Lecture - 1 Digital circuit

- Number systems
- Logic gates
- Logic family
- Combinational & sequential circuit

→ Number system :-

1) Binary No. system
base - 2 or B

digits - 0, 1 ex. $(101)_B$ or $(101)_2$

2) Octal No. system

base - 8 or O
digits - 0, 1, 2 ... 7
ex. $(345)_8$

3) Decimal
base - 10 or D
digits - 0, 1, ..., 9
ex. $(189)_{10}$

4) Hexadecimal No. system

base - 16 or H

digits - 0 ... 9

Symbol A B C D E F

10 11 12 13 14 15

5) Base - 4

Digits 0, 1, 2, 3

ex. $(12)_4$

0, 1, 2, 3

10, 11, 12, 13

20, 21, 22, 23

In Hexadecimal number system

0, 1, 2 ... 9, A, B, C, D, E, F, 10, 11, 12,

19, 1A, 1B ... 1F, 20, 21, 22, ... 29,

2A, 2B, ... 2F

Base - 4 0, 1, 2, 3

Base - 10 0 ... 9

Base - 8 0 ... 7

Note: largest number in any number system is 1 less than its base.

Conversion:-

(1) Decimal to Binary:

Dividend

$(25)_{10} = (\quad)_2$

Quotient | Division | Remainder

Division	Quotient	Remainder
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

$$(25)_{10} = (11001)_2$$

(2)

$(0.625)_{10} = (\quad)_2$

$0.625 \times 2 = 1.25 \rightarrow 1$

$0.25 \times 2 = 0.5 \rightarrow 0$

$0.5 \times 2 = 1.0 \rightarrow 1$

$$(0.625)_{10} = (101)_2$$

$$(25.625)_{10} = (11001.101)_2$$

* Octal to binary :-

~~$(73)_8 = (\quad)_2$~~

* Decimal - octal.

$$(73)_{10} - (\quad)_8$$

8	73	
8	9	1
8	1	1
	0	1

$$(73)_{10} = (111)_8$$

* $(73)_{10} - (\quad)_{16}$

16	73	
16	4	9
	0	4

$$(73)_{10} = (49)_{16} = (111)_8$$

Note :- If base is greter, than number will be smaller, If base is lesser then number will be bigger.

Binary to Decimal.

MSB $\rightarrow$ $(1001)_2$ $\leftarrow$ LSB

$(1001)_2 = (\quad)_{10}$

$$\begin{array}{cccc} (1 & 0 & 0 & 1)_2 \\ 3 & 2 & 1 & 0 = n \end{array}$$

$= 1 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3$

$= 1 + 0 + 0 + 8$

$(1001)_2 = (9)_{10}$

→ binary to decimal:

$(10.101)_2 = (\quad)_{10}$

$$\begin{array}{cccccc} (1 & 0 & . & 1 & 0 & 1)_2 \\ 1 & 0 = n & & -1 & -2 & -3 \end{array}$$

$= 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 0 + 2 + 0.5 + 0 + 0.125$

$(10.101)_2 = (2.625)_{10}$

* Binary to octal:-

$(1101010)_2 = (\quad)_8$

$$\underbrace{001}_{1}\ \underbrace{101}_{5}\ \underbrace{010}_{2} = (152)_8$$

Make group of 3 bit for octal (bcoz using 3 bit you can represent any number in octal)

Binary to hexadecimal

$(10101011)_2 = (\quad)_{16}$

Make group of 4 bit for hexadecimal (bcoz using 4 bit you can represent any number in hexadecimal number system

$$\underbrace{1010}_{A}\ \underbrace{1011}_{B} = (AB)_{16}$$

Convert $(4F)_{16} = (\ldots)_8$

$$\begin{matrix} 4 & F \end{matrix}$$

$= (0100\ \ 1111)_2$

$$= \underbrace{001}_{1}\ \underbrace{001}_{1}\ \underbrace{111}_{7} = (117)_8$$

$(6A)_{16} = (\quad)_{10}$

$(6 \quad A)_{16}$

$= A \times 16^0 + 6 \times 16^1$

$= 10 + 96 = (106)_{10}$

* Gate Question

Q The octal equivalent of $(AB.CD)_{16}$

$(A \quad B \; . \; C \quad D)_{16}$

$(1010 \quad 1011 \; . \; 1100 \quad 1101)_2$

$\underbrace{010}_{2} \; \underbrace{101}_{5} \; \underbrace{011}_{3} \; . \; \underbrace{110}_{6} \; \underbrace{011}_{3} \; \underbrace{010}_{2}$

$(253.632)_8$

Q The binary representation of decimal number (1.375) is

$(1)_{10} = (\ )_2$

$$\begin{array}{c|c|l} 2 & 1 & \\ \hline & 0 & 1 \end{array}$$

$= (1)_2$

$(0.375)_{10} = (\ )_2$

$0.375 \times 2 = 0.75 \rightarrow 0$

$0.75 \times 2 = 1.5 \rightarrow 1$

$0.5 \times 2 = 1.0 \rightarrow 1$

$(0.375)_{10} = (.011)_2$

$(1.375)_{10} = (1.011)_2$

Note = Me double verifying

$(1.011)_2 = (\ )_{10}$

$(1.011)_2 = 1 \times 2^{0} + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$

0 (1) -1 -2 -3

$= 1 + 0 + 0.25 + 0.125$

$= 1 + 0.375 = (1.375)_{10}$

Arithmatic operation

ADDITION

$(23)_{10}$
$+(55)_{10}$
78

answer

if $8 < \underset{base}{10}$ = Valid decimal number

No Carry.

$$\begin{array}{r} 1\,1 \\ 68 \\ +\;46 \\ \hline 114 \end{array} \quad 14$$

$14 > \underset{base}{10}$ It's not

single digit valid decimal

Number so it written as

$$14 = \underset{Base}{10} \times \underset{Carry}{1} + \underset{Result}{4}$$

Base $\times$ Carry + Result

$11 = 10 \times 1 + 1$

addition in Binary

$$\begin{array}{r} \overset{1}{(1\ 0\ 1)_2} \\ +\ (1\ 0\ 0)_2 \\ \hline (1\ 0\ 0\ 1)_2 \end{array}$$

2 < base (2) ✗

2 is not valid binary ~~num~~ digit

$$2 = \underset{\text{Base}}{2} \times \underset{\text{carry}}{1} + \underset{\text{Result}}{0}$$

* Octal

$$\begin{array}{r} \overset{1}{6}\ 7 \\ +\ \ \ 2 \\ \hline 7\ 1 \end{array}$$

$7 + 2 = 9$ ✗ valid

$9 = 8 \times 1 + 1$

* Hexadecimal

$$\begin{array}{r} 2\ \overset{1}{F}\ 1 \\ +\ A\ 3\ A \\ \hline D\ 2\ B \end{array}$$

$F + 3 = 15 + 3 = 18$ ✗ valid

$18 = 16 \times 1 + 2$

Ex. ADD $(63)_7 + (56)_8$

Base different → So we can't direct add them,

So we have to convert both into same number system then only we can add them.

[illegible] $(56)_8 = (\quad)_{10} = (\quad)_7$

$(56)_8$

$6 \times 8^0 + 5 \times 8^1$

$= 6 + 40$

$= (46)_{10}$

7	46	4
7	6	4
	0	6

$= (64)_7$

$$\boxed{\begin{array}{r} {}^{1}\,{}^{1} \\ (64)_7 \\ +\ (63)_7 \\ \hline 1\,6\,0 \end{array}}$$

$7 = 7 \times 1 + 0$

$13 = 7 \times 1 + 6$

$\boxed{\text{ans}} = (160)_7$

Subtraction :-

$$\begin{array}{r} 5\ 6 \\ -\ 2\ 3 \\ \hline 3\ 3 \end{array} \qquad \begin{array}{r} \scriptstyle Base \\ \scriptstyle 10+2 \\ 5\ \cancel{2} \\ -\ 2\ 3 \\ \hline 2\ 9 \end{array} \qquad \begin{array}{r} \scriptstyle Base \\ \scriptstyle 10+0 \\ 6\ 0 \\ -\ 4\ 5 \\ \hline 1\ 5 \end{array}$$

Binary

$$\begin{array}{r} \scriptstyle Base \\ \scriptstyle 2+0 \\ 1\ 1\ \cancel{0} \\ -\ 1\ 0\ 1 \\ \hline 0\ 0\ 1 \end{array}$$

Octal :-

$$\begin{array}{r} \scriptstyle 8+2 \\ 6\ \cancel{2} \\ -\ 4\ 5 \\ \hline 1\ 5 \end{array} \qquad \begin{array}{r} \scriptstyle 8 \\ 5\ \cancel{0} \\ -\ 2\ 5 \\ \hline 3\ 3 \end{array}$$

Hexadecimal :-

$$\begin{array}{r} \scriptstyle 16+A \\ 2\ A \\ 1\ F \\ 1\ \\ \hline 0\ B \end{array}$$

Q $(15)_7 - (26)_7 + (53)_7$ Wrong explaination

Lecture:-12

* Multiplication No system

$$\begin{array}{r} 2\,1\,1 \\ *\ \ 6 \\ \hline 6\,6 \end{array}$$

$$\begin{array}{r} 5 \\ *\,36 \\ \hline 30 \\ +\,150 \\ \hline 180 \end{array}$$

* Binary:-

$$\begin{array}{r} 101 \\ *\ \ 11 \\ \hline 101 \\ 1010 \\ \hline 1111 \end{array}$$

* octal :-

$$\begin{array}{r} 74 \\ *\ 21 \\ \hline 74 \\ 170\times \\ \hline 1774 \end{array}$$

$8 = 8 \times 1 + 0$

$15 = 8 \times 1 + 7$

* Hexadecimal :-

AB $16 \times 1 + 6$

* 22 $16 \times 1 + 5$

156

156 ~~X~~

1 6 B 6

* Complements :-

base's Complement   (base-1)'s Complement

ex.

binary   2's   1's

Decimal   10's   9's

How to find complement

(1) (base-1)'s complement

subtracting each digit from the largest digit in the number system.

(base-1)'s Complement.

ex $(405)_8 \Rightarrow$ 7's Complement

$$\begin{array}{r} 777 \\ 405 \\ \hline 372 \end{array}$$

2) base's Complement:-
by Simply adding 1 to the result of (base-1)'s complement

$(405)_8$ = 8's complement

= 7's Complement + 1

$$= \begin{array}{r} 372 \\ +\ \ 1 \\ \hline 373 \end{array}$$

* Calculate 3's complement for $(301)_4$
(~~base~~-1)'s Complement
~~4's Complement~~
3's Complement

$$\begin{array}{r} 333 \\ 301 \\ \hline 032 \end{array}$$

~~3's Complement:-~~

* Calculate 2's complement for $(1001)_2$

1's complement

1 1 1 1
1 0 0 1
0 1 1 0

0 1 1 0
\+ 1
2's complement 0 1 1 1

1 0 0 1
↓ ↓ ↓
1 0 ← 2's complement

Direct 1's complement & 2's complement

1 0 0 1 ⇒ Reverse all bit
1's complement → 0 1 1 0

1 0 0 1 ⇒
LSB

start from LSB
Look for first "1"
keep that "1" as it is, after first "1" reverse All bit

2's complement
first
1 0 1 0 ←
↓ ↓ ↓ ↓
0 1 1 0

ex

	1	0	0	1
2's Complement →	0	1	1	1

1001

0110

Number Representation

- Magnitude
  - Unsigned Magnitude
  - Signed Magnitude
- Complement
  - 1's Complement
  - 2's Complement

Unsigned Magnitude	Signed Magnitude	1's Complement	2's Complement
→ we can represent only +ve number	→ Both +ve & -ve	→ Both +ve & -ve	→ Both +ve & -ve
→ +6 = 110	→ +6 ~~0110~~ = '0'110 ↓ +ve	→ +6 '0'110	→ +6 = '0' 110

representation of positive Number are same in all Case

Unsigned Magnitude	Signed Magnitude	1's Complement	2's Complement
-6 X	→ +6 '1' 110	→ -6 '0' 110 → +6 ↓ 1's complement 1 001 → -6 ↓ 1's complement 0 110 → 6 is Magnitude -ve This is representation of -6 in 1's complement form	→ -6 '0' 110 ⇒ +6 ↓ 2's complement 1 010 ⇒ -6 ↓ 2's complement 0 110 → 6 is Magnitude -ve This is representation of -6 in 2's Complement
	+6 in 6bit '0' 00110	+6 in 6 bit '0' 00110	+6 in 6 bit
	-6 in 6bit ~~111110~~ '1' 00 110	-6 in 6 bit ~~111001~~ 000 110 ⇒ +6 ↓ 1's Complement 111001 ⇒ -6 or Copy MSB of -6 Representation of 1's Complement 11 '1' 001	-6 in 6 bit 000110 → +6 ↓ 2's complement 111010 → -6 or Copy MSB of -6 Representation of 2's Complement 11 '1' 010

Singed Magnitude	Decimal
0 0 0 0	+0
0 0 0 1	+1
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	−0
1 0 0 1	−1
1 1 1 0	−6
1 1 1 1	−7

disadvantage:-

→ 2 different representation for 0

→ Range

-7 to $+7$ 4 bit

$-(2^{4-1}-1)$ to $+(2^{4-1}-1)$

-7 to $+7$

n bit $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

1's Complement	Decimal
0 0 0 0	+0
0 0 0 1	+1
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	−7
1 0 0 1	−6
1 1 1 0	−1
1 1 1 1	−0

disadvantage:

→ 2 different representation for 0

→ Range

-7 to +7   4 bit

$-(2^{4-1}-1)$ to $+(2^{4-1}-1)$

-7 to +7

n bit $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

2's complement	Decimal
0 0 0 0	+0
0 0 0 1	+1
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	-8
1 0 0 1	-7
1 1 1 0	-2
1 1 1 1	-1

-8 to +7   4 bit

$-(2^{4-1})$ to $+(2^{4-1}-1)$

n bit $-(2^{n-1})$ to $+(2^{n-1}-1)$

* Arithmatic operation using Complement

Q. 5 - 4 = 1

1's complement

+5 ⇒ 0101

+4 ⇒ 0100

↓ 1's

-4 ⇒ 1011

$$\begin{array}{r} 0101 \\ + 1011 \\ \hline \boxed{1}\,0000 \\ \quad\; 1 \\ \hline \boxed{0001} \end{array}$$

5

4

+1

This is in 1's complement form

2's Complement :

Q 5 - 4

+5 = 0101

+4 = 0100

↓ 2's

-4 = 1100

$$\begin{array}{r} 0101 \\ + 1100 \\ \hline \boxed{1}\,\boxed{0001} \end{array}$$

5

4

+1

X discard it

This result is in 2's complement form

Gate X = 01110 & Y = 11001 are 2 5 bit binary number represented in 2's Complement form. The sum of X & Y represented in 2's Complement format using 6 bit is

(a) 100111 (b) 001000

(c) 000111 (d) 101001

X = 0 1 1 1 0 = +14
Y = 1 1 0 0 1 = -7
↓ 2's
0 0 1 1 1 = 7

14
- 7
+ 7

in 6 bit 2's complement form
1 0 0 0 1 1

Lecture :- 3

Codes

↓ Weighted Code

weights
8 4 2 1
1 0 1 0

→ BCD

↓ NON - weighted Code

→ Excess - 3
→ gray Code

BCD is always 4 bit

BCD

Decimal	BCD B_3	B_2	B_1	B_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10 ⋮ 15	Invalid BCD Code			

to represent greater number

$(23)_{10} \rightarrow (0010\ \ 0011)_{BCD}$

Excess -3 Code

Excess - 3 = BCD + 3

Decimal	B_3	B_2	B_1	B_0	E_3	E_2	E_1	E_0
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

it's also called self complementing code.

→ gray code :- (Cyclic code or unit distance code)

	B_3	B_2	B_1	B_0	G_3	G_2	G_1	G_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	1	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	1	0	0
9	1	0	0	1	1	1	0	1
10	1	0	1	0	1	1	1	1
11	1	0	1	1	1	1	1	0
12	1	1	0	0	1	0	1	0
13	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	0

BCD Addition

(a)

23	0010	0011
+16	0001	0110
39	0011	1001
	↓	↓
	3	9

(b)

66 ⇒	0110	0110
+55	0101	0101
121	1011	1011
	11	11

Both Invalid BCD Code

→ to make it valid add $\boxed{0110 = +6}$ to each Invalid BCD Number

	1 1 1		
	1011		1011
	0110		0110
0001	0010		0001
↓	↓		↓
1	2		1

Convert binary to gray code

$$\begin{array}{cccccccc} & \oplus & & \oplus & & \oplus & \\ 1 & & 0 & & 1 & & 1 \\ \downarrow & & & & & & \\ 1 & & 1 & & 1 & & 0 \end{array}$$

$(1011)_2 = (1110)_G$

$$\begin{array}{cccccccc} & \oplus & & \oplus & & \oplus & \\ 1 & & 1 & & 1 & & 1 \\ \downarrow & & & & & & \\ 1 & & 0 & & 0 & & 0 \end{array}$$

$(1111)_2 = (1000)_G$

First ~~bit~~ MSB as it is
then ~~two~~ do x-or operation of 1st
and 2nd bit (from MSB side)
result will be your 2nd bit
of gray code .. same process

$$\begin{array}{cccccccc} & \oplus & & \oplus & & \oplus & \\ 0 & & 1 & & 0 & & 0 \\ \downarrow & & & & & & \\ 0 & & 1 & & 1 & & 0 \end{array}$$

$(0100)_2 = (0110)_G$