Simple Linear Regression

- Simple Linear Regression Model
- Least Squares Method

Simple Linear Regression

- Managerial decisions often are based on the relationship between two or more variables.
- Regression analysis can be used to develop an equation showing how the variables are related.
- The variable being predicted is called the dependent variable and is denoted by y.
- The variables being used to predict the value of the dependent variable are called the independent variables and are denoted by x.

Simple Linear Regression

- Simple linear regression involves one independent variable and one dependent variable.
- The relationship between the two variables is approximated by a straight line.
- Regression analysis involving two or more independent variables is called multiple regression.

Simple Linear Regression Model

- The equation that describes how y is related to x and an error term is called the regression model.
- The simple linear regression model is:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where:

β_0 and β_1 are called parameters of the model,

ε is a random variable called the error term.

Simple Linear Regression Equation

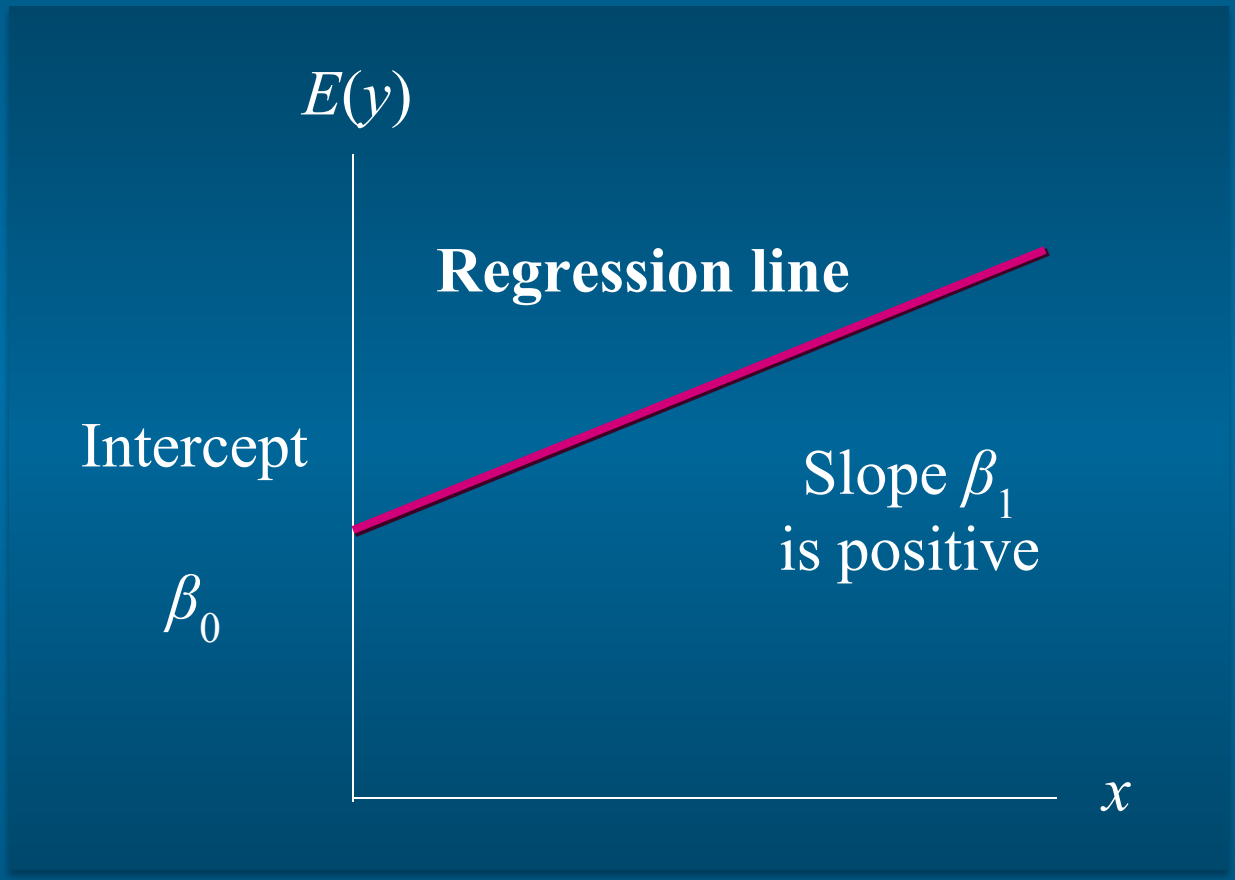
- The simple linear regression equation is:

$$E(y) = \beta_0 + \beta_1 x$$

  - Graph of the regression equation is a straight line.
  - β_0 is the y intercept of the regression line.
  - β_1 is the slope of the regression line.
  - $E(y)$ is the expected value of y for a given x value.

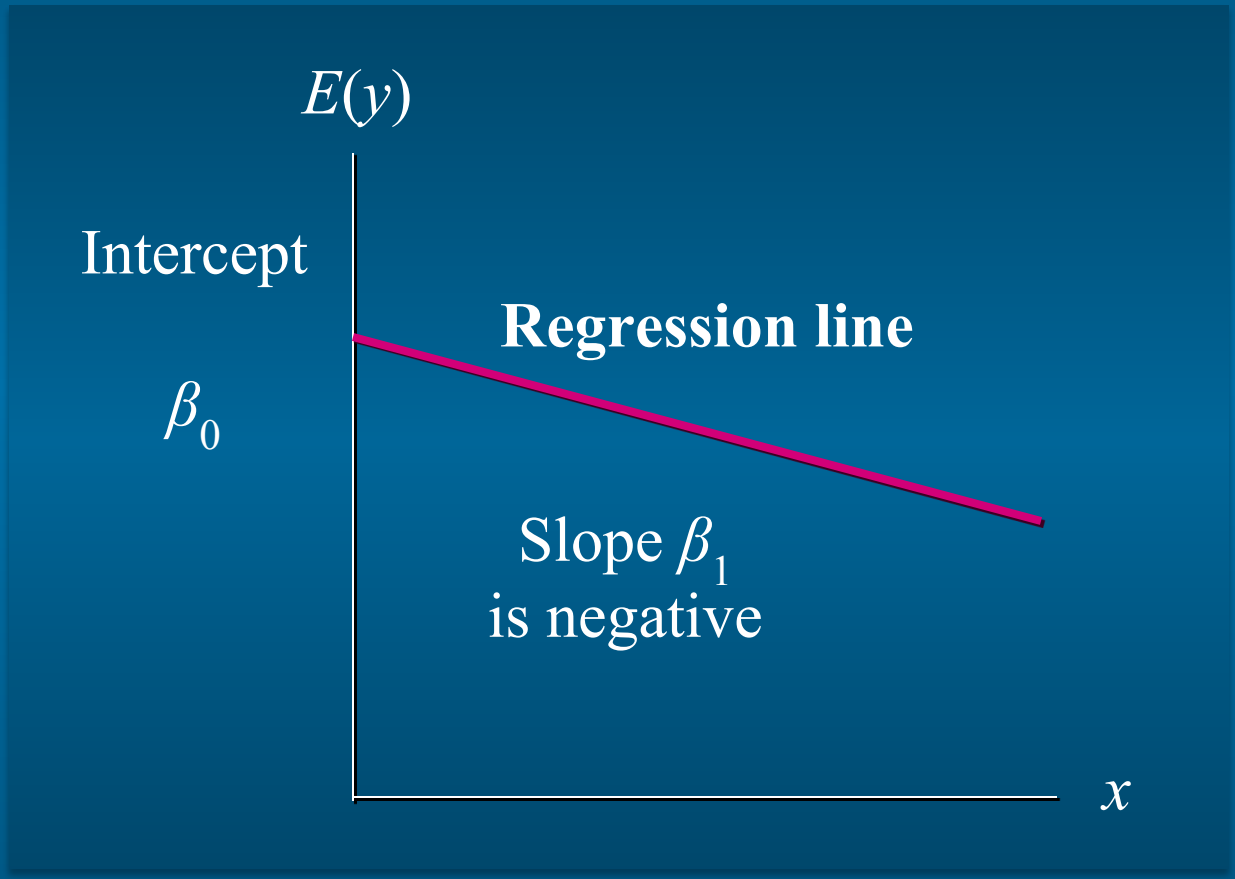
Simple Linear Regression Equation

- Positive Linear Relationship



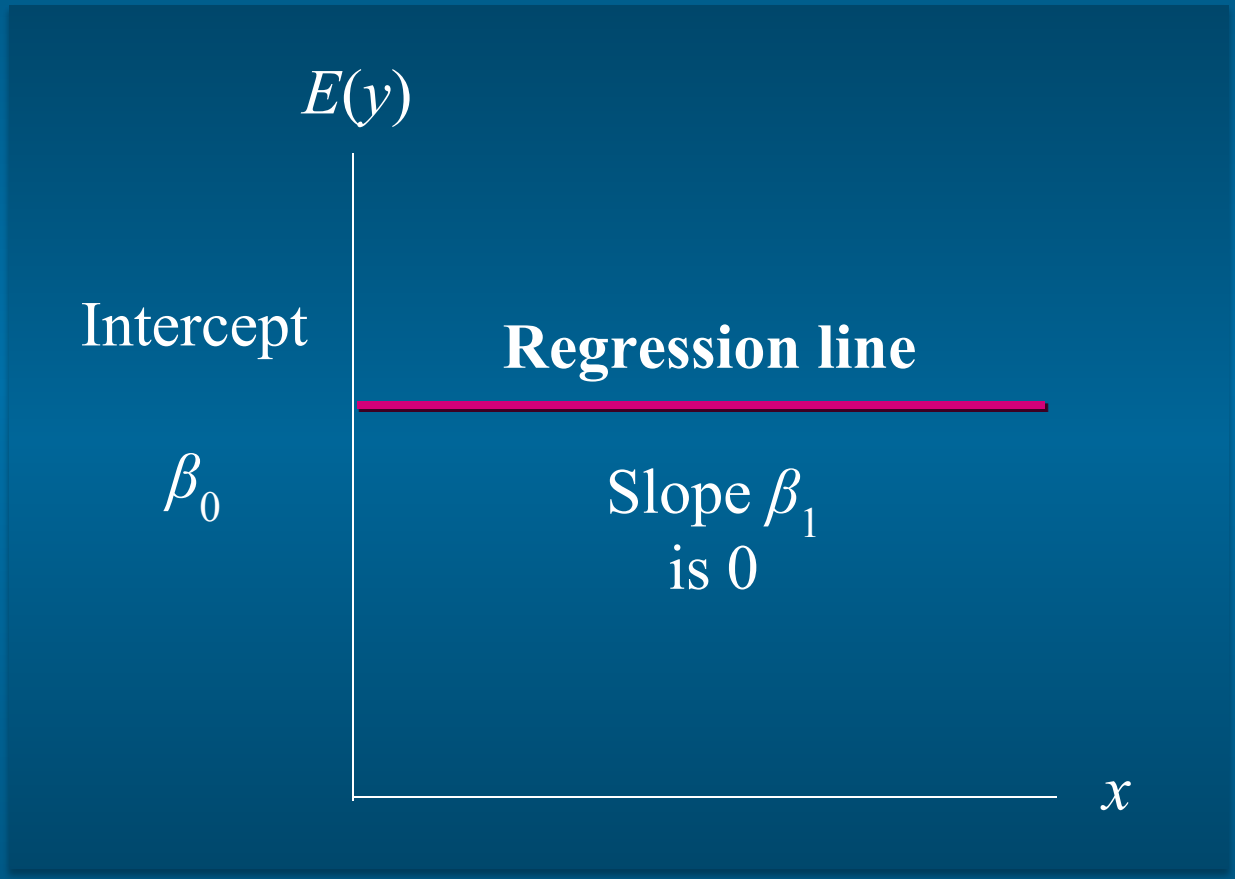
Simple Linear Regression Equation

- Negative Linear Relationship



Simple Linear Regression Equation

- No Relationship



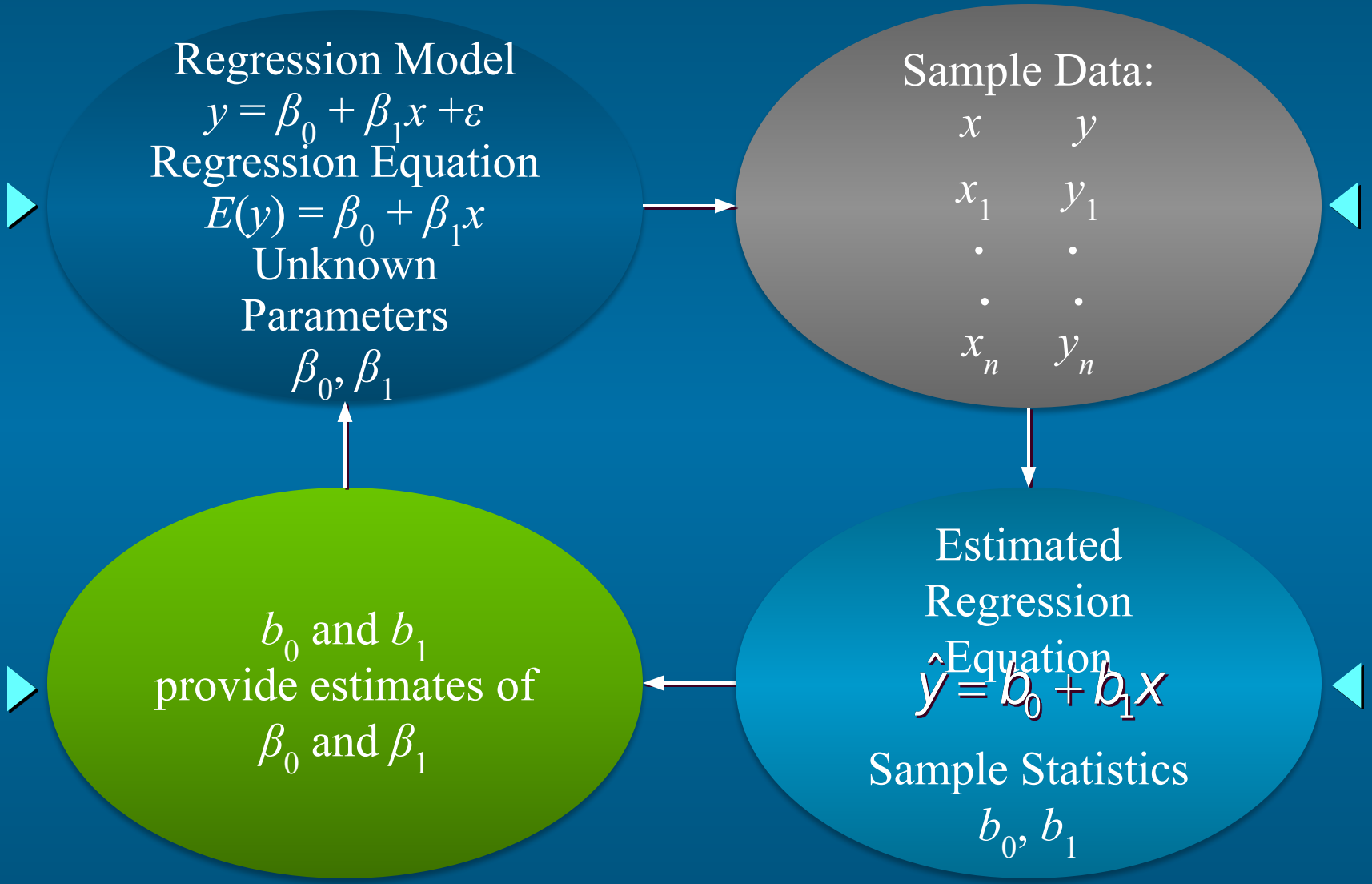
Estimated Simple Linear Regression Equation

- The estimated simple linear regression equation

$$\hat{y} = b_0 + b_1 x$$

  - The graph is called the estimated regression line.
  - b_0 is the y intercept of the line.
  - b_1 is the slope of the line.
  - $\hat{y}$ is the estimated value of y for a given x value.

Estimation Process



Least Squares Method

- Least Squares Criterion

$$\min \Sigma (y_i - \hat{y}_i)^2$$

where:

y_i = observed value of the dependent variable for the ith observation

$\hat{y}_i$ = estimated value of the dependent variable for the ith observation

Least Squares Method

- Slope for the Estimated Regression Equation

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

where:

x_i = value of independent variable for ith observation

y_i = value of dependent variable for ith observation

$\bar{x}$ = mean value for independent variable

$\bar{y}$ = mean value for dependent variable

Least Squares Method

- y-Intercept for the Estimated Regression Equation

$$b_0 = \bar{y} - b_1\bar{x}$$

Simple Linear Regression

- Example: Reed Auto Sales

Reed Auto periodically has a special week-long sale. As part of the advertising campaign Reed runs one or more television commercials during the weekend preceding the sale. Data from a sample of 5 previous sales are shown on the next slide.

Simple Linear Regression

- Example: Reed Auto Sales

Number of TV Ads (x)	Number of Cars Sold (y)
1	14
3	24
2	18
1	17
3	27
$\Sigma x = 10$	$\Sigma y = 100$
$\bar{x} = 2$	$\bar{y} = 20$

Estimated Regression Equation

- Slope for the Estimated Regression Equation

$$b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \frac{20}{4} = 5$$

- y-Intercept for the Estimated Regression Equation

$$b_0 = \bar{y} - b_1\bar{x} = 20 - 5(2) = 10$$

- Estimated Regression Equation

$$\hat{y} = 10 + 5x$$