DATA 605 - Discussion 15

Jai Jeffryes

5/6/2020

Assignment

Pick any exercise in 8.8 of the calculus textbook. Solve and post your solution.

P. 496

In Exercises 17 - 20, use the Taylor series given in Key Idea 8.8.1 to verify the given identity.

Problem 17

$cos(-x) = cos(x)$

The Taylor series given in 8.8.1 for cosine is,

$$cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2^n}}{(2n)!}$$

Substitute $-x$,

$$cos(-x) = \sum_{n=0}^{\infty} (-1)^n \frac{(-x)^{2^n}}{(2n)!}$$

What I really want to do is simply pull out the terms that are different. Is there a way, somehow, to make that legal, formally?

I want to compare the numerators.

$$x^{2^n},\ (-x)^{2^n}$$

Expand the summation for the second of those terms.

$$\sum_{n=0}^{\infty} (-x)^{2^n} = (-x)^{2^0} + (-x)^{2^1} + (-x)^{2^2} + \cdots + (-x)^{2^n}$$

$$= 1 + (x)^2 + (x)^4 + \cdots + (x)^{2n}$$

Since each term in the sum is positive,

$$cos(-x) = \sum_{n=0}^{\infty} (-1)^n \frac{(-x)^{2^n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2^n}}{(2n)!} = cos(x)$$

$

Problem 18

$sin(-x) = -sin(x)$

From Key Idea 8.8.1,

$$sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$sin(-x) = \sum_{n=0}^{\infty} (-1)^n \frac{(-x)^{2n+1}}{(2n+1)!}$$

Compare the numerators,

$$x^{2n+1}, \ (-x)^{2n+1}$$

The second numerator expanded in summation is,

$$\sum_{n=0}^{\infty} (-x)^{2n+1} = (-x)^1 + (-x)^3 + (-x)^5 + \cdots + (-x)^n$$

All of these terms are negative while expansion of the other term gives us,

$$\sum_{n=0}^{\infty} (x)^{2n+1} = x^1 + x^3 + x^5 + \cdots + x^n$$

For every pair of terms, the signs are opposite. Therefore $sin(-x) = -sin(x)$

These aren't rigorous proofs, but I see the patterns.