DATA605 Discussion Week 4

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Jai Jeffryes

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C20

Example **SAR** concludes with an expression for a vector $u \in \mathbb{C}^5$ that we believe will create the vector $v \in \mathbb{C}^5$ when used to evaluate T. That is, $T(u) = v$. Verify this assertion by actually evaluating T with u. If you do not have the patience to push around all these symbols, try choosing a numerical instance of v, compute u, and then compute $T(u)$, which should result in v.

Given

```
# Archetype R
R <- matrix(c(
    -65, 128, 10, -262, 40,
    36, -73, -1, 151, -16,
    -44, 88, 5, -180, 24,
    34, -68, -3, 140, -18,
    12, -24, -1, 49, -5
), nrow = 5, byrow = T)
R
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  -65  128   10 -262   40
## [2,]   36  -73   -1  151  -16
## [3,]  -44   88    5 -180   24
## [4,]   34  -68   -3  140  -18
## [5,]   12  -24   -1   49   -5
```

```
# Beezer assertion. R is non-singular and has an inverse.
R.inv <- solve(R)
R.inv
```

```
##      [,1] [,2] [,3]   [,4] [,5]
## [1,]  -47   92  1.0 -181.0  -14
## [2,]   27  -55  3.5  110.5   11
## [3,]  -32   64 -1.0 -126.0  -12
## [4,]   25  -50  1.5   99.5    9
## [5,]    9  -18  0.5   35.5    4
```

Verify

- Choose numerical instance of v

```
v <- c(22, -3, 100, 5, 17)
v
```

```
## [1]  22  -3 100   5  17
```

- Compute u

```
u <- R.inv %*% v
u
```

```
##          [,1]
## [1,] -2353.0
## [2,]  1848.5
## [3,] -1830.0
## [4,]  1500.5
## [5,]   547.5
```

- Compute $T(u)$ and compare

```
t.u <- R %*% u
t.u
```

```
##      [,1]
## [1,]   22
## [2,]   -3
## [3,]  100
## [4,]    5
## [5,]   17
```

- Compare the two results

```
# Here is v.
v
```

```
## [1]  22  -3 100   5  17
```

```
# Here is T(u). Expected: same as v.
t.u
```

```
##      [,1]
## [1,]   22
## [2,]   -3
## [3,]  100
## [4,]    5
## [5,]   17
```

Conclusion

Expected result is confirmed. Archetype **R** is a surjective linear transformation.