Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

AI5030: Probability and Stochastic Processes

Quiz 2

Date: 28 August 2024

Question	1(a)	1(b)	2	Total
Marks Scored				

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

1. A box contains one coupon labelled 1, two coupons labelled 2, and so on up to ten coupons labelled 10. Two coupons are drawn simultaneously and uniformly at random from the box.

   (a) **(2 Marks)**
   Specify Ω and $\mathbb{P}$ for the experiment, assuming that $\mathscr{F} = 2^{\Omega}$.

   Solution: Notice that there are a total of $1+2+\cdots+10 = 55$ coupons in the box. Writing (i,j) to denote the outcome in which one of the coupons drawn has the label i and the other coupon has label j, we have

$$\Omega = \Big\{(1,2),(1,3),\ldots,(1,10),\\ (2,1),(2,2),\ldots,(2,10),\\ (3,1),(3,2),\ldots,(3,10),\\ \vdots\\ (10,1),(10,2),\ldots,(10,10)\Big\}.$$

   There are 9 outcomes of the form $(1,\cdot)$, 10 outcomes of the form $(j,\cdot)$ for $j \in \{2,\ldots,10\}$. However, noting that the outcome (i,j) is same as (j,i), we note that there are a total of 54 outcomes in Ω.
   Assuming that $\mathscr{F} = 2^{\Omega}$, we then have

$$\mathbb{P}(\{(i,j)\}) = \begin{cases} \dfrac{\binom{i}{1}\cdot\binom{j}{1}}{\binom{55}{2}}, & i \neq j,\\[2ex] \dfrac{\binom{i}{2}}{\binom{55}{2}}, & i = j. \end{cases}$$

   (b) **(2 Marks)**
   Find the probability of the event that the two coupons carry the same number.

Solution: Let E be the event in question. Then, we have $E = \bigcup_{i=2}^{10}\{(i,i)\} = \{(2,2),(3,3),\ldots,(10,10)\}$. Furthermore,

$$\mathbb{P}(E) = \sum_{i=2}^{10}\mathbb{P}(\{(i,i)\}) = \sum_{i=2}^{10}\frac{\binom{i}{2}}{\binom{55}{2}} = \frac{1}{9}.$$

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2. **(1 Mark)**

   Consider the collection
$$\mathscr{D} = \Big\{(-\infty, x] : \, x \in \mathbb{R}\Big\}.$$
   Express the open interval $(2, 3)$ via countable unions, complements, and/or countable intersections of sets in $\mathscr{D}$.

   Solution: Note that $(2,\ 3) = (-\infty,\ 3) \cap (2,\ +\infty)$. Therefore, it suffices to express $(-\infty,\ 3)$ and $(2,\ +\infty)$ via countable unions, complements, and/or countable intersections of sets in $\mathscr{D}$. We have
$$(-\infty,\ 3) = \bigcup_{n=1}^{\infty} \left(-\infty,\ 3 - \frac{1}{n}\right], \qquad (2,\ +\infty) = \Big((-\infty,\ 2]\Big)^c.$$