

DATA 604 Final Project

Introduction to compartmental
models of infectious disease
outbreak

Jai Jeffryes
7/16/2020



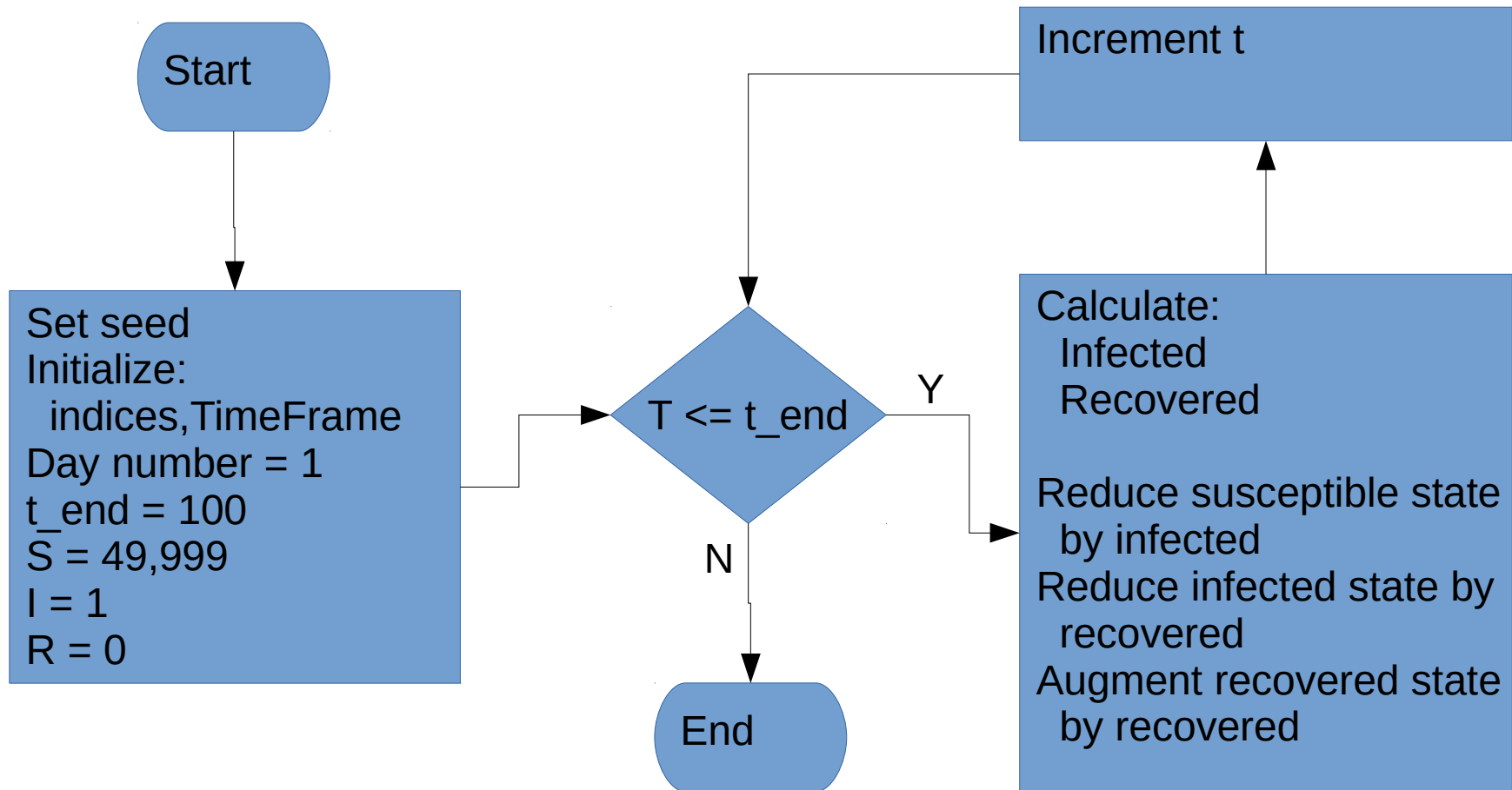
Problem statement

- *With rapidly changing ecology, urbanization, climate change, increased travel and fragile public health systems, epidemics will become more frequent, more complex and harder to prevent and contain. [1]*
- One of the tools available for exploring infectious disease dynamics early in an outbreak is the SIR model. [2]

SIR assumptions

- Population is constant. The model does not consider births and deaths.
- Immunity, once acquired upon recovery, endures forever.
- The population is well mixed. An individual has equal probability of contact with all other members of the population.

Processing flowchart

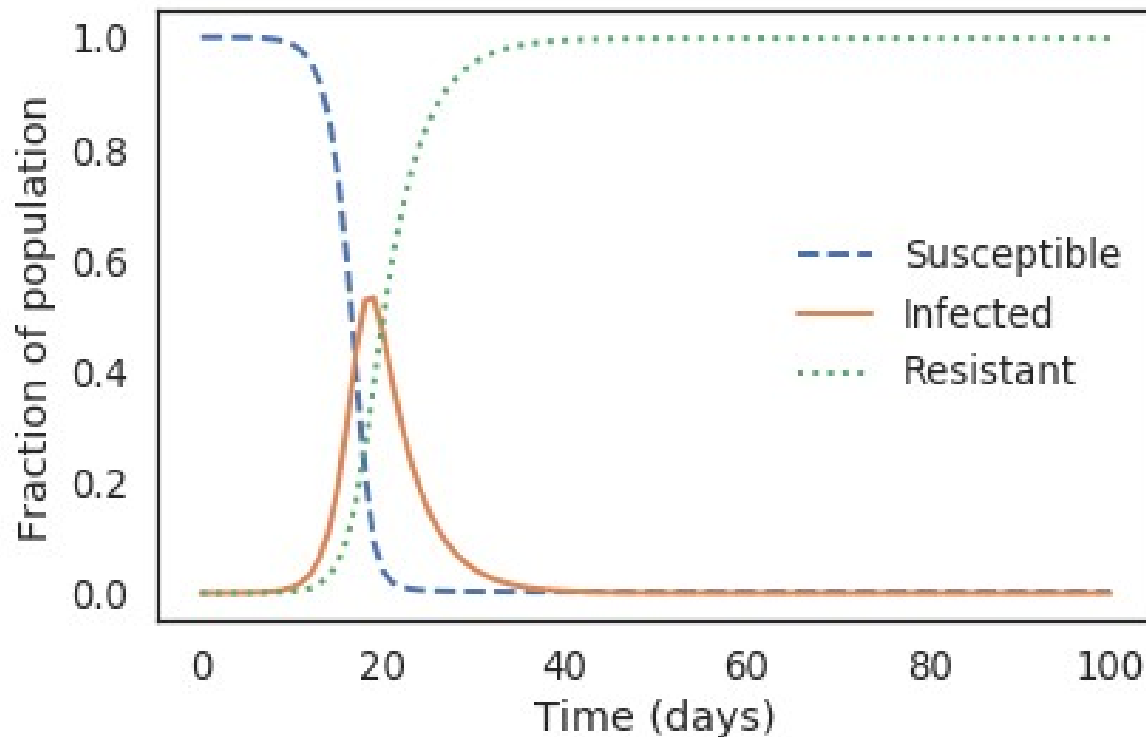


Stock and flow diagram



Simulation output

- $\text{Beta} = 0.05249 * (2.7 + 18)$. (Transmissibility times household size and contacts, discussed in next model.)
- $\text{Gamma} = 1 / 4.5$. (Reciprocal of recovery time.)



Validity and verification

- Literature supported the validity of employment of the SIR model as an abstraction of epidemic disease.
- Prior to the SIR model it was not evident that there is a threshold for infections. Verification of the SIR model historically included successful eradication of smallpox. [3]

Context of contact

- Explore the assumption about mixing. [4]
- Under normal circumstances, non-household daily contacts tend to exceed household contacts. Lockdown measures reduce non-household contacts to a level similar to or below household contacts.
- This project implements such a model and compares the trajectory of infection to that of the simple SIR model.

Refinement - parameters

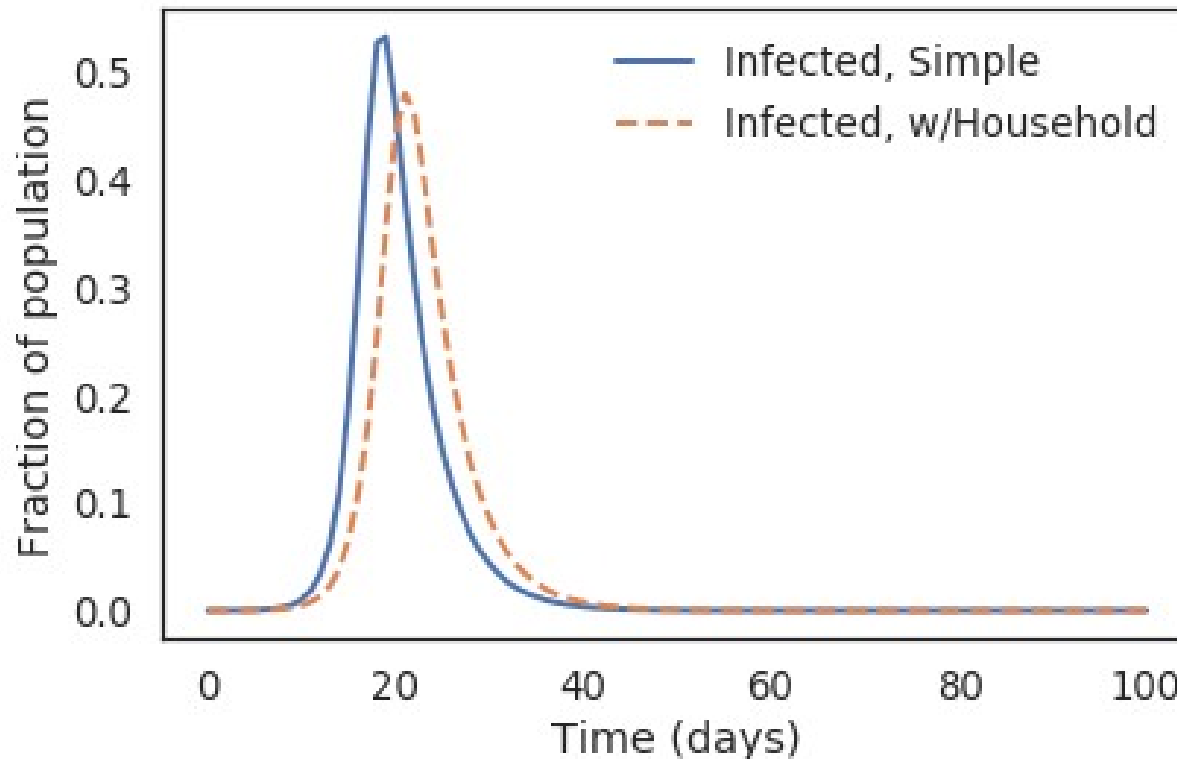
- hs . Average household size.
- β . Average transmission rate between an infected individual and a susceptible one.
- κ . Average number of contacts.
- Cc . Cases who acquire infection via the community.
- Ch . Cases who acquire infection via their household.

Refinement - formulae

- $\kappa - h_s + 1$. The number of non-household contacts.
- $(h_s - 1)C_c$. Maximum number of household acquired infections in a population. Unimplemented.
- $dS/dt = -\beta\kappa IS$. Rate of change in susceptibility, no context.
- $dC_c/dt = \beta c(k - h_s + 1)IS$. Rate of increase in community cases among $(k - h_s + 1)$ non-household contacts.
- $dC_h/dt = \beta h(h_s - 1)IS(1 - C_h/(h_s - 1)C_c)$. Rate of increase in household cases among $h_s - 1$ contacts and constrained by a carrying capacity.
- $dS/dt = -\beta c(k - h_s + 1)IS - \beta h(h_s - 1)IS(1 - C_h/(h_s - 1)C_c)$. The full ODE for rate of change for compartment S. Again, this project does not implement the final term for carrying capacity.

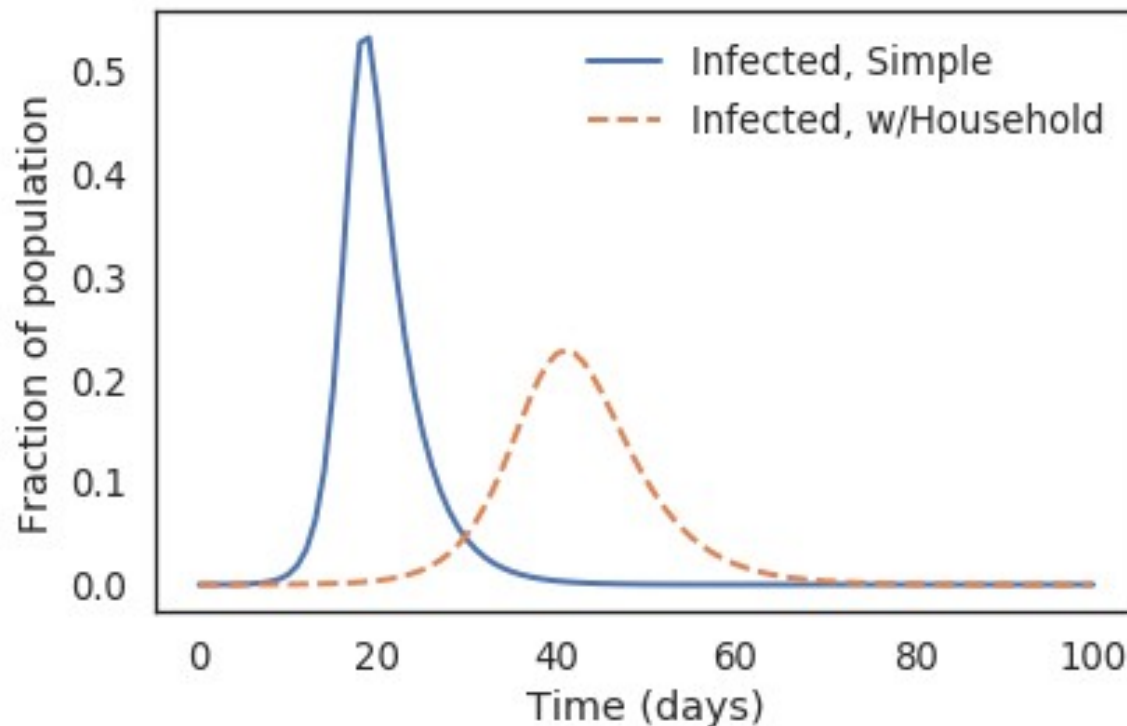
Comparison of infected curves

- This simulation set an average household size lower than community contacts. There is some delay of peak infections and the maximum is slightly lower.



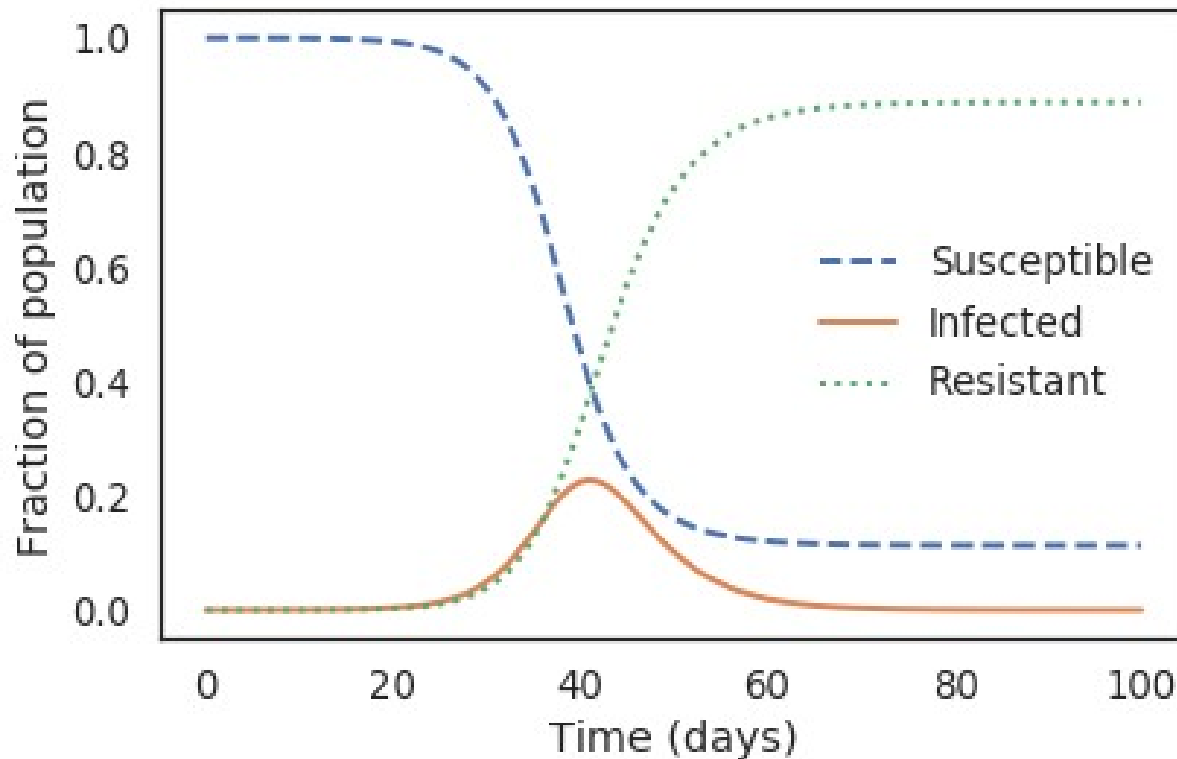
Simulate lockdown

- Reduce total contacts while preserving household contacts.
- Flattened the curve as under policies ordered by New York Governor Andrew Cuomo.



Observation

- An epidemic dies out due to a lack of infected hosts, not depletion of susceptible individuals. This mechanism was not evident until models suggested it. [3]



References

1. Juliet Bedford. [A new twenty-first century science for effective epidemic response.](#) Nature, November 6, 2019.
2. Kermack, W; McKendrick, A.
[A Contribution to the Mathematical Theory of Epidemics.](#)
3. Howard Weiss. [The SIR model and the foundations of public health](#)
4. Paul Dempsey.
[Modifying SEIR models to explore the role of average household size in the era of lockdowns and social distancing](#)

Acknowledgments

- Paul Dempsey for, discussions on LinkedIn about his SEIR paper.
- Dr. Howard Weiss, for correspondence and discussion about SIR models in public health policy and retrospective data collection and modeling issues during the 2020 pandemic crisis.
- Dr. Knut Wittkowski, for discussion on LinkedIn about the pandemic, herd immunity, and public health policy, and for sending me his papers for study.

Background

[Compartmental models in epidemiology \(Wikipedia\)](#).

[The SIR Model for spread of disease](#). Allen Downey's reference for his case study.

[The SIR model and the foundations of public health](#). Howard (Howie) Weiss. Very good orientation.

[Prism site](#). PRISM, the Centre for Research Excellence in Policy Relevant Infectious diseases Simulation and Mathematical Modelling in Australia.

[A User's Guide to Infectious Disease Modeling](#). This manual is an excellent domain orientation.

Paul Dempsey's modified SIR model, exploring household and non-household contributions to case count under lockdown.

[Modifying SEIR models to explore the role of average household size in the era of lockdowns and social distancing](#)

[Paul Dempsey's model](#).

Knut Wittkowski.

[The first three months of the COVID-19 epidemic: Epidemiological evidence for two separate strains of SARS- CoV-2 viruses spreading and implications for prevention strategies](#)

.

[Modeling COVID-19 Spread vs Healthcare Capacity](#). Shiny App to predict COVID-19 cases by clinical outcome. Comes with a well-documented tutorial.

[Code on GitHub](#).

[Forecasting COVID-19](#). Rob Hyndman's assessment of model challenges. Includes a comparison of compartmental and agent-based models.

[Social distancing and mobility reductions have reduced COVID-19 transmission in King County, WA](#). The study estimates the reproductive number from case data. It employs a distinctive SEIR model and documents the equations.

[COVID-19: Modeling Distributions of Incubation and Recovery Times](#). Stock and flow diagrams. Although the flows use inappropriate diagrammatic notation, they convey an idea. The article includes delays for state changes.

[Report 9: Impact of non-pharmaceutical interventions \(NPIs\) to reduce COVID-19 mortality and healthcare demand](#). Neil Ferguson's analysis from March 2020.