Homework III

I. REMARK

- Reading materials: Ch 1-10 in the textbook.
- Don't write just an answer. Please justify your answer.
- "Our greatest weakness lies in giving up. The most certain way to succeed is always to try just one more time"

II. PROBLEM SET

1) **First-order chemical reactions** In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. For the change of δ-glucono lactone into gluconic acid, for example,

$$\frac{dy}{dt} = -0.6y$$

when t is measured in hours. If there are 100 grams of δ-glucono lactone present when $t = 0$, how many grams will be left after the first hour?

2) **Finding volume** Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y = e^x$, and the line $x = \ln 2$ about the line $x = \ln 2$.

3) Evaluate the integral $\displaystyle\int \frac{\sqrt{x+1}}{x}\,dx$

   (*Hint:* Let $x + 1 = u^2$.)

4) Find the value of b for which

$$1 + e^b + e^{2b} + e^{3b} + \cdots = 9.$$

5) Find a parametrization for the line segment joining points $(0, 2)$ and $(4, 0)$ using the angle θ in the accompanying figure as the parameter.
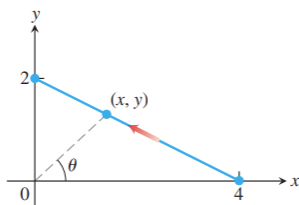
6) The accompanying figure shows the first five of a sequence of squares. The outermost square has an area of $4\ \text{m}^2$. Each of the other squares is obtained by joining the midpoints of the sides of the squares before it. Find the sum of the areas of all the squares.
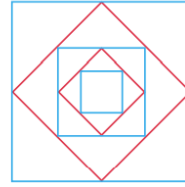
7) Show that the sum of the first $2n$ terms of the series

$$1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + \cdots$$

is the same as the sum of the first n terms of the series

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} + \cdots.$$

Do these series converge? What is the sum of the first $2n + 1$ terms of the first series? If the series converge, what is their sum?

8) Use a geometric series to represent each of the given functions as a power series about $x = 0$, and find their intervals of convergence.

   a. $f(x) = \dfrac{5}{3 - x}$ b. $g(x) = \dfrac{3}{x - 2}$

9) Find the coordinates of the centroid of the curve
$$x = \cos t + t \sin t, \quad y = \sin t - t \cos t, \quad 0 \le t \le \pi/2.$$

10) Find the areas of the regions in Exercises 1–8.
   1. Bounded by the spiral $r = \theta$ for $0 \le \theta \le \pi$