Karnaugh Map - K-map

- For Simplify boolean Expression:

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

normal method

$Y = \bar{A}\bar{B} + A\bar{B} = \bar{B}(\bar{A} + A)$
$= \bar{B}(1) = \bar{B}$

$Y = (A + \bar{B})(\bar{A} + \bar{B})$
$= \bar{B} + A\cdot\bar{A}$
$= \bar{B}$

But if we use K-map we will get direct answer.

→ 2 variable K-map
$f(A,B)$

A \ B	0	1
0	1_{0}	$_{1}$
1	1_{2}	$_{3}$

$Y = \bar{B}$

A \ B	$\bar{B}$	B
$\bar{A}$	1_{0}	$_{1}$
A	1_{2}	$_{3}$

$Y = \bar{B}$

pairing can be 1, 2, 4, 8, 16

Q simplify $F(A,B) = \Sigma m(1)$

A \ B	$\bar{B}$	B
$\bar{A}$	0_{0}	$\boxed{1}_{1}$
A	0_{2}	0_{3}

$F = \bar{A}B$

→ It will give simplest form

$\boxed{\text{K-map gaurantes}}$

Q simplify $F(A,B) = \Sigma m(0,2,3)$

A \ B	$\bar{B}$	B
$\bar{A}$	1 (0)	0 (1)
A	1 (2)	1 (3)

$F = A + \bar{B}$

3 variable K-map

$F(A,B,C) = \Sigma m(0,2,4,6)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	(1)	(3)	1 (2)
A	1 (4)	(5)	(7)	1 (6)

$F = \bar{C}$

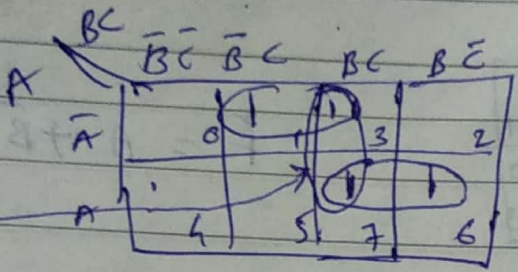
property :- K map is folded

→ $F(A,B,C) = \Sigma m(0,1,3,6)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	1 (1)	1 (3)	(2)
A	(4)	(5)	(7)	1 (6)

$F = \bar{A}\bar{B} + \bar{A}C + AB\bar{C}$

→ $F(A,B,C) = \sum m(1,3,6,7)$



$= \boxed{\bar{A}C + AB}$

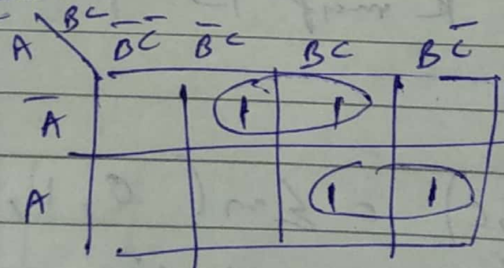
$F \to \bar{A}C + AB + BC$

if we simplify → $\bar{A}C + AB$ (But K map gauranted simplest form 9 right ?)

Consensus theorem

then what happened ?

That's because we have done some mistake



all 1's are covered

→ $F(A,B,C) = \sum m(0,1,5,6,7)$

$F = \bar{A}\bar{B} + AB + \bar{B}C$

$F = \bar{A}\bar{B} + AB + AC$

Expression obtained from K-map is not Unique

→ જ્યાં 1's cover થાય
Java Joie

~~130~~, ~~148~~, ~~127~~, ~~149~~

→ $F(A,B,C) = \Sigma m(0,1,3,5,6,7)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	1 (1)	1 (3)	(2)
A	(4)	1 (5)	1 (7)	1 (6)

$F = C + \bar{A}\bar{B} + AB$

Don't Care Condition

$F(A,B)$

A	B	F
0	0	0
0	1	1
1	0	0
1	1	X

X → uncertain (either 0 or 1)

$f(A,B) = \Sigma m(1) + \Sigma d(3)$

A \ B	$\bar{B}$	B
$\bar{A}$	0 (0)	1 (1)
A	0 (2)	X (3)

$F = B$

→ use X if pair is form, otherwise don't use (અગર ગ્રૂપની બનાવ્યા એ 2ની પેર બને તો બનાવી દેવાની)

* 4 variable K-map

$F(A, B, C, D)$

$2^4 = 16$ combination

$F(A,B,C,D) = \sum m(0,1,3,5,7,8,9,10,11,13,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	1 (3)	0 (2)
$\bar{A}B$	0 (4)	1 (5)	1 (7)	0 (6)
AB	0 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	0 (10)

$F = \bar{B}\bar{C} + D + A\bar{B}$

Q $F(A,B,C,D) = \sum m(0,1,4,5,8,9,13,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	0 (3)	0 (2)
$\bar{A}B$	1 (4)	1 (5)	0 (7)	0 (6)
AB	0 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$	1 (8)	1 (9)	0 (11)	0 (10)

$F = \bar{A}\bar{C} + \bar{B}\bar{C} + ABD$

* $F(A,B) = \Pi M(0,1,3)$

A \ B	$\bar{B}$	B
$\bar{A}$	1 (0)	1 (1)
A	(2)	1 (3)

$\Rightarrow$

A \ B	B	$\bar{B}$
A	0 (0)	0 (1)
$\bar{A}$	(2)	0 (3)

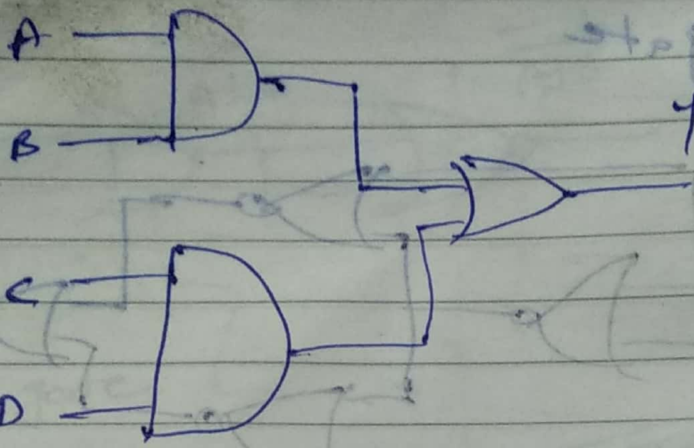
$F = (\bar{B}) \cdot (A)$

$= A\bar{B}$

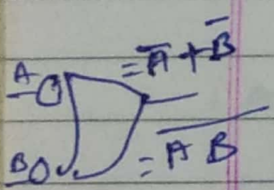
* Implementation of SOP expression
→ only NAND gate
→ first draw 2 level AND-OR ckt

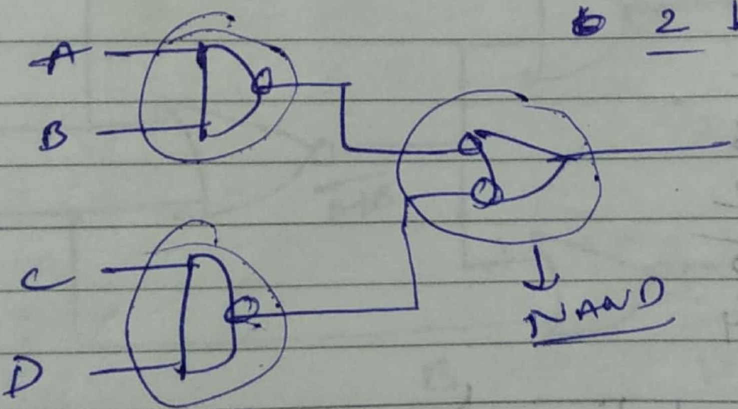
ex Implement ~~A+B~~ AB + CD using minimum no. of NAND gate.

$Y = AB + CD$

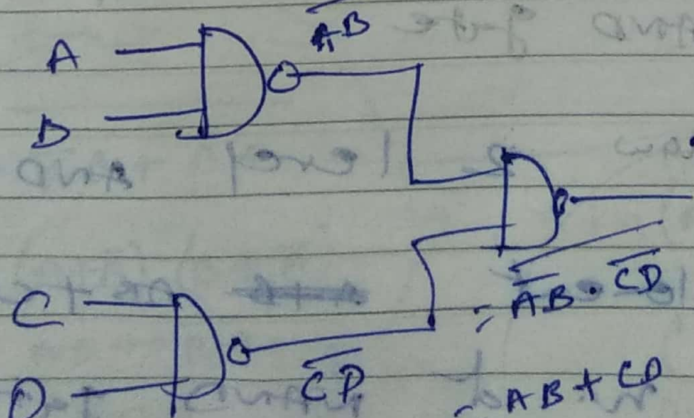


2 bubbles cancle out





↓ NAND



$Y = AB + CD$

$= \overline{\overline{AB}\cdot\overline{CD}}$

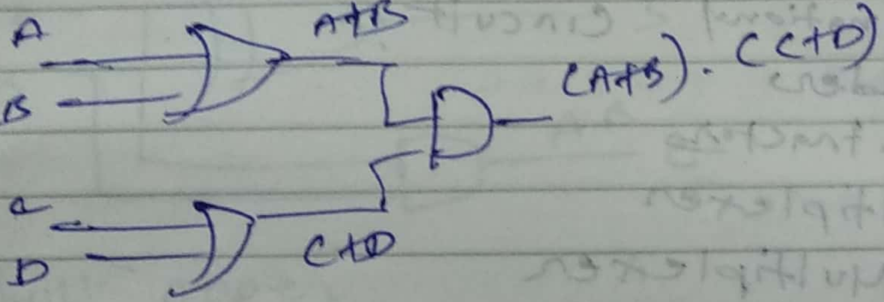
$= AB + CD$

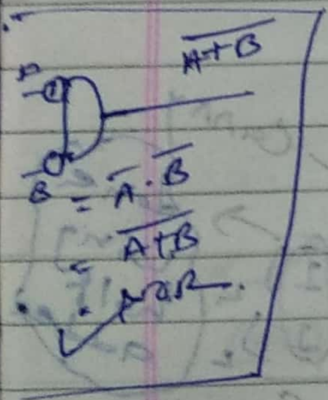
* Implementation of POS expression

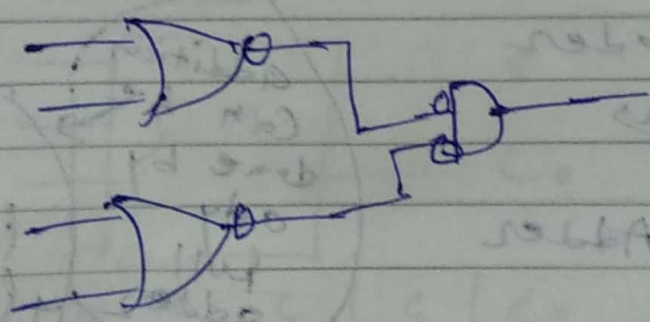
— NOR

- first draw OR – AND ckt

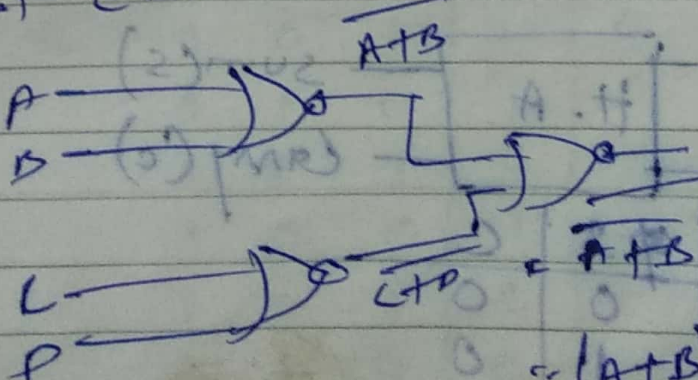
Ex $(A+B)\cdot(C+D)$ using minimum no. of NOR gate







final ckt



$= \overline{\overline{A+B}+\overline{C+D}}$

$= (A+B)\cdot(C+D)$

step-1 Make group according number 1 present in Binary Representation.

Quine-Mccluskey minimization Technique (Tabular Method)

$Y(ABCD) = \Sigma m (0, 1, 3, 7, 8, 9, 11, 15)$

binary

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
3	0	0	1	1
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
11	1	0	1	1
15	1	1	1	1

→ step 1:-

group	Minterms	Binary Rep A B C D
0	M0	0 0 0 0 ✓
1	M1	0 0 0 1 ✓
	M8	1 0 0 0 ✓
2	M3	0 0 1 1
	M9	1 0 0 1 ✓
3	M7	0 1 1 1 ✓
	M11	1 0 1 1 ✓
4	M15	1 1 1 1 ✓

→ step.2:- write match term if one(1) group minterm compare with next(2) and $\boxed{\text{different should be 1 distance only}}$

and if ~~to~~ distance is only 1 then merge that two minterm in step 2, also tick (✓) in step 1 if that minterm is in step 2.

→ Same process in step-3

આ ટીક કરવાનું છે bcoz step 2 માં એ use થયા છે merge કરવામાં so.

step 2

group	Matched pair	B.R ABCD
0	M0-M1	000- ✓
	M0-M8	-000 ✓
1	M1-M3	00-1 ✓
	M1-M9	-001 ✓
	M8-M9	100- ✓
2	M3-M7	0-11 ✓
	M3-M11	-011 ✓
	M9-M11	10-1 ✓
3	M7-M15	-111 ✓
	M11-M15	1-11 ✓

✓ bcoz they used in step-3 for merging and distance is only 1

step-3

group	m.p	B R	
0	M0-M1-M8-M9	-00-	$\bar{B}\bar{C}$
	M0-M8-M1-M9	-00-	
1	M1-M3-M9-M11	-0-1	$\bar{B}D$
	M1-M9-~~M8~~-M11 M3	-0-1	
2	M3-M7-M11-M15	--11	CD
	M3-M11-M7 M15	--11	

prime Emplicant

Question ના બધા જ
લઈ લેવાના
But don't care નહી લેવાના

P.I	Minterm involved	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0, 1, 8, 9	X	X			X	X		
$\bar{B}D$	1, 3, 9, 11		X	X			X	X	
CD	3, 7, 11, 15			X	X			X	X

minterm ની જેટલા હોય એટલા ટીક (X) કરો

Now, દરેક કોલમ ચેક કરો જેના માત્રી એક જ (X) હોય એના પર O (Round) કરો

→ So upper ના example મા જોશો 4 Round થયા હવે એક પણ Round આવે એ Row લઈ લેવાની answer ની. Like

$\bar{B}\bar{C} + CD$

Now હવે ચેક કરો એ Row ને ચેક ટીક (✓) કરો

For example $\bar{B}\bar{C}(0,1,8,9) + CD(3,7,11,15)$

So total 0, 1, 8, 9, 3, 7, 11, 15 all Column Covered

Implicant :- group of 1's
prime Implicant :- largest posible group of 1's
Essential prime Implicant :- Is prime Implicant having atleast 1 minterm can't combined in any other way.

→ So we can say that $\boxed{\bar{B}\bar{C} + CD}$ is answer, otherwise agar cover ના થાત તો cover thay એ row લેવી પડત.

→ Last table ni 3 જ entry છે because step 3 ni 3 જ group છે. AND step-1 & step 2 ni બધા જ minterm ટીક (✓) કરેલા છે એટલે.

(In case agar koi step-2 and step-1 ni ટીક ના પડેલા હોત તો તેને Last વાળા table મા ADD કરવા પડત)

For verification only

→ To check whether answer is correct or not, try with K-map

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	1 (3)	(2)
$\bar{A}B$	(4)	(5)	1 (7)	(6)
AB	(12)	(13)	1 (15)	(14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	(10)

→ $\boxed{CD + \bar{B}\bar{C}}$

ex $f(a,b,c,d) = \sum(0,5,8,9,10,11,14,15)$

* $f(a,b,c,d) = \sum m(0,1,5,9,4,2,15) + d(13,6)$

0	0000
1	0001
2	0010
4	0100
5	0101
6	0110
9	1001
13	1101
15	1111

step-1

group	minterm	variable A B C D
0	0 ✓	0 0 0 0
1	1 ✓	0 0 0 1
	2 ✓	0 0 1 0
	4 ✓	0 1 0 0
2	5 ✓	0 1 0 1
	6 ✓	0 1 1 0
	9 ✓	1 0 0 1
3	13 ✓	1 1 0 1
4	15 ✓	1 1 1 1

step-2

group	matched pair	variable A B C D
0	0,1 ✓	0 0 0 -
	0,2 ✓	0 0 - 0
	0,4 ✓	0 - 0 0
1	1,5 ✓	0 - 0 1
	1,9 ✓	- 0 0 1
	2,6 ✓	0 - 1 0
	4,5 ✓	0 1 0 -
	4,6 ✓	0 1 - 0
2	5,13 ✓	- 1 0 1
	9,13 ✓	1 - 0 1
3	13,15	1 1 - 1

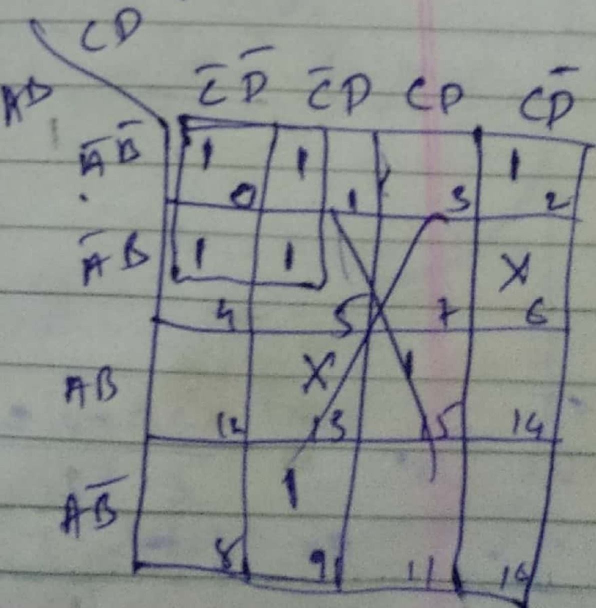
Step-3

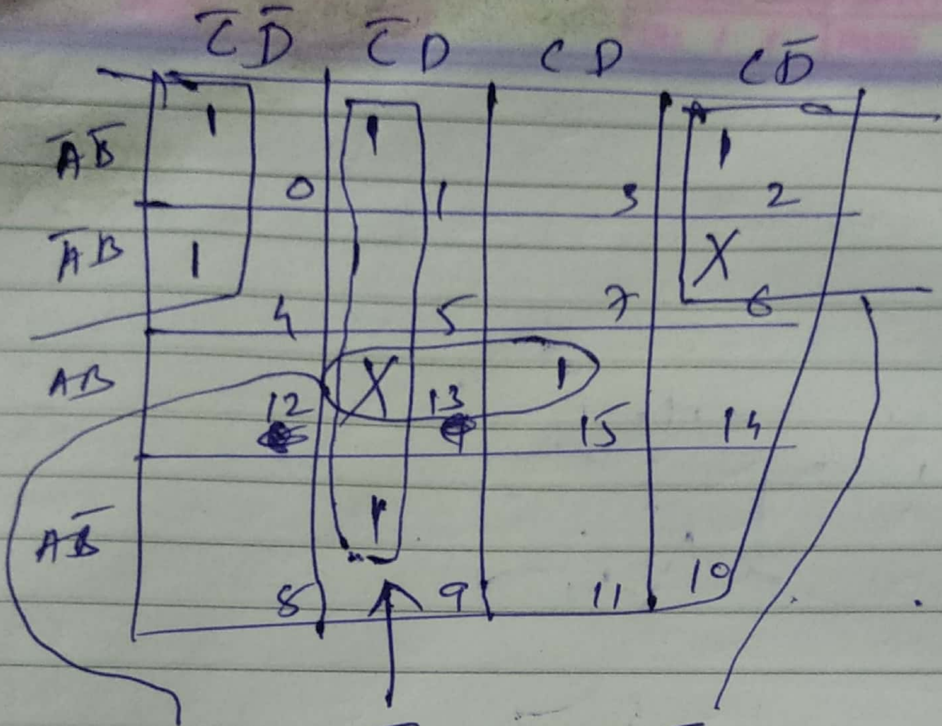
	group	matched pair	variable ABCD
$\bar{A}\bar{C}$	0	0,1,4,5	0 – 0 –
$\bar{A}\bar{D}$		0,2,4,6	0 – – 0
$\bar{A}\bar{C}$		0,4,1,5	0 – 0 –
$\bar{A}\bar{D}$		0,4,2,6	0 – – 0
$\bar{C}D$	1	1,5,9,13	– – 0 1
		~~2,6,9~~,13	–
$\bar{C}D$		1,9,5,13	– – 0 1

	0	1	2	4	5	9	15
$\bar{A}\bar{C}$ (0,1,4,5)	X	X		X	X		
$\bar{A}\bar{D}$ (0,2,4,6)	X		X	X			
$\bar{C}$ ~~B~~D (1,5,9,13)		X			X	X	
ABD (13,15)							X

$\bar{A}\bar{D} + \bar{C}\overset{D}{\cancel{B}} + ABD$

(0,2,4,6,1,5,9,13,15)





$AB\bar{D} + \bar{C}D + \bar{A}\bar{D}$

$F(A,B,C,D,E) = \sum m(0,1,2,3,8,9,16,17,20,21,24,25,28,29,30,31)$

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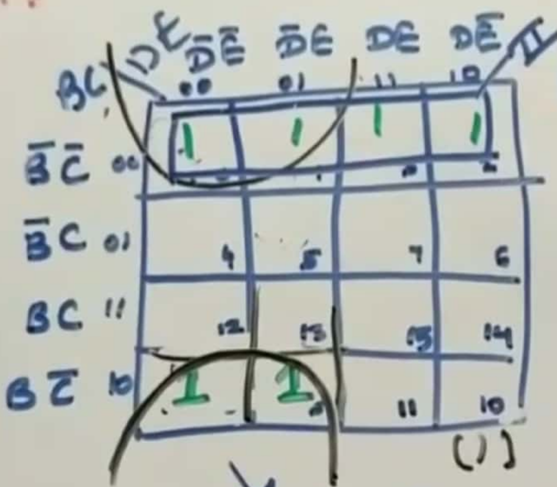
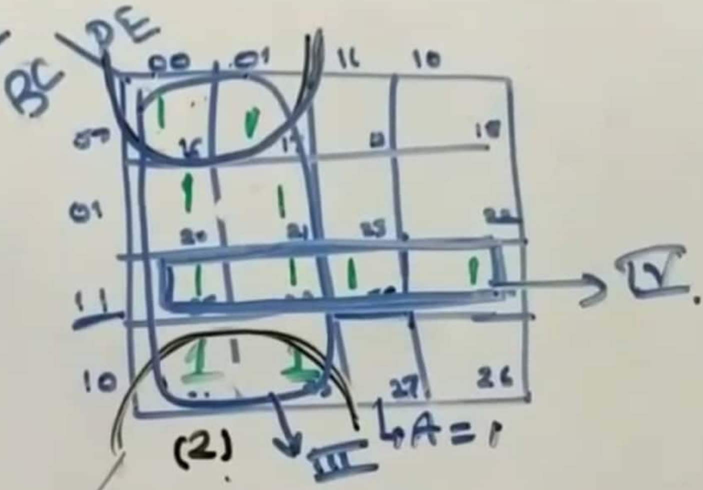


$A = 0$

$(\bar A)$



optn (a)

$A\bar D + \overline{(C+D)} + \overline{\bar A\bar B\bar C} + (\overline{\bar A + \bar B + \bar C})$

$F = \mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV}$

$F = \bar C\bar D + \bar A\bar B\bar C + A\bar D + ABC$

$\rightarrow$ $\boxed{\overline{A+B} = \bar A\cdot\bar B \quad \overline{A\cdot B} = \bar A + \bar B}$

$\bar C\bar D$

$\bar{\bar A}\,\bar{\bar B}\,\bar{\bar C} \rightarrow ABC$

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