

Almost Cost-Free Communication in Federated Best Arm Identification

Workshop on Information Theory and Data Science
Institute for Mathematical Sciences
National University of Singapore

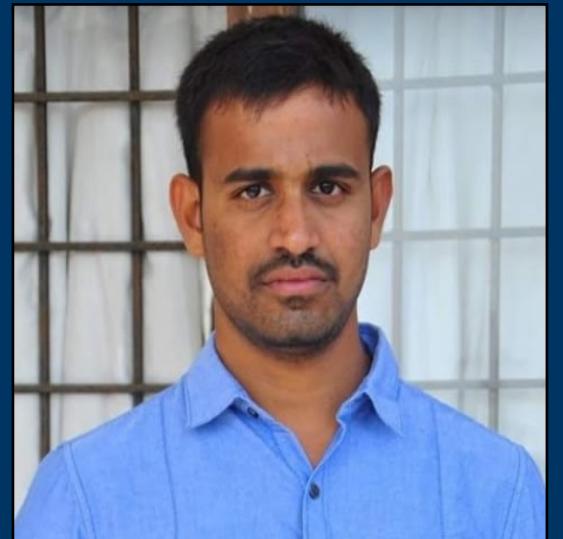
16 January 2023



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Joint Work with

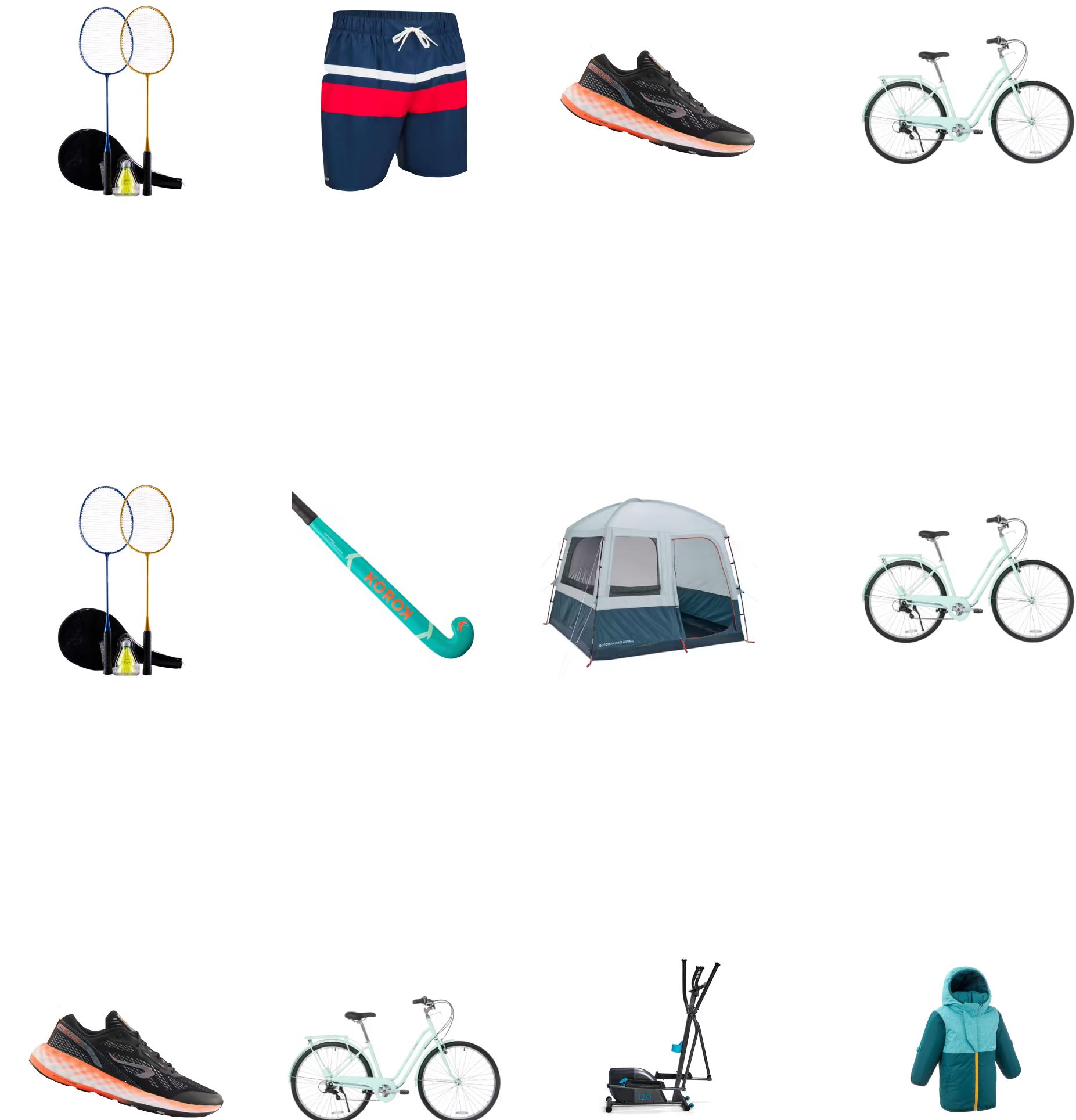
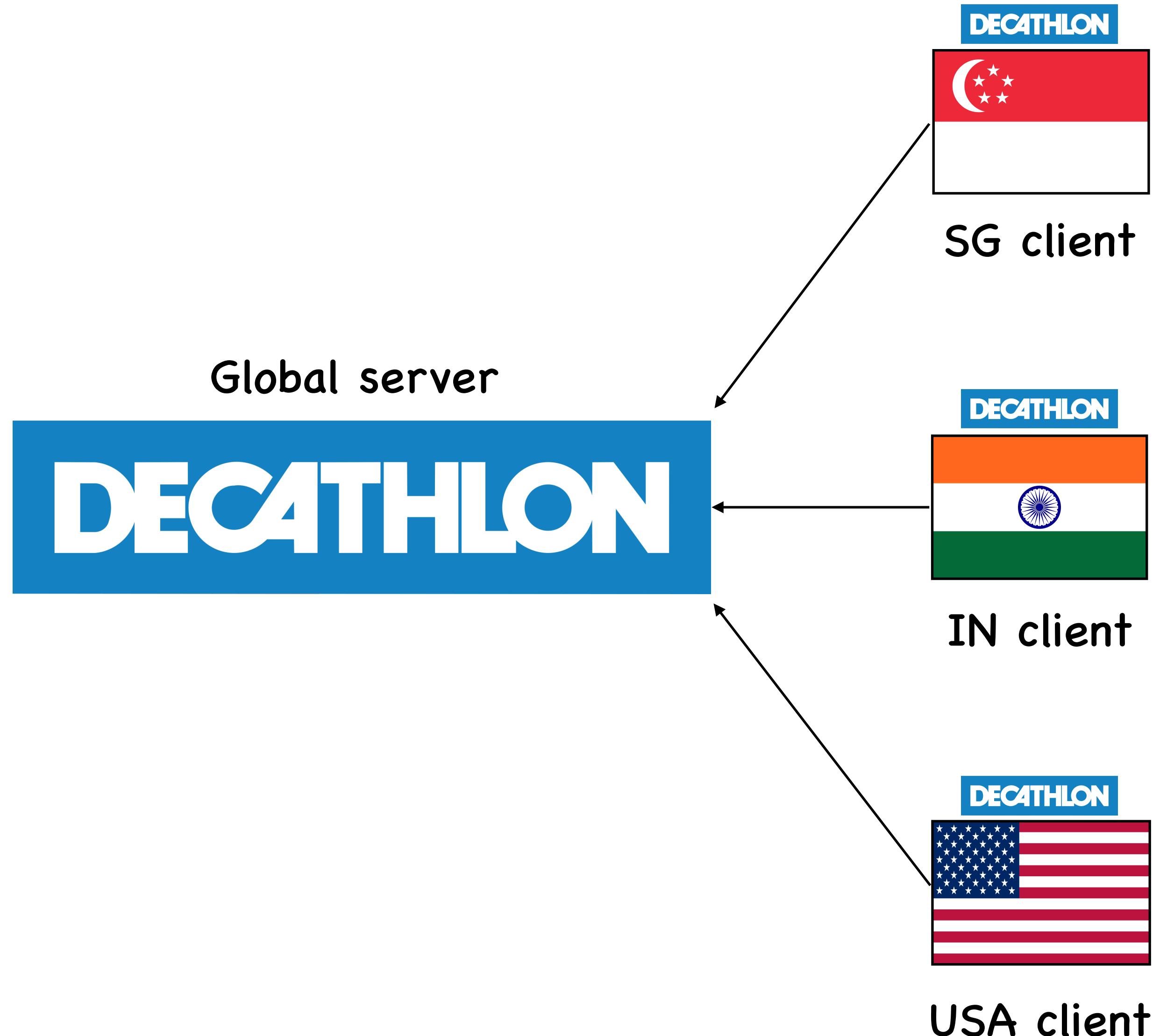


Kota Srinivas Reddy



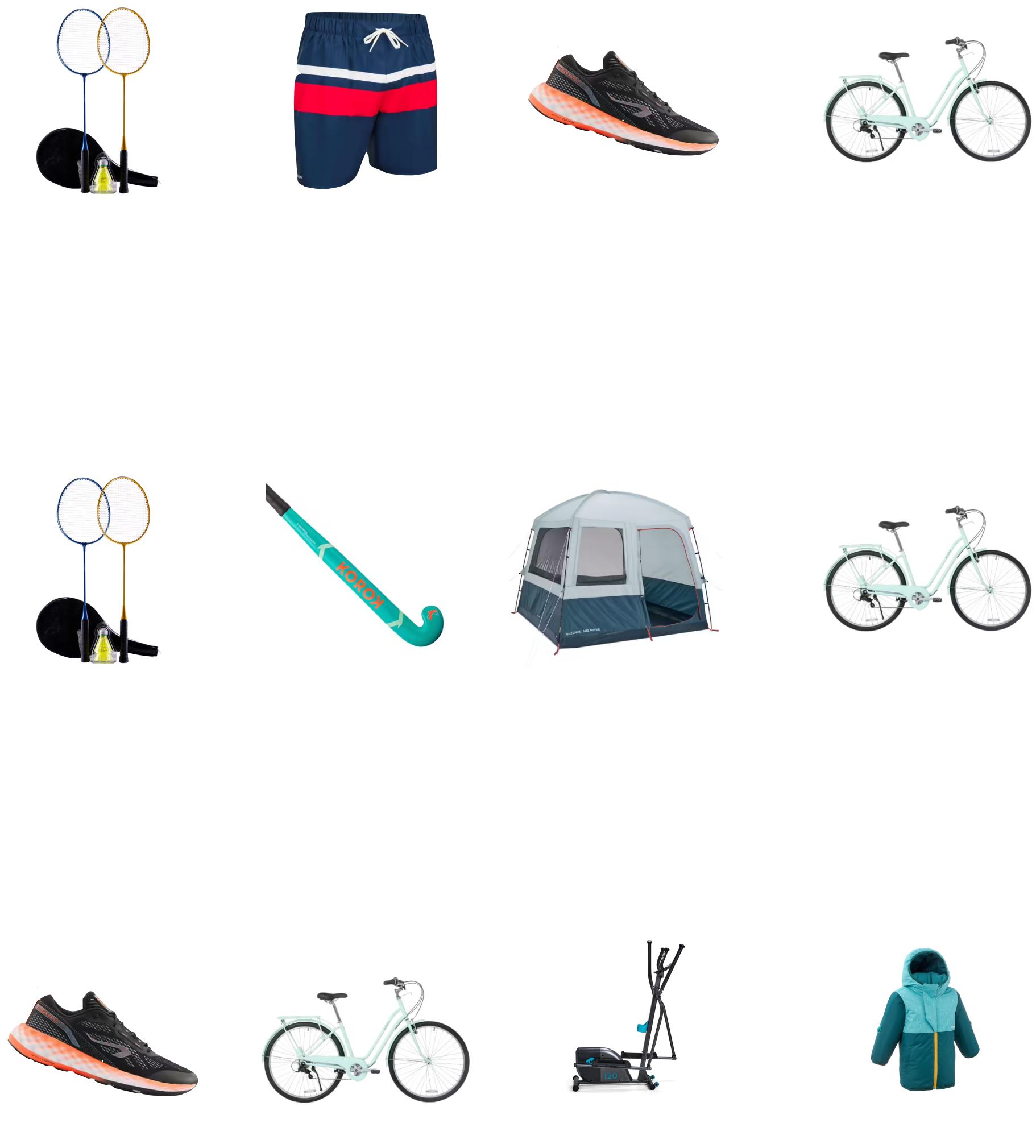
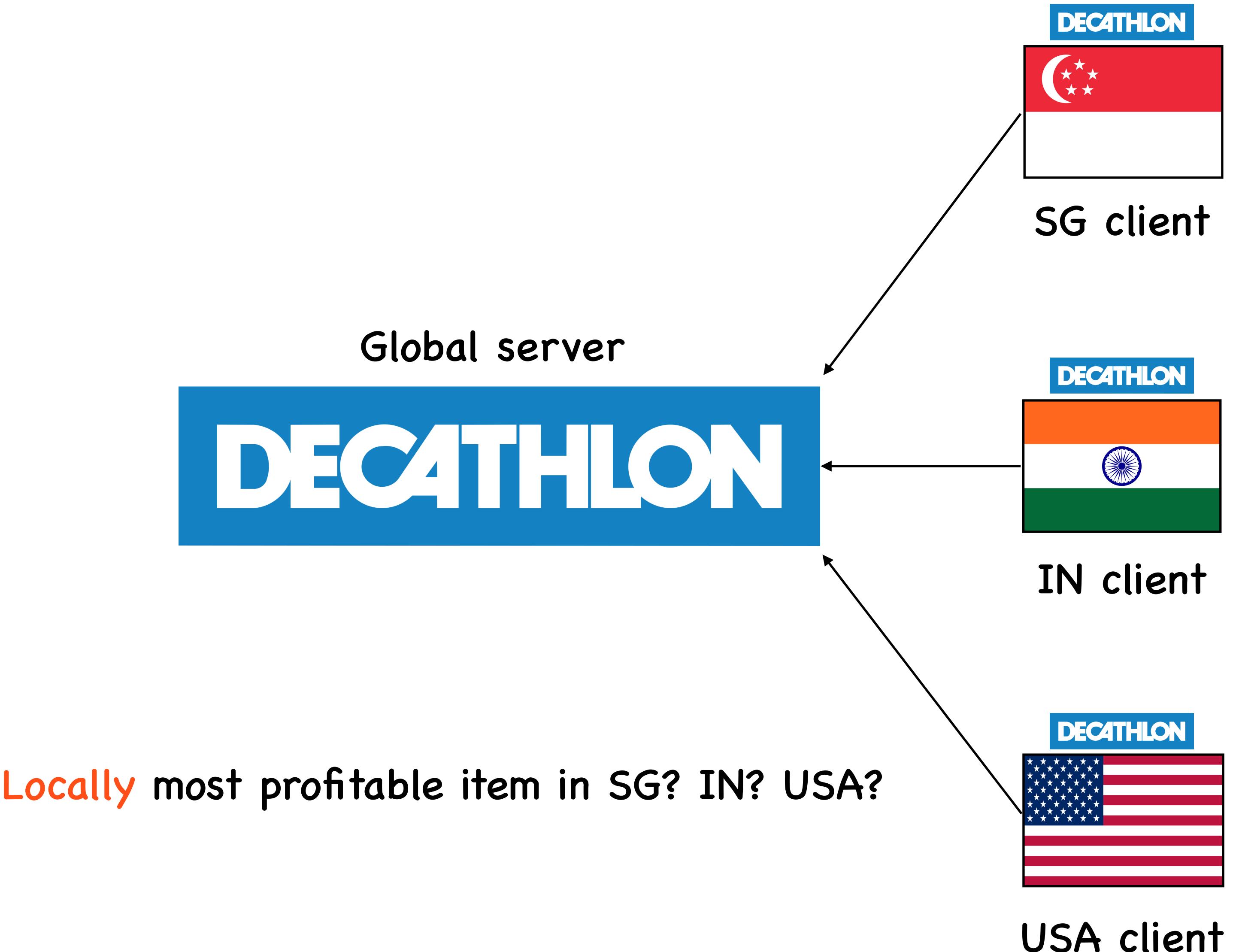
Vincent Y. F. Tan

Federated Best Arm Identification



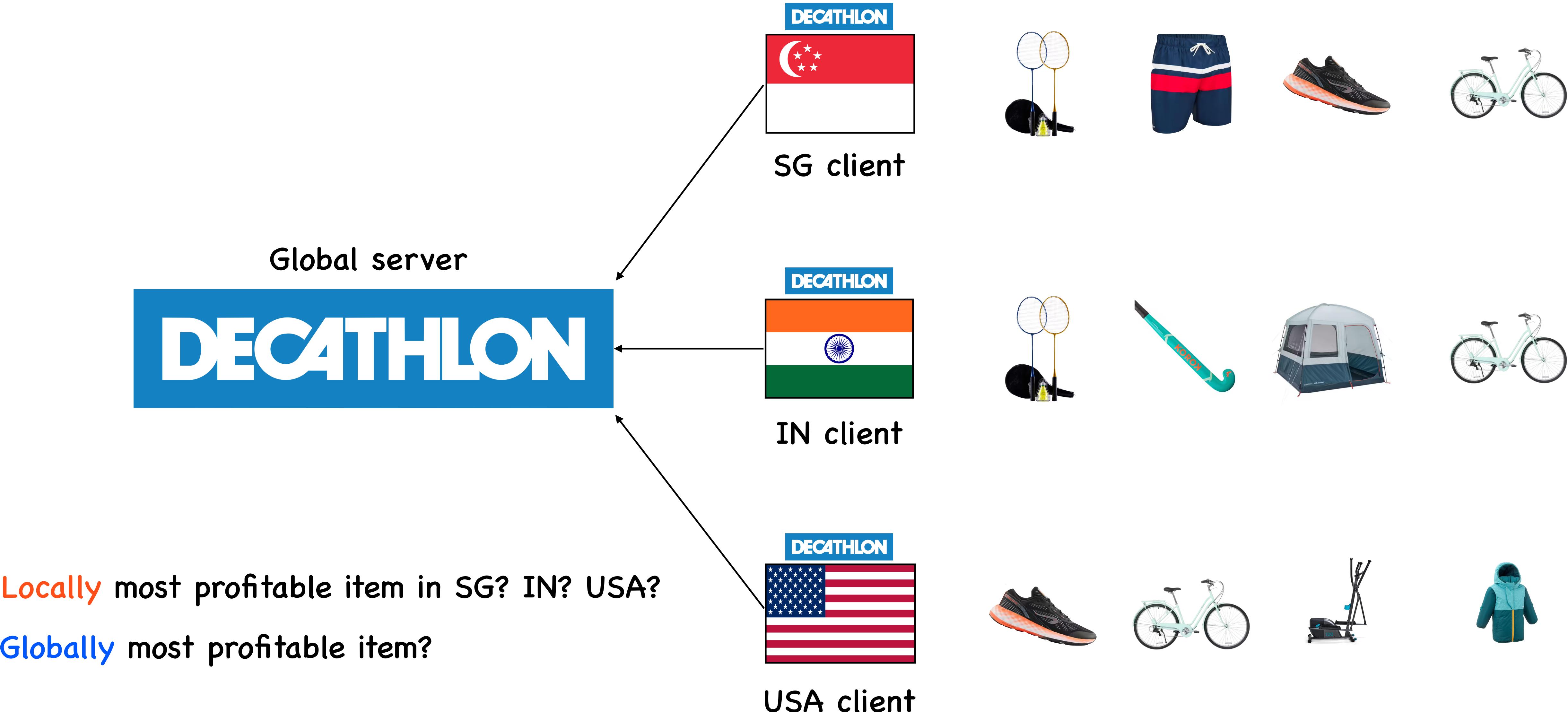
* Pics only for demonstration, courtesy Google

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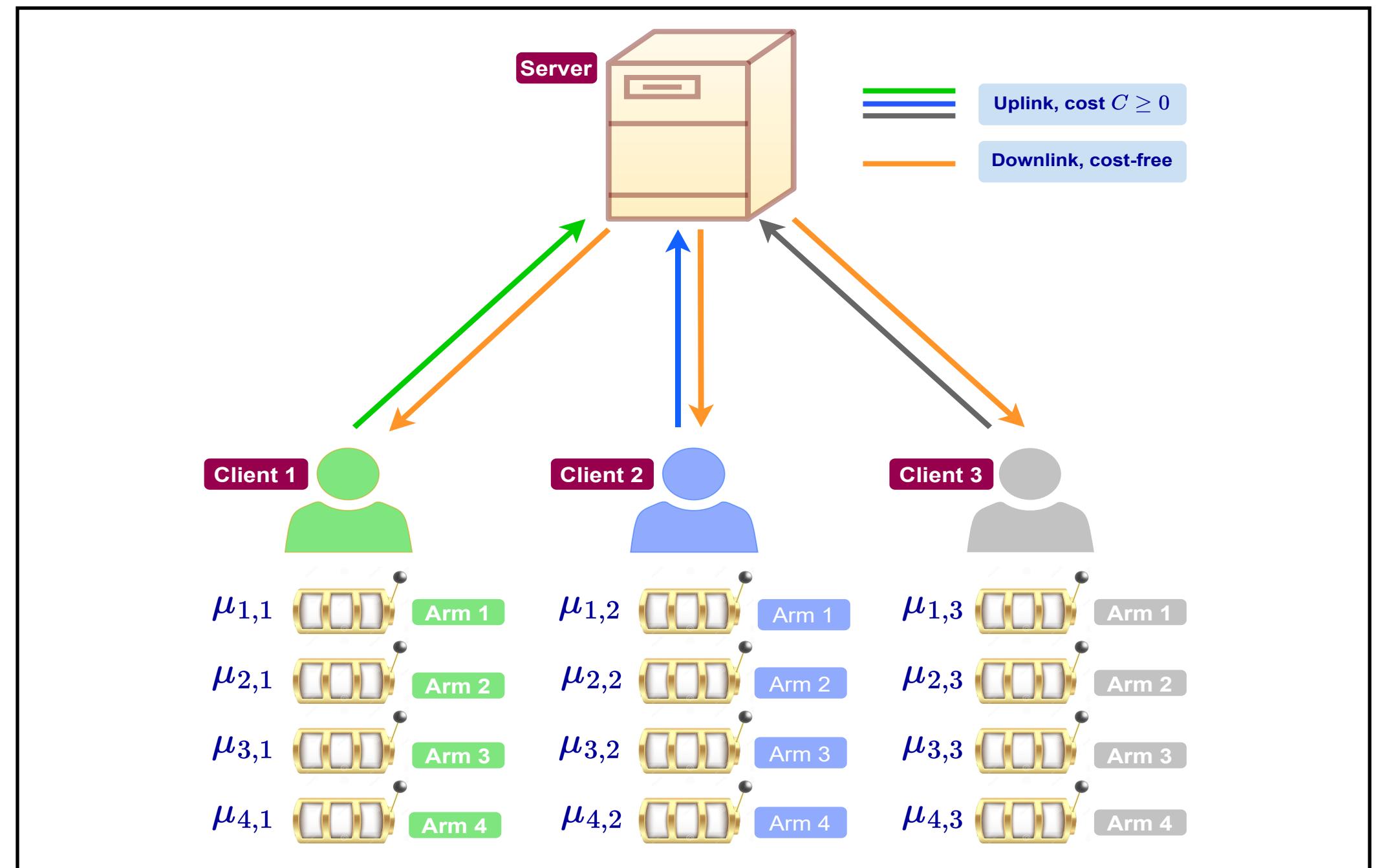
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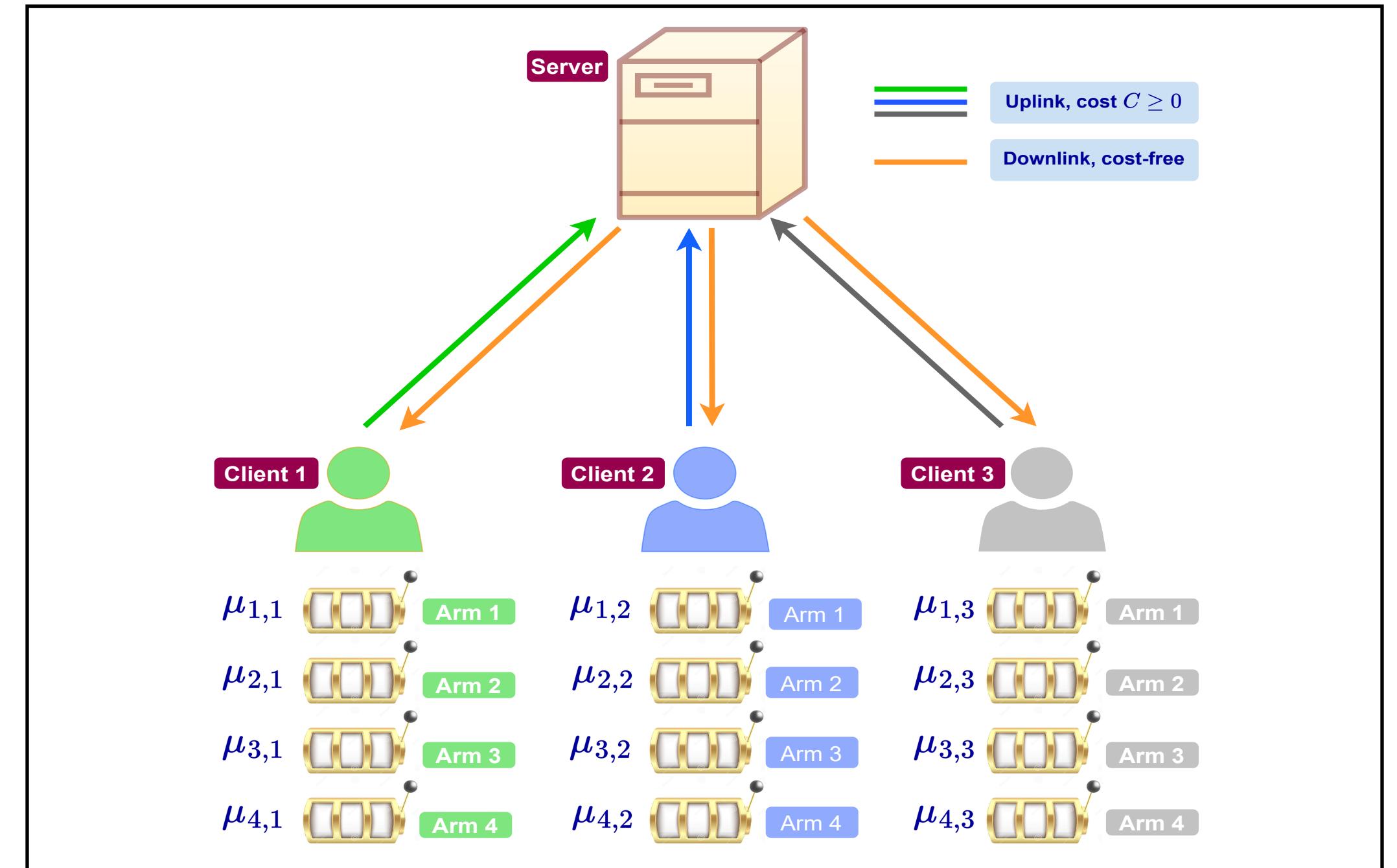
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Problem Abstraction



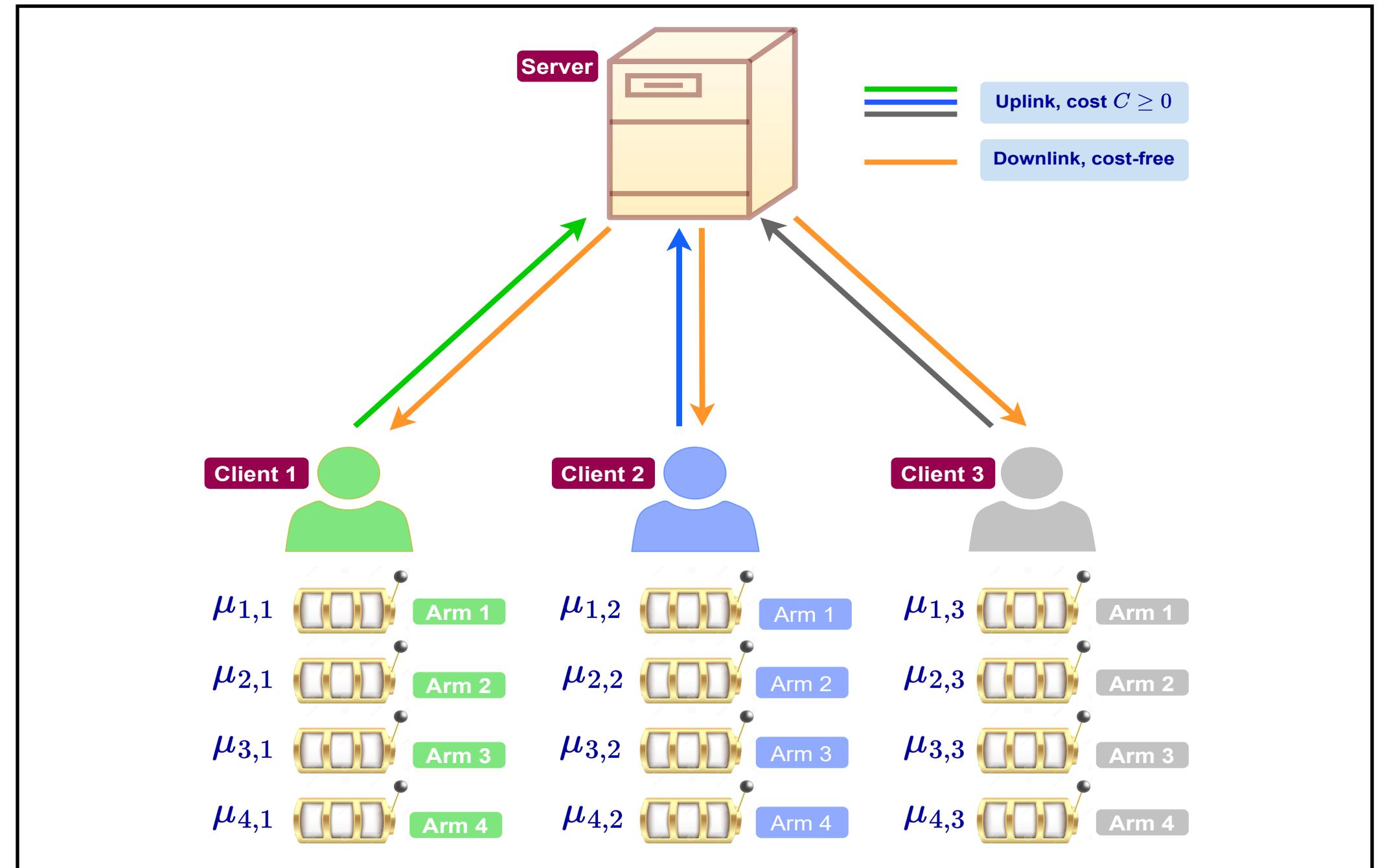
Problem Abstraction

- Single server, M clients



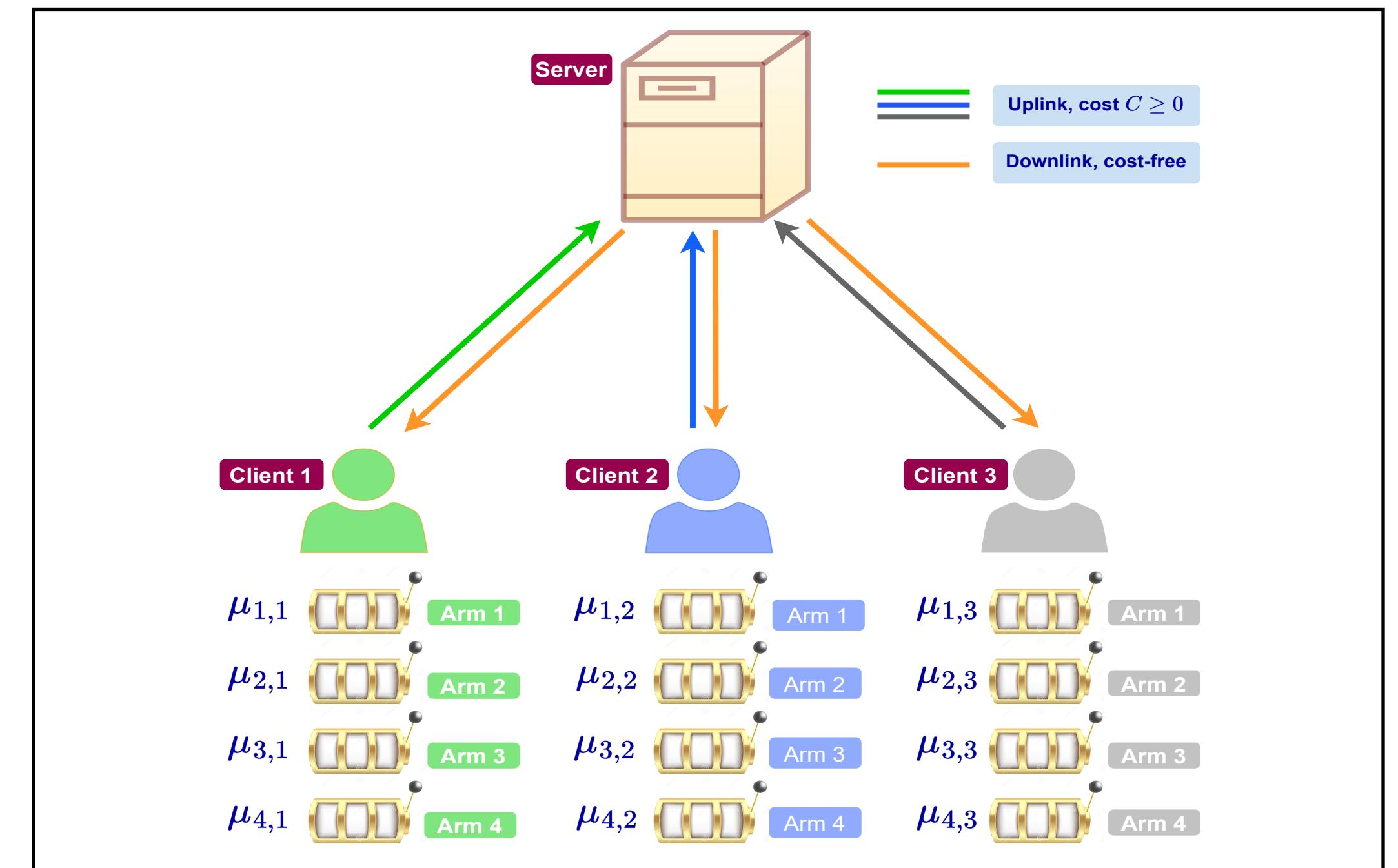
Problem Abstraction

- Single server, M clients
- Each client has access to a K -armed bandit



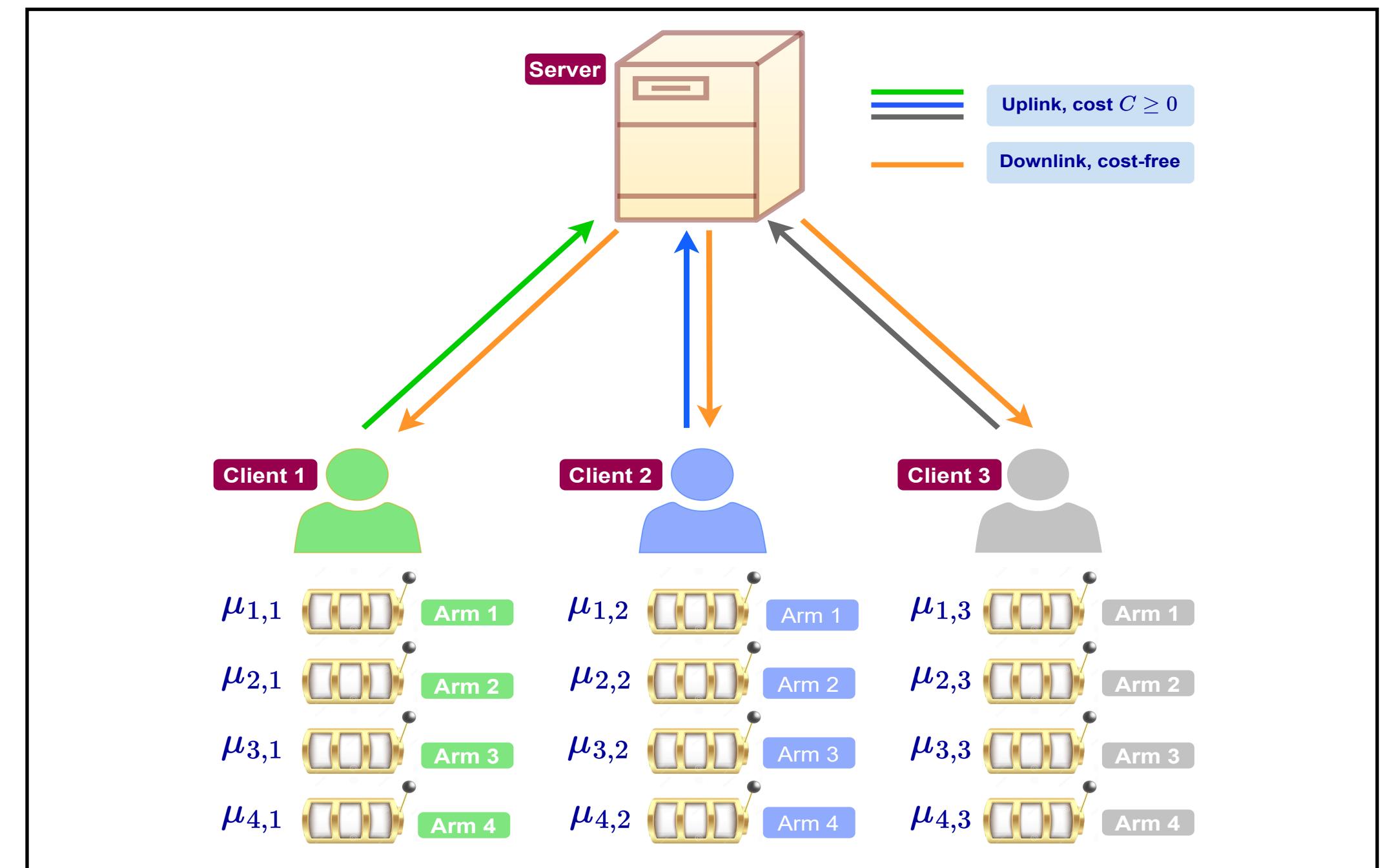
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- Single server, M clients
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- Independent Gaussian rewards from each arm (unit variance)



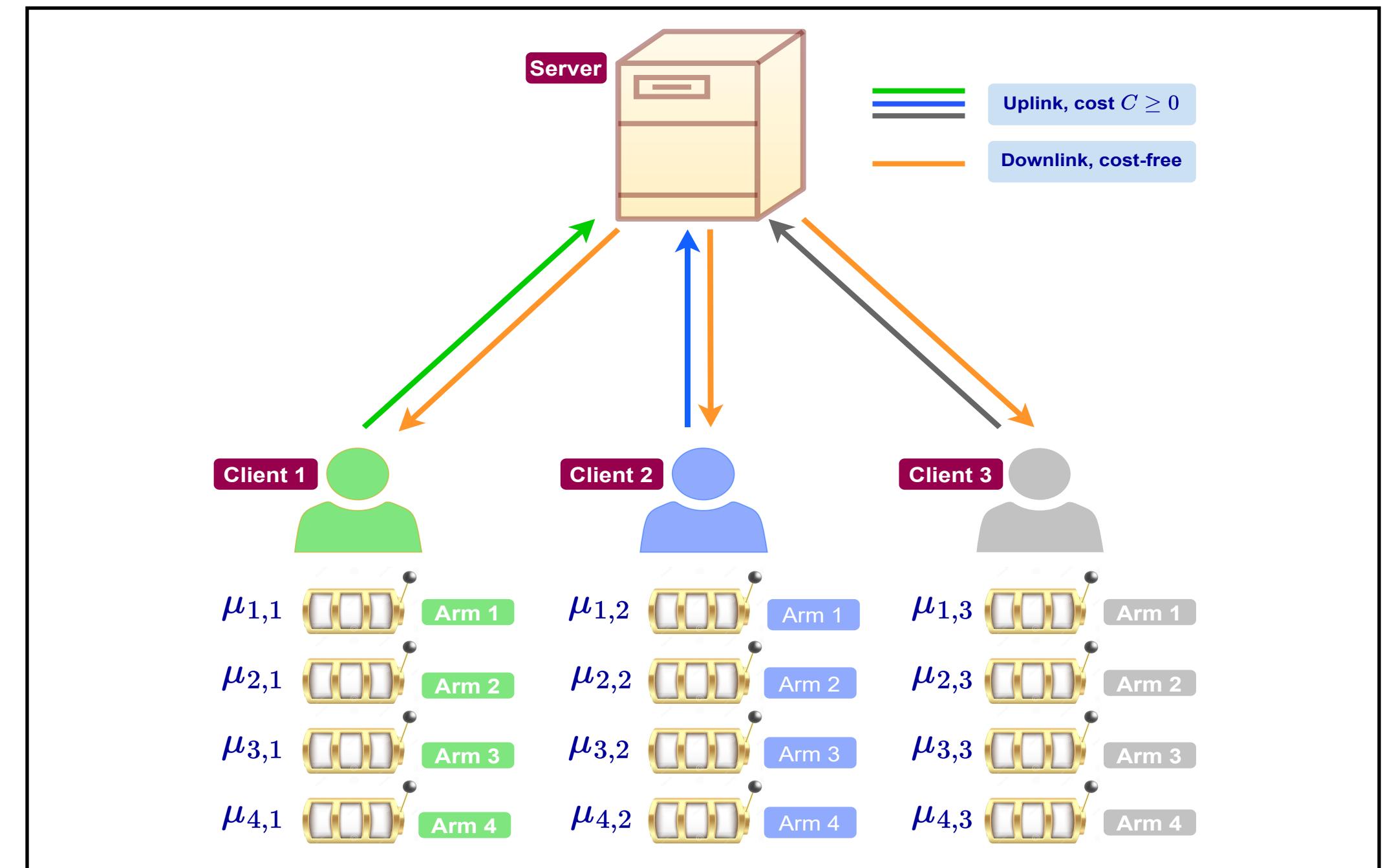
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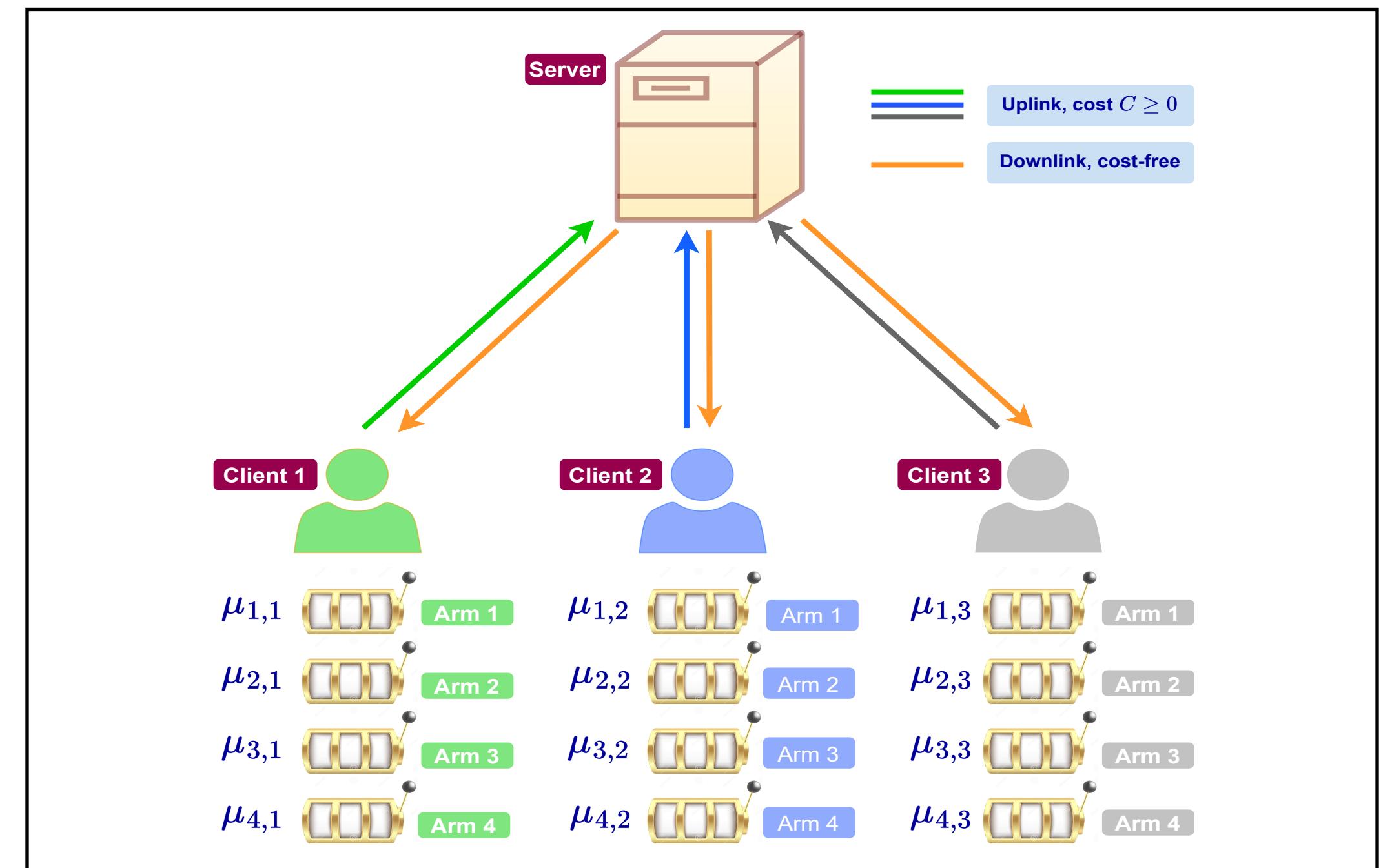
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- Global best arm – $\arg \max_k \frac{1}{M} \sum_{m=1}^M \mu_{k,m}$

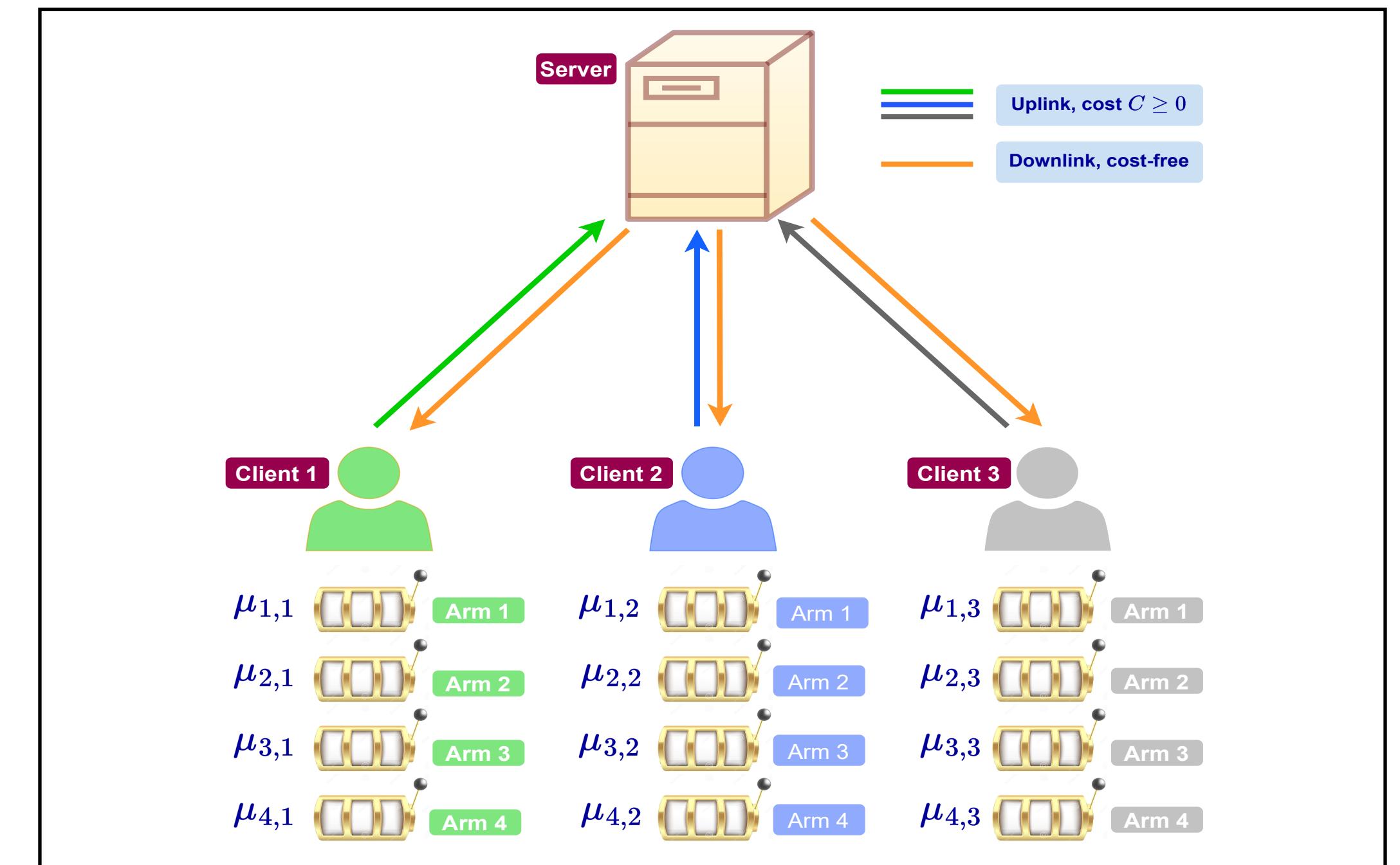


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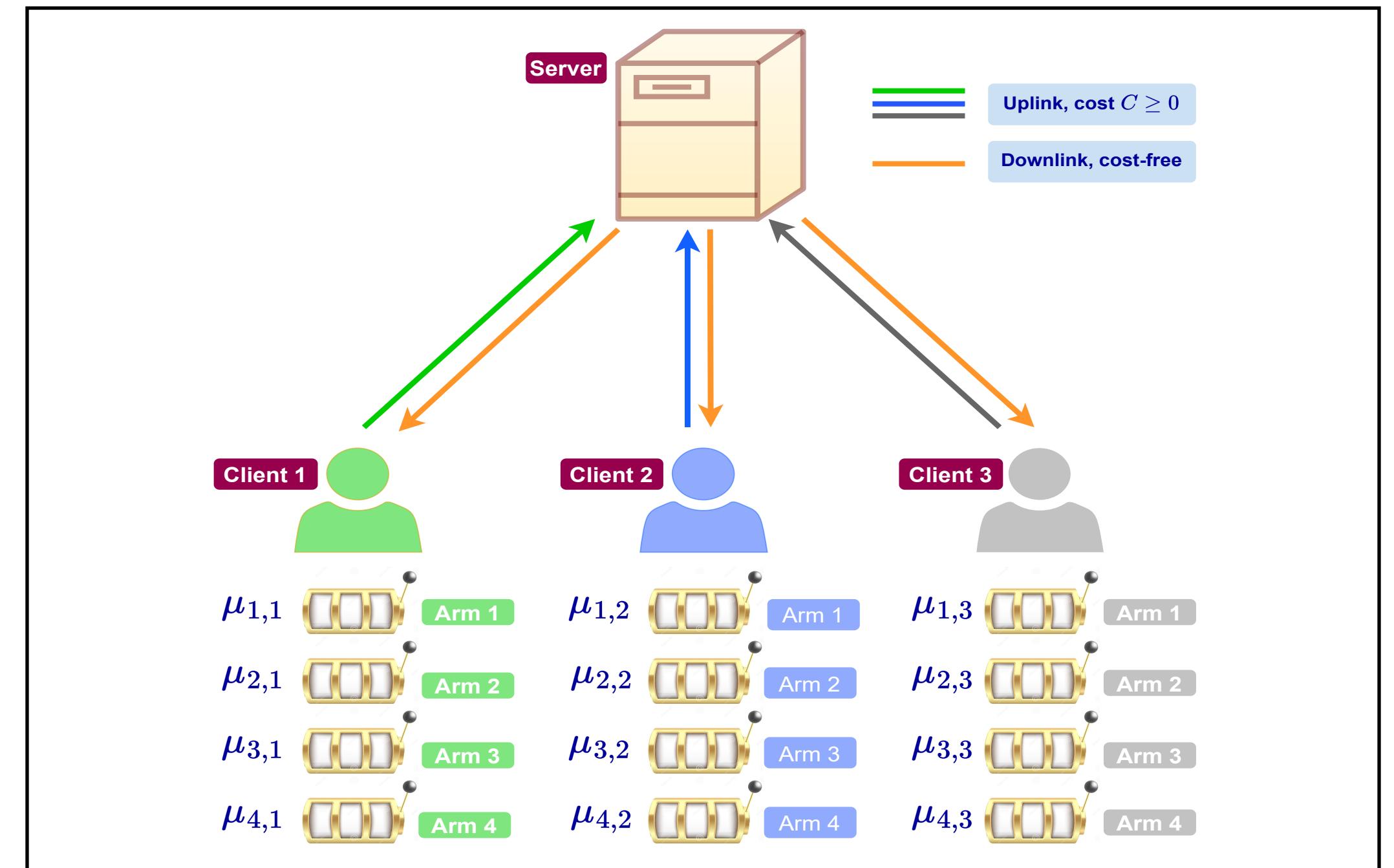
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Downlink cost: nil



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Find local (clients) & global (server) best arms

$$\min \quad \mathbb{E} \left[\underbrace{\text{no. of arm pulls + communication cost}}_{\text{total cost}} \right]$$

s.t. error probability $\leq \delta$

Results at a Glance – 1/3

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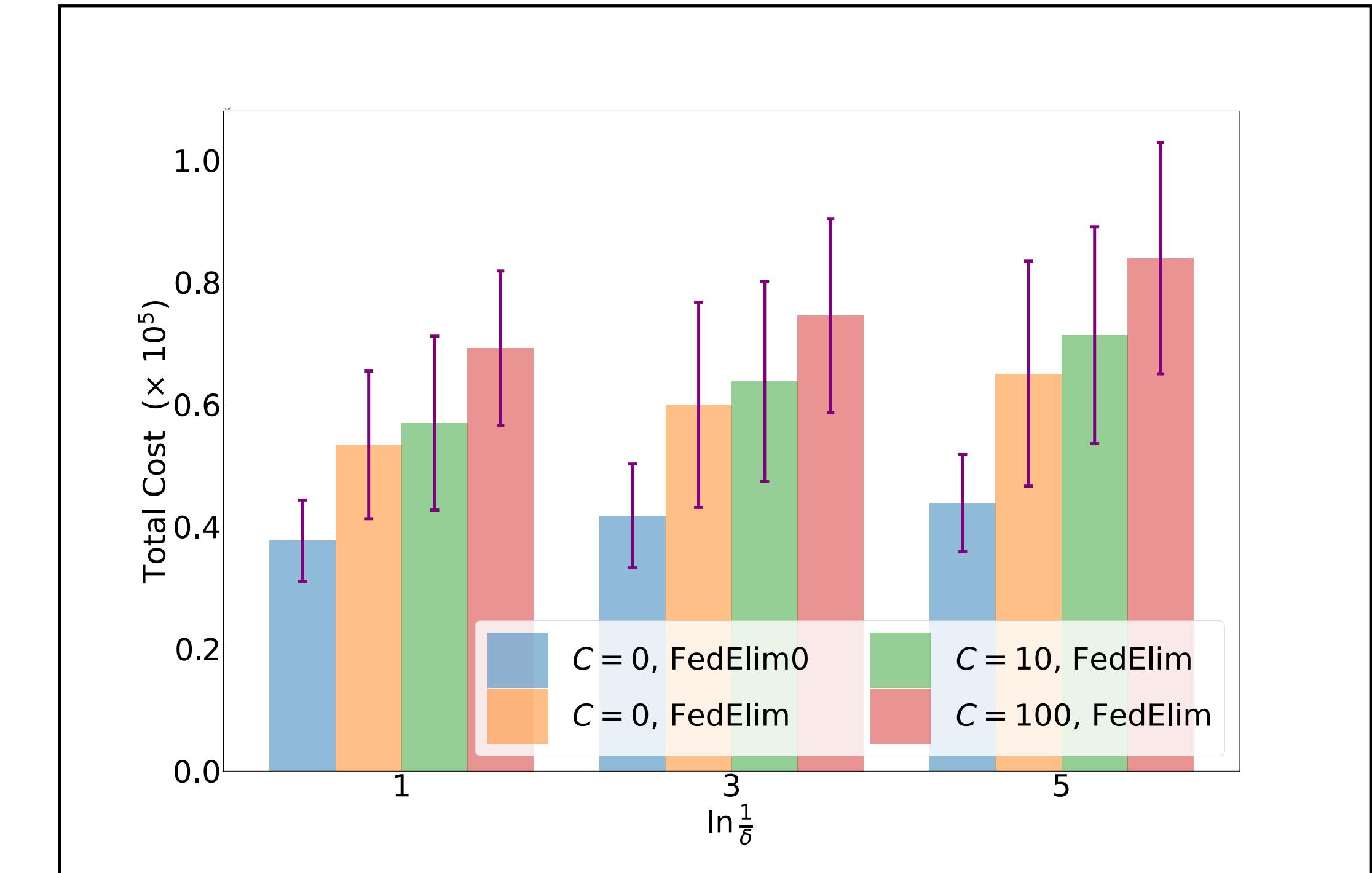
almost cost-free communication

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For any $C > 0$, $\forall \delta$ small,

$$P\left(\text{total cost of FedElim} \lesssim 3 \times \text{total cost of Per}(1) \text{ when } C = 0\right) \geq 1 - \delta$$

Key takeaway 1

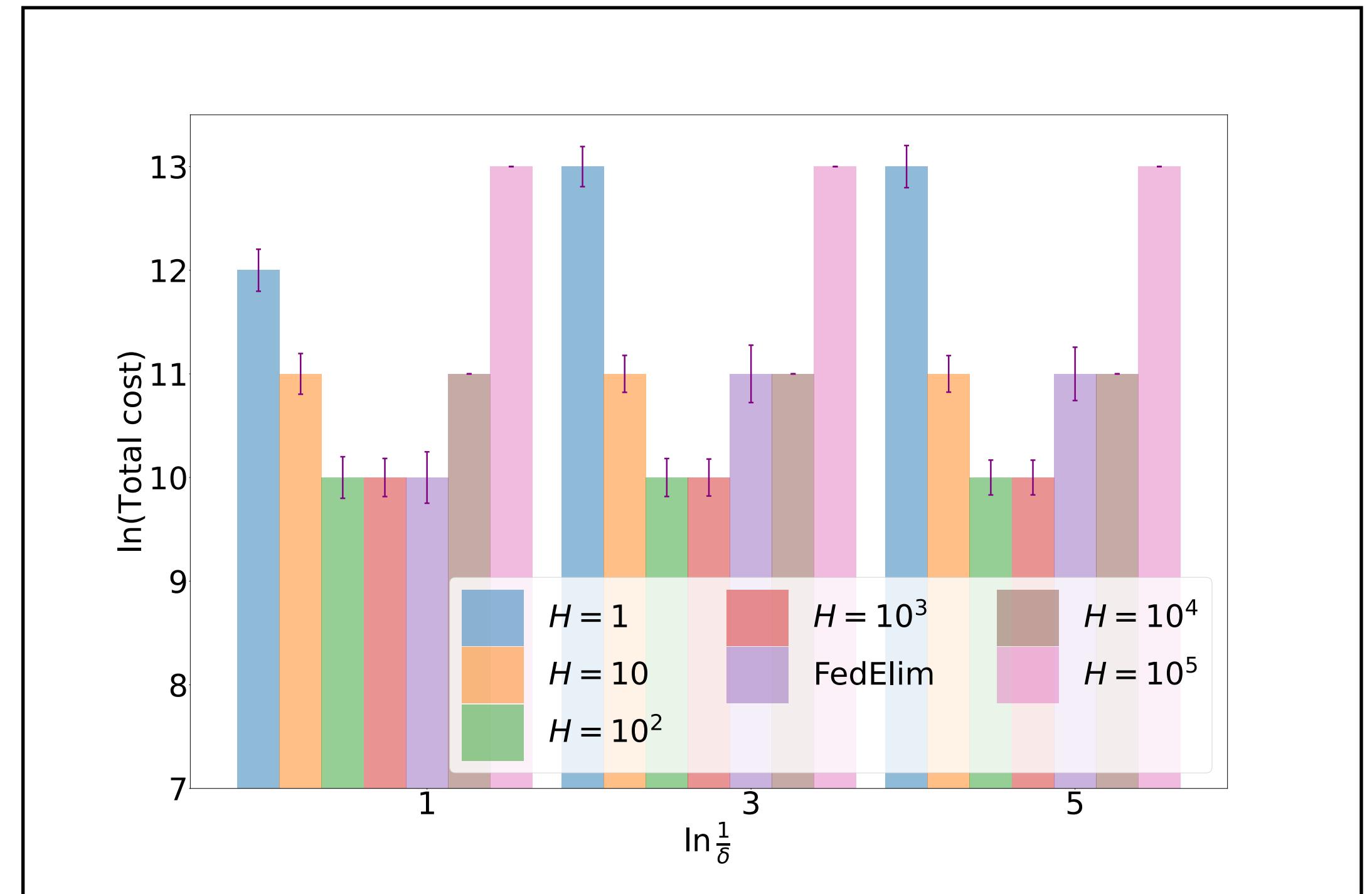
Results at a Glance – 2/3

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FedElim operates at a sweet spot

Key takeaway 2

Results at a Glance – 3/3

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Scheme	No. of arm pulls	Comm. cost	Total cost
Per(H)	$O(T)$	$O(CT/H)$	$O(T+CT/H)$
FedElim*	$2T$	$O(C \log(T))$	$3T$
SupExp	$O(T^2)$	$O(C \log\log(T))$	$O(T^2)$

* for sufficiently small error probabilities

- Performance evaluation on 2 synthetic datasets + MovieLens dataset

FedElim carefully balances no. of arm pulls and comm. cost

Key takeaway 3

Related Works

- Shi, Shen, and Yang (2021) – similar setting, regret minimisation

- Mitra, Hassani, and Pappas (2021)

- Arm sets disjoint across clients
- Only global best arm identification
- Local best arms can be eliminated along the way
- Periodic communication scheme

- Tao, Zhang, and Zhou (2019), Hillel et al. (2013)

- Identical arm distributions at all the clients
- Collaborative learning of the best arm

Federated Multi-armed Bandits with Personalization

Chengshuai Shi
University of Virginia

Cong Shen
University of Virginia

Jing Yang
The Pennsylvania State University

Abstract

A general framework of personalized federated multi-armed bandits (PF-MAB) is proposed, which is a new learning paradigm analogous to the federated learning (FL) framework in supervised learning and enjoys some features of the MAB problem. Under the PF-MAB framework, a mixed bandit learning problem that flexibly balances generalization and personalization is studied. Theoretical analysis for the mixed model is presented. We then propose the Personalized Federated Upper Confidence Bound (PF-UCB) algorithm, where the exploration length is chosen carefully to achieve the desired balance of learning the local model and supplying global information for the federated learning problem. Theoretical analysis proves that PF-UCB achieves an $O(\log(T))$ regret regardless of the degree of personalization. It has a linear dependency as the lower bound. Experiments using both synthetic and real-world datasets confirm the theoretical analysis and demonstrate the effectiveness of the proposed algorithm.

Exploiting Heterogeneity in Robust Federated Best-Arm Identification

Aritra Mitra, George J. Pappas, and Hamed Hassani *

Abstract

We study a federated variant of the best-arm identification problem in stochastic multi-armed bandits: a set of clients, each of whom can sample only a subset of the arms, collaborate via a server to identify the best arm (i.e., the arm with the highest mean reward) with prescribed confidence. For this problem, we propose Fed-SEL, a simple communication-efficient algorithm that identifies the best arm after $\tilde{O}(\sqrt{d})$ rounds of communication at the clients. To study the performance of Fed-SEL, we introduce a notion of arm-heterogeneity that captures the level of dissimilarity between distributions of arms corresponding to different clients. Interestingly, our analysis reveals the benefits of arm-heterogeneity in reducing both the sample- and communication-complexity of Fed-SEL. As a special case of our analysis, we show that for certain heterogeneous problem instances, Fed-SEL outputs the best-arm after just one round of communication. Our findings have the following key implication: unlike federated supervised learning where recent work has shown that statistical heterogeneity can lead to poor performance, one can provably reap the benefits of both local computation and heterogeneity for federated best-arm identification. As our final contribution, we develop variants of Fed-SEL, both for federated and peer-to-peer settings, that are robust to the presence of Byzantine clients, and hence suitable for deployment in harsh, adversarial environments.

2019 IEEE 60th Annual Symposium on Foundations of Computer Science (FOCS)

Collaborative Learning with Limited Interaction:
Tight Bounds for Distributed Exploration in
Multi-Armed Bandits

Chao Tao¹, Qin Zhang¹, Yuan Zhou²

¹Computer Science Department, Indiana University
Email: tao@cs.indiana.edu
²Computer Science Department, Indiana University
Email: qzhangcs@indiana.edu

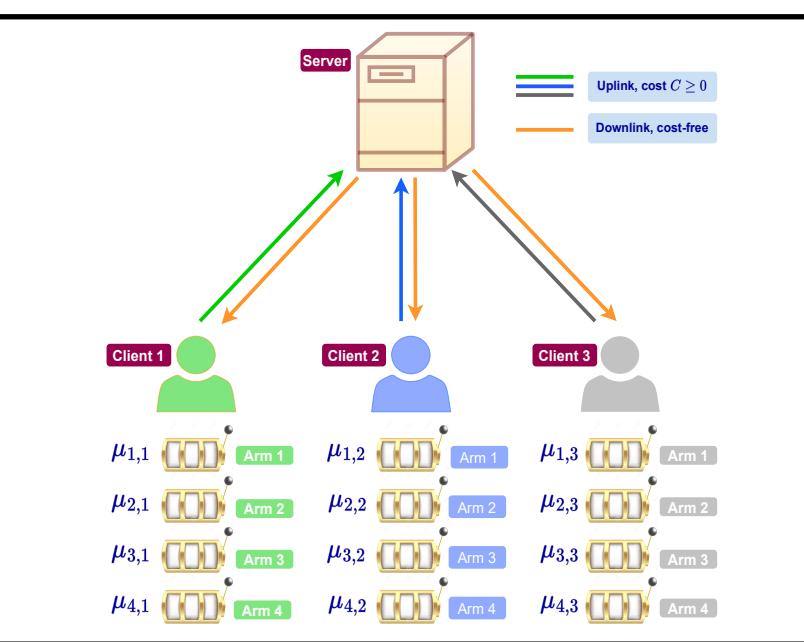
Department of ISE, University of Illinois at Urbana-Champaign
and
Email: yuanz@illinois.edu

The $C = 0$ Case

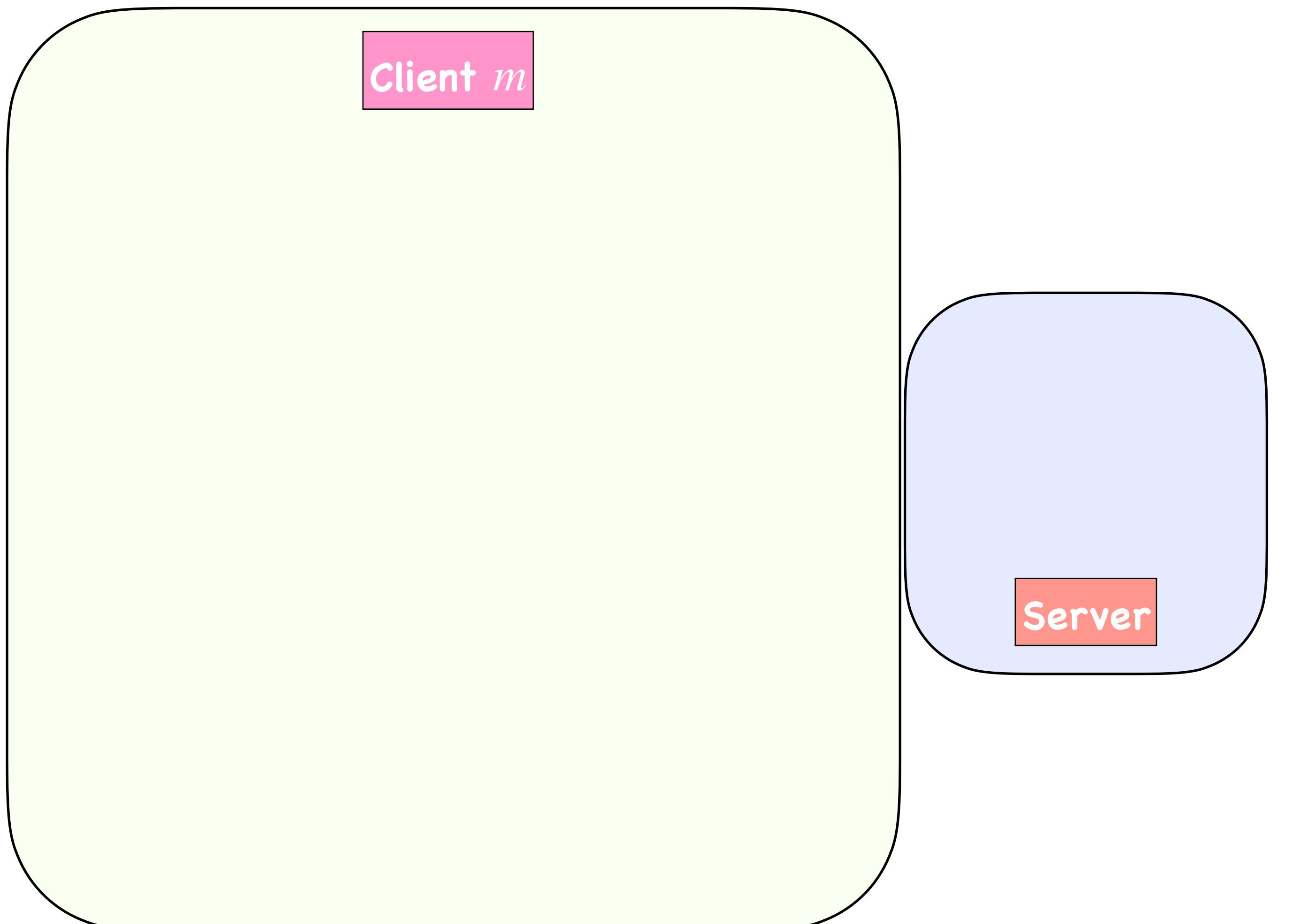
FedElim for $C = 0$ (FedElim0)

$$\begin{aligned} \min \quad & \mathbb{E}[\text{no. of arm pulls} + \text{communication cost}] \\ \text{s.t.} \quad & \text{error probability} \leq \delta \end{aligned}$$

= total cost



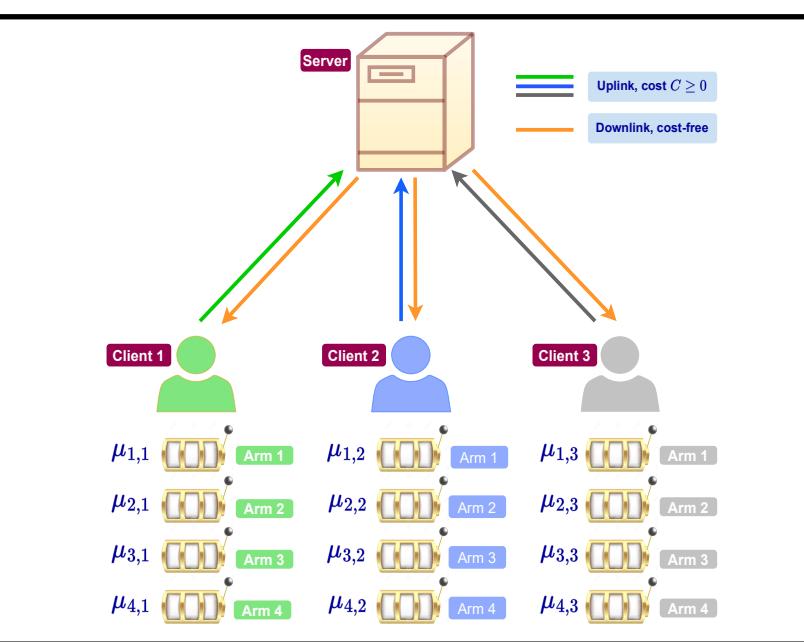
Comm. at **every time instant**



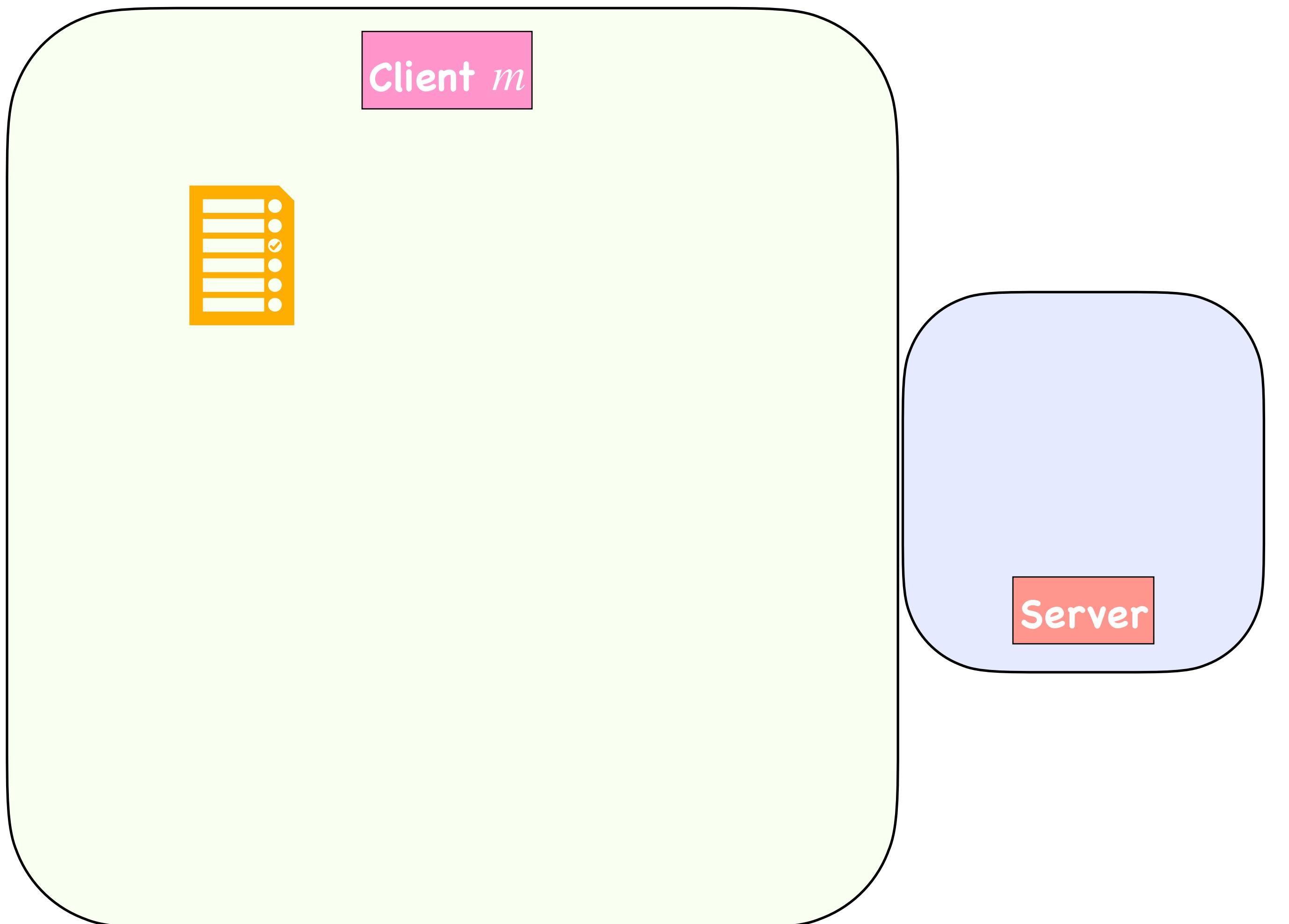
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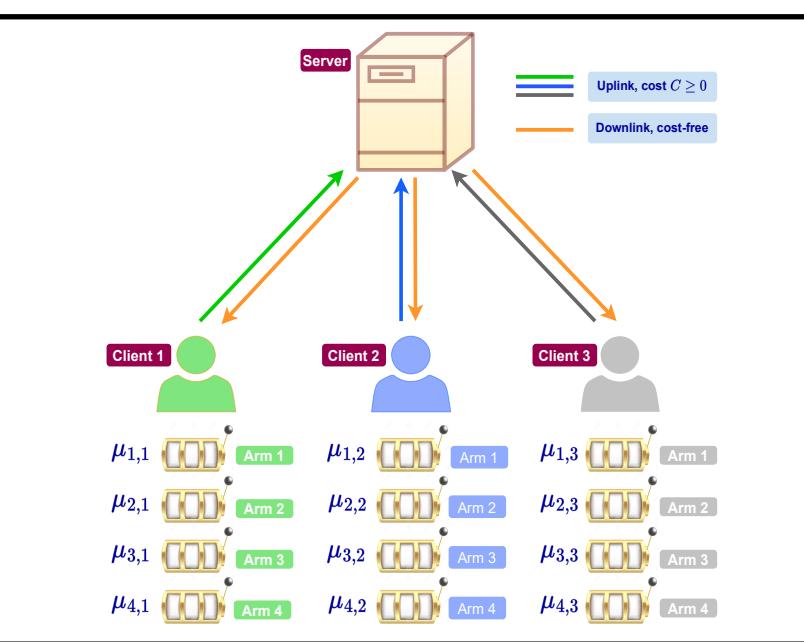
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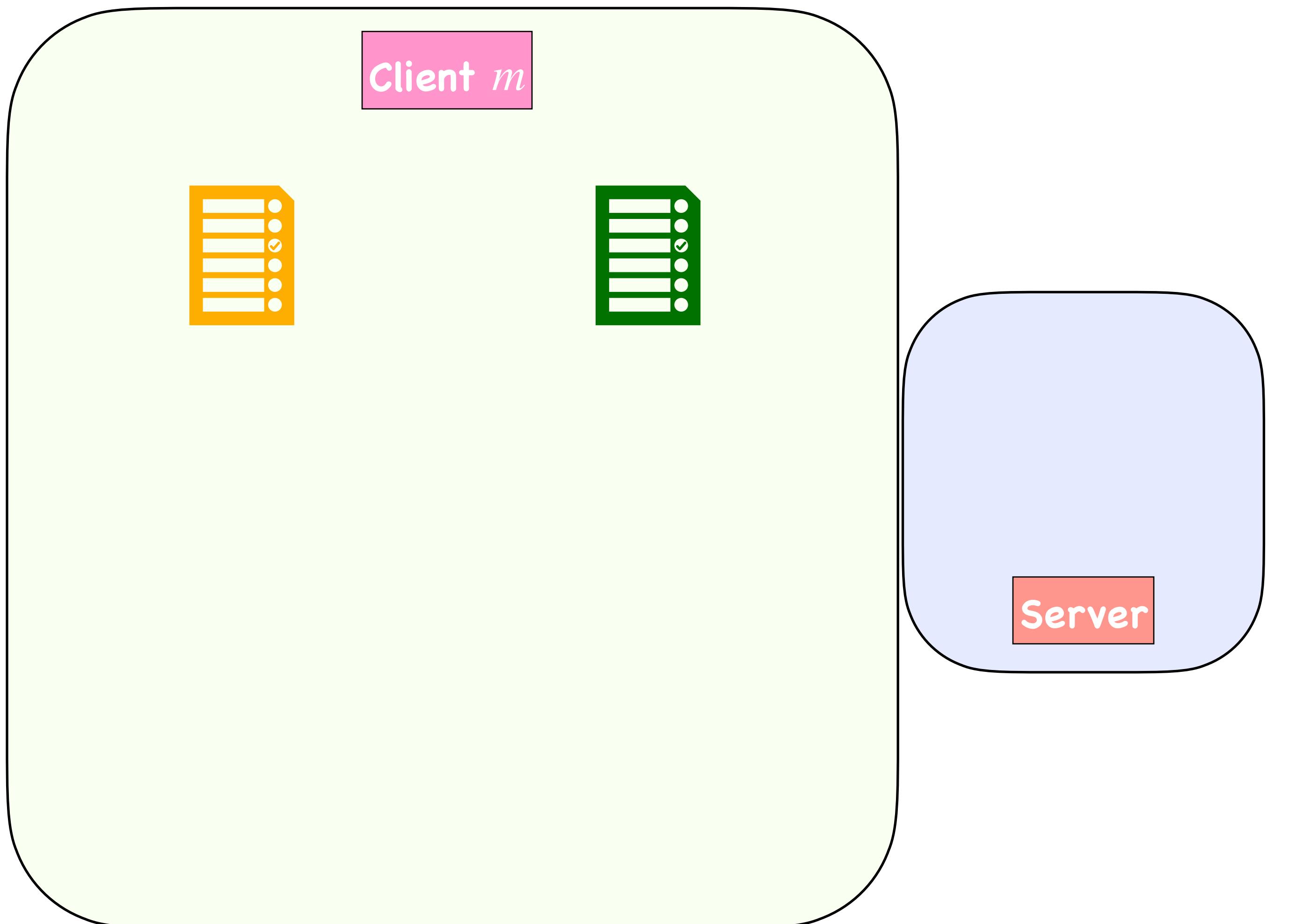
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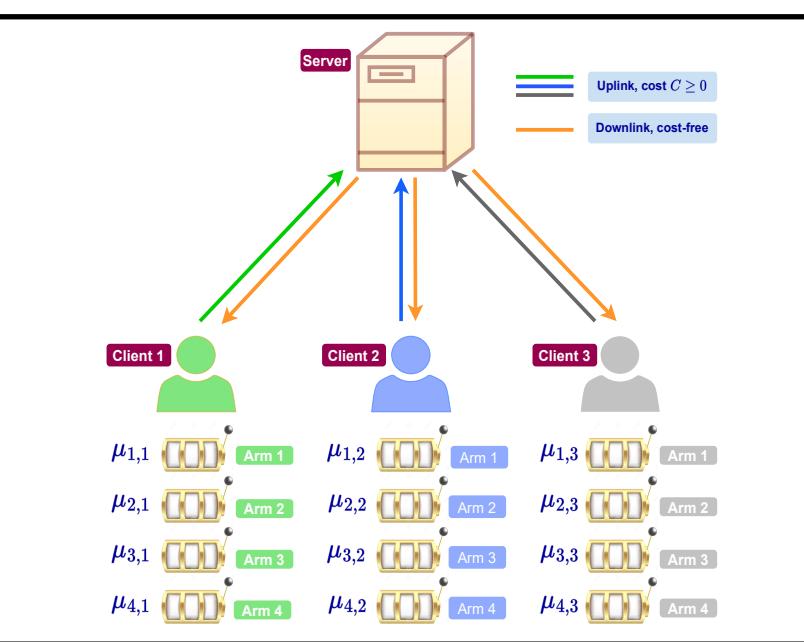


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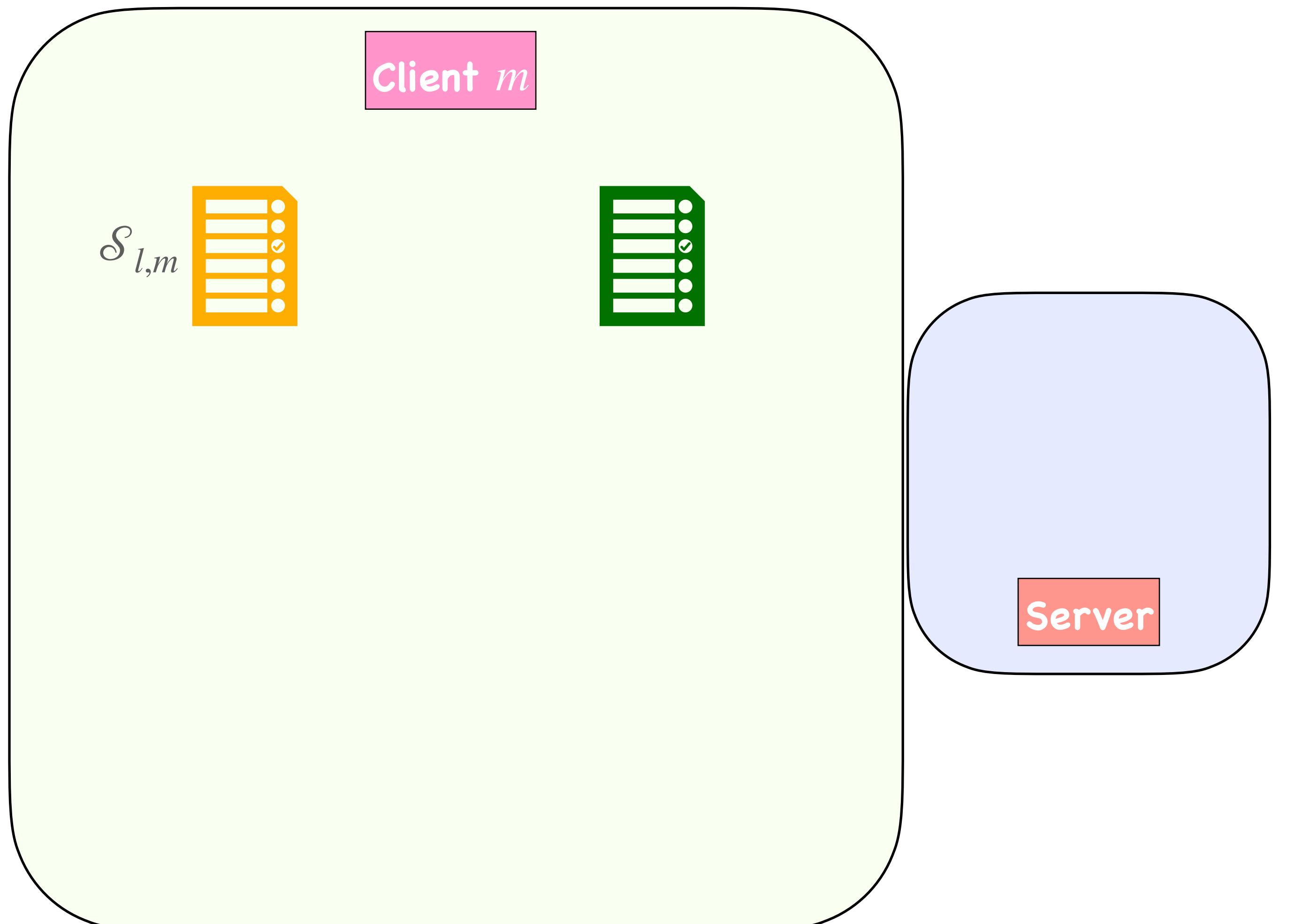
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$\mathcal{S}_{l,m}$

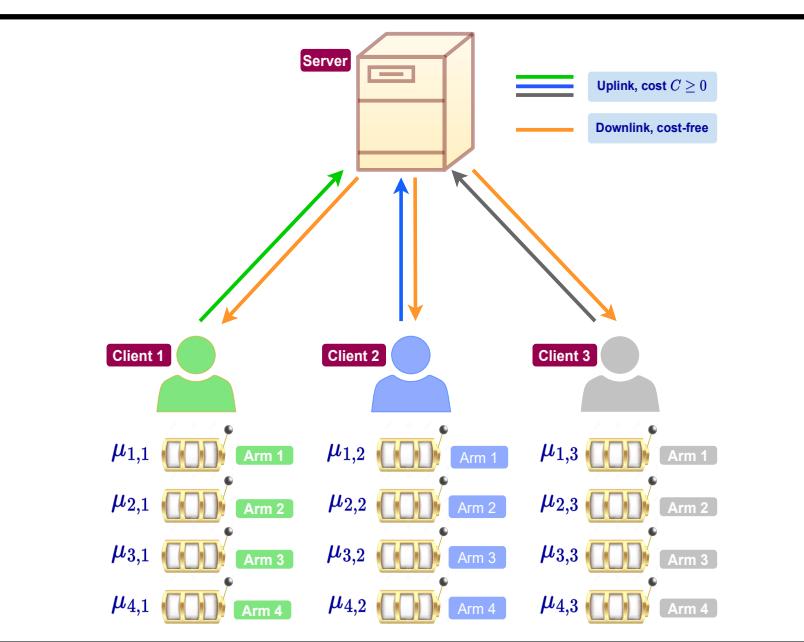
\mathcal{S}_g

Client m

Server

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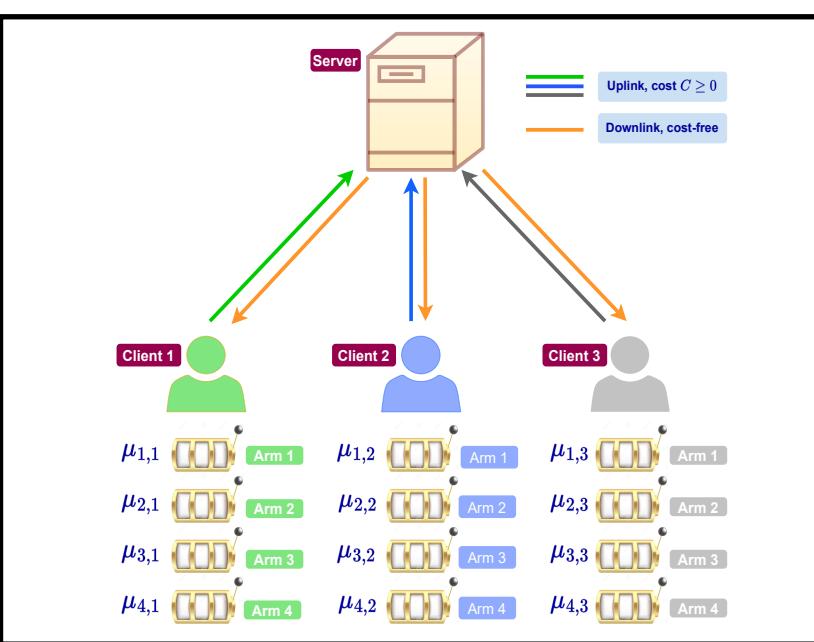


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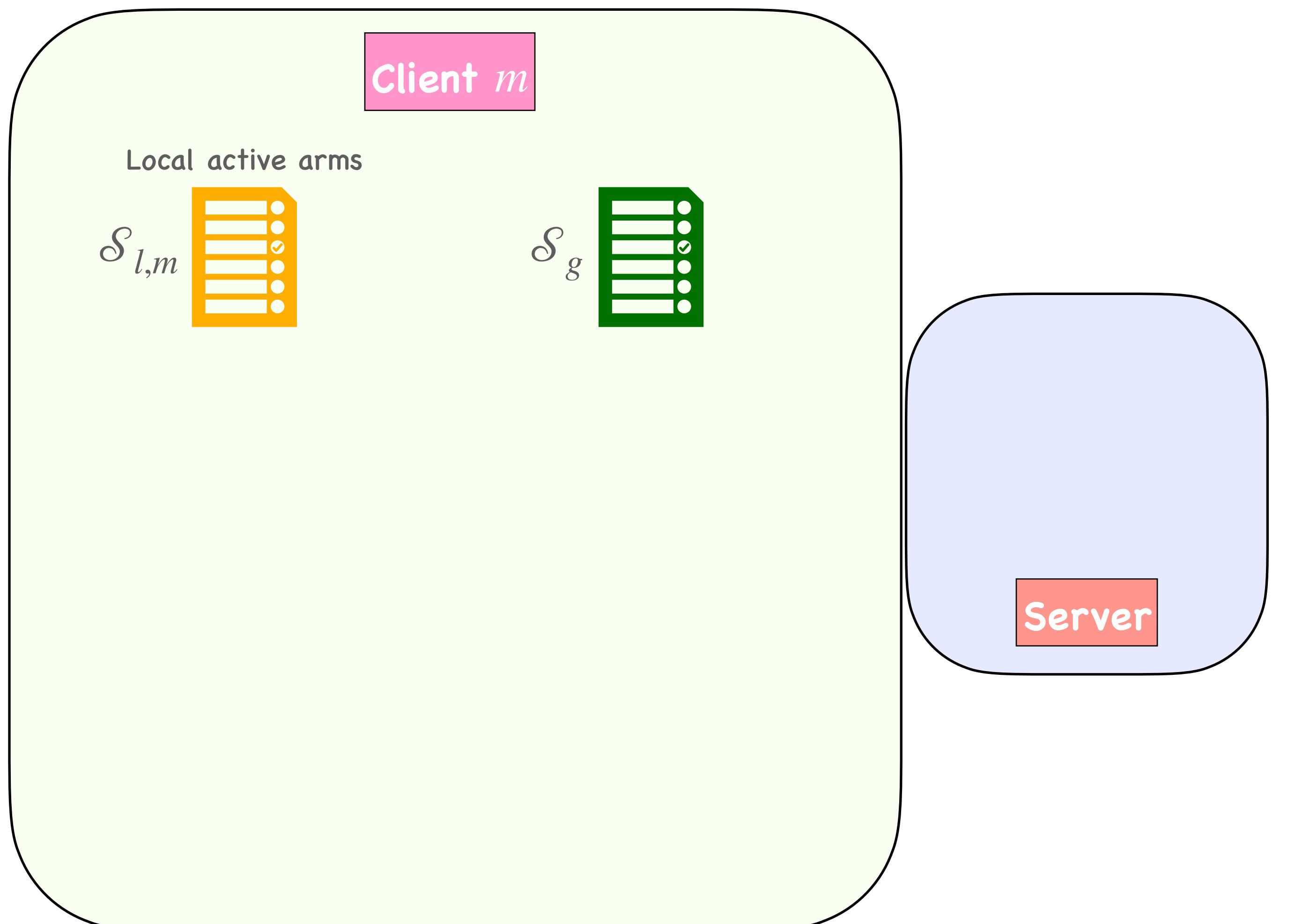
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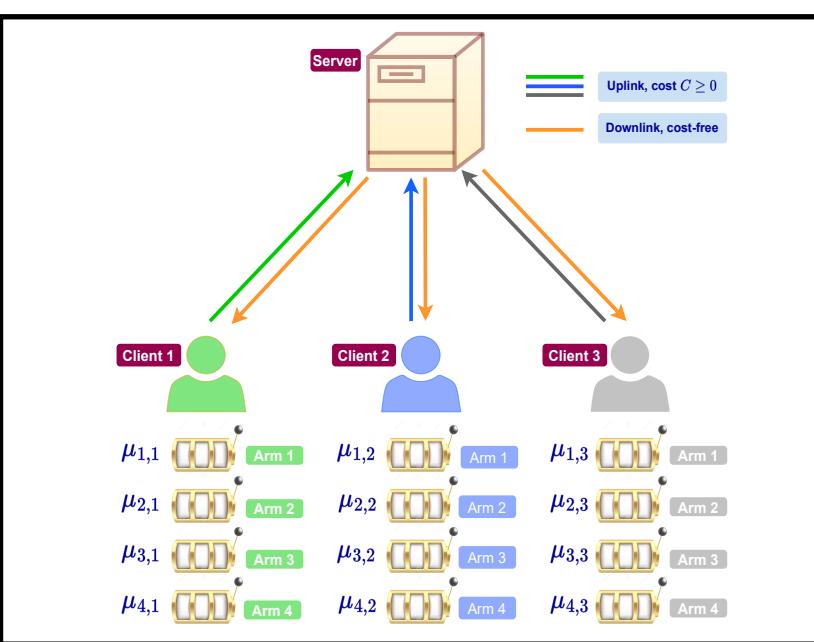
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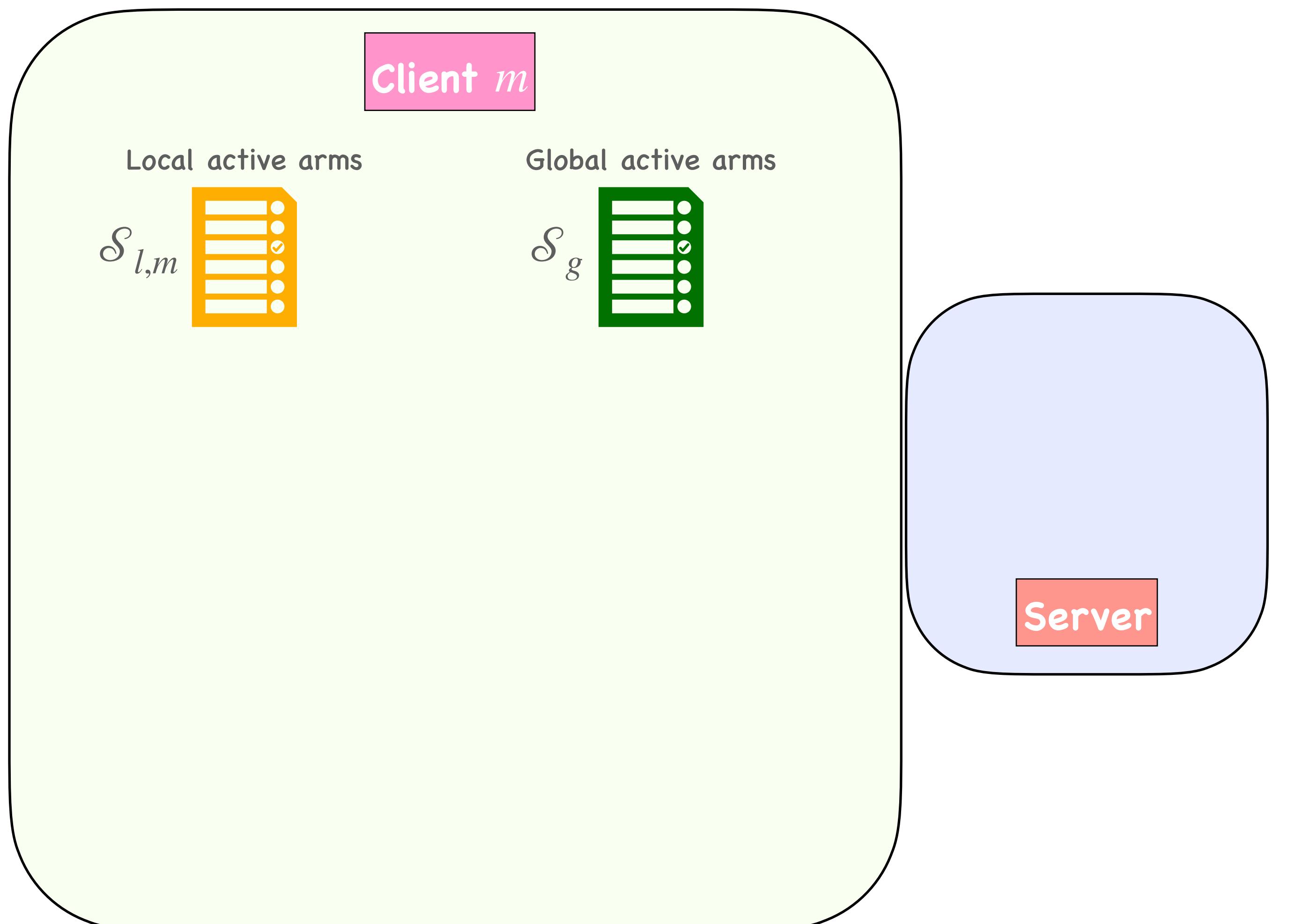
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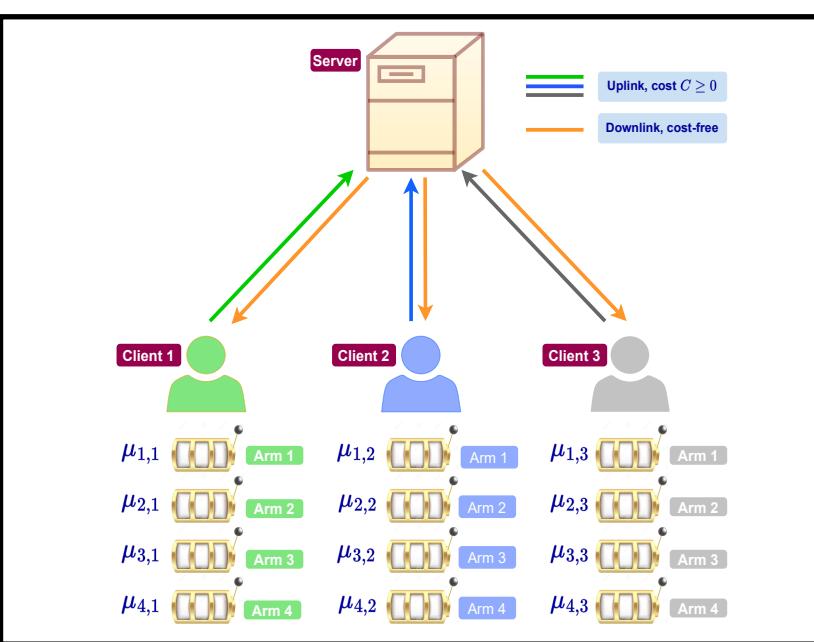


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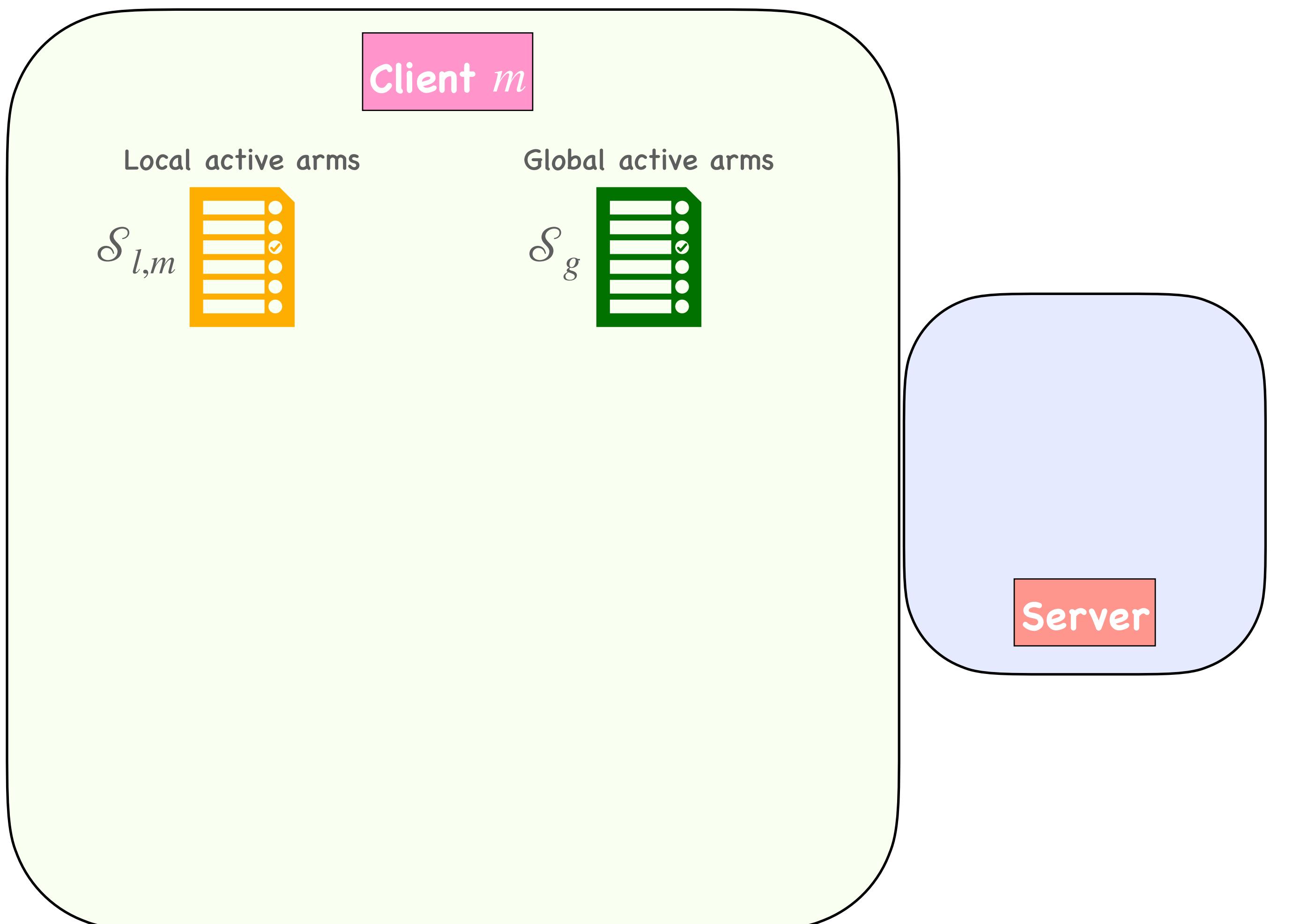


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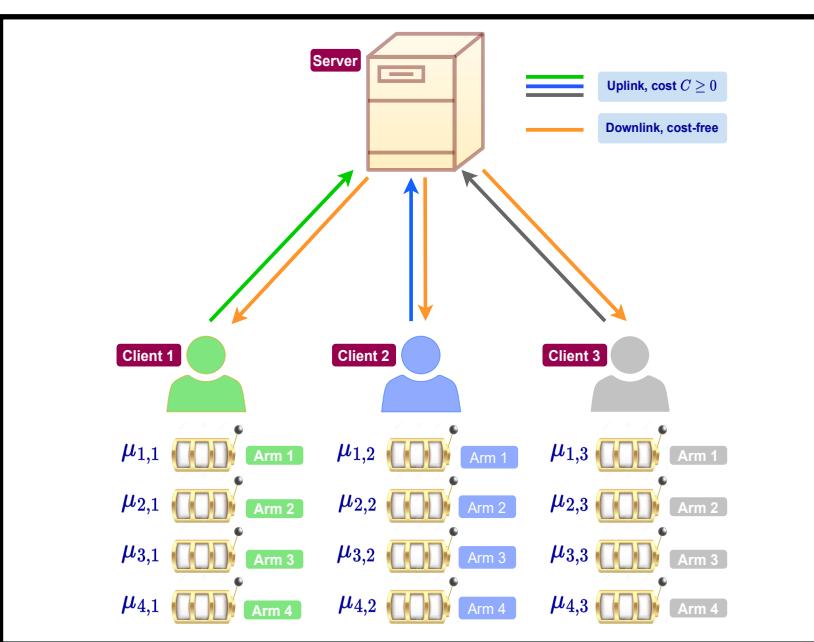
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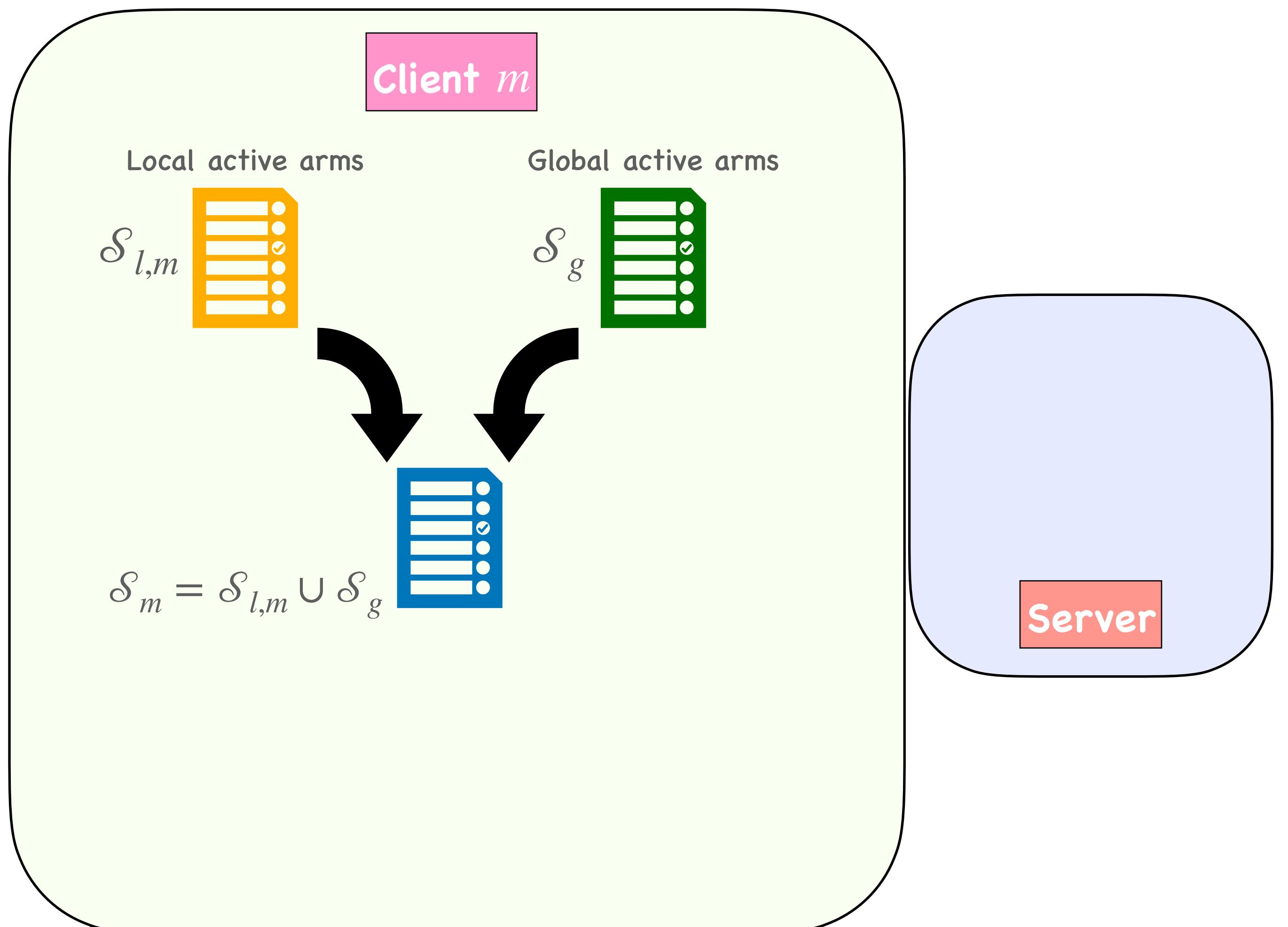


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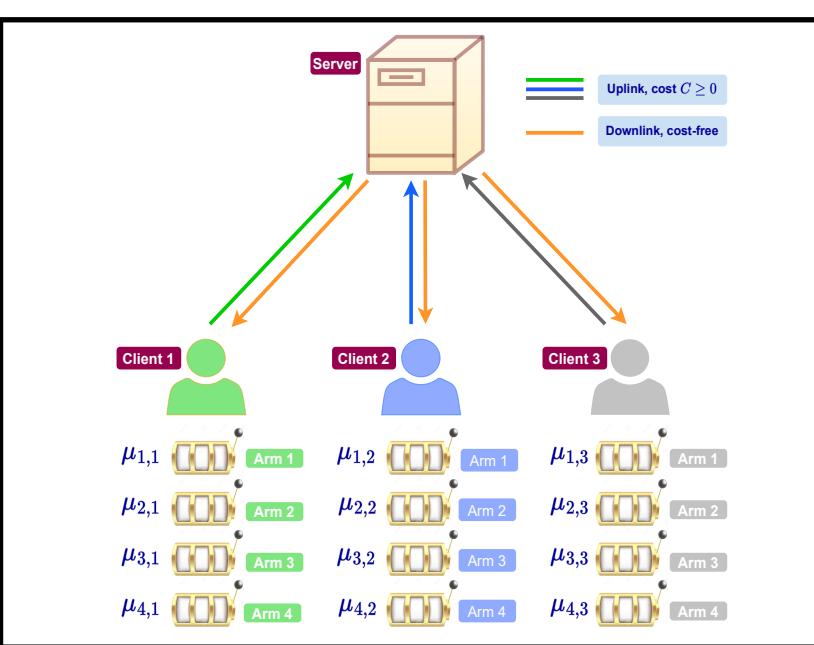
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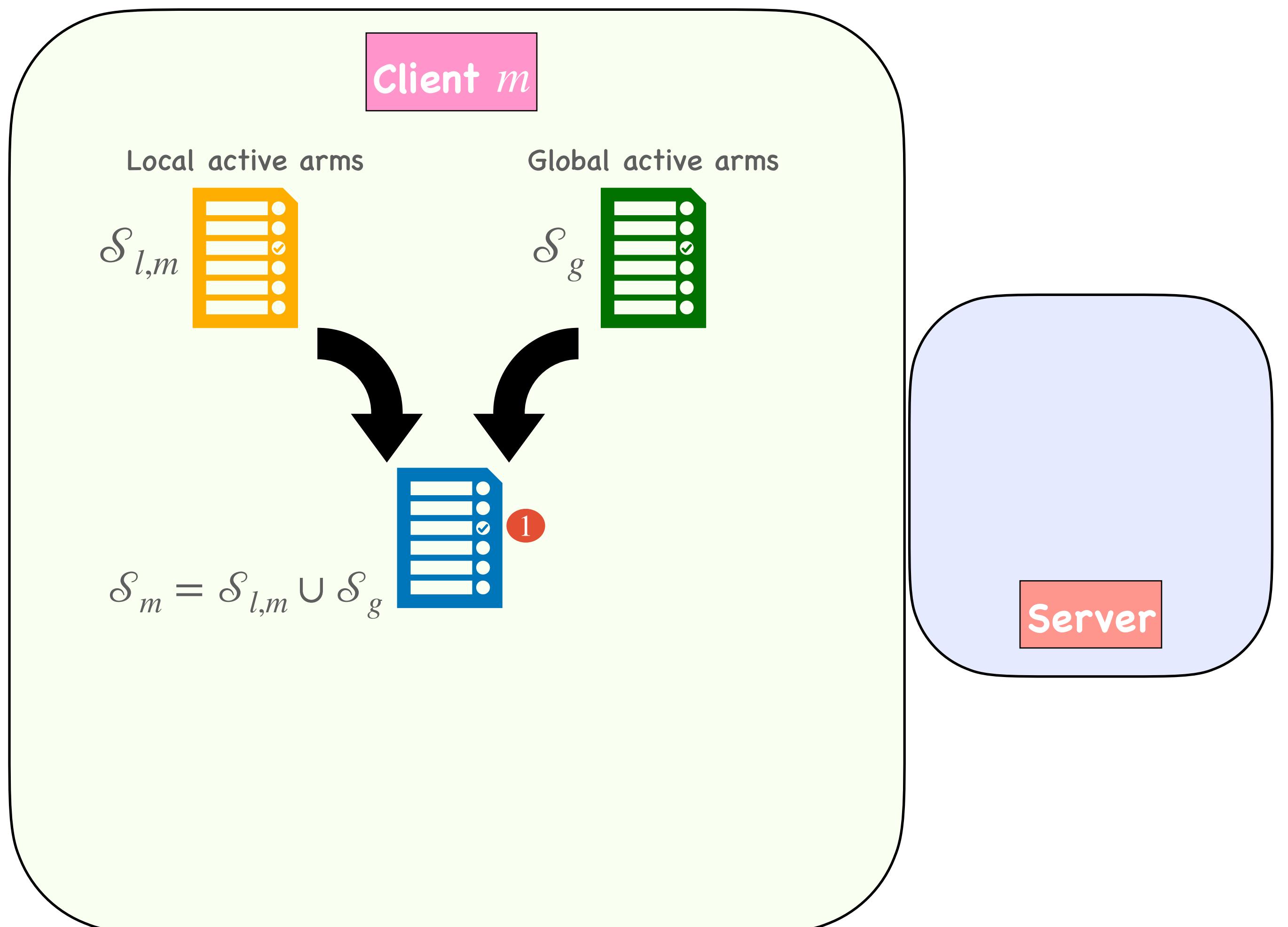
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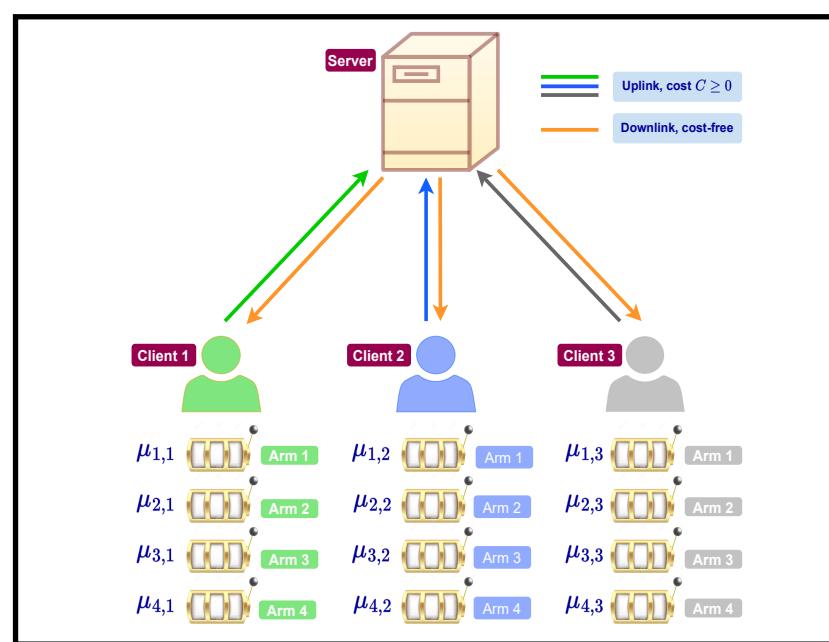
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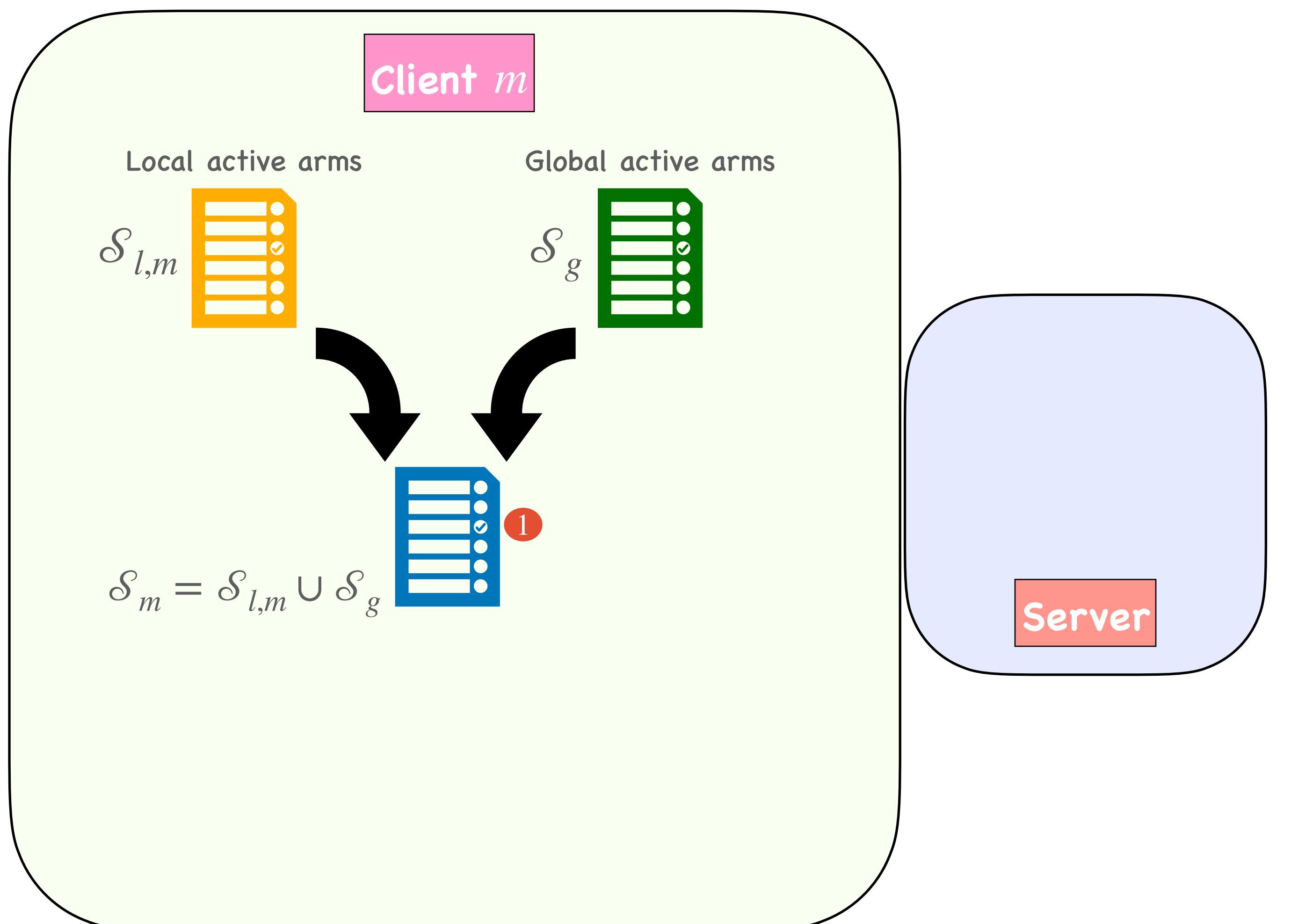
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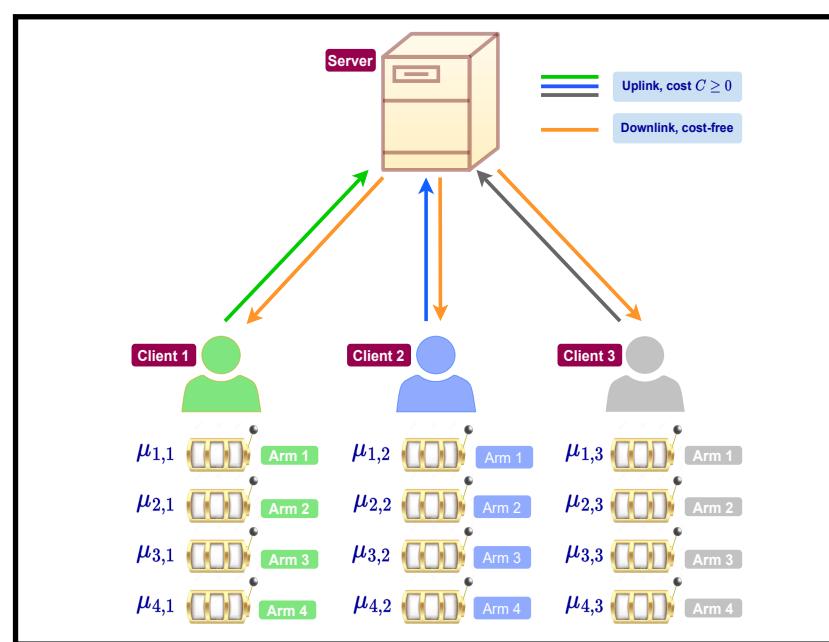
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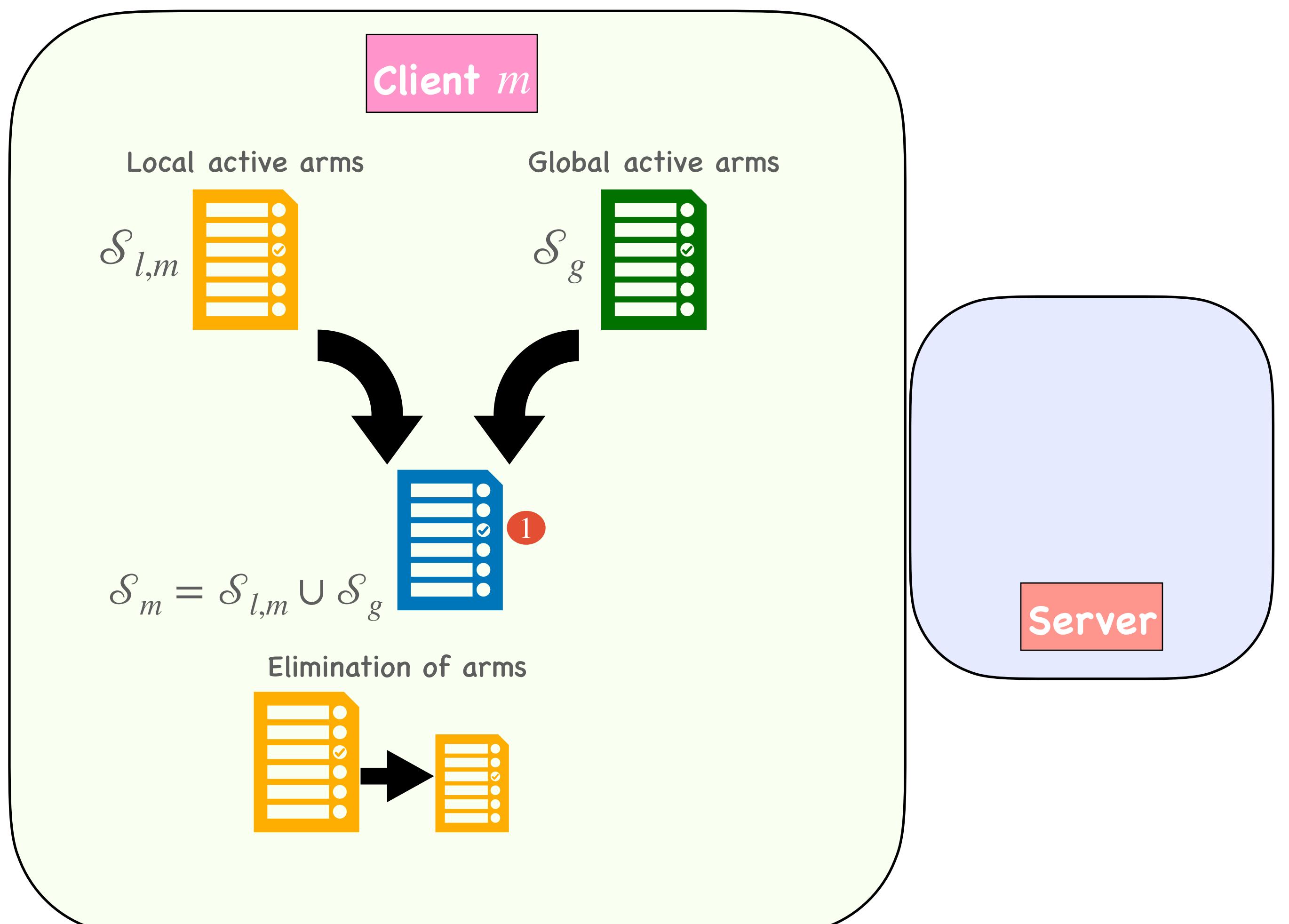
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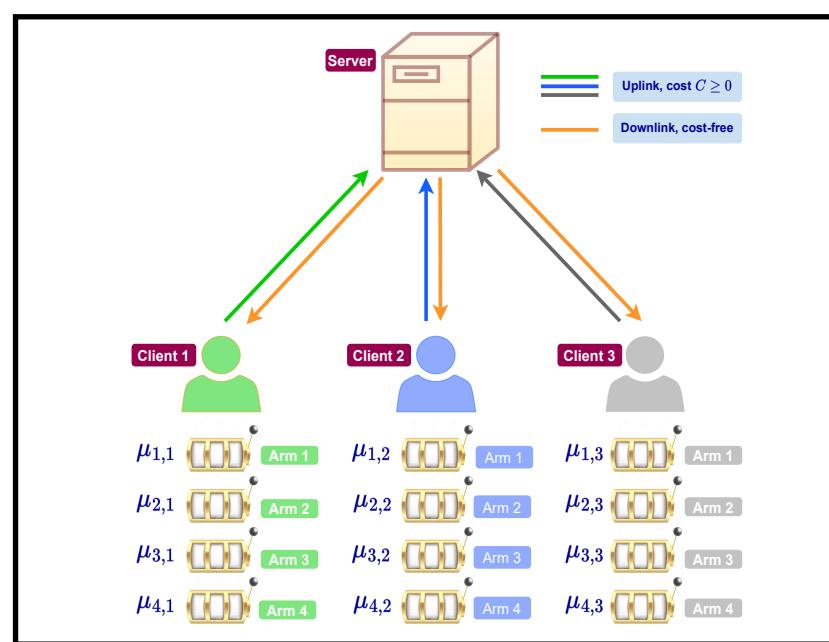
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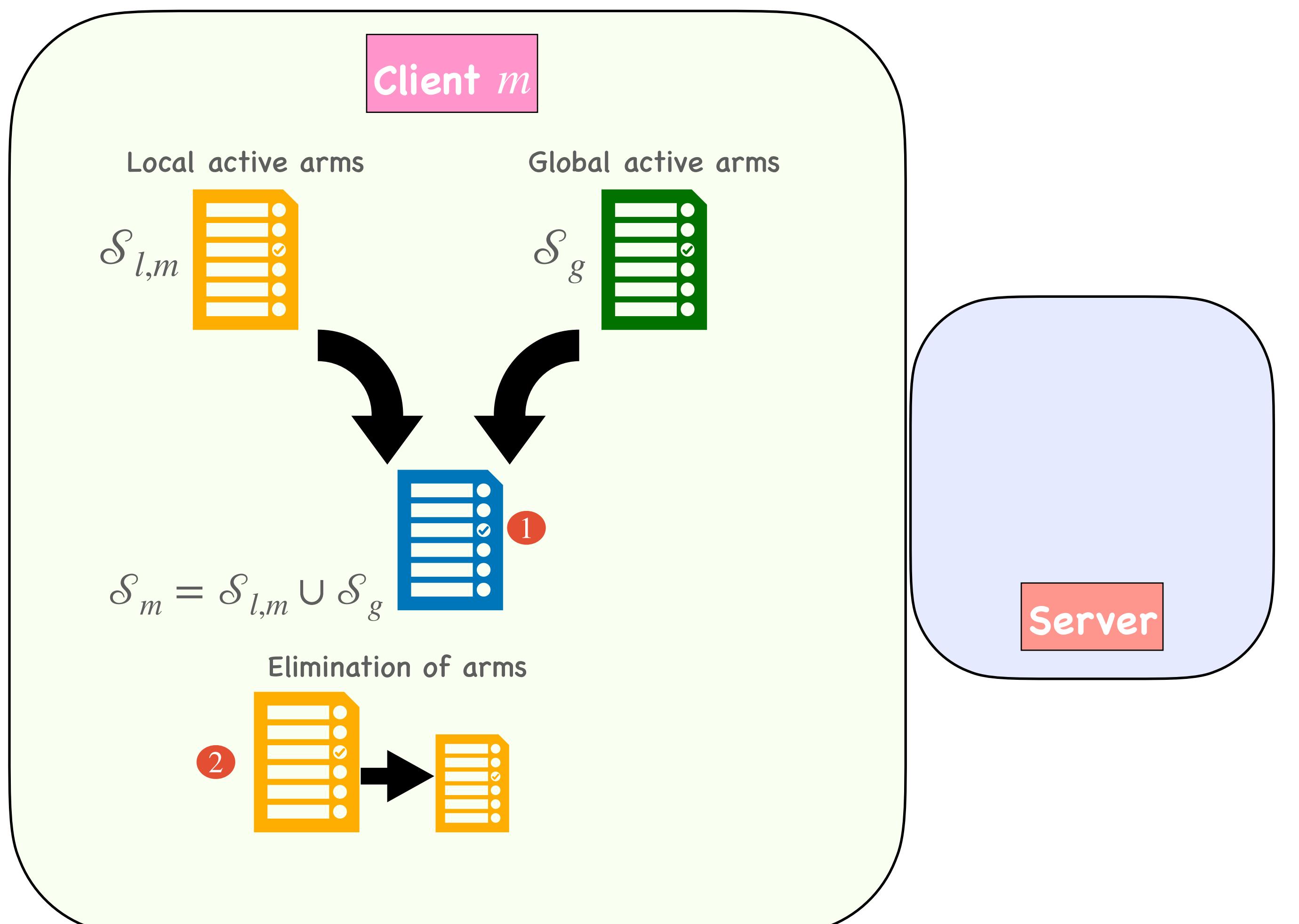
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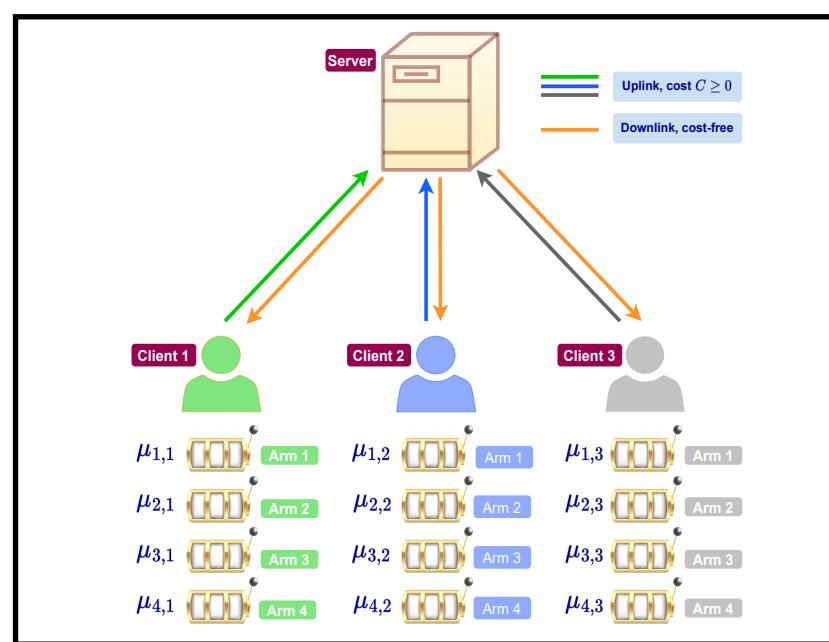
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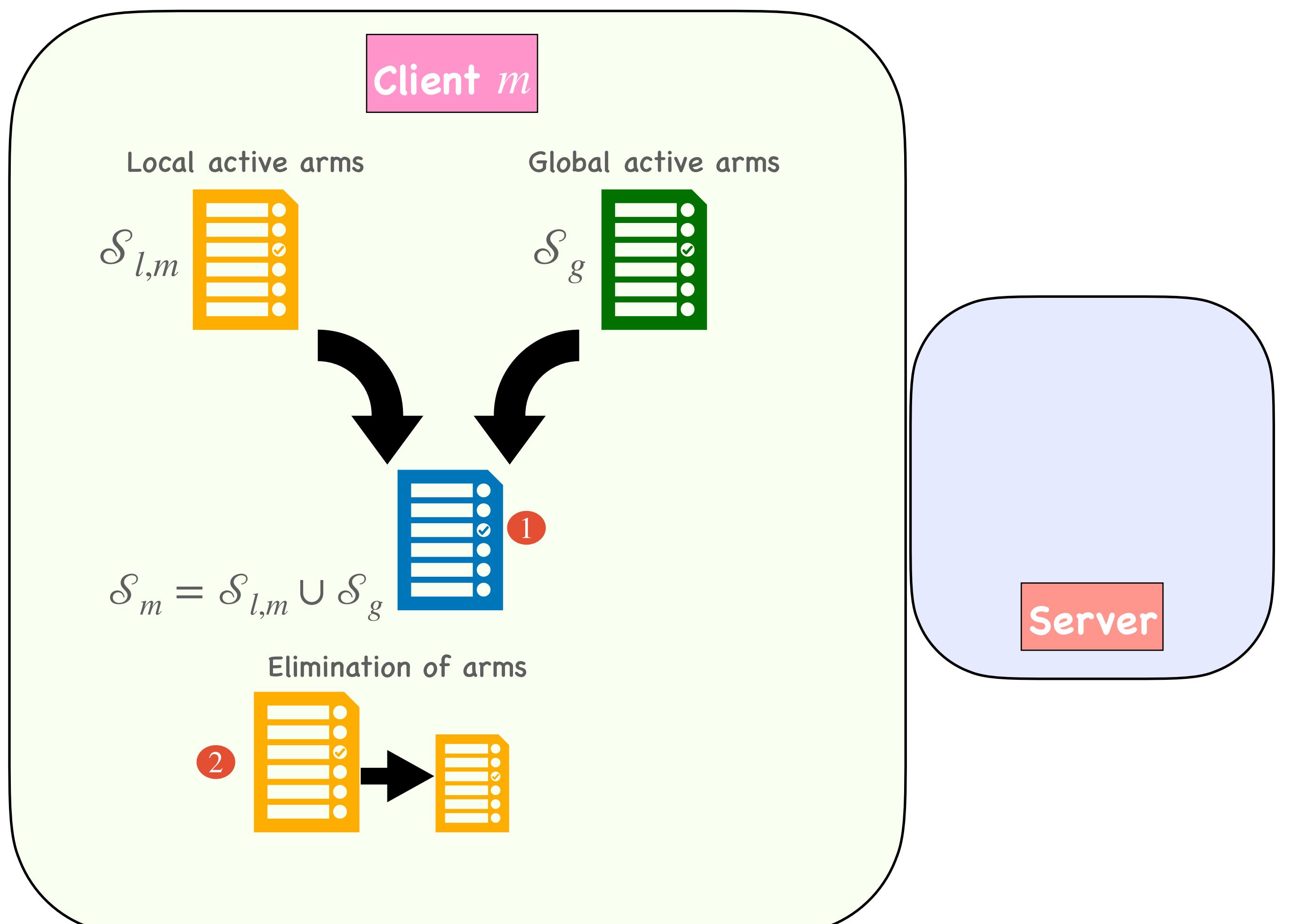
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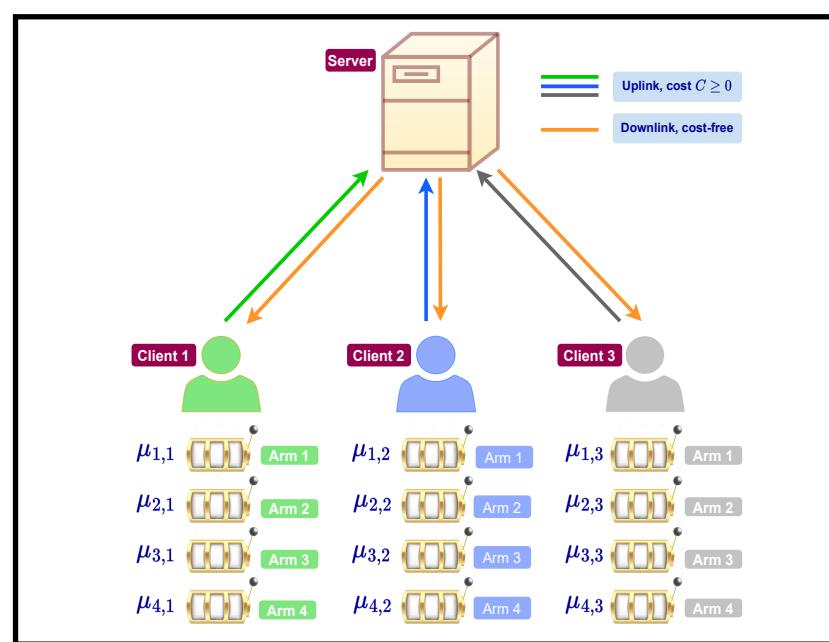
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Update $\mathcal{S}_{l,m}$

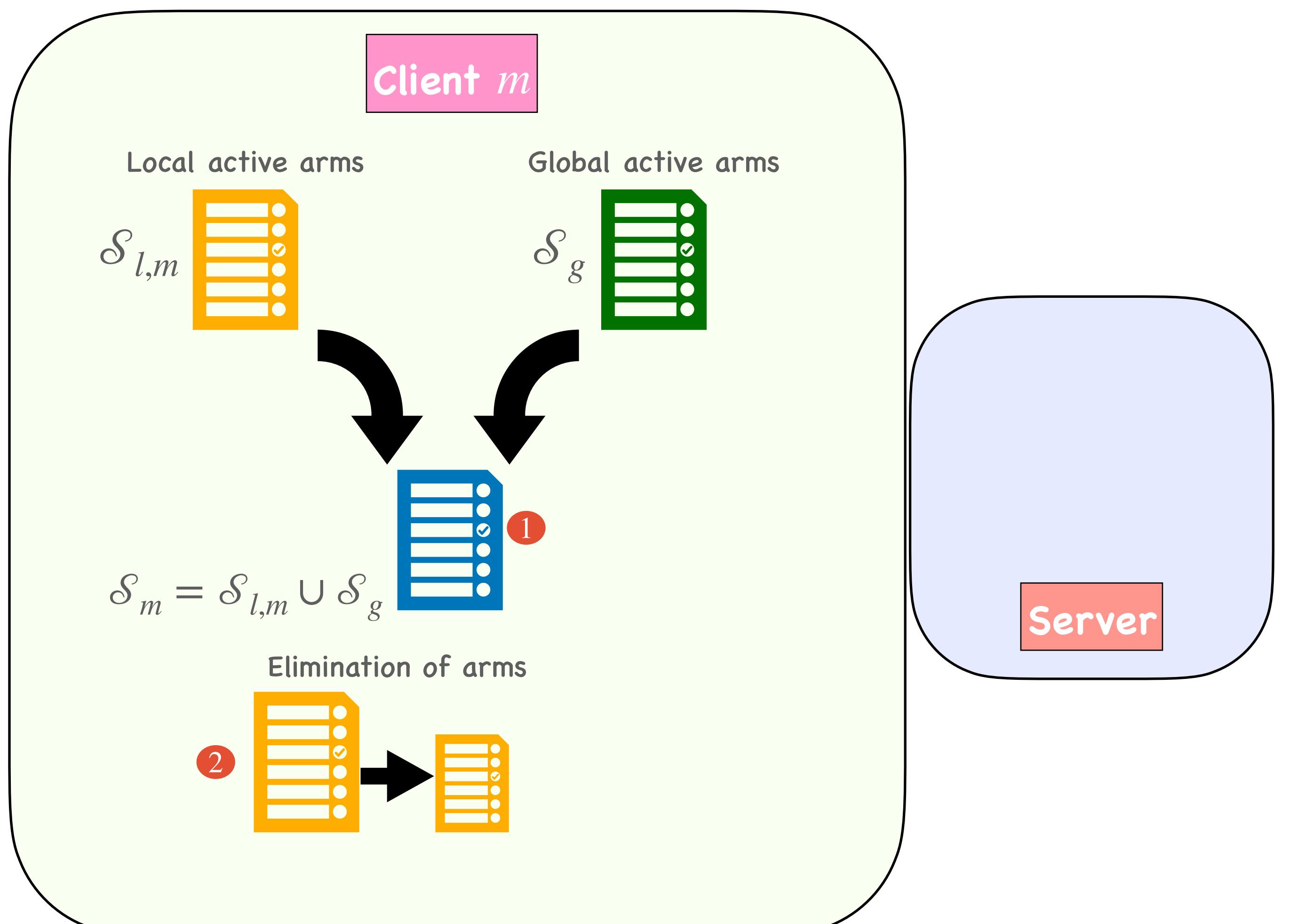
FedElim for $C = 0$ (FedElim0)

$$\begin{aligned} \min \quad & \mathbb{E}[\text{no. of arm pulls} + \text{communication cost}] \\ \text{s.t.} \quad & \text{error probability} \leq \delta \end{aligned}$$

= total cost



Comm. at **every time instant**



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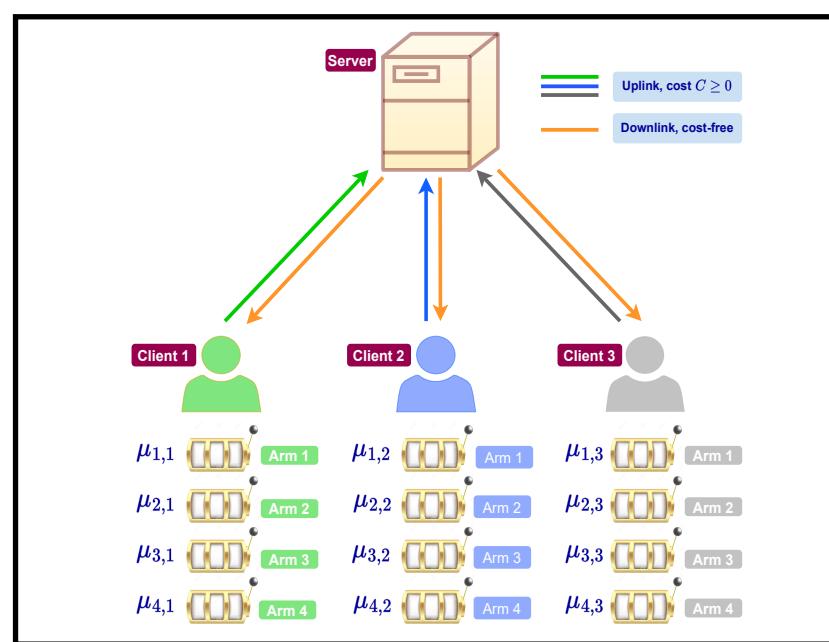
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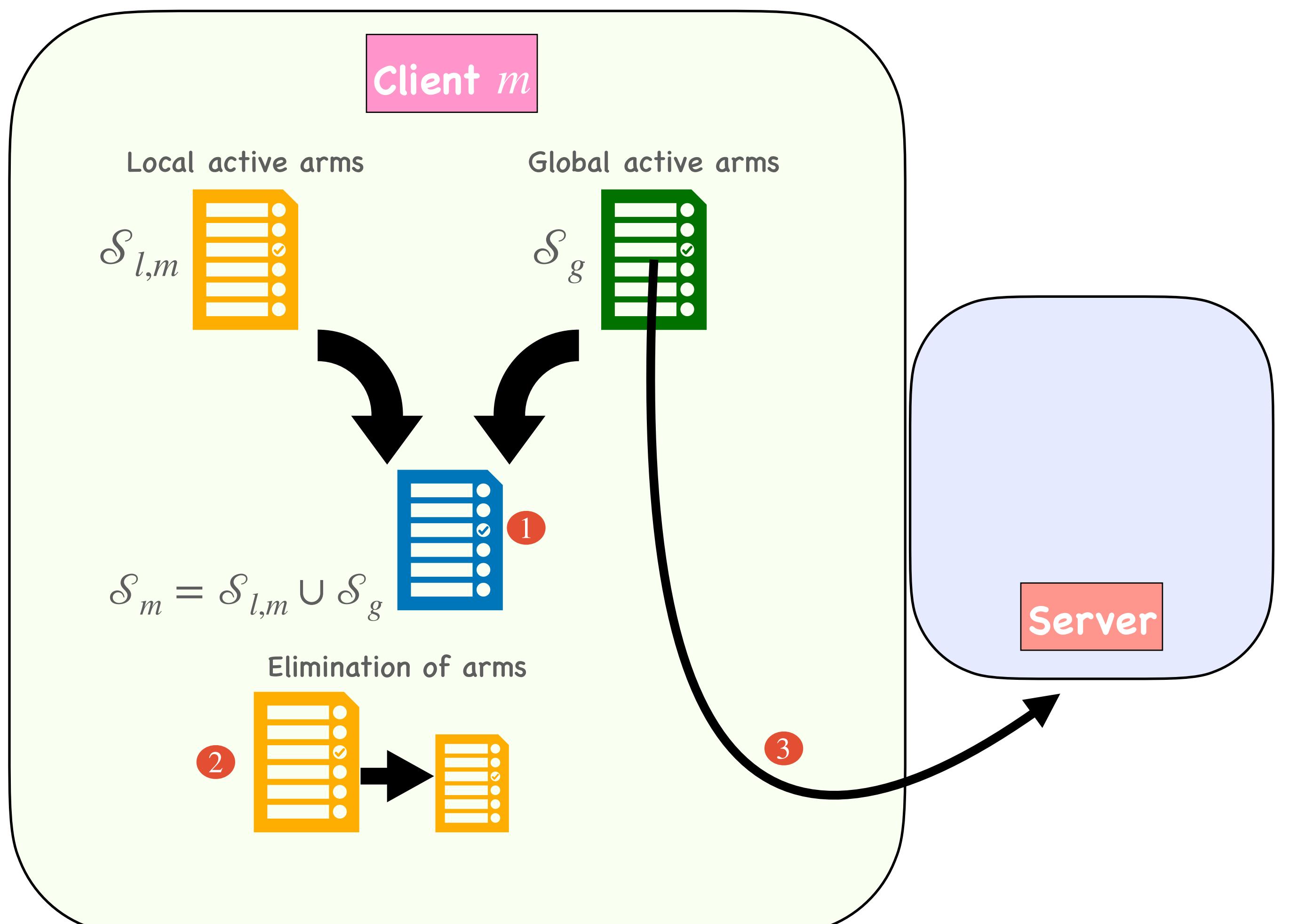
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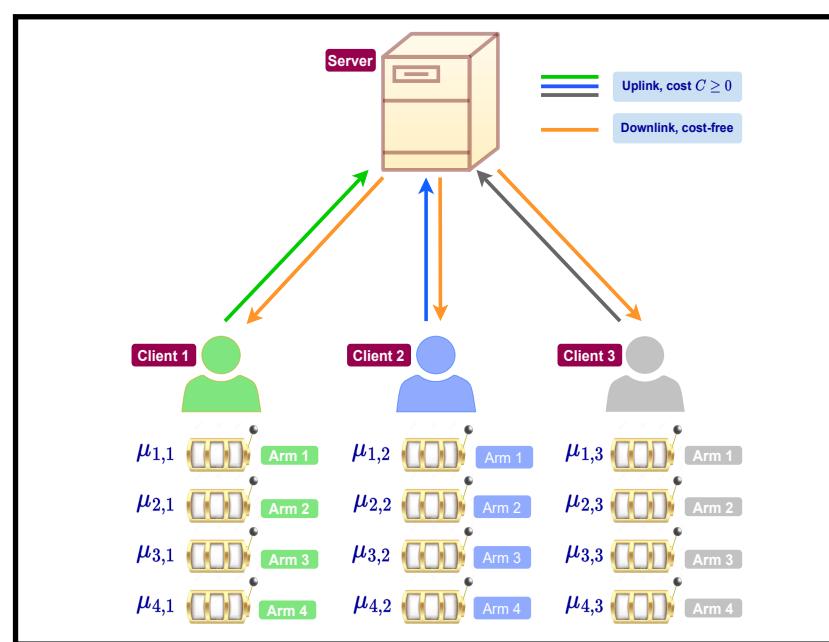
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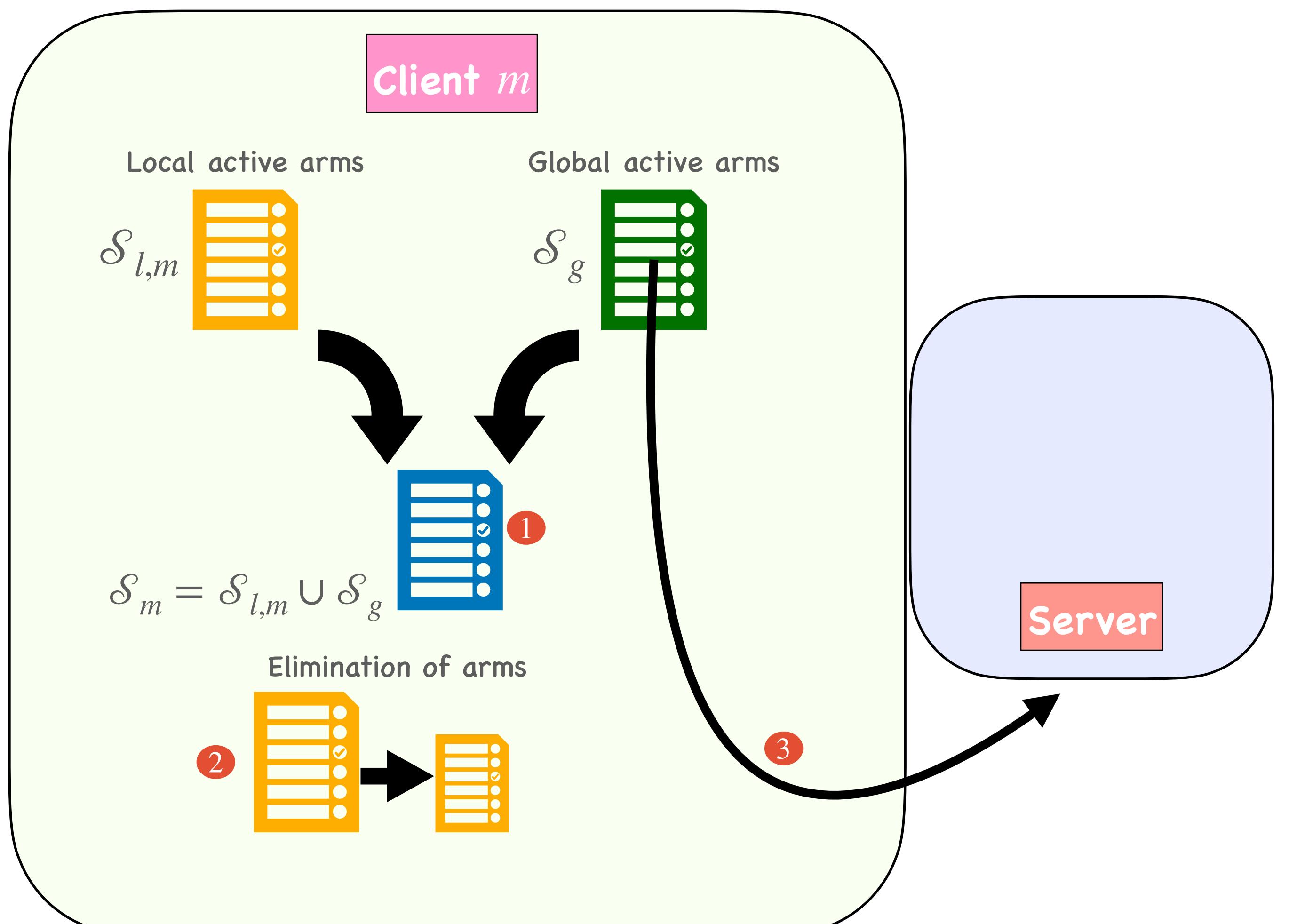
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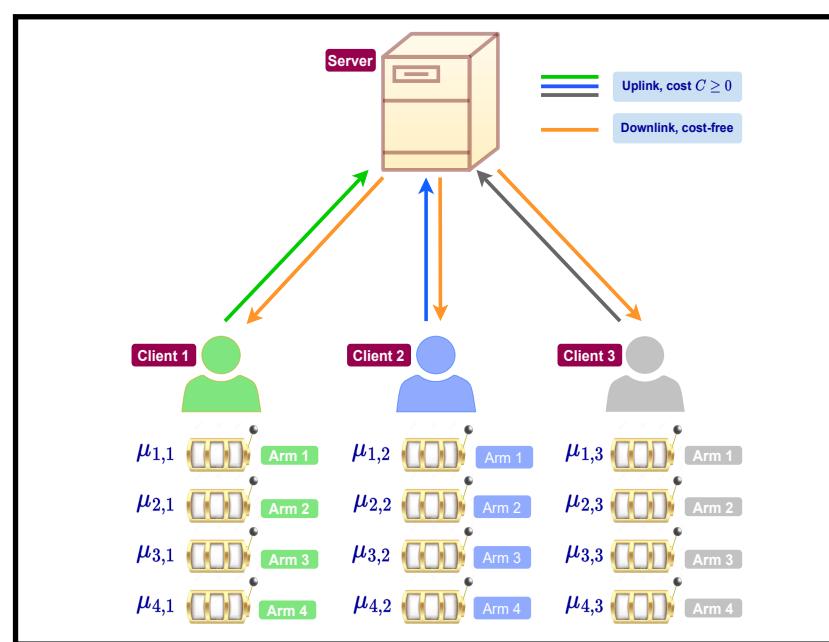
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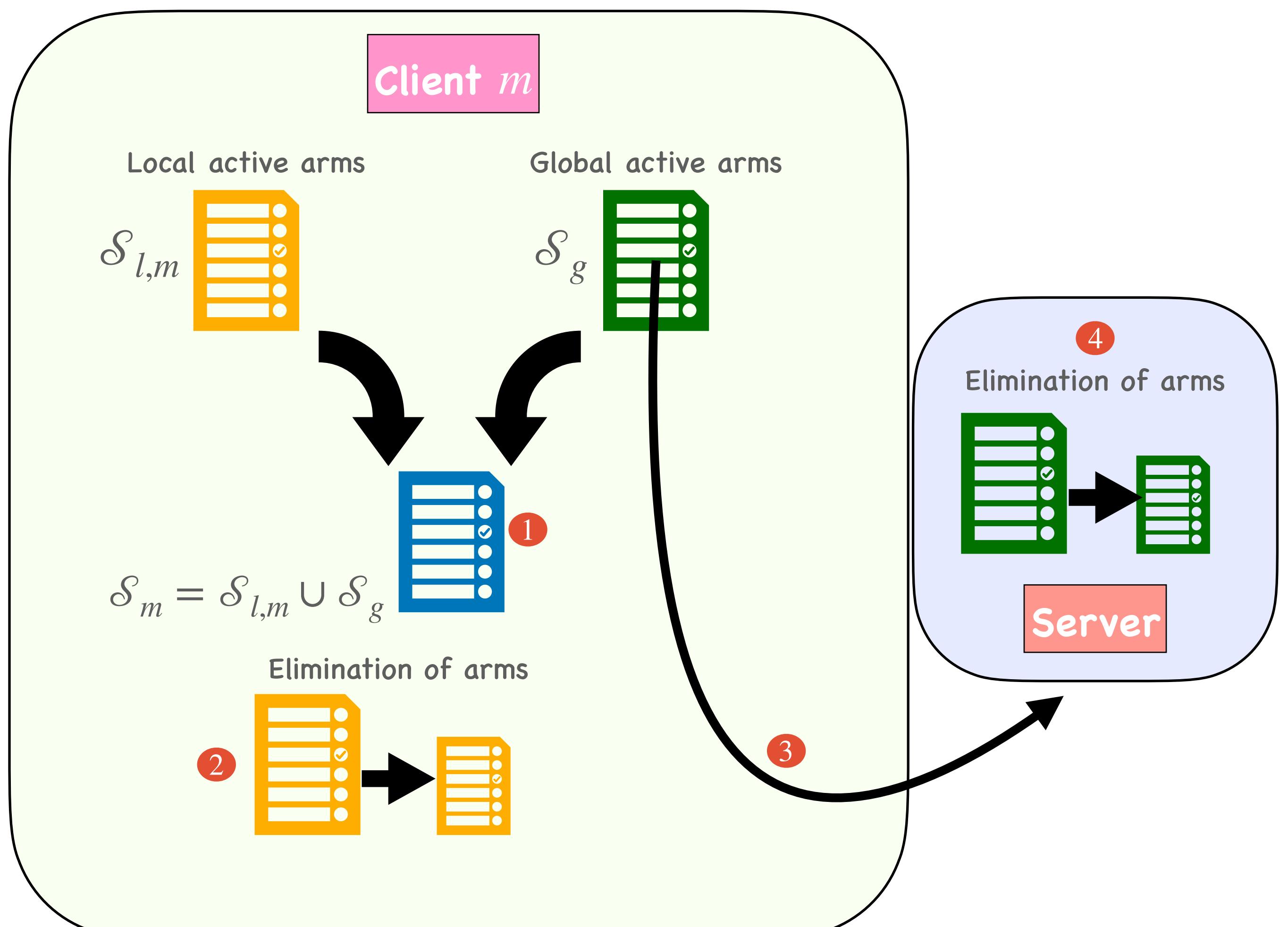
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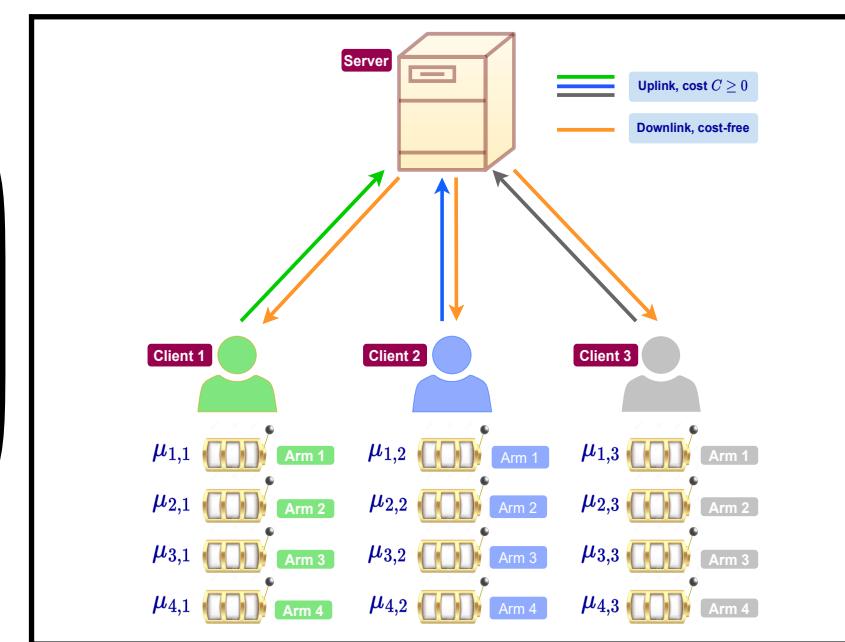
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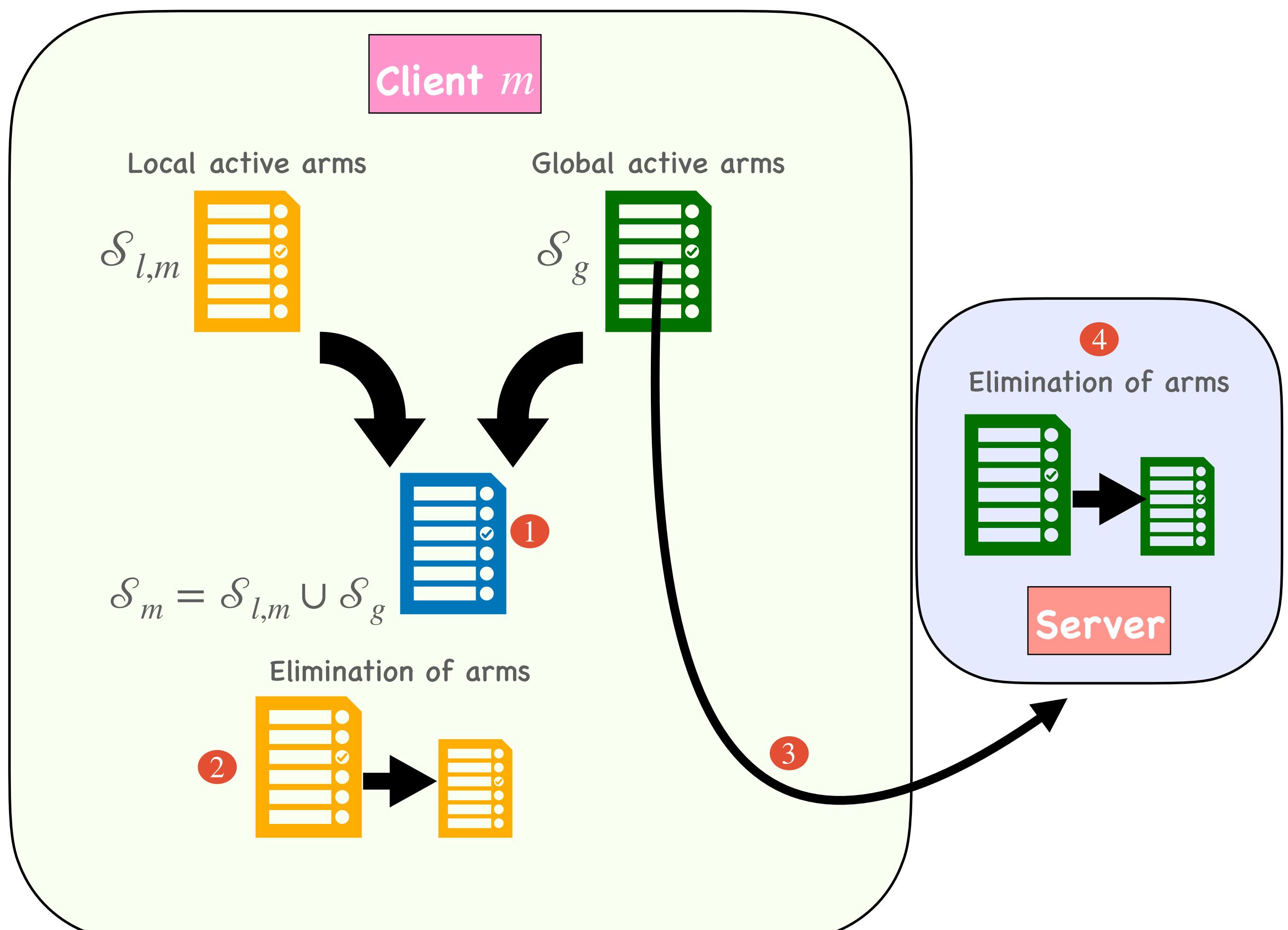
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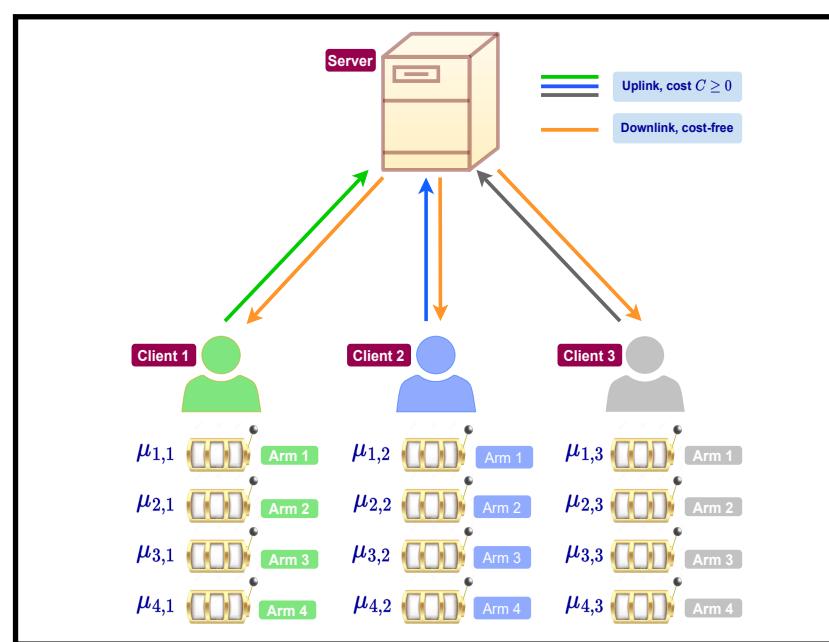
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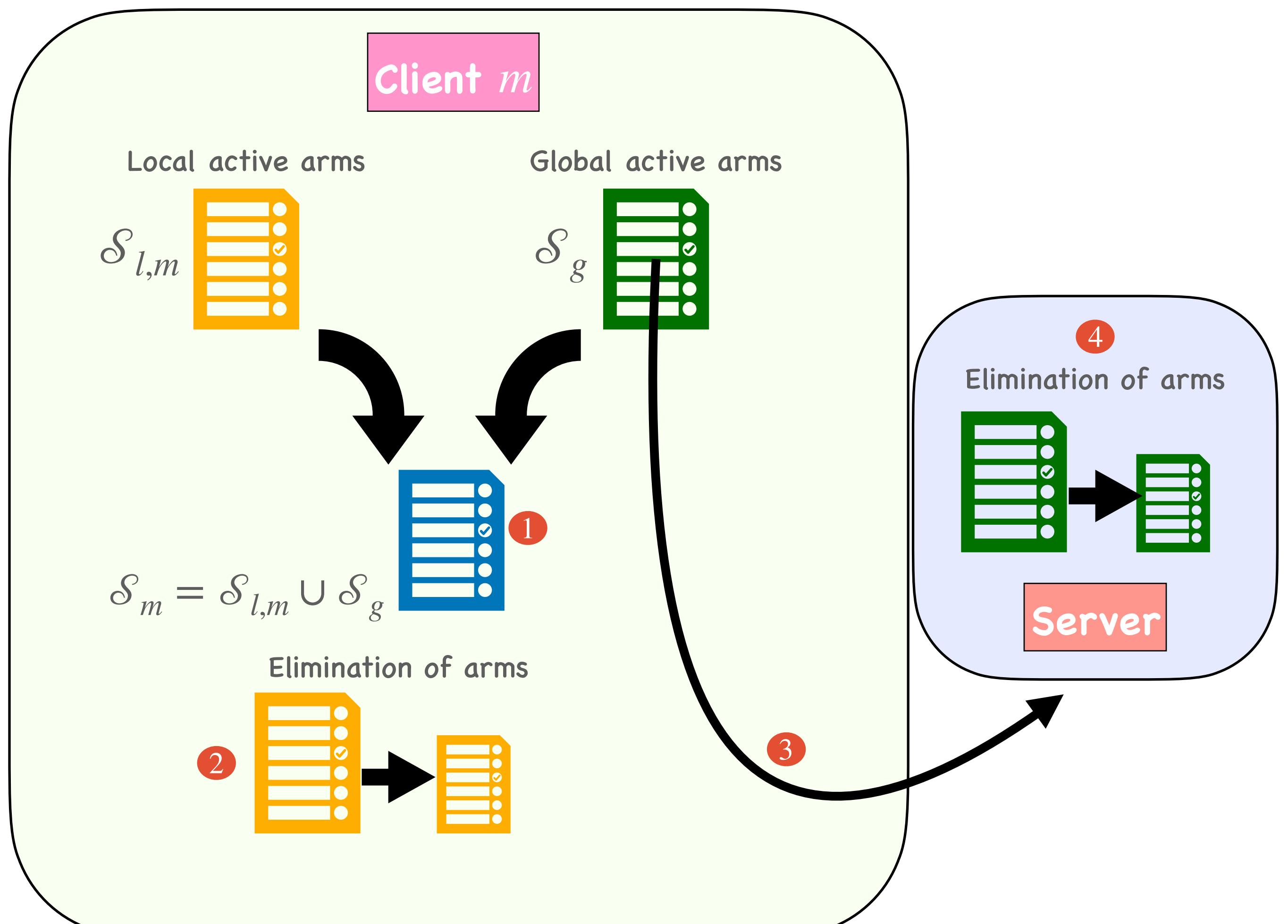
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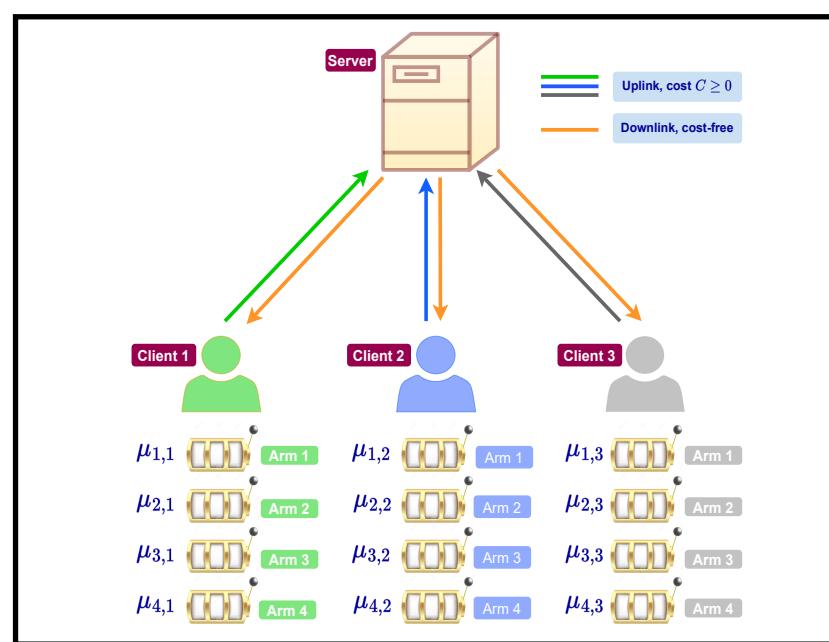
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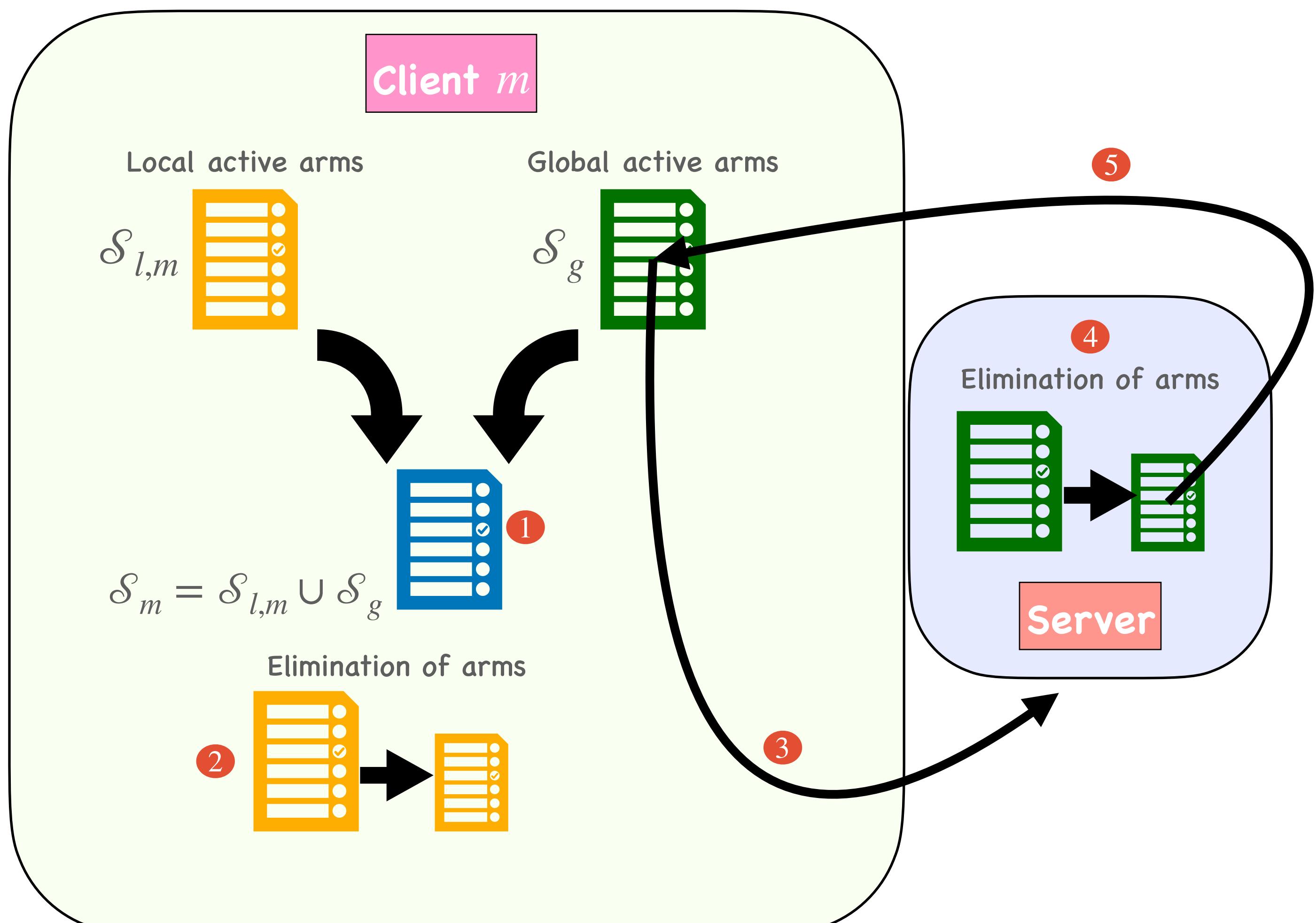
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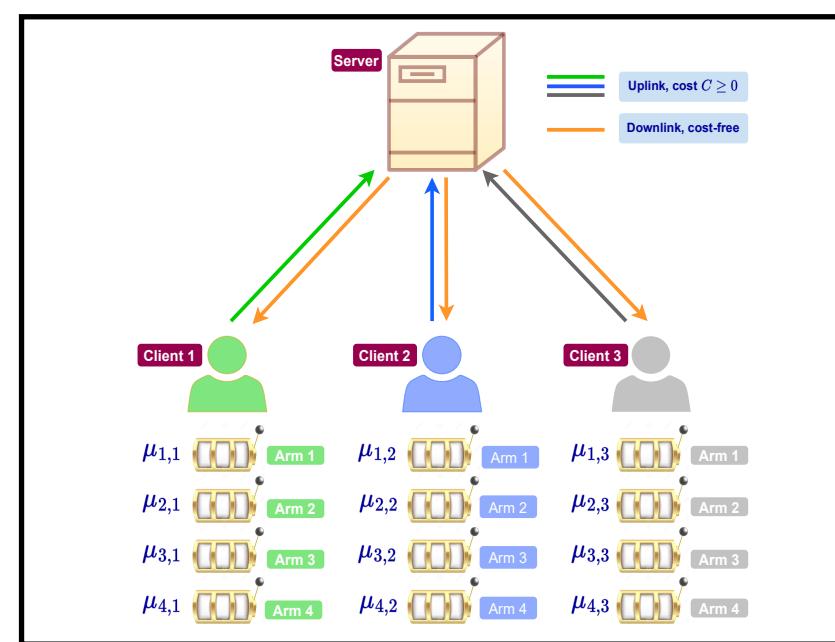
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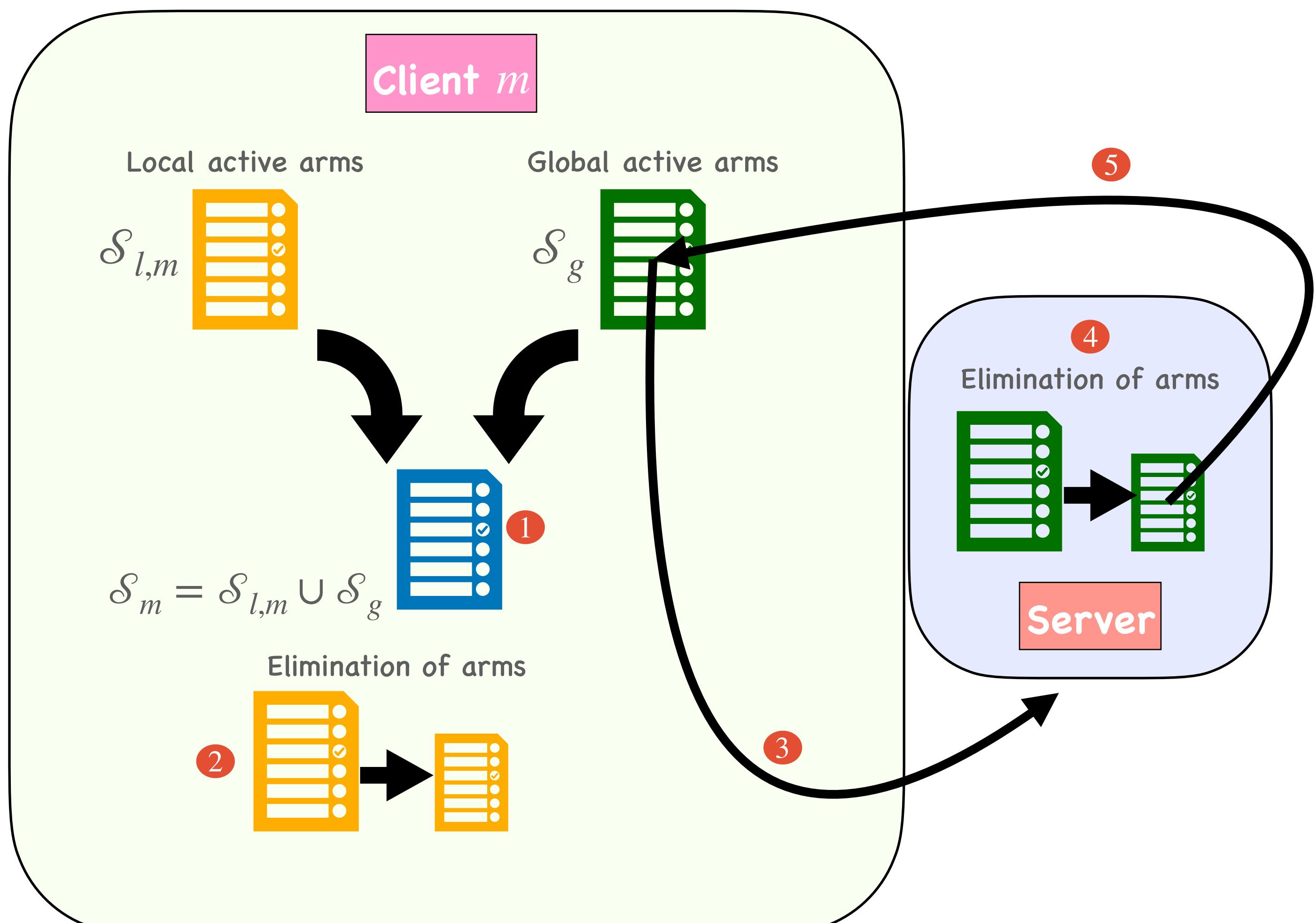
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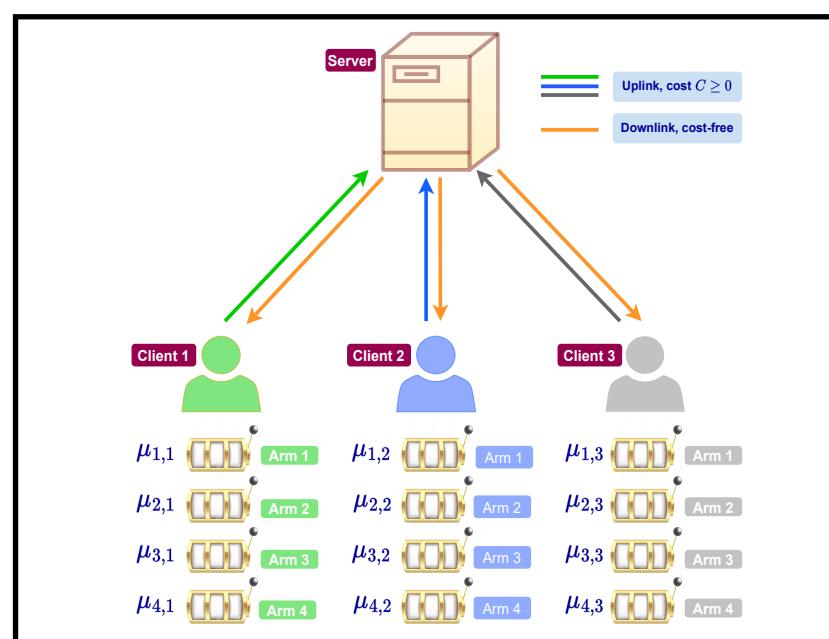
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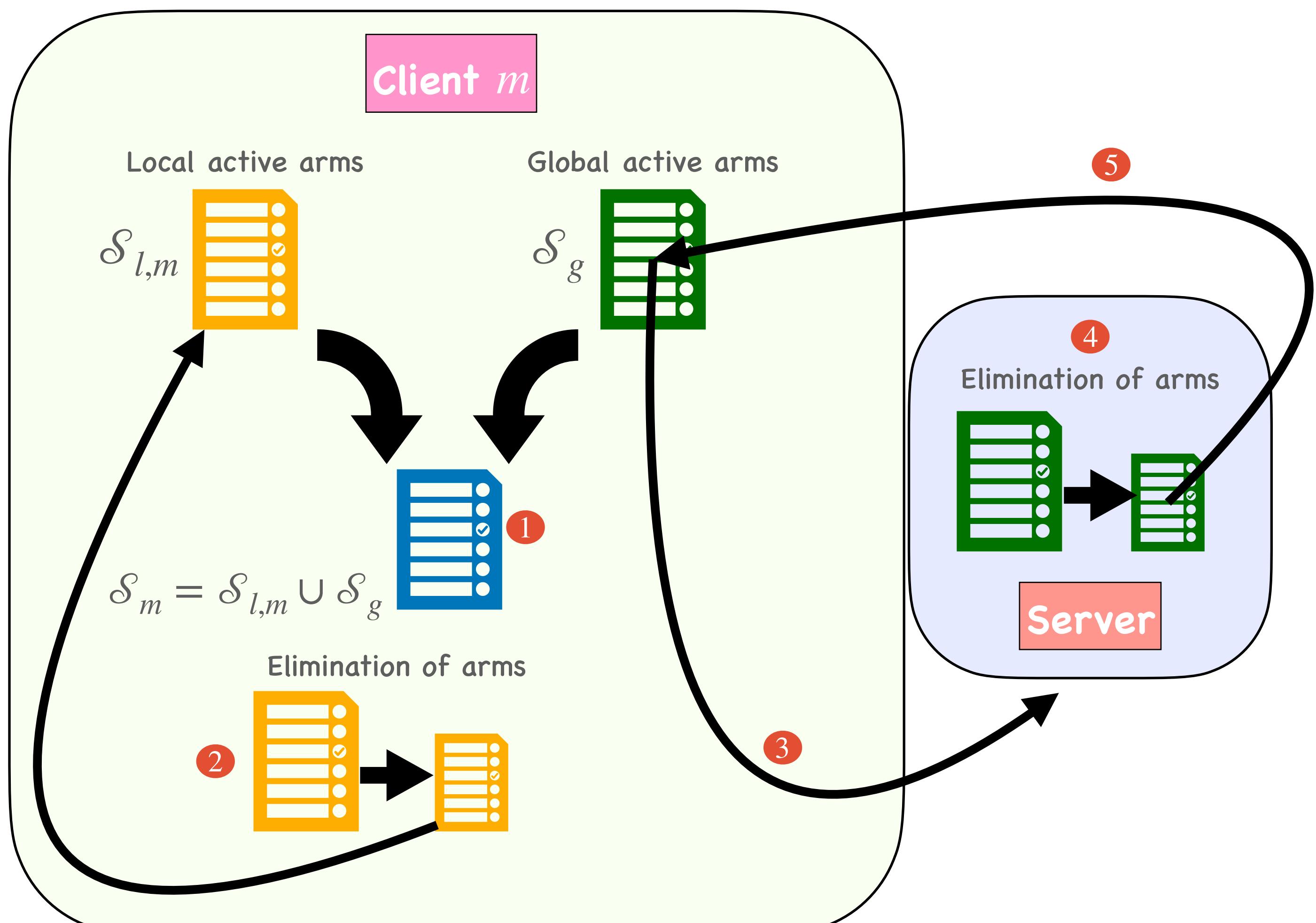
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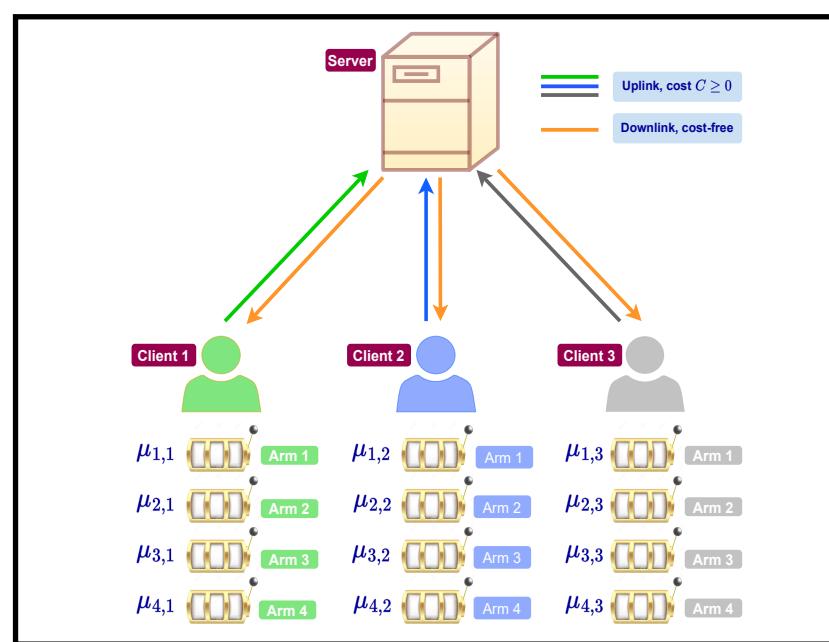
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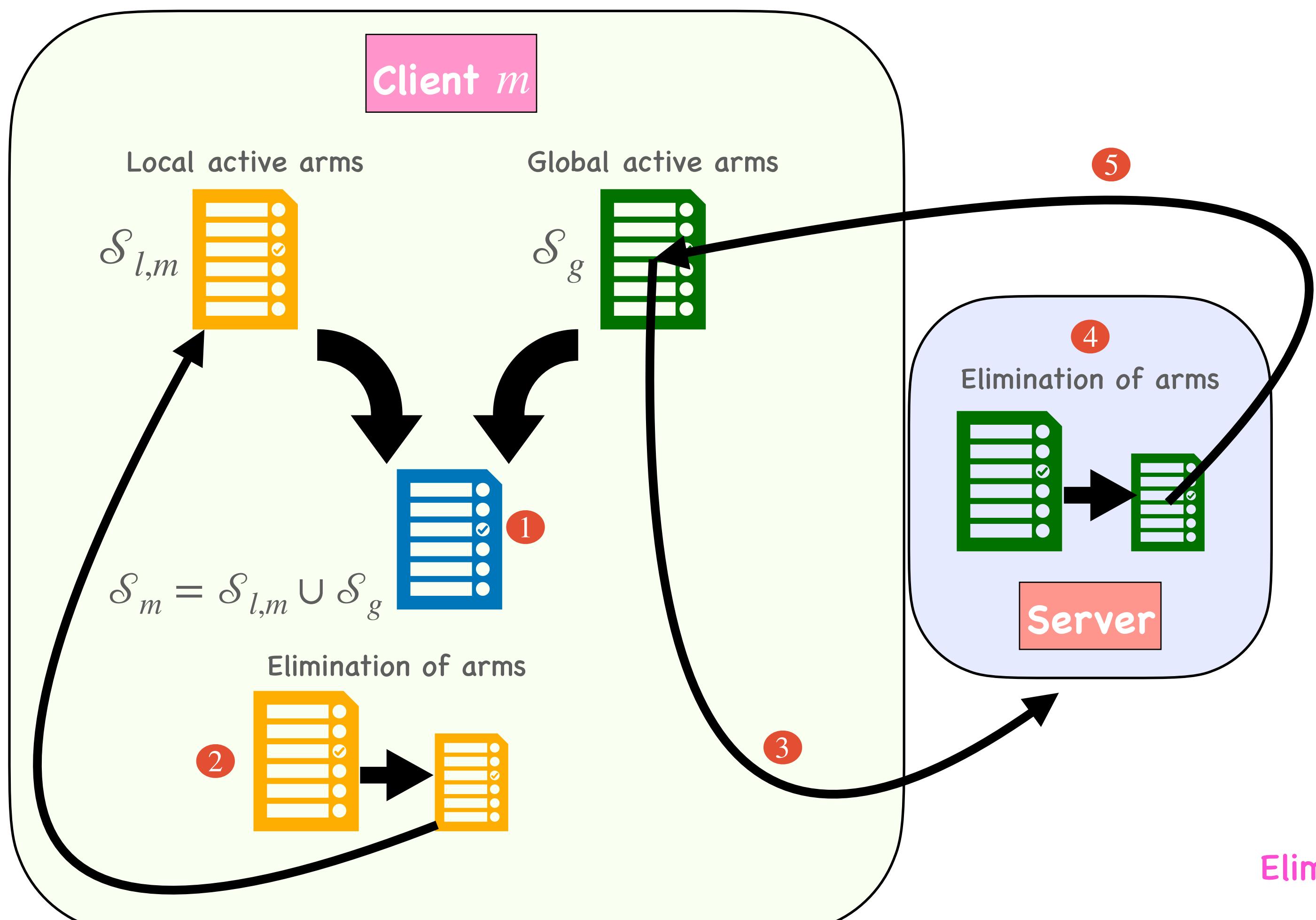
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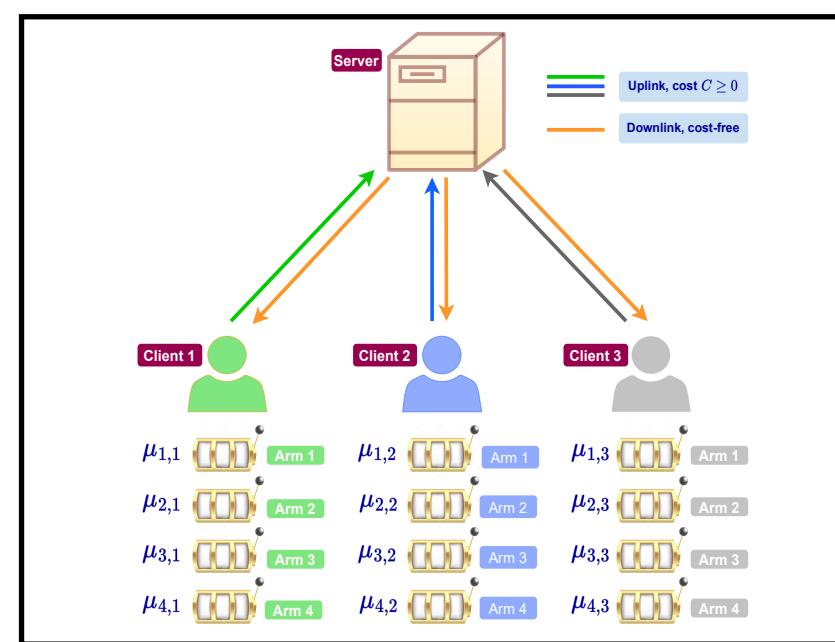
$$\alpha_g(n) = \sqrt{\frac{2 \ln(8Kn^2/\delta)}{Mn}}$$

Elim. at client m continues until $|\mathcal{S}_{l,m}| = 1$ (local best arm)

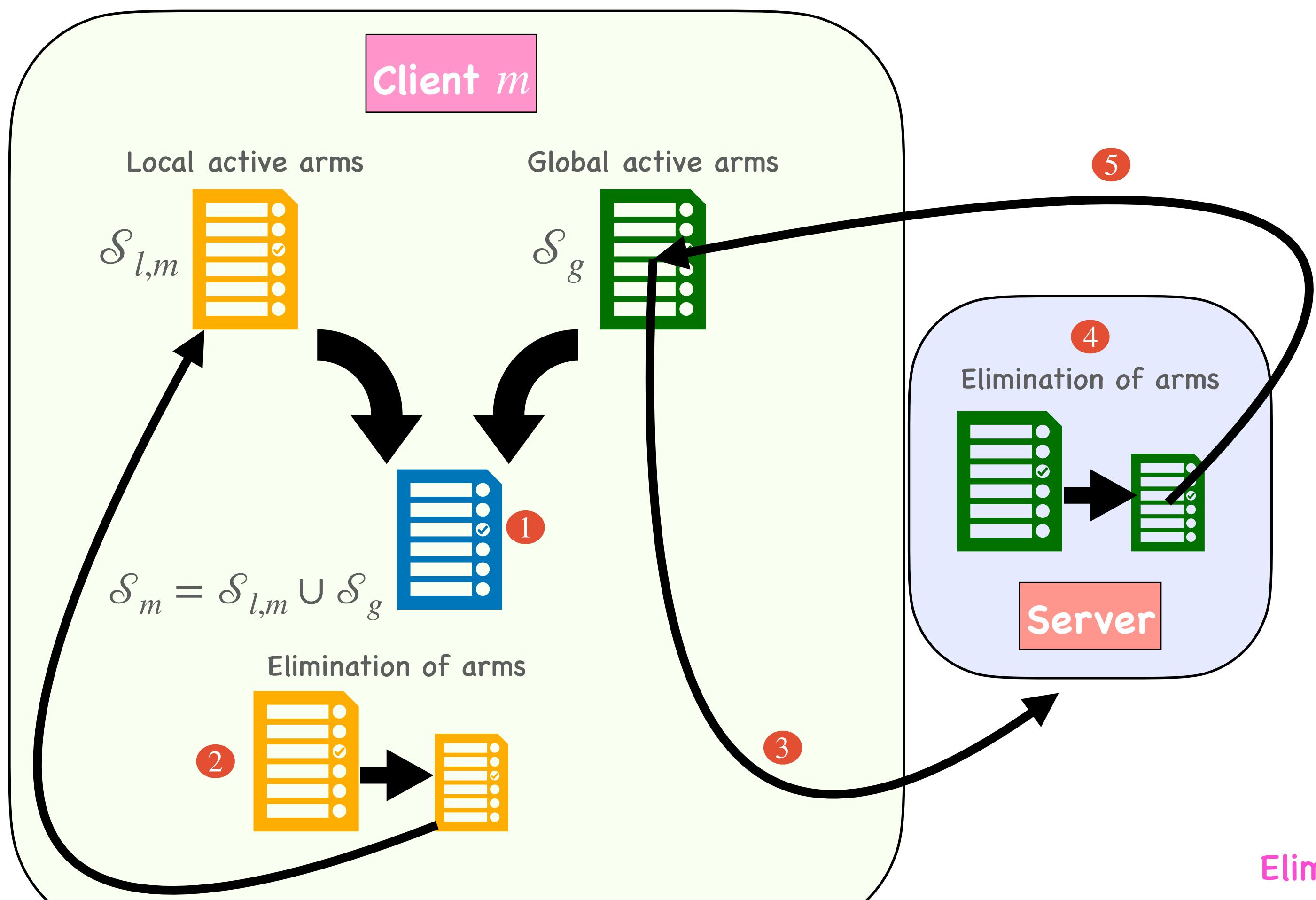
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Comm. with server stops when $|\mathcal{S}_g| = 1$ (global best arm)

Performance of FedElim0 ($C = 0$)

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- Stops in finite time w.p.1

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Performance of FedElim0 ($C = 0$)

- Stops in finite time w.p.1
- Error probability $\leq \delta$
- Total cost of FedElim0 (T_{FedElim0}) satisfies

$$P\left(T_{\text{FedElim0}} \leq \underbrace{\sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, T_k\}}_{=T}\right) \geq 1 - \delta,$$

$$T_{k,m} = 102 \cdot \frac{\ln\left(\frac{64\sqrt{\frac{8KM}{\delta}}}{\Delta_{k,m}^2}\right)}{\Delta_{k,m}^2} + 1, \quad T_k = 102 \cdot \frac{\ln\left(\frac{64\sqrt{\frac{8K}{\delta}}}{M\Delta_k^2}\right)}{M\Delta_k^2} + 1,$$

$\Delta_{k,m} = \max_a \mu_{a,m} - \mu_{k,m}$ (local sub-optimality gap),

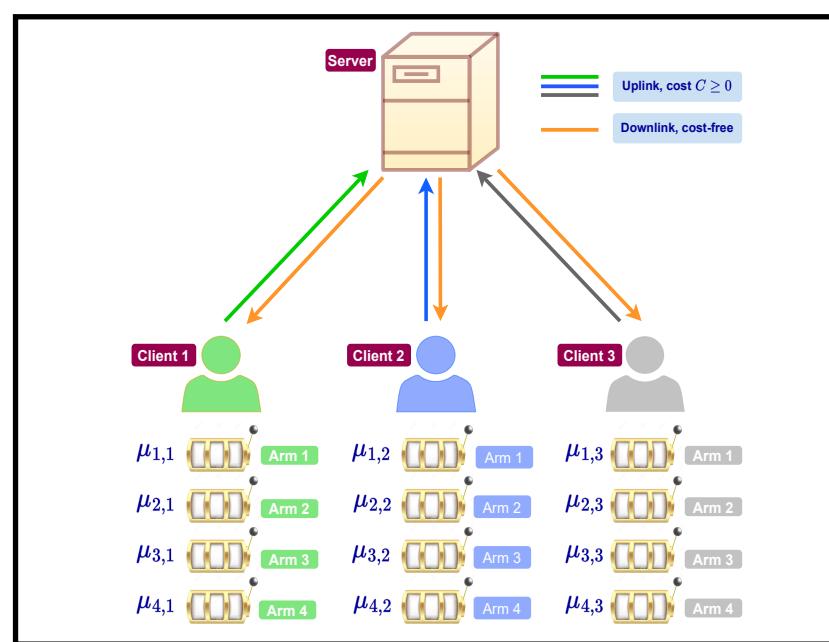
$\Delta_k = \max_a \frac{1}{M} \sum_{m=1}^M (\mu_{a,m} - \mu_{k,m})$ (global sub-optimality gap),

The $C > 0$ Case

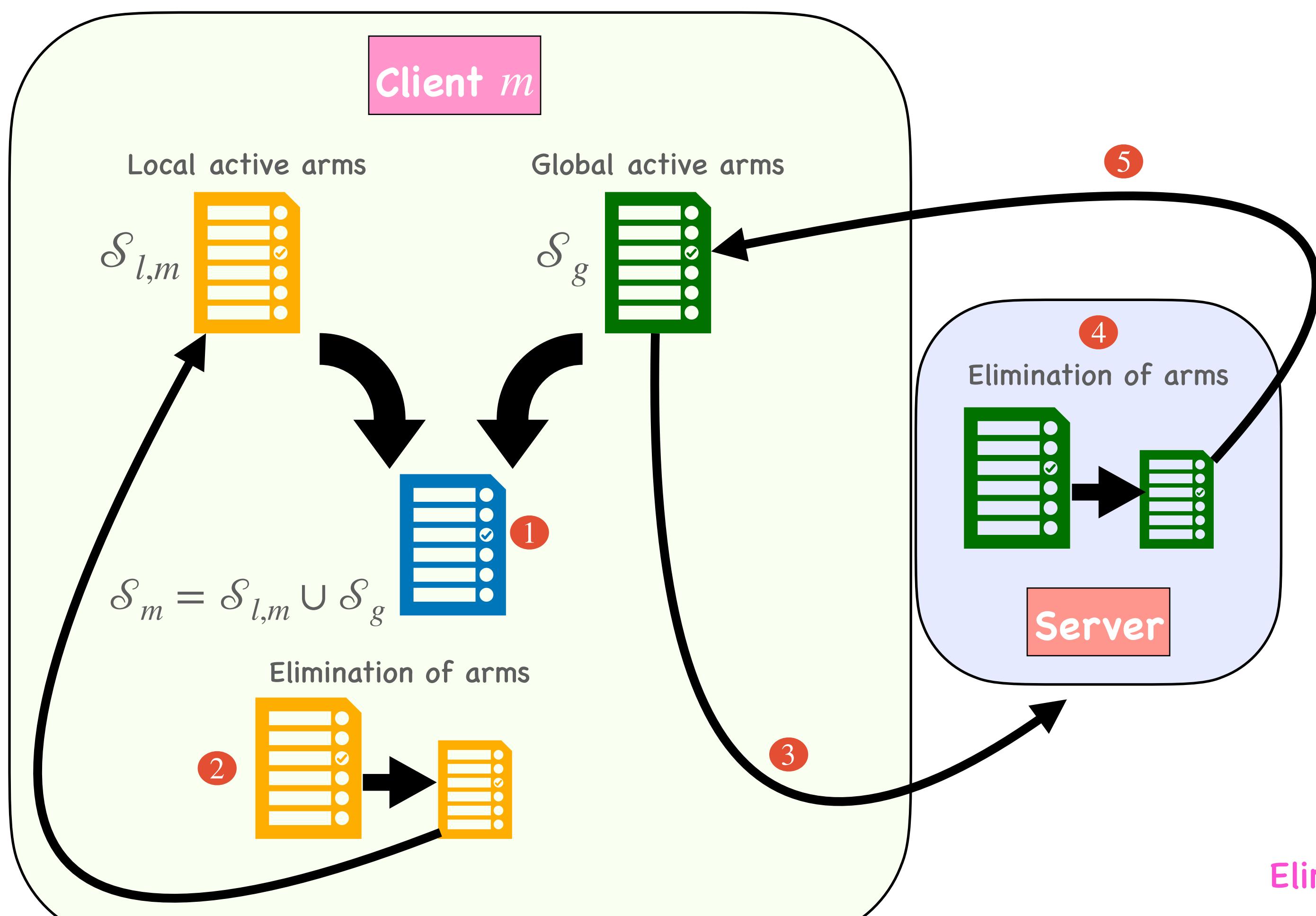
FedElim ($C > 0$)

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= total cost



Comm. only at exponential time instants $n \in \{2^t : t \geq 0\}$



Initialise: $\mathcal{S}_{l,m} = [K]$, $\mathcal{S}_g = [K]$

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Update $\mathcal{S}_{l,m}$
3. Client m sends $\{\hat{\mu}_{k,m}(n) : k \in \mathcal{S}_g\}$ and \mathcal{S}_g to server if $n \in \{2^t\}_{t=0}^\infty$
4. Eliminate $k \in \mathcal{S}_g$ if $\frac{1}{M} \sum_{m=1}^M \hat{\mu}_{k,m}(n) \leq \max_{a \in \mathcal{S}_g} \frac{1}{M} \sum_{m=1}^M \hat{\mu}_{a,m}(n) - 2\alpha_g(n)$
5. Server sends updated \mathcal{S}_g to all clients

$$\alpha_l(n) = \sqrt{\frac{2 \ln(8Kn^2/\delta)}{n}}$$

$$\alpha_g(n) = \sqrt{\frac{2 \ln(8Kn^2/\delta)}{Mn}}$$

Elim. at client m continues until $|\mathcal{S}_{l,m}| = 1$ (local best arm)

Comm. with server stops when $|\mathcal{S}_g| = 1$ (global best arm)

Performance of FedElim ($C > 0$)

$$\tau = \sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, T_k\}$$

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- Stops in finite time w.p.1
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- Error probability $\leq \delta$
- With prob. $1 - \delta$,
 - Total no. of arm pulls (T_{FedElim}) satisfies $T_{\text{FedElim}} \leq \sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, 2T_k\} \leq 2\tau$

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 - Comm cost ($C_{\text{FedElim}}^{\text{comm}}$) satisfies $C_{\text{FedElim}}^{\text{comm}} \leq C \times M \times \sum_{k=1}^K \left\lceil \frac{\ln T_k}{\ln 2} \right\rceil \leq \tau^*$

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* for sufficiently small δ

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- Error probability $\leq \delta$
- With prob. $1 - \delta$,
 - Total no. of arm pulls (T_{FedElim}) satisfies $T_{\text{FedElim}} \leq \sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, 2T_k\} \leq 2\tau$
 - Comm cost ($C_{\text{FedElim}}^{\text{comm}}$) satisfies $C_{\text{FedElim}}^{\text{comm}} \leq C \times M \times \sum_{k=1}^K \left\lceil \frac{\ln T_k}{\ln 2} \right\rceil \leq \tau^*$
 - $T_{\text{FedElim}} + C_{\text{FedElim}}^{\text{comm}} \leq 3\tau$

* for sufficiently small δ

Numerical Results

Synthetic Gaussian Data – 1/3

$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Clients

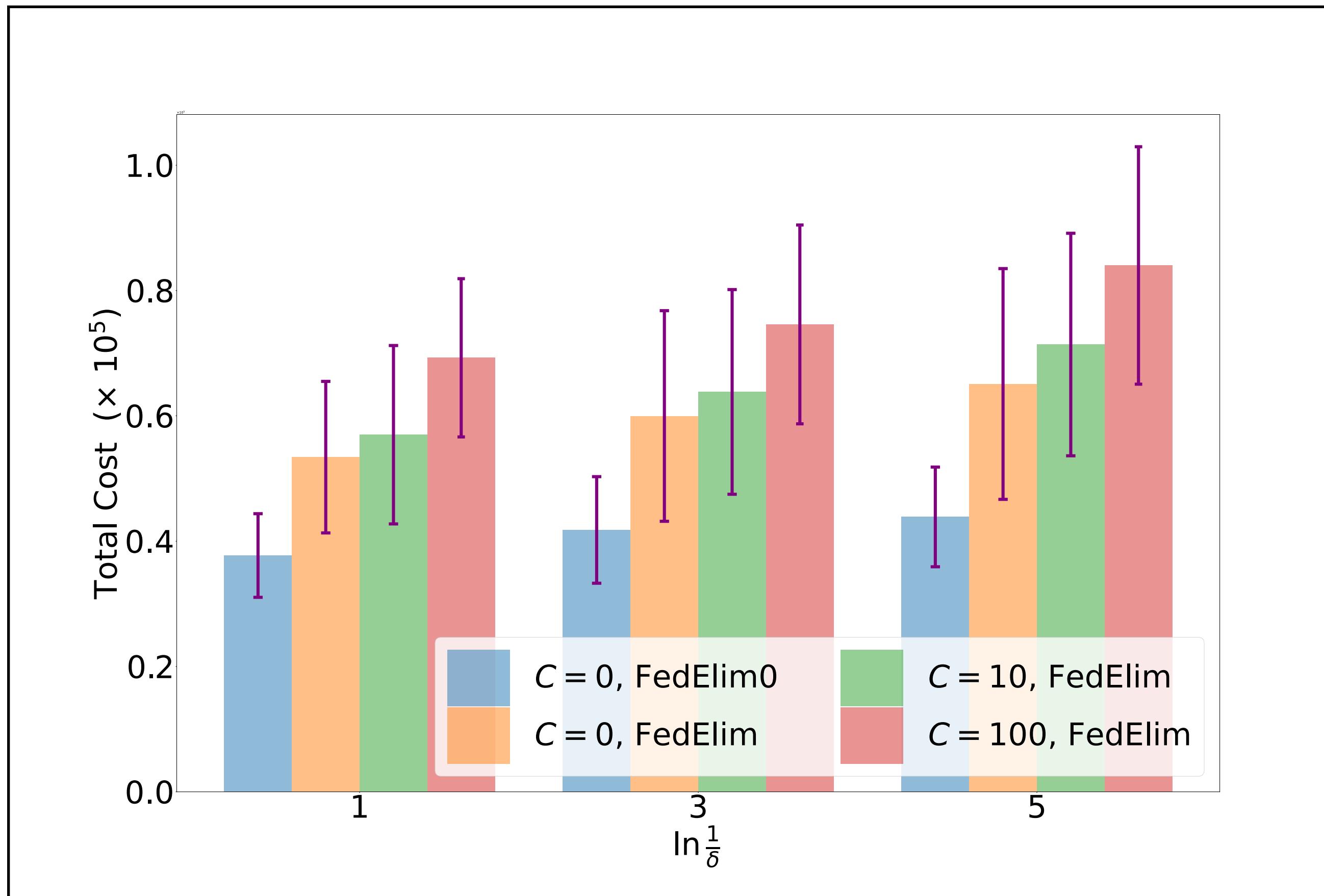
Local best arm of client 1 = 1

Local best arm of client 2 = 2

Local best arm of client 3 = 3

Global best arm = 4

Synthetic Gaussian Data – 1/3

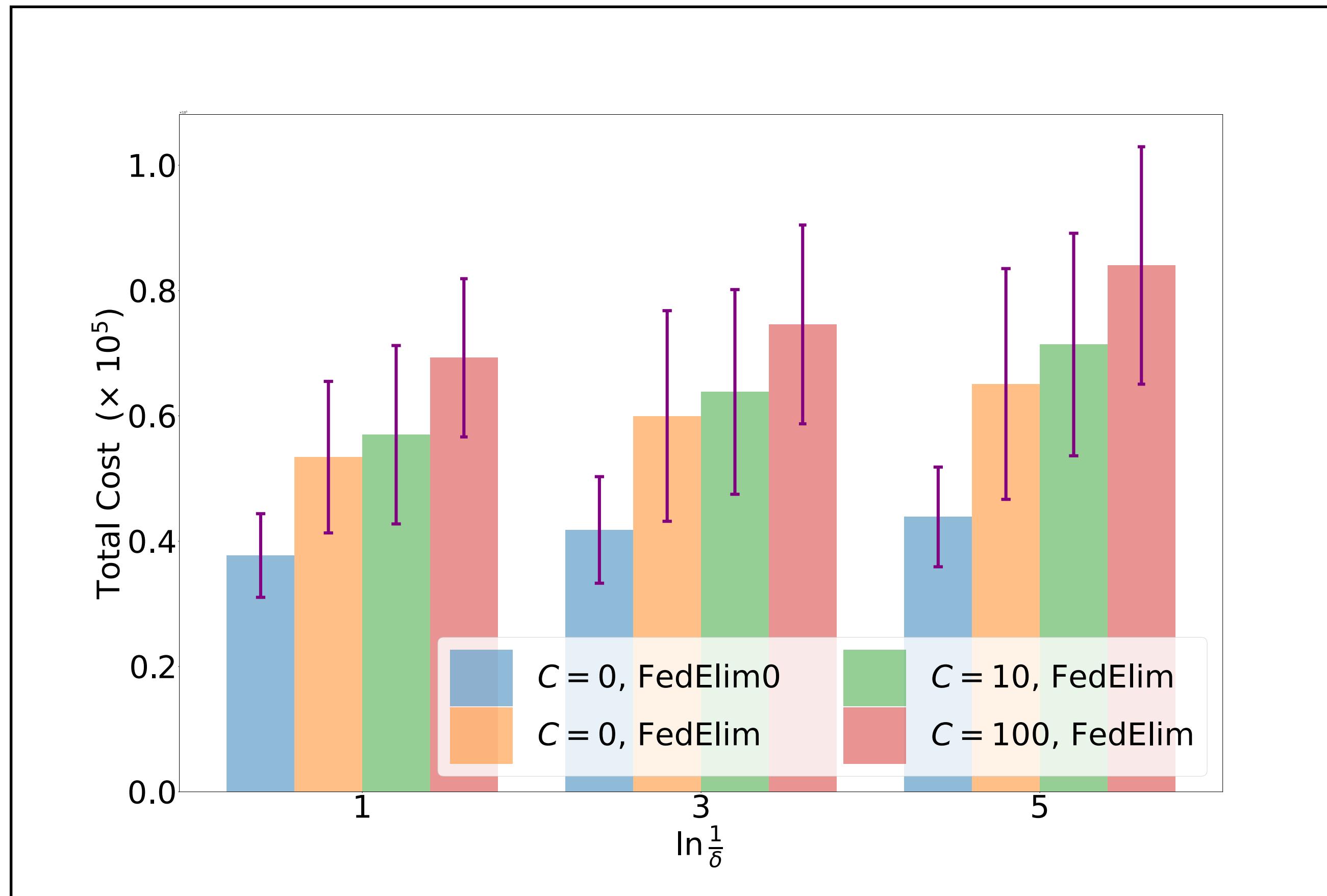


$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Local best arm of client 1 = 1
Local best arm of client 2 = 2
Local best arm of client 3 = 3
Global best arm = 4

Synthetic Gaussian Data – 1/3



Length of blue bar $\leq T$

$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Clients

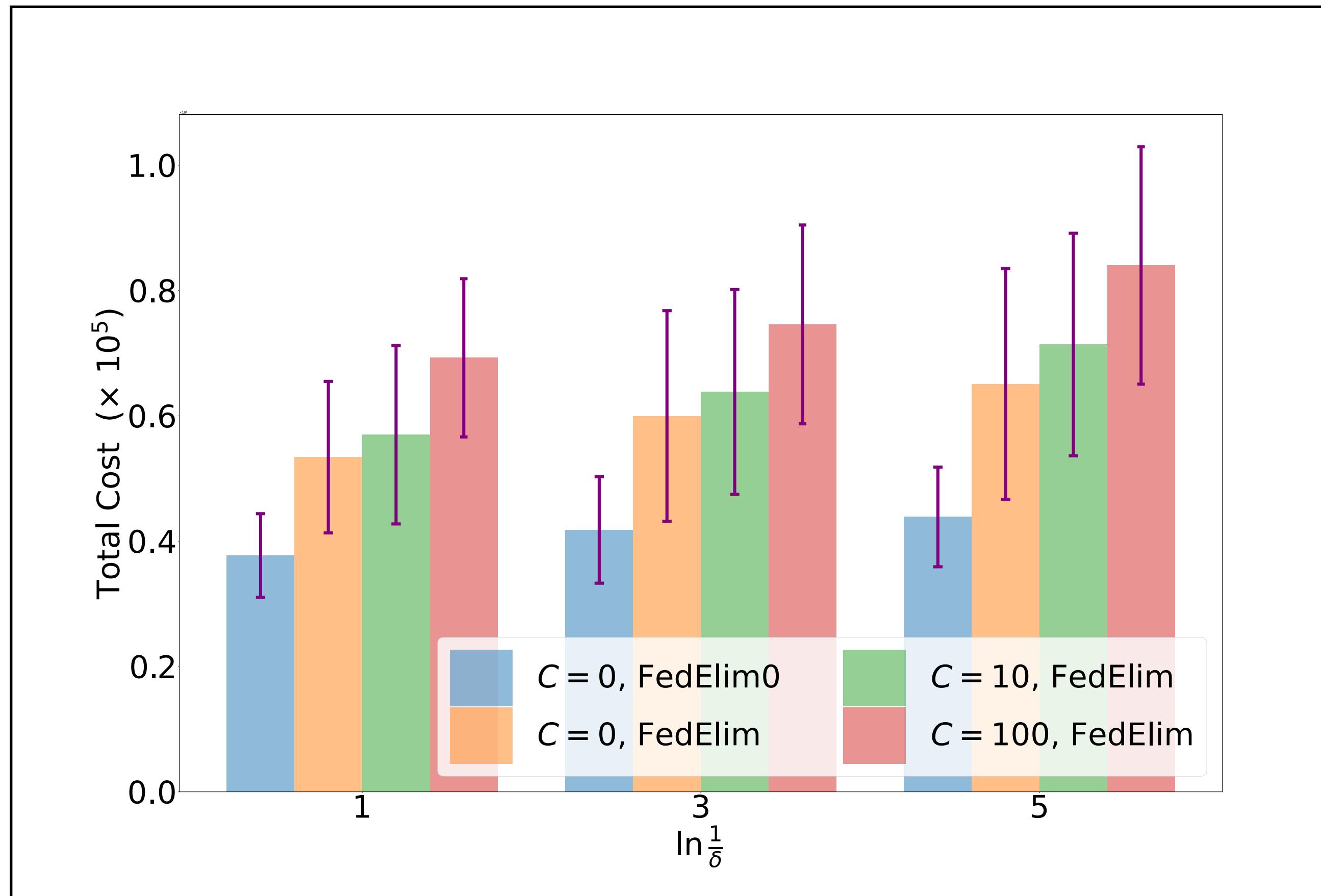
Local best arm of client 1 = 1

Local best arm of client 2 = 2

Local best arm of client 3 = 3

Global best arm = 4

Synthetic Gaussian Data – 1/3



$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Clients

Local best arm of client 1 = 1

Local best arm of client 2 = 2

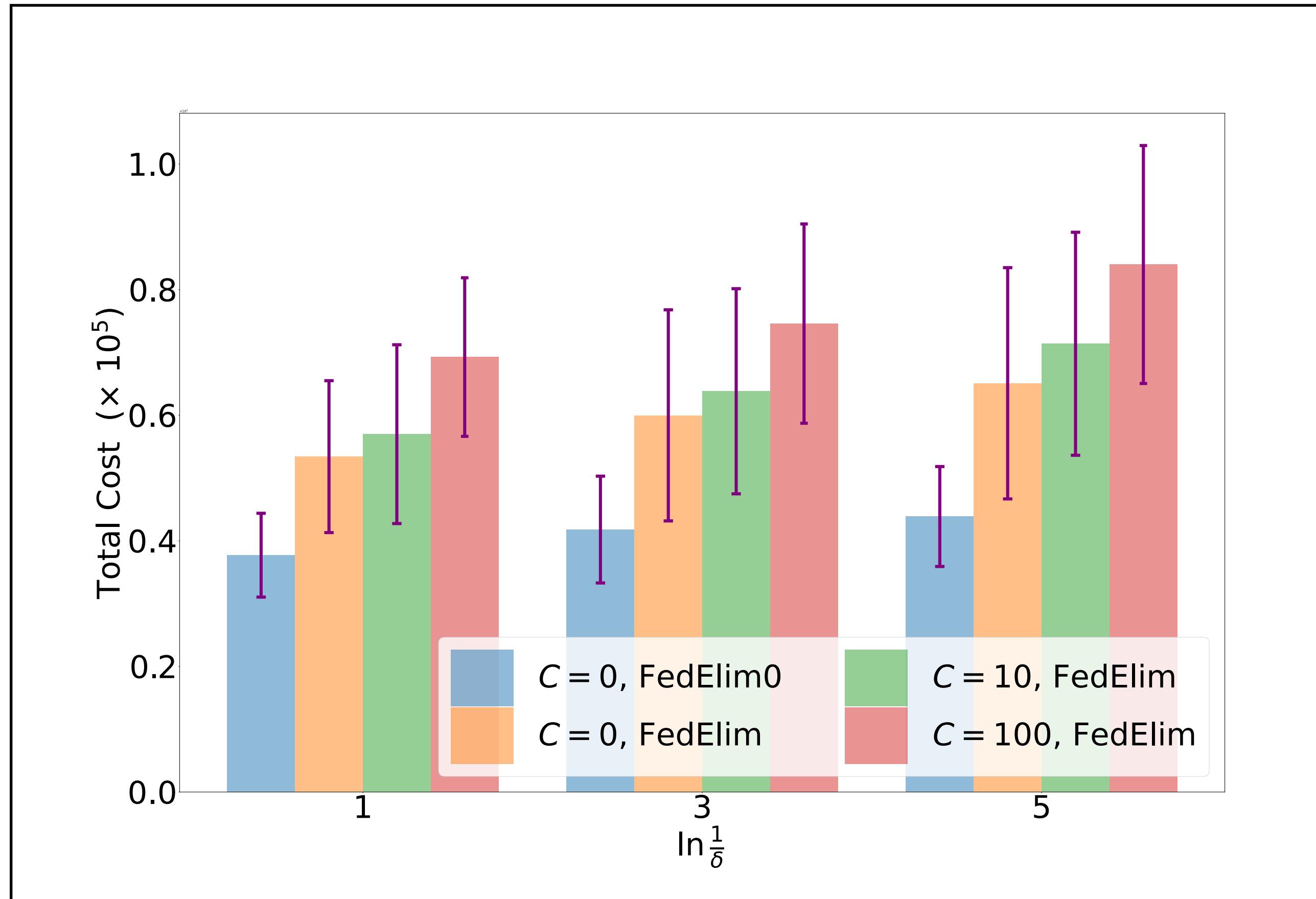
Local best arm of client 3 = 3

Global best arm = 4

Length of blue bar $\leq T$

Length of yellow bar $\leq 2T$

Synthetic Gaussian Data – 1/3



$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

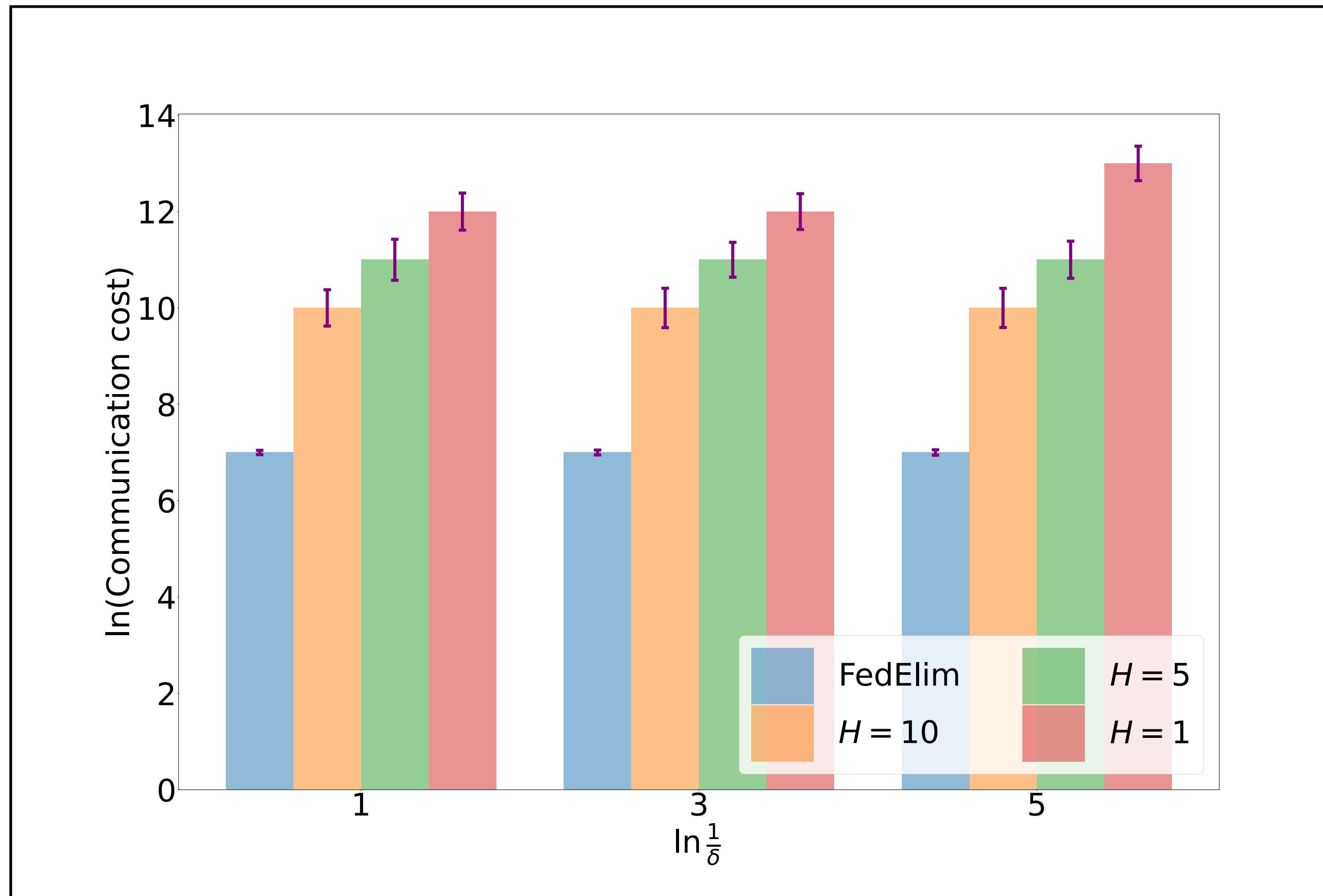
Local best arm of client 1 = 1
Local best arm of client 2 = 2
Local best arm of client 3 = 3
Global best arm = 4

Length of blue bar $\leq T$

Length of yellow bar $\leq 2T$

Length of any bar $\leq 3T$

Synthetic Gaussian Data – 2/3

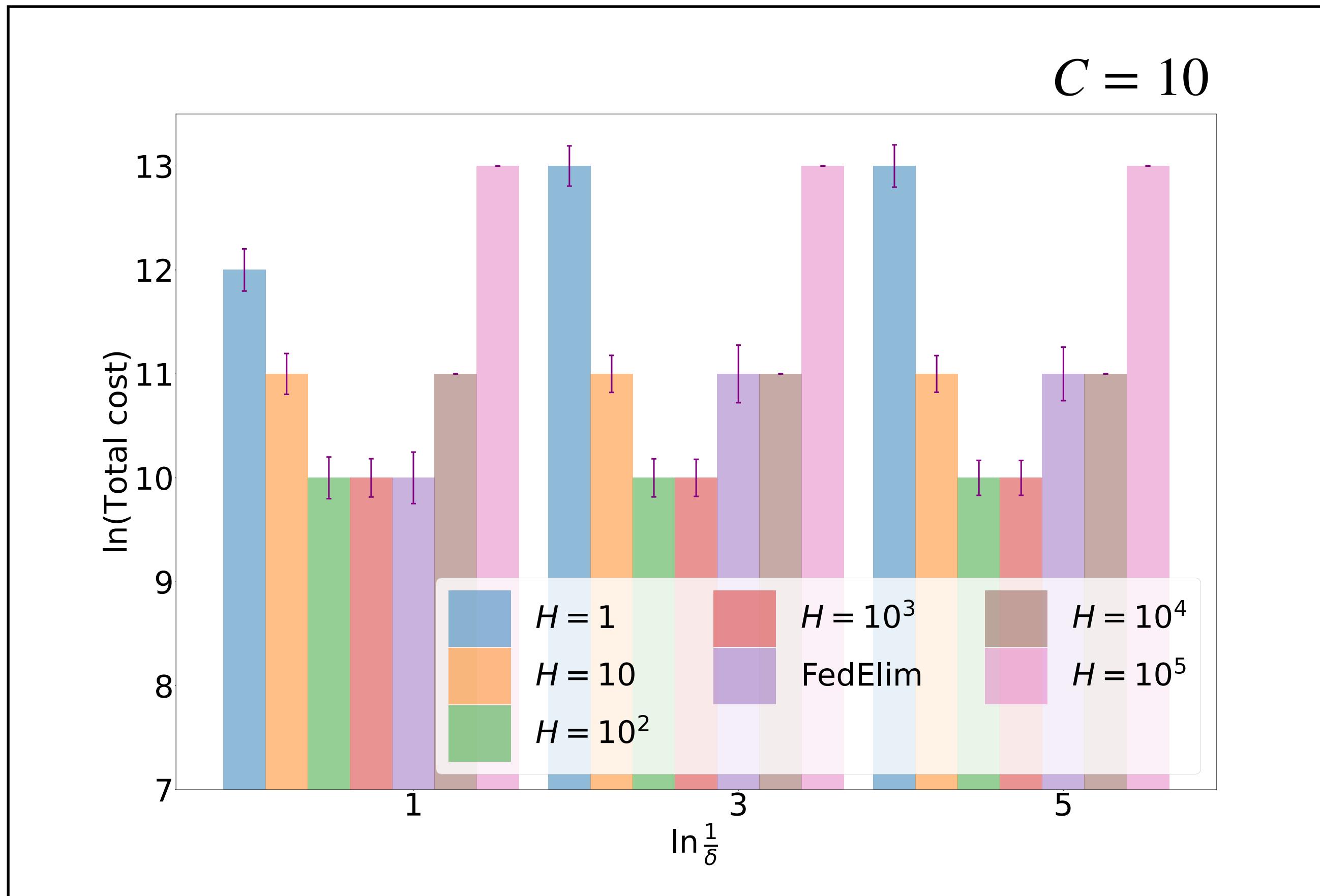


$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Local best arm of client 1 = 1
Local best arm of client 2 = 2
Local best arm of client 3 = 3
Global best arm = 4

Synthetic Gaussian Data – 3/3



$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Clients

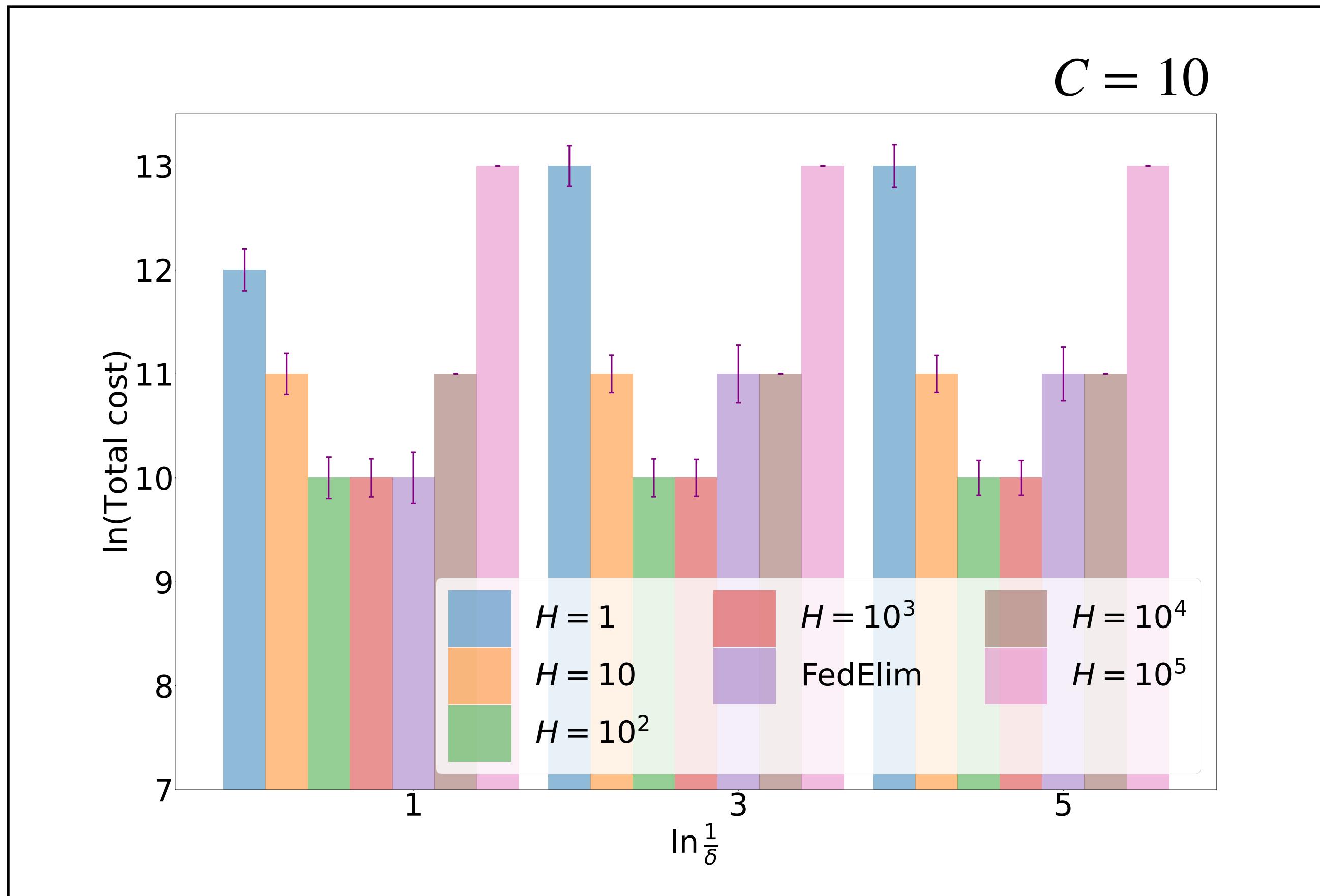
Local best arm of client 1 = 1

Local best arm of client 2 = 2

Local best arm of client 3 = 3

Global best arm = 4

Synthetic Gaussian Data – 3/3



FedElim operates at a “sweet spot”

$$\mu = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.1 & 0.1 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Arms

Clients

Local best arm of client 1 = 1

Local best arm of client 2 = 2

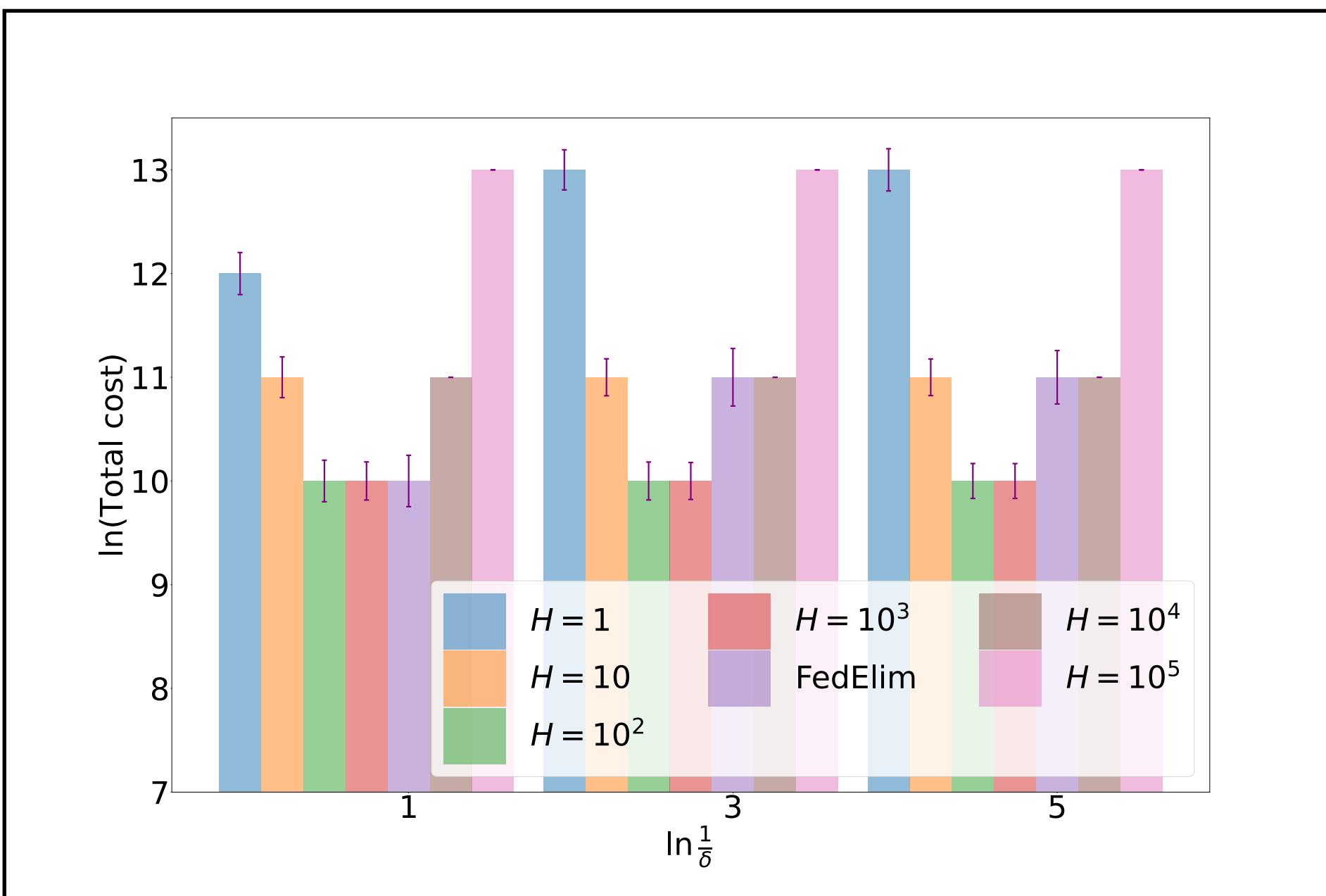
Local best arm of client 3 = 3

Global best arm = 4

$$C = 10$$

$$T_{\text{arm}} = \text{no. of pulls} + 10 \times t$$

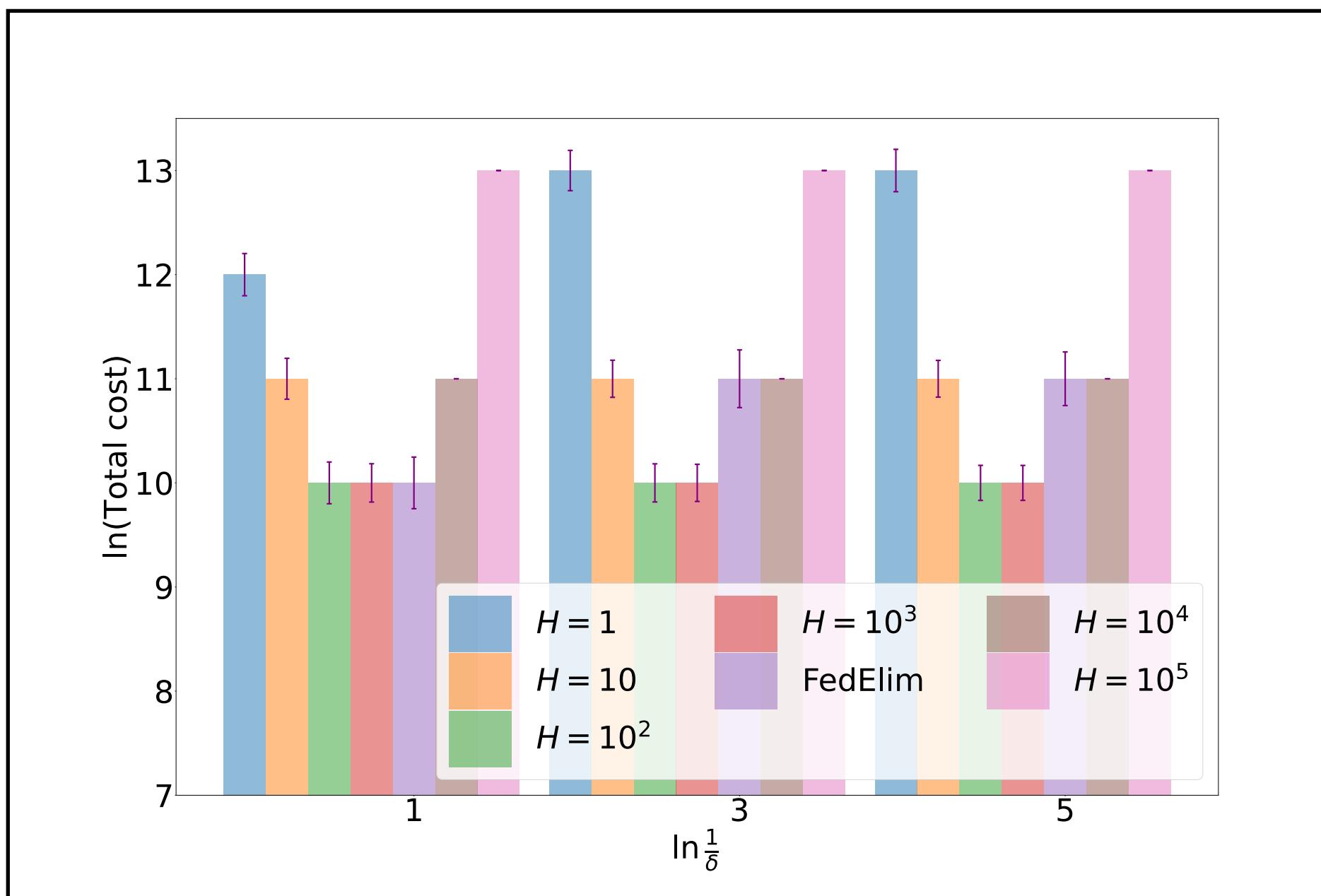
$$T_k = 1000 \text{ (depends on } \delta)$$



$$C = 10$$

$$T_{\text{arm}} = \text{no. of pulls} + 10 \times t$$

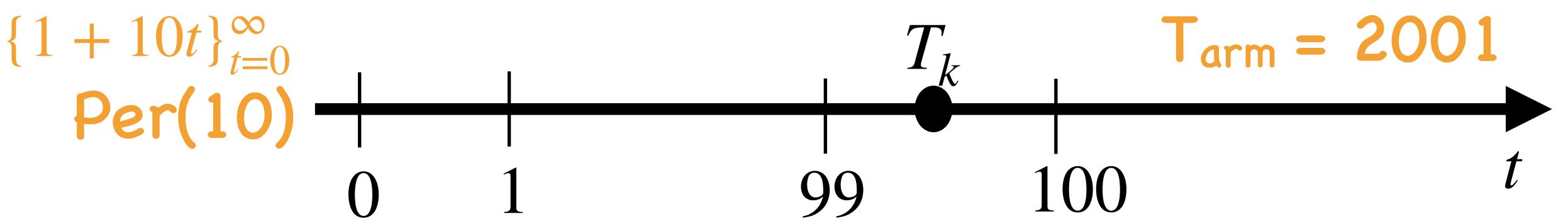
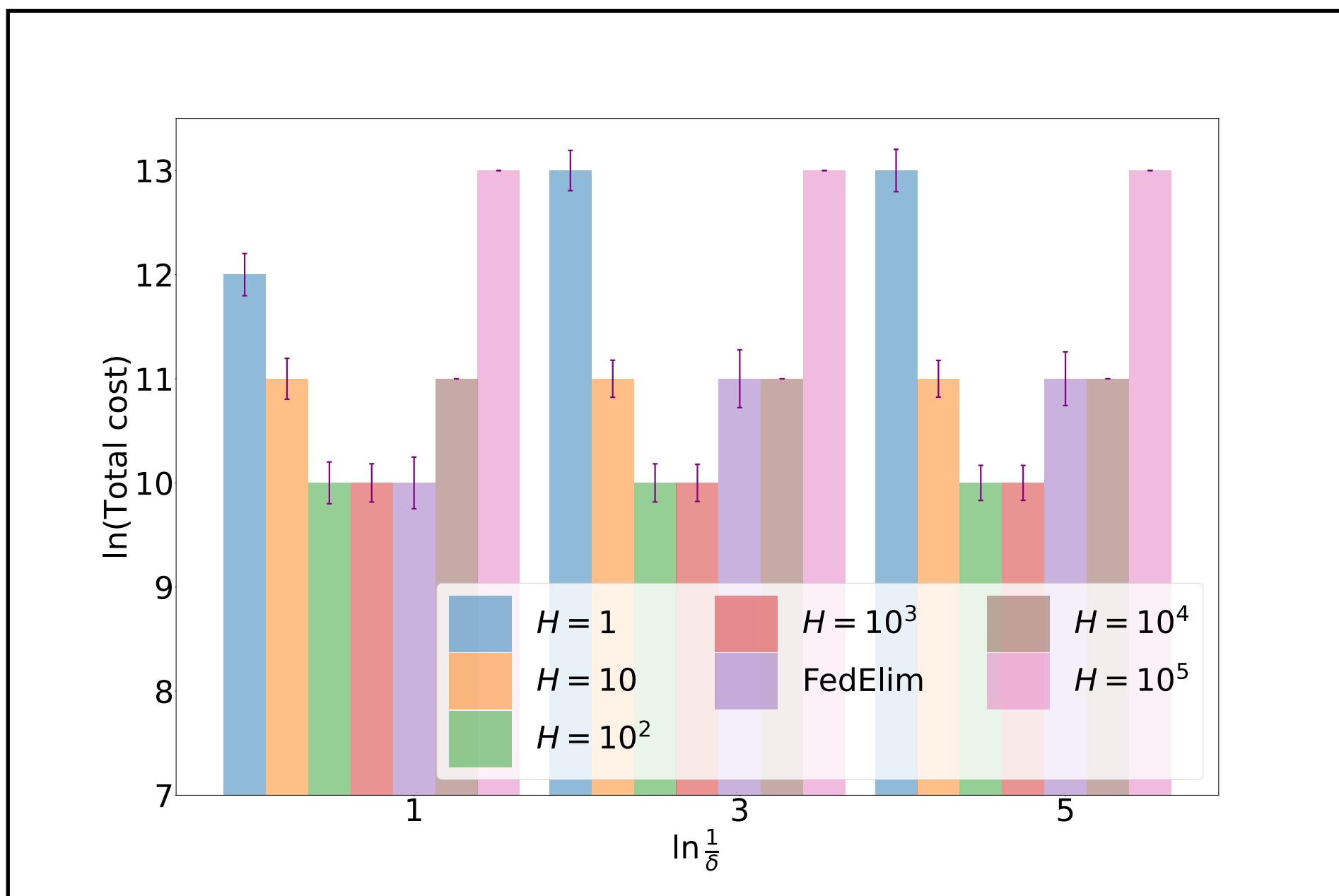
$$T_k = 1000 \text{ (depends on } \delta)$$



$$C = 10$$

$$T_{\text{arm}} = \text{no. of pulls} + 10 \times t$$

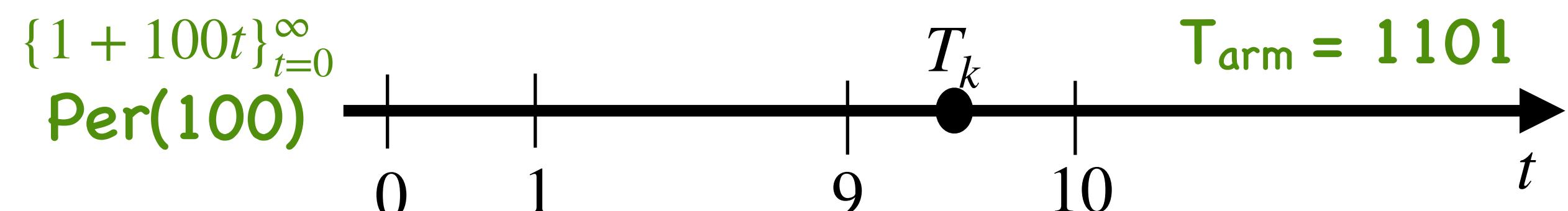
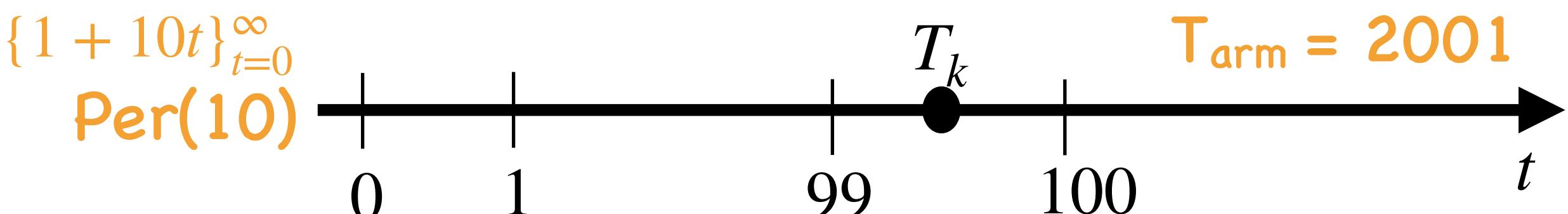
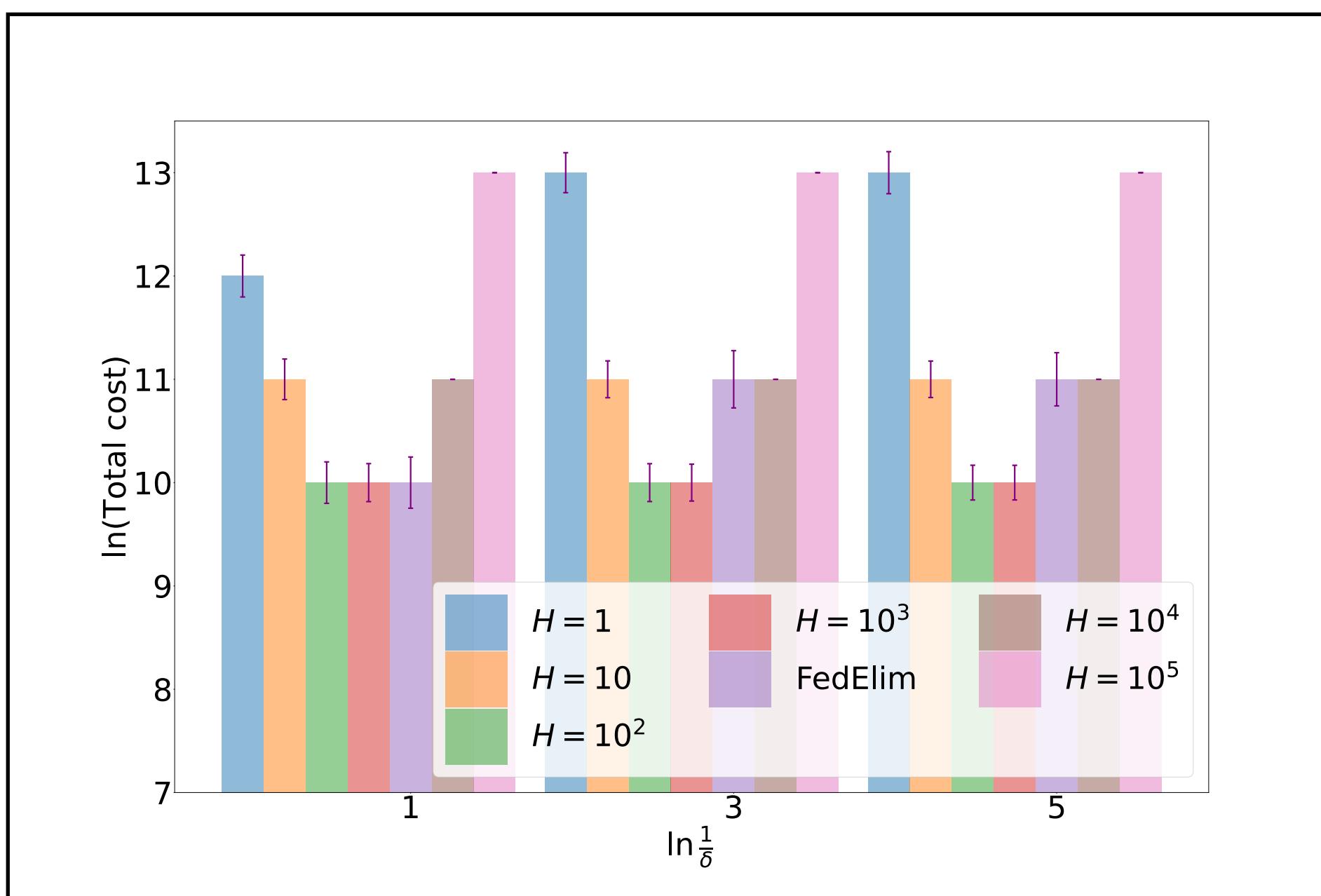
$$T_k = 1000 \text{ (depends on } \delta)$$



$$C = 10$$

$$T_{\text{arm}} = \text{no. of pulls} + 10 \times t$$

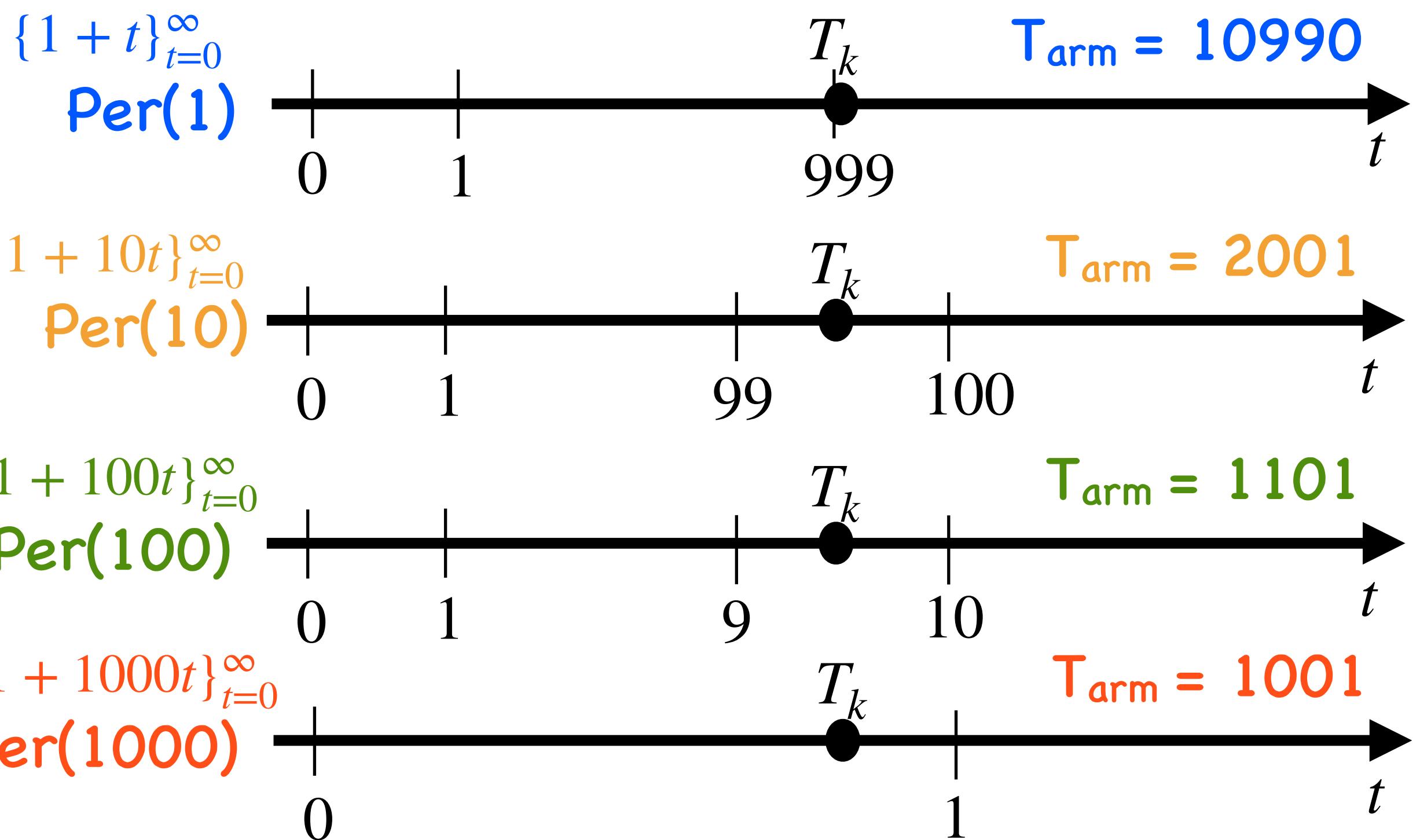
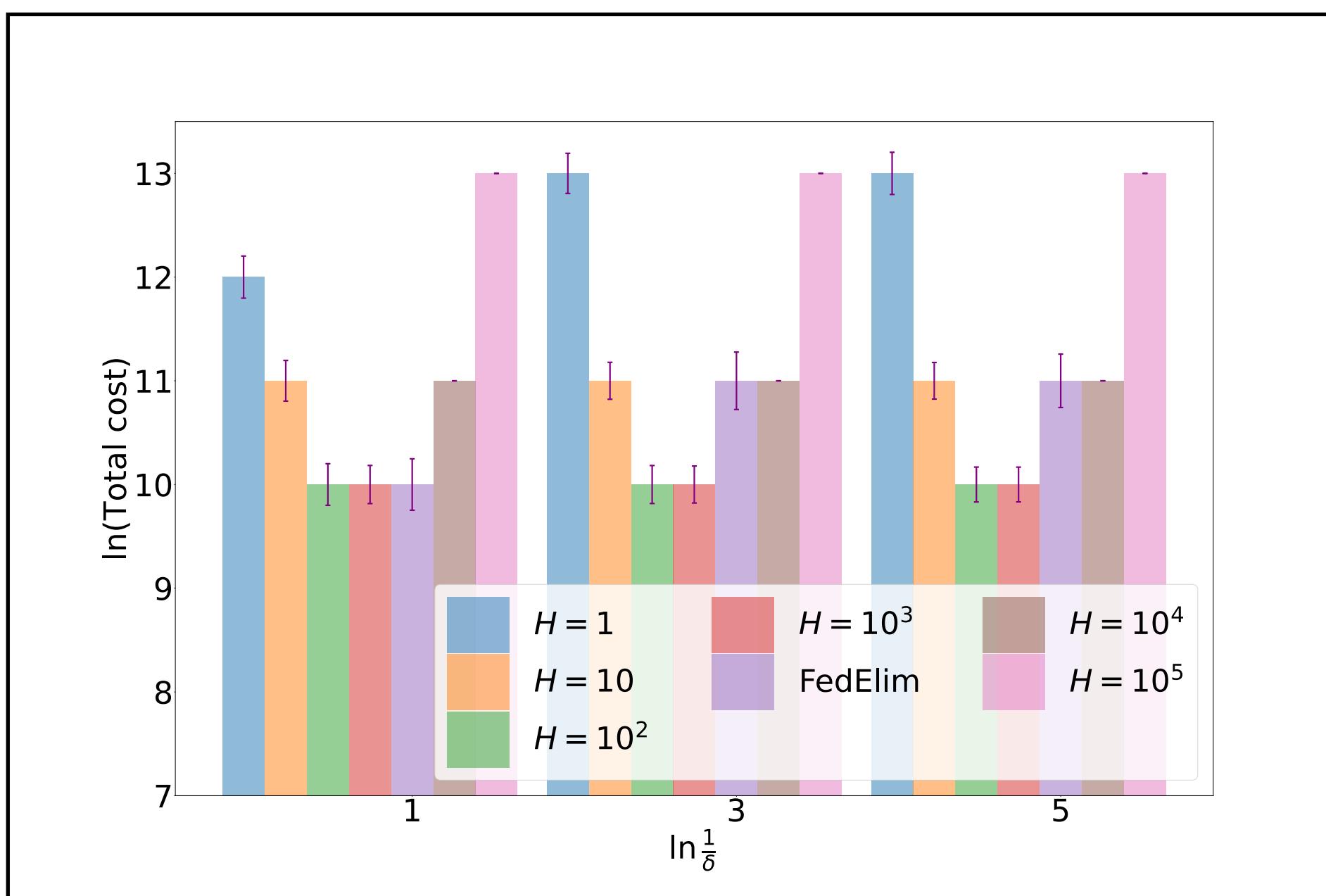
$$T_k = 1000 \text{ (depends on } \delta)$$



$$C = 10$$

$$T_{\text{arm}} = \text{no. of pulls} + 10 \times t$$

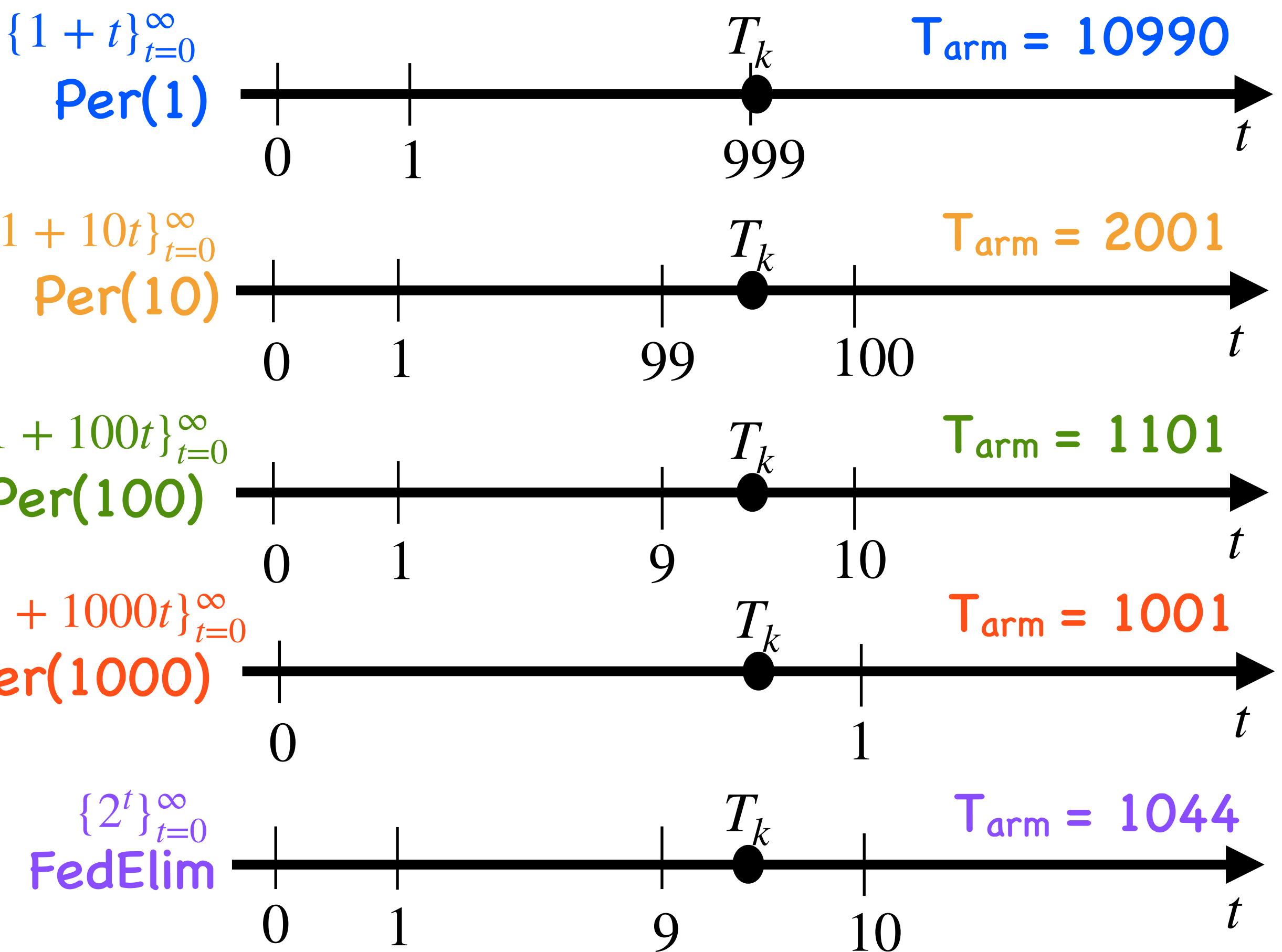
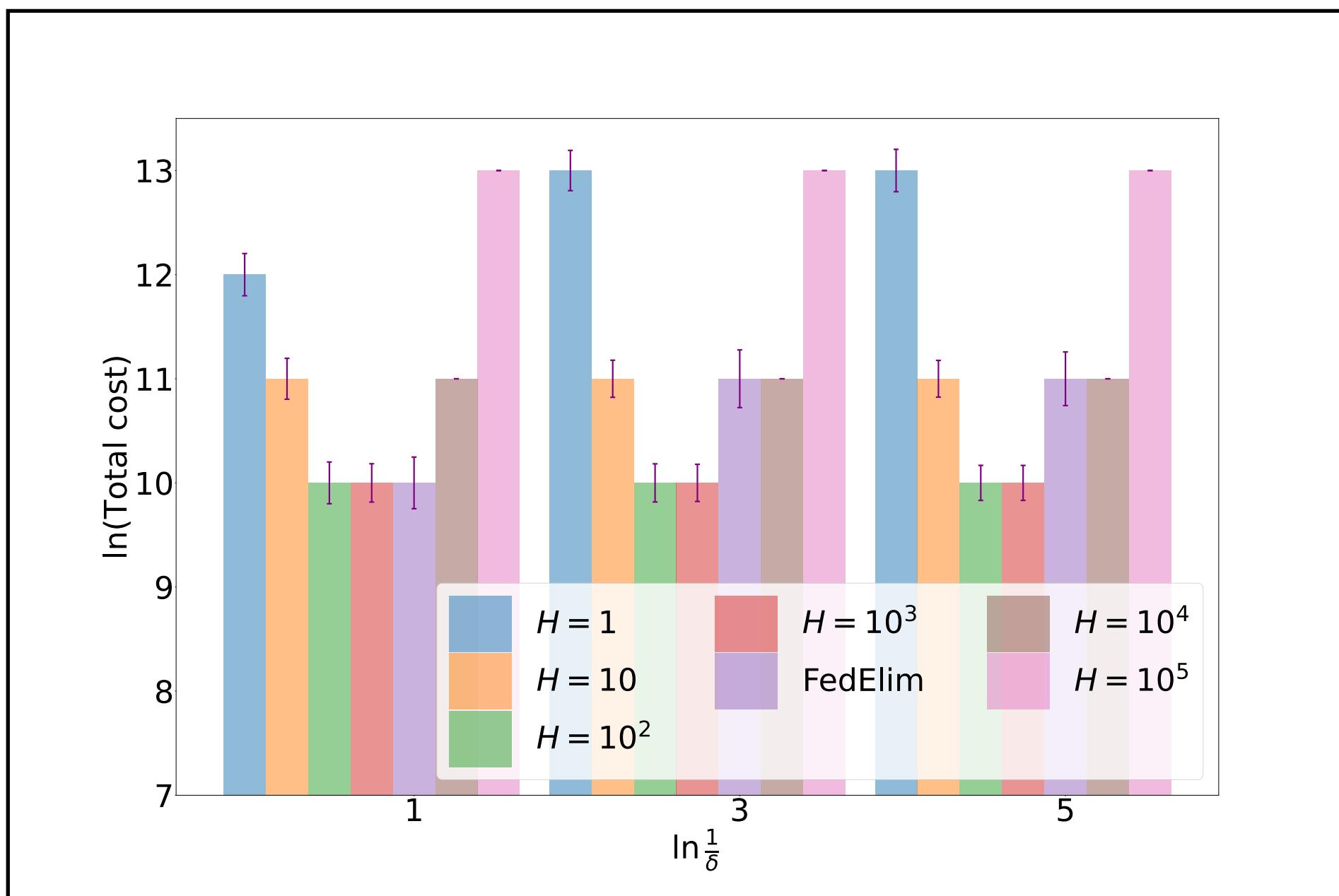
$$T_k = 1000 \text{ (depends on } \delta)$$



$$C = 10$$

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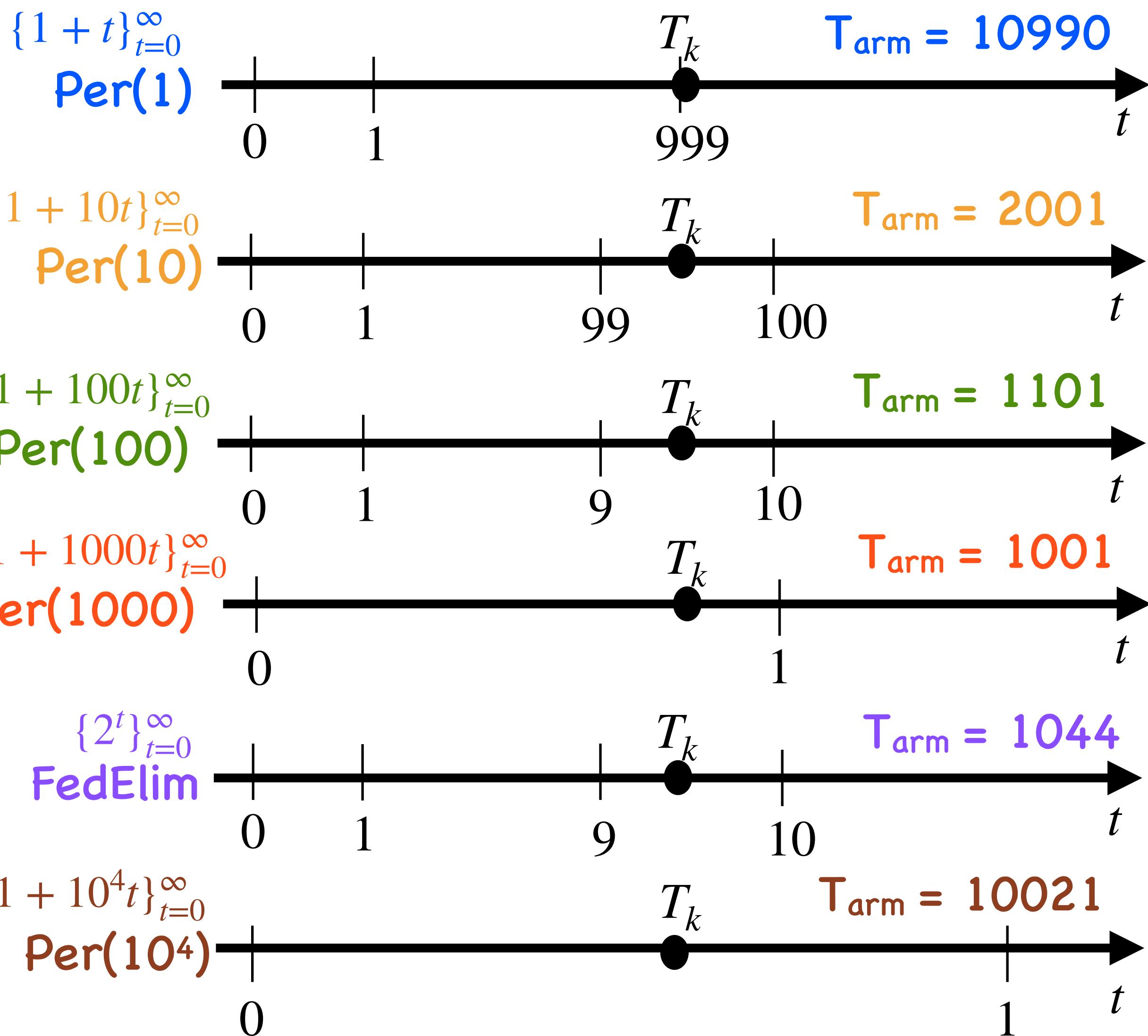
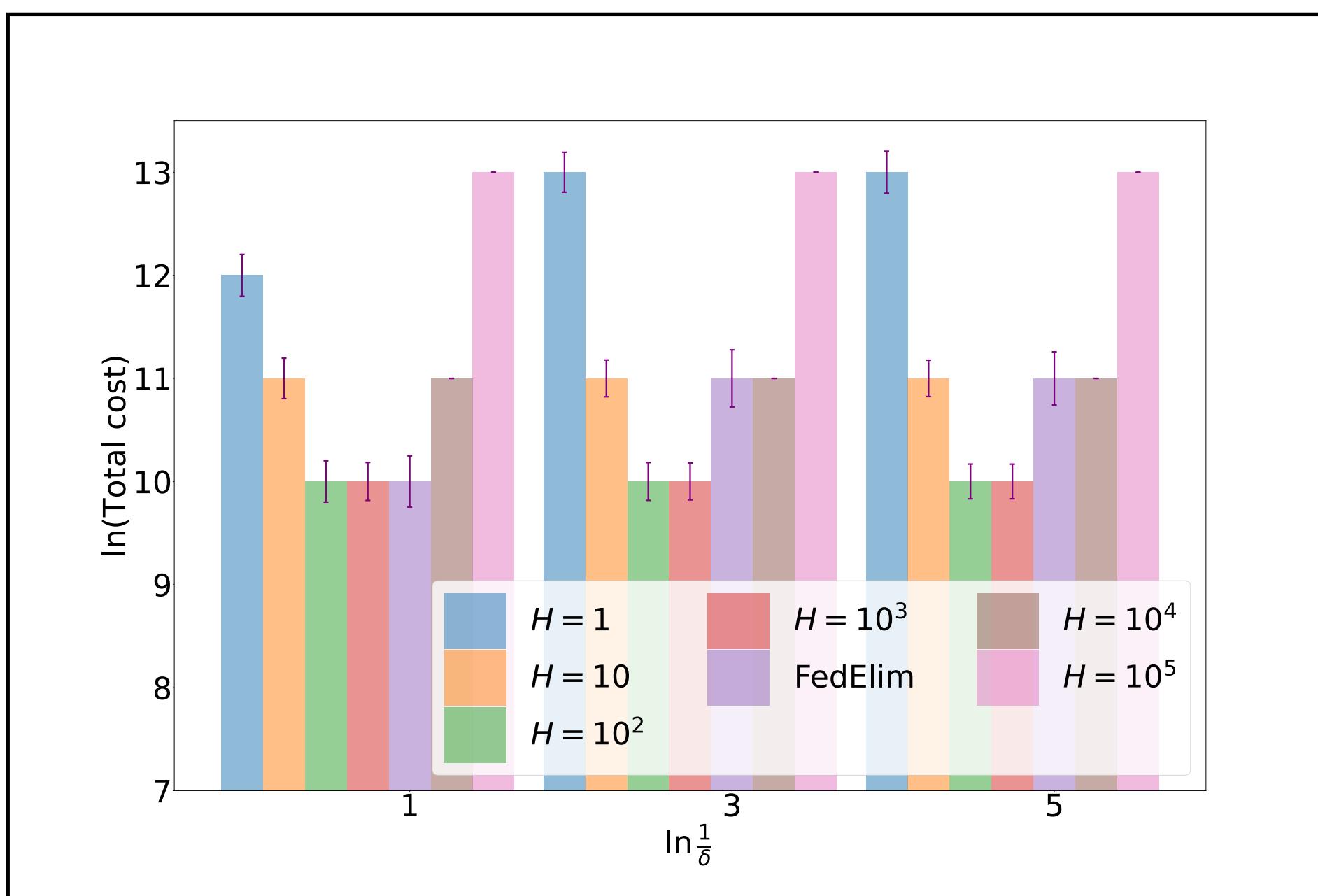
$$T_k = 1000 \text{ (depends on } \delta)$$



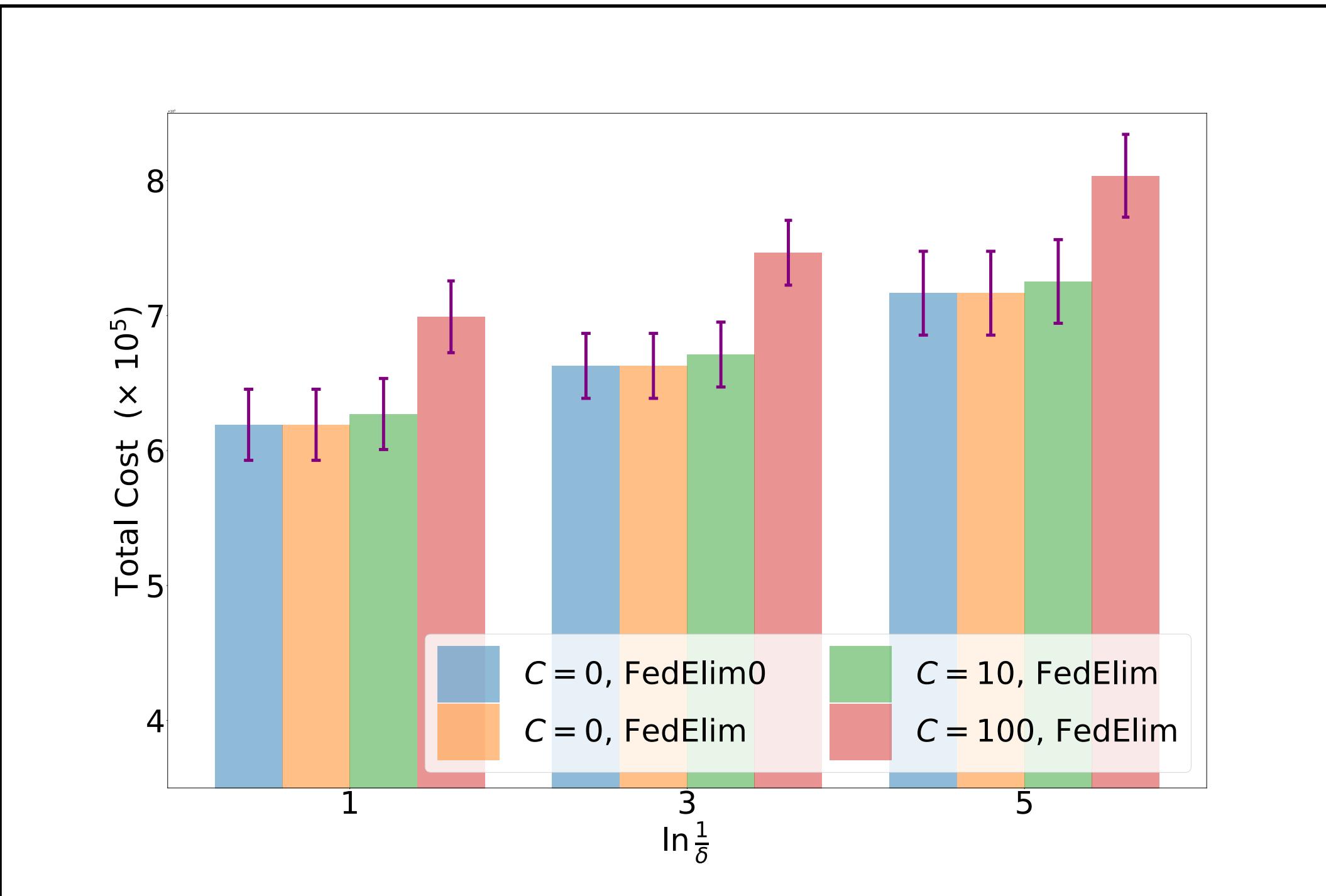
$$C = 10$$

$$T_{\text{arm}} = \text{no. of pulls} + 10 \times t$$

$$T_k = 1000 \text{ (depends on } \delta)$$



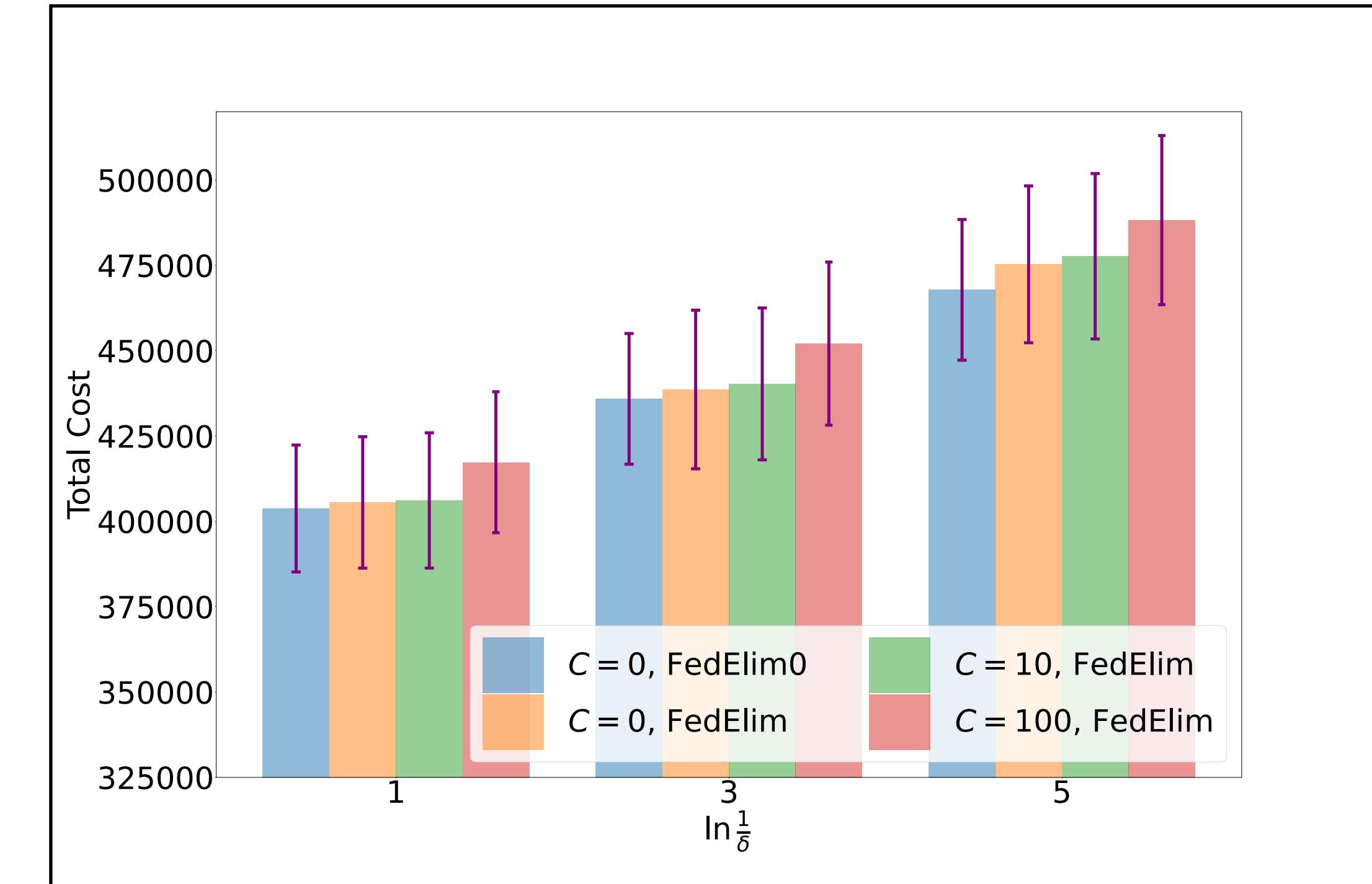
More Datasets – 1/3



MovieLens dataset (Cantador et al. (2011))

38 countries (clients), 20 genres (arms)

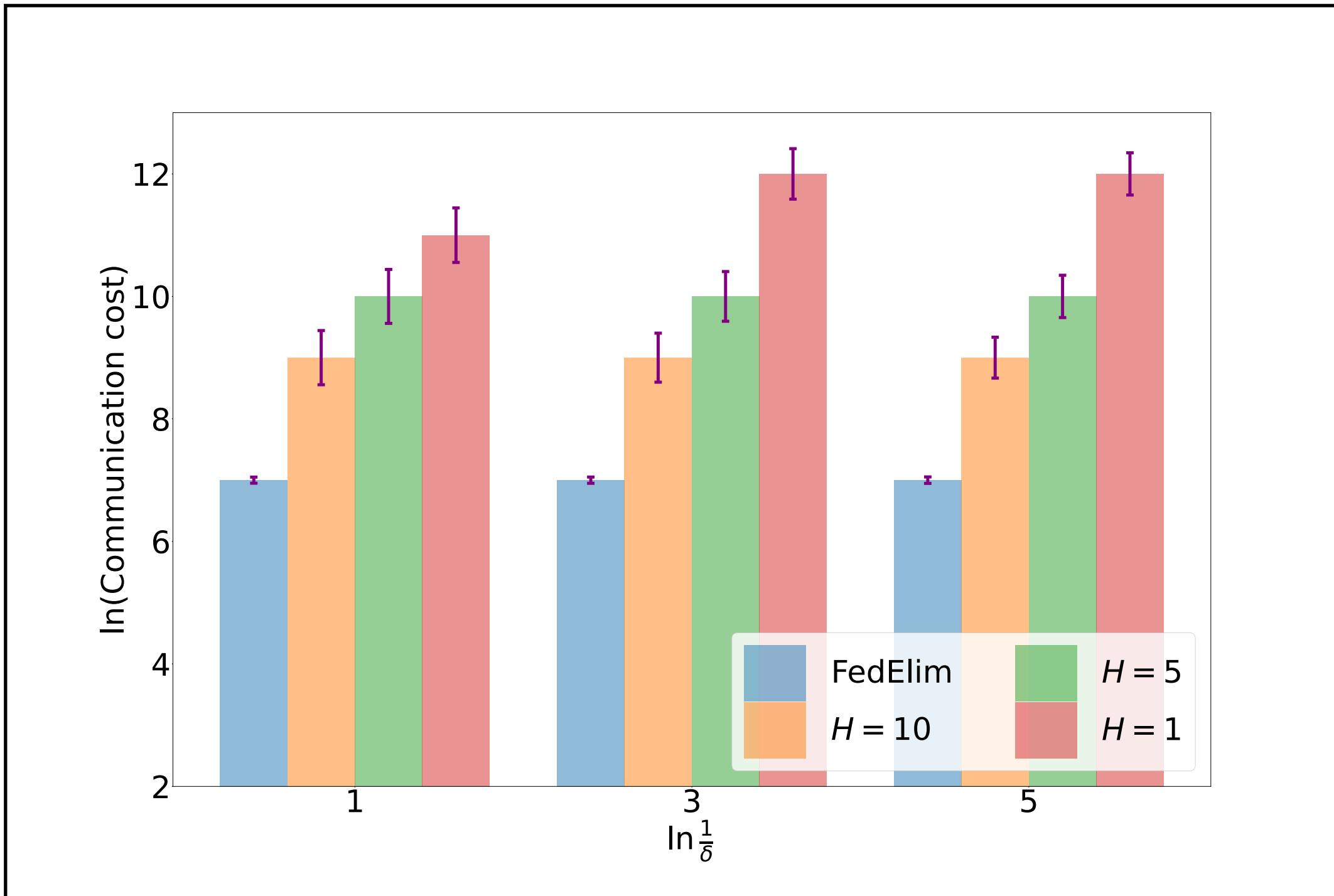
2.04M movie ratings



Bernoulli dataset from Mitra, Hassani, and Pappas (2021)

$$\boldsymbol{\mu} = \begin{bmatrix} 0.9 & 0.85 & 0.1 \\ 0.85 & 0.8 & 0.3 \\ 0.7 & 0.6 & 0.5 \end{bmatrix}$$

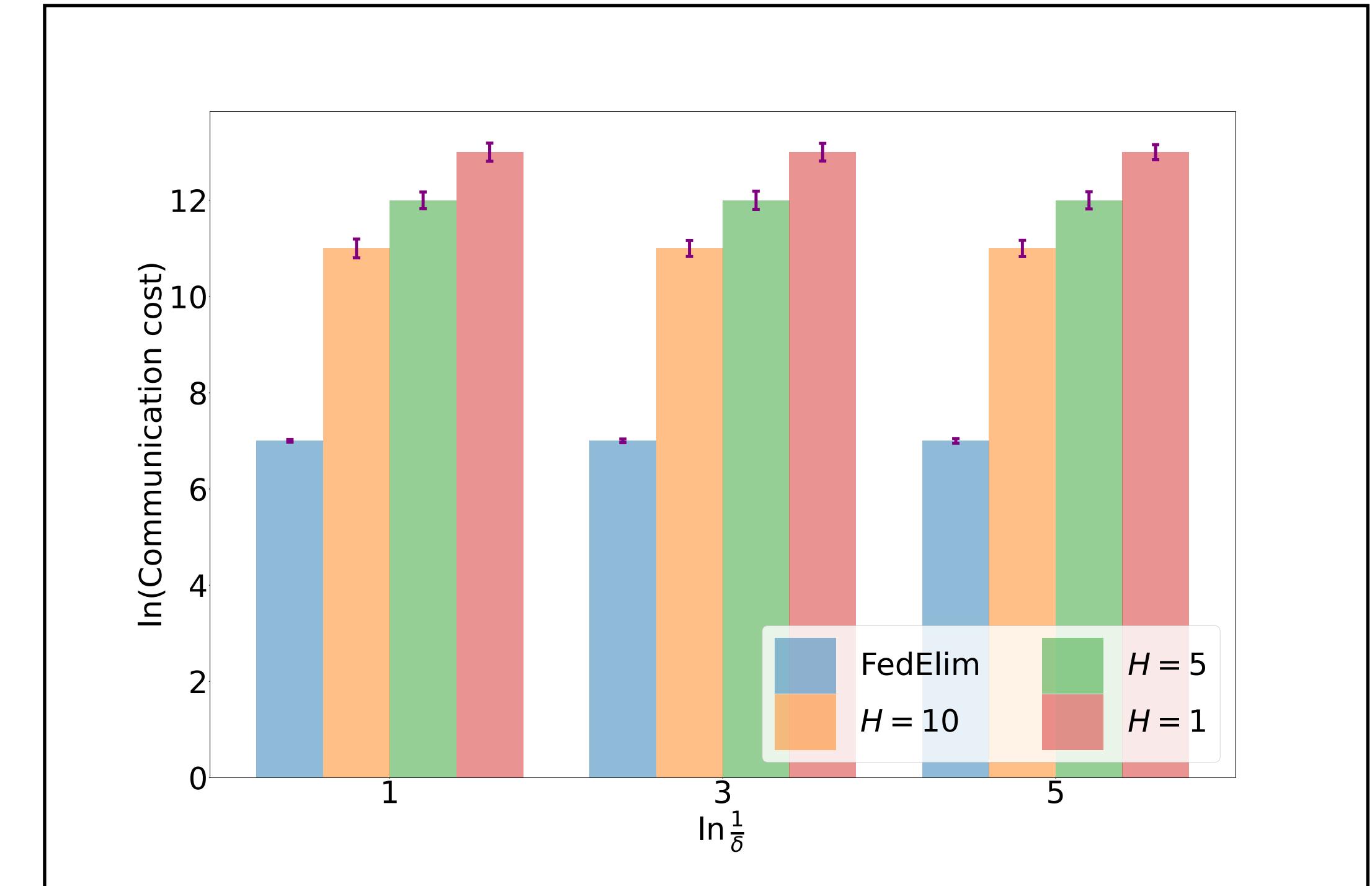
More Datasets – 2/3



MovieLens dataset ([Cantador et al. \(2011\)](#))

38 countries (clients), 20 genres (arms)

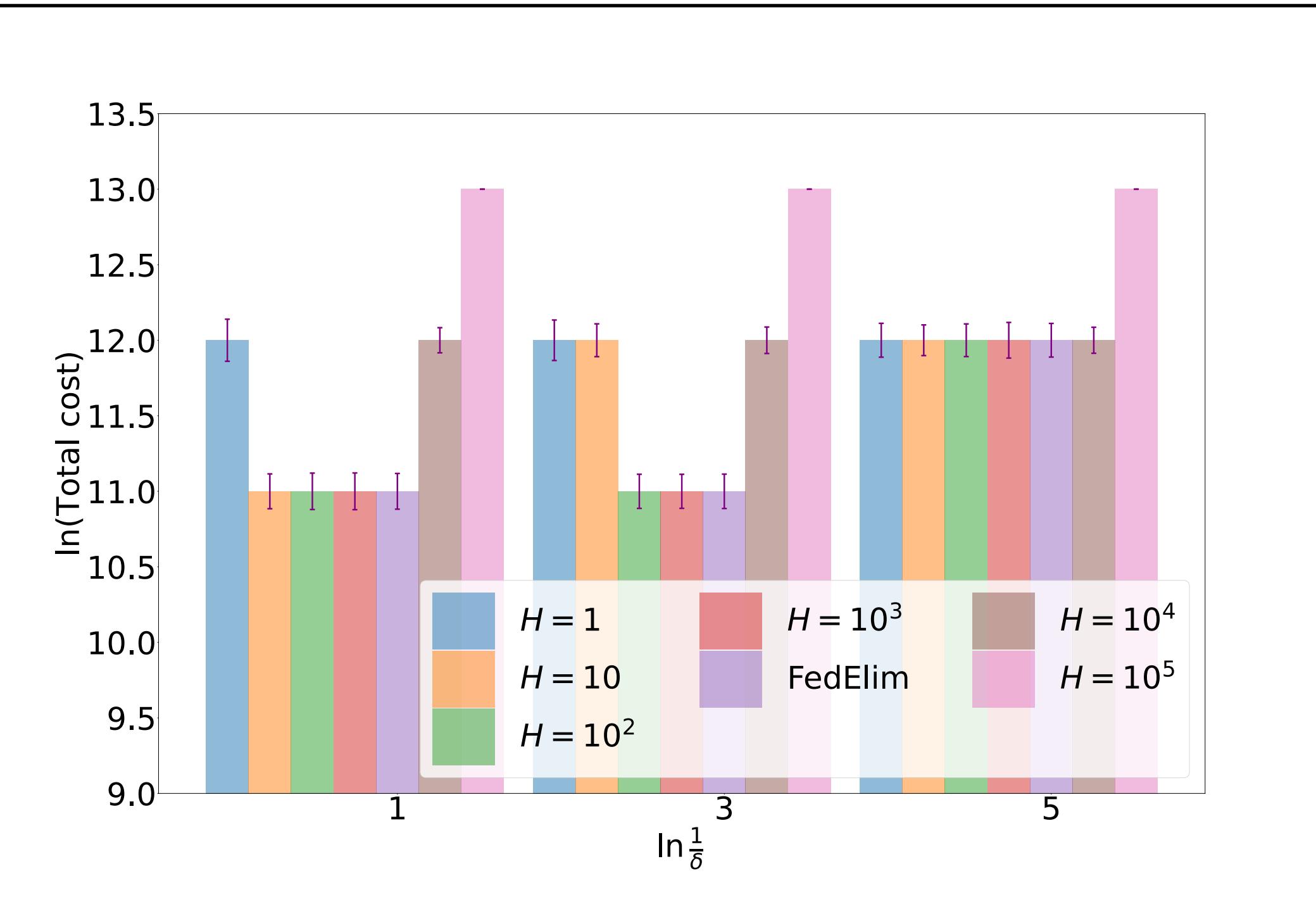
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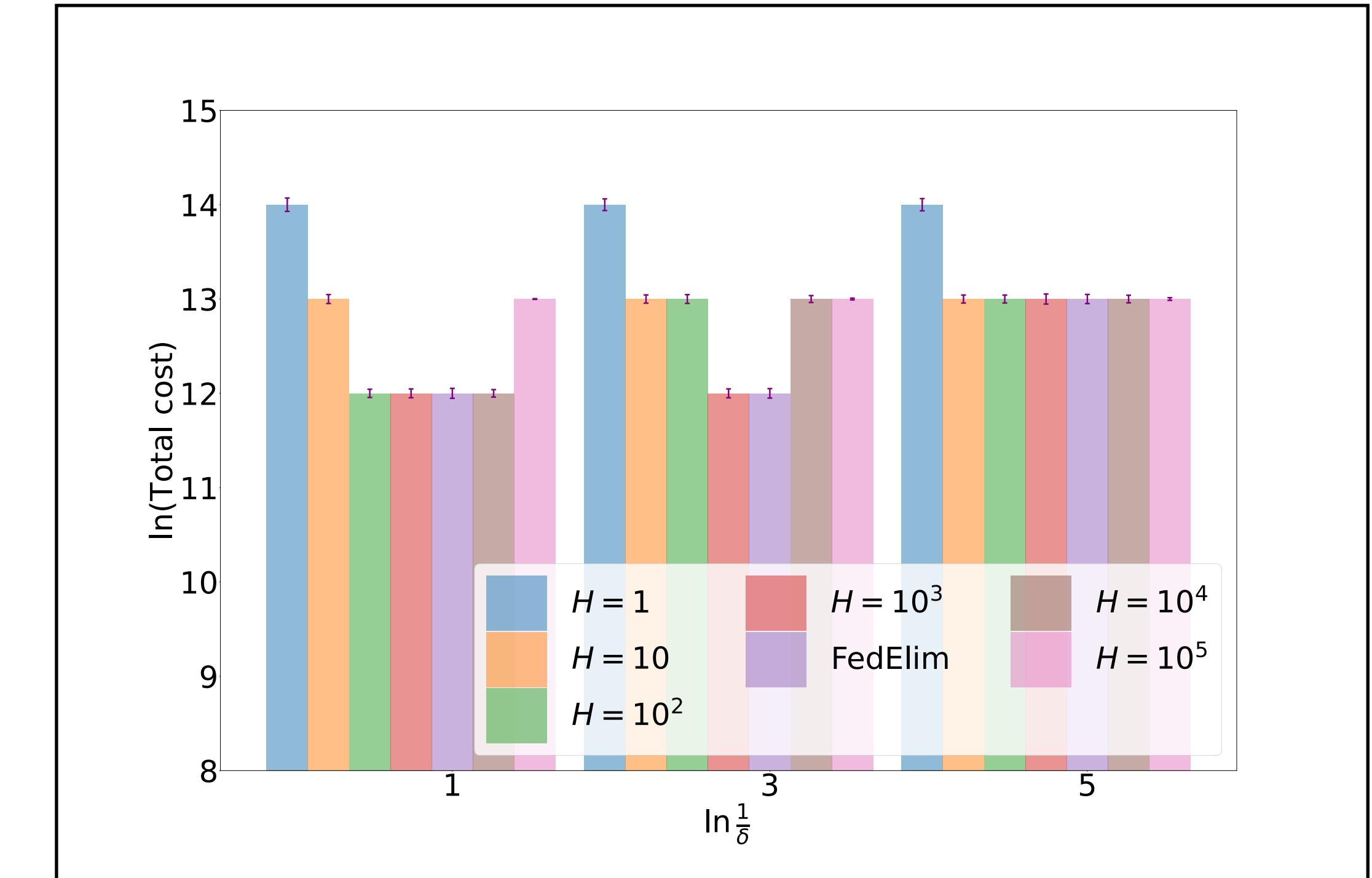
More Datasets – 3/3



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38 countries (clients), 20 genres (arms)

2.04M movie ratings



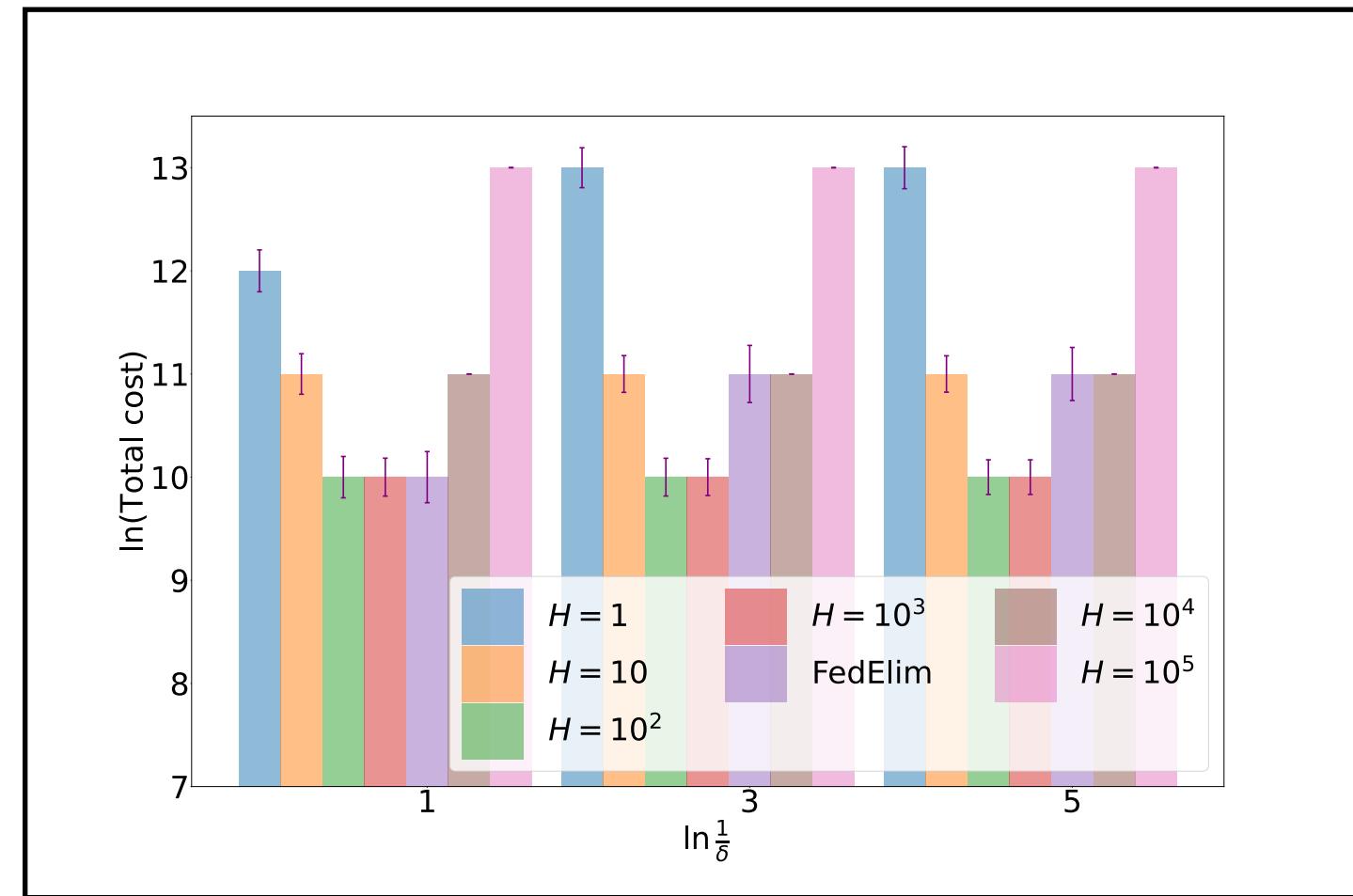
Bernoulli dataset from [Mitra, Hassani, and Pappas \(2021\)](#)

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Comparison with Per(H) and SupExp

$$\tau = \sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, T_k\}$$

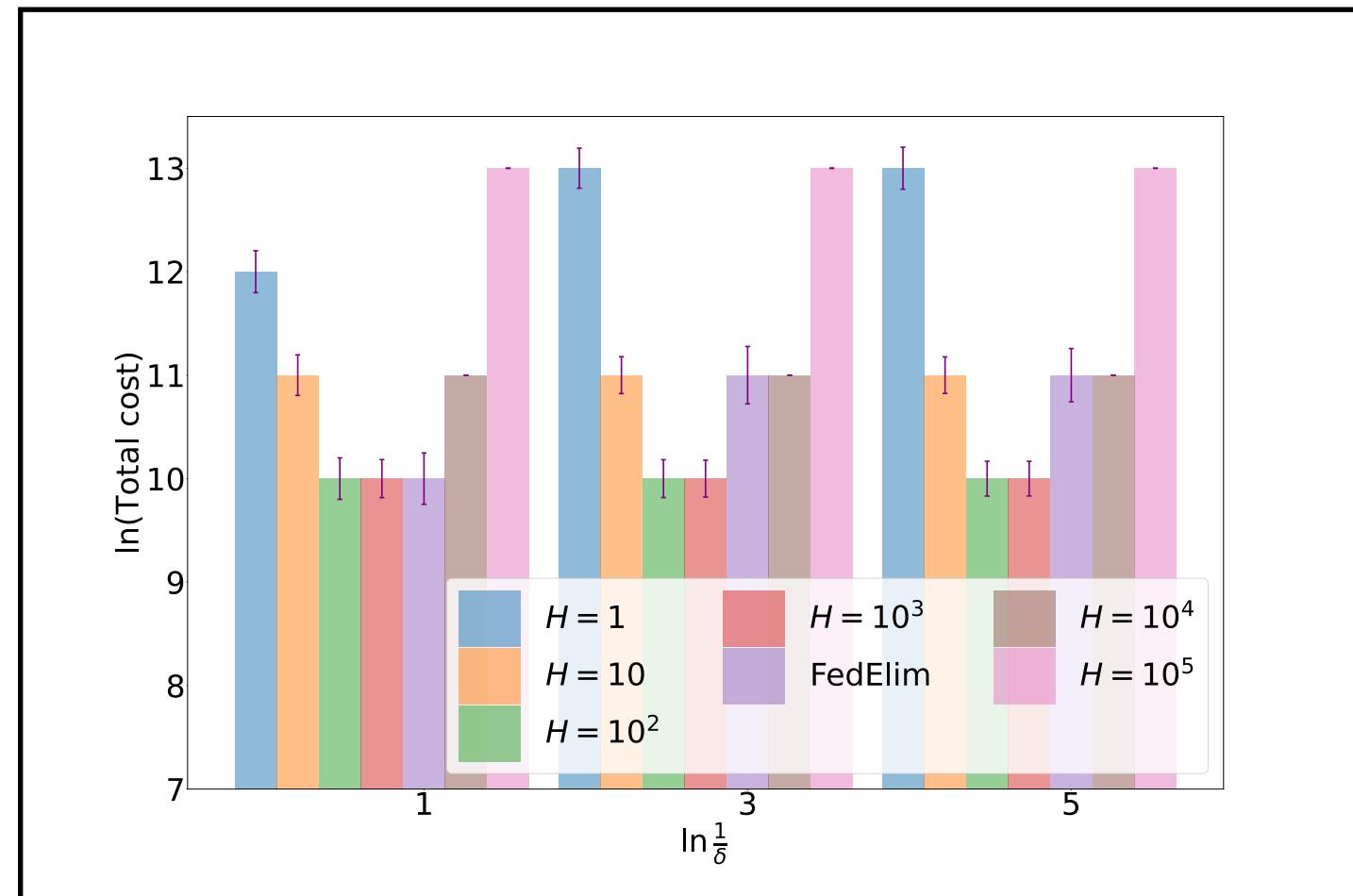
- $\tau_{\text{Per}(H)} \leq \sum_{k=1}^K \sum_{m=1}^M \max \left\{ T_{k,m}, \left\lceil \frac{T_k}{H} \right\rceil H \right\} = O\left(\tau + (H \times M \times K)\right)$
- $c_{\text{Per}(H)}^{\text{comm}} \leq C \times M \times \sum_{k=1}^K \left\lceil \frac{T_k}{H} \right\rceil = O\left(C \times \frac{\tau}{H} + C \times M \times K\right)$



Comparison with $\text{Per}(\mathsf{H})$ and SupExp

$$\tau = \sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, T_k\}$$

- $\tau_{\text{Per}(\mathsf{H})} \leq \sum_{k=1}^K \sum_{m=1}^M \max \left\{ T_{k,m}, \left\lceil \frac{T_k}{\mathsf{H}} \right\rceil \mathsf{H} \right\} = O\left(\tau + (\mathsf{H} \times M \times K)\right)$
- $c_{\text{Per}(\mathsf{H})}^{\text{comm}} \leq C \times M \times \sum_{k=1}^K \left\lceil \frac{T_k}{\mathsf{H}} \right\rceil = O\left(C \times \frac{\tau}{\mathsf{H}} + C \times M \times K\right)$

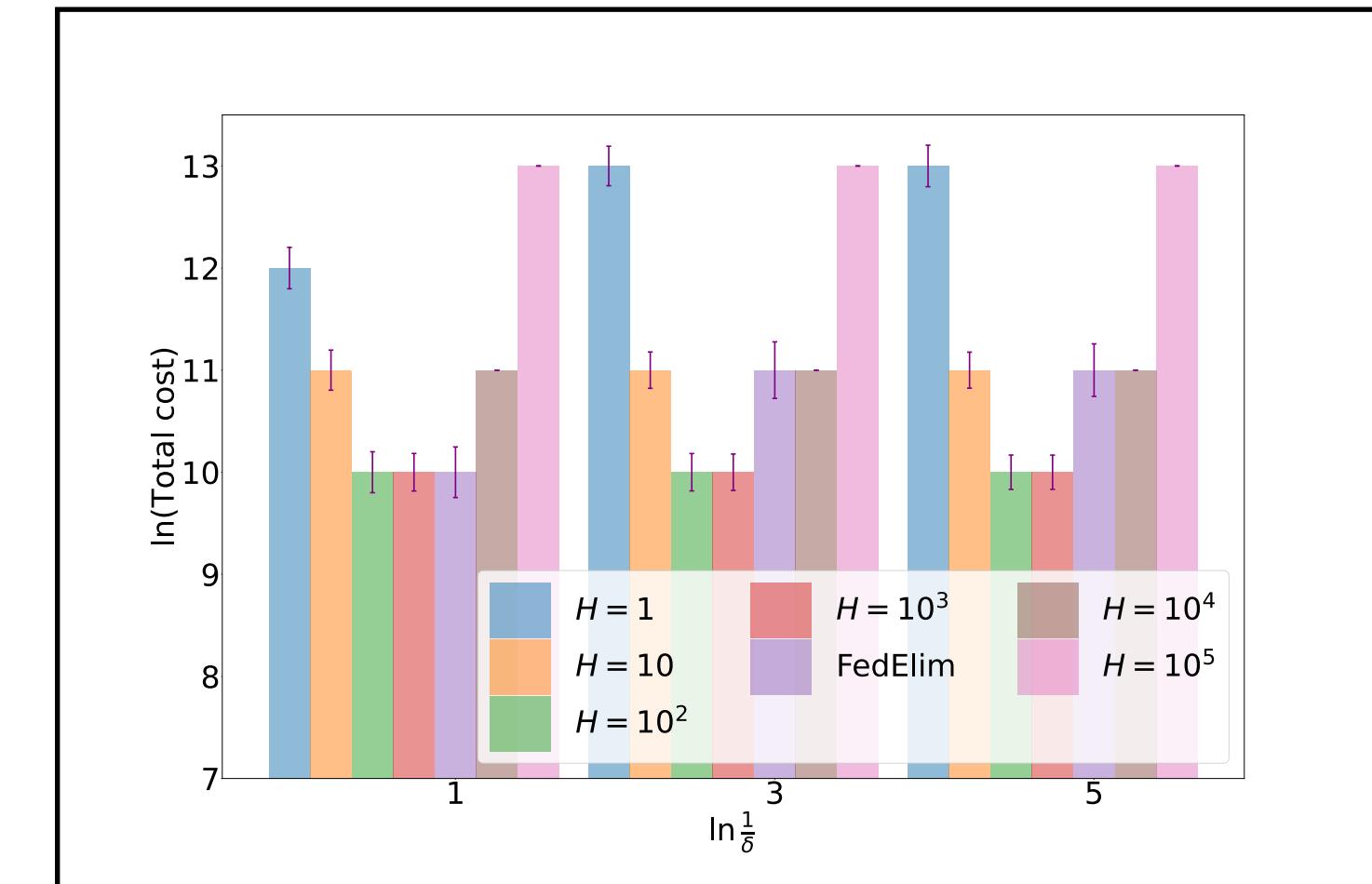


$$\mathsf{H}_{\text{opt}} = \sqrt{\frac{C \cdot \tau}{M \cdot K}}$$

Comparison with $\text{Per}(H)$ and SupExp

$$\tau = \sum_{m=1}^M \sum_{k=1}^K \max\{T_{k,m}, T_k\}$$

- $T_{\text{Per}(H)} \leq \sum_{k=1}^K \sum_{m=1}^M \max \left\{ T_{k,m}, \left\lceil \frac{T_k}{H} \right\rceil H \right\} = O\left(\tau + (H \times M \times K)\right)$
- $C_{\text{Per}(H)}^{\text{comm}} \leq C \times M \times \sum_{k=1}^K \left\lceil \frac{T_k}{H} \right\rceil = O\left(C \times \frac{\tau}{H} + C \times M \times K\right)$



$$H_{\text{opt}} = \sqrt{\frac{C \cdot \tau}{M \cdot K}}$$

Scheme	No. of arm pulls	Comm. cost	Total cost
$\text{Per}(H)$	$O(\tau)$	$O(CT/H)$	$O(\tau + CT/H)$
FedElim^*	2τ	$O(C \log(\tau))$	3τ
SupExp	$O(\tau^2)$	$O(C \log\log(\tau))$	$O(\tau^2)$

High-probability upper bounds

Summary and Future Directions

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- For all δ small, $P(T_{\text{FedElim}} \lesssim 3T_{\text{FedElim0}}) \geq 1 - \delta$

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- FedElim \approx Per(H_{opt})

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- $\text{FedElim} \approx \text{Per}(H_{\text{opt}})$
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 - High-probability upper bound on total cost —> upper bound on expected total cost
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 - Quantisation of uplink information

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Thank You!

More details: arXiv:2208.09215

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