Mathematical notation of the models

In *“Population dynamics of two deer species under predation and changing climate”*

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$i = 1, 2, 3\ \ldots\ N$

X	a matrix of size N × 7
β	a column vector of size 7 × 1
Z	a matrix of size N × K
s	a column vector of size K × 1

where K is the number of knots in the spline. K = 8 in both of the models.

Model structure:

$$y \sim LogNormal(\mu, \sigma)$$

$$\mu = \mathrm{X}\beta + Zs$$

where β contains the intercept and linear coefficients, X contains the variables and a column of 1s for the intercept, Z contains the basis functions and s the penalized spline coefficients

$$\sigma = b_1 e^{-b_2 x_{density}} + e^{C_N}$$

$$C_N = 1_N C$$

$$C_N \sim CAR(\rho, \tau_c, A)$$

where A represents the adjacency matrix and associated metrics, C is the population-level asymptote, 1_N *is a column vector of 1s of size N, and* C_N *is a column vector of individual asymptotes.*

For more on CAR in Stan, see https://github.com/mbjoseph/CARstan

Splines (nonlinear, smooth effects):

$Z \ni R$	basis function matrix
$s = z_s \tau_s$	penalized spline coefficients
$z_s \sim N(0, 1)$	prior for standard. penalized spline coefficients
$\tau_{s1} \sim HalfNormal(0.5)$ (roe deer)	prior for SDs of penalized spline coefficients
$\tau_{s2} \sim HalfNormal(0.25)$ (white-tailed deer)	prior for SDs of penalized spline coefficients

Priors for linear effects:

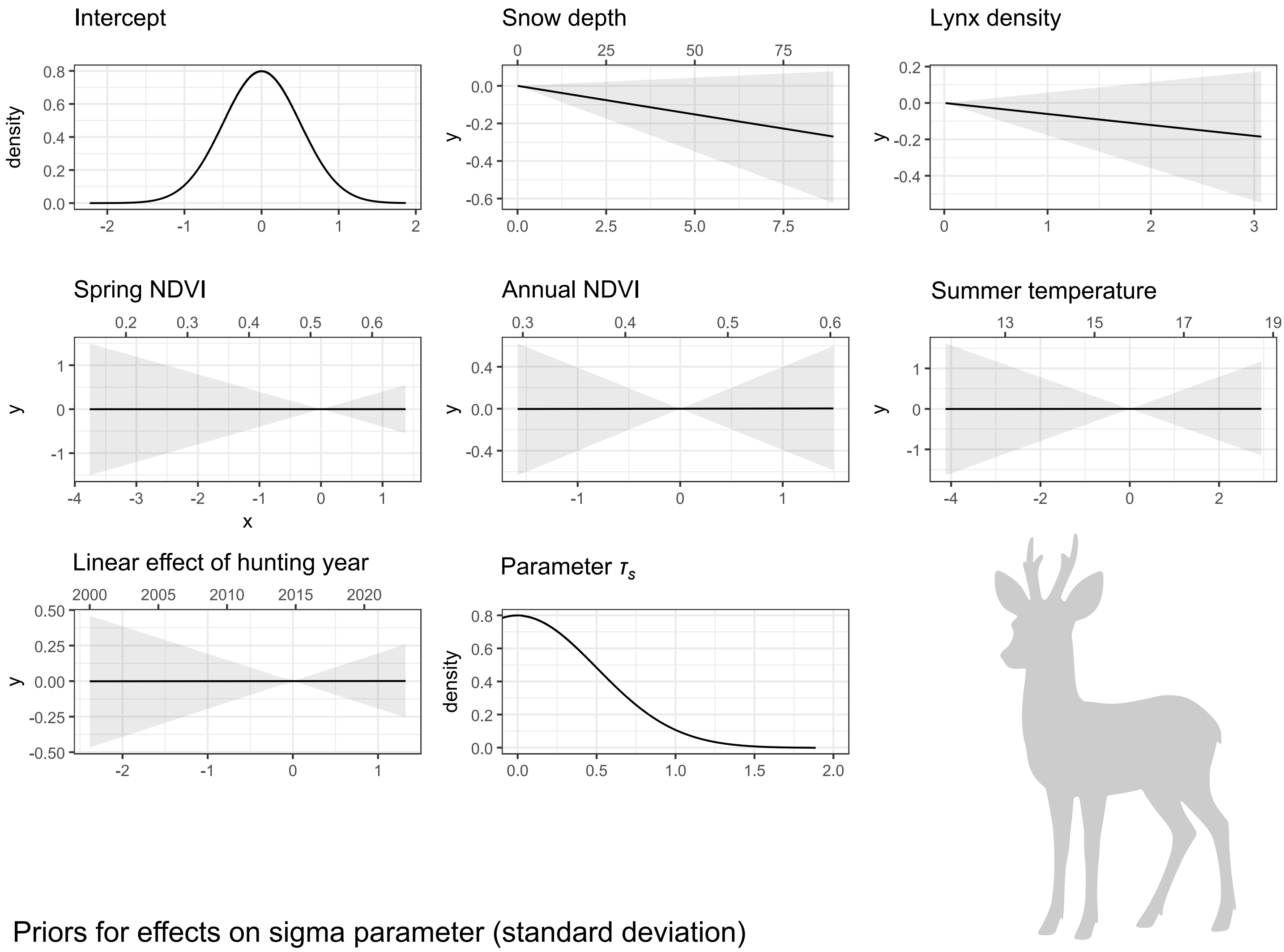
$\beta_0 \sim N(0, 0.5)$	prior for intercept
$\beta_s \sim N(0, 0.1)$	prior for linear effect of the splines
$\beta_{snow} \sim N(-0.03, 0.02)$	prior for snow depth
$\beta_{lynx} \sim N(-0.06, 0.06)$	prior for lynx density
$\beta_{spring} \sim N(0, 0.2)$	prior for spring NDVI
$\beta_{NDVI} \sim N(0, 0.2)$	prior for annual NDVI
$\beta_{summer} \sim N(0, 0.2)$	prior for summer temperature

Priors for standard deviation:

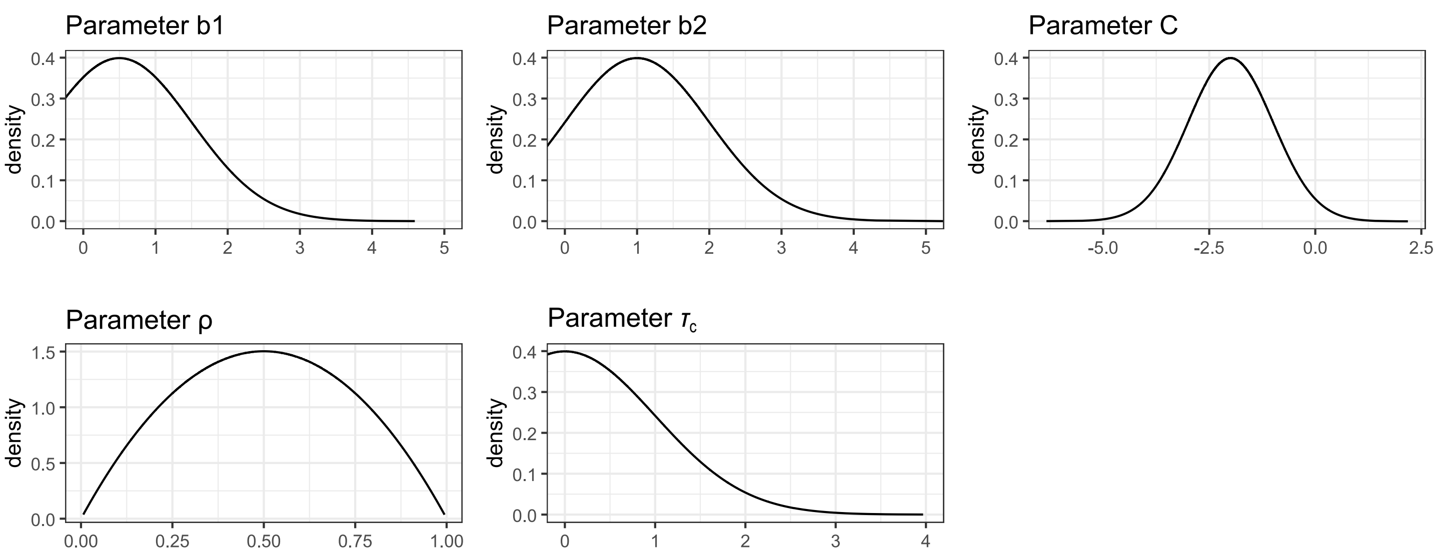
$C \sim N(-2, 1),\ C < 0$	prior for population-level asymptote
$b_1 \sim N(0.5, 1),\ b_1 > 0$	prior for parameter b_1
$b_2 \sim N(1, 1),\ b_2 > 0$	prior for parameter b_2
$\rho \sim Beta(2, 2)$	prior for degree of autocorrelation
$\tau_c \sim HalfNormal(1)$	prior for parameter τ_c

Priors for roe deer model

Priors for effects on mu parameter (mean)



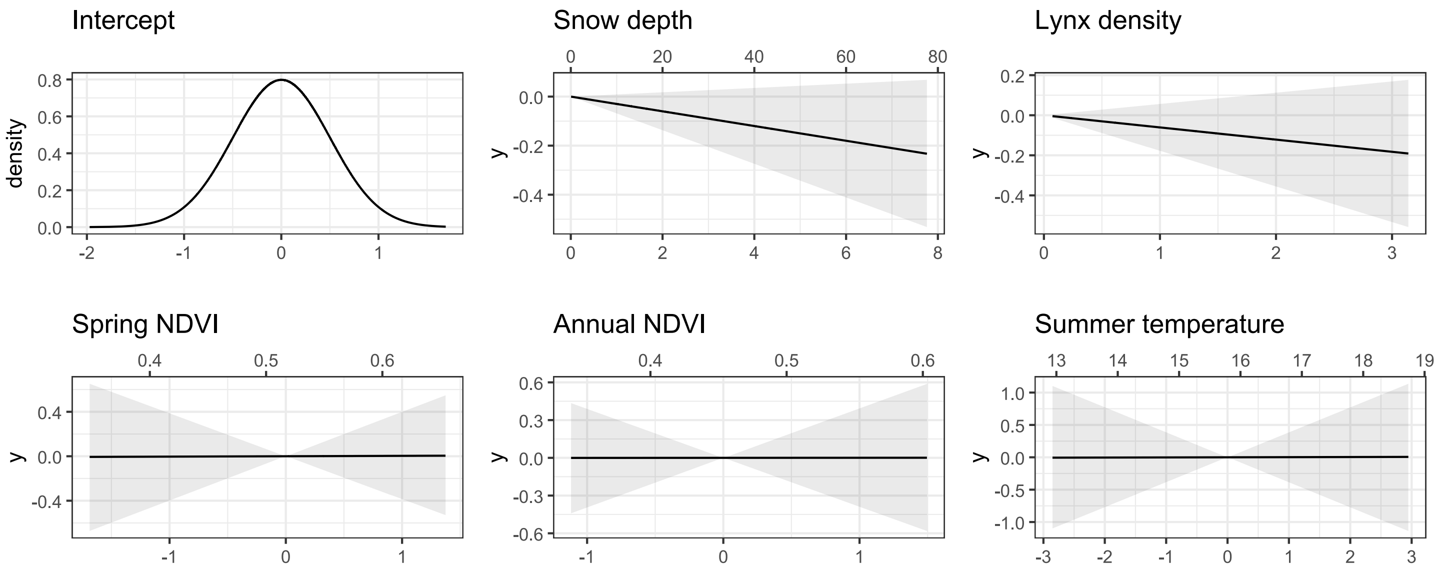
Priors for effects on sigma parameter (standard deviation)

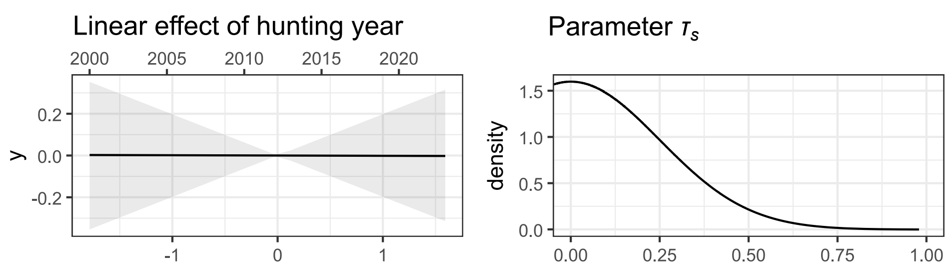


Prior information used in bayesian modelling. Priors are represented as prior distributions or prior effects with 95 % probability intervals. Upper x-axes represent backtransformed units.

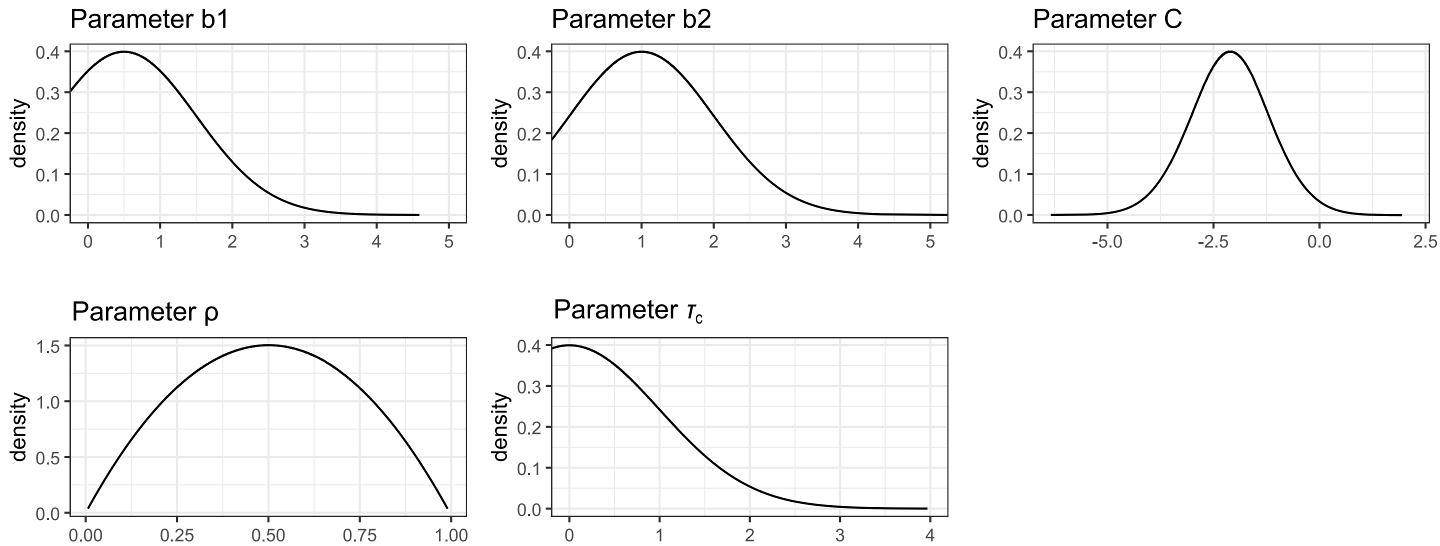
Priors for white-tailed deer model

Priors for effects on mu parameter (mean)





Priors for effects on sigma parameter (standard deviation)



Prior information used in bayesian modelling. Priors are represented as prior distributions or prior effects with 95 % probability intervals. Upper x-axes represent backtransformed units.