We are interested in determining the reproduction number for a given observed outbreak size (assumed to be initiated from a single importation).

First we define



the probability of an outbreak of size  occurring given a reproduction number  [see Farrington 2003]. We assume here a Poisson offspring distribution.

However, only  individuals are observed due to underreporting. Defining a reporting rate  (i.e. the probability of observing an individual),



follows a binomial distribution with parameters  and .

Probability of observing  individuals given  and  follows:



We note that for ,  becomes: , with  and  the Lambert's W function.

Therefore the likelihood of observing  individuals is proportional to the posterior distribution of  assuming a flat posterior distribution:



Given a set of  , with  , of observed outbreak sizes, we have:



We can then numerically calculate this by setting an upper threshold limit, , on .

Given , we search a threshold, , such that



We need to solve this assuming a negative binomial distribution.

**Accounting for unobserved outbreaks**

All of the above assume that we observe the size of all outbreak start with a single case. However, obviously, an outbreak for which no cases are observed will be missed in our sample, and thus we need to correct for the form of censoring.

Therefore, we normalise for the unobserved outbreak

,

And the likelihood/posterior becomes:



As seen above, given a reproduction number , the probability of observing an outbreak is:



Therefore, the distribution of the number of unobserved outbreak follows a negative binomial with parameters .

**What to do when** 

So far the results assume .

Given finite outbreak size observed, we need to calculate the probability of extinction to get the outbreak size when  conditional on extinction.

Actually, the probably of extinction of an outbreak with  can be calculated as:



with  and  the Lambert's W function.

Secondly, the distribution of outbreak size for  conditional on extinction is the same as the unconditional distribution of outbreak size for  . We have



Now, the posterior distribution of  is given by .

If we assume a flat prior for , we have:



Or for : ;

and for : ;