$$\begin{cases} T(n) = 4T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + C_0.n, n \geq 1 \\ T(0) = C_1,\ C_1 > 0 \end{cases} \Longrightarrow T(n) = O(n^\alpha), \alpha = ?$$

$T(\overline{a_k \ldots a_0}) = 4T(\overline{a_k \ldots a_1}) + C_0(2^k a_k + 2^{k-1} a_{k-1} + \ldots + a_0)$

$T(\overline{a_k \ldots a_1}) = 4T(\overline{a_k \ldots a_2}) + C_0(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1)$

Nhân 2 vế cho 4:

$4T(\overline{a_k \ldots a_1}) = 4^2 T(\overline{a_k \ldots a_2}) + 4C_0(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1)$

Tiếp tục:

$4^2 T(\overline{a_k \ldots a_2}) = 4^3 T(\overline{a_k \ldots a_3}) + 4^2 C_0(2^{k-2} a_k + 2^{k-3} a_{k-1} + \ldots + a_2)$

$\vdots$

$4^{k-1} T(\overline{a_k \ldots a_{k-1}}) = 4^k T(\overline{a_k}) + 4^{k-1} C_0(2a_k + a_{k-1})$

$T(\overline{a_k \ldots a_0}) = 4^k T(\overline{a_k}) + C_0\left(2^k a_k + 2^{k-1} a_{k-1} + \ldots + a_0\right) + 4C_0\left(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1\right) + 4^2 C_0\left(2^{k-2} a_k + 2^{k-3} a_{k-1} + \ldots + a_2\right) + \cdots + 4^{k-1} C_0(2a_k + a_{k-1})$

Với: $C_0\left(2^k a_k + 2^{k-1} a_{k-1} + \cdots + a_0\right) +$
$4C_0\left(2^{k-1} a_k + 2^{k-2} a_{k-1} + \cdots + a_1\right) +$
$4^2 C_0\left(2^{k-2} a_k + 2^{k-3} a_{k-1} + \cdots + a_2\right) +$
$\ldots +$
$4^{k-1} C_0(2a_k + a_{k-1})$

$$= C_0 \sum_{i=0}^{k} a_i.\left(\sum_{j=i}^{2k-1} 2^j\right)$$

Mà:

$$\sum_{j=i}^{2k-1} 2^j = \frac{2^i\left(2^{2k-1-i+1} - 1\right)}{2-1}$$
$$= 2^i\left(2^{2k-i} - 1\right)$$
$$= 2^{2k} - 2^i$$

$$C_0 \sum_{i=0}^{k} a_i.\left(2^{2k} - 2^i\right)\ với\ k = log_2 n$$

$$= C_0 \sum_{i=0}^{log_2 n} a_i.\left(2^{2lo\ \ _2 n} - 2^i\right)$$

$$= C_0 \sum_{i=0}^{log_2 n} a_i.\left(2^{log_2 n^2} - 2^i\right)$$

$$= C_0 \sum_{i=0}^{log_2 n} a_i.\left(n^2 - 2^i\right)$$

$$= C_0 n^2 \sum_{i=0}^{log_2 n} a_i - C_0 \sum_{i=0}^{log_2 n} a_i 2^i$$

$$\mathrm{T}(\overline{\mathrm{a_k \dots a_0}}) = \ 4^{\mathrm{k}}\mathrm{T}(\overline{\mathrm{a_k}}) + C_0 n^2 \sum_{i=0}^{log_2 n} a_i - C_0 \sum_{i=0}^{log_2 n} a_i 2^i$$

$$\mathrm{T}(\overline{\mathrm{a_k \dots a_0}}) = \ 2^{2 log_2 n} C_1 + C_0 n^2 \sum_{i=0}^{log_2 n} a_i - C_0 \sum_{i=0}^{log_2 n} a_i 2^i$$

$$\mathrm{T}(\overline{\mathrm{a_k \dots a_0}}) = \ n^2 C_1 + C_0 n^2 \sum_{i=0}^{log_2 n} a_i - C_0 \sum_{i=0}^{log_2 n} a_i 2^i$$

$$\mathrm{T}(\overline{\mathrm{a_k \dots a_0}}) = \ n^2 (C_1 + C_0 \sum_{i=0}^{log_2 n} a_i) - C_0 \sum_{i=0}^{log_2 n} a_i 2^i$$

$$\mathrm{T(n)} = O(n^2)$$

$$\begin{cases} T(n) = 3T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + C_2.n, n \geq 1 \\ T(0) = C_1;\ C_1,\ C_2 > 0 \end{cases}$$

$T(\overline{a_k \ldots a_0}) = 3T(\overline{a_k \ldots a_1}) + C_2(2^k a_k + 2^{k-1} a_{k-1} + \ldots + a_0)$

$T(\overline{a_k \ldots a_1}) = 3T(\overline{a_k \ldots a_2}) + C_2(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1)$

Nhân 2 vế cho 3:

$3T(\overline{a_k \ldots a_1}) = 3^2 T(\overline{a_k \ldots a_2}) + 3C_2(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1)$

Tiếp tục:

$3^2 T(\overline{a_k \ldots a_2}) = 3^3 T(\overline{a_k \ldots a_3}) + 3^2 C_2(2^{k-2} a_k + 2^{k-3} a_{k-1} + \ldots + a_2)$

$\vdots$

$3^{k-1} T(\overline{a_k \ldots a_{k-1}}) = 3^k T(\overline{a_k}) + 3^{k-1} C_2(2a_k + a_{k-1})$

$T(\overline{a_k \ldots a_0}) = 3^k T(\overline{a_k}) + C_2\left(2^k a_k + 2^{k-1} a_{k-1} + \ldots + a_0\right) + 3C_2\left(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1\right) + 3^2 C_2\left(2^{k-2} a_k + 2^{k-3} a_{k-1} + \ldots + a_2\right) + \cdots + 3^{k-1} C_2(2a_k + a_{k-1})$

Với: $3^k T(\overline{a_k}) + C_2\left(2^k a_k + 2^{k-1} a_{k-1} + \ldots + a_0\right) + 3C_2\left(2^{k-1} a_k + 2^{k-2} a_{k-1} + \ldots + a_1\right) + 3^2 C_2\left(2^{k-2} a_k + 2^{k-3} a_{k-1} + \ldots + a_2\right) + \cdots + 3^{k-1} C_2(2a_k + a_{k-1})$

$$= C_2 \sum_{i=0}^{k} a_i . \left(\sum_{j=0}^{k-1} 3^j 2^{i-j} \right)$$

Mà $k = log_2 n$ thì $3^k T(\overline{a_k}) = 3^{log_2 n} T(0) = 3^{log_2 n} C_1 = n^{log_2 3}.C_1$

$$T(\overline{a_k \ldots a_0}) = 3^k T(\overline{a_k}) + C_2 \sum_{i=0}^{k} a_i . \left(\sum_{j=0}^{k-1} 3^j 2^{i-j} \right)$$

$$\Leftrightarrow T(n) = n^{log_2 3}.C_1 + C_2 \sum_{i=0}^{log_2 n} a_i . \left(\sum_{j=0}^{log_2 n - 1} 3^j 2^{i-j} \right)$$

$\Rightarrow T(n) = O(n^{log_2 3})$