

Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)



భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090/EE5817: PROBABILITY AND STOCHASTIC PROCESSES

QUIZ 03

DATE: 12 SEPTEMBER 2025

Question	1	2	Total
Marks Scored			

1. Let $(p_n)_{n \in \mathbb{N}}$ be non-negative numbers with $\sum_{n \in \mathbb{N}} p_n = 1$.
Let $\mathbb{P} : \mathcal{B}(\mathbb{R}) \rightarrow [0, 1]$ be defined as

$$\mathbb{P}(A) := \sum_{n \in \mathbb{N}} p_n \delta_n(A), \quad \text{where for any } n \in \mathbb{N}, \quad \delta_n(A) = \begin{cases} 1, & n \in A, \\ 0, & n \notin A. \end{cases}$$

(a) **(2 Marks)**

Verify that \mathbb{P} is a valid probability measure on $\mathcal{B}(\mathbb{R})$.

(b) **(1 Mark)**

Let $p_n = \frac{1}{4} \cdot \left(\frac{3}{4}\right)^{n-1}$ for all $n \in \mathbb{N}$. If E is the set of even natural numbers, compute $\mathbb{P}(E)$.

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2. Let $(\Omega, \mathcal{F}) = (\mathbb{N}, 2^{\mathbb{N}})$. For each $n \in \mathbb{N}$, let $P_n : \mathcal{F} \rightarrow [0, 1]$ be defined as

$$P_n(A) := \frac{|A \cap \{1, 2, \dots, n\}|}{n}, \quad A \in \mathcal{F}.$$

Given a set $A \in \mathcal{F}$, its density $D(A)$ is defined as

$$D(A) := \lim_{n \rightarrow \infty} P_n(A), \quad \text{provided the limit exists.}$$

Let \mathcal{D} be the collection of all sets whose density is well-defined.

(a) **(1 Mark)**

Show that \mathcal{D} is closed under complements, i.e., if $A \in \mathcal{D}$, then $A^c \in \mathcal{D}$.

(b) **(1 Mark)**

Let $M = \{3k : k = 1, 2, \dots\}$. Find $D(M)$.