EE2703 : Applied Programming Lab
Assignment 8

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1 Introduction

In the previous assignment, we found DFTs of periodic signals using Fourier Transforms. In this assignment, we extend to understand and find the DFTs of non-periodic signals. We find the spectrum of $\sin(\sqrt{2}t)$ and Chirp Signal. In the process we find learn the Gibbs Phenomenon and the use of windowing to reduce the Gibbs Phenomenon. We estimate the frequency of a signal data known to contain a sinusoid of frequency ω_o and some phase lag δ_o.

2 Assignment

2.1 Spectrum of $\sin(\sqrt{2}t)$

First we find the spectrum of using the following code:

```
t = p.linspace(-p.pi,p.pi,65);t=t[:-1]
dt = t[1]-t[0];fmax=1/dt
y = p.sin(p.sqrt(2)*t)
y[0] = 0
y = p.fftshift(y)
Y = p.fftshift(p.fft(y))/64.0
w = p.linspace(-p.pi*fmax,p.pi*fmax,65);w=w[:-1]
p.figure(figsize = (8,8))
p.subplot(2,1,1)
p.plot(w,abs(Y),lw=2)
p.xlim([-10,10])
p.ylabel(r"$|Y|$",size=16)
p.title(r"Spectrum of $\sin\left(\sqrt{2}t\right)$")
```

```
    p.grid(True)
    p.subplot(2,1,2)
    p.plot(w,p.angle(Y),'ro',lw=2)
    p.xlim([-10,10])
    p.ylabel(r"Phase of $Y$",size=16)
    p.xlabel(r"$\omega$",size=16)
    p.grid(True)
    p.savefig("fig10-1.png")
    p.show()
```

After using the above code, we get the following spectrum

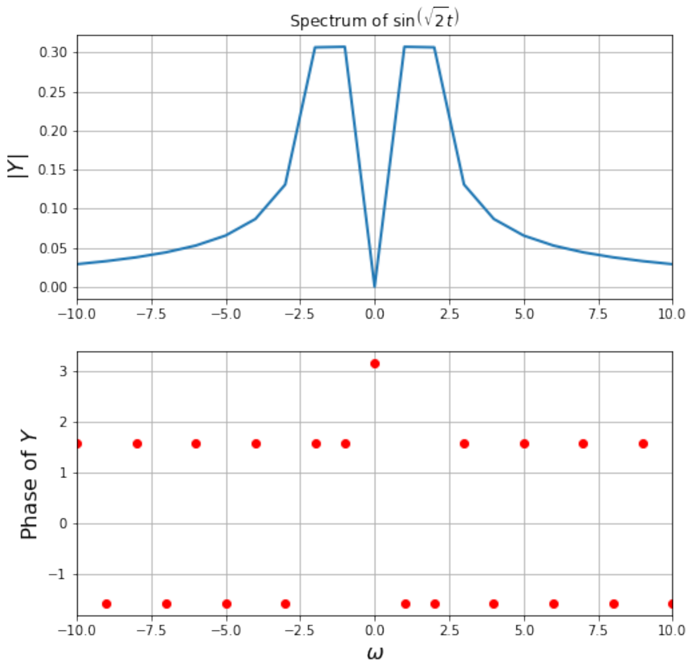

Figure 1: Spectrum of $\sin(\sqrt{2}t)(inaccurate)$

We expect two single peaks. But here we have multiple peaks. This is due to a phenomenon known as Gibbs Phenomenon.

First let us see what function **numpy.fft** is plotting in this case.

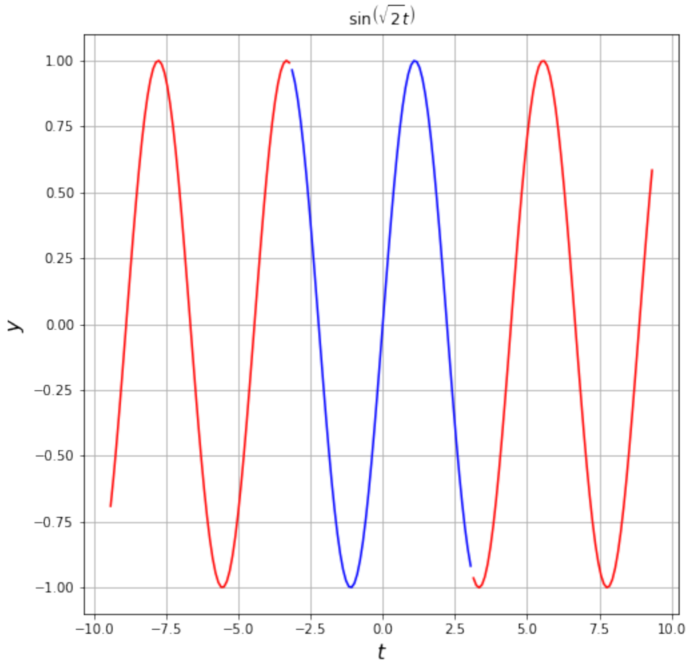

Figure 2: Plot of $\sin(\sqrt{2}t)$

The above plot is the plot of the function $\sin(\sqrt{2}t)$. We divide the above plot into three regions: $(-3\pi, -\pi), (-\pi, \pi), (\pi, 3\pi)$. The fourier transform is found our for the blue curve in Figure 2. Therefore, **numpy.fft** finds the fourier transform of the signal as shown in figure 3.

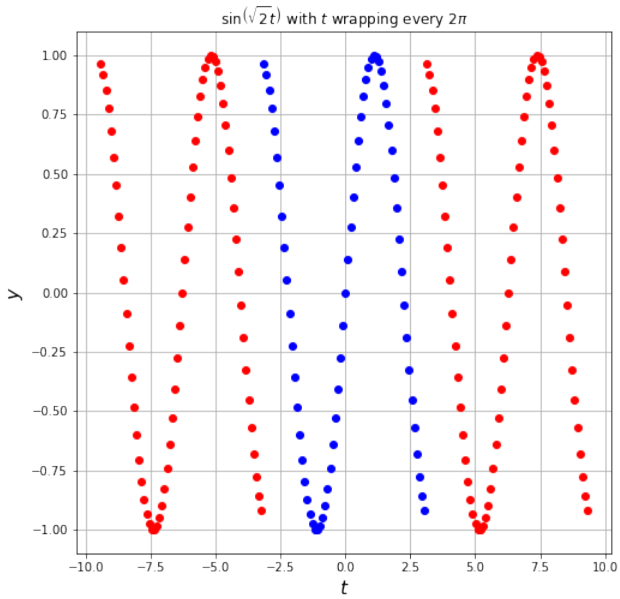

Figure 3: Plot of $\sin(\sqrt{2}t)$ between $[-\pi, \pi]$ (extended)

2.1.1 Gibbs Phenomenon

We can see in Figure 3 that there is a sudden jump at odd multiples of π. This sudden jump in the signal forces magnitude spectrum to decay as $\frac{1}{\omega}$ at these discontinuities. In Figure 4, we see the magnitude plot of function:

$$f(t) = t \qquad t \in (-\pi, \pi)$$
$$f(t+\pi) = f(t)$$

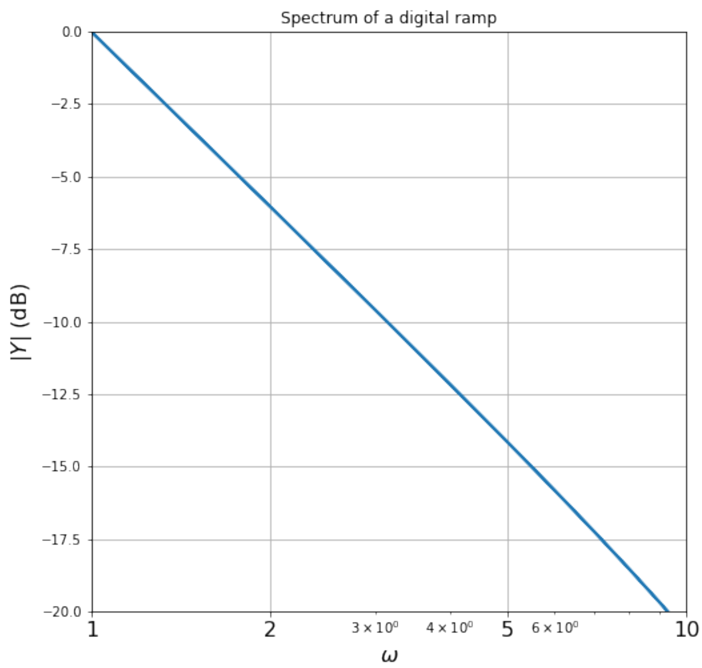

Figure 4: Spectrum of digital ramp

Therefore, due to this slow decaying, the spectrum for $\sin(\sqrt{2}t)$ had multiple peaks.

2.1.2 Windowing

The Gibbs Phenomenon can be reduced by using windowing. Here, we supress the jump in the signal using using a window. It is basically a function which we multiply to the signal to suppress the jump. The window we will be using is Hamming Window.

$$w[n] = 0.54 + 0.46\cos(\frac{2\pi n}{N-1}) \qquad |n| < \frac{N-1}{2}$$

```
    t1 = p.linspace(-PI,PI,65)[:-1]
    t2 = p.linspace(-3*PI,-PI,65)[:-1]
```

```
    t3 = p.linspace(PI,3*PI,65)[:-1]
    n = p.arange(64)
    wnd = p.fftshift(0.54+0.46*p.cos(2*PI*n/63))
    y = p.sin(p.sqrt(2)*t1)*wnd
    p.figure(5,figsize=(8,8))
    p.plot(t1,y,'bo',markeredgecolor='black', markeredgewidth=0.5)
    p.plot(t2,y,'ro',markeredgecolor='black', markeredgewidth=0.5)
    p.plot(t3,y,'ro',markeredgecolor='black', markeredgewidth=0.5)
    p.ylabel("$y$",size=16)
    p.xlabel("$t$",size=16)
    p.title(r"$\sin\left(\sqrt{2}t\right)\times w(t)$ with $t$ wrapping every $2\
pi$ ")
    p.grid(True)
    p.savefig("fig10-5.png")
    p.show()
```

After applying the window on the function $\sin(\sqrt{2}t)$, we get the function as show in Figure 5.

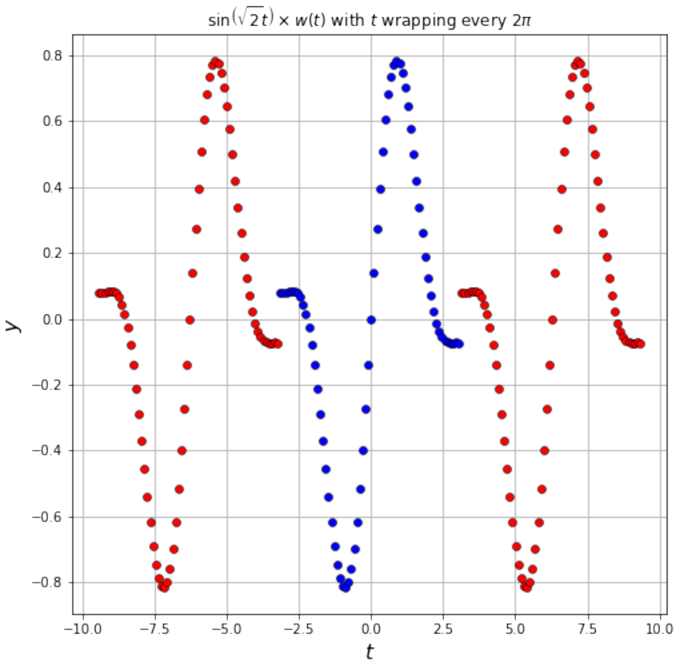

Figure 5: Plot of $\sin(\sqrt{2}t)$ with windowing

We can see that the jump is reduced.

Now let us find the Spectrum using this function.

```
    t = p.linspace(-PI,PI,65);t=t[:-1]
    dt = t[1]-t[0]; fmax=1/dt
    n = p.arange(64)
```

```
    wnd = p.fftshift(0.54+0.46*p.cos(2*PI*n/63))
    y = p.sin(p.sqrt(2)*t)*wnd
    y[0] = 0 # the sample corresponding to -tmax should be set zeroo
    y = p.fftshift(y) # make y start with y(t=0)
    Y = p.fftshift(p.fft(y))/64.0
    w = p.linspace(-PI*fmax,PI*fmax,65);w = w[:-1]
    p.figure(6,figsize=(8,8))
    p.subplot(2,1,1)
    p.plot(w,abs(Y),'b',w,abs(Y),'bo',lw=2)
    p.xlim([-8,8])
    p.ylabel(r"$|Y|$",size=16)
    p.title(r"Spectrum of $\sin\left(\sqrt{2}t\right)\times w(t)$")
    p.grid(True)
    p.subplot(2,1,2)
    p.plot(w,p.angle(Y),'ro',lw=2)
    p.xlim([-8,8])
    p.ylabel(r"Phase of $Y$",size=16)
    p.xlabel(r"$\omega$",size=16)
    p.grid(True)
    p.savefig("fig10-6.png")
    p.show()
```

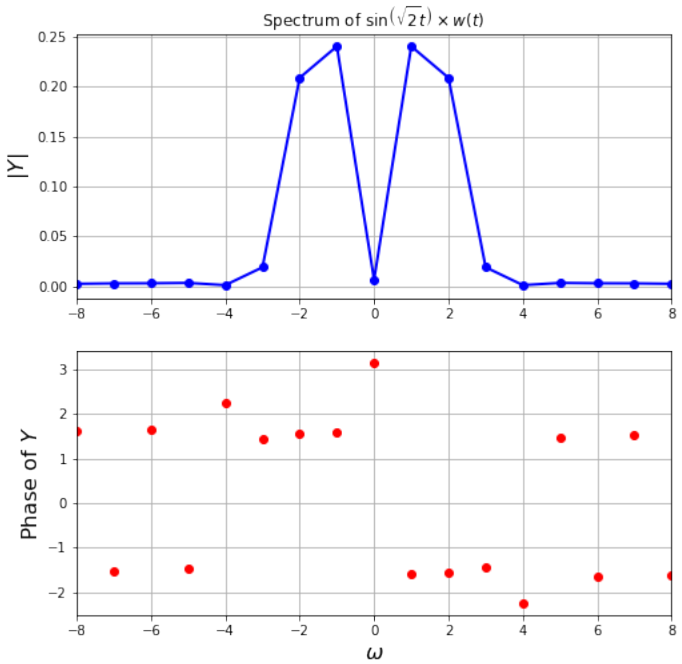

Figure 6: Spectrum of $\sin(\sqrt{2}t)$ after windowing

Now we have single peak but less accurate. If we increase the no. of samples, then we can increase the accuracy. After increasing the number of samples from 64 to 256, we get the spectrum as shown in Figure 7.

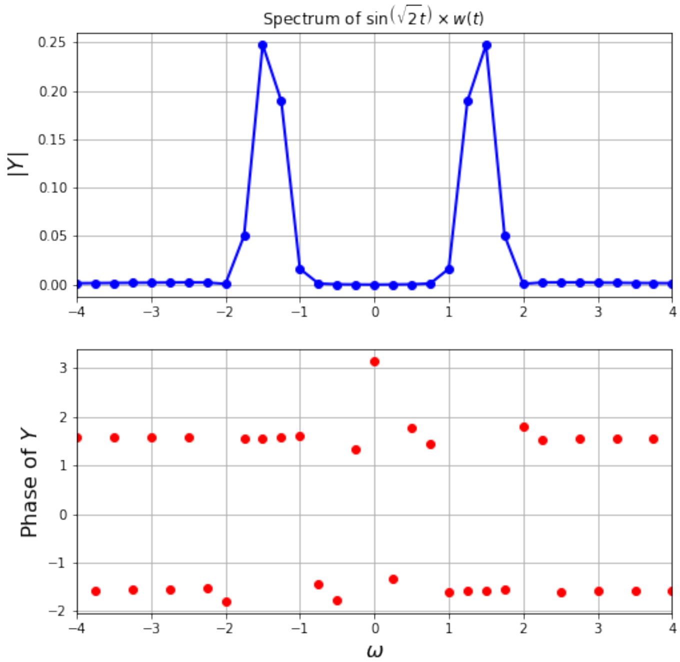

Figure 7: Spectrum of $\sin(\sqrt{2}t)$ after windowing and increasing the samples

Now we can see that the accuracy is much better than the first and second case but not the ideal case. Here, we do not get a single peak at $\omega = \pm\sqrt{2}$, this is because since we multiplied the original signal with the window, the spectrum of the window is also present. This is the reason, the peak is spread out a bit rather than a perfect single peak.

2.2 Question 2: Spectrum of $\cos^3(\omega_o t)$

We find the Spectrum of $\cos^3(\omega_o t)$ for $\omega_o = 0.86$ with and without the Hamming Window. The Spectrum plots are show in Figures 8 and 9. In Figure 8, we have the Spectrum without the Window and in Figure 9, it is with the window.

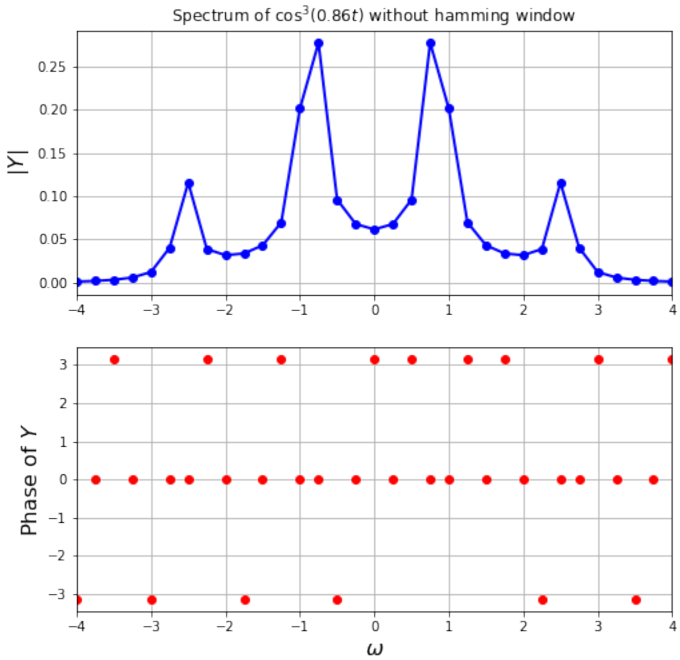

Figure 8: Spectrum of $\cos^3(0.86t)$ without Hamming

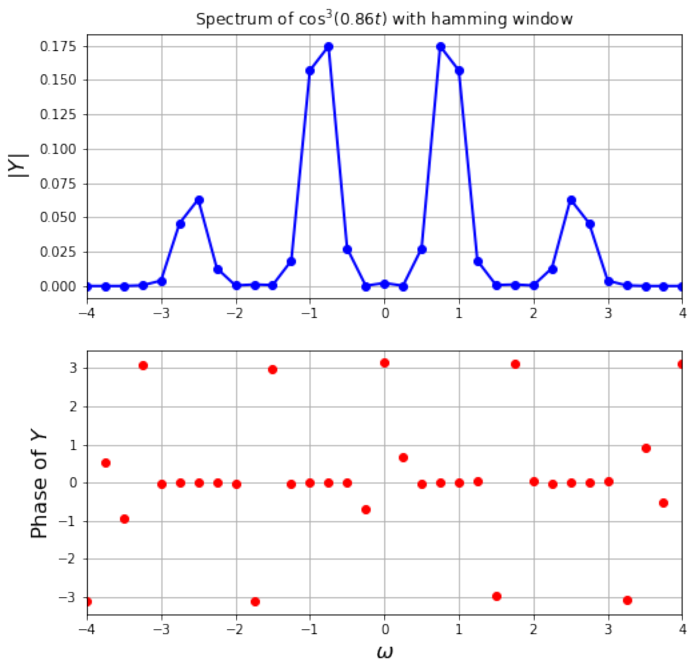

Figure 9: Spectrum of $\cos^3(0.86t)$ with Hamming

The Spectrum with Hamming window has much narrower and discrete peaks than the one without Hamming Window.

2.3 Question 3 & 4: Estimating ω and δ of a given sinusoidal signal data

In this section, we have to estimate the value of ω and δ of a sinusoid from its DFT. We get an input of 128 values with the value of t $\in (-\pi, \pi)$.

Firstly, we find the Spectrum of the given signal same as we have done in the previous cases. We also use the Hamming Window. Now to find the estimate for ω, we take the weighted average. We iterate over the number of samples taken to find the average and take the frequency estimate which has minimum error. Now that we have the frequency, we can find the phase value at this frequency. This phase will be the δ of the given signal. The same thing is done in the case, where we have gaussian noise along with the signal. Now to find the error, we calculate a signal using the estimated values. This signal is subtracted from the original signal and maximum value of this error vector is taken as the error for the estimated values.

$$\omega_{est} = \frac{\sum_{i=0}^{k} |Y(j\omega_i)| * \omega_i}{\sum_{i=0}^{k} |Y(j\omega_i)|}$$

In the above equation, k iterates from 1 to 60 and $\omega > 0$. For the signal data which represents the function $\cos(1.42t + 0.6)$, we have the following estimates.

```
Without Noise:
omega = 1.420893911012074
delta = 0.5951783874041221
With Noise:
omega_noise = 1.4095919969924964
delta_noise = 0.5927214850281324
```

2.4 Question 5: Spectrum of Chirp

In this section, we find the Spectrum of the Chirp Signal. The frequency of the chirp changes with time. The Chirp Signal:

$$\cos\left(16\left(1.5 + \frac{1}{2\pi}\right)t\right)$$

As we can see, the frequency of the signal varies between 16 and 32 with time. The Spectrum of this signal is shown in Figures 10 and 11. In Figure 10, it is without the Hamming Window and in Figure 11, it is with the Hamming Window.

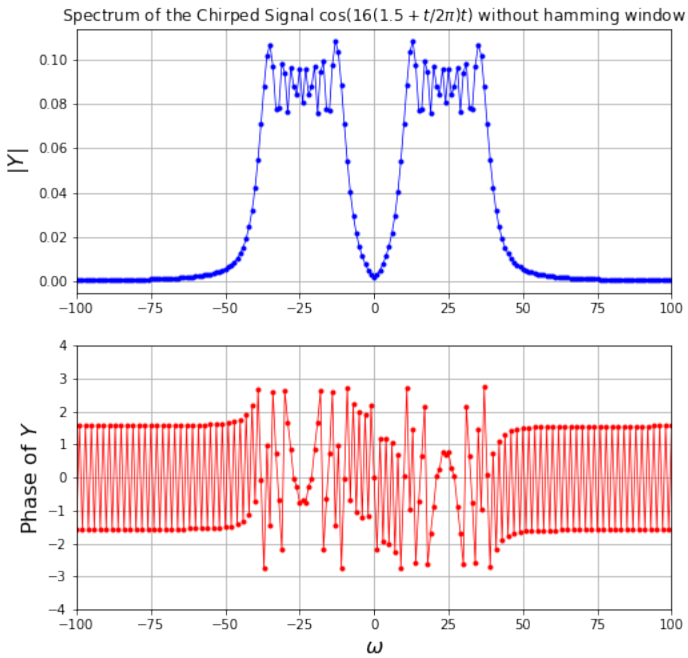

Figure 10: Spectrum of Chirp without Hamming

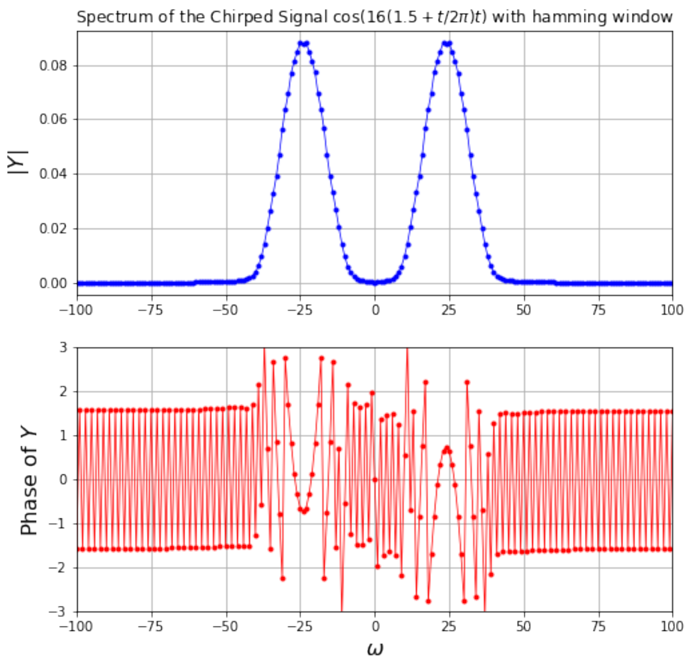

Figure 11: Spectrum of Chirp with Hamming

2.5 Question 6: Time Evolution of DFT

In this section, we take a better look at the what is happening to the phase spectrum of the Chirp. We find the spectrum around a small time interval at the same time instance and plot the spectrum at each time instance on a 3D plot. In Figure 12 and 13, we plot the variation of Spectrum without the Hamming Window and in Figure 14 and 15, we plot with the Hamming Window.

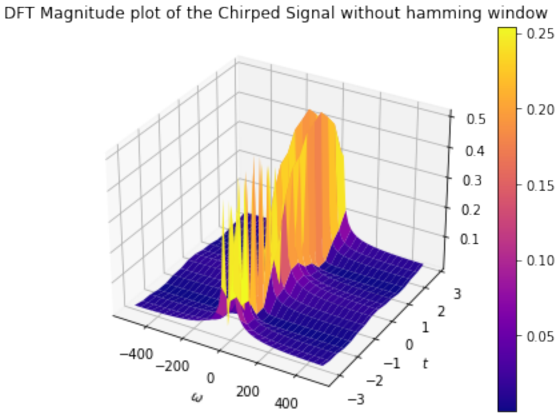

Figure 12: Magnitude Spectrum of Chirp without Hamming

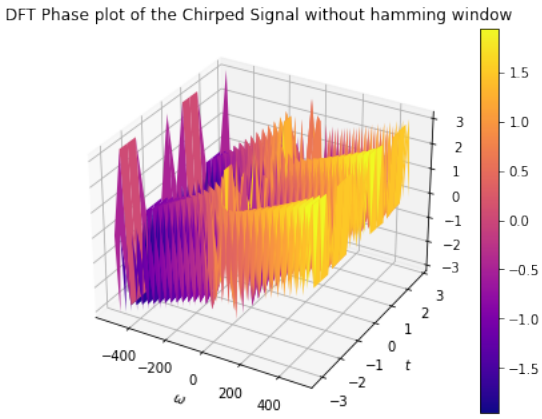

Figure 13: Phase Spectrum of Chirp without Hamming

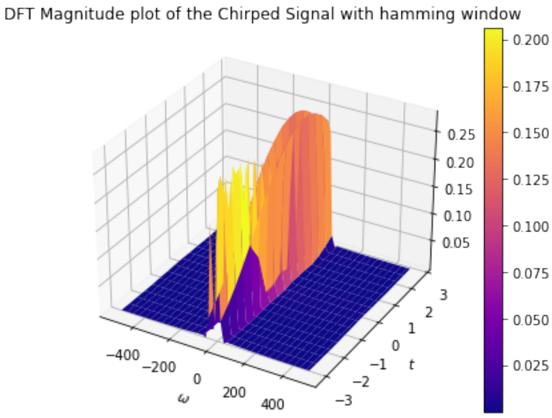

Figure 14: Magnitude Spectrum of Chirp with Hamming

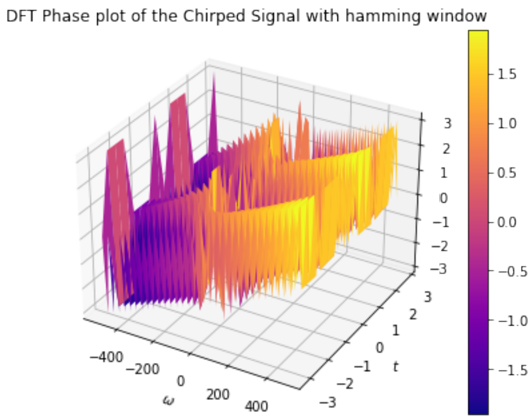

Figure 15: Phase Spectrum of Chirp with Hamming

3 Conclusion

Therefore, we have found out the spectrum of non periodic signals. We got to know what is Gibbs Phenomenon and how to minimize its effect. We also estimated the frequency and phase delay of a given sinusoidal signal.