

Mathematical notation of the models

In “*Population dynamics of two deer species under predation and changing climate*”

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$i = 1, 2, 3 \dots N$

X a matrix of size $N \times 7$

β a column vector of size 7×1

Z a matrix of size $N \times K$

s a column vector of size $K \times 1$

where K is the number of knots in the spline. $K = 8$ in both of the models.

Model structure:

$y \sim \text{LogNormal}(\mu, \sigma)$

$\mu = X\beta + Zs$

where β contains the intercept and linear coefficients, X contains the variables and a column of 1s for the intercept, Z contains the basis functions and s the penalized spline coefficients

$\sigma = b_1 e^{-b_2 x_{\text{density}}} + e^{C_N}$

$C_N = 1_N C$

$C_N \sim \text{CAR}(\rho, \tau_c, A)$

where A represents the adjacency matrix and associated metrics, C is the population-level asymptote, 1_N is a column vector of 1s of size N , and C_N is a column vector of individual asymptotes.

For more on CAR in Stan, see <https://github.com/mbjoseph/CARstan>

Splines (nonlinear, smooth effects):

$Z \ni R$	basis function matrix
$S = Z_S \tau_S$	penalized spline coefficients
$z_S \sim N(0, 1)$	prior for standard. penalized spline coefficients
$\tau_{s1} \sim HalfNormal(0.5)$ (roe deer)	prior for SDs of penalized spline coefficients
$\tau_{s2} \sim HalfNormal(0.25)$ (white-tailed deer)	prior for SDs of penalized spline coefficients

Priors for linear effects:

$\beta_0 \sim N(0, 0.5)$	prior for intercept
$\beta_S \sim N(0, 0.1)$	prior for linear effect of the splines
$\beta_{snow} \sim N(-0.03, 0.02)$	prior for snow depth
$\beta_{lynx} \sim N(-0.06, 0.06)$	prior for lynx density
$\beta_{spring} \sim N(0, 0.2)$	prior for spring NDVI
$\beta_{NDVI} \sim N(0, 0.2)$	prior for annual NDVI
$\beta_{summer} \sim N(0, 0.2)$	prior for summer temperature

Priors for standard deviation:

$C \sim N(-2, 1), C < 0$	prior for population-level asymptote
$b_1 \sim N(0.5, 1), b_1 > 0$	prior for parameter b_1
$b_2 \sim N(1, 1), b_2 > 0$	prior for parameter b_2
$\rho \sim Beta(2, 2)$	prior for degree of autocorrelation
$\tau_c \sim HalfNormal(1)$	prior for parameter τ_c