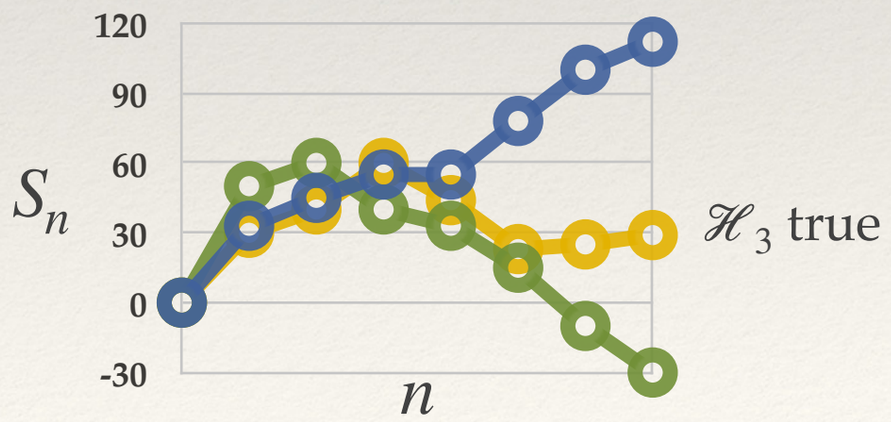

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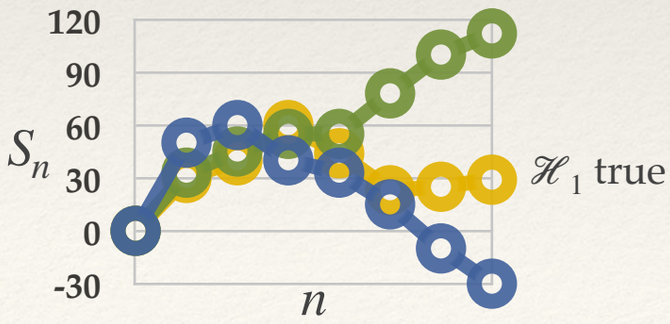

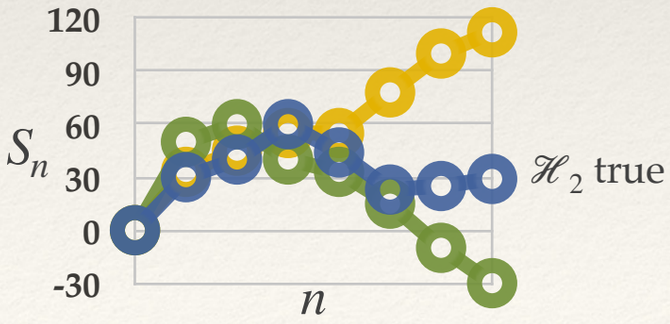

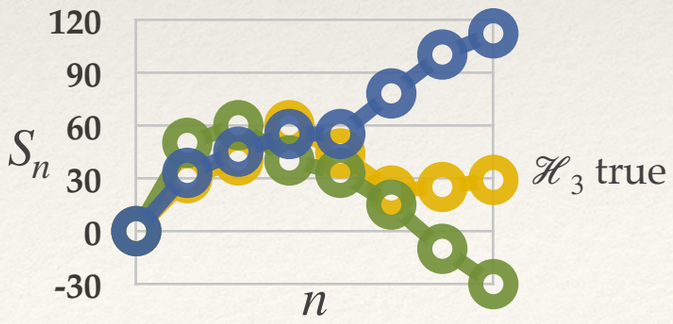

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Back to Rested Markov Bandits

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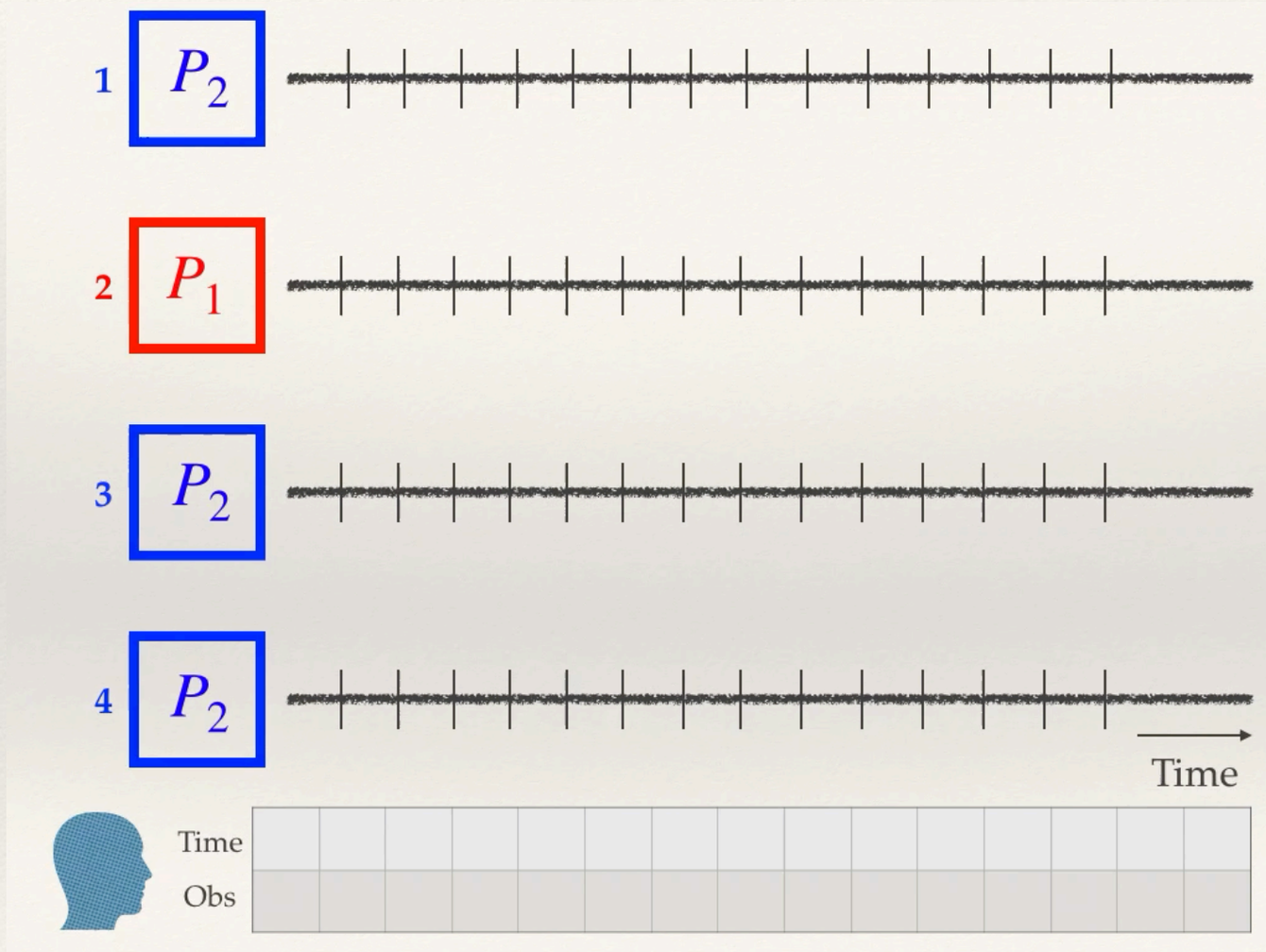

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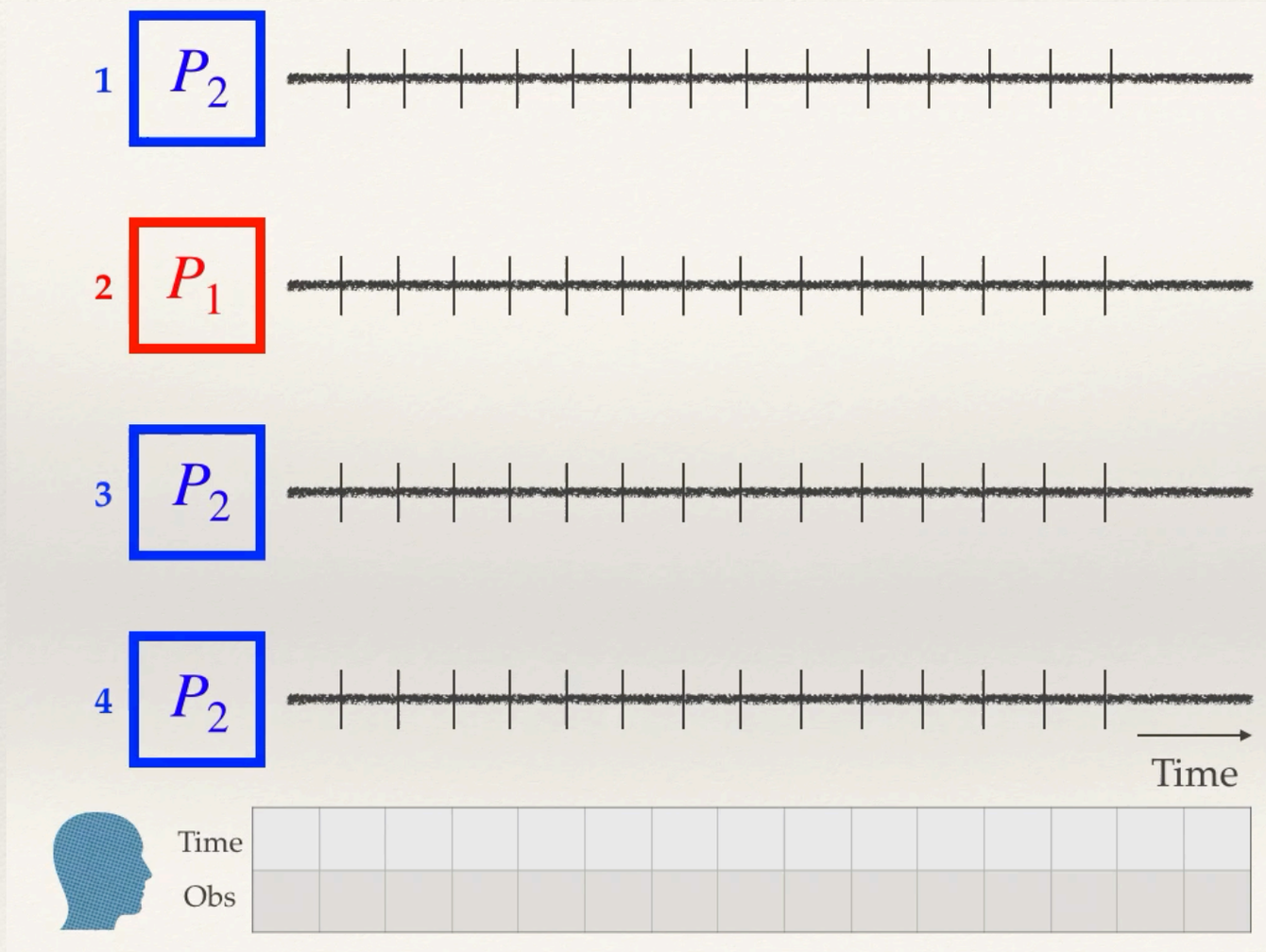

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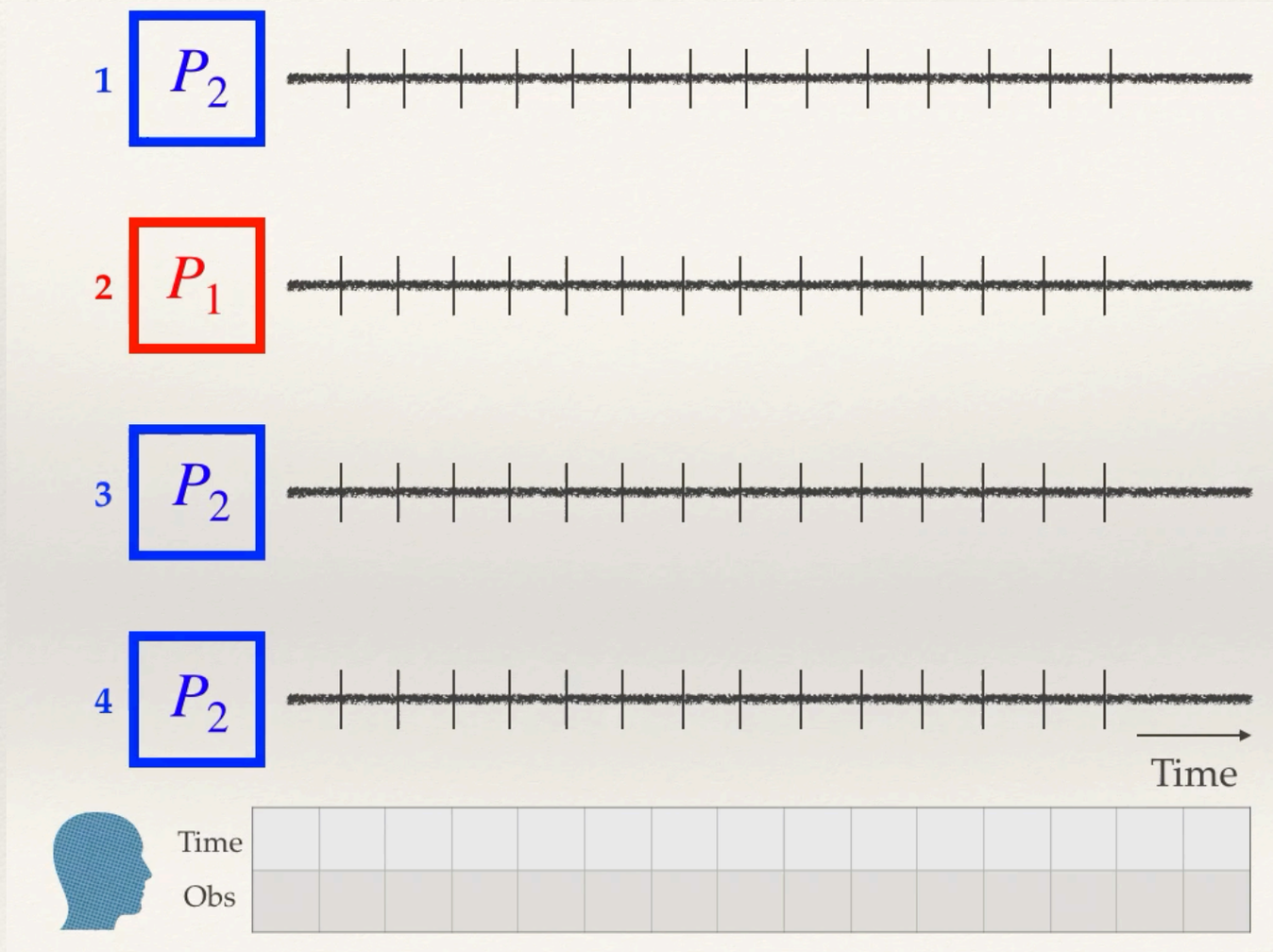

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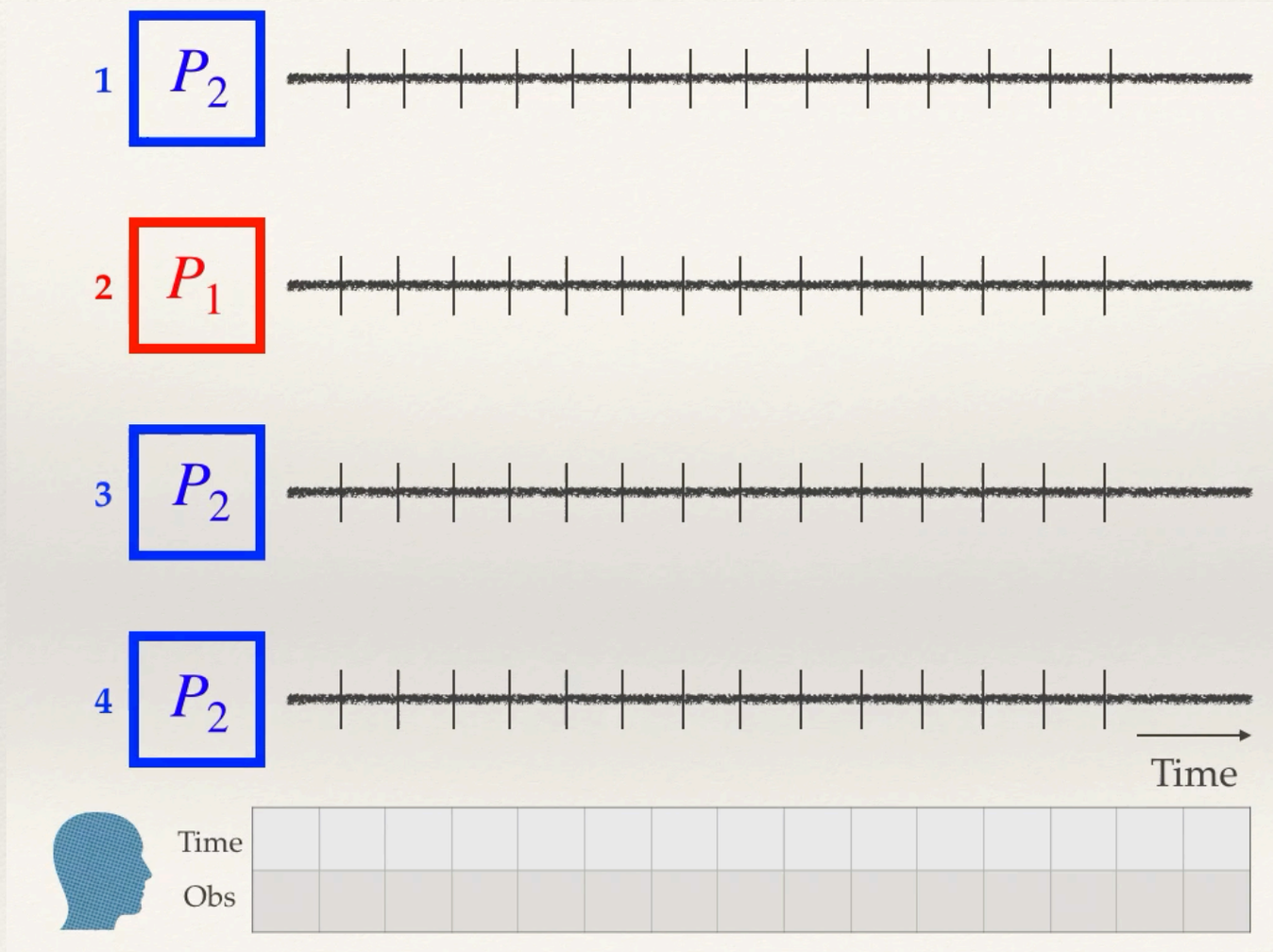

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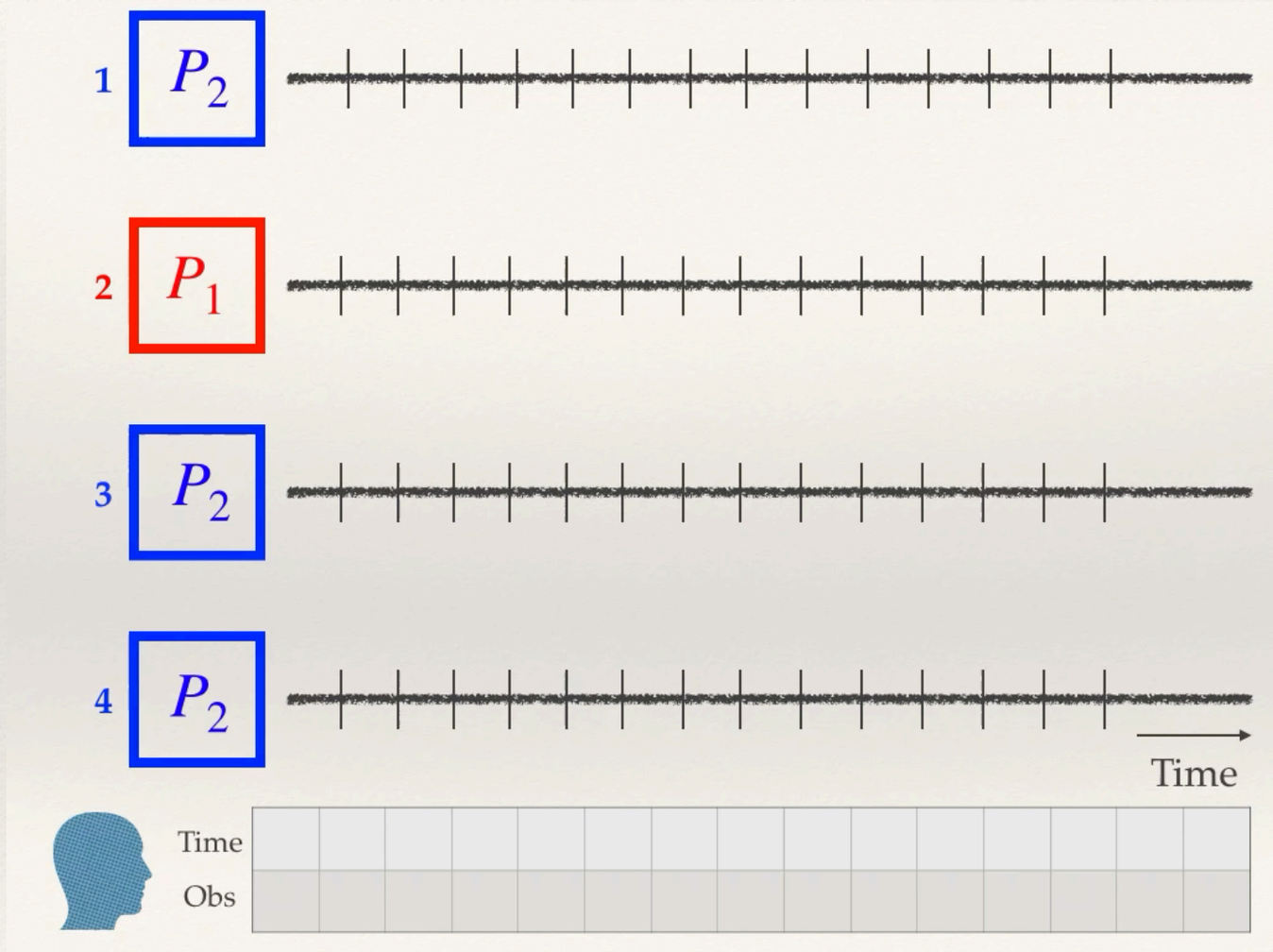

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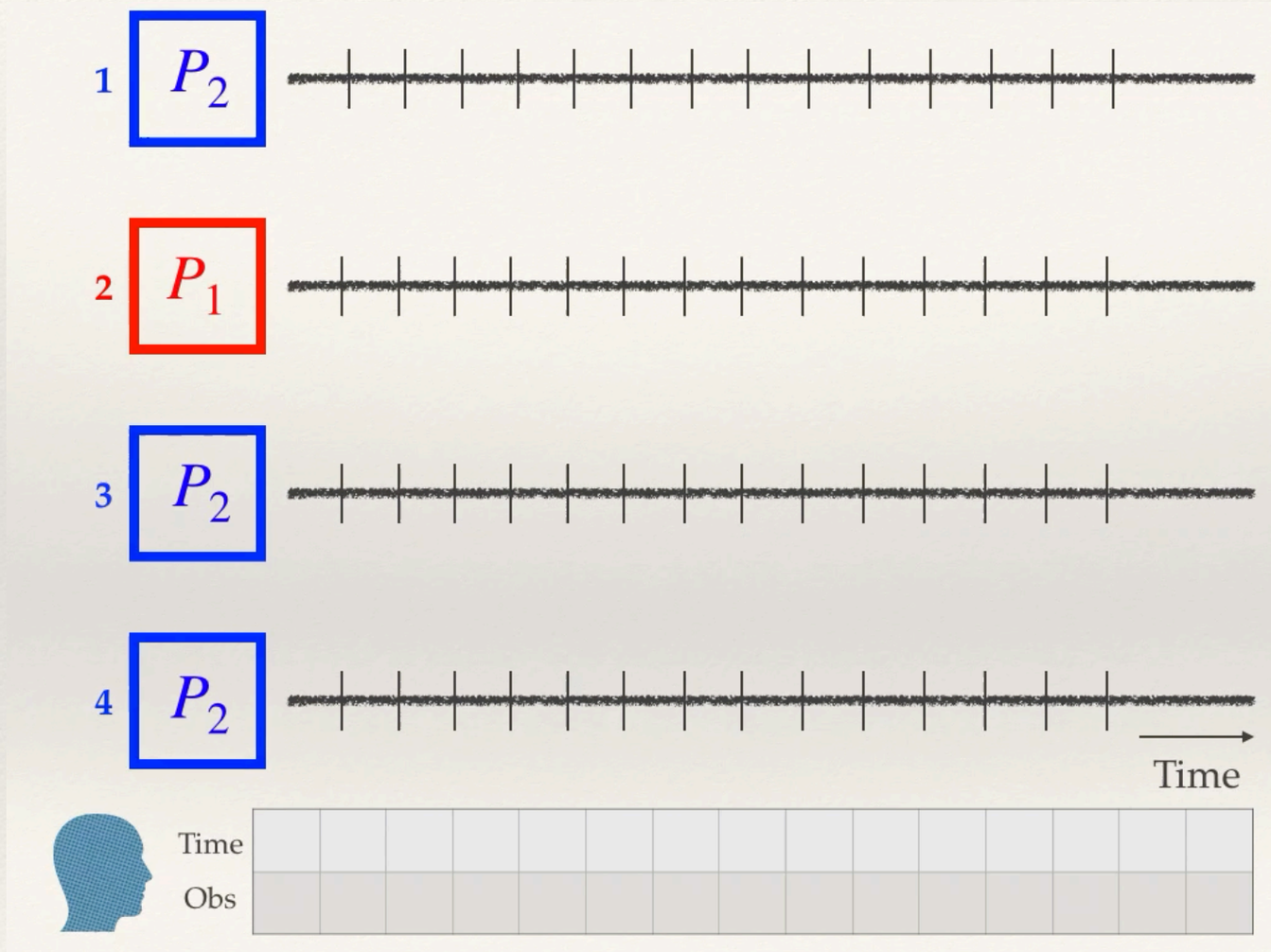

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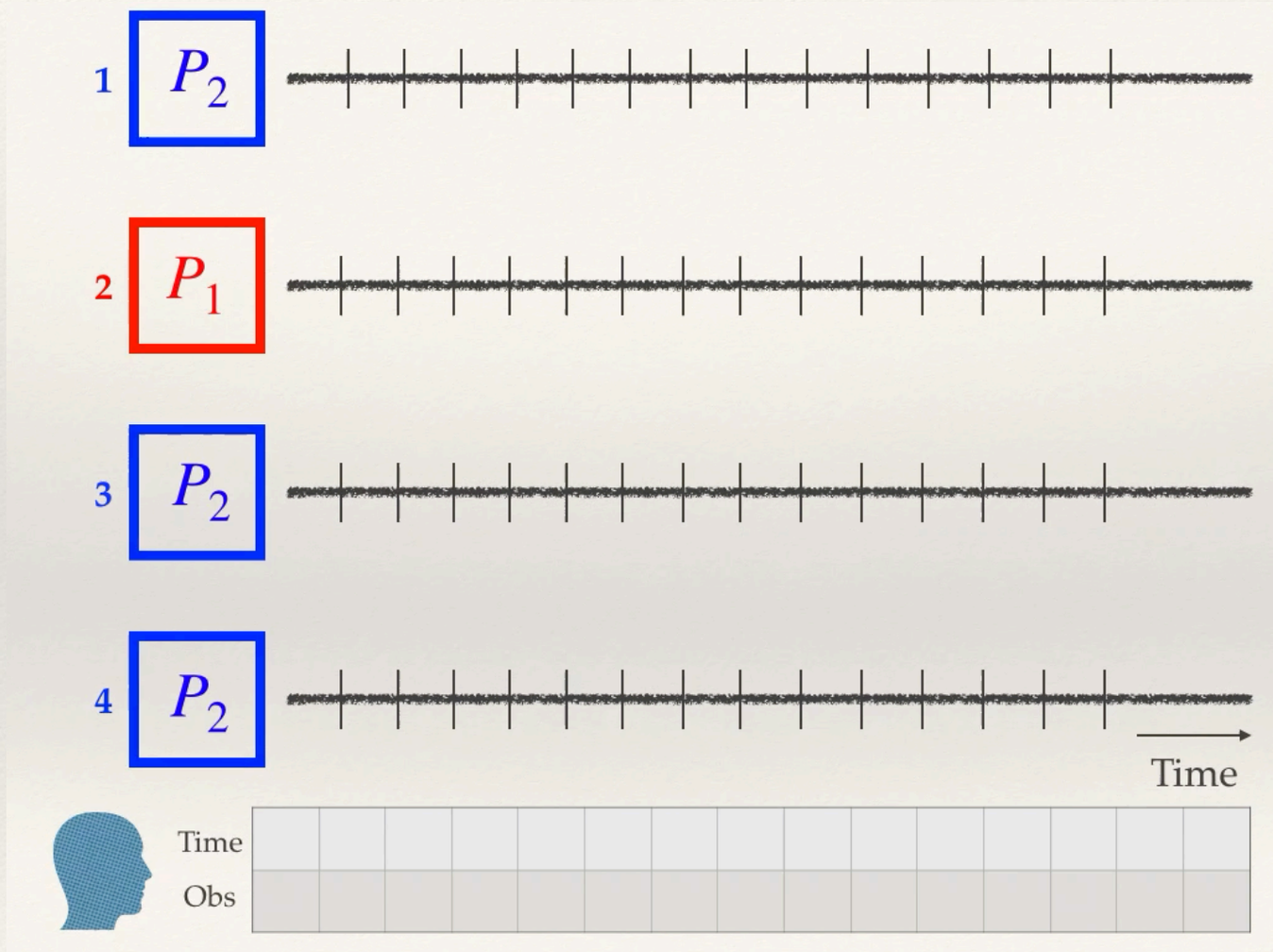

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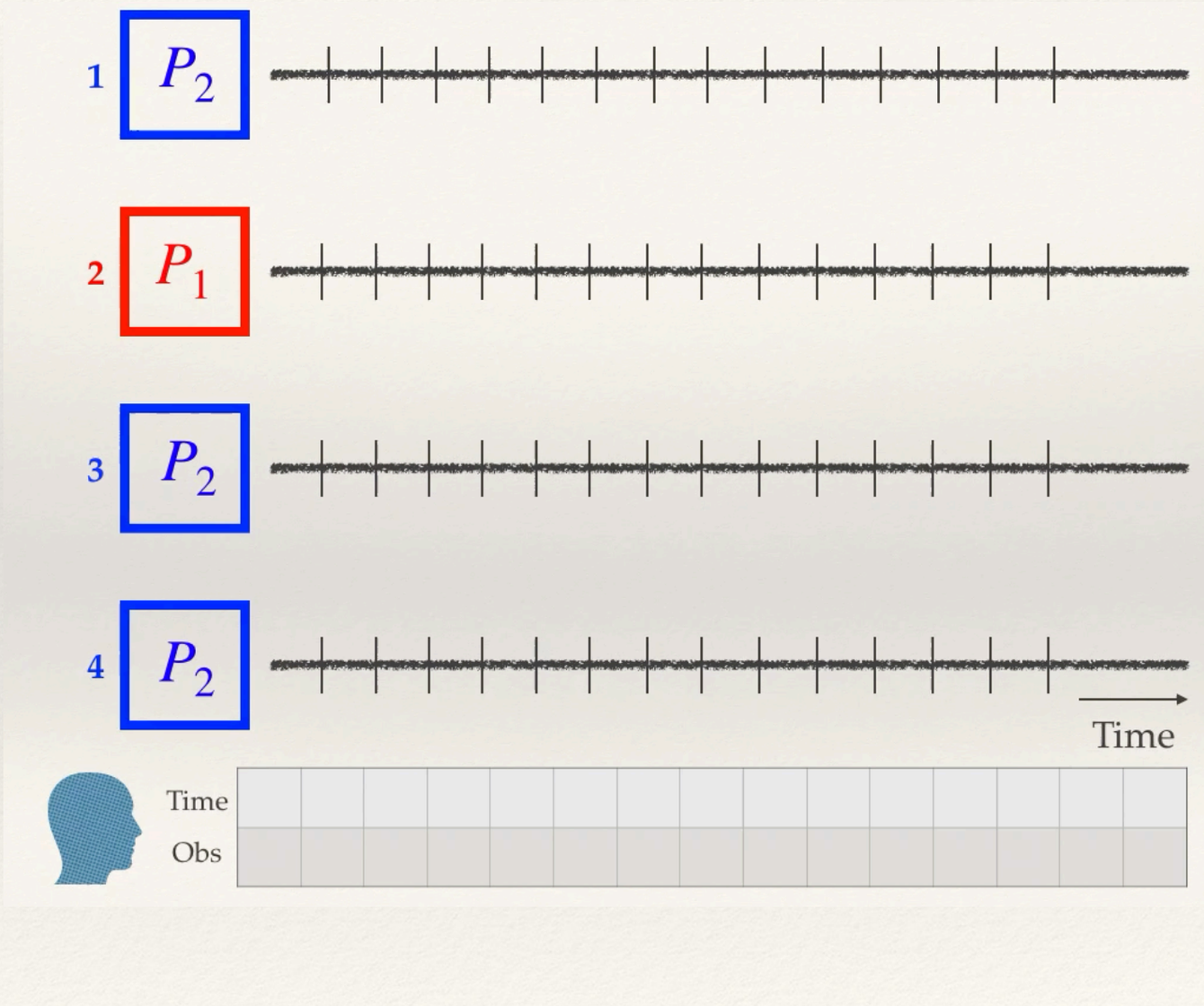

Embedding Our Problem in Chernoff's Framework

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We obtain results for the composite hypotheses case

Our Contributions

- We obtain an asymptotic[1] lower bound on the expected number of samples required to identify the odd arm as a function of error probability
- We propose an asymptotically optimal scheme which meets the desired error probability criterion
- The scheme is a modification of the classical GLRT with forced exploration
- Key challenges in the Markov setting identified
- We believe our results constitute a key first step towards solving the more difficult case of "restless" Markov arms

[1]The asymptotics is as the error tolerance vanishes.

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Basic entity to work with

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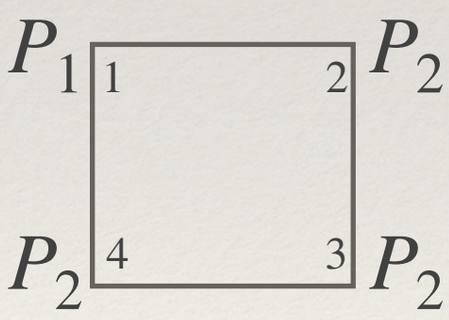

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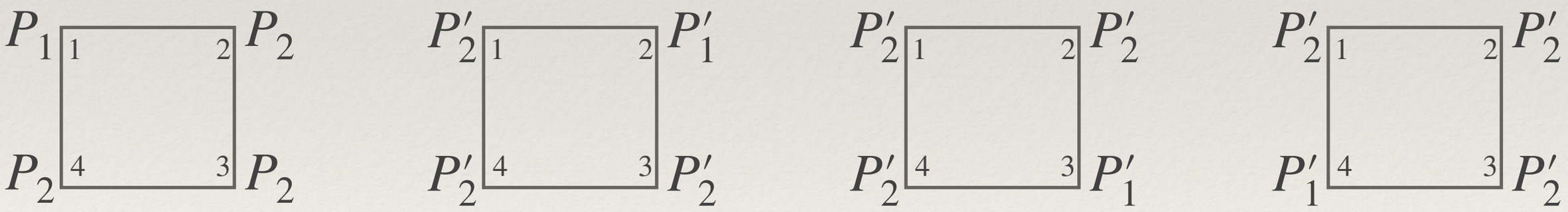

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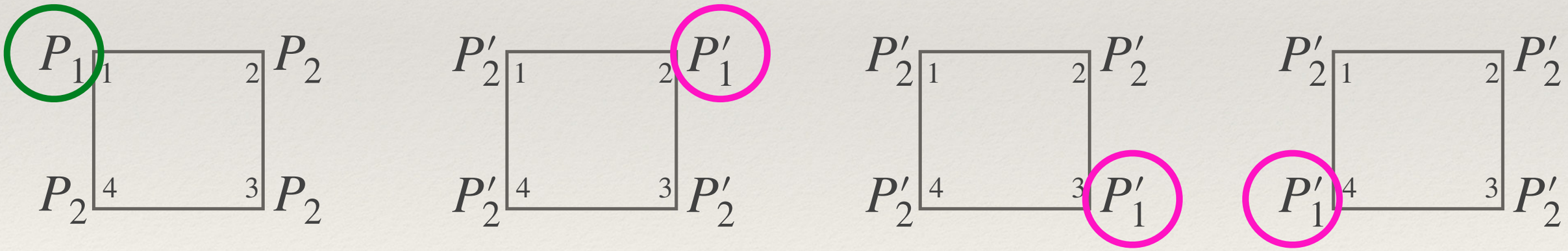

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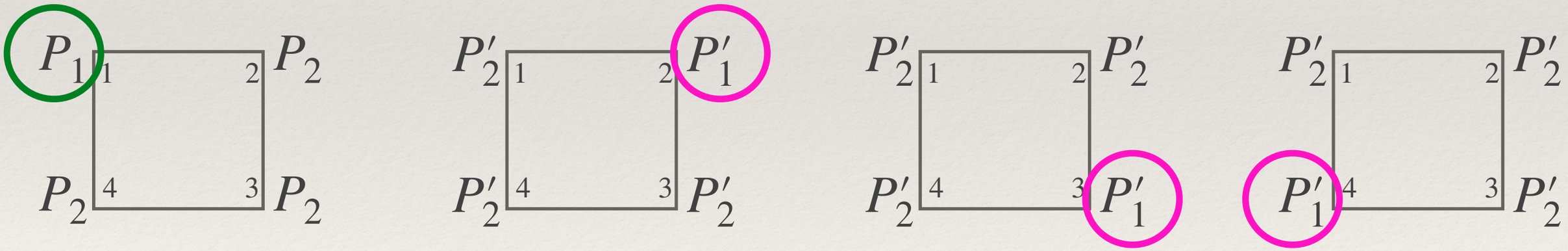
Which alternative configuration is the `nearest' to the true configuration?



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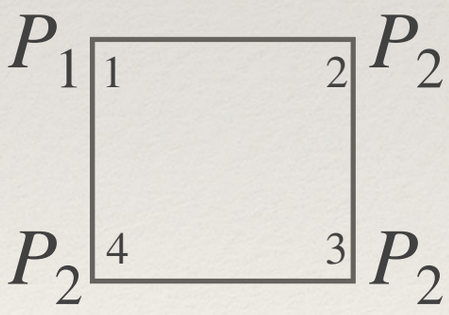

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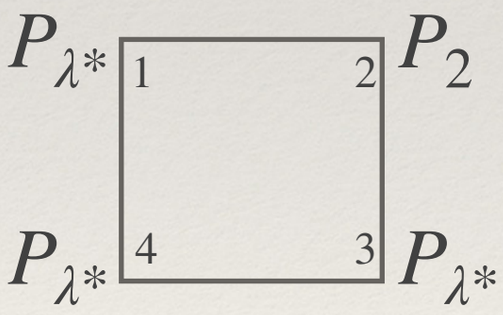

`Nearest' Alternative Configuration

$$\lambda^* = \lambda^*(i, P_1, P_2) = \arg \max_{0 \leq \lambda_1 \leq 1} \left\{ \lambda_1\, D(P_1 || P_{\lambda_1} \mid \mu_1) + (1 - \lambda_1) \frac{(K-2)}{(K-1)} D(P_2 || P_{\lambda_1} \mid \mu_2) \right\}$$

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- Instead of replacing the numerator of $Z_i(n)$ by max as in GLR statistic, we replace it by an *average* computed with respect to an artificial prior on P_1, P_2. We call this *modified GLR statistic*

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    - If coin falls tails, select the next arm according to the optimal distribution for the configuration $(i^*(n), \hat{P}_1^n, \hat{P}_2^n)$

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- **Asymptotic optimality**:

$$\lim_{\delta \downarrow 0} \lim_{\epsilon \downarrow 0} \frac{E[\tau(\pi^\star(\delta))\,|\,(i, P_1, P_2)]}{-\log \epsilon} \leq \frac{1}{D^*(i, P_1, P_2)}$$

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Sequential Controlled Sensing for Composite Multihypothesis Testing

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Abstract

The problem of multi-hypothesis testing with controlled sensing of observations is considered. The distribution of observations collected under each control is assumed to follow a single-parameter exponential family distribution. The goal is to design a policy to find the true hypothesis with minimum expected delay while ensuring that probability of error is below a given constraint. The decision maker can control the delay by intelligently choosing the control for observation collection in each time slot. We derive a policy that satisfies the given constraint on the error probability. We also show that the policy is asymptotically optimal in the sense that it asymptotically achieves an information-theoretic lower bound on the expected delay.

24 Oct 2019 (ArXiv)

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Thank you!