Data 605 - Assignment 14

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Contents

Assignment

This week, we'll work out some Taylor Series expansions of popular functions.

- $f(x) = \frac{1}{(1-x)}$
- $f(x) = e^x$
- $f(x) = ln(1+x)$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as an R-Markdown document.

Problem 1: $f(x) = \frac{1}{(1-x)}$

Am I just doing the Maclaurin Series version of this? The series is defined for $x = 0$. $x - 1$ is undefined because 0 would appear in the denominator.

Find the derivative of the function using the Chain Rule, letting $u = 1 - x$.

$$\frac{d}{dx}(\frac{1}{1-x}) = \frac{d}{dx}(1-x)^{-1} = (-1 \cdot (1-x)^{-2}) \cdot (-1) = \frac{1}{(1-x)^2}$$

Let's extend this to more derivatives and compute their value for $x = 0$.

$$f(x) = \frac{1}{(1-x)},\ f(0) = 1$$

$$f'(x) = \frac{1}{(1-x)^2},\ f'(0) = 1$$

$$f''(x) = \frac{2}{(1-x)^3},\ f''(0) = 2$$

$$f'''(x) = \frac{6}{(1-x)^4},\ f'''(0) = 6$$

$$f''''(x) = \frac{24}{(1-x)},\ f''''(0) = 24$$

$$f^{(n)}(x) = \frac{n!}{(1-x)^{(n+1)}},\ f^{n}(0) = n!$$

Answer

$$f(x) = \frac{1}{(1-x)} = \sum_{n=1}^{\infty} \frac{n!}{(1-x)^{(n+1)}}$$

Problem 2: $f(x) = e^x$

Answering this problem proves that I watched David Jerison's video!

We'll center the Taylor expansion on $c = 0$

$$e^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Consider that the function $f(x) = e^x$ is its own derivative. Thus,

$$f(x) = e^x$$
$$f'(x) = e^x$$
$$f''(x) = e^x$$
$$Etc.$$

$$f^{(n)}(x) = e^x \Big|_{x=0} = 1$$

Therefore, all the numerators in the expansion are 1.

Answer

$$f(x) = e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

This example computes the value of e by setting $x = 1$

$$e = e^1 = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$$

Problem 3: $f(x) = ln(1+x)$

Find the derivative of the function using the Chain Rule, letting $u = 1 + x$.

$$\frac{d}{dx} ln(1+x) = \frac{1}{1+x} \cdot 1 = \frac{1}{1+x}$$

Successive derivatives alternate in polarity. We'll expand around $c = 0$.

$$f(x) = ln(1+x),\ f(0) = 0$$

$$f'(x) = \frac{1}{1+x},\ f'(0) = 1$$

$$f''(x) = \frac{-1}{(1+x)^2},\ f''(0) = -1$$

$$f^{(3)}(x) = \frac{2}{(1+x)^3},\ f^{(3)}(0) = 2$$

$$f^{(4)}(x) = \frac{-6}{(1+x)^4},\ f^{(4)}(0) = -6$$

$$f^{(5)}(x) = \frac{24}{(1+x)^5},\ f^{(5)}(0) = 24$$

$$f^{(n)}(x) = \sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{(n-1)!}{(1+x)^n},\ f^{(n)}(0) = (-1)^{(n-1)}(n-1)!$$

Answer

$$f(x) = ln(1+x) = \sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{(n-1)!}{(1+x)^n}$$