# Algorithms: K Nearest Neighbors

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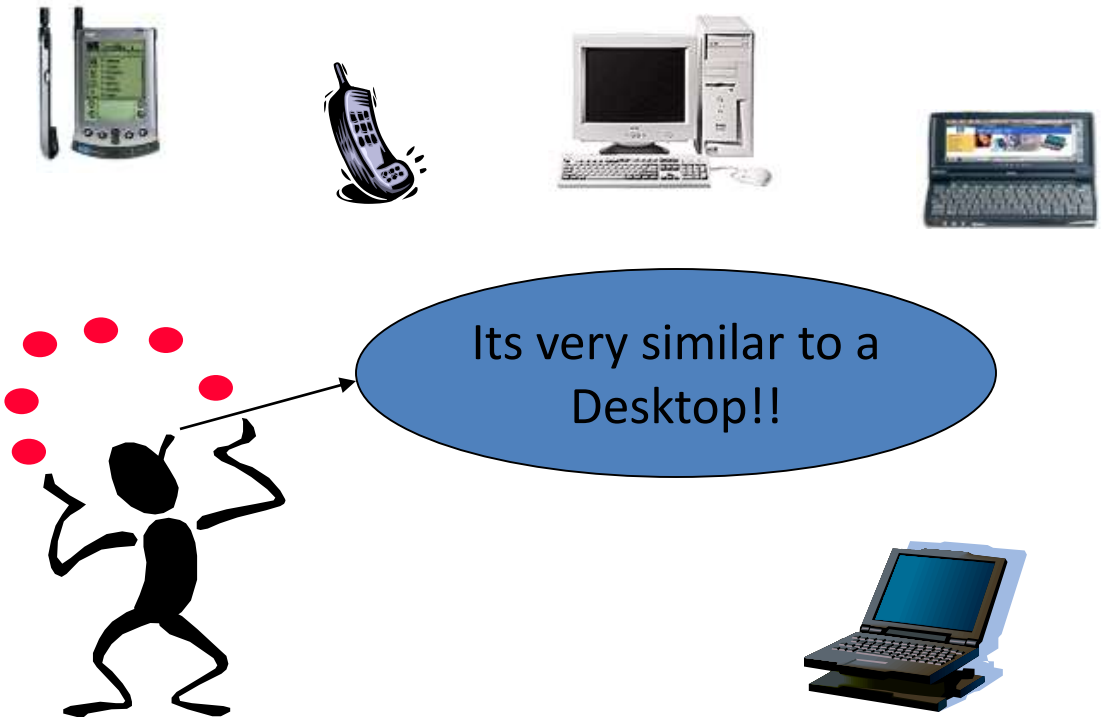
# Algorithms: K Nearest Neighbors

# Simple Analogy..

- Tell me about your friends(*who your neighbors are*) and *I will tell you who you are*.

# Instance-based Learning



# KNN – Different names

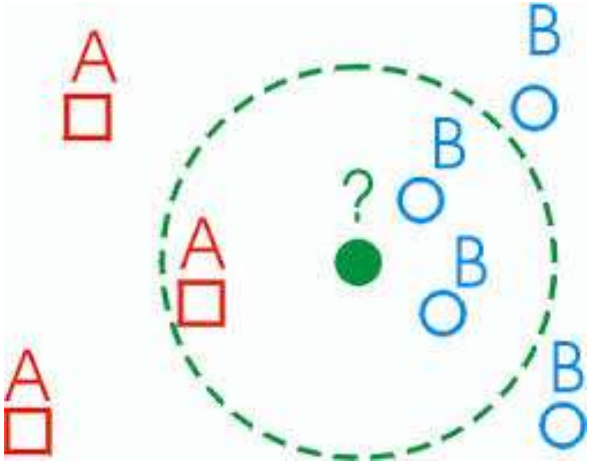
- K-Nearest Neighbors
- Memory-Based Reasoning
- Example-Based Reasoning
- Instance-Based Learning
- Lazy Learning

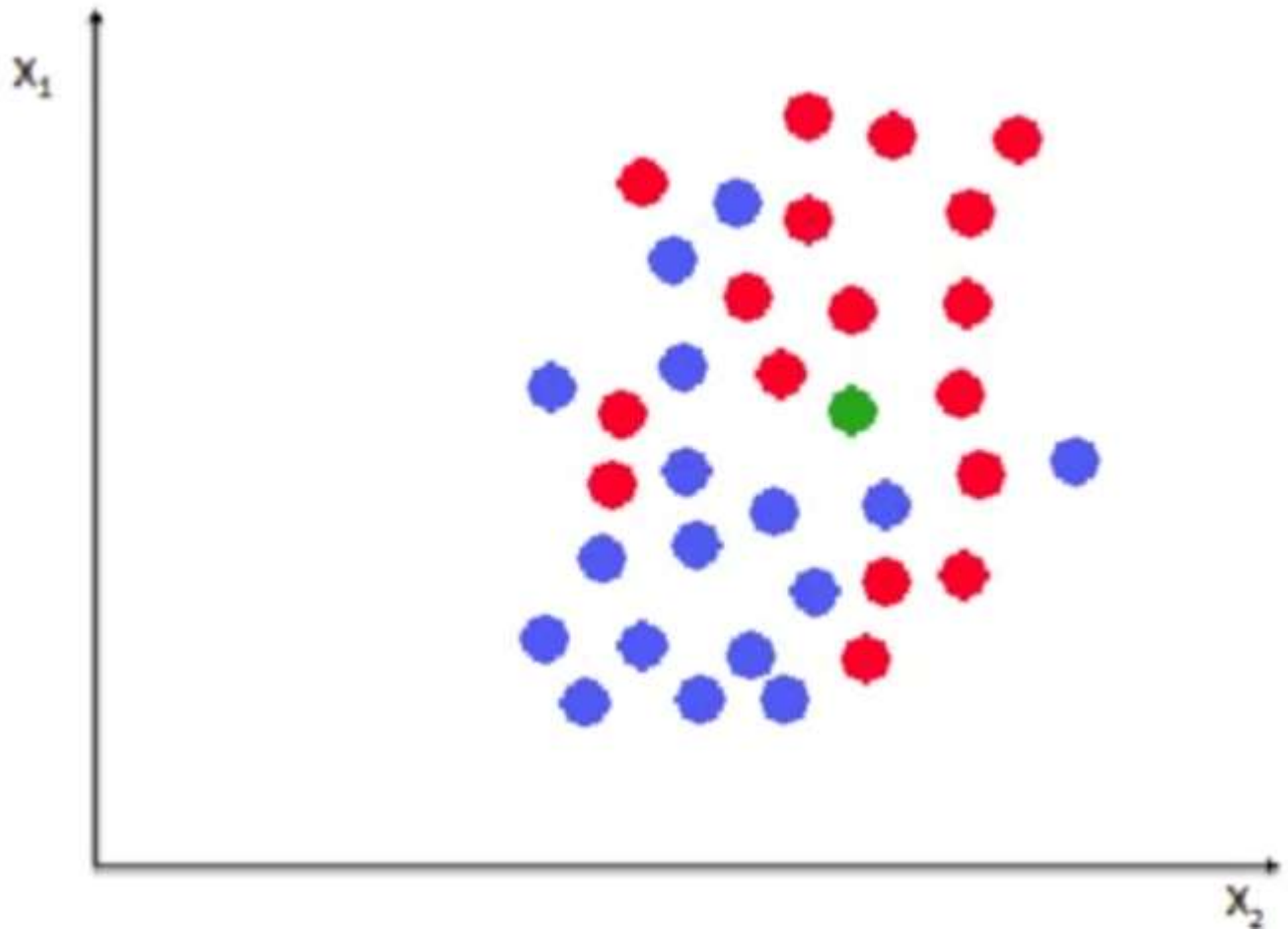
# What is KNN?

- A powerful classification algorithm used in pattern recognition.
- K nearest neighbors stores all available cases and classifies new cases based on a *similarity measure*(e.g **distance function**)
- One of the top data mining algorithms used today.
- A non-parametric lazy learning algorithm (An Instance-based Learning method).

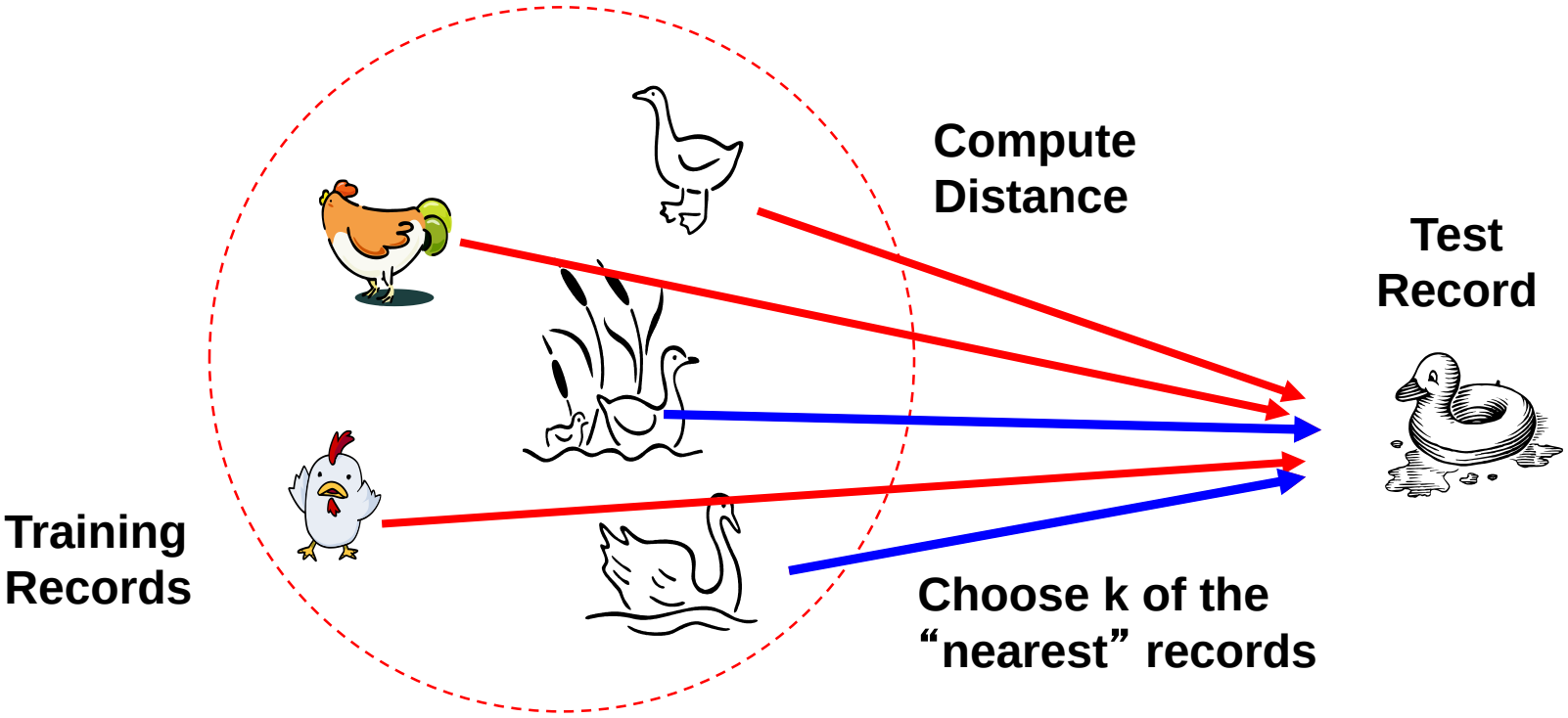
# KNN: Classification Approach

- An object (a new instance) is classified by a majority votes for its neighbor classes.
- The object is assigned to the most common class amongst its K nearest neighbors.(*measured by a distant function* )




$X_1$
$X_2$

# Distance Measure



# Distance measure for Continuous Variables

**Distance functions**

Euclidean $\sqrt{\sum_{i=1}^{k}(x_i - y_i)^2}$

Manhattan $\sum_{i=1}^{k}|x_i - y_i|$

Minkowski $\left(\sum_{i=1}^{k}(|x_i - y_i|)^q\right)^{1/q}$

# Distance Between Neighbors

- Calculate the distance between new example (E) and all examples in the training set.

- *Euclidean* distance between two examples.
  - $X = [x_1, x_2, x_3, .., x_n]$
  - $Y = [y_1, y_2, y_3, ..., y_n]$

  - The Euclidean distance between *X* and *Y* is defined as:

$$D(X,Y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

# K-Nearest Neighbor Algorithm

- All the instances correspond to points in an n-dimensional feature space.
- Each instance is represented with a set of numerical attributes.
- Each of the training data consists of a set of vectors and a class label associated with each vector.
- Classification is done by comparing feature vectors of different K nearest points.
- Select the K-nearest examples to E in the training set.
- Assign E to the most common class among its K-nearest neighbors.

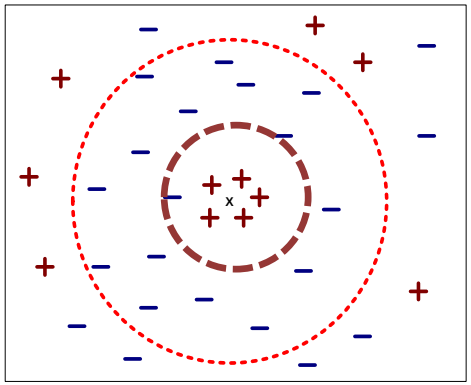
# 3-KNN: Example(1)

| Customer | Age | Income | No. credit cards | Class |
|---|---|---|---|---|
| George | 35 | 35K | 3 | No |
| Rachel | 22 | 50K | 2 | Yes |
| Steve | 63 | 200K | 1 | No |
| Tom | 59 | 170K | 1 | No |
| Anne | 25 | 40K | 4 | Yes |
| John | 37 | 50K | 2 | YES |

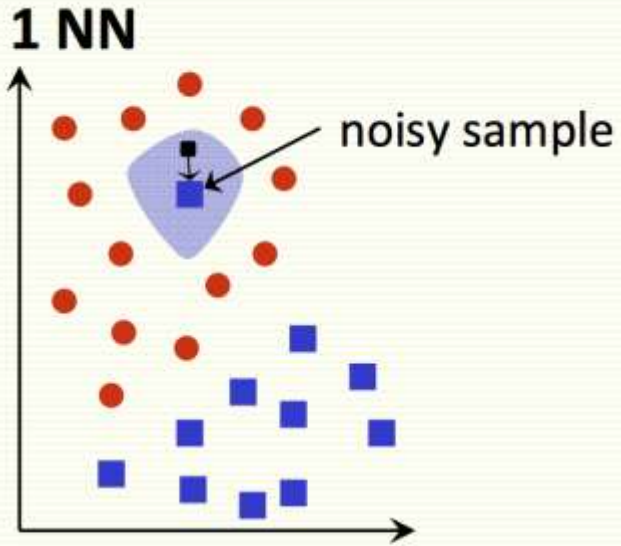
| Distance from John |
|---|
| sqrt $[(35-37)^2+(35-50)^2+(3-2)^2]=15.16$ |
| sqrt $[(22-37)^2+(50-50)^2+(2-2)^2]=15$ |
| sqrt $[(63-37)^2+(200-50)^2+(1-2)^2]=152.23$ |
| sqrt $[(59-37)^2+(170-50)^2+(1-2)^2]=122$ |
| sqrt $[(25-37)^2+(40-50)^2+(4-2)^2]=15.74$ |

# How to choose K?

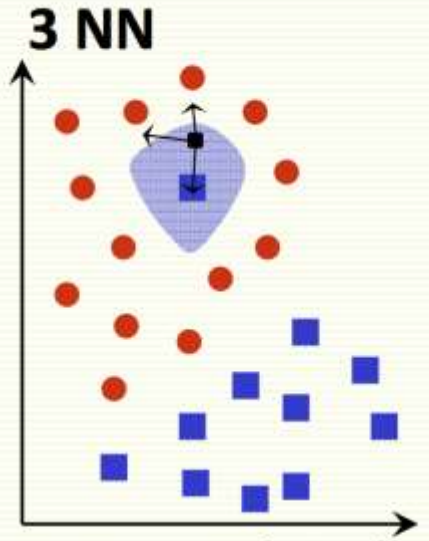
- If K is too small it is sensitive to noise points.
- Larger K works well. But too large K may include majority points from other classes.



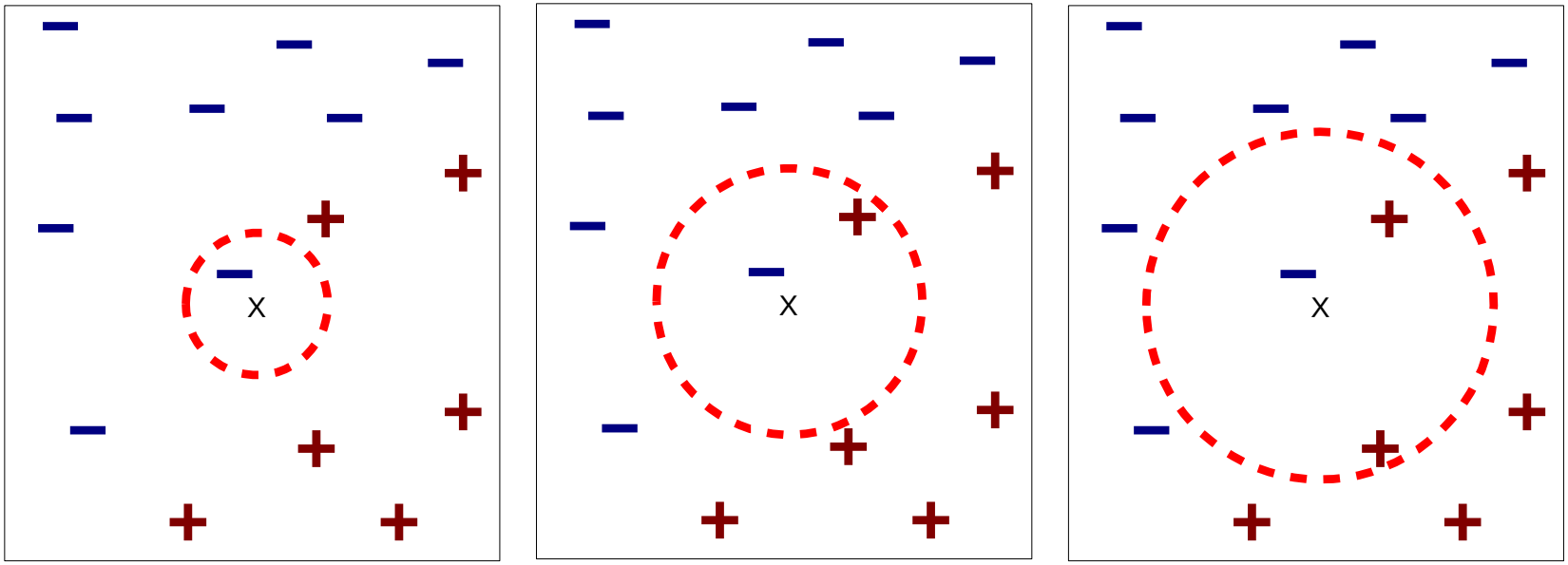
- Rule of thumb is K < sqrt(n), n is number of examples.




every example in the blue shaded area will be misclassified as the blue class




every example in the blue shaded area will be classified correctly as the red class




(a) 1-nearest neighbor (b) 2-nearest neighbor (c) 3-nearest neighbor

K-nearest neighbors of a record x are data points that have the k smallest distance to x

# KNN Feature Weighting

- Scale each feature by its importance for classification

$$D(a,b) = \sqrt{\sum_k w_k (a_k - b_k)^2}$$

- Can use our prior knowledge about which features are more important
- Can learn the weights $\mathbf{w_k}$ using **cross-validation** (to be covered later)

# Feature Normalization

- Distance between neighbors could be dominated by some attributes with relatively large numbers.
  - e.g., income of customers in our previous example.

$$a_i = \frac{v_i - \min v_i}{\max v_i - \min v_i}$$

- Arises when two features are in different scales.

- Important to normalize those features.
  - Mapping values to numbers between 0 – 1.

# Nominal/Categorical Data

- Distance works naturally with numerical attributes.

- Binary value categorical data attributes can be regarded as 1 or 0.

**Hamming Distance**

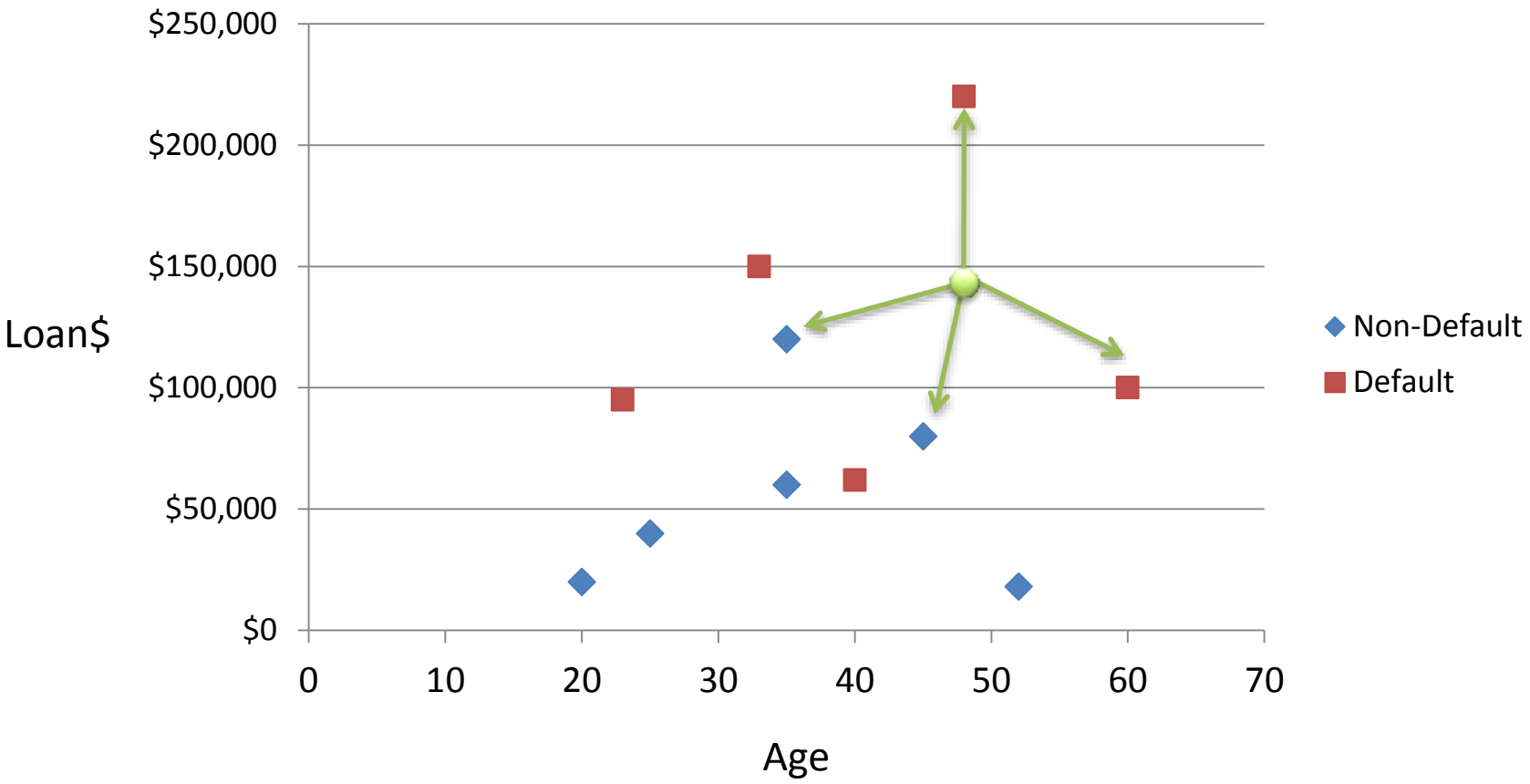
$$D_H = \sum_{i=1}^{k} |x_i - y_i|$$

$$x = y \Rightarrow D = 0$$

$$x \neq y \Rightarrow D = 1$$

| X | Y | Distance |
|---|---|---|
| Male | Male | 0 |
| Male | Female | 1 |

# KNN Classification



# KNN Classification – Distance

| Age | Loan | Default | Distance |
|---|---|---|---|
| 25 | $40,000 | N | 102000 |
| 35 | $60,000 | N | 82000 |
| 45 | $80,000 | N | 62000 |
| 20 | $20,000 | N | 122000 |
| 35 | $120,000 | N | 22000 |
| 52 | $18,000 | N | 124000 |
| 23 | $95,000 | Y | 47000 |
| 40 | $62,000 | Y | 80000 |
| 60 | $100,000 | Y | 42000 |
| 48 | $220,000 | Y | 78000 |
| 33 | $150,000 | Y | 8000 |
| | | | |
| **48** | **$142,000** | **?** | |

Euclidean Distance

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

# KNN Classification – Standardized Distance

| Age | Loan | Default | Distance |
|---|---|---|---|
| 0.125 | 0.11 | N | 0.7652 |
| 0.375 | 0.21 | N | 0.5200 |
| 0.625 | 0.31 | N | 0.3160 |
| 0 | 0.01 | N | 0.9245 |
| 0.375 | 0.50 | N | 0.3428 |
| 0.8 | 0.00 | N | 0.6220 |
| 0.075 | 0.38 | Y | 0.6669 |
| 0.5 | 0.22 | Y | 0.4437 |
| 1 | 0.41 | Y | 0.3650 |
| 0.7 | 1.00 | Y | 0.3861 |
| 0.325 | 0.65 | Y | 0.3771 |
| | | | |
| **0.7** | **0.61** | **?** | |

Standardized Variable

$$X_s = \frac{X - Min}{Max - Min}$$

## Strengths of KNN

- Very simple and intuitive.
- Can be applied to the data from any distribution.
- Good classification if the number of samples is large enough.

## Weaknesses of KNN

- Takes more time to classify a new example.
  - need to calculate and compare distance from new example to all other examples.
- Choosing k may be tricky.
- Need large number of samples for accuracy.