Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5090: Stochastic Processes

Quiz 1

Date: 02 February 2026

Question	1	2	Total
Marks Scored			

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Assume that all random variables are defined with respect to this common space.

1. Let $\{X_n\}_{n\in\mathbb{N}}$ be a sequence of real-valued random variables.

   (a) **(2 Marks)**
   Show from first principles that for every choice of $n \in \mathbb{N}$ and $x \in \mathbb{R}$, the set

   $$\left\{ \sup_{k\geq n} X_k < x \right\} \in \mathscr{F}.$$

   (b) **(1 Mark)**
   Using the result of part (a), justify why

   $$\left\{ \limsup_{n\to\infty} X_n \geq 2 \right\} \in \mathscr{F}.$$

   Show every step of reasoning clearly.

2. **(2 Marks)**

Suppose that $\{X_n\}_{n\in\mathbb{N}}$ is a sequence of **independent**, real-valued random variables, with

$$\mathbb{P}\left(\left\{X_n = \frac{1}{2}\Big(1-\frac{1}{n}\Big)\right\}\right) = \mathbb{P}\left(\left\{X_n = \frac{1}{2}\Big(1+\frac{1}{n}\Big)\right\}\right) = \frac{1}{2}\left(1-\frac{1}{n}\right), \qquad \mathbb{P}(\{X_n = 1\}) = \frac{1}{n}.$$

Does the above sequence convergence almost-surely?
If yes, identify a limit random variable and prove the almost-sure convergence.
If not, justify why the sequence does not converge almost-surely.