Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD   (Tick one)

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भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090: Stochastic Processes
Quiz 2
Date: 04 March 2025

Question	1	2	Total
Marks Scored			

Instructions:

- **Fill in your name and roll number on each of the pages.**

- **You may use any result covered in class directly without proving it.**

- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

1. **(3 Marks)**

   Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of i.i.d. random variables with the common PMF

   $$\mathbb{P}\left(X_1 = \frac{1}{2}\right) = \frac{1}{2} = \mathbb{P}\left(X_1 = -\frac{1}{2}\right).$$

   Show that the above sequence does not converge in probability to any random variable X.

   Note: By definition, we have $X_n \xrightarrow{\text{p.}} X$ if and only if

   $$\lim_{n \to \infty} \mathbb{P}(|X_n - X| > \varepsilon) = 0 \qquad \forall \varepsilon > 0.$$

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2. **(2 Marks)**

   Fix $\alpha > 1$.

   For each $n \in \mathbb{N}$, let $X_n \sim \mathcal{N}\left(0, \frac{1}{n^\alpha}\right)$.

   Show that $X_n \xrightarrow{\text{a.s.}} 0$.

   Hint: Use Borel–Cantelli lemma.