Language Technology

Chapter 11: Dense Vector Representations https://link.springer.com/chapter/10.1007/978-3-031-57549-5_11

Pierre Nugues

Pierre.Nugues@cs.lth.se

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Dimension Reduction

- One-hot encoding with TFIDF can produce very long vectors: Imagine a vocabulary one million words per language with 100 languages.
- A solution is to produce dense vectors using a dimension reduction.
- Such vectors are also called word embeddings
- The reduction is similar to a principal component analysis (PCA) or a singular value decomposition (SVD)
- The embedding of a word can be constant (static) or depend on the context (contextual)



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Reminder: Categorical Values

Linear classifiers only understand numbers

A collection of two documents D1 and D2:

D1: Chrysler plans new investments in Latin America.

D2: Chrysler plans major investments in Mexico.

How to represent these words or these documents?



Reminder: Categorical Values: One-hot encoding

One-hot encoding:

Assigns a unique index to each symbol. The number of indices corresponds to the number of symbols:

```
{'Chrysler': 1, 'in': 2, 'investments': 3,
'major': 4, 'Mexico': 5, 'plans': 6}
```

2 Represent a symbol of index *i* by a unit vector $\mathbf{x} = (x_1, x_2, ..., x_n)$, where *n* is the largest index and all the coordinates are 0, except $x_i = 1$

```
'Chrysler': (1, 0, 0, 0, 0, 0)
'in': (0, 1, 0, 0, 0, 0)
'investments': (0, 0, 1, 0, 0, 0)
```



. . .

Reminder: Categorical Values: Multi-hot encoding

A collection of two documents D1 and D2:

- D1: Chrysler plans new investments in Latin America.
- D2: Chrysler plans major investments in Mexico.

Multi-hot encoding (also called a bag-of-words representation):

- Creates a index of all the symbols (words) in all the documents
- For each document, creates a set of its symbols (word)
- 3 Represents a document by a vector of 0s and 1s. $x_i = 1$ if the word of index i is in the document, or 0 otherwise.

D.	america	chrysler	in	investments	latin	major	mexico	new	plans
1	1	1	1	1	1	0	0	1	1
2	0	1	1	1	0	1	1	0	1

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Reminder: Hashing

One-hot encoding leads to very large dimensions as there are billions of different words.

In the Tatoeba corpus, the number of unigrams, bigrams, and trigrams is (10854, 361766, 1536870). Note that these values change all the time A solution is to hash the symbols and use the remainder of a division (modulo) as index

```
>>> hash('abc')
-6712881850779232724
>>> hash('abc') % 100
76 # Always less than 100
```

Hashing:

- Reduces the vectors size and makes it manageable
- Creates conflicts: two symbols can have the same hash number of the same
- Is usable in classification



Code Example

Experiment: hashing CLD3 n-grams Jupyter Notebook: https://github.com/pnugues/edan20/tree/master/labs_2024



Reminder: Reproducible Hash Codes

```
def reproducible_hash(string):
    11 11 11
    reproducible hash on any string
    Arguments:
       string: python string object
    Returns:
       signed int64
    0.00
    # We are using MD5 for speed not security.
    h = hashlib.md5(string.encode("utf-8"),
                     usedforsecurity=False)
    return int.from_bytes(h.digest()[0:8], 'big',
```

Dense Vectors

We can replace one-hot vectors by dense ones using embeddings

A dense representation is a trainable vector of 10 to 300 dimensions.

The vector parameters are learned in the fitting procedure.

Dimensionality reduction inside a neural network or another procedure.

Example: GloVe file 100d.

Many techniques, often based on language modeling, here CBOW



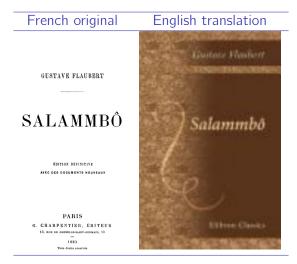
Creating Word Embeddings

- We can derive word embeddings from corpora. Their construction is then similar to that of language models;
- We can also introduce an embedding layer as input to a neural network. The embedding parameters are then trainable.
- We can finally pretrain embeddings with a corpus and fine-tune them on an application.



A Dataset: Salammbô

A corpus is a collection - a body - of texts.





Letter Counts

Characters in Salammbô: A small dataset to explain PCA

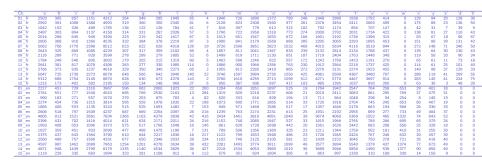
Chapter	French		English		
	# characters	# A	# characters	# A	
Chapter 1	36,961	2,503	35,680	2,217	
Chapter 2	43,621	2,992	42,514	2,761	
Chapter 3	15,694	1,042	15,162	990	
Chapter 4	36,231	2,487	35,298	2,274	
Chapter 5	29,945	2,014	29,800	1,865	
Chapter 6	40,588	2,805	40,255	2,606	
Chapter 7	75,255	5,062	74,532	4,805	
Chapter 8	37,709	2,643	37,464	2,396	
Chapter 9	30,899	2,126	31,030	1,993	
Chapter 10	25,486	1,784	24,843	1,627	
Chapter 11	37,497	2,641	36,172	2,375	
Chapter 12	40,398	2,766	39,552	2,560	
Chapter 13	74,105	5,047	72,545	4,597	
Chapter 14	76,725	5,312	75,352	4,871	
Chapter 15	18,317	1,215	18,031	1,119	

Data set: Jupyter Notebook:

https://github.com/pnugues/pnlp/tree/main/notebooks

Representing Documents with Bags of Characters

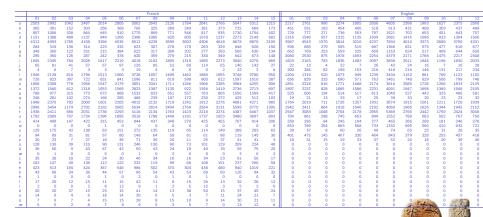
Character counts per chapter, where the fr and en suffixes designate the language, either French or English



Each chapter (document) is modeled by a vector of 40 characters

Transposing the Matrix: Character Counts

Character counts per chapter in French, left part, and English, right part



Each character is modeled by a vector of chapters.

Singular Value Decomposition

There are as many as 40 characters: the 26 unaccented letters from a to z and the 14 French accented letters: à, â, é, è, ê, ë, etc.

Singular value decomposition (SVD) reduces these dimensions, while keeping the resulting vectors semantically close

X is the $m \times n$ matrix of the letter counts per chapter, in our case, m = 30 and n = 40.

We can rewrite X as:

$$X = U\Sigma V^{\mathsf{T}}$$
,

where

- U is a matrix of dimensions $m \times m$,
- Σ , a diagonal matrix of dimensions $m \times n$, and
- V, a matrix of dimensions $n \times n$

The diagonal terms of Σ are called the **singular values** and are traditionally arranged by decreasing value.

To reduce the dimensions, we keep the highest values and set the zero.

Code Example

Jupyter Notebook:

https://github.com/pnugues/pnlp/tree/main/notebooks



Vector Space Model

With the vector space model, we represent:

- Documents in a space of words (the words they contain) or
- Words in a space of documents (the documents that contain them).

Here, the rows are the words in the corpus, and the columns, the documents.

Words\D#	D_1	D_2	D_3	 D_n
w_1	$C(w_1, D_1)$	$C(w_1, D_2)$	$C(w_1, D_3)$	 $C(w_1, D_n)$
<i>W</i> ₂	$C(w_2, D_1)$	$C(w_2, D_2)$	$C(w_2, D_3)$	 $C(w_2, D_n)$
<i>w</i> ₃	$C(w_3, D_1)$	$C(w_3, D_2)$	$C(w_3, D_3)$	 $C(w_3, D_n)$
Wm	$C(w_m, D_1)$	$C(w_m, D_2)$	$C(w_m, D_3)$	 $C(w_m, D_n)$

 $C(w_i, D_j)$ can be Boolean values, counts, $tf \times idf$, or a metric similar to $tf \times idf$ as in **latent semantic indexing**.

We can extend singular value decomposition from characters to

Word Embeddings

A PCA applied to this matrix will result in dense vectors representing the words.

Transposing the matrix, we represent documents with dense vectors We compute the word embeddings with a singular value decomposition, where we truncate the matrix $\mathbf{U}\mathbf{\Sigma}$ to 50, 100, 300, or 500 dimensions. The word embeddings are the rows of this matrix.



Embeddings from Cooccurrences

We replace the documents with counts $C(w_i, w_i)$ in a context window

Words\Words	w_1	<i>W</i> ₂	<i>W</i> 3	 Wn
w_1	$C(w_1, w_1)$	$C(w_1, w_2)$	$C(w_1, w_3)$	 $C(w_1, w_n)$
W_2	$C(w_2, w_1)$	$C(w_2, w_2)$	$C(w_2, w_3)$	 $C(w_2, w_n)$
<i>W</i> ₃	$C(w_3, w_1)$	$C(w_3, w_2)$	$C(w_3, w_3)$	 $C(w_3, w_n)$
Wn	$C(w_n, w_1)$	$C(w_m, D_2)$	$C(w_m, D_3)$	 $C(w_n, D_n)$

We also extend singular value decomposition from characters to words.



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Word Similarity

Contrary to one-hot encoder words, we can measure the similarity of two dense vectors **u** and **v**.

We usually measure the similarity between two embeddings $\bf u$ and $\bf v$ with the cosine similarity:

$$\cos(\mathbf{u},\mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| \, ||\mathbf{v}||},$$

ranging from -1 (most dissimilar) to 1 (most similar) or with the cosine distance ranging from 0 (closest) to 2 (most distant):

$$1 - \cos(\mathbf{u}, \mathbf{v}) = 1 - \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| \, ||\mathbf{v}||}.$$



Pierre Nuques

Embeddings in PyTorch

PyTorch has an embedding class. This is just a lookup table. From PvTorch documentation:

```
embedding = nn.Embedding(10, 3)
input = torch.LongTensor([[1, 2, 4, 5], [4, 3, 2, 9]])
embedding(input)
embedding = nn.Embedding(10, 3, padding_idx=0)
input = torch.LongTensor([[0, 2, 0, 5]])
embedding(input)
```

Loading pretrained embeddings:

```
embeddings = nn.Embedding.from_pretrained(
  torch.FloatTensor(embedding_matrix),
  freeze=False.
  padding_idx=0)
```



Code Example

Jupyter Notebook:

https://github.com/pnugues/pnlp/tree/main/notebooks



Word2vec Embeddings

word2vec is another kind of embeddings that comes in two forms:

CBOW and skipgrams.

CBOW uses a neural network architecture to predict a word given its surrounding context

The set up is similar to fill-the-missing-word questionnaires.

The missing word is called the focus word

CBOW embeddings corresponds to neural network parameters

The embeddings are trained on a corpus



Cloze Test

Guess a missing word given its context. Using the example: Sing, O goddess, the anger of Achilles son of Peleus,

Cloze test: A reader, given the incomplete phrase:

Sing, O ____, the anger of Achilles

has to fill in the blank with the correct word, here goddess.



Word2vec Dataset

Using contexts of five words and training sentences such as: Sing, O goddess, the anger of Achilles son of Peleus,

we generate a training set of contexts deprived of their focus word (X) and the focus word to predict (y):

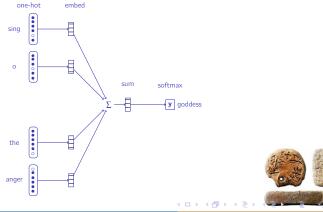
$$X = \begin{bmatrix} sing & o & the & anger \\ o & goddess & anger & of \\ goddess & the & of & achilles \\ the & anger & achilles & son \\ anger & of & son & of \\ of & achilles & of & peleus \end{bmatrix}; \mathbf{y} = \begin{bmatrix} goddess \\ the \\ anger \\ of \\ achilles \\ son \end{bmatrix}$$



CBOW Architecture

We train a neural network to get the CBOW embeddings: N dimension of the embeddings, V size of the vocabulary.

- Embedding table (V, N) and sum;
- 2 Linear layer (N, V) and softmax.



Embedding Bags in PyTorch

EmbeddingBags class creates embedding objects.

```
embedding_bag = nn.EmbeddingBag(MAX_CHARS, 64, mode='sum')
```

Given a list of embeddings (a list of rows) as input, an embedding bag returns the weighted sum of the embeddings.

We specify the weights with a per_sample_weights parameter.

https://pytorch.org/docs/stable/generated/torch.nn.

EmbeddingBag.html



Programming Embedding Bags in PyTorch (I)



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Code Example

Jupyter Notebook:

https://github.com/pnugues/pnlp/tree/main/notebooks



Examples

Words closest to *he*, *she*, *London*, *table*, and *Monday* with CBOW embeddings trained on a corpus of Dickens novels:

```
he ['she', 'they', 'it', 'be', 'that']
she ['he', 'they', 'it', 'i', 'be']
london ['paris', 'england', 'town', 'india', 'dover']
table ['desk', 'counter', 'box', 'sofa', 'ground']
monday ['sunday', 'thursday', 'saturday', 'noon', 'wednesday']
```



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Code Example

Jupyter Notebook:

https://github.com/pnugues/pnlp/tree/main/notebooks



Glove Embeddings

There are many kinds of word embeddings: Global vectors (GloVe) is one of them

We can replace documents by a context of a few words to the left and to the right of the focus word: w_i .

A context C_j is then defined by a window of 2K words centered on the word:

Word: w_i ,

Context: $W_{i-K}, W_{i-K+1}, ..., W_{i-1}, W_{i+1}, ..., W_{i+K-1}, W_{i+K}$

where the context representation uses a bag of words.

We can even reduce the context to a single word to the left or to the right of w_i and use bigrams.

Glove Embeddings

We store counts of word pairs (w_i, w_j) in a matrix:

Words	<i>W</i> ₁	W ₂	W3	 Wn
W_1	X_{11}	X_{12}	X_{13}	 X_{1n}
<i>W</i> ₂	X_{21}	X_{22}	X_{23}	 X_{2n}
<i>W</i> 3	$X_{11} \ X_{21} \ X_{31}$	X_{32}	X_{33}	 X_{3n}
Wn	X_{n1}	X_{n2}	X_{n3}	 X_{nn}

 X_{ij} is the number of times word w_i occurs in the context of word w_i , for instance 10 words to the left and 10 to the right

To train the embeddings, we minimize the loss (simplified):

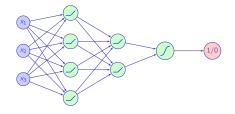
$$J = \sum_{i,j=1}^{V} (\mathbf{w}_i \cdot \mathbf{w}_j - \log X_{ij})^2,$$



where \mathbf{w}_i , resp. \mathbf{w}_i , is the embedding vector of word of index

Using Word Embeddings

We can use word embeddings to replace one-hot vectors as they will make the representation much more compact.



In a text categorization task, for instance, you would use a window of words (for instance the 200 first words of the document), where each word would be represented by its embedding.

The input layer is then called an embedding layer.

The embeddings are trainable parameters that you can initialize pre-trained embeddings or random values.

Popular Word Embeddings

Embeddings from large corpora are obtained with iterative techniques Some popular embedding algorithms with open source programs:

word2vec: https://github.com/tmikolov/word2vec

GloVe: Global Vectors for Word Representation

https://nlp.stanford.edu/projects/glove/

fastText: https://fasttext.cc/

To derive word embeddings, you will have to apply these programs on a very large corpus

Embeddings for many languages are also publicly available. You just download them

gensim is a Python library to create word embeddings from a corpus.

https://radimrehurek.com/gensim/index.html