Abstract

Inverse problems are central to many applications of uncertainty quantification in the natural sciences and in engineering. Often they depend on differential equations which renders them ill–posed and computationally highly demanding. The Bayesian approach is a popular way of dealing with the ill–posedness but itself leads to further computational complexity. We employ Gaussian process emulators to reduce the computational overhead. Stuart & Teckentrup (2018) established convergence rates of approximate posterior distributions based on Gaussian process emulation of the forward model and the potential. We review their work and additionally prove two theorems which state that similar estimates holds for approximation of the likelihood function. Based on this approximation, we establish connections to Bayesian quadrature and Bayesian optimisation. The computational bottleneck of computing Gaussian process approximations is solving linear systems involving covariance matrices, which are densely populated and ill–conditioned. We introduce an algorithm which combines hierarchical matrices with selected Krylov solvers. This algorithm has the potential of computing the solution to these systems in almost linear time. We provide basic examples where this almost linear complexity is attained.