

Dimension 3
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p. 388, c 11

Find the characteristic polynomial of

matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

Solution:

$$P_A(x) = \det(A - xI_3)$$

$$= \begin{vmatrix} 3-x & 2 & 1 \\ 0 & 1-x & 1 \\ 1 & 2 & 0-x \end{vmatrix}$$

$$= (3-x) \begin{vmatrix} 1-x & 1 \\ 2 & 0-x \end{vmatrix} + (2-1) \begin{vmatrix} 0 & 1 \\ 1 & 0-x \end{vmatrix} + (1) \begin{vmatrix} 0 & 1-x \\ 1 & 2 \end{vmatrix}$$

$$= (3-x)((1-x)(0-x)-2) - 2(0(0-x)-1) + (0-(1-x))$$

$$= (3-x)(0-x+x^2-2) - 2(-1) + 1 + x$$

$$= (3-x)(x^2-x-2) + 2 - 1 + x$$

$$= 3x^2 - x^3 - 3x + x^2 - 6 + 3x + 1 + x$$

$$= -x^3 + 4x^2 - 5$$