EE2703 : Applied Programming Lab
End Semester

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1 Introduction

In this problem, we find the magnetic field on the axis due to a loop antenna having the current distribution as follows

$$I = \frac{4\pi}{\mu_o}\cos(\phi)\exp(j\omega t) \tag{1}$$

Here, the ϕ is the polar coordinate in cylindrical coordinate system (r,ϕ,z). The radius of the loop is given to be 10cm and is places in the x-y plane and centered at origin. We have to compute $\vec{B}$ on z-axis from 1cm to 1000cm and then fit the data to $|\vec{B}| = cz^b$.

2 Defining Coordinates

Since we are computing Magnetic Field in space in a specified region, it would be useful to define few vectors and arrays which store the required coordinates of x,y and z. In the problem, we were asked to find the magnetic field on z axis. But from the Current plot in Figure 2, we see that the magnetic field on z axis will be zero. So, we take an assumption and find the Magnetic field on the line x=y=1. So the required coordinate values for x and y are [0,1,2]. The coordinates for z are from 1 to 1000. Hence we define these coordinates using **numpy.linspace**. From these coordinates, we define meshgrid for 3D coordinates.

```
    x = np.linspace(0,2,num=3)
    y = np.linspace(0,2,num=3)
    z = np.linspace(1,1000,num=1000)

    X,Y,Z = np.meshgrid(x,y,z)
```

3 Current Distribution

The current distribution is given in Equation 1. We plot the quiver plot of this current distribution to get an idea of how current is flowing in the loop. Before we plot that, we divide the loop into 100 small sections. We find the coordinates of center of each of these sections and also the angular position. We use this data to find the current flowing through each of these sections and then plot the quiver plot. In Figure 1, we see the center points of each section of wire.

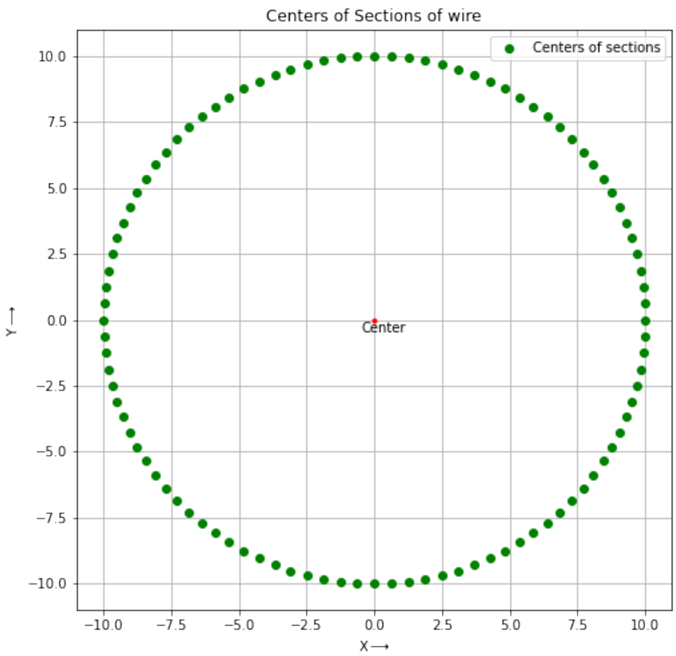

Figure 1: Center positions of the small sections

The python code to compute the current flow in each section of wire:

```
radius = 10
no_of_sections = 100
angle_section = 2*PI/no_of_sections
angles = np.linspace(0,2*PI,no_of_sections+1)[:-1]
cosine = np.cos(angles)
sine = np.sin(angles)
wire_coords = np.array([radius*cosine,radius*sine])
I = np.array([-1e7*sine*cosine,1e7*cosine*cosine])
```

The Quiver plot is shown in Figure 2 below. It is plotted using the I array computed in the above code. The arrow marks point in the direction of current flow. The color of the arrow gives the magnitude of current referenced to the colorbar adjacent to the plot. As we can see, the current flows from bottom to top on both left part and right part of the ring. The magnitude of current is also given by the length of the arrow.

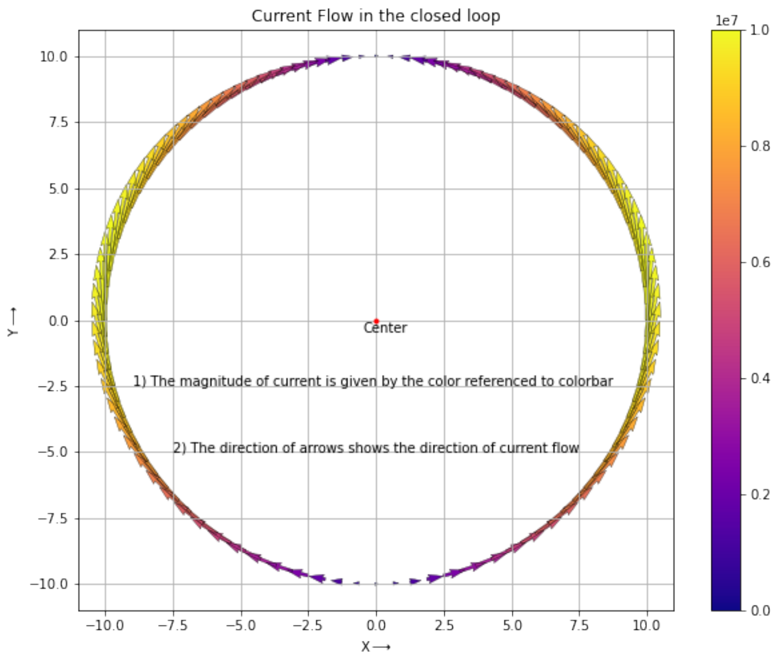

Figure 2: Quiver Plot of current flow

4 Computing Vector Potential

To compute $\vec{B}$, it involves computation of Vector Potential, $\vec{A}$. The Vector Potential $\vec{A}$ given the Current distribution, is computed by

$$\vec{A}(r,\phi,z) = \frac{\mu_o}{4\pi}\int \frac{I(\phi)\hat{\phi}e^{-jkR}ad\phi}{R} \tag{2}$$

Here $\vec{R} = \vec{r} - \vec{r'}$ and $k = \frac{\omega}{c} = 0.1$. $\vec{r}$ is the position vector of the point where we want to find the $\vec{A}$ and $\vec{r'}$ is the position vector of the section due to which the Vector Potential is created. But computation of this integral might require high computation power. This can be approximated to a sum as shown below:

$$\vec{A}_{ijk} = \sum_{l=0}^{N-1} \frac{cos(\phi_l')\exp(-jkR_{ijkl})\vec{dl'}}{R_{ijkl}} \tag{3}$$

Now to find the sum, we should compute the term which is summed in the Equation 3. To find the term, we require to compute the terms R_{ijk} and $\vec{dl'}$. The code to find $\vec{dl'}$ is given below:

```
    dl_length = 2*PI*radius/no_of_sections
    r_ = np.array([radius*cosine,radius*sine])
```

```
    r_ = np.append(r_,np.zeros((1,len(r_[0]))),axis=0).T
    dl = np.array([-dl_length*sine,dl_length*cosine])
    dl = np.append(dl,np.zeros((1,len(dl[0]))),axis=0).T
```

Now that we have computed $\vec{dl'}$, we have to find R_{ijk} and from that we should compute $\vec{A_{ijk}}$ due to the section with index l. To do this we define a function **calc()**. The python code for this function is given below.

```
    def calc(l):
        r = np.moveaxis(np.array((X,Y,Z)).T,[0,1,2],[2,0,1])
        R = np.linalg.norm(r - r_[l],axis=3)
        A = np.exp(-1j*R/radius)/R*cosine[l]*(dl[l].reshape(3,1,1,1))
        return A       # [A in which axis][Point at which A is required (x,y,z)]
```

In the above function, We computed R and from that we also computed $\vec{A_{ijk}}$ due to the section with index l.

Now that we have found out the vector potential due to a section in the space, we have to just add the vector potentials due to all the sections to get the resultant vector potential in the space. So, we just add the output of **calc(l)** for all l. This gives the Vector potential in space. The shape of Vector Potential is 3x3x3x1001. The axis 0 gives the potential in a particular direction and axes 1,2 and 3 give the potential at point (x,y,z) with axis 1 corresponding to x, axis 2 corresponding to y and axis 3 corresponding to z. The code to find the Resultant Vector Potential is given below.

```
    A = np.zeros(calc(0).shape)
    for l in range(no_of_sections):
        A = calc(l)+A
    A_x = A[0]
    A_y = A[1]
```

Thus we have computed the Vector potential in the space. Now we can find the Magnetic field.
Question: Justify why we can use for loop here?
If we use the vectorized form of A which has the Vector potential due to all the sections, we need to sum the elements of the array A which has the same computation as for a for loop. Hence we use for loop to simplify the code.

5 Magnetic Field

Magnetic Field,$\vec{B}$ is computed from Vector Potential, $\vec{A}$ by taking the curl of vector potenital.

$$\vec{B} = \nabla \times \vec{A}$$

We can use Numerical calculations to compute $\vec{B}$. The equation to find the magnetic field on line x=y=1 is

$$B_z(z) = \frac{A_y(2\Delta x, \Delta y, z) - A_y(0, \Delta y, z)}{2\Delta x} - \frac{A_x(\Delta x, 2\Delta y, z) - A_x(\Delta x, 0, z)}{2\Delta y}$$

Note: In the Question PDF, the above equation is divided by 4 rather than 2. But when we solve to find the equation, we get that it is divided by 2. So we divide it by 2 rather than 4. And the equation given in PDF was for Magnetic field on z axis, but in this case we assumed that we are finding Magnetic Field on line x=y=1. Therefore, there is a slight change in the equation as seen. The code to compute Magnetic field and plot it:

```
B=(A_y[2,1,:]-A_x[1,2,:]-A_y[0,1,:]+A_x[1,0,:])/(2.0)
plt.loglog(z,np.abs(B),label='Magnetic Field Variation with z')
plt.title('Variation of Magnetic Field with z on loglog plot')
plt.legend()
plt.xlabel('z $\longrightarrow$')
plt.ylabel('Magnetic Field $\longrightarrow$')
plt.grid()
plt.savefig('fig3.png')
plt.show()
```

The magnetic field computed using the above code is plotted on a loglog graph. The plot is shown in Figure 3.

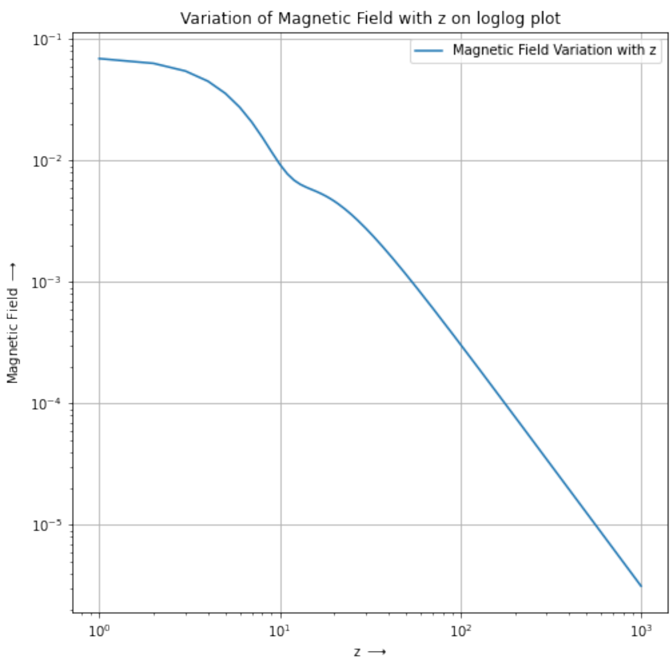

Figure 3: Original Magnetic field on log-log plot

As we can see in the plot, the Magnetic Field is not a constant slope line at the beginning but after z=10, the magnetic field decreases with constant slope in the loglog plot. So we can approximate the magnetic field, with an exponential of z.

Let us assume that Magnetic field is given by

$$B_z = cz^b \tag{4}$$

Taking log on both sides of the above equation, we get

$$\log(B_z) = \log(c) + b\log(z) \tag{5}$$

Now we can use least squares method to find the estimated values of c and b. We can see that the linear region starts approximately at z=20. So we take the samples from z=20 to estimate the coefficients.

```python
A=np.hstack([np.ones((len(B[20:]),1)),(np.log(z[20:])).reshape(len(B[20:]),1)
])
log_c, b = np.linalg.lstsq(A,np.log(np.abs(B[20:])))[0]
c = np.exp(log_c)
```

The estimated value of b and c are

```
Estimate of b: -1.96920706797629
Estimate of c: 2.5861334621525613
```

Using these estimate values,let us plot the estimated Magnetic field and the original magnetic field. The plot is shown in Figure 4.

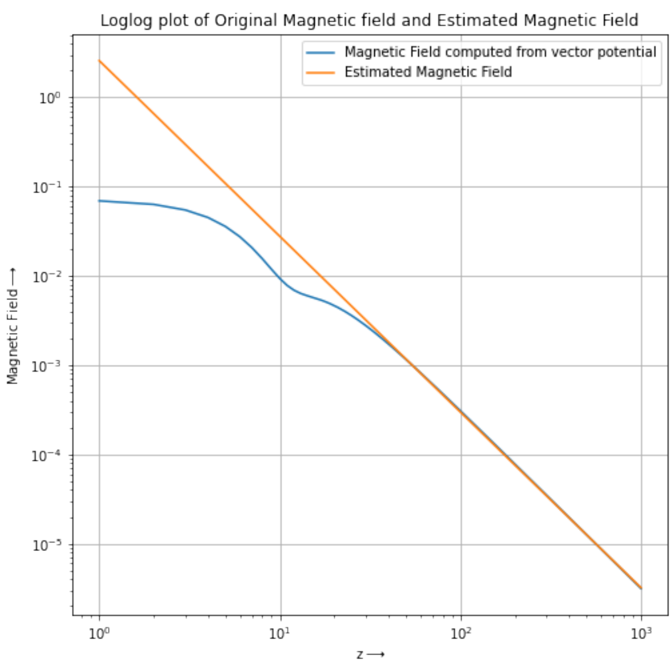

Figure 4: Original and Estimated Magnetic Fields on log-log plot

6 Question 11

We find that the Magnetic field is approximately decaying at a rate of 2. That is

$$B_z \propto \frac{1}{z^2} \tag{6}$$

A static Magnetic field is generated due to a constant current passing through the loop. The Magnetic field in this case is

$$\vec{B} = \frac{\mu_o I r^2}{2(\sqrt{z^2 + r^2})^3} \tag{7}$$

Here we can see that, the Magnetic field is approximately decaying at a rate of 3 as we can see in the denominator. To see it more clearly, let us assume that we are measuring at a point far away from the loop. In this case, $z >> r$. Hence, we can approxiamte it as

$$\vec{B} = \frac{\mu_o I r^2}{2z^3}$$

Thus,we can see the difference between static case and the case given in the problem. In the static case, the Magnetic field decays at a rate of 3 while in the case of this problem, the magnetic field is decaying at a rate of 2. This is due to the sinusoidal variation of current in the given problem. In the static case, there is no variation and hence it has more decaying rate than in this case.

7 Conclusion

In this problem, we found out the current distribution and plotted it on a quiver plot. Then we used the current vectors to compute the Vector Potential due to a small segment of wire. Then we used the super position of Vector Potentials due to all the small segments to compute the resultant Vector Potential. Then, we used Numerical Calculation to compute Magnetic Field and approximated it to a exponential in z. We observe that the magnetic field decays at a rate of 2 with z. We also found out that a static magnetic field decays at a rate of 3.