

EDAN96

Applied Machine Learning

Lecture 9: Training Techniques, Backward Propagation, and
Automatic Differentiation

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Content

Overview and practice of some neural network architectures:

- Logistic loss
- The Module class
- Dense vectors
- A word on data loaders
- Backward propagation
- A word on automatic differentiation

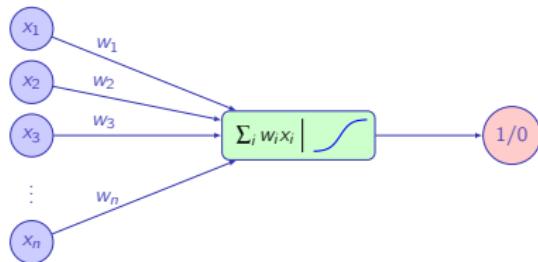
Creating a Network with PyTorch

So far, we have used the `Sequential` class to create networks
For more complex architectures, we need to derive a class from
`nn.Module`

This class must have the `__init__()` and `forward()` methods:

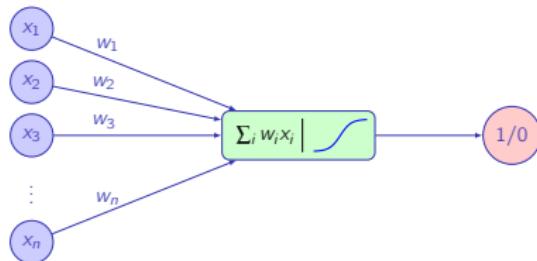
- In the `__init__()` constructor, you declare and create all the trainable parameters
- `forward()` implements the computation of the forward pass

Logistic Regression



```
model = nn.Sequential(  
    torch.nn.Linear(input_dim, 1),  
    torch.nn.Sigmoid())
```

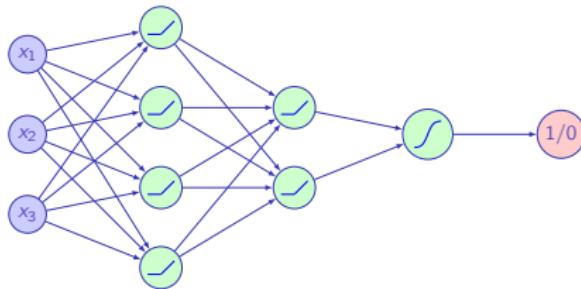
Logistic Regression



```
class Model(nn.Module):
    def __init__(self, input_dim):
        super().__init__()
        self.fc1 = nn.Linear(input_dim, 1)

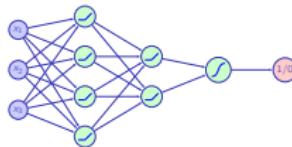
    def forward(self, x):
        x = torch.sigmoid(self.fc1(x))
        return x
```

Neural Networks with Hidden Layers



```
model = nn.Sequential(  
    nn.Linear(input_dim, 4),  
    nn.ReLU(),  
    nn.Linear(4, 2),  
    nn.ReLU()  
    nn.Linear(2, 1),  
    torch.nn.Sigmoid()  
)
```

Neural Networks with Hidden Layers



```
class Model2(nn.Module):
    def __init__(self, input_dim):
        super().__init__()
        self.fc1 = nn.Linear(input_dim, 4)
        self.fc2 = nn.Linear(4, 2)
        self.fc3 = nn.Linear(2, 1)

    def forward(self, x):
        x = torch.relu(self.fc1(x))
        x = torch.relu(self.fc2(x))
        x = torch.sigmoid(self.fc3(x))
        return x
```

Code Example

Experiment: Deriving a class with a Jupyter Notebook:

[https://github.com/pnugues/edan96/blob/main/programs/
8-Salammbó_class_torch.ipynb](https://github.com/pnugues/edan96/blob/main/programs/8-Salammbó_class_torch.ipynb)

Problems with Softmax

The logistic and softmax functions are typical activations of the last layer
They are numerically unstable. Take for instance

$$\text{softmax}(1000.0, 1000.0, 1000.0) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

Now try to compute it from the formula:

$$\text{softmax}(x_i) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

Multiclass in PyTorch

To solve the numerical under or overflows, PyTorch integrates softmax in the cross entropy loss

It uses a specific function called LogSumExp . See

<https://en.wikipedia.org/wiki/LogSumExp>

See also:

<https://gregorygundersen.com/blog/2020/02/09/log-sum-exp/>

The last layer of a multiclass classification is typically a linear layer. For instance:

```
model = nn.Sequential(  
    nn.Linear(input_dim, 5),  
    nn.ReLU(),  
    nn.Linear(5, 3)  
)
```

The layer outputs are called logits

The cross entropy loss uses a logit input in PyTorch: <https://pytorch.org/docs/stable/generated/torch.nn.CrossEntropyLoss.html>

Code Example

Experiment:

The code of a neural network with a sequential model is very close to that of logistic regression.

We just list the layers and activation functions

Jupyter Notebook: https://github.com/pnugues/edan96/blob/main/programs/9-Salammbó_multi_torch.ipynb

Binary Classification from Logits

Binary and multiclass classifications have different architectures in PyTorch.

Maybe a bug in the API design?

We can use the `nn.BCEWithLogitsLoss()` to have a logit input:

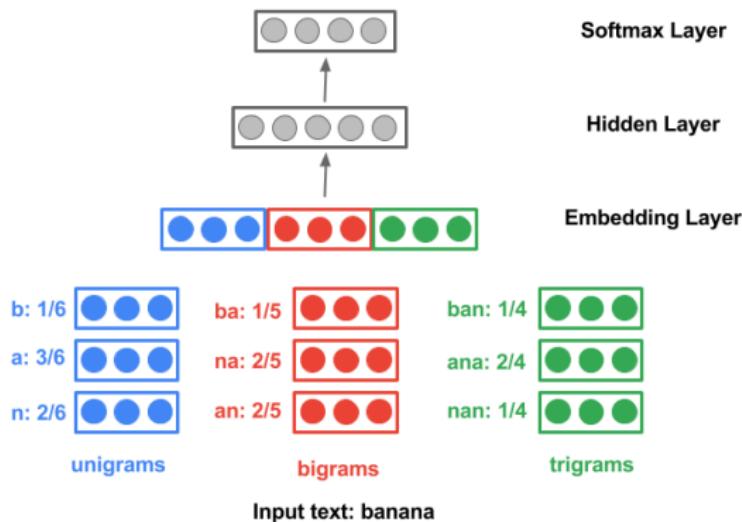
```
model2 = nn.Sequential(  
    nn.Linear(input_dim, 4),  
    nn.ReLU(),  
    nn.Linear(4, 2),  
    nn.ReLU(),  
    nn.Linear(2, 1)  
)
```

Experiment: Deriving a class with a Jupyter Notebook:

[https://github.com/pnugues/edan96/blob/main/programs/
10-Salammbo_class_logits_torch.ipynb](https://github.com/pnugues/edan96/blob/main/programs/10-Salammbo_class_logits_torch.ipynb)

Sum of Embeddings in CLD3

CLD3 computes the weighted sum of embeddings



Categorical Values: Multi-hot encoding

A collection of two documents D1 and D2:

D1: Chrysler plans new investments in Latin America.

D2: Chrysler plans major investments in Mexico.

Multi-hot encoding (also called a bag-of-words representation):

D.	america	chrysler	in	investments	latin	major	mexico	new	plans
1	1	1	1	1	1	0	0	1	1
2	0	1	1	1	0	1	1	0	1

This technique can create extremely large sparse vectors

Dense Vectors

We can replace one-hot vectors by dense ones using embeddings
A dense representation is a trainable vector of 10 to 300 dimensions.
The vector parameters are learned in the fitting procedure.
Dimensionality reduction inside a neural network or another procedure.
Example: GloVe file 100d.
Many techniques, often based on language modeling, here CBOW

Cloze Test

Guess a missing word given its context. Using the example:

Sing, O **goddess**, the anger of Achilles son of Peleus,

Cloze test: A reader, given the incomplete phrase:

Sing, O ----, the anger of Achilles

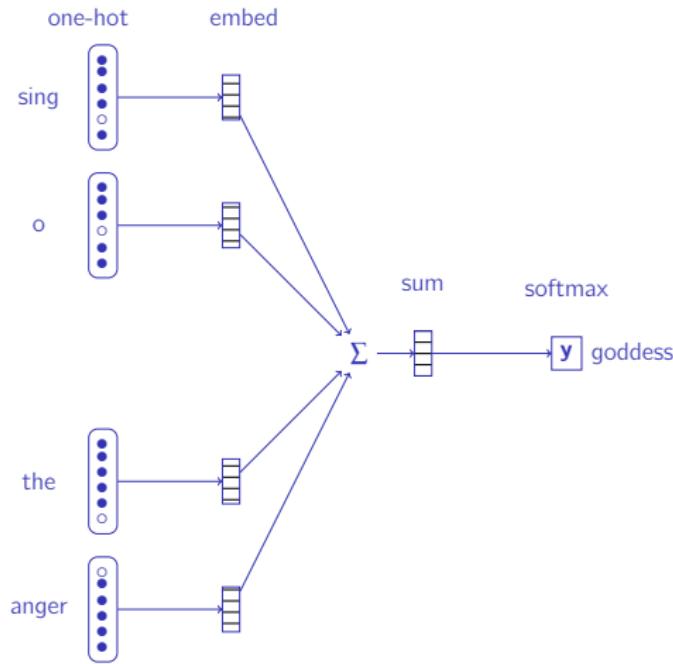
has to fill in the blank with the correct word, here **goddess**.

Easy to create a dataset for a Cloze test

$$X = \begin{bmatrix} \text{sing} & \text{o} & \text{the} & \text{anger} \\ \text{o} & \text{goddess} & \text{anger} & \text{of} \\ \text{goddess} & \text{the} & \text{of} & \text{achilles} \\ \text{the} & \text{anger} & \text{achilles} & \text{son} \\ \text{anger} & \text{of} & \text{son} & \text{of} \\ \text{of} & \text{achilles} & \text{of} & \text{peleus} \end{bmatrix}; \mathbf{y} = \begin{bmatrix} \text{goddess} \\ \text{the} \\ \text{anger} \\ \text{of} \\ \text{achilles} \\ \text{son} \end{bmatrix}$$

CBOW Architecture

Context words one-hot encoded, in practice just an index, followed by an **embedding layer**.



Embeddings in PyTorch

PyTorch has an `Embedding(num_embeddings, embedding_dim, ...)` class

An embedding object is a matrix from which we can extract embedding vectors using an index

This is just a lookup table

```
# Creates trainable vectors of size 64
embedding_chars = nn.Embedding(MAX_CHARS, 64)

# Extracts embeddings in rows 3 and 2,
# corresponding to two characters
embedding_chars(torch.LongTensor([3, 2]))
```

Code Example

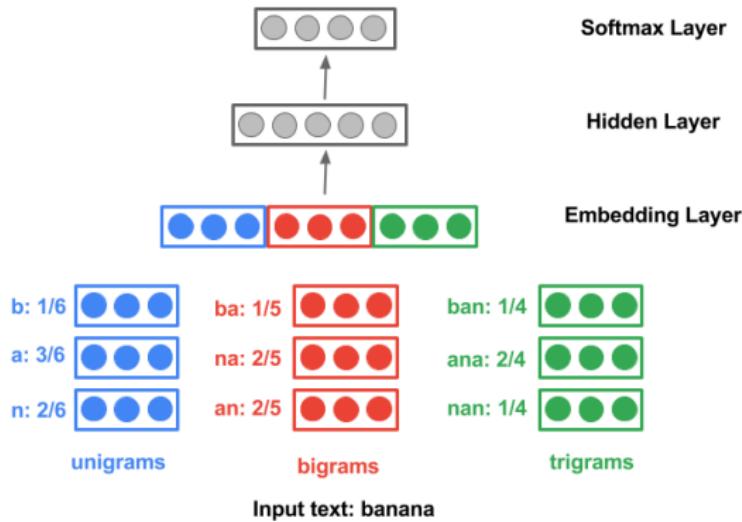
Experiment: Embeddings with a Jupyter Notebook:

[https://github.com/pnugues/edan96/blob/main/programs/
11-pytorch_embeddings.ipynb](https://github.com/pnugues/edan96/blob/main/programs/11-pytorch_embeddings.ipynb)

To create a batch, we would need to pad the character, bigram, and trigram hash codes.

Sum of Embeddings in CLD3

CLD3 computes the weighted sum (mean) of the embeddings



Embedding Bags in PyTorch

EmbeddingBags class creates embedding objects.

```
embedding_bag = nn.EmbeddingBag(MAX_CHARS, 64) # default mean  
embedding_bag = nn.EmbeddingBag(MAX_CHARS, 64, mode='sum')
```

Given a list of embeddings (a list of rows) as input, an embedding bag returns:

- ① The mean,
- ② The sum, or
- ③ The weighted sum of the embeddings.

We specify the weights with a per_sample_weights parameter.

<https://pytorch.org/docs/stable/generated/torch.nn.EmbeddingBag.html>

Programming Embedding Bags in PyTorch (I)

```
embedding_bag = nn.EmbeddingBag(MAX_CHARS, 64, mode='sum')

# Computes the sum of rows 1 and 2 and rows 3 and 4
# The result is a matrix of two rows
embedding_bag(torch.tensor([[1, 2], [3, 4]]))

embedding_bag(torch.tensor([[1, 2], [3, 4]]),
              per_sample_weights=torch.tensor([[0.5, 0.5],
                                              [0.2, 0.8]]))
```

Programming Embedding Bags in PyTorch (II)

With bags of unequal sizes, we have to use a list of offsets

```
embedding_bag(torch.tensor([1, 2, 3, 4]),  
             offsets=torch.tensor([0, 2]),  
             per_sample_weights=torch.tensor([0.5, 0.5, 0.2, 0.8]))
```

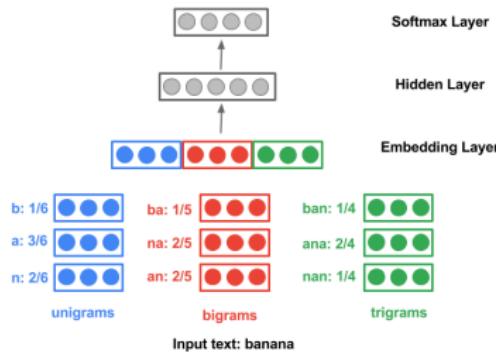
Adding the embeddings

Describe a language detector: Given a string predict the language:

- *Bonjour* → French
- Guten Tag → German

Follow the complete compact language detector (CLD3)

<https://github.com/google/cld3>



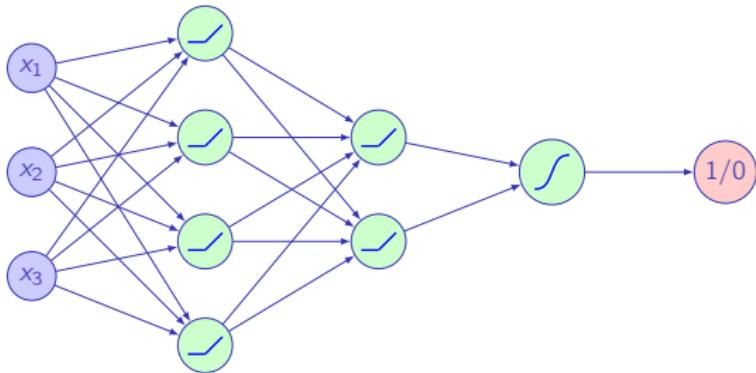
Code Example

Experiment: Classification with embedding bags Jupyter Notebook:
https://github.com/pnugues/pnlp/blob/main/notebooks/11_07_language_detector.ipynb

Data Loaders

- Current datasets have now terabytes of data
- Impossible to fit into memory (even Tatoeba)
- For real world applications, you will have to use or write a data loader that can create smaller, processable batches from your storage
- This involves the Dataset and DataLoader classes
- Beyond the scope of this lecture
- Read on this here: <https://pytorch.org/blog/efficient-pytorch-io-library-for-large-datasets-many-file>

Backpropagation: The Forward Pass



The forward pass:

- ➊ Layer 1 $f^{(1)}(W^{(1)}\mathbf{x})$, where $f^{(1)}$ is the activation function.
- ➋ For the second layer, $f^{(2)}(W^{(2)}f^{(1)}(W^{(1)}\mathbf{x}))$,
- ➌ Last layer (L) and output the prediction:

$$\hat{y} = f^{(L)}(W^{(L)} \dots f^{(2)}(W^{(2)}f^{(1)}(W^{(1)}\mathbf{x})) \dots).$$

- ➍ For the figure $\hat{y} = f^{(3)}(W^{(3)}W^{(2)}W^{(1)}\mathbf{x})$, where $f^{(3)}(x)$ is the logistic function.

Naive Gradient Descent

Try to minimize the difference between the predicted and observed annotations: $\text{Loss}(y, \hat{y})$.

$$\mathbf{w}_{(k+1)} = \mathbf{w}_{(k)} - \alpha_{(k)} \nabla \text{Loss}(\mathbf{w}_{(k)}).$$

We compute the gradient:

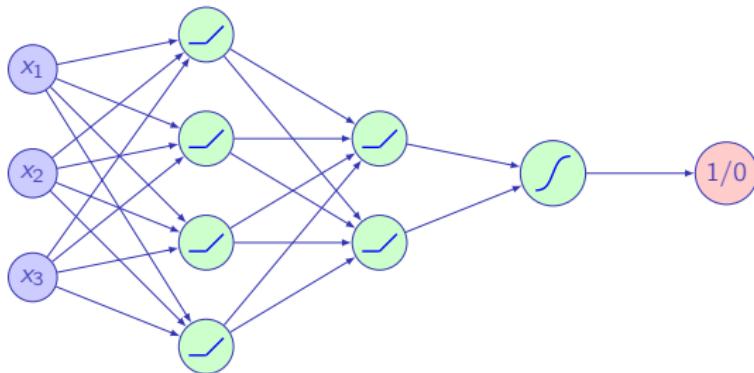
$$\begin{aligned}\frac{\partial \text{Loss}(\mathbf{w})}{\partial w_{ij}^{(l)}} &= \frac{\partial(-y \ln \hat{y} - (1-y) \ln(1-\hat{y}))}{\partial w_{ij}^{(l)}} \\ &= \frac{\partial(-y \ln f^{(3)}(W^{(3)} W^{(2)} W^{(1)} \mathbf{x}) - (1-y) \ln(1-f^{(3)}(W^{(3)} W^{(2)} W^{(1)} \mathbf{x})))}{\partial w_{ij}^{(l)}},\end{aligned}$$

for all the weights $w_{ij}^{(l)}$.

Method impractical in real cases (billions of weights)

Breaking Down the Computation

We first compute the gradient with respect to the inputs.



$$\begin{aligned}\hat{y} &= \mathbf{a}^{(L)}, \\ &= f^{(L)}(\mathbf{z}^{(L)}), \\ &= f^{(L)}(\mathbf{W}^{(L)}\mathbf{a}^{(L-1)})\end{aligned}$$

Gradient with Respect to the Input

For a given layer, we have:

$$\begin{aligned}\mathbf{z}^{(l)} &= \mathcal{W}^{(l)} \mathbf{a}^{(l)}, \\ &= \mathcal{W}^{(l)} f(\mathbf{z}^{(l-1)})\end{aligned}$$

We compute:

$$\frac{\partial \mathbf{z}^{(l)}}{\partial \mathbf{z}^{(l-1)}}$$

and we can show that this relation applies for any pair of adjacent layers l and $l-1$ in the network:

$$\nabla_{\mathbf{z}^{(l-1)}} \mathbf{z}^{(l)} = f^{(l-1)\prime}(\mathbf{z}^{(l-1)})^\top \odot \mathcal{W}^{(l)}.$$

Recurrence Relation

Using the chain rule:

$$\begin{aligned}\nabla_{\mathbf{x}} \text{Loss}(\hat{y}, y) &= \nabla_{\mathbf{x}} \text{Loss}(f^{(L)}(W^{(L)} \dots f^{(2)}(W^{(2)} f^{(1)}(W^{(1)} \mathbf{x})) \dots), y), \\ &= \frac{\partial \text{Loss}(\hat{y}, y)}{\partial \mathbf{z}^{(L)}} \frac{\partial \mathbf{z}^{(L)}}{\partial \mathbf{z}^{(L-1)}} \frac{\partial \mathbf{z}^{(L-1)}}{\partial \mathbf{z}^{(L-2)}} \cdots \frac{\partial \mathbf{z}^{(2)}}{\partial \mathbf{z}^{(1)}} \frac{\partial \mathbf{z}^{(1)}}{\partial \mathbf{x}},\end{aligned}$$

For our network:

$$\begin{aligned}\nabla_{\mathbf{x}} \text{Loss}(y, \hat{y}) &= -\frac{\partial(y \ln \hat{y} + (1-y) \ln(1-\hat{y}))}{\partial \mathbf{x}}, \\ &= -\frac{y - \hat{y}}{\hat{y}(1-\hat{y})} \hat{y}(1-\hat{y}) W^{(3)} W^{(2)} W^{(1)}, \\ &= (\hat{y} - y) W^{(3)} W^{(2)} W^{(1)}, \\ &= (f^{(3)}(W^{(3)} W^{(2)} W^{(1)} \mathbf{x}) - y) W^{(3)} W^{(2)} W^{(1)}.\end{aligned}$$

Gradient with Respect to the Weights

We now compute the gradient with respect to $W^{(l)}$, l being the index of any layer. From the chain rule, for the last layer, L , we have:

$$\nabla_{W^{(L)}} \text{Loss}(y, \hat{y}) = \frac{\partial \text{Loss}(y, \hat{y})}{\partial \mathbf{z}^{(L)}} \frac{\partial \mathbf{z}^{(L)}}{\partial W^{(L)}}$$

and

$$\begin{aligned}\mathbf{z}^{(L)} &= W^{(L)} f^{(L-1)}(\mathbf{z}^{(L-1)}), \\ &= W^{(L)} \mathbf{a}^{(L-1)}.\end{aligned}$$

The partial derivatives of $\mathbf{z}^{(L)}$ with respect to $W^{(L)}$ simply consist of the transpose of $\mathbf{a}^{(L-1)}$. Then, we have:

$$\frac{\partial \mathbf{z}^{(L)}}{\partial W^{(L)}} = \mathbf{a}^{(L-1)\top}.$$

We can show:

$$\nabla_{W^{(l)}} \text{Loss}(y, \hat{y}) = \nabla_{\mathbf{z}^{(l)}} \text{Loss}(y, \hat{y}) \mathbf{a}^{(l-1)\top}.$$

Code Example

Experiment: Checking the gradient with PyTorch Jupyter Notebook:
https://github.com/pnugues/edan96/blob/main/programs/backprop_mse_test.ipynb

Backward Differentiation

A generalization of backpropagation

PyTorch records all the operations in the forward pass

It then computes the graph of derivatives using the chain rule proceeding backwards

- ➊ <https://pytorch.org/blog/overview-of-pytorch-autograd-engine/>
- ➋ <https://docs.pytorch.org/docs/stable/notes/autograd.html>
- ➌ <https://github.com/pytorch/pytorch/blob/master/tools/autograd/derivatives.yaml>

Using PyTorch's example:

$$f(x, y) = \log xy$$

We have:

$$g(x, y) = xy$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial f}{\partial g} \frac{\partial g}{\partial x} &= \frac{1}{xy} y &= \frac{1}{x} \\ \frac{\partial f}{\partial y} &= \frac{\partial f}{\partial g} \frac{\partial g}{\partial y} &= \frac{1}{xy} x &= \frac{1}{y} \end{aligned}$$

Code Example

Experiment: Checking the gradient with PyTorch Jupyter Notebook:
<https://github.com/pnugues/edan96/blob/main/programs/14-autodiff.ipynb>

Further Reading

- ① For a video overview: https://www.youtube.com/playlist?list=PLZHQB0WTQDNU6R1_67000Dx_ZCJB-3pi, especially the two last lectures;
- ② PyTorch https://pytorch.org/tutorials/beginner/blitz/autograd_tutorial.html
- ③ Functions: <https://github.com/pytorch/pytorch/blob/master/tools/autograd/derivatives.yaml>
- ④ For a description of it in Tensorflow, see
<https://www.tensorflow.org/guide/autodiff>
- ⑤ For a description of the tf.gradients class:
https://www.tensorflow.org/api_docs/python/tf/gradients
- ⑥ For a more elaborate description: http://www.cs.toronto.edu/~rgrosse/courses/csc421_2019/slides/lec06.pdf