# Mathematical notation of the models

In *"Population dynamics of two deer species under predation and changing climate"*

Toivonen Pyry[1,2*], Aikio Sami[1], Huitu Otso[1], Mäntyniemi Samu[1], Valtonen Mia[1], Laaksonen Toni[2]

[1]Natural Resources Institute Finland, FI-00790 Helsinki, Finland

[2]Department of Biology, University of Turku, FI-20014 Turku, Finland

$i = 1, 2, 3 \ \ldots \ N$

| | |
|---|---|
| $X$ | a matrix of size N × 7 |
| $\beta$ | a column vector of size 7 × 1 |
| $Z$ | a matrix of size N × K |
| $s$ | a column vector of size K × 1 |

*where K is the number of knots in the spline. K = 8 in both of the models.*

**Model structure:**

$$y \sim LogNormal(\mu, \sigma)$$

$$\mu = \mathrm{X}\beta + Zs$$

*where β contains the intercept and linear coefficients, X contains the variables and a column of 1s for the intercept, Z contains the basis functions and s the penalized spline coefficients*

$$\sigma = b_1 e^{-b_2 x_{density}} + e^{C_N}$$

$$C_N = 1_N C$$

$$C_N \sim CAR(\rho, \tau_c, A)$$

*where A represents the adjacency matrix and associated metrics, C is the population-level asymptote,* $1_N$ *is a column vector of 1s of size N, and* $C_N$ *is a column vector of individual asymptotes.*

*For more on CAR in Stan, see [https://github.com/mbjoseph/CARstan](https://github.com/mbjoseph/CARstan)*

**Splines (nonlinear, smooth effects):**

| | |
|---|---|
| $Z \ni R$ | basis function matrix |
| $s = z_s \tau_s$ | penalized spline coefficients |
| $z_s \sim N(0,1)$ | prior for standard. penalized spline coefficients |
| $\tau_{s1} \sim HalfNormal(0, 0.5)$ (roe deer) | prior for SDs of penalized spline coefficients |
| $\tau_{s2} \sim HalfNormal(0, 0.25)$ (white-tailed deer) | prior for SDs of penalized spline coefficients |

**Priors for linear effects:**

| | |
|---|---|
| $\beta_0 \sim N(0, 0.5)$ | prior for intercept |
| $\beta_s \sim N(0, 0.1)$ | prior for linear effect of the splines |
| $\beta_{snow} \sim N(-0.03, 0.02)$ | prior for snow depth |
| $\beta_{lynx} \sim N(-0.06, 0.06)$ | prior for lynx density |
| $\beta_{spring} \sim N(0, 0.2)$ | prior for spring NDVI |
| $\beta_{NDVI} \sim N(0, 0.2)$ | prior for annual NDVI |
| $\beta_{summer} \sim N(0, 0.2)$ | prior for summer temperature |

**Priors for standard deviation:**

| | |
|---|---|
| $C \sim N(-2, 1) < 0$ | prior for population-level asymptote |
| $b_1 \sim N(0.5, 1) < 0$ | prior for parameter $b_1$ |
| $b_2 \sim N(1, 1) < 0$ | prior for parameter $b_2$ |
| $\rho \sim Beta(2, 2)$ | prior for degree of autocorrelation |
| $\tau_c \sim HalfNormal(0, 1)$ | prior for parameter $\tau_c$ |