Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

भारतीय सांकेतिक विज्ञान संस्थ हैदराबाद
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090: Stochastic Processes

Quiz 6

Date: 22 April 2025

Question	1	2	Total
Marks Scored			

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Assume that all random variables appearing in the problems below are defined w.r.t. $\mathscr{F}$.

1. **(2 Marks)**

   Let P be an irreducible, doubly stochastic matrix on the finite state space $\mathcal{X} = \{1, 2, \ldots, K\}$.
   Given two disjoint sets $A, B \subseteq \mathcal{X}$, $A \cap B = \emptyset$, define the probability flux from A to B as

   $$\Phi(A, B) \coloneqq \sum_{x \in A} \sum_{y \in B} \pi_x \, P_{x,y},$$

   where $\pi = [\pi_x : x \in \mathcal{X}]$ satisfies $\pi = \pi P$.
   Express $\Phi(\{1\}, \{1\}^c)$ in terms of the entries of P.

Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

2. Let P be an irreducible, doubly stochastic TPM on a finite state space $\mathcal{X}$, with $P_{x,x} < 1$ for all $x \in \mathcal{X}$.

   (a) **(1 Mark)**
   Identify a stationary distribution for P.

   (b) **(2 Marks)**
   Consider another TPM $\widetilde{P}$ defined by

$$\widetilde{P}_{x,y} = \begin{cases} 0, & x = y, \\ \dfrac{P_{x,y}}{1 - P_{x,x}}, & x \neq y. \end{cases}$$

   Identify a stationary distribution for $\widetilde{P}$.