Midterm Exam

ZF15697 Calculus in Engineering, Spring 2021
School of BioMedical Convergence Engineering, PNU
Apr. 22. 10:00 - 11:30

I. REMARK

- You are permitted on one page of notes.
- There are a total of 100 points in the exam.
- You must SHOW YOUR WORK to get full credit.

II. PROBLEM SET

1) The function $f(x)$ is given as
$$f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ \frac{1}{2}x + 4, & x > 0 \end{cases}$$

   a) Graph the function $f(x)$.
   b) Graph the function $g(x) = -\frac{1}{2}f(-2(x-3))$.
   c) The function $f(x)$ is one-to-one? If then, what is the inverse of the function? Graph the function $f^{-1}(x)$. (if $f(x)$ is not one-to-one, you don't need to find and draw $f^{1}(x)$.

2) Prove the limit statements (You must use the definition of limit !!!).
$$\lim_{x \to 3}(x-1)^2 = 4$$

3) Find all points (x, y) on the graph of $f(x) = x^2$ with tangent lines passing through the point $(3, 8)$.
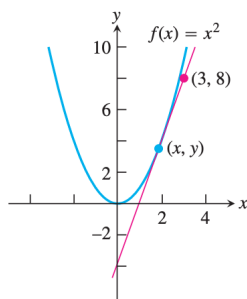
4) The function $f(x)$ is given as
$$f(x) = \frac{1}{3x - 2}$$
   Show that $f'(x) = -3f^2(x)$.
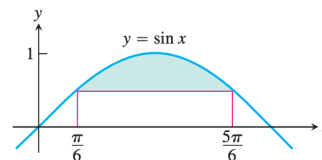
5) The height of a body moving vertically is given by
$$s = -\frac{1}{2}gt^2 + v_0 t + s_0, \quad g > 0.$$
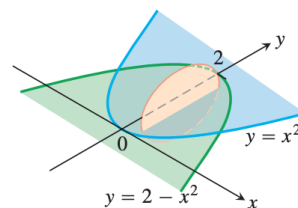   On what invervals is s increasing or decreasing? Find the body's maximum height.

6) Show that
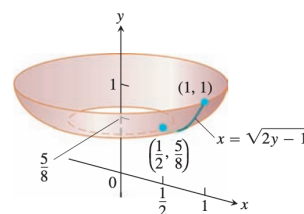$$\lim_{k \to \infty}\left(1 + \frac{r}{k}\right)^k = e^r.$$

7) Find the area of the shaded regions.

8) The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.

9) Find the area of the surfaces generated by revolving the curves about the y-axis.

10) When $a=1$, find the centroid $(\bar{x}, \bar{y})$ of the given shaded area.