Homework II

Due: Oct. 9. (Fri) 23:59 PM

I. REMARK

- Reading materials: notes, book (Foundations of Signal Processing, Ch 2.C and Ch 2.4.4) or book (An Exploration of Random Process for Engineers, Ch 1 and 3)
- Please write whole processes to justify your answer.
- Be healthy!!

II. PROBLEM SET

1) Let x be uniformly distributed over [0,1]. The goal is to obtain the samples of random variable y whose PDF is given as

$$f(y) = \begin{cases} 2y, & \text{if } 0 \leq y \leq 1 \\ 0, & \text{else} \end{cases}$$

   - In MATLAB, "x = rand(0,1,1000)" returns the 1000 random numbers. Draw the histogram.
   - Find CDF of y.
   - Find the function $g()$ such that "y=g(x)" returns the 1000 numbers for the random variable y. Draw the histogram.

2) Download 'rfData1.mat' file. Load the file in MATLAB. The file contains synthetic tumor data recorded over time using an ultrasound system (simulator). In general, tumor cells construct complex vascular system rapidly (angiogenesis) to obtain nutrients and oxygen. Our purpose is to detect the vascular system to clarify the tumor existence and position. The variable $X \in R^{N \times S \times K}$ corresponds to the data where N, S, and K are the numbers of axial samples, lateral samples, and temporal samples, respectively (See the figure below).
   - For imaging, do "imagesc(abs(mean(X,3)))". Tissue has far bigger signal power than red blood cells. Thus, the image would visualize only tissue.
   - Convert the 3D data array X into 2D matrix $\bar{X} \in R^{NS \times K}$ using 'reshape' function. Now, NS is the total number of spatial samples.
   - Perform SVD decomposition using 'svd' function. Now $\bar{X} = U\Sigma V^T$. The option 'econ' is required for saving memory. It would generate rank($\bar{X}$)=K eigenvectors. Check that singular values are arranged in descending order.
   - Draw eigenvalues arranged in descending order.
   - When every time point is regarded as a random variable (K random variables), the number of samples per each random variable is NS. Find NS projections (components) onto first eigenvector $\mathbf{v}_1$. You may use '$\bar{X}V(:,1)$'. Also, find NS projections onto second eigenvector $\mathbf{v}_2$. You may '$\bar{X}V(:,2)$'. Compute their correlation to clarify that they are uncorrelated.
   - Convert the first eigenvector of U, '$U(:,1)$' into 2D matrix where the matrix size is $N \times S$. Let the matrix be $U1$. Use 'imagesc(abs($U1$))' for imaging. Check that the spatial pattern looks general tissue. Repeat everything using the 100th eigenvector of U. Check that the spatial pattern looks vascular system.
   - Plot $V(:,1)$ and $V(:,100)$. More oscillation denotes faster and more heterogeneous motion.