Name:
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Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090: Stochastic Processes

Quiz 3

Date: 25 March 2025

Question	1	2	Total
Marks Scored			

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Assume that all random variables appearing in the problems below are defined w.r.t. $\mathscr{F}$.

1. Suppose that a fair die is rolled sequentially and independently over time.
Let $X_n, n \in \mathbb{N}$, denote the outcome of the die at time n.
Set $T_1 = 1$, and for each $k \in \{2, \dots, 6\}$, let

$$T_k := \inf \left\{ n > T_{k-1} : X_n \in \{1, \dots, 6\} \setminus \{X_{T_1}, \dots, X_{T_{k-1}}\} \right\}$$

denote the first time instant of observing the kth distinct face of the die. Furthermore, let

$$S_1 = T_1 = 1, \qquad S_k := T_k - T_{k-1}, \quad k \in \{2, \dots, 6\}.$$

(a) **(1 Mark)**
Write down the PMF of S_k for any $k \in \{2, \dots, 6\}$.

(b) **(1 Mark)**
Using the PMF of part (a), compute $\mathbb{E}[S_k]$ for any $k \in \{2, \dots, 6\}$.

(c) **(1 Mark)**
Compute the expected time required to observe all six faces of the die.
Hint: Use the result of part (b).

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2. **(2 Marks)**

Consider a filtration $\{\mathscr{F}_k\}_{k\in\mathbb{N}}$ in which $\mathscr{F}_k \subseteq \mathscr{F}_{k+1} \subset \mathscr{F}$ for all $k \in \mathbb{N}$.
Let N be a stopping time w.r.t the filtration $\{\mathscr{F}_k\}_{k\in\mathbb{N}}$. Let

$$\mathscr{F}_N := \Big\{A \in \mathscr{F} : A \cap \{N = k\} \in \mathscr{F}_k \ \forall k \in \mathbb{N}\Big\}.$$

Show that $\mathscr{F}_N$ is a σ-algebra of subsets of Ω.