Data 605 - Assignment 15

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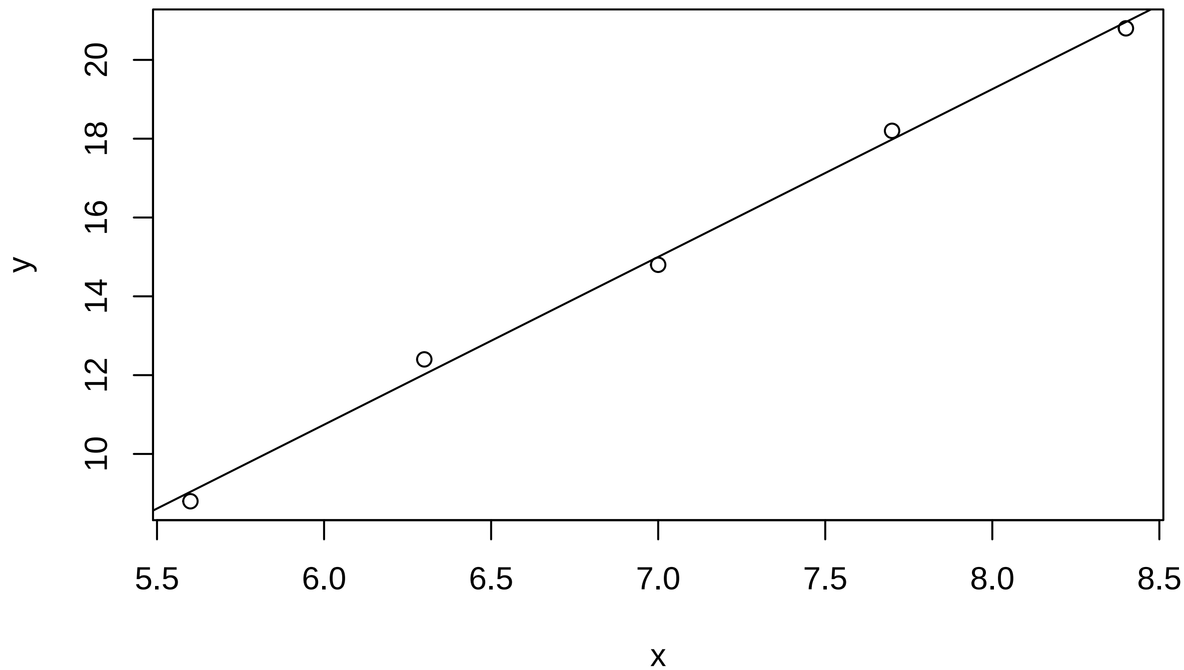
Contents

Problem 1

Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary. (5.6,8.8), (6.3,12.4), (7,14.8), (7.7,18.2), (8.4,20.8)

```
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)
df <- data.frame(cbind(x, y))

df_m <- lm(y ~ x, data = df)
plot(y ~ x, data = df)
abline(df_m)
```




```
df_m
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

Answer

$$y = 4.26x - 14.80$$

Problem 2.

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Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

$$f(x, y) = 24x - 6xy^2 - 8y^3$$

Partial derivatives of f

$$f_x = 24 - 6y^2 \text{ and } f_y = -12xy - 24x^2$$
$$f_{xx} = 0 \text{ and } f_{yy} = -12x - 48y$$
$$f_{xy} = -12y \text{ and } f_{yx} = -12y$$

Find critical points

Seek critical points where f_x and f_y are simultaneously 0.

$$\begin{aligned} f_x = 0 &\implies 0 = 24 - 6y^2 \\ &\implies 6y^2 = 24 \\ &\implies y^2 = \frac{24}{6} = 4 \\ &\implies y = \pm\sqrt{4} = \pm 2 \end{aligned}$$

Therefore, our critical points have two possible values for y. $f_x = 0$ at $y = \pm 2$.

We're looking for x values for the critical points. In f_y, solve for x for each y we found.

Assume $y = 2$

$$\begin{aligned} f_y = 0 \wedge y = 2 &\implies 0 = -12x \cdot 2 - 24 \cdot 4 = -24x - 96 \\ &\implies 96 = -24x \\ &\implies x = -4 \end{aligned}$$

Therefore, $CriticalPoint_1 = (-4, 2)$

Assume $y = -2$

$$\begin{aligned} f_y = 0 \wedge y = -2 &\implies 0 = -12x \cdot -2 - 24 \cdot 4 = 24x - 96 \\ &\implies 96 = 24x \\ &\implies x = 4 \end{aligned}$$

Therefore, $CriticalPoint_2 = (4, 2)$

Second derivative test

Calculate the test value D for each critical point. Refer to the partial derivatives from above. Evaluate the condition of the second derivative test.

$$D = f_{xx} f_{yy} - f_{xy}^2$$

For $CriticalPoint_1 = (-4, 2)$

$$\begin{aligned} D &= 0 \cdot (-12x - 48y) - (-12y)^2 = -144y^2 \\ &= -144 \cdot 2^2 = -576 \end{aligned}$$

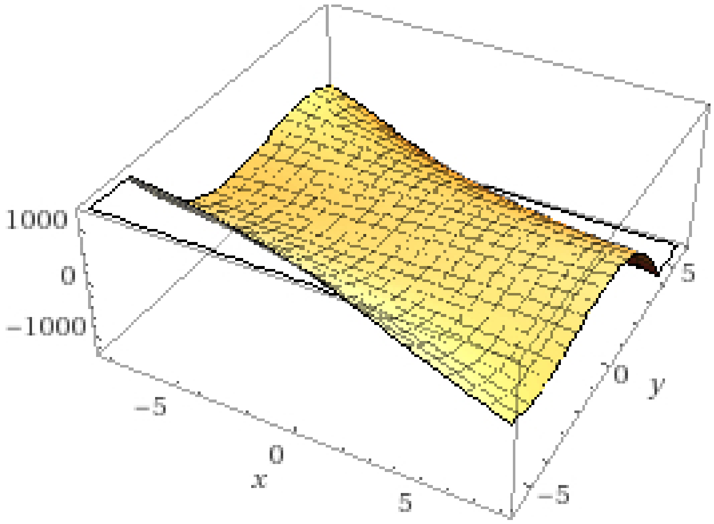
$D = -576 < 0$. Therefore (-4, 2) is a saddle point.

Consider $CriticalPoint_2$

$$\begin{aligned} D &= -144y^2 \\ &= -144 \cdot (-2)^2 = -576 \end{aligned}$$

$D = -576 < 0$. Therefore (4, -2) is a saddle point.

Answer



Critical points:

- (-4, 2) is a saddle point.
- (4, -2) is a saddle point.

Problem 3

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

- Step 1. Find the revenue function $R(x, y)$.
- Step 2. What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

Step 1

The revenue is,

$$R = Price_{house} \times Units_{house} + Price_{name} \times Units_{name}$$

$Price_{house} = x$

$Units_{house} = 81 - 21x + 17y$

$Price_{name} = y$

$Units_{name} = 40 + 11x - 23y$

Therefore,

$$\begin{aligned} R(x, y) &= x \cdot (81 - 21x + 17y) + y \cdot (40 + 11x - 23y) \\ &= x \cdot (81 - 21x + 17y) + y \cdot (40 + 11x - 23y) \\ &= x81 - 21x^2 + 17xy + 40y + 11xy - 23y^2 \end{aligned}$$

Therefore, after collecting terms and rearranging,

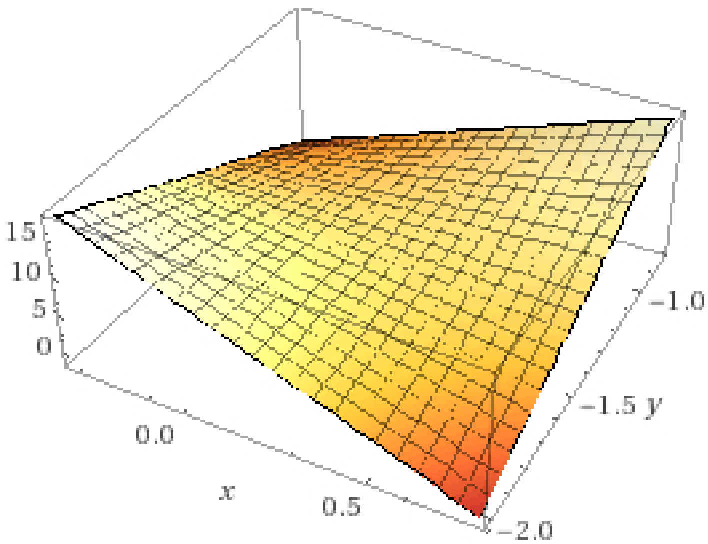
$$R(x, y) = -21x^2 - 23y^2 + 81x + 40y + 28xy$$

Step 2

```
x <- 2.30
y <- 4.10

revenue <- -21*x^2 - 23*y^2 + 81*x + 40*y + 28*x*y
```

Answer



- Step 1: $R(x, y) = -21x^2 - 23y^2 + 81x + 40y + 28xy$
- Step 2: \$116.62

Problem 4

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

Production formula

The conditions of the problem constrain this optimization, such that $x + y = 96$. Therefore, $y = -x + 96$. We substitute this constraint into the cost function.

$$\frac{1}{6}x^2 + \frac{1}{6}(-x + 96)^2 + 7x + 25(-x + 96) + 700$$

$$= \frac{1}{6}x^2 + \frac{1}{6}(x^2 - 192x + 9216) + 7x - 25x + 2400 + 700$$

Collecting terms,

$$= \frac{1}{3}x^2 - 32x + 1536 - 18x + 3100$$

$$= \frac{1}{3}x^2 - 50x + 4636$$

Extreme value

Find the first and second derivatives.

$$C' = \frac{2}{3}x - 50$$

$$C'' = \frac{2}{3}$$

Set the first derivative to 0 to find an extreme value.

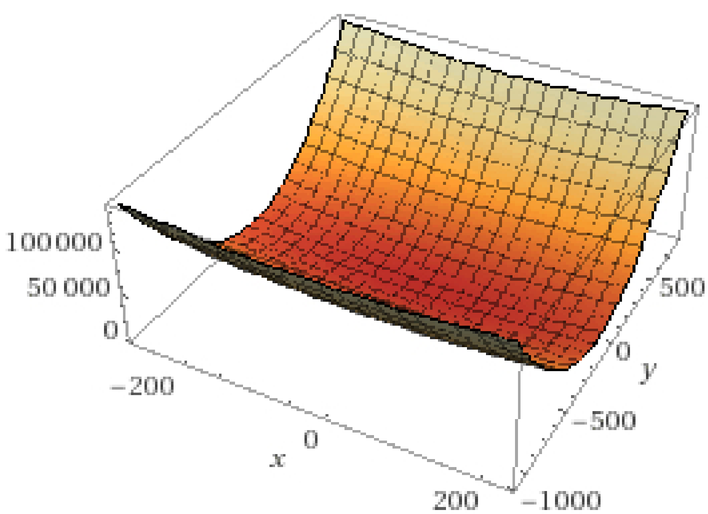
$$0 = \frac{2}{3}x - 50 \implies 50 = \frac{2}{3}x \implies x = \frac{3}{2} \cdot 50 = 75$$

An extreme value is located at (75, 96 - 75), which is (75, 21).

Since C'' is positive, the function is concave up at this extreme value, which is therefore a local minimum and indeed an absolute minimum.

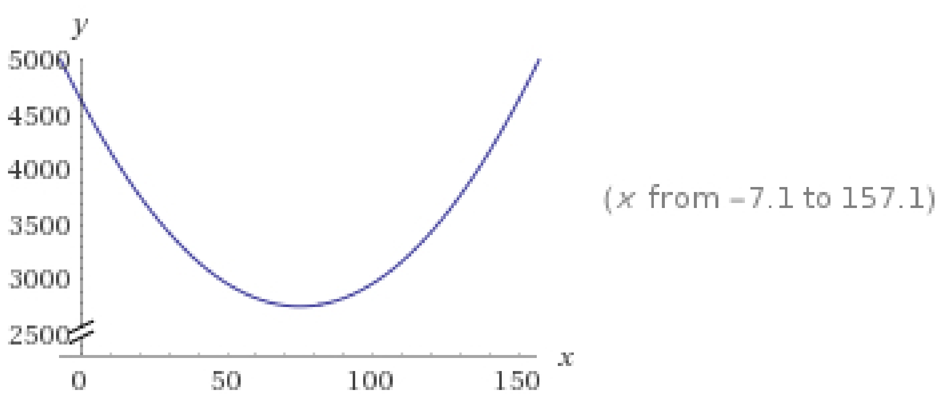
Plots

The cost function $C(x, y)$



Production function

Since $y = -x + 96$, this subset is a diagonal slice across that trough.



The plot illustrates the minimum found for x, 75.

Answer

The production units which minimize weekly costs are:

- Los Angeles: 75.
- Denver: 21.

Problem 5

Evaluate the double integral on the given region.

$$\iint_R (e^{8x+3y}) dA; R : 2 \le x \le 4 \text{ and } 2 \le y \le 4$$

Write your answer in exact form without decimals.

Generalize integration

Several times, we're going to need to integrate the function $f(x) = e^{nx}$. Generalize that operation using u-substition.

$$\int e^{nx} dx$$

Let $u = nx,\ dx = ndx$

$$\int e^{nx} dx = \frac{1}{n} \int e^{nx} ndx = \frac{1}{n} \int e^u du$$

$$= \frac{1}{n} e^u + C = \frac{1}{n} e^{nx} + C$$

We will use this result in upcoming steps.

Express definite integral

$$\int_2^4 \int_2^4 e^{8x+3y} dydx$$

$$= \int_2^4 (\int_2^4 e^{8x} e^{3y} dy) dx$$

We're evaluating the inner integral now. With respect to y, x is a constant. Therefore,

$$= \int_2^4 (e^{8x} \int_2^4 e^{3y} dy) dx$$

$$= \int_2^4 (e^{8x} \cdot \frac{1}{3} e^{3y} \Big|_2^4) dx$$

$$= \int_2^4 \frac{e^{8x}}{3} (e^{12} - e^6) dx$$

Move the constants to the outside of the integral.

$$= \frac{(e^{12} - e^{6})}{3} \int_{2}^{4} e^{8x} dx$$

$$= \frac{(e^{12} - e^{6})}{3} \cdot \frac{1}{8} e^{8x} \Big|_{2}^{4}$$

$$= \frac{1}{24}(e^{12} - e^{6})(e^{32} - e^{16})$$

Answer

$$= \frac{1}{24}(e^{32} - e^{16})(e^{12} - e^{6})$$

Reference

All plots: WolframAlpha.